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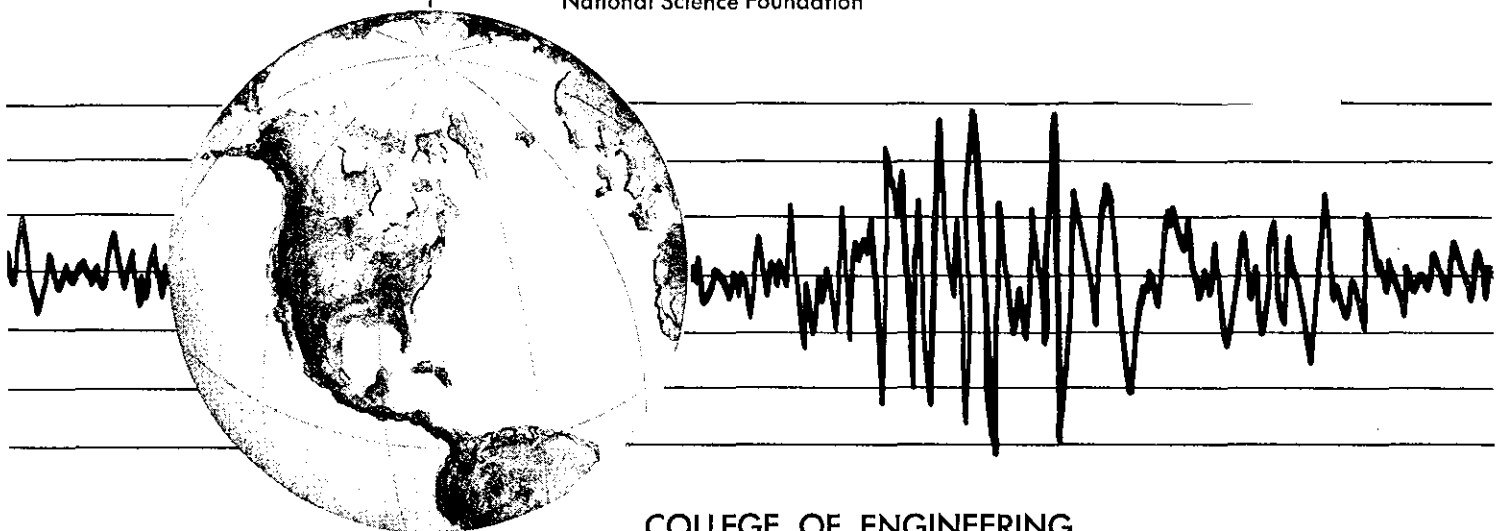
EARTHQUAKE ENGINEERING RESEARCH CENTER

# STRUCTURAL STEEL BRACING SYSTEMS: BEHAVIOR UNDER CYCLIC LOADING

by

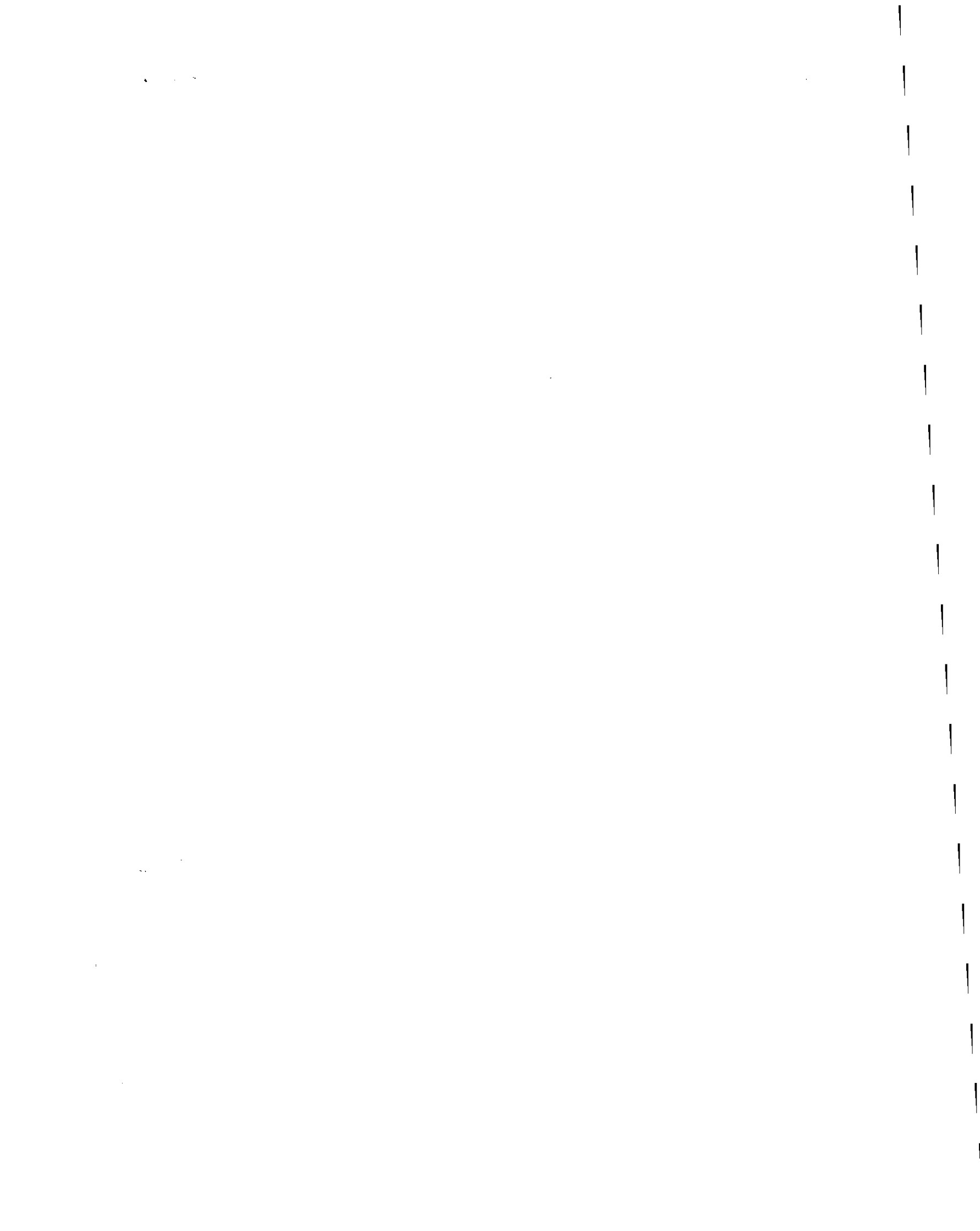
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<p>16. Abstracts A survey is made of existing literature on the performance of steel braced frame structures under cyclic excitations. Particular emphasis is placed on inelastic behavior under extreme credible excitations which may occur during a severe earthquake. The experimental and analytical studies of the behavior of an individual brace are described. The effect of the individual braces on the behavior of the entire structural system is then brought out. The behavior of a concentrically braced frame is discussed with respect to dynamic response to given excitations as well as its quasi-static hysteretic behavior under cyclic load. The advantages and limitations of the two possible approaches to design and correlations between them are indicated.</p> <p>The overall problem is very complex and has not been completely resolved, but a number of plausible design concepts have been advanced. These are reviewed in the report. Most of these are based on static methods of analysis and are intended to assure good dynamic performance of the structure. These approaches are not a substitute for dynamic analysis, but they help simplify the design procedure. Several design concepts, such as the eccentrically connected braced frame, show that braced frames can perform well under extreme excitations. Finally, the limitations of current knowledge are summarized and recommendations for further research are made.</p>				
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June 1976  
Report No. EERC 76-17  
Earthquake Engineering Research Center  
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## ABSTRACT

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The overall problem is very complex and has not been completely resolved, but a number of plausible design concepts have been advanced. These are reviewed in the report. Most of these are based on static methods of analysis and are intended to assure good dynamic performance of the structure. These approaches are not a substitute for dynamic analysis, but they help simplify the design procedure. Several design concepts, such as the eccentrically connected braced frame, show that braced frames can perform well under extreme excitations. Finally, the limitations of current knowledge are summarized and recommendations for further research are made.



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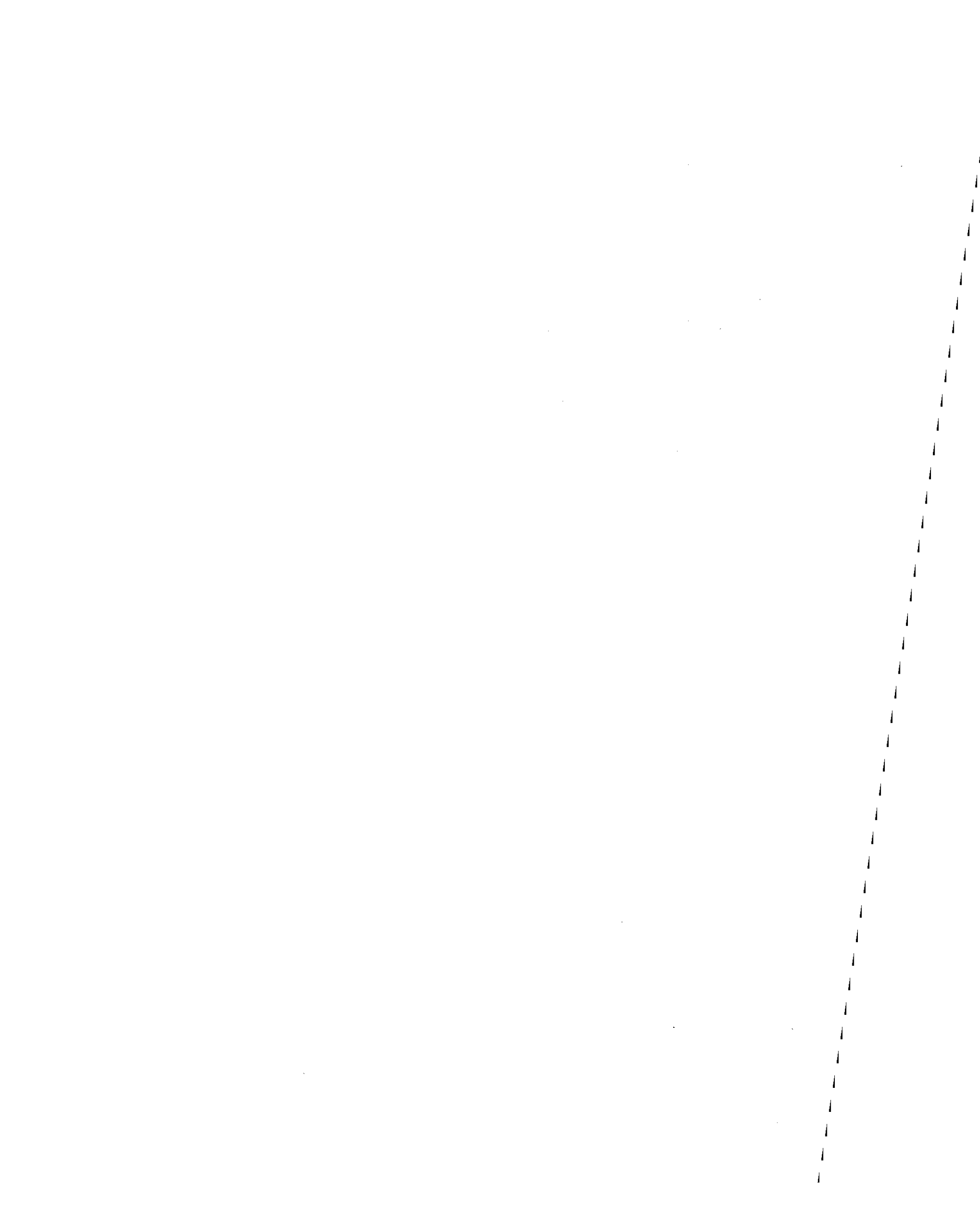
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## I. INTRODUCTION

In the design of structures to resist earthquake excitations, two basic requirements must be met. First, the structure must perform satisfactorily during events which have a moderate to high frequency of occurrence. This is a serviceability requirement and is generally met by requiring that the structure remain elastic. On slender steel moment-resisting frame structures, serviceability requires that story drift be controlled in order to minimize cracking of the interior finish and to assure that  $P-\Delta$  moments do not become critical. In order to minimize drift, consideration is often given to braced steel frames. For tall structures, the bracing provides considerable lateral stiffness to the structure and, therefore, prevents excessive deflection under the serviceability conditions.

The second requirement is to preclude the disaster of an actual collapse due to extreme earthquake excitation. Except for the case of very special structures such as nuclear reactors, it is economically unfeasible to design the structure so that it remains elastic during this extreme excitation. Therefore, in standard building construction, the design of the structure is heavily dependent upon the ability of the structure to absorb and dissipate energy in addition to the strength requirements. It is widely known that many bracing systems do not exhibit ideal energy dissipation characteristics because of buckling of the braces. The intrinsic ability of structural steel to absorb and dissipate energy in such designs is not effectively utilized.

The purpose of this survey is to summarize the research done on steel braced frames pertaining to aseismic design. Particular attention is paid

to the energy dissipation characteristics of braced frames. Based on this summary of past research, possible methods of satisfying both of the design requirements equally well are suggested.

Objective. The objective of this report is to describe the past research and current information about the performance of braced frame structures under cyclic excitation. The inelastic performance under extreme credible excitations will be of primary interest. The research done in Japan will be emphasized because it is less familiar to the American reader and only a few studies have been made in the United States. These U.S. studies have been primarily made at the University of Michigan.

Scope. This study is limited to a survey of existing literature on the performance of braced frames.

## II. CYCLIC BEHAVIOR OF A BRACE

General Behavior. The cyclic behavior of individual bracing elements has been studied both analytically and experimentally [1, 2, 3, 4, 5, 6, 7]. The results of these studies consistently indicate a general cyclic force-deformation relationship of the type shown in Figure 1. This behavior strongly suggests that the plastic rotation is concentrated in a region in the middle of the brace, denoted as H in Figure 1. For theoretical analysis, this general behavior can be broken up into several zones. The first zone, O-A, is generated by monotonically applying a compressive strain to a column. The behavior of this first zone will depend on the slenderness ratio and initial imperfections of the member. For a perfectly straight, slender member, it is theoretically possible to obtain a linear increase in  $\delta$  with axial load. Due to imperfections, real columns show a small amount of lateral deflection right from the start. The increase in  $\delta$  becomes strongly nonlinear while the Euler load is approached. In this range of loading, the lateral deflection,  $\Delta$ , of the center of the brace continues to increase while the compressive load remains nearly constant. For perfectly elastic members the range of nearly constant load carrying capacity is very large. For ductile members, however, instability occurs at some point, such as A, which depends on the geometry of the member and the mechanical properties of the material used. At instability,  $\Delta$  increases at a rate such that the incremental increase in  $P-\Delta$  bending moments is greater than the corresponding incremental increase in resisting moments in the center of the brace. In real members at moderate values of  $\Delta$ , the center of the brace yields because of the induced bending strain.

The second characteristic zone A-B, shown in Figure 1, is dominated by the inelastic bending of the brace due to the  $P-\Delta$  moments induced by the

compressive load  $P$ . The magnitude of  $P$  monotonically decreases with the increasing magnitude of deformation. The magnitude of the load must decrease because the  $P-\Delta$  moments cannot exceed the member's plastic moment capacity. The zone A-B is characterized by very large lateral deflections of the center of the brace and by large inelastic curvature in this center region. Cyclic reversal is shown to take place at point B where the compressive load is decreased. Immediately after decreasing the compressive load, the inelastically strained portion of the brace will again begin to behave elastically.

The third zone B-C of Figure 1 corresponds to elastic unloading of the member. The slope of this zone is much smaller than that of the virgin elastic curve due to the large permanent lateral deflection of the center of the brace, which results in a curved rather than a straight member.

The fourth zone C-D represents a zone of continued elastic bending with the brace lengthening while an increasing tensile load is applied. The lateral deflection  $\Delta$  decreases considerably in the third and fourth zones. This decrease in  $\Delta$  is primarily elastic. The decrease in  $\Delta$  is caused by a decrease in the  $P-\Delta$  moments induced by the decrease in the compressive load and by the change in sign of the  $P-\Delta$  moments with the application of tensile load. Point D is the start of yielding due to  $P-\Delta$  bending moments induced by the tensile load. Since the  $P-\Delta$  bending moment is of opposite sign to the  $P-\Delta$  moment induced by compressive loads, this inelastic bending partially restraightens the brace as it lengthens. The tensile  $P-\Delta$  moments reduce as the brace straightens and, therefore, the tensile load required to sustain yielding must increase as the brace straightens. Thus, the fifth zone D-E has a monotonically increasing tensile load as the brace lengthens.

Point E is the point at which the brace is fully straightened. If the tensile force were removed at this point, the brace would remain essentially

straight and be slightly longer than its original length. The internal bending moments are essentially zero when Point E is reached, and any elongation beyond point E is purely plastic uniaxial elongation. The sixth zone E-F is plastic uniaxial elongation of the brace. This zone is characterized by a nearly constant tensile load  $P$  with increasing elongation  $\delta$  for an elastic perfectly plastic material or by an increasing tensile load  $P$  with increasing elongation for a strain hardening material. Point F is a load reversal point. Thus, the final zone F-G consists of elastic unloading. The elongation will decrease linearly with decreasing tensile load, and the slope is essentially the same as that of the virgin elastic curve.

Figure 1 represents the generalized form of a single cycle of loading on a brace. Subsequent cycles will have the same general characteristics. However, the numerical relationship between axial load  $P$  and axial deformation  $\delta$  may be greatly changed for later cycles. The first of these changes is the translation of the origin or starting point of later cycles to a new location in the  $P$ - $\delta$  space. The translation is caused by permanent uniaxial elongation at the end of the preceding cycle. Secondly, the peak magnitudes of the axial load  $P$  may be quite different for the various zones in later cycles. One reason for this is that the Bauschinger effect lowers the apparent yield stress of the material in later cycles. Another reason is the fact that the brace was first plastically kinked and then plastically restraightened. Hence, because of its strain history, it is not likely to be nearly as straight a brace as the brace before the first cycle. The strain history and the Bauschinger effect may greatly reduce the critical buckling and post-buckling loads in later cycles.

Experimental Studies. Experimental cyclic tests of bracing members exhibiting the same general characteristics as those discussed in connection

with Figure 1 are shown in Figure 2. These curves are based on the work of Wakabayashi et al [1], who performed a number of cyclic push-pull tests on bars with various slenderness ratios. Figure 2 is typical of the results which were obtained.

Igarashi et al [2] used the experimental curves obtained by Wakabayashi to develop some general conclusions on the cyclic behavior of the brace. The first of these conclusions is that stable hysteresis loops (P- $\delta$  curves) can be obtained if the slenderness ratio is less than approximately 30. Stable hysteresis loops are those for which the maximum compressive strength of the brace in the first cycle is obtained in subsequent cycles. The second conclusion is that the residual bending deformation does accumulate as the number of cycles increase and, thus, the maximum compressive strength in each cycle gradually decreases if the slenderness ratio is greater than about 40. Third, the post-buckling strength drops very rapidly for braces with buckling loads which approach the Euler buckling load. Finally, the stiffness of the compressive unloading zone (i.e. slope of zone B-C in Fig. 1) increases as the slenderness ratio decreases and decreases as the axial displacement  $\delta$  increases in magnitude.

Kahn and Hanson [3] reported on a series of cyclic experimental tests made on 1 in. by 1/2 in. steel bars. The lengths of the bars were varied to produce slenderness ratios of 85, 120, and 210. The bars were tested under both dynamic and quasi-static loading conditions. Figure 3 is typical of the cyclic behavior obtained in these tests. It was found that the dynamic hysteretic response was nearly identical to the static response, although the dynamic response was slightly stiffer when the brace was loaded in tension.

Analytical Studies. One method of analytically predicting the cyclic behavior of a brace was proposed by Higgenbotham [4]. In this analytical

model, the brace is assumed to remain elastic when loaded in compression except for a central plastic hinge location. The differential equations governing the lateral deflections of the brace are derived to include large geometric effects. The axial displacement of this model is the sum of the elastic elongation of the brace, the displacement due to the plastic hinge rotation, and the displacement due to the elastic flexural deflection of the brace. Higgenbotham solved the equations for the pinned-end condition and adapted these solutions to other boundary conditions by the use of symmetry and the effective length coefficients. The problem was solved for the actual moment diagram of the brace and for linear approximations of the actual moment diagram. These solutions were compared with each other and with experimental results. The analysis predicted the general cyclic force-displacement behavior of the brace, but quantitative agreement with experimental results did not always have a high degree of accuracy.

Nonaka [5] proposed a similar analytical method which includes the plastic axial deformation of the brace but limits it to small geometric effects. The axial deformation  $\delta$  is expressed as the sum of the components as follows:

$$\delta = \delta^e + \delta^g + \delta^p + \delta^t \quad (1)$$

where  $\delta^e$  is the uniform elastic axial elongation;  $\delta^g$  is the deformation due to the change in geometry caused by lateral deflection;  $\delta^p$  is the plastic axial deformation at the plastic hinge; and,  $\delta^t$  is the deformation due to plastic elongation distributed over the length of the bar. The elastic component  $\delta^e$  is related to the axial load by the linear elastic law

$$\delta^e = \frac{PL}{AE} \quad (2)$$

The geometric component is given as

$$\delta^g = -\alpha \frac{P_E L}{AE} \left( \frac{\theta}{\pi \cosh v} \right)^2 \frac{\sinh 2v}{2v} + 1 \quad (3)$$

where

$$\alpha = \frac{A}{I} \frac{M_p}{P_y}^2 \quad v = \frac{\pi}{2} \sqrt{\left| \frac{P}{P_E} \right|}$$

$$\theta = \frac{v^2}{\tanh v} \quad v = \left| \frac{P_y}{P} \right| - 1 \quad P_E = \frac{\pi^2 EI}{L^2}$$

P is negative for a compressive axial force. On Eq. (3), the hyperbolic functions must be replaced by trigonometric functions when P is a compressive force.

The third component of Eq. (1),  $\delta^p$ , changes value only when plastic action takes place in the central hinge location. It can be calculated by the flow rule associated with a yield condition. Nonaka used the simplified yield condition

$$\left| \frac{P}{P_y} \right| + \left| \frac{M}{M_y} \right| = 1 \quad (4)$$

giving

$$\delta^p = \frac{-4}{\pi^2} \alpha \frac{P_E L}{AE} \theta \quad (5)$$

The final term,  $\delta^t$ , can assume any non-negative and irreversible value.

The only restriction is that  $\Delta \delta^t$  is greater than zero only when  $P = P_y$  and  $M = 0$  are simultaneously satisfied. Since these conditions are jointly satisfied only when the brace is in its straight configuration,  $\Delta \delta^t = 0$  at all other times.

In another paper, Igarashi et al [6] computed axial force-deformation relationships from the same basic concepts used by Nonaka. However, they

used the more accurate yield condition

$$\left( \frac{P}{P_y} \right)^2 + \left| \frac{M}{M_p} \right| = 1 \quad (6)$$

This yield criterion is generally regarded as more realistic, but the resulting equations are more complex than those derived by Nonaka.

The methods of analysis, which have been discussed, are applicable to slender members which buckle elastically or nearly elastically. Members with low slenderness ratios are not controlled by elastic buckling and, as a result, the expansion of the yield zone during cyclic axial load must be considered. Fujimoto et al [7] used the finite element method to study cyclic inelastic buckling behavior. The length of the brace is divided into a number of segments with the assumption that plane sections remain plane at the interfaces. Each segment is further divided into slices. A stiffness array is formulated for the brace using these subdivisions and constraints. At the end of each time step, all slice segments are checked to see if a change in yield state has occurred. If a change has occurred, the stiffness arrays are modified and the analysis is continued. This method is the most generally realistic model proposed, but it requires a large amount of computation for each brace.

Yamada and Tsuji [8] proposed a method which avoids the extremely large amount of computation required by the finite element method but still incorporates some of its advantages. This is done by modeling the brace cross-section by three equivalent bars as shown in Figure 4. These equivalent bars are then divided into 30 increments along the member length. The longitudinal deformation  $\delta$  is composed of the summation of the elongation of the centroidal axis (i.e. center bar) and the deformation due to

the lateral deflection. They were also able to incorporate the Bauschinger effect into their model. The Bauschinger effect is modelled by using the Ramberg-Osgood function

$$\frac{\epsilon}{\epsilon_y} = \frac{\sigma}{\sigma_y} + r \left( \frac{\sigma}{\sigma_y} \right)^n \quad (7)$$

Since the Ramberg-Osgood model usually requires considerable computation when used in frame analysis, Yamada and Tsuji [9] proposed a simpler model shown in Figure 5 to represent the stress-strain curves. This model is based on the overlay or the sublayer technique [10, 11] and consists of a stress-strain curve which is built-up in the virgin state by the sum of an elastic and elastic perfectly plastic material. Once the virgin material experiences an excursion into the plastic region, the elastic perfectly plastic material must be divided into two, as shown in Figure 5b. Figure 5c shows the resulting stress-strain curves for the model. The proposed model and the Ramberg-Osgood model are both compared with experimental results in Figures 6a and 6b. Both models compare reasonably well with the experimental results.

Concluding Remarks on Brace Behavior. The general behavior of a brace under cyclic axial loading is now well understood. However, the analysis of a braced frame structure under cyclic loads requires the assignment of numerical values to the brace behavior. This subject requires further investigation. A number of analytical methods have been proposed to compute these numerical values, but all of these methods have limitations. For example, the methods of Higgenbotham [4] and Nonaka [5] are simple but best suited for slender braces. The methods of Fujimoto [7] and Yamada [8, 9] are generally more useful but computationally more complex. Another point to consider

is that the residual stress distribution will be very different for structural shapes than for bars. However, most of the experimental tests and many of the analytical studies have been made on bars. This causes more uncertainty in the selection process. Thus, the analytical model to be employed for design considerations depends on the brace to be analyzed, the accuracy desired, and the amount of computation that can be accepted.

### III. BEHAVIOR OF A BRACED FRAME

Today most braced frames are designed and constructed as concentrically braced frames; that is, the center line of the brace intersects the center line intersection of the beam and column. In this chapter, the discussion of braced frame behavior will be limited to concentrically braced frames.

The behavior of concentrically braced frames under cyclic loading is of great importance in designing structures which will not collapse during an extreme earthquake. This is the topic of primary concern in this chapter. However, the behavior of a braced frame under monotonic displacement is much better understood and tends to influence ideas about behavior under cyclic loading. Therefore, this chapter will start with a discussion of the monotonic case, follow with a discussion of cyclic behavior, and conclude with a discussion of the research into the cyclic behavior. The general purpose is to consider reasons why braced frames are sometimes unfavorably regarded and to show what has been done to better understand and overcome these problems.

Behavior under Monotonic Deflection. The monotonic behavior of a concentrically braced frame is highly dependent upon the behavior of the brace, but it is also influenced by the bending resistance of the frame. Figure 7 is a plot of the lateral strength,  $H$ , as a function of the monotonically increasing lateral displacement,  $\Delta$ , for three different frame types. The loads are applied very slowly so that dynamic effects are eliminated, and the frame is loaded so the brace is in compression. These three types of frame are (1) a moment resisting frame with a brace added (noted as  $H_{BF}$  in Fig. 7); (2) the same moment resisting frame with no brace (noted as  $H_F$  in Fig. 7); and (3) a braced structure the same size as  $H_{BF}$  but without moment resisting connections (noted as  $H_B$  in Fig. 7). The lateral stiffness of the truss,  $H_B$ ,

is provided by the brace. As a result, the post-buckling strength of the truss decreases with increasing  $\Delta$  in the same way as the compressive strength of the brace decreases with axial shortening. This decrease is shown in the figure.

Figure 7 shows that the moment resisting frame,  $H_F$ , is much less stiff than the truss  $H_B$ . Thus, it will take a much larger deflection than the truss for the same force level, and the  $P-\Delta$  moments due to gravity loads will become increasingly significant. The increasing  $P-\Delta$  moments will cause the lateral force-displacement relationship to depart rapidly from the linear behavior. The moment resisting frame is able to resist increasing lateral load until a sufficient amount of inelastic action has taken place so that instability occurs. Instability is the point at which an increase in lateral displacement necessitates a decrease in lateral resistance. The lateral deflections of the frame are much larger at the instability point than the lateral deflection of the truss at instability.

The bulk of the stiffness of the braced moment resisting frame is provided by the brace. Therefore, the brace carries the greater portion of the lateral loads until the brace buckles. After buckling, the brace loses strength. However, the strength of the braced frame will remain fairly constant because the decrease in brace strength is partially or totally offset by the increase in bending moments of the frame. This results in a structure which is very stiff for small lateral deflections but is ductile with little loss in strength.

Since ductile behavior is very desirable in preventing collapse due to extreme earthquake excitations, it is very tempting to design braced structures so that the brace is capable of carrying only a portion of lateral loads. The rest of the lateral loads are carried by bending moments. This is a desirable concept from a monotonic point of view, and Japanese designers

often employ it in their design practice. This design concept is based primarily on static considerations. Furthermore, it must be noted that the strength of the braced frame,  $H_{BF}$ , is not the simple sum of strengths of the truss,  $H_B$ , and the moment resisting frame,  $H_F$ , because the truss loses much of its strength well before the moment resisting frame achieves its maximum strength. Further implications of this design concept will be discussed later.

Behavior under Cyclic Loading. There are two primary methods of examining the cyclic behavior of braced frames. The first is to analytically or experimentally determine the inelastic dynamic response of the structure. This response determines the strains, deformations, and displacements which the structure must withstand to prevent collapse under the given excitation. If the structure is designed to meet these requirements, it will survive the given excitation. However, earthquake excitations are non-deterministic, and inelastic dynamic analysis is a costly and complex computational procedure.

As a result, another approach is often used to study the cyclic behavior of braced frames. This second approach is to study the quasi-static inelastic hysteretic behavior of the frame. This approach is much easier since it is based on static quantities, such as the stiffness and strength of the structure and the resulting inelastic force-deflection curves. This type of study is valuable because the area enclosed in each of these lateral force-deflection hysteresis loops is the energy dissipated by the frame in that cycle. Since energy dissipation is very important in controlling the inelastic dynamic response of the structure, this quasi-static approach parallels the dynamic analysis approach. The quasi-static energy dissipation is also relevant to the dynamic case since the strain rate induced by earthquake excitation is not large enough to substantially affect the material properties. Thus, the

goal of the quasi-static method is to develop structures which have good energy dissipation characteristics; that is, structures with full hysteretic loops which have large enclosed areas and do not degrade in later cycles. This concept is generally accepted as it fits well with standard design practices. The usual design procedure is to design the structure statically and then check it dynamically.

The quasi-static and dynamic concepts are not identical but they are related since any structure with poor energy dissipation characteristics will usually undergo larger dynamic responses than a similar structure with good energy dissipation characteristics. Dynamic analysis is still necessary as a check to the quasi-static approach because it is the only method of assuring that sufficient energy can be dissipated. However, if a structure exhibits good energy dissipation characteristics, it is more easily designed to satisfy dynamic requirements such as ductility. Most of the past research on braced frames has been performed in a quasi-static method.

The quasi-static general cyclic behavior of a concentrically braced frame can be characterized by a reversed S-shape lateral force-displacement hysteresis loop; that is, the hysteresis loops are pinched. Figure 8b is an example of a moderately pinched hysteresis loop. This pinching is due to the lateral deflections necessary to restraighen the brace after buckling. The degree of pinching is quite significant because it indicates the loss in energy dissipation capabilities of the frame. Energy dissipation is necessary in the design of structures to prevent collapse under extreme earthquake excitations. Therefore, a braced frame with a severely pinched hysteresis loop will have relatively poor energy dissipation characteristics and will usually be expected to sustain larger lateral displacements in an extreme earthquake.

Experimental Studies. Experiments have verified that concentrically braced frames do produce pinched hysteretic curves. Wakabayashi and his associates [1, 12, 13, 14, 15] conducted a number of tests on braced frames. Figures 8a and b are the cyclic hysteretic curves for a brace and the corresponding braced frame from one of the tests. All of these tests produced pinched hysteretic loops which were found to be relatively unaffected by the magnitude of the vertical gravity loads. These tests also showed that the degree of pinching is variable and some braced frames exhibited unstable behavior in later cycles. Unstable hysteretic loops are those in which the maximum strength decreases in successive cycles.

Analytical Studies. The earliest analytical studies of the inelastic behavior of braced frames were based on the assumption of a slip model of brace behavior. The slip model, which is shown in Figure 9, assumes the presence of two inclined braces with each becoming alternately inactive due to buckling during application of cyclic loading. As a result, it can be used only in certain bracing configurations, such as the X and K braces. Tanabashi and Kaneta [16] and Veletsos [17] performed analytical studies on single-story braced frames with the slip model. Workman [18] used the slip model to analyze multi-story braced frames dynamically. These slip model studies are useful because they show the value and limitations of the inelastic behavior of the brace as an energy dissipator. Further, the slip model still remains one of the simplest models for analyzing inelastic braced frame behavior. This model is not too unrealistic for extremely slender braces but it neglects a considerable amount of the inelastic action of braces with moderate and small slenderness ratios. These less slender braces will also dissipate energy in the inelastic compressive zones and, therefore, the actual force-displacement hysteresis loops will not be as

severely pinched as predicted by the slip model.

Wakabayashi et al [13, 14] made a series of theoretical calculations, compared them to experimental results, and found very good correlations between the two. These calculations were based on the relatively simple Nonaka [5] model of brace behavior. The bending members were analyzed by means of the bilinear moment curvature relationship shown in Figure 10. When a member (see Fig. 11) was subjected to combined axial load and bending moment, the moment-curvature relationship was revised by using the yield criterion.

$$M_{pc} = 1.18 M_p \left(1 - \frac{P}{P_y}\right) \text{ with } M_{pc} \leq M_p. \quad (8)$$

Thus the presence of an axial load will lower the yield point. This results in the modified moment-curvature relationship shown in Figure 12. The yield point in this modified relationship is lowered but the upper slope is increased due to the strain hardening of steel.

The structure is then divided into segments and an axial force  $P_0$ , shear force  $Q_0$ , end moment  $M_0$ , and end rotation  $\theta_0$  are assumed for the starting point. The solution then advances from segment to segment until the other end of the structure is reached. The boundary conditions are then checked, and the initial boundary conditions are modified if necessary and resolved. Iteration continues until all boundary conditions are met. The basis for this solution is the expression for the average bending moment of the  $i^{th}$  segment.

$$M_i = M_{i-1} + H \frac{\Delta X_i}{2} + P_{\theta_{i-1}} \frac{\Delta X_i}{2} \quad (9)$$

The average curvature of the  $i^{th}$  segment,  $\phi_i$ , is obtained from the moment-curvature relationship (i.e. Fig. 12). This average curvature is assumed to remain constant over the  $i^{th}$  segment, and the bending moment, deflection, and rotation can be computed at the  $i^{th}$  end by

$$\begin{aligned}
\theta_i &= \theta_{i-1} + \Delta x_i \phi_i \\
Y_{i+1} &= Y_{i-1} + \frac{\Delta x_i}{2} (\theta_{i-1} + \theta_i) \\
M_{i+1} &= M_A + Q_i x_i + P_i Y_i
\end{aligned} \tag{10}$$

These calculations were compared with experimental results [14] showing satisfactory agreement. Figure 13 shows a comparison of the experimental and computed lateral force-deflection hysteretic loops. This method is not cast into matrix form and, therefore, is not suitable for the analysis of large multi-story structures or for inelastic dynamic analysis.

Igarashi and Inoue [2] developed a matrix formulation for analytically computing the inelastic behavior of a braced frame. This is a more complex formulation since it assumes a more complex yield criterion and member behavior. The local coordinate system for a member analyzed by this model is shown in Figure 14. The analyses performed were inelastic static, but since the procedure was formulated in matrix notation, it should be usable for dynamic analysis of very large structures. It should be noted that this method requires the definition of a stability function, such as Euler's buckling load. Therefore, the accuracy of the results is limited by the accuracy of the function. Figure 15 is a comparison of the computed results with the experimental results. The agreement is quite good. A more detailed coverage of this method of analysis is given in Appendix A.

Nilforoushan [19] proposed another method of analyzing the inelastic behavior of a braced frame. This analysis is based on a linear approximation of the general cyclic behavior of the brace as shown in Figure 16. The general cyclic behavior of the brace is approximated by a series of straight lines. The straight lines are chosen to give a good fit to the cyclic behavior determined by other theoretical or experimental means. This method

fits well with the concept of zones of brace behavior discussed earlier since each zone is approximated by one or more straight line segments. An inelastic analysis can then be performed by modifying the structural stiffness matrix whenever a brace moves from one linear zone to another.

Nilforoushan performed dynamic analyses of this type on several structures using the 1940 El Centro acceleration record as the base excitation. This method should be generally applicable to all braced systems; however, the braces used by Nilforoushan were all very slender and, therefore, his parameters may not work well with less slender braces.

All of the methods discussed thus far require the definition of a stability or buckling criterion. This is usually taken as Euler's buckling load. Euler's buckling load is a reasonably good approximation for very slender members but is not very accurate for less slender members. Fujimoto [7] used the sliced segment method, which was discussed earlier, to model the brace behavior and perform a finite element analysis of a frame. This is the most general and accurate analysis proposed to date. However, it takes a fairly large number of equations to model the behavior of a single brace. This makes it impractical to analyze the behavior of most multi-story and multi-bay structures.

#### Concluding Remarks on the Behavior of a Concentrically Braced Frame

The energy dissipation capability of a concentrically braced frame is regarded by some engineers with some skepticism. This skepticism is largely due to the poor energy dissipation characteristics associated with severely pinched hysteresis loops. It is generally agreed that all braced forms exhibit pinched hysteresis loops, but not all are severely pinched. This

doubt can be overcome by performing a dynamic analysis of these structures and/or developing bracing systems which exhibit satisfactory dissipation characteristics.

In either case, it is necessary to have a good inelastic analytic tool for large structures (both static and dynamic). The slip model is easy to use but is of limited accuracy. The method of Fujimoto [7] is very accurate but impractical for very large structures. This leaves the approximate method used by Nilforoushan and the method of Igarashi [2] as the best available approaches. The analytical model to be employed for large structures depends on the type of brace to be analyzed, the accuracy desired, and the acceptable amount of computational effort. It should be noted that none of these methods consider the effect of lateral torsional or local buckling due to load reversals. The less complex methods (Nilforoushan [19] and Igarashi [2]) assume a simplified consideration of end restraint.

#### IV. ALTERNATIVE CONCEPTS OF BRACED FRAME DESIGN

The general behavior of concentrically braced frames and methods for computing this behavior have been discussed in the previous chapters. It is generally accepted that the energy dissipation characteristics of such braced frames sometimes may be unsatisfactory. This unsatisfactory behavior is primarily due to the relatively large lateral deflections necessary to restraighen the brace after it is inelastically kinked during compression. The behavior itself is determined by considering the quasi-static application of cyclic loads. The uncertainty in the acceptability of the behavior of a particular braced structural system can be resolved by a dynamic analyses used in the design process. There have been several stiffness formulations of brace behavior which, although not precise, are suitable for analyzing the quasi-static and dynamic response of the structure. However, because of the complexity and expense of inelastic dynamic analysis, most designers prefer to base their design on the concepts of static analysis. Such an approach is not intended as a substitute for dynamic analysis but is intended to help assure good dynamic performance of the structure. This chapter will be a discussion of several of the static design concepts which have been proposed.

Lateral Loads Carried by the Brace. Japanese designers often employ a concept in which the brace is designed to carry only a prescribed percentage of the design lateral loads. The remaining lateral loads are carried by moment resisting capabilities of the frame. The previous discussion of the general monotonic behavior of a concentrically braced frame adds credibility to this concept since it shows that the braced moment resisting frame is initially very stiff and then, after buckling of the brace, exhibits

rather ductile behavior with very little loss in strength. Igarashi and Inoui [2] performed some cyclic inelastic calculations which also support this approach. Their studies showed that the degree of pinching of the lateral force-deflection hysteretic loops becomes less severe as the percentage of lateral load carried by the brace decreases. This is a very useful design concept, but it has limitations. One limitation is that the percentage of design lateral loads carried by the brace is a constant value, while the actual percentage is known to vary with time, as can be established by an inelastic dynamic analysis. The varying dynamic percentage is the factor which will control the ductility and degree of pinching during a dynamic excitation. Since it does not have a unique value, the degree of actual pinching is hard to predict. The static percentage alone cannot assure good dynamic energy dissipation characteristics. Another problem is that the actual ultimate capacity of the brace and the frame can be predicted only approximately. As a result, the actual static percentage may vary considerably from the design value.

Eccentric Bracing Systems. Fujimoto et al [20] analytically and experimentally studied the behavior of an eccentric K-braced frame, such as that shown in Figure 17a. The braces were eccentrically connected so that the central segment of the beam yielded in shear and bending before the bracing elements could buckle. This appears to be a good design since the energy dissipation characteristics of steel beams in moment resisting frames, which are well understood, are known to be excellent. Fujimoto and his associates studied the eccentric K-braced system and found that it can develop full (unpinched) hysteretic loops without any reduction in ultimate strength. Figure 18 is an example of several of the hysteresis loops that they obtained. These full hysteresis loops should permit the

structure to dissipate a large amount of energy without excessive lateral deflections. It was noted that large deflections occurred in the beams and, hence, considerable damage to the floor slab must be expected. Figure 19 is a series of photographs showing the distorted beams.

Hisatoku [21] studied the case of an inverted Y brace, such as that shown in the sketch of Figure 17b. In this investigation, it was found that either pinched or full lateral force-deflection hysteresis loops were generated depending upon the design of the inverted Y. The loops were pinched if one of the diagonal portions of the Y buckled before the vertical strut yielded. The loops were full if the vertical strut yielded before either of the diagonals buckled. It was also noted that severe distortion occurs at the midspan of the beam for this type of structure.

Another method which has some similarity to the eccentric brace studies of Fujimoto [20] and Hisatoku [21] is the staggered truss system. Gupta [22] analytically studied the earthquake resistance of a staggered truss framing system which was designed by the procedure proposed by Hanson, Goel, and Berg [23]. An illustration of this bracing system is shown in Figure 20. Two interesting features of this framing system are that a given column is braced only at alternate floor levels and that the center panel of the braced levels are always unbraced. As a result, any shear or moment which is transferred through the center panel of the truss must be transferred by shear, bending moments, and axial load of the truss chords. Gupta performed a dynamic analysis on various staggered truss structures and found that they performed well and that all yielding occurred in these central chords. These calculations also indicated large inelastic rotations within the central panel. This is similar to the eccentric K brace and the inverted Y brace systems because the inelastic dissipation is due to yielding by

bending and shear.

A common concept which can be noted from the few studies available in this area is that it is desirable to force yielding in the structure due to bending and shear stress before the brace can buckle. The yielding due to bending and shear will dissipate the energy and prevent the development of severely pinched hysteretic loops, which may result because of kinking and restraightening of the buckled brace. This concept is gaining acceptance but requires more study before it can be generally applied. It should be noted that while energy dissipation characteristics may be greatly improved by this method, satisfactory performance of a given structure can be assured only by considering the dynamic response of that structure.

Stiffness Modification. The dynamic response of a structure, due to an earthquake, depends on the dynamic characteristics of the structure and the excitation. The first natural period of most tall structures fits into a fairly well defined range of about one to several seconds. It is well known that within this range, on the average, the equivalent static design loads (or accelerations) decrease and the corresponding deflections increase as the natural period increases. Thus, this design concept permits the designer to reduce the equivalent static design loads for less stiff, longer period structures. This approach is incorporated in most earthquake design codes, such as the UBC. Since the lateral stiffness of braced frame structures is mostly provided by the brace, it is often suggested that the design lateral loads can be reduced by bracing only at selected levels. This could be accomplished by bracing only alternate levels, bracing only bottom levels, or adopting other configurations.

Goel and Hanson [23] performed an analytical study which included several reduced stiffness bracing systems. Dynamic analyses were performed

on an X-braced frame, a moment-resisting frame, a frame braced only on alternate levels, and a frame braced on all but the bottom level. The member behavior was assumed to be elastic-perfectly plastic with a  $P-\Delta$  modification for the columns. It was found that the fully-braced frame did reduce the lateral displacements and inelastic activity in the columns and girders. The alternate level bracing, the unbraced bottom level, and the moment-resisting frame all had smaller accelerations and larger lateral deflections.

This study lends support to the design concept of reducing stiffness to reduce the earthquake design loads. This approach is valuable and widely used but it is only part of the answer. The idea that equivalent static design loads decrease with increasing period is true only for the average structure subjected to average extreme earthquake excitation. Unusual circumstances can produce unusual results. For example, it is well known that soil layers attenuate and amplify ground motion depending on the frequency of excitation. A soil condition which amplified very low frequency excitations might actually produce an increase in equivalent static design loads for long period structures. It is also worth noting that lateral deflections increase as the stiffness decreases, and  $P-\Delta$  moments may become significant if the stiffness is reduced excessively. While stiffness modification is a valuable design concept, structures should be individually checked to assure that it is applicable.

Tension in Lower Columns. It is well known [23] that braced frames develop high column loads when subjected to lateral loads. Therefore, it is possible that tension may develop in the lower columns. Since foundations are not usually designed to resist large tensions, the designer

would like to avoid or minimize this tension. It has been suggested that tension in lower columns can be avoided or controlled by spreading the bracing over all bays. This idea was studied by Tsuji [24]. A number of bracing systems were analytically studied and it was recommended that bracing configurations, such as those in Figures 21b through 21f, be used. Bracing configurations such as 21a should be avoided because of the high probability of tension in the lower columns. The spreading of bracing over all bays is a useful approach but it may be architecturally impractical and other acceptable solutions must then be sought.

Multi-Phase Bracing. Another rather different bracing system, the multi-phase system, has been analyzed and tested by Shepherd [26]. The bracing in this system is designed by the slip model since very slender braces are used. An X bracing system is used, but each brace is really a dual brace. One of the dual braces is a brace with high yield stress and low ductility; the other brace has low yield stress and high ductility. The stiffness of the system is very high at low levels of excitation since both brace systems are elastic. At high excitation levels the low yield brace yields and the high yield brace fractures, and the stiffness of the structure drops dramatically. This stiffness degradation results in a long period and low design loads for extreme excitations. This design method is similar to the stiffness reduction methods which were discussed earlier. However, it is unique because the two design criteria of serviceability under frequent events and prevention of collapse under extreme events are met by two essentially different structures, a very stiff structure for frequent events and a very flexible structure for extreme excitations. The system proposed by Shepherd is primarily intended for water tower type structures. It would not be suitable for tall building structures.

However, the idea of changing the structural stiffness during extreme earthquakes is very appealing. One such suggestion [27] is that a brace be developed which "yields" without kinking equally well in tension or compression. This might be accomplished by a mechanical means such as a friction type slip connection which maintained its friction load during slippage and worked in both tension and compression. This idea is still in the formulation stage but structures directly using this and similar concepts may become increasingly important in the future.

Summary. This chapter contains a discussion of several ideas proposed as solutions to some of the problems of earthquake resistance of braced frames. These concepts are basically static design methods which are intended to help ensure good performance of the structure but which provide only a part of the answer. It is still necessary to use the concepts of Chapter III to fully assure good structural performance. It is necessary to study and analyze the hysteresis loops to assure that good energy dissipation characteristics are present. It is also necessary to consider the dynamic response to assure that sufficient energy is dissipated. None of the concepts discussed here fully meet these parallel requirements but an understanding of the methods and their limitations may be helpful in producing satisfactory structures. However, it must be noted that the more flexible structures are more susceptible to non-structural damage. Moreover, an erroneous conclusion may be reached by adhering only to the criterion based on the period of the structure. For long durations of strong motions the danger of incremental collapse accentuated by the  $P-\Delta$  effect must be considered especially if the structural resistance and/or strain hardening is low.

## V. SUMMARY AND CONCLUSIONS

It is well known that braced frames are very effective in controlling lateral deflections because of the high lateral stiffness provided by the braces. Moreover, concentrically braced frames develop high axial forces in the braces. When these high axial forces are cyclically applied, the braces alternately buckle, inelastically kink under compression, and restraighthen. Slender braces, which are commonly used in structures, have very low resistance and stiffness during the post buckling kinking and restraighthening phase. Therefore, concentrically braced frames often develop pinched lateral force-deflection hysteretic loops. The degree of pinching depends on a number of factors such as the slenderness ratio of the brace and the extent of the inelastic kinking. The area enclosed within each of these hysteretic loops is the energy dissipated by the structure per cycle. Since this energy dissipation is necessary in the practical design of structures to prevent collapse under extreme excitation and pinched hysteretic loops have smaller enclosed areas, concentrically braced frames are looked upon with some disfavor. However, it may be possible to show analytically that the structure performs satisfactorily during the inelastic dynamic response to extreme credible earthquake excitations. For this purpose, several methods have been proposed to predict analytically the inelastic dynamic response of a braced structure. The more accurate of these methods, because of their complexity, are unsuitable for use in the analysis of large structures, but several of the other methods give a reasonably good overall prediction of the inelastic dynamic response. All of these analytical methods are expensive and time consuming, and

they can be performed as a final check only after the design is complete. Therefore, the designer needs to have design procedures available which would assure good dynamic performance of the structure. Several of these design concepts have been proposed and are being studied. One such concept is to connect the brace eccentrically at the joint. Eccentrically connected braces cause yielding of the girders in bending and shear before the brace buckles. Therefore, the hysteresis loops are not pinched and the energy absorption and dissipation approach the highly regarded characteristics of steel moment resisting frames. Other design procedures which show promise in helping to obtain good cyclic performance of the structure are, (a) the idea of designing braced frames with high moment resisting capability to provide a ductile structure with full hysteresis loops, and (b) the idea of modifying the structural stiffness to minimize the equivalent static design load on the structure. Each of these design concepts is useful, but requires more study before it is fully accepted.

The general behavior and problems of concentrically braced frames under cyclic loads are well understood. However, the analytical methods of quantifying this behavior and the design techniques for circumventing the problems are in need of further research. Some of the more promising areas for further study are as follows:

- 1) Development of more accurate and efficient analytical models of inelastic behavior. The ability to perform an inelastic analysis depends very heavily upon the ability to determine the inelastic behavior of the individual elements. There are a few methods available at this time, but they are all limited in either their ability to be utilized in large structures or in their accuracy. An accurate model is needed,

which is computationally efficient and can include factors such as strain hardening and the Bauschinger Effect.

2) Experimental cyclic tests on braces of larger size. The major part of the available test data on braces has been obtained from cyclic tests on small specimens (primarily rectangular bars) with relatively high slenderness ratios. The existing test data were obtained using the slenderness ratio as a principal variable, and so, the results are assumed to apply to larger specimens. However, the initial imperfections and residual stress distributions are quite different for wide flanges, channels, and tubular sections which are used in construction. These imperfections greatly affect the onset of inelastic behavior for individual members. Thus, cyclic tests on commonly used structural shapes are needed before it will be possible to develop the more accurate analytical methods discussed previously.

3) Further development of new and existing design concepts. Eccentric connection of the bracing system is a particularly promising design approach. Further studies of systems such as that shown in Fig. 22 [28] would be a logical follow-up to the successful studies of Fujimoto [20] and Hisatoku [21]. Experimental and analytical studies are needed to verify the validity of this concept and to further define the key parameters which are applicable to an eccentrically braced system.

Another design approach which is very promising is the coupled braced frame shown in Fig. 23. In this system, the braced walls are designed to remain essentially elastic at all times, while energy dissipation is accomplished by the inelastic bending of the connecting beams. This is analogous to the coupled shear wall concept which is used successfully in reinforced concrete. A study of this system should include

studies of soil-structure interaction and tensile foundation loads as well as of the energy dissipation characteristics of the structure.

New stiffness modification techniques for the entire structural frame also offer challenging possibilities. The bracing system shown in Fig. 24 has been suggested [29] as one such promising system. The unbraced levels reduce the stiffness of the frame and should also reduce the design lateral loads. The bracing would be inserted only where necessary to control lateral deflections. Structures obtained by stiffness modification may have sudden changes in structural properties. Abrupt discontinuities in stiffness, strength, or mass may cause malfunctioning of the structure. Moreover, these more flexible structures would tend to increase structural and non-structural damage potential. As a result, stiffness modification is an area worthy of further study.

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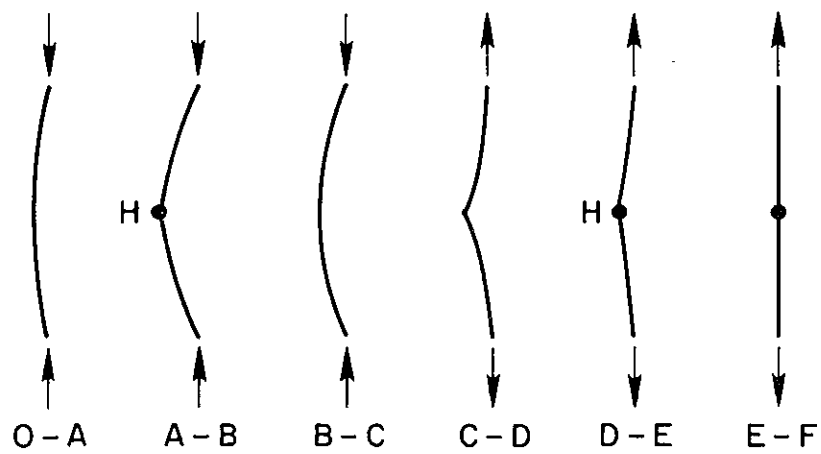
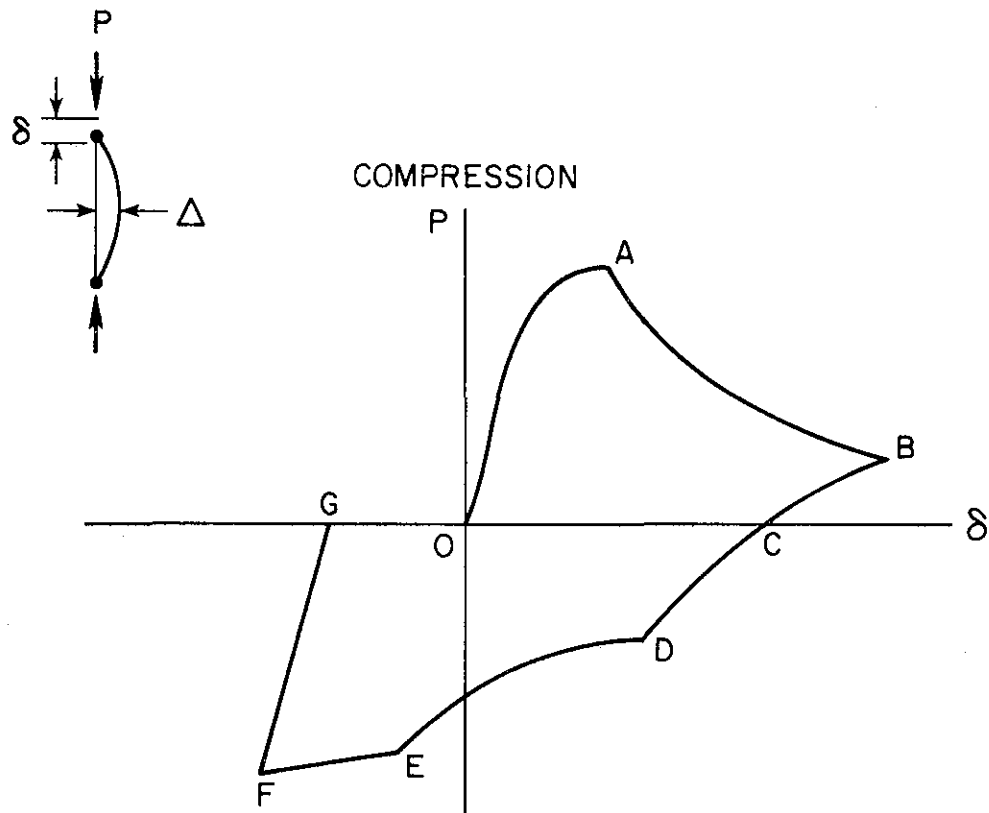


FIGURE 1 - TYPICAL LOAD DEFORMATION RELATIONSHIP OF A SLENDER BAR

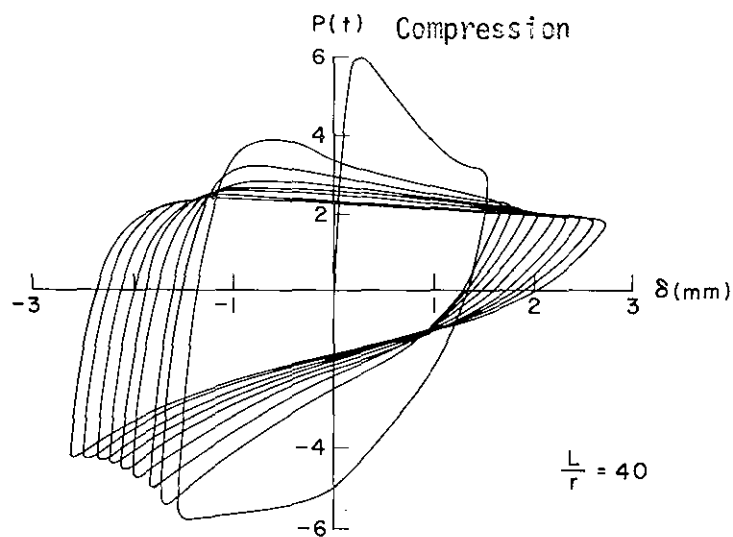
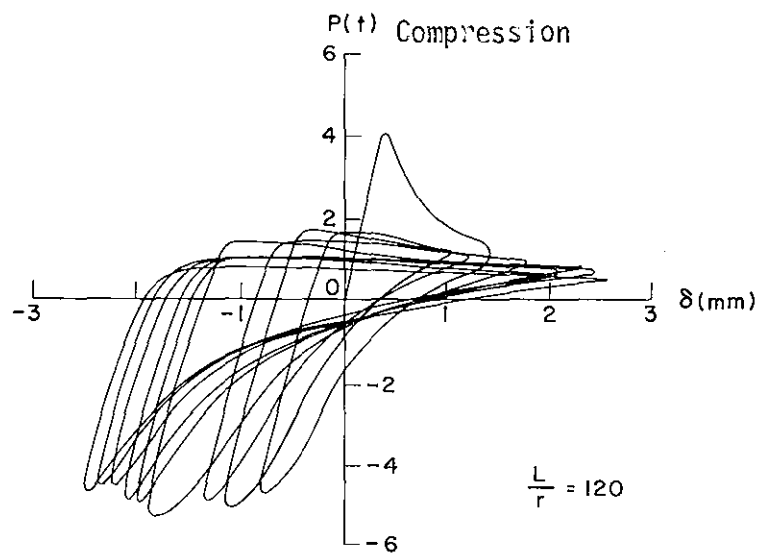


FIGURE 2 - EXPERIMENTAL HYSTERETIC LOOPS OF BARS WITH DIFFERENT SLENDERNESS RATIOS [2]

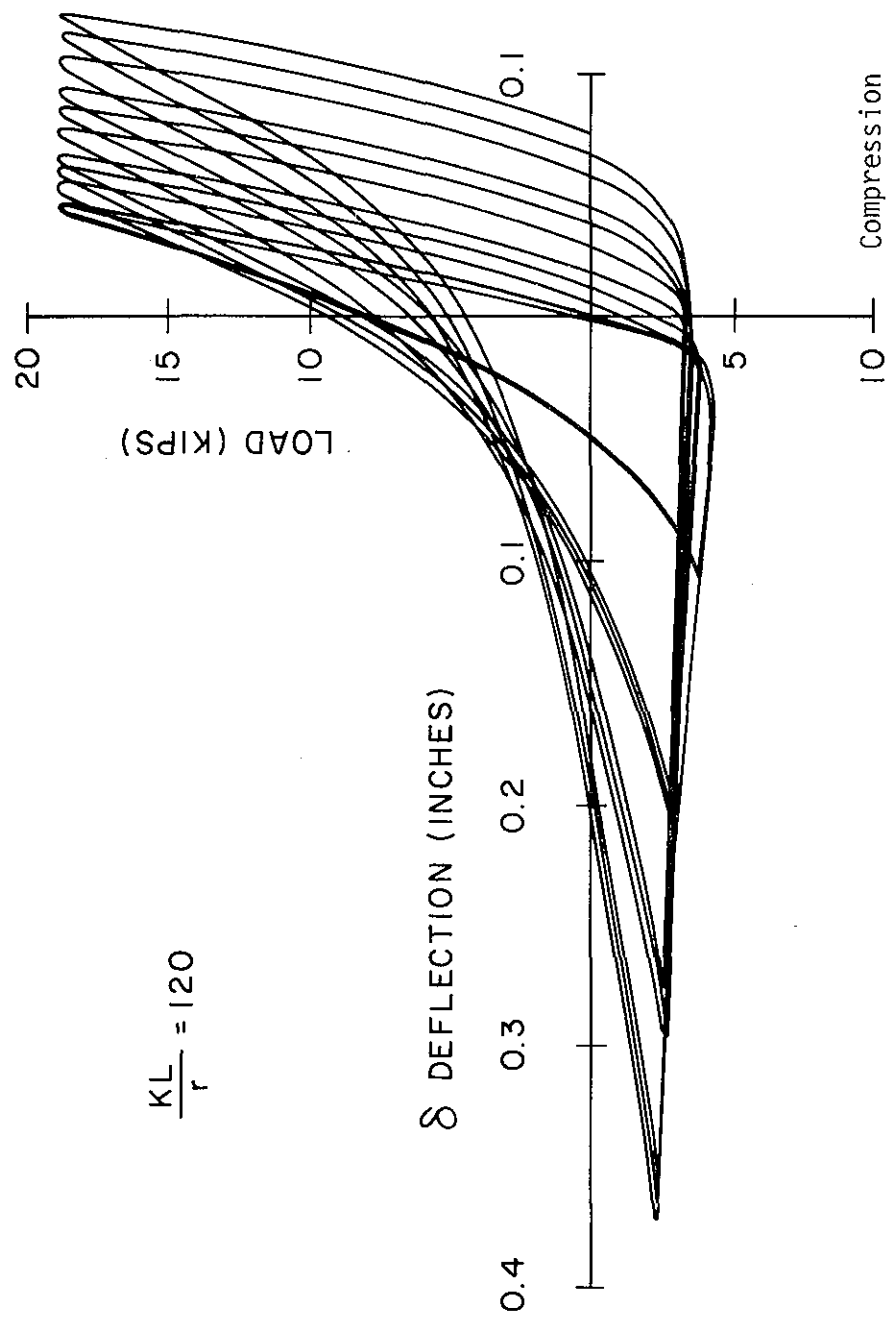


FIGURE 3 - TYPICAL EXPERIMENTAL HYSTERETIC LOOP [3]

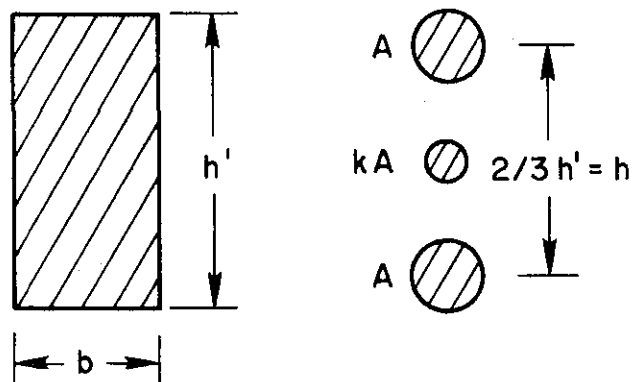
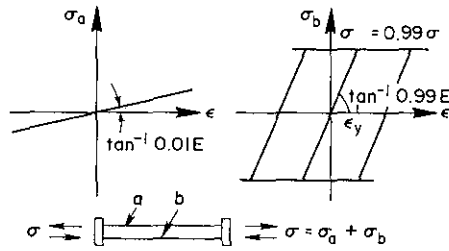
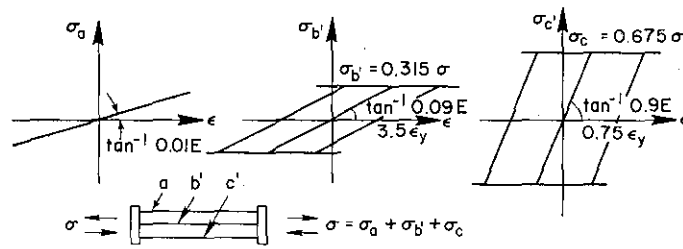


FIGURE 4 - RECTANGULAR CROSS SECTION  
AND EQUIVALENT 3-BAR MODEL



(a) INITIAL MODEL



(b) SUBSEQUENT MODEL

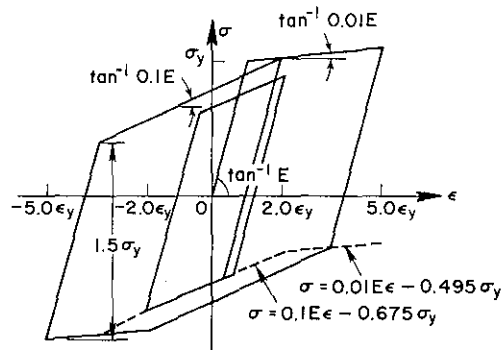
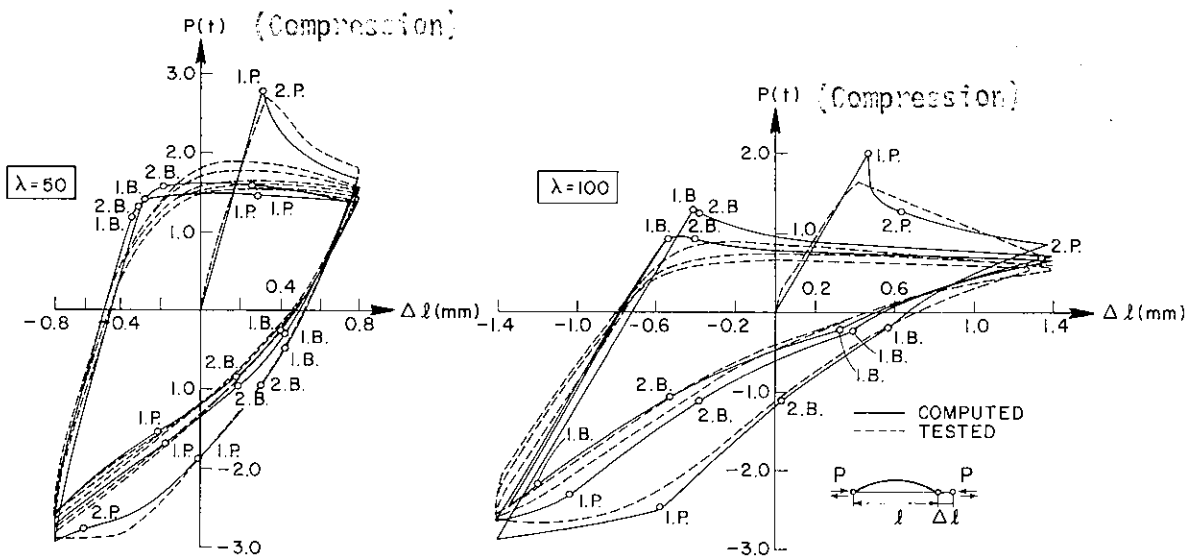
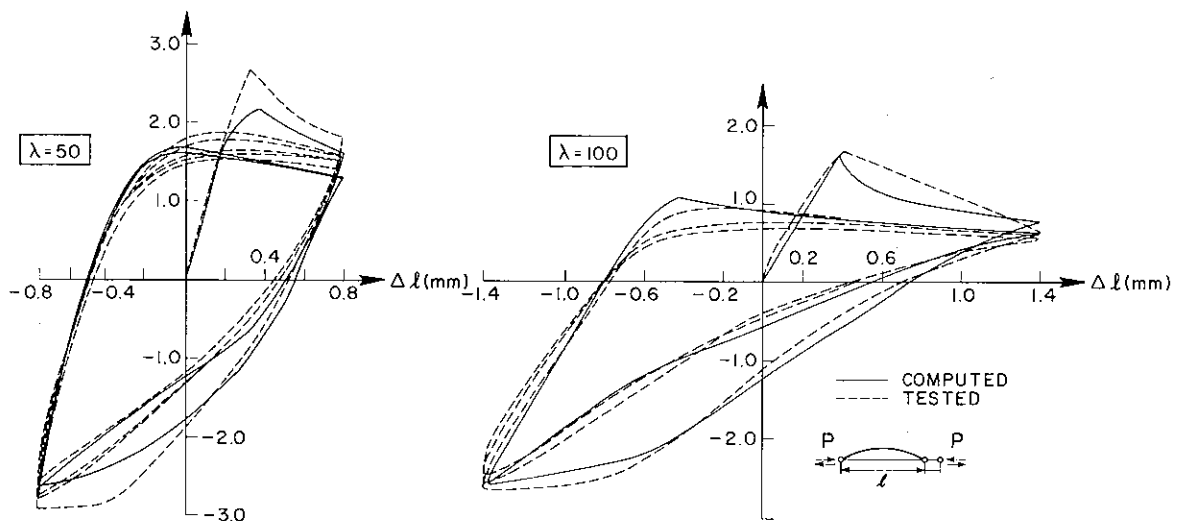


FIGURE 5 - HYSTERETIC LOOPS BY A BAUSCHINGER MODEL [9]



(a) BAUSCHINGER MODEL PER FIG. 5



(b) RAMBERG-OSGOOD FUNCTION

FIGURE 6 - COMPARISON OF BAUSCHINGER MODELS TO TEST RESULTS FOR SLENDER BARS [9]

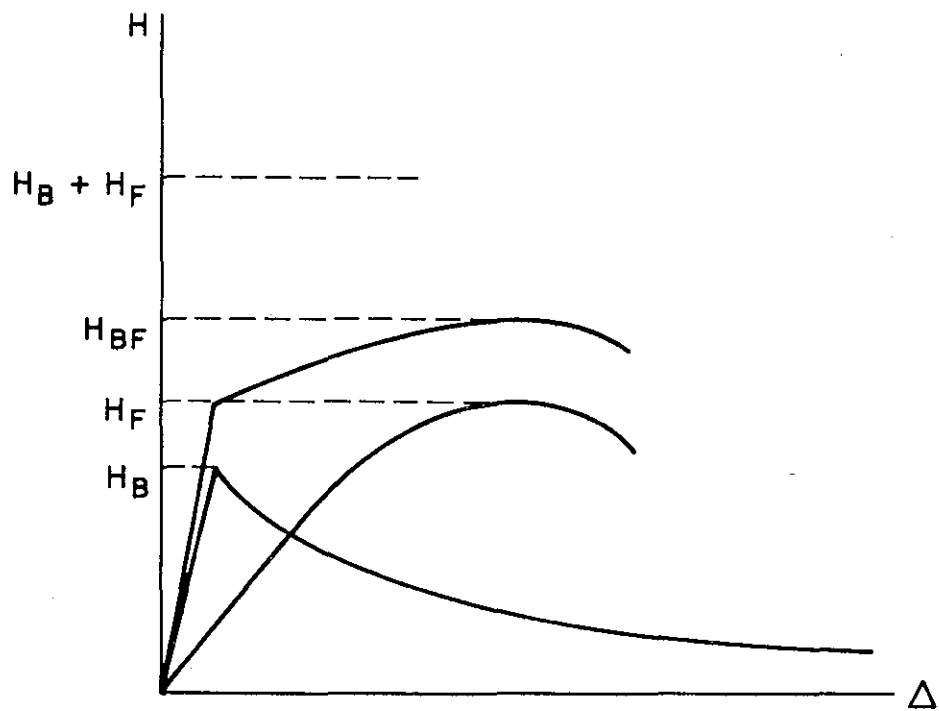
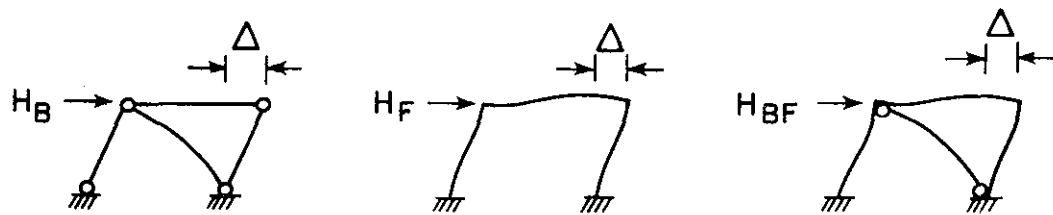
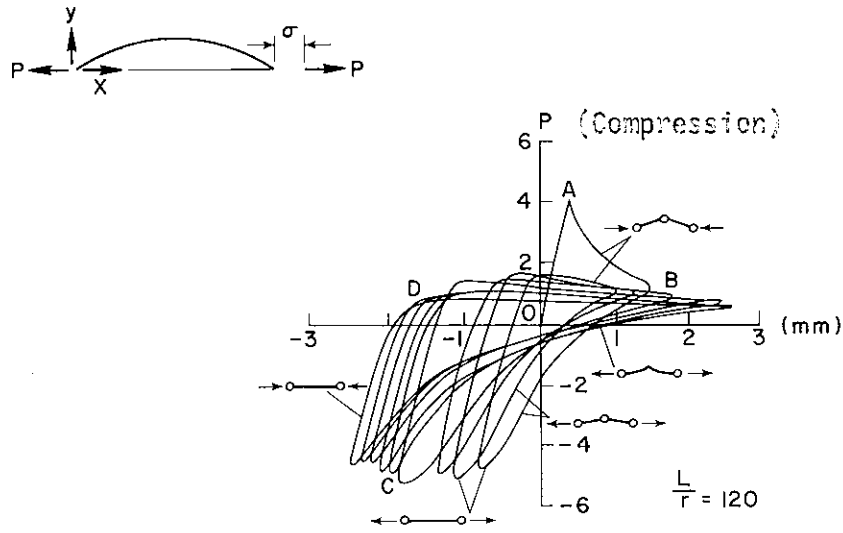
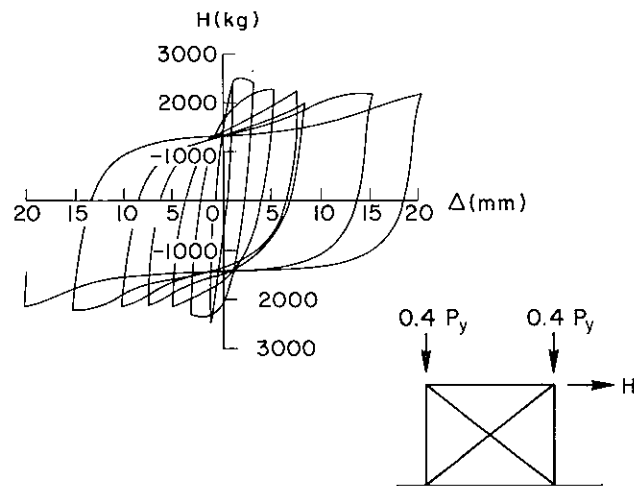


FIGURE 7 - ULTIMATE STATIC STRENGTH OF BRACED FRAMES



(a) AXIAL FORCE-DISPLACEMENT CURVE FOR BRACE



(b) FORCE-DISPLACEMENT CURVES OF BRACED FRAME

FIGURE 8 - HYSTERETIC LOOPS OF BRACE  
AND BRACED FRAME

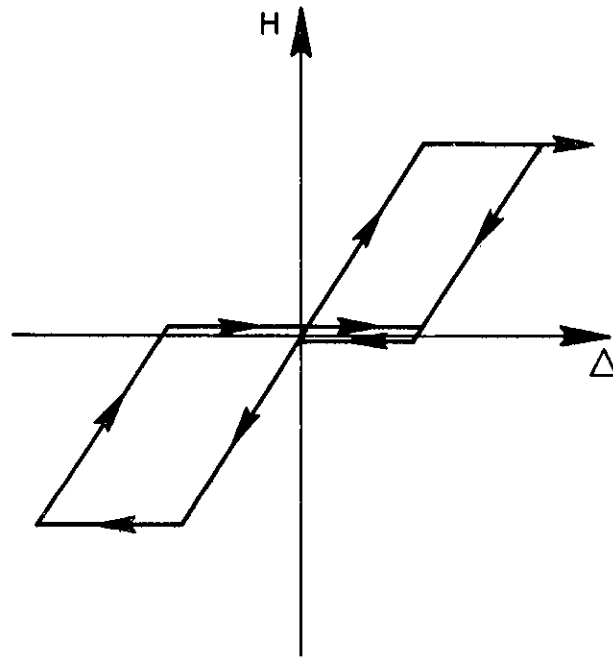


FIGURE 9 - SLIP MODEL OF BRACED FRAME BEHAVIOR

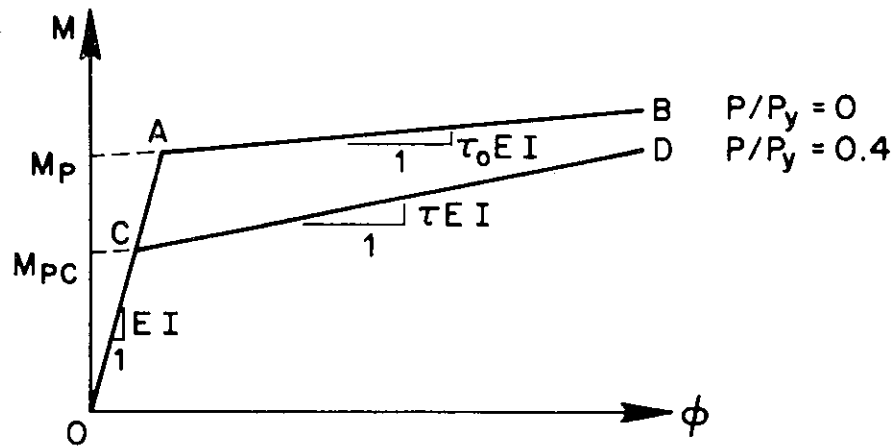


FIGURE 10 - BI-LINEAR MOMENT CURVATURE  
RELATIONSHIP

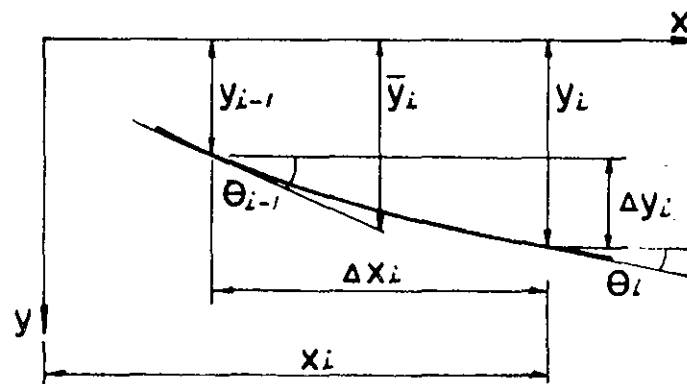
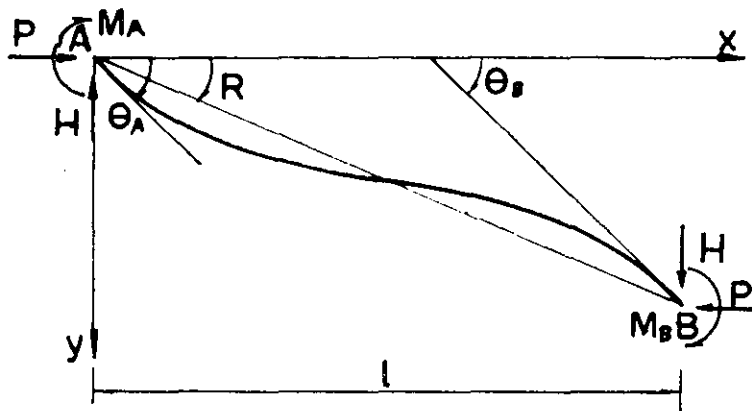
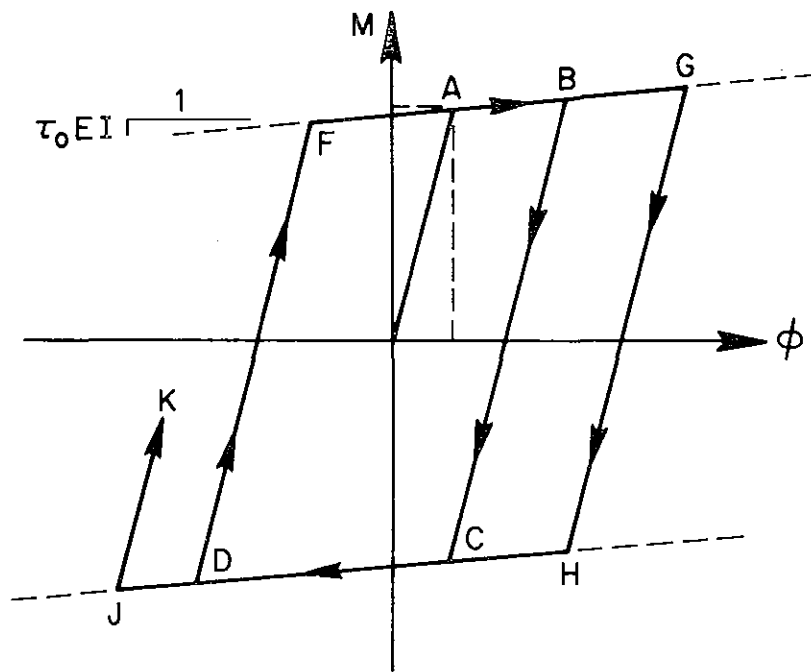
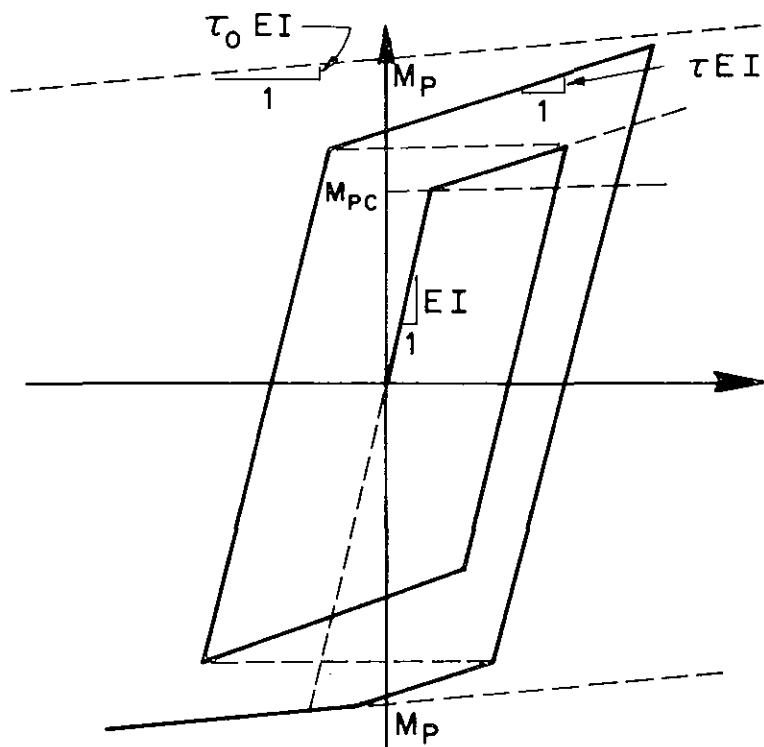


FIGURE 11 - MEMBER SUBJECTED TO AXIAL LOAD  
AND BENDING MOMENT [14]



(a) WITHOUT AXIAL FORCE



(b) WITH AXIAL FORCE

FIGURE 12 - CYCLIC MOMENT CURVATURE RELATIONSHIPS [14]

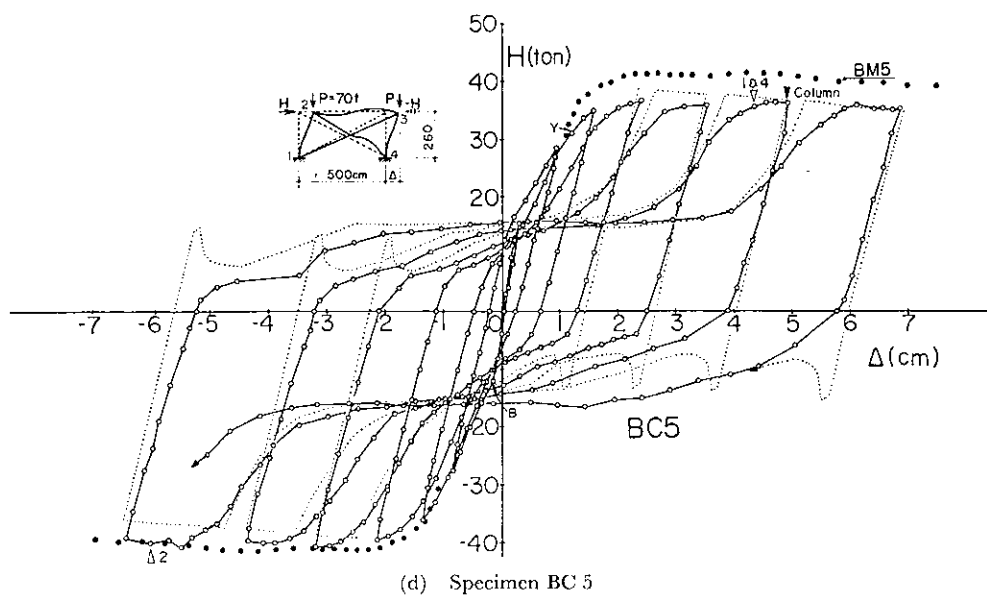
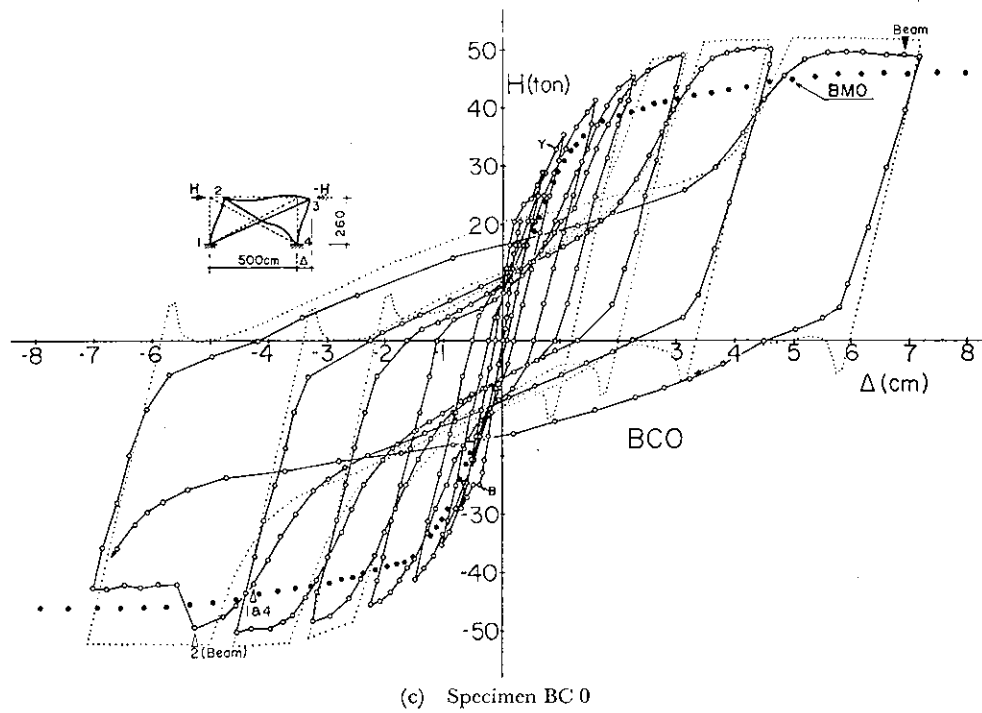
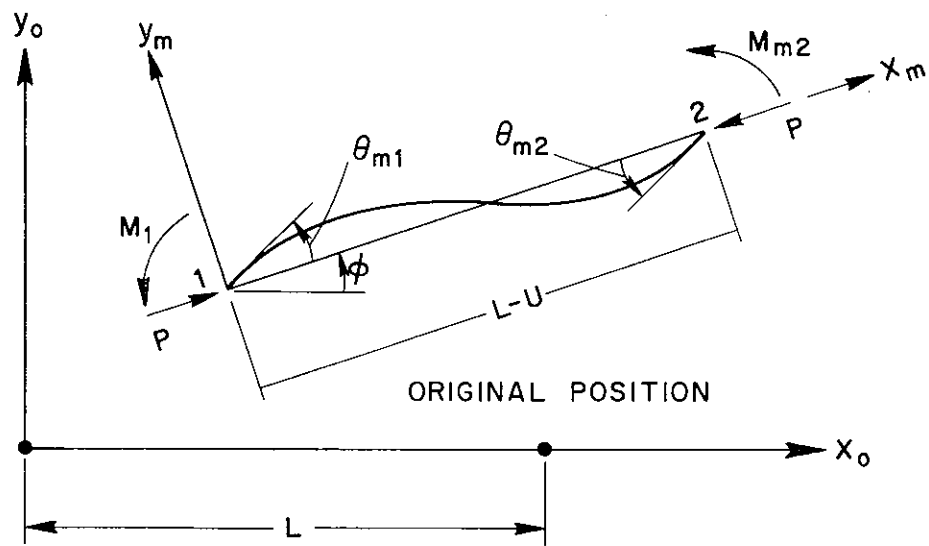
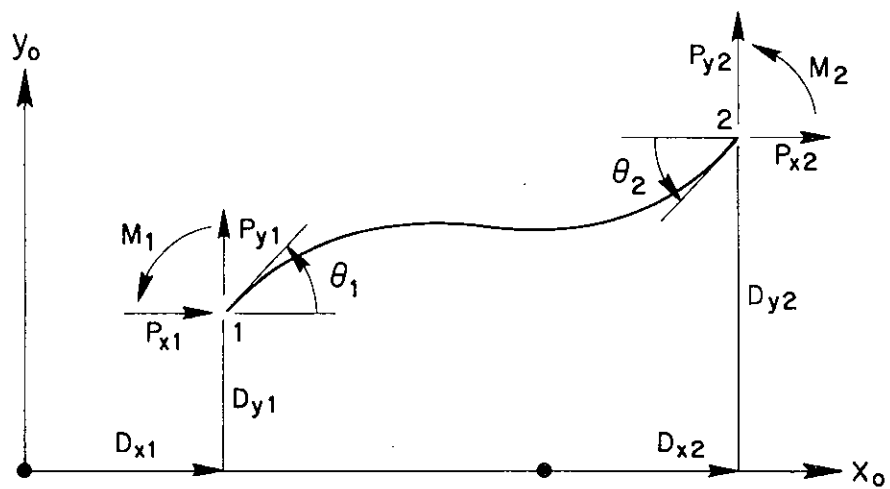


FIGURE 13 - COMPARISON OF CALCULATIONS AND EXPERIMENTAL RESULTS [14]



(a)



(b)

FIGURE 14 - DEFINITION OF DEFORMATIONS  
AND RESULTANTS [2]

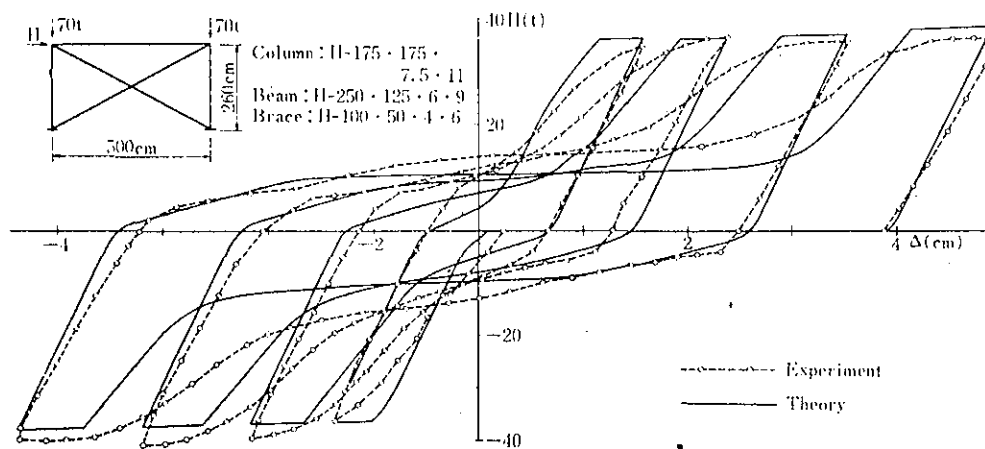
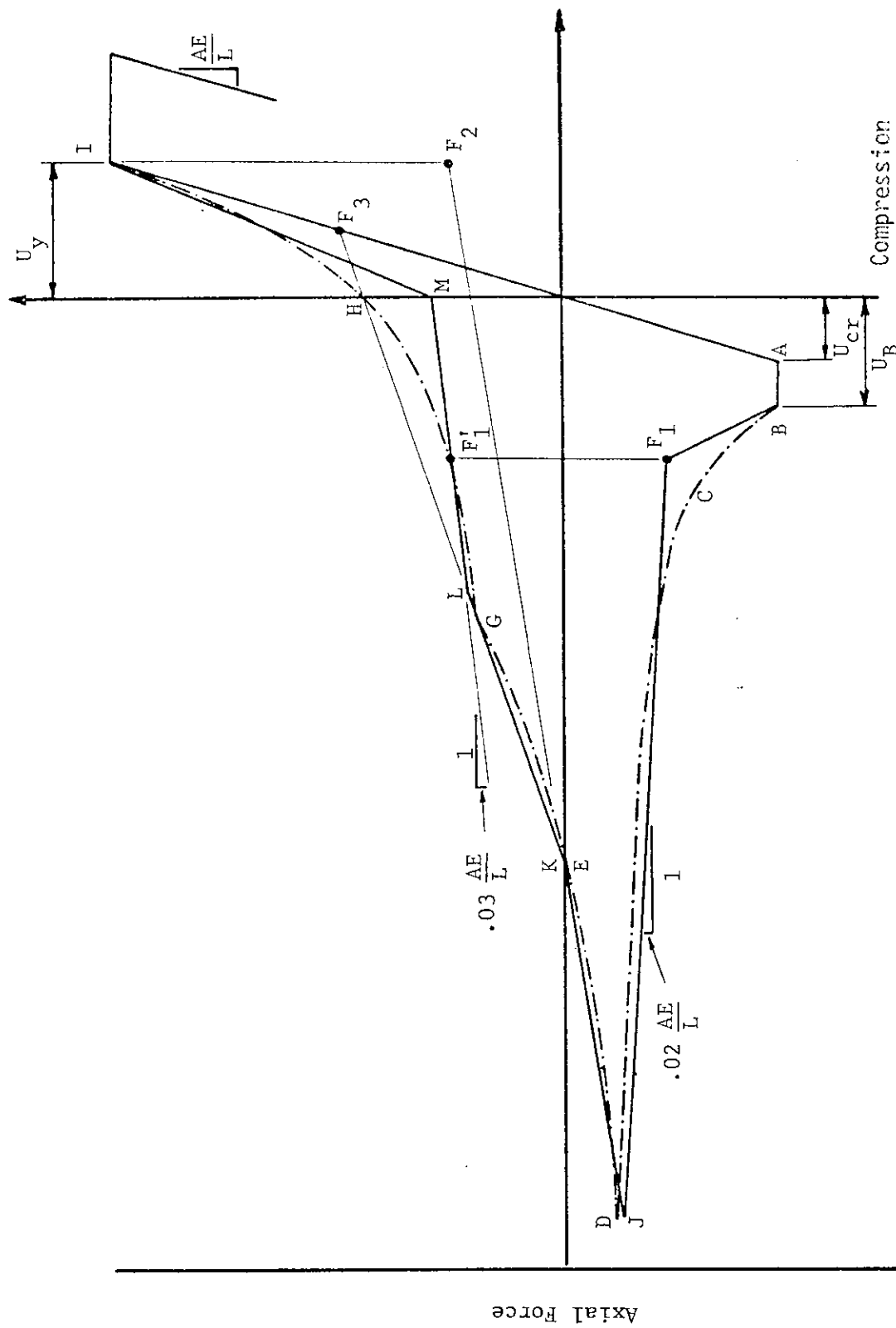
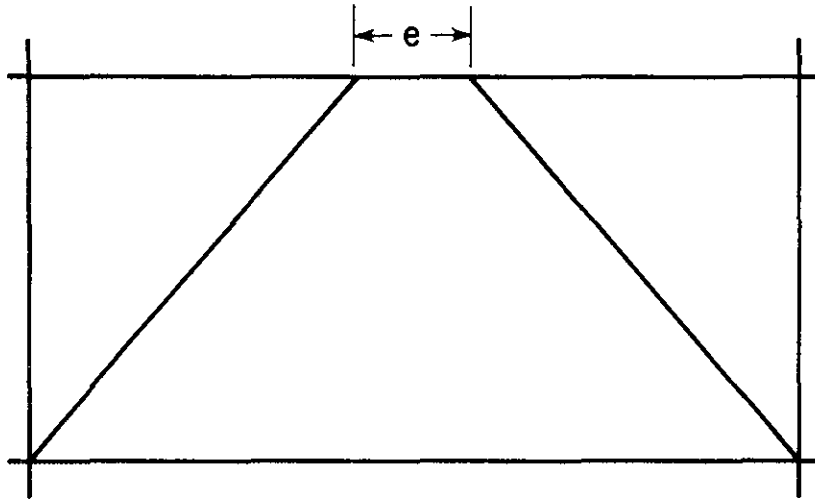


FIGURE 15 - COMPARISON OF CALCULATIONS WITH EXPERIMENTAL RESULTS [2]

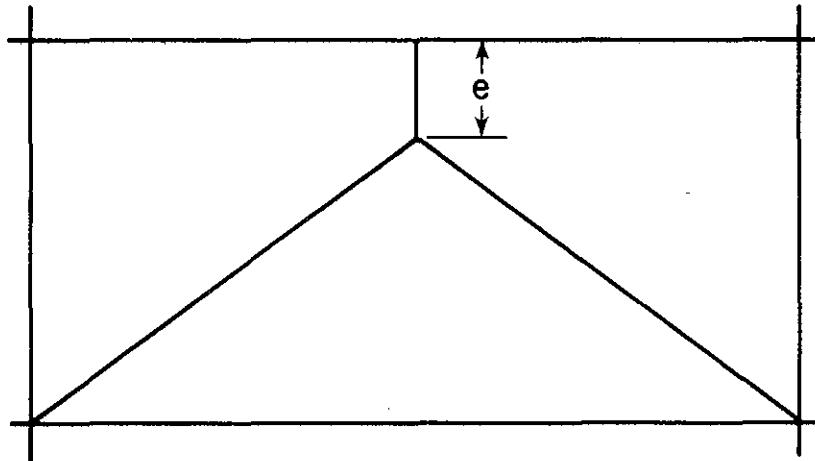


Axial Displacement

FIGURE 16 - LINEAR SIMPLIFICATION OF FORCE-DEFORMATION CURVE [19]



(a) ECCENTRIC K BRACE



(b) INVERTED Y BRACE

FIGURE 17 - ECCENTRICALLY CONNECTED BRACING ELEMENTS

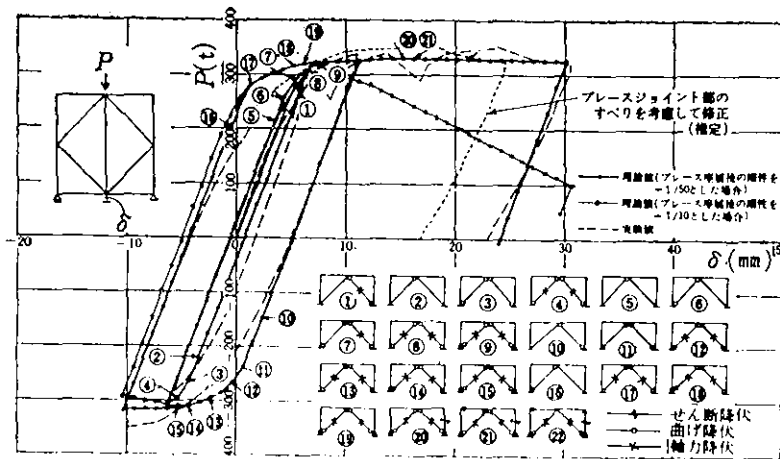


Fig. 4.5 A-1  $P \sim \delta$  RELATION

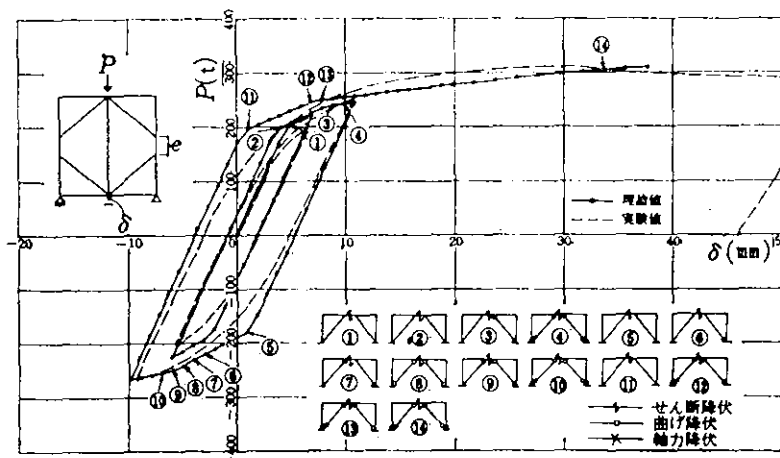


Fig. 4.6 B-2  $P \sim \delta$  RELATION

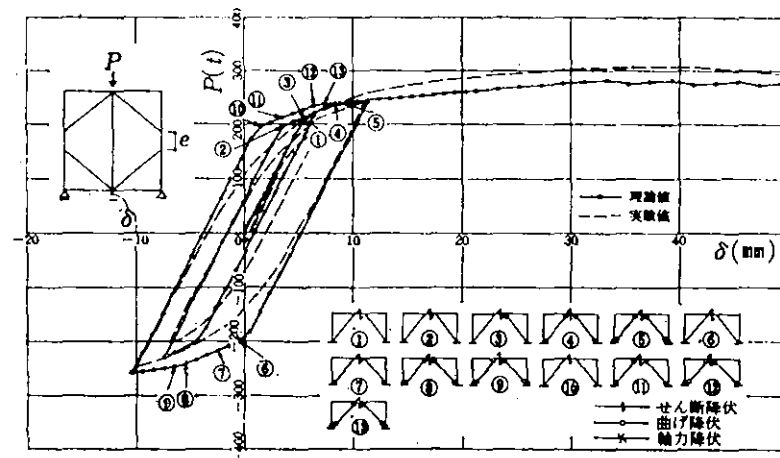


Fig. 4.7 C-4  $P \sim \delta$  RELATION

FIGURE 18 - LOAD-DEFORMATION CURVES FOR ECCENTRIC K BRACED FRAMES [20]

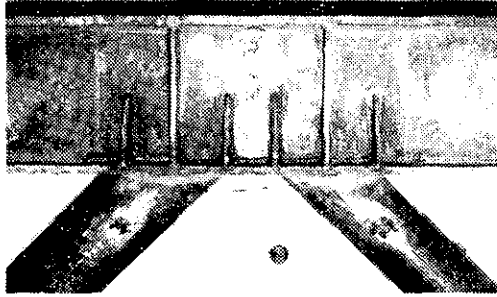


Photo 5.5

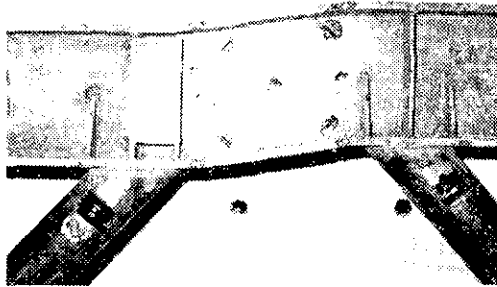


Photo 5.6

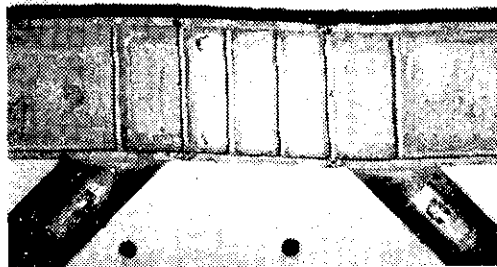


Photo 5.7



Photo 5.8

FIGURE 19 - PHOTOGRAPH OF DEFORMATION IN  
ECCENTRIC CONNECTIONS [20]

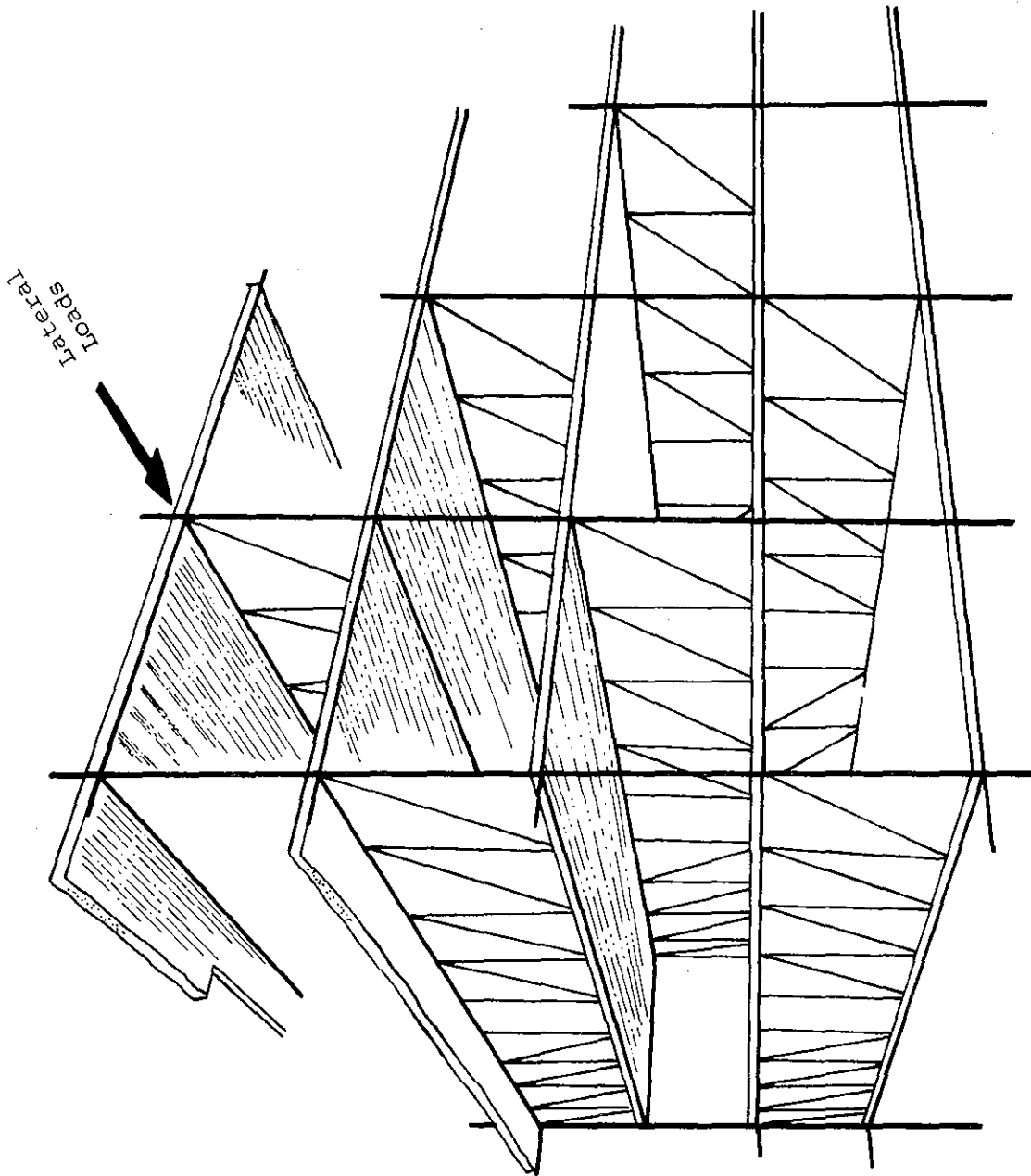


FIGURE 20 - STAGGERED TRUSS FRAMING SYSTEM

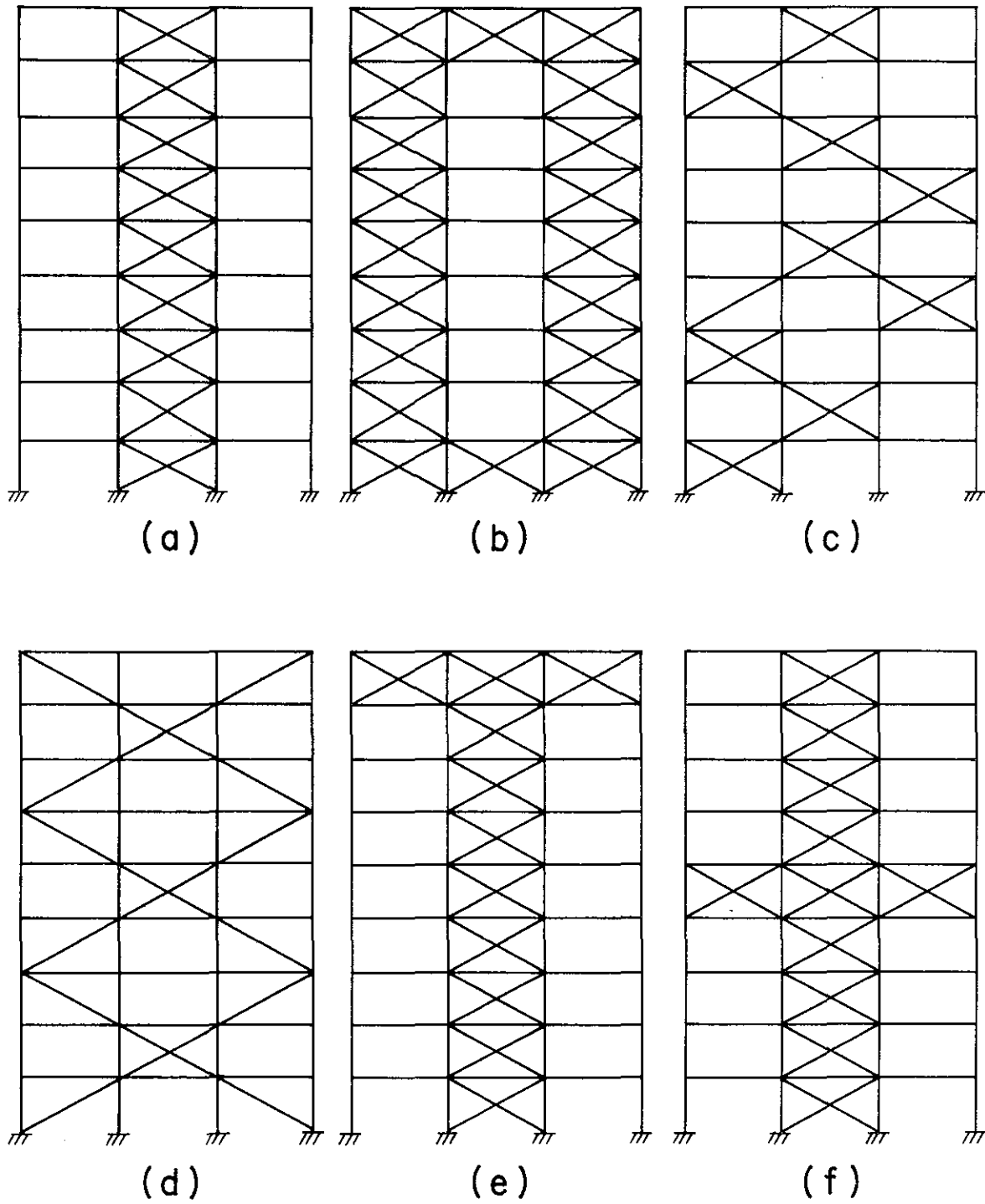


FIGURE 21 - ARRANGEMENTS OF BRACING SYSTEM

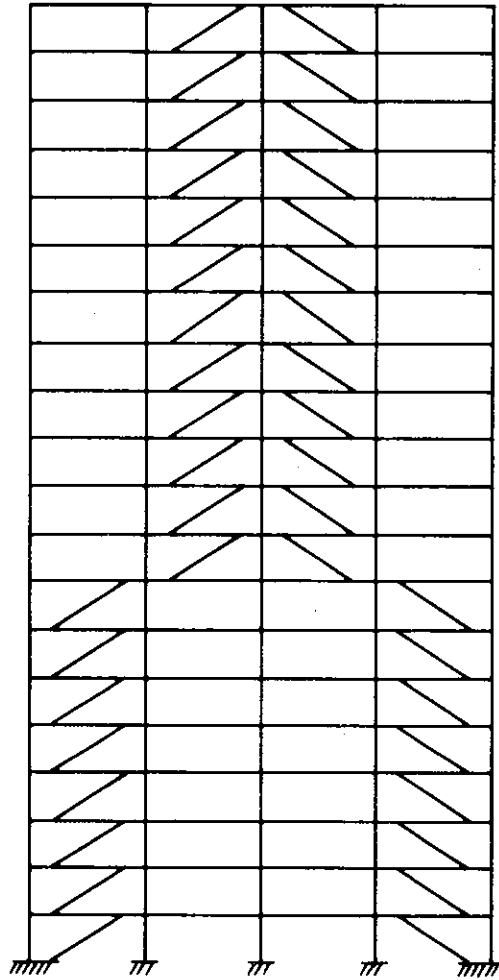


FIGURE 22 - ECCENTRICALLY BRACED SYSTEM

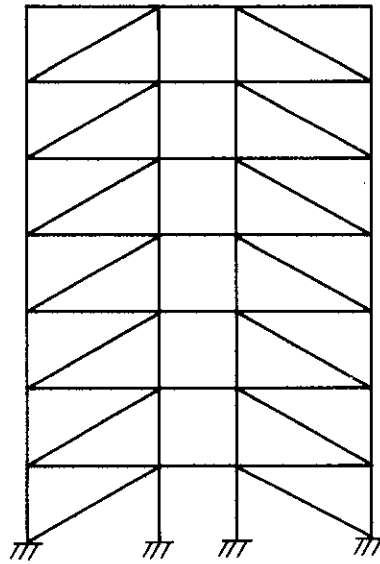


FIGURE 23 - COUPLED BRACED FRAME SYSTEM

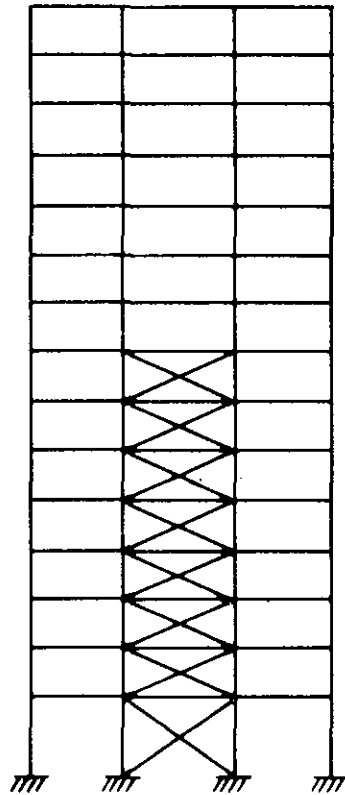


FIGURE 24 - SYSTEM WITH REDUCED STIFFNESS [29]

## APPENDIX A

### Matrix Formulation of Igarashi and Inoui

Igarashi and Inoui [2] developed a matrix formulation for analytically computing the inelastic behavior of a braced frame. Figure 14 of the report shows the local coordinate system which is used in this analysis. The stiffness formulation for a brace member in the elastic zone in dimensionless form and in local coordinates is:

$$\begin{Bmatrix} \Delta P \\ \Delta M_1 \\ \Delta M_2 \end{Bmatrix} = \begin{bmatrix} \beta_0 & -\beta_0\beta_1 & -\beta_0\beta_2 \\ -\beta_0\beta_1 & \frac{S}{\alpha} + \beta_0\beta_1^2 & \frac{SC}{\alpha} + \beta_0\beta_1\beta_2 \\ -\beta_0\beta_2 & \frac{SC}{\alpha} + \beta_0\beta_1\beta_2 & \frac{S}{\alpha} + \beta_0\beta_2^2 \end{bmatrix} \begin{Bmatrix} \Delta u \\ \Delta \theta_{m1} \\ \Delta \theta_{m2} \end{Bmatrix} \quad (A-1a)$$

or expressed in matrix notation

$$\Delta \underline{P}_m = \underline{k}_m^e \cdot \Delta \underline{d}_m \quad (A-1b)$$

where

$$N = P/P_y$$

$$N_E = (\pi^2 EI/L^2)/P_y$$

$$M = M/M_p$$

$$\alpha = A/I (M_p/P_y)^2$$

$$\beta_0 = 1 / \left[ 1 + (\pi^2/\alpha N_E) \left\{ \frac{db_1}{dp} (\theta_{m1} + \theta_{m2})^2 + \frac{db_2}{dp} (\theta_{m1} + \theta_{m2})^2 \right\} \right]$$

$$\beta_1 = (\pi^2/\alpha N_E) [(b_1 + b_2) \theta_{m1} + (b_1 - b_2) \theta_{m2}]$$

$$\beta_2 = (\pi^2/\alpha N_E) [(b_1 - b_2) \theta_{m1} + (b_1 + b_2) \theta_{m2}]$$

S and C are stability functions,

b<sub>1</sub> and b<sub>2</sub> are the corresponding shape functions.

The deflection U and  $\theta$  in dimensional form are expressed by

$$U = (P_y L/AE) u$$

$$\theta = (M_p L/\alpha EI) \theta$$

When inelastic bending occurs, plastic hinges are assumed to occur only at the member ends. The stress-strain relationship obeys Prager's rule as modified by Ziegler, in which strain hardening is taken into account. Thus the modified forms of the stiffness formulation for the plastic zones of behavior are:

- 1) In the case of a single plastic hinge at end 1 of the member in Figure 14a,

$$\Delta_{\underline{m}}^P = [\underline{k}_m^e - \underline{k}_m^e \underline{f}_1 \underline{f}_1^T \underline{k}_m^e \left\{ 1/(g_{11} + \tau|f_1|^2) \right\}] \Delta_{\underline{d}m} \quad (A2)$$

- 2) In the case of a single plastic hinge at end 2 of the member in Figure 14a,

$$\Delta_{\underline{m}}^P = [\underline{k}_m^e - \underline{k}_m^e \underline{f}_2 \underline{f}_2^T \underline{k}_m^e \left\{ 1/(g_{22} + \tau|f_2|^2) \right\}] \Delta_{\underline{d}m} \quad (A3)$$

- 3) In the case of plastic hinges at both ends,

$$\begin{aligned} \Delta_{\underline{m}}^P = & [\underline{k}_m^e - \left\{ (g_{22} + \tau|f_2|^2) \underline{k}_m^e \underline{f}_1 \underline{f}_1^T \underline{k}_m^e \right. \\ & - g_{12} \underline{k}_m^e \underline{f}_1 \underline{f}_2^T \underline{k}_m^e - g_{21} \underline{k}_m^e \underline{f}_2 \underline{f}_1^T \underline{k}_m^e \\ & \left. + (g_{11} + \tau|f_1|^2) \underline{k}_m^e \underline{f}_2 \underline{f}_2^T \underline{k}_m^e \right\} / \\ & \left\{ (g_{11} + \tau|f_1|^2)(g_{22} + \tau|f_2|^2) - g_{12}g_{21} \right\}] \Delta_{\underline{d}m} \quad (A4) \end{aligned}$$

In the preceding equations  $\underline{f}_i$  is the vector which is normal to the yield surface for the  $i^{\text{th}}$  end of the member and  $\tau$  is the coefficient of strains hardening. The term  $|f_i|^2$  is the square of the magnitude of the  $i^{\text{th}}$  normal vector

$$|f_i|^2 = \underline{f}_i^T \underline{f}_i$$

The term  $g_{ij}$  is an elastic stiffness term for these normal directions

$$g_{ij} = \underline{f}_i^T \underline{k}_m^e \underline{f}_j$$

The stiffness matrix  $\underline{k}_m$  can also be expressed in dimensional form  $\underline{K}_m$  by

$$\underline{K}_m = \underline{B}_p \underline{k}_m \underline{B}_d^{-1} \quad (A5)$$

where

$$\underline{B}_p = \begin{Bmatrix} P_y & 0 & 0 \\ 0 & M_p & 0 \\ 0 & 0 & M_p \end{Bmatrix}$$

$$\underline{B}_d = \begin{Bmatrix} \frac{P_y L}{AE} & 0 & 0 \\ 0 & \frac{M_p L}{\alpha EI} & 0 \\ 0 & 0 & \frac{M_p L}{\alpha EI} \end{Bmatrix}$$

The element stiffness must now be transformed into the fixed coordinate system  $(x_0, y_0)$  stiffness  $\underline{K}_0$  by

$$\underline{K}_0 = \underline{T}^T \underline{k}_m \underline{T} + \underline{A} \quad (A6)$$

where

$$\underline{T} = \begin{Bmatrix} 1 & -(Dy_1 - Dy_2)/L & 0 & -1 & (Dy_1 - Dy_2)/L & 0 \\ 0 & 1/L & 1 & 0 & 1/L & 0 \\ 0 & 1/L & 0 & 0 & 1/L & 1 \end{Bmatrix}$$

and

$$\underline{A} = \begin{Bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -P/L & 0 & 0 & P/L & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & P/L & 0 & 0 & -P/L & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{Bmatrix}$$

The element stiffness can now be assembled into the structural stiffness matrix and the solution proceeds in the standard form.

$$\underline{\underline{P}}_0 = \underline{\underline{K}}_0 \underline{\underline{D}}_0 \quad (A6)$$

where

$$\underline{\underline{P}}_0 = \begin{Bmatrix} P_{x_1} \\ P_{y_1} \\ M_1 \\ P_{x_2} \\ P_{y_2} \\ M_2 \end{Bmatrix} \quad \underline{\underline{D}}_0 = \begin{Bmatrix} D_{x_1} \\ D_{y_1} \\ \theta_1 \\ D_{x_2} \\ D_{y_2} \\ \theta_2 \end{Bmatrix}$$

The solution proceeds in an incremental manner where at the end of each step the elements must be checked for yielding. If yielding has occurred, the stiffness is modified by equation A2, A3, or A4, and the solution proceeds through the next step.

Igarashi and Inoue [2] made a number of inelastic calculations by this method with good results. Figure 15 is an example of these calculations. However, it should be noted that this solution requires the definition of a stability function. Stability functions become very approximate for short members, members with high residual stresses, and initially crooked members. As a result, the accuracy of this method is limited to accuracy of this function.

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