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SENSITIVITY ANALYSIS FOR HYSTERETIC DYNAMIC SYSTEMS: THEORY AND APPLICATIONS

by

D. RAY
K. S. PISTER
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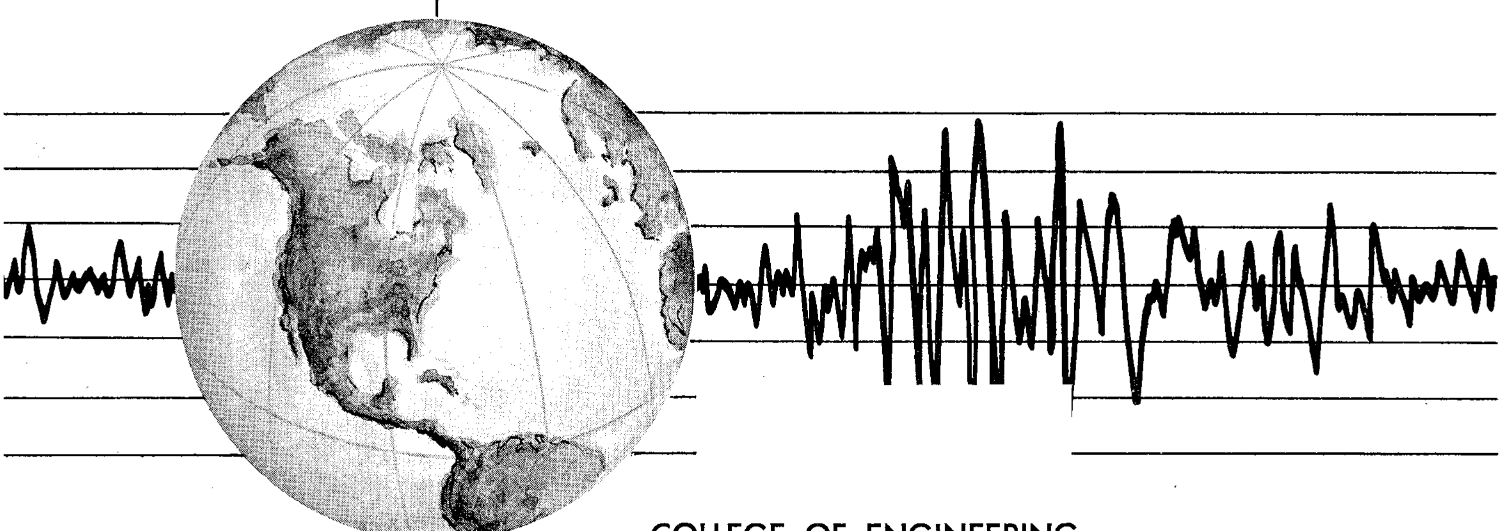

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ABSTRACT

Sensitivity analysis, calculation of the rate of change of response variables with respect to design variables, is a critical component in the process of re-analysis for improvement of trial designs or in seeking an optimum design. This report presents necessary theorems and provides details for numerical computation of sensitivity matrices for spatially discretized structural systems subjected to dynamic excitation. General results are presented for nonlinear (hysteretic) structures and explicit numerical examples illustrate the methodology applied to multi-story shear frames whose force-displacement relationship is bilinear hysteretic.

1. INTRODUCTION

For the purpose of analysis and design for dynamic loads, it is customary to idealize complex structures as spatially discrete dynamic systems whose degrees of freedom are associated with motion of a finite set of nodal points, at which are represented mass, damping and internal restoring force properties of the components of the structure. In general spatial discretization is most easily carried out by employing the finite element method, although in cases such as rigid frames, discretization into beam and column elements follows in an obvious manner. For a wide class of problems the resulting equations governing dynamic behavior of the now-idealized structure can be put in the form

$$\mathcal{D}[\beta, z(\beta,\tau), t] = 0 \; ; \; \tau \in [0,t], \; t \in [0,T] \tag{1.1}$$

with initial conditions

$$z(\beta,0) = 0,$$

where $z(\beta,t)$ denotes the $\bar{N}$-dimensional state vector* of the system at time t, $\mathcal{D}$ is a differential or integro-differential operator defining system dynamics and β is a P-dimensional time-invariant parameter vector characterizing both the properties of the structure as well as those of the forcing function producing motion. For example, we may consider β to be partitioned into sub-vectors which respectively characterize distribution of mass, geometric properties of structural components, constitutive properties of materials and amplitude, frequency and

*In typical application, z is composed of ordered sets of displacements and velocities of nodal points; readers unfamiliar with this representation may find a discussion such as that contained in [1] helpful. State space formulation of the problem facilitates treatment of basic theorems underlying the work as well as unifying results with those of other areas of application.

stationarity characteristics of the exciting force system. Here we will postpone giving examples in favor of describing the general structure of the problem and the significance of sensitivity analysis. Returning to (1.1) we note that the present value of the state vector may depend on the past history (path of evolution) of the dynamic process (such as may occur in inelastic systems) and that T denotes the extent of the time period of interest over which the system is observed.

In typical problems of prediction of structural response the vector β is completely prescribed; i.e., distribution of mass, length and area of members and their mechanical behavior as well as the dynamic excitation function are given and the problem is to obtain the state vector as a function of time (for the prescribed β) by direct numerical integration of (1.1). Here we are interested in an <u>inverse</u> problem associated with (1.1), applications of which are found in mathematical modeling of mechanical behavior of structural components, optimal synthetic design or simply conducting trade-off studies in attempting to achieve an improved (not necessarily optimal) design of a structure. A common element in each of these activities is sensitivity analysis, i.e., the capability to compute efficiently the change in structural response (values of the state vector) associated with changes in the parameter vector. In our problem format, since both z and β are vectors, the resulting set of partial derivatives constitutes a sensitivity matrix of size $\bar{N}$xP whose elements are functions of time and the parameter vector β.

A problem format including both mathematical modeling (parameter identification) and optimal design is the following: Find

$$\min_{\beta} f[\beta, z(\beta,t)] \ , \ t \in [0,T] \tag{1.2}$$

subject to the system dynamics (1.1) and a set of constraints

$$q[\beta, z(\beta,t)] \leq 0, \quad t \in [0,T], \tag{1.3}$$

where the dimension of the vector q is M.

The objective, or cost function (1.2) in the case of a parameter identification problem represents an error measure between a set (or sets) of observed data and the prediction of a hypothesized mathematical model whose parameter vector β is to be adjusted to minimize the error. Equation (1.1) defines the dynamic properties of the test configuration for which data are collected and constraints (1.3) reflect natural or imposed limitations on values of the parameter and state vectors. Readers interested in more specific details of this class of problems in the context of structural mechanics may consult [4]. In design applications undetermined components of β are usually taken to be member cross-sectional areas (or functions thereof), while the components of β associated with mass, member length and constitution, and input are prescribed. Here the objective function (1.2) is some measure of cost of the structure, (1.1) continues to govern dynamic response of the structure and (1.3) prescribes limits on structural response, e.g., maximum stresses or displacements in the structure. Whether dealing with the objective function or constraints, in either the problem of identification or design, it is clear that to calculate changes in cost or constraints, it is necessary (or at least helpful)* to be able to calculate the sensitivity matrix $\frac{\partial z}{\partial \beta}$.

For example, in executing the search procedure for an optimal design

*In direct search methods such calculations are not required. In gradient methods and for trade-off studies, efficient calculation of the sensitivity matrix is demanded.

using a method of feasible directions, computations of gradients of 'active' constraint functions are required, [2]. Referring to (1.3), this calls for the major computational task of evaluating $\frac{\partial q_i}{\partial \beta_j}(\beta, z(\beta, t))$ for each constraint q_i for which (1.3) is an equality. One method of evaluating these gradients is by first obtaining $\frac{\partial z}{\partial \beta}(\beta, t)$, $t \in [0, T]$ as the solution of a system of <u>$(\bar{N}xP)$ matrix equations</u>, called perturbation equations, (see Appendix A) resulting from employing linear perturbation analysis on the dynamic equations of motion (called system equations), [2]. Then, premultiplying this matrix by $\frac{\partial q}{\partial z}(z(\beta, t))$ provides the required gradients. Since the dynamic equations of the system and the perturbation equations have the same form, substantial savings in computation time can be effected by carrying forward the solution of these equations simultaneously; see [1] for examples. In the sequel this approach is associated with the term "first implementable form."

For maximum computational efficiency under certain circumstances it is preferable to employ a second method of evaluating the sensitivity matrix (and gradients of active constraints) in which use is made of the adjoint of the perturbation equation, a vector equation with dimensions $(\bar{N}x1)$; in the sequel this approach is identified with the "second implementable form." To give illustrations in which the choice of implementable form is important, in an optimal design problem the optimal structure is likely to be associated with a set of "tight constraints," i.e., most constraint functions will be active and the full sensitivity matrix is required, necessitating use of the first computational method. On the other hand if a trial design is far from optimal, with few constraints active, or if one wishes to make selective comparisons of design changes, the latter method is obviously superior from a computational standpoint.

For linear system dynamics it is possible to give explicit results for the calculation of sensitivity matrices, as shown for example in [1]. However, for nonlinear systems no such results are available. Therefore, the purpose of this report is to present necessary theorems and provide details enabling numerical computation of sensitivity matrices for discretized dynamic systems whose material properties are hysteretic. The basic theorems for the perturbation equation and its adjoint, applications of which were qualitatively discussed above, are contained in Appendix A. For analytical simplicity these equations are given in state-space representation of the system dynamics. In section 2 the results obtained in Appendix A are transcribed into the more familiar second-order integro-differential format in order to take advantage of symmetry in the mass, damping and tangent stiffness matrices. Explicit representations of sensitivity equations are presented for a multi-story shear frame with a bilinear, hysteretic force-displacement relationship, subjected to earthquake ground motion and for which the parameter vector components are moments of inertia of the column cross-sections of each story. Alternative methods for calculating elements of the sensitivity matrix are illustrated choice of which depends on the number of constraints active at the particular stage of the design process. Section 3 describes a numerical treatment of the material presented in the preceding sections, while a discussion of the results for example problems appears in Section 4.

2. IMPLEMENTABLE SENSITIVITY ANALYSIS

2.1. Introduction

The theorems and corollary for calculating sensitivity matrix elements presented in Appendix A appear in general form. Specialization for particular structural systems and member force-deformation relationships provides the necessary detail to yield corresponding implementable* forms for applications. In this section the structural system chosen for illustrative purposes is a shear-type frame with a bilinear force-deformation relationship between story shear force and relative story drift see Figure A1(b). The first theorem and corollary appearing in the Appendix will be transcribed to yield two implementable forms, with a direct transformation of the result given in Appendix (A-2) producing an alternative numerically favorable to the particular structural model and force-deformation relationship chosen here. Finally, it should be recognized that the selections of structural system and hysteretic model, while simplifying, are not formally restrictive on the field of application of the basic theorems presented in the Appendix. Sensitivity equations for other structural models, such as a rigid frame with flexible girders, or other hysteretic material models, can be obtained in a similar fashion. Before proceeding it should be recalled that the first form provides the full sensitivity matrix by solving a system of ($\bar{N}$xP) matrix equations while in the second method the elements of the sensitivity matrix can be calculated a row at a time by solving a vector equation of dimension ($\bar{N}$x1), where $\bar{N}$ and P are the dimensions of the state vector and design vector, respectively.

2.2. Structural System and Transcription of System Equations:

Structural Model - The form of equations and the ease with

*By implementable is meant computationally feasible.

which one can perform numerical analysis of the dynamic system equations (1.1), (or alternatively (A-6) for the perturbation equations) depends to a large extent on the model chosen for the hysteretic force-deformation relationship of the structure. Although the bilinear model chosen here has certain elements of simplicity, most of the algebraic and computational complications of the sequel are a result of the fact that the tangent-stiffness matrix is discontinuous when a particular structural element first yields, unloads from a plastic state or subsequently yields upon re-loading. The reader should separate this largely "book-keeping" complication from the logical structure underlying sensitivity analysis.

It is assumed that the structure is comprised of columns whose cross-sectional area and elastic section modulus are continuously differentiable functions of the moment of inertia of the cross-section, [2]. Thus, the parameter vector, β, for an N-story, one-bay, shear-type frame as shown in Fig. 1 can be expressed as $\beta = (I_1, I_2, \ldots, I_N)^T \in \mathbb{R}^N_+$, where for each i, I_i is the moment of inertia of the cross-section of a column with respect to its strong axis. As noted in section 2.1, we adopt a bilinear structural model illustrated in Fig. A1(b). Let $t_i(\beta)$, $i \in \{1,2,\ldots,J\}$ with $t_o(\beta) \equiv 0$, be the set of points in time domain $[0,T]$ at which any element of the structure's tangent-stiffness matrix changes. For any $t \in [t_j(\beta), t_{j+1}(\beta)] \subset [0,T]$, $j = 0,1,\ldots,J$, the tangent-stiffness matrix, denoted by $K^j(\beta)$, remains constant and continuously differentiable in β. The transition points, $t_i(\beta)$, $i \in \{1,2,\ldots,J\}$ correspond to changes in any or all element stiffnesses caused, either by yielding of elements, or, by unloading and reloading of yielded elements, or, both.

A Rayleigh damping matrix is considered in the sequel, i.e.,

$$C(\beta) = \tilde{a}_o(\beta)\ M + \tilde{a}_1(\beta)\ K^o(\beta) \tag{2.1}$$

where $\tilde{a}_o(\beta)$ and $\tilde{a}_1(\beta)$ are determined by specification of only two modal damping ratios and solving the system of two equations:

$$W(\beta)\ \tilde{a}(\beta) = 2\xi \tag{2.2}$$

where

$$W(\beta) = \begin{bmatrix} \dfrac{1}{\omega_1(\beta)} & \omega_1(\beta) \\ \\ \dfrac{1}{\omega_2(\beta)} & \omega_2(\beta) \end{bmatrix} \quad ;$$

$\tilde{a}(\beta) \equiv (\tilde{a}_o(\beta), \tilde{a}_1(\beta))^T$, is the undetermined coefficient vector, $\xi \equiv (\xi_1, \xi_2)^T$, is the specified modal damping ratio vector, and M is the diagonal mass matrix, assumed independent of β. The equation of motion of such a system, initially at rest and subsequently subjected to horizontal ground motion of acceleration, $\ddot{u}_g(t) \in C^o[0,T]$, may be expressed as

$$M\ddot{u}(\beta,t) + C(\beta)\ \dot{u}(\beta,t) + K^j(\beta)u(\beta,t) = -\sum_{i=1}^{j} [K^{i-1}(\beta) - K^i(\beta)]\ u(\beta, t_i(\beta))$$

$$- M\hat{1}\ddot{u}_g(t),$$

$$t \in (t_j(\beta), t_{j+1}(\beta)] \subset [0,T], \quad \forall j \in \{0,1,\ldots,J\} \tag{2.3}$$

$$u(\beta,0) = 0$$

$$\dot{u}(\beta,0) = 0$$

where $\ddot{u}(\beta,t)$, $\dot{u}(\beta,t)$ and $u(\beta,t)$ are system acceleration, velocity and displacement vectors relative to the base motion and $\hat{1} \in \mathbb{R}^N$ with all components unity. We now relate the state vector z to the motion u. In (A-4) let

$$z(\beta,t) = \begin{pmatrix} z_1(\beta,t) \\ \hline z_2(\beta,t) \end{pmatrix} \equiv \begin{pmatrix} u(\beta,t) \\ \hline \dot{u}(\beta,t) \end{pmatrix} \quad \text{with } \bar{N} = 2N$$

$$g_i(\beta,z(\beta,s)) \equiv \begin{pmatrix} \theta \\ \hline -M^{-1}K^i(\beta)\,z_2(\beta,s) \end{pmatrix} \qquad , \forall i \in \{0,1,\ldots,J\}$$

and

$$f(\beta,z(\beta,t),t) \quad \begin{pmatrix} z_2(\beta,t) \\ \hline -M^{-1}[C(\beta)\,z_2(\beta,t)+\hat{1}\ddot{u}_g(t)] \end{pmatrix} \tag{2.4}$$

Then, (A-4) yields, after integration, the system equation of motion (2.3) and corresponding initial conditions. Let $F_s(\beta,t)$ be the restoring force vector whose ith component, $F_{si}(\beta,t)$, denotes the spring force associated with the ith degree of freedom; let $F(\beta,t)$ be the vector of story shear force and $x(\beta,t)$ be the relative story drift vector. Define L to be a constant, lower bidiagonal matrix with all diagonal terms 1 and lower first off-diagonal terms -1; all other terms are 0. Then, for a shear-type frame it follows that

$$F_s(\beta,t) = L^T F(\beta,t), \tag{2.5)(i}$$

$$x(\beta,t) = Lu(\beta,t), \tag{2.5)(ii}$$

and $$K^j(\beta) = L^T[k^j]L \tag{2.5)(iii}$$

where,

$$F_s(\beta,t) = \sum_{i=1}^{j} [K^{i-1}(\beta)-K^{i}(\beta)]\ u(\beta,t_i(\beta)) + K^j u(\beta,t),$$
$$t \in (t_j(\beta),t_{j+1}(\beta)] \subset [0,T] \qquad (2.5)(iv)$$

and $[k^j]$ is an (NxN) diagonal matrix whose ith diagonal term represents the current ith story stiffness, which may be either the elastic or post yielding stiffness of the story. Now, combining the results of (2.3), (2.4) and (2.5), we have the following familiar equations of motion:

$$M\ddot{u}(\beta,t) + Cu(\beta,t) + F_s(\beta,t) = -M\hat{1}\ddot{u}_g(t),\ t \in [0,T] \qquad (2.6)$$

2.3. First Implementable Form

Differentiating (2.4) with respect to z and using definitions (A-16), we obtain the 2Nx2N constant matrices

$$A(s) \equiv \frac{\partial f}{\partial z}(\beta,z(\beta,t),t) = \left[\begin{array}{c|c} \theta & I \\ \hline \theta & -M^{-1}C(\beta) \end{array}\right] \qquad (2.7)(i)$$

where θ is an (NxN) null matrix and

I is an (NxN) identity matrix, $\forall s \in (t_i(\beta),t_{i+1}(\beta)]$.

and

$$B_i(s) \equiv \frac{\partial g_i}{\partial z}(\beta,z(\beta,t)) = \left[\begin{array}{c|c} \theta & \theta \\ \hline \theta & -M^{-1}K^i(\beta) \end{array}\right] \qquad (2.7)(ii)$$

for $\forall s \in (t_i(\beta),t_{i+1}(\beta)]$. Using (2.4), (2.7) and performing appropriate integration in (A.6), gives the following transcribed perturbation equation for calculating components of the sensitivity matrix $\frac{\partial u}{\partial \beta}$:

$$M \frac{\partial \ddot{u}}{\partial \beta_\ell}(\beta,t) + C(\beta)\frac{\partial \dot{u}}{\partial \beta_\ell}(\beta,t) + K^j(\beta)\frac{\partial u}{\partial \beta_\ell}(\beta,t)$$

$$= - \sum_{i=1}^{j} [K^{i-1}(\beta)-K^i(\beta)]\{\frac{\partial u}{\partial \beta_\ell}(\beta,t_i(\beta)) + \dot{u}(\beta,t_i(\beta))\frac{\partial t_i}{\partial \beta_\ell}(\beta)\}$$

$$- \sum_{i=1}^{j} [\frac{\partial K^{i-1}}{\partial \beta_\ell}(\beta) - \frac{\partial K^i}{\partial \beta_\ell}(\beta)]\, u(\beta,t_i(\beta)) - \frac{\partial K^j}{\partial \beta_\ell}(\beta)\, u(\beta,t)$$

$$- \frac{\partial C}{\partial \beta_\ell}(\beta)\, \dot{u}(\beta,t), \quad t \in (t_j(\beta),t_{j+1}(\beta)] \subset [0,T],\ \ell \in \{1,2,\ldots,N\}$$

$$\frac{\partial u}{\partial \beta_\ell}(\beta,0) = 0$$

$$\frac{\partial \dot{u}}{\partial \beta_\ell}(\beta,0) = 0 \qquad (2.8)$$

It may be noted that at any transition time $t_i(\beta)$, $i \in \{1,2,\ldots,J\}$ at which stiffness transition is caused by yielding of an element, $\frac{\partial F_s}{\partial \beta_\ell}(\beta,t)$ is a discontinuous function of t with the jump given by

$$\frac{\partial F_s}{\partial \beta_\ell}(\beta,t_i(\beta)^+) - \frac{\partial F_s}{\partial \beta_\ell}(\beta,t_i(\beta)^-) = [K^{i-1}(\beta)-K^i(\beta)]\, \dot{u}(\beta,t_i(\beta))\frac{\partial t_i}{\partial \beta_\ell}(\beta) \qquad (2.9)$$

For those $t_i(\beta)$ at which a transition is caused by reappearance of elastic behavior of elements (due to unloading or reloading of yielded elements) a reversal in direction of motion occurs corresponding to a condition of zero velocity. In these instances the discontinuity in $\frac{\partial F_s}{\partial \beta_\ell}(\beta,t)$ vanishes as can be noted from (2.9). In general, however, when a transition is a combination of both conditions, no such simplification results.

Finally, for convenience let

$$Q_s^{\ell}(\beta,t) = \frac{\partial F_s}{\partial \beta_\ell}(\beta,t) - K^j(\beta)\,\frac{\partial u}{\partial \beta_\ell}(\beta,t), \qquad t \in [0,T], \quad \ell \in \{1,2,\ldots,N\} \tag{2.10}$$

Then, using (2.10), (2.5)(iv) and its derivative with respect to β_ℓ along with (2.9), we obtain from (2.8) the required first implementable form for calculation of the sensitivity matrix $\frac{\partial u}{\partial \beta}$,

$$M\,\frac{\partial \ddot{u}}{\partial \beta_\ell}(\beta,t) + C(\beta)\,\frac{\partial \dot{u}}{\partial \beta_\ell}(\beta,t) + K^j(\beta)\,\frac{\partial u}{\partial \beta_\ell}(\beta,t) = -\,\frac{\partial C}{\partial \beta_\ell}(\beta)\,\dot{u}(\beta,t) - Q_s^{\ell}(\beta,t),$$

$$\frac{\partial u}{\partial \beta_\ell}(\beta,0) = \frac{\partial \dot{u}}{\partial \beta_\ell}(\beta,0) = 0 \qquad t \in [0,T], \quad \ell \in \{1,2,\ldots,N\} \tag{2.11}$$

Evaluation of $Q_s^{\ell}(\beta,t)$, $t \in [0,T]$, will be discussed in section 3 on numerical experimentation.

2.4. Second Implementable Form:

To transcribe the result given by the second theorem in Appendix A-4 to second order form, the following relations must be introduced to describe conditions for stiffness transitions:

yield condition - $x^c_{pyi}(\beta,t)$ and $x^c_{nyi}(\beta,t)$ denote the positive and negative yield drift, respectively, of story i at any time $t \in [0,T]$; then, defining $t_i^*(\beta)$, the time corresponding to first yielding of element i, $i \in \{1,2,\ldots,N\}$, we have,

(a) for $t \leq t_i^*(\beta)$, $i \in \{1,2,\ldots,N\}$

$$x^c_{pyi}(\beta,t) = x_{yi}(\beta), \tag{2.12)(i}$$

$$x^c_{nyi}(\beta,t) = -\,x_{yi}(\beta),$$

where $x_{yi}(\beta)$ is the initial yield drift for element i.

(b) for $t > t_i^*(\beta)$

(i) for $\dot{x}_i(\beta,t) > 0$,

$$x_{pyi}^c(\beta,t) = x_i(\beta,t), \tag{2.12)(ii}$$

$$x_{nyi}^c(\beta,t) = x_i(\beta,t) - 2x_{yi}(\beta),$$

(ii) for $\dot{x}_i(\beta,t) < 0$,

$$x_{pyi}^c(\beta,t) = x_i(\beta,t) + 2x_{yi}(\beta), \tag{2.12)(iii}$$

$$x_{nyi}^c(\beta,t) = x_i(\beta,t).$$

unloading or reloading condition - For any $t_j(\beta)$, $j \in \{1,2,\ldots,J\}$ at which element i, $i \in \{1,2,\ldots,N\}$, unloads or reloads, i.e., the corresponding element stiffness changes from post-yielding stiffness to its elastic stiffness, we have,

$$\dot{x}_i(\beta,t_j(\beta)) = 0 \tag{2.13}$$

Now, from (2.12), if the mth element yields at any $t_i(\beta)$, then

$$\begin{aligned} x_m(\beta,t_i(\beta)) &= x_i(\beta,t_{i_m}(\beta) + 2x_{ym}(\beta), \quad \text{for } \dot{x}_m(\beta,t_i(\beta)) > 0 \\ &= x_i(\beta,t_{i_m}(\beta)) - 2x_{ym}(\beta), \quad \text{for } \dot{x}_m(\beta,t_i(\beta)) < 0 \end{aligned} \tag{2.14}$$

where $t_{i_m}(\beta)$ corresponds to the most recent time at which the mth element is unloaded or reloaded.

Then, using definition (A-15)(i) and (2.4), we have, for $t \in (t_j(\beta),t_{j+1}(\beta)]$,

$$R_\ell^j(\beta,t) = \left(\begin{array}{c} \theta \\ \displaystyle\sum_{i=1}^{j} -M^{-1}[K^{i-1}(\beta)-K^i(\beta)]\dot{u}(\beta,t_i(\beta))\frac{\partial t_i}{\partial \beta_\ell}(\beta) \end{array} \right) \tag{2.15}$$

But using relation (2.5)(ii), (2.5)(iii) and differentiating (2.14) and using (2.13), we have

$$[K^{i-1}(\beta)-K^i(\beta)]\ \dot{u}(\beta,t_i(\beta))\ \frac{\partial t_i}{\partial \beta_\ell}(\beta)$$

$$= U_i(\beta)\ \frac{\partial u}{\partial \beta_\ell}(\beta,t_i(\beta)) + \sum_{m \in N_i} V_{i_m}(\beta)\ \frac{\partial u}{\partial \beta_\ell}(\beta,t_{i_m}(\beta)) + w^i(\beta), \tag{2.16}$$

where, $N_i \subset \{1,2,\ldots,N\}$ is the set of elements which have yielded at $t_i(\beta)$

$$\begin{aligned} U_i(\beta) &= -L^T[\Delta k^i]L \\ V_{i_m}(\beta) &= L^T[\Delta k^i_m]L \quad , \end{aligned} \tag{2.17}$$

and where $[\Delta k^i]$ is an (NxN) diagonal matrix in which all elements, except those yielded at $t_i(\beta)$, are zero, with $\underline{m}$th diagonal,

$$[\Delta k^i]_{mm} = k_{em} - k_{ym}, \quad m \in N_i.$$

Also, $[\Delta k^i_m]$ is an (NxN) diagonal matrix, in which all elements except $\underline{m}$th element, $m \in N_i$, are zero, and $w^i(\beta) = L^T\gamma^i(\beta)$, where $\gamma^i(\beta)$ is an (Nx1) vector with

$$\begin{aligned} \gamma^i_m(\beta) &= 0 \ , \quad m \notin N_i \\ &= \varepsilon_m[k_{em}-k_{ym}]\ \frac{dx_{ym}}{d\beta_m}(\beta_m), \end{aligned} \tag{2.18}$$

with $\varepsilon_m = 0$, ± 1, or ± 2, depending on whether the first positive or negative yield point is the same as the most recent unloading point, i.e., yielding or subsequent positive or subsequent negative yielding, respectively.

Now, defining

$$\underset{(2N\times 2N)}{\bar{U}_i(\beta)} \equiv \left[\begin{array}{c|c} H & H \\ \hline -M^{-1}U_i(\beta) & H \end{array}\right]$$

$$\underset{(2N\times 2N)}{\bar{V}_{i_m}(\beta)} \equiv \left[\begin{array}{c|c} H & H \\ \hline -M^{-1}V_{i_m}(\beta) & H \end{array}\right], \ \forall\, m \in N_i,$$

and

$$\underset{(2N\times 2N)}{\bar{\omega}^{i}(\beta)} = \left(\begin{array}{c} \theta \\ \hline -M^{-1}\omega^{i}(\beta) \end{array}\right) \tag{2.19}$$

from (2.15), we have, for $t \in (t_j(\beta), t_{j+1}(\beta)]$

$$R_\ell^j(\beta,t) = \sum_{i=1}^{j} \bar{w}^i(\beta) + \sum_{i=1}^{j}\Big\{ \bar{U}_i(\beta)\,\frac{\partial z}{\partial \beta_\ell}\,(\beta, t_i(\beta))$$

$$+ H(t-t_i(\beta)) \sum_{m\in N_i} V_{i_m}(\beta)\,\frac{\partial z}{\partial \beta_\ell}\,(\beta, t_{i_m}(\beta))\Big\} \tag{2.20}$$

where $H(\cdot)$ is the Heaviside step function. Now, let us partition $\tilde{p}(t,s)$ in (A-25) as

$$\underset{(2N\times 1)}{\tilde{p}(t,s)} = \left(\begin{array}{c} \underset{(N\times 1)}{\tilde{p}_1(t,s)} \\ \hline \underset{(N\times 1)}{\tilde{p}_2(t,s)} \end{array}\right) \quad , \tag{2.21}$$

Using definitions (2.4), (2.7), (2.17), (2.18), (2.21) and relations (2.20), we have from (A.19), for any $S \in (t_j(\beta), t_{j+1}(\beta)] \subset [0,T]$

$$\ddot{\tilde{p}}_1(t,s) = -\sum_{i=1}^{j}\left\{\{\delta(s-t_i(\beta))\ U_i(\beta)^T + \sum_{m\in N_i} \delta(s-t_{i_m}(\beta))\ V_{i_m}(\beta)^T\right\} M^{-1}\tilde{p}_2(t,t_i)$$

with $\quad \tilde{p}_1(t,t) = 0$ (2.22)

$$\dot{\tilde{p}}_1(t,t) = \frac{\partial q}{\partial u}\,(u(\beta,t))^T$$

and

$$\ddot{\tilde{p}}_2(t,s) - C(\beta)\ M^{-1}\dot{\tilde{p}}_2(t,s) + K^j(\beta)\ M^{-1}\tilde{p}_2(t,s) = -\ \dot{\tilde{p}}_1(t,s)$$

with

$$\tilde{p}_2(t,t) = \dot{\tilde{p}}_2(t,t) = 0 \tag{2.23}$$

and

$$\frac{\partial q}{\partial \beta_\ell}\ (u(\beta,t)) = \int_0^t \dot{\tilde{p}}_2(t,s)^T\ r_\ell(\beta,s)\ ds \tag{2.24}$$

Now, defining

$$\hat{p}(t,s) \equiv M^{-1}\tilde{p}_2(t,s), \quad s \in [0,t], \tag{2.25}$$

the desired second implementable form results:

$$\frac{\partial q}{\partial \beta_\ell}\ (u(\beta,t)) = \int_0^t \dot{\hat{p}}(t,s)\ \bar{r}_\ell(\beta,s)\ ds,\ \ \ell \in \{1,2,\ldots,p\}, \qquad t \in [0,T] \tag{2.26}$$

where $\hat{p}(t,s)$ satisfies, for any $s \in (t_j(\beta), t_{j+1}(\beta)] \subset [0,t]$,

$$M\ddot{\hat{p}}(t,s) - C(\beta)\ \dot{\hat{p}}(t,s) + K^j(\beta)\ \hat{p}(t,s) = -\dot{\tilde{p}}_1(t,s),$$

with $\quad \hat{p}(t,t) = 0$

$$\dot{\hat{p}}(t,t) = 0 \tag{2.27}$$

where $\dot{\tilde{r}}_1(t,s)$ is given by (2.22) with (2.25), and

$$\bar{r}_\ell(\beta,s) \equiv Mr_\ell(\beta,s)$$

$$= - \sum_{i=1}^{j} \left\{ \left[\frac{\partial K^{i-1}}{\partial \beta_\ell}(\beta) - \frac{\partial K^i}{\partial \beta_\ell}(\beta) \right] u(\beta,t_i(\beta)) + \frac{\partial K^j}{\partial \beta_\ell}(\beta)\, u(\beta,s) \right\}$$

$$- \frac{\partial C}{\partial \beta_\ell}(\beta)\, \dot{u}(\beta,s) - \sum_{i=1}^{j} w^i(\beta), \quad s \in [0,t] \qquad (2.28)$$

Thus, (2.26) through (2.22), (2.25) (2.27) and (2.28) constitutes the second implementable form.

2.5. Third Implementable Form:

A third implementable form follows by directly treating the transcribed second order perturbation equation given by (2.8) as the starting point. Let us define, for $j \in \{1,2,...,J\}$, the sets

$$J^j_Y \equiv \{i \in \{1,2,...,j\} \,|\, t_i(\beta) \text{ caused by at least one element yielding}\}$$

$$J^j_u \equiv \{i \in \{1,2,...,j\} \,|\, t_i(\beta) \text{ caused by at least one element unloading or reloading}\}$$

Fact: $J^j_Y \cup J^j_u = \{1,2,...,j\}$ and $J^j_Y \cap J^j_u$ may or may not be a null set.

Then, using (2.5), and differentiating (2.14), with fact (2.13), we have, from (2.8), for any $s \in [0,t]$, $t \in (t_j(\beta),t_{j+1}(\beta)] \subset [0,T]$

$$M \frac{\partial \ddot{u}}{\partial \beta_\ell}(\beta,s) + C(\beta)\frac{\partial \dot{u}}{\partial \beta_\ell}(\beta,s) + [K^o + \sum_{i=1}^{j} H(s-t_i(\beta))[K^i(\beta) - K^{i-1}(\beta)]] \frac{\partial u}{\partial \beta_\ell}(\beta,s)$$

$$= - \sum_{i \in J^j_u} W_i(\beta) \int_0^s \delta(\tau - t_i(\beta)) \frac{\partial u}{\partial \beta_\ell}(\beta,\tau)\, d\tau \qquad (2.29)$$

$$- \sum_{i \in J^j_y} H(s-t_i(\beta)) \left\{ \sum_{m \in N_i} V_{i_m}(\beta) \int_0^s \delta(\tau - t_{i_m}(\beta)) \frac{\partial u}{\partial \beta_\ell}(\beta,\tau)\, d\tau \right\} + \bar{r}_\ell(\beta,s),$$

with

$$\frac{\partial u}{\partial \beta_\ell}(\beta,0) = \frac{\partial \dot{u}}{\partial \beta_\ell}(\beta,0) = 0, \qquad \ell \in \{1,2,\ldots,N\}$$

where $W_i(\beta) = L^T[\tilde{\Delta}k^i] L$ and where $[\tilde{\Delta}k^i]$ is an (NxN) diagonal matrix in which all elements, except those unloaded or reloaded, are zero, with the mth non-zero diagonal given by $[\tilde{\Delta}k^i]_{mm} = k_{ym}-k_{em}$ and all other terms have been defined. Performing the same mathematical operations as used in the proof of the second theorem in Appendix A-4, we have the desired third implementable form:

$$\frac{\partial q}{\partial \beta_\ell}(u(\beta,t)) = \int_0^t \bar{p}(t,s)^T \, \bar{r}_\ell(\beta,s)\, ds, \; t \in [0,T] \tag{2.30}$$

where

$\bar{r}_\ell(\beta,s)$ is given by (2.28)

and $\bar{p}(t,s)$, for any $s \in (t_i(\beta),t_{i+1}(\beta)] \subset [0,T]$, $t \in (t_j(\beta),t_{j+1}(\beta)] \subset [0,T]$, satisfies the equation

$$\begin{aligned} M\ddot{\bar{p}}(t,s) &- C(\beta)\,\dot{\bar{p}}(t,s) + K^i(\beta)\,\bar{p}(t,s) \\ &= -\sum_{k\in J_u^j} W_k^T(\beta)\,\delta(s-t_k(\beta))\left[\int_{t_k(\beta)}^t \bar{p}(t,\tau)\,d\tau\right] \\ &\quad - \sum_{n\in J_y^j}\sum_{m\in N_k} V_{n_m}^T(\beta)\,\delta(s-t_{n_m}(\beta))\left[\int_{t_n(\beta)}^t \bar{p}(t,\tau)\,d\tau\right], \end{aligned}$$

with

$$\begin{aligned} &\bar{p}(t,t) = 0 \\ &\dot{\bar{p}}(t,t) = -M^{-1}\left[\frac{\partial q}{\partial u}(u(\beta,t))\right]^T \end{aligned} \tag{2.31}$$

3. A NUMERICAL TREATMENT OF IMPLEMENTABLE FORMS

3.1. Introduction

This section describes the numerical analysis that has been utilized to compute with the basic model of system dynamics and three forms of equations for sensitivity matrix elements presented in Section 2 for the bilinear hysteretic shear frame. The integration scheme chosen for complete analysis (dynamic analysis and sensitivity analysis) is the familiar linear acceleration step-by-step method, [1]. While it is clear that study of alternative integration schemes for dynamic analysis of nonlinear systems constitutes an important research problem in itself, we have not addressed this issue here in view of the broader objective of the report.

3.2. SOLUTION OF SYSTEM EQUATIONS

Let Δt denote the time step of integration, then $\Delta u(\beta,t)$, the increment in displacement vector in time Δt for any $t \in [0,T]$ such that $t_i(\beta) \notin [t,t+\Delta t]$, for $\forall i \in \{1,2,\ldots,J\}$, is given by the following system of linear algebraic equations:

$$K^*(\beta,t)\ \Delta u(\beta,t) = \Delta r^*(\beta,t) \tag{3.1}$$

where,

$$K^*(\beta,t+\Delta t) = \left(\frac{6}{\Delta t^2}\right) M + \left(\frac{3}{\Delta t}\right) C(\beta) + K(\beta,t+\Delta t)$$

and,

$$\Delta r^*(\beta,t) = -M\hat{1}\ddot{u}_g(t+\Delta t) + Ma(\beta,t) + C(\beta)\ b(\beta,t) - F_s(\beta,t)$$

$K(\beta,t)$ is the structure tangent-stiffness matrix at t, and,

$$a(\beta,t) = \left(\frac{6}{\Delta t}\right) \dot{u}(\beta,t) + 2\ddot{u}(\beta,t)$$

$$b(\beta,t) = 2\dot{u}(\beta,t) + \left(\frac{\Delta t}{2}\right) \ddot{u}(\beta,t).$$

Equation (3.1) can be solved by any of the standard techniques, namely, direct methods based on decomposition of $K^*(\beta,t)$, or, iterative methods. Observing that $K^*(\beta,t)$ in (A.2) is symmetric and tridiagonal, the Jacobi method with Cholesky decomposition [3] is considered the most efficient choice for the computer coding developed for numerical experimentation in the next section. For $t_i(\beta) \in [t,t+\Delta t]$, for $i \in \bar{J}$, $\bar{J} \subset \{1,2,\ldots,J\}$, the system equation reduces to system of nonlinear algebraic equations which is solved by an iterative procedure similar to what may be termed a backward secant method belonging to the general class of Newton-like methods, details of which can be found in [1].

3.3. Solution of First Implementable Form

The solution of (2.11) for $t \in [0,T]$ likewise reduces to a system of linear algebraic equations:

$$\bar{K}^* \frac{\partial \Delta u}{\partial \beta_\ell} (\beta,t) = \bar{R}^\ell(\beta,t) \tag{3.2}$$

where, $\bar{K}^* = (\frac{6}{\Delta t^2}) M + (\frac{3}{\Delta t}) C(\beta) + K(\beta,t+\Delta t)$

and

$$\bar{R}^\ell(\beta,t) = M \frac{\partial a}{\partial \beta_\ell} (\beta,t) + C(\beta) \frac{\partial b}{\partial \beta_\ell} (\beta,t) - \frac{\partial C}{\partial \beta_\ell} (\beta)\, \dot{u}(\beta,t+\Delta t) - Q_s^\ell(\beta,t+\Delta t)$$

<u>Evaluation of $Q_s^\ell(\beta,t+\Delta t)$</u>

Let k_i^0 and k_i^n be stiffnesses of element i corresponding to time t and $t + \Delta t$, respectively; if $Q^\ell(\beta,t+\Delta t)$ is the (Nx1) vector with components defined as:

$$Q_i^\ell(\beta,t+\Delta t) = \frac{\partial F_i}{\partial \beta_\ell} (\beta,t+\Delta t) - k_i^n \frac{\partial \Delta x_i}{\partial \beta_\ell} (\beta,t), \quad \forall \ell \in \{1,2,\ldots,N\} \quad \forall i \in \{1,2,\ldots,N\}\ , \tag{3.3}$$

by (2.5)

$$Q_s^\ell(\beta,t+\Delta t) = L^T Q^\ell(\beta,t+\Delta t) \tag{3.4}$$

Now, let $K(\beta,t)$ change to $K(\beta,t+\Delta t)$:

(i) for $i \in \{1,2,\ldots,r\}$, the element stiffnesses remain unchanged, i.e., $k_i^0 = k_i^n = k_{ei}$, or, $k_i^0 = k_i^n = k_{yi}$,

(ii) for $i \in \{r+1,r+2,\ldots,r+m\}$, elements have yielded, i.e., $k_i^0 = k_{ei}$ and $k_i^n = k_{yi}$

(iii) for $i \in \{r+m+1,r+m+2,\ldots,N\}$, elements have unloaded or reloaded, i.e., $k_i^0 = k_{yi}$ and $k_i^n = k_{ei}$.

Then, $Q_i^\ell(\beta,t+\Delta t)$, $\forall i \in \{1,2,\ldots,N\}$, are computed from

$$
\begin{aligned}
\text{(i)}\quad Q_i^\ell(\beta,t+\Delta t) &= \frac{\partial F_i}{\partial \beta_\ell}(\beta,t) + \frac{\partial k_i^n}{\partial \beta_\ell}\Delta x_i(\beta,t), \quad \text{for } \forall i \in \{1,2,\ldots,r\} \\
&= \frac{\partial F_i}{\partial \beta_\ell}(\beta,t) - (1-\alpha_i)\left\{\frac{\partial k_i^0}{\partial \beta_\ell}(x_i(\beta,t)-\tilde{x}_{yi}^c(\beta,t))\right. \\
&\qquad \left. + k_i^0\left(\frac{\partial x_i}{\partial \beta_\ell}(\beta,t) - \frac{\partial \tilde{x}_{yi}^c}{\partial \beta_\ell}(\beta,t)\right)\right. \\
&\qquad + \frac{\partial k_i^n}{\partial \beta_\ell}\Delta x_i(\beta,t), \quad \text{for } \forall i \in \{r+1,r+2,\ldots,r+m\} \\
&= \frac{\partial F_i}{\partial \beta_\ell}(\beta,t) + \frac{\partial k_i^n}{\partial \beta_\ell}\Delta x_i(\beta,t), \qquad \forall i \in \{r+m+1,r+m+2,\ldots,N\}
\end{aligned}
\tag{3.5}
$$

where,

$$
\begin{aligned}
\tilde{x}_{yi}^c(\beta,t) &= x_{pyi}^c(\beta,t), \quad \text{for } \Delta x_i(\beta,t) > 0 \\
&= x_{nyi}^c(\beta,t), \quad \text{for } \Delta x_i(\beta,t) < 0, \quad \forall i \in \{1,2,\ldots,N\}
\end{aligned}
$$

Thus, (3.4) and (3.5) enable one to evaluate $Q_s^\ell(\beta,t)$, $t \in [0,T]$.

The derivative of the damping matrix $C(\beta)$ with respect to β is obtained by direct differentiation of (2.1); this, of course, requires derivatives of the first two elastic frequencies, $\omega_1(\beta)$, $\omega_2(\beta)$ of the system. Using results from [2]:

$$\frac{\partial \omega_i}{\partial \beta_\ell}(\beta) = \left(\frac{1}{2\omega_i(\beta)}\right)\left(\frac{\phi_i^T(\beta)\left[\frac{\partial K^\circ}{\partial \beta_\ell}(\beta)\right]\phi_i(\beta)}{\phi_i^T(\beta)M\phi_i(\beta)}\right), \quad \begin{array}{l} i = 1,2, \\ \forall \ell \in \{1,2,\ldots,N\}, \end{array} \qquad (3.6)$$

where

$\phi_1(\beta)$ and $\phi_2(\beta)$ are eigen-vectors corresponding to $\omega_1(\beta)$ and $\omega_2(\beta)$, respectively.

The quantities $\frac{\partial F_s}{\partial \beta_\ell}(\beta,t+\Delta t)$, $\frac{\partial a}{\partial \beta_\ell}(\beta,t+\Delta t)$ and $\frac{\partial b}{\partial \beta_\ell}(\beta,t+\Delta t)$ are updated from the recurrence relations:

$$\frac{\partial F_s}{\partial \beta_\ell}(\beta,t+\Delta t) = Q_s(\beta,t) + K(\beta,t+\Delta t)\,\frac{\partial \Delta u}{\partial \beta_\ell}(\beta,t),$$

$$\frac{\partial a}{\partial \beta_\ell}(\beta,t+\Delta t) = \left(\frac{30}{\Delta t^2}\right)\frac{\partial \Delta u}{\partial \beta_\ell}(\beta,t) - \left(\frac{6}{\Delta t}\right)\frac{\partial b}{\partial \beta_\ell}(\beta,t) - 2\,\frac{\partial a}{\partial \beta_\ell}(\beta,t),$$

$$\frac{\partial b}{\partial \beta_\ell}(\beta,t+\Delta t) = \left(\frac{9}{\Delta t}\right)\frac{\partial \Delta u}{\partial \beta_\ell}(\beta,t) - 2\,\frac{\partial b}{\partial \beta_\ell}(\beta,t) - \left(\frac{\Delta t}{2}\right)\frac{2a}{\partial \beta_\ell}(\beta,t),$$
$$\forall \ell \in \{1,2,\ldots,N\} \qquad (3.7)$$

<u>Remark</u>: It is recognized from (3.2) that the solution at any time $t \in [0,T]$ requires prior knowledge of $\Delta u(\beta,t)$ for time step Δt at t, hence prior solution of (3.1). Moreover, (3.1) and (3.2) have the same coefficient matrix $K^*(\beta,t)$ for $t_i(\beta) \notin [t,t+\Delta t]$, $\forall i \in \{1,2,\ldots,J\}$ and $K^*(\beta,t+\Delta t)$ for $t_i(\beta) \in [t,t+\Delta t]$, $\forall i \in \bar{J} \subset \{1,2,\ldots,J\}$. Thus, in the former case, once the necessary decomposition is achieved for solution of (3.1), it can also be utilized for solution of (3.2). In the latter situation, decomposition for (3.2) can be utilized for (3.1) in the next

time step of integration, etc. In either case, recognition of this fact forms the basis of the concept of a "complementary pair" that is essential for efficient solution of both system and sensitivity equations in the first implementable form.

3.4. Solution of Second Implementable Form:

From (2.26), one observes that the differential equation must be integrated backward from s=t to s=0; otherwise, it is a piecewise linear differential equation. Assuming a linear distribution of $\ddot{\hat{p}}(t,s)$ in time interval s and (s-Δs), where Δs is the time step of integration, knowing the state at s, the state at s-Δs can be determined from:

$$\begin{aligned} \hat{p}(t,s-\Delta s) &= \hat{p}(t,s) - \Delta\hat{p}, \\ \dot{\hat{p}}(t,s-\Delta s) &= (\frac{3}{\Delta s})\Delta\hat{p} + \hat{b}(s) \\ \ddot{\hat{p}}(t,s-\Delta s) &= -(\frac{6}{\Delta s^2})\Delta\hat{p} + \hat{a}(s) \end{aligned} \tag{3.8}$$

where,

$$\begin{aligned} \hat{a}(s) &\equiv (\frac{6}{\Delta s})\ \dot{\hat{p}}(t,s) - 2\ddot{\hat{p}}(t,s), \\ \hat{b}(s) &\equiv -2\dot{\hat{p}}(t,s) + \frac{\Delta s}{2}\ \ddot{\hat{p}}(t,s) \end{aligned} \tag{3.9}$$

Then, one needs to solve the system of algebraic equations

$$\hat{K}(s-\Delta s)\Delta\hat{p} = \Delta\hat{d} \tag{3.10}$$

where,

$$\hat{K} \equiv [\frac{6M}{(\Delta s)^2}+\frac{3}{\Delta s}\ C + K(\beta,s-\Delta s)]$$

and $\Delta\hat{d} \equiv K(\beta,s-\Delta s) + M\hat{a}(s) - C\hat{b}(s) + \dot{\hat{p}}_1(t,s-\Delta s)$.

Evaluation of $\dot{\tilde{p}}_1(t,s-\Delta s)$ -

From (2.22), $\tilde{p}_1(t,t) = [\frac{\partial q}{\partial u}(u(\beta,t))]^T$

(i) For $t_i(\beta) \notin [s-\Delta s,s]$, $i \in \{1,2,\ldots,J\}$, $\tilde{p}_1(t,s)$ remains constant, i.e.,

$$\dot{\tilde{p}}_1(t,s-\Delta s) = \dot{\tilde{p}}_1(t,s)$$

(ii) For $t_i(\beta) \in [s-\Delta s,s]$, for any $i \in J^j_Y$, by direct integration of (2.22) from $(s-\Delta s)$ to s, we have

$$\dot{\tilde{p}}_1(t,s-\Delta s) = \dot{\tilde{p}}_1(t,s) + U_i(\beta)^T\hat{p}(t,t_i)$$

(iii) For $t_i(\beta) \in [s-\Delta s,s]$, for any $i \in J^j_u$, $i = k_m$, for some $k \in J^j_Y$ and some $m \in N_k$, then,

$$\dot{\tilde{p}}_1(t,s-\Delta s) = \dot{\tilde{p}}_1(t,s) + \sum_{\substack{m\in N_k \\ i=k_m}} V_{k_m}(\beta)p(t,t_k(\beta))$$

(iv) For $t_i(\beta) \in [s-\Delta s,s]$ for any $i \in J^j_y \cap J^j_u$

$$\dot{\tilde{p}}_1(t,s-\Delta s) = \dot{\tilde{p}}_1(t,s) + U_i(\beta)\hat{p}(t,t_i) + \sum_{\substack{m\in N_k \\ i=k_m}} V_{k_m}(\beta)\hat{p}(t,t_k(\beta))$$

<u>Fact</u>: $U_i(\beta)$, $V_{i_m}(\beta)$, $i \in J^j_Y$, $m \in N_i$ are all symmetric matrices. Then, (2.26) can be integrated by any standard method, such as the trapezoidal rule or Gaussian quadrature. It may be noted that $\bar{r}_\ell(\beta,s)$, $s \in [0,t]$, required for evaluation of (2.26) needs to be calculated according to (2.22) during integration of the system equations of motion and made available for evaluation of (2.26) backward and in parallel with the step-by-step integration of (2.27).

3.5. Solution of Third Implementable Form

The solution of (2.31) is effected in almost the same way as has been done for the previous case, the difference occurring in the treatment at stiffness transition times resulting from the nature of (2.31). It must be noted that for $t_i(\beta) \notin [s-\Delta s, s]$, the forcing function vanishes and integration for the step is carried out with no difficulty, as already described. From (2.31), it is obvious that $\dot{\bar{p}}(t,s)$ is discontinuous in s at $t_i(\beta)$, $i \in J_u^j$. But jumps in $\dot{\bar{p}}(t,s)$ due to these discontinuities are easily computed by integrating (2.31) locally across discontinuity points; then, integration is carried out by resetting the state at these discontinuity points.

4. EXAMPLES AND DISCUSSION

4.1. Introduction

The intent of this section is to illustrate and compare the numerical procedures developed in the preceding sections. The examples chosen involve shear-frame structural models with bilinear force-displacement relationships to illustrate inelastic dynamic response of a class of seismically loaded structures. The problems chosen and the ensuing discussion serve to emphasize three main points concerning the role of sensitivity analysis in the area of seismic design: (1) modification of a given (not necessarily feasible) design to meet performance constraints and to possibly attain an optimal design (2) selection of an algorithm and numerical analysis scheme that is suited to the problem under consideration and (3) estimation of relative costs of various available schemes for carrying out the sensitivity analysis.

In searching for a feasible design of a structure subjected to a strong-motion earthquake in which nonlinear response occurs it is necessary to satisfy performance constraints of the type described in Sec. 1. Typically, such inequalities can be written

$$\tilde{q}(\beta) \equiv \max_{t \in [0,T]} |q(\beta, z(\beta,t))| \leq 0 \tag{4.1}$$

It is important to note that, unlike the problem of meeting constraints for a linear model of a structure, it is impossible to remove time from (4.1). (For a linear system maximum response is usually estimated by the square root of sum of squares of modal maxima, thus removing time as a parameter of the constraint.) Accordingly, the effect of changing the design vector β in (4.1) becomes an important computational problem for design. Depending on the properties of the structure and the earthquake acceleration history, it is evident that more than one maximizer may exist in (4.1), i.e., more than one element may exist in the set $\mathcal{T}$ defined by

$$\mathcal{T} \equiv \{t \in [0,T] \,\big|\, |q(\beta, z(\beta,t))| = \tilde{q}(\beta)\} \tag{4.2}$$

Therefore, it is essential to be able to calculate efficiently values of $\frac{\partial q}{\partial \beta}$ for $t \in \mathcal{T}$ in order to make rational design changes in the structure.

Finally, it may be noted that the second and third implementable forms given in Secs. 2 and 3 are variations of the same basic theorem. Since experience shows that the latter form is numerically more efficient for locating stiffness transition times, the subsequent discussion is limited to applications of the first and third forms.

4.2. Structure and Loading Characteristics

The structural models chosen for illustrative purposes include two- and eight-story shear frames (Fig. 1). As noted in Sec. 2, the parameter vector β consists of moments of inertia of column sections of each story, i.e., $\beta \equiv I$ where, $I = (I_1, I_2, \ldots, I_N)^T$, N=2 for 2-story frame, 8 for 8-story frame. The columns are assumed to be structural steel with yield stress, $\sigma_y = 36$ ksi and elastic modulus, $E = 30 \times 10^3$ ksi; empirical relations among column cross-sectional area, A in in^2, elastic section modulus, S in in^3, and in terms of I in in^4 are assumed as follows [2]:

$$\left.\begin{aligned} A &= 0.80 I^{1/2} \\ S &= 0.78 I^{3/4} \end{aligned}\right\} \tag{4.3}$$

Referring to the Rayleigh damping matrix introduced in Sec. 2, the required modal damping ratios for both first and second modes are taken to be 5% of critical, i.e., $\xi_1 = \xi_2 = 0.05$. The force-deformation relationship between each story shear force and relative drift is assumed bilinear, hysteretic. The post-yield stiffness is assumed to be 10% of the initial stiffness. Basic data of the example frames are shown in Table 1. The maximum stress in an elastic column at story i at any time t is given by

$$\sigma_i(I,t) = \frac{M_i(I,t)}{S_i(I_i)} + \frac{P_i}{A_i(I_i)} \tag{4.4}$$

The relative yield drift, $x_{yi}(I_i)$, for any story i, is obtained by setting $\sigma_i = \sigma_y$ and given by

$$x_{yi}(I_i) = 5.0544 I_i^{-1/4} - 0.1755 P_i I_i^{-3/4} \tag{4.5}$$

The earthquake ground motion selected for numerical experimentation is the North-South component of 1940, the El-Centro earthquake.

If $x_m(\beta,t)$ is the relative drift at story m, m = 1,2,...,N, the results presented are limited to presenting values of the matrix $\frac{\partial x_m}{\partial \beta_\ell}(\beta,t)$, m = 1,2,...,N; ℓ = 1,2,...,N; $t \in [0,T]$; note that $q(u(\beta,t))$ of Sec. 2.4 and 2.5 is identical to $(e^m)^T u(I,t)$, m = 1,2,...,N; where, e^m is an (Nx1)

VARIABLES	2-STORY FRAME		8-STORY FRAME							
STORY	1	2	1	2	3	4	5	6	7	8
STORY HEIGHT (in)	180	180	180	180	180	180	180	180	180	180
STORY MASS ($K\text{-}sec^2/in$)	.208	.208	.208	.208	.208	.208	.208	.208	.208	.208
STORY COLUMN LOAD (K)	80	40	320	280	240	200	160	120	80	40
COLUMN MOMENT OF INERTIA (in^4)	200	120	1880	1880	1280	1280	680	680	380	380
ELEMENT STIFFNESS (K/in)	24.69	14.81	232.10	232.10	158.02	158.02	83.95	83.95	46.91	46.91
YIELD DRIFT (in)	1.08	1.33	0.57	0.60	0.65	0.68	0.78	0.83	0.98	1.06

TABLE 1. BASIC DATA OF EXAMPLE FRAMES

STORY M	TIME AT WHICH $\lvert x_m(I,t)\rvert$ IS MAXIMUM $\hat{t}_m$, sec	COMPONENTS OF $\frac{\partial x_m}{\partial I_\ell}(I,\hat{t}_m) \times 10^5$, $\ell = 1,2,\ldots,8$								$\left\lVert \frac{\partial x_m}{\partial I}(I,\hat{t}_m) \right\rVert_2$ †
		$\ell = 1$	2	3	4	5	6	7	8	
1	5.21	28.37561 [27.95069]	-4.80386 [-4.851986]	-10.26950 [-9.553437]	-10.29089 [-9.353512]	-1.026152 [-.5075799]	3.430299 [2.321951]	-13.91871 [-15.79527]	10.90006 [6.113882]	36.94562351
2	5.23	-1.125191	24.25340	-10.04949	-8.592899	5.676748	5.644728	10.67863	17.16017	
3	5.25	-3.776120	-3.923206	35.09141	-11.53099	-13.70655	8.467581	36.97536	23.30400	
4	6.06	3.474651	.8942530	.1673584	-40.25626	50.49412	7.374568	80.93469	5.293593	
5	2.08	1.214112 [1.105393]	-3.656844 [-3.734916]	-14.68171 [-14.80984]	-12.61568 [-12.66520]	326.5196 [326.7675]	-116.7178 [-117.7419]	-179.8681 [-179.4278]	-99.16624 [-99.34504]	
6	2.06	.8685165	.01847870	-1.738183	-.9113435	-78.21299	196.3976	-88.02047	-9.001918	
7	2.18	1.834193 [1.756549]	-7.993580 [-8.052627]	-20.38371 [-20.47287]	-10.76276 [-10.80641]	-189.9170 [-191.5276]	-38.20529 [-36.48084]	1252.640 [1251.952]	39.61974 [39.38191]	
8	2.42	2.281240	3.867979	7.529473	4.384918	18.81857	-12.02595	173.3286	-195.1931	

†Euclidean norm vector in R^N, i.e., $y \in R^N$, $\lVert y\rVert_2 = \left(\sum_{i=1}^{N} y_i^2\right)^{1/2}$

TABLE 2. DERIVATIVES OF MAXIMUM REL. DRIFTS FOR 8-STORY FRAME

vector given by

$$e_j^m = \begin{cases} 0 & , \quad j \neq m, \ j \neq m-1 \\ -1 & , \quad j = m-1 \\ 1 & , \quad j = m \end{cases} \tag{4.6}$$

Obviously, $\frac{\partial q}{\partial u}$ in Secs. 2.4 and 2.5 is equal to $(e^m)^T$.

4.3. Numerical Results:

Tables 2 and 3 summarize the results for the eight- and two-story frames; the derivatives of the mth story relative drift $x_m(I,t)$ with respect to the design vector I are evaluated for times t_m, at which the relative drift assumes an absolute maximum in [0,T]. Bracketed values have been obtained using the third implementable form while unbracketed values result from use of the first form. Comparison between the two methods of computation are limited to stories 1, 5 and 7 in the eight-story frame. It will be noted in Table 4, in which required drift ductility factors of each story are tabulated, that inelastic deformation occurs in stories 5, 6 and 7; thus a numerical comparison of the two methods is shown for inelastic response quantities.

4.4. Discussion of Results

The computations leading to Tables 2 and 3 were based on an integration algorithm employing a linear acceleration assumption in each time-step, with a time-step of $\Delta t = 0.01$ seconds -- both for dynamic analysis and sensitivity analysis. In view of the similarity in structure of the differential equations appearing in the implementable forms of sensitivity analysis it appears that the integration time-step for sensitivity analysis has as large an acceptable value as that for dynamic analysis, provided that the forcing functions in the former case are adequately

STORY M	TIME AT WHICH $\lvert x_m(I,t) \rvert$ is MAXIMUM, $\hat{t}_m$ sec	COMPONENTS OF $\frac{\partial x_m}{\partial \beta_\ell}(I,\hat{t}_m)\times 10^3$	
		$\ell = 1$	2
1	4.465	.8982965 [.8476618]	19.73316 [18.69639]
2	2.0325	-5.846570 [-5.847385]	28.99335 [28.99365]

TABLE 3. DERIVATIVES OF MAXIMUM REL. DRIFTS FOR 2-STORY FRAME

STORY M	$\max_t \lvert x_m(I,t) \rvert$ (in)	REQUIRED DUCTILITY
1	0.5575	0.9766
2	0.4979	0.8362
3	0.6140	0.9472
4	0.5199	0.7635
5	1.2411	1.5934
6	0.8912	1.0717
7	2.6873	2.7375
8	0.9139	0.8596

TABLE 4, MAXIMUM REL. DRIFT AND REQUIRED DUCTILITIES FOR 8-STORY FRAME represented by the choice of Δt. The results in Tables 2 and 3 support this observation.

We turn next to the question of selecting an algorithm for sensitivity analysis appropriate to the problem at hand. The underlying motivation is that of applications to design. For the optimization problem (1.2) introduced in Sec. 1, with constraint functions given by (4.1), one needs to compute gradients at times when the functions assume extreme values. If the number of active constraints is found to be small (as would occur in early stages of the design process), the second and third implementable forms are superior to the first in that only vector equations need to be solved as opposed to matrix equations of higher dimensionality required by the first form. This means that one can exercise selectivity over the choice of what constraint gradients are to be calculated when the second or third forms are used. On the other hand for a design closer to optimal or a "fully constrained" design, it would be expected that gradients of all constraints would be required -- thus suggesting the use of the first form. It should be noted that if one is interested only in the effect on maximum response of changes in <u>selected</u> members in the structure, use of the second or third forms is obviously called for.

In this report we have stressed the use of certain algorithms for calculation of the gradients of either the cost function or constraint functions which are required in a method of optimal design of structures subjected to dynamic loads. It is natural to seek a comparison of the efficiency of these rather sophisticated methods with a direct finite difference approximation of the gradient calculations. To this end let T_d denote the computation time required to perform the analysis of the dynamic structural system (shear frame), i.e., the time required to solve an (Nxl) dimension vector differential equation over the time interval of interest [0,T]. Then, each component variation of the design vector requires solution of two (Nxl) vector differential equations, resulting in a total computation time for gradient evaluation of $2PT_d$, where P is the dimension of the design vector. An important limitation of the finite difference approach to gradient calculation is that the entire coupled system of gradients must be calculated to obtain any one particular element of the sensitivity matrix unlike forms two and three discussed in this report.

Finally, we turn to a discussion of the computation time required for the forms used in the report. We limit our attention to the case in which P=N, i.e., the dimensions of the design vector and the structural response vector are equal (as in the shear frame studied). In this case the computation time for the finite difference scheme is $2NT_d$. For the first implementable form, employing the "complementary concept" which takes advantage of the fact that the operators for dynamic analysis and sensitivity analysis are the same [1], the computation time is $T_1 = \frac{1}{2} NT_d$, a factor of four faster than the finite difference solution of the same problem.

Continuing, we may compare the cost of the second or third form with the the first, provided that the system response vector is large enough to make the comparison meaningful. Referring to (2.28), let T_r be the time required to compute $\bar{r}_\ell(I,\hat{t}_m)$, $\ell \in \{1,2,\ldots,N\}$ and $\hat{t}_m$ the time at which $x_m(I,t)$ attains a maximum. Using the second or third implementable form, the required computation time is $T_{2,3} \cong (\frac{1}{N} + 1)(\frac{\hat{t}_m}{T})T_d + T_r$, while the same computation using the first form would require $T_1 \simeq \frac{1}{2} NT_d(\frac{\hat{t}_m}{T})$. If the complementary pair concept is employed in using the second or third forms as well, the time is modified to $T_{2,3} \simeq (\frac{1}{N} + \frac{1}{2}) (\frac{\hat{t}_m}{T})T_d + T_r$. The choice between these two computational forms will obviously depend upon the size of the system as well as the characteristics of the particular problem, which will govern the computation of T_r.

4.5. Concluding Remarks

In this report we have presented a method and illustrated its application to a critical problem in the design of structures subjected to dynamic loading. For a general nonlinear force-displacement relationship characteristic of elastic-plastic behavior of materials, we have obtained formulas for calculating the rate of change of structural response variables with respect to parameters incorporated in the structural model. In particular, examples are given for a bileanear model of a shear frame, for which both maximum story drift, as well as the rate of change of maximum story drift with respect to changes in selected column sizes, are calculated. It has been shown that the calculation of such sensitivity matrices can be carried out simultaneously with the dynamic analysis of the system. Unification of this work with an optimization algorithm capable of handling functional constraints (such as maximum over time) is currently in progress. Likewise, adaptation of the basic algorithm for other types of hysteretic structural models as well as incorporation of ground motion parameters is under study.

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APPENDIX A - MATHEMATICAL FORMULATION

A.1 Introduction

Here are presented in state space formulation a mathematical characterization of the bilinear hysteretic force-deformation relationship and two general theorems leading to sensitivity analysis for response quantities relative to a time invariant parameter vector. A corollary to the second theorem provides an alternative more suitable for subsequent numerical treatment found in Sec. 3.

A.2 State-Space Characterization of a Bilinear Hysteretic Model

To illustrate hysteretic behavior of a framed structure, we restrict ourselves to the simplest case, viz. the single story frame in Fig. A1-(a) with bilinear hysteresis, as shown in Fig. A1-(b), where $F = \int_0^t K(\beta, x(s), \dot{x}(s))\dot{x}(s)ds$. The change in slope (loading effect) at point a results from yielding of the columns, while, when the force is reduced at b (unloading effect) the slope increases again. Note that if starting from b the force is reduced to zero, the deformation does not reduce to zero. The residual deformation is given by $x = x_r$.

Now, let the solution of (1.1) or equivalently (2.3) at t, from the initial condition $z_0 = (x_0, \dot{x}_0)$ at time t_0 be denoted by $x(t, t_0, z_0, \beta)$. Then, assuming that we start at $t_0 = 0$ with $z_0 = 0$ and trace out the graph in Fig. A1-(b) as the motion proceeds, we can compute the times $t_1(\beta)$, $t_2(\beta)$, etc., where $x(t, 0, z_0, \beta)$ is equal to x_y, x_2, x_3, etc. recursively, as follows.

(i) To compute $t_1(\beta)$, we set $K(\beta, x, \dot{x}) \equiv k_e(\beta)$ in the equation of motion (2.3) and compute $x(t,0,0,\beta)$, and $\dot{x}(t,0,0,\beta)$. Then $t_1(\beta)$ is obtained as the solution of the equation.

$$\sigma_1^1(\beta, x(t_1(\beta),0,0,\beta), \dot{x}(t_1(\beta),0,0,\beta))$$
$$\triangleq x(t_1(\beta),0,0,\beta) - x_y(\beta) = 0 \tag{A.1}$$

(ii) To compute $t_2(\beta)$, we set $K(\beta,x,\dot{x}) \equiv k_y(\beta)$ in the equation of motion and compute $x(t,t_1(\beta),\ z_1,\beta)$ for $t \geq t_1(\beta)$, where $z_1 = (x(t_1(\beta),0,0,\beta),x(t_2(\beta),0,0,\beta))$. Then $t_2(\beta)$ is obtained as the solution o

$$\sigma_2^2(\beta, x(t_2(\beta),t_1(\beta),z_1,\beta), \dot{x}(t_2(\beta),t_1(\beta),z_1,\beta))$$
$$\triangleq \dot{x}(t_2(\beta),t_1(\beta),z_1,\beta) = 0 \tag{A.2}$$

since the velocity at b must be zero. Thus, if we wish to compute $t_1(\beta)$, $t_2(\beta)$ simultaneously, we must solve an equation of the form

$$\sigma^2(\beta,\zeta^2(\beta,t^2(\beta)) = 0 \tag{A.3}$$

where $t^2(\beta) = (t_1(\beta),t_2(\beta))$ and $\zeta^2(\beta,t^2(\beta),\beta) = (z(t_1(\beta),0,0,\beta),$ $z(t_2(\beta),t_1(\beta),z(t_1(\beta),0,0,\beta),\beta))$, $z = (x,\dot{x})$, and $\sigma^2\colon \mathbb{R}^p \times \mathbb{R}^4$ is defined componentwise as follows: $\sigma_1^2 = \sigma_1^1$ and σ_2^2 is as in (A.2). Thus, we see that we get relations of the type used in Appendix A.3.

A.3 <u>First Theorem:</u>

Let operator $\mathcal{D}$ defined in (1.1) have the representation

$$\mathcal{D}(\beta,z(\beta,\cdot),t) \equiv \dot{z}(\beta,t) - f(\beta,z(\beta,t),t) - \int_0^t g(\tau,\beta,z(\beta,\tau))d\tau,\ t \in [0,T]$$

with $z(\beta,0) = 0$. The function $f\colon \mathbb{R}^p \times \mathbb{R}^{\bar{N}} \times \mathbb{R}^1 \to \mathbb{R}^{\bar{N}}$ is continuous in t and is continuously differentiable in β and z. The function g is defined piecewise, as follows: $g(t,\beta,z) = g_i(\beta,z)$ for $t \in I_i$, where $I_0 = [0,t_1(\beta)]$, $I_i = (t_i(\beta),t_{i+1}(\beta)]$, $i = 1,2,\ldots,J$, and $\bigcup_{i=1}^{J} I_i = [0,T]$. The $t_i(\beta)$ are the instants of time at which some components of $x(\beta,t)$ pass through a slope discontinuity in the appropriate force-deformation hysteresis graph as notes in Appendix A2. It is assumed that there are

J such points in $[0,T]$. The times $t_i(\beta)$ are determined by an implicit relationship, as follows. For $i = 1,2,\ldots,J$, let $t^i(\beta) = (t_1(\beta), t_2(\beta), \ldots, t_i(\beta))^T$ and let $\zeta^i(\beta, t^i(\beta)) = (z(\beta,t_1(\beta)), z(\beta,t_2(\beta)), \ldots, z(\beta,t_i(\beta)))^T$. Then, the force-deformation-hysteretic-graph defines a set of functions $\sigma^i\colon \mathbb{R}^P \times \mathbb{R}^{\bar{N}_i} \to \mathbb{R}^i$, $i = 1,2,\ldots,J$, such that for $i = 1,2,\ldots,J$

$$\sigma^i(\beta, \zeta^i(\beta, t^i(\beta))) = 0 \tag{A.5}$$

It is assumed that the model for hysteretic behavior is such that the $\sigma^i(\cdot,\cdot)$ are continuously differentiable. Under these assumptions, the following results hold.

<u>Theorem</u>: The solution $z(\beta,t)$ of (1.1) of Sec. 1 with operator $\mathcal{D}$ defined by (A.4), is differentiable with respect to β, with $\frac{\partial z}{\partial \beta}(\beta,t)$ computable, column by column, as the solution of the following set of integro-differential equations

$$\begin{aligned}\frac{d}{dt}\left(\frac{\partial z}{\partial \beta_\ell}(\beta,t)\right) &= \frac{\partial f}{\partial z}(\beta,z(\beta,t),t)\frac{\partial z}{\partial \beta_\ell}(\beta,t) + \int_0^t \frac{\partial g}{\partial z}(\tau,\beta,z(\beta,\tau))\frac{\partial z}{\partial \beta}(\beta,\tau)\,d\tau \\ &\quad + \frac{\partial f}{\partial \beta_\ell}(\beta,z(\beta,t),t) + \int_0^t \frac{\partial g}{\partial \beta_\ell}(\tau,\beta,z(\beta,\tau))\,d\tau + G^j\frac{\partial t^j}{\partial \beta_\ell}(\beta), \\ &\quad t \in I_j \subset [0,T],\ \ell \in \{1,2,\ldots,P\}\end{aligned} \tag{A.6}$$

with initial condition

$$\frac{\partial z}{\partial \beta_\ell}(\beta,0) = 0$$

where G^j is a $(\bar{N}\mathrm{x}j)$ matrix with columns G_i^j defined, for $i = 1,2,\ldots,j$, by

$$g(t_i^+(\beta),\beta,z) \triangleq \lim_{\substack{\varepsilon\to 0\\ \varepsilon>0}} g(t_i(\beta)+\varepsilon,\beta,z), \text{ and}$$

$$G_i^j = g(t_i(\beta),\beta,z) - g(t_i^+(\beta),\beta,z) \tag{A.7}$$

For a proof of this theorem, see [1]. The proof is established in exactly the same way as that of the differentiability of the solution of a differential equation with respect to parameters, except that instead of the Bellman-Gronwall lemma, one must make use of the following generalization:

Lemma 1: Let h(t) be a scalar function such that

$$0 \leqq h(t) \leq \lambda + \mu[\int_0^t h(s)ds + \int_0^t \int_0^t h(\tau)d\tau ds] \tag{A.8}$$

where λ and μ are positive constants, $t \geq 0$, and h(t) is continuous in $t \in [0,T]$. Then,

$$h(t) \leq c(\mu)e^{aT} \tag{A.9}$$

where $c(\mu) > 0$ is a constant and $a = \frac{\mu + \sqrt{\mu^2+4\mu}}{2}$, i.e., a is the positive root of the quadratic equation:

$$a^2 - a\mu - \mu = 0 \tag{A.10}$$

For a proof of this lemma, see [1].

A.4 Second Theorem:

Let $q(z(\beta,t))$ be a real-valued function where $z(\beta,t)$ satisfies (1.1); then, $\frac{\partial q}{\partial \beta_\ell}(z(\beta,t))$, $\forall t \in [0,T]$, $\forall \ell \in \{1,2,\ldots,P\}$ are given by

$$\frac{\partial q}{\partial \beta_\ell}(z(\beta,t)) = \int_0^t p(t,s)^T r_\ell(\beta,s)ds, \quad t \in [0,T] \tag{A.11}$$

where p(t,s) satisfies the following integro-differential equation

$$\frac{d}{ds} p(t,s) = -\frac{\partial f}{\partial z}(\beta,z(\beta,s),s)^T p(t,s) - \frac{\partial g}{\partial z}(s,\beta,z(\beta,s))^T (\int_s^t p(t,\tau)d\tau)$$
$$- \sum_{i=1}^{j} S_i^j(\beta)^T \delta(s-t_i(\beta))(\int_{t_{i(\beta)}}^t p(t,\tau)d\tau),$$

$$p(t,t) = \frac{\partial q}{\partial z}(z(\beta,t))^T \qquad \begin{array}{l} s \in [0,t] \\ t \in (t_j(\beta),t_{j+1}(\beta)] \subset [0,T] \end{array} \tag{A.12}$$

and

$$r_\ell(\beta,s) = \frac{\partial f}{\partial \beta_\ell}(\beta,z(\beta,s),s) + \int_0^s \frac{\partial g}{\partial \beta_\ell}(\tau,\beta,z(\beta,\tau))d\tau + m_\ell^i(\beta),$$
$$s \in (t_i(\beta),t_{i+1}(\beta)] \subset [0,t] \tag{A.13}$$

where,

$S_i^j(\beta)$ is the $(\bar{N}x\bar{N})i^{th}$ column of $S^j(\beta)$ matrix partitioned as:

$\underset{(\bar{N}x\bar{N}_j)}{S^j(\beta)} = [S_1^j(\ ) \vdots S_2^j(\ ) \vdots \ \cdots \ \vdots S_j^j]$, defined by

$$S^j(\beta) \equiv -G^j[\frac{\partial \sigma^j}{\partial \zeta^j}\frac{\partial \zeta^j}{\partial t^j}]^{-1}[\frac{\partial \sigma^j}{\partial \zeta^j}]$$

and

$$m_\ell^i(\beta) \equiv -G^i[\frac{\partial \sigma^i}{\partial \zeta^i}\frac{\partial \zeta^i}{\partial t^i}]^{-1}[\frac{\partial \sigma^i}{\partial \beta_\ell}]$$

and $\delta(\cdot)$ is the Dirac δ-function.

Proof:

Differentiating (A.5) with respect to β_ℓ, $\ell \in \{1,2,\ldots,p\}$, we have

$$\underset{(ix1)}{\frac{\partial t^i}{\partial \beta_\ell}(\ell)} = - \underset{(ix\bar{N}i)}{[\frac{\partial \sigma^i}{\partial \zeta^i}} \ \underset{(\bar{N}ixi)}{\frac{\partial \zeta^i}{\partial t^i}]^{-1}} \underset{(ix1)}{(\frac{\partial \sigma^i}{\partial \beta_\ell}} + \underset{(ix\bar{N}i)}{\frac{\partial \sigma^i}{\partial \zeta^i}} \ \underset{(\bar{N}ix1)}{\frac{\partial \zeta^i}{\partial \beta_\ell})} \tag{A.14}$$

Defining

$$y_\ell^i(\beta,t^i(\beta)) \equiv (\frac{\partial z}{\partial \beta_\ell}(\beta),t_1(\beta)),\ \frac{\partial z}{\partial \beta_\ell}(\beta,t_2(\beta)),\ldots.,\ \frac{\partial z}{\partial \beta_\ell}(\beta,t_i(\beta)))^T,$$

and $R_\ell^i(\beta,t) \equiv G^i \frac{\partial t^i}{\partial \beta_\ell}(\beta)$, $t \in (t_j(\beta),t_{j+1}(\beta)]$, $\ell \in \{1,2,\ldots,P\}$ (A.15)(i)

noting that $y_\ell^i(\beta,t^i(\beta)) = \frac{\partial \zeta^i}{\partial \beta_\ell}$ in (A.14), we have,

$$R^i_\ell(\beta,t) = m^i_\ell(\beta) + \sum_{k=1}^{i} S^i_k(\beta) y_\ell(\beta,t_k(\beta)), \quad \ell \in \{1,2,\ldots,P\} \tag{A.15)(ii}$$

where,

$$\underset{(\bar{N}x1)}{m^i_\ell(\beta)} \equiv -G^i\left[\frac{\partial\sigma^i}{\partial\zeta^i}\frac{\partial\zeta^i}{\partial t^i}\right]^{-1}\frac{\partial\sigma^i}{\partial\beta_\ell},$$

and $S^i_k(\beta)$ is an $(\bar{N}x\bar{N})$ k^{th} submatrix of $S^i(\beta)$ defined as

$$\underset{(\bar{N}x\bar{N}i)}{S^i(\beta)} \equiv -G^i\left[\frac{\partial\sigma^i}{\partial\zeta^i}\frac{\partial\zeta^i}{\partial t^i}\right]^{-1}\left[\frac{\partial\sigma^i}{\partial\zeta^i}\right] \text{ and partitioned as}$$

$$\underset{(\bar{N}x\bar{N}i}{S^i(\beta)} = [\underset{(\bar{N}x\bar{N})}{S^i_1(\beta)} \vdots \underset{(\bar{N}x\bar{N})}{S^i_2(\beta)} \vdots \ldots\ldots | \underset{(\bar{N}x\bar{N})}{S^i_i(\beta)}].$$

For simplicity of expression we now define functions

$$B(t) \equiv B_j(t) \quad , \quad t \in (t_j(\beta),t_{j+1}(\beta)]$$

where

$$B_j(t) \equiv \frac{\partial g_j}{\partial z}(t,\beta,z(\beta,t)),$$

$$A(t) \equiv \frac{\partial f}{\partial z}(\beta,z(\beta,t),t) , \tag{A.16}$$

$$\tilde{R}^j(\beta,t) \equiv \sum_{k=1}^{j} S^j_k(\beta) y_\ell(\beta,t_k(\beta))$$

$$r_\ell(\beta,t) \equiv \frac{\partial f}{\partial\beta_\ell}(\beta,z(\beta,t),t) + \int_0^t \frac{\partial g}{\partial\beta_\ell}(\beta,\tau,z(\beta,\tau))d\tau + m^j_\ell(\beta)$$

Using (A.15) and definitions (A.16), (A.6) may now be expressed, for $s \in (t_j(\beta),t_{j+1}(\beta)]$, as

$$\dot{y}_\ell(\beta,s) = A(s)y_\ell(\beta,s) + \int_0^s B(\tau)y_\ell(\beta,\tau)d\tau$$
$$+ \sum_{k=1}^{j} H(s-t_k(\beta))S^j_k(\beta)y_\ell(\beta,t_k(\beta)) + r_\ell(\beta,s).$$

$$y_\ell(\beta,0) = 0 \tag{A.17}$$

Fact: $A(s)$, $B(s)$ and $r_\ell(\beta,s)$ are piecewise continuously differentiable functions of $s \in [0,t]$. Premultiplying by an as yet unspecified $(\bar{N}x\bar{N})$, nonsingular and differentiable matrix function, $\Phi(t,s)$ and integrating from 0 to t, we have, from (A.17)

$$\int_0^t \Phi(t,s)\dot{y}_\ell(\beta,s)ds = \int_0^t \Phi(t,s)A(s)y_\ell(\beta,s)ds + \int_0^t \Phi(t,s)(\int_0^s B(\tau)y_\ell(\beta,\tau)d\tau)ds$$
$$+ \int_0^t \Phi(t,s)(\sum_{i=1}^{j} H(s-t_i(\beta)))\int_0^s S_i^j(\beta)\delta(\tau-t_i(\beta))y_\ell(\beta,\tau)d\tau)ds$$
$$+ \int_0^t \Phi(t,s)r_\ell(\beta,s)ds \qquad (A.18)$$

Now,

$$\int_0^t \Phi(t,s)\dot{y}_\ell(\beta,s)ds = \Phi(t,t)y_\ell(\beta,t) - \int_0^t \dot{\Phi}(t,s)y_\ell(\beta,s)ds \qquad (A.19)(i)$$

and

$$\int_0^t \Phi(t,s)(\int_0^s B(\tau)y_\ell(\beta,\tau)d\tau)ds$$

$$= \int_0^t \Phi(t,s)\int_0^t H(s-\tau)B(\tau)y_\ell(\beta,\tau)d\tau ds$$

$$= \int_0^t [\int_0^t \Phi(t,s)H(s-\tau)ds]B(\tau)y_\ell(\beta,\tau)d\tau$$

$$= \int_0^t [\int_\tau^t \Phi(t,s)ds]B(\tau)y_\ell(\beta,\tau)d\tau$$

$$= \int_0^t [\int_s^t \Phi(t,\tau)d\tau]B(s)y_\ell(\beta,\tau)d\tau \qquad (A.19)(ii)$$

by similar arguments as in (ii)

$$\int_0^t \Phi(t,s)(\sum_{i=1}^{j} H(s-t_i(\beta)))\int_0^s S_i^j(\beta)\delta(\tau-t_i)y_\ell(\beta,\tau)d\tau)ds$$

$$= \int_0^t [\int_{t_i(\beta)}^t \Phi(t,\tau)d\tau] \sum_{i=1}^{j} S_i^j(\beta)\delta(s-t_i)y_\ell(s)ds \qquad (A.19)(iii)$$

Finally, let $\Phi(t,s)$ satisfy

$$\dot{\Phi}(t,s) = -\Phi(t,s)A(s) - [\int_s^t \Phi(t,\tau)d\tau]B(s) - \sum_{i=1}^{j}[\int_{t_i(\beta)}^{t} \Phi(t,\tau)d\tau]S_i^j(\beta)\delta(s-t_i(\beta)),$$

$$\Phi(t,t) = I \tag{A.20}$$

then, using (A.19)(i), (ii) and (iii) and (A.20) in (A.18), we have

$$y_\ell(\beta,t) = \int_0^t \Phi(t,s)r_\ell(\beta,s)ds \tag{A.21}$$

Define

$$p(t,s) \equiv \Phi(t,s)^T[\frac{\partial q}{\partial z}(z(\beta,t))]^T \tag{A.22}$$

we have, then, from (A.20), (A.21) and (A.22),

$$\frac{\partial q}{\partial \beta_\ell}(z(\beta,t)) = \int_0^t p(t,s)^T r_\ell(\beta,s)ds \tag{A.23}$$

where,

$$\dot{p}(t,s) = -A(s)^T p(t,s) - B(s)^T[\int_s^t p(t,\tau)d\tau] - \sum_{i=1}^{j} S_i^j(\beta)^T\delta(s-t_i(\beta))[\int_{t_i(\beta)}^{t} p(t,\tau)d\tau] \tag{A.24}$$

$$p(t,t) = [\frac{\partial q}{\partial z}(z(\beta,t))]^T$$

Q.E.D.

A.5 Corollary:

Let $\frac{\partial q}{\partial \beta_\ell}(z(\beta,t))$ in Sec. A.4 be given by (A.11), (A.12) and (A.13); then, it is also true that

$$\frac{\partial q}{\partial \beta_\ell}(z(\beta,t)) = \int_0^t \dot{p}(t,s)^T r_\ell(\beta,s)ds \;,\; t \in [0,T] \tag{A.25}$$

where,

$\tilde{p}(t,s)$ satisfies the following integro-differential equation:

$$\frac{d^2}{ds^2}\tilde{p}(t,s) + A(s)^T \frac{d}{ds}\tilde{p}(t,s) - B(s)^T\tilde{p}(t,s)$$
$$= \sum_{i=1}^{j} S_i^j(\beta)^T \delta(s-t_i(\beta)) \left(\int_{t_i(\beta)}^{t} [\frac{d}{d\tau}\tilde{p}(t,\tau)]\, d\tau \right),$$

with $\quad s \in [0,t]$

$\quad t \in (t_j(\beta), t_{j+1}(\beta)] \subset [0,T]$

$$\tilde{p}(t,t) = 0$$

$$\frac{d}{ds}\tilde{p}(t,s)\Big|_{s=t} = \frac{\partial q}{\partial z}(z(\beta,t))^T \tag{A.26}$$

where all terms have already been defined.

Proof:

Define the following transformation on $p(t,s)$:

$$\frac{d}{ds}\tilde{p}(t,s) = p(t,s) \quad , \quad s \in [0,t]$$

$$\tilde{p}(t,t) = 0 \tag{A.27}$$

Then, the required result (A.25), (A.26) follows directly.

Q.E.D.

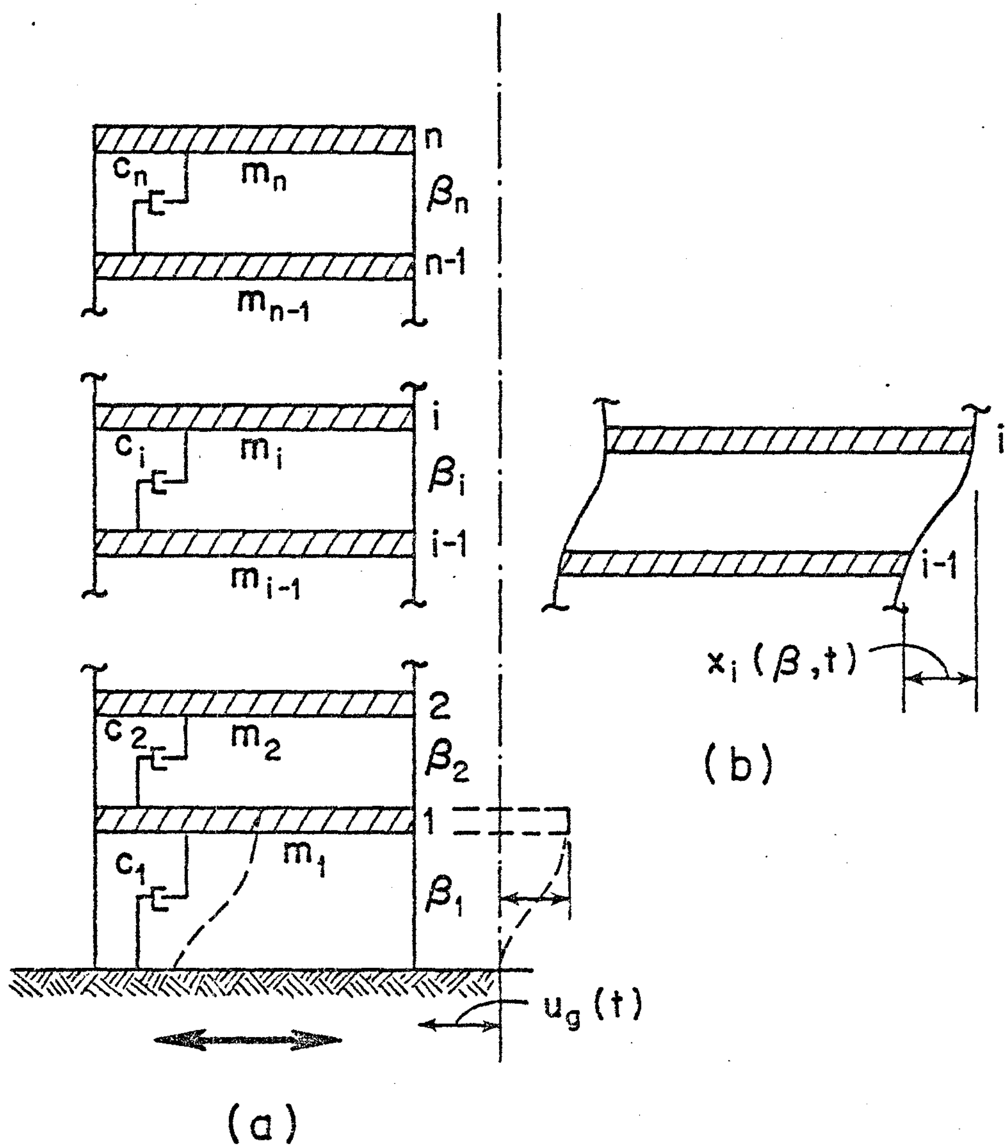

FIG. 1 - STRUCTURAL MODEL

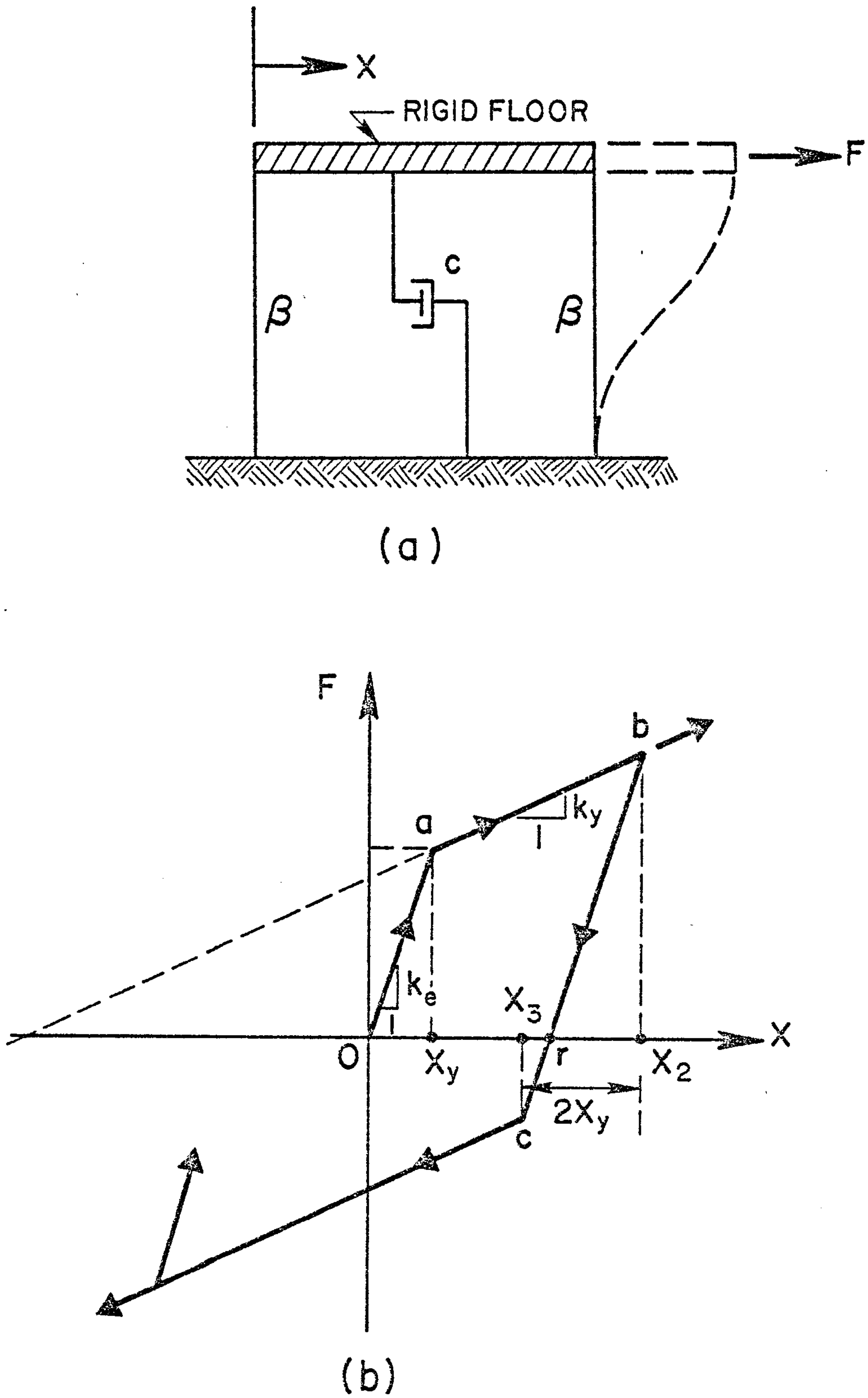

FIG. A-1 - BILINEAR HYSTERETIC MODEL

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