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DYNAMIC INSTABILITY AND ULTIMATE CAPACITY OF INELASTIC SYSTEMS PARAMETRICALLY EXCITED BY EARTHQUAKES--PART I

by

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August, 1973

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ABSTRACT

A procedure of analysis is presented for determining the dynamic instability and response of framed structures subjected to pulsating axial loads, time-dependent lateral forces, or foundation movements. Included in the analytical work are the instability criterion of a structural system, the finite element technique of structural matrix formulation, and the computer solution methods.

Dynamic instability is defined by a region in relation to transverse natural frequency, longitudinal forcing frequency, and the magnitude of axial dynamic force. The axial pulsating load is expressed in terms of static buckling load for ensuring that the applied load is not greater than the buckling capacity of a structural system. Consequently, the natural frequency and static instability analyses are also included. For static instability analysis, both the concentrated and uniformly distributed axial loads have been investigated.

The displacement method has been used in this research for structural matrix formulation for which the elementary matrices of mass, stiffness, and stability have been developed by using the Lagrangian equation. The system matrices have been formulated by using the equilibrium and compabitibility conditions of the constituent members of a system.

Two numerical integration techniques of the fourth-order

Runge-Kutta method and the linear acceleration method have been employed for the elastic and elasto-plastic response of continuous beams, shear buildings, and frameworks. The general considerations are the bending deformation, p-Δ effect, and the effect of girder shears on columns. For the elasto-plastic analysis, the effect of axial load on plastic moment is also included.

A number of selected examples are presented, and the results are illustrated in a series of charts, tables, and figures in which the significant effect of pulsating load on the amplitude of transverse vibration can be observed.

ACKNOWLEDGEMENTS

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I. INTRODUCTION

In recent years the theory of dynamic instability has become one of the newest branches of the structural dynamics and mechanics of deformable solids. The problems, which have been examined on the basis of the classical theory of vibrations and structural dynamics, emphasize the response history resulting from lateral, time-dependent excitations. It is known, when a rod is subjected to the action of longitudinal compressive force varying periodically with time, that for a definite frequency the transverse vibrations of the rod will have a rapidly increasing amplitude. Thus, the study of the formation of this type of vibration and the formulation of the methods for the prevention of their occurence are necessary in the various areas of mechanics, transportation, industrial construction, and structures excited by earthquakes.

A. Purpose of Investigation

The purpose of this study has been to develop an analytical method for determining the behavior of dynamic instability and to study the response of structural systems subjected to longitudinal pulsating loads and lateral dynamic forces or foundation movements. The mathematical formulation, which is general enough for computer analysis of large structural systems, considers geometric and material nonlinearity.

B. Scope of Investigation

The scope of the study may be briefly stated as the derivation of instability criteria and the development of finite element formulation of structural matrices and the numerical methods of computer solution.

In Chapter III, the basic formulation of mass matrix, stiffness matrix, and stability matrix is presented by using the energy concept and finite element technique. The governing differential equation is expressed in terms of system matrix, which is formulated on structural geometric and equilibrium conditions.

In order to evaluate the dynamic instability regions, it is convenient to express the axial load in terms of static buckling load and the longitudinal forcing frequency in terms of natural frequency. Thus, in Chapter IV, the techniques for finding natural frequencies, buckling loads, and instability regions are presented. For the buckling load case, the uniform axial load is also investigated.

Two numerical integration techniques for dynamic response that use the fourth-order Runge-Kutta method and the linear acceleration method are presented in Chapter V in which a comparison of numerical solutions shows the accuracy of the presented methods.

Chapter VI contains examples of the dynamic response of various types of frameworks subjected to axial pulsating loads, lateral forces, or foundation movements.

The elasto-plastic case is given in Chapters VII and VIII for the formulation of member matrices and system matrix, plastic hinge rotations, and numerical solutions.

Two typical computer programs of elastic and elasto-plastic analyses of general types of rigid frames are given in the Appendix.

II. REVIEW OF LITERATURE

A. Structural Dynamics With Longitudinal Excitations

The behavior of structural systems subjected to both lateral and longitudinal excitations is little known. Most of the research work has been concentrated on the problem of an elastic column subjected to a periodically varying axial load for the purpose of searching for the stability criteria of double symmetric columns (1) as well as non-symmetric columns (2).

Sevin E. (3), among other investigators, studied the effect of longitudinal impact on the lateral deformation of initially imperfect columns. Recently, Cheng and Tseng (5) investigated the effect of static axial load on the Timoshenko beam-column systems.

It seems that very little work has been done on either the criteria of dynamic instability or the response behavior of framed structures subjected to dynamic lateral and longitudinal excitations.

B. Structural Dynamics Without Longitudinal Excitations

The conventional structural dynamics problems have been generally solved by using three methods: lumped mass, distributed mass, and consistent mass. Before computer facilities were available, the lumped mass model with a finite degree of freedom had been extensively studied by a number

of investigators. With the advent of computers, research work on multistory structures was initiated by several investigators, namely N.M. Newmark, R.W. Clough, J.A. Blume, (6,7,9-12,17), and later by Cheng (13), E.L. Wilson, and I.P. King, (14,15,16).

For the distributed mass system, the early research work was limited to single members (18) or one-story frames (19). Later Levien and Hartz (20) used the dynamic flexibility matrix method to solve problems of one- and two-story rigid frames, and Cheng (4,13,29) solved free and forced vibrations of continuous beams and rigid frames by using the displacement method. The dispalcement and flexibility methods cited above may be considered exact in the sense that the members must be prismatic and that the structural joints are rigid.

In recent years, the finite element technique has been extensively used for solving structural dynamics problems. The method was initially proposed by Archer (21) for plane frameworks. Cheng (22) recently extended the technique to solve space frame problems. The model of the method is similar to the distributed mass system. The equation of motion, however, is expressed in an explicit form for which the solution effort is much less than that of the distributed mass model.

The fundamental behavior of the dynamic response of elasto-plastic systems can be found in standard texts (23,24). The elasto-plastic analysis method of beams and one-story frames

with distributed mass has appeared in references (25,26) in which the method is limited to simple structures.

For large structures, typical studies can be found in references (27,28). Berg and Dadeppo (27) investigated the response of a multistory elasto-plastic structures subjected to lateral dynamic forces, and Walpole and Sheperd (28) studied the behavior of reinforced concrete frames subjected to earthquake movements.

III. MATRIX FORMULATION OF ELASTIC STRUCTURAL SYSTEMS

The displacement matrix method has been used in the structural system formulation for static and dynamic instability analysis and dynamic response. The formulation involves deriving differential equations, element matrices of stiffness, mass, stability, and the matrix of general structural systems. The structures are plane frameworks in which the joints are rigid and the constituent members are prismatic. As shown in Fig. 3.1, the structure is subjected to time-dependent axial forces, N(t), and lateral dynamic load, F(t), or foundation movement, G(t), and may have a superimposed uniform mass, m, and a concentrated mass, M_i, in addition to its own weight.

For the purpose of investigating large systems, the shears transmitted from girders to columns are taken into consideration,and the members are assumed to have bending deformation only.

A. Governing Differential Equation

Consider an arbitrary member of a structural system as shown in Fig. 3.2. The governing differential equations for such an element can be obtained by using the Lagrangian equation

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_i}\right) - \frac{\partial T}{\partial q_i} + \frac{\partial U}{\partial q_i} - \frac{\partial V}{\partial q_i} = \frac{\partial W}{\partial q_i} = Q_i \tag{3.1}$$

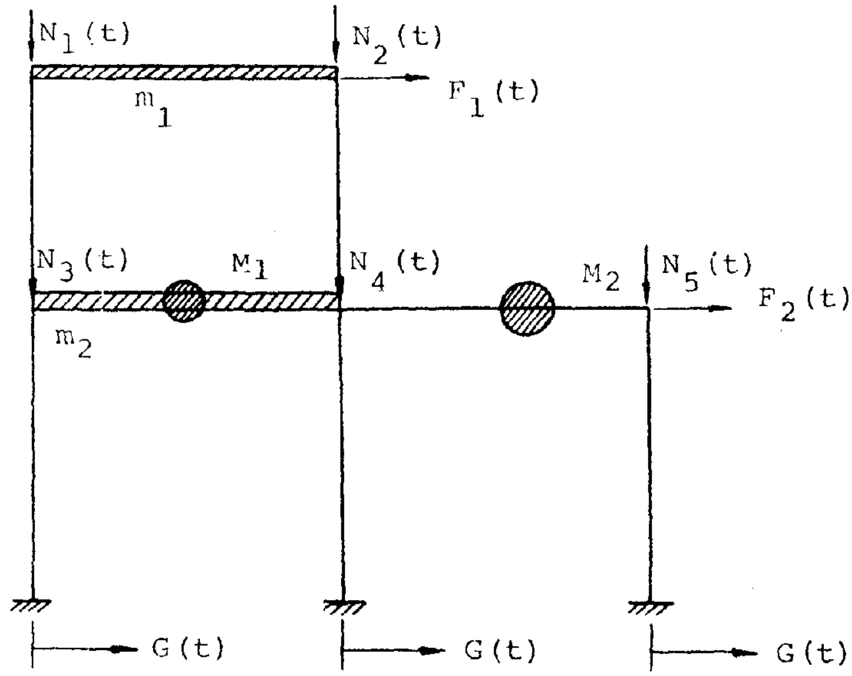


Fig. 3.1 General Problem

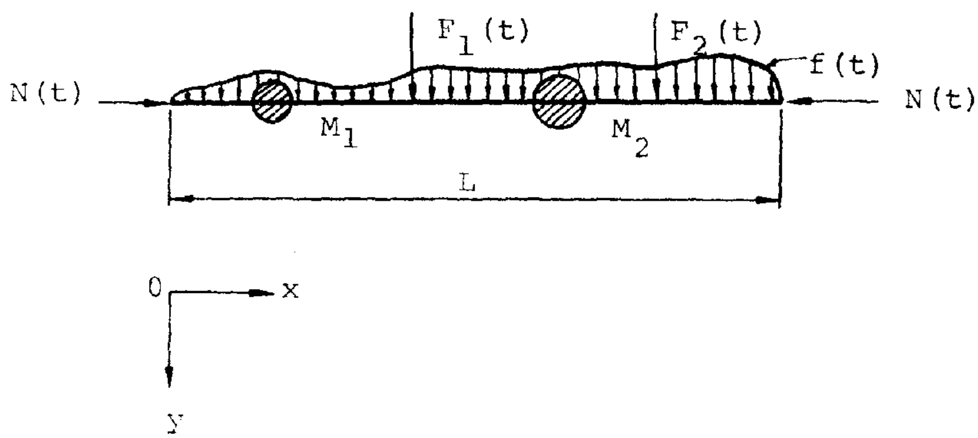


Fig. 3.2 Loading on a Typical Member

in which

T = kinetic energy

U = strain energy of bending

V = potential energy done by axial force

Q_i = generalized forces

q_i = generalized coordinates at node i associated with Q_i

$\dot{q}_i$ = generalized velocities

W = work done by generalized external forces.

Let $\phi(x)$ be the shape function and $q_i(t)$ be the time function of the beam motion, then the displacement of the beam can be expressed as

$$y(x,t) = \sum_{i=1}^{n} q_i(t)\phi_i(x). \tag{3.2}$$

The kinetic energy for lateral displacement of the member is

$$T = \tfrac{1}{2}\int_0^L m\left[\frac{\partial y(x,t)}{\partial t}\right]^2 dx \tag{3.3}$$

where m is the mass per unit length.

The strain energy for bending of the member may be represented by

$$U = \tfrac{1}{2}\int_0^L EI\left[\frac{\partial^2 y(x,t)}{\partial x^2}\right]^2 dx \tag{3.4}$$

where E, and I are Young's elastic modulus and moment of inertia, respectively.

The potential energy for the longitudinal force is

$$V = \frac{1}{2}\int_0^L N(t)\left[\frac{\partial y(x,t)}{\partial x}\right]^2 dx\,. \tag{3.5}$$

By the substitution of Eq. (3.2), one may obtain

$$T = \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{dq_i}{dt}\frac{dq_j}{dt}\int_0^L m\phi_i(x)\phi_j(x)dx \tag{3.6}$$

$$U = \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} q_i q_j \int_0^L EI\frac{d^2\phi_i(x)}{d^2x}\,\frac{d^2\phi_j(x)}{d^2x}\,dx \tag{3.7}$$

$$V = \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} q_i q_j \int_0^L N(t)\frac{d\phi_i(x)}{dx}\,\frac{d\phi_j(x)}{dx}\,dx \tag{3.8}$$

or

$$T = \frac{1}{2}\sum_{i}^{n}\sum_{j}^{n} m_{ij}\dot{q}_i\dot{q}_j = \frac{1}{2}\{\dot{q}\}^T[m_{ij}]\{\dot{q}\} \tag{3.9}$$

$$U = \frac{1}{2}\sum_{i}^{n}\sum_{j}^{n} k_{ij}q_i q_j = \frac{1}{2}\{q\}^T[k_{ij}]\{q\} \tag{3.10}$$

$$V = \frac{1}{2}\sum_{i}^{n}\sum_{j}^{n} s'_{ij}q_i q_j = \frac{1}{2}\{q\}^T[s'_{ij}]\{q\} \tag{3.11}$$

where

$$m_{ij} = \int_0^L m\phi_i(x)\phi_j(x)dx \tag{3.12}$$

$$k_{ij} = \int_0^L EI\phi_i''(x)\phi_j''(x)dx \tag{3.13}$$

$$s'_{ij} = \int_0^L N(t)\phi_i'(x)\phi_j'(x)dx. \tag{3.14}$$

To include the concentrated masses in the formulation of m_{ij}, let us consider masses $M_k(x_k)$ acting at the positions x_k, $k=1,2,\cdots,r$, then Eq. (3.12) should be expressed as

$$m_{ij} = \int_0^L m\phi_i(x)\phi_j(x)dx + \sum_{k=1}^{r} M_k(x_k)\phi_i(x_k)\phi_j(x_k). \tag{3.15}$$

The work done by external forces acting at the generalized coordinate q_i is

$$W = \sum_{i=1}^{n} [\sum_{j=1}^{p} \{F_j(x_j)\phi_i(x_j)\} + \int_0^L f(x,t)\phi_i(x)dx]q_i \tag{3.16}$$

where $F_j(x_j)$ is the concentrated forces acting at positions x_j, $j=1,2,\cdots,p$.

Let $N(t)=(\alpha+\beta\cos\theta t)N_0$, then Eq. (3.14) becomes

$$s'_{ij} = (\alpha+\beta\cos\theta t)s_{ij} \tag{3.17}$$

where

$$s_{ij} = \int_0^L N_0 \phi_i'(x) \phi_j'(x) dx.$$

By substituting Eqs. (3.9, (3.10), (3.11), and (3.17) into Eq. (3.1) and by performing the operation shown in Eq. (3.1), the following governing differential equations of motion can be obtained:

$$[m_{ij}]\{\ddot{q}\} + [k_{ij}]\{q\} - (\alpha+\beta\cos\theta t)[s_{ij}]\{q\} = \{f\} \qquad (3.18)$$

in which the matrices $[m_{ij}]$, $[k_{ij}]$, and $[s_{ij}]$ are the matrices of mass, stiffness, and stability defined in Eqs. (3.12), (3.13), and (3.17), respectively. The term $\{f\}$ is the vector of equivalent generalized external forces. All the elements in $[m_{ij}]$, $[k_{ij}]$, and $[s_{ij}]$ are derived in the next section.

For a structural system, the member matrices are assembled together by using the equilibrium and continuity conditions at nodal points and are discussed in Section C. Similar to Eq. (3.18), the system matrix may be written as

$$[M]\{\ddot{X}\} + [K]\{X\} - (\alpha+\beta\cos\theta t)[S]\{X\} = \{F\} \qquad (3.19)$$

in which $\{X\}$ represents global coordinates; $[M]$, $[K]$, and $[S]$ are the matrices of total structural mass, stiffness, and

stability, respectively, and may be formulated through the procedure of displacement method. Eq. (3.19) is the governing differential equation of motion to be used in this study of the dynamic instability and dynamic response.

B. Derivation of Members Mass, Stiffness, Stability Matrices

For the displacement method, it is generally preferable to formulate the mass matrix, stiffness matrix, and stability matrix of a typical member on the basis of a set of defined local coordinates; then, the system matrices can be formulated by transfering local coordinates to global coordinates by using equilibrium and compatibility conditions.

Let us consider a typical bar shown in Fig. 3.3 in which q_i (i=1,2,3,4) are local coordinates in the positive direction, and Q_i (i=1,2,3,4) are positive local generalized forces corresponding to q_i. The compressive axial force , N(t), is considered to be positive. The displacements, q_i, are due to the application of the generalized forces Q_i. The displacement y(x,t) of the beam section at point , x, and time, t, may be written as

$$y(x,t) = \sum_{i=1}^{4} q_i(t)\phi_i(x). \qquad (3.20)$$

If bending deformation is considered only, then the differential equation of beam deflection is $\phi''''(x)=0$ for which

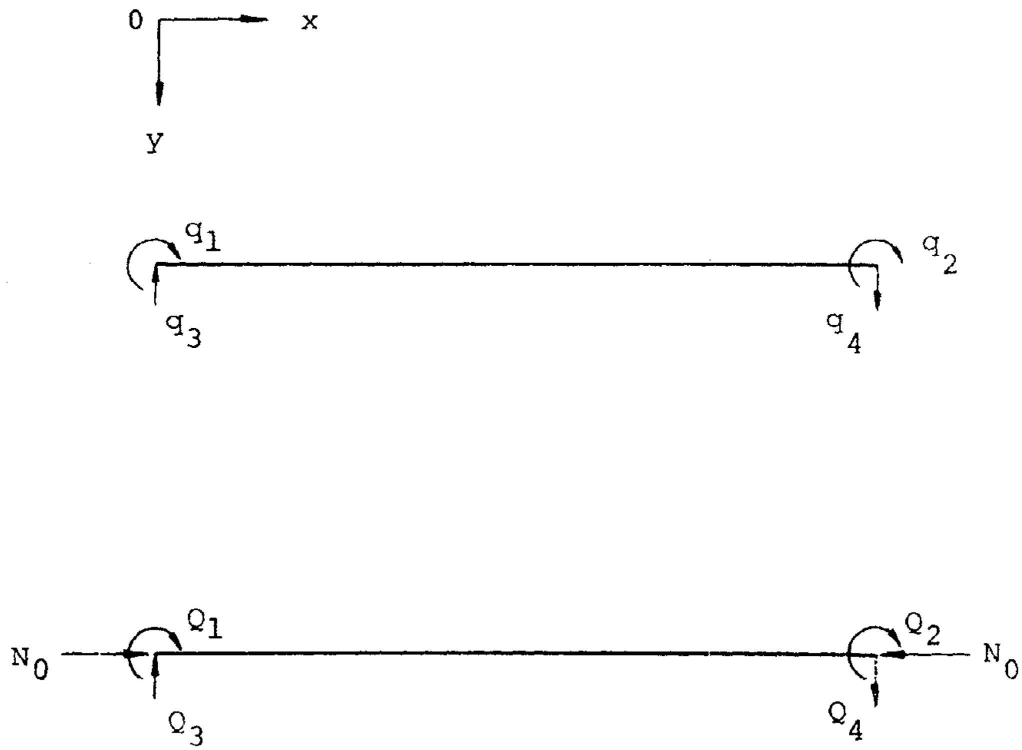


Fig. 3.3 Generalized Local Coordinates and Generalized Forces for a Typical Beam

the solution may be expressed in cubic polynomials

$$\phi(x) = a_1 + a_2x + a_3x^2 + a_4x^3.$$

This is the shape function in Eq. (3.20). Let the coordinates, q_i, in Fig. 3.3 be displaced, one at each time, for a unit displacement; then $\phi(x)$ becomes

$$\phi_1(x) = (x-2x^2/L+x^3/L^2) \tag{3.21}$$

$$\phi_2(x) = (x^3/L^2-x^2/L) \tag{3.22}$$

$$\phi_3(x) = (-1+3x^2/L^2-2x^3/L^3) \tag{3.23}$$

$$\phi_4(x) = (3x^2/L^2-2x^3/L^3). \tag{3.24}$$

Substituting Eqs. (3.21 to 3.24) into Eqs. (3.12 to 3.14) and performing the integration over the bar length, we can obtain $[m_{ij}]$, $[k_{ij}]$, and $[s_{ij}]$ as follows:

$$\begin{Bmatrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \end{Bmatrix}_m = \underbrace{\begin{bmatrix} \frac{4mL^3}{420} & \frac{-3mL^3}{420} & \frac{-22mL^2}{420} & \frac{13mL^2}{420} \\ \frac{-3mL^3}{420} & \frac{4mL^3}{420} & \frac{13mL^2}{420} & \frac{-22mL^2}{420} \\ \frac{-22mL^2}{420} & \frac{13mL^2}{420} & \frac{156mL}{420} & \frac{-54mL}{420} \\ \frac{13mL^2}{420} & \frac{-22mL^2}{420} & \frac{-54mL}{420} & \frac{156mL}{420} \end{bmatrix}}_{[m_{ij}]} \begin{Bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \\ \ddot{q}_3 \\ \ddot{q}_4 \end{Bmatrix} \tag{3.25}$$

$$\begin{Bmatrix} Q_1 \\ \\ Q_2 \\ \\ Q_3 \\ \\ Q_4 \end{Bmatrix}_k = \underbrace{\begin{bmatrix} \frac{4EI}{L} & \frac{2EI}{L} & \frac{-6EI}{L^2} & \frac{-6EI}{L^2} \\ \\ \frac{2EI}{L} & \frac{4EI}{L} & \frac{-6EI}{L^2} & \frac{-6EI}{L^2} \\ \\ \frac{-6EI}{L^2} & \frac{-6EI}{L^2} & \frac{12EI}{L^3} & \frac{12EI}{L^3} \\ \\ \frac{-6EI}{L^2} & \frac{-6EI}{L^2} & \frac{12EI}{L^3} & \frac{12EI}{L^3} \end{bmatrix}}_{[k_{ij}]} \begin{Bmatrix} q_1 \\ \\ q_2 \\ \\ q_3 \\ \\ q_4 \end{Bmatrix} \tag{3.26}$$

$$\begin{Bmatrix} Q_1 \\ \\ Q_2 \\ \\ Q_3 \\ \\ Q_4 \end{Bmatrix}_p = N_0 \underbrace{\begin{bmatrix} \frac{2L}{15} & \frac{-L}{30} & \frac{-1}{10} & \frac{-1}{10} \\ \\ \frac{-L}{30} & \frac{2L}{15} & \frac{-1}{10} & \frac{-1}{10} \\ \\ \frac{-1}{10} & \frac{-1}{10} & \frac{6}{5L} & \frac{6}{5L} \\ \\ \frac{-1}{10} & \frac{-1}{10} & \frac{6}{5L} & \frac{6}{5L} \end{bmatrix}}_{[s_{ij}]} \begin{Bmatrix} q_1 \\ \\ q_2 \\ \\ q_3 \\ \\ q_4 \end{Bmatrix} . \tag{3.27}$$

Note that Q_1, Q_2 and Q_3, Q_4 correspond to moments and shears, respectively; q_1, q_2 and q_3, q_4 correspond to

rotations and displacements, respectively; and $\ddot{q}_1$, $\ddot{q}_2$ and $\ddot{q}_3$, $\ddot{q}_4$ are accelerations due to rotations and displacements, respectively. For convenience, let us rewrite Eqs. (3.25, 3.26, 3.27) in the following condensed forms:

$$\left\{\begin{array}{c} Q_m \\ -- \\ Q_v \end{array}\right\}_m = \left[\begin{array}{c|c} [MMR] & [MMY] \\ \hline [MVR] & [MVY] \end{array}\right]\left\{\begin{array}{c} \ddot{q}_r \\ -- \\ \ddot{q}_s \end{array}\right\} \tag{3.28}$$

$$\left\{\begin{array}{c} Q_m \\ -- \\ Q_v \end{array}\right\}_k = \left[\begin{array}{c|c} [KMR] & [KMY] \\ \hline [KVR] & [KVY] \end{array}\right]\left\{\begin{array}{c} q_r \\ -- \\ q_s \end{array}\right\} \tag{3.29}$$

$$\left\{\begin{array}{c} Q_m \\ -- \\ Q_v \end{array}\right\}_p = \left[\begin{array}{c|c} [SMR] & [SMY] \\ \hline [SVR] & [SVY] \end{array}\right]\left\{\begin{array}{c} q_r \\ -- \\ q_s \end{array}\right\} \tag{3.30}$$

in which the subscripts m, k, and p signify that the moments $\{Q_m\}$ and shears $\{Q_v\}$ are associated with $[m_{ij}]$, $[k_{ij}]$, and $[s_{ij}]$, respectively. The subscripts r and s signify the joint rotations and displacements, respectively.

C. System Matrices of Mass, Stiffness, and Stability

The displacement method of formulating structural

system matrix has been well documented (31,32). Following Cheng's recent work (13), one can rewrite the relationship between the generalized external forces, {F}, and generalized external displacement, {X}, as

$$\left\{\begin{array}{c} F_r \\ \hline F_s \end{array}\right\} = \left(\left[\begin{array}{c|c} [A_m][MMR][A_m]^T & [A_m][MMY][A_v]^T \\ \hline [A_v][MVR][A_m]^T & [A_v][MVY][A_v]^T \end{array}\right] + \left[\begin{array}{c|c} 0 & 0 \\ \hline 0 & [A_{ms}] \end{array}\right] \right) \left\{\begin{array}{c} \ddot{X}_r \\ \hline \ddot{X}_s \end{array}\right\}$$

$$+ \left[\begin{array}{c|c} [A_m][KMR][A_m]^T & [A_m][KMY][A_v]^T \\ \hline [A_v][KVR][A_m]^T & [A_v][KVY][A_v]^T \end{array}\right] \left\{\begin{array}{c} X_r \\ \hline X_s \end{array}\right\}$$

$$- \left[\begin{array}{c|c} [A_m][SMR][A_m]^T & [A_m][SMY][A_v]^T \\ \hline [A_v][SVR][A_m]^T & [A_v][SVY][A_v]^T \end{array}\right] \left\{\begin{array}{c} X_r \\ \hline X_s \end{array}\right\} . \qquad (3.31)$$

Knowing $\{F_r\}$ and $\{F_s\}$, one can find $\{X_r\}$, $\{X_s\}$, $\{\ddot{X}_r\}$, and $\{\ddot{X}_s\}$ from Eq. (3.31) by using the numerical integration presented in Chapter V. Consequently, the member end moments and end shears can be obtained as follows:

$$\left\{\begin{array}{c} Q_m \\ \hline Q_v \end{array}\right\} = \left[\begin{array}{c|c} [MMR]\,[A_m]^T & [MMY]\,[A_v]^T \\ \hline [MVR]\,[A_m]^T & [MVR]\,[A_v]^T \end{array}\right] \left\{\begin{array}{c} \ddot{X}_r \\ \hline \ddot{X}_s \end{array}\right\}$$

$$+ \left[\begin{array}{c|c} [KMR]\,[A_m]^T & [KMY]\,[A_v]^T \\ \hline [KVR]\,[A_m]^T & [KVY]\,[A_v]^T \end{array}\right] \left\{\begin{array}{c} X_r \\ \hline X_s \end{array}\right\}$$

$$- \left[\begin{array}{c|c} [SMR]\,[A_m]^T & [SMY]\,[A_v]^T \\ \hline [SVR]\,[A_m]^T & [SVY]\,[A_v]^T \end{array}\right] \left\{\begin{array}{c} X_r \\ \hline X_s \end{array}\right\} \qquad (3.32)$$

in which

$[A_m]$ = equilibrium matrix relating internal moments to external nodal moments;

$[A_v]$ = equilibrium matrix relating internal shears to external nodal forces;

$[F_r]$ = external nodal moments;

$[F_s]$ = external nodal forces;

$[X_r]$ = global rotations;

$[X_s]$ = global displacements;

$[\ddot{X}_r]$ = acceleration due to global rotations;

$[\ddot{X}_s]$ = acceleration due to global displacement;

$[A_{ms}]$ = diagonal matrix involves the inertial forces due to joint displacements; and

T = transpose of matrix.

Eqs. (3.31, 3.32) have been explained in detail in SUBROUTINE ASATA, ASATB, SATMV shown in the Appendix.

D. Shear Building Subjected to Lateral Forces

In many practical cases, the girder stiffnesses compared with those of columns are sufficiently large. Consequently, the structural joint rotations are very small and only the sway displacements are significant. By neglecting the global coordinates corresponding to the structural joint rotations, one can rewrite Eq. (3.31) as

$$[M]\{\ddot{X}_s\} + [K]\{X_s\} - [S]\{X_s\} = \{F_s\} \tag{3.33a}$$

where

$$[M] = [A_V][MVY][A_V]^T + [A_{ms}]$$

$$[K] = [A_V][KVY][A_V]^T$$

$$[S] = [A_V][SVY][A_V]^T .$$

When the axial load is $N(t)=(\alpha+\beta\cos\theta t)N_0$, then Eq. (3.33a) becomes

$$[M]\{\ddot{X}_s\} + [K]\{X_s\} - (\alpha+\beta\cos\theta t)[S]\{X_s\} = \{F_s\} . \tag{3.33b}$$

IV. STATIC AND DYNAMIC STABILITY

A. Boundary of Dynamic Instability

When a structural framework is subjected to a transverse pulsating load, the framework will generally experience forced vibration with a certain frequency of the excitation. The amplitude of the vibration becomes larger and larger when the forcing frequency approaches the natural frequency of the vibrating system. The behavior is called resonance. However, when the frame is subjected to a pulsating axial load as shown in Eq. (3.19), an entirely different type of resonance is observed. The resonance occurs when a certain relationship exists between the natural frequency, the frequency of longitudinal forces and their magnitude. This resonance is called parametric resonance. The behavior of parametric resonance may be studied by using the governing differential equations of motion, Eq. (3.19).

Let us consider the time dependent axial forces only, then Eq. (3.19) becomes

$$[M]\{\ddot{X}\} + [[K] - (\alpha+\beta\cos\theta t)[S]]\{X\} = 0 \tag{4.1}$$

which represents a system of second-order differential equations with the periodic coefficient of the known Mathieu-Hill type. It has been observed that the Mathieu-Hill equation similar to Eq. (4.1) has periodic solutions with periods

T and 2T ($T=2\pi/\theta$) at the boundaries of the instability region (2). The regions of instability may be determined by finding the periodic solutions of Eq. (4.1) in the form of a trigonometric series. The instability regions are bounded by two solutions with the same period. The stability regions are bounded by two solutions with different periods. The critical values of parameters α, β, and θ contained in Eq. (4.1) are obtained from the condition that Eq. (4.1) has periodic solutions. The stability or instability solutions of Eq. (4.1) correspond to the stability or instability of the structural system. The above-mentioned statement can be illustrated by the following derivation.

For the solution with period 2T, let the trial solution be in the form of a series

$$\{X\} = \sum_{k=1,3,5,\cdots}^{\infty} (A_k \sin\frac{k\theta t}{2} + B_k \cos\frac{k\theta t}{2}) \tag{4.2}$$

in which A_k and B_k are vectors independent of time. By substituting Eq. (4.2) into Eq. (4.1), the following system of matrix equations can be obtain by comparing the coefficients of $\sin\frac{k\theta t}{2}$ and $\cos\frac{k\theta t}{2}$:

$$([K] - (\alpha - \tfrac{1}{2}\beta)[S] - \tfrac{1}{4}\theta^2[M])A_1 - \tfrac{1}{2}\beta[S]A_3 = 0$$

$$([K] - \alpha[S] - \tfrac{1}{4}k^2\theta^2[M])A_k - \tfrac{1}{2}\beta[S](A_{k-2}+A_{k+2}) = 0$$

$$(k = 3,5,7,\cdots\cdots),$$

$$([K] - (\alpha+\tfrac{1}{2}\beta)[S] - \tfrac{1}{4}\theta^2[M])B_1 - \tfrac{1}{2}\beta[S]B_3 = 0$$

$$([K] - \alpha[S] - \tfrac{1}{4}k^2\theta^2[M])B_k - \tfrac{1}{2}\beta[S](B_{k-2}+B_{k+2}) = 0$$

$$(k = 3,5,7,\cdots\cdots).$$

Solutions having the period $2T=4\pi/\theta$ can occur if the following conditions are satisfied:

$$\begin{vmatrix} [K]-(\alpha\pm\tfrac{1}{2}\beta)[S]-\dfrac{\theta^2}{4}[M] & -\tfrac{1}{2}\beta[S] & 0 & \cdots \\ -\tfrac{1}{2}\beta[S] & [K]-\alpha[S]\dfrac{9}{4}\theta^2[M] & -\tfrac{1}{2}\beta[S] & \cdots \\ 0 & -\tfrac{1}{2}\beta[S] & [K]-\alpha[S]-\dfrac{25}{4}\theta^2[M] & \cdots \\ \cdots\cdot & \cdots\cdot & \cdots\cdot & \cdots \end{vmatrix} = 0$$

(4.3)

Similarly, for the solution with period T, let the trial solution be represented by

$$\{X\} = \tfrac{1}{2}B_0 + \sum_{k=2,4,6,\cdots}^{\infty} (A_k\sin\frac{k\theta t}{2}+B_k\cos\frac{k\theta t}{2}). \qquad (4.4)$$

Substituting Eq. (4.4) into Eq. (4.1) yields Eqs. (4.5) and (4.6) for the solution having the period $T=2\pi/\theta$.

For finding the regions of instability as sketched in Fig. 4.1, one may solve Eqs. (4.3), (4.5), and (4.6) for the

critical values of the parameters (α, β, N_0, θ). The first region of instability (Region A) is determined from Eq. (4.3). Similarly, the second region of instability (Region C) is determined from Eqs. (4.5) and (4.6). The stability region (Region B) is confined by Region A and Region C.

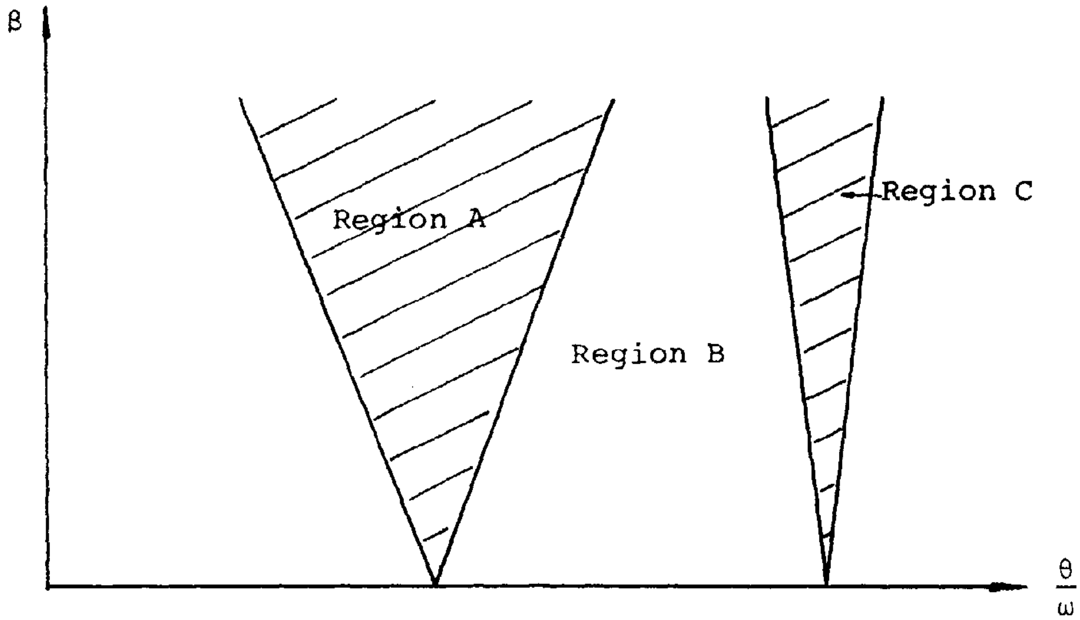


Fig. 4.1 Instability Region

$$\begin{vmatrix} [K]-\alpha[S]-\theta^2[M] & -\frac{1}{2}\beta[S] & 0 & \cdots \\ -\frac{1}{2}\beta[S] & [K]-\alpha[S]-4\theta^2[M] & -\frac{1}{2}\beta[S] & \cdots \\ 0 & -\frac{1}{2}\beta[S] & [K]-\alpha[S]-16\theta^2[M] & \cdots \\ \cdots\cdots & \cdots\cdots & \cdots\cdots & \cdots \end{vmatrix} = 0 \tag{4.5}$$

$$\begin{vmatrix} [K]-\alpha[S] & -\beta[S] & 0 & \cdots \\ -\frac{1}{2}\beta[S] & [K]-\alpha[S]-\theta^2[M] & -\frac{1}{2}\beta[S] & \cdots \\ 0 & -\frac{1}{2}\beta[S] & [K]-\alpha[S]-4\theta^2[M] & \cdots \\ 0 & 0 & -\frac{1}{2}\beta[S] & \cdots \\ \cdots\cdots & \cdots\cdots & \cdots\cdots & \cdots \end{vmatrix} = 0 \qquad (4.6)$$

In practice, only the finite number of terms in the determinant is used for studying the principal instability regions. Thus when the first term of the series of Eq. (4.2) is considered (i.e., $k=1$, $\{X\}=A_1\sin(\theta t/2)+B_1\cos(\theta t/2)$), one may have

$$\left|[K] - (\alpha\pm\tfrac{1}{2}\beta)[S] - \frac{\theta^2}{4}[M]\right| = 0 \qquad (4.7)$$

which corresponds to the first matrix element along the diagonal of Eq. (4.3). The solution of Eq. (4.7) gives the principal regions of dynamic instability. Similarly, from Eqs. (4.5) and (4.6), we may have

$$\left|[K] - \alpha[S] - \theta^2[M]\right| = 0$$

and

$$\begin{vmatrix} [K]-\alpha[S] & -\beta[S] \\ & \\ -\frac{1}{2}\beta[S] & [K]-[S]-\theta^2[M] \end{vmatrix} = 0$$

which give the secondary region (Region C of Fig. 4.1) of dynamic instability. Note that Eq. (4.7) is an eigenvalue equation which can be solved by the conventional method of expanding the determinant equation, Eq. (4.7), into a polynomial equation for the eigenvalue and its associated eigenvector. For this research of studying large structural systems, a different technique of matrix iteration has been used by utilizing computer facilities (32).

B. Static Buckling Load and Natural Frequency

It may be observed from Eq. (4.7) that an instability region is confined by the axial load and the ratio of axial forcing frequency to the natural frequency. In order to ensure that the amount of axial load to be applied is not greater than the elastic buckling capacity of the system, it is essential to express the applied load in terms of buckling load N_0, as αN_0 and βN_0. α and β are fractional numbers less than one. In this section, the techniques of finding static buckling load and natural frequency are discussed.

By observing Eq. (4.1), one may obtain the three groups of eigenvalue problems classified as (a), (b), and (c) that are shown below:

(a). For static buckling case when $\{\ddot{X}\}=0$, $\beta\cos\theta t=0$, then Eq. (4.1) becomes

$$([K] - \alpha[S])\{X\} = 0 \quad \text{or} \quad |[K] - \alpha[S]| = 0 . \tag{4.8}$$

(b). For free vibration of harmonic motions without external axial loads, Eq. (4.1) may be written as

$$[M]\{\ddot{X}\} + [K]\{X\} = 0 . \tag{4.9}$$

Let $\{X\}=\{Ae^{i\omega t}\}$, then Eq. (4.9) becomes

$$|[K] - \omega^2[M]| = 0 \tag{4.10}$$

which gives the natural frequency ω.

(c). For the influence of static axial loads on the natural frequency, one can rewrite Eq. (4.1) as

$$|[K] - \alpha[S] - \omega^2[M]| = 0 \qquad (4\ 11)$$

from which one may observe that the compressive load will decrease the natural frequency, and the tensile force will increase the natural frequency.

Let Eqs. (4.7), (4.8), (4.10), and (4.11) be expressed in a standard eigenvlaue form as

$$\frac{1}{\lambda}\{X\} = [DM]\{X\} \tag{4.12}$$

where [DM] and λ in Eq. (4.7) signify either

$$[DM] = [[K] - (\alpha+\tfrac{1}{2}\beta)[S]]^{-1}[M], \text{ and } \lambda = \theta^2/4 \tag{4.13}$$

or

$$[DM] = [[K] - (\alpha-\tfrac{1}{2}\beta)[S]]^{-1}[M], \text{ and } \lambda = \theta^2/4. \tag{4.14}$$

[DM] and λ in Eq. (4.8) represent

$$[DM] = [K]^{-1}[S], \text{ and } \lambda = \alpha.$$

For Eq. (4.10)

$$[DM] = [K]^{-1}[S], \text{ and } \lambda = \omega^2,$$

and for Eq. (4.11)

$$[DM] = [[K] - \alpha[S]]^{-1}[M], \text{ and } \lambda = \omega^2.$$

The matrix iteration method developed by Cheng (30) has been employed to obtain the eigenvalue λ and its associated eigenvector {X}.

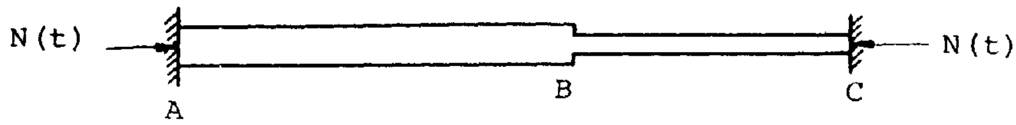
Example 4.1. Consider that the step beam given in Fig. 42a is subjected to an axial force $N(t)=\alpha N_0+\beta N_0\cos\theta t$. The cross

section of segments AB, BC are 8.375"x3.465" and 6.925"x 3.465", respectively. Let $E=30\times10^6$psi, $\gamma=490$ lbs/ft^3, $L_{AB}=144"$, $L_{BC}=96"$. Find the dynamic instability region.

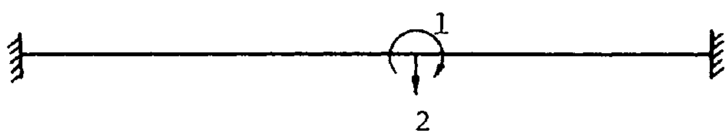
Solution: Using the local coordinates, $\{q\}$, and global coordinates, $\{X\}$, shown in Fig. 4.2b and 4.2c, respectively, one can find the equilibrium matrices $[A_m]$, $[A_v]$ tabulated in Fig. 4.2d and then manipulate Eq. (3.19) for

$$[M]=\begin{bmatrix} \dfrac{4m_{AB}L_{AB}^3+4m_{BC}L_{BC}^3}{420} & \dfrac{-22m_{AB}L_{AB}^2+22m_{BC}L_{BC}^2}{420} \\ \\ \dfrac{-22m_{AB}L_{AB}^2+22m_{BC}L_{BC}^2}{420} & \dfrac{156m_{AB}L_{AB}+156m_{BC}L_{BC}}{420} \end{bmatrix} \tag{4.18}$$

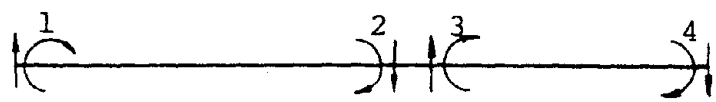
$$[K]=\begin{bmatrix} \dfrac{4EI_{AB}}{L_{AB}}+\dfrac{4EI_{BC}}{L_{BC}} & \dfrac{-6EI_{AB}}{L_{AB}^2}-\dfrac{6EI_{BC}}{L_{BC}^2} \\ \\ \dfrac{-6EI_{AB}}{L_{AB}^2}+\dfrac{6EI_{BC}}{L_{BC}^2} & \dfrac{12EI_{AB}}{L_{AB}^3}-\dfrac{12EI_{BC}}{L_{BC}^3} \end{bmatrix} \tag{4.19}$$



(a) Given Problem



(b) Global Coordinates



(c) Local Coordinates

A_m	P_r \ M	1	2	3	4
		0.	1.	1.	0.

A_v	P_s \ P	1	2	3	4
		0.	1.	-1.	0.

(d) Equilibrium Matrices

Fig. 4.2 Example 4.1

$$[S] = \begin{bmatrix} \dfrac{2L_{AB}}{15} + \dfrac{2L_{BC}}{15} & 0 \\ 0 & \dfrac{6}{5L_{AB}} + \dfrac{6}{5L_{BC}} \end{bmatrix}. \tag{4.20}$$

Thus substituting Eqs. (4.19) and (4.20) into Eq. (4.8) gives the static buckling load N_0=2975. kips. Using Eqs. (4.10) and (4.12) yields the natural frequency ω=28.95 cps. Let α=0., 0.1, 0.2, 0.3, 0.4, 0.5, and β=0., 0.1, 0.2, 0.3, 0.4, 0.5, then one can find various values of θ from Eqs. (4.12), (4.13), (4.14). Expressing θ in terms of θ/ω and then using parameters α and β, one can draw the instability regions shown in Fig. 4.3.(12).

C. Static Buckling Resulting From a Combined Action of Distributed and Concentrated Axial Forces

In the previous section, the static buckling load was assumed to be acting at the structural joints as a concentrated force. However, there are many cases where the longitudinal forces are distributed along the members. Typical examples may be the self-weight of chimneys, the self-weight of slender tall buildings, and the weight of walls attached to columns. The stability matrix for the above mentioned type of structures is different from that in Eq. (3.27).

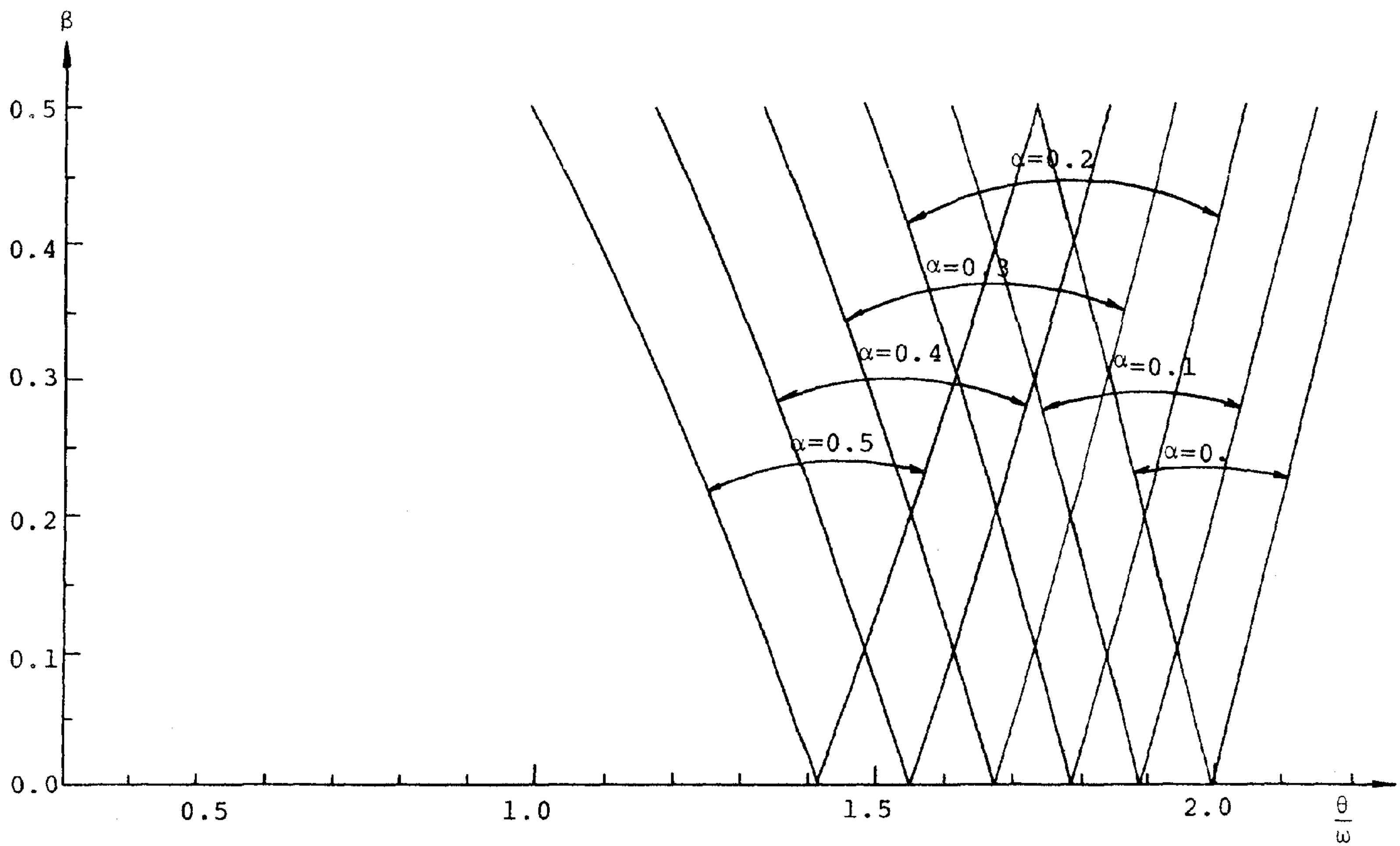


Fig. 4.3 Dynamic Instability Region

It is well known that if a longitudinal compressive force is continuously distributed along a bar, the classical mathematical formulation becomes very sophisticated, because the differential equation of the deflection curve of the buckled bar will no longer be an equation with constant coefficients. Consequently, the direct integration of the equation can only be applied to simple bars, such as cantilever columns. It is the purpose of this section to present the stability matrix due to a combined action of distributed and concentrated axial forces.

1. Formulation of Stability Matrix

Consider the beam of Fig. 4.4a subjected to a concentrated axial force, N, and a uniformly distributed axial load, q. The generalized coordinates, q_i, and generalized forces, Q_i, are shown in Fig. 4.4b and c, respectively. Let N, q, Q_i, q_i are positive as shown, the displacement y(x) of the beam at point x due to q_i and Q_i may be expressed as

$$y(x) = \sum_{i=1}^{4} q_i \phi_i(x). \tag{4.21}$$

For bending deformation only, the shape functions $\phi(x)$ of Eq. (4.21) are the same as Eqs. (3.21, 3.22, 3.23, 3.24) shown below:

$$\left.\begin{aligned}
\phi_1(x) &= (x - 2x^2/L + x^3/L^2) \\
\phi_2(x) &= (x^3/L^2 - x^2/L) \\
\phi_3(x) &= (-1 + 3x^2/L^2 - 2x^3/L^3) \\
\phi_4(x) &= (3x^2/L^2 - 2x^3/L^3)
\end{aligned}\right\} \quad (4.22)$$

The total potential energy, V, due to N and q is given by

$$V = V_N + V_q$$

where V_N is the virtual work done by the axial force N on displacement Δ, and V_q is the virtual work done by the uniformly distributed axial load, q, on displacement Δ; where Δ is the displacement resulting from the displacements q_i. For an element dx shown in Fig. 4.4d one may have

$$d\Delta = ds - dx \quad (4.23)$$

$$ds = dx\{1 + (dy/dx)^2\}^{\frac{1}{2}} \quad (4.24)$$

for small deflection, Eq. (4.24) becomes

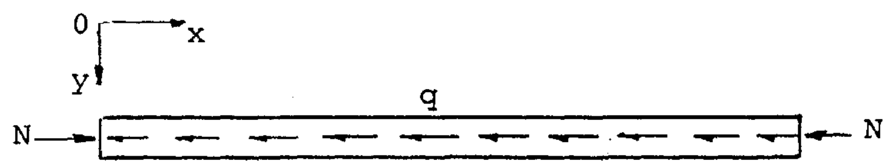
$$ds = dx\{1 + \tfrac{1}{2}(dy/dx)^2\} \quad (4.25)$$

Substituting Eq. (4.25) into Eq. (4.23) and then integrating over the length , one can obtain

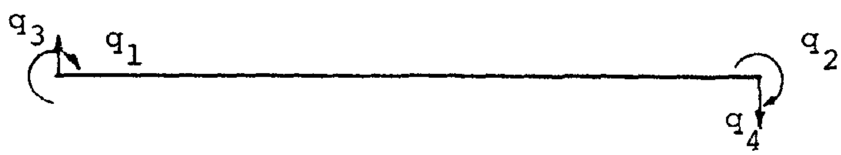
$$\Delta = \tfrac{1}{2}\int_0^L (dy/dx)^2 dx \ .$$

We can now write the work V_N as

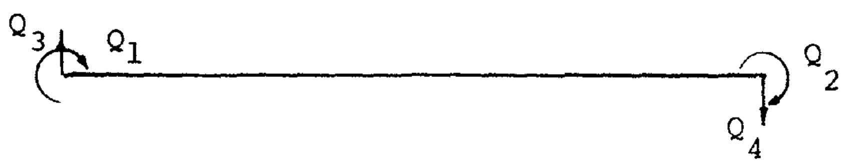
$$V_N = N\Delta = \tfrac{1}{2}N\int_0^L (dy/dx)^2 dx \tag{4.26}$$



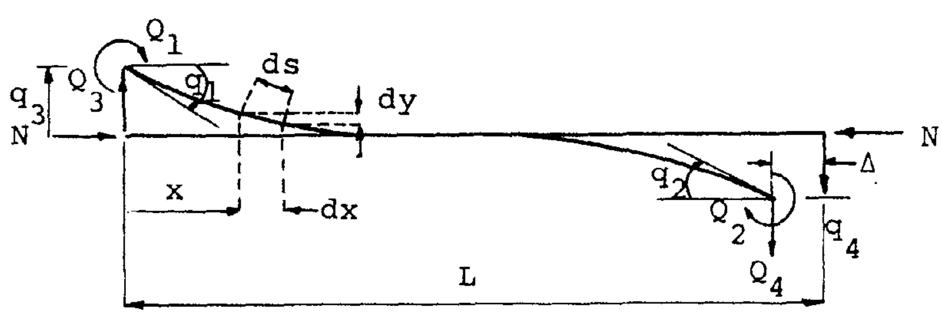
(a) Typical Bar



(b) Local Generalized Coordinates



(c) Local Generalized Forces



(d) Force-Deformation Relationship

Fig. 4.4 Typical Bar Subjected to Concentrated Axial Load N and Uniformly Distributed Load q

From Fig. 4.4d, $d\Delta = ds - dx = \frac{1}{2}(dy/dx)^2 dx$, the work done by the load acting on the right side of x on $d\Delta$ is

$$dV_q = (L-x)d\Delta = q(L-x)\{\tfrac{1}{2}(dy/dx)^2\}dx$$

Therefore, the total work produced by the distributed load over the length is

$$V_q = \int_0^L dV_q = \tfrac{1}{2}\int_0^L q(L-x)(dy/dx)^2 dx. \tag{4.27}$$

The strain energy is

$$U = \tfrac{1}{2}\int_0^L EI\{y''(x)\}^2 dx. \tag{4.28}$$

The virtual work done by forces Q_i on q_i may be written as

$$W = \sum_i Q_i q_i. \tag{4.29}$$

By Lagrange's equation,

$$\partial U/\partial q_i - \partial V/\partial q_i = \partial W/\partial q_i \tag{4.30}$$

upon which the substitution of Eqs. (4.26), (4.27), (4.28), (4.29) leads

$$\{\nabla U\} - \{\nabla V_N\} - \{\nabla V_q\} = \{\nabla W\}. \tag{4.31}$$

From Eq. (4.21),

$$y'(x) = \sum_i q_i \phi_i'(x) \tag{4.32}$$

$$y''(x) = \sum_i q_i \phi_i''(x). \tag{4.33}$$

Thus substituting Eqs (4.32), (4.33) into Eqs (4.26), (4.27) and (4.28), respectively, gives

$$U = \tfrac{1}{2}\sum_i \sum_j k_{ij} q_i q_j = \tfrac{1}{2}\{q\}^T [k_{ij}]\{q\} \tag{4.34}$$

$$V_N = \tfrac{1}{2}\sum_i \sum_j s_{ij} q_i q_j = \tfrac{1}{2}\{q\}^T [s_{ij}]\{q\} \tag{4.35}$$

$$V_q = \tfrac{1}{2}\sum_i \sum_j g_{ij} q_i q_j = \tfrac{1}{2}\{q\}^T [g_{ij}]\{q\} \tag{4.36}$$

in which

$$k_{ij} = \int_0^L EI\phi_i''(x)\phi_j''(x)\,dx$$

$$s_{ij} = \int_0^L N\phi_i'(x)\phi_j'(x)\,dx$$

$$g_{ij} = \int_0^L q(L-x)\phi_i'(x)\phi_j'(x)\,dx.$$

The substitution of Eqs. (4.34), (4.35), and (4.36) into Eq. (4.30) yields the results of Eq. (4.31) as

$$\begin{aligned}\{\nabla U\} &= [k_{ij}]\{q\} \\ \{\nabla V_N\} &= [s_{ij}]\{q\} \\ \{\nabla V_g\} &= [g_{ij}]\{q\} \\ \{\nabla W\} &= \{Q\}\,.\end{aligned} \tag{4.37}$$

Therefore Eq. (4.31) may be rewritten as

$$[k_{ij}]\{q\} - [s_{ij}]\{q\} - [g_{ij}]\{q\} = \{Q\} \tag{4.38}$$

in which $[k_{ij}]$ and $[s_{ij}]$ are exactly the same as Eqs. (3.26) and (3.27). The term $[g_{ij}]$ is the stability matrix due to a uniformly distributed axial load, and can be expressed as follows

$$\begin{Bmatrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \end{Bmatrix} = \underbrace{\begin{bmatrix} \frac{6qL^2}{60} & \frac{-qL^2}{60} & 0 & 0 \\ \frac{-qL^2}{60} & \frac{2qL^2}{60} & \frac{-qL}{10} & \frac{-qL}{10} \\ 0 & \frac{-qL}{10} & \frac{3q}{5} & \frac{3q}{5} \\ 0 & \frac{-qL}{10} & \frac{3q}{5} & \frac{3q}{5} \end{bmatrix}}_{[g_{ij}]} \begin{Bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{Bmatrix} . \tag{4.39}$$

Through the displacement method discussed in Section C

of Chapter III, one can calculate the buckling load of a structure subjected to a simultaneous action of concentrated axial force N and distributed axial load,q. The following examples are selected for the comparison of the numerical solution obtained by the present method with Timoshenko's rigorous mathematical approach (34).

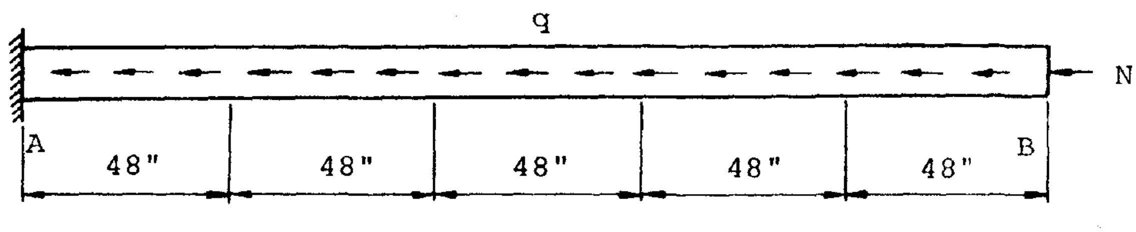
2. Numerical Examples

Example 4.2. Consider the uniform cantilever column shown in Fig. 4.5a with a concentrated axial force , N, acting at end, B, and a uniform load, q, acting along the axis. Find either the critical load, q_{cr}, or the critical load, N_{cr}. Let the member length L=240 in., the uniform cross section $A=24\ in.^2$, $I=96\ in.^4$, and $E=30x10^6$ psi.

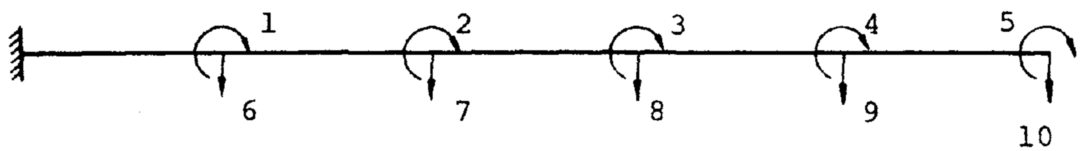
Solution: Let the column be divided into five segments as shown in Fig. 4.5a. The global coordinates and local coordinates are shown in Fig. 4.5b and 4.5c, respectively, from which the equilibrium matrices $[A_m]$ and $[A_v]$ are established as follows:

$[A_m]$ =

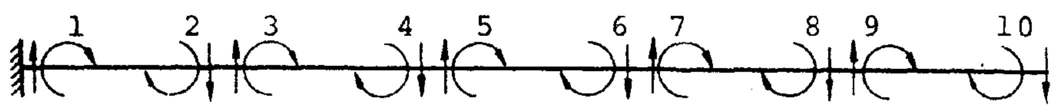
P_r \ M	1	2	3	4	5	6	7	8	9	10
1	0	1	1	0	0	0	0	0	0	0
2	0	0	0	1	1	0	0	0	0	0
3	0	0	0	0	0	1	1	0	0	0
4	0	0	0	0	0	0	0	1	1	1
5	0	0	0	0	0	0	0	0	0	0



(a) Loading



(b) Global Coordinates



(c) Local Coordinates

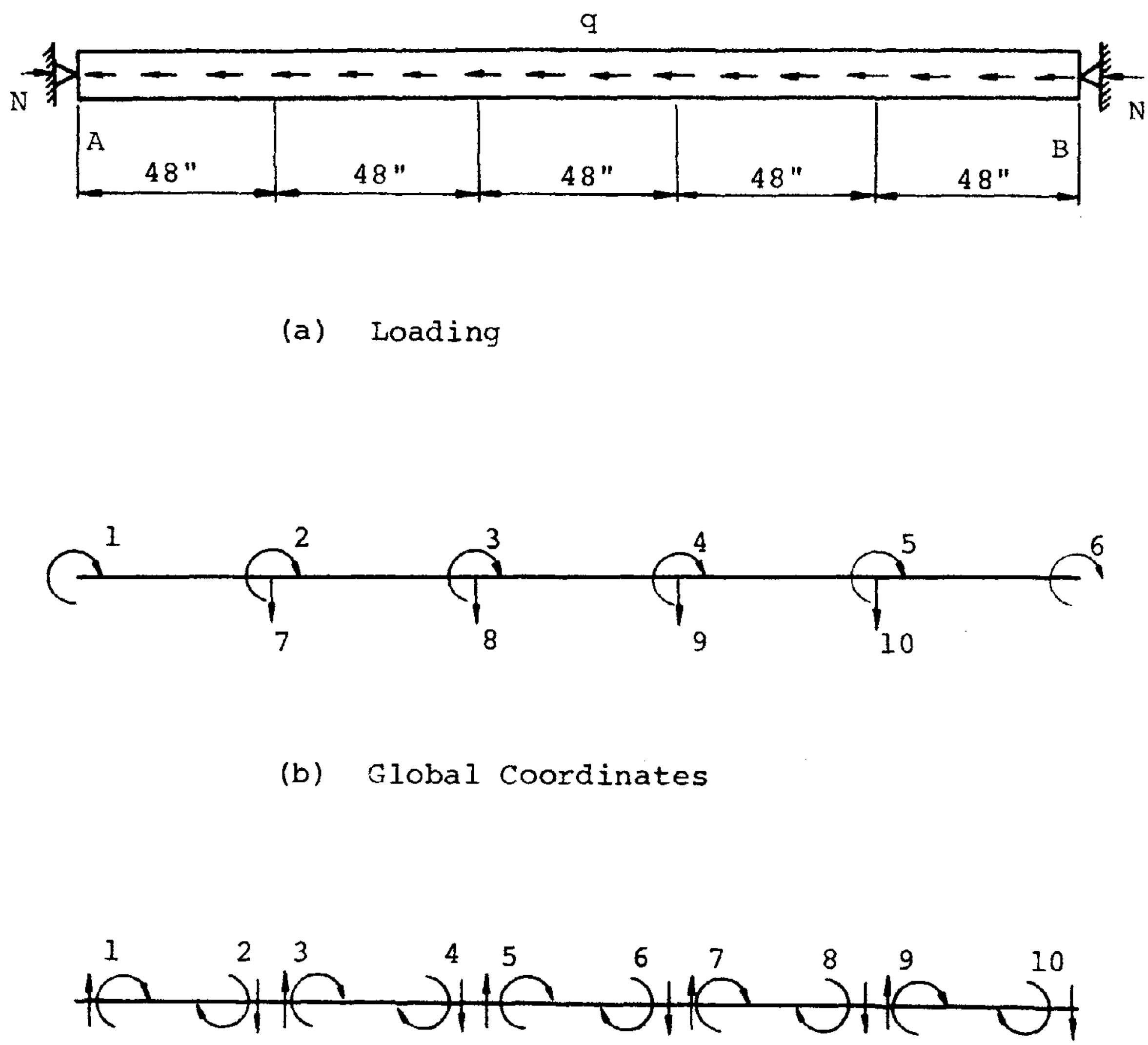
Fig. 4.5 Example 4.2

$$[A_v] = \begin{array}{c|cccccccccc} P_s \backslash P & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ \hline 1 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \\ 3 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 \\ 4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 \\ 5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{array}$$

The eigenvalue equation of this problem is similar to Eq. (4.8) with the inclusion of $[g_{ij}]$. Using the digital computer program based on the matrix iteration method (32) yields the solutions shown in Tables I and II in which the comparison of the present solution with Timoshenko's solution is very satisfactory.

Example 4.3. Consider the simply supported uniform beam shown in Fig. 4.6a with a concentrated axial force, N, acting at both ends, A and B, and a uniform load, q, acting along the axis. Find the critical load, q_{cr}, for given N and critical load, N_{cr}, for given q. Let L=240 in., A=30.2376 in.^{2}, I=192 in.^{4}, and E=30x10^6 psi.

Solution: Let the beam be divided into five segments as shown in Fig. 4.6a. The generalized global coordinates and generalized local coordinates are shown in Figs. 4.6b and 4.6c, respectively, from which the equilibrium matrices $[A_m]$ and $[A_v]$ are established as follows:



(a) Loading

(b) Global Coordinates

(c) Local Coordinates

Fig. 4.6 Example 4.3

$[A_m]$ =

P_r \ M	1	2	3	4	5	6	7	8	9	10
1	1	0	0	0	0	0	0	0	0	0
2	0	1	1	0	0	0	0	0	0	0
3	0	0	0	1	1	0	0	0	0	0
4	0	0	0	0	0	1	1	0	0	0
5	0	0	0	0	0	0	0	1	1	0
6	0	0	0	0	0	0	0	0	0	1

$[A_v]$ =

P_s \ P	1	2	3	4	5	6	7	8	9	10
1	0	1	-1	0	0	0	0	0	0	0
2	0	0	0	1	-1	0	0	0	0	0
3	0	0	0	0	0	1	-1	0	0	0
4	0	0	0	0	0	0	0	1	-1	0

Similar to Example 4.2, the solutions obtained by using the computer program are shown in Tables III and IV in which a very good comparison between the present solution with Timoshenko's solution is shown.

Table I Buckling Load q_{cr} with N Given of Example 4.2

Timoshenko (34)				Present Method
b	$N=bEI/L^2$ (lbs)	a	$q_{cr}=a\pi^2EI/L^3$ (lbs/in.)	q_{cr} (lbs/in.)
$\pi^2/4.$	123,370.00	0.00	0.00	0.00
2.28	114,000.00	0.25	128.51	130.99
2.08	104,000.00	0.50	257.02	269.06
1.91	95,500.00	0.75	385.53	385.42
1.72	86,000.00	1.00	514.04	514.70
0.96	48,000.00	2.00	1,028.08	1,019.80
0.15	7,500.00	3.00	1,542.13	1,538.97
0.00	0.00	3.18	1,634.66	1,632.92

Table II Buckling Load N_{cr} with q Given of Example 4.2

	Timoshenko (34)			Present Method
a	$q=a\pi^2EI/L^3$ (lbs/in.)	b	$N_{cr}=bEI/L^2$ (lbs)	N_{cr} (lbs)
0.00	0.00	$\pi^2/4.$	123,370.00	123,372.92
0.25	128.51	2.28	114,000.00	114,184.90
0.50	257.02	2.08	104,000.00	104,898.80
0.75	385.53	1.91	95,500.00	95,499.12
1.00	514.04	1.72	86,000.00	86,064.87
2.00	1,028.08	0.96	48,000.00	47,358.83
3.00	1,542.13	0.15	7,500.00	7,276.96
3.18	1,634.66	0.00	0.00	0.00

Table III Buckling Load q_{cr} with N Given of Example 4.3

	Timoshenko (34)			Present Method
b	$N=bEI/L^2$ (lbs)	a	$q_{cr}=a\pi^2 EI/L^3$ (lbs/in.)	q_{cr} (lbs/in.)
π^2	986,965.00	0.00	0.00	0.00
8.63	836,000.00	0.25	1,028.09	1,025.64
7.36	736,000.00	0.50	2,056.18	2,057.84
6.08	608,000.00	0.75	3,084.26	3,084.26
4.77	477,000.00	1.00	4,112.35	4,112.62

Table IV Buckling Load N_{cr} with q Given of Example 4.3

	Timoshenko (34)			Present Method
a	$q=a\pi^2 EI/L^3$ (lbs/in.)	b	$N_{cr}=bEI/L^2$ (lbs)	N_{cr} (lbs)
0.00	0.00	π^2	986,965.00	987,110.30
0.25	1,028.09	8.63	863,000.00	862,559.50
0.50	2,056.18	7.36	736,000.00	725,177.20
0.75	3,084.27	6.08	608,000.00	607,814.00
1.00	4,112.35	4.77	477,000.00	476,929.00

V. NUMERICAL INTEGRATION METHODS AND THEIR APPLICATION TO DYNAMIC RESPONSE

In the analysis of dynamic response, an exact or rigorous mathematical approach may be possible for a very simple structure subjected to a force expressable in a mathematical function. For practical problems of complicated structures and loadings, the direct mathematical integration becomes tedious, or, perhaps impossible. Therefore, it is often desirable and sometimes imperative to solve the equations of motion by step-by-step numerical integration procedures which are designed to utilize modern computational techniques.

Two well-known methods, the Runge-Kutta fourth-order method and the linear acceleration method, have been employed in this research for the general dynamic excitation of elastic as well as inelastic structures.

A. Fourth-Order Runge-Kutta Method

Consider the following second-order simultaneous differential equation

$$\{\frac{d^2x}{dt^2}\} = F(t,x,dx/dt) \qquad (5.1)$$

of which the numerical integration by the fourth-order Runge-Kutta method may be expressed as (33)

$$\{X\}_{i+1} = \{X\}_i - (dt)\{X\}_i + \left(\frac{dt}{6}\right)(\{K_1\}+\{K_2\}+\{K_3\}) \qquad (5.2)$$

$$\{\dot{X}\}_{i+1} = \{\dot{X}\}_i - \frac{1}{6}(\{K_1\}+2\{K_2\}+2\{K_3\}+\{K_4\}) \qquad (5.3)$$

where

$$\{K_1\} = (dt)F(t_i, \{X\}_i, \{\dot{X}\}_i)$$

$$\{K_2\} = (dt)F(t_i+\frac{dt}{2}, \{X\}_i+\frac{dt}{2}\{\dot{X}\}_i, \{\dot{X}\}_i+\tfrac{1}{2}\{K_1\})$$

$$\{K_3\} = (dt)F(t_i+\frac{dt}{2}, \{X\}_i+\frac{dt}{2}\{\dot{X}\}_i+\frac{dt}{4}\{K_1\}, \{\dot{X}\}_i+\tfrac{1}{2}\{K_2\})$$

$$\{K_4\} = (dt)F(t_i+dt, \{X\}_i+dt\{\dot{X}\}_i+\frac{dt}{2}\{K_2\}, \{\dot{X}\}_i+\{K_3\}).$$

From either Eq. (3.19) or Eq. (3.31), one can write the acceleration equations as

$$\{\ddot{X}\} = [M]^{-1}(\{F\} - ([K] - (\alpha+\beta\cos\theta t)[S])\{X\}). \qquad (5.4)$$

Because of the similarity between Eq. (5.1) and Eq. (5.4), the solution of Eq. (5.4) can be obtained by applying the fourth-order Runge-Kutta method.

The SUBROUTINE GFMKP in the appended computer programs is based on Eqs. (5.2 and 5.3) for which two examples are selected for the comparison of the numerical solution with the exact solution by direct integration.

Example 5.1. Find x and y of the following simultaneous second-order differential equations by using (a) direct

integration and (b) the fourth-order Runge-Kutta method.

$$\left.\begin{aligned} \frac{d^2x}{dt^2} + \frac{dy}{dt} + x - y &= \sin t \\ \frac{d^2y}{dt^2} + \frac{dx}{dt} + x - y &= 2t^2 \end{aligned}\right\} \tag{5.5}$$

of which the initial conditions are:

x=2., y=-4.5, dx/dt=-1., and dy/dt=-3.5 at t=0.

Solution: (a) Using the given initial conditions one can find the following solution to Eq. (5.5) by the direct integration technique:

$$x = 1+t-2t^2+\frac{2}{3}t^3-\frac{1}{6}t^4+e^{-t}-\sin t$$

$$y = -6-3t-4t^2-\frac{1}{6}t^4+e^{t}-e^{-t}-\frac{1}{2}\sin t-\frac{1}{2}\cos t$$

in which x and y are function of t. Let t be varied in an interval of 0.1 sec., then the values of x and y are tabulated in Table V.

(b) Let Eq. (5.5) be rewritten in the following matrix form:

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{Bmatrix} \ddot{x} \\ \ddot{y} \end{Bmatrix} + \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{Bmatrix} \dot{x} \\ \dot{y} \end{Bmatrix} + \begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix} \begin{Bmatrix} x \\ y \end{Bmatrix} = \begin{Bmatrix} \sin t \\ 2t^2 \end{Bmatrix}. \quad (5.6)$$

Using the computer program GFMKP the solution of x and y in Eq. (5.6) has been found for the interval of time dt=0.004 sec. The result is shown in Table VI. Comparing Table V with Table VI reveals that the difference is negligible. x and y obtained in (a) and (b) are plotted in Fig. 5.1.

Example 5.2. Find x, y, z of the following simultaneous second-order differential equations by using (a) the direct integration method and (b) the fourth-order Runge-Kutta method.

$$\left.\begin{aligned} d^2x/dt^2 + d^2z/dt^2 - x &= 0 \\ d^2y/dt^2 + d^2z/dt^2 - y &= 0 \\ d^2x/dt^2 + y &= 2\cos t \end{aligned}\right\} \quad (5.7)$$

of which the initial conditions are: x=0, y=0, z=3, dx/dy=0, dy/dt=0 and dz/dt=1.5 at t=0.

Solution: (a) The solutions to Eq. (5.7) are obtained by the direct integration method as

$$x = t \sin t$$

$$y = t \sin t$$

$$z = 1.5t - 2t\sin t - 2(1-\cos t) - 3.$$

The numerical values of x, y, z are tabulated in Table VII.

(b) Let Eq. (5.7) be rewritten in matrix form as

$$\begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} \ddot{z} \\ \ddot{y} \\ \ddot{x} \end{Bmatrix} + \begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{Bmatrix} z \\ y \\ x \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 2\cos t \end{Bmatrix}. \tag{5.8}$$

The computer solution of Eq. (5.8) for dt=0.002 sec. is shown in Table VIII. The comparison between the results obtained by these two methods is very satisfactory. Fig. 5.2 shows the function of x, y, z vs time.

B. Linear Acceleration Method

The general expression of numerical integration of a second-order differential equation may be rewritten as (17)

$$\{X\}_t = \{X\}_{t-dt} + \{\dot{X}\}_{t-dt}(dt) + (\tfrac{1}{2}-B')\{\ddot{X}\}_{t-dt}(dt)^2 + B'\{\ddot{X}\}_t(dt)^2 \tag{5.9}$$

$$\{\dot{X}\}_t = \{\dot{X}\}_{t-dt} + \tfrac{1}{2}(\{\ddot{X}\}_{t-dt} + \{\ddot{X}\}_t)(dt) \tag{5.10}$$

in which the parameter B' is chosen to change the form

of the variation of acceleration in the time interval dt. When B'=1/6, the motion solution corresponds to a linear variation of acceleration in the time interval dt, and Eqs. (5.9) and (5.10) become

$$\{X\}_t = \{X\}_{t-dt} + (dt)\{\dot{X}\}_{t-dt} + \frac{1}{3}\{\ddot{X}\}_{t-dt}(dt)^2 + \frac{1}{6}\{\ddot{X}\}_t(dt)^2 \tag{5.11}$$

$$\{\dot{X}\}_t = \{\dot{X}\}_{t-dt} + \tfrac{1}{2}(dt)\{\ddot{X}\}_{t-dt} + \tfrac{1}{2}(dt)\{\ddot{X}\}_t \tag{5.12}$$

in which the subscripts t and t-dt denote the response at time, t, and the previous, t-dt, respectively. Thus the solution method is called a linear acceleration method.

Let the governing differential equation of motion of Eq. (3.19) be rewritten as

$$[M]\{\ddot{X}\} + ([K] - (\alpha+\beta\cos\theta t)[S])\{X\} = \{F\} \tag{5.13}$$

which is actually a nonlinear differential equation, because the stability matrix $(\alpha+\beta\cos\theta t)$ [S] is time dependent. The motion equation may be considered to be linear during a very short time duration, dt, for which Eq. (5.13) can be expressed in an incremental form as

$$[M]\{\Delta\ddot{X}\} + ([K] - (\alpha+\beta\cos\theta t)[S])\{\Delta X\} = \{\Delta F\} \tag{5.14}$$

in which

$\{\Delta\ddot{X}\}$ = incremental acceleration;

$\{\Delta X\}$ = incremental displacement; and

$\{\Delta F\}$ = incremental force.

From Eqs. (5.11) and (5.12) we have

$$\{\Delta\dot{X}\} = \{\dot{X}\}_t - \{\dot{X}\}_{t-dt} = 3/dt\{\Delta X\} + \{B\} \tag{5.15}$$

and

$$\{\Delta\ddot{X}\} = \{\ddot{X}\}_t - \{\ddot{X}\}_{t-dt} = 6/dt^2\{\Delta X\} + \{A\} \tag{5.16}$$

in which

$$\{\Delta X\} = \{X\}_t - \{X\}_{t-dt} \tag{5.17}$$

$$\{A\} = -6/dt\{\dot{X}\}_{t-dt} - 3\{\ddot{X}\}_{t-dt} \tag{5.18}$$

$$\{B\} = -3\{\dot{X}\}_{t-dt} - dt/2\{\ddot{X}\}_{t-dt}. \tag{5.19}$$

Substituting Eqs. (5.15 to 5.19) into Eq. (5.14) yields the following symbolic form:

$$[K']\{\Delta X\} = \{\Delta R\} \tag{5.20}$$

in which

$$[K'] = 6/dt^2[M] + [K] - [S'] \tag{5.21}$$

$$\{\Delta R\} = \{\Delta F\} - [M]\{A\} \tag{5.22}$$

$$[S'] = (\alpha+\beta\cos\theta t)[S]. \tag{5.23}$$

Thus Eq. (5.14) is reduced to the pseudo-static form of Eq. (5.20) from which $\{\Delta X\}$ can be solved as

$$\{\Delta X\} = [K']^{-1}\{\Delta R\} . \tag{5.24}$$

Using the pseudo-static form to find the dynamic response of a structure, one must repeatedly perform the following procedures :

$$\{A\} = -6/dt\{\dot{X}\}_{t-dt} - 3\{\ddot{X}\}_{t-dt}$$

$$\{B\} = -3\{\dot{X}\}_{t-dt} - dt/2\{\ddot{X}\}_{t-dt}$$

$$\{\Delta R\} = \{\Delta F\} - [M]\{A\}$$

$$[K'] = ([K]-[S']+6/dt^2[M])$$

$$\{\Delta X\} = [K']^{-1}\{\Delta R\}$$

$$\{X\}_t = \{X\}_{t-dt} + \{\Delta X\}$$

$$\{\dot{X}\}_t = \{\dot{X}\}_{t-dt} + \{\Delta\dot{X}\} = \{\dot{X}\}_{t-dt} + 3/dt\{\Delta X\} + \{B\}$$

$$\{\ddot{X}\}_t = \{\ddot{X}\}_{t-dt} + \{\Delta\ddot{X}\} = \{\ddot{X}\}_{t-dt} + 6/dt^2\{X\}_{t-dt} + \{A\}$$

in which [S'] is different from time to time. Consequently, the structure is assumed to behave in a linear manner during each time increment, and the nonlinear response is obtained as a sequence of successive increments.

C. Modal Analysis

In analyzing the response of a structural system subjected to dynamic excitation, the governing differential equations of motion are usually composed of a set of coupled differential equations of second order. One of the approaches of solving these coupled equations is to uncouple the equations by using a technique of linear coordinate transformation. The linear transformation is obtained by assuming that the response is a superposition of the normal modes of a system multiplied by corresponding time-dependent generalized coordinates. The solutions to the uncoupled equations can be obtained by using Duhamel's integral. This analysis is called modal analysis (23,24).

D. Application of Numerical Integration Methods to a Structure Subjected to a Ground Acceleration

When a structure is excited by a ground acceleration, the motion equations of Eq. (3.19) may be expressed in terms of the following relative coordinates:

$$
\begin{aligned}
\{X_s\}_{relative} &= \{X_s\} - \{X_g\} \\
\{X_r\}_{relative} &= \{X_r\} \\
\{\ddot{X}_s\}_{relative} &= \{\ddot{X}_s\} - \{\ddot{X}_g\} \\
\{\ddot{X}_r\}_{relative} &= \{\ddot{X}_r\}
\end{aligned}
\tag{5.25}
$$

Table V Values of x and y of Example 5.1 by Direct Integration Method

Time	Direct Integration Method	
sec.	x (inch)	y (inch)
0.0	0.2000000E 01	-0.4500000E 01
0.1	0.1885653E 01	-0.4877422E 01
0.2	0.1745129E 01	-0.5309491E 01
0.3	0.1581950E 01	-0.5796082E 01
0.4	0.1399307E 01	-0.6337336E 01
0.5	0.1200031E 01	-0.6933632E 01
0.6	0.9865822E 00	-0.7585610E 01
0.7	0.7610353E 00	-0.8294145E 01
0.8	0.5250612E 00	-0.9060350E 01
0.9	0.2799199E 00	-0.9885552E 01
1.0	0.2643967E-01	-0.1077128E 02
1.1	-0.2349665E 00	-0.1171918E 02
1.2	-0.5043706E 00	-0.1273120E 02
1.3	-0.7822802E 00	-0.1380931E 02
1.4	-0.1069665E 01	-0.1495567E 02
1.5	-0.1367968E 01	-0.1617241E 02
1.6	-0.1679098E 01	-0.1746175E 02
1.7	-0.2005452E 01	-0.1882587E 02
1.8	-0.2349898E 01	-0.2026689E 02
1.9	-0.2715786E 01	-0.2178680E 02
2.0	-0.3106950E 01	-0.2338741E 02

Table VI Values of x and y of Example 5.1 by Runge-Kutta Method

Time	Runge-Kutta Method	
sec.	x (inch)	y (inch)
0.0	0.2000000E 01	-0.4500000E 01
0.1	0.1885633E 01	-0.4877402E 01
0.2	0.1745090E 01	-0.5309444E 01
0.3	0.1581895E 01	-0.5796010E 01
0.4	0.1399232E 01	-0.6337241E 01
0.5	0.1199939E 01	-0.9933517E 01
0.6	0.9864780E 00	-0.7585473E 01
0.7	0.7609386E 00	-0.8293986E 01
0.8	0.5249753E 00	-0.9060167E 01
0.9	0.2798458E 00	-0.9885345E 01
1.0	0.2638184E-01	-0.1077105E 02
1.1	-0.2350211E 00	-0.1171899E 02
1.2	-0.5044181E 00	-0.1273105E 02
1.3	-0.7823184E 00	-0.1380923E 02
1.4	-0.1069688E 01	-0.1495564E 02
1.5	-0.1367956E 01	-0.1617238E 02
1.6	-0.1679055E 01	-0.1746130E 02
1.7	-0.2005371E 01	-0.1882509E 02
1.8	-0.2349773E 01	-0.2026601E 02
1.9	-0.2715609E 01	-0.2178578E 02
2.0	-0.3106709E 01	-0.2338623E 02

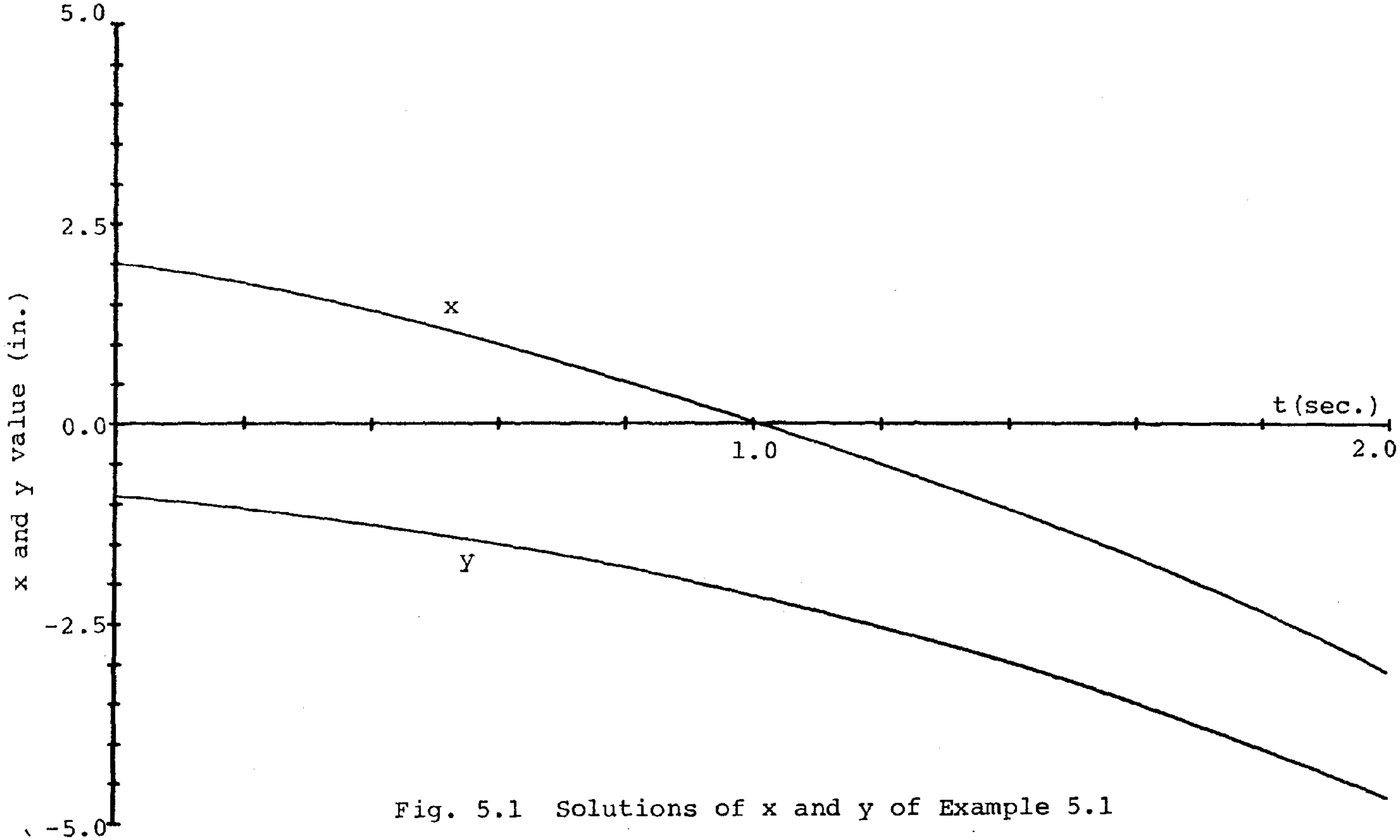


Fig. 5.1 Solutions of x and y of Example 5.1

Table VII Value of x, y, z of Example 5.2 by Direct Integration Method

Time	Direct Integration Method	
sec.	x or y (inch)	z (inch)
0.0	0.0000000E 00	0.3000000E 01
0.1	0.9983279E-02	0.3140006E 01
0.2	0.3973359E-01	0.3260354E 01
0.3	0.8865470E-01	0.3361946E 01
0.4	0.1557643E 00	0.3446251E 01
0.5	0.2397080E 00	0.3515290E 01
0.6	0.3387790E 00	0.3571613E 01
0.7	0.4509431E 00	0.3618241E 01
0.8	0.5738719E 00	0.3658622E 01
0.9	0.7049769E 00	0.3696569E 01
1.0	0.8414487E 00	0.3736200E 01
1.1	0.9802999E 00	0.3781870E 01
1.2	0.1118392E 01	0.3838078E 01
1.3	0.1252545E 01	0.3909409E 01
1.4	0.1379519E 01	0.4000428E 01
1.5	0.1496104E 01	0.4115623E 01
1.6	0.1599156E 01	0.4259301E 01
1.7	0.1685648E 01	0.4435519E 01
1.8	0.1752727E 01	0.4647988E 01
1.9	0.1797756E 01	0.4900013E 01
2.0	0.1818370E 01	0.5194410E 01

Table VIII Values of x, y, z of Example 5.2 by Runge-Kutta Method

Time	Runge-Kutta Method	
sec.	x or y (inch)	z (inch)
0.0	0.0000000E 00	0.3000000E 01
0.1	0.9983249E-02	0.3140024E 01
0.2	0.3973317E-01	0.3260397E 01
0.3	0.8865428E-01	0.3362012E 01
0.4	0.1557640E 00	0.3446340E 01
0.5	0.2397076E 00	0.3515406E 01
0.6	0.3387781E 00	0.3571754E 01
0.7	0.4509427E 00	0.3618406E 01
0.8	0.5738727E 00	0.3658814E 01
0.9	0.7049797E 00	0.3696787E 01
1.0	0.8414543E 00	0.3736449E 01
1.1	0.9803007E 00	0.3782142E 01
1.2	0.1118409E 01	0.3838374E 01
1.3	0.1252581E 01	0.3909723E 01
1.4	0.1379579E 01	0.4000764E 01
1.5	0.1496188E 01	0.4115977E 01
1.6	0.1599264E 01	0.4259674E 01
1.7	0.1685781E 01	0.4435905E 01
1.8	0.1752886E 01	0.4648388E 01
1.9	0.1797944E 01	0.4900426E 01
2.0	0.1818587E 01	0.5194835E 01

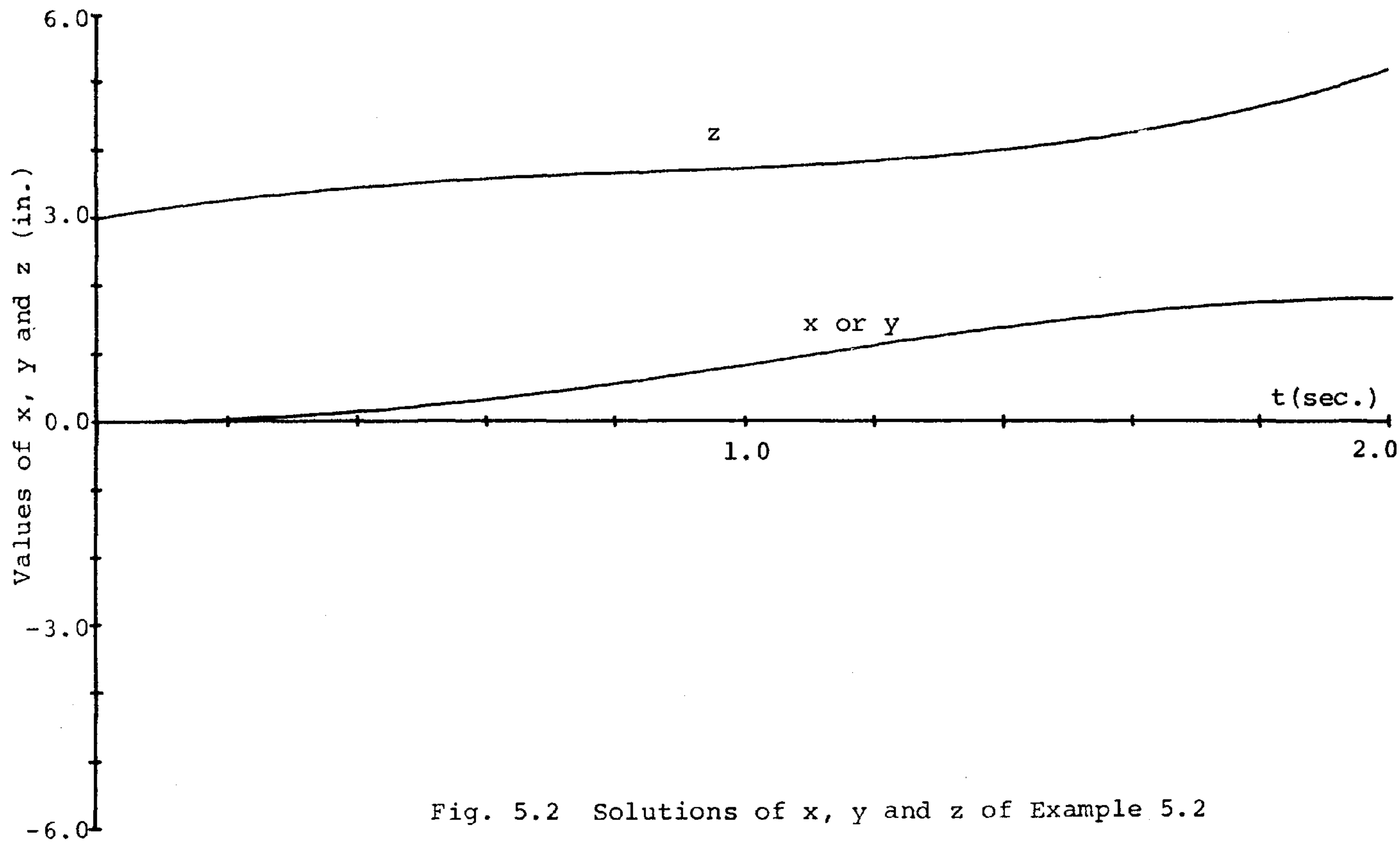


Fig. 5.2 Solutions of x, y and z of Example 5.2

in which

$\{X_g\}$ = ground displacement; and

$\{\ddot{X}_g\}$ = ground acceleration.

Upon substitution of Eq. (5.25) into Eq. (3.19), the motion equations become

$$[M]\left\{\begin{array}{c}\ddot{X}_r\\ \hline \ddot{X}_s\end{array}\right\}_{rel.} + ([K]-(\alpha+\beta\cos\theta t)[S])\left\{\begin{array}{c}X_r\\ \hline X_s\end{array}\right\}_{rel.} = -\ddot{X}_g[M]\left\{\begin{array}{c}0\\ \hline 1\end{array}\right\}. \tag{5.26}$$

If the joint rotations are neglected, then Eq. (5.13) becomes

$$[M]\{\ddot{X}_s\}_{rel.} + [[K] - (\alpha+\beta\cos\theta t)[S]]\{X_s\}_{rel.} = -\ddot{X}_g[M]\{1\}. \tag{5.27}$$

Example 5.3. Consider the shear building shown in Fig. 5.3 subjected to a ground acceleration $\ddot{X}_g=(-8.\pi^2\sin4\pi t)$ in./sec.^{2} The structure is assumed to be stationary at t=0. Find the relative displacements y_1 and y_2.

Solution: Without considering the joint rotations, the diagrams of relative displacements and internal shears are shown in Figs. 5.4a and 5.4b, respectively. The governing differential equations of motion can be established as

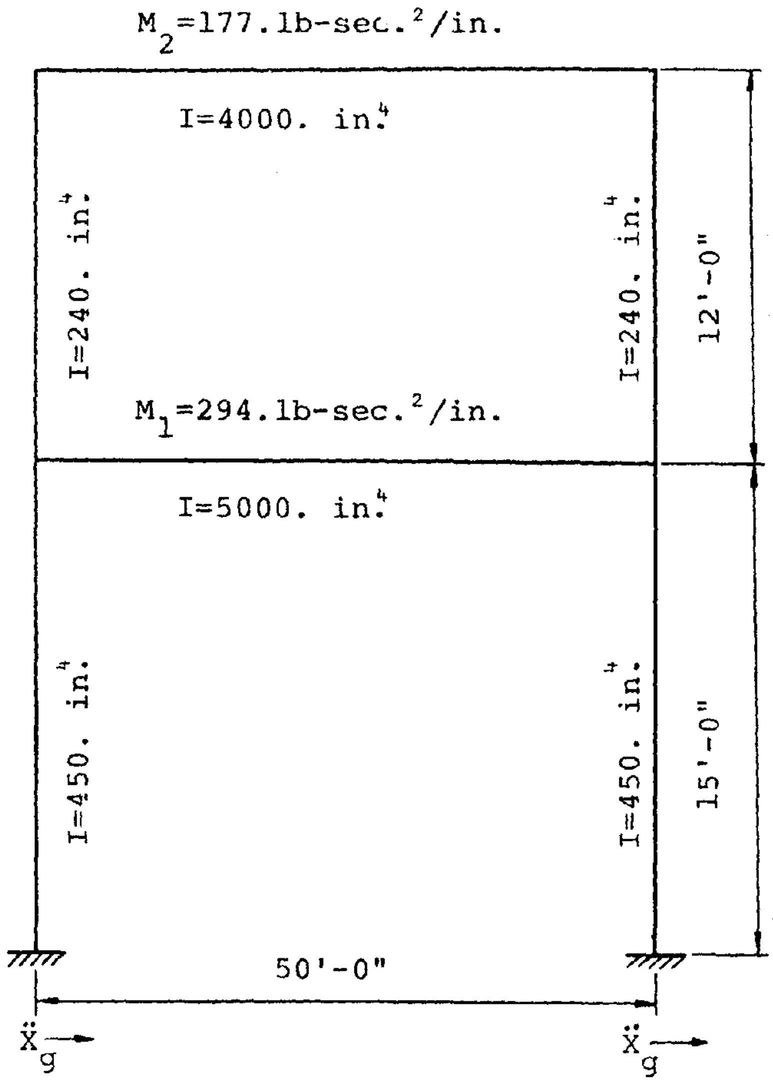
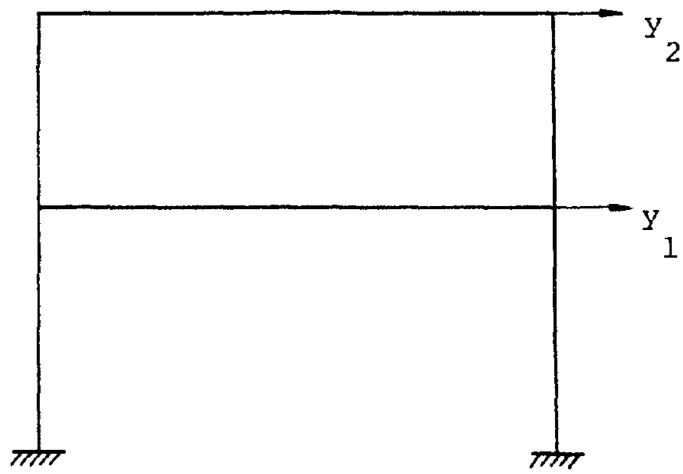
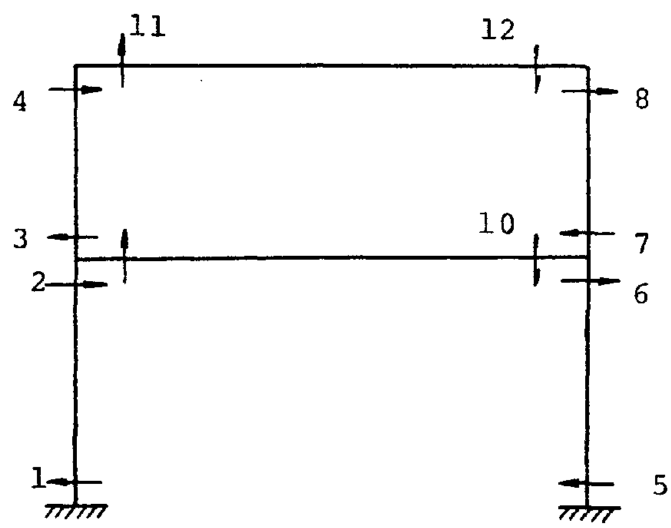


Fig. 5.3 Example 5.3



(a) Relative displacements



(b) Internal Shears

Fig. 5.4 Diagrams for Example 5.3

$$\begin{bmatrix} 0.294 & 0 \\ 0 & 0.177 \end{bmatrix} \begin{Bmatrix} \ddot{y}_1 \\ \ddot{y}_2 \end{Bmatrix} + \begin{bmatrix} 113.4258 & -57.8703 \\ 57.8703 & 57.8703 \end{bmatrix} \begin{Bmatrix} y_1 \\ y_2 \end{Bmatrix}$$

$$= (-8\pi^2 \sin 4\pi t) \begin{Bmatrix} -0.294 \\ -0.177 \end{Bmatrix}. \qquad (5.28)$$

The solutions to Eq. (5.28) by modal matrix method are

$$y_1 = (1.435726\sin\omega_1 t - 0.01522\sin\omega_2 t - 1.118190\sin 4\pi t) \text{ in.}$$

$$y_2 = (2.077400\sin\omega_1 t + 0.01747\sin\omega_2 t - 1.695700\sin 4\pi t) \text{ in.}$$

in which ω_1=10.0493 rad./sec. and ω_2=24.7338 rad./sec. Eq. (5.28) is also solved by the Runge-Kutta method and the linear acceleration method. The results obtained by using these three methods are shown in Tables IX, X, and XI. The values of y_1 and y_2 obtained by the Runge-Kutta method are plotted in Fig. 5.5.

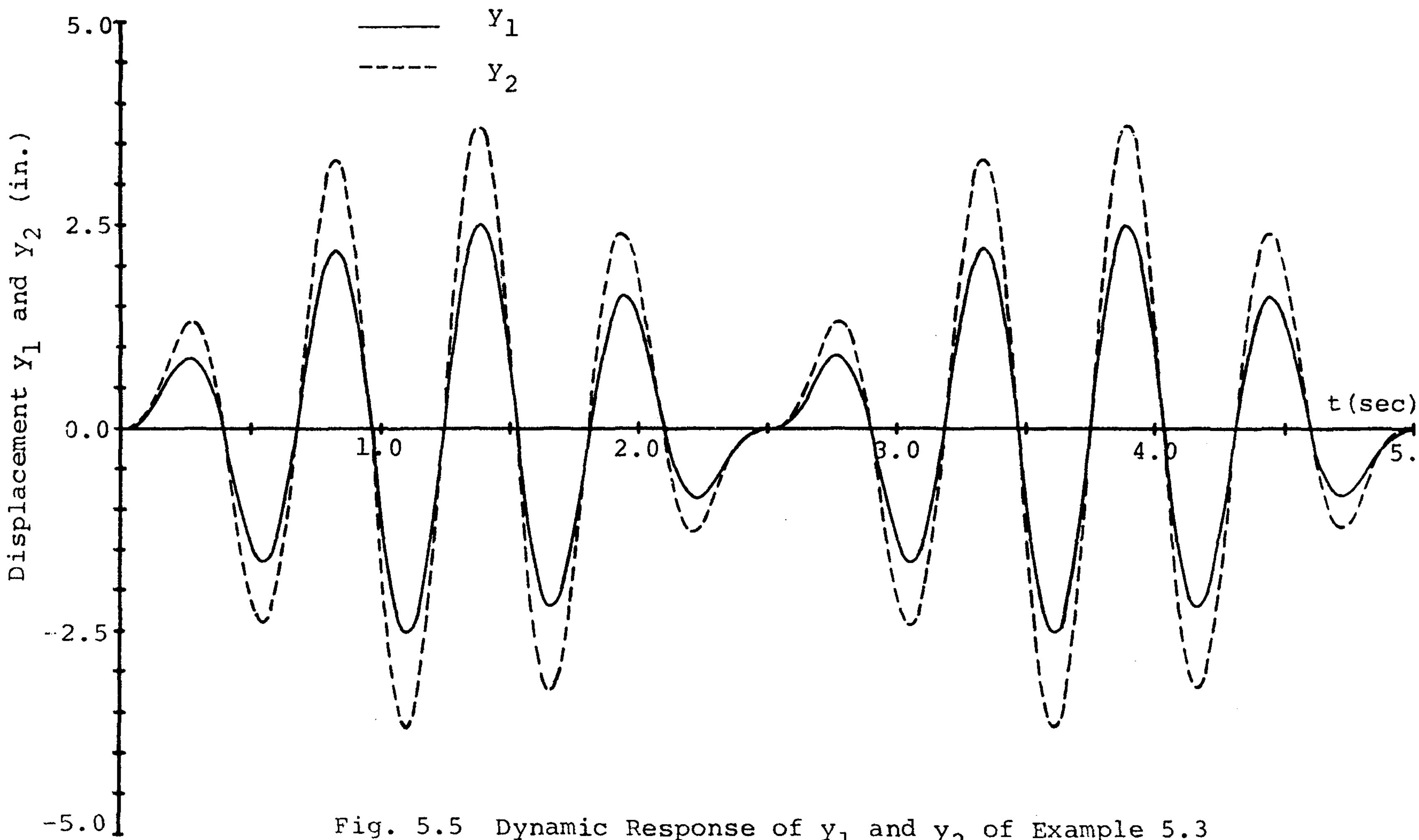


Fig. 5.5 Dynamic Response of y_1 and y_2 of Example 5.3

Table IX Modal Matrix Solution of Example 5.3

Time	Modal Matrix Method	
sec.	y_1(inch)	y_2(inch)
0.00	0.0000000E 00	0.0000000E 00
0.25	0.8464944E 00	0.1220889E 01
0.50	-0.1363268E 01	-0.1980392E 01
0.75	0.1368720E 01	0.1968805E 01
1.00	-0.8336927E 00	-0.1221648E 01
1.25	0.5519118E-03	-0.1810213E-01
1.50	0.8590817E 00	0.1220779E 01
1.75	-0.1358700E 01	-0.1991364E 01
2.00	0.1373142E 01	0.1958570E 01
2.25	-0.8222809E 00	-0.1220102E 01
2.50	-0.5587449E-02	-0.3398962E-01
2.75	0.8692333E 00	0.1222534E 01
3.00	-0.1355897E 01	-0.1998659E 01
3.25	0.1374121E 01	0.1950198E 01
3.50	-0.8128962E 00	-0.1215090E 01
3.75	-0.5842257E-02	-0.4788642E-01
4.00	0.8772812E 00	0.1229915E 01
4.25	-0.1357949E 01	-0.2004099E 01
4.50	0.1373104E 01	0.1948336E 01
4.75	-0.8089312E 00	-0.1207948E 01
5.00	-0.1285170E-01	-0.5452869E-01

Table X Runge-Kutta Solution of Example 5.3

Time	Runge-Kutta Method	
sec.	y_1 (inch)	y_2 (inch)
0.00	0.0000000E 00	0.0000000E 00
0.25	0.8465530E 00	0.1220697E 01
0.50	-0.1362926E 01	-0.1980345E 01
0.75	0.1368614E 01	0.1968013E 01
1.00	-0.8329155E 00	-0.1221237E 01
1.25	0.4706755E-03	-0.1900962E-01
1.50	0.8594314E 00	0.1220535E 01
1.75	-0.1358029E 01	-0.1991096E 01
2.00	0.1372542E 01	0.1957166E 01
2.25	-0.8211390E 00	-0.1219411E 01
2.50	-0.1396582E-02	-0.3526889E-01
2.75	0.8695287E 00	0.1223184E 01
3.00	-0.1355843E 01	-0.1997948E 01
3.25	0.1372791E 01	0.1949301E 01
3.50	-0.8122261E 00	-0.1212588E 01
3.75	-0.7707227E-02	-0.4857550E-01
4.00	0.8767487E 00	0.1231712E 01
4.25	-0.1358685E 01	-0.2002067E 01
4.50	0.1370356E 01	0.1948045E 01
4.75	-0.8087917E 00	-0.1203523E 01
5.00	-0.1628288E-01	-0.5468697E-01

Table XI Linear Acceleration Solution of Example 5.3

Time	Linear Acceleration Method	
sec.	y_1(inch)	y_2(inch)
0.00	0.0000000E 00	0.0000000E 00
0.25	0.8443995E 00	0.1216910E 01
0.50	-0.1357155E 01	-0.1973529E 01
0.75	0.1362345E 01	0.1956882E 01
1.00	-0.8228942E 00	-0.1209591E 01
1.25	-0.6071389E-02	-0.3182024E-01
1.50	0.8656081E 00	0.1225681E 01
1.75	-0.1353907E 01	-0.1989300E 01
2.00	0.1364450E 01	0.1941218E 01
2.25	-0.8034375E 00	-0.1198106E 01
2.50	-0.1523147E-01	-0.5947738E-01
2.75	0.8820280E 00	0.1237475E 01
3.00	-0.1353487E 01	-0.1997902E 01
3.25	0.1362159E 01	0.1931440E 01
3.50	-0.7889202E 00	-0.1180500E 01
3.75	-0.3058952E-01	-0.8214664E-01
4.00	0.8945796E 00	0.1258290E 01
4.25	-0.1360874E 01	-0.2003111E 01
4.50	0.1354414E 01	0.1928658E 01
4.75	-0.7809602E 00	-0.1157982E 01
5.00	-0.4974973E-01	-0.9619385E-01

VI. DYNAMIC RESPONSE OF ELASTIC STRUCTURAL SYSTEMS

The numerical integration techniques described in the preceding chapter will be used herein to study the instability behavior and displacement response of a structure subjected to time dependent axial forces as well as lateral forces or foundation movements. A number of selected examples given below have been studied by using digital computer programs based on the numerical integration techniques described in Chapter V.

A. Numerical Examples

Example 6.1. Consider a beam column shown in Fig. 6.1a subjected to N_t at both ends and periodic lateral force, F_t, at point B. The periodic force, F_t, is shown in Fig. 6.1b, and the axial force is $N_t = (\alpha + \beta\cos\theta t)N_0$. The member properties are :

Cross sectional area: $A_{AB}=30.24$ in.^{2}, $A_{BC}=24.$ in.^{2}

Member length : $L_{AB}=144.$ in., $L_{BC}=96.$ in.

Moment inertia : $I_{AB}=192.$ in.^{4}, $I_{BC}=96.$ in.^{4}

The static buckling load and natural frequency are found to be 2974.80 kips and 181.9423 rad./sec., respectively. The principal dynamic instability region for $N_0=2974.80$ kips, $\omega=181.9423$ rad./sec., $\alpha=0$, and $\beta=0.2$ is shown in Fig. 6.2. Two cases of dynamic response are investigated by using the Runge-Kutta method with time interval dt=0.004 sec. As

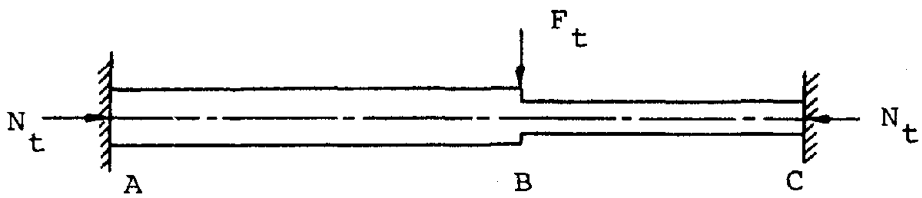
indicated in Fig. 6.2, case A is for θ=251.7872 rad./sec. in the stability region, and case B is for θ=364.00 rad./sec. in the principal instability region. The lateral deflections at point B corresponding to case A and case B are shown in Fig. 6.3.

Example 6.2. Consider a two-story steel framework shown in Fig. 6.4a in which the masses lumped at the floor and the length and moment inertia of the constituent members are given. The columns of the frame are subjected to a time dependent axial force, $N_t=(\alpha+\beta\cos\theta t)N_0$, and the base of the frame is excited by a ground acceleration, $\ddot{X}_g=(-8\pi^2\sin 4\pi t)$ in./sec.^{2} After the static buckling load, N_0, and natural frequency, ω, of the structural system have been found, the principal instability regions for N_0=1001.626 kips, ω=10.0494 rad./sec. are investigated. The results are shown in Fig. 6.5 for various axial loads corresponding to α=0., 0.2, 0.4, and β=0.1, 0.2, 0.3, 0.4, 0.5. Two cases of dynamic response sketched in Fig. 6.5 have been studied in which case A is for α=0., β=0.3, and θ=15.0 rad./sec. in the stability region, and case B is for α=0., β=0.3, and θ=20.1 rad./sec. in the instability region. The Runge-Kutta method with a time interval dt=0.025 sec. has been employed for studying the relative displacements y_1 and y_2. The results associated with case A and case B are shown in Figs. 6.6 and 6.7. These two cases have also been investigated by the linear acceleration method with a time

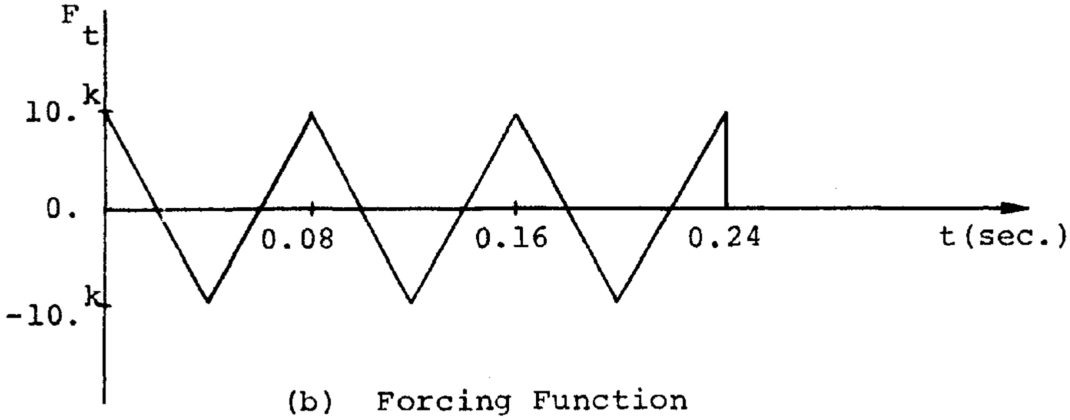
interval dt=0.0125 sec. The relative displacements y_1 and y_2 are shown in Figs. 6.8 and 6.9.

B. Discussion of Results

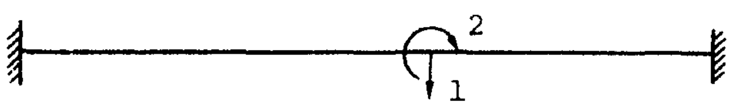
For the cases in the instability region, the deflection response grows exponentially with time. The deflection response associated with the cases in the stability region, however, is quite stable. The results obtained by the Runge-Kutta method agree satisfactorily with those obtained by the linear acceleration method.



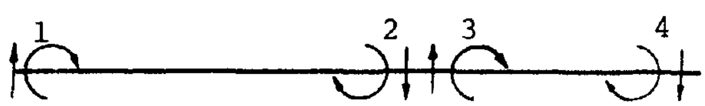
(a) Given Structure



(b) Forcing Function



(c) Global Coordinates



(d) Local Coordinates

Fig. 6.1 Example 6.1

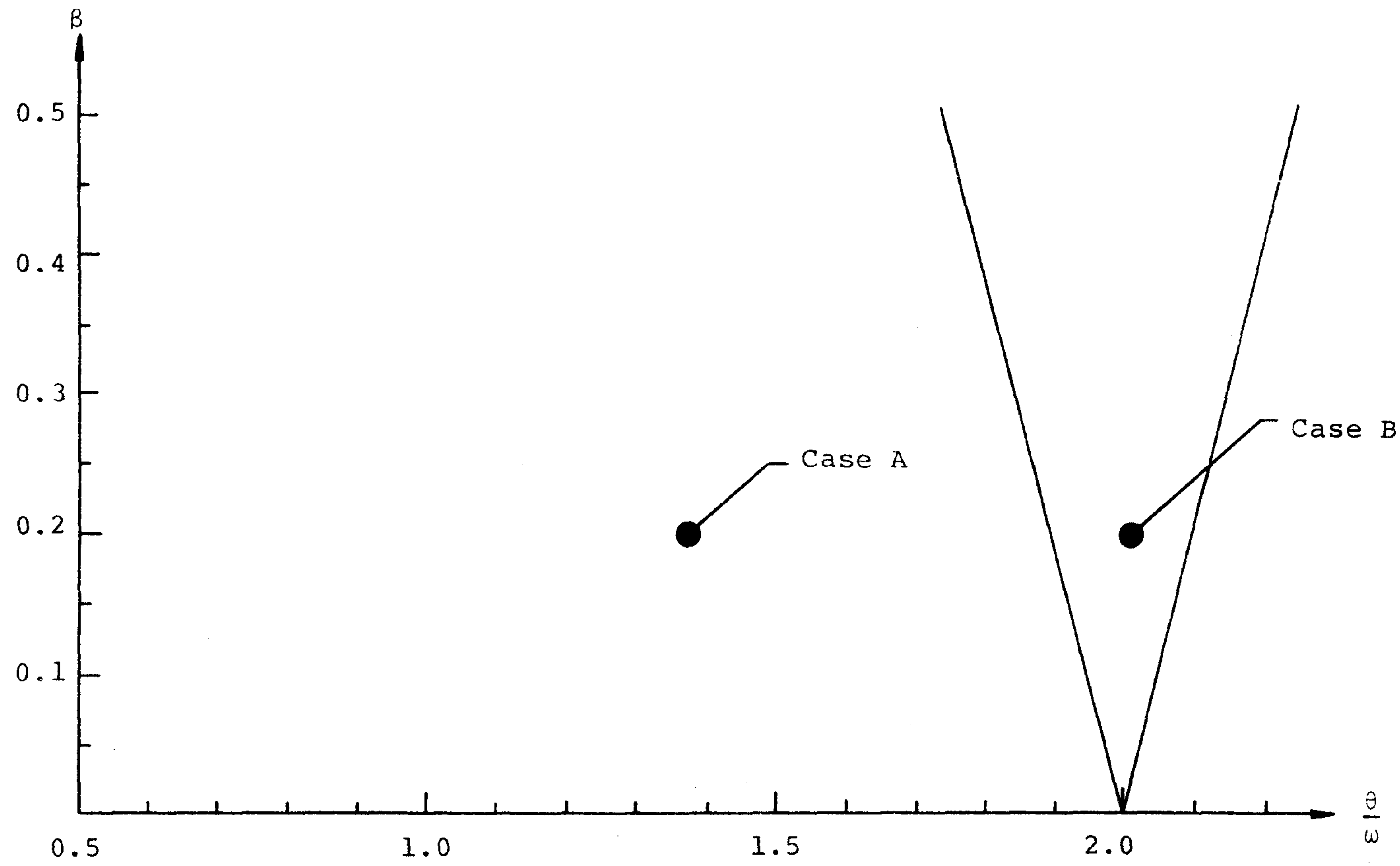


Fig. 6.2 Dynamic Instability Region of Example 6.1

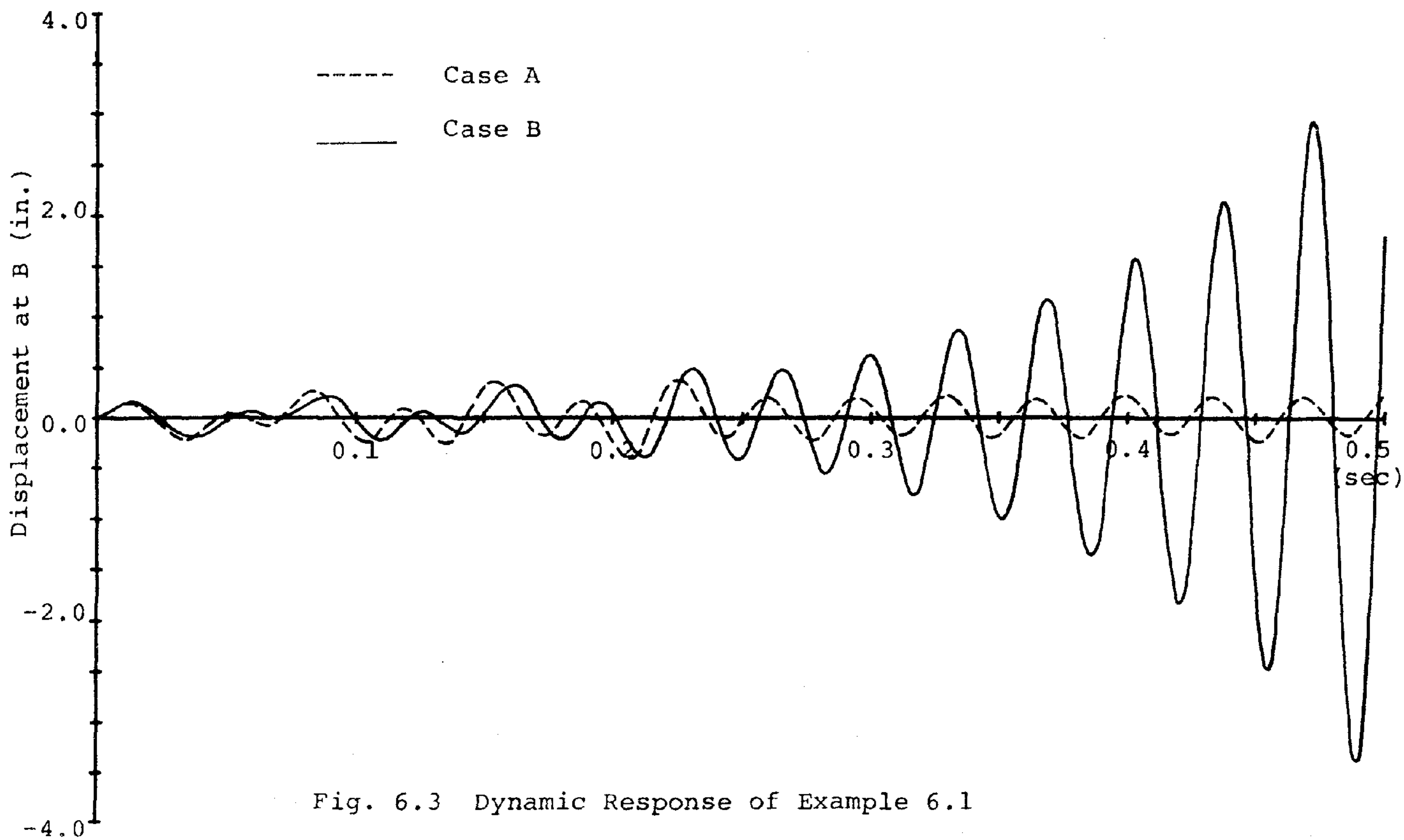
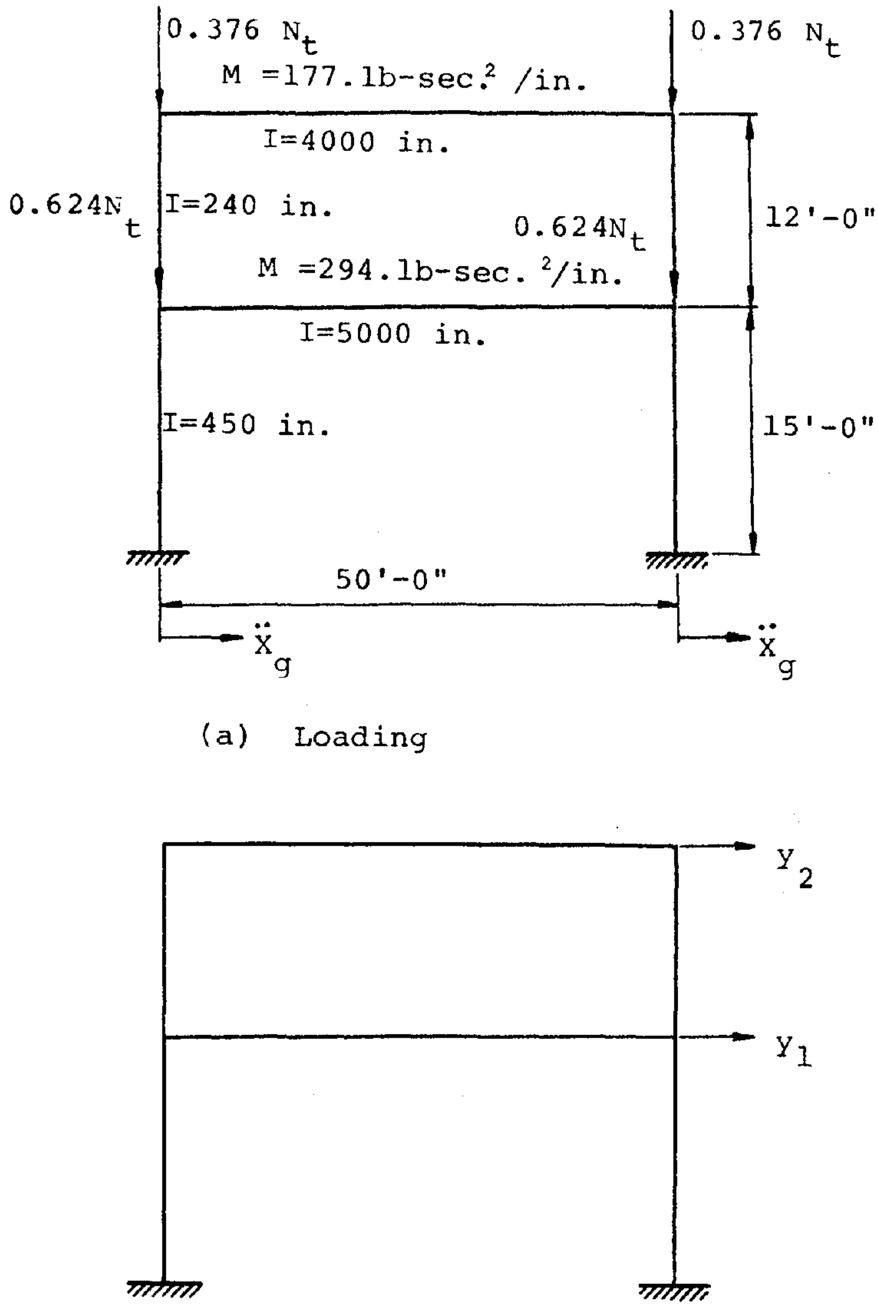


Fig. 6.3 Dynamic Response of Example 6.1



(a) Loading

(b) Relative Displacements

Fig. 6.4 Example 6.2

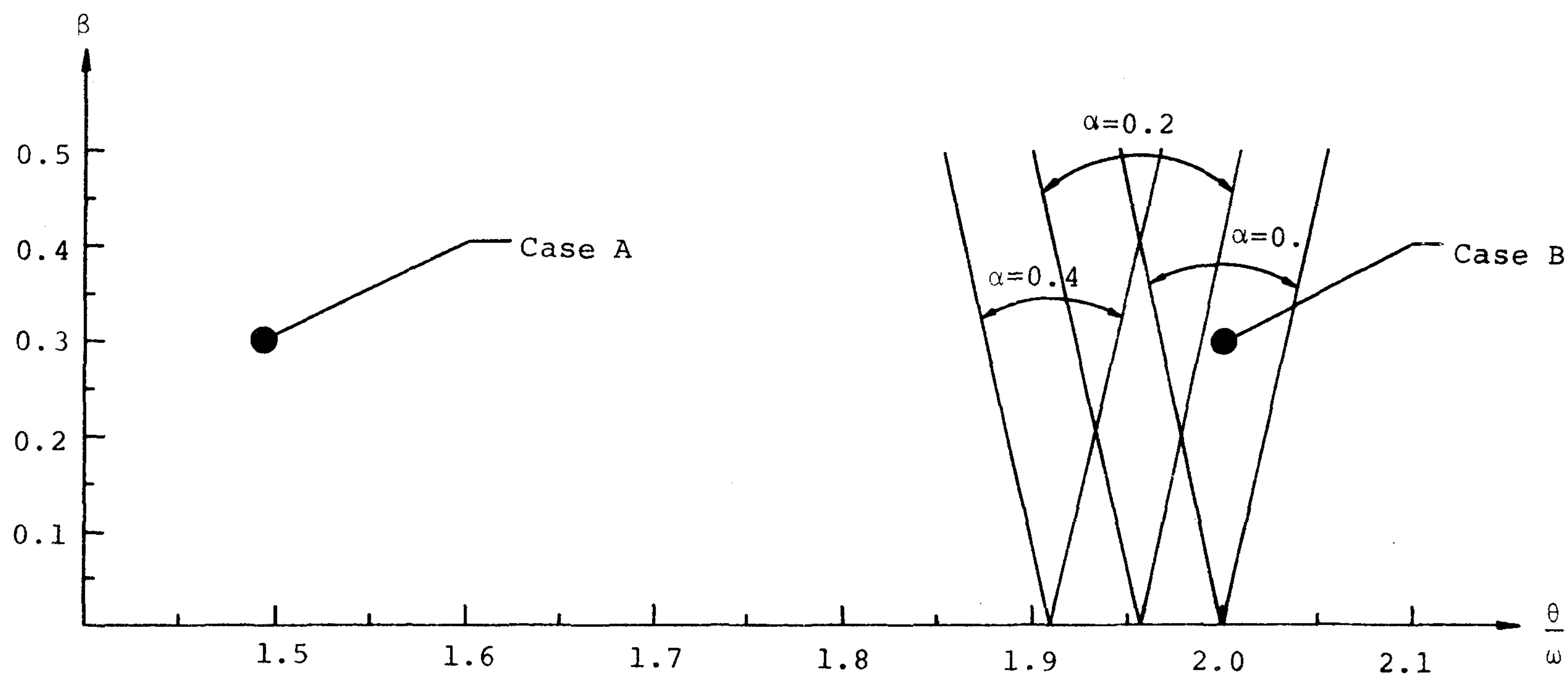


Fig. 6.5 Dynamic Instability Region of Example 6.2

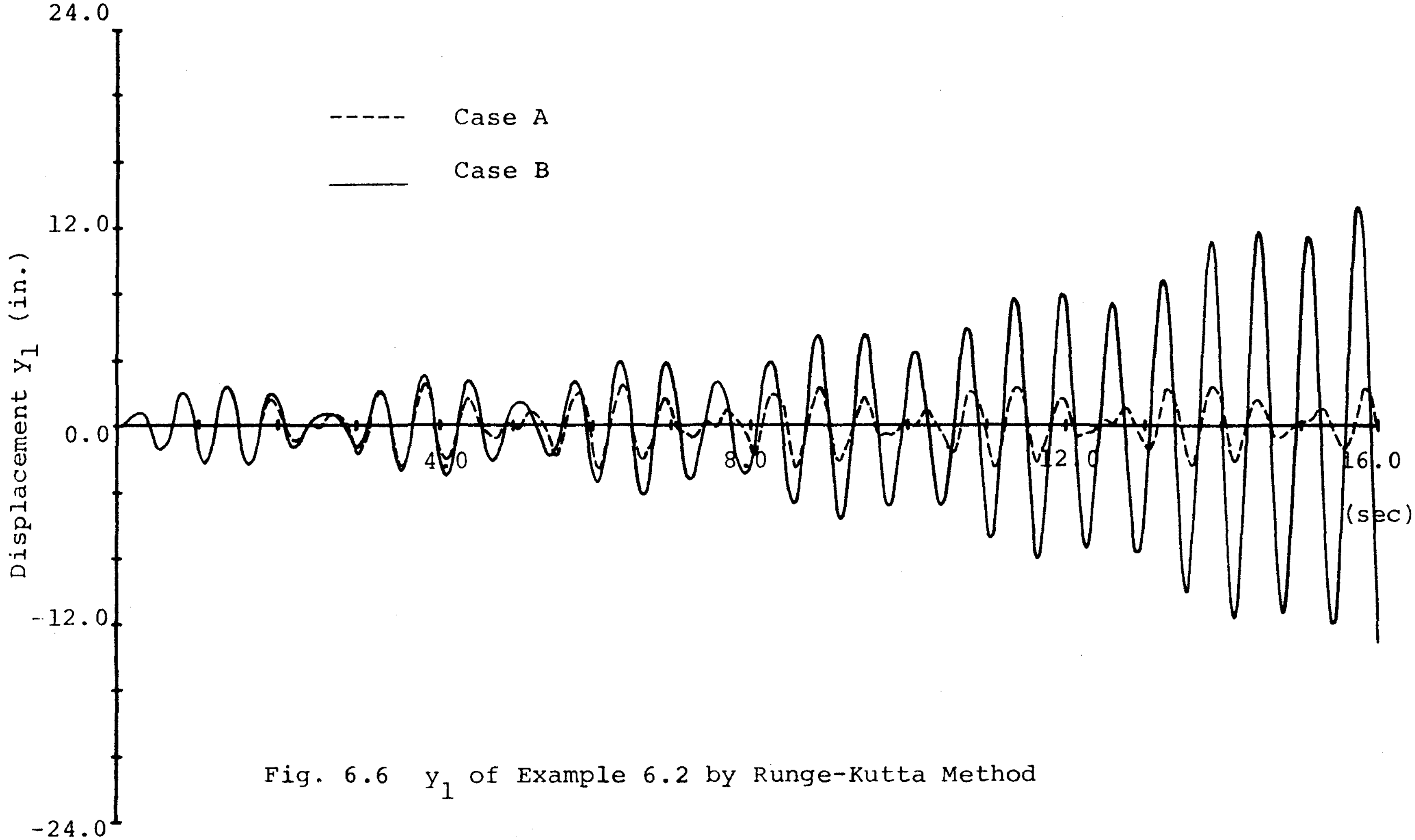


Fig. 6.6 y_1 of Example 6.2 by Runge-Kutta Method

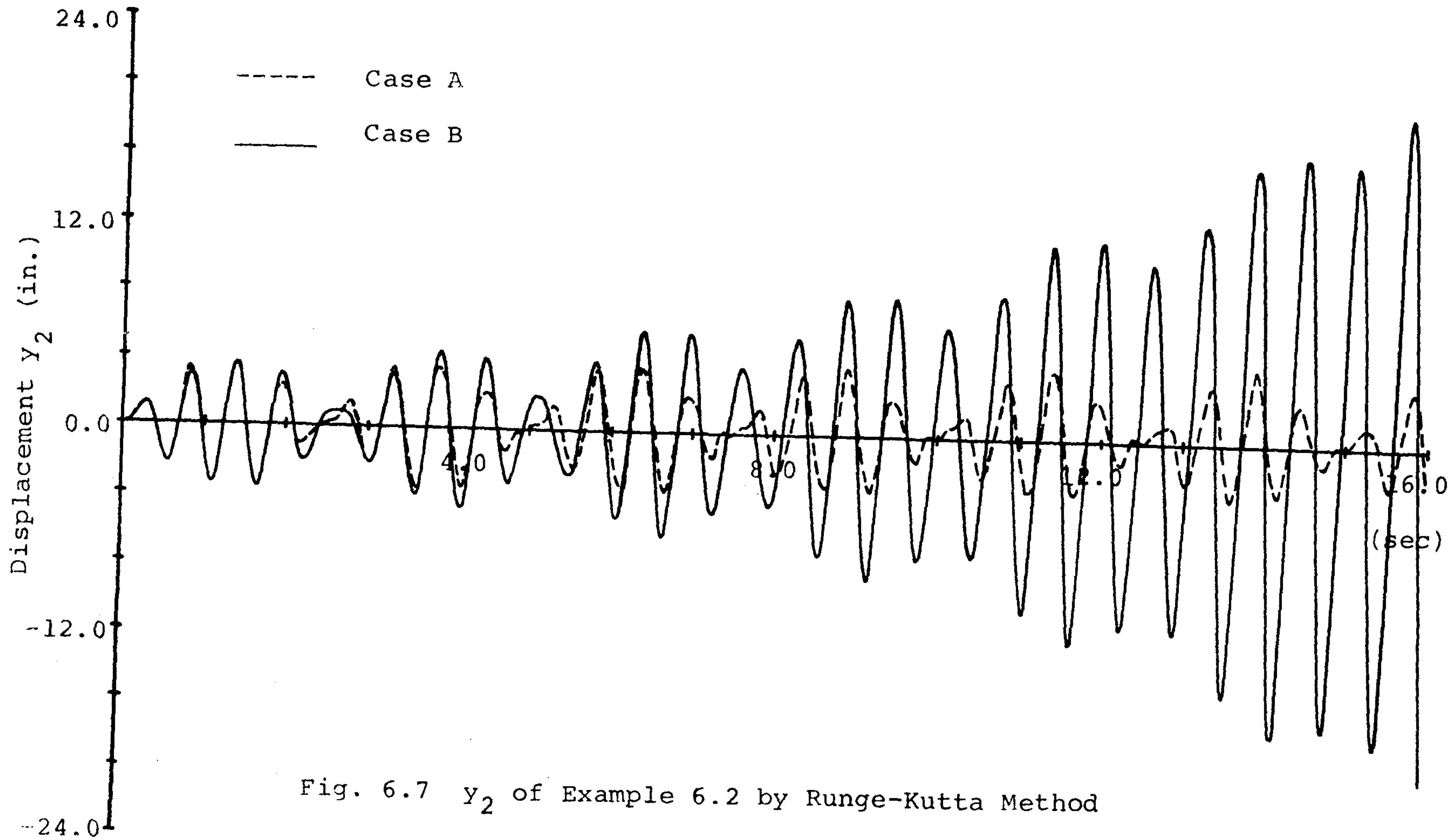


Fig. 6.7 y_2 of Example 6.2 by Runge-Kutta Method

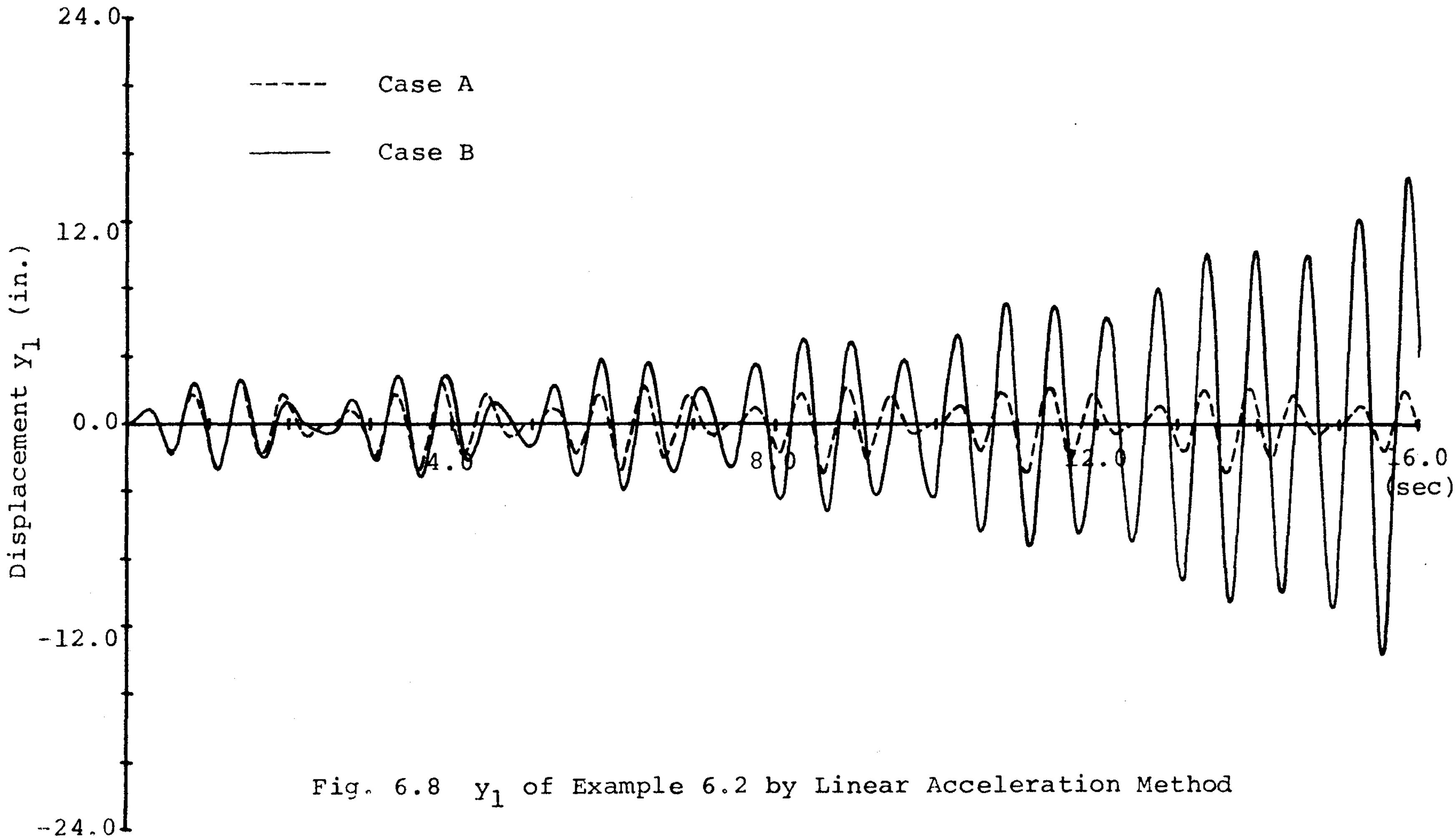


Fig. 6.8 y_1 of Example 6.2 by Linear Acceleration Method

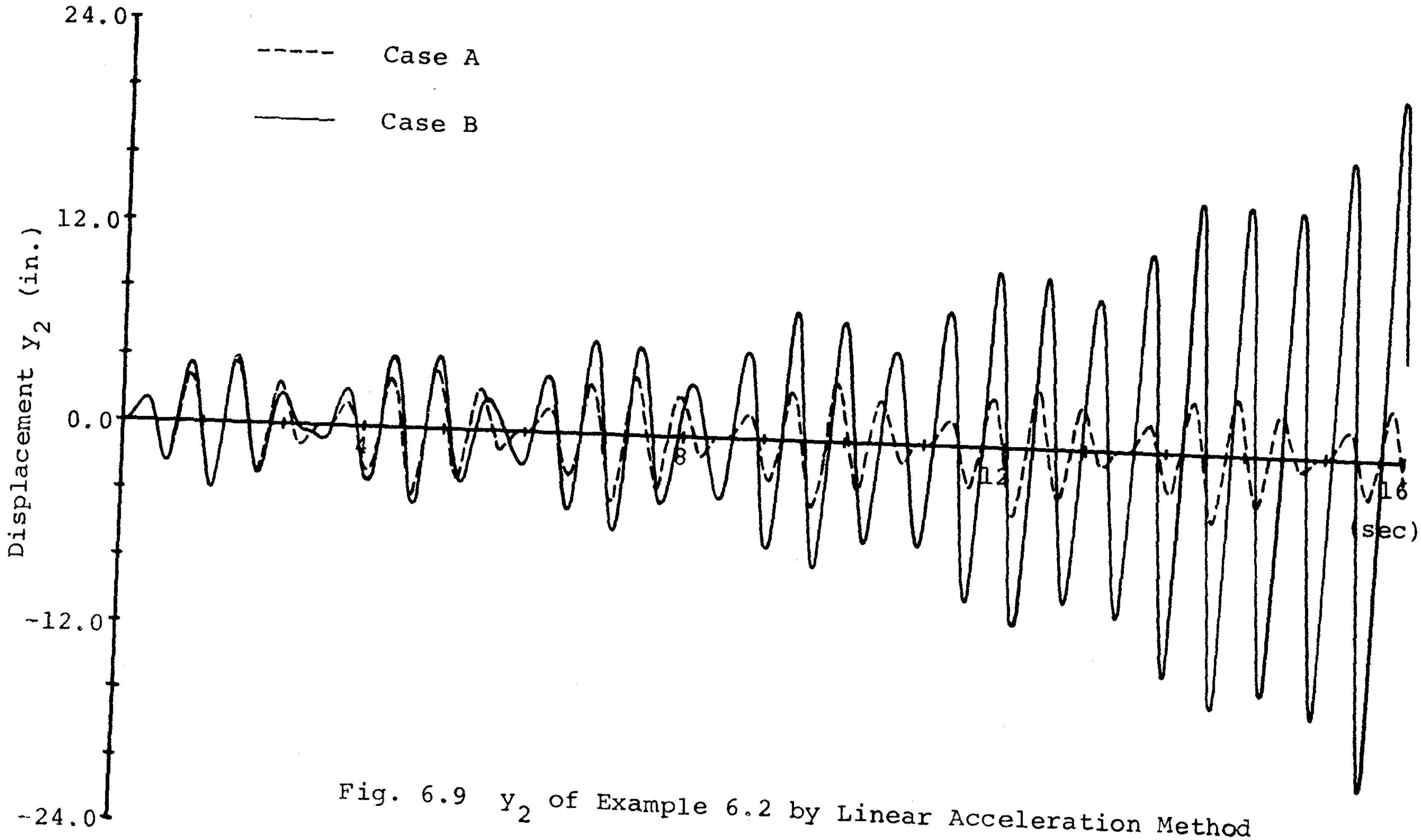


Fig. 6.9 y_2 of Example 6.2 by Linear Acceleration Method

VII. MATRIX FORMULATION FOR ELASTO-PLASTIC STRUCTURAL SYSTEMS

When the deflection of a structural framework becomes sufficiently large, the internal moments of the constituent members may exceed the elastic limit. In such a case, the elastic analysis is no longer applicable, and the structure must be analyzed to include the inelastic deformation. Therefore, the elementary mass, stiffness, and stability matrices of a typical member must be derived to account for the deformation beyond the elastic limit.

A. Idealized Elasto-Plastic Moment-Rotation Characteristics

Let us assume that the constituent members of a frame have ideal elasto-plastic moment-rotation characteristics as shown in Fig. 7.1. The typical moment-rotation diagrm has a linear relationship called elastic branch which varies from zero moment to the reduced plastic moment M_{pc}. The reduced plastic moment is evaluated to account for the effect of axial load on the plastic moment. For any further deformation, the member will have a plastic hinge at which the applied moment is M_{pc}. When the member has reverse deformation, the moment-rotation relationship becomes linear and parallel to the original elastic branch. The elastic behavior remains unchanged until the internal moment reaches M_{pc}. Consequently, a plastic hinge is assumed, and a constant moment is applied at the hinge.

The cyclic process is sketched in Fig. 7.1.

B. Reduced Plastic Moment

The influence of axial force on plastic moment is calculated according to ASCE manuals (35) as

(a) for wide-flange sections

when $0 \leqq P \leqq 0.15P_y$

$$M_{pc} = M_p = F_y Z \tag{7.1}$$

when $0.15p_y \leqq P \leqq P_y$

$$M_{pc} = 1.18[1-(P/P_y)]M_p \tag{7.2}$$

(b) for rectangular section

$$M_{pc} = [1 - (P/P_y)^2]M_p \tag{7.3}$$

where

Z = plastic section modulus;

P_y = $F_y A$;

F_y = yielding stress of steel;

A = cross sectional area;

P = axial force;

M_p = $F_y Z$ = plastic moment; and

M_{pc} = reduced plastic moment.

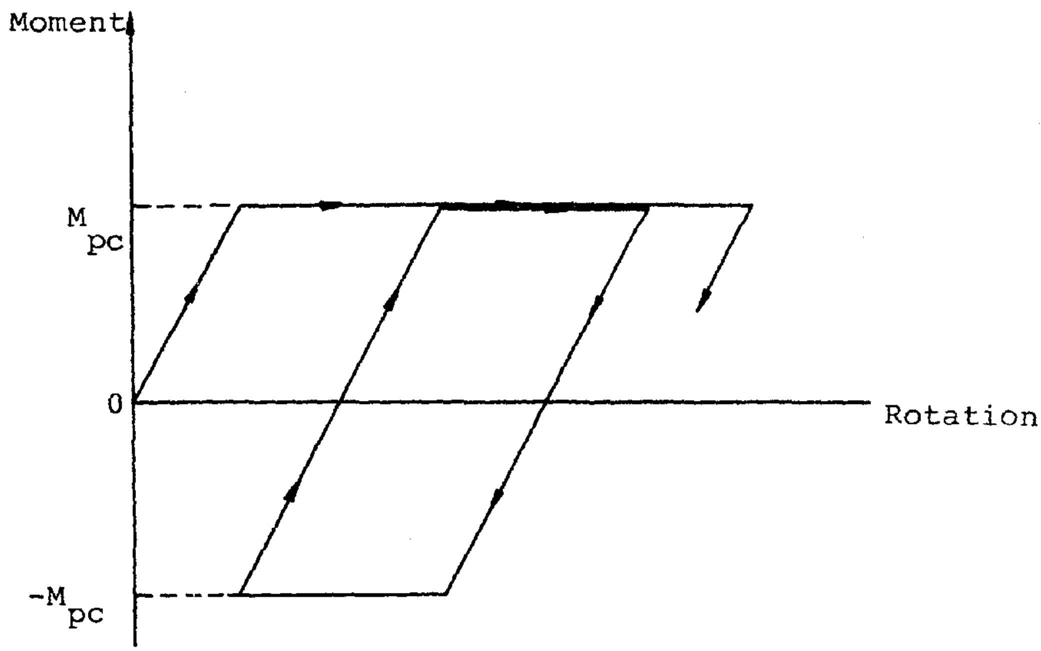


Fig. 7.1 Idealized Moment-Rotation Relationships

C. Modified Elementary Mass, Stiffness, and Stability Matrices

An elastic analysis for dynamic response can only be carried out to the loading stage at which none of the internal moment reaches plastic moment. When an internal moment reaches plastic moment, the frame is then modified by inserting a real hinge at the location with a plastic moment applied at the hinge. Thus the mass, stiffness , and stability matrices of that member must be modified according to the hinge location.

Let the typical member shown in Fig. 7.2 have a hinge at j, then the shape functions of the member are

$$\left.\begin{aligned} \phi_1(x) &= (x^3/2L^2 - 3x^2/2L + x) \\ \phi_2(x) &= 0 \\ \phi_3(x) &= (-x^3/2L^3 + 3x^2/2L^2 - 1) \\ \phi_4(x) &= (3x^2/2L^2 - x^3/2L^3). \end{aligned}\right\} \quad (7.4)$$

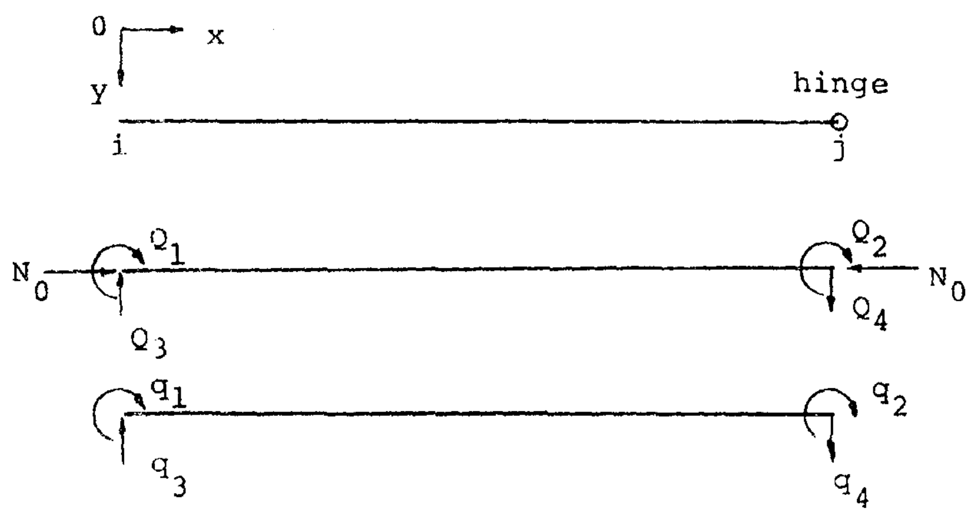


Fig. 7.2 Generalized Local Coordinates and Generalized Forces of a Beam with j End Hinged

Following the same procedure used in Chapter III, one can derive the mass, stiffness, and stability matrices as

$$\begin{Bmatrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \end{Bmatrix}_m = \underbrace{\begin{bmatrix} \frac{8mL^3}{420} & 0. & \frac{-36mL^2}{420} & \frac{11mL^2}{280} \\ 0. & 0. & 0. & 0. \\ \frac{-36mL^2}{420} & 0. & \frac{204mL}{420} & \frac{-39mL}{280} \\ \frac{11mL^2}{280} & 0. & \frac{-39mL}{280} & \frac{99mL}{420} \end{bmatrix}}_{[m_{ij}]} \begin{Bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \\ \ddot{q}_3 \\ \ddot{q}_4 \end{Bmatrix} \tag{7.5}$$

$$\begin{Bmatrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \end{Bmatrix}_k = \underbrace{\begin{bmatrix} \frac{3EI}{L} & 0. & \frac{-3EI}{L^2} & \frac{-3EI}{L^2} \\ 0. & 0. & 0. & 0. \\ \frac{-3EI}{L^2} & 0. & \frac{3EI}{L^3} & \frac{3EI}{L^3} \\ \frac{-3EI}{L^2} & 0. & \frac{3EI}{L^3} & \frac{3EI}{L^3} \end{bmatrix}}_{[k_{ij}]} \begin{Bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{Bmatrix} \tag{7.6}$$

$$\begin{Bmatrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \end{Bmatrix}_p = N_0 \underbrace{\begin{bmatrix} \frac{L}{5} & 0. & \frac{-1}{5} & \frac{-1}{5} \\ 0. & 0. & 0. & 0. \\ \frac{-1}{5} & 0. & \frac{6}{5L} & \frac{6}{5L} \\ \frac{-1}{5} & 0. & \frac{6}{5L} & \frac{6}{5L} \end{bmatrix}}_{[s_{ij}]} \begin{Bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{Bmatrix} . \qquad (7.7)$$

Similarly, let the typical member shown in Fig. 7.3 have a hinge at end i, then the shape functions for the boundary conditions of the member may be derived as

$$\begin{aligned} \phi_1(x) &= 0 \\ \phi_2(x) &= (x^3/2L^2 - x/2) \\ \phi_3(x) &= (-x^3/2L^3 + 3x/2L - 1) \\ \phi_4(x) &= (-x^3/2L^3 + 3x/2L) . \end{aligned} \qquad (7.8)$$

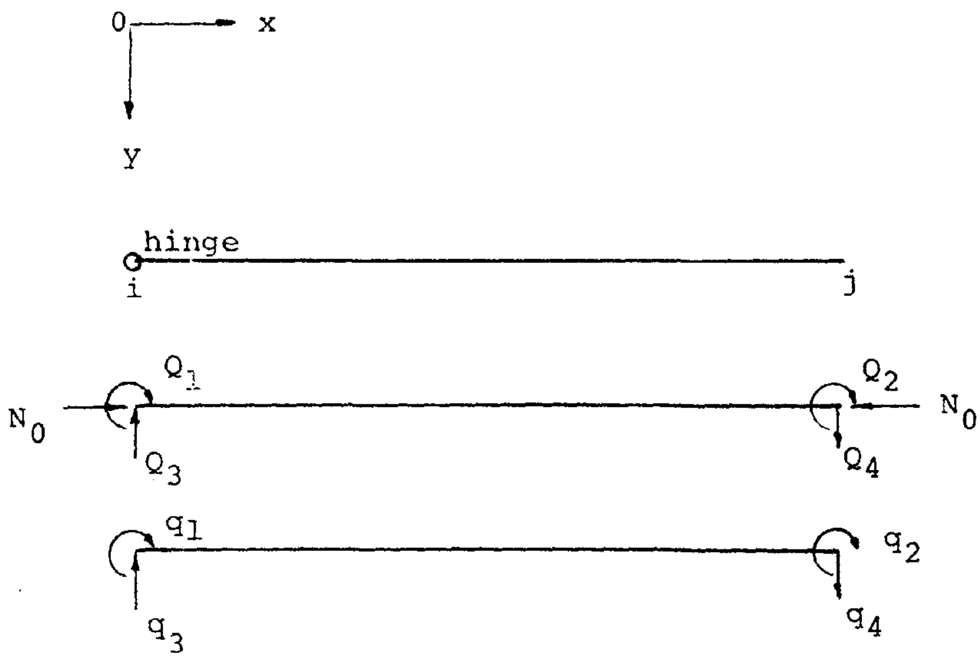


Fig. 7.3 Generalized Local Coordinates and Generalized Forces of a Beam with i End Hinged

Consequently, the mass, stiffness, and stability matrices become

$$\begin{Bmatrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \end{Bmatrix}_m = \underbrace{\begin{bmatrix} 0. & 0. & 0. & 0. \\ 0. & \frac{8mL^3}{420} & \frac{11mL^2}{280} & \frac{-36mL^2}{420} \\ 0. & \frac{11mL^2}{280} & \frac{99mL}{420} & \frac{-39mL}{280} \\ 0. & \frac{-36mL^2}{420} & \frac{-39mL}{280} & \frac{204mL}{420} \end{bmatrix}}_{[m_{ij}]} \begin{Bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \\ \ddot{q}_3 \\ \ddot{q}_4 \end{Bmatrix} \quad (7.9)$$

$$\begin{Bmatrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \end{Bmatrix}_k = \underbrace{\begin{bmatrix} 0. & 0. & 0. & 0. \\ 0. & \frac{3EI}{L} & \frac{-3EI}{L^2} & \frac{-3EI}{L^2} \\ 0. & \frac{-3EI}{L^2} & \frac{3EI}{L^3} & \frac{3EI}{L^3} \\ 0. & \frac{-3EI}{L^2} & \frac{3EI}{L^3} & \frac{3EI}{L^3} \end{bmatrix}}_{[k_{ij}]} \begin{Bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{Bmatrix} \tag{7.10}$$

$$\begin{Bmatrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \end{Bmatrix}_p = N_0 \underbrace{\begin{bmatrix} 0. & 0. & 0. & 0. \\ 0. & \frac{L}{5} & \frac{-1}{5} & \frac{-1}{5} \\ 0. & \frac{-1}{5} & \frac{6}{5L} & \frac{6}{5L} \\ 0. & \frac{-1}{5} & \frac{6}{5L} & \frac{6}{5L} \end{bmatrix}}_{[s_{ij}]} \begin{Bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{Bmatrix} . \tag{7.11}$$

If a member has both ends hinged, then the stiffness and stability matrices become null, and the mass matrix is

$$\begin{Bmatrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \end{Bmatrix}_m = \underbrace{\begin{bmatrix} 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. \\ 0. & 0. & \frac{2mL}{3} & \frac{-mL}{3} \\ 0. & 0. & \frac{-mL}{3} & \frac{2mL}{3} \end{bmatrix}}_{[m_{ij}]} \begin{Bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \\ \ddot{q}_3 \\ \ddot{q}_4 \end{Bmatrix} . \tag{7.12}$$

D. System Matrices of Mass, Stiffness, and Stability

Because the mass, stiffness, and stability matrices of a member with one end or both ends hinged have been modified to account for the boundary conditions, the formulation of system mass, stiffness, and stability matrices can be made by following the same procedure described in Section C of Chapter III.

VIII. DYNAMIC RESPONSE OF ELASTO-PLASTIC STRUCTURES

A. Transfer Matrix for Plastic Moments and Their Associated Shears

When an internal moment at the nodal point of a member is equal to or greater than the reduced plastic moment, then a plastic hinge is assumed to be at the node with a constant moment, M_{pc}, applied at the hinge. Thus the plastic hinge is treated as a real hinge, and the member mass, stiffness, and stability matrices must be modified to satisfy the boundary conditions. If a plastic hinge forms at end i of member ij, the moment M_{pc} at that end must be carried over to end j with the magnitude of $M_{pc}f_{co}$ (f_{co} is the carry-over factor including the effect of axial force). Consequently, $M_{pc}f_{co}$ is treated as the external moment at joint j. The shears, which result from M_{pc} and $M_{pc}f_{co}$ on the member ij, are then transfered to the structural nodes and become the external forces.

Let {FEM}, {FEV} represent the plastic moments M_{pc}, $M_{pc}f_{co}$ and the shears due to M_{pc}, $M_{pc}f_{co}$, respectively; then the transfer matrix can be expressed as

$$\{TF\} = \begin{Bmatrix} TF_r \\ TF_s \end{Bmatrix} = \begin{Bmatrix} [A_m]\{FEM\} \\ [A_v]\{FEV\} \end{Bmatrix} \tag{8.1}$$

where $\{TF\}$ = external load matrix transfered from plastic moments $\{TF_r\} = [A_m]\{FEM\}$, and shears $\{TF_s\} = [A_v]\{FEV\}$.

$\{TF\}$ should be combined with the load matrix $\begin{Bmatrix} F_r \\ F_s \end{Bmatrix}$ in Eq. (3.31) for dynamic response of the elasto-plastic case.

The internal moments and shears can be evaluated from Eq. (3.32) and should be combined with moments $\{FEM\}$ and shears $\{FEV\}$ for the final solution.

B. Calculation of Plastic Hinge Rotation

As discussed previously, when an internal nodal moment reaches the plastic moment capacity, a real hinge is inserted at that node with a constant moment applied at the hinge which is allowed to rotate according to the material behavior shown in Fig. 7.1. When the hinge rotates in the direction of the plastic moment, the moment is assumed to be constant, and the rotation can increase indefinitely. When the hinge rotation is in the opposite direction of the moment, however, the plastic moment is removed, and the member becomes elastic. Thus the plastic hinge rotation must be calculated at each step of the numerical integrations and compared with the previous one, if any, in order to check the change of the sign of rotation. For an entire structural system, the hinge rotations may be otained as follows:

$$\{H_r\} = ([FS]\{Q_m\} - [FY][A_v]^T\{X_s\}) - [A_m]^T\{X_r\} \qquad (8.2)$$

in which the first term of the right side of Eq. (8.2) is

composed of the force-deformation relationship of the constituent members given in Eqs. (3.29, 3.30), and the second term is due to external nodal rotations. The typical element in the first term can be derived from Eqs. (3.29) and (3.30) as

$$\{q_r\} = [KMR+SMR]^{-1}\{Q_m\} + [KMY+SMY][A_v]^T\{X_s\}. \tag{8.3}$$

Thus the elements in Eq. (8.2) are

$$[FS] = \begin{bmatrix} [FM]_1 & & & & \\ & [FM]_2 & & & \\ & & \ddots & & \\ & & & [FM]_i & \\ & & & & \ddots \\ & & & & & [FM]_n \end{bmatrix} \tag{8.4}$$

$$[FM]_i = \frac{1}{DET_i}\begin{bmatrix} \left(\frac{4EI_i}{L_i} - \frac{2N_iL_i}{15}\right) & -\left(\frac{2EI_i}{L_i} + \frac{N_iL_i}{30}\right) \\ -\left(\frac{2EI_i}{L_i} + \frac{N_iL_i}{30}\right) & \left(\frac{4EI_i}{L_i} - \frac{2N_iL_i}{15}\right) \end{bmatrix} \tag{8.5}$$

$$DET_i = \left(\frac{6EI_i}{L_i} - \frac{N_iL_i}{10}\right)\left(\frac{2EI_i}{L_i} - \frac{N_iL_i}{6}\right) \tag{8.6}$$

$$[FY] = \begin{bmatrix} [L]_1 & & & & \\ & [L]_2 & & & \\ & & \ddots & & \\ & & & [L]_i & \\ & & & & \ddots \\ & & & & & [L]_n \end{bmatrix} \tag{8.7}$$

$$[L]_i = \begin{bmatrix} \frac{-1}{L_i} & \frac{-1}{L_i} \\ & \\ \frac{-1}{L_i} & \frac{-1}{L_i} \end{bmatrix} \tag{8.8}$$

Note that $\{Q_m\}$ is the vector of internal nodal moments due to nodal displacements. The subscript i denotes the number of members. The element i of the vector $\{H_r\}$ has value only if a plastic hinge exists at node i.

C. Numerical Examples

Example 8.1 Let Example 6.1 be used to investigate the elasto-plastic dynamic response for $\alpha=0.$, $\beta=0.2$ and $\theta=364.$ rad./sec. The deflections of point B for elastic and elasto-plastic cases are shown in Fig. 8.1.

Example 8.2 Example 6.2 is used to investigate the elasto-plastic dynamic response for $\alpha=0.$, $\beta=0.3$, and

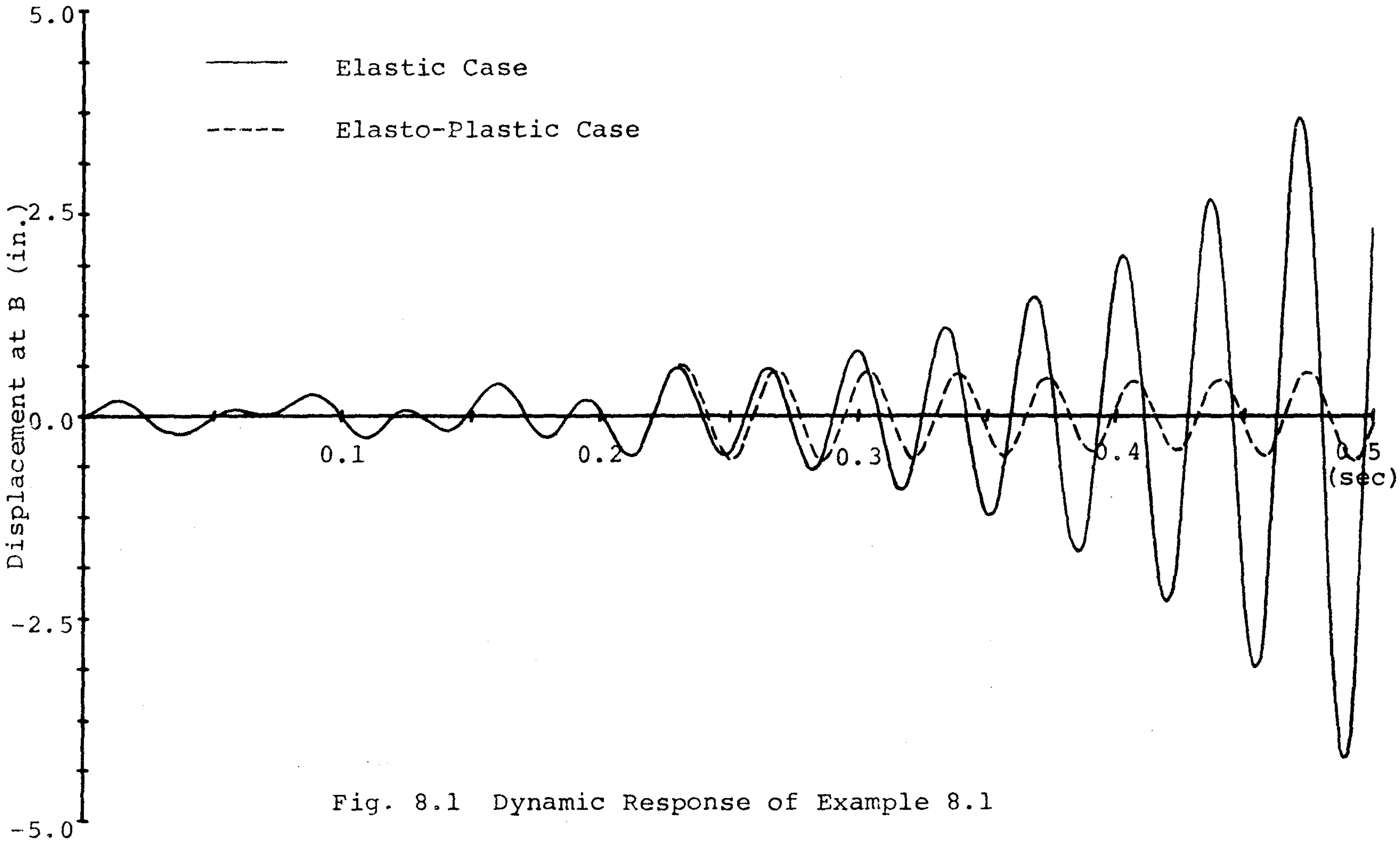


Fig. 8.1 Dynamic Response of Example 8.1

θ=20.1 rad./sec. The lateral deflections of y_1 and y_2 for both elastic and elasto-plastic cases are shown in Figs. 8.2 and 8.3.

D. Discussion of Results

From these two examples, it can be observed that the behavior of elastic case is different from that of the elasto-plastic case. The parametric resonance shows up clearly for the elastic case and cannot be observed for the elasto-plastic case. The reason is that the dynamic instability region is based on the assumption that the structure is elastic.

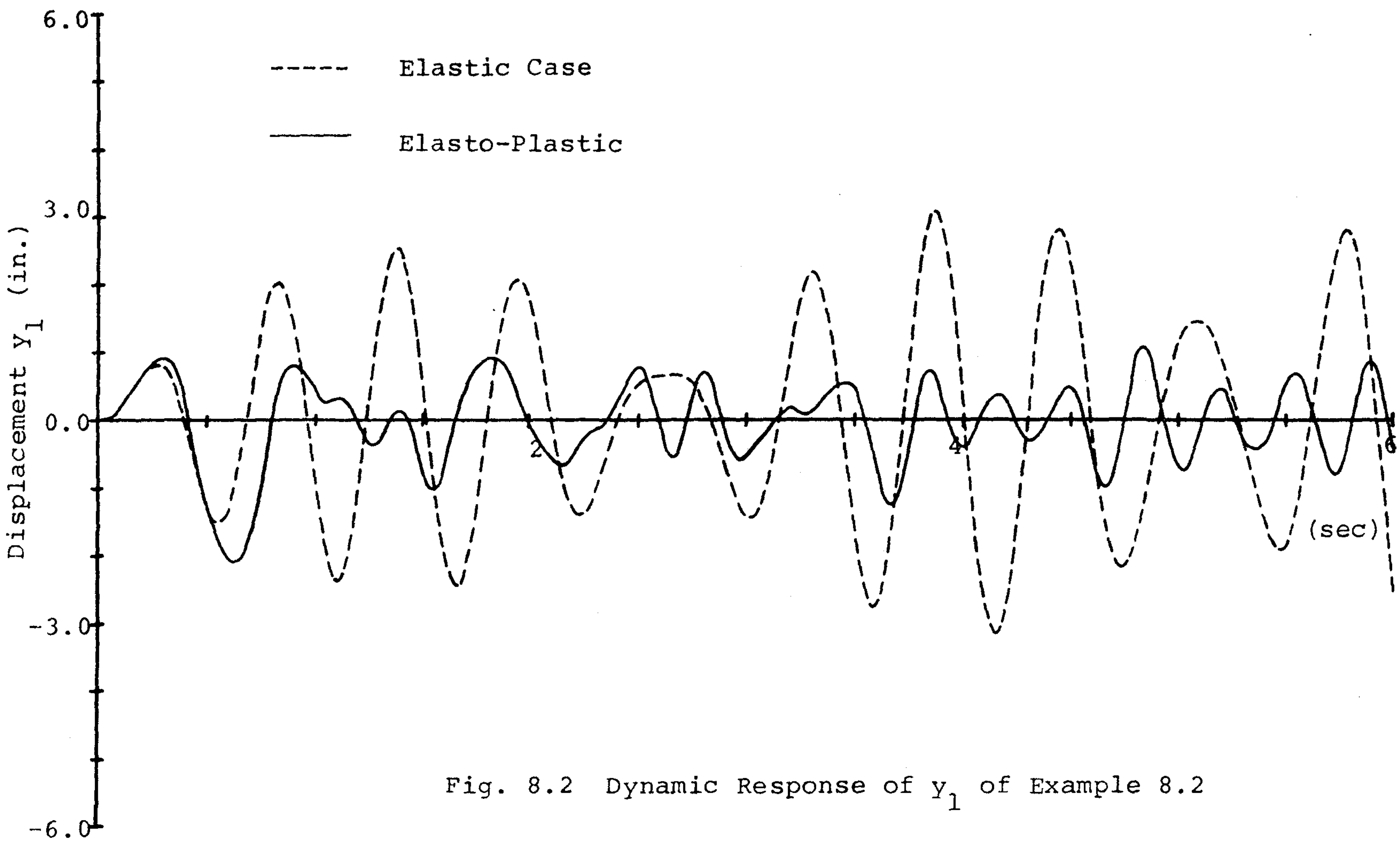


Fig. 8.2 Dynamic Response of y_1 of Example 8.2

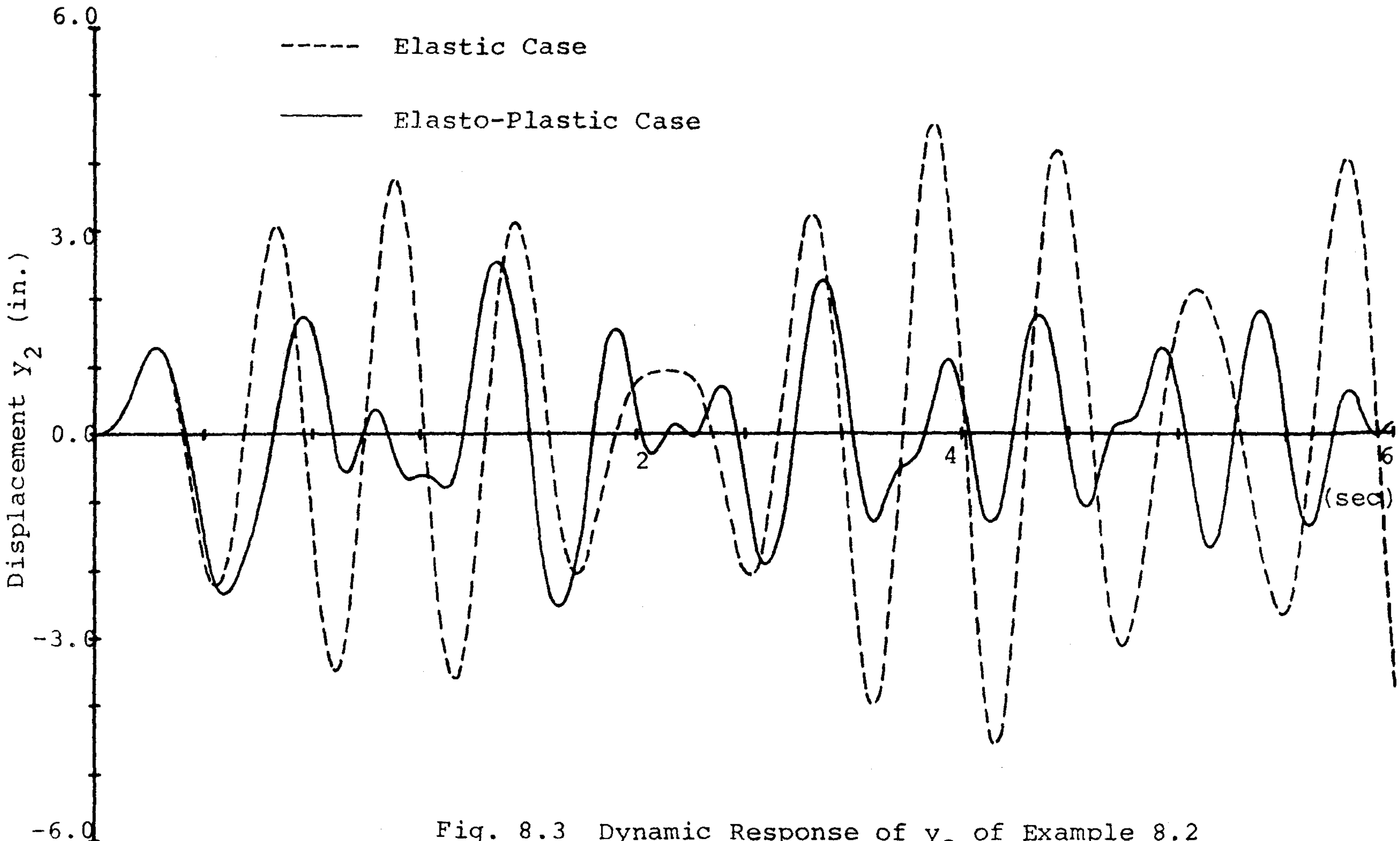


Fig. 8.3 Dynamic Response of y_2 of Example 8.2

IX. SUMMARY AND CONCLUSIONS

An analytical method is presented for determining the behavior of dynamic instability and response of frameworks subjected to longitudinal pulsating loads and lateral dynamic forces or foundation movements. Some of the features of this work are summarized as follows:

1. Dynamic instability criteria are discussed and formulated in relation to the magnitude of axial force, the longitudinal forcing frequency, and the transverse frequency.

2. The displacement method is employed for structural matrix formulation for which the typical member matrices of mass, stiffness, and stability are derived.

3. Eigenvalues of free vibrations and static instability are investigated in this work. The static instability analysis includes both concentrated and uniformly distributed loads.

4. The elastic and elasto-plastic frameworks are analyzed for the response of displacements, internal moments, and shears resulting from dynamic lateral forces or ground accelerations. General considerations include bending deformation, geometric nonlinearity, the effect of girder shears on columns, and the effect of axial loads on plastic moments.

5. Two numerical methods, the fourth-order Runge-Kutta method and the linear acceleration method, are used to solve the nonlinear differential equations of motion. The solutions obtained by these two methods compare very satisfactorily.

6. A number of selected examples are presented from which it can be observed that the deflection response corresponding to the instability region grows exponentially with time.

7. The dynamic instability analysis yields the stability and instability regions from which one may design a structure to avoid the occurrence of parametric resonance.

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APPENDIX

Computer Programs

LIST OF SYMBOLS USED IN COMPUTER PROGRAMS

AFV	= matrix relating girder shears to columns
AFP	= matrix relating vertical forces to axial force in columns
AF	= axial force in column
AM	= matrix $[A_m]$
AV	= matrix $[A_v]$
AMS	= matrix $[A_{ms}]$
AREA	= cross sectional area
A, B, C, D	= constants of K_1, K_2, K_3, K_4 for Runge-Kutta formula
ALPHA	= coefficient of axial load
BETA	= coefficient of axial load
DT	= small increment of time
FY	= yielding stress F_y
PSB	= static buckling load
PT	= time-dependent axial force
PM	= plastic moment ZF_y
PY	= cross sectional area times F_y
NM	= number of members
NP	= number of degrees of freedom
NPR	= number of degrees of freedom in joint rotation
NPS	= number of degrees of freedom in side sway
NVP	= number of vertical forces acting on columns
VA	= α value

VB	=	β value
NPTS	=	number of time steps
XL	=	member length
XM	=	mass per unit length
XI	=	moment of inertia of cross section
XE	=	elastic Young's modulus
X	=	displacement
XT	=	velocity
XTT	=	acceleration
XEM	=	internal end moments
XEV	=	internal end shears
XXM	=	system mass matrix [M]
XXK	=	system stiffness matrix [K]
XXP	=	system stability matrix [S]
T	=	time
ZP	=	plastic modulus
ZETA	=	θ value
NPH	=	new plastic hinge
LPH	=	old plastic hinge
NRH	=	relieved plastic hinge
HR	=	plastic hinge rotation $\{H_r\}$
FEM	=	internal moment due to plastic moment
FEV	=	internal shear due to plastic moment
COFR	=	carry-over factor f_{co}

FRY1, FRY2, FRY3, FRY4 = element of matrix [FY]

FMV1, FMV2, FMV3, FMV4 = element of matrix [L]

FRM1, FRM2, FRM3, FRM4 = element of matrix [FS]

PMR1, PMR2, PMR3, PMR4 = element of submatrix [SMR]

PMY1, PMY2, PMY3, PMY4 = element of submatrix [SMY]

PVR1, PVR2, PVR3, PVR4 = element of submatrix [SVR]

PVY1, PVY2, PVY3, PVY4 = element of submatrix [SVY]

SMR1, SMR2, SMR3, SMR4 = element of submatrix [KMR]

SMY1, SMY2, SMY3, SMY4 = element of submatrix [KMY]

SVR1, SVR2, SVR3, SVR4 = element of submatrix [KVR]

SVY1, SVY2, SVY3, SVY4 = element of submatrix [KVY]

XMR1, XMR2, XMR3, XMR4 = element of submatrix [MMR]

XMY1, XMY2, XMY3, XMY4 = element of submatrix [MMY]

XVR1, XVR2, XVR3, XVR4 = element of submatrix [MVR]

XVY1, XVY2, XVY3, XVY4 = element of submatrix [MVY]

FLOW CHART OF ELASTIC DYNAMIC RESPONSE PROGRAM

START

↓

READ AND WRITE INPUT

↓

SET UP MEMBER MASS MATRIX $[m_{ij}]$
AND MEMBER STIFFNESS MATRIX $[k_{ij}]$

↓

SET UP STRUCTURAL MASS MATRIX [M]
AND STRUCTURAL STIFFNESS MATRIX [K]

↓

INVERT MASS MATRIX [M]

↓

CALCULATE SA^T FOR [M] AND [K]

↓

DEFINE AND WRITE INITIAL CONDITIONS

↓

DO 9999 KK = 2, NPTS

↓

FIND AXIAL FORCE IN COLUMNS

↓

SET UP MEMBER STABILITY MATRIX $[s_{ij}]$

↓

SET UP STRUCTURAL STABILITY MATRIX [S]

↓

CALCULATE SA^T FOR [S]

↓

CALL SUBROUTINE GFMKP

↓

CALCULATE $\{X\}$, $\{\dot{X}\}$, $\{\ddot{X}\}$ BY RUNGE-KUTTA METHOD

↓

CALCULATE INTERNAL FORCES

↓

T = T + DT

↓

9999 CONTINUE

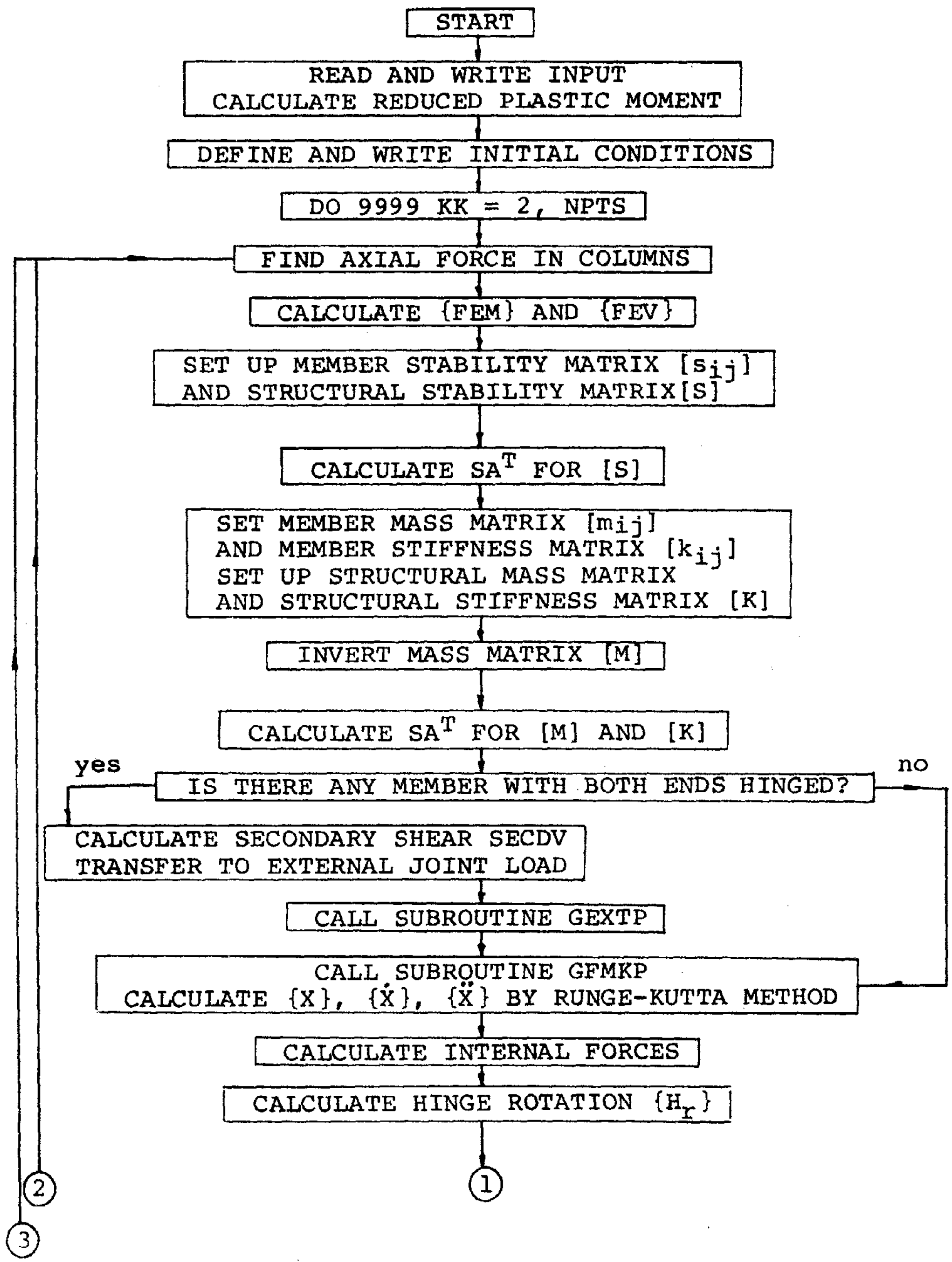
↓

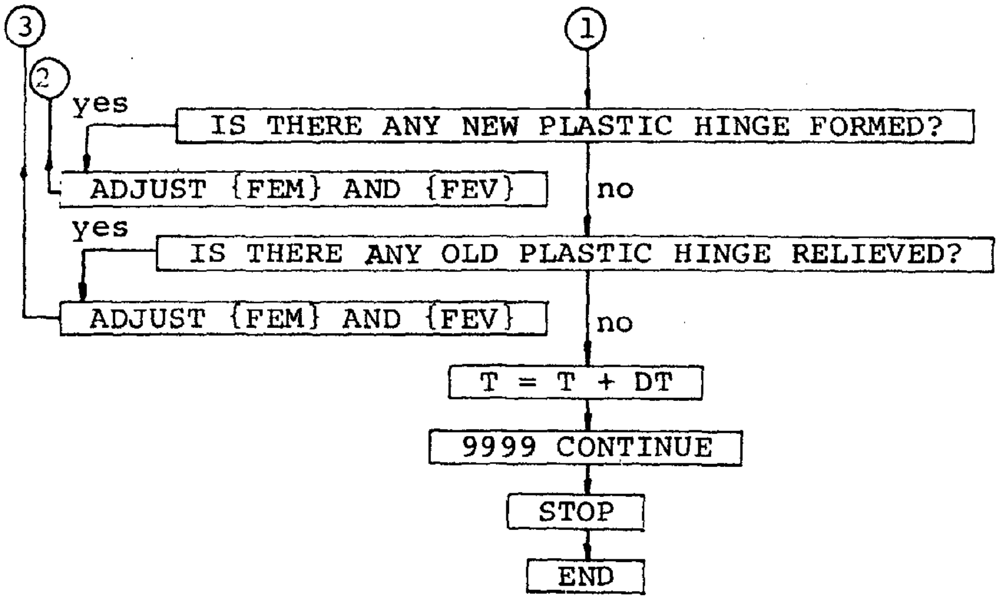
STOP

↓

END

FLOW CHART OF ELASTO-PLASTIC DYNAMIC RESPONSE PROGRAM




3
1
2
yes
IS THERE ANY NEW PLASTIC HINGE FORMED?
ADJUST {FEM} AND {FEV}
no
yes
IS THERE ANY OLD PLASTIC HINGE RELIEVED?
ADJUST {FEM} AND {FEV}
no
T = T + DT
9999 CONTINUE
STOP
END

```
*    ELASTIC DYNAMIC RESPONSE                                   *
      DIMENSION ALPHA(10),PAB(10),BETA(10),RATIO(18)
      DIMENSION AM(12,18),AMS(3,3),AV(12,18)
      DIMENSION XL(10),XI(10),XM(10)
      DIMENSION NPH(18)
      DIMENSION ASAT(10,10),INDEX(50)
      DIMENSION XXK(10,10),XXM(10,10),XXP(10,10)
      DIMENSION SMR1(10),SMR2(10),SMY1(10),SMY2(10)
      DIMENSION SMR3(10),SMR4(10),SMY3(10),SMY4(10)
      DIMENSION XMR1(10),XMR2(10),XMY1(10),XMY2(10)
      DIMENSION XMR3(10),XMR4(10),XMY3(10),XMY4(10)
      DIMENSION PMR1(10),PMR2(10),PMY1(10),PMY2(10)
      DIMENSION PMR3(10),PMR4(10),PMY3(10),PMY4(10)
      DIMENSION SVR1(10),SVR2(10),SVY1(10),SVY2(10)
      DIMENSION SVR3(10),SVR4(10),SVY3(10),SVY4(10)
      DIMENSION PVR1(10),PVR2(10),PVY1(10),PVY2(10)
      DIMENSION PVR3(10),PVR4(10),PVY3(10),PVY4(10)
      DIMENSION XVR1(10),XVR2(10),XVY1(10),XVY2(10)
      DIMENSION XVR3(10),XVR4(10),XVY3(10),XVY4(10)
      DIMENSION XMP(18,10),XVP(18,10),XEM(18)
      DIMENSION XMK(18,10),XMM(18,10),XVK(18,10)
      DIMENSION A(10),B(10),C(10),D(10),XMI(10,10)
      DIMENSION RX(10),RXT(10),RXTT(10)
      DIMENSION X(10),XT(10),XTT(10)
      DIMENSION XA(10),XB(10),XC(10),XD(10)
      DIMENSION XTA(10),XTB(10),XTC(10),XTD(10)
      DIMENSION XAC(10),XTAC(10)
      DIMENSION XEV(18),XVM(18,10),XEVM(18)
      DIMENSION XEMPK(18),XEVPK(18),XEMM(18)
      DIMENSION AF(10),AFV(10,18),AFP(10,18),PT(10)
      DIMENSION AFVV(10),AFPP(10)
      DIMENSION FMV1(10),FMV2(10),FMV3(10),FMV4(10)
      DIMENSION AREA(10),FY(10),PY(10),RDPM(18)
      DIMENSION EDPM(18),PM(18),ZP(10)
      DIMENSION AXIALF(18),PLIMIT(18),REDUC(18)
    1 READ(1,2) NO
      IF(NO) 52,52,3
    3 WRITE(3,1001)
      WRITE(3,500) NO
      READ(1,401) NM,NP,NPR,NPS,NVP
      NEM=2*NM
      READ(1,1009) (NPH(I),I=1,NEM)
      READ(1,400)(XL(I),I=1,NM)
      READ(1,400)(AREA(I),I=1,NM)
      READ(1,400)(XI(I),I=1,NM)
      READ(1,400)(XM(I),I=1,NM)
      READ(1,400)(ZP(I),I=1,NM)
      READ(1,400)(FY(I),I=1,NM)
      READ(1,400)(ALPHA(I),I=1,NM)
      READ(1,400)(BETA(I),I=1,NM)
      READ(1,601) PSB,XE
      READ(1,400) VA,VB,ZETA
```

```
      WRITE(3,700)
      WRITE(3,701)(XL(I),I=1,NM)
      WRITE(3,702)
      WRITF(3,701)(XI(I),I=1,NM)
      WRITE(3,703)
      WRITE(3,701)(XM(I),I=1,NM)
      WRITE(3,704)
      WRITE(3,701)(ALPHA(I),I=1,NM)
      WRITE(3,705)
      WRITE(3,701)(BETA(I),I=1,NM)
      WRITE(3,706)
      WRITE(3,701) PSB,XE
      WRITE(3,4321) VA,VB,ZETA
      WRITE(3,3348)
      DO 3346 I=1,NM
      PY(I)=AREA(I)*FY(I)
      PM(I)=ZP(I)*FY(I)
      WRITE(3,3347) I,AREA(I),FY(I),PY(I),ZP(I),
     &PM(I)
 3346 CONTINUE
      DO 402 I=1,NPR
      DO 402 J=1,NEM
  402 AM(I,J)=0.
      DO 407 I=1,NPS
      DO 407 J=1,NEM
  407 AV(I,J)=0.
      DO 414 I=1,NM
      DO 414 J=1,NEM
  414 AFV(I,J)=0.
      DO 415 I=1,NM
      DO 415 J=1,NVP
  415 AFP(I,J)=0.
  406 READ(1,403) I,J,AMIJ
      IF(I) 404,404,405
  405 AM(I,J)=AMIJ
      GO TO 406
  404 READ(1,403) I,J,AVIJ
      IF(I) 408,408,409
  409 AV(I,J)=AVIJ
      GO TO 404
  408 DO 410 I=1,NPS
      DO 410 J=1,NPS
  410 AMS(I,J)=0.
  413 READ(1,403) I,J,AMSIJ
      IF(I) 411,411,412
  412 AMS(I,J)=AMSIJ
      GO TO 413
  411 READ(1,403) I,J,AFVIJ
      IF(I) 417,417,416
  416 AFV(I,J)=AFVIJ
      GO TO 411
  417 READ(1,403) I,J,AFPIJ
```

```
      IF(I) 418,418,419
  419 AFP(I,J)=AFPIJ
      GO TO 417
  418 WRITE(3,650)
      WRITE(3,603) ((AM(I,J),J=1,NEM),I=1,NPR)
      WRITE(3,651)
      WRITE(3,603) ((AV(I,J),J=1,NEM),I=1,NPS)
      WRITE(3,652)
      WRITE(3,633)((AMS(I,J),J=1,NPS),I=1,NPS)
      WRITE(3,653)
      WRITE(3,603)((AFV(I,J),J=1,NEM),I=1,NM)
      WRITE(3,654)
      WRITE(3,603)((AFP(I,J),J=1,NVP),I=1,NM)
C     FORMULATE MASS & STIFF. MATRIX
      DO 1000 I=1,NM
      MN=I
      CALL        STIFFA(SMR1,SMR2,SMR3,SMR4,
     &SMY1,SMY2,SMY3,SMY4,SVR1,SVR2,SVR3,SVR4,
     &SVY1,SVY2,SVY3,SVY4,XMR1,XMR2,XMR3,XMR4,
     &XMY1,XMY2,XMY3,XMY4,XVR1,XVR2,XVR3,XVR4,
     &XVY1,XVY2,XVY3,XVY4,MN,XE,XI,XL,XM)
 1000 CONTINUE
C     SET UP XXM,XXK
      CALL        ASATA(NPR,NPS,NM,AM,AV,
     &SMR1,SMR2,SMR3,SMR4,SMY1,SMY2,SMY3,SMY4,
     &SVR1,SVR2,SVR3,SVR4,SVY1,SVY2,SVY3,SVY4,XXK)
      CALL        ASATB(NPR,NPS,NM,AM,AV,
     &XMR1,XMR2,XMR3,XMR4,XMY1,XMY2,XMY3,XMY4,
     &XVR1,XVR2,XVR3,XVR4,XVY1,XVY2,XVY3,XVY4,
     &AMS,XXM)
      CALL        ASATM(NP,XXM,XMI)
C     FORMULATE S*AT
      CALL        SATMV(NPR,NPS,NM,SMR1,SMR2,SMR3,
     &SMR4,SMY1,SMY2,SMY3,SMY4,AM,AV,XMK)
      CALL        SATMV(NPR,NPS,NM,SVR1,SVR2,SVR3,
     &SVR4,SVY1,SVY2,SVY3,SVY4,AM,AV,XVK)
      CALL        SATMV(NPR,NPS,NM,XMR1,XMR2,XMR3,
     &XMR4,XMY1,XMY2,XMY3,XMY4,AM,AV,XMM)
      CALL        SATMV(NPR,NPS,NM,XVR1,XVR2,XVR3,
     &XVR4,XVY1,XVY2,XVY3,XVY4,AM,AV,XVM)
C     DEFINE THE INITIAL CONDITION
      READ(1,900)(X(I),I=1,NP)
      READ(1,900)(XT(I),I=1,NP)
      READ(1,900)(XTT(I),I=1,NP)
      DO 671 I=1,NEM
      XEV(I)=0.
  671 CONTINUE
      T=0.
      NPTS=201
      WRITE(3,901)T
      DO 9000 I=1,NP
      WRITE(3,903) X(I),XT(I),XTT(I)
```

```
 9000 CONTINUE
      DT=0.002
      DO 9999 KK=2,NPTS
      DO 930 I=1,NP
      RX(I)=X(I)
      RXT(I)=XT(I)
      RXTT(I)=XTT(I)
  930 CONTINUE
      RT=T
C     FIND AXIAL FORCE
      ZT=ZETA*T
      CZT=COS(ZT)
      DO 655 I=1,NVP
      PT(I)=VA*PSB+VB*PSB*CZT
  655 CONTINUE
      DO 666 I=1,NM
      AFVV(I)=0.
      DO 666 J=1,NEM
      AFVV(I)=AFVV(I)+AFV(I,J)*XEV(J)
  666 CONTINUE
      DO 667 I=1,NM
      AFPP(I)=0.
      DO 667 J=1,NVP
      AFPP(I)=AFPP(I)+AFP(I,J)*PT(J)
  667 CONTINUE
      DO 668 I=1,NM
      AF(I)=AFVV(I)+AFPP(I)
  668 CONTINUE
      DO 1100 I=1,NM
      MN=I
      CALL        STIFPA(PMR1,PMR2,PMR3,PMR4,
     &PMY1,PMY2,PMY3,PMY4,PVR1,PVR2,PVR3,PVR4,
     &MN,XL,AF)
 1100 CONTINUE
      CALL        ASATA(NPR,NPS,NM,AM,AV,
     &PMR1,PMR2,PMR3,PMR4,PMY1,PMY2,PMY3,PMY4,
     &PVR1,PVR2,PVR3,PVR4,PVY1,PVY2,PVY3,PVY4,XXP)
C     CALCULATE  S*AT  FOR   P
      CALL        SATMV(NPR,NPS,NM,PMR1,PMR2,PMR3,
     &PMR4,PMY1,PMY2,PMY3,PMY4,AM,AV,XMP)
      CALL        SATMV(NPP,NPS,NM,PVR1,PVR2,PVR3,
     &PVR4,PVY1,PVY2,PVY3,PVY4,AM,AV,XVP)
C     CALCULATE A,B,C,D VECTOR
      DO 3001 I=1,NP
      XA(I)=RX(I)
      XTA(I)=RXT(I)
 3001 CONTINUE
      TA=RT
      CALL GFMKP(TA,DT,NP,NPR,VA,VB,ZETA,PSB,XA,XXP,
     &XXK,XMI,A)
      TB=RT+DT/2.
      DO 931 I=1,NP
```

```
      XB(I)=RX(I)+(DT/2.)*RXT(I)
      XTB(I)=RXT(I)+0.5*A(I)
  931 CONTINUE
      CALL GFMKP(TB,DT,NP,NPR,VA,VB,ZETA,PSB,XB,XXP,
     &XXK,XMI,B)
      TC=RT+DT/2.
      DO 934 I=1,NP
      XC(I)=RX(I)+(DT/2.)*(RXT(I))+(DT/4.)*(A(I))
      XTC(I)=RXT(I)+0.5*B(I)
  934 CONTINUE
      CALL GFMKP(TC,DT,NP,NPR,VA,VB,ZETA,PSB,XC,XXP,
     &XXK,XMI,C)
      TD=RT+DT
      DO 936 I=1,NP
      XD(I)=RX(I)+DT*RXT(I)+(DT/2.)*B(I)
      XTD(I)=RXT(I)+C(I)
  936 CONTINUE
      CALL GFMKP(TD,DT,NP,NPR,VA,VB,ZETA,PSB,XD,XXP,
     &XXK,XMI,D)
      DO 938 I=1,NP
      X(I)=RX(I)+DT*RXT(I)+(DT/6.)*(A(I)+B(I)+C(I))
      XT(I)=RXT(I)+(1./6.)*(A(I)+2.*B(I)+2.*C(I)+
     &D(I))
  938 CONTINUE
      TAC=RT+DT
      DO 939 I=1,NP
      XAC(I)=X(I)
      XTAC(I)=XT(I)
  939 CONTINUE
      CALL GFMKP(TAC,DT,NP,NPR,VA,VB,ZETA,PSB,XAC,
     &XXP,XXK,XMI,XTT)
      DO 941 I=1,NP
      XTT(I)=XTT(I)/DT
  941 CONTINUE
      T=RT+DT
      WRITE(3,901)T
      DO 9100 I=1,NP
      WRITE(3,903) X(I),XT(I),XTT(I)
 9100 CONTINUE
C     CALCULATE  END FORCES
      DO 890 I=1,NEM
      XEM(I)=0.
      XEV(I)=0.
      DO 890 J=1,NP
      XEM(I)=XEM(I)+(XMK(I,J)-XMP(I,J))*X(J)+
     &XMM(I,J)*XTT(J)
      XEV(I)=XEV(I)+(XVK(I,J)-XVP(I,J))*X(J)+
     &XVM(I,J)*XTT(J)
  890 CONTINUE
      WRITE(3,891) T
      DO 9001 I=1,NEM
      WRITE(3,892) I,XEM(I),XEV(I)
```

```
 9001 CONTINUE
      T=RT+DT
 9999 CONTINUE
    2 FORMAT(I5)
  400 FORMAT(6F10.4)
  401 FORMAT(5I5)
  403 FORMAT(2I5,F10.4)
  500 FORMAT(//10X,'NO. OF PROGRAMS =',I5)
  601 FORMAT(2F10.2)
  603 FORMAT(12F10.4)
  633 FORMAT(2F10.4)
  650 FORMAT(//10X,'AM    MATRIX')
  651 FORMAT(//10X,'AV   MATRIX')
  652 FORMAT(//10X,'AMS MATRIX')
  653 FORMAT(//10X,'AFV    MATRIX')
  654 FORMAT(//10X,'AFP    MATRIX')
  700 FORMAT(//10X,'MEMBER LENGTH')
  701 FORMAT(3E16.7)
  702 FORMAT(//10X,'MEMBER MOMENT INERTIA')
  703 FORMAT(//10X,'MEMBER MASS')
  704 FORMAT(//10X,'ALPHA VALUE')
  705 FORMAT(//10X,'BETA VALUE')
  706 FORMAT(//10X,'LOAD P AND ELASTIC MODULUS')
  892 FORMAT(//2X,'PT',I2,2X,E16.7,4X,E16.7,4X,E16.7
     &,4X,E16.7,4X,E16.7)
  891 FORMAT(//10X,'END MOMENT,END SHEAR AT TIME=',
     &F10.7)
  900 FORMAT(6F10.4)
  903 FORMAT(//10X,E16.7,10X,E16.7,10X,E16.7)
 1001 FORMAT(1H1)
  901 FORMAT(//10X,'X,XT,XTT AT TIME  T=',F10.7)
 1009 FORMAT(6I5)
 3347 FORMAT(//10X,I5,5E16.7)
 3348 FORMAT(//10X,'MEMBER NO.',10X,'AREA',10X,'FY',
     &10X,'PY',10X,'ZP',10X,'PM')
 4321 FORMAT(//10X,'VA=',F10.4,5X,'VB=',F10.4,5X,
     &'ZETA=',F10.4)
   52 STOP
      END
```

```
*   ELASTO-PLASTIC DYNAMIC RESPONSE                              *
      DIMENSION AM(12,12),AMS(4,4),AV(12,12)
      DIMENSION ASAT(10,10),INDEX(50),XL(10),XI(10)
      DIMENSION ALPHA(10),PAB(10),BETA(10),XM(10)
      DIMENSION XXK(10,10),XXM(10,10),XXP(10,10)
      DIMENSION SMR1(10),SMR2(10),SMY1(10),SMY2(10)
      DIMENSION SMR3(10),SMR4(10),SMY3(10),SMY4(10)
      DIMENSION XMR1(10),XMR2(10),XMY1(10),XMY2(10)
      DIMENSION XMR3(10),XMR4(10),XMY3(10),XMY4(10)
      DIMENSION PMR1(10),PMR2(10),PMY1(10),PMY2(10)
      DIMENSION PMR3(10),PMR4(10),PMY3(10),PMY4(10)
      DIMENSION SVR1(10),SVR2(10),SVY1(10),SVY2(10)
      DIMENSION SVR3(10),SVR4(10),SVY3(10),SVY4(10)
      DIMENSION PVR1(10),PVR2(10),PVY1(10),PVY2(10)
      DIMENSION PVR3(10),PVR4(10),PVY3(10),PVY4(10)
      DIMENSION XVR1(10),XVR2(10),XVY1(10),XVY2(10)
      DIMENSION XVR3(10),XVR4(10),XVY3(10),XVY4(10)
      DIMENSION XMP(18,10),XVP(18,10),XEM(18)
      DIMENSION XMK(18,10),XMM(18,10),XVK(18,10)
      DIMENSION A(10),B(10),C(10),D(10),XMI(10,10)
      DIMENSION RX(10),RXT(10),RXTT(10)
      DIMENSION X(10),XT(10),XTT(10)
      DIMENSION XA(10),XB(10),XC(10),XD(10)
      DIMENSION XTA(10),XTB(10),XTC(10),XTD(10)
      DIMENSION XEV(18),XVM(18,10),XEVM(18)
      DIMENSION XEMPK(18),XEVPK(18),XEMM(18)
      DIMENSION DR(18),DY(18),FAVT(18,18),ENDR(18)
      DIMENSION FRM1(10),FRM2(10),FRM3(10),FRM4(10)
      DIMENSION FRY1(10),FRY2(10),FRY3(10),FRY4(10)
      DIMENSION AF(10),AFV(10,18),AFP(10,18),PT(10)
      DIMENSION FMV1(10),FMV2(10),FMV3(10),FMV4(10)
      DIMENSION FEM(12),FEV(12),PE(10),RSFT(10)
      DIMENSION AREA(10),FY(10),PY(10),RDPM(18)
      DIMENSION NRH(18),LPH(18),PRHR(18),NPH(18)
      DIMENSION LPHR(18),LPHRD(18),EDPM(18),PM(18)
      DIMENSION COFR(10),RXEV(20),MNPH(20),MNRH(20)
      DIMENSION SPVY1(10),SPVY2(10),SPVY3(10)
      DIMENSION SAVT(20,20),SECDV(20),SPVY4(10)
      DIMENSION DET(10),AFVV(10),AFPP(10)
      DIMENSION PSE(10),XS(10),ZP(10)
      DIMENSION AXIALF(18),PLIMIT(18)
      DIMENSION REDUC(20),HRATIO(18)
      DIMENSION XAC(10),XTAC(10)
      DIMENSION AMTX(18),HR(18)
    1 READ(1,2) NO
      IF(NO) 52,52,3
    3 WRITE(3,1001)
      WRITE(3,500) NO
      READ(1,401) NM,NP,NPR,NPS,NVP
      NEM=2*NM
      ALOWM=1.05
      ALOWP=0.30
```

```
      READ(1,400)(XL(I),I=1,NM)
      READ(1,400)(AREA(I),I=1,NM)
      READ(1,400)(XI(I),I=1,NM)
      READ(1,400)(XM(I),I=1,NM)
      READ(1,400)(ZP(I),I=1,NM)
      READ(1,400)(FY(I),I=1,NM)
      READ(1,400)(ALPHA(I),I=1,NM)
      READ(1,400)(BETA(I),I=1,NM)
      READ(1,601) PSB,XE
      READ(1,400) VA,VB,ZETA
      WRITE(3,700)
      WRITE(3,701)(XL(I),I=1,NM)
      WRITE(3,702)
      WRITE(3,701)(XI(I),I=1,NM)
      WRITE(3,703)
      WRITE(3,703)
      WRITE(3,701)(XM(I),I=1,NM)
      WRITE(3,704)
      WRITE(3,701)(ALPHA(I),I=1,NM)
      WRITE(3,705)
      WRITE(3,701)(BETA(I),I=1,NM)
      WRITE(3,706)
      WRITE(3,701) PSB,XE
      WRITE(3,4321) VA,VB,ZETA
      WRITE(3,3348)
      DO 3346 I=1,NM
      PY(I)=AREA(I)*FY(I)
      PM(I)=ZP(I)*FY(I)
      WRITE(3,3347) I,AREA(I),FY(I),PY(I),ZP(I),
     &PM(I)
 3346 CONTINUE
C     CALCULATE REDUCED PLASTIC MOMENT
      DO 1007 I=1,NM
      AXIALF(I)=(ALPHA(I)*VA+BETA(I)*VB)*PSB
      PLIMIT(I)=0.15*PY(I)
      REDUC(I)=((ALPHA(I)*VA+BETA(I)*VB)*PSB)/PY(I)
      RDPM(I)=PM(I)*(1.-REDUC(I)*REDUC(I))
      IL=2*I-1
      JR=2*I
      EDPM(IL)=RDPM(I)
      EDPM(JR)=RDPM(I)
 1007 CONTINUE
      WRITE(3,3349)
      DO 3447 I=1,NM
      WRITE(3,3347) I,AXIALF(I),PLIMIT(I),RDPM(I)
 3447 CONTINUE
      DO 402 I=1,NPR
      DO 402 J=1,NEM
  402 AM(I,J)=0.
      DO 407 I=1,NPS
      DO 407 J=1,NEM
  407 AV(I,J)=0.
```

```
      DO 414 I=1,NM
      DO 414 J=1,NEM
  414 AFV(I,J)=0.
      DO 415 I=1,NM
      DO 415 J=1,NVP
  415 AFP(I,J)=0.
  406 READ(1,403) I,J,AMIJ
      IF(I) 404,404,405
  405 AM(I,J)=AMIJ
      GO TO 406
  404 READ(1,403) I,J,AVIJ
      IF(I) 408,408,409
  409 AV(I,J)=AVIJ
      GO TO 404
  408 DO 410 I=1,NPS
      DO 410 J=1,NPS
  410 AMS(I,J)=0.
  413 READ(1,403) I,J,AMSIJ
      IF(I) 411,411,412
  412 AMS(I,J)=AMSIJ
      GO TO 413
  411 READ(1,403) I,J,AFVIJ
      IF(I) 417,417,416
  416 AFV(I,J)=AFVIJ
      GO TO 411
  417 READ(1,403) I,J,AFPIJ
      IF(I) 418,418,419
  419 AFP(I,J)=AFPIJ
      GO TO 417
  418 WRITE(3,650)
      WRITE(3,603)  ((AM(I,J),J=1,NEM),I=1,NPR)
      WRITE(3,651)
      WRITE(3,603)  ((AV(I,J),J=1,NEM),I=1,NPS)
      WRITE(3,652)
      WRITE(3,633)((AMS(I,J),J=1,NPS),I=1,NPS)
      WRITE(3,653)
      WRITE(3,603)((AFV(I,J),J=1,NEM),I=1,NM)
      WRITE(3,654)
      WRITE(3,603)((AFP(I,J),J=1,NVP),I=1,NM)
      DO 561 I=1,NM
      FMV1(I)=-1./XL(I)
      FMV2(I)=-1./XL(I)
      FMV3(I)=-1./XL(I)
      FMV4(I)=-1./XL(I)
  561 CONTINUE
C     DEFINE THE INITIAL CONDITION
      DO 567 I=1,NEM
      NPH(I)=0
      MNPH(I)=0
      NRH(I)=0
      MNRH(I)=0
      LPH(I)=0
```

```
      PRHR(I)=0.
      HRATIO(I)=0.
      LPHRD(I)=0
  567 CONTINUE
      DO 4446 I=1,NPS
      XS(I)=0.
 4446 CONTINUE
      DO 6001 I=1,NEM
      SECDV(I)=0.
 6001 CONTINUE
      DO 9449 I=1,NP
      PSE(I)=0.
 9449 CONTINUE
      DO 560 I=1,NEM
      FEV(I)=0.
      FEM(I)=0.
  560 CONTINUE
      READ(1,900)(X(I),I=1,NP)
      READ(1,900)(XT(I),I=1,NP)
      READ(1,900)(XTT(I),I=1,NP)
      DO 671 I=1,NEM
      XEV(I)=0.
  671 CONTINUE
      WRITE(3,674)
      WRITE(3,900)(XEV(I),I=1,NEM)
      T=0.
      DT=0.004
      KZERO=0
      WRITE(3,901)T
      DO 9000 I=1,NP
      WRITE(3,903) X(I),XT(I),XTT(I)
 9000 CONTINUE
      WRITE(3,1920)(LPH(I),I=1,NEM)
      NPTS=51
      DO 9999 KK=2,NPTS
 1599 WRITE(3,1001)
      WRITE(3,1919) (LPH(I),I=1,NEM)
      DO 930 I=1,NP
      RX(I)=X(I)
      RXT(I)=XT(I)
      RXTT(I)=XTT(I)
  930 CONTINUE
      DO 5682 J=1,NEM
      RXEV(J)=XEV(J)
 5682 CONTINUE
      RT=T
      PRT=RT
      WRITE(3,901) PRT
      DO 1972 I=1,NP
      WRITE(3,903) RX(I),RXT(I),RXTT(I)
 1972 CONTINUE
C     FIND AXIAL FORCE
```

```
      ZT=ZETA*T
      CZT=COS(ZT)
      DO 655 I=1,NVP
      PT(I)=VA*PSB+VB*PSB*CZT
  655 CONTINUE
      DO 666 I=1,NM
      AFVV(I)=0.
      DO 666 J=1,NEM
      AFVV(I)=AFVV(I)+AFV(I,J)*XEV(J)
  666 CONTINUE
      DO 667 I=1,NM
      AFPP(I)=0.
      DO 667 J=1,NVP
      AFPP(I)=AFPP(I)+AFP(I,J)*PT(J)
  667 CONTINUE
      DO 668 I=1,NM
      AF(I)=AFVV(I)+AFPP(I)
  668 CONTINUE
C     CALCULATE  CARRY OVER FACTOR
      DO 1601 I=1,NM
      COFR(I)=(2.*XE*XI(I)/XL(I)+AF(I)*XL(I)/30.)/
     &(4.*XE*XI(I)/XL(I)-2.*AF(I)*XL(I)/15.)
 1601 CONTINUE
      DO 1020 I=1,NM
      IL=2*I-1
      JR=2*I
      MIL=LPH(IL)
      MJR=LPH(JR)
      IF(MIL) 2011,2011,2012
 2011 IF(MJR) 2013,2013,2014
 2013 FEM(IL)=0.
      FEM(JR)=0.
      GO TO 1020
 2014 FEM(IL)=FEM(JR)*COFR(I)
      FEM(JR)=FEM(JR)
      GO TO 1020
 2012 IF(MJR) 2015,2015,2016
 2015 FEM(IL)=FEM(IL)
      FEM(JR)=FEM(IL)*COFR(I)
      GO TO 1020
 2016 FEM(IL)=FEM(IL)
      FEM(JR)=FEM(JR)
 1020 CONTINUE
C     FINDED END SHEAR
      DO 562 K=1,NM
      L=2*K-1
      M=2*K
      FEV(L)=FMV1(K)*FEM(L)+FMV2(K)*FEM(M)
      FEV(M)=FMV3(K)*FEM(L)+FMV4(K)*FEM(M)
  562 CONTINUE
      DO 1000 I=1,NM
      MN=I
```

```
      IL=2*I-1
      JR=2*I
      NIL=LPH(IL)
      NJR=LPH(JR)
      IF(NIL)1011,1011,1002
 1011 IF(NJR) 1003,1003,1004
 1003 CALL        STIFPA(PMR1,PMR2,PMR3,PMR4,
     &PMY1,PMY2,PMY3,PMY4,PVR1,PVR2,PVR3,PVR4,
     &PVY1,PVY2,PVY3,PVY4,MN,XL,AF)
      GO TO 1000
 1004 CALL        STIFPB(PMR1,PMR2,PMR3,PMR4,
     &PMY1,PMY2,PMY3,PMY4,PVR1,PVR2,PVR3,PVR4,
     &PVY1,PVY2,PVY3,PVY4,MN,XL,AF)
      GO TO 1000
 1002 IF(NJR) 1005,1005,1006
 1005 CALL        STIFPC(PMR1,PMR2,PMR3,PMR4,
     &PMY1,PMY2,PMY3,PMY4,PVR1,PVR2,PVR3,PVR4,
     &PVY1,PVY2,PVY3,PVY4,MN,XL,AF)
      GO TO 1000
 1006 CALL        STIFPD(PMR1,PMR2,PMR3,PMR4,
     &PMY1,PMY2,PMY3,PMY4,PVR1,PVR2,PVR3,PVR4,
     &PVY1,PVY2,PVY3,PVY4,MN,XL,AF)
 1000 CONTINUE
      CALL        ASATA(NPR,NPS,NM,AM,AV,
     &PMR1,PMR2,PMR3,PMR4,PMY1,PMY2,PMY3,PMY4,
     &PVR1,PVR2,PVR3,PVR4,PVY1,PVY2,PVY3,PVY4,XXP)
      CALL        SATMV(NPR,NPS,NM,PMR1,PMR2,PMR3,
     &PMR4,PMY1,PMY2,PMY3,PMY4,AM,AV,XMP)
      CALL        SATMV(NPR,NPS,NM,PVR1,PVR2,PVR3,
     &PVR4,PVY1,PVY2,PVY3,PVY4,AM,AV.XVP)
      DO 8000 I=1,NM
      MN=I
      IL=2*I-1
      JR=2*I
      IF(KZERO.EQ.0) GO TO 8101
      IF(LPHRD(IL)-LPH(IL)) 8101,8102,8101
 8102 IF(LPHRD(JR)-LPH(JR)) 8101,8000,8101
 8101 NIL=LPH(IL)
      NJR=LPH(JR)
      IF(NIL)8011,8011,8002
 8011 IF(NJR) 8003,8003,8004
 8003 CALL        STIFFA(SMR1,SMR2,SMR3,SMR4,
     &SMY1,SMY2,SMY3,SMY4,SVR1,SVR2,SVR3,SVR4,
     &SVY1,SVY2,SVY3,SVY4,XMR1,XMR2,XMR3,XMR4,
     &XMY1,XMY2,XMY3,XMY4,XVR1,XVR2,XVR3,XVR4,
     &XVY1,XVY2,XVY3,XVY4,MN,XE,XI,XL,XM)
      GO TO 8000
 8004 CALL        STIFFB(SMR1,SMR2,SMR3,SMR4,
     &SMY1,SMY2,SMY3,SMY4,SVR1,SVR2,SVR3,SVR4,
     &SVY1,SVY2,SVY3,SVY4,XMR1,XMR2,XMR3,XMR4,
     &XMY1,XMY2,XMY3,XMY4,XVR1,XVR2,XVR3,XVR4,
     &XVY1,XVY2,XVY3,XVY4,MN,XE,XI,XL,XM)
```

```
          GO TO 8000
     8002 IF(NJR) 8005,8005,8006
     8005 CALL          STIFFC(SMR1,SMR2,SMR3,SMR4,
         &SMY1,SMY2,SMY3,SMY4,SVR1,SVR2,SVR3,SVR4,
         &SVY1,SVY2,SVY3,SVY4,XMR1,XMR2,XMR3,XMR4,
         &XMY1,XMY2,XMY3,XMY4,XVR1,XVR2,XVR3,XVR4,
         &XVY1,XVY2,XVY3,XVY4,MN,XE,XI,XL,XM)
          GO TO 8000
     8006 CALL          STIFFD(SMR1,SMR2,SMR3,SMR4,
         &SMY1,SMY2,SMY3,SMY4,SVR1,SVR2,SVR3,SVR4,
         &SVY1,SVY2,SVY3,SVY4,XMR1,XMR2,XMR3,XMR4,
         &XMY1,XMY2,XMY3,XMY4,XVR1,XVR2,XVR3,XVR4,
         &XVY1,XVY2,XVY3,XVY4,MN,XE,XI,XL,XM)
     8000 CONTINUE
          IF(KZERO.EQ.0) GO TO 8204
          DO 8201 I=1,NEM
          IF(LPHRD(I)-LPH(I)) 8204,8201,8204
     8201 CONTINUE
          GO TO 8203
     8204 CALL          ASATA(NPR,NPS,NM,AM,AV,
         &SMR1,SMR2,SMR3,SMR4,SMY1,SMY2,SMY3,SMY4,
         &SVR1,SVR2,SVR3,SVR4,SVY1,SVY2,SVY3,SVY4,XXK)
          CALL          ASATB(NPR,NPS,NM,AM,AV,
         &XMR1,XMR2,XMR3,XMR4,XMY1,XMY2,XMY3,XMY4,
         &XVR1,XVR2,XVR3,XVR4,XVY1,XVY2,XVY3,XVY4,
         &AMS,XXM)
          CALL          ASATM(NP,XXM,XMI)
          CALL          SATMV(NPR,NPS,NM,SMR1,SMR2,SMR3,
         &SMR4,SMY1,SMY2,SMY3,SMY4,AM,AV,XMK)
          CALL          SATMV(NPR,NPS,NM,SVR1,SVR2,SVR3,
         &SVR4,SVY1,SVY2,SVY3,SVY4,AM,AV,XVK)
          CALL          SATMV(NPR,NPS,NM,XMR1,XMR2,XMR3,
         &XMR4,XMY1,XMY2,XMY3,XMY4,AM,AV,XMM)
          CALL          SATMV(NPR,NPS,NM,XVR1,XVR2,XVR3,
         &XVR4,XVY1,XVY2,XVY3,XVY4,AM,AV,XVM)
C         RECORD LPH(I)
     8203 DO 1081 I=1,NEM
          LPHRD(I)=LPH(I)
     1081 CONTINUE
C         CHECK IF THERE IS ANY MEMBER BOTH ENDS HINGED
     4444 DO 5011 I=1,NM
          IL=2*I-1
          JR=2*I
          NMIL=LPH(IL)
          NMJR=LPH(JR)
          IF(NMIL) 5011,5011,5012
     5012 IF(NMJR) 5011,5011,5013
     5011 CONTINUE
          GO TO 4445
C         CALCULATE SECONDARY SHEAR
     5013 DO 4020 I=1,NM
          IL=2*I-1
```

```
      JR=2*I
      MMIL=LPH(IL)
      MMJR=LPH(JR)
      IF(MMIL) 4012,4012,4011
 4011 IF(MMJR) 4012,4012,4013
 4013 SPVY1(I)=AF(I)*(-1./XL(I))
      SPVY2(I)=AF(I)*(-1./XL(I))
      SPVY3(I)=AF(I)*(-1./XL(I))
      SPVY4(I)=AF(I)*(-1./XL(I))
      GO TO 4020
 4012 SPVY1(I)=AF(I)*0.
      SPVY2(I)=AF(I)*0.
      SPVY3(I)=AF(I)*0.
      SPVY4(I)=AF(I)*0.
 4020 CONTINUE
      DO 4470 J=1,NPS
      DO 4470 K=1,NM
      L=2*K-1
      M=2*K
      SAVT(M,J)=SPVY3(K)*AV(J,L)+SPVY4(K)*AV(J,M)
      SAVT(L,J)=SPVY1(K)*AV(J,L)+SPVY2(K)*AV(J,M)
 4470 CONTINUE
      DO 4480 I=1,NEM
      SECDV(I)=0.
      DO 4480 J=1,NPS
      SECDV(I)=SECDV(I)+SAVT(I,J)*XS(J)
 4480 CONTINUE
C     TRANSFER SECONDARY SHEAR TO EXTERNAL JOINT
      DO 4564 I=1,NPS
      II=I+NPR
      PSE(II)=0.
      DO 4564 J=1,NEM
      PSE(II)=PSE(II)+AV(I,J)*SECDV(J)
 4564 CONTINUE
      WRITE(3,903)(PSE(LL),LL=1,NP)
 4445 CALL  GEXTP(AM,AV,NP,NPR,NM,FEM,FEV,PSE,RSFT)
C     CALCULATE A,B,C,D VECTOR
      DO 3001 I=1,NP
      XA(I)=RX(I)
      XTA(I)=RXT(I)
 3001 CONTINUE
      TA=RT
      CALL GEMKP(TA,DT,NP,NPR,VA,VB,ZETA,PSB,XA,XXP,
     &XXK,XMI,RSFT,A)
      TB=RT+DT/2.
      DO 931 I=1,NP
      XB(I)=RX(I)+(DT/2.)*RXT(I)
      XTB(I)=RXT(I)+0.5*A(I)
  931 CONTINUE
      CALL GEMKP(TB,DT,NP,NPR,VA,VB,ZETA,PSB,XB,XXP,
     &XXK,XMI,RSFT,B)
      TC=RT+DT/2.
```

```
      DO 934 I=1,NP
      XC(I)=RX(I)+(DT/2.)*(RXT(I))+(DT/4.)*(A(I))
      XTC(I)=RXT(I)+0.5*B(I)
  934 CONTINUE
      CALL GFMKP(TC,DT,NP,NPR,VA,VB,ZETA,PSB,XC,XXP,
     &XXK,XMI,RSFT,C)
      TD=RT+DT
      DO 936 I=1,NP
      XD(I)=RX(I)+DT*RXT(I)+(DT/2.)*B(I)
      XTD(I)=RXT(I)+C(I)
  936 CONTINUE
      CALL GFMKP(TD,DT,NP,NPR,VA,VB,ZETA,PSB,XD,XXP,
     &XXK,XMI,RSFT,D)
      DO 938 I=1,NP
      X(I)=RX(I)+DT*RXT(I)+(DT/6.)*(A(I)+B(I)+C(I))
      XT(I)=RXT(I)+(1./6.)*(A(I)+2.*B(I)+2.*C(I)+
     &D(I))
  938 CONTINUE
      TAC=RT+DT
      DO 939 I=1,NP
      XAC(I)=X(I)
      XTAC(I)=XT(I)
  939 CONTINUE
      CALL GFMKP(TAC,DT,NP,NPR,VA,VB,ZETA,PSB,XAC,
     &XXP,XXK,XMI,RSFT,XTT)
      DO 941 I=1,NP
      XTT(I)=XTT(I)/DT
  941 CONTINUE
      DO 7446 I=1,NPS
      II=I+NPR
      XS(I)=X(II)
 7446 CONTINUE
 4448 T=RT+DT
      WRITE(3,901)T
      DO 9100 I=1,NP
      WRITE(3,903) X(I),XT(I),XTT(I)
 9100 CONTINUE
C     CALCULATE  END FORCES
      DO 890 I=1,NEM
      XEMPK(I)=0.
      XEVPK(I)=0.
      XEMM(I)=0.
      XEVM(I)=0.
      DO 890 J=1,NP
      XEMPK(I)=XEMPK(I)+(XMK(I,J)-XMP(I,J))*X(J)
      XEVPK(I)=XEVPK(I)+(XVK(I,J)-XVP(I,J))*X(J)
      XEMM(I)=XEMM(I)+XMM(I,J)*XTT(J)
      XEVM(I)=XEVM(I)+XVM(I,J)*XTT(J)
  890 CONTINUE
      DO 1890 I=1,NEM
      XEMPK(I)=XEMPK(I)+FEM(I)
      XEVPK(I)=XEVPK(I)+FEV(I)+SFCOV(I)
```

```
 1490 CONTINUE
C      DEFORMATION  CHECK
      DO 450 I=1,NM
      DET(I)=(6.*XE*XI(I)/XL(I)-AF(I)*XL(I)/10.)*
     1(2.*XE*XI(I)/XL(I)-AF(I)*XL(I)/6.)
      FRM1(I)=((4.*XE*XI(I)/XL(I))-(2.*AF(I)*XL(I)/
     &15.))/DET(I)
      FRM4(I)=((4.*XE*XI(I)/XL(I))-(2.*AF(I)*XL(I)/
     &15.))/DET(I)
      FRM2(I)=-((2.*XE*XI(I)/XL(I))+(1.*AF(I)*XL(I)/
     &30.))/DET(I)
      FRM3(I)=-((2.*XE*XI(I)/XL(I))+(1.*AF(I)*XL(I)/
     &30.))/DET(I)
      FRY1(I)=-1./XL(I)
      FRY2(I)=-1./XL(I)
      FRY3(I)=-1./XL(I)
      FRY4(I)=-1./XL(I)
  450 CONTINUE
      DO 460 K=1,NM
      L=2*K-1
      M=2*K
      DR(L)=FRM1(K)*XEMPK(L)+FRM2(K)*XEMPK(M)
      DR(M)=FRM3(K)*XEMPK(L)+FRM4(K)*XEMPK(M)
  460 CONTINUE
      DO 470 J=1,NPS
      DO 470 K=1,NM
      L=2*K-1
      M=2*K
      FAVT(L,J)=FRY1(K)*AV(J,L)+FRY2(K)*AV(J,M)
      FAVT(M,J)=FRY3(K)*AV(J,L)+FRY4(K)*AV(J,M)
  470 CONTINUE
      DO 480 I=1,NEM
      DY(I)=0.
      DO 480 J=1,NPS
      JJ=J+NPR
      DY(I)=DY(I)+FAVT(I,J)*X(JJ)
  480 CONTINUE
      DO 490 I=1,NEM
      ENDR(I)=DR(I)-DY(I)
  490 CONTINUE
      DO 491 I=1,NEM
      AMTX(I)=0.
      DO 499 J=1,NPR
      AMTX(I)=AMTX(I)+AM(J,I)*X(J)
  499 CONTINUE
      HR(I)=ENDR(I)-AMTX(I)
  491 CONTINUE
      DO 492 I=1,NEM
      IF(ABS(HR(I)).LE.0.0000001) HR(I)=0.
  492 CONTINUE
      WRITE(3,550)
      DO 551 I=1,NEM
```

```
      WRITE(3,552) I,HR(I),ENDR(I),AMTX(I),PRHP(I)
  551 CONTINUE
      DO 893 I=1,NEM
      XEM(I)=XEMPK(I)+XEMM(I)
      XEV(I)=XEVPK(I)+XEVM(I)
  893 CONTINUE
      WRITE(3,891) T
      DO 9001 I=1,NEM
      WRITE(3,892) I,XEM(I),XEV(I)
 9001 CONTINUE
C     CHECK : IS THERE ANY NEW PLASTIC HINGE FORMED
      DO 569 I=1,NEM
      IF(MNRH(I)) 7856,7857,7857
 7856 IF(ABS(XEM(I)/EDPM(I)).GT.ALOWM) NPH(I)=1
      GO TO 569
 7857 IF(LPH(I).EQ.1) GO TO 569
      IF(ABS(XEM(I)/EDPM(I)).GT.1) NPH(I)=1
  569 CONTINUE
      DO 590 I=1,NEM
      IF(NPH(I).EQ.0) GO TO 590
      LPH(I)=LPH(I)+NPH(I)
      MNPH(I)=NPH(I)
      IF(XEM(I).LT.0) GO TO 596
      KC=1
      GO TO 581
  596 KC=-1
  581 FEM(I)=KC*EDPM(I)
      NOM=(I+1)/2
      IF(I.EQ.2*NOM) GO TO 582
      IF(LPH(I+1).EQ.0) FEM(I+1)=FEM(I)
      GO TO 590
  582 IF(LPH(I-1).EQ.0) FEM(I-1)=FEM(I)
  590 CONTINUE
      DO 599 I=1,NEM
      IF(NPH(I).EQ.0) GO TO 599
      DO 2599 K=1,NEM
      NPH(K)=0
 2599 CONTINUE
      T=RT
      DO 5678 J=1,NP
      X(J)=RX(J)
      XT(J)=RXT(J)
      XTT(J)=RXTT(J)
 5678 CONTINUE
      DO 5680 J=1,NEM
      XEV(J)=RXEV(J)
 5680 CONTINUE
      DO 5683 J=1,NPS
      JJ=J+NPR
      XS(J)=RX(JJ)
 5683 CONTINUE
      GO TO 1599
```

```
  599 CONTINUE
C     CHECK  IS THERE ANY OLD PLASTIC HINGE RELIEVED
      DO 2005 I=1,NEM
      IF(MNPH(I).EQ.1) GO TO 2005
      IF(LPH(I).EQ.0) GO TO 2005
      IF(HR(I)) 2000,2001,2001
 2000 IF(PRHR(I)) 2004,2003,2003
 2001 IF(PRHR(I)) 2003,2004,2004
 2004 HRATIO(I)=(PRHR(I)-HR(I))/PRHR(I)
      IF(HRATIO(I).GT.ALOWR) GO TO 2003
      GO TO 2005
 2003 NRH(I)=-1
      MNRH(I)=NRH(I)
 2005 CONTINUE
      DO 571 I=1,NEM
      IF(NRH(I).EQ.0) GO TO 571
      WRITE(3,595) I,NRH(I)
      LPH(I)=LPH(I)+NRH(I)
      NOM=(I+1)/2
      IF(I.EQ.2*NOM) GO TO 572
      IF(LPH(I+1).EQ.1) GO TO 573
      FEM(I+1)=0.
      FEM(I)=0.
      GO TO 571
  573 FEM(I)=FEM(I+1)*COFR(NOM)
      GO TO 571
  572 IF(LPH(I-1).EQ.1) GO TO 574
      FEM(I-1)=0.
      FEM(I)=0.
      GO TO 571
  574 FEM(I)=FEM(I-1)*COFR(NOM)
  571 CONTINUE
      DO 594 I=1,NEM
      IF(NRH(I).EQ.0) GO TO 594
      DO 3599 J=1,NEM
      NRH(J)=0
 3599 CONTINUE
      T=RT
      DO 5679 K=1,NP
      X(K)=RX(K)
      XT(K)=RXT(K)
      XTT(K)=RXTT(K)
 5679 CONTINUE
      DO 5681 J=1,NEM
      XEV(J)=RXEV(J)
 5681 CONTINUE
      DO 5684 J=1,NPS
      JJ=J+NPR
      XS(J)=RX(JJ)
 5684 CONTINUE
      DO 5689 L=1,NP
      PSE(L)=0.
```

```
 5689 CONTINUE
      GO TO 1599
  594 CONTINUE
C     CHOOSE THE MAX HINGE ROTATION
      DO 583 I=1,NEM
      IF(LPH(I).EQ.0) GO TO 583
      IF(HR(I)) 585,586,586
  585 IF(PRHR(I)) 587,588,588
  586 IF(PRHR(I)) 588,587,587
  587 IF(ABS(HR(I)).GT.ABS(PRHR(I))) PRHR(I)=HR(I)
      GO TO 583
  588 PRHR(I)=HR(I)
  583 CONTINUE
      DO 580 I=1,NEM
      LPHR(I)=LPH(I)
      NPH(I)=0
      MNPH(I)=0
      NRH(I)=0
      MNRH(I)=0
  580 CONTINUE
      DO 5685 J=1,NPS
      JJ=J+NPR
      XS(J)=X(JJ)
 5685 CONTINUE
      KZERO=KK
      T=RT+DT
 9999 CONTINUE
    2 FORMAT(I5)
  400 FORMAT(6F10.4)
  401 FORMAT(5I5)
  403 FORMAT(2I5,F10.4)
  500 FORMAT(//10X,'NO. OF PROGRAMS =',I5)
  550 FORMAT(//5X,'HINGE ROTATION',18X,'HR',18X,'DF'
     &,10X,'AMTX',13X,'MAX.  PRHR')
  552 FORMAT(//10X,'POINT(',I5,')',10X,4E16.7)
  595 FORMAT(//10X,'PLASTIC HINGE RELIEVED AT POINT',
     &I5,2X,I5)
  601 FORMAT(2F10.2)
  603 FORMAT(12F10.4)
  633 FORMAT(2F10.4)
  650 FORMAT(//10X,'AM   MATRIX')
  651 FORMAT(//10X,'AV  MATRIX')
  652 FORMAT(//10X,'AMS MATRIX')
  653 FORMAT(//10X,'AFV   MATRIX')
  654 FORMAT(//10X,'AFP   MATRIX')
  674 FORMAT(//10X,'INITIAL XEV')
  700 FORMAT(//10X,'MEMBER LENGTH')
  701 FORMAT(3E16.7)
  702 FORMAT(//10X,'MEMBER MOMENT INERTIA')
  703 FORMAT(//10X,'MEMBER MASS')
  704 FORMAT(//10X,'ALPHA VALUE')
  705 FORMAT(//10X,'BETA VALUE')
```

```
  892 FORMAT(//2X,'PT',I5,E16.7,4X,E16.7,4X,E16.7,
     &4X,E16.7,4X,E16.7)
  706 FORMAT(//10X,'LOAD P AND ELASTIC MODULUS')
  901 FORMAT(//10X,'X,XT,XTT AT TIME  T=',F10.7)
  891 FORMAT(//10X,'END MOMENT,END SHEAR AT TIME=',
     &F10.7)
  900 FORMAT(6F10.4)
  903 FORMAT(//10X,E16.7,10X,E16.7,10X,E16.7)
 1001 FORMAT(1H1)
 1920 FORMAT(//10X,'PLASTIC HINGE LOCATION(INITIAL)'
     &,6I5)
 1919 FORMAT(//10X,'PLASTIC HINGE LOCATION',6I5)
 3348 FORMAT(//10X,'MEMBER NO.',10X,'AREA',10X,'FY',
     &10X,'PY',10X,'ZP',10X,'PM')
 4321 FORMAT(//10X,'VA=',F10.4,5X,'VB=',F10.4,5X,
     &'ZETA=',F10.4)
 3347 FORMAT(//10X,I5,5E16.7)
 3349 FORMAT(//10X,'NO. OF MEMBER',2X,'AXIALF',2X,
     &'PLIMIT',2X,'RDPM')
   52 STOP
      END

      SUBROUTINE ASATA(NPR,NPS,NM,AM,AV,
     &AMR1,AMR2,AMR3,AMR4,AMY1,AMY2,AMY3,AMY4,
     &AVR1,AVR2,AVR3,AVR4,AVY1,AVY2,AVY3,AVY4,XXA)
      DIMENSION AMR1(10),AMR2(10),AMR3(10),AMR4(10)
      DIMENSION AMY1(10),AMY2(10),AMY3(10),AMY4(10)
      DIMENSION AVR1(10),AVR2(10),AVR3(10),AVR4(10)
      DIMENSION AVY1(10),AVY2(10),AVY3(10),AVY4(10)
      DIMENSION AM(12,12),AV(12,12),XXA(10,10)
C     FORMULATE FRAME STIFFNESS & STABILITY MATRIX
      DO 419 I=1,NPR
      DO 419 J=1,NPR
      XXA(I,J)=0.
      DO 419 K=1,NM
      L=2*K-1
      M=2*K
      XXA(I,J)=XXA(I,J)+AM(I,L)*(AMR1(K)*AM(J,L)+
     &AMR2(K)*AM(J,M))+AM(I,M)*(AMR3(K)*AM(J,L)+
     &AMR4(K)*AM(J,M))
  419 CONTINUE
      DO 420 I=1,NPS
      DO 420 J=1,NPR
      II=I+NPR
      XXA(II,J)=0.
      DO 420 K=1,NM
      L=2*K-1
      M=2*K
      XXA(II,J)=XXA(II,J)+AV(I,L)*(AVR1(K)*AM(J,L)+
     &AVR2(K)*AM(J,M))+AV(I,M)*(AVR3(K)*AM(J,L)+
     &AVR4(K)*AM(J,M))
  420 CONTINUE
```

```
      DO 421 I=1,NPR
      DO 421 J=1,NPS
      JJ=J+NPR
      XXA(I,JJ)=0.
      DO 421 K=1,NM
      L=2*K-1
      M=2*K
      XXA(I,JJ)=XXA(I,JJ)+AM(I,L)*(AMY1(K)*AV(J,L)+
     &AMY2(K)*AV(J,M))+AM(I,M)*(AMY3(K)*AV(J,L)+
     &AMY4(K)*AV(J,M))
  421 CONTINUE
      DO 422 I=1,NPS
      DO 422 J=1,NPS
      II=I+NPR
      JJ=J+NPR
      XXA(II,JJ)=0.
      DO 422 K=1,NM
      L=2*K-1
      M=2*K
      XXA(II,JJ)=XXA(II,JJ)+AV(I,L)*(AVY1(K)*AV(J,L)
     &+AVY2(K)*AV(J,M))+AV(I,M)*(AVY3(K)*AV(J,L)+
     &AVY4(K)*AV(J,M))
  422 CONTINUE
      RETURN
      END

      SUBROUTINE ASATB(NPR,NPS,NM,AM,AV,AMR1,AMR2,
     &AMR3,AMR4,AMY1,AMY2,AMY3,AMY4,AVR1,AVR2,AVR3,
     &AVR4,AVY1,AVY2,AVY3,AVY4,AMS,XXA)
C     FORMULATE FRAME MASS MATRIX
      DIMENSION AMR1(10),AMR2(10),AMR3(10),AMR4(10)
      DIMENSION AMY1(10),AMY2(10),AMY3(10),AMY4(10)
      DIMENSION AVR1(10),AVR2(10),AVR3(10),AVR4(10)
      DIMENSION AVY1(10),AVY2(10),AVY3(10),AVY4(10)
      DIMENSION AM(12,12),AV(12,12),XXA(10,10)
      DIMENSION AMS(4,4)
      DO 419 I=1,NPR
      DO 419 J=1,NPR
      XXA(I,J)=0.
      DO 419 K=1,NM
      L=2*K-1
      M=2*K
      XXA(I,J)=XXA(I,J)+AM(I,L)*(AMR1(K)*AM(J,L)+
     &AMR2(K)*AM(J,M))+AM(I,M)*(AMR3(K)*AM(J,L)+
     &AMR4(K)*AM(J,M))
  419 CONTINUE
      DO 420 I=1,NPS
      DO 420 J=1,NPR
      II=I+NPR
      XXA(II,J)=0.
      DO 420 K=1,NM
      L=2*K-1
```

```
      M=2*K
      XXA(II,J)=XXA(II,J)+AV(I,L)*(AVR1(K)*AM(J,L)+
     &AVR2(K)*AM(J,M))+AV(I,M)*(AVR3(K)*AM(J,L)+
     &AVR4(K)*AM(J,M))
  420 CONTINUE
      DO 421 I=1,NPR
      DO 421 J=1,NPS
      JJ=J+NPR
      XXA(I,JJ)=0.
      DO 421 K=1,NM
      L=2*K-1
      M=2*K
      XXA(I,JJ)=XXA(I,JJ)+AM(I,L)*(AMY1(K)*AV(J,L)+
     &AMY2(K)*AV(J,M))+AM(I,M)*(AMY3(K)*AV(J,L)+
     &AMY4(K)*AV(J,M))
  421 CONTINUE
      DO 422 I=1,NPS
      DO 422 J=1,NPS
      II=I+NPR
      JJ=J+NPR
      XXA(II,JJ)=AMS(I,J)
      DO 422 K=1,NM
      L=2*K-1
      M=2*K
      XXA(II,JJ)=XXA(II,JJ)+AV(I,L)*(AVY1(K)*AV(J,L)
     &+AVY2(K)*AV(J,M))+AV(I,M)*(AVY3(K)*AV(J,L)+
     &AVY4(K)*AV(J,M))
  422 CONTINUE
      RETURN
      END

      SUBROUTINE ASATM(NP,ASAT,ASATI)
      DIMENSION ASAT(10,10),ASATI(10,10),INDEX(100)
      DO 16 I=1,NP
   16 INDEX(I)=0
   17 AMAX=-1.
      DO 18 I=1,NP
      IF (INDEX(I)) 18,19,18
   19 TEMP=ABS(ASAT(I,I))
      IF(TEMP-AMAX) 18,18,20
   20 ICOL=I
      AMAX=TEMP
   18 CONTINUE
      IF(AMAX) 21, 29, 22
   22 INDEX(ICOL)=1
      PIVOT=ASAT(ICOL,ICOL)
      ASAT(ICOL,ICOL)=1.0
      PIVOT=1./PIVOT
      DO 23 J=1,NP
   23 ASAT(ICOL,J)=ASAT(ICOL,J)*PIVOT
      DO 24 I=1,NP
      IF(I-ICOL) 25,24,25
```

```
   25 TEMP=ASAT(I,ICOL)
      ASAT(I,ICOL)=0.0
      DO 26 J=1,NP
   26 ASAT(I,J)=ASAT(I,J)-ASAT(ICOL,J)*TEMP
   24 CONTINUE
      GO TO 17
   21 DO 27 I=1,NP
      DO 27 J=1,NP
   27 ASATI(I,J)=ASAT(I,J)
      GO TO 28
   29 WRITE(3,100)
  100 FORMAT(//10X,'SINGULAR MATRIX OCCURS')
   28 RETURN
      END

      SUBROUTINE SATMV(NPR,NPS,NM,AMR1,AMR2,AMR3,
     &AMR4,AMY1,AMY2,AMY3,AMY4,AM,AV,AMK)
C     FORMULATE S*AT
      DIMENSION AM(12,18),AV(12,18),AMK(18,10)
      DIMENSION AMR1(10),AMR2(10),AMR3(10),AMR4(10)
      DIMENSION AMY1(10),AMY2(10),AMY3(10),AMY4(10)
      DO 1500 J=1,NPR
      DO 1500 K=1,NM
      L=2*K-1
      M=2*K
      AMK(L,J)=AMR1(K)*AM(J,L)+AMR2(K)*AM(J,M)
      AMK(M,J)=AMR3(K)*AM(J,L)+AMR4(K)*AM(J,M)
 1500 CONTINUE
      DO 600 J=1,NPS
      DO 600 K=1,NM
      L=2*K-1
      M=2*K
      JJ=J+NPR
      AMK(L,JJ)=AMY1(K)*AV(J,L)+AMY2(K)*AV(J,M)
      AMK(M,JJ)=AMY3(K)*AV(J,L)+AMY4(K)*AV(J,M)
  600 CONTINUE
      RETURN
      END

      SUBROUTINE STIFFA(SMR1,SMR2,SMR3,SMR4,
     &SMY1,SMY2,SMY3,SMY4,SVR1,SVR2,SVR3,SVR4,
     &SVY1,SVY2,SVY3,SVY4,XMR1,XMR2,XMR3,XMR4,
     &XMY1,XMY2,XMY3,XMY4,XVR1,XVR2,XVR3,XVR4,
     &XVY1,XVY2,XVY3,XVY4,I,XE,XI,XL,XM)
      DIMENSION XI(10),XL(10),XM(10)
      DIMENSION SMR1(10),SMR2(10),SMY1(10),SMY2(10)
      DIMENSION SMR3(10),SMR4(10),SMY3(10),SMY4(10)
      DIMENSION XMR1(10),XMR2(10),XMY1(10),XMY2(10)
      DIMENSION XMR3(10),XMR4(10),XMY3(10),XMY4(10)
      DIMENSION SVR1(10),SVR2(10),SVY1(10),SVY2(10)
      DIMENSION SVR3(10),SVR4(10),SVY3(10),SVY4(10)
      DIMENSION XVR1(10),XVR2(10),XVY1(10),XVY2(10)
```

```
      DIMENSION XVR3(10),XVR4(10),XVY3(10),XVY4(10)
      SMR1(I)=(4.*XE*XI(I))/XL(I)
      SMR2(I)=(2.*XE*XI(I))/XL(I)
      SMR3(I)=(2.*XE*XI(I))/XL(I)
      SMR4(I)=(4.*XE*XI(I))/XL(I)
      SMY1(I)=(-6.*XE*XI(I))/(XL(I)*XL(I))
      SMY2(I)=(-6.*XE*XI(I))/(XL(I)*XL(I))
      SMY3(I)=(-6.*XE*XI(I))/(XL(I)*XL(I))
      SMY4(I)=(-6.*XE*XI(I))/(XL(I)*XL(I))
      SVR1(I)=(-6.*XE*XI(I))/(XL(I)*XL(I))
      SVR2(I)=(-6.*XE*XI(I))/(XL(I)*XL(I))
      SVR3(I)=(-6.*XE*XI(I))/(XL(I)*XL(I))
      SVR4(I)=(-6.*XE*XI(I))/(XL(I)*XL(I))
      SVY1(I)=(12.*XE*XI(I))/(XL(I)*XL(I)*XL(I))
      SVY2(I)=(12.*XE*XI(I))/(XL(I)*XL(I)*XL(I))
      SVY3(I)=(12.*XE*XI(I))/(XL(I)*XL(I)*XL(I))
      SVY4(I)=(12.*XE*XI(I))/(XL(I)*XL(I)*XL(I))
      XMR1(I)=(4.*XM(I)*XL(I)*XL(I)*XL(I))/420.
      XMR2(I)=(-3.*XM(I)*XL(I)*XL(I)*XL(I))/420.
      XMR3(I)=(-3.*XM(I)*XL(I)*XL(I)*XL(I))/420.
      XMR4(I)=(4.*XM(I)*XL(I)*XL(I)*XL(I))/420.
      XMY1(I)=(-22.*XM(I)*XL(I)*XL(I))/420.
      XMY2(I)=(+13.*XM(I)*XL(I)*XL(I))/420.
      XMY3(I)=(+13.*XM(I)*XL(I)*XL(I))/420.
      XMY4(I)=(-22.*XM(I)*XL(I)*XL(I))/420.
      XVR1(I)=(-22.*XM(I)*XL(I)*XL(I))/420.
      XVR2(I)=(+13.*XM(I)*XL(I)*XL(I))/420.
      XVR3(I)=(+13.*XM(I)*XL(I)*XL(I))/420.
      XVR4(I)=(-22.*XM(I)*XL(I)*XL(I))/420.
      XVY1(I)=(156.*XM(I)*XL(I))/420.
      XVY2(I)=(-54.*XM(I)*XL(I))/420.
      XVY3(I)=(-54.*XM(I)*XL(I))/420.
      XVY4(I)=(156.*XM(I)*XL(I))/420.
      RETURN
      END

      SUBROUTINE STIFFB(SMR1,SMR2,SMR3,SMR4,
     &SMY1,SMY2,SMY3,SMY4,SVR1,SVR2,SVR3,SVR4,
     &SVY1,SVY2,SVY3,SVY4,XMR1,XMR2,XMR3,XMR4,
     &XMY1,XMY2,XMY3,XMY4,XVR1,XVR2,XVR3,XVR4,
     &XVY1,XVY2,XVY3,XVY4,I,XE,XI,XL,XM)
      DIMENSION XI(10),XL(10),XM(10)
      DIMENSION SMR1(10),SMR2(10),SMY1(10),SMY2(10)
      DIMENSION SMR3(10),SMR4(10),SMY3(10),SMY4(10)
      DIMENSION XMR1(10),XMR2(10),XMY1(10),XMY2(10)
      DIMENSION XMR3(10),XMR4(10),XMY3(10),XMY4(10)
      DIMENSION SVR1(10),SVR2(10),SVY1(10),SVY2(10)
      DIMENSION SVR3(10),SVR4(10),SVY3(10),SVY4(10)
      DIMENSION XVR1(10),XVR2(10),XVY1(10),XVY2(10)
      DIMENSION XVR3(10),XVR4(10),XVY3(10),XVY4(10)
      SMR1(I)=(3.*XE*XI(I))/XL(I)
      SMR2(I)=0.
```

```
      SMR3(I)=0.
      SMR4(I)=0.
      SMY1(I)=(-3.*XE*XI(I))/(XL(I)*XL(I))
      SMY2(I)=(-3.*XE*XI(I))/(XL(I)*XL(I))
      SMY3(I)=0.
      SMY4(I)=0.
      SVR1(I)=(-3.*XE*XI(I))/(XL(I)*XL(I))
      SVR2(I)=0.
      SVR3(I)=(-3.*XE*XI(I))/(XL(I)*XL(I))
      SVR4(I)=0.
      SVY1(I)=(3.*XE*XI(I))/(XL(I)*XL(I)*XL(I))
      SVY2(I)=(3.*XE*XI(I))/(XL(I)*XL(I)*XL(I))
      SVY3(I)=(3.*XE*XI(I))/(XL(I)*XL(I)*XL(I))
      SVY4(I)=(3.*XE*XI(I))/(XL(I)*XL(I)*XL(I))
      XMR1(I)=(8.*XM(I)*XL(I)*XL(I)*XL(I))/420.
      XMR4(I)=0.
      XMR2(I)=0.
      XMR3(I)=0.
      XMY1(I)=(-36.*XM(I)*XL(I)*XL(I))/420.
      XMY2(I)=(+11.*XM(I)*XL(I)*XL(I))/280.
      XMY3(I)=0.
      XMY4(I)=0.
      XVR1(I)=(-36.*XM(I)*XL(I)*XL(I))/420.
      XVR2(I)=0.
      XVR3(I)=(+11.*XM(I)*XL(I)*XL(I))/280.
      XVR4(I)=0.
      XVY1(I)=(+204.*XM(I)*XL(I))/420.
      XVY2(I)=(-39.*XM(I)*XL(I))/280.
      XVY3(I)=(-39.*XM(I)*XL(I))/280.
      XVY4(I)=(+99.*XM(I)*XL(I))/420.
      RETURN
      END

      SUBROUTINE STIFFC(SMR1,SMR2,SMR3,SMR4,
     &SMY1,SMY2,SMY3,SMY4,SVR1,SVR2,SVR3,SVR4,
     &SVY1,SVY2,SVY3,SVY4,XMR1,XMR2,XMR3,XMR4,
     &XMY1,XMY2,XMY3,XMY4,XVR1,XVR2,XVR3,XVR4,
     &XVY1,XVY2,XVY3,XVY4,I,XE,XI,XL,XM)
      DIMENSION XI(10),XL(10),XM(10)
      DIMENSION SMR1(10),SMR2(10),SMY1(10),SMY2(10)
      DIMENSION SMR3(10),SMR4(10),SMY3(10),SMY4(10)
      DIMENSION XMR1(10),XMR2(10),XMY1(10),XMY2(10)
      DIMENSION XMR3(10),XMR4(10),XMY3(10),XMY4(10)
      DIMENSION SVR1(10),SVR2(10),SVY1(10),SVY2(10)
      DIMENSION SVR3(10),SVR4(10),SVY3(10),SVY4(10)
      DIMENSION XVR1(10),XVR2(10),XVY1(10),XVY2(10)
      DIMENSION XVR3(10),XVR4(10),XVY3(10),XVY4(10)
      SMR4(I)=(3.*XE*XI(I))/XL(I)
      SMR2(I)=0.
      SMR3(I)=0.
      SMR1(I)=0.
      SMY4(I)=(-3.*XE*XI(I))/(XL(I)*XL(I))
```

```
      SMY3(I)=(-3.*XE*XI(I))/(XL(I)*XL(I))
      SMY2(I)=0.
      SMY1(I)=0.
      SVR4(I)=(-3.*XE*XI(I))/(XL(I)*XL(I))
      SVR3(I)=0.
      SVR2(I)=(-3.*XE*XI(I))/(XL(I)*XL(I))
      SVR1(I)=0.
      SVY1(I)=(3.*XE*XI(I))/(XL(I)*XL(I)*XL(I))
      SVY2(I)=(3.*XE*XI(I))/(XL(I)*XL(I)*XL(I))
      SVY3(I)=(3.*XE*XI(I))/(XL(I)*XL(I)*XL(I))
      SVY4(I)=(3.*XE*XI(I))/(XL(I)*XL(I)*XL(I))
      XMR4(I)=(8.*XM(I)*XL(I)*XL(I)*XL(I))/420.
      XMR1(I)=0.
      XMR2(I)=0.
      XMR3(I)=0.
      XMY4(I)=(-36.*XM(I)*XL(I)*XL(I))/420.
      XMY3(I)=(+11.*XM(I)*XL(I)*XL(I))/280.
      XMY2(I)=0.
      XMY1(I)=0.
      XVR4(I)=(-36.*XM(I)*XL(I)*XL(I))/420.
      XVR3(I)=0.
      XVR2(I)=(+11.*XM(I)*XL(I)*XL(I))/280.
      XVR1(I)=0.
      XVY4(I)=(+204.*XM(I)*XL(I))/420.
      XVY2(I)=(-39.*XM(I)*XL(I))/280.
      XVY3(I)=(-39.*XM(I)*XL(I))/280.
      XVY1(I)=(+99.*XM(I)*XL(I))/420.
      RETURN
      END

      SUBROUTINE STIFFD(SMR1,SMR2,SMR3,SMR4,
     &SMY1,SMY2,SMY3,SMY4,SVR1,SVR2,SVR3,SVR4,
     &SVY1,SVY2,SVY3,SVY4,XMR1,XMR2,XMR3,XMR4,
     &XMY1,XMY2,XMY3,XMY4,XVR1,XVR2,XVR3,XVR4,
     &XVY1,XVY2,XVY3,XVY4,I,XE,XI,XL,XM)
      DIMENSION XI(10),XL(10),XM(10)
      DIMENSION SMR1(10),SMR2(10),SMY1(10),SMY2(10)
      DIMENSION SMR3(10),SMR4(10),SMY3(10),SMY4(10)
      DIMENSION XMR1(10),XMR2(10),XMY1(10),XMY2(10)
      DIMENSION XMR3(10),XMR4(10),XMY3(10),XMY4(10)
      DIMENSION SVR1(10),SVR2(10),SVY1(10),SVY2(10)
      DIMENSION SVR3(10),SVR4(10),SVY3(10),SVY4(10)
      DIMENSION XVR1(10),XVR2(10),XVY1(10),XVY2(10)
      DIMENSION XVR3(10),XVR4(10),XVY3(10),XVY4(10)
      SMR2(I)=0.
      SMR1(I)=0.
      SMR3(I)=0.
      SMR4(I)=0.
      SMY1(I)=0.
      SMY2(I)=0.
      SMY3(I)=0.
      SMY4(I)=0.
```

```
      SVR1(I)=0.
      SVR2(I)=0.
      SVR4(I)=0.
      SVR3(I)=0.
      SVY1(I)=0.
      SVY2(I)=0.
      SVY3(I)=0.
      SVY4(I)=0.
      XMR1(I)=0.
      XMR2(I)=0.
      XMR3(I)=0.
      XMR4(I)=0.
      XMY1(I)=0.
      XMY2(I)=0.
      XMY3(I)=0.
      XMY4(I)=0.
      XVR1(I)=0.
      XVR2(I)=0.
      XVR3(I)=0.
      XVR4(I)=0.
      XVY1(I)=(2.*XM(I)*XL(I))/3.
      XVY2(I)=(-1.*XM(I)*XL(I))/3.
      XVY3(I)=(-1.*XM(I)*XL(I))/3.
      XVY4(I)=(2.*XM(I)*XL(I))/3.
      RETURN
      END

      SUBROUTINE STIFPA(PMR1,PMR2,PMR3,PMR4,
     &PMY1,PMY2,PMY3,PMY4,PVR1,PVR2,PVR3,PVR4,
     &PVY1,PVY2,PVY3,PVY4,I,XL,AF)
      DIMENSION XI(10),XL(10),AF(10)
      DIMENSION PMR1(10),PMR2(10),PMY1(10),PMY2(10)
      DIMENSION PMR3(10),PMR4(10),PMY3(10),PMY4(10)
      DIMENSION PVR1(10),PVR2(10),PVY1(10),PVY2(10)
      DIMENSION PVR3(10),PVR4(10),PVY3(10),PVY4(10)
      PMR2(I)=AF(I)*(-XL(I)/30.)
      PMR3(I)=AF(I)*(-XL(I)/30.)
      PMR4(I)=AF(I)*(2.*XL(I)/15.)
      PMR1(I)=AF(I)*(2.*XL(I)/15.)
      PMY1(I)=AF(I)*(-1./10.)
      PMY2(I)=AF(I)*(-1./10.)
      PMY3(I)=AF(I)*(-1./10.)
      PMY4(I)=AF(I)*(-1./10.)
      PVR1(I)=AF(I)*(-1./10.)
      PVR2(I)=AF(I)*(-1./10.)
      PVR3(I)=AF(I)*(-1./10.)
      PVR4(I)=AF(I)*(-1./10.)
      PVY1(I)=AF(I)*(6./(5.*XL(I)))
      PVY2(I)=AF(I)*(6./(5.*XL(I)))
      PVY3(I)=AF(I)*(6./(5.*XL(I)))
      PVY4(I)=AF(I)*(6./(5.*XL(I)))
      RETURN
```

```
      END

      SUBROUTINE STIFPB(PMR1,PMR2,PMR3,PMR4,
     &PMY1,PMY2,PMY3,PMY4,PVR1,PVR2,PVR3,PVR4,
     &PVY1,PVY2,PVY3,PVY4,I,XL,AF)
      DIMENSION XI(10),XL(10),AF(10)
      DIMENSION PMR1(10),PMR2(10),PMY1(10),PMY2(10)
      DIMENSION PMR3(10),PMR4(10),PMY3(10),PMY4(10)
      DIMENSION PVR1(10),PVR2(10),PVY1(10),PVY2(10)
      DIMENSION PVR3(10),PVR4(10),PVY3(10),PVY4(10)
      PMR1(I)=AF(I)*(1.*XL(I)/5.)
      PMR2(I)=AF(I)*0.
      PMR3(I)=AF(I)*0.
      PMR4(I)=AF(I)*0.
      PMY1(I)=AF(I)*(-1./5.)
      PMY2(I)=AF(I)*(-1./5.)
      PMY3(I)=AF(I)*0.
      PMY4(I)=AF(I)*0.
      PVR1(I)=AF(I)*(-1./5.)
      PVR2(I)=AF(I)*0.
      PVR3(I)=AF(I)*(-1./5.)
      PVR4(I)=AF(I)*0.
      PVY1(I)=AF(I)*(6./(5.*XL(I)))
      PVY2(I)=AF(I)*(6./(5.*XL(I)))
      PVY3(I)=AF(I)*(6./(5.*XL(I)))
      PVY4(I)=AF(I)*(6./(5.*XL(I)))
      RETURN
      END

      SUBROUTINE STIFPC(PMR1,PMR2,PMR3,PMR4,
     &PMY1,PMY2,PMY3,PMY4,PVR1,PVR2,PVR3,PVR4,
     &PVY1,PVY2,PVY3,PVY4,I,XL,AF)
      DIMENSION XI(10),XL(10),AF(10)
      DIMENSION PMR1(10),PMR2(10),PMY1(10),PMY2(10)
      DIMENSION PMR3(10),PMR4(10),PMY3(10),PMY4(10)
      DIMENSION PVR1(10),PVR2(10),PVY1(10),PVY2(10)
      DIMENSION PVR3(10),PVR4(10),PVY3(10),PVY4(10)
      PMR1(I)=AF(I)*0.
      PMR2(I)=AF(I)*0.
      PMR3(I)=AF(I)*0.
      PMR4(I)=AF(I)*(1.*XL(I)/5.)
      PMY1(I)=AF(I)*0.
      PMY2(I)=AF(I)*0.
      PMY3(I)=AF(I)*(-1./5.)
      PMY4(I)=AF(I)*(-1./5.)
      PVR1(I)=AF(I)*0.
      PVR2(I)=AF(I)*(-1./5.)
      PVR3(I)=AF(I)*0.
      PVR4(I)=AF(I)*(-1./5.)
      PVY1(I)=AF(I)*(6./(5.*XL(I)))
      PVY2(I)=AF(I)*(6./(5.*XL(I)))
      PVY3(I)=AF(I)*(6./(5.*XL(I)))
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      PVY4(I)=AF(I)*(6./(5.*XL(I)))
      RETURN
      END

      SUBROUTINE STIFPD(PMR1,PMR2,PMR3,PMR4,
     &PMY1,PMY2,PMY3,PMY4,PVR1,PVR2,PVR3,PVR4,
     &PVY1,PVY2,PVY3,PVY4,I,XL,AF)
      DIMENSION XI(10),XL(10),AF(10)
      DIMENSION PMR1(10),PMR2(10),PMY1(10),PMY2(10)
      DIMENSION PMR3(10),PMR4(10),PMY3(10),PMY4(10)
      DIMENSION PVR1(10),PVR2(10),PVY1(10),PVY2(10)
      DIMENSION PVR3(10),PVR4(10),PVY3(10),PVY4(10)
      PMR1(I)=AF(I)*0.
      PMR2(I)=AF(I)*0.
      PMR3(I)=AF(I)*0.
      PMR4(I)=AF(I)*0.
      PMY1(I)=AF(I)*0.
      PMY2(I)=AF(I)*0.
      PMY3(I)=AF(I)*0.
      PMY4(I)=AF(I)*0.
      PVR1(I)=AF(I)*0.
      PVR2(I)=AF(I)*0.
      PVR3(I)=AF(I)*0.
      PVR4(I)=AF(I)*0.
      PVY1(I)=AF(I)*0.
      PVY2(I)=AF(I)*0.
      PVY3(I)=AF(I)*0.
      PVY4(I)=AF(I)*0.
      RETURN
      END

      SUBROUTINE GEXTP(AM,AV,NP,NPR,NM,FEM,FEV,PSE,
     &PSFT)
      DIMENSION FEV(12),FEM(12),PE(10),RSFT(10)
      DIMENSION AM(12,12),AV(12,12),PSE(10)
      NEM=NM*2
      NPS=NP-NPR
      DO 563 I=1,NPR
      PE(I)=0.
      DO 563 J=1,NEM
      PE(I)=PE(I)+AM(I,J)*FEM(J)
  563 CONTINUE
      DO 564 I=1,NPS
      II=I+NPR
      PE(II)=0.
      DO 564 J=1,NEM
      PE(II)=PE(II)+AV(I,J)*FEV(J)
  564 CONTINUE
      DO 565 I=1,NP
      RSFT(I)=-PE(I)-PSE(I)
  565 CONTINUE
      RETURN
```

```
      END
      SUBROUTINE GEMKP(T,DT,NP,NPR,VA,VB,ZETA,PSR,X,
     &XXP,XXK,XMI,RSFT,G)
      DIMENSION XP(10,10),XMF(10),XXMPK(10),
     &XMPK(10,10),XXK(10,10)
      DIMENSION FT(10),X(10),XXP(10,10),XMI(10,10),
     &G(10),DG(10)
      DIMENSION RSFT(10),SFT(10)
      NPS=NP-NPR
      TS1=0.02
      TS2=0.02
      TD1=0.04
      TD2=0.08
      TD3=0.12
      TD4=0.16
      TD5=0.20
      TD6=0.24
      FO1=10000.
      FO2=10000.
      SLOP1=FO1/TS1
      SLOP2=FO2/TS2
      IF(T-TD1) 9006,9006,9007
 9006 SFT(1)=FO1-SLOP1*T
      GO TO 9008
 9007 IF(T-TD2) 9009,9009,9010
 9009 SFT(1)=-FO2+SLOP2*(T-TD1)
      GO TO 9008
 9010 IF(T-TD3) 9011,9011,9012
 9011 SFT(1)=FO1-SLOP1*(T-TD2)
      GO TO 9008
 9012 IF(T-TD4) 9013,9013,9014
 9013 SFT(1)=-FO2+SLOP2*(T-TD3)
      GO TO 9008
 9014 IF(T-TD5) 9015,9015,9016
 9015 SFT(1)=FO1-SLOP1*(T-TD4)
      GO TO 9008
 9016 IF(T-TD6) 9017,9017,9018
 9017 SFT(1)=-FO2+SLOP2*(T-TD5)
      GO TO 9008
 9018 SFT(1)=0.
 9008 DO 9004 I=1,NPR
      FT(I)=RSFT(I)
 9004 CONTINUE
      DO 9005 I=1,NPS
      IN=NPR+I
      FT(IN)=SFT(I)+RSFT(IN)
 9005 CONTINUE
      DO 906 I=1,NP
      XMF(I)=0.
      XXMPK(I)=0.
      DO 907 J=1,NP
```

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      XMPK(I,J)=0.
      DO 908 K=1,NP
  908 XMPK(I,J)=XMPK(I,J)+XMI(I,K)*(XXP(K,J)-XXK(K,J))
      XXMPK(I)=XXMPK(I)+XMPK(I,J)*X(J)
  907 XMF(I)=XMF(I)+XMI(I,J)*FT(J)
      DG(I)=XMF(I)+XXMPK(I)
      G(I)=DG(I)*DT
  906 CONTINUE
      RETURN
      END
```