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"A General Evaluation Approach to
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and its Application to Nuclear Power"

David Okrent, Project Director

ON THE AVERAGE PROBABILITY DISTRIBUTION OF PEAK GROUND ACCELERATION IN THE U.S. CONTINENT DUE TO STRONG EARTHQUAKES

T. HSIEH
D. OKRENT
G.E. APOSTOLAKIS

UCLA • SCHOOL OF ENGINEERING AND APPLIED SCIENCE

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16. Abstracts The primary objective of this study is to obtain an estimate of the average probabilities of different magnitudes of peak horizontal ground acceleration due to seismic events east of the Rocky Mountains. Past history is used to provide an average seismicity, and future earthquakes are assumed to occur randomly with regard to location and time. Since data for earthquakes of appreciable magnitude in the eastern United States during the 20th Century are limited, the seismic history of various regions around the world having a seismicity and geological structure similar to the eastern United States is lumped with data for the latter, in order to obtain a more representative estimate of average seismicity. Regions included are: Western and Central Europe, Arabian Continent, Southeastern Asia, Brazil, India, Canada, Australia, and Antarctic. For comparison, Western United States is also analyzed for average probabilities of ground accelerations.

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Energy and Kinetics Department
School of Engineering and Applied Science
University of California
Los Angeles, California

PREFACE

This report represents one aspect of a National Science Foundation funded study at UCLA entitled, "A General Evaluation Approach to Risk-Benefit for Large Technological Systems and Its Application to Nuclear Power," (NSF Grant GI-39416). The objectives of this project can be defined to include the following:

1) To make significant strides in the provision of improved bases or criteria for decision-making involving risk to the public health and safety (where a risk involves a combination of a hazard and the probability of that hazard).

2) To make significant strides in the structuring and development of improved, and possibly alternative, general methodologies for assessing risk and risk-benefit for technological systems.

3) To develop improvements in the techniques for the quantitative assessment of risk and benefit.

4) To apply methods of risk and risk-benefit assessment to specific applications in nuclear power (and possibly other technological systems) in order to test methodologies, to uncover needed improvements and gaps in technique, and to provide a partial, selective, independent assessment of the levels of risk arising from nuclear power.

Reports prepared previously under this grant include the following:

1. Mathematical Methods of Probabilistic Safety Analysis, G.E. Apostolakis, UCLA-ENG-7464 (Sept. 1974)
2. Biostatistical Aspects of Risk-Benefit: The Use of Competing Risks Analysis, H.N. Sather, UCLA-ENG-7477 (Sept. 1974)

3. Applying Cost-Benefit Concepts to Projects which Alter Human Mortality, J. Hirshleifer, T. Bergstrom, E. Rappaport, UCLA-ENG-7478 (Nov. 1974)

4. Historical Perspectives on Risk for Large Scale Technological Systems, by W. Baldewicz, G. Haddock, Y. Lee, Prajoto, R. Whitley and V. Denny, UCLA-ENG-7485 (Dec. 1974)

5. A Prediction of the Reliability of the Core Auxiliary Cooling System for a HTGR, K.A. Solomon, D. Okrent and W.E. Kastenberg, UCLA-ENG-7495 (Jan. 1975)

6. Pressure Vessel Integrity and Weld Inspection Procedure, K.A. Solomon, D. Okrent and W.E. Kastenberg, UCLA-ENG-7496 (Jan. 1975)

7. A Survey of Expert Opinion on Low Probability Earthquakes, D. Okrent, UCLA-ENG-7515 (Feb. 1975)

ACKNOWLEDGMENTS

The basic approach taken in this study was suggested by Prof. J.T. Wilson of the University of Michigan. Prof. L. Knopoff of the University of California, Los Angeles, provided valuable advice in the method of choosing areas of the world similar in seismic characteristics to the eastern U.S. Prof. M. Trifunac of California Institute of Technology provided valuable comments on the various seismic correlations employed. The authors gratefully acknowledge all this assistance.

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I. INTRODUCTION

The primary objective of this study is to obtain an estimate of the average probabilities of different magnitudes of peak horizontal ground acceleration due to seismic events in that part of the United States east of the Rockies. In effect, past history is used to provide an average seismicity and future earthquakes are assumed to occur randomly with regard to location and time. This is equivalent to the assumption that the past history of earthquakes in the eastern United States and the fundamental knowledge of earthquake causes in this area are insufficient to definitely "localize" past earthquakes in terms of local tetonic structure. This assumption tends to make any prediction for an "average" site yield a higher probability of accelerations, while, for sites near historical major earthquakes (e.g., New Madrid) the prediction should be low. However, the estimates may provide some basis for judging how much below average the historically low seismicity of some region has been over a very limited period of geological time.

Actually, data for earthquakes of appreciable magnitude in the eastern United States during the 20th century are very limited. Hence, following a suggestion by J. Wilson,[1] the seismic history of various regions around the world having a seismicity and geological structure similar to the eastern United States was lumped with data for the latter, in order to obtain a hopefully more representative estimate of average seismicity.

The following regions (excluding areas of deep sea, volcano, and known seismically active belts) were included:[2]

A.	Western and central Europe	:	10°W to 60°E; 42.5°N to 70°N
	regions no	:	31,36,40,49
B.	Arabian continent	:	34°E to 60°E; 13°N to 35°N
	regions no	:	29,30,37
C.	Southeastern Asia	:	98°E to 109°E; 5°N to 23°N
	regions no	:	24,25
D.	Brazil	:	35°W to 65°W; 5°N to 33°S
	regions no	:	8,35
E.	India	:	70°E to 88°E; 8°N to 25°N
	region no	:	26,33,47
F.	Canada	:	55°W to 110°W; 42°N to 70°N
	region no	:	34
G.	Australia, region no	:	38
H.	Antarctic excluding the peninsula		
	region no	:	50

The combined area of the above, is roughly 1.82×10^7 sq. mi., compared to an area of only 2.8×10^6 sq. mi. for the eastern U.S. itself.

For comparison purposes, that part of the U.S. west of the rockies was also analyzed for average probabilities of various accelerations. Only data from the western U.S. were used in estimating average seismicity in this case.

As it is illustrated below, a very considerable uncertainty in the estimates of this study arises from the differences among published seismic correlations, for example between epicentral intensity and felt area; large uncertainties also arise from the extrapolation of such correlations to large earthquakes.

II. SEISMIC HISTORY

Historical records of earthquakes are available in terms of both earthquake magnitude and Modified Mercalli (MM) Intensity. The magnitude scale, indicating the strain energy released at the earthquake source, is based on instrumental measurement and is relatively reliable. However, the concept was introduced by Richter about four decades ago, and previous seismic data were recorded only in terms of intensity, which is a subjective evaluation and subject to more scatter. For example, the estimated epicentral intensity for the 1811-1812 New Madrid earthquakes varies between MMX and MMXII.

Earthquake data for this study were taken primarily from Gutenberg and Richter[3] and from Rothe.[4] Earthquakes of magnitude 5 or greater for the period 1904-1965 were considered. Presumably, the older portion of this data represents magnitude values converted from historical estimates of intensity.

The data for the western U.S., and for the "lumped" region having seismic and geologic characteristics similar to the eastern U.S. are given for the period 1904-1965 in Tables 1 and 2 respectively. Some recent earthquake data (1963-1974) for the eastern U.S. itself is given in Table 3.

Table 1. Earthquake Magnitude Distribution in the Western U.S. data: 1904 to 1965, area: 0.9×10^6 sq. mi.

Magnitude	No. of Eq.	Avg. No. of Eq. per yr per sq. mi.
A. Incremental		
M = 5.5 (5.25 - 5.75)	55	9.8×10^{-7}
6.0 (5.75 - 6.25)	26	4.7×10^{-7}
6.5 (6.25 - 6.75)	14	2.5×10^{-7}
7.0 (6.75 - 7.25)	9	1.6×10^{-7}
7.5 (7.25 - 7.75)	2	3.6×10^{-8}
8.0 (7.75 - 8.25)	1	1.8×10^{-8}
B. Cumulative		
M ≥ 8.0	1	1.8×10^{-8}
≥ 7.5	3	5.4×10^{-8}
≥ 7.0	8	1.43×10^{-7}
≥ 6.5	23	4.13×10^{-7}
≥ 6.0	45	8.03×10^{-7}
≥ 5.5	79	1.41×10^{-6}
≥ 5.0	129	2.31×10^{-6}

Table 2. Earthquake Magnitude Distribution in the Eastern U.S. plus Similar Areas of the World

data: 1904 to 1965, lumped area: 2.1×10^7 sq. mi.

Magnitude	No. of Eq.	Avg. No. of Eq. per yr. per sq. mi.
A. Incremental		
M = 5.5 (5.25 - 5.75)	48	3.7×10^{-8}
6.0 (5.75 - 6.25)	23	1.8×10^{-8}
6.5 (6.25 - 6.75)	10	7.7×10^{-9}
B. Cumulative		
M $\geq$ 7.0	1	7.7×10^{-10}
$\geq$ 6.5	9	6.94×10^{-9}
$\geq$ 6.0	24	1.85×10^{-8}
$\geq$ 5.5	62	4.78×10^{-8}
$\geq$ 5.0	106	8.17×10^{-8}

Table 3. Earthquake Magnitude Distribution in the Eastern U.S.
data: 1963 to 1974, area: 2.8×10^6 sq. mi.

Magnitude	No. of Eq.	Avg. No. of Eq. per yr. per sq. mi.
A. Incremental		
M = 4.0 (3.75 - 4.25)	42	1.29×10^{-6}
4.5 (4.25 - 4.75)	53	1.53×10^{-6}
5.0 (4.75 - 5.25)	14	4.30×10^{-7}
5.5 (5.25 - 5.75)	8	2.46×10^{-7}
B. Cumulative		
M ≥ 5.5	3	9.24×10^{-8}
M ≥ 5.0	15	4.62×10^{-7}
M ≥ 4.5	55	1.69×10^{-6}
M ≥ 4.0	94	2.89×10^{-6}

III. EARTHQUAKE FREQUENCY DISTRIBUTIONS

In the analysis of earthquake frequency, it has often been assumed that an exponential relationship exists between the number of earthquakes and their associated magnitude or intensity. This relationship can be expressed by the equation[5]

$$\text{Log } N(M) = a - bM \qquad (1)$$

or

$$\text{Log } N(I) = a - bI \qquad (2)$$

where M is magnitude

I is intensity

and N(M) (or N(I)) is the number of earthquakes having a magnitude (intensity) equal to and larger than the specified M(I).

a and b are taken to be constants for a particular seismic region in a specified period of time.

N(M) may be the total number of earthquakes having a magnitude greater than M in the region; or, it may be the number of earthquakes per unit area per unit time. The "b" value (or slope) will remain the same with a change in normalization; however, the value of "a" will change accordingly.

Of course, if the seismicity for a region changes, or if a different set of data are used to assess the seismicity, the "b" value can change.

The b value in Equation (1) has received considerable attention from some seismologists and geologists. For example, Chinnery[6] mentioned that b values lying in the range of 0.8 to 1.0 are found in most parts of the world. Ikegami[7] found that periods of great seismic energy release correspond to periods during which the b value is a minimum and that in Japan the b value can change from 0.77 to 1.25 within 20 years.

The magnitude data of Tables 1 and 2 are plotted in Figure 1. Various methods for "fitting" the data are possible; frequently a least squares method is employed in which a lesser weighting is given to the end data, which frequently deviates from such a straight line, falling somewhat below.

In this study several approaches were applied to "fitting" and extrapolating the data. In Figure 1, the data were fit "by eye." Also plotted in Figure 1 are the limited data of Table 3.

The method of "fitting by eye" in Figure 1 gave the following results:

$$\text{Log } N(M) = -2.81 - 0.55\ M \text{ (western U.S.)} \tag{3}$$

$$\text{Log } N(M) = -2.71 - 0.839\ M \text{ (eastern U.S.)} \tag{4}$$

where N is the number of earthquakes per year per square mile with magnitude M or greater.

The same data for the western U.S. were also examined by the maximum likelihood method with the result (See Appendix 1, also Reference 8)

$$\text{Log } N(M) = -3.73 - 0.51\ M \text{ (western U.S.; max. likelihood)} \tag{5}$$

These low "b" values for the western U.S. were somewhat surprising. Hence, only the more recent data (1934-1965) were plotted (See Figure 2). Using a similar method of "best fit," the following equation with a higher b value was then obtained

$$\text{Log } N(M) = -1.72 - 0.7\ M \text{ (western U.S.; 1934-65)} \tag{6}$$

A similarly higher "b" value is obtained if the method of maximum likelihood is applied to only this more recent data.

For purposes of the study, it was of interest to ascertain the sensitivity of the probabilities of differing accelerations to the data-fitting process. Hence, two other procedures were also employed.

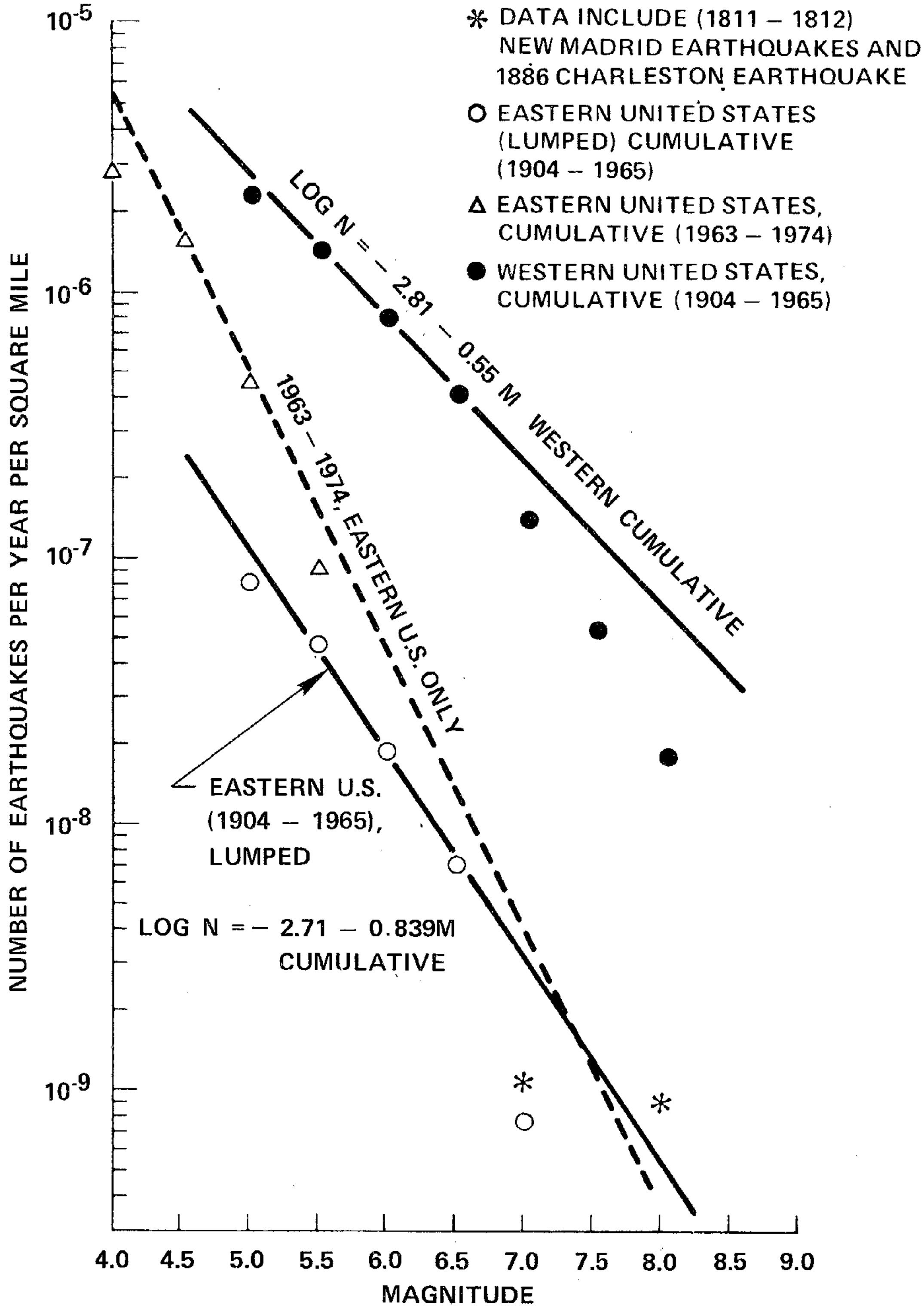

Figure 1. Earthquake Distribution in the U.S.

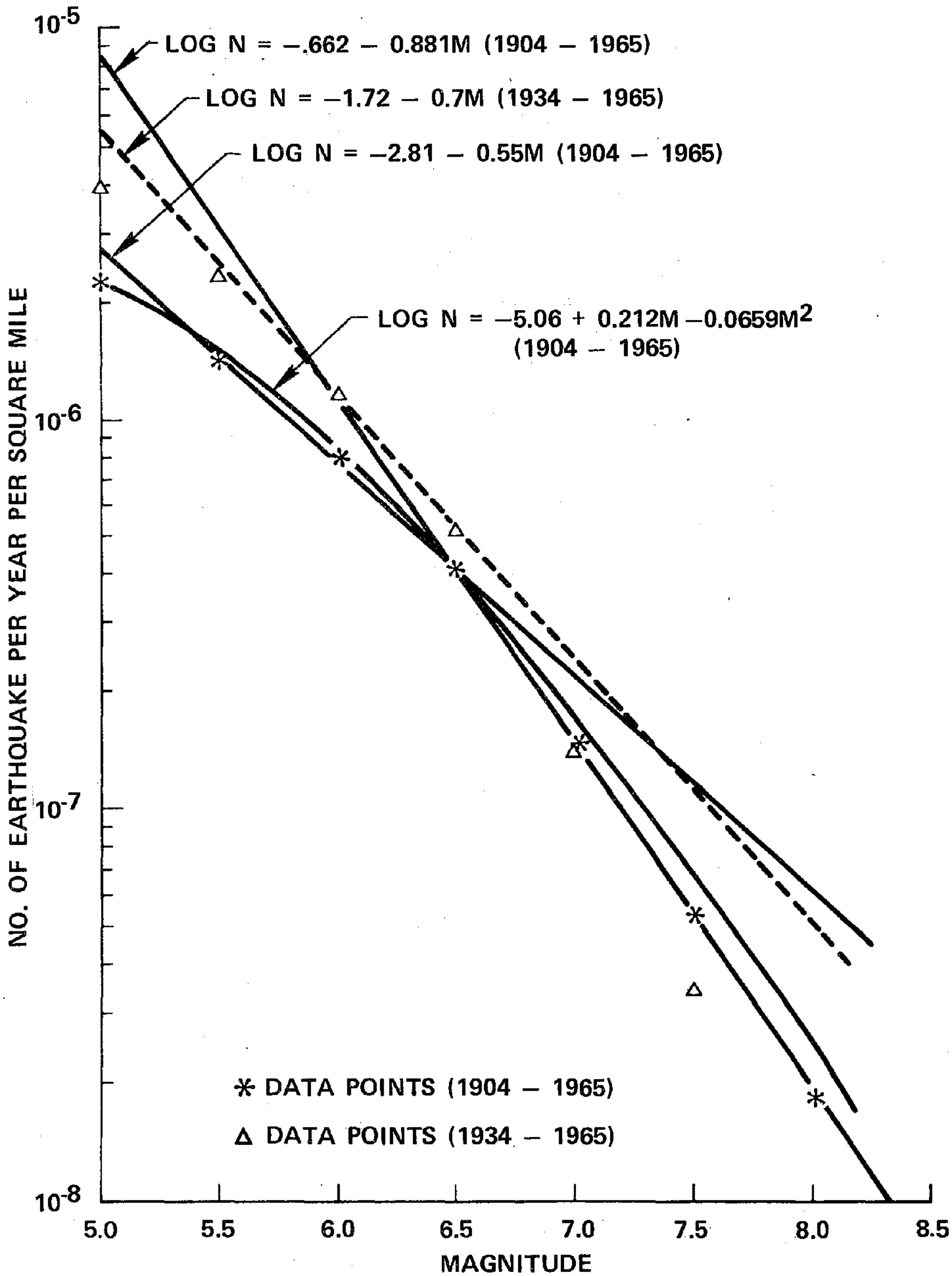

Figure 2. The Seismicity of the Western U.S.

First, a straight line fit was made in which the higher magnitude data were given the heaviest weight; the result obtained for the 1904-1965 data was

$$\text{Log } N(M) = -0.66 - 0.88\ M \tag{7}$$

Also, a polynomial fit was made to the data, as shown on Figure 2. The result obtained for the 1904 to 1965 data was

$$\text{Log } N(M) = -5.06 + 0.212\ M - 0.0659\ M^2 \tag{8}$$

With regard to the eastern United States, few large earthquakes were included in the time period (1904-1965). If the time period is extended back two hundred years, and the Charleston and New Madrid earthquakes are included (with appropriate normalization) the points shown by asterisks are obtained.

To provide additional estimates of historic seismicity for the "lumped data" representation of the eastern U.S., the maximum likelihood method was applied, giving

$$\text{Log } N(M) = -\ 3.91 - 0.64\ M \tag{9}$$

When a polynomial fit was made (more or less neglecting the New Madrid earthquakes) the following result was obtained for the lumped data, eastern U.S.

$$\text{Log } N(M) = -9.22 + 1.37\ M - 0.186\ M^2 \tag{10}$$

The curves of Equation (2) and (10) are compared in Figure 3.

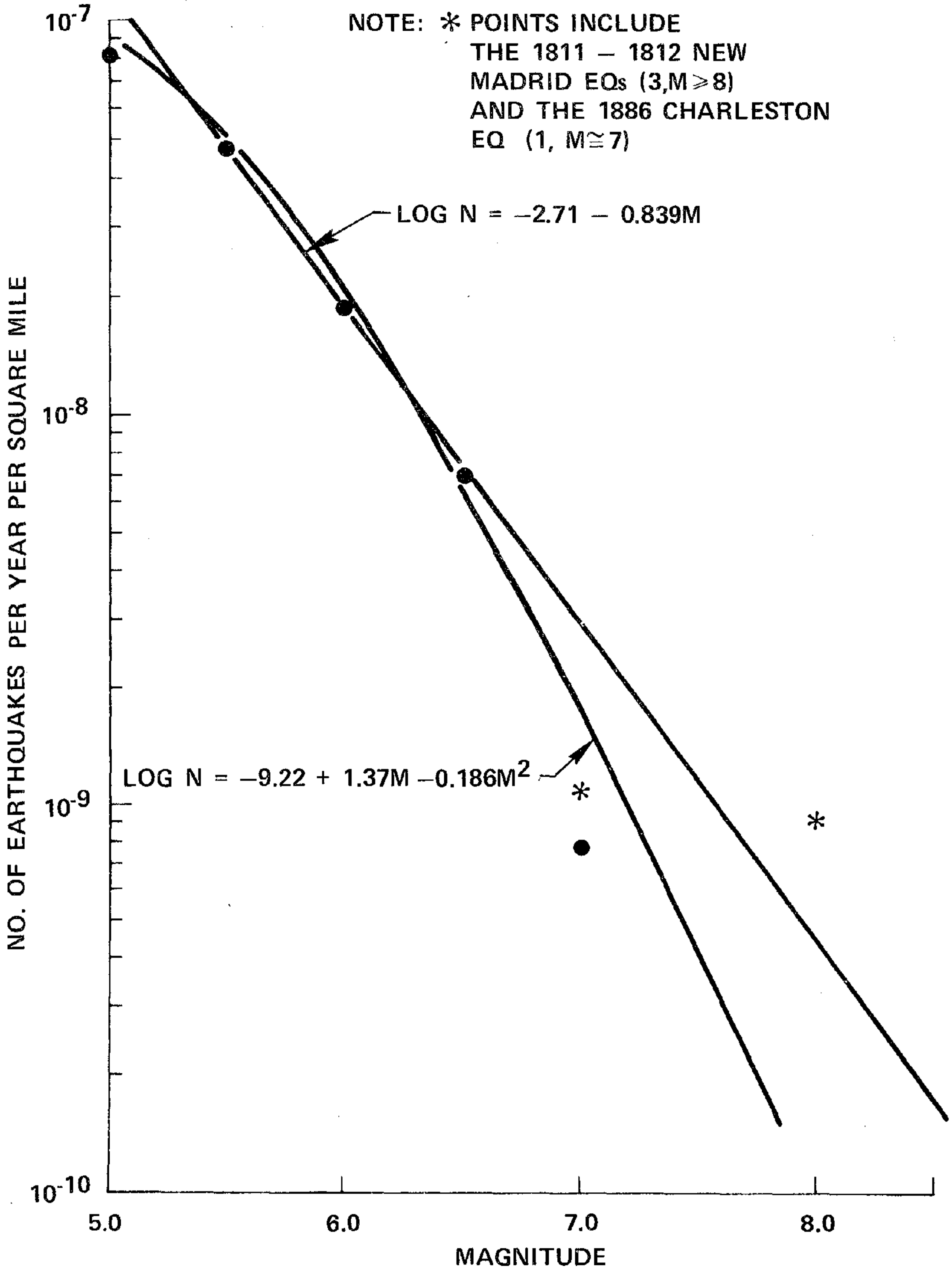

Figure 3. The Seismicity of the Lumped Eastern U.S.

IV. METHODOLOGY FOR PROBABILITY DISTRIBUTIONS OF PEAK GROUND ACCELERATION

To obtain an estimate of the probability of various accelerations, assuming a random distribution of earthquakes, it is necessary to have a basis for defining isoseismal acceleration lines as a function of distance from earthquake source (or some equivalent set of relationship). For the western U.S., Mickey's correlation[9]

$$g = 3.04 \times 10^{-4} \; \frac{10^{0.74M}}{R^{1.4}} \tag{11}$$

was used, where R is the hypocentral distance in kilometers, and

$$R = (D^2 + H^2)^{1/2}$$

where D is the epicentral distance and H is the focal depth (assumed to be 15 km).

The following methodology was then applied. We first divide the interesting range of ground acceleration into some convenient intervals and denote them by g_i. We also discretize the magnitude of earthquake and denote it by m_j. Corresponding to each g_i, the incremental perceptible area a_{ij} due to an earthquake of magnitude m_j can be determined from the empirical correlation (i.e., Mickey). If the total area of the region being considered is A, the probability that an arbitrary point in A will experience g_i when exactly one earthquake with magnitude m_j has occurred is

$$\frac{a_{ij}}{A}$$

Once an earthquake with magnitude m_j has occurred, the probability that any point in A will not experience g_i is then

$$1 - \frac{a_{ij}}{A}$$

The same probability for the case of exactly n earthquakes in one year (or other unit of time) is

$$\left(1 - \frac{a_{ij}}{A}\right)^n$$

Assuming that the occurrence of earthquakes follows a Poisson distribution for the magnitude range under consideration, the probability of exactly n earthquakes with magnitude m_j occurring is

$$\frac{e^{-\mu_j} \mu_j^n}{n!}$$

where μ_j is the expected number of earthquakes of magnitude m_j in one year (or other unit of time). By including the zeroth event, the probability that any site in A will not experience g_i is

$$\sum_{n=0,1,2,\ldots}^{\infty} \left(1 - \frac{a_{ij}}{A}\right)^n \frac{e^{-\mu_j}\mu_j^n}{n!} = e^{-\mu_j} \sum_{n=0,1,2\ldots}^{\infty} \left(1 - \frac{a_{ij}}{A}\right)^n \frac{\mu_j^n}{n!}$$

$$= e^{-\mu_j} e^{(1-a_{ij}/A)\mu_j} = \exp\left(- \frac{a_{ij}\mu_j}{A}\right)$$

Considering all the possible m_j's, the previous probability becomes

$$\prod_j e^{-\frac{a_{ij}\mu_j}{A}} = e^{-\frac{1}{A} \sum_j a_{ij}\mu_j}$$

The probability that a random site will experience g_i in a year is

$$P(g_i) = 1 - e^{-\frac{1}{A} \sum_j a_{ij}\mu_j} \tag{12}$$

which is similar in form to Housner's probability equation.[10,11]

For the eastern U.S. a general correlation between magnitude, acceleration and distance from the source was not available. The following approach was used in this study.

First magnitude was converted into epicentral intensity using the following relationship[6]

$$I_0 = 1.67\ M - 2.0 \tag{13}$$

where I_0 is the epicentral intensity.

Brazee's correlation[12]

$$\mathrm{Log}\ A_I = a_a + b_a\ I_0 \tag{14}$$

was then employed to give the total area A_I with intensity I for an epicentral intensity I_0. The constants a_a and b_a are given in Reference 12.

Acceleration was then derived from intensity using average foundation conditions in the correlation of Coulter, et al.[13] The curve of Reference 13 was represented by the following algebraic relationships.

$$\begin{aligned} \mathrm{Log}\ g &= -3.233 + 0.342\ I\ , \quad \text{for } g \le 0.5g \\ \mathrm{Log}\ g &= -6.53 + 1.129\ I - 0.04663\ I^2\ , \quad \text{for } g > 0.5g \end{aligned} \tag{15}$$

V. RESULTS

Using the previously defined data, methodology and correlations, the average probabilities per year of exceeding a given acceleration were computed for the western and eastern United States.

In Table 4 the results are compared employing four different magnitude-frequency relationships, namely those of Equations (3),(7),(5) and (8). The results differ by only about a factor of two among the four magnitude-frequency relationships.

In Table 5, the results are compared employing three different magnitude-frequency relationships, namely those of Equations (4),(10) and (9). In this case the results vary by about a factor of three at small "g" value and a factor of 5 at large "g" values.

It is of interest to note that although the average seismicity in the western U.S. is more than an order of magnitude greater than the eastern U.S., in terms of earthquakes of various magnitudes (See Fig. 1), the predicted average accelerations are only 3 or 4 times as great (based on best fit distribution). Although the uncertainties in the results of this study are very considerable, this qualitative trend is consistent with previous observations concerning the much smaller rate of attenuation in the eastern U.S.

Table 4. The Annual Probability of Ground Acceleration in the Western U.S.

gr. acc. (cumulative)	I.* $\log N = -2.81-0.55M$	II.** $\log N = -0.662-0.881M$	III.*** $\log N = -3.73-0.513M$	IV. $\log N = -5.06+0.212M-0.0659M^2$
$\geq 0.05g$	3.65×10^{-2}	2.31×10^{-2}	3.66×10^{-2}	2.36×10^{-2}
$\geq 0.10g$	1.33×10^{-2}	7.93×10^{-3}	1.33×10^{-2}	8.50×10^{-3}
$\geq 0.15g$	7.22×10^{-3}	4.18×10^{-3}	7.26×10^{-3}	4.60×10^{-3}
$\geq 0.20g$	4.63×10^{-3}	2.60×10^{-3}	4.67×10^{-3}	2.93×10^{-3}
$\geq 0.25g$	3.27×10^{-3}	1.81×10^{-3}	3.30×10^{-3}	2.06×10^{-3}
$\geq 0.30g$	2.44×10^{-3}	1.33×10^{-3}	2.46×10^{-3}	1.53×10^{-3}
$\geq 0.40g$	1.50×10^{-3}	7.91×10^{-4}	1.52×10^{-3}	9.32×10^{-4}
$\geq 0.50g$	1.00×10^{-3}	5.18×10^{-4}	1.02×10^{-3}	6.20×10^{-4}
$\geq 0.60g$	7.08×10^{-4}	3.63×10^{-4}	7.18×10^{-4}	4.30×10^{-4}
$\geq 0.80g$	3.74×10^{-4}	1.87×10^{-4}	3.80×10^{-4}	2.28×10^{-4}
$\geq 1.00g$	1.94×10^{-4}	9.30×10^{-5}	1.98×10^{-4}	1.16×10^{-4}

*The b value is determined by the lower magnitude earthquakes.

**The b value is determined by the higher magnitude earthquakes.

***The b value is determined by the max. likelihood method.

Table 5. The Annual Probability of Ground Acceleration in the Eastern U.S.

gr. acc. (cumulative)	I. log N=-2.71-0.839M	II.* log N=-9.22+1.37M-0.186M^2	III.** log N=-3.908-0.636M
$\geq$ 0.05g	9.60×10^{-3}	3.78×10^{-3}	1.11×10^{-2}
$\geq$ 0.10g	4.07×10^{-3}	1.32×10^{-3}	4.86×10^{-3}
$\geq$ 0.15g	2.35×10^{-3}	6.85×10^{-4}	2.85×10^{-3}
$\geq$ 0.20g	1.48×10^{-3}	4.03×10^{-4}	1.82×10^{-3}
$\geq$ 0.25g	1.04×10^{-3}	2.68×10^{-4}	1.29×10^{-3}
$\geq$ 0.30g	7.82×10^{-4}	1.95×10^{-4}	9.71×10^{-4}
$\geq$ 0.40g	4.63×10^{-4}	1.07×10^{-4}	5.79×10^{-4}
$\geq$ 0.5g	3.03×10^{-4}	6.70×10^{-5}	3.81×10^{-4}
$\geq$ 0.6g	2.32×10^{-4}	5.00×10^{-5}	2.93×10^{-4}
$\geq$ 0.8g	1.15×10^{-4}	2.32×10^{-5}	1.45×10^{-4}
$\geq$ 1.0g	5.44×10^{-5}	1.05×10^{-5}	6.91×10^{-5}

*The new Madrid EQs are neglected (more or less).

**The b value is determined by the max. likelihood method.

VI. SOME OTHER COMPARATIVE RESULTS

The correlation between local intensity and acceleration is one obvious source of uncertainty in predictions such as those of this study. Ambraseys[14] has reviewed this matter recently; Trifunac and Brady[15] are finding an empirical relationship between acceleration and intensity which differs appreciably from the correlation of Coulter, et al.[13] used in this study. An available way of illustrating the uncertainties introduced in going from intensity to acceleration was to compare the results obtained by using the differing correlations of Reference 13 for soft, average and firm foundations.

The soft and firm foundation correlations of Reference 13 was represented algebraically as follows:

Soft:

$$\text{Log } g = -2.821 + 0.3333\ I \tag{16}$$

Firm:

$$\text{Log } g = -3.341 + 0.303\ I \tag{17}$$

Comparative results employing the correlations of Equations (16),(15) and (17) are given in Table 6.

Another comparison is possible by employing Algermissen's relationship[16]

$$\text{Log } N(I) = 6.302 - 0.9024\ I \tag{18}$$

between intensity and frequency of earthquakes greater than I., instead of the magnitude-frequency relationship derived from the lumped data, and a magnitude-epicentral intensity conversion.

Comparative results employing Algermissen's intensity distribution and the magnitude-frequency relation of Equation (4) are given in Table 7.

Table 6. Probability of Ground Acceleration per Year in the Eastern U.S. for Different Site Conditions[13]

Ground Accel. (incremental)*	Soft	Average	Firm
0.1g	7.7×10^{-3}	3.6×10^{-3}	1.4×10^{-3}
0.2g	1.9×10^{-3}	6.5×10^{-4}	1.5×10^{-4}
0.5g	2.0×10^{-4}	4.6×10^{-5}	7.9×10^{-6}
1.0g	3.5×10^{-5}	1.0×10^{-5}	6.9×10^{-7}

*The increment includes a band of accelerations around that specified, e.g., the probability at 0.1g is equal to the average of the two probabilities 0.05g to 0.1g and 0.1g to 0.15g.

Table 7. Probability of Ground Acceleration per Year in the Eastern U.S. Using Algermissen's Intensity-Frequency Correlation or Equation (4)

Ground Accel. (incremental)*	Algermissen	Equation (4)
0.1g	5.5×10^{-3}	3.6×10^{-3}
0.2g	3.4×10^{-4}	6.5×10^{-4}
0.5g	9.1×10^{-6}	4.6×10^{-5}
1.0g	1.1×10^{-6}	1.0×10^{-5}

*See Table 6

VII. PROBABILITY DISTRIBUTION IN THE EASTERN U.S. BY EXTREME VALUE APPROXIMATION

In the above probability calculations, two assumptions have been made. First, the number of earthquakes occurring per unit time follows a Poisson distribution. Second, particularly for the eastern U.S., since the data (even the lumped one) cover only up to magnitude 7 for the period of 1904 to 1965, linear extrapolation on a semi-log paper has been made. It is of interest to compare the results with some different approach such as extreme value method.[17]

Gumbel's extreme value theory has been frequently applied to the analysis of seismic risk.[18,19] The cumulative distribution function of annual largest earthquake with magnitude M smaller than or equal to m in a region is

$$F(M \leq m) = e^{-\alpha e^{-\beta m}} \tag{19}$$

The parameters α and β are estimate from the best fit to the equation

$$\ell n(-\ell n\ F(m)) = \ell n\ \alpha - \beta m \tag{20}$$

If all the annual largest earthquake magnitudes in n years are arranged in order of increasing values, F(m) is given by[19]

$$F_i(m) = \frac{i}{n+1} \tag{21}$$

These points should be close to a straight line on an extreme value paper. The reference gives a method of testing the fitness. The probability density function of annual largest earthquake with average magnitude m_j between $m_{j-1/2}$ and $m_{j+1/2}$ is

$$P(m_j) = F_{j+1/2}(m) - F_{j-1/2}(m) \tag{22}$$

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An approximation is made in applying the extreme value method by neglecting all the contribution of more than one earthquake to the probability, which is plausible in a region of very low seismicity. The probability of ground acceleration of g_i at any site in the region is

$$P(g_i) = \sum_j P(g_i/m_j)\ P(m_j) \tag{23}$$

where the first factor $P(g_i/m_j)$ is the probability of experiencing g_i at a point when an earthquake of magnitude m_j has occurred, which can be evaluated as

$$P(g_i/m_j) = \frac{a_{ij}}{A} \tag{24}$$

as discussed previously. The second factor $P(m_j)$ is the magnitude distribution in one year and is derived from the extreme value distribution function $F(m)$.

A model for the occurrence of largest earthquake proposed by Epstein et al.[20] seems to be applicable for the low seismicity eastern U.S. They derived a cumulative distribution function similar to Equation (19) and obtained a set of relations as

$$\left.\begin{aligned} \ln \alpha &= a \ln 10 \\ \beta &= b \ln 10 \end{aligned}\right\} \tag{25}$$

where a and b are the constants of the well-known cumulative linear relation discussed previously. For the eastern U.S., using the lumped seismic data from 1930 to 1965, we found (Fig. 4)

$$\left.\begin{aligned} \ln \alpha &= 10.5 \\ \beta &= 1.95\ , \end{aligned}\right\}$$

comparing to

$$\left.\begin{aligned} \ln \alpha &= 10.6 \\ \beta &= 1.93 \end{aligned}\right\}$$

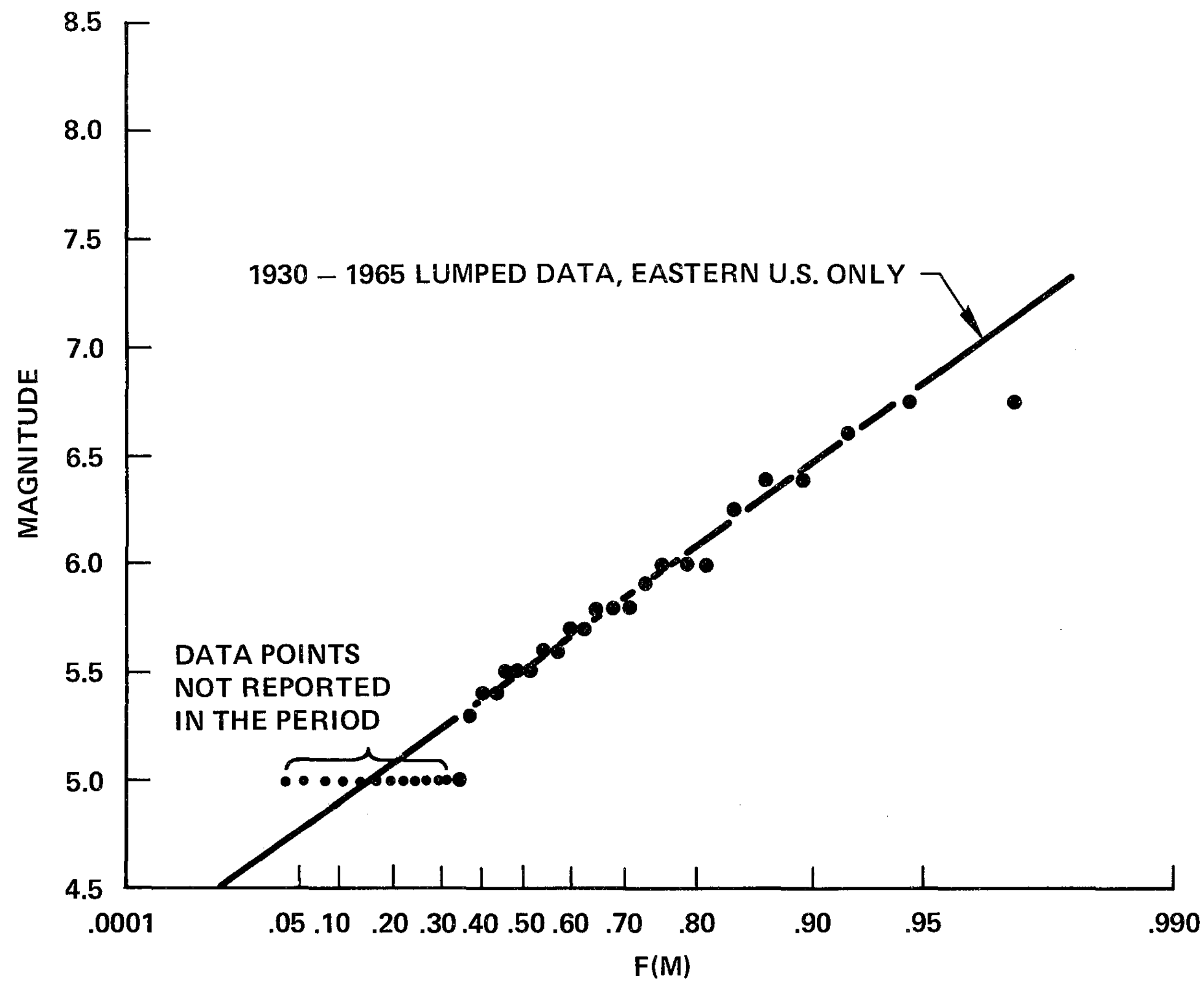

Figure 4. Extreme Value Approach for the Eastern U.S. (lumped).

obtained by using Equations (25) and (4). Since Equation (4) was based on 1904 to 1965 data instead of 1930 to 1965 data which were used for obtaining α and β, a probability calculation similar to that as given in Table 5, but using 1930 to 1965 data, was repeated. A brief comparison among these results is given in Table 8.

Table 8. Probability of Ground Acceleration per Year in the Eastern United States Based on Extreme Value and "b" Value Approaches

Ground Accel. (incremental)*	Extreme Value Approx.	Linear Magnitude Dist. (best fit)	
	1930 to 1965 data	1930 to 1965 data	1904 to 1965 data
0.1g	1.5×10^{-3}	2.9×10^{-3}	3.6×10^{-3}
0.2g	2.6×10^{-4}	4.7×10^{-4}	6.5×10^{-4}
0.5g	1.8×10^{-5}	3.1×10^{-5}	4.6×10^{-5}
1.0g	4.0×10^{-6}	6.8×10^{-6}	1.0×10^{-5}

*The increment includes a band of accelerations around that specified, e.g., the probability at 0.1g is equal to the average of the two probabilities 0.05 to 0.1g and 0.1g to 0.15g.

VIII. DISCUSSION

The estimated probabilities for the larger accelerations must be considered to have very large uncertainties. It appears to be doubtful that the combination of correlations used, whether between magnitude, acceleration and felt area, or intensity, acceleration and felt area are really extrapolable for accelerations above about 0.3 to 0.5g. It may well be that the isoseismal areas of high intensity are much smaller than those estimated by applying Brazee's correlation to large events. Much more work is needed in this aspect.

The probabilities obtained from Algermissen's intensity distribution[16] were presented for the purpose of comparison only. Table 7 indicates that the difference is small for low acceleration, becoming larger for higher acceleration but still within one order of magnitude difference for the highest acceleration. It should be emphasized that the intensity distribution was extrapolated from Algermissen's work and not obtained directly from seismic data analysis.

The site condition has smaller effects at low accelerations, but may change the probability numbers significantly either in an upward direction (for a softer site) or in a downward direction (for a firm site) for high accelerations (Table 6), assuming the applicability of Coulter's curve.[13] Further work is needed to evaluate the import of the measurements of Trifunac[15] which yield higher accelerations on a firmer foundation, given the same estimated MM intensity. Differences in the dominant frequency for firm or soft foundation conditions may be present. Acceleration versus frequency, as well as velocity and displacement, may be significant in the evaluation of the effects of vibratory motion on soils or structures; however our study does not delve into this aspect.

Table 8 indicates that the linear extrapolation on a semi-log paper gives about the same results as the extreme value approximation (within a factor of 2 or 3). The probabilities obtained from the extreme value approximation are slightly lower than that from the former method; this may be ascribed to the omission of all but the largest earthquake per year in the formulation of Equation (23).

An interesting conclusion which can be drawn from this study is that the probabilities of ground accelerations in the western U.S. may be higher by only a factor of about three than those in the eastern U.S. Although the seismicity differs more than one order of magnitude (Tables 1 through 3 and Figure 1), the difference of geological attenuation appears to bring the probabilities closer for the two regions (Tables 4 and 5).

As noted previously, the correlations among earthquake magnitude, felt area, and ground acceleration involve uncertainties which are not small. For example, calculations of probability of ground acceleration in the western U.S., using different correlations such Milne's,[21] Esteva's,[22] Housner's,[10] and Mickey's,[9] indicated that there is about one decade difference in the probabilities for ground accelerations up to about 0.3g. The uncertainties are even worse for higher ground accelerations primarily because of the fact that Milne's and Housner's correlations are not applicable for accelerations greater than 0.5g. Esteva's correlation gives the lowest probabilities for accelerations up to 0.5g, and predicts overall probabilities even lower than that estimated for the eastern U.S.

The corresponding relationship for the eastern U.S. is further worsened by the fact that in most literature the correlations are presented in terms of Modified Mercalli Intensity (MMI) and felt area. An additional correlation between MMI and ground acceleration must be applied, which introduces more uncertainty.

Nuttli[23] presented a few direct correlations between earthquake magnitudes (M=7.2, 6.2, and 5.7) and ground acceleration for a portion of the central U.S. The correlation which covers an interesting range of ground acceleration is for M=7.2. (For ground acceleration above 0.45g, it involves a wide band of uncertainty.)

In the following, we compare this and several other correlations with Brazee's correlation:

1. Brazee's felt areas are about 3 to 4 times of that of the Nuttli's.[23] (M = 7.2, 0.1g to 0.4g, wave freq. = 0.3 hz, hard rock.)

2. For MMI greater than XI, Brazee's correlation is based on some extrapolation. If the 1811 - 1812 New Madrid earthquake epicentral intensities were XII, and the attenuation curve of the TVA Nuclear Power Plant Site Study[24] is used, Brazee's correlation gives larger felt area (a factor of about 10). However, the Preliminary Safety Analysis Report (PSAR) for the Grand Gulf Nuclear Power Station[25] indicated that the epicentral intensities of the 1811 - 1812 New Madrid earthquakes could be as low as XI; if this is the case, Brazee's felt area is about the same as would be predicted using the TVA attenuation curve. If the felt area listed in Table C.3.3 of the Grand Gulf Station PSAR means MMI IV, then Brazee's felt area is about 3.5 times larger. Nuttli[26] reported that the epicentral intensities of the New Madrid earthquakes were X to XI, and that the isoseismal

line covered an area of 600,000 km^2 for MMI VII and greater, and an area of 2,500,000 km^2 for MMI V and greater. If the epicentral intensities were XI, Brazee's corresponding felt areas are about 2.5 to 4 times of that of Nuttli's. However, if the epicentral intensities were X, Brazee's felt areas are smaller by a factor of about 1.5 to 3.

3. Bollinger[27] provided a representative felt area — intensity relationship of

$$\text{Log } A(\text{sq. mi.}) = 2.3(\pm 0.3) + 0.4(\pm 0.3)\ I_o \qquad (26)$$

for the southeastern U.S. region. Comparison shows that Brazee's felt areas are about the same, if the felt area means an iso-seimal line with intensity I, and are much smaller if the felt area means an isoseismal line with intensity close to IV. Bollinger[27] indicates that felt areas can scatter as much as three orders in magnitude.

4. Milne[21] gave a relationship between acceleration and distance in the eastern Canada. Comparison shows that Brazee's felt areas are about 2 to 5 times larger.

5. Draft WASH-1400[28] reports much lower probabilities of ground acceleration (about a factor of 100) in the eastern U.S. then obtained in this study. Their work is based primarily on a paper by Cornell, et al.[29]

The differences appear to be attributable in part, as follows:

a. Reference 29 uses felt areas which are smaller (about a factor of 2) than Brazee's correlation.

b. Reference 29 uses an MMI value corresponding to ground acceleration roughly one unit higher on the scale than used in this study. For example, 0.1g corresponds to MMI = 7.4, based on Cornell's correlation, where 0.1g corresponds to MMI = 6.5 based on Coulter's curve.

c. Reference 29 does not appear to have assigned a nonzero probability to higher intensities (e.g., MM IX).

More specifically, Cornell employs the following relationship among intensity, epicentral intensity and distance:

$$I_{site} = 2.6 + I_0 - 1.3 \ln R \tag{27}$$

On the other hand, Brazee's correlation has the form given previously in Equation (14).

For an epicentral intensity of IX and an intensity VIII, the ratio of felt areas is

$$\frac{\text{Brazee}}{\text{Cornell}} = \frac{3020}{790} = 3.8$$

while for an intensity VII, the ratio of felt areas is

$$\frac{\text{Brazee}}{\text{Cornell}} = \frac{16{,}200}{3720} = 4.4$$

For an epicentral intensity of VIII and an intensity VII, the ratio of felt areas is

$$\frac{\text{Brazee}}{\text{Cornell}} = \frac{2770}{790} = 3.5$$

6. A comparison between Brazee's correlation and the data in Coffman, et al.[30] is also made. Generally speaking, while the felt area showed great scatter in Reference 30, the median agreed reasonably well with Brazee's correlation, if an isoseismal of intensity III was taken as the limit in the latter.

APPENDIX I

MAXIMUM LIKELIHOOD ESTIMATION OF THE PARAMETERS OF A DISTRIBUTION FROM GROUPED DATA

The distribution function of the magnitude of earthquakes which is used in the text (see Equation (1)) is

$$
\begin{aligned}
H(m) \equiv P[\text{EQ. magn.} \geq M] &= 10^{-b(M-M_0)} \quad , \quad M \geq M_0 \\
&= 0 \quad\quad\quad\quad\quad , \quad M < M_0
\end{aligned}
\tag{I.1}
$$

where M_0 is the lowest magnitude of interest (changing the value of M_0 for a particular region affects the value of a in Equation (1)).

When the observations of earthquake magnitudes are grouped into intervals (see Tables 1,2 and 3), the maximum-likelihood estimator (MLE) of b, say $\hat{b}$, is calculated as follows:

Let π_i denote the interval of magnitudes $(M_{i+1} - M_i)$. Then the probability that one earthquake has magnitude in π_i is

$$
\begin{aligned}
I_i &\equiv H(M_i) - H(M_{i+1}) \\
&= 10^{-b(M_i-M_0)} - 10^{-b(M_{i+1}-M_0)}
\end{aligned}
\tag{I.2}
$$

and the probability that exactly n_i earthquakes have magnitudes in π_i is $I_i^{n_i}$.

If the total number of magnitude intervals is k+1, the likelihood function L is defined as the probability that out of n earthquakes, n_0 will have magnitudes in π_0, n_1 in $\pi_1, \ldots, n_k$ in π_k $\left(\sum_{i=0}^{k} n_i = n\right)$. This probability is readily found to be (multinomial distribution):

$$
L = \frac{n!}{\prod_{i=0}^{k} n_i!} \prod_{i=0}^{k} I_i^{n_i} \tag{I.3}
$$

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The MLE $\hat{b}$ is chosen to be that value of b which maximizes L or the logarithm of L, (the calculations are facilitated if log L is used and clearly the value of $\hat{b}$ will be the same). Therefore, $\hat{b}$ is the solution of the equation

$$\frac{d \log L}{db} = 0 \qquad (I.4)$$

Using Equations (I.3) and (I.4) we get

$$\frac{d \log L}{db} = \frac{d}{db}\left(\sum_{i=0}^{k} n_i \log I_i\right) = 0$$

or

$$\sum_{i=0}^{k} \frac{n_i}{I_i} \frac{dI_i}{db} = 0$$

or, using Equation (I.2),

$$\sum_{i=0}^{k} \frac{n_i\left[-(M_i-M_0)10^{-b(M_i-M_0)} + (M_{i+1}-M_0)10^{-b(M_{i+1}-M_0)}\right]}{\left[10^{-b(M_i-M_0)} - 10^{-b(M_{i+1}-M_0)}\right]} = 0 \qquad (I.5)$$

Having estimated $\hat{b}$ from Equation (I.5), the corresponding value of a can be determined from Equation (1).

The general theory of maximum likelihood estimation can be found in Reference 31, and details on the present application in Reference 8.

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