# ULTIMATE STRENGTH OF MASONRY SHEAR WALLS: PREDICTIONS VS TEST RESULTS 

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#### Abstract

This study compares the ability of four different equations to predict the ultimate shear stress in masonry walls failing in shear. Experimental data on fully-grouted reinforced shear walls from four different sources are compared with the predictions from the four equations. Wall characteristics from 62 test specimens were used as input to the four predictive equations. The ultimate strength predictions were then compared to the actual measured strength of the 62 test walls.

Two of the equations (the existing Uniform Building Code equation for shear strength of masonry walls and the Architectural Institute of Japan's equation for predicting the shear strength of reinforced concrete shear walls) were found to be inadequate for the prediction of ultimate shear strength of masonry walls. An equation proposed by Shing et al. was found to predict shear strength well for only limited ranges of variables, primarily because excessive weight is given to the contributions of horizontal reinforcement to strength. An equation proposed by Matsumura was found to be the best predictor of the four equations examined, but it lacks the consistency needed to use it as a basis for design.

The conclusions drawn from the present study indicate the possibility of developing reliable predictive formulations using both rational analysis and an empirical approach.

Key Words: Masonry; predicted strength; reinforced walls; shear strength; shear walls; test strength; ultimate load


## PREFACE

SI units are used in this report. However, U.S. Customary units are also specified to conform with current practices of the masonry industry in the United States. Codes and standards, construction specifications and tolerances, and nominal and actual sizes of standard masonry units manufactured in the United States are all currently measured in U.S. Customary Units. This system of measure was therefore used as a supplement to aid the masonry industry and standards organizations in utilizing the results of this investigation.

## 1. INTRODUCTION

The last decade has seen the rapid evolution of reinforced masonry as an engineered construction material, increasingly allowing masonry buildings to be considered as viable alternatives to concrete or steel structures. By stimulating competition and reducing building costs, improved masonry design enhances U.S. construction productivity in both domestic and international markets. Increased safety for occupants, especially during earthquakes, is another benefit the nation reaps from the improved procedures being developed for engineered masonry buildings. Federally funded research has contributed significantly to the rapid progress that has been made in recent years.

As part of the ongoing effort to improve masonry technology and to make masonry design and analysis methodologies comparable to those of concrete and steel, limit state design procedures are being developed. Major progress towards understanding the ultimate behavior of masonry walls has been made, in large part through research conducted under the auspices of the Joint U.S.Japan Technical Coordinating Committee on Masonry Research (better known as JTCCMAR). JTCCMAR coordinates masonry research on material behavior and seismic response analysis and design in the United States and Japan. The U.S. research is coordinated by the Technical Coordinating Committee for Masonry Research (TCCMAR).

Since its inception in 1984, a substantial amount of TCCMAR research has been sponsored by the National Science Foundation (NSF) under the National Earthquake Hazards Reduction Program (NEHRP), which was established in accordance with the National Earthquake Hazards Reduction Act passed by the U.S. Congress in 1977. That legislation assigned the National Bureau of Standards (now known as the National Institute of Standards and Technology or NIST) the mission to assist in the development of improved design procedures for buildings subject to earthquakes. The NIST Masonry Research Program is coordinated with the JTCCMAR programs.

A recent NIST report titled "Review of Research Literature on Masonry Shear Walls" [1] reviews the existing literature on experimental research on masonry shear walls conducted during the past 15 years. The report recommends that the accuracy and reliability of proposed formulations for predicting masonry shear wall strength under lateral and gravity loads be assessed. The present study implements this recommendation in part, through comparisons of the experimentally observed shear strengths of fully-grouted walls with predictions according to four different equations for evaluating shear strength.

A detailed description of the predictive equations and the data sets used in this study are included in Section 3. The methodology used in the comparison is
described in Section 4. Section 5 presents and analyzes the results of the comparative study. Conclusions drawn from the study and possible topics for additional investigations are described in Sections 6 and 7.

## 2. OBJECTIVE

The objective of this study was to evaluate the accuracy of empirical equations in predicting in-plane shear strength of fully grouted concrete and clay masonry walls. Accuracy was assessed by comparing predicted strengths to actual tested strengths. Four equations were checked against four sets of experimental data from independent research sources. Two of the equations assessed here are part of existing code provisions. The other two empirical equations are proposed formulas which have been developed using experimental data. These experimental data sets were among the four sets used in this study to evaluate the accuracy of predicted strength. By crosschecking an equation from one source against data from another source and against the much larger combined data set, the accuracy and consistency of the equations were assessed, within the ranges of parameters used in the experiments.

## 3. SCOPE

The report titled "Review of Technical Literature on Masonry Shear Walls" [1] guided the selection of the predictive equations and experimental data sets to be used. Results from some 700 independent tests, along with accompanying documentation regarding the geometric and material properties of the test specimens, methods of testing, and variation of design parameters, are included in that report. Test results of laterally loaded specimens of fullygrouted reinforced masonry were selected for this study. Partially-grouted masonry walls were not included in the present study.

### 3.1 Experimental Data Sets

The data sets included in this study were limited to results from fully-grouted reinforced masonry walls which were subjected to reverse cyclic lateral loads until failure in the shear mode occurred. Three data sets were obtained from tests in which the top and bottom surfaces of the specimens were rotationally fixed. One data set, obtained from tests conducted by Shing et al, used specimens which were rotationally free at the top (i.e. cantilever walls). Both clay and concrete unit masonry walls were represented in the tests. The scope of the experiments and the range of test variables were other factors taken into consideration in the selection of the data. The data sets selected for this study were assembled from the following experimental programs:

1) tests conducted by Shing et al. at the University of Colorado under the U.S.-Japan Joint Technical Committee on Masonry Research (JTCCMAR) program [2,3],
2) tests conducted by Matsumura at Kanagawa University in Japan [4],
3) tests conducted by Okamoto et.al. at Japan's Building Research Institute, Ministry of Construction, in connection with the JTCCMAR program [5], and
4) masonry research conducted by Sveinsson et al. at the University of California at Berkeley [6].

Each of the experimental studies used displacement-controlled tests consisting of multiple cycles of reversed loadings. Predefined load-displacement histories characterized by increasing amplitudes to failure were used. The common loading procedure and the use of similar loading rates in each of the four studies produced comparable tests. The load-displacement histories are described in detail in the cited references.

Ultimate shear strength was defined as the average of the two peak shear forces attained in the two opposite directions of cyclic loading. Shear strength was calculated using gross area based on actual dimensions. Data from specimens that were reported to have failed in bending were eliminated. The final data set consisted of 62 separate tests. The data subsets finally selected from the studies listed above are identified in the text by the letters $\mathrm{S}, \mathrm{M}, \mathrm{O}$ and B, respectively.

Relevant properties of the specimens are listed in Table 1. Definitions of symbols used in column headings of the table are included in section 3.3. The four groups are identified by the suffix ( $S, M, O$, or $B$ ) appended to the specimen identification tag in the second column. There are $10 \mathrm{~S}, 18 \mathrm{M}, 90$, and 25 B specimens. Specimens 21-S, 22-S, WSR2-M, WSR4-M, WSR5-M, WSR6-M, WSR1-O, WSR4-O, WSR7-O, and the nine B specimens designated by $B R$ are single-wythe walls constructed with hollow brick units. The four $B$ specimens designated by DBR are two-wythe grouted walls built with solid brick units. Specimen WSRC-O is a reinforced concrete shear wall which was included primarily because one of the empirical formulas examined in this study was developed for reinforced concrete shear walls. Its inclusion does not significantly affect the evaluations of accuracy performed in this study. The

| TEST NUMBER | SPECIMEN LABEL | $\begin{gathered} \mathrm{h} \\ \mathrm{~mm} \end{gathered}$ | $\begin{gathered} \mathrm{L} \\ \mathrm{~mm} \end{gathered}$ | $\begin{gathered} \mathrm{t} \\ \mathrm{~mm} \end{gathered}$ | $\begin{gathered} \mathrm{d} \\ \mathrm{~mm} \end{gathered}$ | $\begin{aligned} & \hline \mathrm{sh} \\ & \mathrm{~mm} \end{aligned}$ | r | rd | $\begin{aligned} & \mathrm{fim} \\ & \mathrm{MPa} \end{aligned}$ | fyh MPa |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 3－S | 1829 | 1829 | 143 | 1727 | 406 | 1.00 | 1.08 | 20.67 | 385.84 |
| 2 | 4－S | 1829 | 1829 | 143 | 1727 | 406 | 1.00 | 1.06 | 17.91 | 385.84 |
| 3 | 5－S | 1829 | 1829 | 143 | 1727 | 406 | 1.00 | 1.06 | 17.01 | 385.84 |
| 4 | 7－S | 1829 | 1829 | 143 | 1727 | 408 | 1.00 | 1.06 | 20.67 | 385.84 |
| 5 | 9－5 | 1829 | 1829 | 143 | 1727 | 408 | 1.00 | 1.08 | 20.87 | 385.84 |
| 6 | 13－S＇ | 1829 | 1829 | 143 | 1727 | 408 | 1.00 | 1.08 | 22.74 | 461.63 |
| 7 | 14－S | 1829 | 1829 | 143 | 1727 | 406 | 1.00 | 1.08 | 22.74 | 385.84 |
| 8 | 16－S | 1829 | 1829 | 143 | 1727 | 408 | 1.00 | 1.00 | 17.23 | 481.63 |
| 9 | 21－S | 1829 | 1829 | 137 | 1727 | 406 | 1.00 | 1.08 | 28.18 | 385.84 |
| 10 | 22－S | 1829 | 1829 | 137 | 1727 | 406 | 1.00 | 1.08 | 26.18 | 385.84 |
| 11 | KW4－1－M | 1800 | 1590 | 150 | 1500 | 400 | 1.13 | 1.20 | 21.80 | 385.00 |
| 12 | KW3－1－M | 1800 | 1190 | 150 | 1100 | 400 | 1.51 | 1.64 | 21.80 | 385.00 |
| 13 | KW3S－1－ | 1800 | 1190 | 150 | 1100 | 400 | 1.51 | 1.64 | 21.80 | 385.00 |
| 14 | KW2－1－M | 1800 | 790 | 150 | 700 | 400 | 2.28 | 2.57 | 21.80 | 385.00 |
| 15 | WS2－M | 1800 | 1190 | 190 | 1095 | 400 | 1.51 | 1.64 | 22.30 | 385.00 |
| 16 | WS4－M | 1900 | 1190 | 190 | 1095 | 400 | 1.51 | 1.64 | 22.30 | 385.00 |
| 17 | WS5－M | 1800 | 1190 | 190 | 1095 | 400 | 1.51 | 1.84 | 22.30 | 385.00 |
| 18 | WS9－M | 1800 | 1190 | 190 | 1095 | 400 | 9.51 | 1.64 | 22.30 | 385.00 |
| 19 | WS10－M | 1800 | 1190 | 190 | 1095 | 400 | 1.51 | 1.64 | 22.30 | 385.00 |
| 20 | WSe－2－M | 1800 | 1180 | 190 | 1095 | 400 | 1.51 | 1.64 | 29.00 | 385.00 |
| 21 | WSB21－M | 1800 | 1190 | 190 | 1095 | 400 | 1.51 | 1.64 | 26.10 | 385.00 |
| 22 | WS822－M | 1800 | 1190 | 190 | 1095 | 400 | 1.51 | 1.64 | 27.40 | 385.00 |
| 23 | WSE3－M | 1800 | 1190 | 190 | 1095 | 400 | 1.51 | 1.64 | 26.40 | 385.00 |
| 24 | WSB4－M | 1800 | 1190 | 190 | 1095 | 400 | 1.51 | 1.64 | 31.40 | 385.00 |
| 25 | WSR2－M | 1700 | 1110 | 190 | 1005 | 378 | 1.53 | 1.69 | 28.60 | 385.00 |
| 28. | WSR4－M | 1700 | 1110 | 190 | 1005 | 378 | 1.53 | 1.69 | 28.60 | 385.00 |
| 27 | WSA5－M | 1700 | 1110 | 190 | 1005 | 378 | 1.53 | 1.69 | 28.60 | 385.00 |
| 28 | WSRE－M | 1700 | 1110 | 190 | 1005 | 378 | 1.53 | 1.69 | 28.60 | 385.00 |
| 29 | WS1－0 | 1800 | 2000 | 190 | 1905 | 400 | 0.80 | 0.84 | 17.91 | 354.44 |
| 30 | WS4－0 | 1800 | 1200 | 190 | 1105 | 400 | 1.50 | 1.63 | 22.81 | 354.44 |
| 31 | WS7－0 | 1800 | 800 | 190 | 705 | 400 | 2.25 | 2.55 | 17.91 | 354.44 |
| 32 | WSNT－O | 1800 | 1200 | 190 | 1105 | 400 | 1.50 | 1.63 | 22.81 | 354.44 |
| 33 | WSN2－O | 1800 | 1200 | 190 | 1105 | 400 | 1.50 | 1.63 | 22.81 | 354.44 |
| 34 | WSA1－0 | 1800 | 2000 | 190 | 1905 | 400 | 0.90 | 0.94 | 26.73 | 354.44 |
| 35 | WSR4－0 | 1800 | 1200 | 190 | 1105 | 400 | 1.50 | 1.63 | 25.16 | 354.44 |
| 36 | WSR7－0 | 1800 | 800 | 190 | 705 | 400 | 2.25 | 2.55 | 21.35 | 354.44 |
| 37 | WSRC－O | 1800 | 1200 | 190 | 1105 | 400 | 1.50 | 1.63 | 26.73 | 354.44 |
| 38 | CB13－B | 1422 | 1219 | 194 | 1143 | 284 | 1.17 | 1.24 | 23.14 | 406.51 |
| 39 | CB15－B | 1422 | 1219 | 194 | 1143 | 284 | 1.17 | 1.24 | 23.14 | 406.51 |
| 40 | CB17－B | 1422 | 1219 | 143 | 1143 | 284 | 1.17 | 1.24 | 15.83 | 437.52 |
| 41 | CB18－B | 1422 | 1219 | 143 | 1143 | 284 | 1.17 | 1.24 | 15.83 | 437.52 |
| 42 | CB20－B | 1422 | 1219 | 143 | 1143 | 474 | 1.17 | 1.24 | 15.13 | 437.52 |
| 43 | CB21－B | 1422 | 1219 | 143 | 1143 | 474 | 1.17 | 1.24 | 15.13 | 437.52 |
| 44. | C823－8 | 1422 | 1219 | 143 | 1143 | 203 | 1.17 | 1.24 | 15.13 | 437.52 |
| 45 | CB24－B | 1422 | 1218 | 143 | 1143 | 399 | 1.17 | 1.24 | 15.13 | 437.52 |
| 46 | CB25－B | 1422 | 1219 | 143 | 1143 | 474 | 1.17 | 1.24 | 15.13 | 437.52 |
| 47 | CB26－B | 1422 | 1210 | 143 | 1143 | 474 | 1.17 | 1.24 | 15.13 | 437.52 |
| 48 | BR19－B | 1422 | 1210 | 143 | 1143 | 474 | 1.17 | 1.24 | 20.11 | 437.52 |
| 49 | BR20－B | 1422 | 1218 | 143 | 1143 | 237 | 1.17 | 1.24 | 20.11 | 437.52 |
| 50 | BR21－B | 1422 | 1219 | 143 | 1143 | 474 | 1.17 | 1.24 | 20.11 | 437.52 |
| 51 | BR22－B | 1422 | 1219 | 143 | 1143 | 237 | 1.17 | 1.24 | 20.11 | 437.52 |
| 52 | 日R23－B | 1422 | 1219 | 143 | 1143 | 474 | 1.17 | 1.24 | 20.11 | 437.52 |
| 53 | BR24－8 | 1422 | 1219 | 143 | 1143 | 237 | 1.17 | 1.24 | 20.11 | 437.52 |
| 54 | BR25－B | 1422 | 1219 | 143 | 1143 | 474 | 1.17 | 1.24 | 20.11 | 437.52 |
| 55 | BR20－B | 1422 | 1210 | 143 | 1143 | 237 | 1.17 | 1.24 | 20.11 | 437.52 |
| 56. | BR27－B | 1422 | 1219 | 143 | 1143 | 284 | 1.17 | 1.24 | 20.11 | 409.96 |
| 57. | BR28－B | 1422 | 1219 | 143 | 1143 | 129 | 1.17 | 1.24 | 20.11 | 416.85 |
| 58 | BR30－B | 1422 | 1219 | 143 | 1143 | 203 | 1.17 | 1.24 | 27.62 | 437.52 |
| 59 | D日RAS－B | 1422 | 1219 | 254 | 1143 | 711 | 1.17 | 1.24 | 17.11 | 406.51 |
| 60 | DBR日－B | 1422 | 1219 | 254 | 1143 | 237 | 1.17 | 1.24 | 17.11 | 465.08 |
| 61. | D8R10－B | 1422 | 1219 | 254 | 1143 | 711 | 1.17 | 1.24 | 17.11 | 406.51 |
| 62 ： | D日R12－B | 1422 | 1219 | 254 | 1143 | 356 | 1.17 | 1.24 | 17.11 | 398.24 |

TABLE 1. CONT'D PROPERTIES OF SPECIMENS

| TEST NUMBER | SPECIMEN LABEL | $\begin{aligned} & \mathrm{F} \mathrm{Ve} \\ & \mathrm{MPa} \\ & \hline \end{aligned}$ | $\begin{aligned} & \text { fyvi } \\ & \mathrm{MPa} \end{aligned}$ | $\begin{aligned} & \text { fov } \\ & \mathrm{MPa} \end{aligned}$ | ph | pve | pvi | pv | $\begin{aligned} & \text { SIGMAO } \\ & (\mathrm{MPa}) \end{aligned}$ | ALPHA |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 3-S | 496.08 | 498.08 | 496.08 | 0.00122 | 0.00148 | 0.00667 | 0.00741 | 1.86 | 1 |
| 2 | 4-S | 496.08 | 496.08 | 496.08 | 0.00122 | 0.00148 | 0.00867 | 0.00741 | 0.00 | 1 |
| 3 | 5-S | 498.08 | 498.08 | 488.08 | 0.00122 | 0.00148 | 0.00687 | 0.00741 | 0.69 | 1 |
| 4 | 7-S | 486.08 | 408.08 | 498.08 | 0.00122 | 0.00148 | 0.00867 | 0.00741 | 0.69 | 1 |
| 5 | 9-S | 440.98 | 440.98 | 440.96 | 0.00122 | 0.00077 | 0.00344 | 0.00383 | 1.86 | 1 |
| 6 | 13-S | 447.85 | 447.85 | 447.85 | 0.00222 | 0.00109 | 0.00489 | 0.00543 | 1.86 | 1 |
| 7 | 14-5 | 447.85 | 447.85 | 447.85 | 0.00122 | 0.00109 | 0.00489 | 0.00543 | 1.86 | 1 |
| 8 | 18-5 | 496.08 | 406.08 | 486.08 | 0.00222 | 0.00148 | 0.00667 | 0.00741 | 1.86 | 1. |
| 9 | 21-S | 447.85 | 447.85 | 447.85 | 0.00128 | 0.00114 | 0.00512 | 0.00568 | 1:83 | 1 |
| 10 | 22-S | 447.85 | 447.85 | 447.85 | 0.00128 | 0.00114 | 0.00512 | 0.00568 | 0.69 | 1 |
| 11 | KW4-1-M | 385.00 | 385.00 | 385.00 | 0.00118 | 0.00426 | 0.00134 | 0.00943 | 0.49 | 0.5 |
| 12 | KW3-1-M | 385.00 | 385.00 | 385.00 | 0.00118 | 0.00434 | 0.00140 | 0.00946 | 0.49 | 0.5 |
| 13 | KW35-1- | 385.00 | 385.00 | 385.00 | 0.00118 | 0.00434 | 0.00140 | 0.00946 | 0.49 | 0.5 |
| 14 | KW2-1-M | 385.00 | 385.00 | 385.00 | 0.00118 | 0.00541 | 0.00155 | 0.01148 | 0.48 | 0.5 |
| 15 | WS2-M | 385.00 | 385.00 | 385.00 | 0.00000 | 0.00254 | 0.00111 | 0.00571 | 1.96 | 0.5 |
| 16 | WS4-M | 385.00 | 385.00 | 385.00 | 0.00167 | 0.00254 | 0.00111 | 0.00571 | 1.98 | 0.5 |
| 17 | WS5-M | 385.00 | 385.00 | 385.00 | 0.00334 | 0.00254 | 0.00111 | 0.00571 | 1.96 | 0.5 |
| 18 | WS9-M | 385.00 | 385.00 | 385.00 | 0.00334 | 0.00448 | 0.00111 | 0.00959 | 1.98 | 0.5 |
| 19 | WS10-M | 385.00 | 385.00 | 385.00 | 0.00868 | 0.00448 | 0.00111 | 0.00959 | 1.98 | 0.5 |
| 20 | WS9-2-M | 385.00 | 385.00 | 385.00 | 0.00334 | 0.00448 | 0.00111 | 0.00859 | 1.96 | 0.5 |
| 21 | WSB21-M | 385,00 | 385.00 | $385: 00$ | 0.00334 | 0.00448 | 0.00111 | 0.00959 | 1.96 | 0.5 |
| 22 | WS822-M | 385.00 | 385.00 | 385.00 | 0.00400 | 0.00448 | 0.00111 | 0.00959 | 1.98 | 0.5 |
| 23 | WSB3-M | 385.00 | 385.00 | 385.00 | 0.00353 | 0.00473 | 0.00117 | 0.01013 | 1.96 | 0.5 |
| 24 | WSB4-M | 385.00 | 385.00 | 385.00 | 0.00334 | 0.00448 | 0.00111 | 0.00959 | 1.98 | 0.5 |
| 25 | WSR2-M | 385.00 | 385.00 | 385.00 | 0.00000 | 0.00272 | 0.00121 | 0.00612 | 1.98 | 0.5 |
| 26 | WSR4-M | 385.00 | 385.00 | 385.00 | 0.00167 | 0.00272 | 0.00121 | 0.00612 | 1.96 | 0.5 |
| 27 | WSR5-M | 385.00 | 385.00 | 385.00 | 0.00334 | 0.00272 | 0.00121 | 0.00812 | 1.88 | 0.5 |
| 28 | WSRE-M | 385.00 | 385.00 | 385.00 | 0.00868 | 0.00272 | 0.00121 | 0.00812 | 1.96 | 0.5 |
| 29 | WS1-0 | 386.55 | 371.18 | 378.80 | 0.00167 | 0.00149 | 0.00292 | 0.00509 | 0.00 | 0.5 |
| 30 | WS4-0 | 386.55 | 371.18 | 378.86 | 0.00167 | 0.00249 | 0.00316 | 0.00674 | 1.98 | 0.5 |
| 31 | WS7-0 | 386.55 | 371.18 | 378.88 | 0.00167 | 0.00374 | 0.00351 | 0.00878 | 0.00 | 0.5 |
| 32 | WSN1-0 | 386.55 | 371.18 | 378.88 | 0.00167 | 0.00248 | 0.00318 | 0.00674 | 3.82 | 0.5 |
| 33 | WSN2-O | 386.55 | 371.18 | 378.86 | 0.00187 | 0.00249 | 0.00318 | 0.00874 | 5.87 | 0.5 |
| 34 | WSA1-0 | 386.55 | 363.05 | 374.80 | 0.00167 | 0.00149 | 0.00292 | 0.00509 | 0.00 | 0.5 |
| 35 | WSR4-0 | 386.55 | 371.18 | 378.86 | 0.00187 | 0.00249 | 0.00316 | 0.00874 | 0.00 | 0.5 |
| 36 | WSR7-0 | 386.55 | 371.18 | 378.86 | 0.00167 | 0.00374 | 0.00351 | 0.00879 | 0.00 | 0.5 |
| 37 | WSRC-O | 388.55 | 371.18 | 378.80 | 0.00187 | 0.00249 | 0.00318 | 0.00674 | 2.15 | 0.5 |
| 38 | CB13-B | 465.08 | 465.08 | 485.08 | 0.00281 | 0.00085 | 0.00000 | 0.00169 | 1.88 | 0.5 |
| 39 | CB15-B | 465.08 | 465.08 | 485.08 | 0.00281 | 0.00085 | 0.00000 | 0.00169 | 3.01 | 0.5 |
| 40 | CB17-B | 390.88 | 390.68 | 300.66 | 0.00394 | 0.00222 | 0.00000 | 0.00444 | 2.76 | 0.5 |
| 41 | CB18-B | 409.96 | 409.96 | 400.98 | 0.00394 | 0.00074 | 0.00423 | 0.00444 | 2.78 | 0.5 |
| 42 | C820-8 | 300.68 | 390.66 | 390.66 | 0.00197 | 0.00222 | 0.00000 | 0.00444 | 2.76 | 0.5 |
| 43 | CB21-B | 409.96 | 409:96 | 409.96 | 0.00197 | 0.00074 | 0.00423 | 0.00444 | 2.76 | 0.5 |
| 44 | C823-B | 390.68 | 300.88 | 390.86 | 0.00075 | 0.00222 | 0.00000 | 0.00444 | 2.78 | 0.5 |
| 45 | CB24-B | 380.66 | 390.68 | 390.68 | 0.00272 | 0.00222 | 0.00000 | 0.00444 | 2.76 | 0.5 |
| 48 | CB25-B | 390.66 | 390.68 | 390.88 | 0.00197 | 0.00222 | 0.00000 | 0.00444 | 1.74 | 0.5 |
| 47 | C828-B | 390.66 | 390.60 | 390.68 | 0.00197 | 0.00222 | 0.00000 | 0.00444 | 2.76 | 0.5 |
| 48 | BR10-8 | 390.68 | 380.68 | 380.68 | 0.00197 | 0.00222 | 0.00000 | 0.00444 | 2.76 | 0.5 |
| 49 | BR20-B | 390.68 | 390.68 | 390.66 | 0.00492 | 0.00222 | 0.00000 | 0.00444 | 2.76 | 0.5 |
| 50 | BR21-B | 380.86 | 390.66 | 380.60 | 0.00197 | 0.00222 | 0.00394 | 0.00674 | 2.76 | 0.5 |
| 51 | BR22-B | 437.52 | 437.52 | 437.52 | 0.00492 | 0.00115 | 0.00394 | 0.00459 | 2.78 | 0.5 |
| 52 | BR23-B | 409.98 | 409.88 | 409.08 | 0.00197 | 0.00074 | 0.00423 | 0.00444 | 2.78 | 0.5 |
| 53 | BR24-B | 409.96 | 409.96 | 409.96 | 0.00492 | 0.00074 | 0.00423 | 0.00444 | 2.76 | 0.5 |
| 54 | BR25-B | 390.66 | 300.68 | 390.66 | 0.00197 | 0.00222 | 0.00000 | 0.00148 | 2.78 | 0.5 |
| 55 | BR28-B | 390.68 | 390.68 | 390.68 | 0.00492 | 0.00222 | 0.00000 | 0.00444 | 2.76 | 0.5 |
| 56 | BR27-8 | 390.86 | 380.86 | 380.66 | 0.00254 | 0.00222 | 0.00000 | 0.00444 | 2.78 | 0.5 |
| 57 | BR28-B | 409.86 | 409.96 | 409.80 | 0.00835 | 0.00222 | 0.00000 | 0.00444 | 2.78 | 0.5 |
| 58 | BR30-B | 300.68 | 390.66 | 390.60 | 0.00100 | 0.00222 | 0.00000 | 0.00444 | 2.76 | 0.5 |
| 59 | D8R8S-B | 465.08 | 465.08 | 485.08 | 0.00055 | 0.00065 | 0.00000 | 0.00129 | 1.52 | 0.5 |
| 60 | OBR9-B | 465.08 | 465.08 | 465.08 | 0.00277 | 0.00065 | 0.00000 | 0.00129 | 2.29 | 0.5 |
| 81 | DBR10-B | 465.08 | 465.08 | 465.08 | 0.00055 | 0.00065 | 0.00000 | 0.00129 | 2.29 | 0.5 |
| 62 | OBR12-B | 465.08 | 485.08 | 485.08 | 0.00059 | 0.00085 | 0.00000 | 0.00129 | 2.29 | 0.5 |

remaining specimens are single-wythe walls built with hollow concrete block units.

Subsequent columns of Table 1 specify the geometric and material properties of the specimens, reinforcement ratios, and axial loads. The tabulated compressive strengths of masonry were obtained by prism tests. Matsumura and Shing et al used three-course prisms. Sveinsson et al. used three-course prisms with h/t ratios of 2, and six course prisms with h/t ratios of 4 . The average of the results obtained from the two types of prisms was used as the value of compressive strength for the Sveinsson et al. specimens. Okamoto et al did not report the type of prism used. All researchers used tension tests to determine the yield strengths of the reinforcing steel. The U.S. Customary Units of the data presented by Shing et al and Sveinsson et al, as well as the Metric Units of data presented by Okamota et al, were converted to SI units. The final column in Table 1 gives the experimentally-determined ultimate shear strength of the specimens. Table A1 in Appendix A duplicates Table 1 using U.S. Customary Units.

### 3.2 Predictive Equations

Four equations for the prediction of the ultimate shear strength of fully-grouted reinforced masonry shear walls in which shear is the characteristic mode of failure were selected for study. Two of the equations are of Japanese origin (equations M and J below), and two are of American origin ( S and U ). Two of the equations ( $U$ and $J$ ) are currently prescribed by codes and two ( $S$ and $M$ ) are proposed. There are other proposed formulations which use different functional forms for the effect of various parameters on strength than those examined in this study. For example, equations derived by Blondet et al [9] and Leiva [10] show good correlation with test results from University of California at Berkeley and abroad. These equations deserve further verification against a more diverse exeperimental data base such as used in this study. The original formats of each of the four equations were modified by introducing a common notation and consistent units. The equations shown below are in Sl units. Appendix $B$ describes the conversion of the original forms of the predictive equations into the common format and consistent units used in this report. The definitions of the symbols used in the transformed equations and in the rest of this document are given in section 3.3.

The equations selected include:

1) two related equations proposed by Shing et al which were developed from the data of experiments performed by Shing et al and reported in references [2] and [3] (these two equations are
combined into one comprehensive equation for the purposes of this study),
2) a proposed equation developed by Matsumura based on his and other experimental data and reported in reference [4],
3) an equation prescribed by the Architectural Institute of Japan for predicting the shear strength of concrete shear walls [5]; the data developed by Okamoto et al, as reported in [5], was used by the same authors to examine the ability of this equation to predict the shear strength of masonry walls, and,
4) the equation from the 1988 edition of the Uniform Building Code [7] currently in use for predicting the nominal (ultimate) shear strength of masonry walls as part of the strength design provisions for masonry.

The transformed versions of the above equations will be referred to throughout this paper as $S, M, J$, and $U$, respectively. The standard form used in the transformed equations is:

$$
\begin{aligned}
v_{u} & =v_{m} \\
& +v_{s} \\
& +v_{q}
\end{aligned}
$$

in which the term $\mathrm{v}_{\mathrm{m}}$ represents the contribution to shear strength provided by the masonry and vertical steel, $v_{s}$ represents the contribution of the horizontal steel, and $v_{a}$ represents the contribution of the axial load. The transformed versions of the four equations are:

Equation S :

$$
\begin{aligned}
v_{u} & =\left(0.0217 \rho_{v} f_{v v}+0.166\right) \sqrt{f_{m}^{\prime}} \\
& +\left(\frac{L-2 d^{\prime}}{s_{h}}-1\right) \frac{s_{h}}{L} \rho_{h} f_{y h} \\
& +\left(0.0217 \sigma_{o}\right) \sqrt{f_{m}^{\prime}}
\end{aligned}
$$

Equation M:

$$
\begin{aligned}
v_{u} & =\left[\left(\frac{0.76}{r_{d}+0.7}+0.012\right)\left(4.04 \rho_{v e}\right) \sqrt{f_{m}^{\prime}}\right] \frac{d}{L} \\
& +\left[0.01575\left(\rho_{h} f_{y h}\right)^{\frac{1}{2}} \sqrt{f_{m}^{\prime}}\right] \frac{\delta d}{L} \\
& +\left(0.175 \sigma_{o}\right) \frac{d}{L}
\end{aligned}
$$

Equation J:

$$
\begin{aligned}
v_{u}= & {\left[4.64 \rho_{v e}^{0.23}\left(0.01 f_{m}^{\prime}+0.176\right)\left(\frac{1}{r_{c}+0.12}\right)\right] \frac{d}{L} } \\
+ & {\left[0.739\left(\rho_{h} f_{y h}\right)^{\frac{1}{2}}+0.739\left(\rho_{v i} f_{y v i}\right)^{\frac{1}{2}}\right] \frac{d}{L} } \\
+ & \left(0.0875 \sigma_{o}\right) \frac{d}{L} \\
& \left(r_{c}=1+\langle a r-1\rangle-\langle a r-3\rangle\right)
\end{aligned}
$$

Equation U:

$$
\begin{aligned}
v_{u}= & 0.083 C_{d} \sqrt{f_{m}^{\prime}} \\
+ & \rho_{h} f_{y h} \\
& \left(C_{d}=2.4+1.6\left\langle a r_{d}-1\right\rangle-1.6\left\langle a r_{d}-0.25\right\rangle\right)
\end{aligned}
$$

Note that equation $U$ does not consider the effect of axial load on shear strength (there is no third term in this case).

### 3.3 Notation

The definitions of the terms used in this paper are given below. Actual rather than nominal dimensions are used in these definitions.
$A=(L)(t):$ gross horizontal area of wall $\left(\mathrm{mm}^{2}\right)$
$A_{h}=$ area of horizontal reinforcement in one layer $\left(\mathrm{mm}^{2}\right)$
$A_{\mathrm{vi}}=\quad$ area of vertical reinforcement in one interior core $\left(\mathrm{mm}^{2}\right)$,
$A_{\mathrm{ve}}=\quad$ area of vertical reinforcement in one end core $\left(\mathrm{mm}^{2}\right)$
d $\quad=\quad L-d^{\prime}=$ distance of centroid of vertical reinforcement in an end core to the opposite face of wall (mm)
$d^{\prime} \quad=\quad$ cover of the centroid of vertical reinforcement in an end core (mm)
$\mathrm{f}_{\mathrm{m}}^{\prime} \quad=\quad$ compressive strength of masonry from prism tests (MPa)
$f_{y h}=$ yield strength of horizontal reinforcement (MPa)
$f_{y v} \quad=\quad$ average yield strength of vertical reinforcement (MPa)
$f_{y v i}=\quad$ yield strength of vertical reinforcement in interior cores (MPa)
$h \quad=$ height of wall (mm)
$\mathrm{L}=\quad$ length of wall (mm)
$\mathrm{M}=$ maximum bending moment that occurs simultaneously with shear force $V(N-m)$
$\mathrm{Q}=\quad$ axial load on masonry wall (N)
$r \quad=\quad h / L=$ aspect ratio of wall
$r_{c}=a \operatorname{a}$ discontinuous function of ar (see Appendix B)
$r_{d}=h / d=r L / d$
$\mathrm{s}_{\mathrm{h}} \quad=\quad$ spacing between layers of uniformly distributed horizontal reinforcement (mm)
$\mathbf{s}_{\mathrm{vi}} \quad=\quad$ spacing of vertical reinforcement in the interior cores (mm)
t , $=$ thickness of wall (mm)
$V=$ shear force on horizontal section of wall (N)
$V_{u} \quad=\quad$ ultimate shear force on horizontal section (N)
$\mathrm{v}_{\mathrm{m}}=$ contribution of vertical reinforcement to ultimate shear strength (MPa)
$v_{\mathrm{a}} \quad=$ contribution of axial load to ultimate shear strength (MPa)
$v_{\mathrm{a}}=$ contribution of the horizontal steel to ultimate shear strength (MPa)
$\mathrm{v}_{\mathrm{u}}=\mathrm{V}_{\mathrm{u}} / \mathrm{tL}:$ nominal ultimate shear stress ( MPa )
$a=\mathrm{M} / \mathrm{VLr}=\mathrm{M} / \mathrm{Vdr}_{\mathrm{d}}$
$\delta=1.0$ for inflection point at mid-height, 0.6 for cantilever type bending
$\rho_{h} \quad=\quad A_{h} / s_{h} t\left(=\Sigma A_{h} /\right.$ ht for subset $\left.B\right)=$ horizontal reinforcement ratio
$\rho_{\mathrm{v}} \quad=\quad\left(2 \mathrm{~A}_{\mathrm{ve}}+\sum \mathrm{A}_{\mathrm{vi}}\right) / \mathrm{tL}=$ total vertical reinforcement ratio
$\rho_{\mathrm{vo}} \quad=\quad \mathrm{A}_{\mathrm{ve}} / \mathrm{tL}=$ ratio of vertical reinforcement in one end core
$\rho_{\mathrm{vi}}=\mathrm{A}_{\mathrm{v} i} / \mathrm{s}_{\mathrm{vi}} \mathrm{t}=$ ratio of uniformly distributed vertical reinforcement in the interior cores
$\sigma_{\mathrm{o}}=\mathrm{Q} / \mathrm{A}=$ nominal axial stress on wall (MPa)

## 4. METHODOLOGY

Each equation was used to predict the ultimate shear strength of all 62 test specimens, using appropriate specimen characteristics such as wall dimensions, masonry strength, steel area, and axial load. The predicted strength was then compared to the actual tested strength. The comparisons were grouped by equation and by pairings of equation and data subset. For example, predicted values versus test results for equation $S$ were grouped into five sets: equation $S$ versus test results from all specimens ( $S+M+O+B$ ), subset $S$, subset $M$, and so on. These grouping of comparisons are referred to in this report using the format $X-Y$, where $X$ is the equation identification and $Y$ is the data set. For example, S-M refers to the comparison of the predictions by equation $S$ with test results of subset $M$, while $M$-S refers to the comparison of predictions by equation $M$ with results of subset $S$.

Figures 1 through 5 show plots of test-versus-predicted strength. The straight line represents perfect correlation. Points deviating from this line indicate both the scatter in test results and approximations in the predictive formulation. The spread of points above and below this line illustrates tendencies to over- or under-predict as well as general scatter in the test results.

Normalized plots were created by plotting on the $y$ axis the ratio of test to predicted strength for each of the 62 specimens, which are identified by numbers along the $x$ axis. These plots (Figures 6 through 9) are useful in identifying specific pairs or groups of specimens for further investigation.

A sample mean, $x_{m}$, deviation, $\mathbf{s}$, and variation, $v$, were calculated for each group of comparisons, using the formulae:

$$
\begin{equation*}
s=\sqrt{\frac{\sum_{1}^{n}\left(x_{i}-y_{i}\right)^{2}}{(n-1)}}, \quad x_{m}=\frac{\sum x_{i}}{n}, \quad v=\frac{s}{x_{m}} \tag{5}
\end{equation*}
$$

where $\quad x_{i}=$ ith test value
$y_{i}=\quad$ ith predicted value
$\mathrm{n} \quad=\quad$ sample size


FIGURE 1:EXPERIMENTAL VS PREDICTED STRENGTH: EQUATION S


FIGURE 2: EXPERIMENTAL VS PREDICTED STRENGTH: EQUATION M


FIGURE 3: EXPERIMENTAL VS PREDICTED STRENGTH: EQUATION J




FIGURE 4: EXPERIMENTAL VS PREDICTED STRENGTH: EQUATION U

Equation $S$ vs All Tests


Equation $J$ vs All Tests


Equation M vs All Tests


Equotion U vs All Tests


FIGURE 5: EXPERIMENTAL VS PREDICTED STRENGTH: ALL 62 DATA POINTS

The equation for calculating deviation is similar to that used for standard deviation, but the numerical value of deviation cannot be used in statistical analysis because the data points being evaluated do not represent repetitive tests and the scatter is due to multiple causes. Likewise, variation is defined in the same way as coefficient of variation in statistics, but for the same reasons, does not have the same meaning. However, those indicators as defined and calculated here are useful for making comparisons of the predictive accuracy of the four equations.

To carry the comparisons of the equations one step further, the relative contribution of each of the three strength terms ( $v_{m}, v_{s}$, and $v_{q}$ ) was examined. The values of each of the three terms contributing to the prediction of each equation for all 62 specimens were tabulated (Table 2). Histograms were produced that presented information from all four equations on one plot for each of the three terms, and for the combined ultimate shear strength (Figures 10 through 13).

The effects of specific parameters were investigated by identifying specimens that essentially differed in only one parameter. The ratios of test to predicted values for these similar specimens were compared (Figures 6 through 9). Similar ratios indicate that the predictive equation effectively accounts for the varying parameter. Divergent ratios indicate the opposite.

## 5. RESULTS AND ANALYSIS

### 5.1 Results

Table 2 lists the actual strengths determined from tests of all 62 specimens, and the predicted strengths ( $v_{m}, v_{s}, v_{q}$ and the sum $v_{u}$ ) obtained from each formula. Graphical comparisons of predicted and test results are shown in Figures 1 through 4, each of which contains four data plots. Figure 5 shows the test/prediction plot for each of the four equations for all 62 data points.

Figures 1 through 5 vividly illustrate the fact that none of the equations is able to precisely predict the ultimate shear strength of all the specimens. However, part of the scatter is due to the variability of strength inherent in masonry construction. A study by Blume and Proulx [8] suggests the magnitude of inherent variation that can be expected. Test results from 84 diagonally-loaded shear walls, in replicate groups of 4 and 5 , gave a range of coefficient of variation (standard deviation divided by sample mean) of $3-18 \%$. The large spread in the coefficient of variation is attributed primarily to the small sample size. The average coefficient of variation for all the replicate tests was $8 \%$. The Blume and Proulx study suggests that variation of about $10 \%$ between

| EST | . | Vm |  |  |  | Ve |  |  |  | Vq |  |  |  | Vu | $\mathrm{Vm}+\mathrm{V}_{\mathrm{q}}$ | \% ${ }^{\text {. }}$ | Vu |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| NUMBER |  | MPa |  |  |  | MPa |  |  |  | MPa |  |  |  | MPa |  |  | TESTS |
|  | $s$ | M | J | U | s | M | J | U | $s$ | M | J | u | s | M | $J$ | $u$ | MPa |
| 1 | 1.12 | 1.08 | 0.33 | 0.45 | 0.31 | 0.28 | 1.75 | 0.47 | 0.18 | 0.31 | 0.15 | 0.00 | 1.61 | 1.68 | 2.24 | 0.92 | 1.74 |
| 2 | 1.04 | 1.02 | 0.31 | 0.42 | 0.31 | 0.26 | 1.75 | 0.47 | 0.00 | 0.00 | 0.00 | 0.00 | 1.35 | 1.28 | 2.08 | 0.89 | 1.35 |
| 3 | 1.04 | 1.02 | 0.31 | 0.42 | 0.31 | 0.26 | 1.75 | 0.47 | 0.06 | 0.11 | 0.06 | 0.00 | 1.42 | 1.39 | 2.12 | 0.89 | 1.47 |
| 4 | 1.12 | 1.09 | 0.33 | 0.45 | 0.31 | 0.28 | 1.75 | 0.47 | 0.07 | 0.11 | 0.06 | 0.00 | 1.50 | 1.48 | 2.14 | 0.92 | 1.85 |
| 5 | 0.92 | 0.80 | 0.29 | 0.45 | 0.31 | 0.28 | 1.34 | 0.47 | 0.18 | 0.31 | 0.15 | 0.00 | 1.42 | 1.48 | 1.78 | 0.92 | 1.83 |
| 6 | 1.04 | 1.04 | 0.33 | 0.47 | 0.68 | 0.43 | 1.74 | 1.03 | 0.19 | 0.31 | 0.15 | 0.00 | 1.92 | 1.78 | 2.22 | 1.50 | 1.91 |
| 7 | 1.04 | 1.04 | . 0.33 | 0.47 | 0.31 | 0.28 | 1.51 | 0.47 | 0.19 | 0.31 | 0.15 | 0.00 | 1.55 | 1.84 | 1.99 | 0.95 | 178 |
| 8 | 1.02 | 1.00 | 0.30 | 0.41 | 0.88 | 0.38 | 1.98 | 1.03 | 0.17 | 0.31 | 0.15 | 0.00 | 1.87 | 1.68 | 2.44 | 1.44 | 2.05 |
| 9 | . 1.13 | 1.13 | 0.36 | 0.51 | 0.33 | 0.32 | 1.55 | 0.49 | 0.21 | 0.32 | 0.16 | 0.00 | 1.68 | 1.77 | 2.07 | 1.00 | 178 |
| 10 | 1.13 | 1.13 | . 0.36 | 0.51 | 0.33 | 0.32 | 1.55 | 0.48 | 0.08 | 0.11 | 0.06 | 0.00 | 1.54 | 1.57 | 1.96 | 1.00 | 1.56 |
| 11 | 1.14 | 1.43 | 0.44 | 0.71 | 0.28 | 0.47 | 0.97 | 0.45 | 0.05 | 0.08 | 0.04 | 0.00 | 1.48 | 1.97 | 1.45 | 1.17 | 1.60 |
| 12 | 1.14 | 1.15 | 0.43 | 0.58 | 0.23 | 0.46 | 0.96 | 0.45 | 0.05 | 0.08 | 0.04 | 0.00 | 1.43 | 1.69 | 1.43 | 1.03 | 1.72 |
| 13 | 1.14 | 1.15 | 0.43 | 0.58 | 0.23 | 0.46 | 0.96 | 0.45 | 0.05 | 0.08 | 0.04 | 0.00 | 1.43 | 1.69 | 1.43 | 1.03 | 1.87 |
| 14 | 1.22 | 0.85 | 0.38 | 0.47 | 0.12 | 0.44 | 0.95 | 0.45 | 0.05 | 0.08 | 0.04 | 0.00 | 1.39 | 1.37 | 1.37 | 0.92 | 1.81 |
| 15 | 1.01 | 0.88 | 0.39 | 0.58 | 0.00 | 0.00 | 0.44 | 0.00 | 0.20 | 0.32 | 0.16 | 0.00 | 1.21 | 1.30 | 0.99 | 0.58 | 1.70 |
| 16 | 1.01 | 0.98 | 0.39 | 0.58 | 0.32 | 0.55 | 0.98 | 0.84 | 0.20 | 0.32 | 0.16 | 0.00 | 1.53 | 1.85 | 1.53 | 1.23 | 1.89 |
| 17 | 1.01 | 0.88 | 0.39 | 0.58 | 0.65 | 0.78 | 1.22 | 1.29 | 0.20 | 0.32 | 0.16 | 0.00 | 1.88 | 2.07 | 1.78 | 1.87 | 2.28 |
| 18 | 1,16 | 1.17 | 0.44 | 0.58 | 0.65 | 0.78 | 1.22 | 1.29 | 0.20 | 0.32 | 0.16 | 0.00 | 2.01 | 2.26 | 1.81 | 1.87 | 2.29 |
| 19 | 1.18 | 1.17 | 0.44 | 0.58 | 1.30 | 1.10 | 1.54 | 2.57 | 0.20 | 0.32 | 0.16 | 0.00 | 2.68 | 2.58 | 2.13 | 3.15 | 2.23 |
| 20 | 1.33 | 1.33 | 0.51 | 0.66 | 0.85 | 0.89 | 1.22 | 1.29 | 0.23 | - 0.32 | 0.16 | 0.00 | 2.20 | 2.53 | 1.89 | 1.95 | 2.58 |
| 21 | 1.26 | 1.28 | 0.48 | 0.63 | 0.65 | 0.84 | 1.22 | 1.28 | 0.22 | 0.32 | 0.16 | 0.00 | 2.12 | 2.42 | 1.85 | 1.92 | 2.24 |
| 22 | 1.29 | 1.28 | 0.49 | 0.85 | 0.78 | 0.94 | 1.29 | 1.54 | 0.22 | 0.32 | 0.16 | 0.00 | 2.29 | 2.55 | 1.94 | 2.18 | 2.63 |
| 23 | 1.29 | 1.29 | 0.49 | 0.63 | 0.69 | 0.87 | 1.25 | 1.36 | 0.22 | 0.32 | 0.16 | 0.00 | 2.19 | 2.47 | 1.90 | 1.99 | 2.43 |
| 24 | 1.38 | 1.38 | 0.54 | 0.89 | 0.85 | 0.92 | 1.22 | 1.29 | 0.24 | 0.32 | 0.16 | 0.00 | 2.27 | 2.62 | 1.91 | 1.98 | 2.59 |
| 25 | 1.18 | 1.10 | 0.45 | 0.84 | 0.00 | 0.00 | 0.48 | 0.00 | 0.23 | 0.31 | 0.16 | 0.00 | 1.39 | 1.41 | 1.08 | 0.64 | 2.18 |
| 26 | 1.18 | 1.10 | 0.45 | 0.84 | 0.30 | 0.61 | 0.99 | 0.64 | 0.23 | 0.31 | 0.16 | 0.00 | 1.69 | 2.02 | 1.59 | 1.29 | 1.95 |
| 27 | 1.18 | 1.10 | 0.45 | 0.64 | 0.61 | 0.86 | 1.21 | 1.29 | 0.23 | 0.31 | 0.16 | 0.00 | 1.98 | 2.27 | 1.82 | 1.93 | 1.71 |
| 28 | 1.18 | 1.10 | 0.45 | 0.64 | 1.21 | 1.22 | 1.53 | 2.57 | 0.23 | 0.31. | 0.16 | 0.00 | 2.60 | 2.63 | 2.13 | 3.21 | 2.04 |
| 29 | . 0.88 | 1.10 | 0.31 | 0.72 | 0.42 | 0.49 | 1.28 | 0.59 | 0.00 | 0.00 | 0.00 | 0.00 | 1.30 | 1.58 | 1.59 | 1.31 | 2.68 |
| 30 | 1.06 | 100 | 0.39 | 0.58 | 0.30 | 0.53 | 1.28 | 0.59 | 0.20 | 0.32 | 0.16 | 0.00 | 1.56 | 1.84 | 1.81 | 1.19 | 1.97 |
| 31 | 1.01 | 0.69 | 0.32 | 0.42 | 0.16 | 0.45 | 1.25 | 0.58 | 0.00 | 0.00 | 0000 | 0.00 | 1.18 | 1.14 | 1.57 | 1.01 | 2.04 |
| 32 | 1.06 | 1.00 | 0.39 | 0.59 | 0.30 | 0.53 | 1.28 | 0.59 | 0.41 | 0.63 | 0.32 | 0.00 | 1.76 | 2.16 | 1.97 | 1.19 | 2.40 |
| 33 | 1.06 | 1.00 | 0.39 | 0.59 | 0.30 | 0.53 | 1.28 | 0.59 | 0.61 | 0.95 | 0.47 | 0.00 | 1.97 | 2.48 | 2.12 | 1.19 | 2.61 |
| 34 | 1.07 | 1.34 | 0.39 | 0.88 | 0.42 | 0.60 | 1.27 | 0.59 | 0.00 | . 0.00 | 0.00 | 0.00 | 1.49 | 1.94 | 1.66 | 1.47 | 3.12 |
| 35 | 1.11 | 1.05 | 0.41 | 0.62 | 0.30 | 0.56 | 1.26 | 0.58 | 0.00 | 0.00 | 0.00 | 0.00 | 1.41 | 1.81 | 1.67 | 1.22 | 2.32 |
| 36 | 1.10 | 0.78 | 0.35 | 0.48 | 0.16 | 0.49 | 1.25 | 0.59 | 0.00 | 0.00 | 0.00 | 0.00 | 1.28 | 1.25 | 1.60 | 1.05 | 2.04 |
| 37 | 1.14 | 1.08 | 0.43 | 0.64 | 0.30 | 0.58 | 1.28 | 0.59 | 0.24 | 0.35 | 0.17 | 0.00 | 1.69 | 2.00 | 1.86 | 1.24 | 2.18 |
| 38 | 0.88 | 0.88 | 0.31 | 0.72 | 0.73 | 0.78 | 0.74 | 1.14 | 0.20 | 0.31 | 0.15 | 0.00 | 1.81 | 1.95 | 1.21 | 1.88 | 1.95 |
| 39 | 0.88 | 0.88 | 0.31 | 0.72 | 0.73 | 0.76 | 0.74 | 1.14 | 0.31 | 0.49 | 0.25 | 0.00 | 1.93 | 2.13 | 1.30 | 1.88 | 2.38 |
| 40 | 0.81 | 0.97 | 0.32 | 0.80 | 1.10 | 0.77 | 0.91 | 1.72 | 0.24 | 0.45 | 0.23 | 0.00 | 2.15 | 2.19 | 1.45 | 2.32 | 2.48 |
| 41 | 0.82 | 0.70 | 0.25 | 0.80 | 1.10 | 0.77 | 1.82 | 1.72 | 0.24 | 0.45 | 0.23 | 0.00 | 2.18 | 1.92 | 2.30 | 2.32 | 2.48 |
| 42 | 0.79 | 0.95 | 0.31 | . 0.58 | 0.42 | 0.53 | 0.64 | 0.86 | 0.23 | 0.45 | 0.23 | 0.00 | 1.44 | 1.83 | 1.18 | 1.44 | 2.38 |
| 43 | 0.80 | 0.68 | 0.24 | 0.58 | 0.42 | 0.53 | 1.58 | 0.86 | 0.23 | 0.45 | 0.23 | 0.00 | 1.45 | 1.87 | 2.02 | 1.44 | 2.23 |
| 44 | 0.79 | 0.95 | 0.31 | 0.58 | 0.23 | 0.33 | 0.40 | 0.33 | 0.23 | 0.45 | 0.23 | 0.00 | 1.28 | 1.73 | 0.94 | 0.91 | 1.92 |
| 45 | 0.79 | 0.95 | 0.31 | 0.58 | 0.85 | 0.63 | 0.78 | 1.19 | 0.23 | 0.45 | 0.23 | 0.00 | 1.68 | 2.03 | 1.29 | 1.77 | 2.43 |
| 46 | 0.79 | 0.95 | 0.31 | 0.58 | 0.42 | 0.53 | 0.84 | 0.86 | 0.15 | 0.28 | 0.14 | 0.00 | 1.36 | 1.77 | 1.10 | 1.44 | 1.96 |
| 47 | 0.78 | 0.95 | 0.31 | 0,56 | 0.42 | 0.53 | 0.64 | 0.86 | 0.23 | 0.45 | 0.23 | 0.00 | 1.44 | 1.93 | 1.18 | 1.44 | 2.40 |
| 48 | 0.91 | 1.08 | 0.38 | 0.87 | 0.42 | 0.61 | 0.64 | 0.88 | 0.27 | 0.45 | 0.23 | 0.00 | 1.60 | 2.16 | 1.23 | 1.53 | 1.84 |
| 49 | 0.81 | 1.00 | 0.36 | 0.67 | 1.46 | 0.97 | 1.02 | 2.15 | 0.27 | 0.45 | 0.23 | 0.00 | 2.65 | 2.52 | 1.60 | 2.82 | 1.82 |
| 50 | 1.00 | 1.09 | 0.30 | 0.87 | 0.42 | 0.61 | 1.50 | 0.86 | 0.27 | 0.45 | 0.23 | 0.00 | 1.69 | 2.16 | 2.09 | 1.53 | 2.35 |
| 51 | 0.94 | 0.90 | 0.31 | 0.87 | 1.46 | 0.97 | 1.93 | 2.15 | 0.27 | 0.45 | 0.23 | 0.00 | 2.67 | 2.32 | 2.48 | 2.82 | 2.40 |
| 52 | 0.02 | 0.79 | 0.28 | 0.67 | 0.42 | 0.61 | 1.58 | 0.86 | 0.27 | 0.45 | 0.23 | 0.00 | 1.61 | 1.85 | 2.08 | 1.53 | 2.03 |
| 53 | 0.92 | 0.79 | 0.28 | 0.67 | 1.48 | 0.97 | 1.93 | 2.15 | 0.27 | 0.45 | 0.23 | 0.00 | 2.65 | 2.21 | 2.44 | 2.82 | 2.20 |
| 54 | 0.80 | 1.09 | 0.38 | 0.87 | 0.42 | 0.81 | 0.64 | 0.86 | 0.27 | 0.45 | 0.23 | 0.00 | 1.49 | 2.18 | 1.23 | 1.53 | 2.18 |
| 55 | 0.91 | 1.09 | 0.36 | 0.87 | 1.48 | 0.97 | 1.02 | 2.15 | 0.27 | 0.45 | 0.23 | 0.00 | 2.65 | 2.52 | 1.60 | 2.82 | 2.14 |
| 56 | 0.91 | 1.08 | 0.36 | 0.87 | 0.87 | 0.88 | 0.71 | 1.04 | 0.27 | 0.45 | 0.23 | 0.00 | 1.85 | 2.22 | 1.29 | 1.71 | 2.25 |
| 57 | 0.82 | 1.09 | 0.36 | 0.87 | 2.04 | 1.08 | 1.13 | 2.85 | 0.27 | 0.45 | 0.23 | 0.00 | 3.22 | 2.82 | 1.71 | 3.32 | 2.27 |
| 58 | 1.07 | 1.28 | 0.43 | 0.78 | 0.31 | 0.51 | 0.48 | 0.44 | 0.31 | 0.45 | 0.23 | 0.00 | 1.69 | 2.25 | 1.12 | 1.22 | 2.89 |
| 59 | 0.74 | 0.70 | 0.25 | 0.82 | 0.07 | 0.29 | 0.33 | 0.22 | 0.14 | 0.25 | 0.12 | 0.00 | 0.94 | 1.24 | 0.70 | 0.84 | 1.49 |
| 60 | 0.74 | 0.70 | 0.25 | 0.62 | 0.88 | 0.69 | 0.79 | 1.29 | 0.21 | 0.38 | 0.19 | 0.00 | 1.82 | 1.77 | 1.22 | 1.91 | 1.63 |
| 81 | 0.74 | 0.70 | 0.25 | 0.62 | 0.07 | 0.29 | 0.33 | 0.22 | 0.21 | 0.38 | 0.19 | 0.00 | 1.01 | 1.38 | 0.77 | 0.84 | 1.73 |
| 62 | 0.74 | 0.70 | 0.25 | 0.62 | 0.14 | 0.30 | 0.34 | 0.23 | 0.21. | 0.38 | 0.19 | 0.00 | 1.08 | 1.37 | 0.77 | 0.85 | 1.88 |

test and predicted strength could be attributed to the inherent variability of masonry construction.

Table 3 presents the deviations which were calculated for each of the comparisons. The deviations from the data subsets must be considered closely in evaluating the deviation from the total data set because sample sizes and test scatter of the subsets are different. The correlations with subsets, especially cross-correlations, are meaningful in assessing inconsistencies in the predictive equations.

Table 3 - Deviation (s), Mean ( $x$ ), and Variation ( $v$ ) in predicted vs test strengths (MPa)


The magnitude of the deviations in Table 3 can be considered in light of the Blume and Proulx study. An examination of the range of tested strengths (presented in Table 1) and deviations is instructive. The actual tested strengths varied from 1.35-3.12 MPa (196-453 psi) with $85 \%$ falling between 1.52-2.59 $\mathrm{MPa}(220-376 \mathrm{psi})$. The average strength of the 62 specimens is 2.10 MPa (305 psi). A deviation of $0.69 \mathrm{MPa}(100 \mathrm{psi})$ is about $1 / 3$ of the average strength (eight deviations in Table 3, exclusive of comparisons with all the tests, were greater than 0.69 MPa ( 100 psi ). A deviation of this magnitude cannot be attributed to natural variation in strength of masonry construction. However, a deviation of 0.15 or 0.17 MPa (the best in this study) is about $7 \%$ of the average value, and is certainly an acceptable deviation in light of the Blume and Proulx study.

The deviations in Table 3 show that some equations are more successful in predicting ultimate shear strength than others. The table also shows that some data sets are more accurately predicted than others. An evaluation of the data sets to identify the factors that contribute to the success of the predictions follows.

### 5.2 Analysis of Data

Table 3 (deviation) shows that none of the four equations was very successful in predicting the strengths of data set $O$. The plots for the comparisons against data set $O$ shown in Figures 1 through 4 indicate that all four equations underestimated the actual test strengths. This consistent underprediction suggests that the specimens of data set 0, for whatever reason, developed higher than normal strengths. To determine if this was indeed the case, additional comparisons were made. Specimens WS1 and WSR1 from data set O (test numbers 29 and 34) share similar characteristics with specimen KW4-1 (test number 11) of data set $M$ except for magnitude of axial stress and $\rho_{v}$. The two specimens of group 0 had zero axial stress and $\rho_{v}=0.005$; the specimen of group M had an axial stress of $0.49 \mathrm{MPa}(71 \mathrm{psi})$ and $\rho_{v}=0.009$. The ultimate shear strengths reported from the actual test results were 2.67 and $3.12 \mathrm{MPa}(388$ and 453 psi ) for the 0 specimens, but only $1.60 \mathrm{MPa}(232$ $\mathrm{psi})$ for the M specimen. The fact that these two group O specimens, in spite of the absence of axial load and the lighter vertical reinforcement, developed about twice as much strength as the group $M$ specimen confirms the possibility that the specimens of group O generally may have developed substantially higher strengths than the specimens of group $M$ of similar construction. Similar comparisons with the results of data set $S$ indicate the same trend le.g. compare results of 22-S and WS1-0). The overstrength of these specimens makes the results from data set $O$ less typical than the other data sets in evaluating predictive capabilities of the four equations.

It should be noted that data set B includes specimens with variations in type of anchorage of horizontal reinforcement and the distribution of horizontal and vertical bars in addition to the variation of parameters tabulated in Appendix A. These variations contribute to the scatter evident in the plots of test-versuspredicted strengths.

The normalized plots of Figures 6 through 9, representing equations S, M, J, and $U$, respectively, show the ratios of test strength to predicted strength ( $\mathrm{vut}_{\mathrm{ut}} / \mathrm{v}_{\mathrm{up}}$ ) according to the test number. The four data subsets are identified by their symbols appearing at the top in these figures. Data subsets $\mathrm{S}, \mathrm{M}, \mathrm{O}$ and B correspond to test numbers 1-10, 11-28, 29-37 and 38-62, respectively. Prediction underestimates and overestimates can be readily identified according to whether the line plotting the ratio of test to predicted value falls above or below the unity line, respectively. These plots clearly show the tendency of all the equations to underestimate the unusually high test strength of data set $O$, and also confirm the erratic nature of predictions of the results of data set B.

Table 3 shows that three of the four equations ( $\mathrm{S}, \mathrm{M}$, and J) were more successful in predicting the strengths of data set $S$ than of any other. Plausible reasons for the generally small deviations are (1) the horizontal and vertical reinforcement in the specimens of group $S$ were distributed more uniformly than was typical in the specimens of the other data sets, and (2) several parameters ( $r, d, d^{\prime}$, and $s_{h}$ ) in the $S$ test series were not varied, and the range of most of those that were varied ( $\rho_{\mathrm{h}}, \rho_{\mathrm{v}}, \sigma_{0}$ ) was narrow relative to the other test series. Equation $U$ consistently underestimated data set $S$ by considerable margins. Equation $U$ gives substantially less weight to the strength component $v_{m}$ than equations $S$ and $M$ (Figure 10). It also ignores the contribution of axial load on strength $\left(\mathrm{v}_{\mathrm{q}}=0\right)$.

## Equation S

This formula was developed to fit the test data of group $S$ using regression analysis. Consequently, predictions by equation $S$ were in closer agreement with results of data set $S$ than any other comparison.

Shing et al $[2,3]$, note that experimental observations indicate that the postcracking strength of masonry increases with vertical steel and axial load, mainly through resistance at the compression face, aggregate interlock and dowel action. These contributions to $v_{u}$ are represented by the first and third terms of equation $S$. The contribution of horizontal steel, reflected by the second term of equation $S$, takes into account the ineffectiveness of the top and bottom layers of horizontal steel due to insufficient embedment length to develop their yield capacity following diagonal shear rupture.


FIGURE 7. EQUATION M STRENGTH RATIOS EXPERIMENTAL/PREDICTED


TEST NUMBER

FIGURE 8. EQUATION J STRENGTH RATIOS EXPERIMENTAL/PREDICTED


FIGURE 9. EQUATION U STRENGTH RATIOS EXPERIMENTAL/PREDICTED


Shing et al demonstrated that their proposed formula is in better agreement with their test results than the UBC formula, which they showed to be consistently more conservative. This conclusion is verified in this study, which showed the deviation of predictions by equation $U$ of data set $S$ to be five times the deviation of predictions by equation $S$ of the same data set 10.70 versus 0.15 MPa or 102 versus 21 psi respectively), with equation U consistently underpredicting the strength of the specimens of group S .

## Equation M

In reference [4], Matsumura describes the development of the formula presented in this paper as equation $M$ (original form is shown in Appendix $B$ ). Matsumura developed this formula by utilizing his test results (data set $M$ in this study) as well as test results reported by other researchers in Japan. He, like Shing et al, used regression analysis to determine the appropriate functional forms of the parameters.

Overall, equation $M$ is the most successful of the four equations. Equations $M$ and $S$ are comparable in accuracy in predicting data sets $S$ and $M$, but equation $M$ is the most successful of the four equations in predicting the test results of group B specimens.

## Equation J

Equation J is based on a formula published in the Reinforced Concrete Design Standard of the Architectural Institute of Japan for predicting the ultimate shear strength of concrete shear walls. The predicted strength for specimen WSRC-O, the concrete wall, was about $85 \%$ of test strength. Predictions of masonry shear wall strength by equation J were less successful.

There are notable similarities between equations $M$ and $J$ in the types of parameters considered to have an effect on shear response. A major difference is that equation J considers the contribution of interior vertical bars, which equation M neglects. Another significant difference is that equation M includes the square root of the compressive strength of masonry as a multiplier in the term representing the contribution of horizontal reinforcement; equation J does not.

Equation $J$ was less successful in predicting the strengths of data set $S$ (deviation $=0.46 \mathrm{MPa}$ or 68 psi ) than equations S and M . The weight given to the contribution of interior vertical reinforcement (as expressed in steel ratio $\rho_{\mathrm{vi}}$ ) by equation J partially explains this difference in predictive success. Equation $J$ gives equal weight to ratios of horizontal and interior vertical reinforcement, $\rho_{\mathrm{h}}$ and $\rho_{\mathrm{vi}}$, in the determination of steel contribution to strength.

None of the other equations specifically include the effect of $\rho_{\mathrm{vi}}$ in the predicted strength. Equation $S$ includes $\rho_{\mathrm{v}}$ in the $\mathrm{v}_{\mathrm{m}}$ term, which incorporates the contribution of all vertical steel. Equation M only includes the steel in the end cores, as measured by $\rho_{\mathrm{ve}}$, in its expression for masonry strength. Equation $U$ does not consider the contribution of vertical steel at all. The $v_{s}$ terms in Table 2 show that for data set $S$, predictions of steel contribution to strength by equation $J$ (average 1.67 MPa or 242 psi ) far exceed the predictions of the other three equations (average 0.43 MPa or 62 psi ). The effect of the $\rho_{\mathrm{vi}}$ term in causing the overprediction of data set $S$ is confirmed by examining the magnitudes of $\rho_{\mathrm{vi}}$ in all four data sets.

The ranges of $\rho_{\mathrm{h}}$ and $\rho_{\mathrm{vi}}$ in data set S are 0.0012 to 0.0022 and 0.0034 to 0.0067 , respectively. Because of the relative ratios of horizontal and interior vertical steel in these specimens, the contribution of the interior vertical reinforcement according to equation $J$ will be 2 to 3 times that of the horizontal reinforcement. Figure 8 shows that equation J consistently overpredicted the strength of data set $S$, while it tended to underpredict the strengths of the other data sets. Only 8 specimens in the other data sets have $\rho_{\mathrm{vi}}$ in excess of the minimum value of 0.0034 used in the S series. Equation $J$ predictions of these specimens with lower $\rho_{\mathrm{vi}}$ were, on average, even less successful than predictions for data set $S$ (deviations of $0.56,0.77$, and 0.84 MPa or 82, 111, and 122 psi for data sets $M, O$, and $B$, respectively). The strengths of 4 of the non-S high $-\rho_{\text {vi }}$ specimens are overpredicted by equation $J$. The overestimation of one data subset and underestimation of the others indicates that the adoption of this equation for masonry shear walls through corrections based on regression constants is not possibie.

## Equation U

Equation $U$ was the least successful of the four equations in predicting ultimate shear strength. This equation, the formula for ultimate shear strength of masonry shear walls specified in the 1988 Uniform Building Code [7], does not consider the effect of axial load. For three out of the four data subsets ( $\mathrm{S}, \mathrm{M}$, and $O$ ) equation $U$ underestimates the test results, with the exception of three specimens (numbers 19, 27, and 28) having high $\rho_{h}$, to which equation $U$ is more sensitive than the other equations.

The closest correlations of equation $U$, with data sets $S$ and $M$, correspond to deviations which are 2 to 5 times those corresponding to equations $S$ and $M$. In most instances, equation $U$ gives strength estimates that are overly conservative, with deviations in excess of 0.69 MPa (100 psi) in every case (see Table 3). In addition, predictions based on equation $U$ generally are not consistent with test results.

### 5.3 Analysis of Strength Prediction

The deviations in Table 3 clearly show that equation $M$ was the most successful predictor of actual shear strengths. The deviation calculated from the combined data sets, 0.39 MPa ( 58 psi ), is significantly less than the deviations for the other three equations, $0.58,0.69$, and $0.81 \mathrm{MPa}(84,100$, and 118 psi) for equations $S$, J, and U, respectively. Formula $S$ was comparatively successful in predicting the $S$ and $M$ data sets, but it was only slightly better than equation U in predicting data sets O and B . Equation M was the most successful of the four equations in predicting data set $B$, with a deviation of $0.34 \mathrm{MPa}(50 \mathrm{psi})$, and was by far more successful than equations $S$ and $U$ in predicting the unexpectedly high strength of data set $O$.

The M-M deviation was almost twice the $\mathrm{S}-\mathrm{S}$ deviation $(0.33$ and 0.14 MPa , or 48 psi and 28 psi respectively). This may be attributed to the fact that equation M was calibrated using other data in addition to data subset M , while the equation $S$ was calibrated using only data subset $S$. Additionally, a wider range of parameters was used in the $M$ series of specimens compared to the S series. The M-S comparison was very successful, with a deviation similar to that for the S-S comparison ( 0.17 and 0.15 MPa or 24 and 21 psi , respectively). The S-M deviation, 0.39 MPa ( 56 psi ), was comparable to the $\mathrm{M}-\mathrm{M}$ deviation of 0.33 MPa ( 48 psi ).

To carry the comparisons between formulae one step further, the contribution of the individual terms ( $v_{m}, v_{s}, v_{q}$ ) were examined. Figures 10 through 13 are histograms comparing the magnitude of the predicted $v_{m}, v_{s}, v_{q}$, and $v_{u}$ stresses, respectively, for all 62 tests, following the numerical order of specimens listed in Table 1. Data subsets $S, M, O$ and $B$ are identified by their symbols in the figures.

Figure 10 shows consistently lower estimates of $v_{m}$ by equation J relative to the predictions by the other equations. Values of $v_{m}$ from equation $U$ are low relative to predictions by formulas $S$ and $M$. Values predicted by equations $S$ and $M$ generally exhibit comparable trends for $v_{m}$ predictions. Figure 11 shows the contribution of $v_{s}$ terms. The predictions according to equation $S$ and $M$ are generally comparable in

FIGURE 10. PREDICTED Vm STRESSES, MPa


FIGURE 11. PREDICTED Vs STRESSES, MPa


FIGURE 12. PREDICTED Vq STRESSES, MPa


FIGURE 13. PREDICTED Vu STRESSES, MPa

trend and magnitude, while $\mathrm{v}_{\mathrm{s}}$ contributions according to equations J and U vary widely. The $v_{s}$ terms of the four equations are the least similar in form. Figure 12 shows the contribution of the $v_{\mathrm{q}}$ terms. Formula $U$ does not include a term to account for axial load effect, indicated on the plot by a value of zero for all predictions by equation $U$. The axial load effect in equations $J$ and $S$ is lower than in equation $M$. Equations $J$ and $M$ have identical forms for this term ( $\mathrm{v}_{\mathrm{q}}=$ constant $\sigma_{0} \mathrm{~d} / \mathrm{L}$ ), but the $J$ value of the constant is exactly half the $M$ value. The $v_{q}$ term in equation $S$ is dependent on $\left(f_{m}^{\prime}\right)^{1 / 2}$. Figure 13, which plots the sum of the three terms, $v_{u}$, shows that no trends or similarities between equations can be easily identified from the single value of predicted ultimate strength.

The normalized plots (Figures 6 through 9) together with other data can be used to examine the stability or consistency of the predictive formulas. If two tests identical except in one parameter can be identified for which a predictive equation yields contradictory results, then the weight or even the functional form of that parameter in the formula becomes suspect. In the following paragraphs a selected number of cases are examined in this manner, placing emphasis on equations $S$ and $M$, both of which were developed from masonry data and are proposed for use in masonry design.

Consider the stress ratios based on equations $S$ and $M$ which are plotted in Figures 6 and 7. Specimens 25 and 28 from data set $M$ are identical in all parameters except $\rho_{\mathrm{h}}$. The values of $\rho_{\mathrm{h}}$ are 0 and 0.00668 , respectively. (Note that 0.00762 was the highest value of $\rho_{\mathrm{h}}$ included in the test specimens.) The test results for these two specimens, 2.18 and $2.04 \mathrm{MPa}(316$ and 296 psi ), are nearly the same. However the strength predictions, approximately equivalent by either equation, are $1.4 \mathrm{MPa}(200 \mathrm{psi})$ and $2.6 \mathrm{MPa}(380 \mathrm{psi})$ for the two specimens, an underestimate in the first case and an overestimate in the second. The inconsistent predictions for specimens 25 and 28 imply that in some cases (e.g. high $\rho_{h}$ ) equations $S$ and $M$ can overestimate the effect of horizontal reinforcement on ultimate shear strength by a substantial amount.

Equations $S$ and $M$ desensitize the effect of horizontal reinforcement in different ways. In equation $\mathrm{S}, \mathrm{v}_{\mathrm{s}}$ is proportional to ( $\rho_{\mathrm{h}} \mathrm{f}_{\mathrm{yh}}$ ) times a factor, (L $2 d^{\prime}-s_{h} / / L$, which varies from 0.26 (test no. $36, r=2.25$ ) to 0.77 (test no. $29, r=0.90$ ). As $r$ and $s_{h}$ decrease, the factor increases toward unity. Conversely, the effect of horizontal reinforcement decreases with increasing $r$ and $s_{h}$, and may conceivably become zero or negative. For instance, assume $r=3$ and $s_{h}=h / 3$ (i.e. three levels of horizontal bars). Then, $L=s_{h}$, resulting in a factor which is negative. The average value of the factor is 0.55 for the 62 tests. Equation $M$ desensitizes the effect of horizontal reinforcement by using the expression $0.1575\left(\rho_{\mathrm{h}} \mathrm{f}_{\mathrm{yh}} \mathrm{f}_{\mathrm{m}}\right)^{1 / 2}$. Thus, as $\rho_{\mathrm{h}}$ increases, its effect on
strength increases at a decreasing rate. Both equations, however, tend to overestimate the effect of horizontal reinforcement on ultimate strength in more heavily reinforced walls.

Results for specimens from data set B provide additional information useful in identifying unsuccessful parametric forms. Consider the S-B comparison, as shown in Figure 6. Recall that the deviation of this set of predictions was relatively high, ( 0.64 MPa or 93 psi , Table 3 ). Specimens numbered 49, 51, 53, 55, 57 have identical axial load and are nearly identical in the other parameters. Equation $S$ overestimates the ultimate strengths of these specimens by $11 \%$ to $38 \%$ (Figure 6). By comparison, equation M overestimates the strengths of the same specimens by $-4 \%$ (underestimate) to $31 \%$ (Figure 7). Comparing averages, equations $S$ and $M$ overestimate the five tests by $27 \%$ and $11 \%$, respectively. Recall that the M-B correlation shows considerably less scatter (Table 3) than predictions by other equations for this data set and that the deviation of the M-B comparison was 0.34 MPa ( 59 psi ). The difference in the predictive accuracy of these two equations is mainly due to the differences in the effect of the $v_{s}$ term on estimated strength. For these specimens, the range of differences in predictions for the $\mathrm{v}_{\mathrm{m}}$ term was only from 0.14 to -0.18 MPa ( 20 to -26 psi ). This difference was calculated as S prediction - M prediction. The difference in predicted effect of the $\mathrm{v}_{\mathrm{a}}$ term (axial load) was only $0.19 \mathrm{MPa}(27 \mathrm{psi})$. However the difference in the predicted contribution of horizontal steel strength to ultimate strength ranged from 0.50 to $0.96 \mathrm{MPa}(72$ to 139 psi$)$. Equation S predicted values from 1.47 to 2.03 MPa ( 213 to 295 psi ) for the horizontal steel strength term for these specimens, while equation $M$ predicted contributions from 0.97 to 1.07 MPa (141 to 156 psi ). This demonstrates that equation $S$ gives excessive weight to the effect of horizontal steel on the ultimate strength of these specimens.

The predictions for the six double-wythe brick walls in data set $B$ (specimen numbers 59-62) were also examined. The results from these specimens indicated that the effect of horizontal steel on ultimate strength is not accurately modeled by either equation. Equation $S$, and to a lesser extent equation $M$, underestimate the strengths of specimens 59,61 , and 62 , which have low $\rho_{\mathrm{h}}(0.08$ to $0.11 \%)$, and overestimate the strength of specimen 60 , which has a high $\rho_{\mathrm{h}}(0.35 \%)$.

The above observations demonstrate that both equations $S$ and $M$ need to be examined with the aim of rendering predicted strength less sensitive to the horizontal reinforcement ratio.

## 6. CONCLUSIONS

The following conclusions can be drawn on the basis of the ultimate strength comparisons discussed above. Equation $U$ does not adequately predict ultimate shear strength for the range of parameters represented by the masonry walls included in this study. Equation $J$ is less consistent than equations $S$ and $M$ primarily because it gives excessive weight to the contribution of interior vertical bars in resisting shear forces. Equation $S$ can predict shear strength well for only limited ranges of variables, primarily because it tends to overestimate the effect of horizontal reinforcement on strength. Of the four equations examined, equation $M$ is generally the closest predictor of ultimate strength, but it lacks consistency. The parametric form of the horizontal steel ratio, $\rho_{h}$, has been identified as contributing to this inconsistency. The need to re-evaluate the effect of horizontal steel on strength (the $v_{s}$ term) is indicated for both equations $S$ and $M$. However, such re-evaluation cannot be carried out without a simultaneous re-evaluation of the weight given to $v_{m}$ and $\mathrm{v}_{\mathrm{q}}$ terms in contributing to predicted strength.

## 7. RECOMMENDATIONS

The experimental information compiled in this document and in other sources could be used to develop improved empirical relationships for closer and more consistent prediction of ultimate strength. The need for such improvements is underscored by the fact that the range of parameters encountered in actual masonry construction is wider than that of the test specimens examined in this study.

## 8. REFERENCES

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TABLE A1. PROPERTIES OF SPECIMENS


TABLE A1 CONT'D PROPERTIES OF SPECIMENS

|  | SPECIME | fyve | fyvi | fov | ph | pvo | pvi | pv | SIGMAO | ALPHA |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| NUMBER | LABEL | Psi. | p\&i | psi |  |  | psi |  | psi |  |
| 1 | 3-5 | 72000 | 72000 | 72000 | 0.00122 | 0.00148 | 0.00667 | 0.00741 | 0 | - 1 |
| 2 | 4-S | 72000 | 72000 | 72000 | 0.00122 | 0.00148 | 0.00867 | 0.00741 | 0 | 1 |
| 3 | 5-S | 72000 | 72000 | 72000 | 0.00122 | 0.00148 | 0.00667 | 0.00741 | 0 | 1 |
| 4 | 7-S | 72000 | 72000 | 72000 | 0.00122 | 0.00148 | 0.00667 | 0.00741 | 0 | 1 |
| 5 | 9-S | 64000 | 64000 | 64000 | 0.00122 | 0.00077 | 0.00344 | 0.00383 | 0 | 1 |
| 6 | 13-S | 65000 | 65000 | 65000 | 0.00222 | 0.00109 | 0.00489 | 0.00543 | 0 | 1 |
| 7 | 14-S | 65000 | 65000 | 65000 | 0.00122 | 0.00109 | 0.00489 | 0.00543 | 0 | 1 |
| 8 | 16-S | 72000 | 72000 | 72000 | 0.00222 | 0.00148 | 0.00667 | 0.00741 | 0 | 1 |
| $\theta$ | 21-S | 65000 | 65000 | 65000 | 0.00128 | 0.00114 | 0.00512 | 0.00568 | 0 | 1 |
| 10 | 22-S | 65000 | 65000 | 65000 | 0.00128 | 0.00114 | 0.00512 | 0.00568 | 0 | 1 |
| 11 | KW4-1-M | 55878 | 55878 | 55878 | 0.00118 | 0.00426 | 0.00134 | 0.00943 | 0 | 0.5 |
| 12 | KW3-1-M | 55878 | 55878 | 55878 | 0.00118 | 0.00434 | 0.00140 | 0.00946 | 0 | 0.5 |
| 13 | KW3S-1- | 55878 | 55878 | 55878 | 0.00118 | 0.00434 | 0.00140 | 0.00946 | 0 | 0.5 |
| 14 | KW2-1-M | 55878 | 55878 | 55878 | 0.00118 | 0.00541 | 0.00155 | 0.01148 | 0 | 0.5 |
| 15 | WS2-M | 55878 | 55878 | 55878 | 0.00000 | 0.00254 | 0.00111 | 0.00571 | 0 | 0.5 |
| 16 | WS4-M | 55878 | 55878 | 55878 | 0.00167 | 0.00254 | 0.00111 | 0.00571 | 0 | 0.5 |
| 17 | WS5-M | 55878 | 55878 | 55878 | 0.00334 | 0.00254 | 0.00111 | 0.00571 | 0 | 0.5 |
| 18 | WS9-M | 55878 | 55878 | 55878 | 0.00334 | 0.00448 | 0.00111 | 0.00959 | 0 | 0.5 |
| 19 | WS10-M | 55878 | 55878 | 55878 | 0.00668 | 0.00448 | 0.00111 | 0.00959 | 0 | 0.5 |
| 20 | WS9-2-M | 55878 | 55878 | 55878 | 0.00334 | 0.00448 | 0.00111 | 0.00959 | 0 | 0.5 |
| 21 | WSB21- | 55878 | 55878 | 55878 | 0.00334 | 0.00448 | 0.00111 | 0.00959 | 0 | 0.5 |
| 22 | WSB22- | 55878 | 55878 | 55878 | 0.00400 | 0.00448 | 0.00111 | 0.00959 | 0 | 0.5 |
| 23 | WSB3-M | 55878 | 55878 | 55878 | 0.00353 | 0.00473 | 0.00117 | 0.01013 | 0 | 0.5 |
| 24 | WSB4-M | 55878 | 55878 | 55878 | 0.00334 | 0.00448 | 0.00111 | 0.00959 | 0 | 0.5 |
| 25 | WSR2-M | 55878 | 55878 | . 55878 | 0.00000 | 0.00272 | 0.00121 | 0.00812 | 0 | 0.5 |
| 28 | WSR4-M | 55878 | 55878 | 55878 | 0.00167 | 0.00272 | 0.00121 | 0.00612 | 0 | 0.5 |
| 27 | WSR5-M | 55878 | 55878 | 55878 | 0.00334 | 0.00272 | 0.00121 | 0.00612 | 0 | 0.5 |
| 28 | WSR6-M | 55878 | 55878 | 55878 | 0.00668 | 0.00272 | 0.00121 | 0.00612 | 0 | 0.5 |
| 29 | WS1-O | 56103 | 53872 | 54987 | 0.00187 | 0.00149 | 0.00292 | 0.00509 | 0 | 0.5 |
| 30 | WS4-0 | 58103 | 53872 | 54987 | 0.00167 | 0.00249 | 0.00316 | 0.00874 | 0 | 0.5 |
| 31 | WS7-0 | 56103 | 53872 | 54987 | 0.00167 | 0.00374 | 0.00351 | 0.00879 | 0 | 0.5 |
| 32 | WSN1-O | 56103 | 53872 | 54987 | 0.00167 | 0.00249 | 0.00316 | 0.00674 | 0 | 0.5 |
| 33 | WSN2-O | 56103 | 53872 | 54987 | 0.00167 | 0.00249 | 0.00316 | 0.00674 | 0 | 0.5 |
| 34 | WSR1-O | 56103 | 52693 | 54398 | 0.00167 | 0.00149 | 0.00292 | 0.00509 | 0 | 0.5 |
| 35 | WSR4-0 | 56103 | 53872 | 54987 | 0.00167 | 0.00249 | 0.00316 | 0.00674 | 0 | 0.5 |
| 36 | WSR7-O | 58103 | 53872 | 54987 | 0.00167 | 0.00374 | 0.00351 | 0.00879 | 0 | 0.5 |
| 37 | WSRC-O | 56103 | 53872 | 54987. | 0.00167 | 0.00249 | 0.00316 | 0.00674 | 0 | 0.5 |
| 38 | CB13-8 | 67500 | 67500 | 67500 | 0.00281 | 0.00085 | 0.00000 | 0.00169 | 0 | 0.5 |
| 39 | CB15-B | 67500 | 67500 | 67500 | 0.00281 | 0.00085 | 0.00000 | 0.00169 | 0 | 0.5 |
| 40 | CB17-8 | 56700 | 56700 | 56700. | 0.00394 | 0.00222 | 0.00000 | 0.00444 | 0 | 0.5 |
| 41 | CB18-B | 59500 | 58500 | 59500. | 0.00394 | 0.00074 | 0.00423 | 0.00444 | 0 | 0.5 |
| 42 | CB20-B | 58700 | 58700 | 58700 | 0.00197 | 0.00222 | 0.00000 | 0.00444 | 0 | 0.5 |
| 43 | CB21-B | 59500 | 59500 | 59500 | 0.00197 | 0.00074 | 0.00423 | 0.00444 | 0 | 0.5 |
| 44 | CB23-B | 54700 | 50700 | 56700 | 0.00075 | 0.00222 | 0.00000 | 0.00444 | 0 | 0.5 |
| 45 | CB24-B | 58700 | 56700 | 56700 | 0.00272 | 0.00222 | 0.00000 | 0.00444 | 0 | 0.5 |
| 46 | CB25-B | 58700 | 58700 | 56700 | 0.00197 | 0.00222 | 0.00000 | 0.00444 | 0 | 0.5 |
| 47 | CB26-B | 56700 | 50700 | 50700 | 0.00197 | 0.00222 | 0.00000 | 0.00444 | 0 | 0.5 |
| 48 | BR19-B | 56700 | 56700 | 56700 | 0.00197 | 0.00222 | 0.00000 | 0.00444 | 0 | 0.5 |
| 48 | BR20-B | 50700 | 56700 | 58700 | 0.00492 | 0.00222 | 0.00000 | 0.00444 | 0 | 0.5 |
| 50 | BR21-B | 56700 | 58700 : | 56700. | 0.00197 | 0.00222 | 0.00394 | 0.00674 | 0 | 0.5 |
| 51 | BR22-B | 63500 | 63500 | 83500 | 0.00482 | 0.00115 | 0.00394 | 0.00459 | 0 | 0.5 |
| 52 | BR23-B | 59500 | 59500 | 59500 | 0.00197 | 0.00074 | 0.00423 | 0.00444 | 0 | 0.5 |
| 53 | BR24-B | 59500 | 59500 | $59500^{\circ}$ | 0.00482 | 0.00074 | 0.00423 | 0.00444 | 0 | 0.5 |
| 54 | BR25-B | 58700 | 56700 | 58700 | 0.00197 | 0.00222 | 0.00000 | 0.00148 | 0 | 0.5 |
| 55 | BR20-B | 58700. | 58700 | 58700 | 0.00492 | 0.00222 | 0.00000 | 0.00444 | 0 | 0.5 |
| 58 | BR27-B | 58700 | 56700 | 56700 | 0.00254 | 0.00222 | 0.00000 | 0.00444 | 0 | 0.5 |
| 57 | BR2B-B | 59500 | 58500 | 58500 | 0.00635 | 0.00222 | 0.00000 | 0.00444 | 0 | 0.5 |
| 58 | BR30-B | 58700 | 56700 | 58700 | 0.00100 | 0.00222 | 0.00000 | 0.00444 | 0 | 0.5 |
| 59 | D8R8S-B | 67500 | 67500 | 67500 | 0.00055 | 0.00065 | 0.00000 | 0.00129 | 0 | 0.5 |
| 60 | DBR9-B | 67500 | 07500 | 67500 | 0.00277 | 0.00065 | 0.00000 | 0.00129 | 0 | 0.5 |
| 61 | DBR10-8 | 67500 | 67500 | 67500 | 0.00055 | 0.00065 | 0.00000 | 0.00129 | 0 | 0.5 |
| 62 | DBR12-B | 67500 | 67500 | 67500 | 0.00059 | 0.00065 | 0.00000 | 0.00129 | 0 | 0.5 |


| TEST NUMBER | SPECIMEN Label |  |  |  |  |  |  |  |  |  |  |  |  |  |  | $V_{m}+V_{s}$ | Vq＊＊ | Vu TEST |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $s$ | M | J | U | s | M | J | u | s | M | J | $u$ | s | M | $J$ | U | psi |
| 1 | 3－S | 162 | 158 | 49 | 68 | 46 | 40 | 254 | 68 | 27 | 45 | 22 | 0 | 234 | 243 | 325 | 134 | 253 |
| 2 | 4－S | 154 | 147 | 45 | 81 | 46 | 38 | 254 | 68 | 0 | 0 | 0 | 0 | 197 | 185 | 299 | 130 | 196 |
| 3 | 5－S | 151 | 147 | 45 | 61 | 46 | 38 | 254 | 68 | 9 | 17 | 8 | 0 | 206 | 202 | 307 | 130 | 214 |
| 4 | 7－S | 182 | 158 | 48 | 68 | 46 | 40 | 254 | 68 | 10 | 17 | 8 | 0 | 218 | 215 | 311 | 134 | 240 |
| 5 | 9－S | 134 | 130 | 42 | 66 | 46 | 40 | 194 | 68 | 27 | 45 | 22 | 0 | 206 | 215 | 259 | 134 | 237 |
| 8 | 13－S | $15 \pm$ | 151 | 48 | 69 | $\theta 9$ | 63 | 253 | 149 | 28 | 45 | 22 | 0 | 279 | 259 | 323 | 218 | 278 |
| 7 | 14－S | 151 | 151 | 48 | 69 | 46 | 42 | 220 | 68 | 28 | 45 | 22 | 0 | 225 | 238 | 290 | 137 | 258 |
| 8 | 16－S | 148 | 145 | 44 | 80 | 99 | 54 | 287 | 149 | 24 | 45 | 22 | 0 | 272 | 244 | 353 | 208 | 298 |
| 9 | 21－S | 164 | 165 | 52 | 74 | 48 | 47 | 225 | 72 | 31 | 46 | 23 | 0 | 243 | 258 | 300 | 146 | 260 |
| 10 | 22－S | 164 | 165 | 52 | 74 | 48 | 47 | 225 | 72 | 1.1 | 17. | 8 | 0 | 223 | 228 | 285 | 146 | 226 |
| 11 | KW4－1－M | 166 | 207 | 64 | 103 | 42 | 68 | 141 | 88 | 7 | 12 | 6 | 0 | 215 | 287 | 211 | 168 | 232 |
| 12 | KW3－1－M | 166 | 167 | 63 | 84 | 34 | 66 | 140 | 66 | 7 | 12 | 6 | 0 | 207 | 245 | 208 | 150 | 250 |
| 13 | KW3S－1－ | 168 | 167 | 63. | 84 | 34 | 66 | 140 | 68 | 7 | 12 | 6 | 0 | 207 | 245 | 208 | 150 | 271 |
| 14 | KW2－1－M | 177 | 124 | 56 | 67 | 18 | 64 | 138 | 66 | 7 | 11 | 8 | 0 | 202 | 198 | 199 | 133 | 234 |
| 15 | WS2－M | 148 | 143 | 56 | 84 | 0 | 0 | 65 | 0 | 29 | 40. | 23 | 0 | 176 | 189 | 143 | 84 | 247 |
| 16 | WS4－M | 146 | 143 | 56 | 84 | 47 | 80 | 144 | 93 | 29 | 48 | 23 | 0 | 223 | 268 | 222 | 178 | 274 |
| 17 | WS5－M | 146 | 143 | 56 | 84. | 94 | 113 | 176 | 187 | 29 | 46 | 23 | 0 | 270 | 301 | 255 | 271 | 331 |
| 18 | WS8－M | 169 | 169 | 64 | 84 | 94 | 113 | 176 | 187 | 29 | $40^{\circ}$ | 23 | 0 | 292 | 328 | 263 | 271 | 332 |
| 19 | WS10－M | 169 | 169 | 64 | 84 | 188 | 159 | 223 | 373 | 29 | 46. | 23 | 0 | 386 | 374 | 309 | 458 | 425 |
| 20 | WS9－2－M | 182 | 183 | 74 | 20 | 94 | 128 | 176 | 187 | 33 | 48 | 23 | 0 | 320 | 367 | 274 | 283 | 378 |
| 21 | WSB21－M | － 182 | 183 | 70 | 91. | 94 | 122 | 176 | 187 | 32 | 46 | 23 | 0 | 308 | 351 | 268 | 278 | 325 |
| 22 | WSB22－M | 187 | 188 | 72 | 94 | 113 | 137 | 187 | 224 | 32 | 48. | 23 | 0 | 332 | 370 | 282 | 317 | 382 |
| 23 | WSB3－M | 187 | 187 | 71 | 92 | 99 | 126 | 181 | 197 | 32 | 48. | 23 | 0 | 318 | 359 | 275 | 289 | 353 |
| 24 | WSB4－M | 200 | 201 | 78 | 100. | 94 | 134 | 178 | 187 | 35 | 48 | 23. | 0 | 329 | 380 | 278 | 287 | 376 |
| 25 | WSR2－M | 169 | 159 | 65 | 23 | 0 | 0 | 66 | 0 | 33 | 45 | 23. | 0 | 201 | 204 | 153 | 93 | 316 |
| 28 | WSR4－M | 169 | 159 | 65 | 93 | 44 | 89 | 144 | 93 | 33 | 45. | 23 | 0 | 245 | 293 | 231 | 187 | 283 |
| 27 | WSR5－M | 189 | 159 | 65 | 93 | 88 | 128 | 176 | 187 | 33 | 45 | 23. | 0 | 289 | 330 | 264 | 280 | 248 |
| 28 | WSR6－M | 169 | 159 | 65 | 93 | 176 | 178 | 222 | 373 | 33 | 45 | 23 | 0 | 377 | 382 | 309 | 486 | 296 |
| 29 | WS1－O | 128 | 159 | 48 | 104 | 81 | 71 | 185 | 86 | 0. | 0 | 0 | 0 | 188 | 230 | 231 | 190 | 388 |
| 30 | WS4－O | 153 | 144 | 56 | 88 | 44 | 77 | 183 | 88 | 29 | 48 | 23 | 0 | 227 | 268 | 262 | 172 | 285 |
| 31 | WS7－0 | 148 | 100 | 47 | 61 | 23 | 66 | 181 | 88 | 0 | 0 | 0 | 0 | 169 | 168 | 228 | 147 | 296 |
| 32 | WSN1－O | 153 | 144 | 56 | 88 | 44 | 77 | 183 | 86 | 59 | 92 ． | 46 | 0 | 256 | 313 | 285 | 172 | 349 |
| 33 | WSN2－O | 153 | 144 | 58 | 88 | 44 | 77 | 183 | 88 | 88 | 137 | 69 | 0 | 285 | 359 | 308 | 172 | 379 |
| 34 | WSA1－0 | 156 | 194 | 57. | 127 | 61 | 87 | 184 | 86 | 0 | 0 | 0 | 0 | 216 | 281 | 241 | 213 | 453 |
| 35 | WSR4－O | 181 | 152 | 60 | 90 | 44 | 81 | 183 | 86 | 0 | 0 | 0. | 0 | 205 | 233 | 243 | 176 | 338 |
| 36 | WSR7－O | 180 | 110 | 51 | 67. | 23 | 72 | 181 | 86 | 0 | 0 | 0 | 0 | 182 | 181 | 232 | 153 | 296 |
| 37 | WSRC－O | 168 | 158 | 62 | 93 | 44 | 84 | 183 | 86 | 35 | 50 | 25 | 0 | 245 | 291 | 270 | 179 | 317 |
| 38 | CB13－B | 128 | 128 | 45 | 105 | 106 | 110 | 108 | 166 | 28. | 45 | 22 | 0 | 263 | 283 | 175 | 270 | $2 \mathrm{B3}$ |
| 39 | CB15－B | 128 | 128 | 45 | 105 | 108 | 110 | 108 | 188 | 46 | 72 | 36 | 0 | 280 | 310 | 189 | 270 | 345 |
| 40 | CB17－B | 118 | 141 | 46 | 86 | 160 | 112 | 132 | 250 | 35 | 66 | 33 | 0 | 312 | 318 | 211 | 338 | 357. |
| 41 | CB18－B | 118 | 101 | 36 | 89 | 160 | 112 | 265 | 250 | 35 | 66 | 33 | 0 | 314 | 279 | 333 | 336 | 357 |
| 42 | CB20－B | 115 | 138 | 45 | 85 | 61 | 77 | 93 | 125 | 34 | 66 | 33 | 0 | 209 | 281 | 171 | 210 | 342 |
| 43 | CB21－B | 116 | 99 | 35 | 85 | 61 | 77 | 226 | 125 | 34 | 66 | 33. | 0 | 211 | 242 | 294 | 210 | 324. |
| 44 | CB23－B | 115. | 138 | 45 | 85 | 34 | 48 | 58 | 48 | 34 | 68 | 33. | 0 | 182 | 251 | 136 | 132 | 278 |
| 45 | CB24－B | 115 | 138 | 45 | 85 | 94 | 91 | 110 | 173 | 34 | 68 | 33 | 0 | 243 | 294 | 188 | 257 | 353 |
| 46 | CB25－B | 115 | 138 | 45 | 85 | 61 | 77 | 93 | 125 | 21 | 41 | 21 | 0 | 197 | 257 | 159 | 210 | 285 |
| 47 | CB26－B | 115 | 138 | 45 | 85 | 81 | 77 | 93 | 125 | 34 | 68 | 33 | 0 | 208 | 281 | 171 | 210 | 349 |
| 48 | BR19－8 | 133 | 159 | 52 | 97 | 61 | 88 | 93 | 125 | 38 | 66 | 33 | 0 | 232 | 314 | 178 | 222 | 267 |
| 49 | BR20－8 | 133 | 159 | 52 | 97 | 213 | 141 | 148 | 312 | 39. | 66 | 33 | 0 | 384 | 365 | 233 | 410 | 278 |
| 50 | BR21－B | 145 | 159 | 52 | 97 | 61 | 89 | 218 | 125 | 38 | 66 | 33 | 0 | 245 | 314 | 303 | 222 | 341 |
| 51 | BR22－B | 136 | 130 | 45 | 97. | 213 | 141 | 280 | 312 | 39 | 66 | 33 | 0 | 388 | 337 | 357 | 410 | 348 |
| 52 | BR23－B | 134 | 114 | 41 | 97 | 61 | 89 | 226 | 125 | 39 | 66 | 33 | 0 | 233 | 289 | 299 | 222 | 295 |
| 53 | BR24－B | 134. | 114 | 41 | 97. | 213 | 141 | 280 | 312 | 39 | 60 | 33 | 0 | 385 | 321 | 353 | 410 | 320 |
| 54 | BR25－B | 116 | 159 | 52 | 97 | 61 | 89 | 93 | 125 | 39 | 66 | 33 | 0 | 218 | 314 | 178 | 222 | 316 |
| 55 | BR26－B | 133 | 159 | 52 | 97 | 213 | 141 | 148 | 312 | 39 | 86 | 33 | 0 | 384 | 365 | 233 | 410 | 311 |
| 56 | BR27－B | 133 | 159 | 52 | 97 | 97 | 98 | 103 | 151 | 39 | 68 | 33 | 0 | 268 | 323 | 188 | 249 | 327. |
| 57 | BR28－B | 134 | 159 | 52 | 97 | 295 | 156 | 164 | 384 | 39 | 66 | 33 | 0 | 468 | 381 | 249 | 482 | 330 |
| 58 | BR30－B | 155 | 188 | 63 | 114 | 45 | 74 | 67 | 64 | 46 | 60 | 33 | 0 | 248 | 326 | 162 | 178 | 391 |
| 59 | DBR8S－B | 107 | 101 | 36. | 90 | 10 | 42 | 48 | 33 | 20 | 36. | 18 | 0 | 137 | 179 | 102 | 123 | 218 |
| 60 | D日R9－B | 107 | 101 | 38 | 90 | 127 | 101 | 114 | 187 | 30 | 55 | 27 | 0 | 264 | 258 | 178 | 277 | 237 |
| 61 | DBR10－B | 107 | 101 | 36 | 90 | 10 | 42 | 48 | 33 | 30 | 55 | 27 | 0 | 147 | 198 | 111 | 123 | 251 |
| 62 | D日月12－B | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

Table A3 - Deviation (s), Mean ( $x$ ), and Variation ( $v$ ) in predicted vs test strengths (psi)

| DATA SET | STATS | EQUATION |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | S | M | J | U |
| S | s | 21.19 | 23.95 | 67.63 | 101.74 |
|  |  | 246.01 | 246.01 | 246.01 | 246.01 |
|  | $v$ | 0.09 | 0.10 | 0.28 | 0.41 |
| M | S | 56.46 | 48.19 | 81.71 | 108.13 |
|  | x | 308.42 | 308.42 | 308.42 | 308.42 |
|  | $v$ | 0.18 | 0.18 | 0.27 | 0.35 |
| 0 | S | 145.14 | 145.14 | 111.32 | 183.89 |
|  | x | 344.41 | 344.41 | 344.41 | 344.41 |
|  | $v$ | 0.42 | 0.42 | 0.32 | 0.53 |
| B | S | 93.32 | 93.32 | 122.64 | 109.14 |
|  | $\times$ | 311.03 | 311.03 | 311.03 | 311.03 |
|  | $v$ | 0.30 | 0.16 | 0.39 | 0.35 |
| TOTAL | S | 84.47 | 57.62 | 100.44 | 118.00 |
|  | x | 304.64 | 304.64 | 304.64 | 304.64 |
|  | $v$ | 0.28 | 0.19 | 0.33 | 0.39 |

The original forms of the equations studied in this report are presented here, with explanations of the reformulations required to achieve common format and consistent units.

## Equation S

Proposed equations (15) and (16) in Reference [3] by Shing et al. were combined to form equation $S$ which was examined in this study. The original equations are:

$$
\begin{aligned}
V_{n} & =V_{m}+V_{s} \\
V_{m} & =\left[0.0018\left(\rho_{v} f_{v}+\sigma_{c}\right)+2\right] A \sqrt{f_{m}^{\prime}} \\
V_{s} & =\left[\frac{L-2 d^{\prime}}{s-1}\right] A_{h} f_{v}
\end{aligned}
$$

The notation was changed to the common forms defined in section 3.3 of this paper, by introducing symbols $f_{y v}$ and $f_{y h}$ for the yield strengths of vertical and horizontal steel in lieu of $f_{y}$ in the original equations. The notations $s$ and $\sigma_{c}$ were changed to $\mathrm{s}_{\mathrm{h}}$ and $\sigma_{0}$, respectively. The other symbols are unchanged.

The equations were transformed from force to stress units by dividing by the gross area, A. The two terms of equation (15) above become the first and third terms of equation S after conversion from U.S. Customary units to SI units. After division by $A$, equation (16) above becomes the second term of equation $S$. Equation $S$ as introduced in section 3.2 accounts for the conversion fomr U.S. Customary units to SI units. Note that $A_{h} / \mathrm{A}=\mathrm{s}_{\mathrm{h}} \rho_{\mathrm{h}} / \mathrm{L}$.

## Equation M

Matsumura proposed a formula for predicting ultimate shear force on a masonry shear wall in Reference [4]. This equation, numbered (5) in the reference, is:

$$
\begin{equation*}
V_{u}=\left[k_{u} k_{p}\left(\frac{0.76}{\frac{h}{d}+0.7}+0.012\right) \sqrt{f_{m(g)}^{\prime}}+\left(0.18 \gamma \sigma \sqrt{P_{H} \sigma_{y} \sigma_{m(g)}^{\prime}}+0.2 \sigma_{o(a)}\right)\right] 10^{3} t j \tag{5}
\end{equation*}
$$

In this equation, $k_{\mathrm{p}}=1.16 \rho_{\mathrm{t}}^{0.3}$. For brick and fully grouted concrete block masonry walls coefficients $k_{u}$ and $\gamma$ are equal to 1.0 and are eliminated from the equation. In accordance with the definitions in section 3.3, the notation is changed as follows:

$$
\rho_{t}=\rho_{v e} ; f_{m(\theta)}^{\prime}=f_{m}^{\prime} ; P_{H}=\rho_{h} ;{ }_{H} \sigma_{y}=f_{y h} ; \sigma_{o(\theta)}=\sigma_{o} ; j=0.875 d^{\prime} ;(h / d)=r_{d}
$$

After division of both sides by gross area ( $\mathrm{t} L$ ), the three terms of equation (5) above become the three terms of equation $M$.

## Equation J

The original form of equation $J$ is given by equation (1) of reference [5], which cites the Reinforced Concrete Standards of the Architectural Institute of Japan as the source of the equation:

$$
\begin{equation*}
T_{S H}=\left[\frac{0.053 P_{t e}^{0.23}\left(f_{m}^{\prime}+180\right)}{\left(\frac{M}{Q D}\right)+0.12}+2.7 \sqrt{\sigma_{w h} P_{w e}}+0.1 \sigma_{o e}\right] \frac{B_{e j}}{B D} \tag{1}
\end{equation*}
$$

The notation is changed in accordance with the notation of section 3.3 as follows:

$$
P_{t e}=\rho_{\mathrm{ve}} ; \quad \underline{D}=L ; \quad \sigma_{o e}=\sigma_{o} ; \quad B_{\mathrm{e}}=B=t
$$

where $M / Q D$ is bounded as follows: $1 \leq M / Q D \leq 3$

The notation is changed in accordance with the notation of section 3.3 as follows:

$$
P_{t e}=\rho_{v e} ; D=L ; \sigma_{o e}=\sigma_{o} ; B_{e}=B=t ; M / Q D=r_{c}
$$

However, since M/QD is discontinuous; $r_{r}$ is expressed as a discontinuous function of ar to satisfy the bounds on $M / Q D$,

$$
r_{c}=1+\langle a r-1\rangle-\langle a r-3\rangle
$$

where for any real number, $\langle\mathrm{a}\rangle=0$ for $\mathrm{a} \leq 0$ and $\langle a\rangle=\mathrm{a}$ for a$\rangle>0$. Note that for cantilever walls (test series $S$ ), $M / Q D=h / L=r(a=1)$, and for walls with the top and bottom rotationally fixed (test series $M, O$, and $B), M / Q D=h / 2 L=r / 2(\alpha=$ 1/2).

In the term 2.7( $\left.\sigma_{w h} \mathrm{P}_{\mathrm{wo}}\right)^{1 / 2}, \sigma_{\mathrm{wh}}$ is the yield strength of shear reinforcement ( $\mathrm{kg} / \mathrm{cm}^{2}$ ) and $P_{w e}$ is the ratio of shear reinforcement. Both horizontal reinforcement and interior vertical bars are treated as shear reinforcement (vertical bars in the two exterior cores are excluded). Accordingly, the second term in the above equation is separated into the sum of two terms to distinguish between horizontal and vertical shear reinforcement and their respective shear strengths. After conversion to the notation adopted in this report, the second term becomes:

$$
2.7 \sqrt{\sigma_{w h} P_{w e}}=2.7 \sqrt{\rho_{y h} f_{y h}}+2.7 \sqrt{\rho_{y v i} f_{y v i}}
$$

which, after conversion from the cgs system to U.S. Customary Units, becomes identical to the second term of equation J.

Substituting 0.875 d for j , the first and third terms of equation (1) above become the first and third terms of equation $J$ after conversion to SI units is made.

## Equation U

The original form of equation $U$ is given by combining equations (12-13), (12-14), and (12-15) of the 1988 Uniform Building Code [7].

$$
\begin{align*}
V_{n} & =V_{m}+V_{s}  \tag{12-13}\\
V_{m} & =C_{d} A_{m v} \sqrt{f_{m}^{\prime}}  \tag{12-14}\\
V_{s} & =A_{m v} \rho_{n} f_{V} \tag{12-15}
\end{align*}
$$

The notation is changed to conform with the notation in section 3.3 as follows:

$$
A_{m v}=A ; \rho_{n}=\rho_{h} ; f_{y}=f_{v h} ; V_{n}=V_{u}
$$

$C_{d}$ in the above equation is a discontinuous function of $M / V_{u} d$,

$$
\begin{aligned}
& C_{d}=2.4 \text { for }(M / V d) \leq 0.25 \\
& C_{d}=1.2 \text { for }(M / V d) \succeq 1.00
\end{aligned}
$$

or expressed in fúnctional form,

$$
\left.C_{d}=2.4+1.6\left\langle a r_{d}-1\right\rangle-1.6<a r_{d}-0.25\right\rangle
$$

The numerical coefficient 0.083 in the first term of equation $U$ is obtained by conversion of the UBC equation (12-14) to SI units. Note that the UBC equations do not consider axial load to contribute to strength, thus there is no third term in equation $U$.


[^0]:    ELECTRONIC FORM

