Publication No. R75-34 Order No. 513

Seismic Design Decision Analysis

Report No. 21

PROBABILISTIC AND STATISTICAL MODELS FOR SEISMIC RISK ANALYSIS

by

Daniele Veneziano

July 1975

Sponsored by National Science Foundation
Research Applied to National Needs (RANN)
Grant GI-27955X3

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Abstract

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Preface

This is the 21st in a series of reports under the general title of Seismic Design Decision Analysis. The overall aim of the research is to develop data and procedures for balancing the increased cost of more resistant construction agains the risk of losses during possible future earthquakes. The research has been sponsored by the Earthquake Engineering Program of NSF-RANN under Grant GI-27855X3. A list of previous reports follows this preface.

The analysis presented herein is oriented to the risk of failure (i.e inadequate preformenace) in a single, complex structure. A nuclear power plant is used as an example - because of the work that already appears elsewhere in the literature concerning the behavior of such a facility. However, the theory applies equally well to important non-nuclear facilities.

Dr. Robert V. Whitman, Professor of Civil Engineering is principal investigator for the overall research project, and the author is grateful to Professor R.V. Whitman for his encouragement in pursuing this effort and for his continuous helpful advice. Appreciation is also expressed for the critcal comments expressed by Professor C.A. Cornell on an earlier draft of this report.

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1. Whitman, R.V., C.A.Cornell, E.H.Vanmarcke, and J.W.Reed, "Methodology and Initial Damage Statistics," Department of Civil Engineering Research Report R72-17, M.I.T., March 1972.

2. Leslie, S.K. and J.M.Biggs, "Earthquake Code Evolution and the Effect of Seismic Design on the Cost of Buildings," Department of Civil Engineering Research Report R72-20, M.I.T., May 1972.

3. Anagnostopoulos, S.A., "Non-Linear Dynamic Response and Ductility Requirements of Building Structures Subjected to Earthquakes," Department of Civil Engineering Research Report R72-54, M.I.T., September 1972.

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9. Whitman, R.V.,J.M.Biggs,J.Brennan III,C.A.Cornell,R.de Neufville, and E.H. Vanmarcke, "Summary of Methodology and Pilot Application," Department of Civil Engineering Research Report R73-58, M.I.T., October 1973.

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17. de Neufville, R.(Being written)

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19. Wong, E.,"Correlation Between Earthquake Damage and Strong Ground Motion," R75-24, Order No. 505, May 1975.

20. Munroe, T. and C. Blair,"Economic Impact in Seismic Design Decision Analysis: A Preliminary Investigation," Department of Civil Engineering Research Report R75-25, Order No. 506, June 1975.

BIBLIOGRAPHIC DATA SHEET	1. Report No. MIT-CE-Rt7-34	2.	3. Recipient's Accession No.
4. Title and Subtitle SEISMIC DESIGN DECISION ANALYSIS Probabalistic and Statistical Models for Seismic Risk Analysis			5. Report Date July 1975 6.
7. Author(s) Daniele Veneziano			8. Performing Organization Rept. No. Order No. 513
9. Performing Organization Name and Address Division of Constructed Facilities Department of Civil Engineering Massachusetts Institute of Technology Cambridge, Massachusetts o2139			10. Project/Task/Work Unit No. 11. Contract/Grant No. GI-27955X3
12. Sponsoring Organization Name and Address Office of Advanced Technology Applications, RANN National Science Foundation Washington, D.C. 20550			13. Type of Report & Period Covered Task 14.

15. Supplementary Notes

No. 21 in a series

16. Abstracts

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17. Key Words and Document Analysis. 17a. Descriptors

Reliability, Risk, Probability, Statistics
Engineering, Civil Engineering, Systems Analysis, System Engineering
Earthquakes, Nuclear Power Plants

17b. Identifiers/Open-Ended Terms

Seismic Design Decision Analysis
Seismic Risk
Inductive Uncertainty

17c. COSATI Field/Group

18. Availability Statement	19. Security Class (This Report)	21. No. of Pages
Release unlimited.	UNCLASSIFIED	156
	20. Security Class (This Page) UNCLASSIFIED	22. Price 6.75

FORM NTIS-35 (REV. 3-72) USCOMM-DC 14952-P72

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21. **Number of Pages.** Insert the total number of pages, including this one and unnumbered pages, but excluding distribution list, if any.

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I. Introduction

The seismic risk to which an engineering system is exposed depends on two factors:

- the future seismic demand at the site ("load"); and
- the (future) seismic capacity of the system ("resistance").

Description of the seismic load requires information at two different time scales: at a macroscale, about the sequence of earthquake occurrences near the site; at a microscale, about the detailed time history of the ground motion for each future occurrence during the lifetime of the system. In a state of uncertainty the earthquake sequence can be modeled as a realization of a random point process, and the indicidual microscale time histories as realizations of continuous random processes.

The seismic capacity of the system can be described by a "resistance vector" (assume a finite dimensional model), which collects the seismic response characteristics and the performance criteria of the system as a whole, as well as of its various subsystems and components. Some of the dynamic characteristics may be time- or response-dependent; for example, the structural stiffness and viscous damping.

The amount of information which is required for a complete probabilistic description of both seismic demand and capacity is not within present knowledge and analysis capability. Nevertheless, one can formulate simplified, yet meaningful, models which demand much less information. In these simplified models, the seismic load at the site is generally described by:

1. The mean earthquake occurrence rate, λ, as a partial characterization of the random point process. (Under the common assumption of Poisson arrivals, λ characterizes completely the occurrence process.)
2. The marginal probability distribution of a scalar (possibly vector) "intensity parameter" Y, which replaces in approximation the continuous random process model of the ground motion. Y might measure (or include) the Modified Mercalli intensity at the site I, the peak ground acceleration a, the peak ground velocity v, or any other motion parameter which is correlated with the system's performance.

Also basic to these simplified models is the description of the seismic resistance through a random damage function of intensity, D(Y), which accounts implicitly for all the possible consequences of malfunctioning and failures of any part of the system.

Clearly, there is a whole theory behind the quantification of local seismicity parameters such as λ and the distribution of Y (engineering seismology); similarly, there is a whole theory behind stochastic damage analysis (system reliability, random vibration), which can be used to calculate the damage function D(Y).

The price for this simplified description of demand and capacity is that seismic risk statements can only involve the mean rate of events producing given damages (e.g., see Eq. 2). But this is not a critical limitation; although neither the complete time characteristics of the damage process, nor the exact nature of damage are given by the analysis, mean damage rates provide enough information for the practical evaluation of seismic risk, and for comparison with other natural threats. Clearly, if the occurrence of seismic events follows a Poisson process and the resistance of the system does not depend on time, the occurrence of damaging events is also a Poisson process. Under the weaker condition that strong earthquakes occur as an approximately Poisson process (which is a frequent assumption in engineering seismic risk analysis), the probability of experiencing extensive damage during a period of time T is well approximated by the expected number of such rare and highly damaging events in T.

Formally, the analysis of seismic risk proceeds as follows. Let $F_Y(y) = P\{Y \leq y | \text{seismic event}\}$ be the cumulative distribution function (CDF) of the site intensity measure whenever an earthquake occurs. Then the mean rate of events with site intensity larger than y is:

$$\lambda_y = \lambda \left[1 - F_Y(y)\right] . \tag{I.1}$$

If $F_{D|Y}(d|y) = P\{D \leq d | Y=y\}$ denotes the CDF of the damage caused by an earthquake with site intensity y, the mean rate of events which cause damage in excess of d is:

$$\lambda_D(d) = \lambda \int_{\text{all } y} \left[1 - F_{D|Y}(d|y)\right] dF_Y(y) . \tag{I.2}$$

In many cases it is not the whole function $\lambda_D(\cdot)$ which is of interest, but only $\lambda_D(d_f)$, i.e. the mean rate of events damaging the system beyond a critical level d_f. Such events are called "failures." Then the mean failure rate is, from Equation (2):

$$\lambda_f = \lambda_D(d_f) = \lambda \int_{\text{all } y} P_f(y) \; dF_Y(y) , \tag{I.3}$$

where $P_f(y) = 1 - F_{D|Y}(d_f|y)$ = probability of failure at intensity y.

This report is concerned with the seismic risk analysis of engineering systems in the sense of Equation (3). Some attention is also given to conparing the results from Equation (3) with past proposed approximations. One approximate procedure, which has been used without due caution, is based on the following reasoning. If ground motions more severe than the design earthquake (e.g., in nuclear reactor design, the so-called Safe Shutdown Earthquake) occur with mean rate $\lambda_{DES} = \lambda[1-F_Y(y_{DES})]$, and if $P_f(y_{DES})$ is the probability of failure at the design intensity, then the mean failure rate can be calculated as:

$$\lambda_f = \lambda_{DES} \cdot P_f(y_{DES}) . \qquad (I.4)$$

Equation (4) yields unconservative (too small) estimates of λ_f <u>for any given</u> y_{DES}. In fact,

$$\lambda_f = \lambda \int_{all\,y} P_f(y) \cdot dF_Y(y) \geq \lambda \int_{y > y_{DES}} P_f(y) \cdot dF_Y(y) \geq \lambda \cdot P_f(y_{DES}) \int_{y > y_{DES}} dF_Y(y) ,$$

and the last expression equals the approximation $\lambda_{DES}\, P_f(y_{DES})$. The unconservatism of using Equation (4) instead of (3) is quantified in the present study, and correction factors are calculated, which depend on the seismic risk model and model parameters.

Several of the assumptions made in this study are common to the technical literature on engineering seismic risk, but the model as a whole is original. To the author's knowledge, statistical uncertainty (although not new to seismic risk formulations) was never extended to both damand and capacity parameters, and indeed not even to capacity parameters alone. These extentions are conceptually and sometimes numerically important. Finally, sensitivity analyses of λ_f in Equation (3) of the type presented here were never reported.

The presentation is organized as follows. First, sources of uncertainty (on the seismic demand and on the seismic resistance) and types of uncertainty (deductive and inductive) are briefly reviewed; see Sections II and III. In Section IV a few probabilistic models are studied, in which the functions $P_f(y)$ and $F_Y(y)$ in Equation (3) are given analytical form. The effects of statistical (inductive) uncertainty on the parameters of the probabilistic models are studied analytically and numerically in Section V. Additional numerical results are collected in the Appendices. Finally, Section VI discusses the choice of the parameters for mean failure rate calculation, and presents some numerical examples.

II. Analysis of Uncertainty: Sources

As expressed by Equations (I.2) and (I.3), seismic risk of engineering facilities depends on the unknown seismic demand at the site (function F_Y) and on the unknown seismic resistance of the system (functions $F_{D|Y}$ and P_f). Information related to these functions is briefly reviewed here, and will be used in Sections IV and V to construct probabilistic and statistical seismic risk models. With regard to seismic demand, emphasis is on data and models for Eastern U.S. regions.

II.1 Uncertainty on the Seismic Demand

A common assumption, which has obtained repeated validation from historical records (Richter, 1958; Allen et al, 1965; Esteva, 1968) is that in any given region the instrumental magnitude M has exponential distribution:

$$\Pr\{M > m\} = 1 - F_M(m) = e^{-\beta m} \tag{II.1}$$

This is a consequence of Richter's "linear" frequency-magnitude law, which establishes that the log number of earthquakes exceeding magnitude m, $\log_{10} n_m$, decays linearly with m:

$$\mathrm{Log}_{10}\, n_m = a - bm \quad , \tag{II.2}$$

a and $b = \beta/\ln 10$ being regional constants.

Both from theoretical considerations (Rosenblueth, 1964; Rosenblueth and Esteva, 1966) and from statistical evidence, it appears however that Equations (1) and (2) have a limited magnitude range of validity. The upper limit, m_1, varies from region to region, but in all cases is smaller than 9. If in addition events of small size (say, with $M<m_o$) are neglected, the distribution (1) assumes the doubly truncated exponential form (Cornell and Vanmarcke, 1969; Cornell, 1971):

$$\Pr\{M > m\} = \begin{cases} 1 & , \quad m \leq m_o ; \\ 1 - K_{m_1} \cdot \left[1 - e^{-\beta(m - m_o)}\right] & , \quad m_o < m < m_1 ; \\ 0 & , \quad m \geq m_1 ; \end{cases} \tag{II.3}$$

where $K_{m_1} = [1-e^{-\beta(m_1-m_o)}]^{-1}$ is a normalization constant.

Although the parameter β in equations (1) and (3) varies from region to region, values reported from different parts of the United States show remarkable consistency (see Table 1).

Other nonlinear frequency-magnitude relationships have been proposed. Among others: the "bilinear law" (see, e.g., Esteva, 1974):

$$Pr\{M > m\} = \begin{cases} \alpha_1 \exp(-\beta_1 m) & , m \leq \bar{m} ; \\ \alpha_2 \exp(-\beta_2 m) & , m > \bar{m} ; \end{cases} \qquad \text{(II.4)}$$

where $\beta_2 \geq \beta_1$ and $\alpha_2 = \alpha_1 e^{(\beta_2 - \beta_1)\bar{m}}$; and the (here, truncated) "quadratic law" (Shlien and Toksöz, 1970; Merz and Cornell, 1973):

$$Pr\{M > m\} = \begin{cases} 1 & , m \leq m_o ; \\ 1 - K_{m_1} \cdot \left[1 - e^{\beta_1(m - m_o) + \beta_2(m^2 - m_o^2)}\right] & , m_o < m < m_1 ; \\ 0 & , m \geq m_1 ; \end{cases} \qquad \text{(II.5)}$$

where $K_{m_1} = [1 - e^{\beta_1(m_1 - m_o) + \beta_2(m_1^2 - m_o^2)}]^{-1}$. (A condition on β_1, β_2 and m_o is clearly needed to ensure that Equation II.5 is an appropriate, i.e. non-increasing, complementary CDF.)

Equations (4) and (5) generalize Richter's linear law (1); both have been reported to fit well empirical complementary CDF's.

While the value of β is quite stable throughout the United States, there is evidence of large regional variability in the upper bound magnitude m_1. The question of the upper size limitation is often discussed in terms of Modified Mercalli (MM) epicentral intensity, since most of the historical data are available in this form.

A number of relationships have been proposed between Richter's magnitude and epicentral intensity I_o. Some of them, in the linear form

$$M = a_1 + a_2 I_o \quad , \qquad \text{(II.6)}$$

are collected in Table 2. The parameters a_i (a_2 in particular) are quite stable from region to region.

Due to the linearity of Equation (6), the frequency-epicentral intensity law is of the same type as the assumed frequency-magnitude law. For example, from the exponential magnitude distribution (3) and from Equation (6), it follows that

$$\Pr\{I_o > i\} = \begin{cases} 1 & , \quad i \leq i_o ; \\ 1 - K_{i_1} \cdot \left[1 - e^{-\beta_{I_o}(i - i_o)}\right] & , \quad i_o < i < i_1 ; \\ 0 & , \quad i \geq i_1 ; \end{cases} \qquad (II.7)$$

where $i_j = (m_j - a_1)/a_2 \quad ; \quad j=0,1$,

$\beta_{I_o} = a_2\beta$,

$K_{i_1} = \{1 - \exp[-\beta_{I_o}(i_1 - i_o)]\}^{-1}$.

For typical values of (i_1-i_0) and β_{I_o}, K_{i_1} is very close to 1, and a good approximation to equation (7) is:

$$\Pr\{I_o > i\} = \begin{cases} 1 & , \quad i \leq i_o ; \\ e^{-\beta_{I_o}(i - i_o)} & , \quad i_o < i < i_1 ; \\ 0 & , \quad i \geq i_1 . \end{cases} \qquad (II.8)$$

β_{I_o} can be estimated from β and a_2 if these parameters are known (see Table 1 and 2). In other cases β_{I_o} , or more generally the linear frequency-intensity law:

$$\ln \lambda_i = \alpha_o - \beta_{I_o} i$$

(λ_i=mean annual rate of events with epicentral intensity in excess of i), have been estimated directly from data on epicentral intensity. Table 3 collects some proposed values for α_o and β_{I_o}. The variability of β_{I_o} from region to region (or from author to author) is explained in part by the subjective assessment of epicentral intensities, and by the inclusion/exclusion of early, incomplete data. As to the parameter α_o, it clearly depends on the seismic region and on its extention. The estimates in Table 3 are therefore reported with the only purpose of indicating typical values.

The question of whether an upper bound intensity i_1 , or an upper bound magnitude m_1 can be established with good confidence in a given region is quite controversial. Upper bound magnitudes: m_1=8.7 for the whole world, m_1=8.5 for California (Housner, 1970), m_1=7.4 for Central United States (M & H Engineering, 1974) have been proposed. Housner (1970) has tentatively suggested the following functional dependence of m_1 on the seismicity parameters in equation (2):

$$m_1 = m_{1_c} \frac{b_c}{b} - \frac{1}{b}\left(a_c - a\right) \quad , \tag{II.9}$$

where m_{1_c}=8.5=magnitude upper bound for California;

(a_c,b_c)=(5.5,0.9)=seismicity parameters for California;

(a,b)=seismicity parameters for a generic region.

The upper bound epicentral intensities: i_1=10 for the New Madrid zone, i_1=9 for the Matcog area (M&H Engineering, 1974), i_1=6.3-8.7 for various sources in the Boston area (Cornell and Merz, 1974), and the "maximum creditable" value i_1=10 for the Mississippi Valley area (Howe and Mann, 1973) have also been proposed.

In the Eastern United States, where regional seismicity is weakly correlated with the known geological structure, and where bursts of activity often alternate with periods of quiescence, the arguments against adopting moderate upper bounds (say, i_1=6-7) are rather convincing (Chinnery and Rogers, 1973; Howell, 1973; Nuttli, 1974; Housner, 1970). In Section V it will be shown that, depending on the resistance characteristics of the system, the mean failure rate λ_f in Equation (I.3) may not be sensitive to i_1. When applicable, this is a most welcome result, due to the large uncertainty on and the open controversy about the upper bound intensity.

For the purpose of seismic risk analysis one needs a measure of site intensity. Throughout this study, such measure is taken to be either Modified Mercalli intensity I, or alternatively, peak ground acceleration, a. Other motion parameters, such as peak ground velocity or displacement could be used instead, without altering the procedure, or the results to any significant degree.

A widely used relationship between site intensity I, epicentral intensity I_o, and epicentral or focal distance R is (see, e.g., Cornell, 1968):

$$I = C_1 + C_2 I_o - C_3 \ln R + \varepsilon \quad , \tag{II.10}$$

where C_1,C_2,C_3 are regional constants, and ε is a random error term. For the Northeastern United States, Cornell and Merz (1974) used a more general attenuation law, of the form:

$$I = \begin{cases} I_o + \varepsilon & , \quad R < R_o \quad ; \\ C_1 + C_2 I_o - C_3 \ln R + \varepsilon & , \quad R \geq R_o \quad ; \end{cases} \qquad \text{(II.11)}$$

with parameters: R_o=10 miles

$C_1 = \begin{cases} 2.6 \text{ for sites with rock foundations} \\ 3.1 \text{ for "average" soil conditions} \end{cases}$

C_2=1.0

C_3=1.3

The standard deviation of the zero-mean, normal error term ε was estimated to be about 0.2 for rock foundation sites and about 0.5 when including all possible soil conditions at the site. The value C_3=1.3 was found to agree quite losely with data from Eastern United States regions.

Due to a higher absorbtion of wave evergy, in the western states intensity attenuates much faster with distance (Algermissen, 1972; Brazee, 1972; Bollinger, 1973); for those regions a value of about 2.0 or 2.5 might be appropriate for the coefficient C_3 in Equation (11).

Given the geometry of the active sources, their geographical location with respect to the site, the mean rate of earthquake occurrences, the spatial distribution of the epicenter, and the probability distribution of the epicentral intensity for each source (the last distribution in the form, say, of Equation 8), one can calculate the frequency-intensity law at the site through repeated application of Equation (11) (see Cornell, 1968, 1971; Cornell and Merz, 1974). For a set of sources with no intensity upper bound, the exponential distribution of epicentral intensity:

$$\Pr\{I_o > i\} = \begin{cases} 1 & , \quad i \leq i_o \quad ; \\ e^{-\beta_{I_o}(i - i_o)} & , \quad i > i_o \quad , \end{cases} \qquad \text{(II.12)}$$

and a deterministic attenuation law (ε=0 in Equation 11), the complementary CDF of the site intensity is:

$$\Pr\{I > i\} \propto e^{-\beta_I (i - i_{o_{SITE}})} \quad , \quad i > i_{o_{SITE}} \quad , \qquad \text{(II.13)}$$

where $$i_{o_{SITE}} = \begin{cases} i_o & , \text{ for } R_{min}<R_o, \\ C_1+i_o-1.3 \ln R_{min} & , \text{ for } R_{min}\geq R_o, \end{cases}$$

R_{min} = minimum distance of the site from the active sources,

β_I = $\beta_{I_o}/C_2 \approx \beta_{I_o}$

C_1, i_o, R_o = constants; same as in Equation (11).

Typical results are shown in Figure 1 (from Cornell and Merz, 1974), where the annual probability that Boston experiences an earthquake of intensity i or more is plotted versus i. This probability is contributed by 8 separate seismic sources located at variable distance from the city of Boston. Each curve corresponds to a different set of parameters values, but in all cases it is β_{I_o}=1.10 and C_2=1; i.e., β_I=1.10. The upper curves, denoted UB12 and RANDOM 12, are for the case of an upper bound epicentral intensity i_1=12 for all sources. The slope of these curves is almost identical with that of equation (13), with β_I=1.10 (see the line between dots in Figure 1). The increase of negative slope at high intensities is due to the upper bound on I_o. The randomness of the attenuation law (a standard deviation σ_ε=0.2 was used in Equation 11) has no appreciable effect on the slope of the risk curve. In obtaining curve CA 12 it was assumed that the upper bound epicentral intensity was 12 for two sources, and was variable in the range 6.3-7.3 for the other 6 sources. The remaining curves result from smaller upper bounds on I_o, this reduction causing a rapid risk drop at smaller levels of site intensity.

Within the intensity range shown, the risk curves in Figure 1 are well approximated either by straight lines with an "effective" slope parameter β_I=1.1 (curves UB12 and RANDOM 12) or β_I=1.77 (curve CA 12), or by truncated straight lines with an effective β_I between 1.70 and 2.00 (remaining curves). These and other "nonlinear" seismic risk models will be studied in Section IV.

Results of a similar kind are shown in Figure 2 (from Liu and Dougherty,1975) for a site at variable distance from the San Andreas fault. For the calculation of the site intensity risk curve, the magnitude distribution (3) was used, with parameters m_o=4.5, $m_1=\infty$, β=0.87 ln 10, and a mean occurrence rate over the entire fault length (644 Km) of 6.33 events/year. The attenuation law, expressed in terms of magnitude and focal distance was taken to be:

$$I = C_1 + C_2 M - C_3 \ln R$$
$$= 8.16 + 1.45\,M - 2.46 \ln R \ .$$

Again, a linear relationship between $\log_{10}$ risk and I, with slope $-b/C_2=-0.6$ (slope of the line between dots in Figure 2) provides a good approximation to the risk curves for all but very small site intensities.

A parameter which is often used as a measure of seismic demand at the site is peak ground acceleration. Empirical relationships have been established among a, M and R, and between a and I, so that seismic risk curves in terms of a can be evaluated (in approximation) either from known frequency-magnitude relations, or from site intensity risk curves. According to the best information presently available (Esteva, 1970,1974; Esteva and Villaverde, 1973; Donovan, 1973,1974; see also Newmark, 1974) the model

$$a = b_1 e^{b_2 M} \left[L(R)\right]^{-b_3} \qquad \text{(II.14)}$$

is in satisfactory agreement with the empirical data if, for a in g's and L(R)=a linear function of focal distance in Km, the parameters b_i are given the values in Table 4. (Formally identical relationships have been suggested for peak ground velocity.)

Several proposed relations between log acceleration and MM intensity are shown in Figures 3 and 4. The solid line in Figure 4 used the most extensive set of data.

Due to the approximate linearity of ln a in M (equation 14) and in I (Figures 3 and 4), the considerations about the exponential decay of the site intensity distribution (Equation 13 and related comments) hold also for ln a, after replacing C_2 by b_2. For example, for a set of seismic sources with magnitude distribution (1), the probability that the peak ground acceleration a is exceeded during any one event is proportional to $\exp\{-\ln a \cdot \beta/b_2\}$.

In this study, both seismic demand and seismic resistance are characterized in terms of MM intensity. For design, however, it is desirable to measure intensity through actual characteristics of the motion, such as peak ground acceleration. The relationships sketched in Figures 3 and 4 were fit to very dispersed data (see, e.g., Newmark, 1974, and Ambraseys, 1974). How to account for this dispersion when passing from MMI to ln a is not clear: simply "adding" it to the variability of MM intensity (say, in the attenuation law) generates very large ln a uncertainties.

What is more important, is that there are ways to calculate risk in terms of peak ground acceleration which are more efficient, in the sense of producing less dispersed results. One such way is to first convert epicentral intensities into magnitudes (the empirical relationships in Table 2 show little dispersion; see, e.g., Chinnery and Rogers, 1973), and then use an attenuation law giving acceleration as a (random) function of magnitude and distance. The plots in Figures 3 and 4 should therefore be regarded as best estimates of ln a given MM site intensity, not as functional relationships, and caution should be exercised in their use.

II.2 Uncertainty on the Seismic Resistance

It is generally believed that the uncertainty in the seismic resistance of engineering facilities contributes marginally to the overall risk (Ferry Borges, 1956; Rosenblueth, 1964; Vanmarcke and Cornell, 1969), and that even a seismic risk model with deterministic resistance produces valuable results. The present study reaches different conclusions, particularly when the analysis includes statistical uncertainties. It appears, in fact, that a substantial fraction of total risk may come from moderate intensity earthquakes which, although associated individually with small failure probabilities, are much more frequent than large and statistically more destructive events.

Three different approaches have been pursued to estimate the probability distribution of system damage (this includes the probability of "failure," if failure is defined as a particular damage state) for given seismic intensity: (a) random vibration theory; (b) simulation of artificial ground motions and repeated deterministic analysis of the system's response; (c) direct analysis of damage statistics from past earthquakes. The main advantages and limitation of each approach are:

(a) Random vibration analysis generally requires simple models, both of the ground motion (e.g., a pseudo-stationary Gaussian process) and of the system (e.g., linear elastic, with known parameters). Apart from these limitations, random vibration techniques are most powerful, in that they characterize the system's response as a random process, from which the probabilities of reaching various damage states can be calculated (approximately); see, e.g., Vanmarcke, (1969) and Cornell (1971). Unfortunately, most structural systems become highly nonlinear near collapse, or even after moderate damage. In addition, if the size of the earthquake is known in terms of MM intensity or of peak acceleration, it is not easy to relate these parameters to a random ground motion process.

(b) Simulation methods (see, among others, Housner and Jennings, 1965; Hou, 1968) do not impose such strict limitations on the input and system models; however, by their very nature, they generate information on low probability events at prohibitive computational costs. Simulation methods also become impractical if the behavior of the system is itself uncertain.

(c) In recent years, much has been learned from the analysis of actual damage statistics; although data on severe damage probabilities are still scarce for some categories of buildings, information is becoming available at an (unfortunately) high rate. Lack of statistically relevant data is indeed the major limitation of this approach. Advantages over (a) and (b) are that no assumption is made on the seismic load or on the system behavior, and that direct correlations are obtained between intensity parameters (say, I or a), and damage.

In this study, the damage-statistics approach (c) is followed, with consideration both of the estimated damage probabilities, and of the uncertainty on such estimates due to limited data processing. Information and models of seismic damage are reviewed in the remainder of this section.

Much information can be found in recent literature on the Mean Damage Ratio (MDR=expected repair cost over total property value) for various categories of buildings, exposed to ground motions of given intensity. Mean damage ratio functions (of MMI) have also been fitted to the data, or estimated subjectively. Although the damage statistics for some building categories (such as wooden frame and masonry constructions) are of less direct interest to this study,they are also reviewed briefly, since they provide further insight into the general dependence of seismic damage on intensity and on seismic design.

Figure 5 (adapted from Mann, 1974) summarizes the information available on wooden frame dwellings. The solid curve was proposed by Steinbrugge, McClure and Snow (1969), as a result of a very extensive effort which combined field data, past experience and subjective judgement. The damage values suggested by Friedman and Roy (1969) are also judgemental; they were estimated by extrapolating data on dwellings' damage from the 1957 San Francisco earthquake, the 1952 Kern County earthquake and the 1933 Long Beach earthquake. These data are not strictly comparable with the remaining data points in Figure 5, since they make no distinction between types of dwelling construction (e.g., frame versus brick), or existance of chimney. For wooden frame dwellings and in the intensity range 5 to 8, the MDR varies by a factor of approximately 4 per unit of intensity.

Data for ordinary and for reinforced masonry construction are summarized in Figure 6 (from Mann, 1974). In this case the MDR for a given intensity is very sensitive to the quality of construction and to the use of reinforcement or not. Apart from the rapid decay of the expected damage at low intensity levels, the dependence of MDR on I is approximately exponential (as for wooden frame construction), now with a factor of about 3 per unit of intensity for ordinary masonry, and of about 2.75 per unit of intensity for reinforced masonry. The MDR for weak masonry is from 3 to 10 times the MDR for high quality masonry, depending on the ground motion intensity. At high intensity levels, reinforcement has the effect of reducing the mean damage ratio by a factor or approximately 5.

Damage statistics for high-rise buildings with steel-framed, concrete-framed, and concrete-shear-wall structural systems have become available only in the very recent past. Reliable information was collected after the 1971 San Fernando earthquake (Steinbrugge et al, 1971; Whitman et al, 1973 a,b; Whitman, 1973). The most extensive of these surveys (Whitman, 1973) documented 368 buildings with 5 stories or more, calssified by age, by structural material, and by height. Most of these buildings experienced a motion of intensity 7. At that intensity, old (pre-1933) buildings, designed under no seismic requirement, experienced a MDR about 2% greater than recent (post-1947) construction, designed for the Uniform Building Code seismic zone 3 (UBC 3). On the average, steel frame buildings performed better than concrete-structured buildings. Figure 7 (from Whitman, 1973) displays MDR data for high-rise buildings from the San Fernando as well as from other earthquakes. While data are differentiated by UBC zone, all heights and all types of construction (steel and concrete) are lumped together. The data denoted "Japan" are from the 1968 Higashi-Matsuyama and from the 1968 Tokachi-Oki earthquakes and refer to buildings designed for lateral forces about 2 to 3 times greater than those for UBC zone 3.

Based in part on these empirical data, curves relating the MDR of high-rise buildings to MM intensity have been proposed by several authors. Figure 8 (from Whitman, 1973) shows mean damage ratio functions evaluated subjectively (by S.B.Barnes and Associates, Los Angeles) for 13-story concrete frame buildings designed in compliance with various UBC zones, and for a "Superzone" S, with twice the lateral force required for zone 3. Similar subjective estimates have been made for other structural systems. Figure 9 (also from Whitman, 1973) compares estimates for Concrete Shear Wall (CSW), Concrete Moment-Resisting Frame (CMF), Steel Moment-Resisting Frame (SMF), and Steel Braced Frame (SBF) structural systems. By combining these subjective estimates with empirical data on high-rise buildings, Whitman (1973) suggested the mean damage ratios shown in Figure 10 (solid lines) as applicable to the population of constructions mentioned above.

For high-rise buildings (5-stories or more) in Los Angeles, Whitman and Hong (1973) proposed the dashed lines in Figure 10 (the dotted continuations are extrapolations beyond the available data).

In the analysis of data from the 1971 San Fernando earthquake, Benjamin (1974) found no statistically significant difference between the mean damage ratios of high-rise reinforced concrete and steel constructions. He also observed that log MDR is approximately linear in MMI, and suggested the straight lines (a) and (b) in Figure 10 as probable bounds to the actual log MDR-I relationship.

The degree of correlation between "aseismic" design provisions and effective damage protection is rather controversial. In some cases (see, e.g., McMahon, 1974, for damage to high-rise buildings during the 1972 Managua earthquake; and Pique, 1975, for damage statistics from the 1974 Lima earthquake), comparable mean damage ratios were found for buildings designed for different UBC zones. However, the probability of high damage and collapse were notably and consistently reduced by seismic protection, particularly in shear-wall constructions.

At the other extreme, cases were reported (e.g., Hong and Reed, 1972, on the 1965 Puget Sound, Washington, earthquake) where aseismic protection was apparently very effective. The same conclusions were arrived at by Mann (1974), who compared the performance of skeleton framed buildings designed for UBC zones 0 and 3, during various earthquakes (see Figure 11, where Class A refers to steel and Class B to reinforced concrete constructions).

Evident, but not so extreme, beneficial effects of aseismic design were found by Whitman (1973) for high-rise buildings (see Figures 8 and 10), and by Crumlish and Wirth (1967) for school buildings in California and in Washington.

In all cases, as Whitman (1973) suggested, greater benefits are expected in stiff buildings, if the increased design lateral force does not impare severely the ductility of the system, and if the seismic resistances of various portions of the structure are comparable. Similarly, Newmark (1974) pointed out that construction details, selection of materials, placement of reinforcement and of stiffeners, quality control of welds and connections, more than the general compliance with aseismic provisions are essential to reach high ductility factors and therefore to resist strong ground motions.

Some information is available also on the conditional CDF of damage, $F_{D|I}(d|i)$, i.e. the function which is used in Equation (I.2).

Benjamin (1974) found that for broad classes of buildings (ranging from wooden frame dwellings, to light industrial constructions, to high-rise buildings) the damage data for given intensity fit well both lognormal and gamma distributions. If a lognormal model is used, then (log MDR|I) has normal distribution. Benjamin also found that the variance of (Log MDR|I) is approximately constant with I. For light industrial buildings he estimated:

$\sigma_{\text{Log MDR}|I} = 0.295$ for model (a) in Figure 10;

$\sigma_{\text{Log MDR}|I} = 0.225$ for model (b) in Figure 10.

The damage statistics reported by Whitman (1973) also indicate that $\sigma_{\text{Log MDR}|I}$ is not sensitive to I; the same statistics are consistent with a normal distribution of (Log MDR|I).

As indicated previously, the log mean damage ratio varies almost linearly with the MM intensity. For the developments in Sections IV and V it is not the absolute value of $\sigma_{\text{Log MDR}|I}$ which has importance, but the ratio

$$\beta_D = \frac{b_D}{\sigma_{\text{Log MDR}|I}} , \qquad \text{(II.15)}$$

where b_D is the slope of the linear relationship:

$$E\left[\text{Log MDR}\,|\,I\right] = a_D + b_D I . \qquad \text{(II.16)}$$

Table 5 collects some statistics and some subjective evaluations of the parameters b_D and β_D. In a strict sense, the values of b_D and β_D from Newmark (1974) and Vanmarcke (1971) cannot be compared with those from Benjamin (1974) and Whitman (1973), because they refer to given peak ground acceleration a, instead of MMI. Newmark suggested values of $\sigma_{\ln(\text{response})|a}$ for ordinary buildings and for nuclear reactor structures and equipment (parameter BETA in his Table 3). If the level of response is proportional to a, and a varies by a factor 2 per unit of MMI as suggested by Figure 4 (but see earlier comments on Figures 3 and 4), the response varies also by a factor 2 per unit of intensity; so that one can estimate β_D in equation (15) as:

$$\beta_D \approx \frac{\ln 2}{\sigma_{\ln(\text{response})|a}} .$$

This relationship was used to calculate the β_D values in Table 5 from Newmark's estimates of $\sigma_{\ln(\text{response})|a}$.

The estimates of b_D and β_D from Vanmarcke (1971) were found as follows. If E[log MDR|I] is linear in I (see Equation 16) and if ln a has functional relationship with I (again, see Figure 4 and related comments):

$$\ln a = I \cdot \ln 2 - 7.3 \ , \qquad \text{(II.17)}$$

then E[log MDR|a] is linear in ln a, say:

$$E\left[\text{Log MDR} \mid a\right] = a_{D,a} + b_{D,a} \ln a$$
$$= a_D + \ln 2 \cdot b_{D,a} \cdot I \ ,$$

whence:

$$b_D = b_{D,a} \cdot \ln 2 \ . \qquad \text{(II.18)}$$

It is also:

$$\sigma_{\text{Log MDR}|I} = \sigma_{\text{Log MDR}|\ln a} \ , \qquad \text{(II.19)}$$

where ln a is given by Equation (17). Given b_D, a and $\sigma_{\text{Log MDR}|\ln a}$ — estimates of these parameters can be obtained from the data in Vanmarcke (1971) , — b_D and β_D can be calculated from Equations (18), (19) and (15).

A critical question is how all this information on the damage statistics and on the resistance distribution of ordinary buildings relates to the behavior of special constructions or of new structural typologies. The problem arises, for example, in the seismic risk analysis of nuclear power plants, to which the following considerations are primarily addressed. Statistical data on seismic damage to nuclear power facilities are practically missing, so that the procedure of extracting information from historical records no longer applies. Also the analytical approaches (say, of the random vibration type), which were found somewhat inaccurate for damage prediction of ordinary buildings (Whitman,1973), encounter major difficulties here, due to the complexity of nuclear reactor systems, to the sequentiality of accidental events leading to "failures," to the built-in redundancy, and to the different levels of resistance of various subsystems and components.

Nevertheless, some general conclusions can be drawn on the seismic frequency of specific initiating events. In fact, for each initiating event a single subsystem or component is involved directly, and some damage characteristics of ordinary structures can be assumed to hold, at least qualitattively (e.g., the approximate linearity of the expected log "damage" as a function of MMI). From Table 5, a range of values for β_D in Equation (15) can be established (the values from Newmark were suggested specifically for nuclear reactor structures and equipment). The question remains to be answered, what is a reasonable value for the expected subsystem or component damage at a given MMI (this would determine the parameter a_D in Equation 16), and what damage level d_f should be associated with "accident initiation." In the context of the risk model introduced in Section IV, the last two questions reduce to a single question; for example, what is the seismic intensity at which there is 50% change of accident initiation? Newmark (1974) estimated that at the design value of peak ground acceleration the ratio

$$\frac{\ln(\text{response at failure}) - E[\ln(\text{response})]}{\sigma_{\ln(\text{response})}}$$

for nuclear power plant structures and equipment exceeds by about 0.63 and 0.66, respectively, the same ratio for ordinary buildings designed for UBC zone 3. This indication will be used in Section VI to relate the seismic risk of ordinary buildings to the seismic risk of reactor structures and equipment.

SEISMIC REGION	β	COMMENTS
* Southern New England (Chinnery and Rogers,1973)	2.19(±0.12)	1800-1959; 135 events
* New Jersey (Isacks and Oliver, 1964)	2.17	
* Central Mississippi River Valley (Nuttli, 1974)	2.00(±0.25)	1833-1972; 250,000 Km^2
* North and Central America (Shlien and Toksöz, 1970)	2.26	1963-1968
* Southern California (Albee and Smith, 1967)	1.94	1934-1963; 10,126 events; 296,000 Km^2
* California (Housner, 1970)	2.07	
* Various Parts of the World (Evernden, 1970; Esteva, 1968; Ferry Borges and Castaheta, 1971)	1.61-2.88	
* World		
(Gutenberg and Richter,1941)	2.30	
(Housner, 1970)	2.07	1904-1946

Table II.1

Values of β in Equations (II.1) and (II.3) for Different Seismic Regions

SEISMIC REGION	$M = a_1 + a_2 I_o$
* Southern California (Gutenberg and Richter,1956)	$M = 1 + \frac{2}{3} I_o$
* Southern New England (Chinnery and Rogers, 1973)	$M = 1.2 + 0.6\ I_o$
* Eastern United States - shallow eqs. (Howell, 1973)	$M = 1.3 + 0.6\ I_o$
* Washington and Oregon (Algermissen, 1969)	$M = 0.82 + 0.69\ I_o$
* ——— (Algermissen et al, 1969)	$M = 1.14 + 0.62\ I_o$

Table II.2

Proposed Relationships Between

Magnitude and Epicentral Intensity

SEISMIC REGION	α_o	β_{I_o}
* Southern New England (same for Boston area, southern New Hampshire and Hartford area; Chinnery and Rogers, 1973)		1.31 (±0.07)
* Southeastern United States (Southern Appalachian, Central Virginia and South Carolina-Georgia zones; Bollinger, 1973)	6.93	1.36
* Northeast United States 1928-1967 (Cornell and Merz, 1974)		1.05
* Boston area 1630-1970 (Cornell and Merz, 1974)	3.62	1.10
* New Madrid Zone 1870-1970 (M&H Engineering, 1974)	7.64	1.43
* Matcog Area 1870-1970 (M&H Engineering, 1974)	5.02	1.34
* Mississippi Valley (McClain and Myers, 1970)	3.41	0.93
* Mississippi Valley-St.Lawrence (Algermissen, 1969)	6.24	1.17
* Central United States (Liu and Fagel, 1972)	4.49	1.15
* California (Algermissen, 1969)	9.03	1.24
* World 1534-1974 (Cornell and Merz, 1974)		1.35

Table II.3

Parameters of the Linear Frequency-Epicentral Intensity Law:

$$\ln \lambda_i = \alpha_o - \beta_{I_o} i$$

λ_i = mean annual rate of events in the entire seismic region with epicentral intensity in excess of i

	b_1	b_2	b_3
Esteva (1970)	1.26	0.80	2.00
Donovan(1973)	1.35	0.58	1.52
Donovan(1974)	1.10	0.50	1.32

Table II.4 Coefficients b_i in the acceleration-magnitude-focal distance relation (II.14)

		b_D (**)	β_D
Benjamin (1974)	model (a) in Fig.10	0.484	1.64
	model (b) in Fig.10	0.347	1.54
Whitman (1973)	Post-1947 Buildings		
	San Fernando, I=6	≈ 1.13	≈ 1.88
	San Fernando, I=7	≈ 1.13	≈ 2.13
	San Fernando, I=7.5	≈ 0.91	≈ 1.90
Newmark (1974) (*)			
	Nuclear Reactor Structure		1.33
	Nuclear Reactor Equipment		1.16
Vanmarcke(1971) (*)			
	I≈6.7	≈ 0.68	≈ 1.24
	I≈7.8	≈ 0.59	≈ 1.53

Table II.5 Values of b_D and β_D in equations (II.15) and (II.16)

(*) These values were not obtained from equations (II.15) and (II.16); see explanation in the text.

(**) More than one value of b_D is given for proposed nonlinear functions $E[\log MDR|I]$. The values correspond to local linearization around the indicated MM intensity.

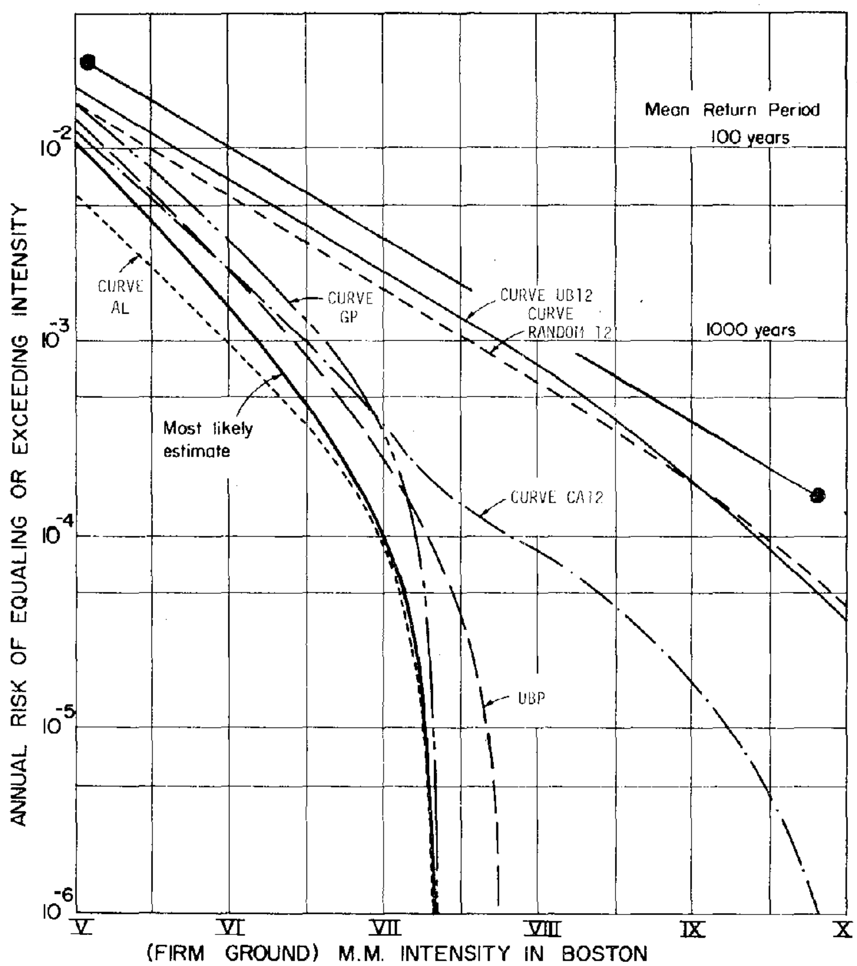


Figure II.1

Influence of Different Assumptions on the Seismic Risk In Boston

(from Cornell and Merz, 1974)

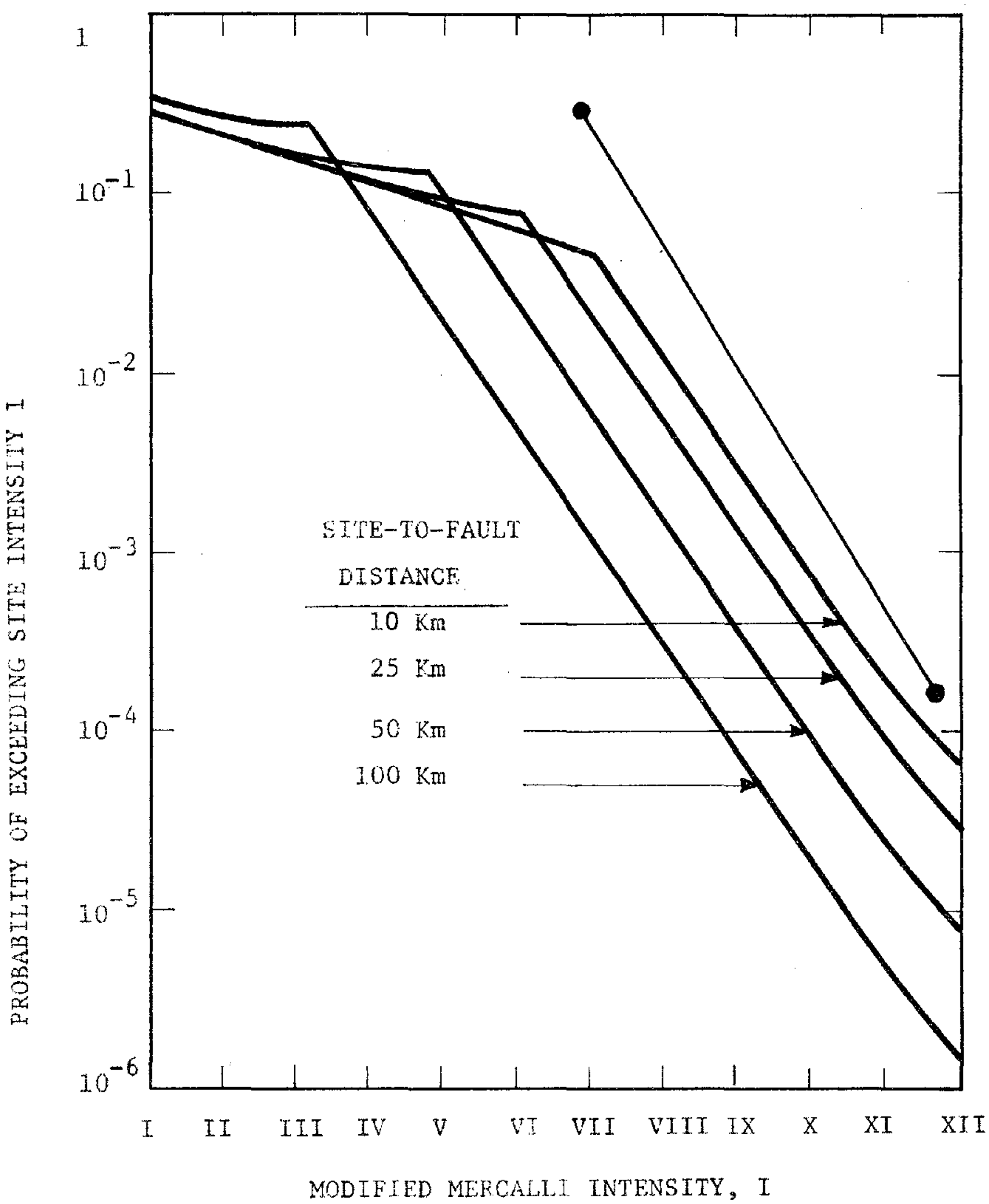


Figure II.2

Probability of Exceeding Site Intensity I
When an Earthquake Occurs Along the San Andreas Fault
(from Liu and Dougherty, 1975)

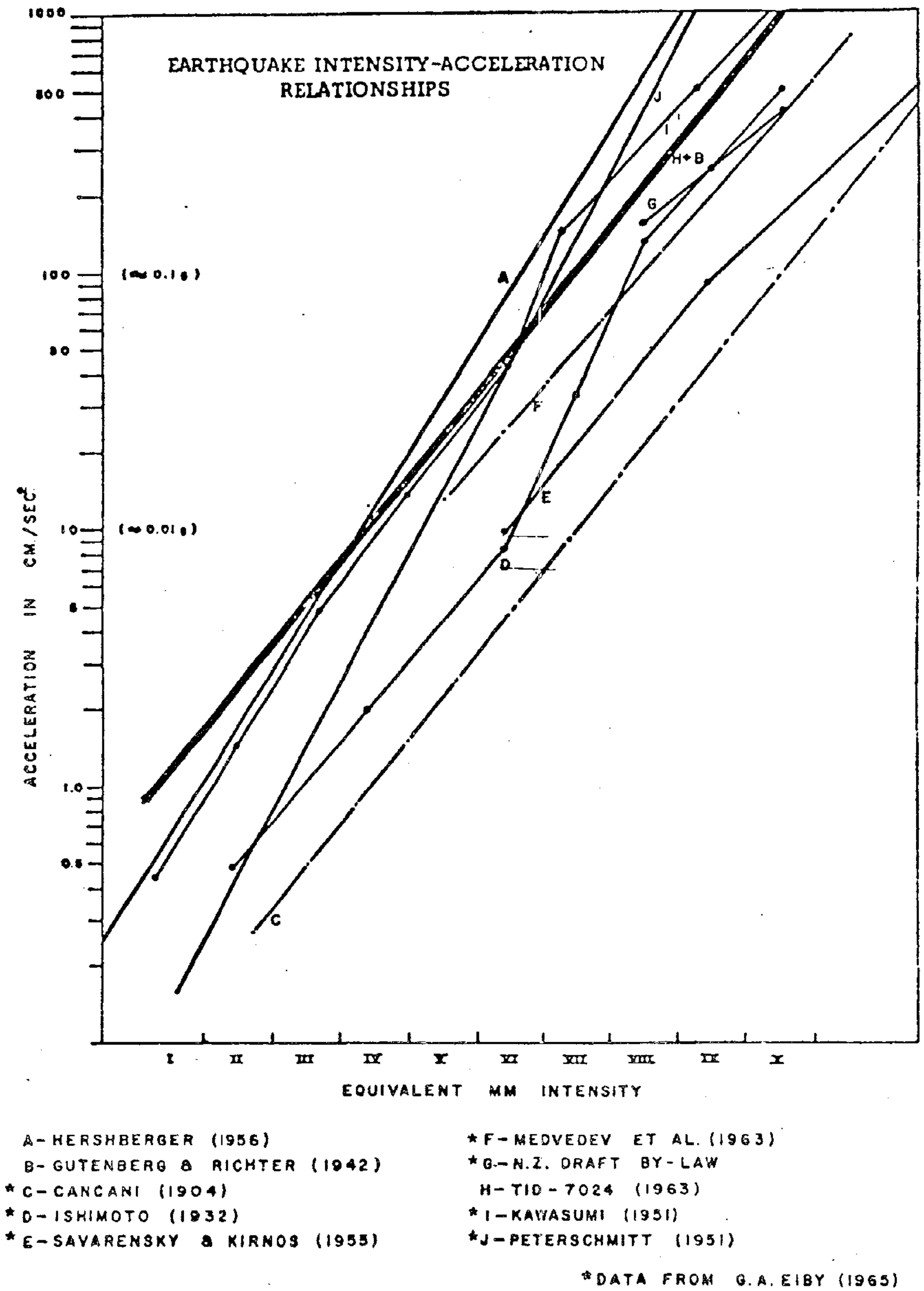


Figure II.3 Relationships between MM Intensity and Peak Ground Acceleration

(from Linehan, 1970)

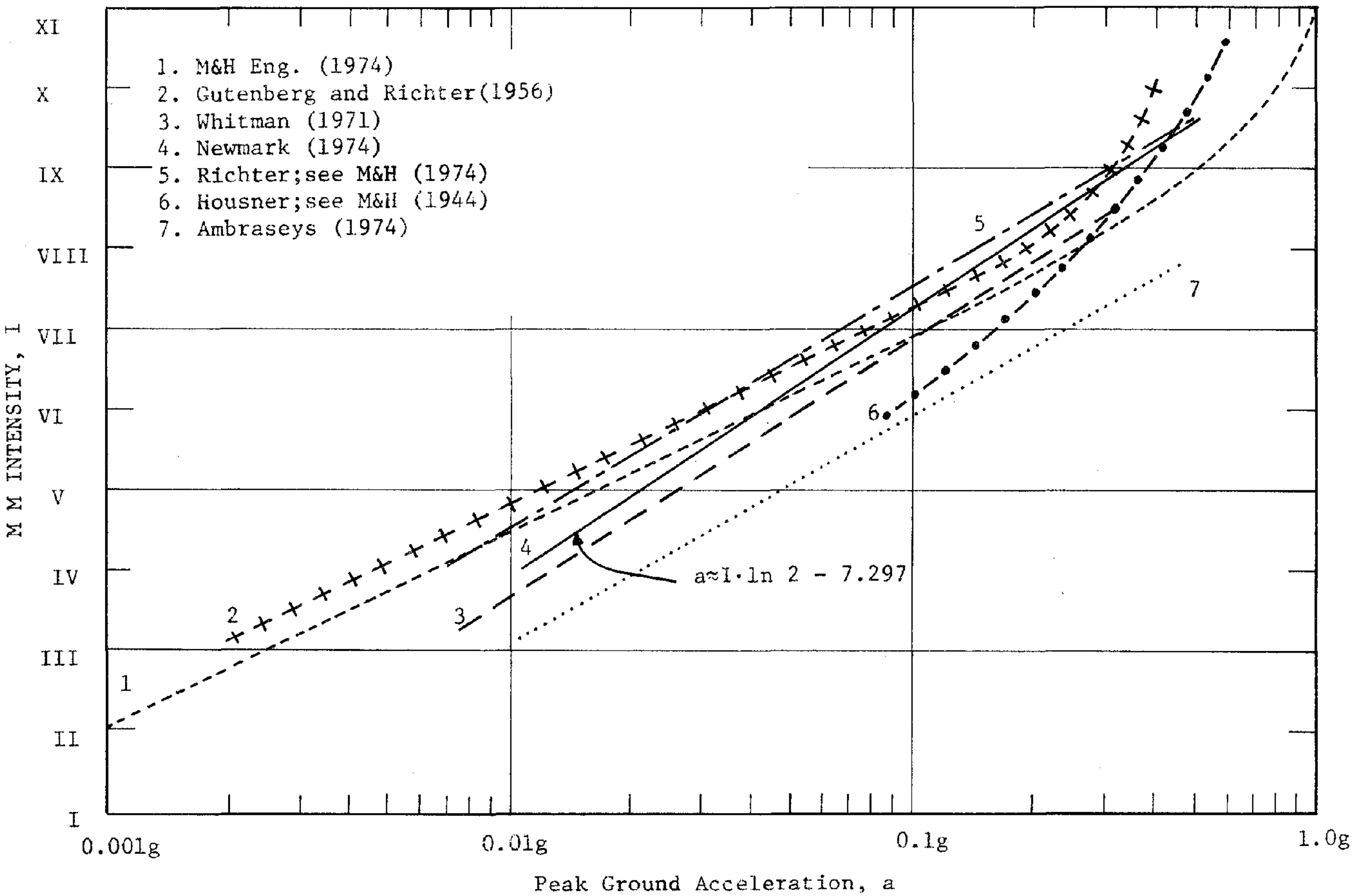


Figure II.4 Proposed Relationships Between MMI and Peak Ground Acceleration.
Curve 7 used European Data

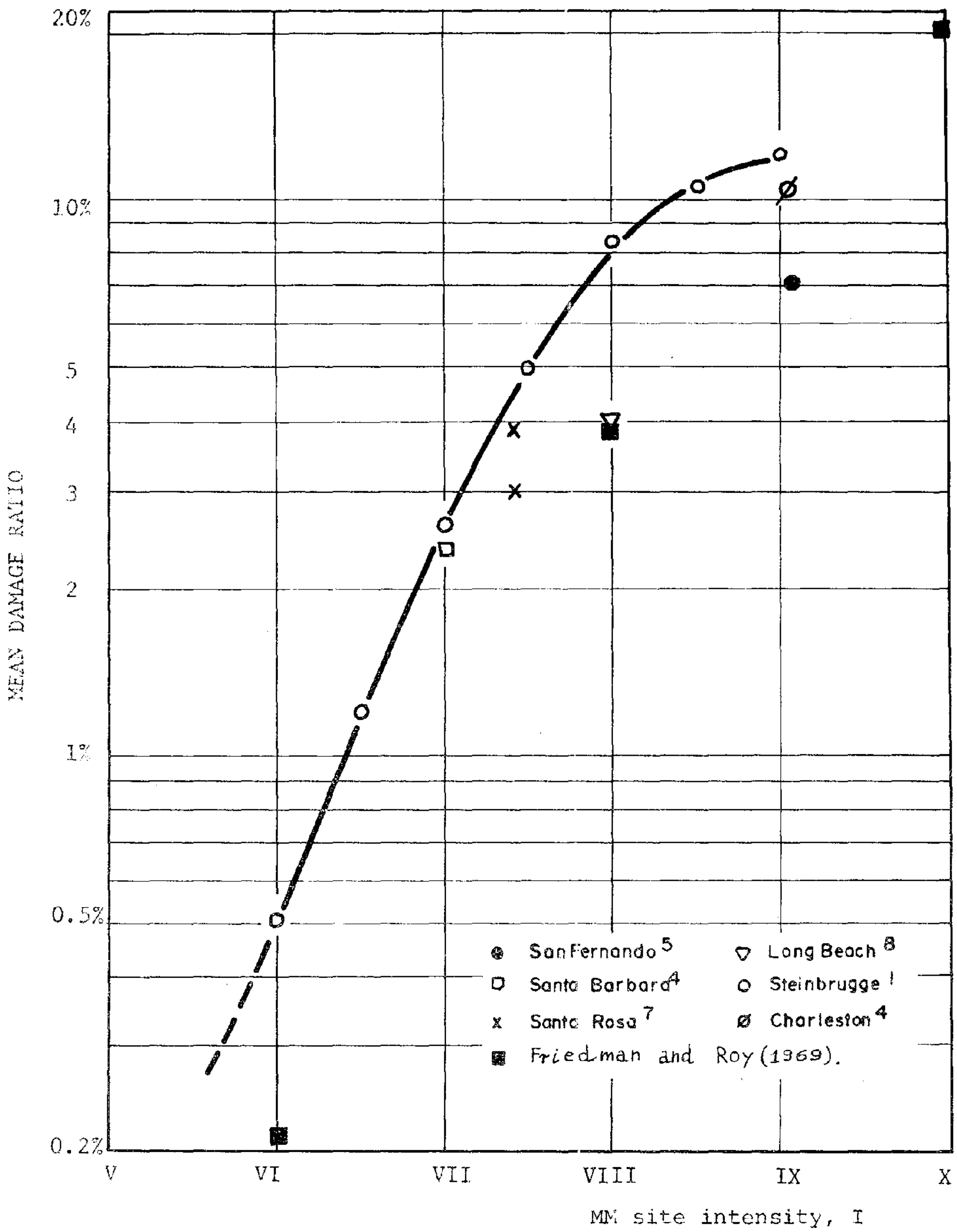
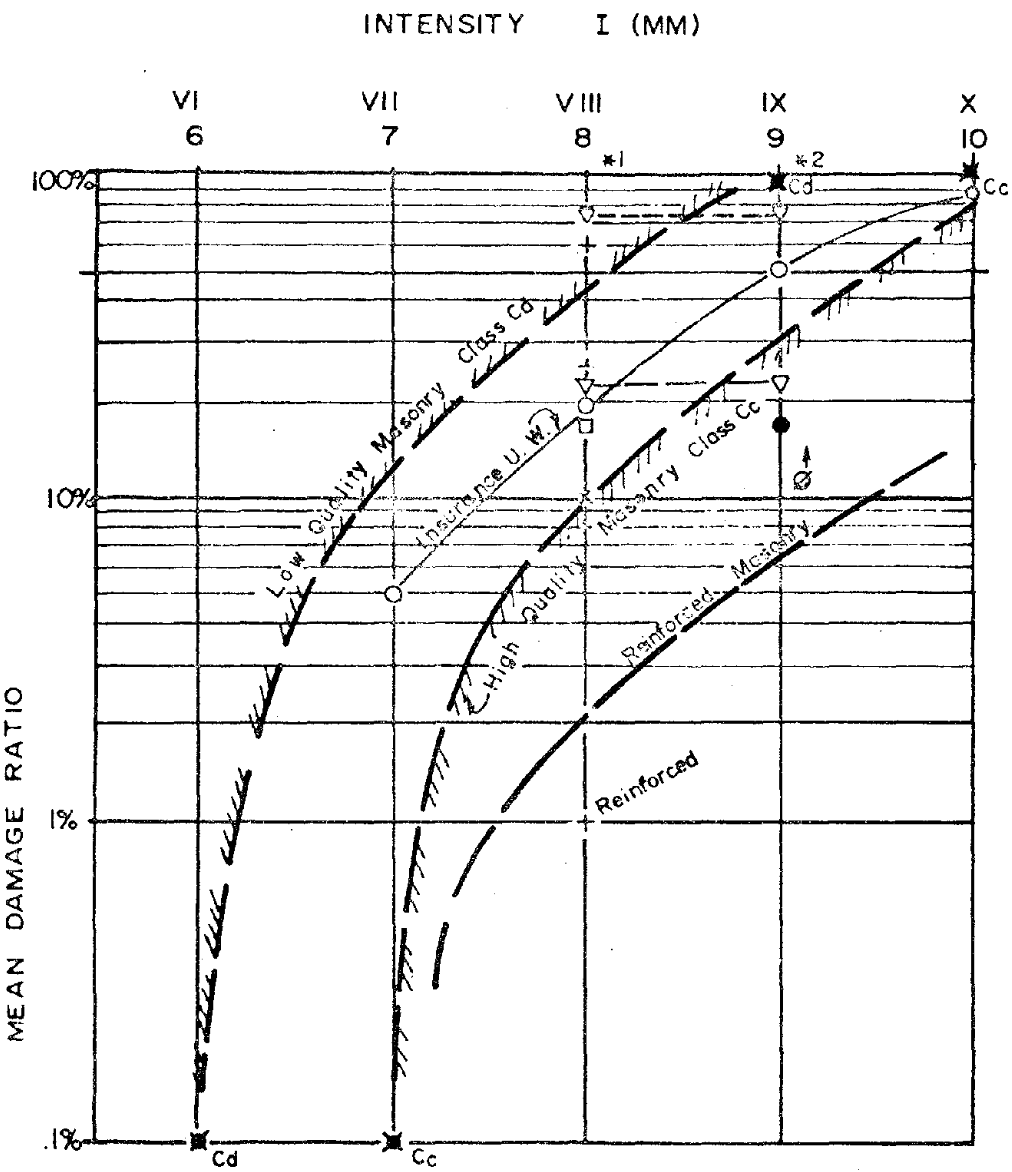


Figure II.5 Mean Damage Ratio versus Intensity for Wooden Frame Dwellings

(from Mann, 1974)

The solid curve is after Steinbrugge et al (1969).



REFERENCE EARTHQUAKES

- O Insuranance Underwriter
- ✱ Richter [9]
- ▽ Long Beach - Martel [8]
- ▽ Long Beach - Hodgson (Schools) [10]
- □ Santa Barbara - Freeman [4]
- + Kerns Co. - Steinbrugge - Moran [6]
- × Santa Rosa - Steinbrugge [7]
- ● San Fernando - Steinbrugge [5]
- Ø Charleston - Freeman [4]

*1 Occasional Collapse's

*2 Collapse of weaker structures and serious of better masonry

Cc, Cd } Types of masonry as described by Richter.

Figure II.6 Mean Damage Ratio versus Intensity for Ordinary and Reinforced Masonry Constructions

(after Mann, 1974)

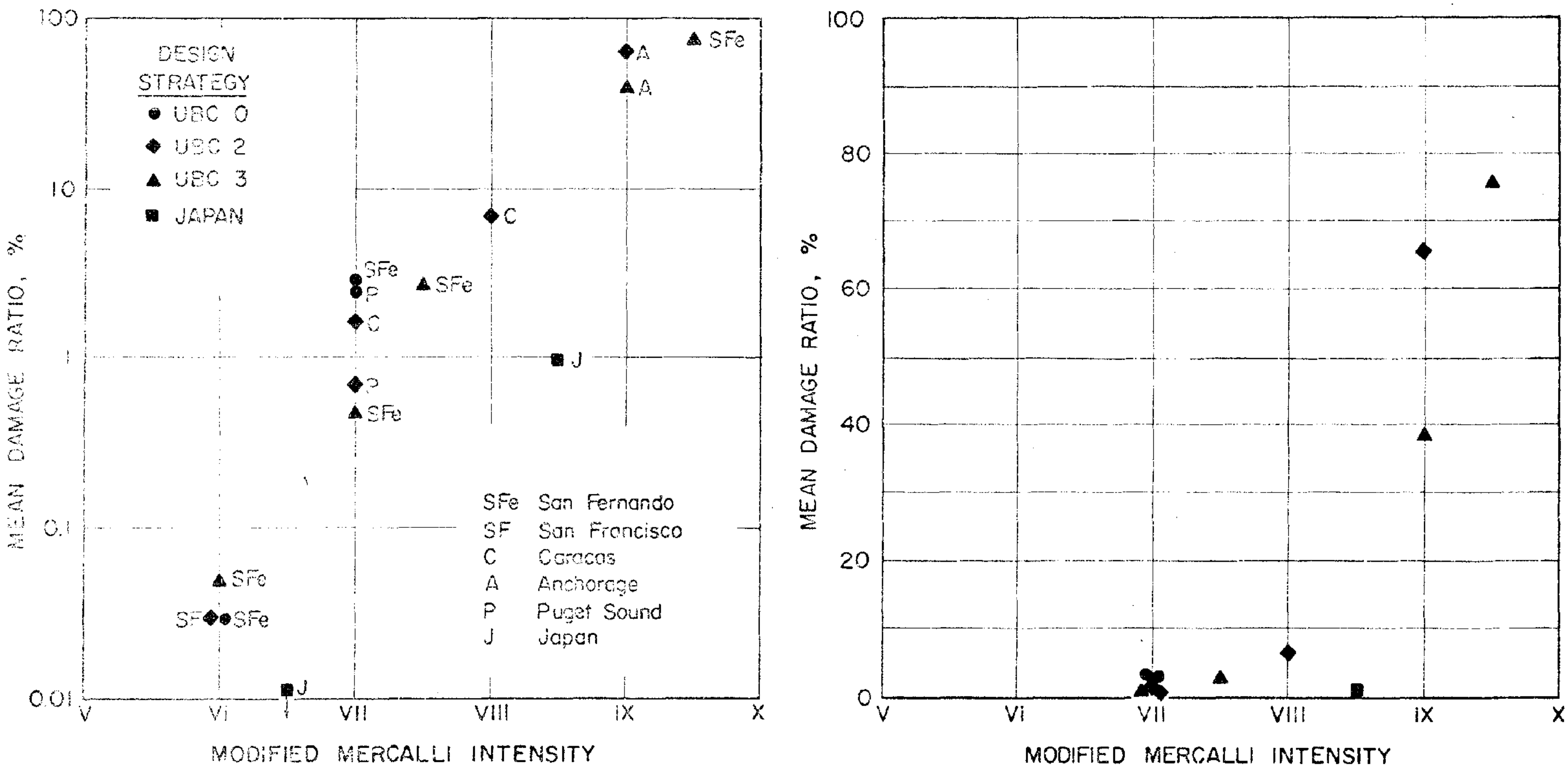


Figure II.7 Data on the Mean Damage Ratio for High-Rise Buildings
(after Whitman, 1973)

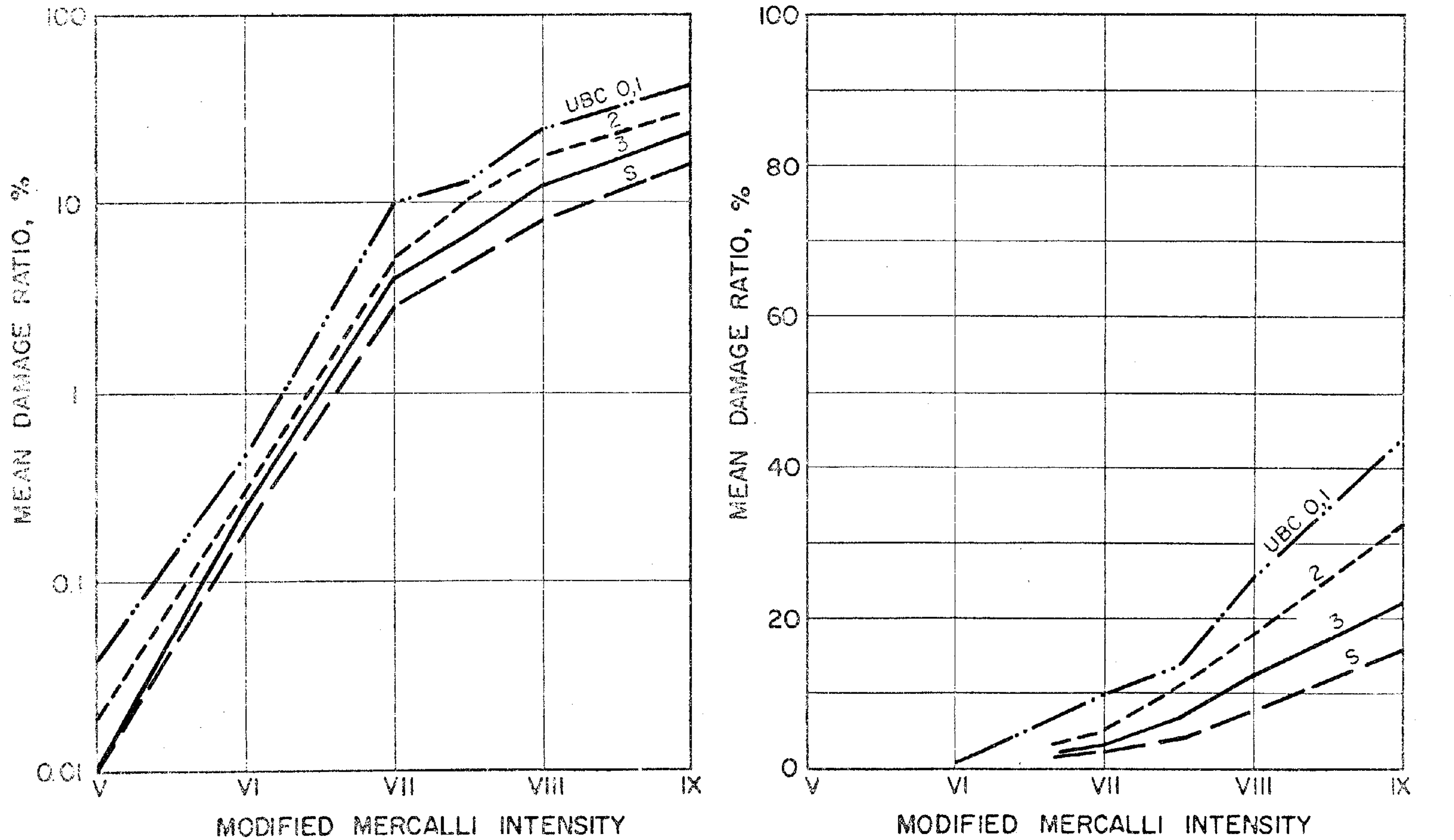


Figure II.8 Subjective Mean Damage Ratios
for High-Rise Concrete Frame Buildings (after Whitman,1973)

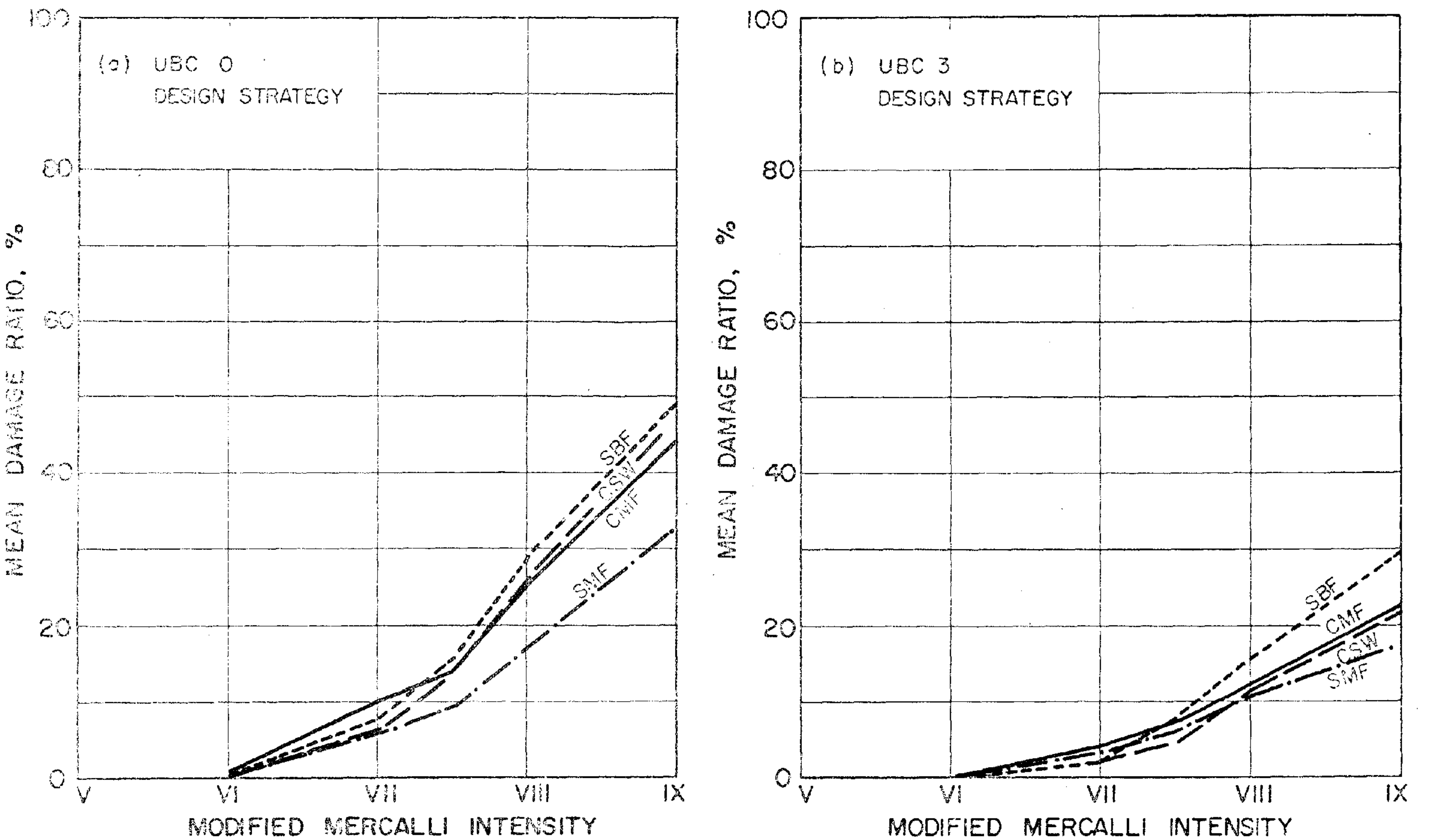


Figure II.9 Subjective Mean Damage Ratios for High-Rise Buildings with Different Structural Systems (after Whitman, 1973)

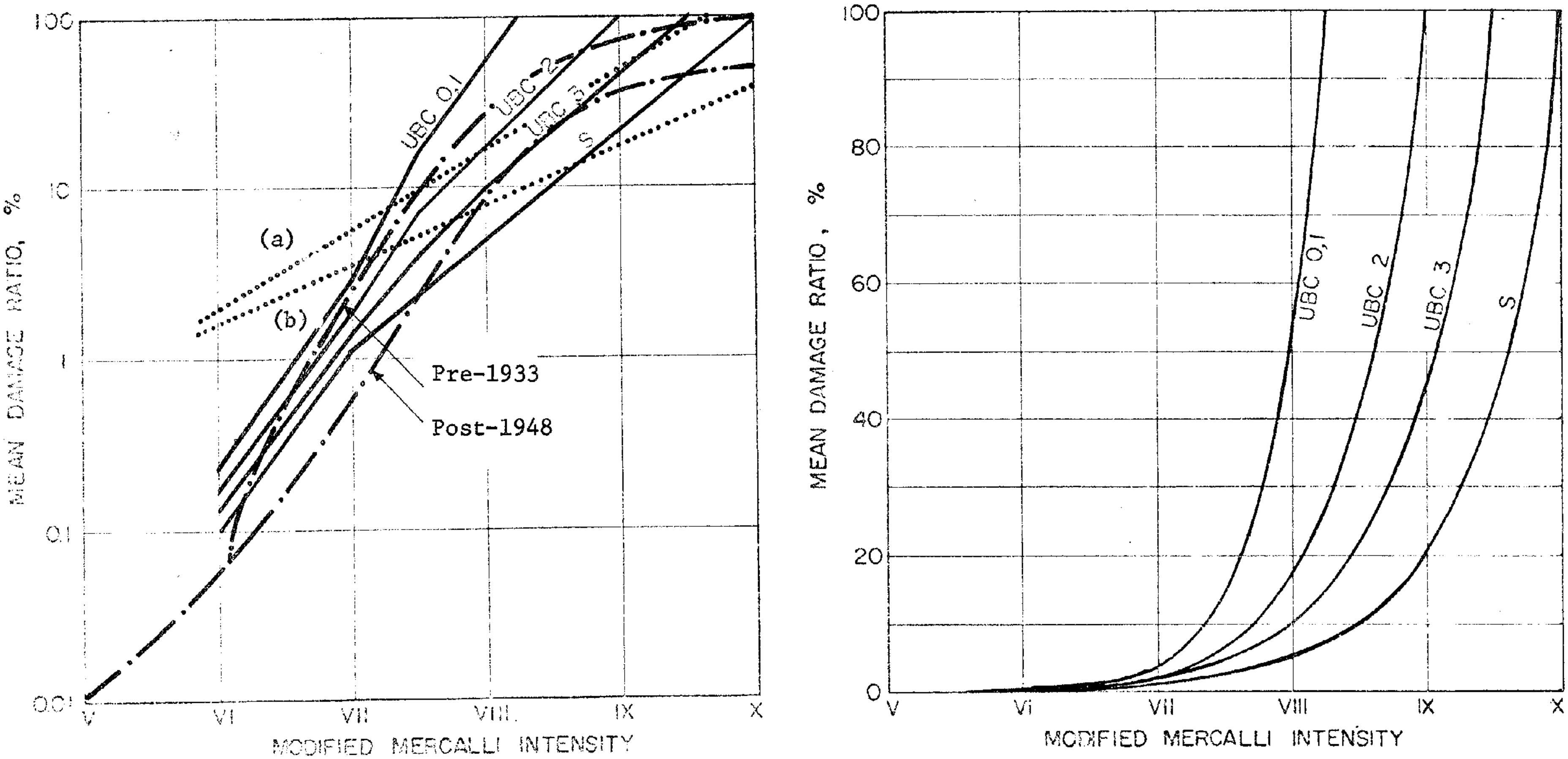
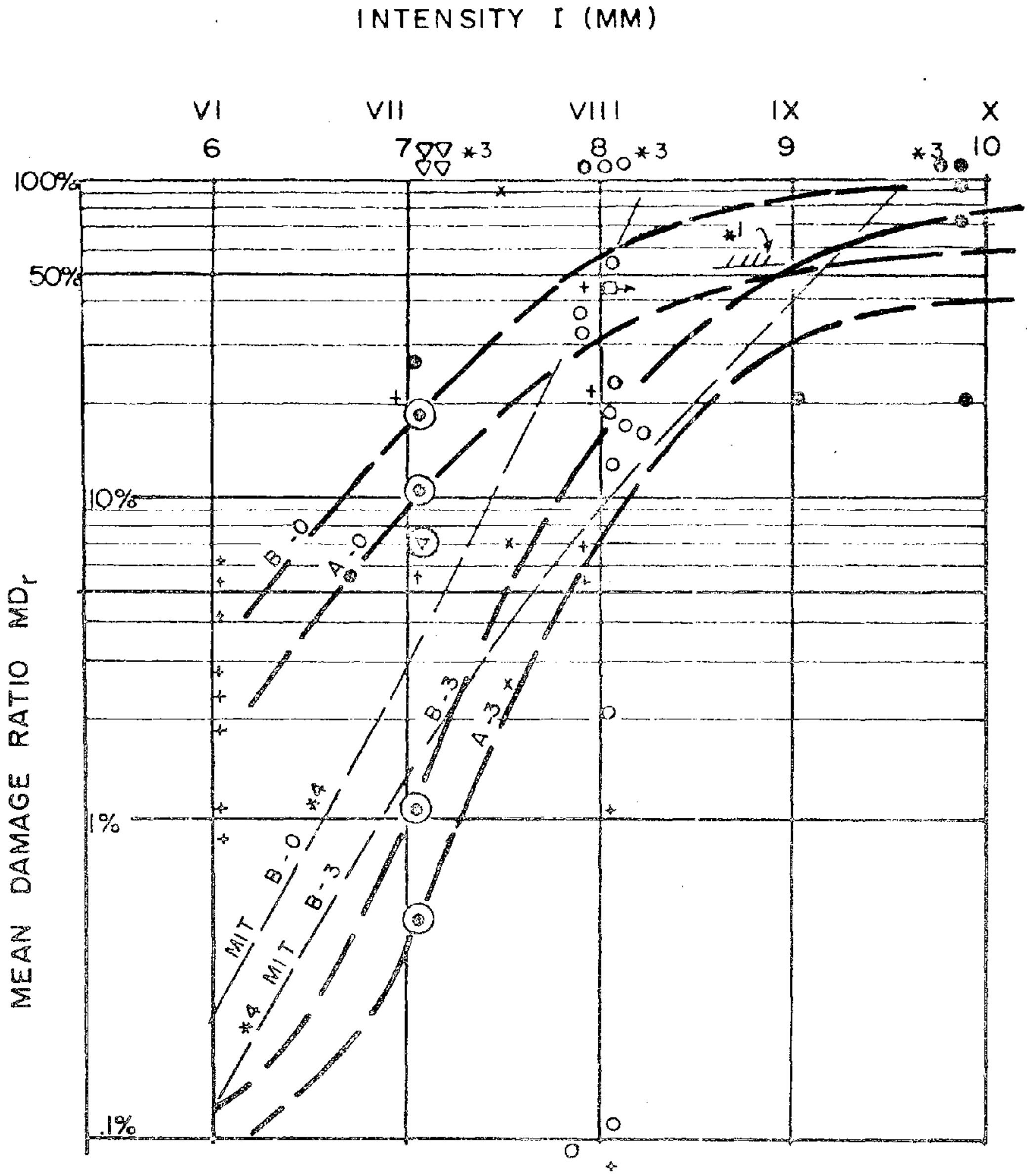


Figure II.10 Solid Lines: Proposed Mean Damage Ratios for High-Rise Buildings Designed for Different UBC Zones (after Whitman, 1973);

-Dashed Lines: Mean Damage Ratios for High-Rise Buildings in Los Angeles (after Whitman and Hong, 1973);

-Dotted lines: Bounds to the Mean Damage Ratios for Light Industrial Constructions (after Benjamin, 1974)



REFERENCE QUAKES

- ⊛ San Fernando
- O Anchorage
- + Kerns
- ☐ Santa Barbara
- X Santa Rosa
- ▽ Caracas
- ////*1 San Francisco

*1 No reports have been found of major skeleton frame collapse during the 1906 San Francisco or 1923 Toyko quakes.

*2 Designed for Zone 3.

⊛ ▽ Average damage ratios

B-O Class B – with no earthquake loading.

A-3 Class A – with Zone 3 earthquake loading.

*3 Collapses shown above 100% line.

*4 Reference 25

Figure II.11 Mean Damage Ratios for Skeleton Frame Structures (after Mann, 1974)

* Class A - Steel frame structures

* Class B - Reinforced concrete frame structures

III. ANALYSIS OF UNCERTAINTY: TYPES

The frequency-magnitude and the frequency-intensity laws presented in Section II (Equations II.1 to II.8 and equation II.13) are idealized relationships, fitted to historical data. The same is true for the intensity-expected damage curves in Figures II.5, II.6, II.8-11. The quantatitive limitation of statistical information is not the only problem of seismic inference. Additional difficulties are due to often present biases and to the incompleteness of historical records. Uncertainties on MMI data include: the uncertainty on epicentral intensity, which may be higher than the intensity at the closest inhabited center; the uncertainty on the mean rate of events in the low-to-moderate intensity range: the older the record, the less complete the data; the uncertain effect of neglected local soil conditions; the uncertain effect of aftershocks, which are typically removed from the statistics; the uncertainty on the epicenter location and on the focal depth.

There is also reason to believe that damage statistics collected through questionnaires are inaccurate and biased. On the other hand, direct subjective evaluations of damage, such as those in Figures II.8-10, differ from author to author.

Because of all these sources of uncertainty, only limited confidence can be placed on any one probabilistic model which is estimated from statistical data, or which relies on professional judgement. In some cases (e.g., in the estimation of the seismicity parameter b for California) the data base is so large that statistical uncertainty can be neglected in the context of the overall accuracy of the analysis. In other cases (e.g., in the estimation of the seismic parameters in a low-seismicity region, or in establishing the resistance distribution of a new piece of equipment) statistical variability may be a major source of uncertainty and risk.

In Sections IV and V, seismic risk models will be classified into two categories: (i) models which result from best data fitting (or from other statistical estimation procedures), and which do not include inductive uncertainty. These models are called "<u>probabilistic</u>," and will be studied in Section IV; (ii) models which incorporate inductive uncertainty; these models are called "<u>statistical</u>," and will be studied in Section V.

Although probabilistic models can be viewed as limit cases of their statistical counterparts, as the amount of information "tends to infinity," they are considered separately on account of their greater simplicity. Also, most of the seismic models proposed in the past have been of the "probabilistic" type (for exceptions see Benjamin, 1968; and Esteva, 1969). It is found appropriate, therefore, to quantify the effects of statistical uncertainty through penalty factors on the "probabilistic" mean failure rate.

The theory of statistical prediction (of future random events, under limited information on the generating probabilistic mechanism) has been developed mainly in the last decade (Thatcher, 1964; Aitchison and Sculthorpe, 1965; Guttman,1970). Different methods and different terminologies are used, depending on the meaning of probability, and on the inference school (frequentist, likelihood, fiducial, Bayesian). Preference is given here to the Bayesian viewpoint, but the numerical results can be readily given a frequentist, or a likelihood, or a fiducial interpretation. The general methodology and some specific results to be used in Section V are reviewed next. For a more detailed account of the theory and for applications in the area of reliability, see Veneziano (1974, 1975).

Consider a random vector $\underline{X}$ (for the case of interest here, $\underline{X}$ might include some measures of site intensity for the next earthquake and some resistance parameters of the facility at risk), with distribution function $F_{\underline{X}}(\cdot)$. Suppose that the type of distribution in known (this assumption can be released, see Veneziano, 1974), but that uncertainty exists on some of the parameters (for example, on the mean value vector, on the covariance matrix, etc.) If $\underline{\Theta}$ is the vector of unknown parameters, with Bayesian distribution $F_{\underline{\Theta}}(\cdot)$,and $F_{\underline{X}|\underline{\Theta}}(\cdot)$ is the conditional CDF of $\underline{X}$, the unconditional distribution of $\underline{X}$ is, from the total probability theorem:

$$F_{\underline{X}}(\underline{x}) = \int_{all\ \underline{\theta}} F_{\underline{X}|\underline{\Theta}=\underline{\theta}}(\underline{x}) \quad dF_{\underline{\Theta}}(\underline{\theta}) \ . \qquad \text{(III.1)}$$

In general $F_{\underline{X}}(\cdot)$ and $F_{\underline{X}|\underline{\Theta}}(\cdot)$ differ both in the parameters, and in the distribution type. In fact, it is precisely this condition which differentiates probabilistic from Bayesian-statistical models.

The probability distribution $F_{\underline{\Theta}}(\cdot)$ in Equation (1) can be either the "prior" distribution $F'_{\underline{\Theta}}(\cdot)$, or the "posterior" distribution, $F''_{\underline{\Theta}}(\cdot)$, the latter including information in addition to that already contained in $F'_{\underline{\Theta}}$. If z denotes this additional information, $F''_{\underline{\Theta}}$ can be found from Bayes' theorem:

$$dF''_{\underline{\Theta}}(\underline{\theta}) \propto dF'_{\underline{\Theta}}(\underline{\theta}) \cdot \ell(\underline{\theta}|z) \ ,$$

where $l(\underline{\Theta}|z) \propto f_{z|\underline{\Theta}}(z|\underline{\Theta})$ is the likelihood function of the experiment which generates z.

For applications in Section V, consider the special case of a normal random variable $X \sim N(\mu, \sigma^2)$, with unknown mean μ and/or unknown variance σ^2. Information on the unknown parameter(s) is provided by a prior distribution and by the random sample $\underline{z}=\{X_1, X_2, \ldots, X_n\}$ from the unknown population of X. The problem of finding the predictive distribution of X, Equation (1), was discussed, among others, by Raiffa and Schlaifer (1961) and by Guttman(1970). The results given below are for the case of the unknown parameter(s) having conjugate prior distribution (Raiffa and Schlaifer, 1961). Under this condition the Bayesian results are numerically identical with the frequentist results obtained by Proshan (1953), after an appropriate redefinition of the sufficient sample statistics.

(a) μ unknown, σ^2 known

For a normal prior distribution of μ: $\mu \sim N(\mu'; \sigma'^2 = \sigma^2/n')$, the posterior distribution of μ is also normal:

$$\mu \sim N\left(\mu'' ; \sigma''^2 = \sigma^2/n''\right) ,$$

where $$\mu'' = \frac{n'\mu' + n\hat{\mu}}{n'+n} \quad ; \quad \hat{\mu} = \frac{1}{n}\sum_{i=1}^{n} X_i \quad ; \quad n'' = n' + n .$$

From Equation (1), X has normal posterior predictive distribution:

$$X \sim N\left(\mu'' ; \sigma^2(1 + 1/n'')\right). \qquad \text{(III.2)}$$

(b) μ known, σ^2 unknown

From Raiffa and Schlaifer (1961) the family of conjugate distributions of the precision parameter $h=1/\sigma^2$ is Gamma-2: for a prior density in the Gamma-2 form:

$$f'_h(h) \propto h^{n'/2-1} \cdot \exp\left(-\tfrac{1}{2} h n' s'^2\right) \qquad , \; h > 0 \quad ; n', s'^2 > 0 ,$$

the posterior density of h is:

$$f''_h(h) \propto h^{n''/2-1} \cdot \exp\left(-\tfrac{1}{2} h n'' s''^2\right) ,$$

where $n'' = n + n'$,

$$s''^2 = \frac{n's'^2 + n s^2}{n' + n} ,$$

and $s^2 = \frac{1}{n}\sum_{i=1}^{n}(X_i - \mu)^2$.

In this case, Equation (1) yields a predictive $t_{n''}$-distribution for $y=(X-\mu)/s''$, with density:

$$f_Y(y) = \frac{1}{\sqrt{n''\pi}} \frac{\Gamma\left(\frac{n''+1}{2}\right)}{\Gamma(n''/2)} \left(1 + y^2/n''\right)^{-(n''+1)/2} . \qquad \text{(III.3)}$$

(c) μ and σ^2 unknown

From Raiffa and Schlaifer (1961), the conjugate family is now Normal-Gamma. If the prior parameters are $[n', \mu', (n'-1)\, S'^2]$, this means that

$$f'_{\mu,\sigma^2}(\mu,\sigma^2) \propto \sigma^{-(n'+2)} \cdot \exp\left\{-\left[(n'-1)S'^2 + n'(\mu-\mu')^2\right]/2\sigma^2\right\} .$$

Given the sample $\{X_1,\ldots,X_n\}$, it is found that the posterior distribution of (μ,σ^2) is also Normal-Gamma, with parameters $[n'', \mu'', (n''-1)S''^2]$, where

$$n'' = n' + n$$

$$\mu'' = (n'\mu' + n\mu)/n''$$

$$(n''-1)S''^2 = (n'-1)S'^2 + (n-1)S^2 + \frac{n'n}{n''}(\hat{\mu} - \mu')^2$$

and $\hat{\mu}$, S^2 are the sample statistics:

$$\hat{\mu} = \frac{1}{n}\sum_{i=1}^{n} X_i \quad ; \quad S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \hat{\mu})^2 .$$

From Equation (1) Aitchison and Sculthorpe (1965) found that

$$Y = \left(\frac{n''}{n''+1}\right)^{1/2} \frac{x - \mu''}{S}$$

has $t_{n''-1}$-distribution; i.e. that the prediction density of y has the form (III.3), with $(n''-1)$ replacing n''.

In Section V it will be shown that replacing the suggested normal distribution of the seismic resistance expressed in terms of ln a or of MMI (see, e.g., Newmark, 1974, and Benjamin, 1974) by a (predictive) t-distribution may increase considerably the calculated risk.

IV. Probabilistic Seismic Damage Models

The information on seismic risk and on seismic resistance of engineering systems reviewed in Section II is used here for mean failure rate calculations. A simple, yet realistic model is presented first, for which closed-form results are readily obtained. Thereafter, more sophisticated models are introduced and studied numerically. In all cases inductive uncertainty is neglected. Statistical (inductive) versions of the same models will be considered in Section V.

IV.1 Linear Gaussian Model

Consider the "linear" damage model in Figure 1 (lower part). D denotes the actual damage or the actual damage ratio, and d_f is the value of D at "failure." For each given MM intensity I, the probability distribution of Log D is assumed Normal (as suggested by Benjamin, 1974), with mean value $a_D+b_D I$ (see Equation II.16) and variance σ_D^2. Then the probability of failure for an earthquake of site intensity I is:

$$P_f(I) = \Phi\left[(d_f - a_D - b_D I)/\sigma_D\right] \quad , \qquad \text{(IV.1)}$$

where $\Phi[\cdot]$ is the standard normal CDF. $P_f(I)$ is also the probability that the resistance (with respect to the threshold damage d_f) is less than I, meaning that the probability distribution of the resistance R (in units of MMI) is normal:

$$R \sim N(\mu_R \; ; \; \sigma_R^2) \quad , \qquad \text{(IV.2)}$$

with mean value: $\mu_R = (d_f - a_D)/b_D$;

and standard deviation: $\sigma_R = \sigma_D / b_D = \beta_D^{-1}$. (see Equation II.15)(*)

Typical values of β_D are given in Table II.5. In terms of the normalized intensity I_N, defined:

$$I_N = (I - \mu_R)/\sigma_R$$

(*)The use of Equation (2) in the following calculations is numerically correct, but the reader may disagree on its interpretation as a resistance distribution.

(I_N measures the algebraic distance of I from the mean resistance in units of σ_R), R is a standard normal variate, R~N(0,1). Let λ_i be the mean rate of events with site intensity larger than i. In its simplest form, the model assumes that λ_i varies exponentially with i (See Figure 1, upper part):

$$\lambda_i = \lambda \; e^{-\beta_I \cdot i} . \tag{IV.3a}$$

For typical values of β_I see Equation (II.13) and related comments, Figures II.1, II.2, and Table II.3. Alternatively, in terms of the normalized intensity $i_N=(i-\mu_R)/\sigma_R$, Equation (3a) can be written:

$$\lambda_{i_N} = \lambda_o \; e^{-\beta_N \cdot i_N} , \tag{IV.3b}$$

where $\lambda_o=\lambda e^{-\beta_I \mu_R}$ is the mean rate of events with site intensity larger than the mean resistance, and

$$\beta_N = \beta_I \sigma_R = \beta_I / \beta_D .$$

For $\beta_I=\beta_{I_o}$=slope of the frequency-epicentral intensity relation (see Equation II.12 and lines between dots in Figures II.1 and II.2), and using Tables II.3 and II.5, β_N is found to vary between 0.60 and 1.20, with typical value of about 0.90.

The mean failure rate, λ_f, can be calculated from Equation (I.3), which in the present case becomes:

$$\lambda_f = \frac{\lambda_o}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\beta_N i_N - i_N^2/2} \, d\, i_N = \lambda_o \, e^{\beta_N^2/2} . \tag{IV.4}$$

The quantity $\gamma_{DET}=e^{\beta_N^2/2}$ can be interpreted as a penalty factor for uncertain resistance (i.e., with respect to the "deterministic" case $\sigma_R=\sigma_D=0$); it increases with σ_R and with β_I (see Equation 3b), and is 1 whenever either of these parameters is zero. Typical values of γ_{DET} are in the range 1.20 to 2.05. However, it will be shown later in this section that γ_{DET} may increase when one allows for upper truncation or for other nonlinearities in the frequency-epicentral intensity law.

The exact mean failure rate, λ_f, can be compared with sometimes used approximations of the form (I.4), rewritten here:

$$\lambda_{f_P} = P \cdot \lambda_o \cdot e^{-\beta_N F_{R,P}} , \tag{IV.5}$$

where $F_{R,P}$ is the P-fractile of the resistance distribution (here, of the standard normal distribution; i.e., $F_{R,P}=\Phi,p$). Recall that λ_{f_p} is the product between $\lambda_o \exp(-\beta_N F_{R,P})$, which is the mean rate of events with intensity $I>i_p=\Phi,p\sigma_R+\mu_R$, and P, which is the probability of failure if an earthquake of intensity i_p occurs. Values of P between 10^{-1} and 10^{-2} (USAEC Reactor Safety Study, WASH-1400, Preliminary Report) and between 10^{-2} and 10^{-4} (Newmark, 1974) have been used. In Figure 2, the ratio

$$\gamma_P = \lambda_f/\lambda_{f_P} = \frac{1}{P}\ e^{\beta_N^2/2 + \beta_N \Phi,P} \tag{IV.6}$$

is plotted versus β_N for selected values of P. For $\beta_N=1$ it is $\gamma_{10^{-1}}=4.6$; $\gamma_{10^{-2}}=16.1$; $\gamma_{10^{-3}}=75$; $\gamma_{10^{-4}}=400$. The curve $\gamma_{DET}=\frac{1}{2}\gamma_{0.5}$ gives the factor of unconservatism when the resistance is assumed deterministic and equal to its mean value μ_R.

IV.2 Nonlinear Gaussian Models

Five cases are considered: (a) truncated linear frequency-intensity law; (b,c) truncated linear first and second derivatives of the frequency-intensity law; (d) quadratic frequency-intensity law; (e) logarithmic frequency-intensity law.

(a) Truncated-Linear λ_{i_N}

The resistance of the system is modeled as in the previous case (Figure 1, lower part), but now the frequency-site intensity curve is truncated at the upper bound intensity level i_1 (See Figure 3, curve b). As observed in Section II, this is a good approximation to calculated site intensity risk curves when the epicentral intensity is bounded (See Figure II.1 and related comments).

For mathematical convenience, let i_1 be the algebraic distance of the upper bound from the mean resistance, in units of standard deviations of resistance. In this case, $i_1 \to \infty$ for untruncated site intensity, $i_1=0$ for truncation at the mean value of resistance. Formally, the mean rate of events with (normalized) site intensity in excess of i_N is:

$$\lambda_{i_N} = \begin{cases} \lambda_o\ e^{-\beta_N i_N} & , \text{ for } i_N \le i_1 , \\ 0 & , \text{ for } i_N > i_1 , \end{cases} \tag{IV.7}$$

which replaces the risk law (3b). Using Equation (7), the mean failure rate is:

$$\lambda_{f,i_1} = \frac{\lambda_o}{\sqrt{2\pi}} \int_{-\infty}^{i_1} e^{-\beta_N i_N - i_N^2/2}\, di_N$$

$$= \lambda_o\, e^{\beta_N^2/2}\, \Phi\,(i_1 + \beta_N)\,. \tag{IV.8}$$

For λ_{f_p} defined as in Equation (5) ($F_{R,P}=\Phi,_p$=P-fractile of the standard normal distribution), the ratio

$$\gamma_{P,i_1} = \lambda_{f,i_1}/\lambda_{f_P} = \frac{1}{P}\, e^{\beta_N \Phi,_P + \beta_N^2/2} \cdot \Phi\,(i_1 + \beta_N) \tag{IV.9}$$

is plotted in Figure 4 as a function of i_1, for β_N=1.0, and for selected values of P. It is emphasized that λ_{f_p} is calculated as if the risk curve were not truncated at i_1, which fact makes γ_{p,i_1} defined also for $i_1<\Phi,_p$.

As $i_1 \to \infty$, the ratio (9) approaches γ_p in Equation (6) and Figure 2. Indeed, values of λ_{f,i_1} very close to λ_f are found for $i_1>0$, meaning that truncation of the risk curve above μ_R has little effect on the calculated mean failure rate. This in an interesting conclusion, which shows that λ_f is not always sensitive to the decay of the seismic risk curve in its upper "tail," as commonly believed. From Equation (8) it is apparent that the upper truncation point for which the mean failure rate becomes half the value for no truncation is: $i_1=-\beta_N$ ($i_1=-1$ in Figure 4). The penalty factor $\gamma_{DET,i_1}=\frac{1}{2}\gamma_{0.5,i_1}$ applies when λ_f is approximated by λ_o (i.e., when assuming σ_R=0 and when using the untruncated linear model, Equation 3b).

Notice that all the curves in Figure 4 are obtained by simple vertical translation of the curve γ_{DET,i_1}. The same being true for any fixed β_N, it is convenient to plot the factors γ_{DET,i_1} for several values of β_N (Figure 5), and to tabulate separately the factors by which γ_{DET,i_1} must be multiplied to calculate γ_{P,i_1}. (This is done in Table 1 for selected values of P.)

Example. Five linear approximations to the risk curves in Figure II.1 are shown in Figure 6. Some of them are untruncated, and correspond to sources with no (or very high) upper truncation of the epicentral intensity. The other curves are truncated, being approximations to those frequency-site intensity relationships in Figure II.1 which used small or moderate upper bounds on the epicentral intensity. The values of β_N, λ_o and i_1 for the five approximations are given in columns 2,3 and 4 of Table 2, respectively. For a normal resistance distribution with mean $\mu_R=8$ and standard deviation $\sigma_r=0.8$ (see Figure 6), the exact mean failure rates, from Equation (8), are given in column 5 of Table 2. The same values could be found from: $\lambda_{f,i_1}=\lambda_o \cdot \gamma_{DET,i_1}$, the last factor being plotted in Figure 5. Finally, the last two columns of Table 2 refer to the approximation (5), where $F_{R,P}=\Phi,_p$ and P=0.1, 0.01, respectively. The numbers in parenthesis are the factors of unconservatism, γ_{p,i_1}, associated with the approximations (see Equation 9, or Figures 2,4,5 and Table 1). It is observed that:

(i) truncation of the frequency-site intensity law has a small effect on the mean failure rate for $i_1>0$. However, the effect would increase markedly for truncation values $i_1<-\beta_N$; the latter is the case for very reliable systems (for high μ_R).

(ii) truncation of the frequency-epicentral intensity law is more important, primarily because it reduces the mean rate λ_o. (At the same time it increases β_N and causes a sudden drop of the risk curve at the site);

(iii) the factors of unconservatism associated with the approximation (5) are not sensitive to truncation of either the epicentral, or the site intensity laws.

Since different assumptions on the frequency-epicentral intensity law have sizable consequences on the mean failure rate through variations of λ_o and β_N, statistical uncertainty on these seismicity parameters will be considered in Section V.

It has been observed (Cornell, 1975) that the truncated linear model (7) is logically unsatisfactory because it associates a finite mean rate (namely, $\lambda_o e^{-\beta_N i_1}$) to events with site intensity equal to the upper bound i_1. (This does not mean, however, that the model should be avoided as a mathematical approximation.) Several other models with upper truncation can be formulated, which do not display this singularity; four of them are considered next.

(b) Truncated Linear $d\lambda_{i_N}/diN$

By truncating the first derivative of the linear risk function (7) at i_1, and by imposing the condition $\lambda_{i_1}=0$ one obtains the following frequency-site intensity law:

$$\lambda_{i_N} = \begin{cases} \lambda_o \left(e^{-\beta_N i_N} - e^{-\beta_N i_1} \right) & , i_N < i_1 \\ 0 & , i_N \geqslant i_1 \end{cases} \qquad \text{(IV.10)}$$

(see a representative plot in Figure 3, curve c). When used in Equation (I.3), the risk function (10) yields the following mean failure rate (compare with Equations 4 and 8):

$$\lambda_{f,i_1} = \lambda_o \left[e^{\beta_N^2/2} \cdot \Phi\left(i_1 + \beta_N\right) - e^{-\beta_N i_1} \cdot \Phi(i_1) \right]. \qquad \text{(IV.11)}$$

(c) Truncated Linear $d^2\lambda_{i_N}/d^2 i_N$

One might still argue that the model (10) implies a discontinuity in the mean rate "density" at i_1 (from the value $\lambda_o\beta_N e^{-\beta_N i_1}$ to zero), and therefore that it is also physically unsound. A "better" model might be obtained by truncating higher order derivatives of the risk function. Truncation of the second derivative at i_1 generates the following model (for a representative plot, see Figure 3, curve d):

$$\lambda_{i_N} = \begin{cases} \lambda_o \left[\left(e^{-\beta_N i_N} - e^{-\beta_N i_1} \right) - \beta_N (i_1 - i_N) e^{-\beta_N i_1} \right] & , i_N < i_1 \\ 0 & , i_N \geqslant i_1 \end{cases} \qquad \text{(IV.12)}$$

which gives the mean failure rate:

$$\lambda_{f,i_1} = \lambda_o \left\{ e^{-\beta_N^2/2} \cdot \Phi\left(i_1 + \beta_N\right) - e^{-\beta_N i_1} \left[\left(1 + \beta_N i_1\right) \Phi(i_1) + \beta_N \phi(i_1) \right] \right\}, \qquad \text{(IV.13)}$$

where $\phi(\cdot)$ is the standard normal density function.

In both models (10) and (12), λ_o is the mean rate of events with site intensity in excess of the mean resistance, for the case of no truncation, $i_1 \to \infty$ (see Figure 3).

A comparison of the three "linear" models with truncation, Equations (7), (10) and (12) is straightfoward in terms of the mean failure rates, Equations (8), (11) and (13). However, when using different truncated "linear" models to approximate the actual nonlinear frequency-intensity law, one would conceivably select different values for i_1, and possibly for λ_o. In so doing, one would reduce the difference between the risks calculated from the various models. In the remainder of this study, no further consideration will be given to models of the type (10) and (12).

(d) Quadratic

A quadratic law for Richter magnitude was proposed by Shlien and Toksöz (1970), and by Merz and Cornell (1973). A quadratic model is used here to approximate frequency-site intensity curves (such as those in Figure II.1 and II.2). Let then:

$$\lambda_{i_N} = \lambda_0 \, e^{-\alpha_N i_N^2 - \beta_N i_N} \quad ; \alpha_N \geq 0 , \tag{IV.14}$$

where, as in the linear case, λ_o is the mean rate of events with site intensity exceeding the mean resistance value, and α_N, β_N are known parameters. Equation (14) is an appropriate risk function only if it is non-increasing; i.e., only for $i_N > -\beta_N / 2\alpha_N$.

For the case of no upper bound site intensity, the mean failure rate can be calculated analytically:

$$\lambda_f = \frac{\lambda_o}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\alpha_N i_N^2 - \beta_N i_N} \cdot e^{-i_N^2/2} \, di_N = \frac{\lambda_o}{\sqrt{2\alpha_N + 1}} \, e^{\beta_N^2/(4\alpha_N + 2)} . \tag{IV.15}$$

Integration over the entire real axis violates the condition that λ_{i_N} should be a non-increasing function of i_N. However, if failure events caused by earthquakes with normalized site intensity $i_N < -\beta_N / 2\alpha_N$ are negligible, the mean failure rate (15) is numerically accurate. When $\alpha_N = 0$, Equation (15) reproduces the mean failure rate for the linear law, Equation (4).

It is interesting to compare the exact mean failure rate, Equation (15), with a conservative approximation obtained from a tangent linearization of the quadratic law. Linearization around $i_N = i_N^*$ yields:

$$\lambda_{i_N, i_N^*} = \lambda_{o, i_N^*} \cdot e^{-\beta_{N, i_N^*} \cdot i_N} , \tag{IV.16}$$

where

$$\lambda_{o, i_N^*} = \lambda_o \, e^{\alpha_N i_N^{*2}} \quad ; \qquad \beta_{N, i_N^*} = \beta_N + 2\alpha_N i_N^* ,$$

and the following upper bound to the mean failure rate:

$$\lambda_{f, i_N^*} = \lambda_o \cdot \exp\left\{ \beta_N^2/2 + 2\alpha_N \beta_N i_N^* + \alpha_N (2\alpha_N + 1) i_N^{*2} \right\} . \tag{IV.17}$$

The tangent approximation which produces the least upper bound is found for $i_N^*=-\beta_N/(2\alpha_N+1)$, being:

$$\min_{i_N^*}\left(\lambda_{f,i_N^*}\right) = \min_{i_N^*} \frac{\lambda_{0,i_N^*}}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \exp\left(-\beta_{N,i_N^*}\cdot i_N - i_N^2/2\right) di_N \qquad \text{(IV.18)}$$
$$= \lambda_0 \exp\left[\beta_N^2/(4\alpha_N+2)\right] .$$

This choice of i_N^* corresponds to $\beta_{N,i_N^*}=-i_N^*$ in Equation (16), which means that the maximum contributions to the risk for the quadratic and the linear tangent laws occur both at $i_N=i_N^*$, and that such maximum contributions coincide (evaluate the integrands in Equations 15 and 18 for $i_N=i_N^*$).

The ratio

$$\min_{i_N^*} \lambda_{f,i_N^*}/\lambda_f = (2\alpha_N+1)^{1/2}$$

is the factor of conservatism for the "best" tangent approximation. Risk curves with $\beta_N=1.6$ and $\alpha_N=0.2,0.3$ are shown in Figure 7. The factor of conservatism is 1.18 for $\alpha_N=0.2$, and 1.26 for $\alpha_N=0.3$, showing that in this (realistic) range of α_N values the tangent approximation produces accurate results. These calculations also suggest that accurate linear approximations to nonlinear seismic risk curves can be obtained in general by choosing the point of tangency, i_N^*, so that the derivative at i_N^*, $-\beta_{N,i_N^*}$, equals i_N^*.

(e) Logarithmic

Consider the 3-parameters frequency-site intensity relationship:

$$\lambda_{i_N} = \lambda\, e^{d\left[\ln(i_1-i_N) - \ln c\right]} \quad ; \; c, d, \lambda > 0 , \quad i_N \le i_1 , \qquad \text{(IV.19)}$$

where i_1 is the intensity upper bound, c is the value of (i_1-i_N) for which $\lambda_{i_N}=\lambda$, and λ and d are a location and a scale parameter on semilog paper, respectively. (Note that λ is a redundant parameter, which is introduced only for mathematical convenience.)

The "logarithmic" law (19) (plotted in Figure 8 for c=1 and d=1,2,3) corresponds to the Pareto distribution of $(i_1-i_N)^{-1}$:

$$1 - F_{(i_1-i_N)}(i) \;\propto\; i^d \quad ; \; i \geq 0 , \; d > 0 .$$

Reasons for using risk functions in the form (19) are:

(i) they include an upper bound intensity;

(ii) the mean rate of events with site intensity in excess of i_N is a decreasing function of i_N.

Neither of these properties is enjoyed by the quadratic law (14).

For a resistance distribution $R\sim N(0;1)$, the mean failure rate

$$\lambda_f = \frac{\lambda}{\sqrt{2\pi}} \int_{-\infty}^{i_1} \left(\frac{i_1 - i_N}{c}\right)^d e^{-i_N^2/2}\, d i_N \tag{IV.20}$$

is a function of i_1, c and d. The linear approximation with slope numerically equal to the intensity i_N^* at the point of tangency is found for

$$i_N^* = \frac{i_1 - \sqrt{i_1^2 + 4d}}{2} \qquad \left(= -\beta_{N, i_N^*}\right), \tag{IV.21}$$

with associated mean failure rate:

$$\begin{aligned} \lambda_{f,i_N^*} &= \lambda\left[(i_1 - i_N^*)/c\right]^d \exp\left[-d i_N^*/(i_1 - i_N^*) + \beta^2_{N, i_N^*}/2\right] \\ &= \lambda\left[\left(i_1 + \sqrt{i_1 + 4d}\right)/2c\right]^d \exp\left(\tfrac{3}{2} i_N^{*2}\right). \end{aligned} \tag{IV.22}$$

The factor $\lambda_{f,i_N^*}/\lambda_f$, by which Equation (12) is a conservative approximation to (20), depends only on d and i_1; it is plotted in Figure 9 for d=1(1)5 and for i_1 values in the range (-3,3). For truncation intensities which are not much smaller than the mean resistance, the linear approximation (22) is quite accurate. Clearly, in the actual linearization of convex risk curves one should not use a tangent approximation, if not to calculate upper bounds for λ_f. Figure 9 shows, however, that for logarithmic risk functions the tangent upper bound is itself quite close to the exact mean failure rate.

IV.3 Linear Gamma Model

Suppose now that the normalized resistance R (zero mean, unit variance) has shifted Gamma distribution, with density:

$$f_R(r) = \begin{cases} 0 & , r < -D , \\ \dfrac{D[D(r+D)]^{D^2-1}}{\Gamma(D^2)} e^{-D(r+D)} & , r \geq -D , \end{cases} \tag{IV.23}$$

D=positive constant

Plots of the density (23) are shown in Figure 10. Being σ_R=1, the Gamma density (23) reduces to a shifted exponential when D=1, and to the standard normal N(0;1) as $D\to\infty$.

For the untruncated linear law (3b), one finds a mean rate of failure

$$\lambda_f = \frac{\lambda_o D}{\Gamma(D^2)} \int_{-D}^{\infty} e^{-\beta_N i_N} \left[D(i_N + D)\right]^{D^2-1} e^{-D(i_N+D)} di_N$$

$$= \lambda_o \; e^{\beta_N D} \left(\frac{D}{D+\beta_N}\right)^{D^2}. \qquad \text{(IV.24)}$$

The ratio between the mean failure rate for Gamma (Eq. 24) and for normal (Eq.4) resistance distribution:

$$\frac{\lambda_{f_G}}{\lambda_{f_N}} = \left(\frac{D}{D+\beta_N}\right)^{D^2} e^{\beta_N D - \beta_N^2/2} \qquad \text{(IV.25)}$$

is plotted in Figure 11 as a function of β_N, for D=1,2,3,5,8. The ratio (25) is generally smaller than 1, due to the Gamma density (23) vanishing for $R<-D$. This shows the importance of the left tail of the resistance distribution, a fact which will be fully emphasized in the following section. However, with the exception of the rather artificious cases when $D \leq 1$, and for typical values of β_N (say, $\beta_N<1.5$) the mean failure rate does not change significantly if one replaces the normal resistance model by a Gamma model with the same first two moments.

FRACTILE P	β_N							
	0.60	0.80	1.00	1.20	1.40	1.60	1.80	2.00
0.5	2	2	2	2	2	2	2	2
10^{-1}	4.63	3.59	2.77	2.15	1.66	1.29	0.995	0.770
10^{-2}	24.77	15.55	9.77	6.13	3.85	2.42	1.52	0.954
10^{-3}	156.6	84.42	45.50	24.53	13.22	7.13	3.84	2.07
10^{-4}	1074	510.4	242.6	115.3	54.80	26.05	12.38	5.88
10^{-5}	7738	3298	1405	598.8	255.2	108.7	46.34	19.75

Table IV.1 Factors by which the values γ_{DET,i_1} in Figure IV.4 must be multiplied to obtain the ratio γ_{P,i_1} in Equation (IV.9)

Col. 1	2	3	4	5	6	7
RISK CURVE	$\beta_N=\beta_I\cdot\sigma_R$	$\lambda_o^{(*)}$	i_1	$\lambda_{f,i_1}^{(*)}$	$\lambda_{f,P=0.1}^{(*)}(\gamma_{0.1,i_1})$	$\lambda_{f,P=0.01}^{(*)}(\gamma_{0.01,i_1})$
1	0.88	5.9 - 4	∞	8.7 - 4	1.8 - 4 (4.7)	4.5 - 5 (19.1)
2	1.55	8.6 - 5	∞	2.9 - 4	6.3 - 5 (4.6)	3.2 - 5 (9.0)
3	1.55	8.6 - 5	0	2.7 - 4	6.1 - 5 (4.4)	3.1 - 5 (8.6)
4	1.58	2.6 - 5	∞	9.1 - 5	2.0 - 5 (4.6)	1.0 - 5 (8.8)
5	1.58	2.6 - 5	-0.625	5.3 - 5	1.4 - 5 (3.9)	7.3 - 6 (7.3)

(*) Notation: $X\text{-}n = X\cdot 10^{-n}$

Table IV.2 Mean Failure Rates for the Risk Curves and the Resistance Distribution in Figure IV.6

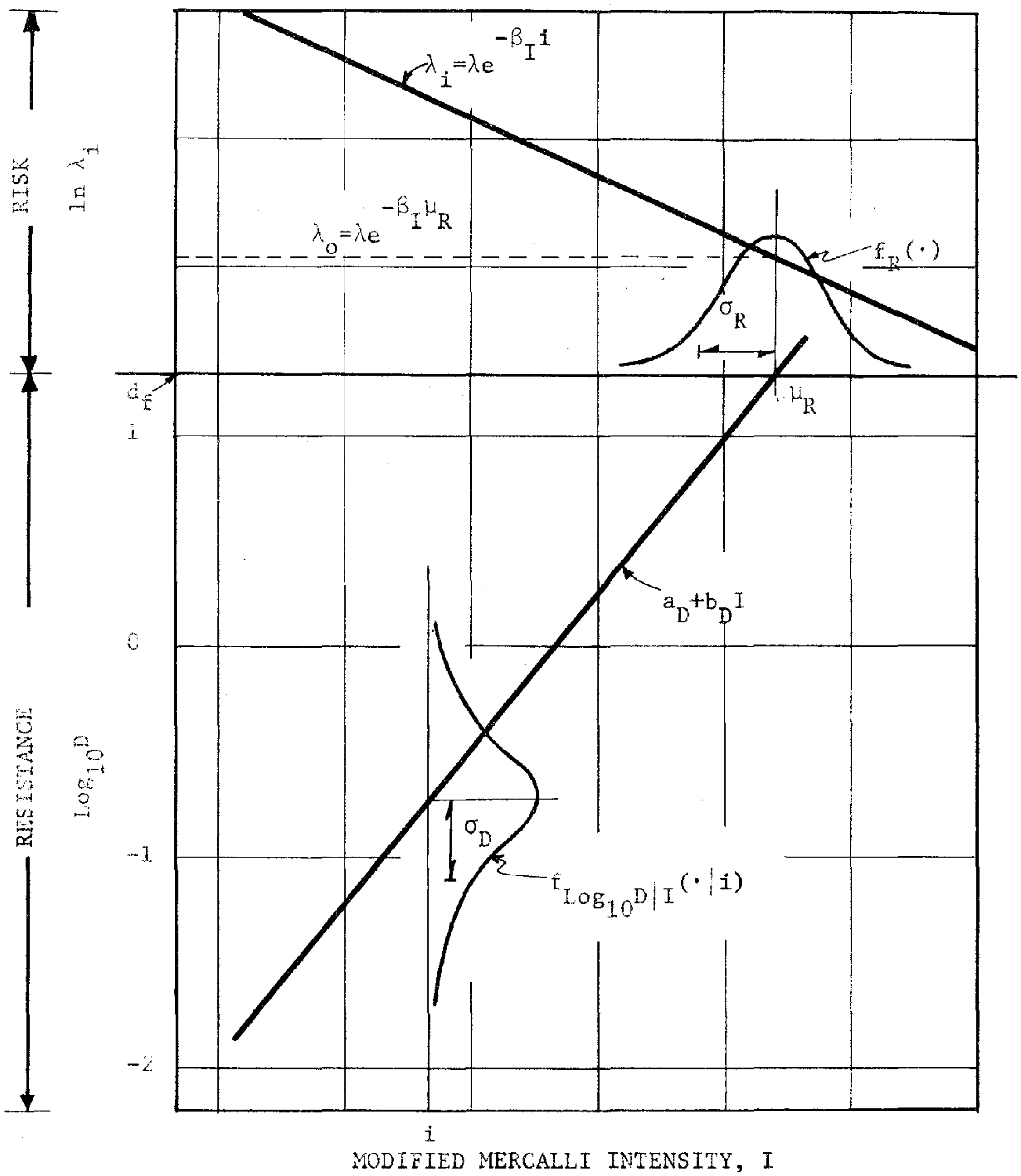


Figure IV.1 "Linear" Seismic Risk Model

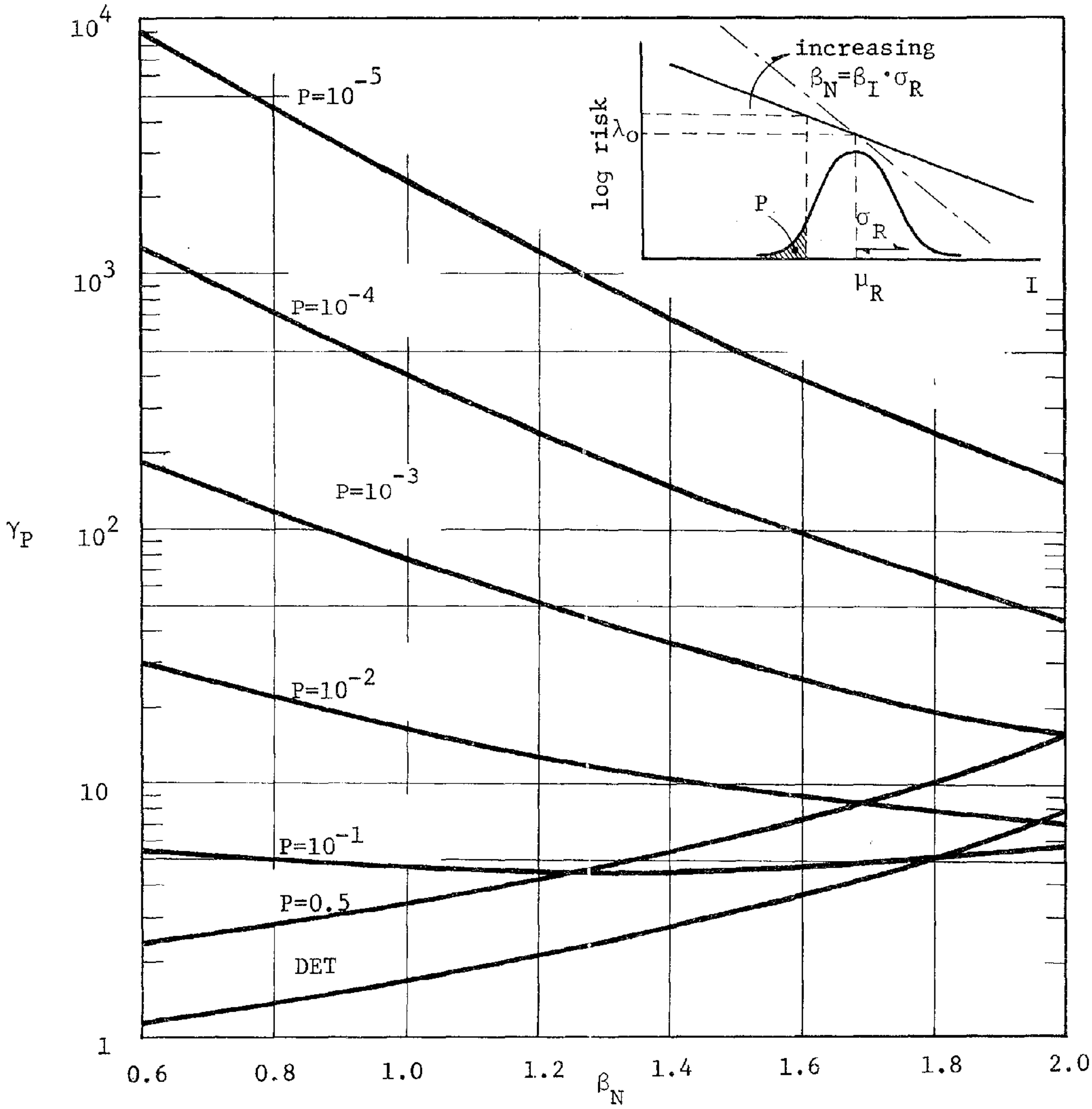


Figure IV.2 Untruncated Linear Frequency-Intensity Law

Ratio $\gamma_P=\lambda_f/\lambda_{f_p}$ in Equation (IV.6)

$\gamma_{DET}=\lambda_f/\lambda_o$

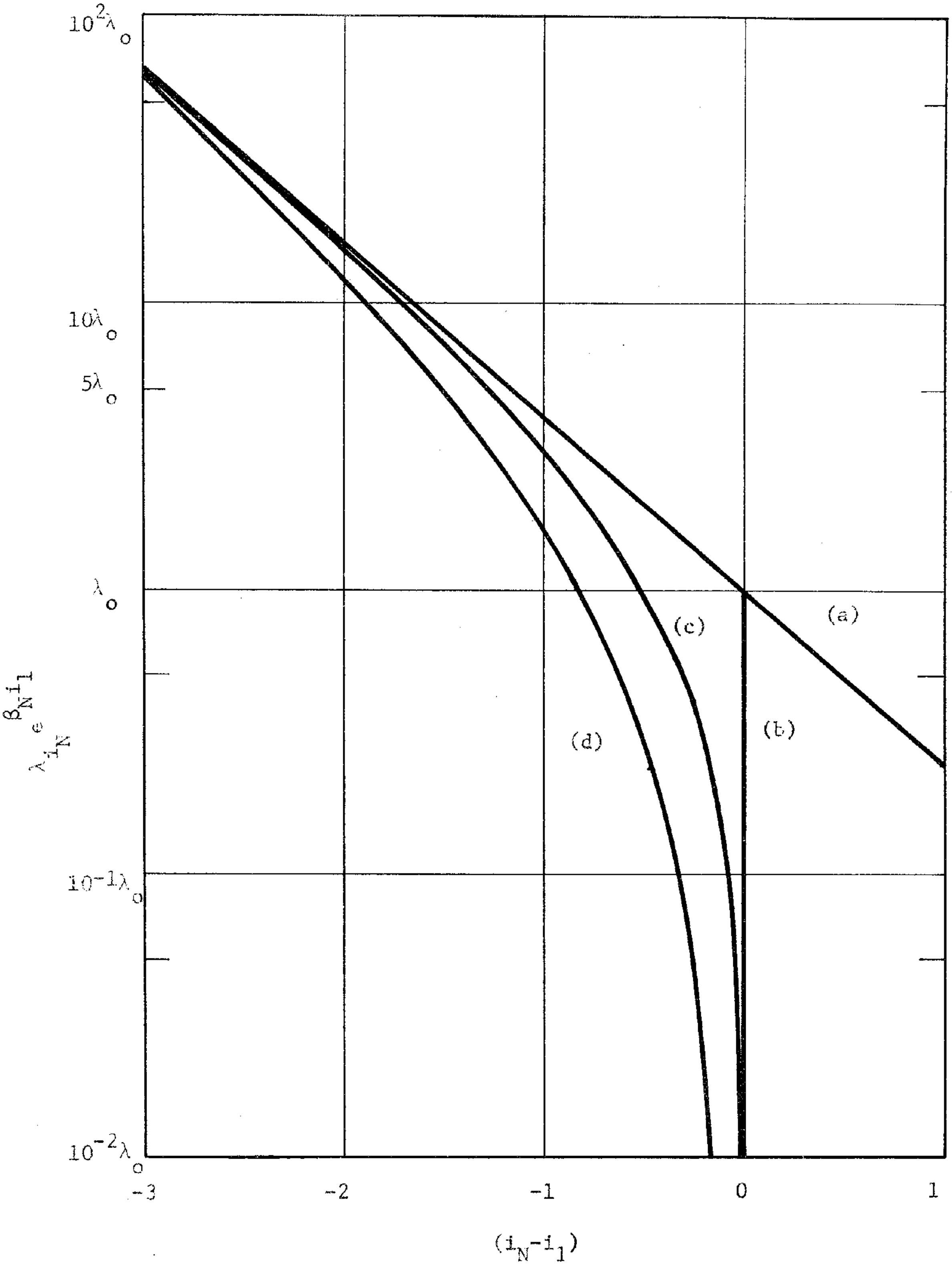


Figure IV.3 Untruncated and Truncated "Linear" Risk Models;

See Section IV.2

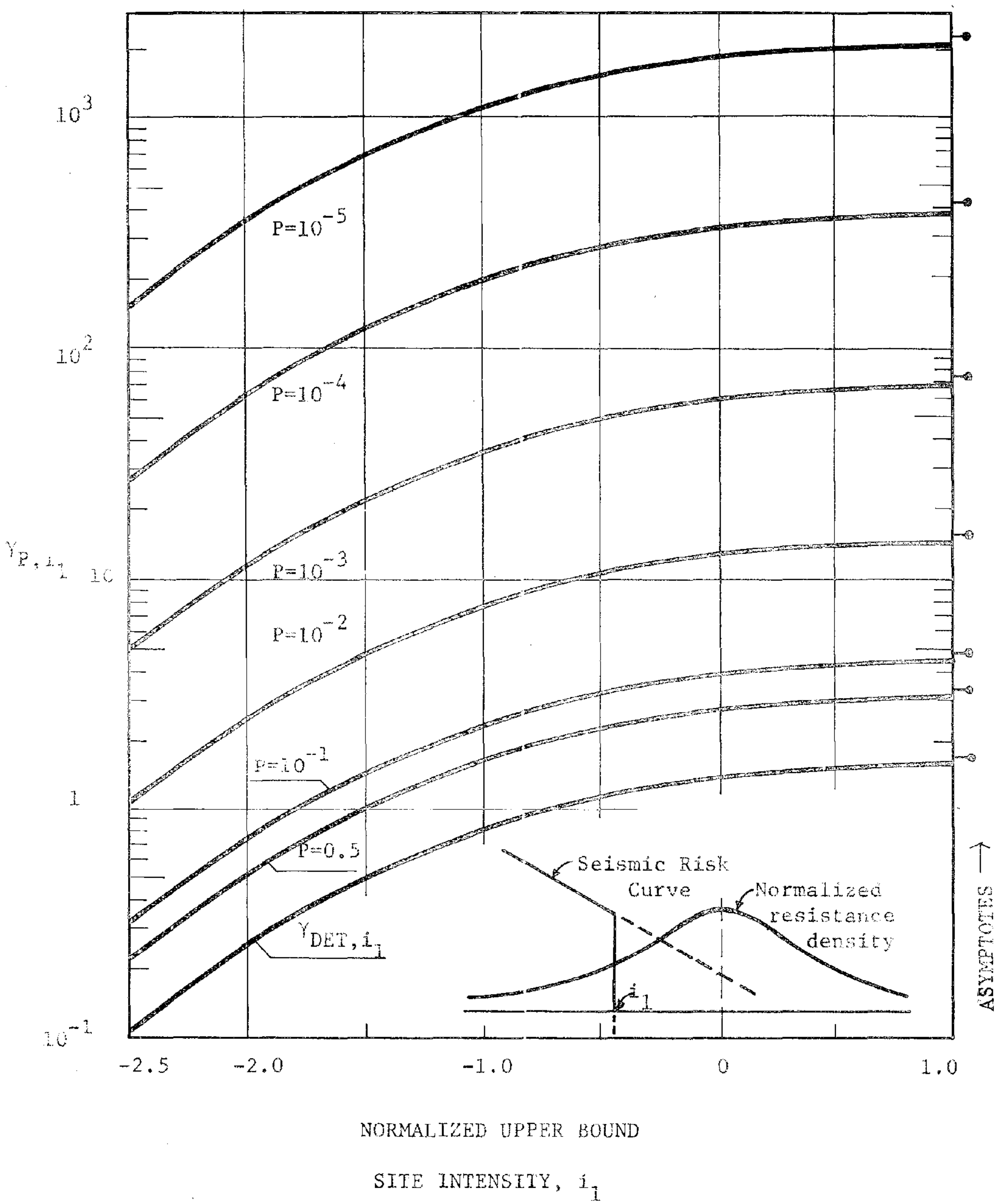


Figure IV.4 Truncated Linear Risk Curve; Ratio

$\gamma_{P,i_1} = \lambda_{f,i_1} / \lambda_{f_P}$ (Equation IV.9) $\beta_N = 1.0$

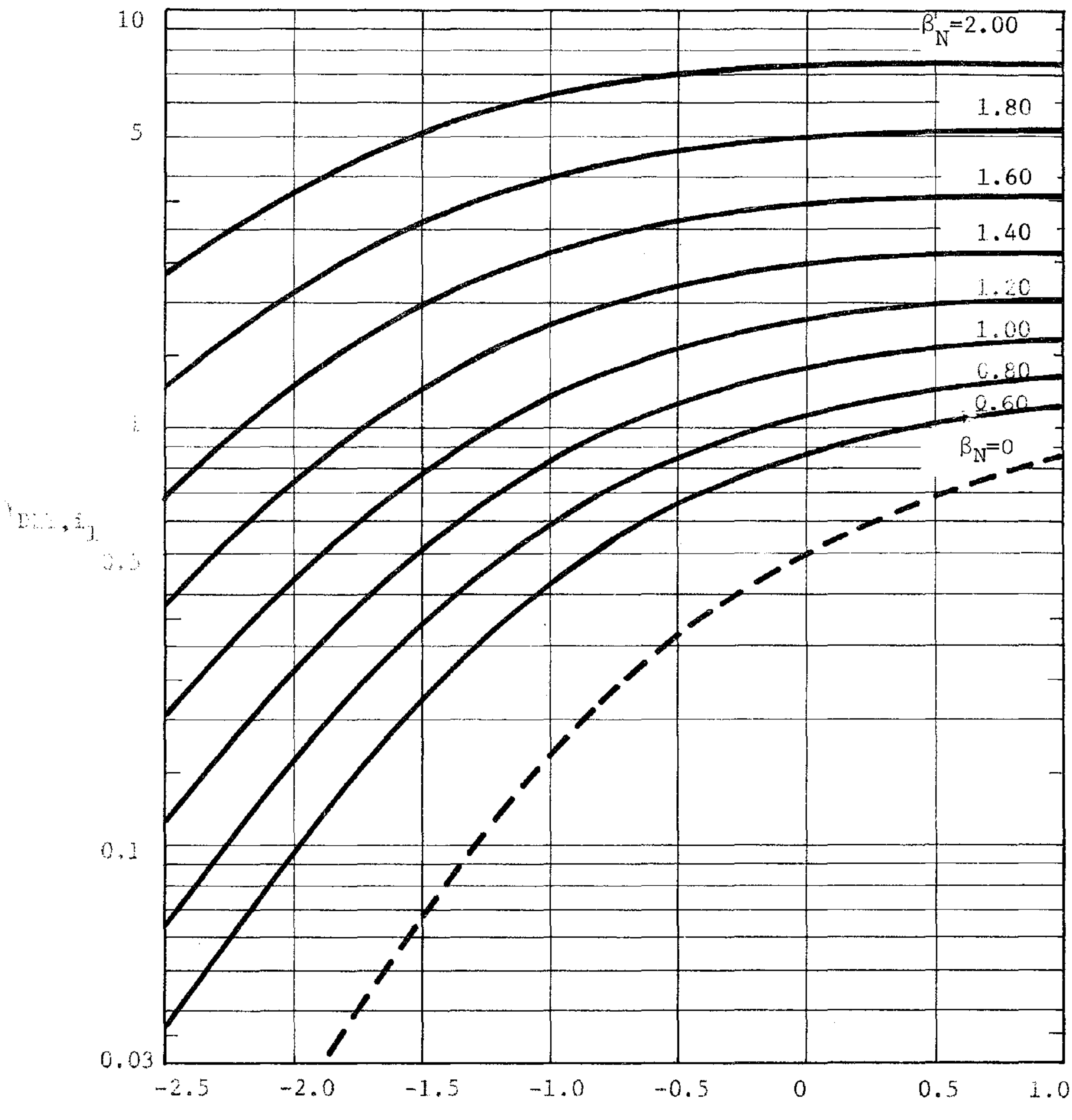


Figure IV.5 - Truncated Linear Risk Law. Ratios $\gamma_{DET,i_1} = \lambda_{f,i_1}/\lambda_o = e^{\beta_N^2/2}\,\Phi(i_1+\beta_N)$ for different values of β_N

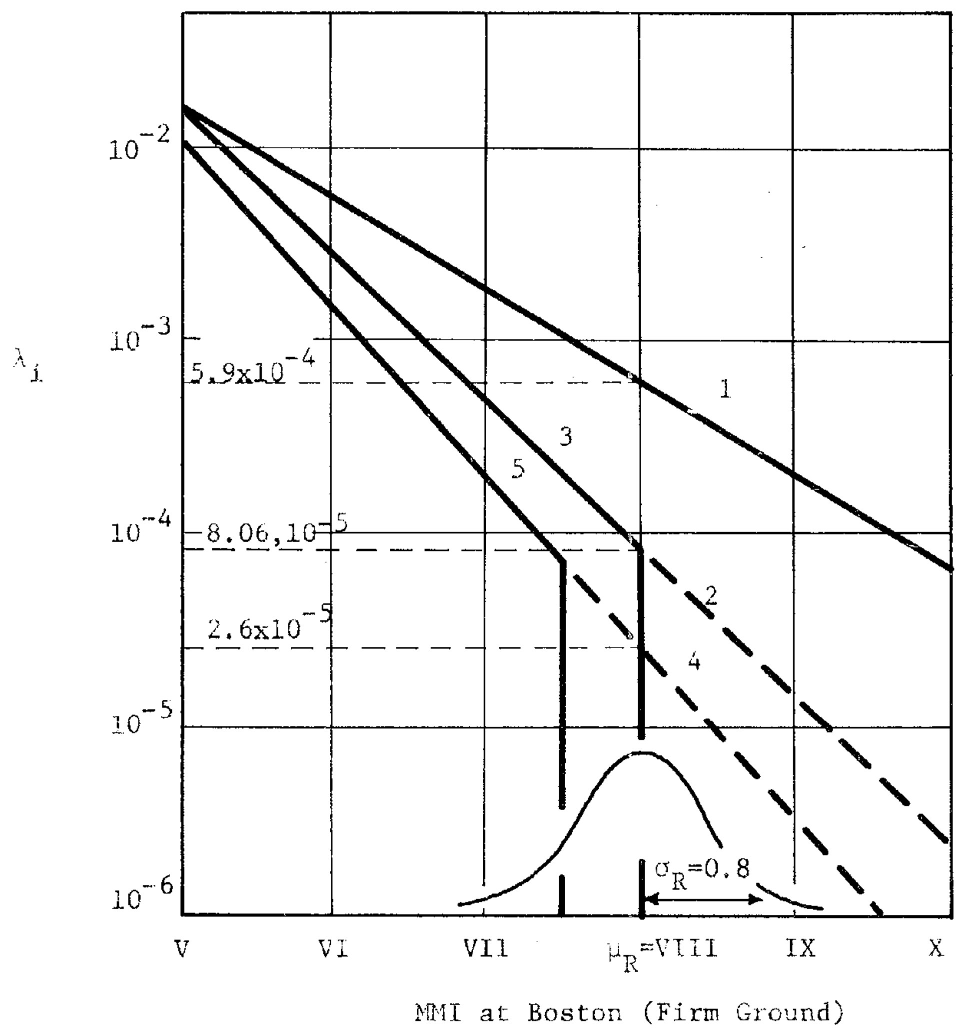


Figure IV.6 Linear and Truncated Linear Approximations to the Risk Curves in Figure II.1

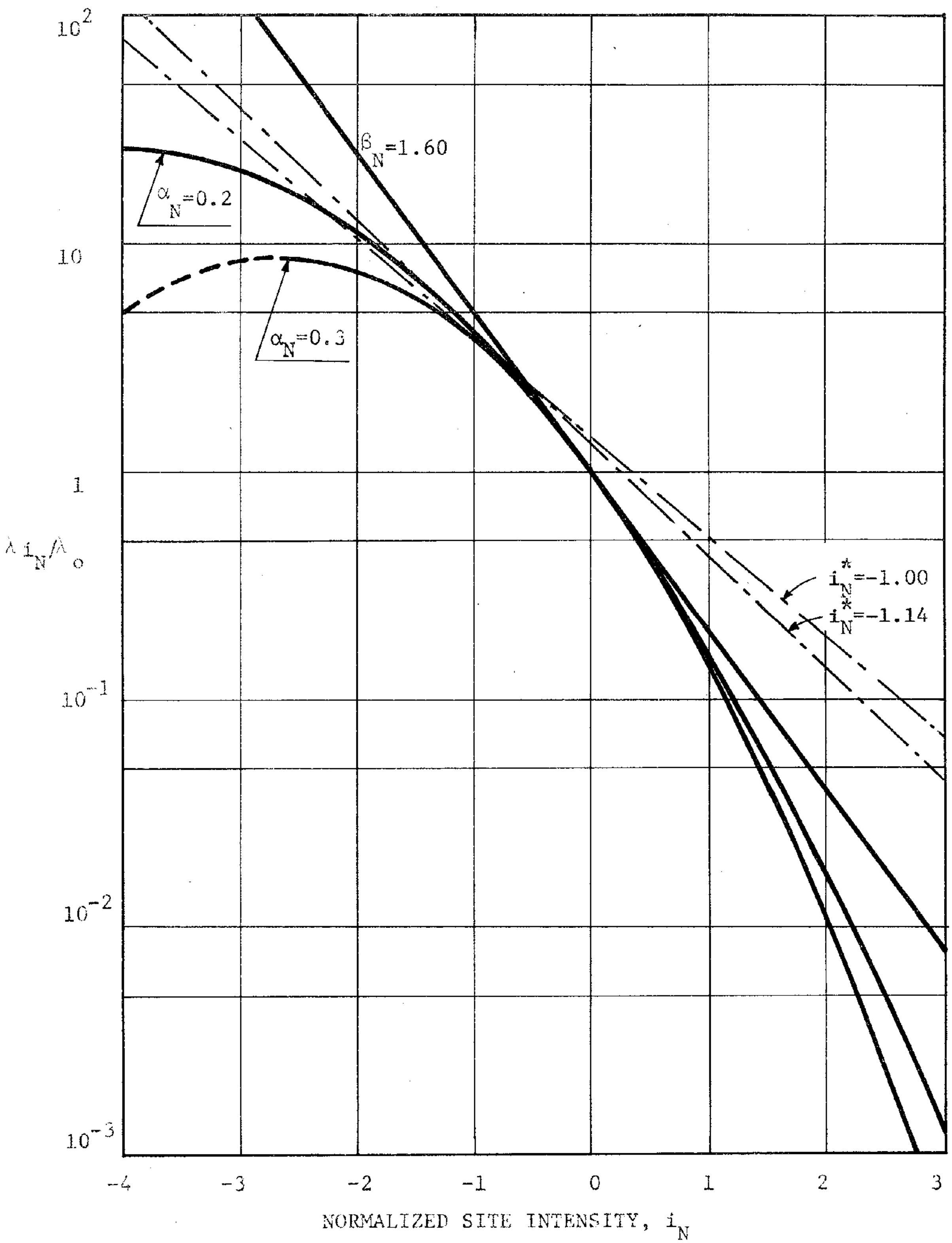


Figure IV.7 Quadratic Law (IV.14);

α_N=0.2, 0.3; β_N=1.60. The dashed-dotted lines are linearizations around $i_N^*=-\beta_N/(2\alpha_N+1)$; see Equation (IV.18)

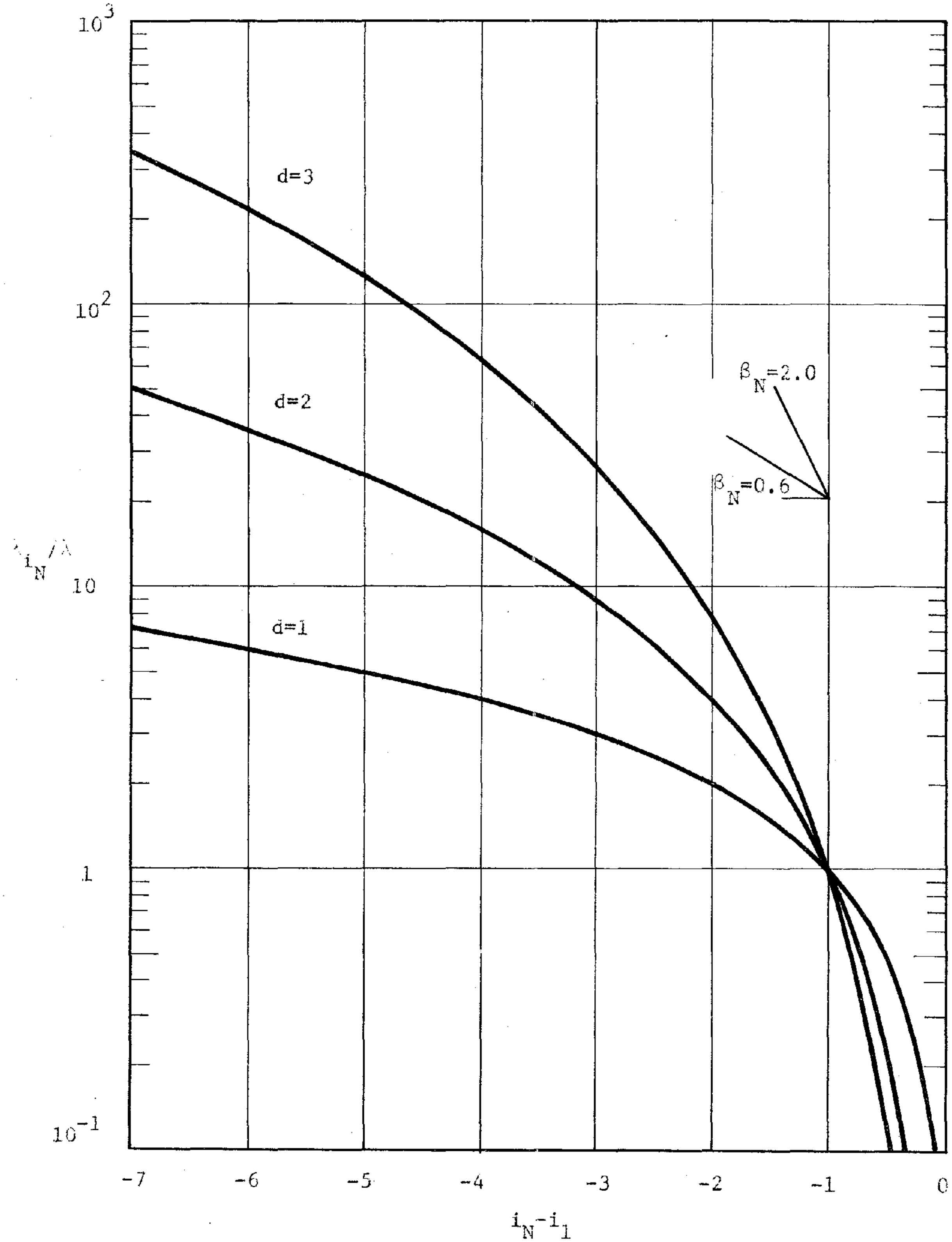


Figure IV.8 Logarithmic Frequency-Intensity Law (IV.19); C=1

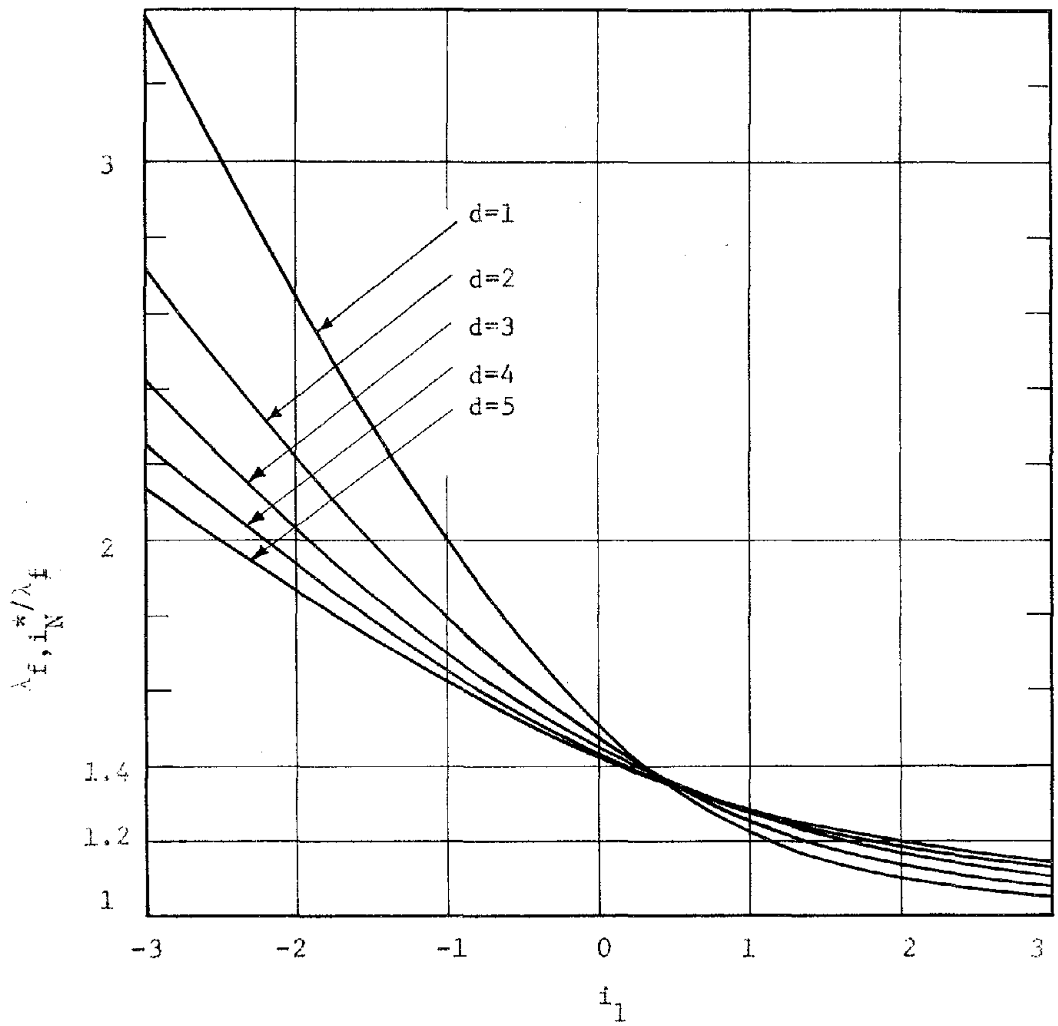


Figure IV.9 Logarithmic Law; Conservatism of the Tangent Approximation (IV.22)

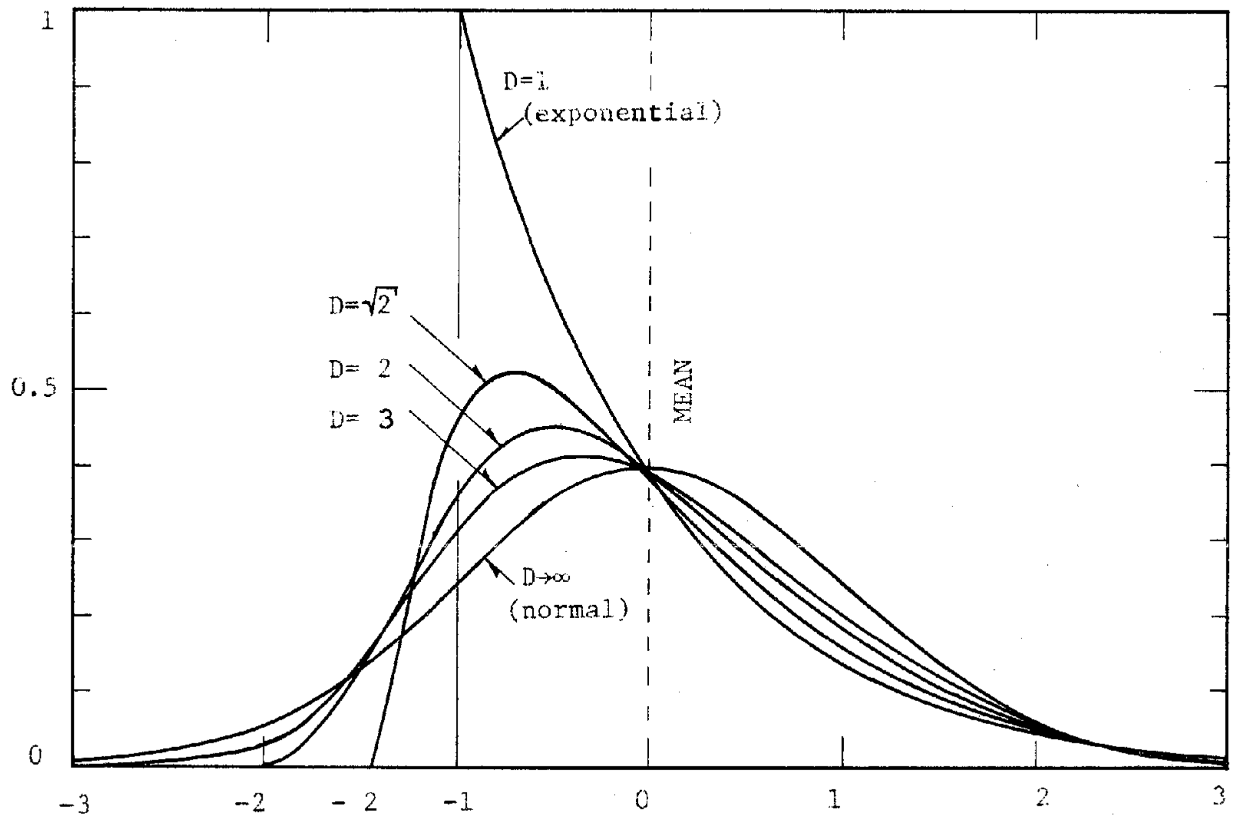


Figure IV.10 Shifted Gamma Densities, Eq.(IV.23).
In all cases the mean is zero and the variance is 1.

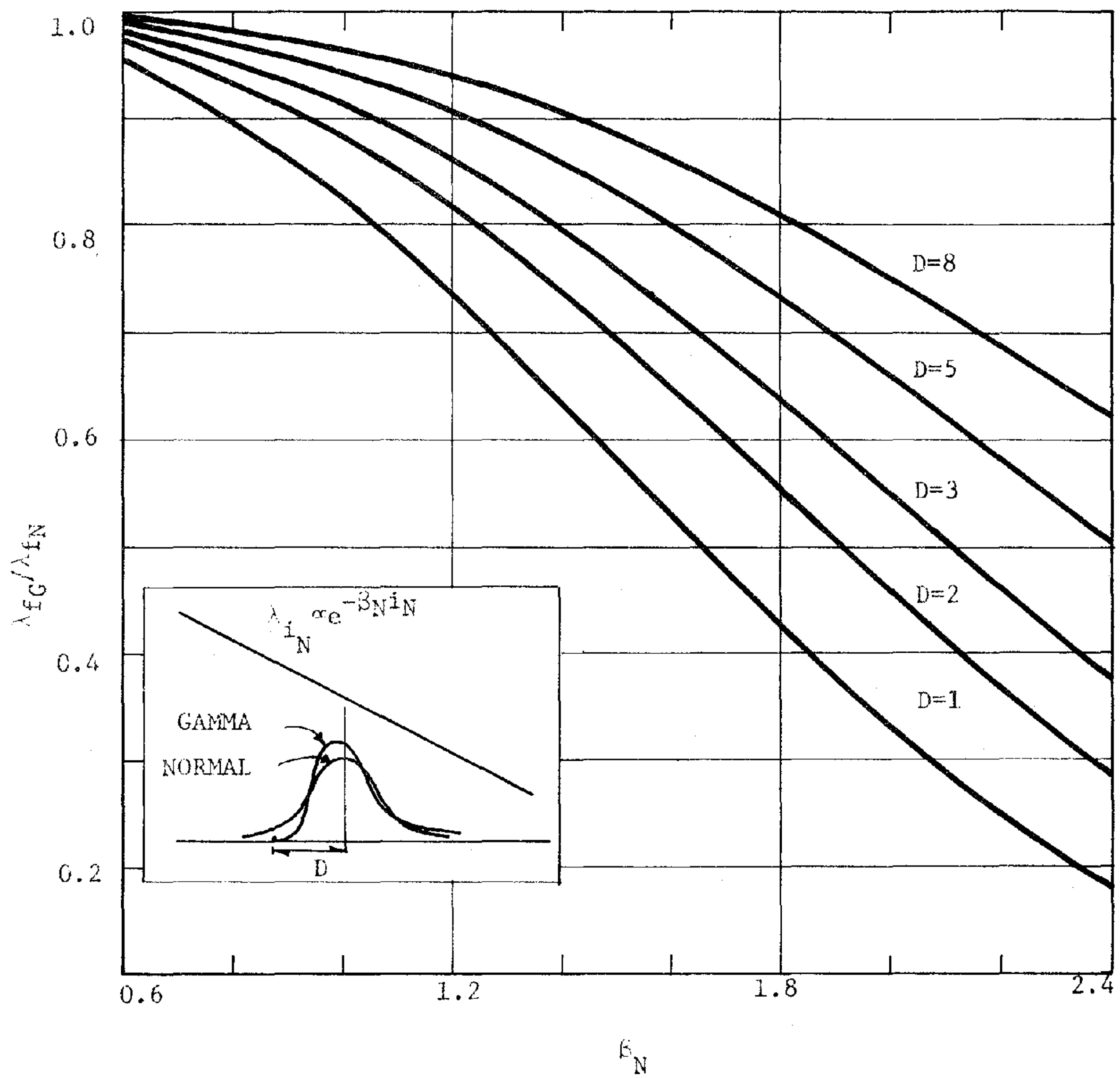


Figure IV.11 Ratio between the Mean Failure Rate for Gamma and for Normal Resistance Distributions, Eq. (IV.25)

V. Statistical Seismic Damage Models

The models analyzed in the last section are intended to be "best" estimates from statistical data. Unfortunately, the information available on the seismic risk at a site, and even more on the resistance distribution, is far from supporting conclusively any particular model. As a result, both the "correct" type of the model (e.g., whether the risk law is linear, or logarithmic, or other; whether the resistance distribution is normal, or Gamma, or other) and the "correct" parameters values (e.g., the mean occurrence rate λ_o and the slope β_N of the linear model) remain uncertain. In the same sense, the parameters of the normal or Gamma resistance distributions are essentially unknown. A few models, in which inductive uncertainties on the parameters are taken into consideration are studied in this section. Some of the numerical results are reported in Appendices A and B.

V.1 Uncertainty on Demand Parameters

In Section IV it was shown that the linear law:

$$\lambda_{i_N} = \lambda_o \, e^{-\beta_N i_N} , \qquad (V.1)$$

or a truncated version of it provide accurate approximations to calculated nonlinear risk curves at a site. The parameters λ_o and β_N depend on the regional seismicity, on the assumed upper bound epicentral intensity, and on the attenuation law. Following the general Bayesian approach in Section III, λ_o and/or β_N are considered now to be random variables, with given probability distribution. In each of the cases studied, the effect of inductive uncertainty is quantified through multiplicative penalty factors on the mean failure rate under perfect statistical information.

(a) Linear Gaussian Model; λ_o unknown; β_N known

Let N(0;1) be the probability distribution of the (standardized) resistance R. If λ_o in Equation (1) has lognormal distribution:

$$\ln \lambda_o \sim N\left(\mu_{\ln \lambda_o} ; \sigma^2_{\ln \lambda_o}\right) \qquad (V.2)$$

and β_N is known, the mean failure rate is

$$\lambda_{f|\beta_N} = \frac{E[\lambda_o]}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\beta_N i_N - i_N^2/2}\, d i_N = \exp\left\{\mu_{\ln\lambda_o} + \left[\sigma^2_{\ln\lambda_o} + \beta_N^2\right]/2\right\}, \quad \text{(V.3)}$$

which means that statistical uncertainty on λ_o increases λ_f by the (penalty) factor:

$$\gamma_{\lambda_o} = \frac{\lambda_{f|\beta_N}}{\lambda_{f|\lambda_o,\beta_N}} = e^{\sigma^2_{\ln\lambda_o}/2}. \quad \text{(V.4)}$$

γ_{λ_o} is plotted versus $e^{\sigma_{\ln\lambda_o}}$ in Figure 1. (Notice that $e^{\sigma_{\ln\lambda_o}}$ is the ratio between the values of λ_o at $(E[\ln\lambda_o]+\sigma_{\ln\lambda_o})$ and at $E[\ln\lambda_o]$). In the range $e^{\sigma_{\ln\lambda_o}}=0.4$ to 1.1 (which corresponds to 1-sigma uncertainty factors on λ_o of 1.5 to 3), the penalty γ_{λ_o} varies from 1.1 to 1.8.

(b) Linear Gaussian Model; λ_o known; β_N unknown

Now let λ_o be known, and β_N have normal distribution $N(\mu_{\beta_N};\sigma^2_{\beta_N})$. For $R\sim N(0;1)$ the mean failure rate is found to be:

$$\lambda_{f|\lambda_o} = \int_{-\infty}^{\infty} \lambda_{f|\lambda_o,\beta_N} \cdot dF_{\beta_N}(\beta_N)$$

$$= \frac{\lambda_o}{\sigma_{\beta_N}\sqrt{2\pi}} \int_{-\infty}^{\infty} \exp\left\{\frac{1}{2}\left[\beta_N^2 - (\beta_N - \mu_{\beta_N})^2/\sigma^2_{\beta_N}\right]\right\} d\beta_N = \frac{\lambda_o}{\sqrt{1-\sigma^2_{\beta_N}}}\, e^{\frac{1}{2}\mu^2_{\beta_N}/(1-\sigma^2_{\beta_N})}. \quad \text{(V.5)}$$

(In all practical situations, it is $\sigma^2_{\beta_N}<1$.)

The associated penalty factor with respect to the case $\sigma_{\beta_N}=0$ is:

$$\gamma_{\beta_N} = \frac{\lambda_{f|\lambda_o}}{\lambda_{f|\lambda_o,\beta_N}} = \left(1-\sigma^2_{\beta_N}\right)^{-1/2} e^{\frac{1}{2}\mu^2_{\beta_N}\,\sigma^2_{\beta_N}/(1-\sigma^2_{\beta_N})}. \quad \text{(V.6)}$$

In Figure 2 this factor is plotted versus μ_{β_N} for $\sigma_{\beta_N}=0.1(0.1)0.5$.

Typical values of $\mu_{\beta N}$ are between 0.8 and 1.6, and of $\sigma_{\beta N}$ between 0.1 and 0.3. This implies a typical $\gamma_{\beta N}$ range of 1.01 to 1.2 (the upper limit of this range is, however, very sensitive to the assumed maximum for $\sigma_{\beta N}$).

(c) Linear Gaussian Model; λ_o and β_N unknown

In general, both λ_o and β_n are unknown. For the marginal distributions given above and under the condition of independence the mean failure rate is

$$\lambda_f = \frac{1}{\sqrt{1-\sigma^2_{\beta_N}}} \exp\left\{\mu_{\ln\lambda_o} + \tfrac{1}{2}\sigma^2_{\ln\lambda_o} + \tfrac{1}{2}\mu^2_{\beta_N}\sigma^2_{\beta_N}/(1-\sigma^2_{\beta_N})\right\}, \qquad (V.7)$$

with associated penalty factor:

$$\gamma_{\lambda_o,\beta_N} = \lambda_f / \lambda_{f|\lambda_o,\beta_N} = \gamma_{\lambda_o} \cdot \gamma_{\beta_N} \quad ; \qquad (V.8)$$

γ_{λ_o} and γ_{β_N} as in Equations (4) and (6).

In most practical cases the assumption of independence between λ_o and β_N is not appropriate. Let then $\ln\lambda_o$ and β_N have generic bivariate normal distribution

$$\begin{bmatrix}\ln\lambda_o\\ \beta_N\end{bmatrix} \sim N\left(\begin{bmatrix}\mu_{\ln\lambda_o}\\ \mu_{\beta_N}\end{bmatrix} ; \begin{bmatrix}\sigma^2_{\ln\lambda_o} & \rho\,\sigma_{\ln\lambda_o}\sigma_{\beta_N}\\ \rho\,\sigma_{\ln\lambda_o}\sigma_{\beta_N} & \sigma^2_{\beta_N}\end{bmatrix}\right). \qquad (V.9)$$

A convenient visualization of what this distribution implies (and a convenient means of selecting the parameters of the covariance matrix) is suggested in Figure 3. In the figure, a (normalized) intensity level i_d is defined, such that the mean rate $\lambda_{i_d}=\lambda_o e^{-\beta_N i_d}$ is independent of β_N. For example, in the case of Figure II.1, the condition of independence might be satisfied at MMI 4 or 5, which implies a value $(4-\mu_R)/\sigma_R$ or $(5-\mu_R)/\sigma_R$ for i_d (μ_R and σ_R are parameters of the resistance distribution).

If the mean and the variance of $\ln\lambda_{i_d}$ are denoted simply μ_d and σ^2_d, the joint distribution of $\ln\lambda_{i_d}$ and β_N is:

$$\begin{bmatrix}\ln\lambda_{i_d}\\ \beta_N\end{bmatrix} \sim N\left(\begin{bmatrix}\mu_d\\ \mu_{\beta_N}\end{bmatrix} ; \begin{bmatrix}\sigma^2_d & 0\\ 0 & \sigma^2_{\beta_N}\end{bmatrix}\right), \qquad (V.10)$$

with an implied joint distribution of $\ln\lambda_o$ and β_N:

$$\begin{bmatrix}\ln\lambda_o\\ \beta_N\end{bmatrix} \sim N\left(\begin{bmatrix}\mu_d + i_d\mu_{\beta_N}\\ \mu_{\beta_N}\end{bmatrix} ; \begin{bmatrix}\sigma^2_d + i^2_d\sigma^2_{\beta_N} & i_d\sigma^2_{\beta_N}\\ i_d\sigma^2_{\beta_N} & \sigma^2_{\beta_N}\end{bmatrix}\right). \qquad (V.11)$$

In general it is $i_d<0$ and $\ln\lambda_o$ and β_N are negatively correlated.

From the joint distribution (9), the conditional distribution of $(\ln\lambda_o|\beta_N)$ is easily found:

$$(\ln\lambda_o|\beta_N) \sim N\left(\mu_{\ln\lambda_o} + \rho\frac{\sigma_{\ln\lambda_o}}{\sigma_{\beta_N}}(\beta_N-\mu_{\beta_N}) ; (1-\rho^2)\,\sigma^2_{\ln\lambda_o}\right).$$

Then, using Equation (3):

$$\lambda_{f|\beta_N} = \exp\left\{\mu_{\ln\lambda_o} + \rho\frac{\sigma_{\ln\lambda_o}}{\sigma_{\beta_N}}(\beta_N-\mu_{\beta_N}) + \tfrac{1}{2}\left[(1-\rho^2)\sigma^2_{\ln\lambda_o} + \beta^2_N\right]\right\}. \qquad (V.12)$$

Finally, integration with respect to β_N yields the unconditional mean failure rate:

$$\lambda_f = \int_{-\infty}^{\infty} \lambda_{f|\beta_N} \cdot f_{\beta_N}(\beta_N)\, d\beta_N$$

$$= \frac{1}{\sqrt{1-\sigma^2_{\beta_N}}} \exp\left\{\mu_{\ln\lambda_o} + \frac{1}{2}\sigma^2_{\ln\lambda_o} + \frac{1}{2}\left(\mu_{\beta_N} + \rho\,\sigma_{\ln\lambda_o}\sigma_{\beta_N}\right)^2 / \left(1-\sigma^2_{\beta_N}\right)\right\}, \quad \text{(V.13)}$$

which can be written, in the notation of Equation (11):

$$\lambda_f = \frac{1}{\sqrt{1-\sigma^2_{\beta_N}}} \exp\left\{\mu_d + i_d\mu_{\beta_N} + \frac{1}{2}\left(\sigma_d^2 + i_d^2\sigma^2_{\beta_N}\right) + \frac{1}{2}\left(\mu_{\beta_N} + i_d\sigma^2_{\beta_N}\right)^2 / \left(1-\sigma^2_{\beta_N}\right)\right\}. \quad \text{(V.14)}$$

From this equation, and after some algebra, one can express the penalty factor on λ_f due to statistical uncertainty of λ_o and β_N:

$$\gamma_{\lambda_o,\beta_N} = \gamma_{\lambda_{i_d}} \cdot \gamma_{\beta_N} \cdot \gamma_{i_d}, \quad \text{(V.15a)}$$

where: $\gamma_{\lambda_{i_d}}$ is given by Equation (4) with λ_{i_d} in place of λ_o; see also plots in Figure 1;

γ_{β_N} is given by Equation (6) and is plotted in Figure 2;

$$\gamma_{i_d} = \exp\left\{\frac{1}{2} i_d\, \sigma^2_{\beta_N}\left(i_d + 2\mu_{\beta_N}\right) / \left(1-\sigma^2_{\beta_N}\right)\right\}. \quad \text{(V.15b)}$$

For $i_d=0$, λ_o and β_N are independent, so that $\lambda_{i_d}=\lambda_o$ and Equation (15a) reproduces the results (7) and (8). If the mean rate λ_{i_d} is known with certainty (which means that the uncertainty on λ_o is totally explained by β_N) the penalty factor (15a) reduces to: $\gamma_{\lambda_o,\beta_N} = \gamma_{\beta_N}\cdot\gamma_{i_d}$. If in addition it is $i_d=0$, λ_o becomes known and $\gamma_{\lambda_o,\beta_N} = \gamma_{\beta_N}$.

The factor γ_{i_d} depends on i_d, μ_{β_N} and σ_{β_N}. Plots of γ_{i_d} versus μ_{β_N} for i_d=-1(-1)-8 and σ_{β_N}=0.1(0.1)0.5 are shown in Figure 4. For small $|i_d|$, γ_{i_d} is not sensitive to μ_{β_N} and σ_{β_N}, and is generally smaller than 1. As $|i_d|$ increases γ_{i_d} also increases, with high penalties for combinations: σ_{β_N} large, μ_{β_N} small. For μ_{β_N} in the range 0.8 to 1.6; σ_{β_N} in the range 0.1 to 0.3, and for $i_d=-5$, γ_{i_d} has values between 1.05 and 2.3. (More will be said on the selection of the seismicity parameters in Section VI.)

(d) Linear Gamma Models

Suppose now that the normalized resistance (zero mean, unit variance) has the shifted Gamma density (IV.23) and that the seismic risk at the site has the linear form (1), with one or both parameters unknown.

* β_N known, and $\ln \lambda_o \sim N(\mu_{\ln \lambda_o}; \sigma^2_{\ln \lambda_o})$. Integration of the conditional mean failure rate with respect to λ_o yields:

$$\lambda_{f|\beta_N} = \int_0^\infty \lambda_{f|\lambda_o,\beta_N} \cdot f_{\lambda_o}(\lambda_o)\, d\lambda_o$$

$$= \left(\frac{D}{D+\beta_N}\right)^{D^2} \exp\left\{\beta_N D + \mu_{\ln\lambda_o} + \frac{1}{2}\sigma^2_{\ln\lambda_o}\right\} . \qquad \text{(V.16)}$$

By comparison with the mean failure rate for λ_o and β_N known, Equation (IV.24), the penalty factor γ_{λ_o} is found to be the same as for normal resistance, i.e., Equation (4).

* If λ_o is known, and β_N has Gamma distribution G(K,r) with density:

$$f_{\beta_N}(\beta_N) = \frac{r(r\beta_N)^{K-1} e^{-r\beta_N}}{\Gamma(K)} , \qquad \text{(V.17)}$$

the use of Equations (IV.24) gives the following expression for the mean failure rate:

$$\lambda_{f|\lambda_o} = \frac{\lambda_o r^K}{\Gamma(K)} \int_0^\infty \frac{\beta_N^{K-1}}{\left(1+\frac{\beta_N}{D}\right)^{D^2}} e^{(D-r)\beta_N}\, d\beta_N$$

$$= \lambda_o \left(\frac{r}{r-D}\right)^K \sum_{n=0}^{\infty} \frac{(D^2;1;n)(K;1;n)}{n!\,[D(r-D)]^n} , \qquad \text{(V.18)}$$

where the symbol $(m;d;\nu)$ denotes

$(m;d;\nu)=m(m+d)(m+2d)\ldots(m+(\nu-1)d) \quad ; \quad \nu=1,2,\ldots$

For a generic density function $f_{\beta N}(\beta_N)$, the same mean failure rate must be calculated numerically from

$$\lambda_{f|\lambda_o} = \lambda_o \int_{all\ \beta_N} e^{\beta_N D} \left(\frac{D}{D+\beta_N}\right)^{D^2} \cdot f_{\beta_N}(\beta_N)\, d\beta_N .$$

However, for practical purposes, it is not very important which distribution one assumes for β_N, since $\lambda_{f|\lambda_o}$ is typcially close to the conditional mean rate $\lambda_{f|\lambda_o,\beta_N=\mu_{\beta N}}$ (see Equation IV.24). This qualitative conclusion is in agreement with earlier results for normal resistance and normal distribution of β_N ; see, e.g., Figure 2.

* Assume now that λ_o and β_N are correlated random variables, with distribution: $\beta_N \sim G(K,r)$ and $(\ln \lambda_o | \beta_N) \sim N(\mu_o + i_d(\beta_N - \mu_{\beta N}); \sigma_d^2)$. As already discussed for the normal model, this corresponds to $\ln \lambda_{i_d} = (\ln \lambda_o - \beta_N i_d)$ being independent of β_N, with distribution: $N(\mu_o - i_d \mu_{\beta N}; \sigma_d^2)$.

Conditional on given β_N the mean failure rate is:

$$\lambda_{f|\beta_N} = \left(\frac{D}{D+\beta_N}\right)^{D^2} \exp\left\{\mu_o - i_d \mu_{\beta_N} + \frac{1}{2}\sigma_d^2 + (i_d + D)\beta_N\right\}. \tag{V.19}$$

Integration with respect to β_N yields the unconditional mean failure rate:

$$\lambda_f = \int_0^\infty \lambda_{f|\beta_N} \cdot f_{\beta_N}(\beta_N)\, d\beta_N$$

$$= \exp\left\{\mu_o - i_d \mu_{\beta_N} + \frac{1}{2}\sigma_d^2\right\} \cdot \left(\frac{\lambda}{\lambda - D - i_d}\right)^K \sum_{n=0}^{\infty} \frac{(D^2;1;n)(K;1;n)}{n![D(\lambda - D - i_d)]^n}. \tag{V.20}$$

If λ_o and β_N are independent, then $i_d = 0$ and $\lambda_f = \gamma_{\lambda_o} \cdot \lambda_{f|\lambda_o}$, where $\gamma_{\lambda_o} = e^{\frac{1}{2}\sigma_d^2}$ is the penalty factor for statistical uncertainty on λ_o, and $\lambda_{f|\lambda_o}$ is given by Equation (18).

V.2 Uncertainty on the Resistance Parameters

One way of introducing uncertainty on the resistance parameters is to treat σ_D and the constants a_D and b_D in Equation (II.16) (see also Figure IV.1) as Bayesian random variables. In this study, the simpler approach is followed, of quantifying statistical uncertainty directly on the parameters μ_R and σ_R^2 of R. Under the assumption that the actual distribution of R is normal, use will be made of the statistical prediction results in Section III.

(a) μ_R unknown, σ_R^2 known

Let $\mu_R \sim N(\hat{\mu}_R, \sigma_R^2/n)$. In a Bayesian approach this distribution corresponds, for example, to a random sample of size n being available from the population of R, and to a noninformative prior distribution of μ_R (to n'=o in the results of Section III). The same distribution would result from the model in Figure IV.1, if σ_D and b_D were known, and a_D were estimated from n independent data points.

Under these conditions the predictive distribution of R is, from Equation (III.2), $N(\hat{\mu}_R; (1+1/n)\sigma_R^2)$, and the reduced variable $R' = (R - \hat{\mu}_R)/\sigma_R\sqrt{1+1/n}$ has standard normal distribution. If λ_o is the mean rate of events with site intensity larger than $\hat{\mu}_R$, the mean failure rate is:

$$\lambda_{f|\sigma_R} = \lambda_o \, e^{\beta_N^2 (1+1/n)/2} , \qquad \text{(V.21)}$$

where $\beta_N = \beta_I \cdot \sigma_R$. This corresponds to a penalty factor for statistical uncertainty on μ_R:

$$\gamma_{\mu_R} = \frac{\lambda_{f|\sigma_R}}{\lambda_{f|\mu_R,\sigma_R}} = e^{\beta_N^2/2n} . \qquad \text{(V.22)}$$

(See plots in Figure 5.)

(b) σ_R^2 unknown, μ_R known or unknown

In Section III it was shown that if μ_R is known, σ_R has noninformative prior distribution ($n'=0$) and a sample of size n is given from the population of R, the predictive distribution of the reduced variable $R'=(R-\mu_R)/S$ is t with n degrees of freedom (S^2 is the sample variance). Under the same conditions, but with μ_R also unknown, it was found that

$$R' = \left(\frac{n}{n+1}\right)^{1/2} \frac{R-\hat{\mu}_R}{S}$$

has t_{n-1} predictive Bayesian distribution. This fact allows one to study jointly the two cases when μ_R is known or unknown.

Let the reduced resistance R' be distributed like t_ν (i.e., $\nu=n$, or $\nu=n-1$), with density:

$$f_{R'}(r) = C_\nu \left(1 + \frac{r}{\nu}\right)^{-(\nu+1)/2} ; \qquad C_\nu = \frac{1}{\sqrt{\nu\pi}} \, \frac{\Gamma[(\nu+1)/2]}{\Gamma(\nu/2)} . \qquad \text{(V.23)}$$

Then the mean failure rate from earthquakes with (normalized) site intensity between i_{N0} and i_{N1} is:

$$\lambda_{f;\nu,i_{N_0},i_{N_1}} = \lambda_o \, C_\nu \int_{i_{N_0}}^{i_{N_1}} \left(1 + \frac{i_N^2}{\nu}\right)^{-(\nu+1)/2} \cdot e^{-\beta_N i_N} \, di_N , \qquad \text{(V.24)}$$

where $\beta_N=\beta_I S$ if μ_R is known, and $\beta_N=\beta_I S(1+\frac{1}{n})^{1/2}$ if μ_R is unknown.

As i_{N0} tends to $-\infty$, $\lambda_{f;\nu,i_{N0},i_{N1}}$ diverges for any finite ν. It therefore becomes important to establish a right truncation point i_{N0} for the resistance distribution ν (or indeed for the validity of the model as a whole), and to exclude failure events caused by earthquake loads of smaller size. The "natural" trunaction at MM intensity zero might be used for this purpose, but a higher truncation point is often more appropriate. In fact, failures at very small site intensities, say for $I \leq 3$, are due primarily to factors other than the seismic load, for example, to very poor design, or to wrong selection of materials, or to gross construction errors. In other cases, the simple knowledge that the system survived previously applied loads (seismic or other) guarantees truncation (or rapid decay) of the resistance density at low intenwity levels. The importance of i_{N0} in the calculation of λ_f is apparent from Figure 6, where the integrand in Equation (24):

$$g(i_N, \nu, \beta_N) = \left(1 + \frac{i_N^2}{\nu}\right)^{-(\nu+1)/2} e^{-\beta_N i_N} \qquad \text{(V.25)}$$

is plotted versus i_N for $\beta_N=1$ and for a set of ν values. The reason for studying this function is that it shows the relative contribution to the risk from events with various site intensities. As $\nu \to \infty$ $g(i_N, \nu, \beta_N)$ approaches $\exp\{-\frac{1}{2} i_N^2 - \beta_N i_N\}$, i.e. the integrand for normal resistance densities (see Equation IV.4).

Noticeable features of the function (25) are:

* For $i_N=0$, it is $g(0,\nu,\beta_N)\equiv 1$;

* For $\beta_N < \frac{\nu+1}{2\sqrt{\nu}}$, $g(\cdot,\nu,\beta_N)$ has a relative maximum at

$$i_N = \frac{-(\nu+1) + \sqrt{(\nu+1)^2 - 4\nu\beta_N^2}}{2\beta_N} \qquad \text{(V.26a)}$$

(as $\nu \to \infty$, this expression approaches $-\beta_N$),and a relative minimum at

$$i_N = \frac{-(\nu+1) - \sqrt{(\nu+1)^2 - 4\nu\beta_N^2}}{2\beta_N} \quad . \qquad \text{(V.26b)}$$

When $\beta_N=1$ (as in Figure 6), the relative maximum occurs at -1 for all ν, and the relative minimum is at $-\nu$. For $\beta_N < \frac{\nu+1}{2\sqrt{\nu}}$ the function (25) decreases monotonically with i_N.

* $\lim_{i_N \to -\infty} g(i_N,\nu,\beta_N) = \infty$ for all finite ν;

  $\lim_{i_N \to \infty} g(i_N,\nu,\beta_N) = 0$ for all ν.

* For $\nu\to\infty$ the relative maximum at $-\beta_N$ is the absolute maximum, about which the function is symmetric.

For positive i_N the functions $g(i_N, \nu, \beta_N)$ are practically the same for all ν. A completely different situation is found at low levels of intensity, particularly for small ν (i.e., for large statistical uncertainty on the resistance parameters). In this case the risk contribution may even increase with decreasing intensity, well within realistic ranges of i_N values. In other words, for small ν the model suggests that if failure occurs at intensity i_{N0} or higher, it is most likely that the event was caused either by an earthquake with very low site intensity (close to i_{N0}), or by an earthquake with site intensity close to the value in Equation (26a). One should associate the former failure events with systems having "very poor performance" (systems of this kind are infrequent, but they rarely escape seismic failure, due to the high frequency of small size shocks), and the latter failure events with rare, but highly destructive earthquakes, having intensity levels close to (but smaller than) the mean resistance of the system. Smaller risk is associated with earthquakes of intermediate size, or with ground motions having site intensity larger than the mean resistance of the system.

While i_{N0} in Equation (24) depends mainly on the truncation of the resistance distribution, i_{N1} depends on the site intensity upper bound. The curves in Figure 6 show that the mean failure rate in Equation (24) is not sensitive to i_{N1}, provided that truncation is above the mean resistance; instead, λ_F may be quite sensitive to i_{N0}, particularly for small ν. This is a qualitatively new result in engineering seismic risk analysis, showing that combinations other than "high demand-average resistance" may dominate the damage statistics, and warning about the possible presence of an intensity range below the mean resistance, with rather uniform contribution to the total risk.

Results from the numerical integration of Equation (24) (a resistance density normalization factor $[1-t_\nu(i_{N0})]^{-1}$ was included in the calculations) are displayed in Figures 7 through 12. In all cases the mean failure rate $\lambda_{f;\nu,i_{N_0},i_{N1}}$ is normalized with respect to the mean failure rate for no statistical uncertainty ($\nu\to\infty$) and for an unbounded intensity range; i.e., with respect to

$$\lambda_{f;\infty,-\infty,\infty} = \lambda_0 e^{\beta_N^2/2}.$$

For $i_{N0}=-8$, the ratio $\dfrac{\lambda_{f;\nu,-8,\infty}}{\lambda_{f;\infty,-\infty,\infty}}$ is plotted in Figure 7 for ν=1,3,5,7,10,20, as a function of β_N. The effect of statistical uncertainty on λ_f increases dramatically with β_N, and in all cases is non-negligible. It should be said, however, that $i_{N0}=-8$ is a rather conservative value for the lower bound. It would result, for example, from a known mean resistance $\mu_R=10$ (MMI scale), from an estimated standard deviation $S_R=1$, and from a resistance truncation point at MMI=2. For the same values of μ_R and S_R, but the lower truncation point moved to 5, one should use the value $i_{N0}=-5$. A second argument in favor of a higher truncation point comes from the nonlinearity of the empirical log mean-damage-ratio as a function of intensity (See Section II.2 and Figures II.5 through II.11). In the present "linear" resistance model (See Figure IV.1), the rapid decrease of log MDR at low intensities can be approximately accounted for through more severe truncations of the resistance distribution.

Figures 8,9 and 10 contain plots of the ratio $\frac{\lambda_{f;\nu,i_{N_0},\infty}}{\lambda_{f;\infty,-\infty,\infty}}$ for $i_{N0}=-4(-1)-8$ and for $\nu=1$ (Figure 8), $\nu=5$ (Figure 9) and $\nu=10$ (Figure 10). These figures confirm previous observations on the high sensitivity of the mean failure rate to the lower truncation point, particularly for large values of β_N.

The effects of varying the upper limit of integration in Equation(24) (this limit coincides with the upper truncation point in the frequency-site intensity law) are quantified in Figures 11 and 12 for $\nu=5$ and $\nu=10$, respectively. In both cases it is $i_{N0}=-8$. The upper curves in these figures are for $i_{N1}=\infty$. It is seen that for $i_{N1}>0$ the upper truncation has no appreciable effect on λ_f; also, for any given i_{N1}, the effect of truncation decreases with β_N. For example, for $i_{N0}=-8$, the values of i_{N1} in Table 1 are required to reduce the mean failure rate $\lambda_{f;\nu,-8,\infty}$ by a factor of 2.

β_N	$\nu=5$	$\nu=10$
0.8	-1.5	-1.0
1.0	-2.6	-1.4
1.2	-5.0	-1.9
1.4	-6.5	-2.7
1.6	-7.0	-4.3

Table V.1 Values of i_{N1} such that $\frac{\lambda_{f;\nu,-8,\infty}}{\lambda_{f;\nu,-8,i_{N_1}}} = 2$.

Tables of the ratio $\frac{\lambda_{f;\nu,i_{N_0},i_{N_1}}}{\lambda_{f;\infty,-\infty,\infty}}$ are collected in Appendix A, for ν=5,10,20; i_{N0}=-8(1)-3; i_{N1}=4(-1)i_{N0}; and β_N=0.6(0.2)2.0.

V.3 UNCERTAINTY ON BOTH DEMAND AND RESISTANCE PARAMETERS

Consider the linear risk model

$$\lambda_i = \lambda_{i_o} e^{-\beta_I (i-i_o)} \tag{V.27}$$

and the normal resistance model:

$$R = \mu_R + \varepsilon_R \quad , \quad \varepsilon_R \sim N(o;\sigma_R^2) \ , \tag{V.28}$$

μ_R, σ_R^2 known or unknown.

For a seismic risk law in the form (27) - as opposed to the equivalent form (1) - it is reasonable to assume that the seismic demand parameters, λ_{i0} and β_I, are independent of the resistance parameters, μ_R and σ_R^2. In the remainder of the present section two cases are considered: (a) λ_{i0}, β_I and μ_R unknown, σ_R^2 known; and (b) λ_{i0}, β_I and σ_R^2 unknown, μ_R known or unknown.

(a) λ_{i0}, β_I, μ_R unknown ; σ_R^2 known

Let $\beta_N=\beta_I \cdot \sigma_R$; $i_d=(i_o-\hat{\mu}_R)/\sigma_R$ ($\hat{\mu}_R$ is an estimate of the mean of R), and $\lambda_{i_d}=\lambda_{i_o}$ have independent normal distribution:

$$\begin{bmatrix} \ln \lambda_{i_d} \\ \beta_N \\ \mu_R \end{bmatrix} \sim N\left(\begin{bmatrix} \mu_d \\ \mu_{\beta_N} \\ \hat{\mu}_R \end{bmatrix} ; \begin{bmatrix} \sigma_d^2 & o & o \\ o & \sigma_{\beta_N}^2 & o \\ o & o & \sigma_R^2/n \end{bmatrix} \right). \tag{V.29}$$

Implied by the model (29) is that R~N ($\hat{\mu}_R$,σ^2(1+1/n), and that the parameter of the frequency-site intensity law, written now:

$$\lambda_{i_N} = \lambda_o e^{-\beta_N i_N} ,$$

where $\lambda_o=\lambda_{i_0} e^{-\beta_I(\hat{\mu}_R - i_o)}$ is the mean rate of events with site intensity greater than $\hat{\mu}_R$, and

$i_N=(i-\hat{\mu}_R)/\sigma_R$,

have the distribution (11) , independent of R. The mean failure rate is still given by Equation (14), after replacing

μ_{β_N} by μ_{β_N} $(1+1/n)^{1/2}$

$\sigma^2_{\beta_N}$ by $\sigma^2_{\beta_N}$ $(1+1/n)$, and

i_d by i_d $(1+1/n)^{-1/2}$

The penalty factor for statistical uncertainty becomes:

$$\gamma_{\lambda_o,\beta_N,\mu_R} = \frac{\lambda_{f|\sigma_R}}{\lambda_{f|\lambda_o,\beta_N,\mu_R,\sigma_R}} = \gamma_{\lambda_{i_d}} \cdot \gamma_{\beta_N} \cdot \gamma_{i_d} \cdot \gamma_{\mu_R} \quad , \qquad \text{(V.30)}$$

where: $\lambda_{f|\lambda_o,\beta_N,\mu_R,\sigma_R}$ is the mean failure rate for no statistical uncertainty when all the parameters equal their mean values;

$\gamma_{\lambda_{i_d}}$ is given by Equation (4), with $\sigma^2_{\ln\lambda_o}$ replaced by σ^2_d (see also Figure 1);

γ_{β_N} is given by Equation (6), with $\mu_{\beta_N}(1+1/n)^{1/2}$ in place of μ_{β_N} and $\sigma^2_{\beta_N}(1+1/n)$ in place of $\sigma^2_{\beta_N}$ (see also Figure 2);

γ_{i_d} is given by Equation (15b) with the replacements above and, in addition, $i_d(1+1/n)^{-1/2}$ instead if i_d (see also Figure 4);

γ_{μ_R} is given by Equation (22).

For $n\to\infty$ the factor (30) approaches the factor $\gamma_{\lambda_o,\beta_N}$ in Equation (15a). The only partial factor in Equation (30) which does not depend on n is $\gamma_{\lambda_{i_d}}$. To exemplify the dependence of the remaining partial factors - and so of $\gamma_{\lambda_o,\beta_N,\mu_R}$ - on n, consider the realistic case:

$\mu_{\beta_N}=1.4$; $\sigma_{\beta_N}=0.2$; $i_d=-5$. The factors γ_{β_N}, γ_{i_d}, γ_{μ_R} and their product are given in Table 2 for $n=\infty,10,5$.

n	γ_{β_N}	γ_{i_d}	γ_{μ_R}	$\gamma_{\beta_N}\,\gamma_{i_d}\,\gamma_{\mu_R}$
∞	1.063	1.258	1	1.337
10	1.083	1.248	1.103	1.491
5	1.107	1.232	1.217	1.659

Table V.2 Partial penalty factors in Equation (V.30); $\mu_{\beta_N}=1.4$; $\sigma_{\beta_N}=0.2$ and $i_d=-5$.

The increase of $\gamma_{\lambda_o,\beta_N,\mu_R}$ with decreasing n is due primarily to the factor γ_{μ_R}.

(b) λ_{i_0}, β_I, σ_R^2 unknown; μ_R known or unknown

Consider now the case when the normalized resistance R' (see Section V.2b for definition) has t_ν-distribution (23), as a result of uncertainty on σ_R^2, and possibly on μ_R. λ_o and β_I are Bayesian random variables, independent of R'. Let:

β_N=as in Equation (24);

$$i_N = \begin{cases} (i-\mu_R)/S & \text{, if } \mu_R \text{ is known;} \\ (1+1/n)^{-1/2}(i-\hat{\mu}_R)/S & \text{, if } \mu_R \text{ is unknown;} \end{cases}$$

$$i_d = \begin{cases} (i_o-\mu_R)/S & \text{, if } \mu_R \text{ is known;} \\ (1+1/n)^{-1/2}(i_o-\hat{\mu}_R)/S & \text{, if } \mu_R \text{ is unknown;} \end{cases}$$

$\lambda_{i_d} = \lambda_{i_o}$.

The joint distribution of ln λ_{i_d} and β_N is assumed to be normal, as in Equation (29). If i_{N0} is the lower truncation point of the resistance distribution and i_{N1} is the upper truncation point of the risk curve, the mean failure rate is:

$$\lambda_{f,i_{N_0},i_{N_1}} = \frac{c_\nu}{1-t_\nu(i_{N_0})} \cdot E[\lambda_{i_d}] \int_{-\infty}^{\infty} f_{\beta_N}(\beta_N) \int_{i_{N_0}}^{i_{N_1}} \left(1+\frac{i_N^2}{\nu}\right)^{-(\nu+1)/2} e^{-\beta_N(i_N-i_d)} \, di_N \, d\beta_N$$

$$= \frac{c_\nu}{1-t_\nu(i_{N_0})} \cdot E[\lambda_{i_d}] \int_{i_{N_0}}^{i_{N_1}} \left(1+\frac{i_N^2}{\nu}\right)^{-(\nu+1)/2} \int_{-\infty}^{\infty} e^{-\beta_N(i_N-i_d)} f_{\beta_N}(\beta_N) \, d\beta_N \, di_N ,$$

where c_ν is the constant in Equation (23); $t_\nu(\cdot)$ is the CDF of t_ν;

$$E[\lambda_{i_d}] = \exp\{\mu_d + \sigma_d^2/2\} \quad ; \quad \beta_N \sim N(\mu_{\beta_N} ; \sigma_{\beta_N}^2).$$

Integration with respect to β_N yields:

$$\lambda_{f,i_{N_0},i_{N_1}} = \frac{c_\nu}{1-t_\nu(i_{N_0})} e^{\mu_d + \sigma_d^2/2} \int_{i_{N_0}}^{i_{N_1}} \left(1+\frac{i_N^2}{\nu}\right)^{-(\nu+1)/2}$$

$$\cdot \exp\left\{-\mu_{\beta_N}(i_N-i_d) + \frac{1}{2}\sigma_{\beta_N}^2 (i_N-i_d)^2\right\} di_N \, . \qquad \text{(V.31)}$$

The penalty factor $\gamma_{\lambda i_d,\beta_N,\sigma_R}$ (or $\gamma_{\lambda i_d,\beta_N,\mu_R,\sigma_R}$), which is the ratio between $\lambda_{f,i_{N_o},i_{N_1}}$ in Equation (31) and the limit mean failure rate:

$$\lim_{\substack{i_{N_o}\to-\infty \\ i_{N_1}\to\infty \\ \nu\to\infty}} \lambda_{f,i_{N_o},i_{N_1}} = \exp\{\mu_d+\mu_{\beta N}\cdot i_d+\mu_{\beta N}^2/2\}$$

, depends on $\sigma_d^2, \mu_{\beta N}, \sigma_{\beta N}^2, i_d, \nu, i_{N_o}$ and i_{N_1}:

$$\gamma_{\lambda i_d,\beta_N,\mu_R,\sigma_R} = \frac{c_\nu}{1-t_\nu(i_{N_o})}\, e^{\sigma_d^2/2}\, e^{-\mu_{\beta_N}^2/2} \int_{i_{N_o}}^{i_{N_1}} \left(1+\frac{i_N^2}{\nu}\right)^{-(\nu+1)/2} \cdot \exp\left\{-\mu_{\beta_N} i_N + \frac{1}{2}\sigma_{\beta_N}^2 (i_N - i_d)^2\right\} di_N. \quad \text{(V.32)}$$

Since the dependence on σ_d^2 is only through the multiplicative factor $e^{\sigma_d^2/2}$, it is convenient to tabulate the ratio $\gamma_{\lambda i_d,\beta_N,\mu_R,\sigma_R}/e^{\sigma_d^2/2}$.

This is done in Appendix B for the following values of the remaining parameters:

$\mu_{\beta N}$=0.6,1.0,1.4,1.8

$\sigma_{\beta N}$=0.1,0.2,0.3

i_d =-3,-6

ν =5,10

i_{N_o}=-8(1)-3

i_{N_1}=4(-1)i_{N_o}

It is interesting to compare the exact penalty, Equation (32), with the approximation by partial factor multiplication; i.e. with $\gamma_{\lambda i_d}\cdot\gamma_{\beta N}\cdot\gamma_{i_d}\cdot\gamma_{\mu_R,\sigma_R}$, where

$$\gamma_{\lambda i_d}\cdot\gamma_{\beta_N}\cdot\gamma_{i_d} = \gamma_{\lambda_o,\beta_N} \qquad \text{(Equation 15a);}$$

$$\gamma_{\mu_R,\sigma_R} = \frac{\lambda_{f;\nu,i_{N_o},i_{N_1}}}{\lambda_{f;\infty,-\infty,\infty}} \qquad \text{is tabulated in Appendix A.}$$

For example, using the following set of parameters' values:

Load parameters: $i_d = -6$; $\mu_{\beta_N} = 1.4$; $\sigma_{\beta_N} = 0.2$; $i_{N_1} = 0$;
Resistance parameters: $\nu = 5$; $i_{N_0} = -5$;

one finds, from Appendix B:

$$\gamma_{\lambda\, i_d, \beta_N, \mu_R, \sigma_R} = 2.699\ e^{\sigma_d^2/2}$$

(meaning a mean failure rate: $2.699\ e^{\sigma_d^2/2} \cdot \lambda_0 e^{\mu_{\beta_N}^2/2}$). From the partial factor approximation one would find instead:

$$\gamma_{\lambda\, i_d} = e^{\sigma_d^2/2} \quad ; \quad \gamma_{i_d} = 1.492 \quad ;$$

$$\gamma_{\beta_N} = 1.063 \quad ; \quad \gamma_{\mu_R, \sigma_R} = 2.107 \ ;$$

and

$$\gamma_{\lambda\, i_d, \beta_N, \mu_R, \sigma_R} \approx 3.342\ e^{\sigma_d^2/2}.$$

A more extensive comparison is shown in Tables 3 a,b,c,d, where the following parameters' values are considered:

Table 3a: $i_d = -6$; $\mu_{\beta_N} = 1.4$; $\sigma_{\beta_N} = 0.2$; $i_{N_1} = 0$; $\nu = 5$; $i_{N_0} = -8(1)-3$;

Table 3b: Same as Table 3a, except for $i_d = -3$;
Table 3c: Same as Table 3a, except for $\sigma_{\beta_N} = 0.3$;
Table 3d: Same as Table 3b, except for $\sigma_{\beta_N} = 0.3$.

Although generally conservative, the approximation by partial factors gives penalties which are sometimes smaller than the exact values. One can explain this as follows: in the approximation, the effect of β_N unknown is calculated under the assumption of normal resistance distribution, while in the exact calculation the same distribution is of t-type (t_5 in Table 3). If $|i_d|$ is small (Tables 3b and 3d) and, at the same time, $|i_{N_0}|$ is large, replacing the normal distribution by the t_5-distribution increases the risk from low intensity earthquakes (See Figure 6), and introduces unconservatism in the approximation. The approximation by partial factors is, instead, conservative for larger $|i_d|$ values, because in this case the sensitivity of the mean failure rate to β_N is smaller if the resistance distribution is t, than if the resistance distribution is normal.

For some numerical evaluation of λ_f when both demand and resistance parameters are unknown, see Section VI.

i_{N_o}	BY PARTIAL FACTORS	EXACT
-8	11.74	8.16
-7	7.20	5.15
-6	4.78	3.61
-5	3.34	2.70
-4	2.39	2.07
-3	1.66	1.55

(a)

i_{N_o}	BY PARTIAL FACTORS	EXACT
-8	7.97	9.54
-7	4.89	5.20
-6	3.24	3.22
-5	2.27	2.19
-4	1.62	1.56
-3	1.13	1.10

(b)

i_{N_o}	BY PARTIAL FACTORS	EXACT
-8	22.09	9.56
-7	13.55	6.36
-6	8.99	4.80
-5	6.29	3.89
-4	4.49	3.22
-3	3.13	2.63

(c)

i_{N_o}	BY PARTIAL FACTORS	EXACT
-8	8.81	13.59
-7	5.40	6.24
-6	3.58	3.52
-5	2.51	2.31
-4	1.79	1.64
-3	1.25	1.17

(d)

Table V.3 Penalty factors: $\gamma\lambda_{i_d,\beta_N,\mu_R,\sigma_R}/e^{\sigma_d^2/2}$.

Parameters' values:

Table a: $i_d=-6$; $\mu_{\beta N}=1.4$; $\sigma_{\beta N}=0.2$; $i_{N_1}=0$; $\nu=5$

Table b: Same as Table a, except for $i_d=-3$;

Table c: Same as Table a, except for $\sigma_{\beta N}=0.3$;

Table d: Same as Table b, except for $\sigma_{\beta N}=0.3$.

The exact values are from Appendix B.

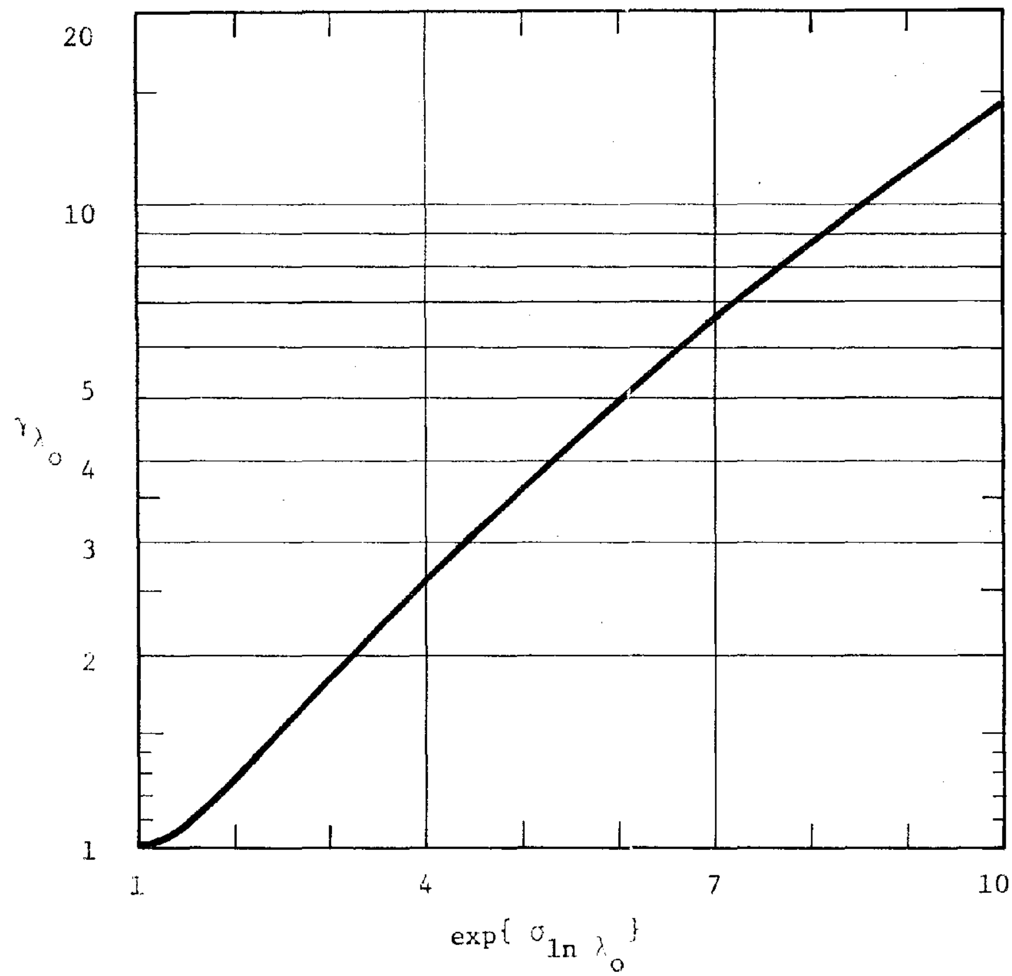


Figure V.1 Penalty factor γ_{λ_o} for statistical uncertainty on the seismicity parameter λ_o; Equation (V.4).

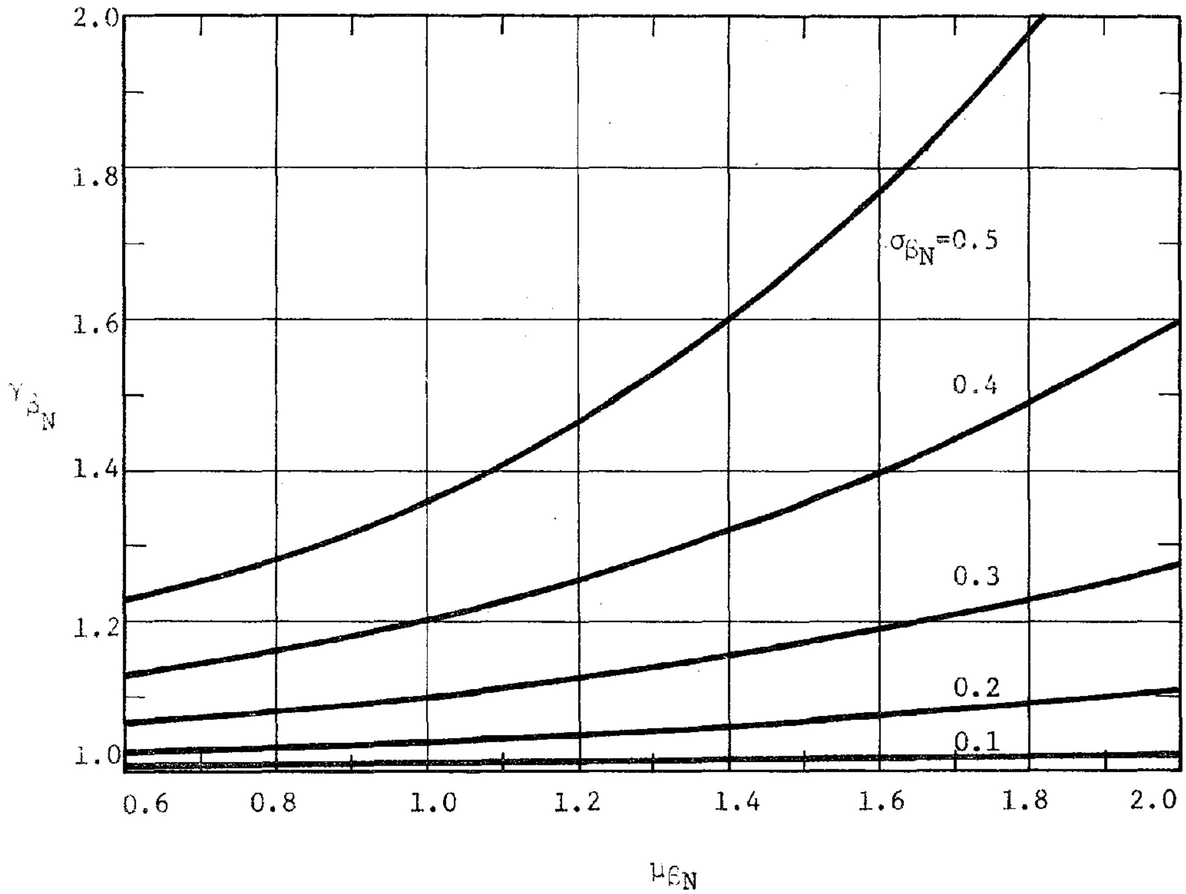
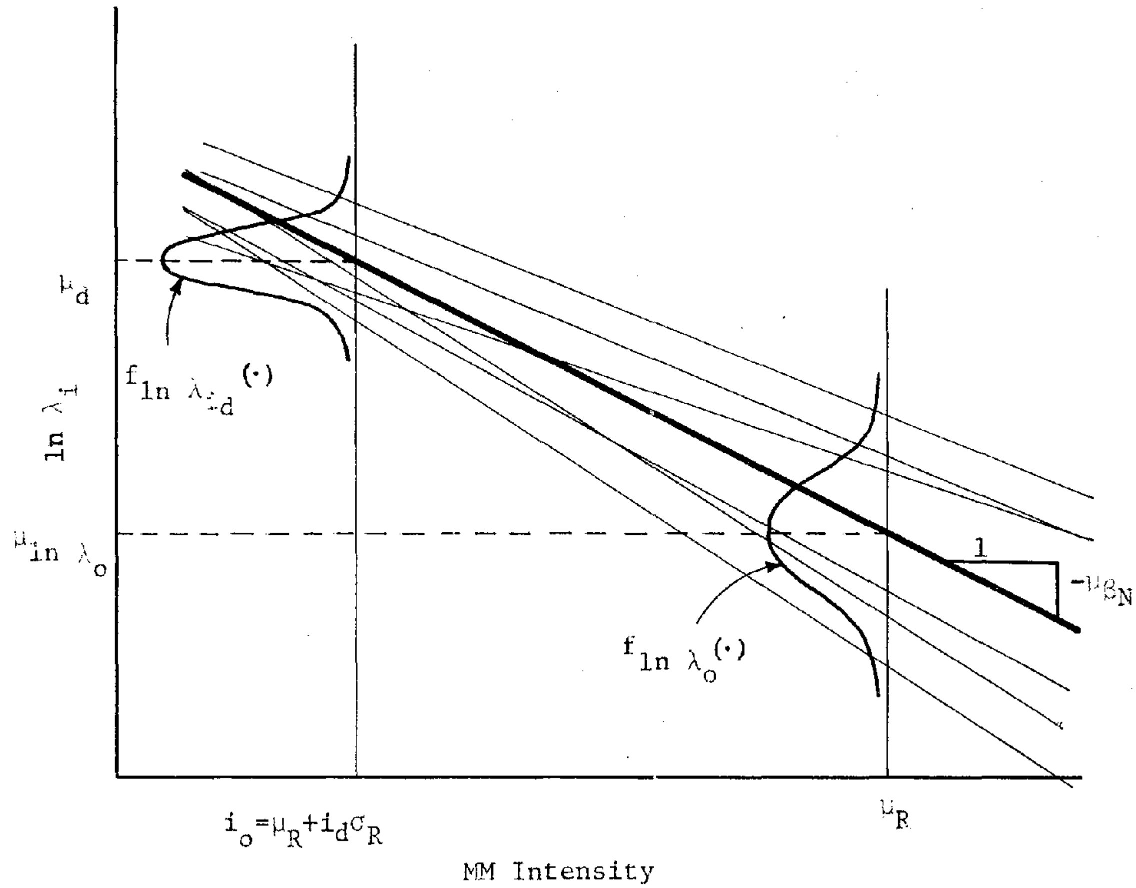


Figure V.2 Penalty factor for statistical uncertainty on the seismicity parameter β_N; Equation (V.6)



<u>Figure V.3</u> Seismic risk model for both λ_o and β_N unknown

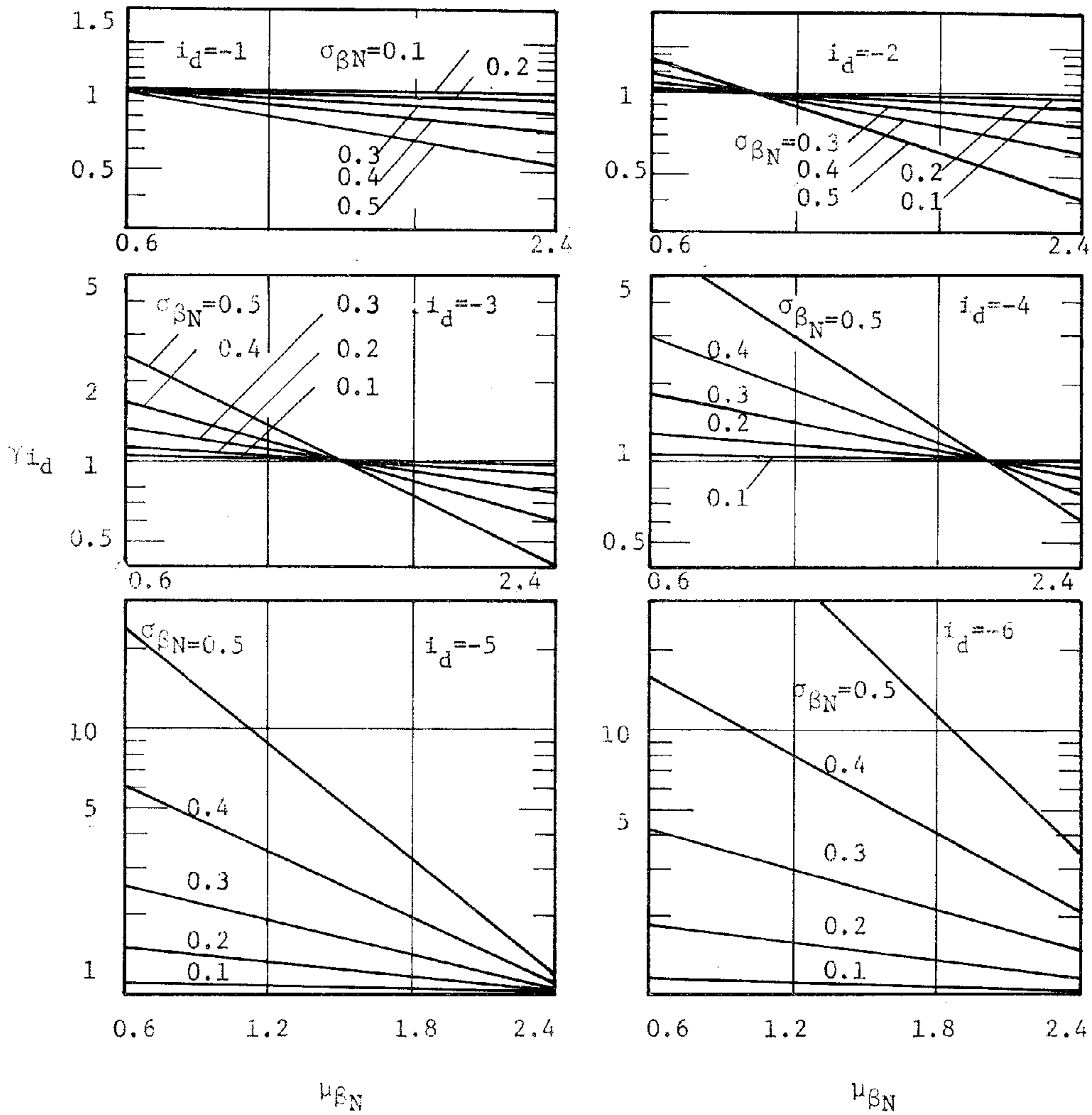


Figure V.4 Penalty factor γ_{i_d} (Equation V.15b). β_N, or β_N and λ_o, unknown (cont.)

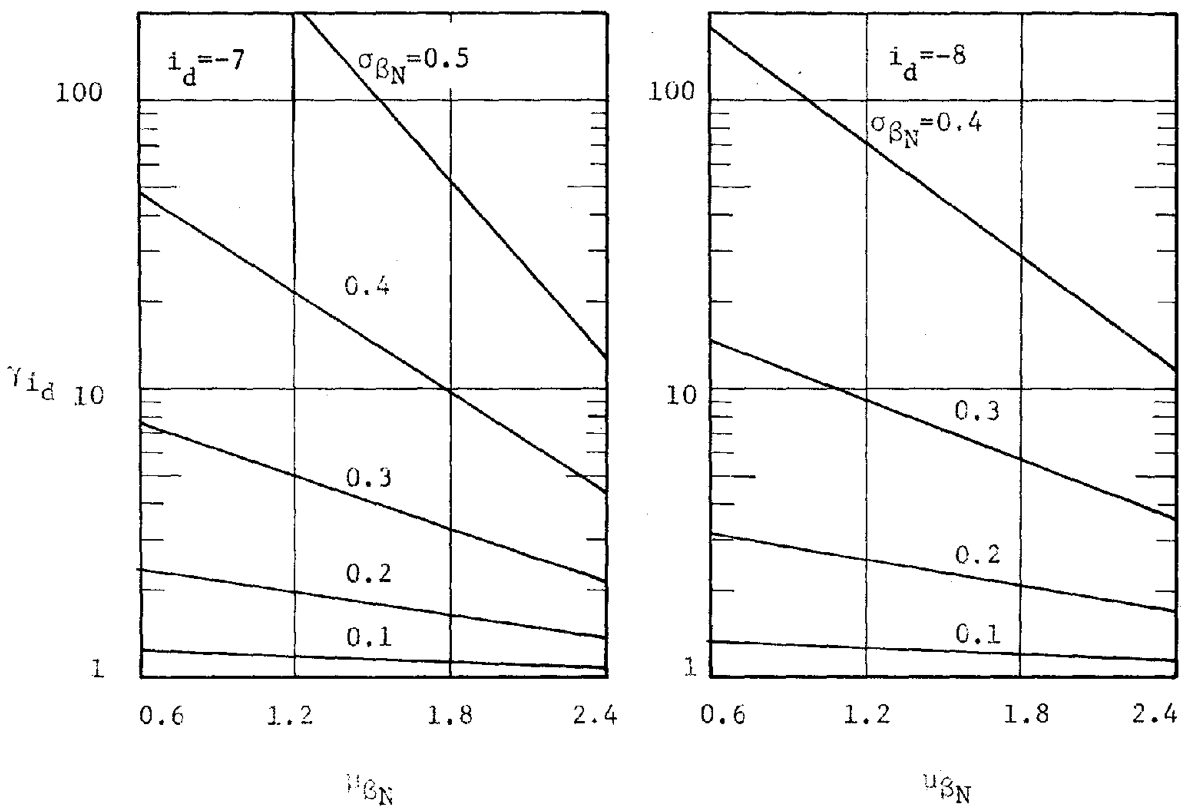


Figure V.4 (cont) Penalty factor γ_{i_d} (Equation V.15 b)

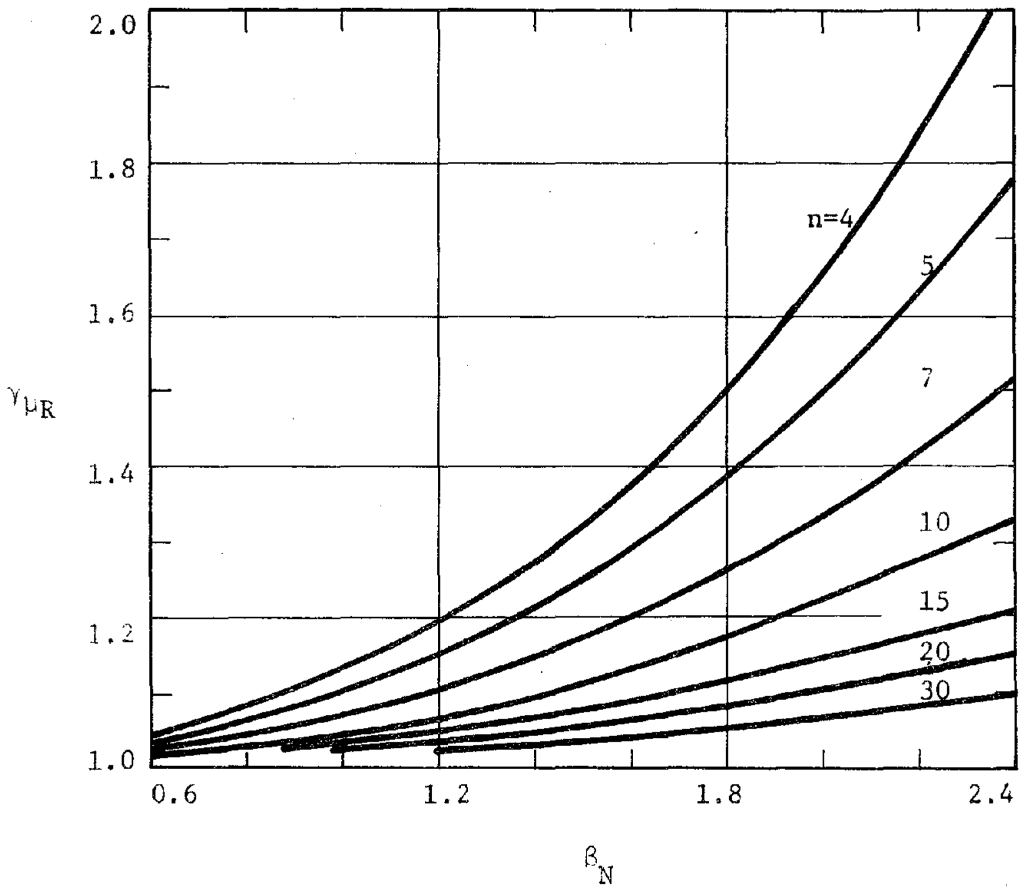


Figure V.5 Penalty factor γ_{μ_R} (Equation V.22) for unknown mean resistance. n=available sample size (for noninformative prior distribution of μ_R).

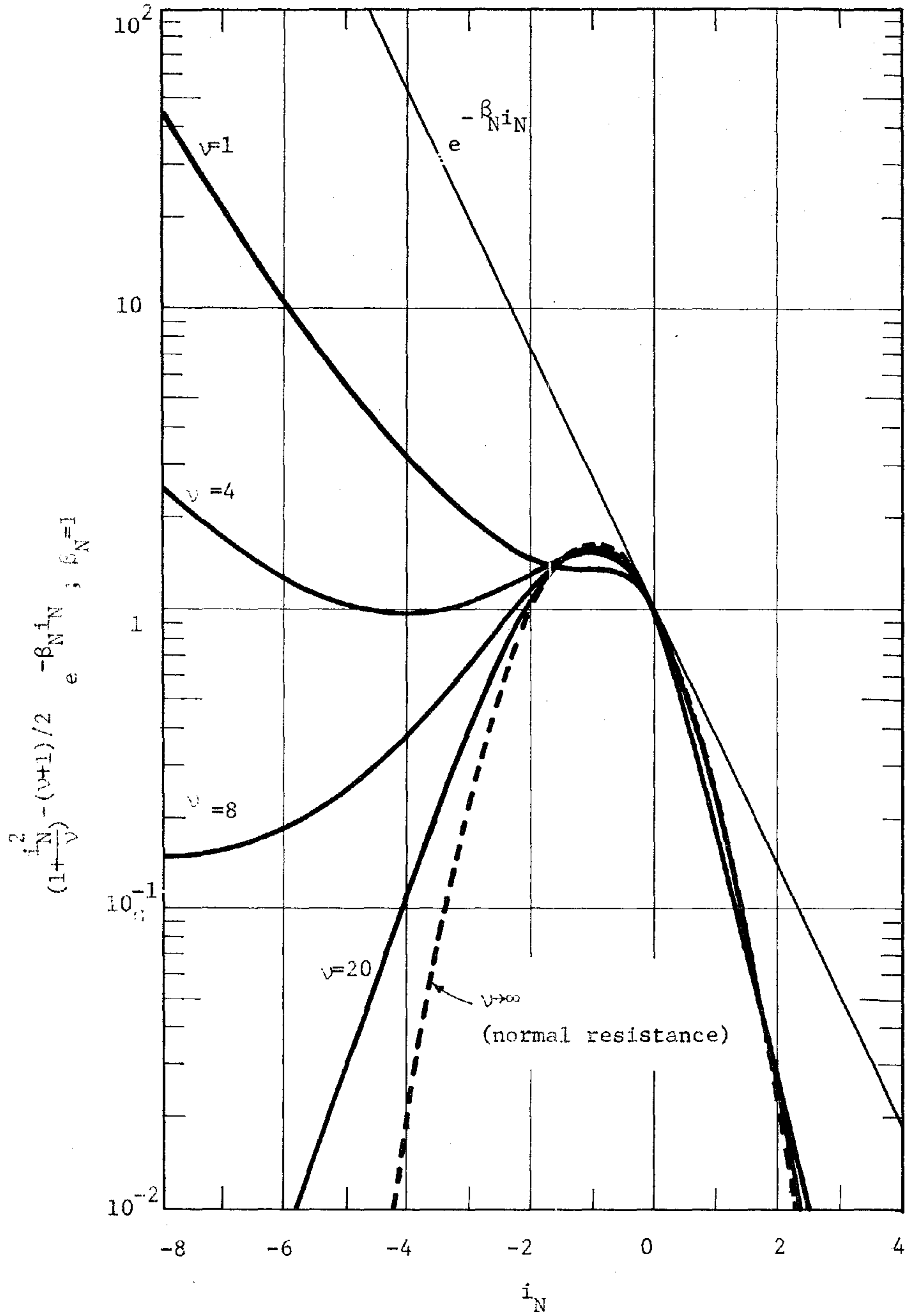
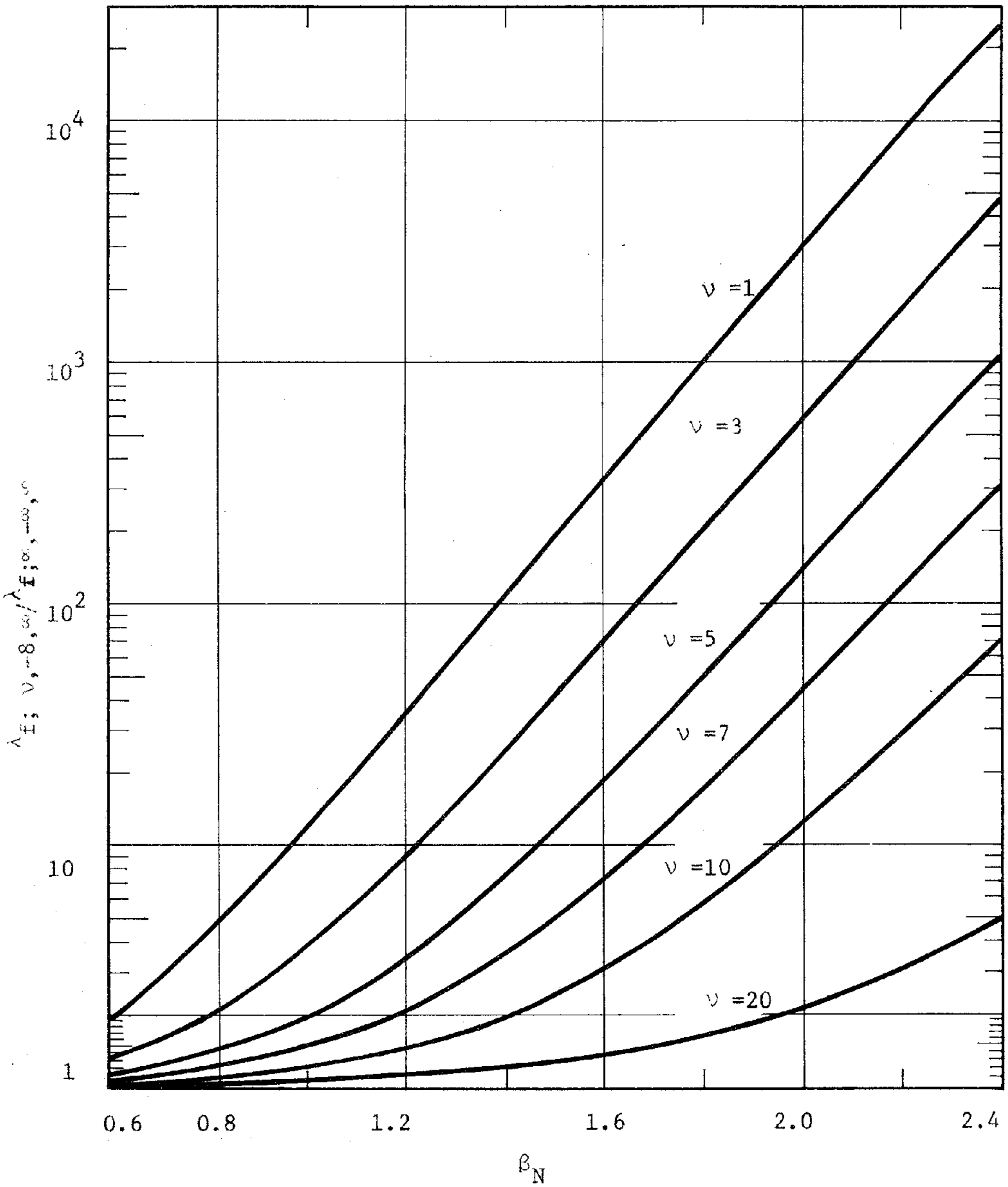
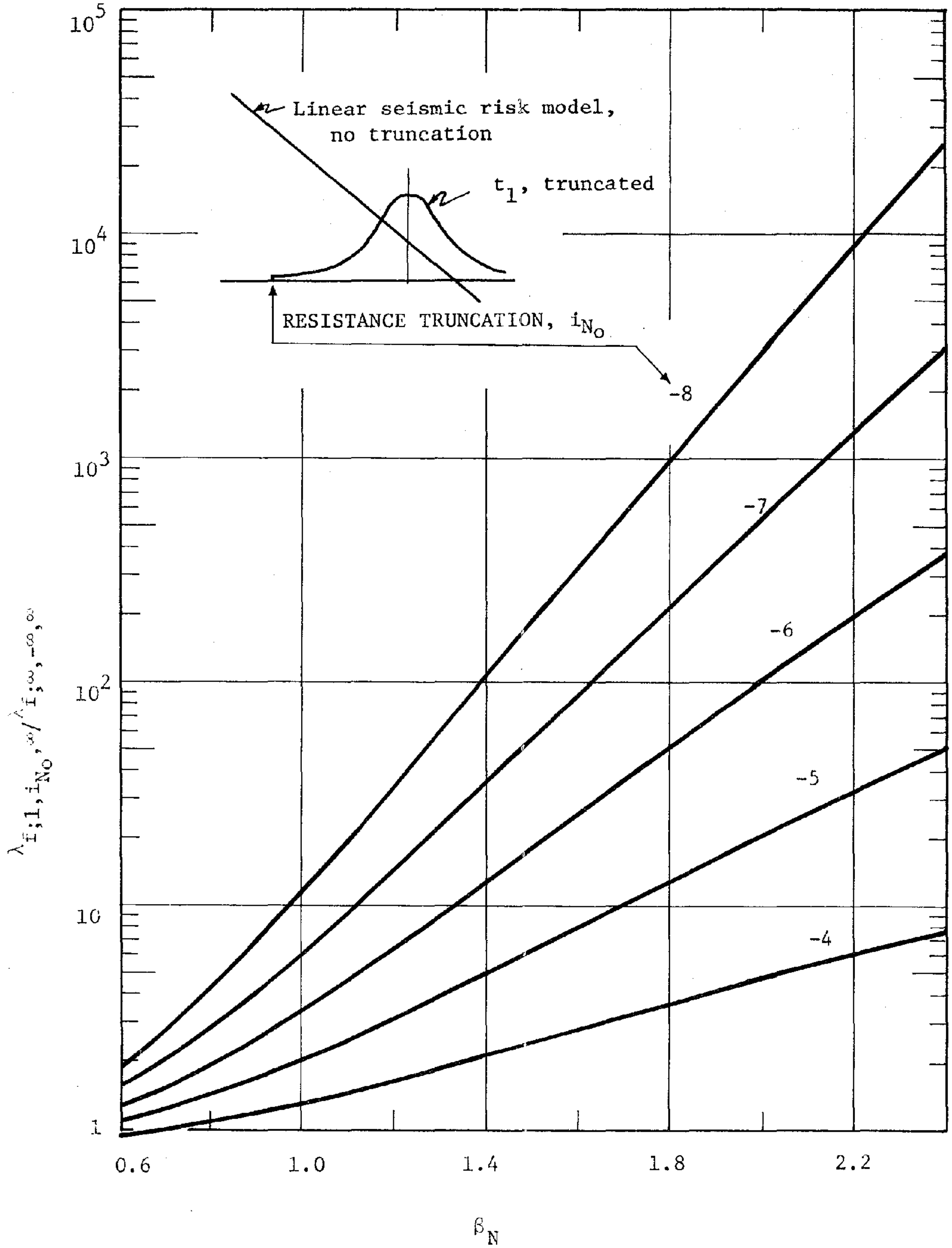


Figure V.6 σ_R^2 unknown; μ_R known or unknown. Plots of the integrand in Equation (V.24) for β_N=1 and ν=1,4,8,20,∞.



<u>Figure V.7</u> Penalty factor for σ_R^2 unknown and μ_R known or unknown. Resistance truncation point i_{N_c}=-8.



<u>Figure V.8</u> Penalty factor for C_R^2 unknown and μ_R known or unknown; ν=1; variable lower resistance truncation point.

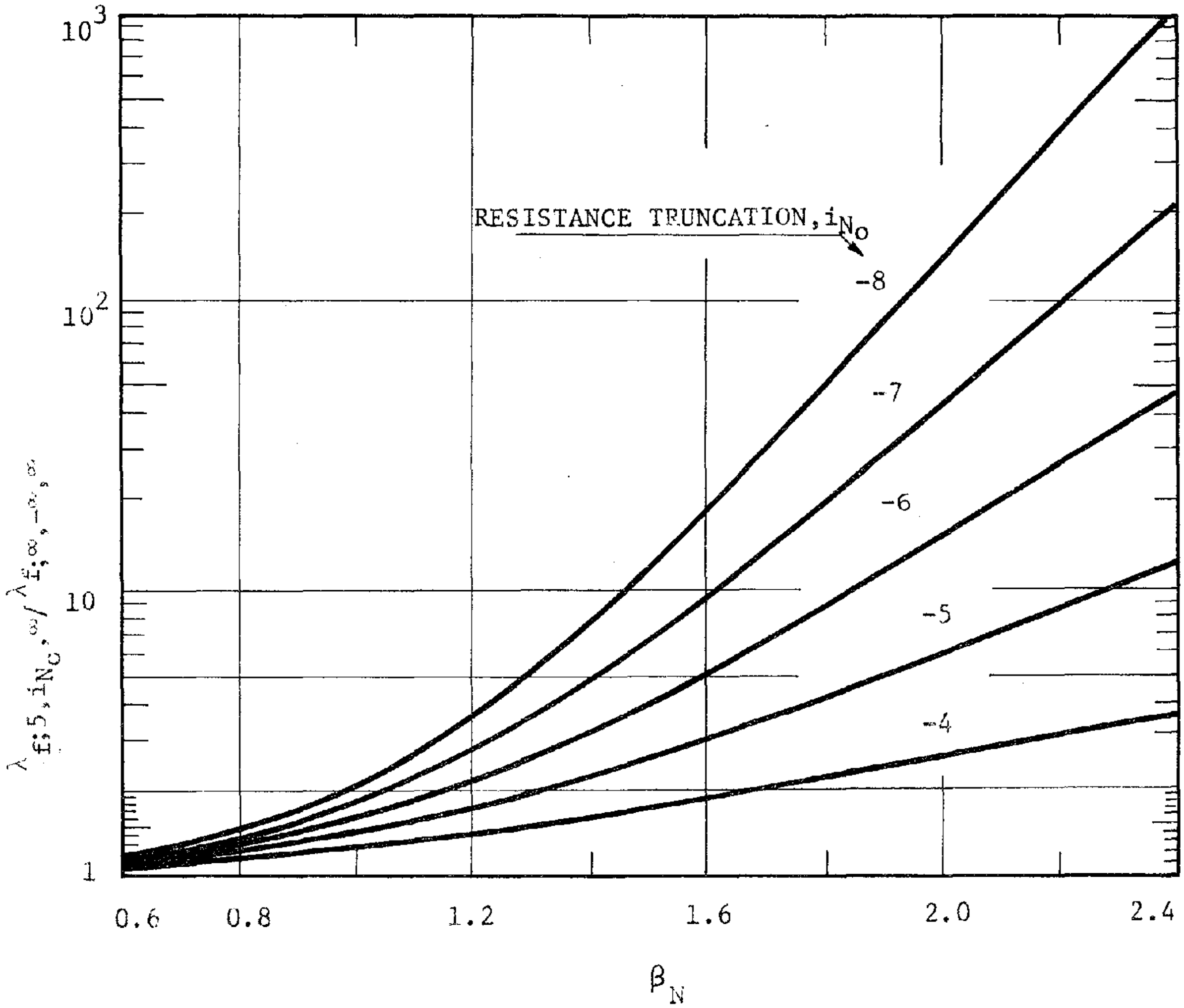


Figure V.9 Penalty factor for σ_R^2 unknown and μ_R known or unknown. ν=5; variable lower resistance truncation point.

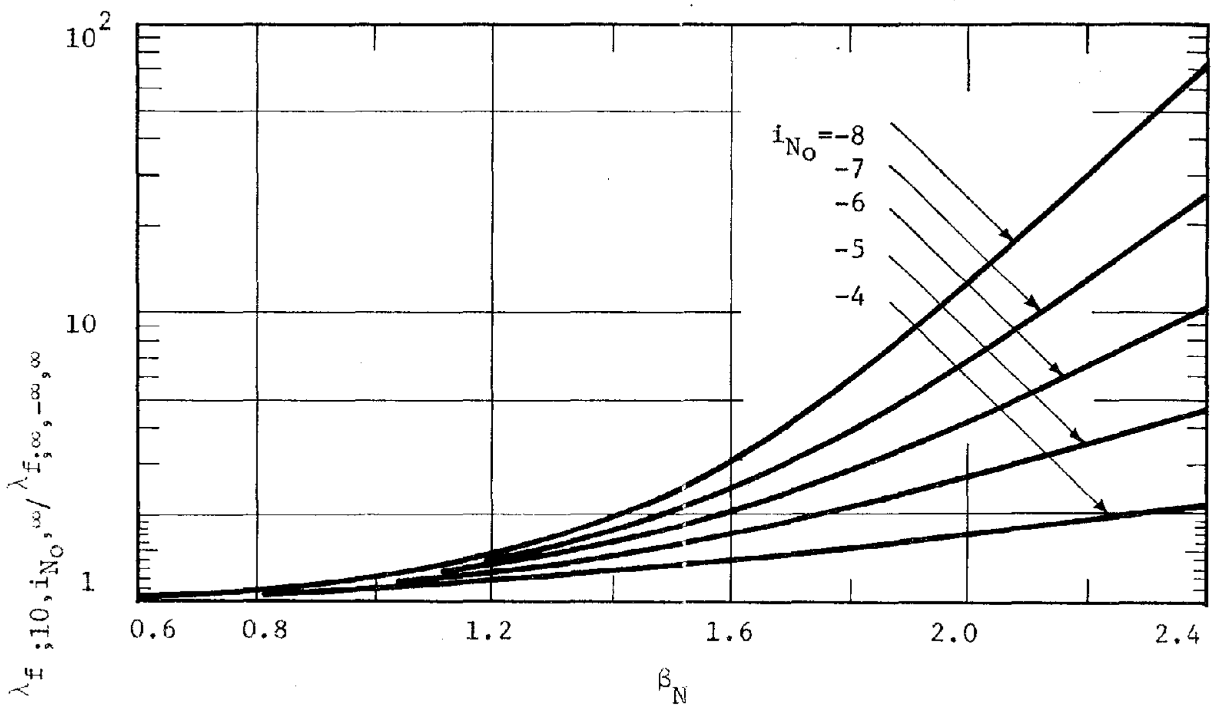


Figure V.10 Penalty factor for σ_R^2 unknown and μ_R known or unknown. ν=10; variable lower resistance truncation point.

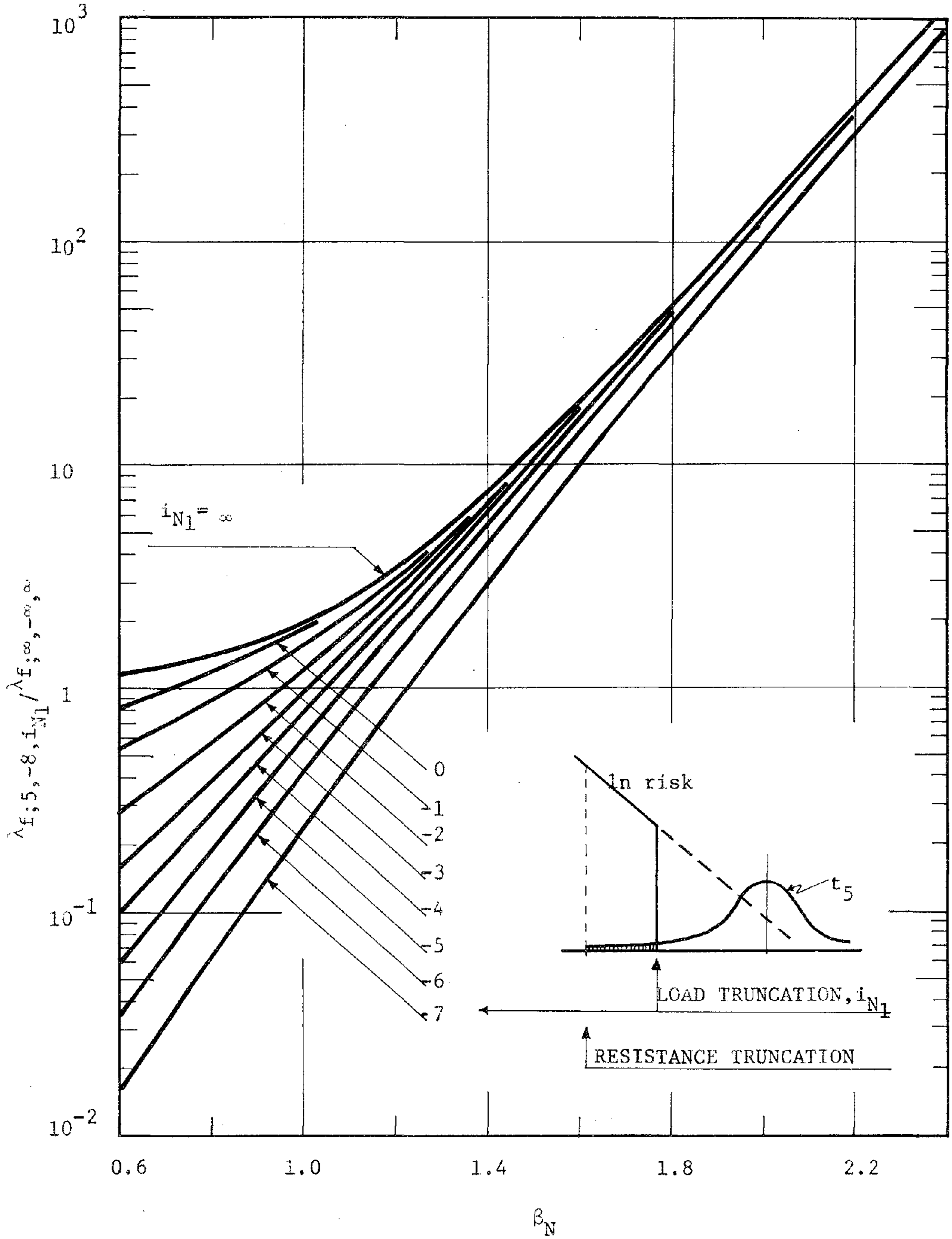


Figure V.11 Penalty factor for σ_R^2 unknown and μ_R known or unknown. ν=5; variable upper bound on site intensity, i_{N1}.

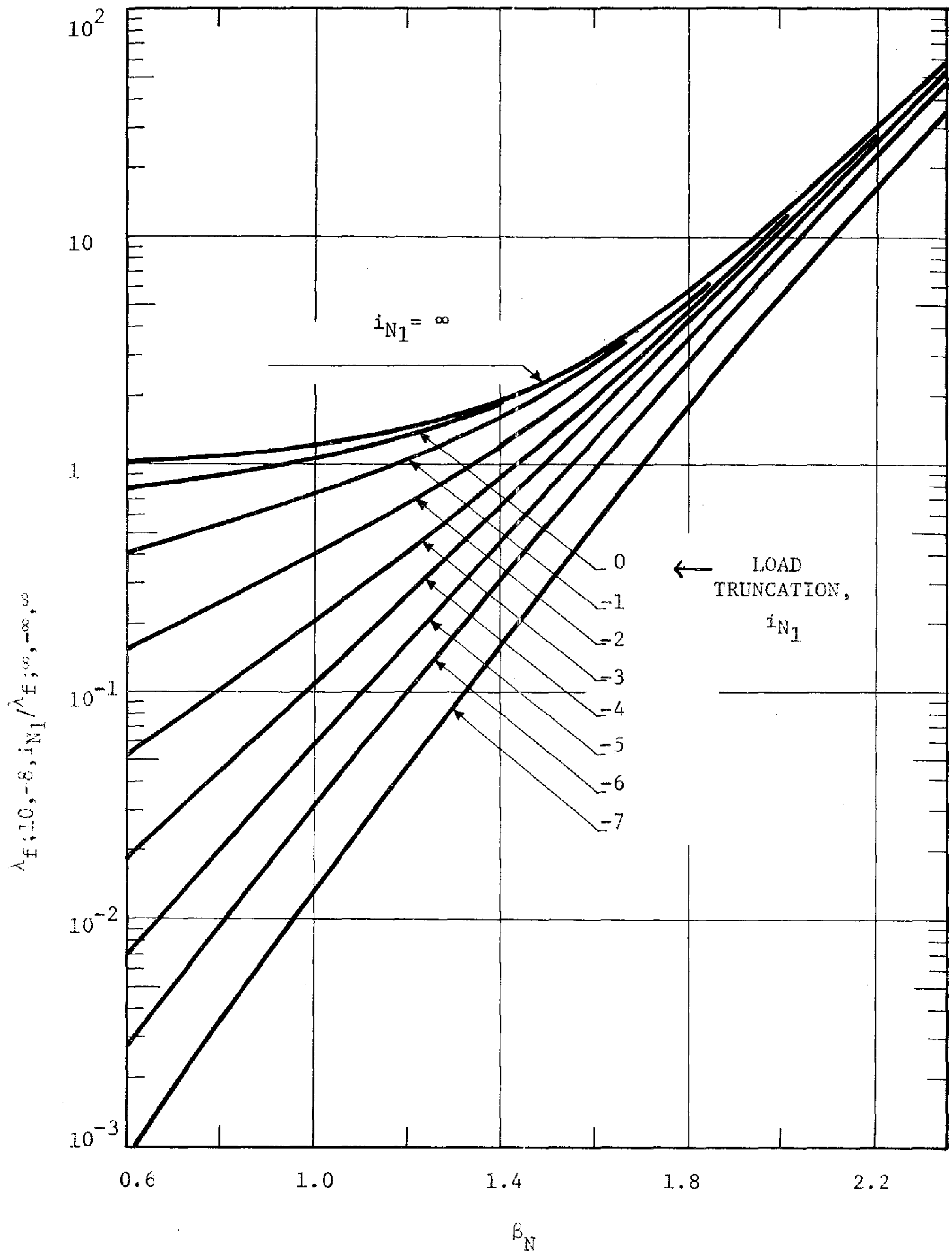


Figure V.12 Penalty factor for σ_R^2 unknown and μ_R known or unknown. $\nu=10$; variable upper bound on site intensity, i_{N_1}.

VI. PARAMETERS SELECTION AND RISK EVALUATION

The information summarized in Section II is used now to select the parameters of the seismic risk model (or their Bayesian distribution).As an application example, the seismic risk of typical nuclear power plants located in Eastern United States regions is calculated, and compared with approximations from Equation (I.4).

VI.1 SELECTION OF RESISTANCE PARAMETERS

The resistance parameters of the probabilistic Gaussian model in Figure IV.1 are μ_R and σ_R. If a statistical Gaussian model is used instead (see Section V.1), the following information must be provided:

n, $\hat{\mu}_R$ and σ_R if μ_R is unknown and σ_R is known;

ν, $\hat{\mu}_R$, S_R and i_{N_0} if σ_R is unknown and μ_R is known or unknown

(for μ_R known, $\hat{\mu}_R=\mu_R$).

In consideration of the limited information available on resistance parameters, the last assumption - σ_R and μ_R unknown - seems to be the most realistic one.

(a) S_R

Estimates of σ_R are available for ordinary civil and industrial constructions ($\sigma_R=\beta_D^{-1}$ and estimates of β_D are given in Table II.5), in which case S_R varies typically between 0.50 and 0.65. Higher values are found using data from Newmark (1974) and from Vanmarcke (1971). From Newmark's data one calculates $S_R=0.75$ for nuclear reactor structures and ordinary civil constructions and $S_R=0.86$ for nuclear equipment. These values refer to seismic demand and resistance expressed originally in units of log peak ground acceleration, and then converted to MMI, through the solid line relationship in Figure (II.4). In the same sense, the data in Vanmarcke (1971) suggest S_R values between 0.65 and 0.70 for ordinary civil constructions. Reasonable values of S_R might be:

$$S_R \text{ in the range} \begin{cases} (0.65,\ 0.75) & \text{for nuclear reactor structures;} \\ (0.70,\ 0.85) & \text{for nuclear reactor equipment.} \end{cases} \qquad \text{(VI.1)}$$

(b) $\hat{\mu}_R$

For ordinary buildings $\hat{\mu}_R$ can be estimated, for example, as the intensity at which the appropriate line in Figure II.10 reaches the critical MDR value, d_f. For reactor systems and components it was concluded by the USAEC Nuclear Reactor Safety Study WASH-1400 (draft report) that the probability of failure under the Safe Shutdown Earthquake (MM intensity i_{SSE}) is in the range 10^{-1} to 10^{-2}(*). For normal resistance distribution this implies an estimated mean value of R:

$$\hat{\mu}_R \text{ in the range} \begin{cases} (i_{SSE}+0.9,\ i_{SSE}+1.62), & \text{for } S_R=0.70, \\ (i_{SSE}+1.09, i_{SSE}+1.97), & \text{for } S_R=0.85. \end{cases} \qquad \text{(VI.2)}$$

For most nuclear power plants either in operation or under construction in the Eastern United States the peak ground acceleration for the SSE, a_{SSE}, is about 0.17g. This value corresponds to a Modified Mercalli intensity i_{SSE} of approximately 8 (see Figure II.4) and to the following ranges for $\hat{\mu}_R$:

$$\hat{\mu}_R \text{ in the range} \begin{cases} (8.9, 9.6) & \text{for } S_R=0.70\ , & \text{(a)} \\ (9.1, 10) & \text{for } S_R=0.85 & \text{(b)} \end{cases} \qquad \text{(VI.3)}$$

(c) ν

The "confidence parameter" ν is not easy to establish because the information on R is rarely in the form of a statistical sample. It is suggested that values in the range 5 to 10 (corresponding to "equivalent sample sizes" from 6 to 11)

(*) Newmark (1974) suggested failure probabilities for nuclear reactor equipment under the design earthquake of the order 10^{-2} to 10^{-4}, or smaller may be appropriate.

For a set of independent N(0;1) variables $Y_1,\ldots,Y_n$ the statistic $n\,\frac{\hat{\mu}}{S}$ has Student's t distribution with $\nu=(n-1)$ degrees of freedom. This means that if $\hat{\mu}_R$ is chosen to be the center value of the intervals (VI.3), the same intervals contain μ_R at the following confidence levels:

0.74 for $\nu=(n-1)=5$

0.88 for $\nu=(n-1)=10$

Arguments of this kind can be used to select an appropriate value (or range of values) for ν.

(d) i_{N_o}

This is another parameter which is difficult to establish with high confidence. If one defines "seismic failures" to be those triggered by ground motions of intensity VI or larger, then i_{N_o} should be given the following values ($i_{N_o} \approx (6-\hat{\mu}_R)/S_R$):

$$i_{N_o} \text{ in the range } \begin{cases} (-5.1,-4.1) & \text{for } S_R=0.70 \text{ and } \hat{\mu}_R \text{ in the range (3a);} \quad \text{(a)} \\ (-4.7,-3.6) & \text{for } S_R=0.85 \text{ and } \hat{\mu}_R \text{ in the range (3b).} \quad \text{(b)} \end{cases} \quad \text{(VI.4)}$$

If the threshold intensity is lowered to V, the range (4a) becomes (-6.5,-5.5), and the rnage (4b) becomes (-5.9,-4.8).

VI.2 SELECTION OF SEISMIC DEMAND PARAMETERS

The exact selection of seismic demand parameters can be done only with reference to a specific grographical location. However, using regional seismic information and typical attenuation laws, ranges of parameters' values can be estimated, sometimes over large areas.

For complete characterization, the linear frequency-site intensity law (IV.3b) requires knowledge of λ_o (mean rate of events with site intensity greater than $\hat{\mu}_R$) and β_N if the model is probabilistic; of i_d, μ_d=E[ln λ_{i_d}], μ_{β_N}, $\sigma_d=\sigma_{\ln \lambda i_d}$, and σ_{β_N} if the model is statistical. For truncated linear risk laws, the upper bound site intensity i_{N_1} must also be given. The analysis in Section V did not allow for statistical uncertainty on i_{N_1}. If the intensity upper bound is larger than the mean resistance, the effect of this uncertainty is negligible; if instead the upper bound is smaller than $\hat{\mu}_R$, the mean failure rate calculated in previous sections and tabulated in the appendices for different values of i_{N_1} can be used to establish the effect of i_{N_1} uncertainty.

The following ranges of seismic demand parameters are consistent with recent risk calculations for Massachusetts (Cornell and Merz, 1974; Tong et al, 1975). With obvious caution, the same ranges can be considered typical for many regions in the Eastern states. Figures 1,2 and 3 (solid lines) are from Tong et al (1975). They give the annual seismic risk in MMI at five different sites in Massachusetts, under different assumptions on the geometry of the seismic sources. In all cases the maximum epicentral intensity I_o was assumed not to exceed 8.7.

(a) i_d

The site intensity i at which λ_i and β_I can be considered independent of one another is approximately V; see Figures VI.1,2,3, and Figure II.1. From the ranges of mean resistance values (3), the normalized intensity $i_d \approx (i-\hat{\mu}_R)/S_R$ is then:

$$i_d \text{ in the range } \begin{cases} (-6.5,-5.5), & \text{for } S_R=0.70 \text{ and } \hat{\mu}_R \text{ in the range (3a);} \quad \text{(a)} \\ (-5.9,-4.8), & \text{for } S_R=0.85 \text{ and } \hat{\mu}_R \text{ in the range (3b)} \quad \text{(b)} \end{cases} \qquad \text{(VI.5)}$$

(b) μ_d, σ_d

At all sites within Massachusetts analyzed by Cornell and Merz (1974) and by Tong et al (1975), and under all the assumptions made by the same authors about the seismic sources and the regional seismic parameters, the mean annual rate of seismic events with site intensity larger than V was found between 0.7×10^{-2} and 2.3×10^{-2}. One may therefore assume:

$$e^{\mu_d} \approx 1.3 \times 10^{-2}, \qquad \text{(VI.6)}$$

and σ_d values between 0.1 and 0.5.

(c) μ_{β_N}, σ_{β_N}

In Section IV (see Figure IV.6 and Table IV.2) $\beta_N = \beta_I \cdot \sigma_R$ was found to have values in the range 0.90 to 1.60. Linearization of the curves in Figures 1,2 and 3 gives β_I values of about 1.45 to 2.30, corresponding to

$$\beta_N \text{ in the range } \begin{cases} (1.0, 1.6), & \text{for } S_R = 0.70 \quad \text{(a)} \\ (1.2, 2.0), & \text{for } S_R = 0.85 \quad \text{(b)} \end{cases} \qquad \text{(VI.7)}$$

Appropriate values of μ_{β_N} and σ_{β_N} for Massachusetts might then be:

$$\mu_{\beta_N} = 1.4; \qquad \sigma_{\beta_N} = 0.2.$$

(d) i_{N1}

The upper bound site intensity varies from region to region and, within each seismic region, from site to site. For Massachusetts, values between 7.5 and 8.7 seem reasonable (see Figures VI.1,2,3 and Figure II.1). If the resistance parameters are in the ranges (3), these values correspond to

$$i_{N1} \text{ in the range } \begin{cases} (-3, 0), & \text{for } S_R = 0.70; \\ (-3, -0.5), & \text{for } S_R = 0.85. \end{cases} \qquad \text{(VI.8)}$$

Upper bounds cannot be established with certainty; indeed, according to a few seismologists (see, e.g., Chinnery and Rogers, 1973), epicentral MM intensities as high as X are possible in Massachusetts. An appropriate practical upper bound for site intensity might be $i_{N1} = -1$.

Although not treated explicitly here, statistical uncertainty on i_{N1} can be incorporated (approximately) into the analysis by assigning (Bayesian) probabilities to a discrete set of i_{N1} values, and by weighting the associated risks through the same probabilities. The tables in Appendices A and B would be helpful for this type of analysis.

VI.3 MEAN FAILURE RATE CALCULATIONS

From the preceeding discussion, the following "best" parameters' estimates are suggested:

* For nuclear power plant resistance: $\mu_R=9.5$; $S_R=0.75$; $\nu=10$; $i_{N_o}=-5$; (VI.9)

* For seismic risk at a Massachusetts site (see dashed lines in Figures 1,2,3):

$$i_d=-6\ ; \qquad e^{\mu_d}=1.3 \times 10^{-2}$$

$$\sigma_d=0.4; \qquad \mu_{\beta_N}=1.40 \qquad \text{(VI.10)}$$

$$\sigma_{\beta_N}=0.2; \qquad i_{N_1}=-1$$

For an untruncated linear risk function, $\mu_{\ln \lambda_o}$ satisfies:

$$\exp\{\mu_{\ln \lambda_o}\} = e^{\mu_d} \cdot e^{\mu_{\beta_\lambda} \cdot i_d} = 0.29 \times 10^{-5},$$

and the mean annual failure rate is:

$$\lambda_f = \exp\{\mu_{\ln \lambda_o}\} \cdot \exp\{\mu_{\beta_N}^2/2\} \cdot \gamma_{\lambda_{i_d}, \beta_N, \mu_R, \sigma_R} .$$

Using the tables in Appendix B it is $\gamma_{\lambda_{i_d}, \beta_N, \mu_R, \sigma_R}/e^{\sigma_d^2/2}=1.468$, and

$$\lambda_f=0.29 \times 10^{-5} \times 2.66 \times 1.08 \times 1.468 = 1.23 \times 10^{-5} \qquad \text{(VI.11)}$$

The values (9),(10) correspond to a probability of failure 0.046 for an event with site intensity 8, and to a mean annual rate 0.48×10^{-4} of exceeding the same intensity. Using the approximation (I.4) one finds $\lambda_f \approx 0.22 \times 10^{-5}$, which is about 5.6 times smaller than the estimate (11).

Conservative estimate of λ_f

Consider the following "pessimistic" parameter values:

$$\begin{aligned} &\mu_R=9.5 \quad &; \quad &S_R=0.75 \\ &\nu=5 \quad &; \quad &i_{N_o}=-5 \\ &i_d=-6 \quad &; \quad &e^{\mu_d}=2.0\times10^{-2} \\ &\sigma_d=0.5 \quad &; \quad &\mu_{\beta_N}=1.40 \\ &\sigma_{\beta_N}=0.3 \quad &; \quad &i_{N_1}=4 \end{aligned} \qquad \text{(VI.12)}$$

The associated mean annual failure rate is:

$$\lambda_f=0.446 \times 10^{-5} \times 2.66 \times 1.133 \times 4.404 = 5.92 \times 10^{-5} \qquad \text{(VI.13)}$$

The difference between the estimates (11) and (13) is due mainly to lowering ν and to increasing σ_{β_N}. Using $i_{N_1}=-1$, and the values (12) for the remaining parameters, one finds $\lambda_f=3.97 \times 10^{-5}$.

For an upper bound site intensity 8, for which $i_{N_1}=-2$, the mean failure rates (11) and (13) become: $\lambda_f=0.75 \times 10^{-5}$, and $\lambda_f=2.71 \times 10^{-5}$, respectively. Changing i_{N_o} by ±1 produces a change of about 10% in the estimate (11), and a change of about 20% in the estimate (13).

Other sensitivity analyses are easily made with the aid of the tables in Appendices A and B.

The preceeding calculations refer to the mean annual rate of accident initiation in a specific mode (e.g. by break of a pipe in the primary coolant system). Many different events may trigger an accident sequence, and eventually lead to core melt and to radioactive releases. The probabilistic analysis of all such sequences is complicated by two factors: (i) the statistical correlation between different failure events (which, should they occur, would be caused by the same ground motion), and (ii) the redundancy of nuclear reactor systems and safety devices. The inclusion of these features in the seismic risk analyses of complex systems is possible within the methodology proposed and illustrated in this study. In fact, the probability (IV.1) - interpreted as the resistance CDF - should account implicitly for all possible failure modes. This is the case, for example, when $P_f(I)$ is estimated from historical records of seismic damage, such as those reviewed in Section II.2. Instead, the calculation of $P_f(I)$ for the whole system is not an easy task when starting from the failure probabilities of subsystems or components, as is usually the case in structural reliability theory. The explicit consideration of this problem is left, however, for future efforts.

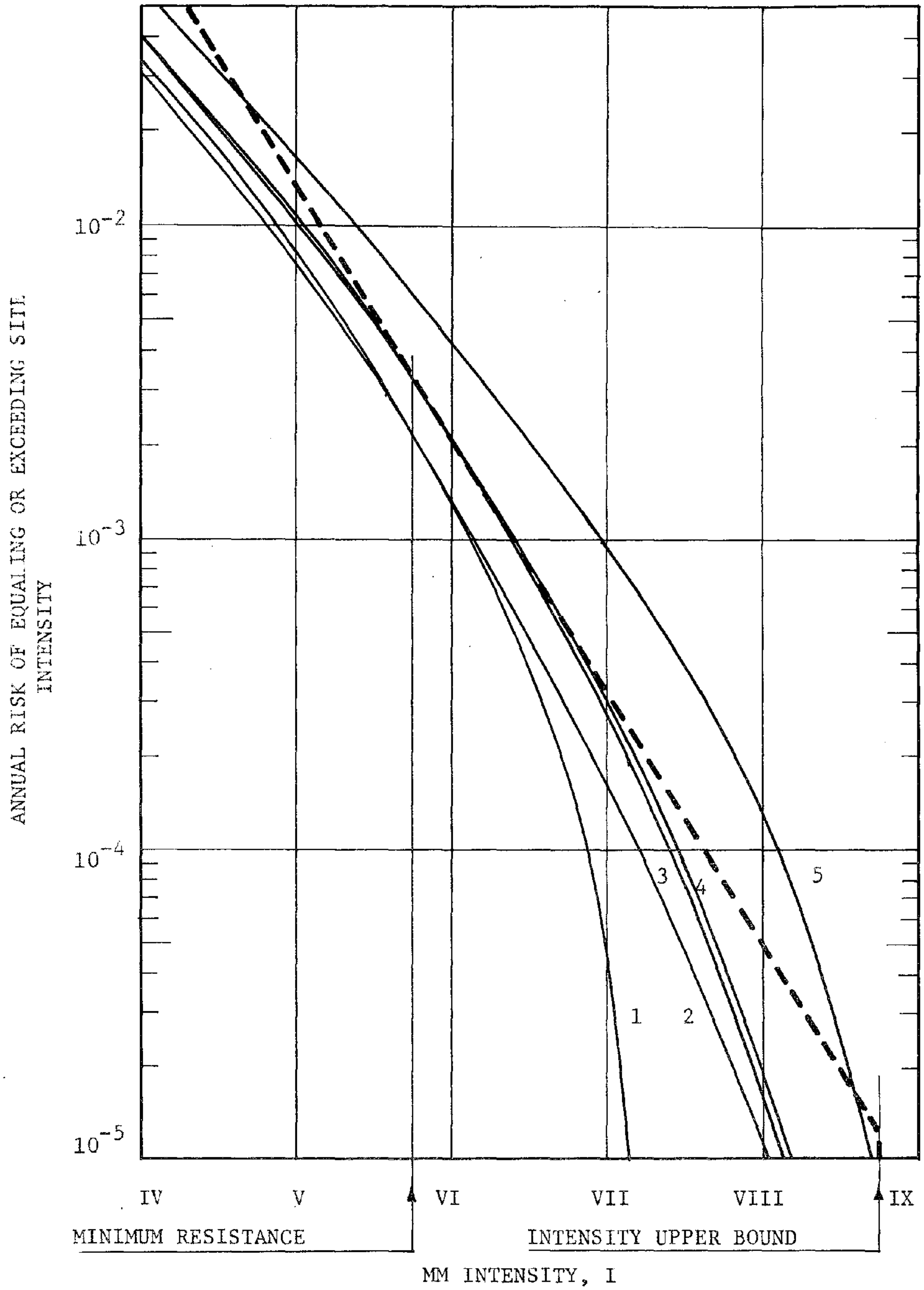


Figure VI.1 Solid Lines: Annual probability of equaling or exceeding intensity I at five sites in Massachusetts (after Tong et al, 1975). Dashed line: truncated linear approximation

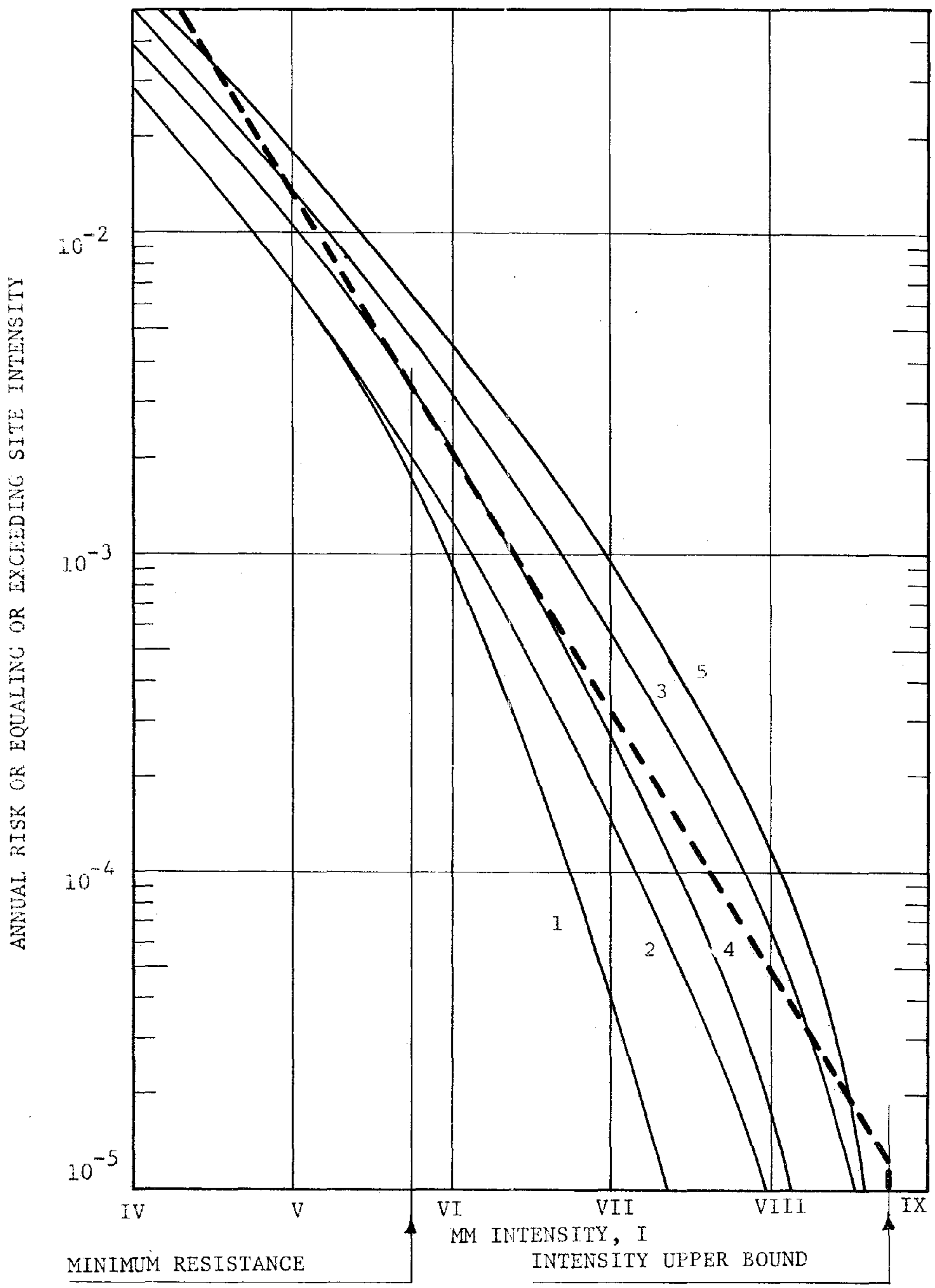


Figure VI.2 Solid lines: Annual probability of equaling or exceeding intensity I at five sites in Massachusetts (after Tong et al, 1975)

Dashed lines: truncated linear approximation

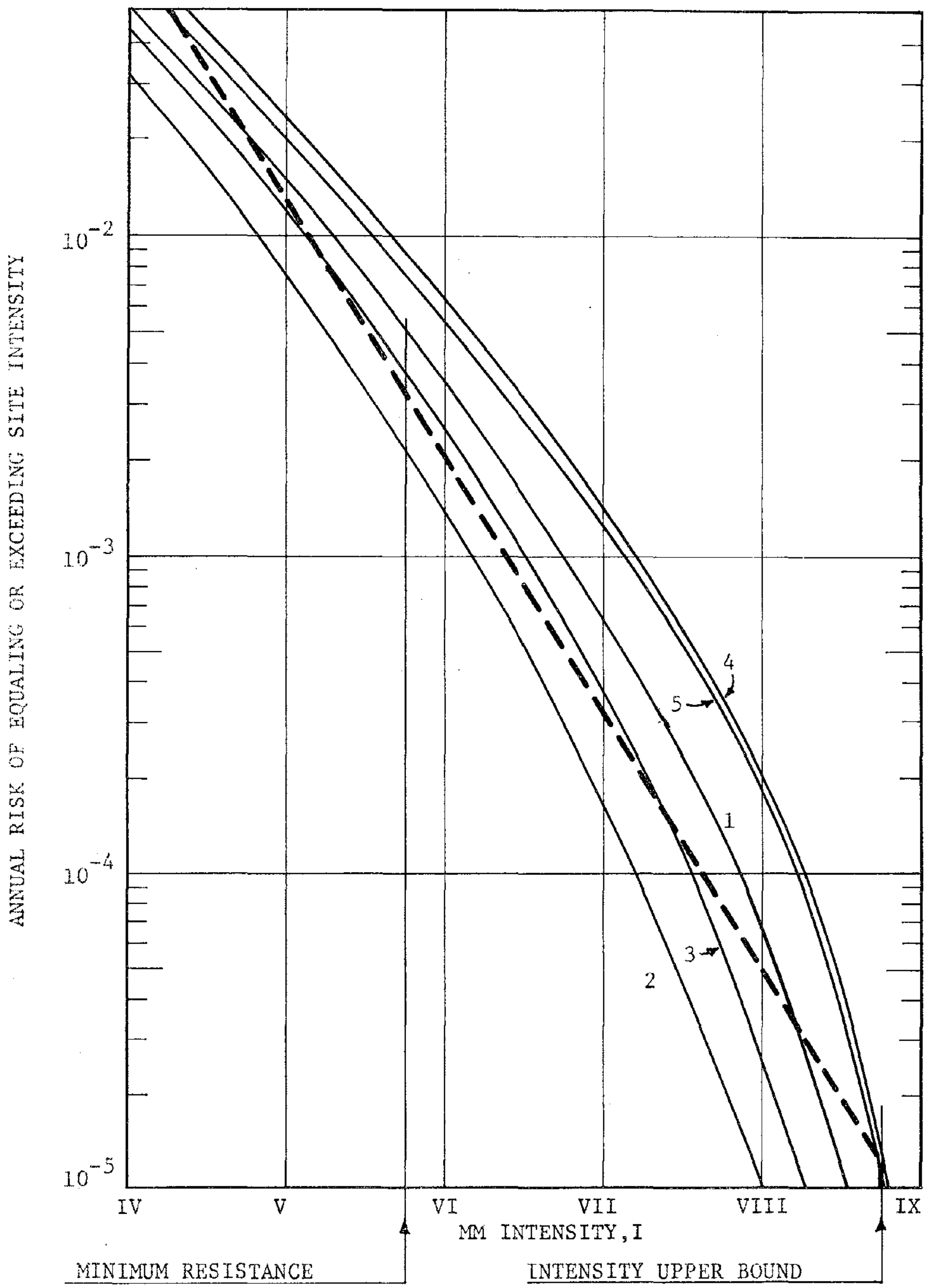


Figure VI.3 Solid lines: Annual probability of equaling or exceeding intensity I at five sites in Massachusetts (after Tong et al, 1975) Dashed lines: truncated linear approximation

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MOST FREQUENTLY-USED SYMBOLS

a,b	constants in the frequency-magnitude law, Eq.(II.2)
	(a is also used for peak ground acceleration, depending on context)
a_1,a_2	constants in the epicentral intensity-magnitude law, Eq.(II.6)
a_D,b_D,β_D	parameters of the linear intensity-mean damage ratio model, Equations (II.15) and (II.16).
b_1,b_2,b_3	constants in the acceleration-magnitude-distance relation (II.14)
D	damage
(M)DR	(mean) damage ratio
d_f	critical level of damage or of damage ratio
I,i	MM site intensity
I_o,i	MM epicentral intensity
I_N,i_N	same as I,i; normalized
i_d	same as i_o, normalized
i_o	MM intensity, such that λ_{i_o} is independent of β_N; see Figure V.3
i_1	MM intensity upper bound
M,m	Richter's magnitude
m_1	magnitude upper bound
P_f	failure probability
R	seismic resistance (also used for epicentral or focal distance; see context)
R'	normalized seismic resistance; see Section V.2.b
Y,y	measure of seismic intensity
y_{DES}	design seismic intensity
β	b ln 10=constant in the frequency-magnitude law
β_{I_o},β_I	decay parameters of the epicentral and site intensity distributions, Eqs.(II.7) and (II.13)
β_N	same as β_I, normalized
γ_{DET}	penalty factor for uncertain resistance

$\gamma_{\theta_1,\ldots\theta_m}$	penalty factor for $\theta_1,\ldots,\theta_m$ unknown; see Section V
Φ,ϕ	standard normal CDF and PDF
λ_{DES}	mean rate of events with intensity larger than the design intensity
λ_f	mean failure rate
λ_{f_p}	approximate mean failure rate; see Eq. (IV.5)
λ_{i_d}	mean rate of events with normalized MM intensity larger than i_d
λ_o	mean rate of events with site intensity larger than the expected resistance (see Section IV.2 for more precise definition in the case of nonlinear risk models)
μ,σ^2	mean, variance
$\hat{\mu},s^2$	unbiased estimates of μ and σ^2
$\underline{\Theta},\underline{\theta}$	vector of unknown parameters; see Section III

APPENDIX A

PENALTY FACTORS FOR UNKNOWN σ_R AND KNOWN OR UNKNOWN μ_R

Unknown σ_R and known or unknown μ_R; tables of the ratio $\dfrac{\lambda_{f;\nu,i_{N_o},i_{N_1}}}{\lambda_{f;\infty,-\infty,\infty}}$;

see Equation (V.24). The tables are for $\nu=5,10,20$; $\beta_N=0.6(0.2)2.0$;

$$i_{N_o}=-8(1)-3; \quad i_{N_1}=4(-1)i_{N_o}.$$

** $\nu = 5 \quad \beta_N = 0.60$ **

i_{N_1} \ i_{N_0}	-8.0	-7.0	-6.0	-5.0	-4.0	-3.0
4.0	1.155	1.139	1.121	1.098	1.064	1.010
3.0	1.154	1.138	1.120	1.097	1.063	1.009
2.0	1.146	1.131	1.113	1.090	1.056	1.001
1.0	1.099	1.084	1.065	1.042	1.008	0.953
0.0	0.895	0.879	0.861	0.838	0.803	0.746
-1.0	0.542	0.527	0.508	0.484	0.449	0.388
-2.0	0.287	0.271	0.253	0.229	0.192	0.129
-3.0	0.160	0.144	0.126	0.101	0.064	0.0
-4.0	0.096	0.080	0.062	0.037	0.0	0.0
-5.0	0.059	0.043	0.025	0.0	0.0	0.0
-6.0	0.034	0.019	0.0	0.0	0.0	0.0
-7.0	0.016	0.0	0.0	0.0	0.0	0.0

** $\nu = 5 \quad \beta_N = 0.80$ **

i_{N_1} \ i_{N_0}	-8.0	-7.0	-6.0	-5.0	-4.0	-3.0
4.0	1.390	1.330	1.271	1.208	1.133	1.032
3.0	1.389	1.329	1.271	1.208	1.133	1.031
2.0	1.386	1.325	1.267	1.204	1.129	1.027
1.0	1.354	1.294	1.235	1.173	1.097	0.995
0.0	1.190	1.130	1.071	1.008	0.933	0.829
-1.0	0.851	0.790	0.732	0.668	0.591	0.484
-2.0	0.554	0.494	0.435	0.371	0.293	0.183
-3.0	0.373	0.313	0.254	0.190	0.112	0.0
-4.0	0.262	0.202	0.143	0.079	0.0	0.0
-5.0	0.184	0.123	0.064	0.0	0.0	0.0
-6.0	0.120	0.059	0.0	0.0	0.0	0.0
-7.0	0.060	0.0	0.0	0.0	0.0	0.0

** ν = 5 β_N =1.00 **

i_{N_1}	i_{N_0} = -8.0	-7.0	-6.0	-5.0	-4.0	-3.0
4.0	1.960	1.733	1.553	1.393	1.236	1.059
3.0	1.960	1.733	1.553	1.393	1.236	1.059
2.0	1.958	1.731	1.550	1.391	1.234	1.057
1.0	1.937	1.711	1.530	1.371	1.213	1.037
0.0	1.811	1.584	1.404	1.244	1.086	0.908
-1.0	1.496	1.269	1.088	0.928	0.769	0.588
-2.0	1.163	0.936	0.755	0.595	0.435	0.250
-3.0	0.916	0.690	0.508	0.348	0.187	0.0
-4.0	0.730	0.503	0.322	0.161	0.0	0.0
-5.0	0.569	0.342	0.161	0.0	0.0	0.0
-6.0	0.408	0.181	0.0	0.0	0.0	0.0
-7.0	0.227	0.0	0.0	0.0	0.0	0.0

** ν = 5 β_N =1.20 **

i_{N_1}	i_{N_0} = -8.0	-7.0	-6.0	-5.0	-4.0	-3.0
4.0	3.439	2.619	2.083	1.696	1.382	1.091
3.0	3.439	2.619	2.083	1.696	1.382	1.091
2.0	3.438	2.618	2.082	1.695	1.381	1.090
1.0	3.425	2.605	2.070	1.683	1.368	1.077
0.0	3.331	2.511	1.975	1.588	1.273	0.981
-1.0	3.049	2.229	1.693	1.306	0.990	0.695
-2.0	2.689	1.869	1.333	0.945	0.628	0.330
-3.0	2.364	1.544	1.008	0.619	0.302	0.0
-4.0	2.064	1.244	0.707	0.319	0.0	0.0
-5.0	1.746	0.926	0.389	0.0	0.0	0.0
-6.0	1.357	0.537	0.0	0.0	0.0	0.0
-7.0	0.820	0.0	0.0	0.0	0.0	0.0

** ν = 5 β_N = 1.40 **

i_{N_1}	i_{N_0}: -8.0	-7.0	-6.0	-5.0	-4.0	-3.0
4.0	7.476	4.617	3.087	2.183	1.581	1.123
3.0	7.476	4.617	3.087	2.183	1.581	1.123
2.0	7.476	4.616	3.086	2.183	1.581	1.123
1.0	7.469	4.609	3.079	2.175	1.573	1.115
0.0	7.401	4.541	3.011	2.107	1.505	1.047
-1.0	7.158	4.298	2.768	1.864	1.261	0.800
-2.0	6.783	3.923	2.393	1.488	0.884	0.419
-3.0	6.370	3.509	1.979	1.074	0.469	0.0
-4.0	5.903	3.043	1.512	0.607	0.0	0.0
-5.0	5.297	2.437	0.906	0.0	0.0	0.0
-6.0	4.392	1.531	0.0	0.0	0.0	0.0
-7.0	2.861	0.0	0.0	0.0	0.0	0.0

** ν = 5 β_N = 1.60 **

i_{N_1}	i_{N_0}: -8.0	-7.0	-6.0	-5.0	-4.0	-3.0
4.0	18.809	9.191	4.985	2.952	1.845	1.153
3.0	18.809	9.191	4.985	2.952	1.845	1.153
2.0	18.809	9.191	4.985	2.952	1.845	1.153
1.0	18.804	9.187	4.980	2.948	1.840	1.148
0.0	18.758	9.140	4.934	2.901	1.793	1.101
-1.0	18.555	8.938	4.731	2.698	1.590	0.896
-2.0	18.179	8.561	4.354	2.321	1.212	0.513
-3.0	17.673	8.055	3.848	1.814	0.703	0.0
-4.0	16.973	7.355	3.148	1.113	0.0	0.0
-5.0	15.862	6.244	2.036	0.0	0.0	0.0
-6.0	13.827	4.209	0.0	0.0	0.0	0.0
-7.0	9.619	0.0	0.0	0.0	0.0	0.0

** ν = 5 β_N =1.80 **

i_{N_1}	i_{N_0}: -8.0	-7.0	-6.0	-5.0	-4.0	-3.0
4.0	50.866	19.694	8.546	4.142	2.179	1.175
3.0	50.866	19.694	8.546	4.142	2.179	1.175
2.0	50.866	19.694	8.546	4.142	2.179	1.175
1.0	50.863	19.691	8.544	4.140	2.177	1.172
0.0	50.832	19.660	8.512	4.109	2.146	1.141
-1.0	50.670	19.498	8.350	3.946	1.983	0.976
-2.0	50.305	19.133	7.985	3.581	1.616	0.606
-3.0	49.708	18.536	7.388	2.982	1.016	0.0
-4.0	48.697	17.525	6.376	1.969	0.0	0.0
-5.0	46.731	15.558	4.409	0.0	0.0	0.0
-6.0	42.325	11.152	0.0	0.0	0.0	0.0
-7.0	31.176	0.0	0.0	0.0	0.0	0.0

** ν = 5 β_N =2.00 **

i_{N_1}	i_{N_0}: -8.0	-7.0	-6.0	-5.0	-4.0	-3.0
4.0	140.982	43.606	15.134	5.938	2.588	1.184
3.0	140.982	43.606	15.134	5.938	2.588	1.184
2.0	140.982	43.606	15.133	5.938	2.588	1.184
1.0	140.980	43.605	15.132	5.937	2.586	1.183
0.0	140.960	43.585	15.112	5.917	2.566	1.162
-1.0	140.835	43.460	14.987	5.792	2.441	1.036
-2.0	140.495	43.120	14.646	5.451	2.099	0.690
-3.0	139.815	42.439	13.966	4.769	1.415	0.0
-4.0	138.407	41.031	12.557	3.358	0.0	0.0
-5.0	135.054	37.678	9.202	0.0	0.0	0.0
-6.0	125.858	28.480	0.0	0.0	0.0	0.0
-7.0	97.384	0.0	0.0	0.0	0.0	0.0

** $\nu = 10$ $\beta_N = 0.60$ **

i_{N_1}	i_{N_0}: -8.0	-7.0	-6.0	-5.0	-4.0	-3.0
4.0	1.054	1.053	1.051	1.047	1.036	1.007
3.0	1.053	1.052	1.050	1.046	1.036	1.007
2.0	1.047	1.046	1.044	1.040	1.030	1.001
1.0	0.998	0.997	0.995	0.991	0.981	0.951
0.0	0.786	0.785	0.783	0.779	0.768	0.738
-1.0	0.419	0.418	0.416	0.412	0.401	0.369
-2.0	0.158	0.157	0.155	0.151	0.140	0.106
-3.0	0.053	0.052	0.050	0.046	0.034	0.0
-4.0	0.019	0.018	0.016	0.012	0.0	0.0
-5.0	0.007	0.006	0.004	0.0	0.0	0.0
-6.0	0.003	0.002	0.0	0.0	0.0	0.0
-7.0	0.001	0.0	0.0	0.0	0.0	0.0

** $\nu = 10$ $\beta_N = 0.80$ **

i_{N_1}	i_{N_0}: -8.0	-7.0	-6.0	-5.0	-4.0	-3.0
4.0	1.115	1.112	1.106	1.095	1.072	1.018
3.0	1.115	1.111	1.105	1.095	1.071	1.018
2.0	1.111	1.108	1.102	1.091	1.068	1.014
1.0	1.079	1.075	1.070	1.059	1.035	0.981
0.0	0.909	0.905	0.900	0.889	0.865	0.810
-1.0	0.555	0.552	0.546	0.535	0.511	0.454
-2.0	0.252	0.249	0.243	0.232	0.208	0.149
-3.0	0.104	0.100	0.095	0.084	0.059	0.0
-4.0	0.045	0.041	0.035	0.024	0.0	0.0
-5.0	0.020	0.017	0.011	0.0	0.0	0.0
-6.0	0.009	0.006	0.0	0.0	0.0	0.0
-7.0	0.004	0.0	0.0	0.0	0.0	0.0

** $\nu = 10$ $\beta_N = 1.00$ **

i_{N_1}	i_{N_0}					
	-8.0	-7.0	-6.0	-5.0	-4.0	-3.0
4.0	1.229	1.216	1.198	1.171	1.123	1.030
3.0	1.229	1.216	1.198	1.171	1.123	1.030
2.0	1.227	1.214	1.197	1.169	1.121	1.028
1.0	1.207	1.194	1.176	1.148	1.100	1.007
0.0	1.075	1.062	1.044	1.017	0.969	0.875
-1.0	0.747	0.734	0.716	0.689	0.640	0.544
-2.0	0.407	0.394	0.377	0.349	0.300	0.203
-3.0	0.206	0.193	0.175	0.148	0.098	0.0
-4.0	0.108	0.095	0.077	0.049	0.0	0.0
-5.0	0.059	0.045	0.028	0.0	0.0	0.0
-6.0	0.031	0.018	0.0	0.0	0.0	0.0
-7.0	0.013	0.0	0.0	0.0	0.0	0.0

** $\nu = 10$ $\beta_N = 1.20$ **

i_{N_1}	i_{N_0}					
	-8.0	-7.0	-6.0	-5.0	-4.0	-3.0
4.0	1.454	1.408	1.356	1.290	1.194	1.043
3.0	1.454	1.408	1.355	1.289	1.194	1.043
2.0	1.454	1.407	1.355	1.289	1.193	1.042
1.0	1.441	1.394	1.342	1.276	1.181	1.029
0.0	1.343	1.296	1.244	1.178	1.083	0.930
-1.0	1.049	1.002	0.950	0.884	0.788	0.634
-2.0	0.683	0.636	0.584	0.517	0.422	0.266
-3.0	0.419	0.372	0.320	0.254	0.157	0.0
-4.0	0.262	0.215	0.163	0.096	0.0	0.0
-5.0	0.165	0.118	0.066	0.0	0.0	0.0
-6.0	0.099	0.052	0.0	0.0	0.0	0.0
-7.0	0.047	0.0	0.0	0.0	0.0	0.0

** $\nu=10$ $\beta_N=1.40$ **

i_{N_1}	i_{N_0} = -8.0	-7.0	-6.0	-5.0	-4.0	-3.0
4.0	1.933	1.771	1.623	1.471	1.290	1.053
3.0	1.933	1.771	1.623	1.471	1.290	1.053
2.0	1.932	1.770	1.623	1.470	1.290	1.053
1.0	1.925	1.762	1.615	1.462	1.282	1.045
0.0	1.854	1.692	1.545	1.392	1.212	0.974
-1.0	1.601	1.439	1.291	1.139	0.958	0.719
-2.0	1.220	1.058	0.910	0.757	0.576	0.336
-3.0	0.887	0.724	0.577	0.424	0.243	0.0
-4.0	0.644	0.482	0.335	0.182	0.0	0.0
-5.0	0.463	0.300	0.153	0.0	0.0	0.0
-6.0	0.310	0.148	0.0	0.0	0.0	0.0
-7.0	0.162	0.0	0.0	0.0	0.0	0.0

** $\nu=10$ $\beta_N=1.60$ **

i_{N_1}	i_{N_0} = -8.0	-7.0	-6.0	-5.0	-4.0	-3.0
4.0	3.025	2.484	2.083	1.743	1.414	1.059
3.0	3.025	2.484	2.083	1.743	1.414	1.059
2.0	3.025	2.484	2.082	1.742	1.414	1.059
1.0	3.021	2.480	2.078	1.738	1.409	1.054
0.0	2.972	2.431	2.030	1.690	1.361	1.005
-1.0	2.761	2.221	1.819	1.479	1.150	0.793
-2.0	2.379	1.838	1.437	1.096	0.767	0.409
-3.0	1.973	1.433	1.031	0.691	0.361	0.0
-4.0	1.613	1.072	0.671	0.330	0.0	0.0
-5.0	1.283	0.742	0.340	0.0	0.0	0.0
-6.0	0.943	0.402	0.0	0.0	0.0	0.0
-7.0	0.541	0.0	0.0	0.0	0.0	0.0

** $\nu = 10$ $\beta_N = 1.80$ **

i_{N_1}	i_{N_0}: -8.0	-7.0	-6.0	-5.0	-4.0	-3.0
4.0	5.668	3.930	2.874	2.145	1.567	1.056
3.0	5.668	3.930	2.874	2.145	1.567	1.056
2.0	5.668	3.930	2.874	2.145	1.567	1.056
1.0	5.665	3.927	2.872	2.142	1.565	1.054
0.0	5.633	3.895	2.840	2.110	1.533	1.021
-1.0	5.464	3.726	2.671	1.941	1.363	0.851
-2.0	5.095	3.357	2.301	1.571	0.994	0.480
-3.0	4.618	2.880	1.825	1.095	0.517	0.0
-4.0	4.102	2.364	1.309	0.579	0.0	0.0
-5.0	3.524	1.786	0.730	0.0	0.0	0.0
-6.0	2.794	1.055	0.0	0.0	0.0	0.0
-7.0	1.738	0.0	0.0	0.0	0.0	0.0

** $\nu = 10$ $\beta_N = 2.00$ **

i_{N_1}	i_{N_0}: -8.0	-7.0	-6.0	-5.0	-4.0	-3.0
4.0	12.293	6.908	4.236	2.727	1.750	1.043
3.0	12.293	6.908	4.236	2.727	1.750	1.043
2.0	12.293	6.908	4.236	2.727	1.750	1.043
1.0	12.291	6.906	4.235	2.725	1.749	1.041
0.0	12.271	6.886	4.214	2.705	1.728	1.020
-1.0	12.140	6.755	4.084	2.574	1.598	0.889
-2.0	11.796	6.411	3.739	2.230	1.253	0.543
-3.0	11.257	5.872	3.200	1.691	0.714	0.0
-4.0	10.544	5.159	2.488	0.978	0.0	0.0
-5.0	9.567	4.182	1.510	0.0	0.0	0.0
-6.0	8.057	2.672	0.0	0.0	0.0	0.0
-7.0	5.385	0.0	0.0	0.0	0.0	0.0

** $\nu=20$ $\beta_N=0.60$ **

i_{N_1} \ i_{N_0}	-8.0	-7.0	-6.0	-5.0	-4.0	-3.0
4.0	1.022	1.022	1.022	1.022	1.018	1.002
3.0	1.022	1.022	1.022	1.021	1.018	1.001
2.0	1.017	1.017	1.017	1.016	1.013	0.996
1.0	0.967	0.967	0.967	0.966	0.963	0.946
0.0	0.751	0.751	0.751	0.751	0.747	0.730
-1.0	0.377	0.377	0.377	0.377	0.373	0.354
-2.0	0.114	0.114	0.114	0.114	0.110	0.090
-3.0	0.024	0.024	0.024	0.023	0.020	0.0
-4.0	0.004	0.004	0.004	0.004	0.0	0.0
-5.0	0.001	0.001	0.001	0.0	0.0	0.0
-6.0	0.000	0.000	0.0	0.0	0.0	0.0
-7.0	0.000	0.0	0.0	0.0	0.0	0.0

** $\nu=20$ $\beta_N=0.80$ **

i_{N_1} \ i_{N_0}	-8.0	-7.0	-6.0	-5.0	-4.0	-3.0
4.0	1.045	1.045	1.044	1.043	1.035	1.005
3.0	1.044	1.044	1.044	1.042	1.035	1.005
2.0	1.042	1.041	1.041	1.039	1.032	1.002
1.0	1.009	1.008	1.008	1.006	0.999	0.969
0.0	0.836	0.835	0.835	0.834	0.826	0.795
-1.0	0.475	0.475	0.475	0.473	0.466	0.433
-2.0	0.170	0.170	0.169	0.168	0.160	0.127
-3.0	0.043	0.043	0.043	0.041	0.034	0.0
-4.0	0.010	0.009	0.009	0.007	0.0	0.0
-5.0	0.002	0.002	0.002	0.0	0.0	0.0
-6.0	0.000	0.000	0.0	0.0	0.0	0.0
-7.0	0.000	0.0	0.0	0.0	0.0	0.0

** $\nu=20$ $\beta_N=1.00$ **

i_{N_1} \ i_{N_0}	-8.0	-7.0	-6.0	-5.0	-4.0	-3.0
4.0	1.080	1.079	1.078	1.074	1.060	1.007
3.0	1.080	1.079	1.078	1.074	1.060	1.007
2.0	1.078	1.078	1.077	1.073	1.058	1.006
1.0	1.057	1.057	1.056	1.052	1.037	0.984
0.0	0.923	0.923	0.922	0.918	0.903	0.850
-1.0	0.589	0.588	0.587	0.583	0.568	0.514
-2.0	0.247	0.247	0.246	0.242	0.227	0.172
-3.0	0.076	0.076	0.075	0.071	0.056	0.0
-4.0	0.020	0.020	0.019	0.015	0.0	0.0
-5.0	0.005	0.005	0.004	0.0	0.0	0.0
-6.0	0.001	0.001	0.0	0.0	0.0	0.0
-7.0	0.000	0.0	0.0	0.0	0.0	0.0

** $\nu=20$ $\beta_N=1.20$ **

i_{N_1} \ i_{N_0}	-8.0	-7.0	-6.0	-5.0	-4.0	-3.0
4.0	1.135	1.134	1.131	1.122	1.093	1.008
3.0	1.135	1.134	1.131	1.122	1.093	1.008
2.0	1.135	1.133	1.130	1.121	1.092	1.007
1.0	1.122	1.120	1.117	1.108	1.079	0.994
0.0	1.022	1.021	1.018	1.008	0.980	0.894
-1.0	0.722	0.721	0.718	0.708	0.680	0.593
-2.0	0.354	0.353	0.350	0.341	0.312	0.224
-3.0	0.131	0.130	0.127	0.117	0.088	0.0
-4.0	0.043	0.042	0.038	0.029	0.0	0.0
-5.0	0.014	0.013	0.009	0.0	0.0	0.0
-6.0	0.005	0.003	0.0	0.0	0.0	0.0
-7.0	0.001	0.0	0.0	0.0	0.0	0.0

** $\nu = 20$ $\beta_N = 1.40$ **

i_{N_1}	i_{N_0} = -8.0	-7.0	-6.0	-5.0	-4.0	-3.0
4.0	1.225	1.220	1.211	1.190	1.136	1.005
3.0	1.225	1.220	1.211	1.190	1.136	1.005
2.0	1.224	1.220	1.211	1.190	1.136	1.004
1.0	1.216	1.212	1.203	1.182	1.128	0.996
0.0	1.145	1.141	1.132	1.110	1.057	0.925
-1.0	0.886	0.882	0.873	0.852	0.798	0.665
-2.0	0.504	0.500	0.491	0.470	0.416	0.282
-3.0	0.224	0.219	0.210	0.189	0.135	0.0
-4.0	0.089	0.084	0.075	0.054	0.0	0.0
-5.0	0.035	0.030	0.021	0.0	0.0	0.0
-6.0	0.013	0.009	0.0	0.0	0.0	0.0
-7.0	0.004	0.0	0.0	0.0	0.0	0.0

** $\nu = 20$ $\beta_N = 1.60$ **

i_{N_1}	i_{N_0} = -8.0	-7.0	-6.0	-5.0	-4.0	-3.0
4.0	1.373	1.359	1.334	1.288	1.191	0.996
3.0	1.373	1.359	1.334	1.288	1.191	0.996
2.0	1.373	1.359	1.334	1.288	1.191	0.995
1.0	1.369	1.354	1.330	1.283	1.186	0.991
0.0	1.319	1.305	1.281	1.234	1.137	0.941
-1.0	1.104	1.090	1.065	1.019	0.922	0.725
-2.0	0.721	0.707	0.683	0.636	0.539	0.341
-3.0	0.381	0.367	0.342	0.296	0.199	0.0
-4.0	0.183	0.168	0.144	0.097	0.0	0.0
-5.0	0.085	0.071	0.047	0.0	0.0	0.0
-6.0	0.039	0.024	0.0	0.0	0.0	0.0
-7.0	0.014	0.0	0.0	0.0	0.0	0.0

** ν =20 β_N =1.80 **

	i_{N_0}					
i_{N_1}	-8.0	-7.0	-6.0	-5.0	-4.0	-3.0
4.0	1.632	1.587	1.524	1.425	1.257	0.978
3.0	1.632	1.587	1.524	1.425	1.257	0.978
2.0	1.632	1.587	1.524	1.425	1.257	0.978
1.0	1.630	1.584	1.521	1.422	1.254	0.975
0.0	1.597	1.552	1.488	1.389	1.222	0.943
-1.0	1.424	1.379	1.316	1.217	1.049	0.769
-2.0	1.055	1.010	0.947	0.848	0.680	0.399
-3.0	0.658	0.612	0.549	0.450	0.282	0.0
-4.0	0.376	0.330	0.267	0.168	0.0	0.0
-5.0	0.208	0.162	0.099	0.0	0.0	0.0
-6.0	0.108	0.063	0.0	0.0	0.0	0.0
-7.0	0.045	0.0	0.0	0.0	0.0	0.0

** ν =20 β_N =2.00 **

	i_{N_0}					
i_{N_1}	-8.0	-7.0	-6.0	-5.0	-4.0	-3.0
4.0	2.112	1.973	1.816	1.613	1.333	0.950
3.0	2.112	1.973	1.816	1.613	1.333	0.950
2.0	2.112	1.973	1.816	1.613	1.333	0.950
1.0	2.110	1.972	1.814	1.612	1.332	0.949
0.0	2.089	1.951	1.793	1.591	1.311	0.928
-1.0	1.956	1.818	1.660	1.458	1.177	0.794
-2.0	1.612	1.474	1.316	1.114	0.834	0.449
-3.0	1.165	1.027	0.869	0.667	0.386	0.0
-4.0	0.779	0.641	0.483	0.281	0.0	0.0
-5.0	0.498	0.360	0.202	0.0	0.0	0.0
-6.0	0.296	0.158	0.0	0.0	0.0	0.0
-7.0	0.138	0.0	0.0	0.0	0.0	0.0

APPENDIX B

PENALTY FACTORS FOR UNKNOWN DEMAND AND RESISTANCE PARAMETERS

Tables of the ratio $\gamma_{\lambda i_d, \beta_N, \mu_R, \sigma_R} / e^{\sigma_d^2/2}$ for unknown demand and resistance parameters (see Equation V.32). The tables are for:

$\sigma_{\beta_N}=0.1, 0.2, 0.3$

$i_d=-3,-6$

$\nu=5,10$

$\mu_{\beta_N}=0.6, 1.0, 1.4, 1.8$

$i_{N_o}=-8(1)-3$

$i_{N_1}=4(-1)i_{N_o}$

$**\sigma_{\beta_N}=0.10 \quad -i_d=3.00 **$

$** \nu = 5 \quad \mu_{\beta_N}=0.60 **$

	i_{N_0}					
i_{N_1}	-8.0	-7.0	-6.0	-5.0	-4.0	-3.0
4.0	1.192	1.175	1.156	1.132	1.098	1.043
3.0	1.191	1.174	1.154	1.130	1.096	1.042
2.0	1.182	1.165	1.146	1.122	1.088	1.034
1.0	1.130	1.113	1.094	1.070	1.035	0.981
0.0	0.914	0.897	0.877	0.853	0.818	0.761
-1.0	0.550	0.533	0.513	0.488	0.452	0.391
-2.0	0.291	0.274	0.255	0.229	0.193	0.129
-3.0	0.164	0.147	0.127	0.102	0.065	0.0
-4.0	0.100	0.083	0.063	0.037	0.0	0.0
-5.0	0.062	0.045	0.025	0.0	0.0	0.0
-6.0	0.037	0.020	0.0	0.0	0.0	0.0
-7.0	0.017	0.0	0.0	0.0	0.0	0.0

$**\sigma_{\beta_N}=0.10 \quad -i_d=3.00 **$

$** \nu = 5 \quad \mu_{\beta_N}=1.00 **$

	i_{N_0}					
i_{N_1}	-8.0	-7.0	-6.0	-5.0	-4.0	-3.0
4.0	2.027	1.777	1.584	1.420	1.260	1.084
3.0	2.027	1.776	1.584	1.420	1.260	1.084
2.0	2.025	1.774	1.582	1.417	1.258	1.081
1.0	2.003	1.752	1.559	1.395	1.236	1.059
0.0	1.869	1.618	1.425	1.261	1.101	0.923
-1.0	1.544	1.293	1.100	0.935	0.774	0.593
-2.0	1.207	0.956	0.763	0.597	0.436	0.251
-3.0	0.960	0.709	0.516	0.350	0.187	0.0
-4.0	0.773	0.522	0.329	0.163	0.0	0.0
-5.0	0.610	0.359	0.166	0.0	0.0	0.0
-6.0	0.444	0.193	0.0	0.0	0.0	0.0
-7.0	0.251	0.0	0.0	0.0	0.0	0.0

** σ_{β_N}=0.10 $-\dot{i}_d$=3.00 **

** ν= 5 μ_{β_N}=1.40 **

	$\dot{i}_{N_o}$					
$\dot{i}_{N_1}$	-8.0	-7.0	-6.0	-5.0	-4.0	-3.0
4.0	7.946	4.772	3.142	2.208	1.599	1.141
3.0	7.946	4.772	3.142	2.208	1.599	1.141
2.0	7.946	4.772	3.142	2.208	1.599	1.140
1.0	7.938	4.763	3.134	2.200	1.591	1.132
0.0	7.866	4.692	3.062	2.128	1.519	1.059
-1.0	7.615	4.441	2.811	1.877	1.267	0.805
-2.0	7.236	4.062	2.432	1.497	0.886	0.420
-3.0	6.822	3.648	2.018	1.082	0.470	0.0
-4.0	6.355	3.180	1.550	0.614	0.0	0.0
-5.0	5.742	2.567	0.936	0.0	0.0	0.0
-6.0	4.806	1.631	0.0	0.0	0.0	0.0
-7.0	3.175	0.0	0.0	0.0	0.0	0.0

** σ_{β_N}=0.10 $-\dot{i}_d$=3.00 **

** ν= 5 μ_{β_N}=1.80 **

	$\dot{i}_{N_o}$					
$\dot{i}_{N_1}$	-8.0	-7.0	-6.0	-5.0	-4.0	-3.0
4.0	55.273	20.626	8.735	4.181	2.193	1.187
3.0	55.273	20.626	8.735	4.181	2.193	1.187
2.0	55.273	20.626	8.735	4.181	2.193	1.187
1.0	55.270	20.623	8.732	4.178	2.190	1.184
0.0	55.237	20.590	8.699	4.145	2.157	1.151
-1.0	55.071	20.423	8.532	3.978	1.990	0.981
-2.0	54.702	20.055	8.163	3.609	1.619	0.607
-3.0	54.104	19.456	7.565	3.009	1.018	0.0
-4.0	53.090	18.443	6.551	1.994	0.0	0.0
-5.0	51.100	16.452	4.559	0.0	0.0	0.0
-6.0	46.544	11.895	0.0	0.0	0.0	0.0
-7.0	34.651	0.0	0.0	0.0	0.0	0.0

** σ_{β_N}=0.10 $-i_d$=3.00 **

** ν=10 μ_{β_N}=0.60 **

i_{N_1}	i_{N_0} -8.0	-7.0	-6.0	-5.0	-4.0	-3.0
4.0	1.088	1.087	1.085	1.081	1.071	1.042
3.0	1.088	1.087	1.085	1.080	1.070	1.041
2.0	1.080	1.079	1.078	1.073	1.063	1.034
1.0	1.027	1.026	1.024	1.019	1.009	0.980
0.0	0.802	0.801	0.799	0.795	0.784	0.753
-1.0	0.423	0.422	0.420	0.416	0.405	0.372
-2.0	0.159	0.158	0.156	0.151	0.140	0.106
-3.0	0.054	0.053	0.051	0.046	0.034	0.0
-4.0	0.019	0.018	0.016	0.012	0.0	0.0
-5.0	0.007	0.006	0.004	0.0	0.0	0.0
-6.0	0.003	0.002	0.0	0.0	0.0	0.0
-7.0	0.001	0.0	0.0	0.0	0.0	0.0

** σ_{β_N}=0.10 $-i_d$=3.00 **

** ν=10 μ_{β_N}=1.00 **

i_{N_1}	i_{N_0} -8.0	-7.0	-6.0	-5.0	-4.0	-3.0
4.0	1.258	1.244	1.225	1.197	1.148	1.055
3.0	1.258	1.244	1.225	1.196	1.148	1.055
2.0	1.256	1.242	1.223	1.194	1.146	1.053
1.0	1.233	1.219	1.200	1.172	1.123	1.030
0.0	1.094	1.080	1.061	1.032	0.984	0.890
-1.0	0.755	0.741	0.722	0.694	0.645	0.549
-2.0	0.412	0.397	0.378	0.350	0.301	0.203
-3.0	0.210	0.196	0.177	0.148	0.099	0.0
-4.0	0.112	0.097	0.078	0.050	0.0	0.0
-5.0	0.062	0.047	0.029	0.0	0.0	0.0
-6.0	0.033	0.019	0.0	0.0	0.0	0.0
-7.0	0.014	0.0	0.0	0.0	0.0	0.0

** σ_{β_N}=0.10 $-i_d$=3.00 **

** ν=10 μ_{β_N}=1.40 **

	i_{N_0}					
i_{N_1}	-8.0	-7.0	-6.0	-5.0	-4.0	-3.0
4.0	1.985	1.805	1.648	1.491	1.308	1.071
3.0	1.985	1.805	1.648	1.491	1.308	1.071
2.0	1.984	1.804	1.648	1.490	1.308	1.070
1.0	1.976	1.796	1.639	1.482	1.299	1.062
0.0	1.901	1.722	1.565	1.407	1.225	0.987
-1.0	1.640	1.461	1.304	1.146	0.963	0.724
-2.0	1.255	1.075	0.918	0.760	0.577	0.336
-3.0	0.921	0.741	0.584	0.427	0.243	0.0
-4.0	0.678	0.498	0.341	0.184	0.0	0.0
-5.0	0.494	0.315	0.158	0.0	0.0	0.0
-6.0	0.337	0.157	0.0	0.0	0.0	0.0
-7.0	0.180	0.0	0.0	0.0	0.0	0.0

** σ_{β_N}=0.10 $-i_d$=3.00 **

** ν=10 μ_{β_N}=1.80 **

	i_{N_0}					
i_{N_1}	-8.0	-7.0	-6.0	-5.0	-4.0	-3.0
4.0	5.970	4.042	2.918	2.165	1.580	1.069
3.0	5.970	4.042	2.918	2.165	1.580	1.069
2.0	5.970	4.042	2.918	2.164	1.580	1.068
1.0	5.968	4.039	2.915	2.162	1.578	1.066
0.0	5.933	4.005	2.881	2.128	1.543	1.031
-1.0	5.759	3.831	2.707	1.954	1.369	0.856
-2.0	5.386	3.458	2.334	1.580	0.995	0.480
-3.0	4.909	2.981	1.857	1.103	0.518	0.0
-4.0	4.392	2.464	1.340	0.586	0.0	0.0
-5.0	3.806	1.878	0.754	0.0	0.0	0.0
-6.0	3.052	1.124	0.0	0.0	0.0	0.0
-7.0	1.928	0.0	0.0	0.0	0.0	0.0

** σ_{β_N}=0.10 $-i_d$=6.00 **

** ν= 5 μ_{β_N} =0.60 **

	i_{N_0}					
i_{N_1}	-8.0	-7.0	-6.0	-5.0	-4.0	-3.0
4.0	1.317	1.301	1.283	1.260	1.226	1.171
3.0	1.315	1.300	1.281	1.258	1.224	1.169
2.0	1.305	1.289	1.271	1.248	1.214	1.159
1.0	1.242	1.227	1.209	1.186	1.152	1.096
0.0	0.992	0.976	0.958	0.934	0.900	0.841
-1.0	0.581	0.566	0.547	0.523	0.487	0.425
-2.0	0.298	0.283	0.264	0.240	0.203	0.138
-3.0	0.163	0.147	0.128	0.104	0.067	0.0
-4.0	0.096	0.081	0.062	0.038	0.0	0.0
-5.0	0.059	0.043	0.025	0.0	0.0	0.0
-6.0	0.034	0.019	0.0	0.0	0.0	0.0
-7.0	0.016	0.0	0.0	0.0	0.0	0.0

** σ_{β_N}=0.10 $-i_d$=6.00 **

** ν= 5 μ_{β_N} =1.00 **

	i_{N_0}					
i_{N_1}	-8.0	-7.0	-6.0	-5.0	-4.0	-3.0
4.0	2.110	1.881	1.700	1.540	1.381	1.200
3.0	2.109	1.880	1.699	1.540	1.381	1.200
2.0	2.107	1.878	1.697	1.537	1.378	1.197
1.0	2.080	1.851	1.670	1.511	1.352	1.170
0.0	1.925	1.696	1.515	1.355	1.196	1.012
-1.0	1.559	1.330	1.148	0.988	0.828	0.641
-2.0	1.190	0.961	0.779	0.619	0.457	0.266
-3.0	0.927	0.698	0.517	0.356	0.193	0.0
-4.0	0.735	0.506	0.324	0.163	0.0	0.0
-5.0	0.572	0.343	0.161	0.0	0.0	0.0
-6.0	0.411	0.182	0.0	0.0	0.0	0.0
-7.0	0.229	0.0	0.0	0.0	0.0	0.0

**σ_{β_N}=0.10 $-i_d$=6.00 **

** ν= 5 μ_{β_N}=1.40 **

	i_{N_0}					
i_{N_1}	-8.0	-7.0	-6.0	-5.0	-4.0	-3.0
4.0	7.661	4.765	3.233	2.328	1.719	1.248
3.0	7.661	4.765	3.233	2.328	1.719	1.248
2.0	7.660	4.764	3.232	2.327	1.719	1.247
1.0	7.651	4.755	3.222	2.317	1.709	1.237
0.0	7.568	4.672	3.139	2.234	1.626	1.153
-1.0	7.286	4.390	2.857	1.952	1.342	0.867
-2.0	6.871	3.975	2.442	1.536	0.925	0.446
-3.0	6.432	3.535	2.002	1.096	0.484	0.0
-4.0	5.950	3.054	1.520	0.613	0.0	0.0
-5.0	5.338	2.441	0.908	0.0	0.0	0.0
-6.0	4.431	1.534	0.0	0.0	0.0	0.0
-7.0	2.897	0.0	0.0	0.0	0.0	0.0

**σ_{β_N}=0.10 $-i_d$=6.00 **

** ν= 5 μ_{β_N}=1.80 **

	i_{N_0}					
i_{N_1}	-8.0	-7.0	-6.0	-5.0	-4.0	-3.0
4.0	51.464	19.884	8.713	4.304	2.320	1.285
3.0	51.464	19.884	8.713	4.304	2.320	1.285
2.0	51.464	19.884	8.713	4.303	2.320	1.285
1.0	51.461	19.881	8.710	4.300	2.317	1.282
0.0	51.423	19.843	8.672	4.262	2.278	1.243
-1.0	51.235	19.655	8.484	4.074	2.090	1.053
-2.0	50.832	19.252	8.081	3.671	1.685	0.644
-3.0	50.198	18.618	7.446	3.035	1.048	0.0
-4.0	49.155	17.575	6.403	1.991	0.0	0.0
-5.0	47.168	15.587	4.414	0.0	0.0	0.0
-6.0	42.756	11.175	0.0	0.0	0.0	0.0
-7.0	31.584	0.0	0.0	0.0	0.0	0.0

** σ_{β_N}=0.10 $-i_d$=6.00 **

** ν=10 μ_{β_N}=0.60 **

	i_{N_0}					
i_{N_1}	-8.0	-7.0	-6.0	-5.0	-4.0	-3.0
4.0	1.217	1.217	1.215	1.211	1.200	1.171
3.0	1.217	1.216	1.214	1.210	1.199	1.170
2.0	1.208	1.207	1.205	1.201	1.191	1.161
1.0	1.144	1.143	1.141	1.137	1.126	1.097
0.0	0.883	0.882	0.881	0.876	0.866	0.834
-1.0	0.456	0.455	0.453	0.449	0.438	0.405
-2.0	0.166	0.165	0.164	0.159	0.148	0.113
-3.0	0.054	0.053	0.052	0.047	0.036	0.0
-4.0	0.019	0.018	0.016	0.012	0.0	0.0
-5.0	0.007	0.006	0.004	0.0	0.0	0.0
-6.0	0.003	0.002	0.0	0.0	0.0	0.0
-7.0	0.001	0.0	0.0	0.0	0.0	0.0

** σ_{β_N}=0.10 $-i_d$=6.00 **

** ν=10 μ_{β_N}=1.00 **

	i_{N_0}					
i_{N_1}	-8.0	-7.0	-6.0	-5.0	-4.0	-3.0
4.0	1.374	1.360	1.343	1.315	1.266	1.171
3.0	1.373	1.360	1.342	1.315	1.266	1.171
2.0	1.371	1.358	1.340	1.312	1.264	1.168
1.0	1.344	1.330	1.313	1.285	1.236	1.141
0.0	1.182	1.169	1.152	1.124	1.075	0.979
-1.0	0.801	0.788	0.770	0.743	0.693	0.595
-2.0	0.425	0.412	0.394	0.366	0.317	0.216
-3.0	0.210	0.197	0.179	0.151	0.102	0.0
-4.0	0.109	0.095	0.078	0.050	0.0	0.0
-5.0	0.059	0.046	0.028	0.0	0.0	0.0
-6.0	0.031	0.018	0.0	0.0	0.0	0.0
-7.0	0.013	0.0	0.0	0.0	0.0	0.0

$\sigma_{\beta_N}=0.10$ $-i_d=6.00$

$\nu=10$ $\mu_{\beta_N}=1.40$

i_{N_1} \ i_{N_0}	-8.0	-7.0	-6.0	-5.0	-4.0	-3.0
4.0	2.066	1.902	1.754	1.602	1.419	1.175
3.0	2.066	1.902	1.754	1.602	1.419	1.175
2.0	2.066	1.902	1.754	1.601	1.419	1.174
1.0	2.056	1.892	1.744	1.591	1.409	1.164
0.0	1.970	1.806	1.658	1.505	1.323	1.078
-1.0	1.676	1.512	1.364	1.211	1.028	0.782
-2.0	1.254	1.090	0.942	0.789	0.606	0.357
-3.0	0.899	0.735	0.587	0.434	0.251	0.0
-4.0	0.649	0.485	0.337	0.184	0.0	0.0
-5.0	0.465	0.301	0.153	0.0	0.0	0.0
-6.0	0.312	0.148	0.0	0.0	0.0	0.0
-7.0	0.164	0.0	0.0	0.0	0.0	0.0

$\sigma_{\beta_N}=0.10$ $-i_d=6.00$

$\nu=10$ $\mu_{\beta_N}=1.80$

i_{N_1} \ i_{N_0}	-8.0	-7.0	-6.0	-5.0	-4.0	-3.0
4.0	5.819	4.060	3.002	2.272	1.688	1.161
3.0	5.819	4.060	3.002	2.272	1.688	1.161
2.0	5.819	4.059	3.002	2.271	1.688	1.161
1.0	5.816	4.056	2.999	2.268	1.684	1.157
0.0	5.776	4.017	2.960	2.229	1.645	1.118
-1.0	5.581	3.821	2.764	2.033	1.449	0.921
-2.0	5.173	3.413	2.356	1.625	1.040	0.510
-3.0	4.666	2.907	1.849	1.118	0.533	0.0
-4.0	4.134	2.374	1.317	0.586	0.0	0.0
-5.0	3.548	1.789	0.731	0.0	0.0	0.0
-6.0	2.817	1.057	0.0	0.0	0.0	0.0
-7.0	1.760	0.0	0.0	0.0	0.0	0.0

σ_{β_N}=0.20 $-i_d$=3.00

ν= 5 μ_{β_N} =0.60

	i_{N_0}					
i_{N_1}	-8.0	-7.0	-6.0	-5.0	-4.0	-3.0
4.0	1.318	1.295	1.271	1.245	1.210	1.157
3.0	1.315	1.292	1.269	1.242	1.207	1.154
2.0	1.302	1.279	1.256	1.230	1.195	1.141
1.0	1.233	1.210	1.187	1.160	1.125	1.070
0.0	0.975	0.952	0.928	0.901	0.865	0.809
-1.0	0.574	0.551	0.528	0.500	0.463	0.402
-2.0	0.306	0.283	0.259	0.232	0.194	0.130
-3.0	0.178	0.155	0.131	0.103	0.065	0.0
-4.0	0.114	0.090	0.067	0.039	0.0	0.0
-5.0	0.075	0.052	0.028	0.0	0.0	0.0
-6.0	0.047	0.024	0.0	0.0	0.0	0.0
-7.0	0.023	0.0	0.0	0.0	0.0	0.0

σ_{β_N}=0.20 $-i_d$=3.00

ν= 5 μ_{β_N} =1.00

	i_{N_0}					
i_{N_1}	-8.0	-7.0	-6.0	-5.0	-4.0	-3.0
4.0	2.261	1.919	1.687	1.506	1.341	1.164
3.0	2.260	1.919	1.686	1.505	1.341	1.164
2.0	2.257	1.915	1.683	1.502	1.337	1.160
1.0	2.227	1.886	1.653	1.472	1.307	1.130
0.0	2.068	1.726	1.494	1.313	1.147	0.969
-1.0	1.711	1.369	1.137	0.955	0.789	0.607
-2.0	1.362	1.020	0.788	0.605	0.438	0.252
-3.0	1.114	0.772	0.539	0.357	0.188	0.0
-4.0	0.926	0.584	0.352	0.169	0.0	0.0
-5.0	0.758	0.416	0.183	0.0	0.0	0.0
-6.0	0.575	0.233	0.0	0.0	0.0	0.0
-7.0	0.342	0.0	0.0	0.0	0.0	0.0

$**\sigma_{\beta_N}=0.20 \quad -i_d=3.00 **$

$** \nu = 5 \quad \mu_{\beta_N}=1.40 **$

	i_{N_0}					
i_{N_1}	-8.0	-7.0	-6.0	-5.0	-4.0	-3.0
4.0	9.636	5.292	3.320	2.289	1.657	1.197
3.0	9.636	5.292	3.319	2.289	1.657	1.197
2.0	9.635	5.291	3.319	2.288	1.656	1.196
1.0	9.625	5.281	3.308	2.277	1.646	1.185
0.0	9.540	5.196	3.223	2.192	1.560	1.099
-1.0	9.265	4.921	2.948	1.917	1.285	0.820
-2.0	8.873	4.529	2.555	1.524	0.890	0.422
-3.0	8.457	4.113	2.139	1.108	0.472	0.0
-4.0	7.987	3.642	1.669	0.637	0.0	0.0
-5.0	7.351	3.007	1.033	0.0	0.0	0.0
-6.0	6.319	1.974	0.0	0.0	0.0	0.0
-7.0	4.345	0.0	0.0	0.0	0.0	0.0

$**\sigma_{\beta_N}=0.20 \quad -i_d=3.00 **$

$** \nu = 5 \quad \mu_{\beta_N}=1.80 **$

	i_{N_0}					
i_{N_1}	-8.0	-7.0	-6.0	-5.0	-4.0	-3.0
4.0	71.400	23.779	9.338	4.301	2.237	1.225
3.0	71.400	23.778	9.338	4.301	2.236	1.225
2.0	71.399	23.778	9.338	4.301	2.236	1.224
1.0	71.396	23.775	9.335	4.297	2.233	1.221
0.0	71.357	23.736	9.296	4.258	2.194	1.182
-1.0	71.175	23.554	9.114	4.076	2.011	0.997
-2.0	70.794	23.173	8.732	3.694	1.628	0.610
-3.0	70.193	22.572	8.131	3.092	1.024	0.0
-4.0	69.174	21.552	7.111	2.071	0.0	0.0
-5.0	67.106	19.484	5.042	0.0	0.0	0.0
-6.0	62.068	14.445	0.0	0.0	0.0	0.0
-7.0	47.626	0.0	0.0	0.0	0.0	0.0

** σ_{β_N}=0.20 $-i_d$=3.00 **

** ν=10 μ_{β_N} =0.60 **

i_{N_1}	i_{N_0} -8.0	-7.0	-6.0	-5.0	-4.0	-3.0
4.0	1.204	1.203	1.200	1.196	1.185	1.157
3.0	1.203	1.201	1.199	1.194	1.184	1.155
2.0	1.192	1.190	1.188	1.184	1.173	1.144
1.0	1.120	1.119	1.116	1.112	1.101	1.072
0.0	0.852	0.851	0.848	0.844	0.833	0.802
-1.0	0.436	0.434	0.432	0.427	0.416	0.383
-2.0	0.161	0.160	0.157	0.153	0.141	0.107
-3.0	0.055	0.054	0.051	0.047	0.035	0.0
-4.0	0.021	0.019	0.017	0.012	0.0	0.0
-5.0	0.009	0.007	0.005	0.0	0.0	0.0
-6.0	0.004	0.002	0.0	0.0	0.0	0.0
-7.0	0.001	0.0	0.0	0.0	0.0	0.0

** σ_{β_N}=0.20 $-i_d$=3.00 **

** ν=10 μ_{β_N} =1.00 **

i_{N_1}	i_{N_0} -8.0	-7.0	-6.0	-5.0	-4.0	-3.0
4.0	1.354	1.334	1.311	1.280	1.230	1.137
3.0	1.353	1.334	1.311	1.280	1.230	1.137
2.0	1.350	1.331	1.308	1.277	1.227	1.134
1.0	1.320	1.300	1.278	1.247	1.196	1.103
0.0	1.155	1.135	1.112	1.081	1.031	0.937
-1.0	0.783	0.764	0.741	0.710	0.659	0.563
-2.0	0.427	0.407	0.384	0.353	0.302	0.204
-3.0	0.224	0.204	0.182	0.150	0.099	0.0
-4.0	0.125	0.105	0.083	0.051	0.0	0.0
-5.0	0.074	0.054	0.031	0.0	0.0	0.0
-6.0	0.042	0.023	0.0	0.0	0.0	0.0
-7.0	0.020	0.0	0.0	0.0	0.0	0.0

** σ_{β_N}=0.20 $-i_d$=3.00 **

** ν =10 μ_{β_N} =1.40 **

	i_{N_0}					
i_{N_1}	-8.0	-7.0	-6.0	-5.0	-4.0	-3.0
4.0	2.162	1.917	1.728	1.555	1.366	1.128
3.0	2.162	1.917	1.728	1.555	1.366	1.128
2.0	2.161	1.916	1.727	1.554	1.365	1.127
1.0	2.150	1.905	1.716	1.543	1.354	1.116
0.0	2.062	1.817	1.628	1.455	1.266	1.028
-1.0	1.776	1.531	1.342	1.169	0.980	0.740
-2.0	1.377	1.132	0.943	0.770	0.580	0.338
-3.0	1.041	0.796	0.607	0.434	0.244	0.0
-4.0	0.797	0.553	0.364	0.190	0.0	0.0
-5.0	0.607	0.362	0.173	0.0	0.0	0.0
-6.0	0.433	0.189	0.0	0.0	0.0	0.0
-7.0	0.244	0.0	0.0	0.0	0.0	0.0

** σ_{β_N}=0.20 $-i_d$=3.00 **

** ν =10 μ_{β_N} =1.80 **

	i_{N_0}					
i_{N_1}	-8.0	-7.0	-6.0	-5.0	-4.0	-3.0
4.0	7.051	4.416	3.057	2.227	1.621	1.107
3.0	7.051	4.416	3.057	2.227	1.621	1.107
2.0	7.051	4.416	3.057	2.226	1.621	1.107
1.0	7.048	4.412	3.054	2.223	1.617	1.103
0.0	7.007	4.372	3.013	2.183	1.577	1.063
-1.0	6.817	4.182	2.823	1.993	1.387	0.871
-2.0	6.431	3.796	2.437	1.606	1.000	0.483
-3.0	5.952	3.316	1.958	1.127	0.520	0.0
-4.0	5.432	2.797	1.438	0.607	0.0	0.0
-5.0	4.825	2.190	0.831	0.0	0.0	0.0
-6.0	3.994	1.359	0.0	0.0	0.0	0.0
-7.0	2.636	0.0	0.0	0.0	0.0	0.0

$** \sigma_{\beta_N} = 0.20 \quad -i_d = 6.00 **$

$** \nu = 5 \quad \mu_{\beta_N} = 0.60 **$

	i_{N_0}					
i_{N_1}	-8.0	-7.0	-6.0	-5.0	-4.0	-3.0
4.0	2.013	1.997	1.979	1.957	1.924	1.869
3.0	2.007	1.991	1.973	1.950	1.917	1.862
2.0	1.978	1.962	1.944	1.921	1.888	1.833
1.0	1.837	1.821	1.803	1.780	1.747	1.690
0.0	1.371	1.355	1.337	1.313	1.278	1.217
-1.0	0.723	0.707	0.689	0.665	0.628	0.560
-2.0	0.336	0.320	0.301	0.277	0.239	0.167
-3.0	0.172	0.156	0.137	0.112	0.074	0.0
-4.0	0.099	0.082	0.064	0.039	0.0	0.0
-5.0	0.060	0.044	0.025	0.0	0.0	0.0
-6.0	0.035	0.019	0.0	0.0	0.0	0.0
-7.0	0.016	0.0	0.0	0.0	0.0	0.0

$** \sigma_{\beta_N} = 0.20 \quad -i_d = 6.00 **$

$** \nu = 5 \quad \mu_{\beta_N} = 1.00 **$

	i_{N_0}					
i_{N_1}	-8.0	-7.0	-6.0	-5.0	-4.0	-3.0
4.0	2.712	2.475	2.293	2.134	1.971	1.776
3.0	2.711	2.474	2.292	2.133	1.970	1.775
2.0	2.703	2.466	2.284	2.125	1.961	1.766
1.0	2.644	2.407	2.225	2.065	1.902	1.706
0.0	2.357	2.120	1.938	1.778	1.614	1.415
-1.0	1.782	1.545	1.363	1.202	1.036	0.832
-2.0	1.280	1.042	0.860	0.699	0.531	0.322
-3.0	0.963	0.725	0.543	0.381	0.213	0.0
-4.0	0.751	0.513	0.331	0.169	0.0	0.0
-5.0	0.582	0.345	0.162	0.0	0.0	0.0
-6.0	0.421	0.183	0.0	0.0	0.0	0.0
-7.0	0.238	0.0	0.0	0.0	0.0	0.0

** σ_{β_N}=0.20 $-i_d$=6.00 **

** ν = 5 μ_{β_N} =1.40 **

	i_{N_0}					
i_{N_1}	-8.0	-7.0	-6.0	-5.0	-4.0	-3.0
4.0	8.332	5.325	3.784	2.875	2.248	1.733
3.0	8.332	5.325	3.783	2.875	2.247	1.733
2.0	8.330	5.323	3.781	2.873	2.245	1.731
1.0	8.308	5.301	3.760	2.852	2.224	1.709
0.0	8.156	5.149	3.608	2.699	2.071	1.554
-1.0	7.715	4.708	3.167	2.258	1.628	1.107
-2.0	7.153	4.146	2.604	1.694	1.063	0.536
-3.0	6.624	3.617	2.075	1.165	0.532	0.0
-4.0	6.095	3.088	1.546	0.635	0.0	0.0
-5.0	5.462	2.454	0.912	0.0	0.0	0.0
-6.0	4.551	1.543	0.0	0.0	0.0	0.0
-7.0	3.008	0.0	0.0	0.0	0.0	0.0

** σ_{β_N}=0.20 $-i_d$=6.00 **

** ν = 5 μ_{β_N} =1.80 **

	i_{N_0}					
i_{N_1}	-8.0	-7.0	-6.0	-5.0	-4.0	-3.0
4.0	53.384	20.544	9.303	4.877	2.829	1.698
3.0	53.384	20.544	9.303	4.877	2.829	1.698
2.0	53.383	20.544	9.303	4.876	2.829	1.697
1.0	53.377	20.537	9.296	4.870	2.822	1.691
0.0	53.307	20.467	9.227	4.800	2.752	1.620
-1.0	53.016	20.176	8.935	4.508	2.459	1.324
-2.0	52.472	19.632	8.391	3.963	1.913	0.772
-3.0	51.711	18.871	7.630	3.201	1.148	0.0
-4.0	50.568	17.728	6.486	2.056	0.0	0.0
-5.0	48.516	15.675	4.432	0.0	0.0	0.0
-6.0	44.087	11.245	0.0	0.0	0.0	0.0
-7.0	32.844	0.0	0.0	0.0	0.0	0.0

** σ_{β_N}=0.20 $-i_d$=6.00 **

** ν=10 μ_{β_N}=0.60 **

	i_{N_0}					
i_{N_1}	-8.0	-7.0	-6.0	-5.0	-4.0	-3.0
4.0	1.922	1.922	1.920	1.916	1.906	1.876
3.0	1.919	1.918	1.916	1.912	1.902	1.873
2.0	1.894	1.893	1.892	1.888	1.877	1.848
1.0	1.749	1.748	1.747	1.743	1.732	1.702
0.0	1.265	1.264	1.263	1.259	1.248	1.215
-1.0	0.592	0.591	0.589	0.585	0.573	0.537
-2.0	0.195	0.194	0.192	0.188	0.176	0.137
-3.0	0.059	0.058	0.056	0.052	0.039	0.0
-4.0	0.019	0.018	0.016	0.012	0.0	0.0
-5.0	0.007	0.006	0.004	0.0	0.0	0.0
-6.0	0.003	0.002	0.0	0.0	0.0	0.0
-7.0	0.001	0.0	0.0	0.0	0.0	0.0

** σ_{β_N}=0.20 $-i_d$=6.00 **

** ν=10 μ_{β_N}=1.00 **

	i_{N_0}					
i_{N_1}	-8.0	-7.0	-6.0	-5.0	-4.0	-3.0
4.0	1.961	1.948	1.930	1.902	1.852	1.750
3.0	1.961	1.947	1.929	1.902	1.852	1.749
2.0	1.954	1.940	1.922	1.895	1.845	1.742
1.0	1.892	1.879	1.861	1.833	1.784	1.680
0.0	1.595	1.581	1.563	1.536	1.485	1.381
-1.0	0.996	0.983	0.965	0.937	0.886	0.778
-2.0	0.483	0.470	0.452	0.424	0.373	0.262
-3.0	0.223	0.210	0.192	0.164	0.112	0.0
-4.0	0.111	0.097	0.080	0.052	0.0	0.0
-5.0	0.059	0.046	0.028	0.0	0.0	0.0
-6.0	0.032	0.018	0.0	0.0	0.0	0.0
-7.0	0.014	0.0	0.0	0.0	0.0	0.0

$**\sigma_{\beta_N}=0.20 \quad -i_d=6.00 **$

$** \nu=10 \quad \mu_{\beta_N}=1.40 **$

	i_{N_0}					
i_{N_1}	-8.0	-7.0	-6.0	-5.0	-4.0	-3.0
4.0	2.581	2.411	2.262	2.109	1.920	1.653
3.0	2.581	2.411	2.262	2.109	1.920	1.653
2.0	2.579	2.409	2.260	2.107	1.918	1.651
1.0	2.557	2.387	2.238	2.085	1.896	1.628
0.0	2.399	2.229	2.080	1.927	1.738	1.469
-1.0	1.940	1.770	1.621	1.468	1.278	1.007
-2.0	1.367	1.197	1.049	0.895	0.705	0.431
-3.0	0.939	0.769	0.620	0.467	0.276	0.0
-4.0	0.663	0.493	0.345	0.191	0.0	0.0
-5.0	0.473	0.302	0.154	0.0	0.0	0.0
-6.0	0.319	0.149	0.0	0.0	0.0	0.0
-7.0	0.170	0.0	0.0	0.0	0.0	0.0

$**\sigma_{\beta_N}=0.20 \quad -i_d=6.00 **$

$** \nu=10 \quad \mu_{\beta_N}=1.80 **$

	i_{N_0}					
i_{N_1}	-8.0	-7.0	-6.0	-5.0	-4.0	-3.0
4.0	6.360	4.534	3.470	2.736	2.132	1.554
3.0	6.360	4.534	3.470	2.736	2.132	1.554
2.0	6.360	4.533	3.470	2.736	2.132	1.554
1.0	6.353	4.526	3.463	2.729	2.125	1.547
0.0	6.280	4.454	3.391	2.657	2.053	1.474
-1.0	5.977	4.150	3.087	2.353	1.749	1.169
-2.0	5.425	3.599	2.536	1.801	1.196	0.613
-3.0	4.816	2.989	1.926	1.192	0.586	0.0
-4.0	4.230	2.404	1.341	0.606	0.0	0.0
-5.0	3.624	1.798	0.735	0.0	0.0	0.0
-6.0	2.890	1.063	0.0	0.0	0.0	0.0
-7.0	1.826	0.0	0.0	0.0	0.0	0.0

$**\sigma_{\beta_N}=0.30 \quad -i_d=3.00 **$

$** \ \nu = 5 \quad \mu_{\beta_N}=0.60 **$

	i_{N_0}					
i_{N_1}	-8.0	-7.0	-6.0	-5.0	-4.0	-3.0
4.0	1.586	1.548	1.516	1.485	1.449	1.397
3.0	1.580	1.541	1.510	1.479	1.442	1.391
2.0	1.553	1.515	1.483	1.452	1.416	1.364
1.0	1.441	1.402	1.371	1.340	1.303	1.250
0.0	1.094	1.055	1.024	0.992	0.954	0.898
-1.0	0.625	0.586	0.554	0.522	0.482	0.421
-2.0	0.339	0.300	0.268	0.236	0.195	0.131
-3.0	0.210	0.171	0.139	0.106	0.065	0.0
-4.0	0.145	0.106	0.074	0.041	0.0	0.0
-5.0	0.104	0.065	0.033	0.0	0.0	0.0
-6.0	0.071	0.032	0.0	0.0	0.0	0.0
-7.0	0.039	0.0	0.0	0.0	0.0	0.0

$**\sigma_{\beta_N}=0.30 \quad -i_d=3.00 **$

$** \ \nu = 5 \quad \mu_{\beta_N}=1.00 **$

	i_{N_0}					
i_{N_1}	-8.0	-7.0	-6.0	-5.0	-4.0	-3.0
4.0	2.783	2.209	1.891	1.678	1.503	1.327
3.0	2.782	2.207	1.889	1.677	1.502	1.325
2.0	2.774	2.200	1.882	1.669	1.495	1.318
1.0	2.727	2.153	1.835	1.622	1.447	1.270
0.0	2.514	1.940	1.621	1.408	1.233	1.054
-1.0	2.098	1.523	1.205	0.991	0.814	0.631
-2.0	1.727	1.152	0.833	0.619	0.442	0.254
-3.0	1.476	0.901	0.583	0.368	0.190	0.0
-4.0	1.287	0.712	0.394	0.179	0.0	0.0
-5.0	1.108	0.533	0.215	0.0	0.0	0.0
-6.0	0.894	0.319	0.0	0.0	0.0	0.0
-7.0	0.575	0.0	0.0	0.0	0.0	0.0

$**\sigma_{\beta_N}=0.30 \quad -i_d=3.00 **$

$** \nu = 5 \quad \mu_{\beta_N}=1.40 **$

i_{N_1}	i_{N_0} -8.0	-7.0	-6.0	-5.0	-4.0	-3.0
4.0	13.726	6.373	3.656	2.441	1.769	1.306
3.0	13.726	6.373	3.656	2.440	1.769	1.305
2.0	13.724	6.371	3.654	2.439	1.767	1.304
1.0	13.707	6.354	3.637	2.422	1.750	1.286
0.0	13.594	6.241	3.524	2.308	1.637	1.172
-1.0	13.275	5.922	3.205	1.989	1.316	0.848
-2.0	12.859	5.506	2.788	1.572	0.898	0.425
-3.0	12.439	5.087	2.369	1.152	0.477	0.0
-4.0	11.965	4.612	1.894	0.677	0.0	0.0
-5.0	11.290	3.936	1.218	0.0	0.0	0.0
-6.0	10.072	2.719	0.0	0.0	0.0	0.0
-7.0	7.354	0.0	0.0	0.0	0.0	0.0

$**\sigma_{\beta_N}=0.30 \quad -i_d=3.00 **$

$** \nu = 5 \quad \mu_{\beta_N}=1.80 **$

i_{N_1}	i_{N_0} -8.0	-7.0	-6.0	-5.0	-4.0	-3.0
4.0	111.653	30.482	10.482	4.517	2.317	1.296
3.0	111.653	30.482	10.482	4.517	2.317	1.296
2.0	111.652	30.481	10.482	4.516	2.316	1.295
1.0	111.647	30.476	10.476	4.511	2.311	1.290
0.0	111.595	30.424	10.425	4.459	2.259	1.237
-1.0	111.384	30.213	10.214	4.248	2.047	1.023
-2.0	110.981	29.810	9.810	3.844	1.642	0.614
-3.0	110.376	29.205	9.205	3.238	1.034	0.0
-4.0	109.347	28.176	8.175	2.207	0.0	0.0
-5.0	107.144	25.972	5.970	0.0	0.0	0.0
-6.0	101.177	20.004	0.0	0.0	0.0	0.0
-7.0	81.177	0.0	0.0	0.0	0.0	0.0

** σ_{β_N}=0.30 $-i_d$=3.00 **

** ν=10 μ_{β_N}=0.60 **

	i_{N_0}					
i_{N_1}	-8.0	-7.0	-6.0	-5.0	-4.0	-3.0
4.0	1.448	1.446	1.443	1.438	1.426	1.399
3.0	1.445	1.442	1.439	1.434	1.423	1.395
2.0	1.422	1.420	1.417	1.412	1.401	1.373
1.0	1.307	1.304	1.301	1.296	1.285	1.257
0.0	0.946	0.944	0.941	0.936	0.924	0.894
-1.0	0.458	0.456	0.453	0.447	0.435	0.402
-2.0	0.166	0.163	0.160	0.155	0.142	0.108
-3.0	0.059	0.056	0.053	0.047	0.035	0.0
-4.0	0.024	0.021	0.018	0.013	0.0	0.0
-5.0	0.011	0.009	0.006	0.0	0.0	0.0
-6.0	0.005	0.003	0.0	0.0	0.0	0.0
-7.0	0.002	0.0	0.0	0.0	0.0	0.0

** σ_{β_N}=0.30 $-i_d$=3.00 **

** ν=10 μ_{β_N}=1.00 **

	i_{N_0}					
i_{N_1}	-8.0	-7.0	-6.0	-5.0	-4.0	-3.0
4.0	1.548	1.516	1.485	1.449	1.396	1.303
3.0	1.547	1.515	1.484	1.448	1.395	1.302
2.0	1.541	1.509	1.478	1.442	1.389	1.296
1.0	1.492	1.460	1.429	1.393	1.340	1.247
0.0	1.271	1.239	1.208	1.172	1.118	1.024
-1.0	0.837	0.805	0.774	0.738	0.684	0.588
-2.0	0.459	0.426	0.395	0.359	0.305	0.206
-3.0	0.254	0.221	0.190	0.154	0.100	0.0
-4.0	0.154	0.122	0.091	0.054	0.0	0.0
-5.0	0.100	0.067	0.037	0.0	0.0	0.0
-6.0	0.063	0.031	0.0	0.0	0.0	0.0
-7.0	0.033	0.0	0.0	0.0	0.0	0.0

$**\sigma_{\beta_N}=0.30 \quad -i_d=3.00 **$

$** \nu=10 \quad \mu_{\beta_N}=1.40 **$

	i_{N_0}					
i_{N_1}	-8.0	-7.0	-6.0	-5.0	-4.0	-3.0
4.0	2.550	2.140	1.882	1.679	1.479	1.239
3.0	2.549	2.140	1.881	1.678	1.478	1.239
2.0	2.548	2.138	1.880	1.677	1.477	1.238
1.0	2.530	2.120	1.862	1.659	1.459	1.220
0.0	2.413	2.003	1.745	1.542	1.342	1.102
-1.0	2.080	1.671	1.412	1.209	1.009	0.767
-2.0	1.657	1.247	0.989	0.786	0.585	0.341
-3.0	1.319	0.909	0.650	0.447	0.246	0.0
-4.0	1.073	0.663	C.4C5	0.201	0.0	0.0
-5.0	0.871	0.462	0.203	0.0	0.0	0.0
-6.0	0.668	0.258	0.0	0.0	0.0	0.0
-7.0	0.410	0.0	0.0	0.0	0.0	0.0

$**\sigma_{\beta_N}=0.30 \quad -i_d=3.00 **$

$** \nu=10 \quad \mu_{\beta_N}=1.80 **$

	i_{N_0}					
i_{N_1}	-8.0	-7.0	-6.0	-5.0	-4.0	-3.0
4.0	9.637	5.185	3.318	2.340	1.697	1.179
3.0	9.637	5.185	3.318	2.340	1.697	1.179
2.0	9.636	5.185	3.318	2.340	1.697	1.179
1.0	9.631	5.179	3.312	2.334	1.691	1.173
0.0	9.577	5.126	3.259	2.281	1.638	1.119
-1.0	9.357	4.906	3.039	2.061	1.418	0.898
-2.0	8.949	4.498	2.630	1.652	1.009	0.487
-3.0	8.466	4.014	2.147	1.169	0.525	0.0
-4.0	7.942	3.490	1.623	0.645	0.0	0.0
-5.0	7.297	2.846	0.978	0.0	0.0	0.0
-6.0	6.319	1.867	0.0	0.0	0.0	0.0
-7.0	4.451	0.0	0.0	0.0	0.0	0.0

** σ_{β_N}=0.30 $-i_d$=6.00 **

** ν= 5 μ_{β_N} =0.60 **

	i_{N_0}					
i_{N_1}	-8.0	-7.0	-6.0	-5.0	-4.0	-3.0
4.0	4.559	4.543	4.526	4.506	4.479	4.436
3.0	4.502	4.486	4.469	4.449	4.421	4.378
2.0	4.332	4.316	4.299	4.279	4.251	4.205
1.0	3.779	3.763	3.746	3.725	3.695	3.644
0.0	2.462	2.445	2.428	2.405	2.371	2.307
-1.0	1.070	1.053	1.035	1.011	0.972	0.894
-2.0	0.415	0.398	0.380	0.355	0.315	0.230
-3.0	0.189	0.172	0.153	0.128	0.087	0.0
-4.0	0.102	0.085	0.066	0.041	0.0	0.0
-5.0	0.061	0.044	0.025	0.0	0.0	0.0
-6.0	0.036	0.019	0.0	0.0	0.0	0.0
-7.0	0.017	0.0	0.0	0.0	0.0	0.0

** σ_{β_N}=0.30 $-i_d$=6.00 **

** ν= 5 μ_{β_N} =1.00 **

	i_{N_0}					
i_{N_1}	-8.0	-7.0	-6.0	-5.0	-4.0	-3.0
4.0	4.622	4.370	4.188	4.029	3.862	3.647
3.0	4.611	4.360	4.177	4.018	3.851	3.637
2.0	4.564	4.312	4.130	3.971	3.803	3.588
1.0	4.334	4.082	3.900	3.740	3.572	3.355
0.0	3.531	3.279	3.096	2.936	2.765	2.540
-1.0	2.306	2.054	1.871	1.709	1.535	1.297
-2.0	1.463	1.211	1.027	0.865	0.687	0.442
-3.0	1.028	0.776	0.592	0.429	0.250	0.0
-4.0	0.779	0.527	0.342	0.179	0.0	0.0
-5.0	0.600	0.348	0.163	0.0	0.0	0.0
-6.0	0.437	0.184	0.0	0.0	0.0	0.0
-7.0	0.253	0.0	0.0	0.0	0.0	0.0

$**\sigma_{\beta_N} = 0.30 \quad -i_d = 6.00 **$

$** \nu = 5 \quad \mu_{\beta_N} = 1.40 **$

	i_{N_0}					
i_{N_1}	-8.0	-7.0	-6.0	-5.0	-4.0	-3.0
4.0	10.076	6.873	5.318	4.404	3.744	3.152
3.0	10.074	6.872	5.316	4.403	3.743	3.151
2.0	10.063	6.860	5.305	4.391	3.731	3.139
1.0	9.980	6.778	5.222	4.309	3.648	3.055
0.0	9.558	6.355	4.800	3.886	3.224	2.627
-1.0	8.628	5.425	3.869	2.954	2.289	1.683
-2.0	7.691	4.488	2.931	2.015	1.348	0.732
-3.0	6.970	3.767	2.210	1.293	0.623	0.0
-4.0	6.350	3.147	1.589	0.672	0.0	0.0
-5.0	5.679	2.476	0.918	0.0	0.0	0.0
-6.0	4.762	1.558	0.0	0.0	0.0	0.0
-7.0	3.204	0.0	0.0	0.0	0.0	0.0

$**\sigma_{\beta_N} = 0.30 \quad -i_d = 6.00 **$

$** \nu = 5 \quad \mu_{\beta_N} = 1.80 **$

	i_{N_0}					
i_{N_1}	-8.0	-7.0	-6.0	-5.0	-4.0	-3.0
4.0	57.150	22.086	10.727	6.272	4.114	2.802
3.0	57.150	22.085	10.727	6.272	4.113	2.802
2.0	57.148	22.083	10.724	6.270	4.111	2.799
1.0	57.122	22.057	10.699	6.244	4.085	2.773
0.0	56.931	21.866	10.507	6.052	3.893	2.579
-1.0	56.321	21.256	9.897	5.442	3.280	1.960
-2.0	55.422	20.357	8.997	4.541	2.376	1.047
-3.0	54.390	19.324	7.964	3.507	1.339	0.0
-4.0	53.057	17.991	6.631	2.171	0.0	0.0
-5.0	50.889	15.823	4.462	0.0	0.0	0.0
-6.0	46.431	11.364	0.0	0.0	0.0	0.0
-7.0	35.069	0.0	0.0	0.0	0.0	0.0

** σ_{β_N}=0.30 $-i_d$=6.00 **

** ν =10 μ_{β_N} =0.60 **

	i_{N_0}					
i_{N_1}	-8.0	-7.0	-6.0	-5.0	-4.0	-3.0
4.0	4.485	4.484	4.483	4.479	4.471	4.448
3.0	4.454	4.453	4.451	4.448	4.439	4.416
2.0	4.312	4.311	4.309	4.306	4.297	4.274
1.0	3.744	3.743	3.741	3.738	3.729	3.702
0.0	2.375	2.374	2.372	2.368	2.358	2.323
-1.0	0.928	0.927	0.925	0.921	0.909	0.867
-2.0	0.256	0.255	0.253	0.249	0.236	0.190
-3.0	0.067	0.066	0.064	0.060	0.047	0.0
-4.0	0.020	0.019	0.017	0.013	0.0	0.0
-5.0	0.007	0.006	0.004	0.0	0.0	0.0
-6.0	0.003	0.002	0.0	0.0	0.0	0.0
-7.0	0.001	0.0	0.0	0.0	0.0	0.0

** σ_{β_N}=0.30 $-i_d$=6.00 **

** ν =10 μ_{β_N} =1.00 **

	i_{N_0}					
i_{N_1}	-8.0	-7.0	-6.0	-5.0	-4.0	-3.0
4.0	3.860	3.846	3.828	3.801	3.749	3.636
3.0	3.855	3.840	3.822	3.795	3.744	3.630
2.0	3.815	3.800	3.782	3.755	3.703	3.590
1.0	3.578	3.563	3.545	3.518	3.466	3.351
0.0	2.744	2.729	2.711	2.684	2.631	2.512
-1.0	1.470	1.455	1.437	1.409	1.356	1.229
-2.0	0.607	0.592	0.574	0.546	0.492	0.361
-3.0	0.248	0.234	0.216	0.188	0.133	0.0
-4.0	0.116	0.101	0.083	0.055	0.0	0.0
-5.0	0.061	0.046	0.028	0.0	0.0	0.0
-6.0	0.032	0.018	0.0	0.0	0.0	0.0
-7.0	0.014	0.0	0.0	0.0	0.0	0.0

** σ_{β_N} =0.30 $-i_d$ =6.00 **

** ν =10 μ_{β_N} =1.40 **

	i_{N_0}					
i_{N_1}	-8.0	-7.0	-6.0	-5.0	-4.0	-3.0
4.0	4.059	3.878	3.728	3.574	3.375	3.066
3.0	4.058	3.877	3.727	3.573	3.374	3.065
2.0	4.048	3.867	3.718	3.563	3.364	3.055
1.0	3.963	3.782	3.632	3.478	3.279	2.969
0.0	3.525	3.344	3.194	3.040	2.840	2.528
-1.0	2.556	2.375	2.226	2.071	1.870	1.553
-2.0	1.600	1.420	1.270	1.115	0.913	0.591
-3.0	1.013	0.832	0.683	0.527	0.325	0.0
-4.0	0.688	0.507	0.358	0.202	0.0	0.0
-5.0	0.486	0.305	0.155	0.0	0.0	0.0
-6.0	0.330	0.150	0.0	0.0	0.0	0.0
-7.0	0.181	0.0	0.0	0.0	0.0	0.0

** σ_{β_N} =0.30 $-i_d$ =6.00 **

** ν =10 μ_{β_N} =1.80 **

	i_{N_0}					
i_{N_1}	-8.0	-7.0	-6.0	-5.0	-4.0	-3.0
4.0	7.694	5.750	4.677	3.937	3.298	2.625
3.0	7.694	5.750	4.677	3.937	3.298	2.625
2.0	7.692	5.748	4.674	3.935	3.296	2.623
1.0	7.665	5.721	4.648	3.908	3.269	2.596
0.0	7.467	5.523	4.450	3.710	3.071	2.396
-1.0	6.831	4.887	3.814	3.074	2.435	1.757
-2.0	5.917	3.973	2.900	2.160	1.519	0.836
-3.0	5.086	3.142	2.069	1.329	0.688	0.0
-4.0	4.400	2.456	1.382	0.642	0.0	0.0
-5.0	3.758	1.814	0.740	0.0	0.0	0.0
-6.0	3.017	1.073	0.0	0.0	0.0	0.0
-7.0	1.944	0.0	0.0	0.0	0.0	0.0