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LATERAL-TORSIONAL RESPONSE OF STRUCTURES SUBJECTED TO SEISMIC WAVES

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ABSTRACT

The behavior of coupled lateral-torsional systems subjected to seismic waves is investigated analytically. This report presents the numerical results of a parametric study for structures subjected to seismic waves. Case studies are provided to show the contribution of each of the selected parameters to the rotational response of the systems. These parameters are: geometric eccentricity, aspect-ratio of the foundation mat, damping ratio, and the ratio of the rotational to translational frequencies. Dynamic eccentricity is selected as an index to represent the level of the response. The sensitivity due to the deviation of the input spectrum is investigated. Accidental eccentricities due to seismic waves are also evaluated. Design concerns are given on how the design eccentricity should be considered based on this study.

Key words: accidental eccentricity; building codes and standards; design eccentricity; dynamic eccentricity; parametric study; seismic waves; structural response.

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1. INTRODUCTION

There has been evidence of serious damage in buildings due to torsional effects in earthquakes [1-2]. Therefore, it is necessary that adequate torsional resistance be provided for building systems. Current design provisions in building codes or standards related to torsional effects [3-5] are mainly based on engineering judgment. These provisions are not necessarily theoretically sound, e.g., it is known that by taking accidental eccentricity as five percent of the building dimension in design may not be conservative [5-9] but this figure is widely used as the accidental eccentricity in present building codes. To avoid structural failures in earthquakes, research is needed to simulate the real behavior of structural systems subjected to seismic waves.

Recently, analytical models have been developed specifically for determining dynamic response of coupled lateral-torsional structural systems. For example, the effects of response due to geometric eccentricity between center of rigidity and center of mass have been investigated extensively by Kan and Chopra for the linear and non-linear systems [7-8]. The response of torsionally coupled elastic systems have been studied by Kung and Pecknold [9] based on a probabilistic ground motion model. Dynamic eccentricity has been estimated by Tso and Dempsey in terms of geometric eccentricity [10]. However, soil-structure interaction (SSI) effects were not considered in these studies. To include SSI effects, a simple approach [11] was proposed that made use of the impedance functions and input motions as computed in references 12 and 13.

In this report, an analytical result will be presented to illustrate the effects of a few parameters pertinent to the characteristics of a structural system. The formulation presented in reference 11 will be briefly described. The parameters selected in the study are: geometric eccentricity, aspect-ratio of the foundation mat, damping ratio, and the ratio of the rotational to translational frequencies. The sensitivity of the response to variation in the ground motion spectrum is also investigated. Based on these analtyical results, discussions are given on how the design eccentricity should be considered in the related seismic design provisions.

2. THE APPROACH

A one-story structural system with geometric eccentricity equal to e at the first floor is shown in figure 1. The structure is subjected to earthquake excitations at the ground floor (foundation). The equations of motion may be written as,

$$m_t \ddot{U}_{yt} + e\, m_t \ddot{U}_{\phi t} + K_y U_{yd} + C_y \dot{U}_{yd} = 0 \tag{1}$$

$$m_t e \ddot{U}_{yt} + I_{\phi t} \ddot{U}_{\phi t} + K_\phi U_{\phi d} + C_\phi \dot{U}_{\phi d} = 0 \tag{2}$$

$$m_b \ddot{U}_{yb} + m_t \ddot{U}_{yt} + m_t e \ddot{U}_{\phi t} = f_y(t) \tag{3}$$

$$m_t e \ddot{U}_{yt} + I_{\phi t} \ddot{U}_{\phi t} + I_{\phi b} \ddot{U}_{\phi b} = f_\phi(t) \tag{4}$$

where m, C, K, and U are equal to the mass, damping, stiffness, and displacement, respectively. Subscript t or b denotes that the term is related to the first or the ground floor; subscript y or ϕ denotes that the term is related to the translational or rotational movements; subscript d denotes tht the term is equal to the difference between the related terms of first floor and the ground floor; e.g., $U_{yd} = U_{yt} - U_{yb}$; $I_{\phi t}$ and $I_{\phi b}$ are the rotational mass moments of inertia taken with respect to the Z-axis located at the center of rigidity, as shown in figure 1 for the first and ground floors, respectively; f_y and f_ϕ are the earthquake excitation forces at the foundation.

"U" and "f" may be transformed into the frequency domain, i.e.,

$$\{U\} = \sum_{S=0} \{U_s\} \exp\,(i\, \omega_s\, t) \tag{5a}$$

and

$$\{f\} = \Sigma_{S=0} \{f_s\} \exp\,(i\, \omega_s\, t) \tag{5b}$$

where ω_s is the frequency. Let the excitation forces between the soil and foundation, f(s) be expressed as [12-13]:

$$\{f(s)\} = [K_f]\, [\{U_f\} - \{U_f^*\}] \tag{6}$$

where (K_f) represents the impedance matrix, (U_f) represents the foundation motion, and (U_f^*) represents the input motion. The subscript f denotes terms related to foundation mat. Equations 1-4 can be rearranged and written in matrix form for each ω_s as:

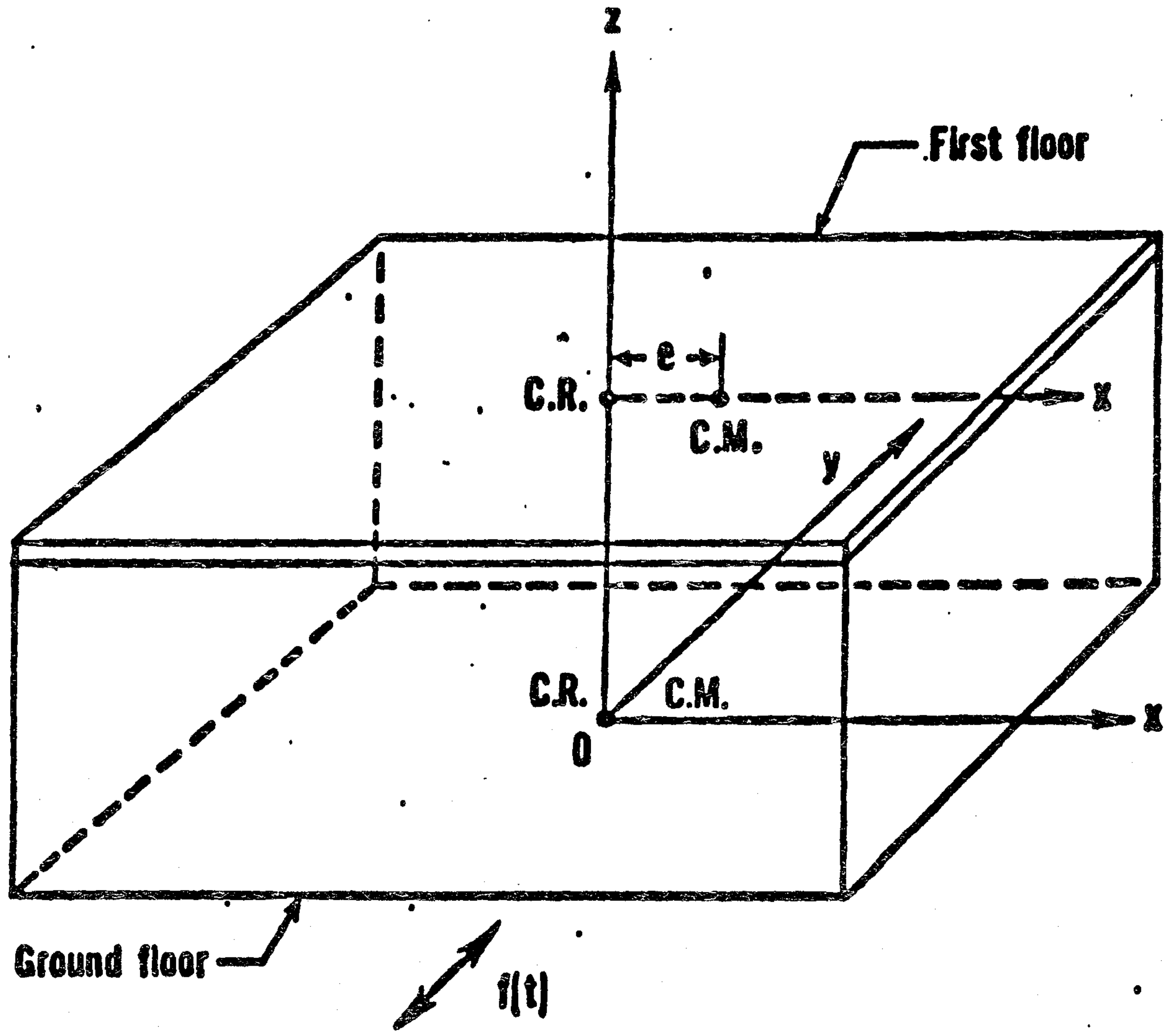

Figure 1. A sketch of a one-story building system subjected to earthquake excitations at the foundation mat

$$\begin{bmatrix} -m_t\omega_s^2 & -em_t\,\omega_s^2 & -m_b\omega_s^2-K_{fy} & 0 \\ -em_t\omega_s^2 & -I_{\phi t}\omega_s^2 & 0 & -I_{\phi b}\omega_s^2 \\ & & & -K_{f\phi} \\ -m_t\omega_s^2+k_y & -e\omega^2 m_t & -K_y-c_y i\omega_s^2 & 0 \\ +C_y i\omega_s & & & \\ -em_t\omega_s^2 & -I_{\phi t}\omega_s^2+K_\phi & 0 & -K_\phi \\ & +C_\phi i\omega_s & & -C_\phi i\omega_s \end{bmatrix} \begin{bmatrix} U_{yt} \\ U_{\phi t} \\ \\ U_{yb} \\ \\ U_{\phi b} \\ \\ \end{bmatrix} = \begin{bmatrix} -K_{fy}U_{fy}^* \\ -K_{f\phi}U_{f\phi}^* \\ \\ 0 \\ \\ 0 \\ \\ \end{bmatrix} \qquad (7)$$

The response of the structure can be calculated once the impedance functions and the input motions of the foundation are known. With the Fast Fourier Transform technique, the dynamic response of the system may be found in the time domain (e.g., ref. 14). For the problem under investigation, the dynamic eccentricity, E(t), is considered to be equal to the torsional moment, M(t), divided by the transverse shear, V(t).

3. ANALYTICAL RESULTS OF THE PARAMETRIC STUDY

Dynamic eccentricity is an important variable for indicating the behavior of a coupled rotational-translational system. Therefore, it is selected as an index to represent the structural response in the investigation. The effects on dynamic eccentricity due to various parameters are illustrated with a few case studies. The analytical results are given in this section. Unless otherwise specified, the general properties of the system are taken as:

$m_t = 3.63 \times 10^7$ kg, $m_b = 1.85 \times 10^8$ kg, $I_{\phi t} = 3.03 \times 10^9$ kg-m^2,
$I_{\phi b} = 1.52 \times 10^{10}$ kg-m^2, $K_y = 1.44 \times 10^9$ N/m, $K_\phi = 2.70 \times 10^{11}$ N-m/rad,
C_y and C_ϕ = 2 percent of the critical damping ratios.

The soil is assumed to have a damping ratio equal to 0.05, with Poisson's ratio equal to 0.33 and the shear modulus equal to 2.15×10^8 N/m^2. The impedance functions and input motions are taken from references 12 and 13. In these figures, U_s^* represents the free field motions. The functions are shown in figure 2 through 5 for foundation mats with dimensions equal to 25.9m x 25.9m (85 ft x 85 ft) and 51.2m x 12.8m (168 ft x 42 ft), respectively. These curves are used as input functions in the analysis. The input spectrum is based on curve a as shown in figure 6 for all cases except otherwise noted. Analytical results are discussed below for each of the parameters.

Geometric Eccentricity - Dynamic eccentricities of the first floor for cases with geometric eccentricity ratios , e/r, equal to 0.1, 0.2 and 0.32 are plotted in figure 7, where r represents the radius of gyration. These curves are obtained by solving equation 7. The foundation mat is assumed to be 25.9m x 25.9m. It is shown in the figure that the dynamic eccentricity becomes larger as the geometric eccentricity increases. It is also shown that the frequencies of the system are affected by the geometric eccentricity values. Accidental eccentricity due to seismic waves can be found by solving equation 7 with e = 0 [11]. The accidental eccentricity corresponding to this case shown in figure 7 is given in figure 8 by the dotted line.

Aspect-Ratio of the Foundation Mat - To illustrate the effects due to the aspect-ratio of the foundation mat, the dynamic eccentricities are plotted in figure 9 for a case with the same structural properties as the previous case except for the dimensions of the foundation mat. The foundation mat selected here is of rectangular shape with dimensions equal to 51.2m x 12.8m. These two cases have about the same mat area, but the input functions are different as shown in figures 2 through 5. Hence, the dynamic eccentricities are different in terms of magnitude and frequency. The accidental eccentricities for this case are also shown in figure 8. As expected, the results shown in figure 8 indciate that the eccentricity increases for cases with a larger aspect-ratio.

Damping Ratio of the Structural Systems - The rotational effects due to damping ratios are shown in figure 10. Dynamic eccentricity for the case with e/r equal to 0.2 as shown in figure 7 is replotted here. For this case the damping ratio is equal to 0.02. The corresponding case with a damping ratio equal to 0.07 is also shown in figure 10. It can be seen that the difference between these cases in terms of maximum dynamic eccentricity is not significant.

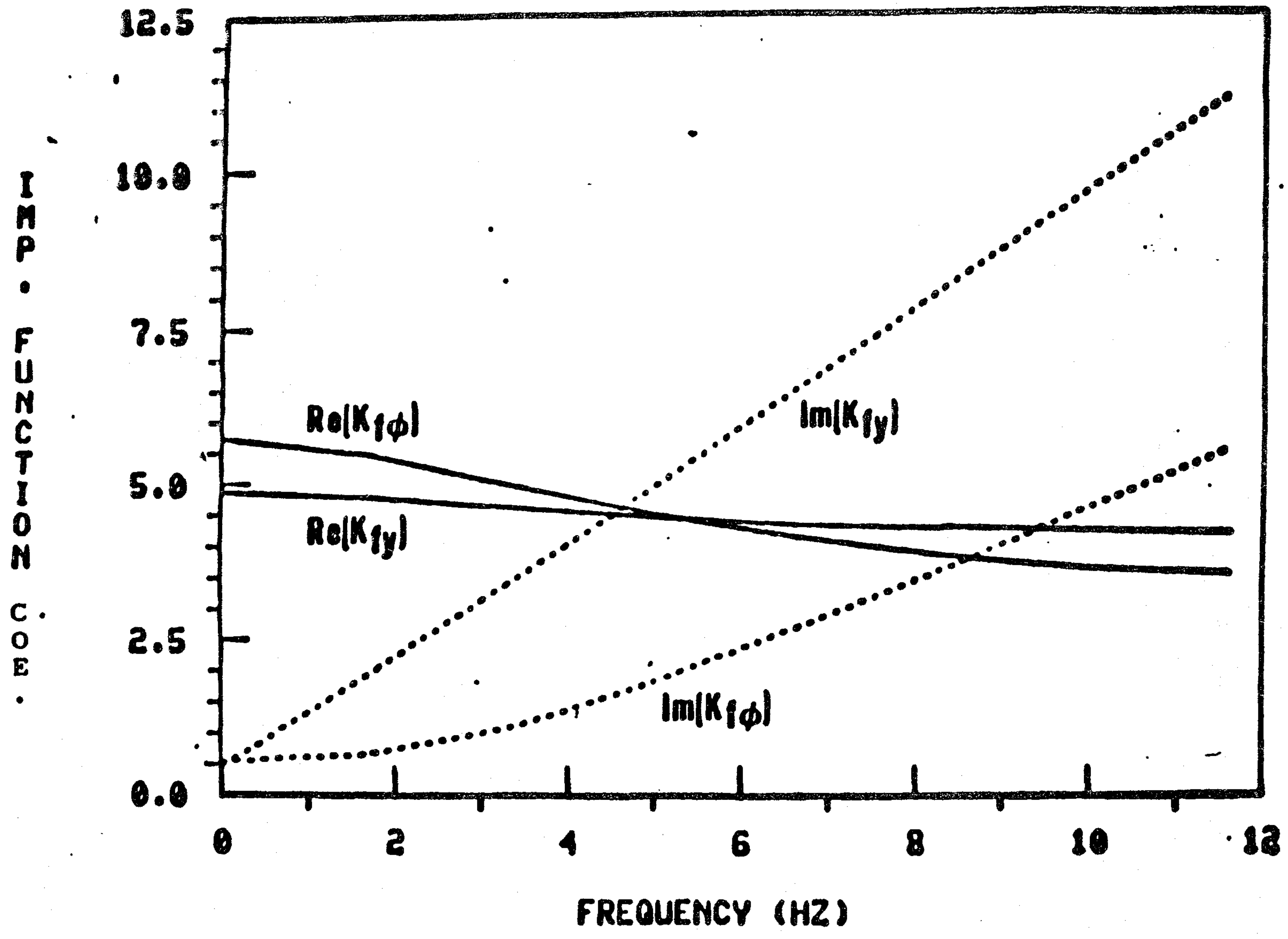

Figure 2. The impedance functions for a foundation mat with dimensions equal to 85 ft x 85 ft

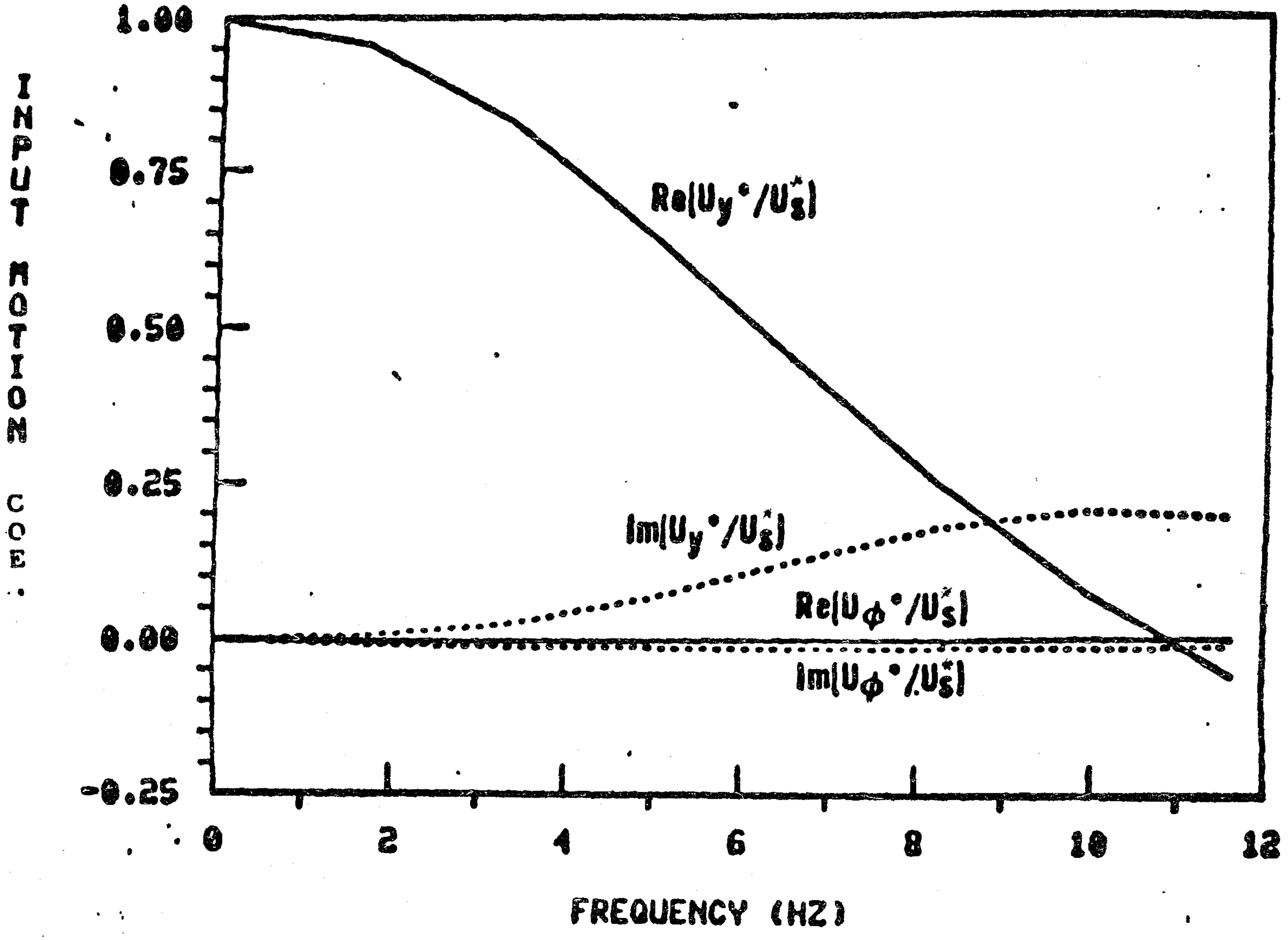

Figure 3. The input motions for a foundation mat with dimensions equal to 85 ft x 85 ft

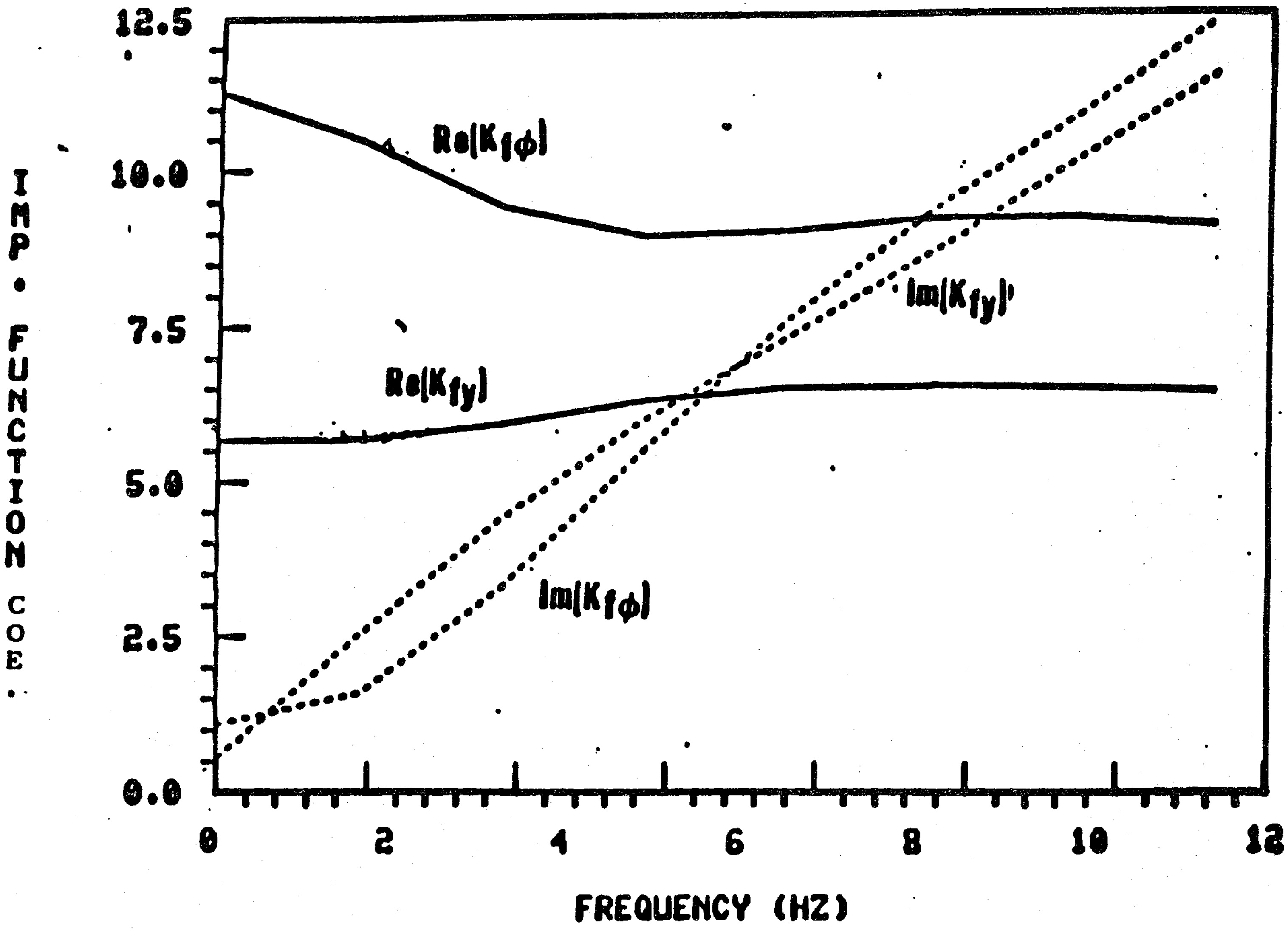

Figure 4. The impedance functions for a foundation mat with dimensions equal to 168 ft x 42 ft

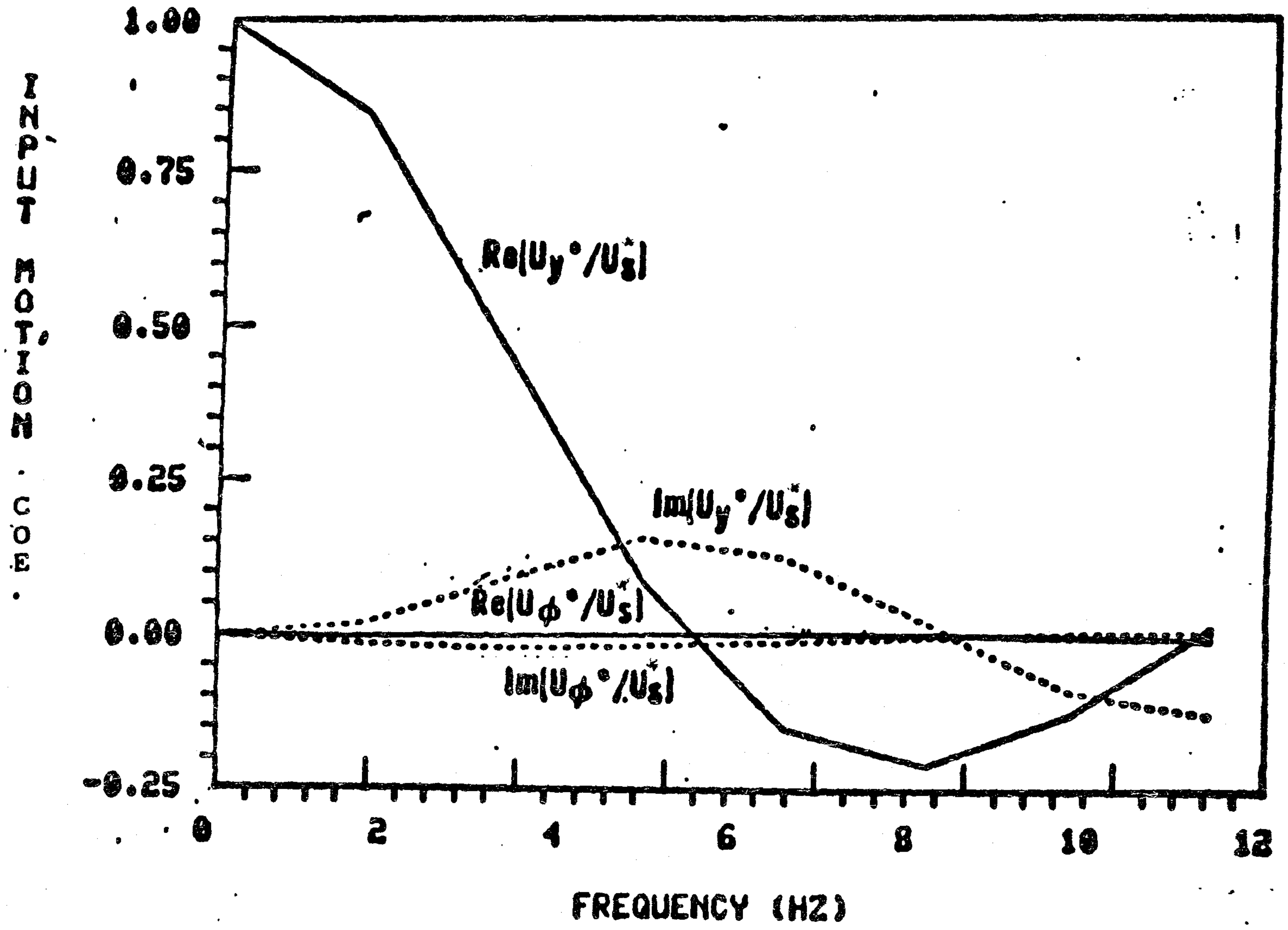

Figure 5. The input motions for a foundation mat with dimensions equal to 168 ft x 42 ft

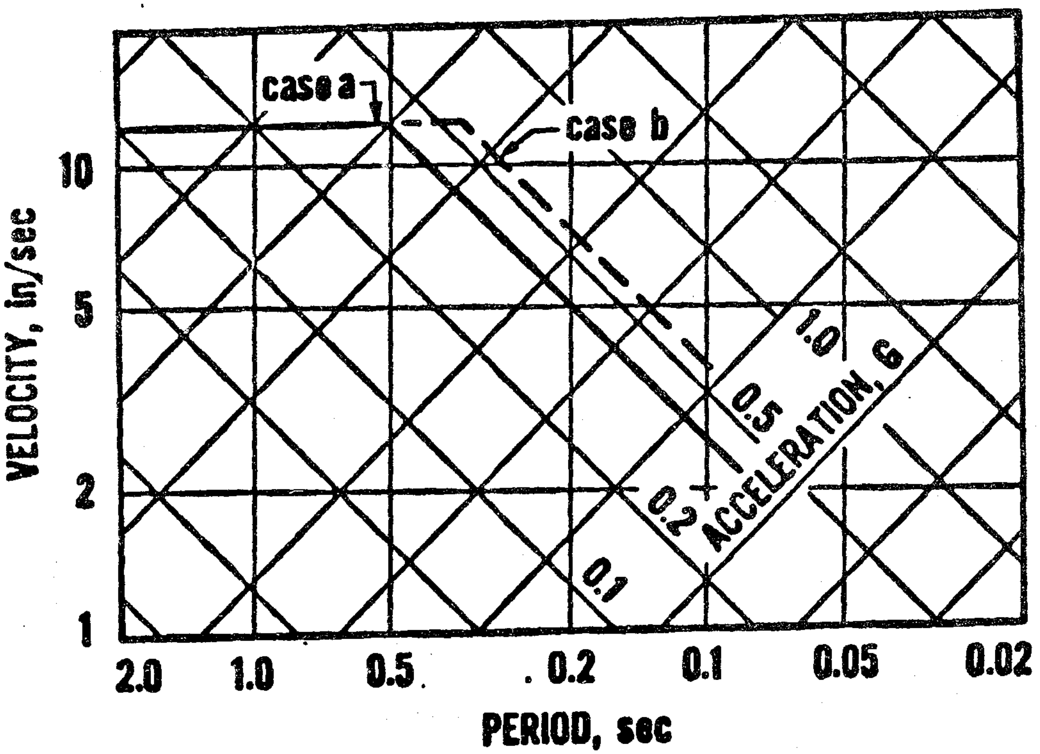

Figure 6. Spectra assumed as the free-field motions

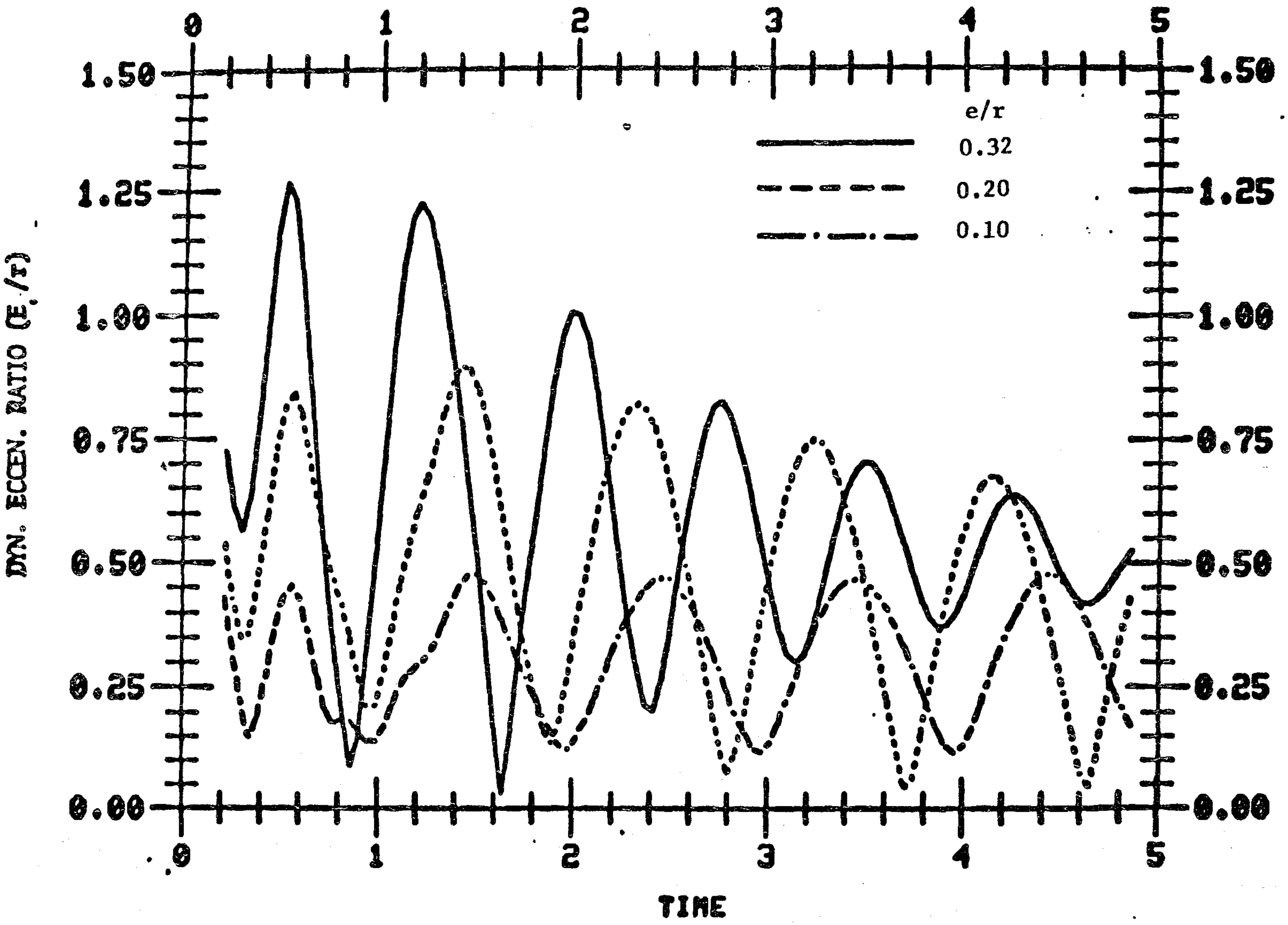

Figure 7. Dynamic eccentricities for a case with foundation mat equal to 85 ft x 85 ft for e/r = 0.1, 0.2, and 0.32, respectively

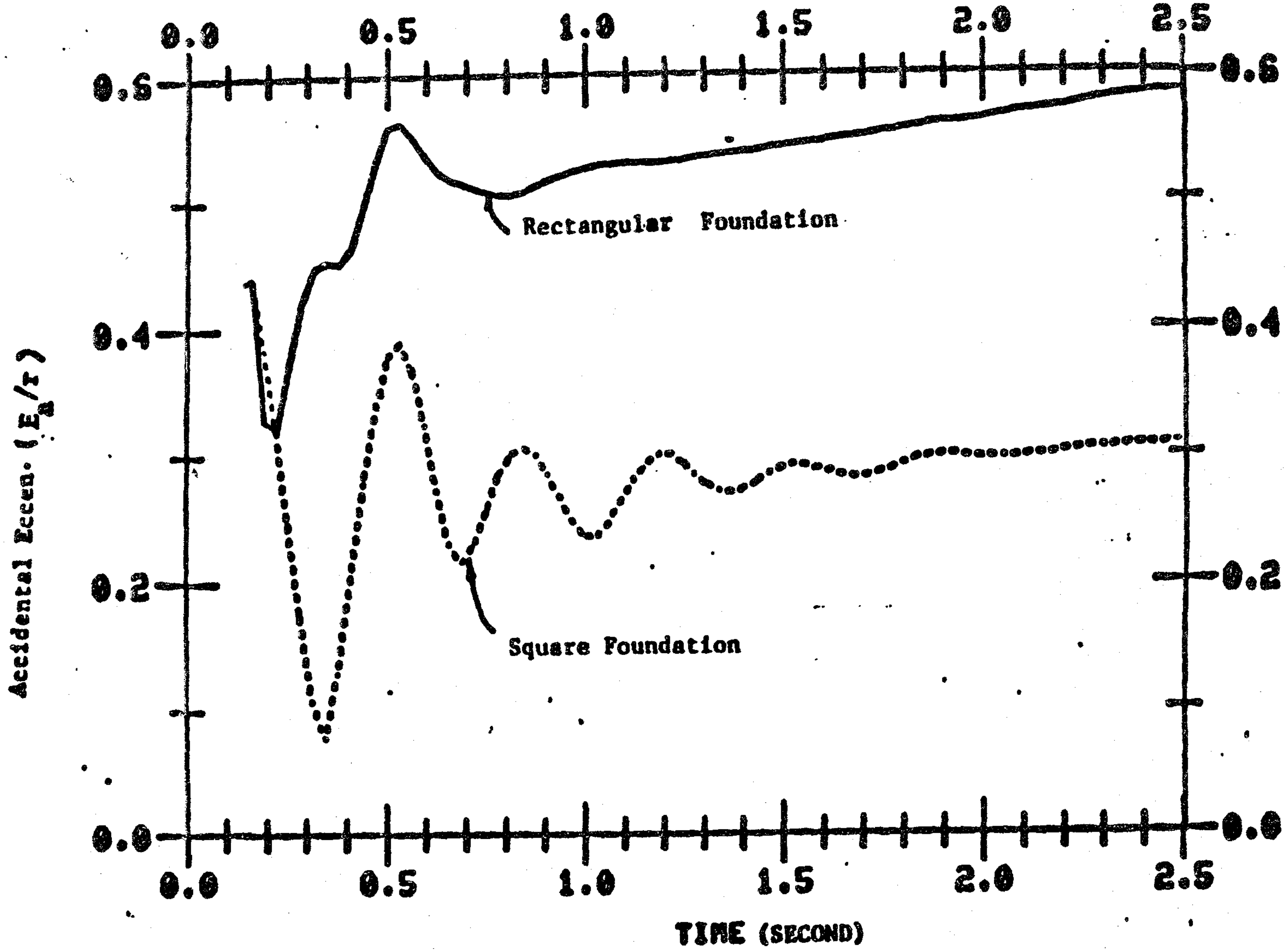

Figure 8. Accidental eccentricities for the selected cases

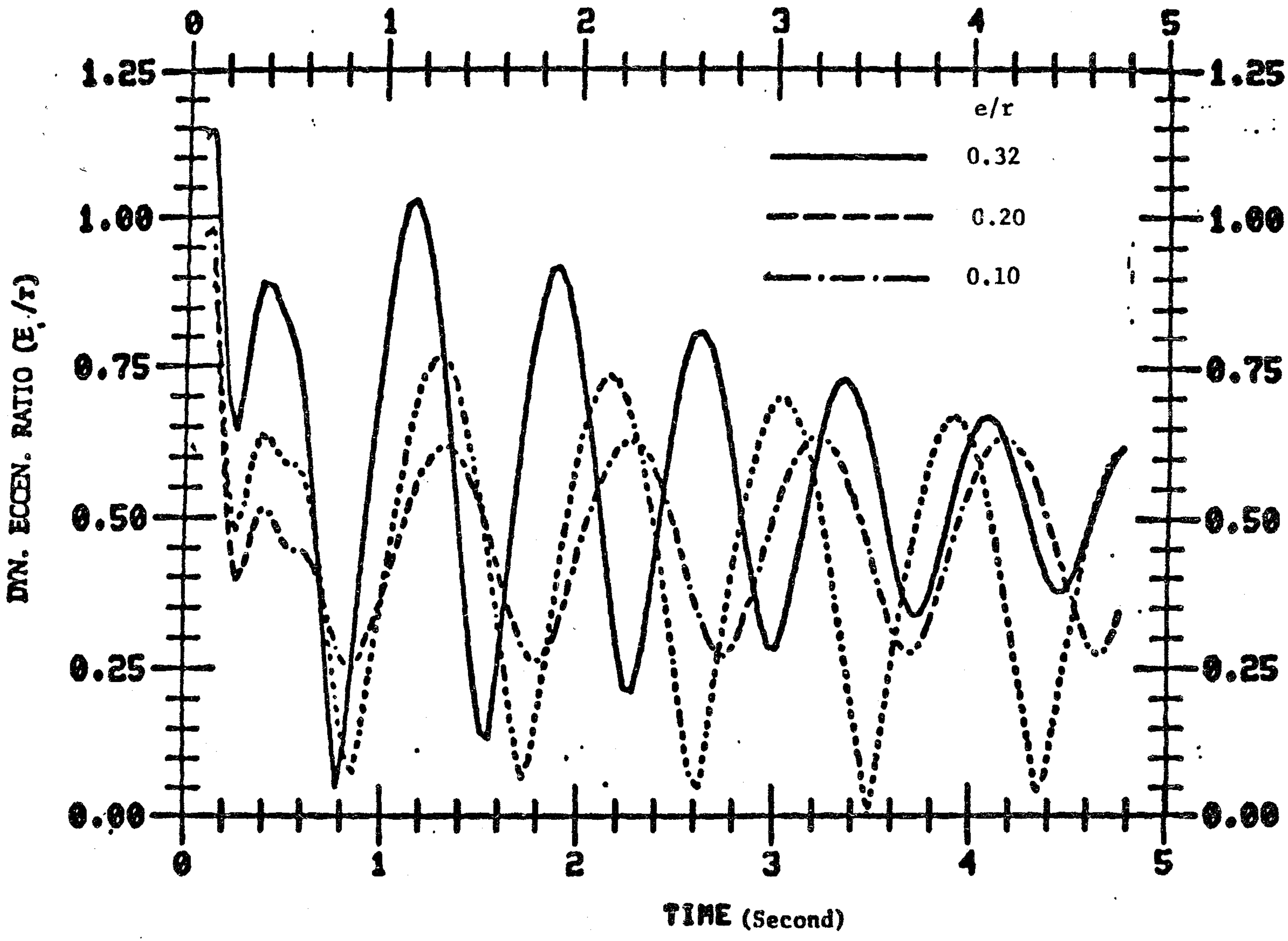

Figure 9. Dynamic eccentricities for a case with the foundation mat equal to 168 ft x 42 ft for e/r = 0.1, 0.2, and 0.32, respectively

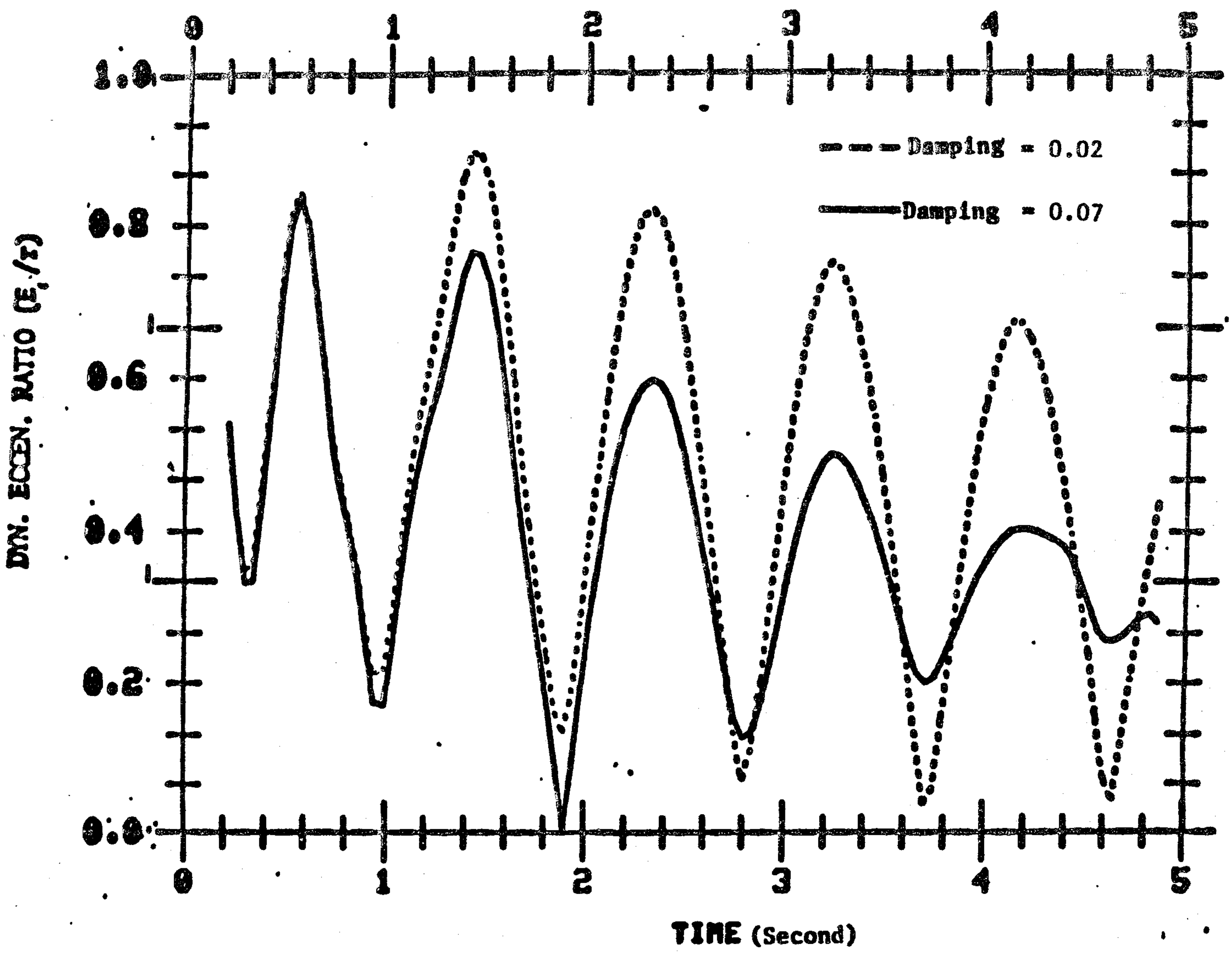

Figure 10. Dynamic eccentricities for cases with different damping ratios

Ratio of the Uncoupled Rotational Frequency to Translational Frequency - Dynamic eccentricities are shown in figure 11 as a function of the ratio between the uncoupled rotational frequency to translational frequency. The curve with a solid line in the figure is a replot of the case with e/r = 0.2 in figure 7. The ratio is approximately 1.5 for this case. The curve with a dotted line represents a case with the same system except with a higher $I_{\phi t}$. For this case, $I_{\phi t}$ is selected such that the rotational frequency is one-third higher than the case with a solid line. As shown in figure 11, the magnitude of the eccentricity is lower for a larger ratio of the rotational to translational frequencies, but the frequency of the dynamic eccentricity is higher if the ratio is higher. In reference 8 and 9, similar effects are found despite the fact that the SSI effect was not considered in those studies.

Ground Motion Spectrum - The sensitivity of dynamic eccentricity due to a change in the spectrum as shown in figure 6 describing ground motion are shown in figures 12 and 13. Figure 12 shows dynamic eccentricities for e/r = 0.1, 0.2, and 0.32,respectively, based on the spectrum shown in case b in figure 6. A replot of the corresponding case for e/r = 0.32 in figures 7 and 12 is shown in the figure 13 in which, it can be seen that the difference in terms of dynamic eccentricity is negligible due to the change in input spectrum as represented by curves a and b in figure 6.

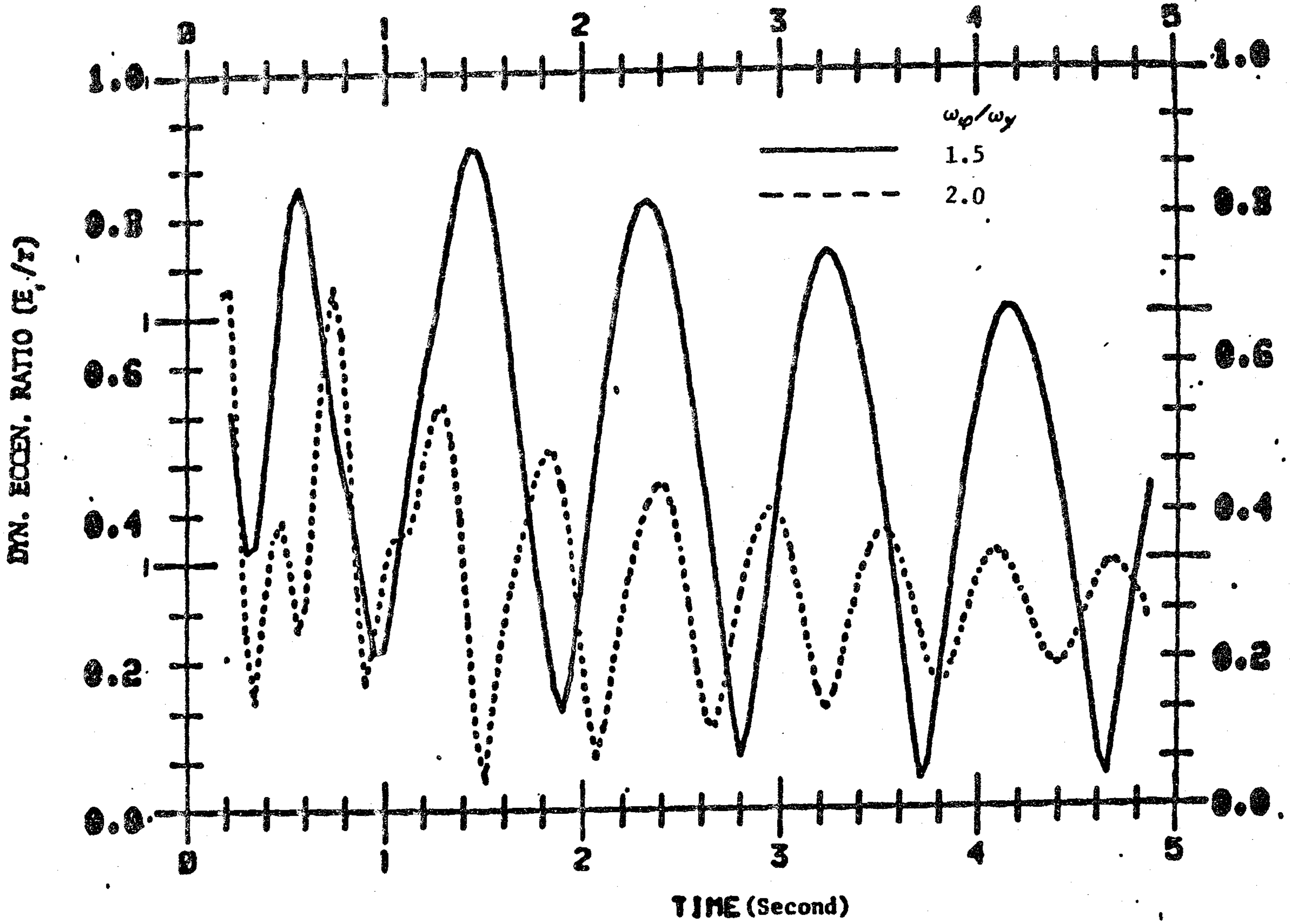

Figure 11. Dynamic eccentricities for cases with different ratios of rotational to translational frequencies

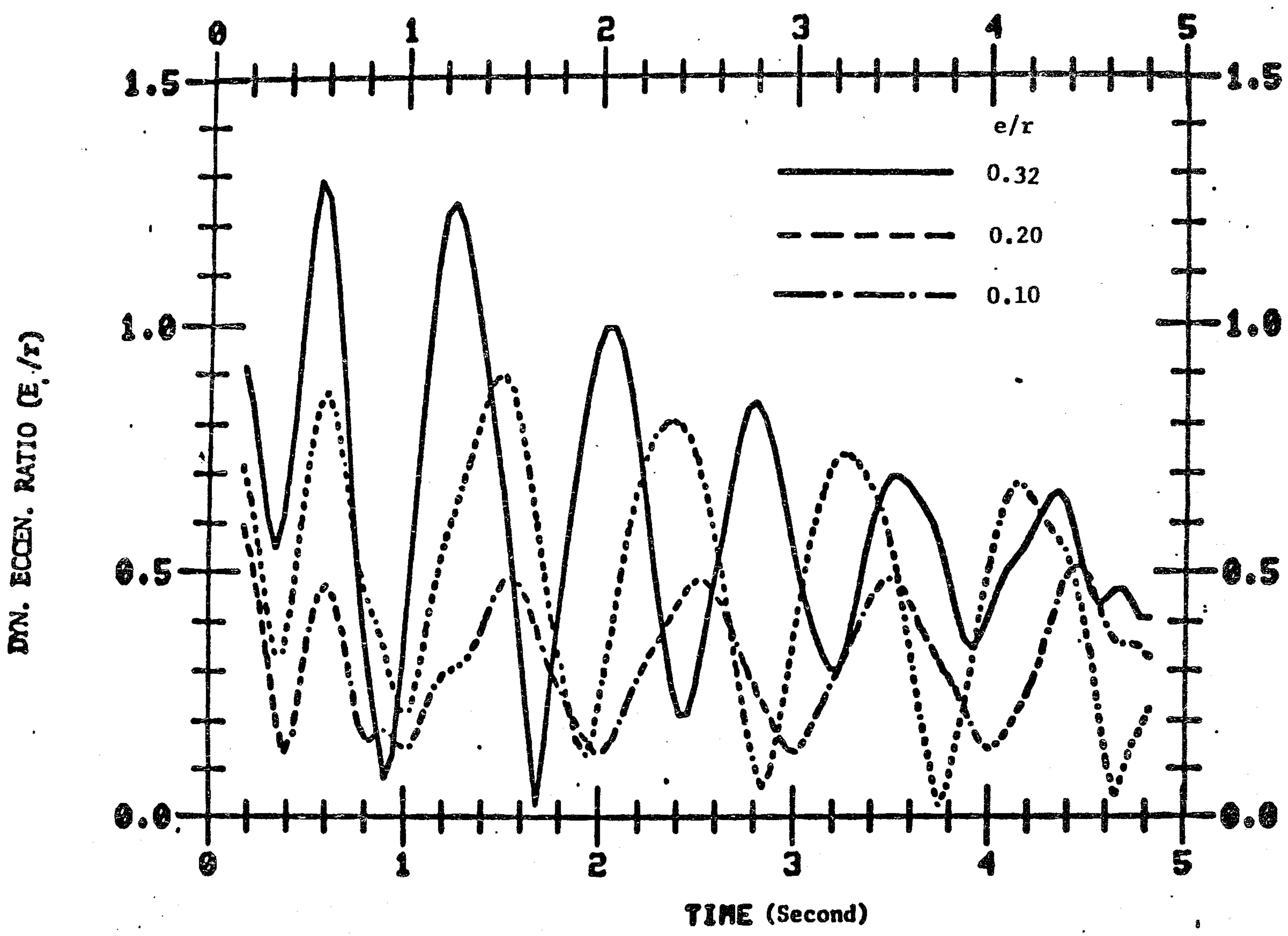

Figure 12. Dynamic eccentricities for a case with the same conditions as the case represented in figure 7 except the input spectrum is represented by case b in figure 6

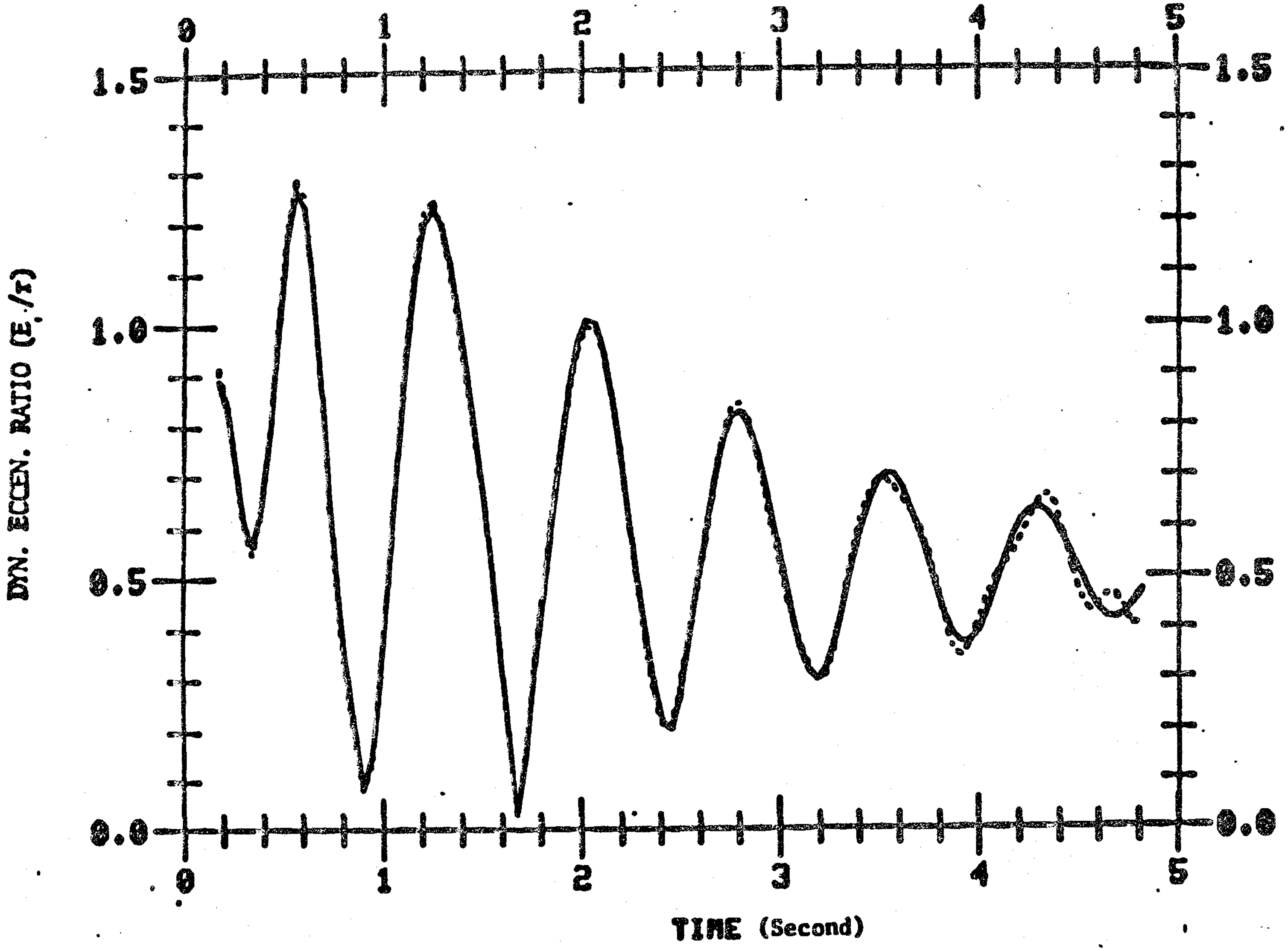

Figure 13. Comparison of the bases shown in figures 7 and 12 for e/r = 0.32

4. DESIGN CONCERNS

In the present seismic building codes or provisions, a design eccentricity is used to represent the level of the rotational response. To select the design eccentricity properly, dynamic effects on structural systems should be considered. In this study, dynamic eccentricity has been found for a structure subjected to seismic waves. In general, the maximum shear and torsion will not occur at the same time. The maximum dynamic eccentricity always occurs at the time of maximum torsion but not the maximum shear. For this reason, the use of dynamic eccentricity as a reference index not only is a proper approach but can lead to an economical design. To illustrate this point, V(t) and M(t) for the case with $e/r = 0.2$ shown in figure 7 are plotted in figure 14. The vertical axis in the figure represents the ratios of $V(t)/V_{max}$ or $M(t)/M_{max}$, where V_{max} and M_{max} are the maximum shear and torsion values, respectively. In this case, the V(t) corresponding to M_{max} is about 88 percent of the V_{max}. Thus, it would be overly conservative to design a structure based on both M_{max} as well as V_{max} as in the present practice.

The term "accidental eccentricity" is used in the literature to cover the torsional effects due to several factors. In reference 3 it is stated that "these factors include rotational component of ground motion about a vertical axis; unforeseeable differences between computed and actual values of stiffness, yield strengths and dead-load masses; and unforeseeable unfavorable distribution of live-load masses." Actually the "rotational component of ground motion about a vertical axis" can be computed with an assumption that the free-field seismic motion is known. For the approach presented here, the accidental eccentricity can be determined as a special case to the general solution of the problem. It is shown that the eccentricity is larger for a structure with a larger aspect ratio of the mat. On the other hand, the effect of accidental eccentricity on the response of symmetric buildings cannot be ignored. The solutions obtained here verify the concepts presented in reference 6.

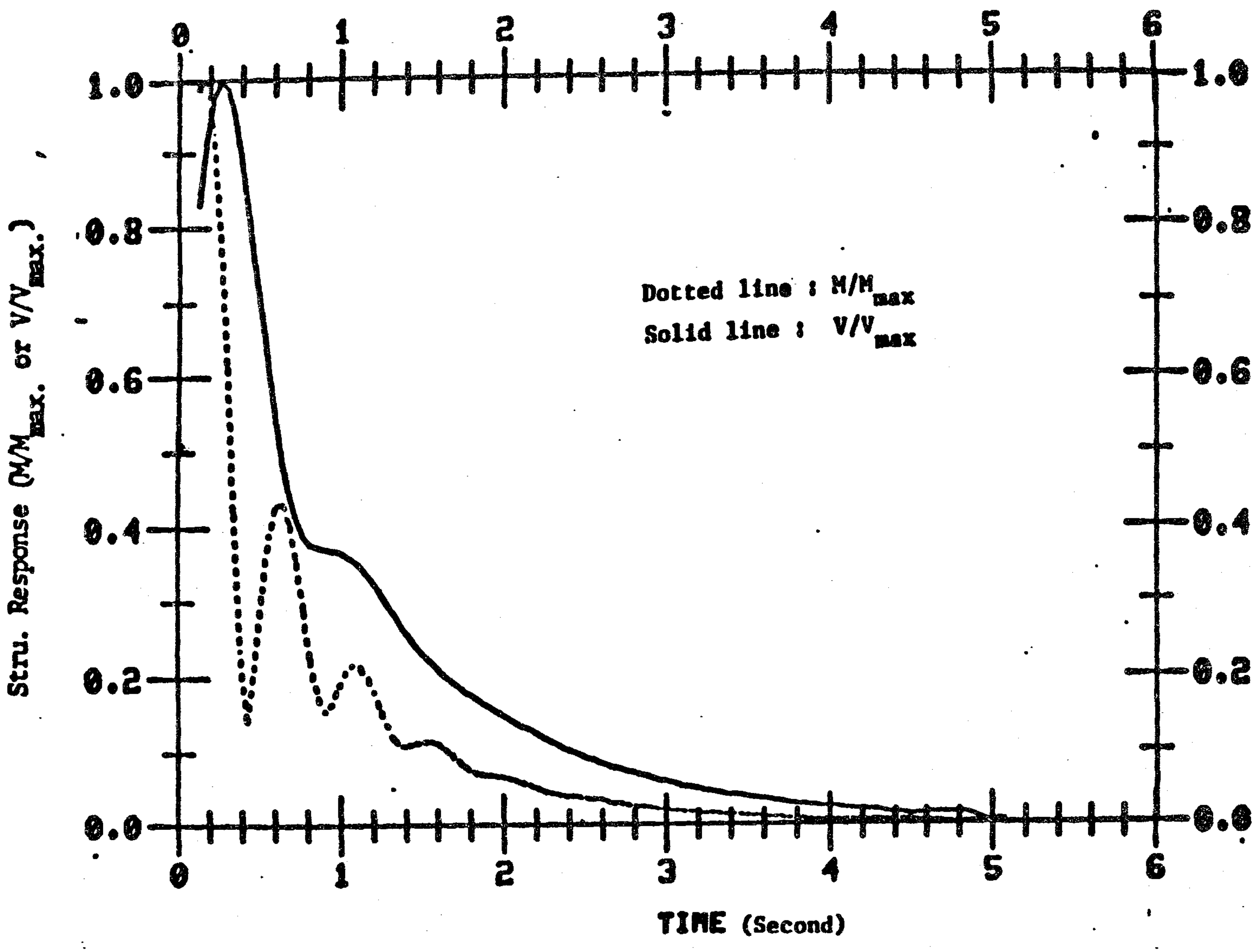

Figure 14. M(t) and V(t) for the case shown in figure 7 with e/r = 0.20

5. CONCLUSION

Analytical results of a parametric study are given for a structure subjected to seismic waves. For the accidental eccentricity due to seismic waves, the results shown here verify the concept presented earlier in reference 7, i.e., the accidential eccentricities on symmetric buildings cannot be ignored. Based on the selected index, dynamic eccentricity, it is also shown in this study that the rotational response of a system depends greatly on geometric eccentricity, the dimensions of the foundation mat and the ratio of rotational frequency to translational frequency. The maximum dynamic eccentricity of the system is not affected significantly by the damping of the system, nor by the variation of the input spectrum as shown in figure 6. It is important that all parameters essential to the rotational responses of the system shall be considered in determining the design eccentricity. The related design provisions in current building codes which consider only the effects of the geometric eccentricity should be updated. To develop consistent [15] seismic provisions based on this approach, further study of various structures under specific conditions is needed.

6. ACKNOWLEDGMENTS

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