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UILU-ENG-76-2024

CIVIL ENGINEERING STUDIES Structural Research Series No. 434



COMPUTED BEHAVIOR OF REINFORCED CONCRETE COUPLED SHEAR WALLS

by T. TAKAYANAGI and W. C. SCHNOBRICH

A Report on a Research Project Sponsored by THE NATIONAL SCIENCE FOUNDATION Research Grant ATA 7422962

> UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN URBANA, ILLINOIS L DECEMBER 1976

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17. Key Words a 7b. Identifiers/	determ each c charac compar	ined by inelastic prope onstituent member are e teristics of coupled sh ed with the available t nt Analysis. 17a. Descriptors d shear walls, Nonlinea esis loops, Equations o	rties. The stablished t ear wall sys est results. r dynamic ar f motion	alysis, Ea	s of each ysteresis the specif computed	inember are loops to ic results are
17. Key Words a 7 b. Identifiers/	determ each c charac compar	ined by inelastic prope onstituent member are e teristics of coupled sh ed with the available t nt Analysis. 170. Descriptors d shear walls, Nonlinea esis loops, Equations o	rties. The stablished t ear wall sys est results.	alysis, Ea	rthquake e	engineering,
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17. Key Words a 17. Identifiers/	determ each c charac compar	ined by inelastic prope onstituent member are e teristics of coupled sh ed with the available t nt Analysis. 170. Descriptors d shear walls, Nonlinea esis loops, Equations o	rties. The stablished t ear wall sys est results. Ir dynamic ar f motion	alysis, Ea	rthquake e	engineering,
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ACKNOWLEDGMENT

The writers wish to express their sincere gratitude to Professor Mete A. Sozen for his invaluable comments and help.

Deep appreciation is due Professor S. Otani of the University of Toronto for his numerous suggestions and help.

Thanks are also due Dr. J. D. Aristizabal-Ochoa, Mr. J. M. Lybas, and Mr. D. P. Abrams for providing the authors with information from their test results.

The work was supported by a National Science Foundation Grant No. ATA 7422962. The support is gratefully acknowledged.

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TABLE OF CONTENTS

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		9 8					Page
CHAPTE	R		·				
1.	INTR	DUCTION	, , , , , , , , , , , , , , , , , , ,	· · · · · · · · · · · · · · ·	· • • • • • • • • •	• • • • • • • • • • •	. 1
	1.1 1.2 1.3	Object and Scor Review of Previ Notation	be ious Researc	h	· · · · · · · · · · · ·		. 1 . 3 . 6
2.	MECH	ANICAL MODEL	· • • • • • • • • • •	• • • • • • • • • • • • •	مەنبە ، مەنبە ، مەنبە ،		. 12
۰.	2.1	Structural Syst Mechanical Mode	tem. Is of Conne	cting Beam a	nd Wall	• • • • • • • • • • •	. 12 . 12
3.	FORC	E-DEFORMATION RE	ELATIONSHIPS	OF FRAME EL	EMENTS		. 15
	3.1 3.2 3.3 3.4	Material Proper Moment-Curvatur Deformational F Deformational F	rties re Relations Properties o Properties o	hip of a Sec f Wall Subel f the Rotati	tion ements onal Sprin	 gs	. 15 . 17 . 19
_		Positioned at t	the Beam End	S	• • • • • • • • • • •		. 24
4.	ANAL'	YTICAL PROCEDURE			•••••	•••••	. 35
	4.1 4.2 4.3 4.4 4.5 4.6	Introductory Re Basic Assumptic Stiffness Matri Structural Stif Static Analysis Dynamic Analysis	emarks ons ix of a Memb ffness Matri s is	er X	· · · · · · · · · · · · · · · · · · ·	· · · · · · · · · · · · · · · · · · ·	. 35 . 36 . 37 . 47 . 49 . 50
5.	HYST	ERESIS RULES					. 60
	5.1 5.2	Hysteresis Rule Modifications o	es by Takeda, of Takeda's	et al Hysteresis R	Rules		. 60 . 60
6.	ANAL	YTICAL RESULTS.			• • • • • • • • • • •		. 63
	6.1 6.2 6.3 6.4 6.5	Model Structure Static Analysis Preliminary Ren Dynamic Analys Effects of Assu	es s of Structu narks of Dyn is of Struct umed Analyti	re-l amic Analysi ure-l cal Conditio	s	· · · · · · · · · · · · · · · · · · ·	. 63 . 64 . 71 . 73
	6.6	on Dynamic Resp Dynamic Analys	oonse is of Struct	ure-2			. 81 . 86

iv

		Page
7	. SUMMARY AND CONCLUSIONS	91
	7.1 Object and Scope 7.2 Conclusions	91 92
LIST	OF REFERENCES	97
APPE	NDIX	
A	CALCULATIONS OF WALL STIFFNESS PROPERTIES IN THE COMPUTER PROGRAM	177
В	COMPUTER PROGRAM FOR NONLINEAR RESPONSE ANALYSIS OF COUPLED SHEAR WALLS	188

1. 1914

. •

LIST OF TABLES

Table		Page
6.1	Assumed Material Properties	.100
6.2	Stiffness Properties of Constituent Elements	.101
6.3	Summary of Assumed Analytical Conditions for Dynamic Runs	.102
6.4	Mode Shapes and Frequencies of Structure-1	.104
6.5	Maximum Responses of Structure-1 in Comparison with Test Results	.105
6.6	Effect of the Numerical Integration Scheme on the Maximum Responses of Structure-1	.107
6.7	Effect of the Choice of Stiffness Matrix for the Calculation of Damping Matrix on the Maximum Responses of Structure-1	.108
6.8	Effect of the Arrangement of Wall Subelements on the Maximum Responses of Structure-1	.109
6.9	Effects of the Pinching Action and Strength Decay of Beams on the Maximum Responses of Structure-1	.110
6.10	Mode Shapes and Frequencies of Structure-2	.112
6.11	Maximum Responses of Structure-2 in Comparison with Test Results	.113

LIST OF FIGURES

Figure		Page
2.1	Mechanical Model of Coupled Shear Wall System	.114
2.2	Connecting Beam Model	.115
2.3	Wall Member Model	.115
3.1	Idealized Stress-Strain Relationship of Concrete	.116
3.2	Idealized Stress-Strain Relationship of Reinforcement	.116
3.3	Distributions of Stress and Strain over a Cross Section	.117
3.4	Moment-Curvature Relationships of a Wall Member Section	.118
3.5	Moment-Curvature Relationship of a Connecting Beam Section	.119
3.6	Idealized Moment-Curvature Relationships of a Wall Member Section	.120
3.7	Idealized Moment-Curvature Relationship of a Connecting Beam Section	.121
3.8	Bond Slip Mechanism	.122
3.9	Idealized Moment-Rotation Relationship of a Rotational Spring	.123
4.1	Force and Displacement Components at Wall Member Ends	.124
4.2	Cantilever Beam with Three Subelements	.124
4.3	Force and Displacement Components at Connecting Beam Ends	.125
4.4	Moment Distribution of a Connecting Beam	.125
4.5	Deformed Configuration of a Connecting Beam	.126
5.1	Takeda's Hysteresis Loops	.127
5.2	Hysteresis Loops with Effect of Changing Axial Force	.128

Figure		Page
5.3	Hysteresis Loops with Effects of Pinching Action and Strength Decay	.129
6.1	Cross-Sectional Properties of Wall and Beam	.130
6.2	Sequence of Cracking and Yielding of Structure-1 under Monotonically Increasing Load	.131
6.3	Relationship of Base Axial Force and Top Vertical Displacement of Walls of Structure-1 under Monotonically Increasing Load	.132
6.4	Base Moment-Top Displacement Relation of Structure-1 under Monotonically Increasing Load	.133
6.5	Redistribution of Base Shear in Walls of Structure-1 under Monotonically Increasing Load	.134
6.6	Ratio of Top Displacement due to Coupling Effect to Total Top Displacement of Structure-1 under Monotonically Increasing Load	.134
6.7	Base Moment Distribution Pattern of Structure-1 under Monotonically Increasing Load	.135
6.8	Moment Distribution Patterns in Members of Structure-1 at the End of Monotonically Increasing Load	.136
6.9	Base Moment-Top Displacement Relation of Structure-1 under Cyclic Loading	.137
6.10	Waveforms of Base Accelerations	.138
6.11	Initial Mode Shapes of Structure-1	.139
6.12	Maximum Responses of Structure-1, Run-1, in Comparison with Test Results	.140
6.13	Response Waveforms of Structure-1, Run-1	.142
6.14	Response Waveforms of Structure-1, Run-2	.150
6.15	Response Waveforms of Structure-1, Run-3	.151
6.16	Response History of Base Moment-Top Displacement Relationship of Structure-1, Run-1	.152
6.17	Response Waveforms of Internal Forces of Structure-1, Run-1	.153

е

Figure	Page
6.18	Hysteresis Loops of a Beam Rotational Spring of Structure-1, Run-1157
6.19	Hysteresis Loops of a Wall Subelement at Base of Structure-1, Run-1158
6.20	Sequence of Cracking and Yielding for Dynamic Response of Structure-1, Run-1
6.21	Ratio of Coupling Base Moment to Total Base Moment at Peak Responses of Structure-1, Run-1
6.22	Ratio of Top Displacement due to Coupling Effect to Total Top Displacement at Peak Responses of Structure-1, Run-1161
6.23	Coupling Effect of Walls on Displacement Distribution at the Maximum Response of Structure-1, Run-1
6.24	Response Waveforms of Structure-1, Run-5163
6.25	Response Waveforms of Structure-1, Run-6164
6.26	Response Waveforms of Structure-1, Run-7165
6.27	Response Waveforms of Structure-1, Run-8
6.28	Response Waveforms of Structure-1, Run-9167
6.29	Response Waveforms of Structure-2, Run-10168
6.30	Sequence of Cracking and Yielding for Dynamic Response of Structure-2, Run-10
A.1	Primary Curve for Hysteresis Loops of a Wall Member Section
A.2	Evaluation of $rac{\partial M}{\partial n}$ for Hysteresis Loops of a Wall Member Section
A.3	Axial Force-Axial Strain Relationship of a Wall Member Section
A.4	Idealized Axial Force-Axial Strain Relationship of a Wall Member Section186
A.5	Relationship of Slope of Axial Force-Axial Strain Line and Curvature for a Wall Member Section
B.1	Flow Diagram of Computer Program for Nonlinear Response Analysis of Coupled Shear Walls

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CHAPTER 1

INTRODUCTION

1.1 Object and Scope

The coupled shear wall is considered to be a very efficient structural system to resist horizontal movements due to earthquake motions. It is not possible to investigate thoroughly through model tests the influence of the many possible variations in the various parameters that control the response of coupled shear walls. The models are too expensive in terms of both time and money. Furthermore, it is not always possible to record when all the events of interest take place. On the other hand, most of the papers dealing with the analysis of coupled shear walls are based on elastic member properties. Those papers where inelastic member properties are allowed are primarily for the case of monotonically increasing loads. In view of the scarcity of data, it is necessary to investigate the nonlinear response behavior of coupled shear walls due to strong earthquake motions.

The study is intended to develop an analytical model which can trace the response history and the failure mechanism of coupled shear walls under dynamic and static loads and to see the characteristics of coupled shear walls behavior under these loads.

Although there are many configurations and variations of shear wall systems in use, the analytical model is discussed only with reference to reinforced concrete coupled shear walls, two walls with connecting beams under horizontal earthquake motions and static loadings.

To predict the actual behavior of coupled shear walls during strong motion earthquakes, the dynamic structural properties in the highly inelastic range are taken into consideration. Inelastic properties such as cracking and crushing of the concrete, and yielding and bond slip of reinforcing steel complicate the problem. Therefore, idealizations and simplifications of the mechanical models for the constituent members are considered necessary in the analytical procedure. The basic model used in the study is composed of flexural line elements, both for the walls and the connecting beams.

These constituent flexural elements incorporate their hysteretic properties utilizing the test data available. The suitable hysteresis loops to each constituent member are established by modifying Takeda's hysteresis rules (1970)* to include the specific characteristics of coupled shear walls.

The instantaneous nonlinear characteristics of the structure and the failure process of each constituent member under strong earthquake motions are estimated by numerically integrating the equation of motion in a step-by-step procedure. Also the failure mechanism of the structure under static loads is traced by constantly increasing lateral load at small increments.

The computed results are compared with the available test results by Aristizabal-Ochoa (1976).

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^{*} References are arranged in alphabetical order in the List of References. The number in parentheses refers to the year of publication.

1.2 Review of Previous Research

Analyses of coupled shear walls have been performed by many investigators. No attempt will be made to cite all such reported investigations. Only a few of the early and directly applicable studies are referred to here.

A typical approach to the shear wall problem is the so-called laminae method. In this method the discrete system of connecting beams is replaced by a continuous connecting medium of equivalent stiffness. Beck (1962) and Rosman (1964) analyzed coupled shear walls under lateral loads based on this idealization. Coull (1968) extended this assumption to take account of the shearing deformations of the walls. Later Tso and Chan (1971) used this method to determine the fundamental frequency of coupled shear wall structures. Such a determination is, of course, essential in the application of the response spectrum technique. All the papers mentioned above are based on linearly elastic properties of the members.

Paulay (1970) used the laminae method to trace the failure mechanism of coupled shear walls under monotonically increasing loads by introducing plastic hinges at the ends of each lamina as well as at the base of wall during the process of loading. Although the laminae method has the advantage of being relatively simple to apply, this method cannot treat the expansion of inelastic action over the length of the wall members.

The use of two dimensional plane stress elements with the finite element method is another way of approaching the analysis of coupled shear walls. Girijarallabhan (1969) used the element method in an attempt to define more precise stress distributions of coupled shear walls.

Yuzugullu (1972) analyzed single-story shear walls and infilled frames by using the finite element method, including in that analysis the inelastic properties of reinforced concrete elements. Naturally this approach is quite time-consuming for a multistory coupled shear wall system. Such an analysis requires a very large number of elements. Furthermore, difficulties arise in the wall element to beam element connection. In order to avoid the use of plane stress elements for the connecting beams, some means of establishing the rotational degree of freedom at the wall connection must be introduced. One possibility is a rigid arm from the wall center to the beam connection.

Instead of using the element method, inelastic beam models in which each member is represented by a flexural line element were developed to save the computing time and to simplify the mechanical model. Several inelastic beam model techniques have been extensively used in the analysis of the nonlinear response behavior of frame subjected to base excitations.

Clough, et al. (1965) proposed the two component model to represent a bilinear nondegrading hysteresis. The member consists of a combined elastic member and an elasto-plastic member. Aoyama, et al. (1968) developed the four component model to represent the trilinear nondegrading hysteresis loop. In this model the idealized beam has an elastic member and three elasto-plastic members in parallel. The four component model and the two component model are based on the same concept. These models are generally called multicomponent models. The multicomponent model has some difficulties when applied to a degrading hysteresis system.

Giberson (1967) proposed the equivalent spring model which is generally called the one component model. In this model rotational

springs, which represent only inelastic behavior of the beam, are introduced at both ends of the beam. The rest of the beam, between the ends, is considered to be elastic. This model has no coupling term in the inelastic part of the flexibility matrix. In other words, the inelastic rotation at one end is related only to the moment at the same end and is independent of the moment at the other end. The inflection point is assumed to be fixed at the same location during the response behavior. This assumption is not realistic because the location of an inflection point is expected to change during the real response behavior of the beam. But this model is considered to be more versatile than the multicomponent model, since the rotational spring car take care of any kind of hysteresis loop.

Takizawa (1973) developed the prescribed flexibility distribution model which is based on the assumption of a distribution pattern of cross sectional flexural flexibility along the member axis. In his paper he used a parabolic curve as the flexural flexibility distribution. The inflection point is not necessarily fixed in this model.

Otani (1972) presented the combined two cantilever beam model. The beam consists of two cantilever beams whose free ends are placed at the inflection point. The beam is not allowed to be subjected to any change of the moment distribution which produces a serious sudden movement of the inflection point. But this model has very natural correspondence between the actual phenomena and the available hysteresis data based on the test result.

Hsu (1974) investigated the inelastic dynamic response of the single shear wall experimentally and analytically. In the analytical part of

his study, he assumed a divided element beam model in which the beam is divided into several elements and each element has a uniform flexural rigidity changeable based on the hysteresis loop. In this model it is easy to handle a local concentration of inelastic action of the member by arranging elements finely at the location of interest.

1.3 Notation

The symbols used in this text are defined where they first appear. A convenient summary of the symbols used is given below.

- A_s = area of the tensile reinforcement
 A's = area of the compressive reinforcement
 b = width of the cross section
 c = depth of the neutral axis
 c' = distance from the neutral axis to the point of
- the maximum tensile stress of the concrete

 c_1 , c_2 = coefficients for the damping matrix

[C] = damping matrix

- - d = distance from the extreme compressive fiber to
 the center of tensile reinforcement
 - d' = distance from the extreme compressive fiber
 to the center of compressive reinforcement

D = total depth of a section or diameter of a reinforcing bar D_c = cracking displacement of the unit length cantilever beam D_v = yielding displacement of the unit length cantilever beam

6.

 D_u = ultimate displacement of the unit length cantilever beam

D(M) = free end displacement of a cantilever beam

 $E_s = modulus$ of elasticity of the steel

E_h = modulus to define stiffness in strain hardening range
 of the steel

 E_v = inelastic modulus of the reinforcement after yielding

EA_i = inelastic axial rigidity of a section

EI = initial flexural rigidity

 EI_{p} = elastic flexural rigidity of a section

EI_i = inelastic flexural rigidity of a section

 $f_{c} = stress of the concrete$ $f_{c}' = compressive strength of the concrete$ $f_{t} = tensile strength of the concrete$ $f_{s} = stress of the steel or stress of the tensile reinforcement$ $f_{s}' = stress of the compressive reinforcement$ $f_{y} = yield stress of the steel$ $f_{u} = ultimate stress of the steel$ f(M) = flexibility resulting from the bond slippage of tensile reinforcement of a beam $[f_{AB}] = flexibility matrix of a cantilever beam$ $GA_{e} = elastic shear rigidity of a section$ [K] = structural stiffness matrix $[K_{ij}] = submatrices used in Eq. (4.16) (i, j = 1 or 2)$

 $[K_{AB}]$ = stiffness matrix of a cantilever beam

[K_c] = instantaneous structural stiffness matrix

which is evaluated at the end of current step

[K_a] = elastic structural stiffness matrix

[K_i] = inelastic structural stiffness matrix

 $[K_w]$ = stiffness matrix of a wall member

 ℓ_i = length of the subselement i

L = length of a beam or development length of the bond stress

 ΔL = elongation of the reinforcment

m = bending moment of a section

 Δm = increment of bending moment

 m_i = lumped mass at the story i

M = bending moment

 M_{c} = cracking moment

 $M_v = yielding moment$

 M_u = moment at concrete strain equal to 0.004

 $M(\phi, n)$ = bending moment function

 ΔM = increment of moment

 ${\bigtriangleup M}_A$, ${\bigtriangleup M}_B$ = incremental moments at the ends of a member

 ΔM_c , M_b = incremental end moments of the flexible element of a connecting beam

 $\{\Delta M\}$ = incremental joint moment vector

[M] = diagonal mass matrix

n = axial force of a section

 Δn = increment of axial force

N = axial load acting on a section

 $N(\phi, \epsilon)$ = axial force function

 $\Delta N_{A},\ \Delta N_{B}$ = incremental shear forces at the ends of a connecting beam

or incremental axial forces at the ends of a wall member

- $\{\Delta N\}$ = incremental joint vertical force vector
- ΔP_A , ΔP_B = incremental shear forces at the ends of a wall member
 - $\{\Delta P\}$ = incremental story lateral force vector

 - R_c = rotation at which the cracking moment is developed
 - $R_{\rm v}$ = rotation at which the yielding moment is developed
 - R_{ij} = rotation at which the ultimate moment is developed
 - SD(M) = instantaneous stiffness of the unit length cantilever beam based on the flexural rigidity
 - ST(M) = instantaneous stiffness of the unit length cantilever beam based on the flexural and shear rigidities

 $\Delta t = time interval$

 $[T_{AB}]$ = transformation matrix of a cantilever beam

u = average bond stress

 ΔU_A , ΔU_B = incremental lateral displacement at the ends of a wall member { ΔU } = incremental story lateral displacement vector or incremental story displacement vector relative to the base

 $\{\Delta \dot{U}\} = \text{incremental story velocity vector relative to the base} \\ \{\Delta \ddot{U}\} = \text{incremental story acceleration vector relative to the base} \\ \{\dot{U}\} = \text{relative story velocity vector at the end of previous step} \\ \{\ddot{U}\} = \text{relative story acceleration vector at the end of previous step} \\ \{\ddot{U}\} = \text{relative story acceleration vector at the end of previous step} \\ \Delta V = \text{increment of the free end displacement of a cantilever beam}$

 $\Delta V_{\texttt{f}}$ = increment of the free end displacement of a cantilever beam only due to the flexural rigidity ΔV_A , ΔV_B = incremental vertical displacement of a member $\{\Delta V\}$ = incremental joint vertical displacement vector $\{\Delta \ddot{X}\}$ = incremental base acceleration vector Z = constant which defines the descending slope of the stress-strain curve of the concrete β = constant of the Newmark β method β_i = damping factor of the ith mode $\gamma = (\omega_i / \omega_{e})^{1/2}$ ε = axial strain of a section $\Delta \varepsilon$ = increment of axial strain $\epsilon_{\rm c}$ = strain of the concrete or concrete strain at the extreme compressive fiber ε_0 = strain at which f' is attained ε_{+} = strain at which f_{+} is attached $\boldsymbol{\epsilon}_s$ = strain at the steel or strain in the tensile reinforcement ϵ_{s}^{\prime} = strain in the compressive reinforcement ε_v = strain at which f is attained $\boldsymbol{\epsilon}_{h}$ = strain at which strain hardening of the steel commences $\boldsymbol{\epsilon}_{u}$ = strain at which \boldsymbol{f}_{u} is attached η = distance from the neutral axis of a section $\Delta \Theta$ = increment of rotation $\Delta \theta^{\, t}_A, \ \Delta \theta^{\, t}_B$ = incremental rotations at the ends of a member $\Delta \theta_{A},~\Delta \theta_{B}$ = incremental rotations at the rigid link ends of a simply supported beam

11

 $\Delta \theta_{C}$, $\Delta \theta_{D}$ = incremental and rotations of the combined spring-flexible element of a connecting beam

- $\{\Delta\theta\}$ = incremental joint rotation vector
 - λ = ratio of the length of a rigid link to that of a flexible element for a connecting beam

 ϕ = curvature

 ϕ_{c} = curvature at cracking

 ϕ_v = curvature at yielding

 φ_{μ} = curvature at concrete strain equal to 0.004

 $\Delta \varphi$ = increment of curvature

 $\{\psi\}$ = first mode shape vector

 ω_{j} = circular frequency of the jth mode

 $\omega_{\textbf{p}}$ = first mode circular frequency in the elastic stage

 $\boldsymbol{\omega}_{\mathbf{i}}$ = first mode circular frequency in the inelastic stage

CHAPTER 2

MECHANICAL MODEL

2.1 Structural System

The lateral resistance of coupled shear walls results primarily from three sources of structural actions: the flexural rigidity of the walls, the flexural rigidity of the connecting beams and the moment effect of the couple growing out of the axial rigidity of the two walls.

The mechanical model chosen to represent the coupled shear walls is shown in Fig. 2.1. The walls and the connecting beams are replaced by massless line members at their centroidal axes. The wall members have flexural, axial and shear rigidities as their resistances. The connecting beam members have flexural and shear rigidities. The axial rigidity of the connecting beam is assumed to be infinite since the horizontal displacements of both walls are practically identical.

Three displacement components are considered at each wall-beam joint: horizontal displacement, vertical displacement and rotation. The righthand screw rule is adopted to describe the positive directions of these displacement components as shown in Fig. 2.1.

The internal subelements or degrees of freedom are condensed out of the stiffness matrix before the system equations are written so that only horizontal story movements appear in the final equations. The mass of each story is assumed to be concentrated at each floor level. In the analysis the wall is considered to be fixed at the base.

2.2 Mechanical Models of Connecting Beam and Wall

A mechanical model of the connecting beams used in the study is the one which Otani (1972) developed based on inelastic actions of a cantilever beam. This model is quite suitable for the connecting beams of a coupled shear wall system, since the contraflexure point is practically fixed at the center of the beam span during its response.

The connecting beams are taken as individual beams connected to the walls through a rigid link and a rotational spring as shown in Fig. 2.2. The rotational spring takes care of any beam end rotation which is produced by the steel bar elongation and concrete compression in the joint core area as well as the inelastic flexural and shear actions over the beam length. Such inelastic flexural action is expected to be localized near the beam ends because of the antisymmetric moment distribution over the beam length. The action within the joint core could have been treated by the effective length concept in which the clear span length of beam is arbitrarily expanded into the joint core to allow for flexural and slip action in the joint core. But it was judged much simpler to consider the joint core as a rigid link and to let the rotational spring take care of the inelastic and other actions of the joint core area. The beam itself is considered to be a flexural member with uniform elastic rigidity along its length.

The wall is also considered to act initially as a beam with a linear variation of strain over the cross section. To use two-dimensional plane stress elements for the walls was judged less desirable, since such an approach would have been much more expensive computationally without any compensating increase in accuracy. It is in fact probable that while accounting for cracking and nonlinear action of the plane stress elements with current concepts and methods the system would not reproduce experimental results as well as line elements can.

The wall members are exposed to a more general moment distribution than are the connecting beams. In addition, the location of the contraflexure point might shift significantly from a change in the moment distribution and the change of axial force during the response might cause a change of moment capacity in the wall members. Therefore the inelastic flexural behavior in the wall can be expected to expand along the length of the member rather than be localized. In order to allow the inelastic action to cover a partial length of a wall member, the member is further divided into several subelements as shown in Fig. 2.3. The stress resultants at the centroid of the subelements are used as the control factors for the determination of the nonlinear properties of the subelements. The degree of subdivision decreases with story height since the major inelastic action is expected at the base.

CHAPTER 3

FORCE-DEFORMATION RELATIONSHIPS OF FRAME ELEMENTS

3.1 Material Properties

Inelastic force-deformation relationships for the wall subelements and corresponding relationships for the rotational springs placed at the connecting beam ends are based on idealized stress-strain relationships for concrete and steel. These inelastic force-deformation relationships are used as the primary curves for the hysteresis loop.

(a) Stress-Strain Relationship for Concrete

A parabola combined with a straight line in the form used by Otani (1972) is also adopted here for the stress-strain relationship of concrete. Accordingly,

$$f_{c} = 0 \qquad \varepsilon_{c} \leq \varepsilon_{t}$$

$$f_{c} = f'_{c} \left[2 \frac{\varepsilon_{c}}{\varepsilon_{0}} - \left(\frac{\varepsilon_{c}}{\varepsilon_{0}} \right)^{2} \right] \qquad \varepsilon_{t} \leq \varepsilon_{c} \leq \varepsilon_{0}$$

$$f_{c} = f'_{c} \left[1 - Z(\varepsilon_{c} - \varepsilon_{0}) \right] \qquad \varepsilon_{0} \leq \varepsilon_{c}$$

$$(3.1)$$

and

$$\varepsilon_{t} = \varepsilon_{0} [1 - (1 - f_{t}/f_{c})^{\frac{1}{2}}]$$
(3.2)

$$f_t = -6.0 (f_c^{\dagger})^{\frac{1}{2}}$$
 (3.3)

where

 f_c = stress of the concrete f_c^{*} = compressive uniaxial strength of the concrete f_t = tensile strength of the concrete ε_t = strain of the concrete ε_0 = strain at which f' is attained

 ε_t = strain at which f_t is attained

Z = constant which defines the descending slope

of the stress-strain curve. The value of 100 was used in this analysis.

Justification for the use of these relations can be found in Otani's thesis. A typical example of the proposed curve is shown in Fig. 3.1.

(b) Stress-Strain Relationship of Steel

A piecewise linear stress-strain relationship is assumed for the reinforcing steel. Accordingly,

$$f_{s} = E_{s} \varepsilon_{s} \qquad \varepsilon_{s} \leq \varepsilon_{y}$$

$$f_{s} = f_{y} \qquad \varepsilon_{y} \leq \varepsilon_{s} \leq \varepsilon_{h}$$

$$f_{s} = f_{y} + E_{h}(\varepsilon_{s} - \varepsilon_{h}) \qquad \varepsilon_{h} \leq \varepsilon_{s} \leq \varepsilon_{u}$$

$$f_{s} = f_{u} \qquad \varepsilon_{u} \leq \varepsilon_{s}$$

$$(3.4)$$

where

 $f_{s} = stress of the steel$ $f_{y} = yield stress of the steel$ $f_{u} = ultimate stress of the steel$ $\varepsilon_{s} = strain of the steel$ $\varepsilon_{y} = strain at which f_{y} is attained$ $\varepsilon_{h} = strain at which strain hardening commences$ $\varepsilon_{u} = strain at which f_{u} is attained$ $E_{s} = modulus of elasticity of the steel$ $E_{h} = modulus to define stiffness in strain hardening range$

The numerical value of E_s is assumed to be 29,000 kip/in.² in the analysis. The representative stress-strain curve of the steel is shown in Fig. 3.2. The stress-strain relations represented by Eqs. (3.4) are assumed to be symmetric about the origin.

3.2 Moment-Curvature Relationship of a Section

The primary moment-curvature curve for a monotonically increasing moment can be derived based on the geometry of the section, the existing axial load, the deformational properties of concrete and steel mentioned in Section 3.1, and the assumption that a linear variation of strain exists across the cross ection. This linear variation is maintained throughout the entire loading.

The relationship of curvature of a section to strain can be expressed by utilizing the assumption of linear strain distribution. This is shown in Fig. 3.3. The relation takes the following forms.

$$\phi = \varepsilon_{c}/c$$

$$= \varepsilon_{s}'/(c - d')$$

$$= \varepsilon_{s}/(d - c)$$
(3.5)

where

 ϕ = curvature

 ε_{c} = concrete strain at the extreme compressive fiber ε_{s}^{\prime} = strain in the compressive reinforcement ε_{s} = strain in the tensile reinforcement d' = distance from the extreme compressive fiber to the center of compressive reinforcement 18 · .

d = distance from the extreme compressive fiber

to the center of tensile reinforcement

c = depth of the neutral axis

The equilibrium equation of the resultant forces can be expressed as follows:

$$\int_{-C'}^{C} f_{c} b dx + A_{s}' f_{s}' - A_{s} f_{s} = N$$
(3.6)

where

f's = stress of the compressive reinforcement
fs = stress of the tensile reinforcement
b = width of the cross section
A's = area of the compressive reinforcement
As = area of the tensile reinforcement
N = axial load acting on the section
c' = distance from the neutral axis to the point
 of the maximum tensile stress of the concrete

The bending moment M at the depth x can be calculated by the following equation.

$$M = \int_{-c'}^{c} f_{c} bndn + (x - c) \int_{-c'}^{c} f_{c} b dx + A_{s}^{i} f_{s}^{i} (x - d') + A_{s} f_{s} (d - x) + N(x - \frac{D}{2})$$
(3.7)

where

D = total depth of the section η = distance from the neutral axis

The stresses f_c , f'_s and f_s can be calculated by Eqs. (3.1) and (3.4) for

given strains $\boldsymbol{\epsilon}_c,~\boldsymbol{\epsilon}_s^{*}$ and $\boldsymbol{\epsilon}_s,$ respectively.

It is difficult to solve Eqs. (3.5) and (3.6) directly for the unknowns ε_c and c, because the solution may not be available in a closed form. Therefore a recommended procedure is that Eqs. (3.5) and (3.6) are solved for c with given ε_c and N by the iteration method. The moment M and curvature ϕ can be derived by Eqs. (3.5) and (3.7) with a calculated c and a given ε_c . The bending moment M is evaluated along the plastic centroid of the section. The moment-curvature curve can be drawn by the series of calculated M and ϕ for different values of ε_c .

Flexural cracking of a reinforced concrete section subjected to both flexural and axial load is assumed to occur when the stress at the extreme tensile fiber of the section exceeds the tensile strength of concrete. Flexural yielding is considered to occur when the tensile reinforcement yields in tension. If the tensile reinforcement is arranged in many layers, the stiffness change occurs gradually starting with the initiation of yielding of the furthest layer of reinforcement and proceeding until yielding occurs in the closest layer to the neutral axis of the section. Because of the requirement of the hysteresis rules used in this analysis, a single value of the yield moment is to be given. Therefore the yield moment is defined as the moment corresponding to the development of the yield strain at the centroid of the reinforcing working in tension.

Typical examples of moment-curvature curves for a wall section and a beam section are shown in Fig. 3.4 and Fig. 3.5, respectively.

3.3 Deformational Properties of Wall Subelements

The inelastic moment-curvature relationships of the wall subelements are used as the primary curves in establishing the hysteresis loops.

The stress resultants computed at the centroid of each subelement are used in the determination of the instantaneous stiffness of the subelement so that each subelement can be subjected to a different stage of nonlinearity.

Each subelement has three types of rigidities: flexural, axial and shear. The instantaneous flexural rigidity of each subelement is defined as the slope of the idealized moment-curvature curve at the point which is located by the history of inelastic action in the subelement.

To simplify the problem this idealized moment-curvature relationship is determined by trilinearizing the original moment-curvature curve. The slopes in the three stages of this idealized moment-curvature relatinship are defined as follows:

$$\phi = M/(\frac{M_{c}}{\phi_{c}}) \qquad M \leq M_{c}$$

$$\phi = M/(\frac{M_{y} - M_{c}}{\phi_{y} - \phi_{c}}) + \phi_{c} \qquad M_{c} \leq M \leq M_{y}$$

$$\phi = M/(\frac{M_{u} - M_{y}}{\phi_{u} - \phi_{y}}) + \phi_{y} \qquad M_{y} \leq M$$

$$(3.8)$$

where

M = bending moment $M_{c} = cracking moment$ $M_{y} = yielding moment$ $M_{u} = moment at concrete strain equal to 0.004$ $\phi = curvature$ $\phi_{c} = curvature at cracking$ $\phi_{y} = curvature at yielding$ $\phi_{u} = curvature at concrete strain equal to 0.004$

A series of idealized moment-curvature relationships for different values of constant axial force are shown in Fig. 3.6. Actually the axial force on a section is not constant and is subject to change in the process of loading. The moment-curvature curve of a section under a changing axial load is traced by appropriate shifts or movements between the series of moment-curvature curves for constant axial loads as shown by the dashed line in Fig. 3.6. It is assumed that the axial force is small enough that the interaction curve is in the linear range, about the zero axial force axis. Cases where the axial compressive forces are near or above the balance point are not considered.

The axial rigidity is affected by cracking depth and any inelastic conditions of the steel and concrete. With an aim to simplifying the problem, it is assumed that the axial rigidity is only related to the curvature and axial strain of the section. Therefore the bending moment and axial force of a section are correlated to each other. A procedure to calculate the instantaneous inelastic flexural and axial rigidities of a section, in which the effect of axial force on the moment-curvature curve and the effect of curvature on the axial force-axial strain curve are taken into account, is developed in this study.

The moment is assumed to be a function of curvature and axial force, while the axial force is a function of curvature and axial strain.

$$\begin{array}{l} m = M(\phi, n) \\ n = N(\phi, \varepsilon) \end{array}$$
 (3.9)

where

m = bending moment of a sectionn = axial force of a sectionM = bending moment function

- N = axial force function
- ϕ = curvature of a section
- ε = axial strain of a section

The incremental forms of moment m and axial force n can be expressed by differentiating Eq. (3.9).

$$\Delta m = \frac{\partial M}{\partial \phi} \Delta \phi + \frac{\partial M}{\partial n} \Delta n \qquad (3.10)$$

$$\Delta n = \frac{\partial N}{\partial \phi} \Delta \phi + \frac{\partial N}{\partial \varepsilon} \Delta \varepsilon$$
 (3.11)

where

 Δm = increment of bending moment Δn = increment of axial force $\Delta \phi$ = increment of curvature

 $\Delta \epsilon$ = increment of axial strain

After substituting Eq. (3.11) for Δn in Eq. (3.10), the following equations can be derived in a matrix form:

$$\begin{cases} \Delta m \\ \Delta n \end{cases} = \begin{bmatrix} \frac{\partial M}{\partial \phi} + \frac{\partial M}{\partial n} \frac{\partial N}{\partial \phi} & \frac{\partial M}{\partial n} \frac{\partial N}{\partial \varepsilon} \\ \frac{\partial N}{\partial \phi} & \frac{\partial N}{\partial \varepsilon} \end{bmatrix} \begin{cases} \Delta \phi \\ \Delta \varepsilon \end{cases}$$
(3.12)

The stiffness matrix as given above is not symmetric because of the assumption of Eq. (3.9). In order to save computing time and to simplify the construction of the structural stiffness matrix, it is desirable to reestablish symmetry in the stiffness matrix. To eliminate this lack of symmetry in the stiffness matrix, Eq. (3.12) is rewritten by taking an inverse of Eq. (3.12). Then the inverse is used to express $\frac{\Delta m}{\Delta \varphi}$ by $\frac{\partial M}{\partial \varphi}$ and
a modification factor and $\frac{\Delta n}{\Delta \varepsilon}$ by $\frac{\partial N}{\partial \varepsilon}$ and a modification factor as follows:

$$\begin{cases} \Delta m \\ \Delta n \end{cases} = \begin{bmatrix} \frac{\partial M}{\partial \phi} \left(\frac{1}{1 - \frac{\partial M}{\partial n} \Delta n} \right) & 0 \\ 0 & \frac{\partial N}{\partial \varepsilon} \left(\frac{1}{1 - \left(\frac{\partial N}{\partial \phi} \middle/ \frac{\partial M}{\partial \phi} \right) \left(\frac{\Delta m}{\Delta n} - \frac{\partial M}{\partial n} \right)} \right) \end{bmatrix} \begin{cases} \Delta \phi \\ \Delta \varepsilon \end{cases}$$
(3.13)

It is assumed that the ratio of the increment of axial force over that of moment $\frac{\Delta n}{\Delta m}$ does not change markedly during the loading process. Therefore the previous step value of $\frac{\Delta n}{\Delta m}$ is used for the matrix terms in Eq. (3.13) to avoid the necessity of an iteration process.

The value of $\frac{\partial M}{\partial \phi}$ can be derived from the idealized moment-curvature hysteresis loop for the corresponding axial force acting on the section. The value of $\frac{\partial N}{\partial \varepsilon}$ can be calculated by referring to the idealized axial force-axial strain curve for a given curvature. The detailed procedure for evaluating $\frac{\partial M}{\partial \phi}$, $\frac{\partial N}{\partial \varepsilon}$, $\frac{\partial N}{\partial \phi}$ and $\frac{\partial M}{\partial n}$ in the computer program is schematically explained in Appendix A.

The current effective flexural rigidity EI_i and current effective axial rigidity EA_i are considered as

$$EI_{i} = \frac{\partial M}{\partial \phi} \left(\frac{1}{1 - \frac{\partial M}{\partial n} \frac{\partial n}{\partial m}} \right)$$
(3.14)

$$\mathsf{EA}_{i} = \frac{\partial \mathsf{N}}{\partial \varepsilon} \left(\frac{1}{1 - \left(\frac{\partial \mathsf{N}}{\partial \phi} \middle/ \frac{\partial \mathsf{M}}{\partial \phi}\right) \left(\frac{\Delta \mathsf{m}}{\Delta \mathsf{n}} - \frac{\partial \mathsf{M}}{\partial \mathsf{n}}\right)} \right)$$
(3.15)

in which $\frac{\partial M}{\partial \phi}$ and $\frac{\partial N}{\partial \varepsilon}$ are considered as pseudo-rigidities. The current effective flexural rigidity represents the slope of the moment-curvature relationship, including the effect of a changing axial force. The pseudo-

flexural rigidity is the slope of the moment-curvature relationship with a constant axial force acting.

The evaluation of the shear deformation of a member in an inelastic range is complicated with the existence of both axial force and moment. In addition, the shear deformation is considered to be of a secondary effect to the entire deformation while the flexural deformation is dominant. Therefore it is considered acceptable to employ the assumption that the inelastic values of shear rigidity reduce in direct proportion to those of flexural rigidity. The equation stating this assumption can be expressed in the form,

$$GA_{i} = \frac{EI_{i}}{EI_{e}} GA_{e}$$
(3.16)

where

GA_i = inelastic shear rigidity
GA_e = elastic shear rigidity
EI_i = inelastic flexural rigidity
EI_e = elastic flexural rigidity

These rigidities of the wall subelements are used for the development of the member stiffness in the analysis.

3.4 <u>Deformational Properties of the Rotational Springs Positioned</u> at the Beam Ends

Rotational springs are placed at the ends of each connecting beam to take care of the rotation due to inelastic flexural action in the beam, bond slippage at the ends of the beam, and shear deformation within the span of the beam. Inelastic flexural action in the connecting beam is assumed to be localized at the ends of the beam since the beam is exposed to antisymmetric moment distribution along its length. There is a natural correspondence between the deformational properties of the rotational springs and the fixed and moment-free end displacement relationship of a cantilever beam, since end rotations of a simply supported member subjected to an antisymmetric moment distribution can be related to the deformations of two cantilevers as discussed by Otani (1972). Therefore the deformational properties of the rotational springs in the inelastic region can be derived by calculating the moment-displacement curve of a cantilever whose span is half the length of the connecting beam span. This assumes the point of contraflexure is fixed at midspan of the connecting beam. To make the procedure applicable to beams with arbitrary length, a cantilever with unit length is considered in the analysis.

(a) Idealized Moment Curvature Relationship

An idealized moment-curvature relationship for the connecting beams is developed to compute the free end displacement of a cantilever beam. The moment-curvature relationship is idealized by three straight lines as shown in Fig. 3.7.

$$\phi = \frac{M}{EI} \qquad M \leq M_{c}$$

$$\phi = \frac{\Phi_{y}}{M_{y}} M \qquad M_{c} \leq M \leq M_{y}$$

$$\phi = \Phi_{y} \left[1 + \frac{1}{EI_{y}} \left(\frac{M}{M_{y}} - 1\right)\right] \qquad M_{y} \leq M$$

$$(3.17)$$

where

EI = initial flexural rigidity

For a given moment, the curvature is calculated by Eq. (3.17).

(b) <u>Rotation due to Inelastic Flexural Action Based on Idealized</u> Moment-Displacement Relationship of a Cantilever Beam

As the bending moment is distributed linearly over the length of the cantilever replacement of the connecting beam with zero moment at the free end and the maximum moment at the fixed end, the curvature distribution can be defined for a given fixed end moment by Eq. (3.17). Displacement at the free end of the cantilever beam is then calculated from the curvature distribution by computing the first moment of the curvature diagram about the free end.

The free end displacement D(M) can be expressed as the function of the fixed end moment M by equations of the form

$$D(M) = \frac{L^{2}}{3} \frac{M}{EI} \qquad M \leq M_{c}$$

$$D(M) = \frac{L^{2}}{3} [(1-\alpha^{3}) \phi_{y} \frac{M}{M_{y}} + \alpha^{2} \phi_{c}] \qquad M_{c} \leq M \leq M_{y}$$

$$D(M) = \frac{L^{2}}{6} [(2+\beta)(1-\beta)\{\beta + \frac{1}{EI_{y}}(1-\beta)\} + \beta(1+\beta) - 2\alpha^{3}] \frac{\phi_{y}}{\beta} + \frac{L^{2}}{3} \alpha^{2} \phi_{c} \qquad M_{y} \leq M$$

$$(3.18)$$

where

L = length of the cantilever beam

$$\alpha \stackrel{::}{=} \frac{M_{c}}{M}$$
$$\beta = \frac{M_{y}}{M}$$

With the moment-displacement relationship of a cantilever beam with unit length available, the relationship for a cantilever beam with any length can be derived by simply multiplying the relationship for a unit length cantilever by the square of the length for the desired span since the free end displacement is always proportional to the square of the length of the cantilever.

The idealized moment-displacement curve of a unit length cantilever is calculated by trilinearizing the original curve, that is, connecting the origin, cracking, yielding and ultimate points successively by straight lines. The ultimate moment is defined as the point when the extreme compressive fiber strain reaches 0.004.

The cracking, yielding and ultimate displacements of the unit length cantilever can thus be expressed as:

$$D_{c} = \frac{M_{c}}{3EI}$$

$$D_{y} = \frac{1}{3} [(1 - \alpha_{y}^{3}) \phi_{y} + \alpha_{y}^{2} \phi_{c}]$$

$$D_{u} = \frac{1}{6} [(2 - \beta_{u})(1 - \beta_{u})\{\beta_{u} + \frac{1}{EI_{y}}(1 - \beta_{u})\}$$

$$+ \beta_{u}(1 + \beta_{u}) - 2\alpha_{u}^{3}]\frac{\phi_{y}}{\beta_{u}} + \frac{1}{3} \alpha_{u}^{2} \phi_{c}$$

$$(3.19)$$

where

 $D_{c} = \text{cracking displacement of the unit length cantilever}$ $D_{y} = \text{yielding displacement of the unit length cantilever}$ $D_{u} = \text{ultimate displacement of the unit length cantilever}$ $\alpha_{y} = \frac{M_{c}}{M_{y}}$ $\alpha_{u} = \frac{M_{c}}{M_{u}}$

Slopes in the three stages of the idealized moment-displacement relationship are defined as follows:

$$SD(M) = \frac{M_{c}}{D_{c}} \qquad M \leq M_{c}$$

$$SD(M) = \frac{M_{y} - M_{c}}{D_{y} - D_{c}} \qquad M_{c} \leq M \leq M_{y}$$

$$SD(M) = \frac{M_{u} - M_{y}}{D_{u} - D_{y}} \qquad M_{y} \leq M$$

$$(3.20)$$

where

 $\beta_u = \frac{M_y}{M_u}$

SD(M) = instantaneous stiffness of the cantilever beam of unit length

The incremental rotation of the rotational spring due to inelastic flexural action can be expressed approximately by the instantaneous stiffness SD(M) since inelastic flexural action is assumed to be localized at the beam end. Accordingly,

$$\Delta \theta = \frac{L}{2SD(M)} \Delta M \qquad (3.21)$$

where

 $\Delta \theta$ = increment of rotation ΔM = increment of moment L = length of beam

Equation (3.21) is used as a part of the instantaneous moment-rotation relationship of the rotational springs in the analysis.

(c) Rotation due to Inelastic Shear Deformation

In addition to the flexural deformation of the connecting beams, rotation due to shear deformation of the beams is also taken into account in this study. The ratio of the shear displacement to the total displacement of a cantilever beam is considered as a modifying factor to be applied to the instantaneous stiffness SD(M) which originally included only the inelastic flexural deformation.

Based on the reasoning discussed in Sec. 3.3, it is assumed that the inelastic shear rigidity reduces in direct proportion to the inelastic flexural rigidity.

The incremental free end displacement due to both shear and flexural deformations in a cantilever beam that result from a given incremental triangular moment distribution can be expressed as follows:

$$\Delta V = \left(\frac{L}{GA_{i}} + \frac{L^{3}}{3EI_{i}}\right) \frac{\Delta M}{L}$$
(3.22)

where

 ΔV = increment of the free end displacement

L = length of the cantilever beam

 ΔM = increment of the fixed end moment

The ratio of the incremental displacement based solely on flexural rigidity to that based on both flexural and shear rigidities is considered to remain constant during any stage of inelastic action. The inelastic flexural rigidity EI_i is assumed to be uniformly distributed along the length of the cantilever beam, although the actual inelastic flexural rigidity is likely to develop near the fixed end of the cantilever beam. Therefore the instantaneous stiffness of the cantilever can be modified

for the case which includes shear deformations as well as flexural deformations by simply multiplying SD(M) by the ratio of the flexural displacement to the sum of flexural and shear displacements. The displacement ratio is

$$\frac{\Delta V_{f}}{\Delta V} = \frac{1}{\frac{3EI_{i}}{GA_{i}L^{2}} + 1} = \frac{1}{\frac{3EI_{e}}{GA_{e}L^{2}} + 1}$$
(3.23)

since

$$\frac{3EI_{i}}{GA_{i}} = \frac{3EI_{e}}{GA_{e}}$$

where

EI_e = elastic flexural rigidity
GA_e = elastic shear rigidity

Thus the stiffness can be expressed as:

$$ST(M) = SD(M) \frac{\Delta V_{f}}{\Delta V}$$
 (3.24)

where

 ΔV_{f} = incremental displacement due only to flexural rigidity

SD(M) = instantaneous stiffness based on flexural rigidity

ST(M) = instantaneous stiffness based on flexural and shear rigidity

For the case when the rotation due to shear deformation is considered in the analyses, the instantaneous stiffness ST(M) is used instead of SD(M) in Eq. (3.21).

(d) Rotation due to Bond Slippage at the Ends of the Beams

Rotation due to the slip of the tensile reinforcement of the beam along its embedded length is considered as an additional flexibility factor for the rotational spring at the ends of a beam.

Bond stress is assumed to be constant along the embedded length of the reinforcement. Therefore the tensile force of the reinforcement is transmitted into the concrete in such a way that the steel stress decreases linearly with distance in from the wallface.

It is assumed that the reinforcement embedment length is sufficient to provide the maximum tensile stress that occurs in the response calculations. The development length L can be computed from the equilibrium of forces as follows:

$$L = \frac{A_s f_s}{\pi D u}$$
(3.25)

where

A_s = cross sectional area of the tensile reinforcement
f_s = stress of the reinforcement at the face of wall
D = diameter of a reinforcing bar
u = average bond stress

The strain hardening portion for the reinforcement is idealized by a line which connects the yield point and the point at the maximum strength. The elongation of the reinforcement over the development length is calculated by integrating the strain over the length.

If the stress of the reinforcement exceeds the yield stress f_y , the development length is divided into two parts, as shown in Fig. 3.8. This is done to accommodate the change in the reinforcement's axial rigidity. Therefore the integration of the strain must be performed separately over the two parts of the development length, that is, from the point of zero stress to that of the yield stress and from the point of the yield stress

to that of the maximum stress.

The elongations of the reinforcement are calculated as:

$$\Delta L = \frac{Lf_s}{2E_s} \qquad f_s \leq f_y \qquad (3.26)$$

$$\Delta L = \frac{f_y^2 L}{2f_s E_s} + (1 - \frac{f_y}{f_s})(\frac{f_y}{E_s} + \frac{f_s - f_y}{2E_y}) L \qquad f_y \le f_s \qquad (3.27)$$

where

 ΔL = elongation of the reinforcement

- E_c = Young's modulus of the reinforcement
- E_y = inelastic modulus of the reinforcement
 after yielding is developed

$$f_y$$
 = yielding stress of the reinforcement

The elongation can be rewritten by substituting Eq. (3.25) for L in Eqs. (3.26) and (3.27), and by replacing A_s by $\frac{\pi}{4} D^2$. The result is

$$\Delta L = \frac{1}{8} \frac{D}{E_s u} f_s^2 \qquad f_s \le f_y \qquad (3.28)$$

$$\Delta L = \frac{D}{4u} \left[\frac{f_y}{E_s} \left(f_s - \frac{f_y}{2} \right) + \frac{\left(f_s - f_y \right)^L}{2E_y} \right] \qquad f_y \le f_s \qquad (3.29)$$

It is assumed that the compressive reinforcement does not slip and the concrete in the joint is rigid. Therefore the rotation due to bond slippage can be expressed as follows:

$$R = \frac{\Delta L}{d - d'}$$
(3.30)

where

R = rotation due to the slip at the ends of a beam

d = depth of the tensile reinforcement

d' = depth of the compressive reinforcement

In order to have a rotation-moment relationship rather than the rotation-stress one, the relation between bending moment and stress is assumed in the form

$$f_{s} = \frac{f_{y}}{M_{y}} M$$
(3.31)

where

M = bending moment at the end of a beam

 M_v = yielding moment at the end of a beam

By using Eq. (3.28) through Eq. (3.31), the rotation-moment relationship can be expressed as follows:

$$R = \frac{1}{8} \frac{D}{E_{s} u} \left(\frac{f_{y}}{M_{y}}\right)^{2} M^{2} \frac{1}{d - d^{*}} \qquad M \leq M_{y} \qquad (3.32)$$

$$R = \frac{Df_{y}^{2}}{4u} \left[\frac{1}{E_{s}} \left(\frac{M}{M_{y}} - \frac{1}{2}\right) + \frac{1}{2E_{y}} \left(\frac{M}{M_{y}} - 1\right)^{2}\right] \frac{1}{d - d^{*}} \qquad M_{y} \leq M \qquad (3.33)$$

The idealized form of the rotation-moment relationship is obtained by trilinearizing the original curve, that is, connecting the origin, cracking, yielding and ultimate moments successively for simplification of the problem.

These break points for the trilinearization can be expressed as follows:

$$R_{c} = \frac{D}{8E_{s} u} \left(\frac{f_{y}}{M_{y}}\right)^{2} M_{c}^{2} \frac{1}{d - d'}$$

$$R_{y} = \frac{D}{8E_{s} u} f_{y}^{2} \frac{1}{d - d'}$$

$$R_{u} = \frac{Df_{y}^{2}}{4u} \left[\frac{1}{E_{s}} \left(\frac{M_{u}}{M_{y}} - \frac{1}{2}\right) + \frac{1}{2E_{y}} \left(\frac{M_{u}}{M_{y}} - 1\right)^{2}\right] \frac{1}{d - d'}$$

$$(3.34)$$

where

 R_c = rotation at which the cracking moment is developed R_y = rotation at which the yielding moment is developed R_u = rotation at which the ultimate moment is developed

The flexibilities in the three stages of the idealized rotationmoment relationship are defined as follows:

$$f(M) = \frac{R_{c}}{M_{c}} \qquad M \leq M_{c}$$

$$f(M) = \frac{R_{y} - R_{c}}{M_{y} - M_{c}} \qquad M_{c} \leq M \leq M_{y}$$

$$f(M) = \frac{R_{u} - R_{y}}{M_{u} - M_{y}} \qquad M_{y} \leq M$$

$$(3.35)$$

where

f(M) = flexibility resulting from the bond slippage of

tensile reinforcement of a beam

The incremental rotation of the rotational spring due to bond slippage can be expressed by the flexibility f(M), as follows:

$$\Delta \theta = f(M) \Delta M \tag{3.36}$$

Equation (3.36) is used as a part of the instantaneous moment-rotation relationship of a rotational spring in the analysis.

The calculated moment-rotation curve of a rotational spring including flexural and shear actions over the beam length and bond slip in the joint core is compared with the test result by Abrams (1976) in Fig. 3.9.

CHAPTER 4

ANALYTICAL PROCEDURE

4.1 Introductory Remarks

This chapter describes a method of analysis for reinforced concrete coupled shear wall structures subjected to static loads and dynamic base excitations. The analytical procedure is developed to study the behavior of a structural system as well as that of its constituent members even when that system is loaded into a highly inelastic range.

The constituent member stiffnesses are evaluated based upon the force-deformation relationships of the rotational springs of the beam and the subelements of the wall as described in Chapter 3. The instantaneous structural stiffness matrix is developed by assembling the constituent member stiffnesses and then condensing out all degrees-of-freedom except those for the horizontal story movements. Only those degrees-of-freedom remain in the final equations.

The mass of the structure is considered to be concentrated at each floor level so that the lumped mass concept can be used in the analysis. The damping matrix is evaluated as the sum of a part proportional to the mass matrix and a part proportional to the structural stiffness matrix.

The inelastic behavior of the structure under static loads is evaluated by applying a known set of lateral loads to the structure. These loads are applied in very small increments. The inelastic dynamic response and failure process of the structure under dynamic base motions are calculated by numerically integrating the equations of motion with a step-by-step procedure, Tung and Newmark (1954).

The effect of load history in each constituent element is taken care of by using a set of hysteresis rules. These rules are an adaptation of those presented by Takeda, et al. (1970). A computer program has been developed to apply the analytical procedure to the analysis of coupled wall structures. The program is briefly explained in Appendix B.

4.2 Basic Assumptions

In this section the basic assumptions used in the analysis in order to simplify the solution of the problem are presented.

(1) The analysis is limited to plane frame problems. Out-of-plane action is ignored in the analysis. Three independent displacements are considered at each joint: two mutually perpendicular translations in a plane and one rotation about an axis normal to the plane.

(2) The right-hand screw rule is adopted to describe the global coordinate system as well as the member coordinate system.

(3) Every member in the structure is considered as a massless line member represented by its centroidal axis.

(4) Geometric nonlinearity is ignored in the analysis. Small deformations are assumed in the analysis so that the calculation of inelastic response of the structure can be based on the initial configuration.

(5) The idealized frame is assumed to be fixed at the base of the structure which rests on an infinitely rigid foundation.

(6) The mass of the structure is assumed to be lumped at each story level.

(7) The inelastic deformation of each constituent member is assumed to follow the Takeda's hysteresis rules.

(8) The instantaneous nonlinear characteristics of the structure are assumed to be constant within a time interval or a load step interval.

(9) Shear deformation in a joint core is ignored in the analysis.

(10) Only horizontal base motion is considered as the external dynamic force applied to the structure.

(11) The axial elongation of the connecting beams is ignored so that the two walls move horizontally at the same rate.

(12) $P-\Delta$ effect is ignored in the analysis.

4.3 Stiffness Matrix of a Member

This section describes the ways to develop the stiffness matrix of each constituent member of the structure such as the connecting beams and walls based upon the force-deformation relationships of frame elements mentioned in Chapter 3.

(a) Wall Member

A wall member has axial force, shear force and bending moment as its force components. Vertical displacement, horizontal displacement and rotation are the displacement components at the ends of each wall member. These member forces and displacements, together with their positive directions, are shown in Fig. 4.1.

Each wall member is considered to consist of several subelements so that each subelement can be subjected to a different stage of inelastic action. The stiffness properties of each subelement are assumed to be constant over the length of that element. A wall member that consists of three subelements is adopted here as an example to explain the derivation of a member's stiffness matrix. This represents a small-enough structure to be easily explained by solving an example problem.

It is necessary to consider the wall member as a cantilever beam for the evaluation of the member stiffness matrix. The configuration of the cantilever beam as well as its coordinate system is shown in Fig. 4.2. The flexibility matrix of the cantilever beam can be derived by using the transformation matrix and the flexibility matrix of each element as follows:

$$[f_{AB}] = [T_{CB}]^{T} [f_{AC}][T_{CB}] + [T_{DB}]^{T} [f_{CD}][T_{DB}] + [f_{DB}]$$
(4.1)

where

 $[f_{AB}]$ = flexibility matrix of the cantilever beam $[f_{AC}], [f_{CD}]$ and $[f_{DB}]$ = flexibility matrices of the elements 1, 2 and 3, respectively

 $[T_{CB}]$ and $[T_{DB}]$ = transformation matrices of the elements 1 and 2, respectively

 $[T_{CB}]^{T}$ and $[T_{DB}]^{T}$ = transpose matrices of $[T_{CB}]$ and $[T_{DB}]$, respectively The matrices which appeared in Eq. (4.1) can be expressed as:

$$[f_{AB}] = \begin{bmatrix} \frac{L}{EA_{i}} & 0 & 0 \\ 0 & \frac{L^{3}}{3EI_{i}} + \frac{L}{GA_{i}} & -\frac{L^{2}}{2EI_{i}} \\ 0 & -\frac{L^{2}}{2EI_{i}} & \frac{L}{EI_{i}} \end{bmatrix}$$
(4.2)

$$\begin{bmatrix} f_{AC} \end{bmatrix} = \begin{bmatrix} \frac{k_{1}}{EA_{11}} & 0 & 0 \\ 0 & \frac{k_{1}^{3}}{3EI_{11}} + \frac{k_{1}}{6A_{11}} & -\frac{k_{1}^{2}}{2EI_{11}} \\ 0 & -\frac{k_{1}^{2}}{2EI_{11}} & \frac{k_{1}}{EI_{11}} \end{bmatrix}$$

$$\begin{bmatrix} f_{CD} \end{bmatrix} = \begin{bmatrix} \frac{k_{2}}{EA_{12}} & 0 & 0 \\ 0 & \frac{k_{2}^{3}}{3EI_{12}} + \frac{k_{2}}{6A_{12}} & -\frac{k_{2}^{2}}{2EI_{12}} \\ 0 & -\frac{k_{2}^{2}}{2EI_{12}} & \frac{k_{2}}{EI_{12}} \end{bmatrix}$$

$$\begin{bmatrix} f_{DB} \end{bmatrix} = \begin{bmatrix} \frac{k_{3}}{EA_{13}} & 0 & 0 \\ 0 & \frac{k_{3}^{3}}{3EI_{13}} + \frac{k_{3}}{6A_{13}} & -\frac{k_{3}^{2}}{2EI_{13}} \\ 0 & -\frac{k_{2}^{2}}{2EI_{13}} & \frac{k_{3}}{2EI_{13}} \\ 0 & -\frac{k_{2}^{2}}{2EI_{13}} & \frac{k_{3}}{EI_{13}} \end{bmatrix}$$

$$\begin{bmatrix} T_{CB} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -L+k_{1} & 1 \\ \end{bmatrix}$$

$$\begin{bmatrix} T_{DB} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -L+k_{1} & 1 \\ \end{bmatrix}$$

$$(4.4)$$

.

where

L = length of the cantilever beam
EA _i = instantaneous equivalent axial rigidity
of the cantilever beam
GA _i = instantaneous equivalent shear rigidity
of the cantilever beam
EI _i = instantaneous equivalent flexural rigidity
of the cantilever beam
EA _{il} , EA _{i2} and EA _{i3} = instantaneous axial rigidities of
elements 1, 2 and 3, respectively
GA _{il} , GA _{i2} and GA _{i3} = instantaneous shear rigidities of
elements 1, 2 and 3, respectively
EI _{il} , EI _{i2} and EI _{i3} = instantaneous flexural rigidities of
elements 1, 2 and 3, respectively
ℓ_1 , ℓ_2 and ℓ_3 = lengths of elements 1, 2 and 3, respectively

These element rigidities EI_{in} , EA_{in} and GA_{in} (n = element number) are calculated from Eqs. (3.14), (3.15) and (3.16) of Section 3.3, respectively. The stiffness matrix $[K_{AB}]$ of the cantilever beam is calculated by computing the inverse of the flexibility matrix $[f_{AB}]$.

The stiffness matrix of a wall member can be developed by using a conventional matrix formula as follows:

$$[K_w] = \begin{bmatrix} T_{AB} K_{AB} T_{AB}^T - T_{AB} K_{AB} \\ - K_{AB} T_{AB}^T K_{AB} \end{bmatrix}$$
(4.5)

where

 $[K_w]$ = stiffness matrix of the wall member of size, six by six $[K_{AB}]$ = stiffness matrix of the cantilever beam of size,

three by three

 $[T_{AB}]$ = transformation matrix of the cantilever beam

$$[T_{AB}] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -L & 1 \end{bmatrix}$$

 $[T_{AB}]^{T}$ = transpose matrix of $[T_{AB}]$

The incremental member end forces are related to the incremental member end displacements through the stiffness matrix $[K_w]$ as follows:

where

 ΔN_A and ΔN_B = incremental axial forces at the ends of a wall member ΔP_A and ΔP_B = incremental shear forces at the ends of a wall member ΔM_A and ΔM_B = incremental moments at the ends of a wall member ΔV_A and ΔV_B = incremental vertical displacements at the ends of a wall member

$$\Delta \Theta_A$$
 and $\Delta \Theta_B$ = incremental lateral displacements
at the ends of a wall member
 $\Delta \Theta_A$ and $\Delta \Theta_B$ = incremental rotations at the ends
of a wall member

These member end displacements and forces are also considered as the joint displacements and the contribution to the joint equilibrium from the wall members, respectively, since the global coordinate system has also been adopted as the local coordinates. The stiffness matrix $[K_w]$ of a wall member is used as that member's contribution to the formulation of the total structural stiffness matrix.

(b) Beam Member

A beam member has shear force and bending moment as its force components, with vertical displacement and rotation as its displacement components. These are specified at the member ends in the normal manner.

The connecting beam is considered as an individual beam connected to each wall through a rigid link and a rotational spring. The rotational spring takes care of the beam end rotation due to bond slip in the joint core as well as the inelastic flexural and shear action over the beam length. The linear flexible beam element spans between the rotational springs. The configuration of the connecting beam and the beam end forces and displacements are shown in Fig. 4.3.

The flexibility matrix for a simply supported connecting beam system, excluding for the time being the rigid links to the wall centerlines, can be calculated by simply adding the flexibilities of the rotational springs to those due to flexural actions in the flexible element. The flexibility matrix is therefore expressed as:

$$\begin{bmatrix} f_{CC} & f_{CD} \\ f_{DC} & f_{DD} \end{bmatrix} = \begin{bmatrix} \frac{L}{6EI} - \frac{L}{6EI} \\ -\frac{L}{6EI} & \frac{L}{6EI} \end{bmatrix} + \begin{bmatrix} \frac{L}{2ST(M_{C})} + f(M_{C}) & 0 \\ 0 & \frac{L}{2ST(M_{D})} + f(M_{D}) \end{bmatrix}$$
(4.7)

where

L = length of the flexible element EI = elastic flexural rigidity of the flexible element

 $\frac{L}{2ST(M_{C})}$ and $\frac{L}{2ST(M_{D})}$ = rotational flexibilities due to the inelastic flexural and shear actions over the beam length, defined in Eqs. (3.21) and (3.24)

 $f(M_{C})$ and $f(M_{D})$ = rotational flexibilities due to the bond slip in the joint core, defined in Eq. (3.36)

 $\rm M_{C}$ and $\rm M_{D}$ = end moments of the flexible element

The first matrix on the right-hand side of Eq. (4.7) is a slightly modified version of the normal flexibility matrix of a simple beam. The reason the first matrix is not in the normally recognized form is that part of the elastic flexibility coefficients of the diagonal elements have been assigned to the element $\frac{L}{2ST(M)}$ in the second matrix. This has been done for computational ease. In the second matrix the flexibility constants $\frac{L}{2ST(M)}$ and f(M) are functions of the existing moment level and the history of the rotational spring.

The incremental end rotations of the combined spring-flexible element system are related to its incremental end moments through the combined flexibility matrix as

$$\begin{cases} \Delta \Theta_{C} \\ \Delta \Theta_{D} \end{cases} = \begin{bmatrix} f_{CC} & f_{CD} \\ f_{DC} & f_{DD} \end{bmatrix} \begin{cases} \Delta M_{C} \\ \Delta M_{D} \end{cases}$$
(4.8)

where

$$\Delta \Theta_{C}$$
 and $\Delta \Theta_{D}$ = incremental end rotations of the
combined spring-flexible element
 ΔM_{C} and ΔM_{D} = incremental end moments of the
flexible element

It should be noted that the interaction effect of the rotations between the ends C and D exemplified by the off diagonal terms depends solely on the elasticity of the flexible element.

Equation (4.8) is converted to the stiffness form by inverting the rotational flexibility matrix as follows:

$$\begin{cases} \Delta M_{\rm C} \\ \Delta M_{\rm D} \end{cases} = \begin{bmatrix} \kappa_{\rm CC} & \kappa_{\rm CD} \\ \kappa_{\rm DC} & \kappa_{\rm DD} \end{bmatrix} \begin{cases} \Delta \theta_{\rm C} \\ \Delta \theta_{\rm D} \end{cases}$$
(4.9)

Incremental moments ΔM_A and ΔM_B at the ends of the rigid links are related to the incremental moments ΔM_C and ΔM_D at the ends of the flexible element through a transformation matrix as follows:

$$\begin{cases} \Delta M_{A} \\ \Delta M_{B} \end{cases} = \begin{bmatrix} 1+\lambda & \lambda \\ \lambda & 1+\lambda \end{bmatrix} \begin{cases} \Delta M_{C} \\ \Delta M_{D} \end{cases}$$
(4.10)

where

 λ = ratio of the length of a rigid link to that of a flexible element

The distribution of moment over the length of a connecting beam is shown in Fig. 4.4. Incremental rotations $\Delta \theta_{C}$ and $\Delta \theta_{D}$ at the ends of the interior flexible element are related to incremental rotations $\Delta \theta_{A}^{\prime}$ and $\Delta \theta_{B}^{\prime}$ at the rigid link ends of a simply supported beam in the same way as Eq. (4.10).

$$\begin{cases} \Delta \Theta_{C} \\ \Delta \Theta_{D} \end{cases} = \begin{bmatrix} 1+\lambda & \lambda \\ \lambda & 1+\lambda \end{bmatrix} \begin{cases} \Delta \Theta_{A} \\ \Delta \Theta_{B} \end{cases}$$
(4.11)

The instantaneous moment-rotation relationship of a simply supported beam made up of the rigid links, rotational springs and flexible element can be expressed by combining Eqs. (4.9), (4.10) and (4.11) as follows:

$$\begin{cases} \Delta M_{A} \\ \Delta M_{B} \end{cases} = \begin{bmatrix} 1+\lambda & \lambda \\ \lambda & 1+\lambda \end{bmatrix} \begin{bmatrix} K_{CC} & K_{CD} \\ K_{DC} & K_{DD} \end{bmatrix} \begin{bmatrix} 1+\lambda & \lambda \\ \lambda & 1+\lambda \end{bmatrix} \begin{cases} \Delta \theta_{A}^{\dagger} \\ \Delta \theta_{B}^{\dagger} \end{cases}$$
(4.12)

It should be noted that no shear forces nor vertical displacements at the ends of the beam member are involved in Eq. (4.12). In order to include the member end shear forces and vertical displacements in the final equation, the incremental end rotations $\Delta \theta_A^{\dagger}$ and $\Delta \theta_B^{\dagger}$ of a simply supported beam member should be expressed in terms of incremental end rotations $\Delta \theta_A$ and $\Delta \theta_B$ and incremental end vertical displacements ΔV_A and ΔV_B of the beam member using the equation

$$\begin{cases} \Delta \Theta_{A}^{\prime} \\ \Delta \Theta_{B}^{\prime} \end{cases} = \begin{bmatrix} \frac{1}{L(1+2\lambda)} & 1 & \frac{-1}{L(1+2\lambda)} & 0 \\ \frac{1}{L(1+2\lambda)} & 0 & \frac{-1}{L(1+2\lambda)} & 1 \end{bmatrix} \begin{cases} \Delta V_{A} \\ \Delta \Theta_{A} \\ \Delta V_{B} \\ \Delta \Theta_{B} \end{cases}$$
(4.13)

The deformed configuration of the connecting beam from which these relationships are readily observed is shown in Fig. 4.5.

Similarly, the incremental member end shear forces ΔN_A and ΔN_B can be expressed by the incremental member end moments ΔM_A and ΔM_B in the form

$$\begin{pmatrix} \Delta N_{A} \\ \Delta M_{A} \\ \Delta M_{B} \\ \Delta M_{B} \end{pmatrix} = \begin{bmatrix} \frac{1}{L(1+2\lambda)} & \frac{1}{L(1+2\lambda)} \\ 1 & 0 \\ \frac{-1}{L(1+2\lambda)} & \frac{-1}{L(1+2\lambda)} \\ 0 & 1 \end{bmatrix} \begin{pmatrix} \Delta M_{A} \\ \Delta M_{B} \end{pmatrix}$$
(4.14)

The final force-displacement relation of a connecting beam is obtained by combining Eqs. (4.12), (4.13) and (4.14) into the following form

$$\begin{cases} \Delta N_{A} \\ \Delta M_{A} \\ \Delta N_{B} \\ \Delta M_{B} \\ \Delta M_{B} \end{pmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ L(1+2\lambda) & L(1+2\lambda) \\ 1 & 0 \\ -1 & -1 \\ L(1+2\lambda) & L(1+2\lambda) \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1+\lambda & \lambda \\ \lambda & 1+\lambda \end{bmatrix} \begin{bmatrix} K_{CC} & K_{CD} \\ K_{DC} & K_{DD} \end{bmatrix}$$

$$\times \begin{bmatrix} 1+\lambda & \lambda \\ \lambda & 1+\lambda \end{bmatrix} \begin{bmatrix} 1+\lambda & \lambda \\ L(1+2\lambda) & 1 & -\frac{1}{L(1+2\lambda)} & 0 \\ \frac{1}{L(1+2\lambda)} & 0 & -\frac{1}{L(1+2\lambda)} & 1 \end{bmatrix} \begin{pmatrix} \Delta V_{A} \\ \Delta \theta_{A} \\ \Delta V_{B} \\ \Delta \theta_{B} \end{pmatrix}$$
(4.15)

where

 $\Delta N_A \text{ and } \Delta N_B = \text{ incremental shear forces at the ends} \\ \text{of a connecting beam} \\ \Delta M_A \text{ and } \Delta M_B = \text{ incremental moments at the ends of a connecting beam} \\ \Delta V_A \text{ and } \Delta V_B = \text{ incremental vertical displacements at the ends of a connecting beam} \\ \Delta \theta_A \text{ and } \Delta \theta_B = \text{ incremental rotations at the ends of a connecting beam} \\ \Delta \theta_A \text{ and } \Delta \theta_B = \text{ incremental rotations at the ends of a connecting beam} \\ \Delta \theta_A \text{ and } \Delta \theta_B = \text{ incremental rotations at the ends of a connecting beam} \\ \Delta \theta_A \text{ and } \Delta \theta_B = \text{ incremental rotations at the ends of a connecting beam} \\ \Delta \theta_A \text{ and } \Delta \theta_B = \text{ incremental rotations at the ends of a connecting beam} \\ \Delta \theta_A \text{ and } \Delta \theta_B = \text{ incremental rotations at the ends of a connecting beam} \\ \Delta \theta_A \text{ and } \Delta \theta_B = \text{ incremental rotations at the ends of a connecting beam} \\ \Delta \theta_A \text{ and } \Delta \theta_B = \text{ incremental rotations at the ends of a connecting beam} \\ \Delta \theta_A \text{ and } \Delta \theta_B = \text{ incremental rotations at the ends of a connecting beam} \\ \Delta \theta_A \text{ and } \Delta \theta_B = \text{ incremental rotations at the ends of a connecting beam} \\ \Delta \theta_A \text{ and } \Delta \theta_B = \text{ incremental rotations at the ends of a connecting beam} \\ \Delta \theta_A \text{ and } \Delta \theta_B = \text{ incremental rotations at the ends of a connecting beam} \\ \Delta \theta_A \text{ and } \Delta \theta_B = \text{ incremental rotations at the ends of a connecting beam} \\ \Delta \theta_A \text{ and } \Delta \theta_B \text{ and } \Delta \theta_B$

With the global coordinate system also adopted as the local coordinate system for the connecting beam, these member end displacements and forces are also considered as the joint displacements and the contribution to the joint forces from the connecting beam, respectively. The stiffness matrix in Eq. (4.15) is used as the beam contribution to the formulation of the structural stiffness matrix.

4.4 Structural Stiffness Matrix

The instantaneous structural stiffness matrix is developed by combining all the instantaneous stiffness matrices of the wall subelements and the beams then condensing out a number of the degrees-of-freedom so that only horizontal story movements appear in the final form of the equations.

The formulation of the full-size structural stiffness matrix is accomplished by adding force contributions from all the members in a structure at each story and joint. The force-displacement relation of a structure is expressible in the form

$$\begin{cases} \Delta P \\ \Delta N \\ \Delta M \end{cases} = \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix} \begin{cases} \Delta U \\ \Delta V \\ \Delta \theta \end{cases}$$
(4.16)

where

 K_{11} = submatrix of size, I by I

 K_{12} = submatrix of size, I by 2J

 K_{21} = submatrix of size, 2J by I

 K_{22} = submatrix of size, 2J by 2J

I = number of stories

J = number of joints

 ΔP = incremental story lateral force vector

 ΔN = incremental joint vertical force vector

 ΔM = incremental joint moment vector

 ΔU = incremental story lateral displacement vector

- ΔV = incremental joint vertical displacement vector
- $\Delta \theta$ = incremental joint rotation vector

The external vertical forces and moments at the joints in the structure are assumed to be zero, since only lateral loads are considered in this analysis. Thus static condensation is used. First Eq. (4.16) can be rearranged as follows:

$$\{\Delta P\} = [K_{11}]\{\Delta U\} + [K_{12}] \left\{ \begin{array}{c} \Delta V \\ \Delta \Theta \end{array} \right\}$$

$$\{ 0 \} = [K_{21}]\{\Delta U\} + [K_{22}] \left\{ \begin{array}{c} \Delta V \\ \Delta \Theta \end{array} \right\}$$

$$(4.17)$$

On solving Eq. (4.18) for the vertical displacement ΔV and rotation vector $\Delta \theta$, the solution can be written as

$$\begin{pmatrix} \Delta V \\ \Delta \theta \end{pmatrix} = - [K_{22}]^{-1} [K_{21}] \{ \Delta U \}$$
 (4.19)

By substituting Eq. (4.19) for the vertical displacement and rotation vector in Eq. (4.17), the incremental lateral displacement-force relationship of the structure can be expressed in the form

$$\{\Delta P\} = \left[[K_{11}] - [K_{12}] [K_{22}]^{-1} [K_{21}] \right] \{\Delta U\}$$
(4.20)

The instantaneous structural stiffness matrix is defined as

$$[K] = [K_{11}] - [K_{12}][K_{22}]^{-1}[K_{21}]$$
(4.21)

where

[K] = instantaneous structural stiffness matrix of size, number of stories by number of stories

Having computed the incremental lateral displacements, the incremental vertical displacements and rotations of the joints can be calculated from Eq. (4.19). Incremental member forces can then be computed from the incremental member end forces versus displacement relationships such as Eqs. (4.6) and (4.15). Finally, current values of the displacements and member forces are evaluated by adding the computed incremental values to the accumulated values from the previous step.

4.5 Static Analysis

An application of the analytical procedure just described to a static load case is discussed in this section. The static load applied to the structure can be either a monotonically increasing load or a cyclic load. However, as noted earlier, only lateral loads are considered as the external loads on the structural system in this analysis. The appropriate lateral loads are applied to each story level of the structure. These loads are applied in small load increments, increasing up to the maximum load. It is assumed that the load distribution shape over the height of the structure does not change during the loading process although the magnitudes of the loads are monotonically increasing or decreasing.

Equation (4.20) of the incremental lateral displacement-force relationships is solved for the lateral story displacements under a set of lateral loads by a step-by-step procedure. The load increment is chosen to be small enough to avoid any significant calculation error due to overshooting in the hysteresis loops.

The structural stiffness is assumed to be constant during the load increment. Story and joint displacements and member forces are calculated at the end of each load increment. If a member force exceeds its limiting value, the member stiffness is modified at the beginning of the next load increment in accordance with the hysteresis rules. The failure mechanism of the structure and the inelastic structural stiffness properties are studied in the analysis of the structure under static loads.

4.6 Dynamic Analysis

The equations of motion of the structure are expressed by the equilibrium conditions on the inertia forces, damping forces, and resisting forces at each story. To calculate the inertia forces, damping forces, and resisting forces at each story, the mass matrix, damping matrix, and

instantaneous structural stiffness matrix must be evaluated respectively. The instantaneous structural stiffness is defined in Eq. (4.21).

(a) Mass Matrix

The lumped mass concept in which all the mass of a story is concentrated at its floor level is assumed in the analysis. Inertia moments and vertical inertia forces at joints are ignored in the analysis. Only lateral inertia forces at the story levels are considered in the calculations of the dynamic response due to base excitations. A consistent mass matrix is therefore considered unnecessary and a diagonal mass matrix in which off-diagonal terms are zero is developed in the form

$$\begin{bmatrix} M \end{bmatrix} = \begin{bmatrix} m_1 & 0 \\ m_2 & \\ 0 & m_I \end{bmatrix}$$
(4.22)

where

[M] = mass matrix of size, number of stories by number of stories $m_1, m_2...m_1$ = lumped mass at each story level

I = number of stories

(b) Damping Matrix

A viscous type damping is adopted in this analysis because of its mathematical simplicity. This simplification is rationalized on the grounds that the damping force phenomenon is not fully understood with present knowledge. With this assumption the damping forces are considered to be proportional to the relative velocities which are measured at each floor relative to the base of the structure. The damping matrix is made up of a part which is proportional to the mass matrix and a part which is proportional to the instantaneous structural stiffness matrix. The matrix can therefore be expressed as

$$[C] = c_1[M] + c_2(K]$$
(4.23)

where

[C] = damping matrix of size, number of stories by number of stories

 c_1 and c_2 = constants which are determined from given damping factors

The damping matrix [C] can be diagonalized by using the normal mode shape vectors, because the damping matrix is a linear combination of the mass and stiffness matrices and the mode shape vectors are orthogonal with respect to the mass matrix as well as the stiffness matrix. By considering this property of the assumed damping matrix, modal damping factors can be expressed in terms of the constants c_1 and c_2 , and modal circular frequencies in the form

$$\beta_{i} = \frac{1}{2} \left(\frac{c_{1}}{\omega_{i}} + c_{2} \omega_{i} \right)$$
 (4.24)

where

 β_i = damping factor of the ith mode

 ω_i = circular frequency of the ith mode

The derivation of Eq. (4.24) can be found in many textbooks on structural dynamics, Clough and Penzien (1975).

The constants c_1 and c_2 in Eq. (4.23) can be determined by introducing the first and second mode damping factors β_1 and β_2 as well as the first and second mode undamped circular frequencies ω_1 and ω_2 into Eq. (4.24) as

$$\beta_{1} = \frac{1}{2} \left(\frac{c_{1}}{\omega_{1}} + c_{2} \omega_{1} \right)$$

$$\beta_{2} = \frac{1}{2} \left(\frac{c_{1}}{\omega_{2}} + c_{2} \omega_{2} \right)$$
(4.25)

By solving Eqs. (4.25) for c_1 and c_2 , the constants c_1 and c_2 are expressed by the first and second mode damping factors and circular frequencies as follows:

$$c_{1} = \frac{2\omega_{1}\omega_{2}(\beta_{2}\omega_{1} - \beta_{1}\omega_{2})}{\omega_{1}^{2} - \omega_{2}^{2}}$$

$$c_{2} = \frac{2(\beta_{1}\omega_{1} - \beta_{2}\omega_{2})}{\omega_{1}^{2} - \omega_{2}^{2}}$$
(4.26)

The first and second mode damping factors β_1 and β_2 are selected based on engineering judgment prior to the calculation of c_1 and c_2 . Once c_1 and c_2 have been determined, higher mode damping factors are automatically assigned by Eq. (4.24).

If the damping matrix is considered to be proportional to only the stiffness matrix, the constant c_2 is calculated by the first of Eqs. (4.25) assuming the constant c_1 to be zero. Thus

$$c_2 = \frac{2\beta_1}{\omega_1} \tag{4.27}$$

Similarly, the constant c_1 is calculated by the following expression for the case where the damping matrix is assumed to be proportional to only the mass matrix.

$$c_1 = 2\beta_1 \omega_1 \tag{4.28}$$

Larger damping factors are automatically assigned to the higher modes for the case where the damping matrix is assumed proportional to just the stiffness matrix. On the other hand, smaller damping factors are automatically assigned to the higher modes for the case with the damping matrix assumed proportional to the mass matrix.

A damping matrix proportional to the stiffness matrix is mainly used in this analysis, since it is effective in reducing the amount of higher frequency components in the structural responses. In this case, the damping matrix is simply expressed in the form

$$[C] = c_2 [K]$$
 (4.29)

The stiffness matrix [K] in Eq. (4.29) can be defined either by the initial stiffness values or by the current instantaneous stiffness values.

If a damping matrix proportional to the initial stiffness matrix is considered in the analysis, the damping matrix would remain unchanged during any inelastic structural response. Naturally this gives overestimated values to the damping matrix. Such overestimations might be acceptable in the analysis, because the damping effect should be expected to become larger when any inelastic action is occurring in the structure.

If the damping matrix proportional to the instantaneous stiffness matrix is considered in the analysis, the damping matrix changes during the response to reflect the current structural stiffness. Therefore the value of c_2 in Eq. (4.29) is likewise changed in the manner described in the following paragraphs in order to keep within reasonable damping factor values.

It is assumed that the first mode component is the dominant factor in the response of the structure. The first mode circular frequency of

an elastic stage can be expressed through Rayleigh's method in the form

$$\omega_{e}^{2} = \frac{\{\psi\}[K_{e}]\{\psi\}^{\mathsf{T}}}{\{\psi\}[M]\{\psi\}^{\mathsf{T}}}$$
(4.30)

where

 ω_e = first mode circular frequency of the elastic stage [K_e] = elastic structural stiffness matrix { ψ } = first mode shape vector of the elastic stage

The first mode shape is not significantly changed after inelastic structural action has taken place in the response. Therefore the first mode shape vector of the elastic stage is also used in the inelastic stage. The first mode circular frequency while in the inelastic stage is expressed as follows:

$$\omega_{\mathbf{i}}^{2} = \frac{\{\psi\}[\mathsf{K}_{\mathbf{i}}]\{\psi\}^{\mathsf{T}}}{\{\psi\}[\mathsf{M}]\{\psi\}^{\mathsf{T}}}$$
(4.31)

where

 ω_i = first mode circular frequency of the inelastic stage [K_i] = inelastic structural stiffness matrix

The relationship between these two frequencies, ω_e and ω_i , can be found from Eqs. (4.30) and (4.31) to be

$$\omega_i^2 = \gamma \omega_e^2 \tag{4.32}$$

where

$$\gamma = \frac{\{\psi\}[\mathsf{K}_{i}]\{\psi\}^{'}}{\{\psi\}[\mathsf{K}_{e}]\{\psi\}^{\mathsf{T}}}$$

т

When any inelastic structural action has taken place in the response, the constant c_2 in Eq. (4.27) is evaluated in the form

$$c_2 = \frac{2\beta_1}{\omega_i}$$
(4.33)

This equation is then rewritten by substituting Eq. (4.32) for $\omega_{\rm i}$ with the result being

$$c_{2} = \frac{2\beta_{1}}{\omega_{e}} \gamma^{-\frac{1}{2}}$$
(4.34)

Thus the constant c_2 is changed by a factor of $\gamma^{-\frac{1}{2}}$ during the motion in accordance with the change in the stiffness matrix in order to keep the damping factor within reasonable values, otherwise the instantaneous damping matrix is underestimated.

(c) Equation of Motion

The equation of motion is developed in incremental form assuming that the properties of the structure are constant within each time interval.

The inelastic structural responses and failure processes under a strong base motion are evaluated by numerically integrating the equations of motion while using a step-by-step procedure. The Newmark β method is used in this solution of the equations of motion.

The incremental form of the equations of motion is expressed as

$$[M]{\Delta U} + [C]{\Delta \dot{U}} + [K]{\Delta U} = -[M]{\Delta X}$$
(4.35)

where

[M] = diagonal mass matrix defined in Eq. (4.22)

[C] = instantaneous damping matrix defined in Eq. (4.29),

which is evaluated at the end of the previous time step

[K] = instantaneous structural stiffness matrix defined in

Eq. (4.21), which is evaluated at the end of the previous time step

- $\{\Delta \ddot{U}\}$ = incremental story acceleration vector, relative to the base $\{\Delta \ddot{U}\}$ = incremental story velocity vector, relative to the base
- $\{ \Delta U \}$ = incremental story displacement vector, relative to the base
- $\{ \Delta X \}$ = incremental base acceleration vector

The incremental relative velocity $\{\Delta \dot{U}\}$ and acceleration $\{\Delta \ddot{U}\}$ are expressed in the Newmark β method as

$$\{\Delta \dot{\mathbf{U}}\} = \frac{1}{2\beta\Delta t} \{\Delta \mathbf{U}\} - \frac{1}{2\beta} \{\dot{\mathbf{U}}\} - (\frac{1}{4\beta} - 1) \Delta t\{\ddot{\mathbf{U}}\}$$
(4.36)

$$\{\Delta \ddot{U}\} = \frac{1}{\beta(\Delta t)^2} \{\Delta U\} - \frac{1}{\beta\Delta t} \{\dot{U}\} - \frac{1}{2\beta} \{\ddot{U}\}$$
(4.37)

where

- $\Delta t = time interval$
- β = a constant which is indicative of the variation of acceleration over the time interval usually chosen between 1/4 and 1/6, and influences the rate of convergence, the stability of the analysis and the amount of error in the Newmark β method.
- {U
 } = relative story velocity vector at the end of the previous
 time step
- {U} = relative story acceleration vector at the end of the previous time step

There are two basic ways to solve the equations of motion with direct integration. One is termed the explicit method. With that approach the accelerations are calculated from the equations of motion and then integrated for the displacements and velocities. The other method is termed the implicit method, in which case the equations of motion are combined with the time integration operators so that displacements are calculated directly. The advantage and disadvantage of both methods when applied to dynamic structural problems were discussed by Belytschko (1976). For the particular problem under investigation in this study, an implicit method is used, since the bandwidth of the stiffness matrix is small and an iteration procedure is not needed. The equations can be solved by Gaussian elimination or any such decomposition procedure. Unless some structural changes occur this decomposition remains in force for the successive time steps. But the implicit method may be more sensitive to error unless the small time interval is used.

The incremental story displacement $\{\Delta U\}$ can be expressed in terms of the response values and structural properties at the end of the previous step by combining Eqs. (4.35), (4.36) and (4.37) in the form

$$\{\Delta U\} = [A]^{-1}\{B\}$$
(4.38)

where

$$\begin{bmatrix} A \end{bmatrix} = \frac{1}{\beta(\Delta t)^2} \begin{bmatrix} M \end{bmatrix} + \frac{1}{2\beta\Delta t} \begin{bmatrix} C \end{bmatrix} + \begin{bmatrix} K \end{bmatrix}$$
$$\{B\} = \begin{bmatrix} M \end{bmatrix} \{ \frac{1}{2\beta} \{ \ddot{U} \} + \frac{1}{\beta\Delta t} \{ \dot{U} \} - \{ \Delta \ddot{X} \} \}$$
$$+ \begin{bmatrix} C \end{bmatrix} \{ (\frac{1}{4\beta} - 1) \Delta t \{ \ddot{U} \} + \frac{1}{2\beta} \{ \dot{U} \} \}$$

If the constant β is chosen to be 1/6 in Eqs. (4.36), (4.37) and (4.38), these equations can be interpreted as the linear acceleration method. If the constant β is assumed to be 1/4, these equations are equivalent to the constant average acceleration method. Both values of β are studied in the analysis.

The stability of the solution requires the time interval Δt to be less than 1/6 of the highest mode period. Therefore to be on the
conservative side and also to minimize the overshoots of the section capacities, the constant time interval Δt is chosen to be 1/10 of the period of the highest elastic mode in the analysis.

The incremental relative velocities are calculated from Eq. (4.36) for the given incremental relative displacements which have been evaluated by Eq. (4.38). The incremental relative accelerations are then calculated from the following equation which is a modified form of Eq. (4.35) and is based on the current structural properties.

$$\{\Delta \ddot{U}\} = -[M]^{-1}\{[C_c]\{\Delta \dot{U}\} + [K_c]\{\Delta U\} + [M]\{\Delta \ddot{X}\}\}$$
(4.39)

where

[K_c] = instantaneous structural stiffness matrix which is evaluated at the end of the current step

Equation (4.37) is not used to calculate the incremental relative accelerations, since the acceleration response is very sensitive to changes in the stiffness properties of the structure. Therefore more accurate results can be obtained by computing the incremental acceleration based on the updated structural properties rather than the previous ones.

The residual forces due to changes in the member stiffnesses that develop within a time interval are applied to the subsequent time step.

CHAPTER 5

HYSTERESIS RULES

5.1 Hysteresis Rules by Takeda, et al.

The hysteresis rules used in this analysis are an adaptation of those presented by Takeda, et al. (1970). The hysteresis rules for a trilinear primary curve are used for the beam rotational spring and the wall subelement. Some modifications were applied to the rules originally set down by Takeda. The modifications are discussed in Section 5.2. The detailed rules of Takeda's hysteresis are given by Otani (1972). Therefore in this study only the basic concept of the hysteresis rule is presented.

The primary curve of the hysteresis loop is established by connecting the origin, cracking point, yielding point and ultimate point successively by straight lines, thus forming the trilinearized curve. No limit on the third slope is considered for the primary curve. The primary curve is assumed to be symmetric about its origin. The loading curve is basically directed toward the previous maximum point on the primary curve in that direction. The slope of unloading curve is degraded depending on the maximum deflection reached in either direction. A typical example including several hysteresis loops is shown in Fig. 5.1.

5.2 Modifications of Takeda's Hysteresis Rules

The original Takeda's hysteresis rules have to be modified to deal with some specific problems that appear in the response behavior of coupled shear walls.

(a) <u>Shifting of Primary Curve due to the Axial Force</u> in the Wall Subelement

For the wall subelements the curves of the moment-curvature relations for different values of axial force are trilinearized as shown in Fig. 3.6. Cracking and yielding levels are shifted in accordance with the value of axial force. It is assumed that the axial force is small enough that the interaction curve is in the linear range, about the zero axial force axis.

The working moment-curvature curve is chosen to be the one corresponding to the present level of axial force. The pseudo-flexural rigidity $\frac{\partial M}{\partial \phi}$ in Eq. (3.14) of Section 3.3 is considered as the slope of the working moment-curvature curve, and it follows Takeda's hysteresis rules. The real flexural rigidity EI₁ in Eq. (3.14) can be obtained by multiplying $\frac{\partial M}{\partial \phi}$ by the factor which reflects the effect of transferring from one moment-curvature curve to another due to the change of axial force. Actual hysteresis loops for a wall subelement are shown by the thick solid curves in Fig. 5.2. The detailed procedure for evaluating $\frac{\partial M}{\partial \phi}$ and $\frac{\partial M}{\partial n}$ in the computer program is discussed in Appendix A.

(b) Pinching Behavior and Strength Decay of Connecting Beam

The primary curves for the rotational springs at the ends of each connecting beam are trilinearized and are assumed to follow Takeda's hysteresis rules but again with several modifications. Two sources that require the modifications are considered in this report. The first source is a pinching action that results from the compression reinforcement yielding before the concrete cracks, that had developed while that concrete had been in tension, can close. The other modification is a beam strength decay due to changes in the shear resisting mechanism.

Once the rotational spring has exceeded the cracking moment, the spring will, on subsequent cycles, demonstrate a pinching effect around the origin with only the reinforcement providing any resistance until the previous tension side cracks have been closed by compression.

The original hysteresis rules have therefore been modified to take care of this pinching effect. This is done in the way that whenever a working hysteresis loop is located in the positive rotation-negative moment range or the negative rotation-positive moment range, an additional spring, whose stiffness is based on only the reinforcement resistance, is installed in series with the original rotational spring.

After the formation of flexure-shear cracks in the beam, the shear carrying mechanism is considered to be shifted from the concrete cross section to a combination of the compressed concrete above the crack and the transverse reinforcement. Under repeated load, the increase of permanent strain in the transverse reinforcement after yielding induces distortion of the concrete section and causes the shear strength to decay as a result.

After the rotational spring has exceeded the yielding moment, a strength decay is introduced in the hysteresis loops on subsequent cycles. The rate of the strength decay is assumed to proportionally increase with rotation for simplification of the problem. A guideline is introduced in the hysteresis loops to include the effect of strength decay in the computer program. After the working hysteresis loop has exceeded the guideline, it goes parallel to the third slope of the original primary curve.

Hysteresis loops which include the effects of both the pinching action and the strength decay are illustrated in Fig. 5.3.

CHAPTER 6

ANALYTICAL RESULTS

6.1 Model Structures

The procedure described in Chapter 4 has been applied to the tenstory coupled shear wall models tested on the University of Illinois earthquake simulator by Aristizabal-Ochoa (1976). The dimensions of the models are shown in Fig. 2.1. The models are made up of two shear walls, each 1 by 7 in. in cross section, and having a height of 90 inches. The walls are joined at each of the floor levels by 1 by 1.5 in. connecting beams spanning the 4 in. spacing between the walls. A weight of 0.5 kip is placed at each floor level.

Two types of models are studied here. These are a weak beam model and a strong beam model. In further discussion they are referred to as structure-1 and structure-2, respectively. The main difference between these two models is the amount of steel reinforcement used in the connecting beams.

Material properties assumed for the models are listed in Table 6.1. The cross-sectional properties of the constituent elements of the models are shown in Fig. 6.1. The stiffness properties of the beam rotational springs and wall subelements were calculated by the procedure described in Chapter 3. These calculated stiffness properties are listed in Table 6.2. The analysis of a structure-1 type is considered to be a primary objective in this study.

6.2 Static Analysis of Structure-1

The inelastic structural behavior and failure mechanism of structure-1 responding to static loads as determined by the procedure described in this study are reported in this section. The results of this static analysis are used as the preliminary or backbone information for the subsequent dynamic analysis. The first mode shape of structure-1 is used to establish the static load distribution, because the first mode is expected to be the major contributor to the response under dynamic loads. The first mode shape is shown in Fig. 6.11.

The static load is increased monotonically at small load increments without changing its distribution pattern. The load increment used in the analysis is 1/300 of the maximum static load. The effect of inelastic axial rigidity of the wall as well as the effect of axial force on inelastic flexural rigidity is included in the analysis.

(a) Failure Mechanism

The sequence of cracking and yielding of constituent elements under the monotonically increasing load is presented in Fig. 6.2.

First cracking appears in the connecting beams at levels 3 and 4. Cracking then progresses to the adjacent lower and upper levels of connecting beams. After all connecting beams have developed cracks, cracking then starts in the lower part of the tension wall and propagates into the upper levels followed by cracking in the lower part of the compression wall. This in turn is followed by yielding of some of the connecting beams beginning at the intermediate levels and proceeding further into the lower and upper levels.

Finally, yielding occurs at the base of the tension wall, then at the base of the compression wall. After yielding has developed at the base of both walls the structure loses practically all its resisting capability against any further load increases. Cracking develops over the height of the tension wall while the cracking system expands up to level 5 of the compression wall.

(b) Effect of Inelastic Axial Rigidity

Axial rigidity of a wall section is considered to change reflecting the levels of curvature and axial strain existing in the wall as explained in Section 3.3. In Fig. 6.3 the relationship between axial force at the base and vertical displacement of the top level of a wall is presented to explain the effect of inelastic axial rigidity on the wall section's behavior. The case of elastic axial rigidity is also shown in Fig. 6.3 to serve as a base for comparison with the case of inelastic axial rigidity. The dead load of the structure is not considered in the calculations. The maximum base axial force is 8.2 kips in the figure. This corresponds to a base moment of 150 kip-in.

In the case where inelastic axial rigidity is assumed in the analysis, the tension wall displays a quite different stiffness curve from that of the compression wall. The curve of the tension wall is softened markedly by the opening of flexural cracks about the base axial force of 2 kips. When the maximum tensile axial force is reached, the top vertical displacement for the case of inelastic axial rigidity is 3.3 times as much as it would be if the axial rigidity remained elastic. The curve for the elastic axial rigidity is symmetric about the origin. For the

compression wall the curves for inelastic axial rigidity and for elastic axial rigidity are practically the same. This means that for all practical purposes the compression wall can be assumed to behave elastically in the axial direction.

(c) Base Moment-Horizontal Displacement Relationship

To study the overall behavior of the structure under a monotonically increasing load, the relationships of base moment to horizontal displacement at the top of the wall for different assumed conditions of axial rigidity of the wall are compared with the test results in Fig. 6.4. Base moment is defined as the sum of the flexural moments of the individual walls and the coupling moment due to the axial forces in the walls.

The curve of the test results is considered to be a pseudo-static curve based on the first mode component of the dynamic responses recorded in the test. The curve of inelastic axial rigidity includes the effect of axial force changes on the inelastic flexural rigidity and the effect of curvature changes on the inelastic axial rigidity in the walls. For the curves of elastic axial rigidity the elastic axial rigidity, which is constant in the process of loading, is assumed for the wall section and no effect of axial force on the flexural rigidity is considered in the walls.

The curve of reduced elastic axial rigidity is obtained by simply reducing the elastic axial rigidity of the walls by a factor while all other assumed conditions are the same as would be the case for elastic axial rigidity. This reduction factor is calculated based on the fact that the tension wall has a fairly small axial rigidity due to the

opening of flexural cracks in contrast to the compression wall where little flexural cracking exists as mentioned in the previous section. A reduction factor of 1.65 is assumed based on the observation that the vertical displacement of the top story for the case of inelastic axial rigidity is 3.3 times as much as that displacement would be if the axial rigidity remained elastic. This effect of inelastic axial rigidity in the tension wall must be averaged over both walls to arrive at the reduced elastic axial rigidity case. Therefore the axial rigidity of the walls is reduced to 12,700 kips for the case of reduced elastic axial rigidity.

As shown in Fig. 6.4, the analysis with inelastic axial rigidity produces a curve which lies close to the pseudo-static curve from the test although the calculated result is slightly stiffer than the pseudostatic curve. Also the curve for the case of reduced elastic axial rigidity is in satisfactory agreement. No appreciable difference exists between the curve with inelastic axial rigidity and that for reduced elastic axial rigidity except for the trailing part of the curve after wall yielding has been initiated.

Cracking and yielding of the walls and beams start at about same loading levels for all three cases. Cracking of the walls and beams starts at very low levels of loading. Yielding of the connecting beams is initiated at a base moment of 112 kip-in. followed by the yielding at the base of the wall at a base moment of about 175 kip-in. After yielding at the base of the wall, a marked change in structural stiffness occurs and the structure loses its main resisting system against any further load increases.

(d) Redistribution of Base Shear in Walls

Redistribution of base shear between the two walls during the process of loading is studied. The results are shown in Fig. 6.5. A part of the shear from the tension wall is transferred to the compression wall through the connecting beams due to the change in the flexural rigidity of the walls. The transferred shear at each level is accumulated down to the base. This causes a significant difference in the shears at the base in the two walls.

As shown in Fig. 6.5, the base shear is equally distributed between the two walls in the elastic stage. When cracking in the tension wall is initiated, suddenly the base shear in the tension wall starts shifting to the compression wall. The shifting of the base shear continues up to the point that only 28% of the total base shear is distributed to the tension wall while the remaining majority being in the compression wall. But when yielding in the walls is initiated, the base shear starts to reestablish back equally between the two walls so that the share to the tension wall increases. The redistribution of shear in the walls causes a compression force in the connecting beams so that the strength of the connecting beam might be increased.

(e) Coupling Effects of Walls

The coupling action of the two walls joined through the connecting beams is the most distinctive feature in the behavior of the coupled shear wall system. The influence of the coupling effects of the walls on the horizontal displacement of the top story and on the base moment are studied here.

Horizontal displacement at each level is caused by the two sources of structural actions. One is the flexural and shear deformations of the individual walls, and the other is the story rotation due to the contraction of the compression wall and the elongation of the tension wall. This is considered to be the coupling action of the two walls. The ratio of the top displacement due to the coupling effect to the total top displacement changes during the process of loading. The variation in the ratio at succeeding levels of deformation is illustrated in Fig. 6.6. The initial ratio of 65% abruptly reduces to 40% with cracking of the walls and beams. After being reduced to 40% the ratio gradually starts to increase until the time of the initiation of beam yielding. At this point the axial rigidity reduces faster than the flexural rigidity. When yielding of the connecting beams starts, the ratio shifts to a gradual decrease. This occurs because no significant increase of axial force in the walls can be introduced at this stage.

A significant portion of the horizontal displacement is caused by the coupling action even late in the loading sequence when large displacements exist. For example, at the total top displacement of 1.75 in. still 30% of this total top displacement is caused by the coupling actions.

Moment at each floor level also consists of both the coupling moment due to the axial forces in the walls and the flexural moment due to the bending of the individual walls. The variations in the ratios of the coupling moment and those of the flexural moment in the walls at the base to the total base moment are illustrated in Fig. 6.7. These

ratios are changing during the process of loading. The ratio of the coupling moment to the total moment starts at 71%, then decreases with the process of inelastic action in the structural members. This decrease continues up to the initiation of yielding in the wall. Inelastic action of the connecting beams is a major contributor to this decrease. The inelastic action of the walls works as softening factors of this tendency. Actually after the walls yield, the ratio starts increasing. At the initiation of yielding in the wall, the coupling moment shares 55% of the total base moment. This is the smallest share held by the coupling moment during the loading.

(f) Flexural Moment Redistribution in Walls at the Base

Furthermore, the flexural moment of the walls is considered to be the sum of a flexural moment of the compression wall and that of the tension wall as shown in Fig. 6.7. At the beginning, the flexural moment is equally distributed between the compression wall and the tension wall. As inelastic action of the walls takes place, the tension wall starts losing its share of the flexural moment. Finally, the tension wall's contribution represents only 20% of the total flexural moment. The shift of the flexural moment from the tension wall to the compression wall reflects the early deterioration of the stiffness properties of the tension wall as such deterioration precedes that in the compression wall.

Moment distribution patterns in all the members at the end of the loading are shown in Fig. 6.8. The concentration of flexural moment on the compression wall, especially at the lower levels, is clearly observed in this figure.

(g) Pinching Action and Strength Decay of Connecting Beams

The effects of pinching action and strength decay of the connecting beams on the overall structural behavior are discussed next. The base moment-top story displacement relationships under a cyclic loading are shown in Fig. 6.9. There are two curves, which have different assumed conditions, presented in Fig. 6.9. One curve includes the effect of pinching action and strength decay of the connecting beams, while the other curve does not include either of these effects.

In the first cycle there is no significant difference between the two curves except a slight pinching action in the curve that includes that effect. But in the second cycle the curve with the pinching action and strength decay included requires more displacement to reach the same level of base moment as that which had been experienced in the previous cycle. Naturally the overall structural stiffness of the case with pinching action and strength decay included decreases significantly in comparison with the case when such action is ignored.

6.3 Preliminary Remarks of Dynamic Analysis

Nonlinear response histories of structure-1 and structure-2 are calculated for selected prescribed base motions. The selected base motions used are adopted from the measured base motions used in the model tests with the earthquake simulator. The base motions for structure-1 and structure-2 are referred to as base motion-1 and base motion-2, respectively. The waveforms of these base motions are the acceleration signals of the El Centro (1940) NS component. The original time axes are compressed by a factor of 2.5 and the amplitudes of acceleration

are modified relative to the original record as appropriate to the model tests. Only the first 3 sec of recorded base motion from the model tests are used in the calculations, because the maximum responses and most of the damage to the structures take place within this time interval. The waveforms of base motion are shown in Fig. 6.10. The maximum accelerations of the base motions are listed below.

	Maximum Acceleration, g	Duration Time, sec	
Base Motion-1	0.41	3.0	
Base Motion-2	0.91	3.0	

The damping matrix is assumed to be proportional to the stiffness matrix with a damping factor for the first mode of 2% of critical. The time interval used in the response calculations is 0.00035 sec. This time interval requires 8,600 steps for the calculation of the response history of the structure to the 3 seconds of input base motion.

The effects of various assumed analytical conditions, such as the deterioration of axial rigidity due to the opening of cracks and the change of inelastic flexural rigidity taking account of the changing axial force in the wall section, the numerical integration scheme, the use of the stiffness matrix for the calculation of the damping matrix, the arrangement of wall subelements, and the pinching action and strength decay of connecting beams, are all studied. The assumed analytical conditions for dynamic runs are summarized in Table 6.3.

Initial mode shapes of structure-1 were computed and the results are shown in Fig. 6.11. Only the first three modes are presented since

the dynamic response of the structure is expected to be produced almost totally from these first three mode components. The first mode shape shows that all levels oscillate in the same phase. The second mode shape indicates that only one node is formed about level eight. The third mode shape shows that two nodes are formed about levels five and nine. Initial mode shapes of structure-2 are very much like those of structure-1 and are presented later in Section 6.6.

6.4 Dynamic Analysis of Structure-1

Three cases in which different analytical conditions are assumed are calculated for the response history of structure-1 subjected to base motion-1. These calculated responses are compared with the test results. These three cases are referred to as run-1, run-2 and run-3, respectively. Run-1 includes the effect of axial force on the inelastic flexural rigidity and the effect of curvature on the axial rigidity of the wall section. Run-2 and run-3 do not include these effects. Instead, linear elastic axial rigidity of the wall section is assumed for run-2, and reduced elastic axial rigidity of the wall section, as discussed in Section 6.2, is assumed for run-3. All other analytical conditions are the same for these three runs. Analytical conditions for each run are listed in Table 6.3. The pinching action and strength decay of the connecting beams are considered in the analysis for these runs, and the current stiffness matrix is used for the calculation of the damping matrix.

(a) Change of Modal Properties during Dynamic Response

Modal properties associated with the first three modes were computed before and after the run for run-1. These are listed in Table 6.4 and

illustrate the change of structural properties that occur during the dynamic motion. Although the mode shapes have not significantly changed, the frequencies have been considerably reduced showing the large deterioration of structural stiffness that has taken place during the dynamic motion.

(b) Maximum Calculated Response Compared with Test Results

The maximum responses from run-1, run-2 and run-3 are compared with the corresponding test values in Table 6.5. Also the maximum responses of run-1 and those of the test are presented in Fig. 6.12. The maximum responses for run-1 are fairly consistent with the test results except for shear in the lower levels and acceleration of the top floor. Run-2 and run-3 predict the maximum responses recorded in the tests to about the same level of accuracy as run-1 but with some exceptions. For example, the maximum displacements of run-2 are considerably smaller than those of the test and the other two runs. The maximum moments of run-3 are slightly smaller than those of the test and the other two runs. A major difference appears in the first mode frequency computed for the structure based on conditions of the structure at the end of the run. This frequency is 10% larger than the corresponding values for the test and the other two runs. This difference is caused by the deterioration of the axial rigidity of the wall section during the dynamic motion. The variable rigidity is not adequately treated in run-2 since the elastic axial rigidity of the wall section is assumed to remain constant throughout run-2.

(c) Calculated Response Waveforms Compared with Test Results

The response waveforms of run-1 are shown in Fig. 6.13. Several of the waveforms are compared with corresponding waveforms from the test. The overall features of the response waveforms of run-1 are similar to those of the test. The elongations of the fundamental period are observed in the response waveforms of run-1 and are fairly consistent with those of the test. The times when the maximum response of the top floor displacement and the base moment occur are comparable to the times recorded for the test. These occur at about 2.4 seconds. The response waveform of the base shear is governed by the first mode component but with some contributing influence of the second mode. The response waveforms of base moment and displacement are smooth and governed almost totally by the first mode component. The response waveforms of acceleration contain higher mode components, especially at the lower levels. At level eight, which is the position of the node for the second mode, the second mode component is not visible in the acceleration waveform.

The response waveforms of base shear, base moment, and horizontal displacement of the top floor for run-2 and run-3 are shown in Fig. 6.14 and Fig. 6.15, respectively. The response waveforms of run-3 are quite similar to those of run-1. The elongation of the fundamental period of run-2 is less than those of run-1 and run-3 showing that run-2 does not predict the structural damage properly.

(d) <u>Response History of Base Moment-Top Floor</u> Displacement Relationship

The values of base moment and top floor displacement were recorded at each time interval in run-1. These are plotted against each other

in Fig. 6.16 in order to see the overall structural history during the dynamic motion. Softening of the stiffness of the structure can be observed in this figure showing the effects of inelastic action, such as cracking and yielding of the various members and the strength decay of connecting beams, on the overall structural behavior. Also the dominance of the first mode components in the makeup of the structural response is seen in this figure through the relatively narrow width of band.

(e) Response Waveforms of Internal Forces

The response waveforms for the flexural moments of the beam rotational springs at several levels, the total flexural moment at the base of the two walls and the axial force of a wall at the base as recorded in run-1 are shown in Fig. 6.17. The first mode component governs all response waveforms of the internal forces with the slight second mode component present. This means that each member behaves in the same way as the structural system does.

(f) <u>Hysteresis Loops of a Beam Rotational Spring</u> and a Wall Subelement

The hysteresis loops for the beam rotational spring at level six and those for a wall subelement at the base, which were computed in run-1 about the time the system underwent its maximum response, are shown in Figs. 6.18 and 6.19, respectively.

The numerical value of the reduced rotational spring stiffness used in the analysis to produce the pinching action in the hysteresis loops is 28 kip-in. This value is calculated based on only the resistance of the reinforcing. The guideline used to establish the effect of strength decay of a connecting beam is determined by

connecting the following two points with a straight line. One point is located at 7/10 of the yielding moment at the yielding rotation. The other is placed at 6/10 of the moment level of the primary curve at an abscissa of twice the yielding rotation. These points are selected based on the test results by Abrams (1976).

Pinching action and strength decay are observed in the hysteresis loops of the beam rotational spring. These effects enhance the softening action on the rotational spring. The hysteresis loops of a wall subelement are made up of smooth curves rather than piecewise straight lines used in the case of the beam rotational springs. These curves account for the shifting from one moment-curvature relationship for a constant axial force to another moment-curvature relationship for a different constant axial force reflecting the change that is occurring in axial force as the element responds to the motion. On the tension side of the loops, softening of the slope of hysteresis loops in comparison to the slope of a primary curve is observed. The primary curve represents the idealized moment-curvature relationship for a constant axial force calculated based on the dead load. On the compression side of the loops, the slope of the hysteresis loops becomes stiffer than that of the primary curve, again due to the presence of the axial forces. Now they are adding a stiffening effect.

On the tension side of the loops an inflection point is observed, at which the slope suddenly starts increasing after the curve has been tracing a relatively flat portion. This inflection point can be explained by the following sequence of events. The increase in the tensile force in the tension wall, which has been the cause of the flat portion, is moderated due to yielding of the connecting beams. Then the axial force

in the walls becomes nearly constant as the beams are no longer supplying the increase. Then the slope for the wall appears to become stiffer again as it ceases to slide down between the curves for different axial forces but remains following the moment-curvature curve for a constant axial force.

(g) Failure Mechanism

The sequences of cracking and yielding of all constituent elements were recorded during run-1. Those data are shown in Fig. 6.20. First, cracking of the connecting beams starts at level 2 and develops to the upper levels, later coming back to catch level 1. After cracking of all the connecting beams has been completed, cracking of wall is initiated at the base, then propagates to the upper levels. Once cracking of the wall elements has progressed to approximately one-half the height of the structure, yielding of the connecting beams begins at the intermediate levels and proceeds to the upper and lower levels except level 1 where no yielding of the beam ever occurs. In the meantime the upper portion of the walls develops some cracking so that all levels of the walls are finally cracked. During the formation of yielding in the connecting beams, the wall yields at the base for a tensile force. Yielding of the tensile wall at the base does not mean that the structural system loses its resistance to further load, since yielding of both walls does not occur at the same time. At the time when yielding of the tension wall occurs the compression wall is still capable of sustaining the additional forces applied to the structural system.

Times when cracking and yielding of the various members occurred as recorded in the calculations are briefly summarized below.

Time, sec	Location of Cracking and Yielding
0.42-0.47	Cracking of Connecting Beam
0.60-0.82	Cracking of Wall in the Lower Levels
0.92-1.20	Cracking of Wall in the Upper Levels
0.96-1.20	Yielding of Connecting Beam
1.10-1.20	Yielding of Wall at the Base for
	a Tensile Force

All the cracking and yielding of the various members are initiated within the first 1.2 seconds. This indicates that the structure was damaged in the early stages of the motion.

Damage ratios, that is, the ratio of the maximum deformation to the yielding deformation, of the members are listed below.

Connect	ting Beam	at the	<u>Left End</u>	Left Side Wall at the Base
Floor Level	Damage Ratio	Floor Level	Damage Ratio	Damage Ratio
10	1.8	5	2.8	1.1
9	2.0	4	3.3	
8	2.5	3	2.6	
7	2.3	2	1.9	
6	3.3	1	0.9	
	Averag	e 2.3		

Only the damage ratios of the left half of the structure are listed here since there is no significant difference between the damage ratios of the left half of the structure and those of the right half of the structure.

The connecting beams in the intermediate levels, such as levels 4, 5 and 6, are the most severely damaged among the members.

(h) Coupling Effects of Walls

The coupling effects of the walls on the base moment and on the displacements of the system are discussed next. The ratios of the coupling base moment due to the axial forces in the walls to the total base moment have been calculated from their computed values and the magnitude of these ratios recorded at peaks in the response waveforms of run-1 are plotted in Fig. 6.21. The ratio changes in the process because of inelastic action in the members. The ratio starts at 60% but suddenly decreases to 53% when yielding of the connecting beams is initiated. This results from the connecting beams losing their capacity to carry any additional shears after yield has started in the beams. For all practical purposes then the axial forces stop increasing in the walls. After yielding of the connecting beams has formed, the moment ratio gradually reduces to 50%.

The ratios of the horizontal displacement at the top due to just the coupling effect to the total horizontal displacement at the top due to all effects were calculated at the peaks in the response waveforms of run-1, and the results are plotted in Fig. 6.22. The ratio starts at 50%, then gradually reduces to 32% because of the inelastic action of the members during the system's response. The deterioration of flexural rigidity of the walls and the moderation of the axial force buildup in the walls after the connecting beams yield are considered to be the major contributions to the reduction of this ratio.

The displacement distribution due to the coupling effect and the total displacement distribution over the height of the structure at the maximum response are presented in Fig. 6.23. The fairly large coupling effect on the displacement is observed especially at the upper levels.

6.5 Effects of Assumed Analytical Conditions on Dynamic Response

The effects of various assumed analytical conditions on the maximum response and the response waveforms are discussed in this section. Already the effects of the axial force change on the inelastic flexural rigidity and the influence on the inelastic axial rigidity due to the opening of cracks in the wall section have been discussed. In the previous section, comparison was made between the elastic axial rigidity case and the reduced axial rigidity case. Therefore the effects of the numerical integration scheme, the choice of the stiffness matrix for the calculation of damping matrix, the arrangement of wall subelements and the pinching action and strength decay of connecting beams are studied here.

Because run-3 in which the reduced axial rigidity was assumed for the wall section successfully reproduced the nonlinear response history of structure-1, the result of run-3 is used as a standard response history against which the response histories of the different assumed conditions are compared. Only the response waveforms of base shear, base moment and top displacement for each run are presented in Fig. 6.24 through Fig. 6.28. Assumed analytical conditions for each run are summarized in Table 6.3.

(a) Effect of the Numerical Integration Scheme

The Newmark β method is used for the solution of the equations of motion. The use of the constant β of 1/4 in the Newmark β method is equivalent to the constant average acceleration method. The use of the constant β of 1/6 is equivalent to the linear acceleration method. The Newmark β method with β of 1/4 is an unconditionally stable scheme. This has been proven even for nonlinear systems by Belytschko and Schoeberle (1975).

As the time intervals used are increased, most numerical integration procedures produce results with some period elongation and amplitude decay. The Newmark β method with β of 1/4 is the most accurate scheme showing the least distortion of period and amplitude as discussed by Bathe and Wilson (1973). Therefore the stability and accuracy of the calculated results can be checked by comparing the case for the constant β of 1/6 with that of 1/4. The constant β of 1/6 is used for run-3. The constant β of 1/4 is assigned to run-4. All other conditions are the same for these two runs.

The maximum responses of run-3 and run-4 are listed in Table 6.6. All the maximum responses of run-4 are quite consistent with those of run-3. This indicates that the choice of numerical integration scheme to be applied to this problem which has a very small time interval, such as 0.00035 sec, has no effect on the solution of the equations of motion. Therefore the computed results can be reliable as far as the stability and accuracy are concerned. The response waveforms of run-4 are not presented, since there is no visible difference between the waveforms of run-3 and those of run-4.

(b) Effect of the Choice of Stiffness Matrix for the Calculation of Damping Matrix

The damping matrix is assumed to be proportional to the stiffness matrix as discussed in Section 4.6. The stiffness matrix for the calculation of the damping matrix can be based on either the initial member stiffness or the updated member stiffness. The effect of the choice of which stiffness matrix should be used for the calculation of damping matrix are studied here by looking at the maximum responses and the response waveforms.

The updated stiffness matrix is used for the calculation of the damping matrix in run-3 while the initial stiffness is used in run-5. All other assumed conditions are the same for both runs. The maximum responses of run-3 and those of run-5 are listed in Table 6.7. The response waveforms of run-3 and those of run-5 are shown in Fig. 6.15 and in Fig. 6.24, respectively.

There are no significant differences in the maximum responses between the two runs. The maximum top displacement of run-3 is larger than that of run-5 while the maximum base moment of run-3 is smaller than that of run-5 showing that more inelastic actions take place in run-3 than in run-5. The elongation of the fundamental period at the end of the dynamic motion in run-3 is slightly larger than that in run-5. This is explained by the fact that if the initial stiffness is used for the damping matrix the damping factor is overestimated after the inelastic actions take place in the members. For the case of run-5 the first mode damping factor is overestimated by a factor of 1.5 at the end of the run.

(c) Effect of the Arrangement of Wall Subelements

Wall subelements can be arranged arbitrarily in a wall member making up that member from up to 7 subelements. If the subelements can be arranged coarsely, less computing time is required. To save on computing time can be a significant factor in the nonlinear dynamic analysis of a multistory structure. The effect of the number and arrangement of wall subelements on the maximum responses and the waveforms are studied here.

The subelement arrangement of run-3 which is shown in Fig. 2.1 is considered as the fine grid. A coarse arrangement in which only one subelement is assigned to each wall member, except the first story where two subelements are assigned, was used for run-6. In run-6 one subelement of 2 in. length is placed next to the base to take care of a possible hinge forming at the base. All other assumed conditions are the same for both runs.

The maximum responses of run-3 and of run-6 are listed in Table 6.8. The response waveforms of run-3 and those of run-6 are shown in Fig. 6.15 and Fig. 6.25, respectively. Although the maximum responses of run-6 are slightly larger than those of run-3, there is no significant difference in the maximum responses between run-3 and run-6. Also the response waveforms of the two runs are almost identical. For the analysis of structure-1 the coarse arrangement of wall subelements provides reasonable results. This means that the inelastic actions of the connecting beams are more important factors for the entire structural behavior than those of the walls in the analysis of structure-1 since the walls have not yielded at the base under compression in this particular problem.

(d) Effects of the Pinching Action and Strength Decay of Connecting Beams

Pinching action and strength decay are ever present characteristics of the connecting beams in a coupled shear wall system as shown by Abrams (1976). The effects of the pinching action and strength decay of the connecting beams on the maximum responses and the response waveforms of the structure under investigation are discussed here.

Four different assumed conditions or variations of the pinching action and strength decay are analyzed for the dynamic response of structure-1. Run-3 includes the effects of pinching action and strength decay. Run-7 includes only the strength decay effect, not the pinching action effect. Run-8 includes only the pinching action effect, not the strength decay effect. Run-9 includes none of these effects. All other assumed conditions are the same for the four runs.

The maximum responses of the four runs are listed in Table 6.9. The response waveforms of run-3 are shown in Fig. 6.15. The response waveforms of run-7, run-8 and run-9 are shown in Figs. 6.26, 6.27 and 6.28, respectively. There are no significant differences among the maximum accelerations of these four runs. The maximum displacements of run-8 and those of run-9 are smaller than those of run-3 by 20%. The maximum displacements of run-7 are smaller than those of run-3 by 10%. This shows that the pinching action and the strength decay, especially the strength decay, are the cause of large displacements. The maximum shears in the lower levels of run-8 and those of run-9 are larger than those of run-3 by 20% while the maximum shears of run-7 show a good agreement with those of run-3. This indicates that strength decay

contributes to the decrease of the maximum shears in the lower levels. From a practical standpoint there is no significant difference among the maximum moments of all the four runs.

The first mode frequency after completion of run-8 and that after run-9 are larger than the corresponding frequency of run-3 by 22% while the first mode frequency of run-7 is larger than that of run-3 by only 7%.

The response waveforms of run-7 are fairly consistent with those of run-3. The response waveforms of run-8 and those of run-9 show a similarity among themselves but have quite different features from those of run-3. For example the periods of the waveforms of run-8 and those of run-9 during the third second are shorter than those of run-3, and the displacement response of run-8 and that of run-9 are reduced, particularly within the third second so that the maximum displacement appears about 1.1 sec rather than about 2.4 sec.

These phenomena, mentioned above, can be explained by the fact that the deterioration of the beam stiffness is enhanced by pinching action and strength decay, especially strength decay.

6.6 Dynamic Analysis of Structure-2

The nonlinear response history of structure-2 subjected to base motion-2 is calculated and discussed in this section. Structure-2 has stronger connecting beams than does structure-1 and it is subjected to a more severe base motion than is structure-1. The calculated maximum responses are compared with those of the test. The dynamic response analysis of structure-2 is referred to as run-10.

The reduced elastic axial rigidity is assumed for the wall section in run-10, since the assumption of the reduced elastic axial rigidity

successfully reproduced the elongation of the period due to the deterioration of axial rigidity of the walls for structure-1 as mentioned in Section 6.4. The effect of axial force on the inelastic flexural rigidity and the effect on inelastic axial rigidity due to the opening of cracks in the wall cannot be properly included in this particular case because the procedure as developed in Section 3.3 does not actually apply. The strength of the connecting beams is of such a magnitude as to allow the axial force to build up in the wall elements to a level above the balance point load of the interaction diagram. Thus the assumption of a linear variation about the zero axial force axis is no longer a valid approximation. Strictly speaking, some additional modifications would have to be made to make the procedures truly applicable to a structure-2 makeup.

All the assumed analytical conditions for run-10 are listed in Table 6.3. The waveform of base motion-2 is shown in Fig. 6.10.

(a) Modal Properties of Structure-2

Modal properties associated with the first three modes of structure-2 were computed before the run and after the run. These properties are listed in Table 6.10 to show the change of structural properties computed to develop during the dynamic motion. The mode shapes of structure-2 are quite similar to those of structure-1 and have not significantly changed during the dynamic motion as was observed in the case of structure-1. On the other hand, the fundamental frequency is reduced to approximately 60% of the initial fundamental frequency during the dynamic motion.

(b) <u>Maximum Calculated Responses in Comparison with the</u> <u>Test Results</u>

The maximum responses of run-10 are compared with those of the test in Table 6.11. The maximum accelerations of run-10 are larger than those of the test, particularly in the top three levels. The maximum displacements of run-10 show a good agreement with those of the test although the test results are slightly larger than the calculated values. The maximum calculated shears of run-10 are larger than those of the test for all levels. The maximum base shear of run-10 is 17% larger than that of the test. The maximum moments of run-10 are larger than those of the test. The maximum base moment of run-10 is 16% larger than that of the test.

These differences on the maximum responses can be explained by the fact that crushing of the concrete at the base of the wall appeared in the test, and this could not be properly treated in the analysis. The fundamental frequency after run of run-10 is quite consistent with that of the test.

(c) Response Waveforms

Response waveforms of run-10 are shown in Fig. 6.29. The response waveforms of base moment and displacements are smooth and are dominated by the first mode component. The maximum top displacement is obtained at 1.97 sec which is consistent with the test. The response waveforms of accelerations show higher mode components, especially at the lower levels. At the higher levels, particularly at level 8, the first mode component becomes more distinguishable in the acceleration waveform. The response waveform of base shear is governed by the first mode component with some influence of the second mode component.

(d) Failure Mechanism

The sequence of cracking and yielding of each constituent member was recorded in run-10 and the result is shown in Fig. 6.30. Only a half of the structural system is shown in the figure, since any kind of inelastic action takes place symmetrically about the center of the structure in the analysis as used because of the assumed analytical conditions.

First cracking of the connecting beams starts at the lower levels, then propagates to the upper levels. After cracking has formed in all connecting beams, cracking of the wall is initiated at the base and propagates to the upper levels. After cracking of the walls has developed up to about level 6, yielding of the connecting beams starts at level 4 and proceeds to the upper and lower levels. During this development of yielding in the connecting beams, both walls yield at the base.

Times at which cracking and yielding of the various members occurred are briefly summarized below.

<u>Time, sec</u>	Location of Cracking and Yielding	
0.39-0.46	Cracking of Connecting Beam	
0.47-0.63	Cracking of Wall in the Lower Levels	
0.94-1.11	Yielding of Connecting Beam	
0.95	Yielding of Both Walls at the Base	
1.07-1.11	Cracking of Wall in the Upper Levels	

All cracking and yielding occurs within the first 1.2 seconds. The structure is damaged in this early stage of the dynamic motion. This was also observed in the case of structure-1.

· ·	Connecti	Wall at the Base		
Floor Level	Damage Ratio	Floor Level	Damage Ratio	
10	4.3	5	3.3	7.4
9	4.5	4	4.5	
8	2.9	3 ·	3.8	
7	3.4	2	3.9	
6	3.5	1	4.3	
	Average	3.8		

Damage ratios of the members are listed below.

The damage ratios of the members of structure-2 are considerably higher than occur in comparable members of structure-1. The wall at the base was very severely damaged and a hinge formed. The concentration of damage at the base of wall is primarily because of the strong connecting beams used in the structure.

CHAPTER 7

SUMMARY AND CONCLUSIONS

7.1 Object and Scope

The main objective of this study is the development of an analytical model which can trace the response history and the failure mechanism of coupled shear walls under dynamic as well as static loads.

The mechanical model of the coupled shear wall system used in this study is based on flexural line elements representing the walls and the connecting beams (Chapter 2). Rotational springs are considered at the ends of each connecting beam. Each wall member is further subdivided into several subelements in order to allow inelastic action to propagate through a story height. These constituent element models incorporate the assumed hysteretic properties of the system. Suitable hysteresis loops to each constituent element are established by modifying Takeda's hysteresis rules (1970) to include the specific characteristics of the coupled shear wall systems analyzed in this study. Factors influencing the hysteresis rules include such effects as the pinching action and strength decay of the connecting beam and the axial force effect on the moment-curvature relations for the wall subelements (Chapter 5).

A procedure to evaluate the inelastic stiffness properties of each constituent element based on the material properties of that element is presented (Chapter 3). The analytical procedure is developed to study the nonlinear behavior of coupled shear wall systems subjected to dynamic loads and static loads (Chapter 4). This procedure is applied to the

ten-story coupled shear wall models tested by Aristizabal-Ochoa (1976). These model structures are analyzed for static loads as well as dynamic loads and are compared with the test results (Chapter 6). The effects of various assumed analytical conditions on the maximum responses and the response waveforms of the model structure subjected to dynamic loads are discussed (Chapter 6).

7.2 Conclusions

(a) <u>Conclusions Related to the Static Analyses</u> of the Model Structure

The nonlinear structural behavior and failure mechanism of structure-1 subjected to static loads which are distributed over the height of the structure in accordance with the first mode shape are analyzed in Section 6.2.

The following statements summarize the conclusions made from the static analysis of structure-1.

(1) The inelastic action of the connecting beams occurs prior to that of the walls. Yielding of the connecting beams is initiated in the intermediate levels and then propagates to the upper and lower levels.

(2) It is necessary to assume the form of the axial inelastic rigidity in the wall section in order to reproduce the overall structural behavior observed in the test. The use of the reduced elastic axial rigidity in the wall section, in which the effect of inelastic axial rigidity is averaged over the height of the wall as well as over the compression and tension walls, produces a good comparison with the case which fully includes the effect of inelastic axial rigidity.

(3) A large portion of the shear in the tension wall is transferred to the compression wall due to the early initiation of inelastic action in the tension wall prior to any development in the compression wall. This results in only 28% of the total shear at the base being distributed to the tension wall at the time of initiation of wall yielding.

(4) The coupling between the walls exerts a considerable influence on the horizontal displacements and on the base moment. For example 30% of the total horizontal displacement of the top story is caused by coupling action when the top displacement reaches a level of 1.75 in. Also 55% of the total base moment is shared by the coupling moment at the time of initiation of wall yielding.

(5) The flexural moment of the wall is concentrated in the compression wall reflecting the early deterioration of stiffness properties of the tension wall prior to those of the compression wall. This occurs in such a way that only approximately 20% of the total flexural moment is contributed by the tension wall during the final stages of loading.

(6) Pinching action and strength decay of the connecting beams produce larger displacements of the structure in subsequent cycles and consequently accelerate the deterioration of the structural stiffness.

(b) <u>Conclusions Related to the Dynamic Analyses</u> of the Model Structures

The nonlinear response histories of the model structures, structure-1 and structure-2, subjected to the strong base motions have been analyzed assuming various analytical conditions and are compared with the test

results in Sections 6.3 through 6.6. Structure-2 has relatively much stronger connecting beams than does structure-1 but also is subjected to stronger base motion than structure-1.

The following statements summarize the conclusions made from the dynamic analyses of structure-1 and structure-2.

(1) Mode shapes of the structures have not changed significantly during the dynamic motion. Frequencies of the structure have decreased considerably reflecting the significant reduction of structural stiffness during the dynamic motion.

(2) The analytical models for structure-1 satisfactorily reproduce the maximum responses and the response waveforms, especially the elongation of the period due to the deterioration of structural stiffness, that were recorded during the test.

(3) Comparison of the calculated response of structure-2 with that of the test is not as good as is the case for structure-1 because the combination of moment and axial force lies outside the limits set when developing the analytical model. The analytical model cannot properly treat the crushing of concrete at the base of wall as observed in the test.

(4) Inelastic actions of the connecting beams play a major role in controlling the structural response since the beam strength controls the axial forces that develop in the wall, and the wall moment capacity is affected by the changes of these axial forces in the walls.

(5) The members of structure-2 are more severely damaged than are those of structure-1 because of a stronger base motion applied to structure-2. The damage is concentrated more at the base of the wall
than in the connecting beams for structure-2. The damage occurs mainly in the connecting beams for the case of structure-1 reflecting the weaker connecting beam used for structure-1.

(6) Inelastic action of the connecting beams occurs prior to any such action in the walls. Yielding of the connecting beams starts at the intermediate levels, then propagates to the upper and lower levels as observed in the case of static loads.

(7) The response waveform of base shear is governed by the first mode component but with some influence of the second mode component. The response waveforms of base moment and displacement are smooth and are governed by the first mode component. The response waveforms of acceleration contain higher mode components, especially those for the lower levels.

(8) The response waveforms of internal forces, such as the flexural moments of the connecting beams, the total flexural moment at the base of the two walls and the axial force in the wall at the base, are governed by the first mode component.

(9) There are fairly large coupling effects between the two walls. These have a major influence on the base moment and top displacement in the dynamic response. For example, 50% of the base moment and 32% of the top displacement are caused by the coupling action of the two walls at the last peak of the response waveforms. The coupling effect on the base moment decreases during the dynamic motion primarily due to inelastic action in the connecting beams. The coupling effect on the top displacement also reduces during the dynamic motion. This is partly the

result of increased wall contribution due to the deterioration of the flexural stiffness properties of the wall while the decay of the connecting beam strength holds the couple forces down.

(10) It is necessary to include the effects of inelastic axial rigidity of the wall section and pinching action and strength decay of the connecting beams in the calculations in order to reproduce the maximum displacement response and the elongation of the period that were evident at the end of the tests. The strength decay has a larger effect on the maximum displacement response and on the elongation of the period than does any pinching action. To assume the reduced elastic axial rigidity in the wall section is a simple way to include the effect of inelastic axial rigidity of the wall section.

(11) The use of different numerical integration schemes shows no significant effect on either the maximum or the waveforms in the dynamic response even though significant inelastic action is involved.

(12) The use of the updated stiffness matrix for the calculation of the damping matrix increases slightly the inelastic actions of the structure during the dynamic motion as compared to the case where the initial stiffness matrix is used.

(13) To use the coarse arrangement of wall subelements produces a slightly larger dynamic response of strucrure-l in comparison to the case with the fine arrangement.

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	Table 6.1	Assumed	Material	Properties
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Properties						
Concrete						
Compressive Strength f', ksi	4.5					
Tensile Strength f _t , ksi	0.403					
Strain at f'	0.003					
Strain at f _t	0.00013					
Steel Reinforcement						
Young's Modulus, ksi	29,000					
Yield Stress f _v , ksi	72					
Ultimate Stress f _u , ksi	83					
Yield Strain ε _ν	0.00248					
Strain Hardening Strain ε _h	0.01					
Ultimate Strain ε _u	0.08					

	Structure-1	Structure-2
Wall Subelement		
Elastic Axial Rigidity, kip	21,000	21,000
Elastic Shear Rigidity, kip	7,610	7,610
Moment-Curvature Relationship of Level 6 to 10 (Primary Curve)		
First Slope, kip-in. ²	85,700	85,700
Second Slope, kip-in. ²	28,500	50,500
Third Slope, kip-in. ²	420	1,000
Cracking Moment, kip-in.	4.5	6.5
Yielding Moment, kip-in.	20.0	39.0
Moment-Curvature Relationship of Base to Level 6 (Primary Curve)		
First Slope, kip-in. ²	85,700	85,700
Second Slope, kip-in. ²	50,500	50,500
Third Slope, kip-in. ²	1,000	1,000
Cracking Moment, kip-in.	6.5	6.5
Yielding Moment, kip-in.	39.0	39.0
Beam Rotational Spring		
Moment-Rotation Relationship		
First Slope, kip-in.	622	810
Second Slope, kip-in.	127	220
Third Slope, kip-in.	14	15
Cracking Moment, kip-in.	0.15	0.15
Yielding Moment, kip-in.	1.56	2.90

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Table 6.2 Stiffness Properties of Constituent Elements

•	General Condit	ions for All Runs	
	Damping Factor	$\beta_1 = 0.02$	
	Time Interval	0.00035	
	Duration Time,	, sec 3.0	
	Number of Step	os 8,600	
	Types of Structure and Base Motion	Effect of N* on EI _i ** and Effect of ϕ *** on EA _i **** in Wall	β in the Newmark β Method
Run-1	Structure-1 Base Motion -1	Included	$\frac{1}{6}$
Run-2	do	Not Included (Elastic Axial Rigidity)	do
Run-3	do	Not Included (Reduced Axial Rigidity)	do
Run-4	do	do	$\frac{1}{4}$
Run-5	do	do	$\frac{1}{6}$
Run-6	do	do	do
Run-7	do	do	do
Run-8	do	do .	do
Run-9	do	do	do
Run-10	Structure-2 Base Motion-2	do	do

ſab	le	6.3	Summary	of	Assumed	Ana	lytical	Conditions	for	Dynamic	Runs
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* N = axial force

** EI_i = inelastic flexural rigidity

*** ϕ = curvature

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**** EA; = inelastic axial rigidity

	Stiffness Matrix for Calculation of Damping Matrix	Arrangement of Wall Subelement	Pinching Action of Beam	Strength Decay of Beam
Run-1	Current	Fine	Included	Included
Run-2	do	do	do	do
Run-3	do	do	do	do
Run-4	do	do	do	do
Run-5	Initial	do	do	do
Run-6	Current	Coarse	do	do
Run-7	do	Fine	Not Included	do
Run-8	do	do	Included	Not Included
Run-9	do	do	Not Included	do
Run-10	do	do	Included	Included

Table 6.3 (continued)

	Be	Before Run-1			After Run-1			
	First Mode	Second Mode	Third Mode	First Mode	Second Mode	Third Mode		
(a) Mode Shap Level	e							
10	1.44*	-0.63	0.31	1.43	-0.65	0.34		
9	1.27	-0.33	0.03	1.27	-0.31	-0.01		
8	1.10	-0.02	-0.21	1.10	0.03	-0.28		
7	0.92	0.26	-0.31	0.92	0.31	-0.34		
6	0.75	0.48	-0.23	0.74	0.49	-0.19		
5	0.57	0.59	-0.01	0.56	0.56	0.07		
4	0.41	0.59	0.22	0.39	0.52	0.29		
3	0.26	0.48	0.34	0.24	0.40	0.38		
2	0.13	.0.30	0.29	0.12	0.23	0.30		
1	0.04	0.11	0.13	0.03	0.08	0.12		
(b) Frequency	1							
Hz	5.0	21	48	2.7	13	32		

Table 6.4 Mode Shapes and Frequencies of Structure-1

* Modal participation factors are included in mode shapes.

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	Test Results	Inelastic Axial Rigidity (Run-1)	Elastic Axial Rigidity (Run-2)	Reduced Elastic Axial Rigidity (Run-3)		
Acceleration, g						
Level 10	1.66	1.36	1.35	1.41		
9	1.12	1.04	1.00	1.05		
8	0.75	0.77	0.81	0.79		
7	0.73	0.73	0.75	0.75		
6	0.85	0.82	0.73	0.87		
5	0.86	0.83	0.70	0.85		
4	0.82	0.77	0.66	0.84		
3	0.71	0.66	0.60	0.69		
2	0.57	0.53	0.51	0.53		
1	0.47	0.50	0.43	0.45		
Base	0.41	0.41	0.41	0.41		
Displacement, in.						
Level 10	1.16	1.16	0.90	1.11		
9	1.00	1.03	0.80	0.98		
8	0.86	0.89	0.70	0.84		
7	0.71	0.75	0.60	0.70		
6	0.58	0.60	0.49	0.56		
5	-	0.46	0.37	0.43		
4	-	0.32	0.27	0.30		
3	-	0.20	0.17	0.19		
2	-	0.09	0.09	0.09		
1	-	0.03	0.03	0.03		

Table 6.5 Maximum Responses of Structure-1 in Comparison with Test Results

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· · · · · · · · · · · · · · · · · · ·	Test Results	Inelastic Axial Rigidity (Run-1)	Elastic Axial Rigidity (Run-2)	Reduced Elastic Axial Rigidity (Run-3)
Shear, kip				
Level 9	0.83	0.65	0.65	0.69
8	1.37	1.21	1.17	1.23
7	1.69	1.67	1.52	1.57
6	1.88	1.85	1.72	1.78
5	1.91	1.94	1.85	1.88
4	1.94	2.08	2.01	1.89
3	2.12	2.26	2.22	2.10
2	2.15	2.31	2.40	2.29
1	2.37	2.73	2.57	2.47
Base	2.54	2.92	2.71	2.60
Moment, kip-in.				
Level 9	.7.5	5.9	5.9	6.2
8	19.9	16.6	16.4	17.2
7	34.6	30.4	30.0	31.3
6	51.5	45.3	45.3	47.0
5	69.2	61.8	60.6	63.3
4	86.0	78.4	76.8	79.7
3	102.1	91.4	93.1	94.9
2	118.9	109.7	110.9	108.5
1	135.4	126.9	128.8	121.7
Base	151.5	148.0	148.5	140.6
Frequency, Hz				
Before Run	•			
lst Mode	4.8	5.0	5.0	4.3
2nd Mode	20.0	21.2	21.2	20.1
After Run				
lst Mode	2.7	2.7	3.0	2.7
2nd Mode	13.0	13.2	13.6	13.6

Table 6.5 (continued)

	β^* of $\frac{1}{6}$ (Run-3)	$\beta \text{ of } \frac{1}{4}$ (Run-4)		β of <u>1</u> (Run-3)	$\beta \text{ of } \frac{1}{4}$ (Run-4)
Acceleration,	g		Shear, kip		
Level 10	1.41	1.41	Level 9	0.69	0.69
9	1.05	1.05	8	1.23	1.23
8	0.79	0.79	7	1.57	1.57
. 7	0.75	0.74	6	1.78	1.78
6	0.87	0.87	5	1.88	1.88
5	0.85	0.85	4	1.89	1.90
4	0.84	0.84	3	2.10	2.12
3	0.69	0.69	2	2.29	2.32
2	0.53	0.53	1	2.47	2.51
1	0.45	0.45	Base	2.60	2.60
Base	0.41	0.41			·
Displacement,	in.		Moment, kip-	in.	
Level 10	1.11	1.10	Level 9	6.2	6.2
9	0.98	0.97	8	17.2	17.2
8	0.84	0.83	7	31.3	31.3
7	0.70	0.70	6	47.0	47.0
6	0.56	0.56	5	63.3	63.3
5	0.43	0.43	4	79.7	79.7
4	0.30	0.30	3	94.9	94.9
3	0.19	0.19	2	108.5	108.5
2	0.09	0.09	1	121.7	121.7
1	0.03	0.03	Base	140.6	141.3
Frequency, Hz	Ве	fore Run	lst Mode	4.3	4.3
			2nd Mode	20.1	20.1
	Af	ter Run	lst Mode	2.7	2.7
			2nd Mode	13.6	13.6

Table 6.6Effect of the Numerical Integration Scheme
on the Maximum Responses of Structure-1

* The constant of β in the Newmark β Method.

3	Updated Stiffness Matrix (Run-3)	Initial Stiffness Matrix (Run-5)		Updated Stiffness Matrix (Run-3)	Initial Stiffness Matrix (Run-5)
Acceleration	, g		Shear, kip		
Level 10	1.41	1.33	Level 9	0.69	0.67
.9	1.05	1.04	8	1.23	1.17
8	0.79	0.78	7	1.57	1.54
7	0.75	0.72	6	1.78	1.79
6	0.87	0.77	5	1.88	1.87
5	0.85	0.79	4	1.89	1.93
4	0.84	0.77	3	2.10	2.20
3	0.69	0.64	2	2.29	2.45
2	0.53	0.52	1	2.47	2.61
1.	0,45	0.44	Base	2.60	2.72
Base	0.41	0.41			
Displacement	, in.		Moment, kip-	·in.	
Level 10	1.11	1.03	Level 9	6.2	6.0
9.	0.98	0.92	8	17.2	16.6
8	0.84	0.80	7	31.3	30.2
7	0.70	0.68	6	47.0	46.1
6	0.56	0.55	5	63.3	62.8
5	0.43	0.43	4	79.7	79.0
4	0.30	0.31	3	94.9	94.0
3	0.19	0.19	2	108.5	107.6
2	0.09	0.10	1	121.7	122.3
1	0.03	0.03	Base	140.6	145.2
Frequency, Ha	z Bo	efore Run	lst Mode	4.3	4.3
			2nd Mode	20.1	20.1
	Α	fter Run	lst Mode	2.7	2.8
			2nd Mode	13.6	13.8

Table 6.7 Effect of the Choice of Stiffness Matrix for the Calculation of Damping Matrix on the Maximum Responses of Structure-1

	Fine Arrangement (Run-3)	Coarse Arrangement (Run-6)		Fine Arrangement (Run-3)	Coarse Arrangement (Run-6)
Acceleration,	g		Shear, kip		
Level 10	1.41	1.42	Level 9	0.69	0.70
9	1.05	1.06	8	1.23	1.25
8	0.79	0.78	7	1.57	1.59
7	0.75	0.76	6	1.78	1.79
6	0.87	0.87	5	1.88	1.89
. 5	0.85	0.85	4	1.89	1.91
4	0.84	0.80	3	2.10	2.07
3	0.69	0.69	2	2.29	2.28
2	0.53	0.53	. 1	2.47	2.48
1	0.45	0.45	Base	2.60	2.64
Base	0.41	0.41			
Displacement,	, in.		Moment, kip	p-in.	
Level 10	1.11	1.17	Level 9	6.2	6.3
9	0.98	1.03	8	17.2	17.4
8	0.84	0.88	7	31.3	31.7
7	0.70	0.73	6	47.0	47.6
6	0.56	0.58	5	63.3	64.3
5	0.43	0.44	4	79.7	80.7
4	0.30	0.31	3	94.9	96.0
3	0.19	0.19	2	108.5	109.5
2	0.09	0.10	1	121.7	123.1
1	0.03	, 0.03	Base	140.6	140.8
Frequency, Ha	z Bet	fore Run	lst Mode	4.3	4.3
			2nd Mode	20.1	20.1
	Af	ter Run	lst Mode	2.7	2.7
			2nd Mode	13.6	13.4

Table 6.8 Effect of the Arrangement of Wall Subelements on the Maximum Responses of Structure-1

	Both Effects Included (Run-3)	Only Strength Decay Included (Run-7)	Only Pinching Action Included (Run-8)	Both Effects Not Included (Run-9)
Acceleratio	on, g			
Level 10	1.41	1.40	1.41	1.40
. 9	1.05	1.05	1.05	1.05
8	0.79	0.78	0.79	0.78
. 7	0.75	0.73	0.73	0.73
6	0.87	0.85	0.83	0.84
. 5	0.85	0.83	0.86	0.87
4	0.84	0.80	0.84	0.81
3	0.69	0.66	0.69	0.68
2	0.53	0.54	0.54	0.53
1	0.45	0.45	0.46	0.45
Base	0.41	0.41	0.41	0.41
Displacemer	ıt, in.			
Level 10	1.11	1.01	0.89	0.88
9	0.98	0.90	0.78	0.78
8	0.84	0.79	0.67	0.67
7	0.70	0.67	0.56	0.56
6	0.56	0.55	0.44	0.45
5	0.43	0.43	0.34	0.34
4	0.30	0.30	0.24	0.24
3	0.19	0.19	0.16	0.16
2	0.09	0.10	0.08	0.08
1	0.03	0.03	0.02	0.02

Table 6.9Effects of the Pinching Action and Strength Decay
of Beams on the Maximum Responses of Structure-1

	E	Both Effects Included (Run-3)	Only Strength Decay Included (Run-7)	Only Pinching Action Included (Run-8)	Both Effects Not Included (Run-9)
Shear, k	ip				<u> </u>
Leve1	9	0.69	0.68	0.69	0.68
	8	1.23	1.22	1.23	1.22
	7	1.57	1.58	1.57	1.58
	6	1.78	1.80	1.78	1.80
	5	1.88	1.89	1.88	1.89
	4	1.89	1.93	1.88	1.90
	3	2.10	2.19	2.17	2.14
	2	2.29	2.43	2.53	2.49
	1	2.47	2.60	2.77	2.73
Ba	se	2.60	2.70	2.92	2.88
Moment,	kip-i	in.			
Leve1	9	6.2	6.1	6.2	6.1
	8	17.2	17.1	17.2	17.1
	7	31.3	31.1	31.3	31.1
	6	47.0	47.0	47.0	47.0
	5	63.3	63.5	63.3	63.5
	4	79.7	80.2	79.7	80.2
	3	94.9	95.7	94.9	95.7
	2	108.5	110.1	108.5	110.1
	1	121.7	124.4	121.7	124.4
Ba	se	140.6	145.2	140.8	141.1
Frequenc	cy, Ha	Z			
Before	e Run				
lst	Mode	4.3	4.3	4.3	4.3
2nd	Mode	20.1	20.1	20.1	20.1
After	Run				
lst	Mode	2.7	2.9	3.3	3.3
2nd	Mode	13.6	13.9	14.5	14.5

Table 6.9 (continued)

		Before Run-10			After Run-10		
		First Mode	Second Mode	Third Mode	First Mode	Second Mode	Third Mode
(a)	Mode Shape Level	2					
	10	1.46*	-0.66	0.32	1.43	-0.63	0.31
	9	1.27	-0.32	0.02	1.28	-0.33	0.03
	8	1.08	0.02	-0.23	1.12	-0.04	-0.20
	7	0.89	0.31	-0.32	0.96	0.22	-0.29
	6	0.70	0.52	-0.23	0.79	0.42	-0.23
	5	0.53	0.63	0.00	0.63	0.54	-0.05
	4	0.37	0.61	0.24	0.47	0.56	0.17
	3	0.22	0.49	0.36	0.31	0.49	0.32
	2	0.11	0.30	0.31	0.18	0.33	0.31
	1	0.03	0.11	0.14	0.07	0.15	0.17
(b)	Frequency						
	Hz	4.4	21	48	2.5	12	31

Table 6.10 Mode Shapes and Frequencies of Structure-2

* Modal participation factors are included in mode shapes.

2 	Test Results	Calculated Results (Run-10)		Test Results	Calculated Results (Run-10)
Acceleration,]		Shear, kip		
Level 10	1.84	2.22	Level 9	0.92	1.28
9	1.24	1.71	8	1.52	1.96
8	1.00	1.30	7	1.84	2.59
7	1.10	1.12	6	2.14	2.93
6	1.31	1.32	5	2.43	3.10
5	1.27	1.43	4	2.72	3.23
4	1.19	1.41	3	2.97	3.40
3	1.10	1.27	2	3.42	4.00
2	1.00	0.98	1	3.74	4.44
1	0.94	0.89	Base	3.92	4.54
Base	0.91	0.91			
Displacement,	in.		Moment, kip-	·in.	
Level 10	2.05	1.97	Level 9	8.4	11.5
9	1.92	1.78	8	22.2	26.5
8	1.69	1.58	7	38.8	49.7
7	1.43	1.37	6	56.2	76.0
6	1.18	1.14	5	77.6	103.5
5	0.94	0.92	4	99.8	131.4
4	0.71	0.69	3	121.9	158.1
3	0.50	0.47	2	149.0	185.3
2	0.28	0.27	1	174.9	211.4
1	0.13	0.11	Base	199.1	234.3
Frequency, Hz	Be	fore Run	lst Mode	4.5	4.4
			2nd Mode	19.0	20.8
	Af	ter Run	lst Mode	2.5	2.5
			2nd Mode	12.0	12.0

Table 6.11 Maximum Responses of Structure-2 in Comparison with Test Results







Fig. 2.2 Connecting Beam Model



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Fig. 2.3 Wall Member Model







Fig. 3.2 Idealized Stress-Strain Relationship of Reinforcement



Fig. 3.3 Distributions of Stress and Strain over a Cross Section



Fig. 3.4 Moment-Curvature Relationships of a Wall Member Section



Fig. 3.5 Moment-Curvature Relationship of a Connecting Beam Section



Fig. 3.6 Idealized Moment-Curvature Relationships of a Wall Member Section



Fig. 3.7 Idealized Moment-Curvature Relationship of a Connecting Beam Section



Fig. 3.8 Bond Slip Mechanism



Fig. 3.9 Idealized Moment-Rotation Relationship of a Rotational Spring











Fig. 4.3 Force and Displacement Components at Connecting Beam Ends



Fig. 4.4 Moment Distribution of a Connecting Beam



Fig. 4.5 Deformed Configuration of a Connecting Beam



Fig. 5.1 Takeda's Hysteresis Loops



TENSION

Fig. 5.2 Hysteresis Loops with Effect of Changing Axial Force



Fig. 5.3 Hysteresis Loops with Effects of Pinching Action and Strength Decay



Wall Flexural Reinforcement From Base To Level 6



Fig. 6.1 Cross-Sectional Properties of Wall and Beam




Sequence of Cracking and Yielding of Structure-1 Fig. 6.2 under Monotonically Increasing Load

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Fig. 6.3 Relationship of Base Axial Force and Top Vertical Displacement of Walls of Structure-1 under Monotonically Increasing Load



Top Floor Displacement, in.

Fig. 6.4 Base Moment-Top Displacement Relation of Structure-1 under Monotonically Increasing Load



Fig. 6.5 Redistribution of Base Shear in Walls of Structure-1 under Monotonically Increasing Load



Fig. 6.6 Ratio of Top Displacement due to Coupling Effect to Total Top Displacement of Structure-1 under Monotonically Increasing Load



Bose Moment, kip-in.

Fig. 6.7 Base Moment Distribution Pattern of Structure-1 under Monotonically Increasing Load



Fig. 6.8 Moment Distribution Patterns in Members of Structure-1 at the End of Monotonically Increasing Load



Fig. 6.9 Base Moment-Top Displacement Relation of Structure-1 under Cyclic Loading



Fig. 6,10 Waveforms of Base Accelerations

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Fig. 6.11 Initial Mode Shapes of Structure-1



Fig. 6.12 Maximum Responses of Structure-1, Run-1, in Comparison with Test Results





Fig. 6.13 Response Waveforms of Structure-1, Run-1



Fig. 6.13 (continued)



Fig. 6.13 (continued)



Fig. 6.13 (continued)



Fig. 6.13 (continued)



Fig. 6.13 (continued)

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Fig. 6.14 Response Waveforms of Structure-1, Run-2



Fig. 6.15 Response Waveforms of Structure-1, Run-3



Fig. 6.16 Response History of Base Moment-Top Displacement Relationship of Structure-1, Run-1



Fig. 6.17 Response Waveforms of Internal Forces of Structure-1, Run-1



Time, sec.

Fig. 6.17 (continued)



Time , sec.

Fig. 6.17 (continued)



Fig. 6.17 (continued)



Fig. 6.18 Hysteresis Loops of a Beam Rotational Spring of Structure-1, Run-1



Fig. 6.19 Hysteresis Loops of a Wall Subelement at Base of Structure-1, Run-1



Fig. 6.20 Sequence of Cracking and Yielding for Dynamic Response of Structure-1, Run-1





Fig. 6.21 Ratio of Coupling Base Moment to Total Base Moment at Peak Responses of Structure-1, Run-1



Fig. 6.22 Ratio of Top Displacement due to Coupling Effect to Total Top Displacement at Peak Responses of Structure-1, Run-1







Fig. 6.24 Response Waveforms of Structure-1, Run-5



Fig. 6.25 Response Waveforms of Structure-1, Run-6



Fig. 6.26 Response Waveforms of Structure-1, Run-7

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Fig. 6.27 Response Waveforms of Structure-1, Run-8


Fig. 6.28 Response Waveforms of Structure-1, Run-9













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Fig. 6.30

Sequence of Cracking and Yielding for Dynamic Response of Structure-2, Run-10

APPENDIX A

CALCULATIONS OF WALL STIFFNESS PROPERTIES IN THE COMPUTER PROGRAM

The detailed procedure to evaluate the wall stiffness properties in the computer program is schematically discussed in this appendix. The properties to be evaluated are $\frac{\partial M}{\partial \phi}$, $\frac{\partial M}{\partial n}$, $\frac{\partial N}{\partial \varepsilon}$, and $\frac{\partial N}{\partial \phi}$. These quantites are first defined in Eqs. (3.10) and (3.11) of Section 3.3.

The instantaneous flexural rigidity and axial rigidity of a wall subelement are derived from these properties as shown in Eq. (3.13) of Section 3.3. These properties are varied nonlinearly and correlated mutually in the process of inelastic action of a subelement making the problem highly complicated.

It is desirable to linearize these properties in a piecewise fashion for simplicity, because the hysteresis rules are already developed around a trilinearized version of the primary curve.

A.1 Calculation of Pseudo-Flexural Rigidity $\frac{\partial M}{\partial \phi}$

The procedure to develop an idealized moment-curvature relationship for a constant axial force is discussed in Section 3.3. The series of idealized moment-curvature relationships for different values of constant axial force are shown in Fig. 3.6.

A major difficulty in the calculation of $\frac{\partial M}{\partial \phi}$ is that there are an infinite number of moment-curvature curves corresponding to different axial forces and axial force variations although the hysteresis rules require a single moment curvature curve.

To avoid this difficulty, a moment-curvature relationship for a specified axial force is chosen to be the primary curve of the hysteresis loop. The dead load of the wall is considered as the specified axial force for which the primary curve is chosen. The primary curve is shown by the thick solid lines in Fig. A.1.

In an actual loading process, the moment-curvature curve may depart from this primary curve due to changes in the axial force. In establishing the various curves the yielding curvature for any moment-curvature curve is assumed to be the same regardless of axial force level, Fig. A.1.

If the moment, axial force, and previous loading history are known at a stage in the loading process, the location for the present loading level can be specified as shown in Fig. A.1.

If the location of the present loading level happens to be at point A, the referring point A' on the primary curve can be located by projecting vertically down from point A to the intersection with the primary curve as shown in Fig. A.1. The moment at the referring point A' is calculated from an equation of the form:

$$m_{A^{1}} = m_{A} - \left(\frac{\partial M}{\partial n}\right) \Delta n$$
 (A.1)

where

 m_A , = moment at the referring point A' m_A = moment at the point A Δn = difference between the present axial force and axial force for the primary curve

In this procedure, whether the cracking point or the yielding point has been exceeded or not can be checked by referring to the hysteresis

loop of the primary curve.

The value $\frac{\partial M}{\partial \phi}$ for an arbitrary moment and axial force can be calculated by locating the referring point on the hysteresis loop of the primary curve, then modifying the slope of hysteresis loop at the referring point.

The modification of $\frac{\partial M}{\partial \phi}$ can be expressed as follows:

$$K = K_0 \left[1 + \frac{\Delta(\frac{\partial M}{\partial n})(n - n_0)}{\Delta m} \right]$$
(A.2)

where

 $K = \frac{\partial M}{\partial \phi} \text{ for arbitrary moment and axial force}$ $K_0 = \frac{\partial M}{\partial \phi} \text{ at the referring point on the primary curve}$ $\Delta(\frac{\partial M}{\partial n}) = \text{ increment of } \frac{\partial M}{\partial n} \text{ at the referring point on the primary curve}$ $\Delta m = \text{ increment of moment at the referring point on the primary curve}$ n = axial force (compression is positive)

n₀ = axial force for which the primary curve is evaluated
 (usually dead load)

A.2 Calculation of $\frac{\partial M}{\partial n}$

The value $\frac{\partial M}{\partial n}$ can be evaluated by examinating the idealized momentcurvature relationship in Fig. 3.6. For simplicity sake, it is assumed that $\frac{\partial M}{\partial n}$ is a linear function of the moment level. The evaluation of $\frac{\partial M}{\partial n}$ at an arbitrary point on the hysteresis loop is schematically explained in Fig. A.2.

First, the values of $\frac{\partial M}{\partial n}$ at specified moment levels of the primary curve are determined by taking an average of the values $\frac{\partial M}{\partial n}$ for different

axial force levels at each specified point. The levels of zero moment, cracking moment and yielding moment are chosen as these specified levels.

The value $\frac{\partial M}{\partial n}$ is assumed to be zero at the zero moment level. After the yielding level, the value $\frac{\partial M}{\partial n}$ is considered to be constant. The value $\frac{\partial M}{\partial n}$ at an arbitrary point in the hysteresis loop can be evaluated by linearly interpolating the values $\frac{\partial M}{\partial n}$ at specified levels such as the cracking and yielding levels.

The procedure to evaluate $\frac{\partial M}{\partial n}$ can be applied to the unloading curve as well as the loading curve with one exception. The exception to this procedure is that before cracking is initiated by the loading, the value $\frac{\partial M}{\partial n}$ is always zero, taking account of the characteristics of the momentcurvature relation of the reinforced concrete section.

A.3 Calculation of Pseudo-Axial Rigidity $\frac{\partial N}{\partial \varepsilon}$

The axial force-axial strain relationship corresponding to a given curvature can be calculated by using the procedure explained in Section 3.2. In the procedure, the axial strain is determined by taking an average of the axial strain distribution over the cross section. A series of axial force-axial strain relationship curves are shown in Fig. A.3. There are an infinite number of such axial force-axial strain curves corresponding to different values of curvature.

It is assumed that the relations between axial force, axial strain, and curvature are kept to be always the same regardless of the loading history. This means that the deterioration of axial rigidity depends only on the axial force and curvature. Therefore, if the axial force and curvature at any stage in the loading process are known, the present location in the loading process can be specified without knowing the previous history of loading. The slope of the axial force-axial strain curve at that location is considered as the instantaneous pseudo-axial rigidity $\frac{\partial N}{\partial c}$.

To simplify the problem, the axial force-axial strain curve is slightly modified. In the computer program each axial force-axial strain curve is represented by a straight line with a different slope as shown in Fig. A.4.

The variation of the slope $\frac{\partial N}{\partial \varepsilon}$ with respect to curvature ϕ can be expressed approximately by a third order equation in the curvature, ϕ , as shown in Fig. A.5. It is assumed that if a curvature ϕ exceeds the limiting value, the value $\frac{\partial N}{\partial \varepsilon}$ becomes constant.

The slope, $\frac{\partial N}{\partial \epsilon}$, is expressed as follows:

$$0 \le |\phi| \le b$$

$$\frac{\partial N}{\partial \varepsilon} = a(b - |\phi|)^3 + c$$

$$b \le |\phi|$$

$$\frac{\partial N}{\partial \varepsilon} = c$$

(A.3)

where

a, b, and c = coefficient of the third order equation $|\phi|$ = absolute value of curvature

The coefficient a, b and c should be evaluated so that the approximate expression of the $\frac{\partial N}{\partial \varepsilon}$ - ϕ curve in Fig. A.5 is made as close to the real one as possible.

The range of axial force-axial strain curves is limited by two boundary lines A and B as shown in Fig. A.3. If an axial force-axial strain curve exceeds either boundary line, A or B, the curve is then assumed to travel along the line A or B depending on which line is exceeded.

Line A corresponds to the situation when the tensile stress due to moment is overcome by the compressive stress due to compressive force. There is no tensile stress on that section. Line B corresponds to a full cracking stage in which only reinforcing bars exist on a section after cracking.

Line A can be approximately straight until a concrete crushing is initiated. Also line B can be straight until reinforcement yielding happens.

The equation of the idealized axial force-axial strain lines in Fig. A.4 can be expressed as follows:

Within the range limited by two boundary lines A and B

$$N = \frac{\partial N}{\partial \varepsilon} \varepsilon + \left(\frac{\partial N}{\partial \phi}\right) \times \phi$$
(A.4)
$$N = K_{1} \varepsilon \quad (compression)$$

$$N = K_{2} \varepsilon \quad (tension)$$

where

Otherwise

N = axial force (function of
$$\phi$$
 and ε)
 ε = axial strain
 $\frac{\partial N}{\partial \varepsilon}$ = slope of N - ε line that is defined in Eq. (A.3)

 $\left(\frac{\partial N}{\partial \phi}\right)_{\epsilon=0}$ = differentiation of axial force with respect to curvature at zero axial strain

 ϕ = curvature

 $K_1 = slope of boundary line A$

 K_2 = slope of boundary line B

A.4 Calculation of
$$\frac{\partial N}{\partial \Phi}$$

The expression of $\frac{\partial N}{\partial \phi}$ can be obtained by simply differentiating Eq. (A.4) with respect to curvature ϕ .

The results of this differentiation are expressed as follows: Within the range limited by two boundary lines A and B

b <u><</u> φ	$\frac{\partial N}{\partial \phi} = d$	
$0 \le \phi \le b$	$\frac{\partial N}{\partial \phi} = -3a(b - \phi)^2 \varepsilon + d$	
-b <u><</u> ¢ <u><</u> 0	$\frac{\partial N}{\partial \phi} = 3a(b + \phi)^2 \epsilon - d$	(A.5)
φ <u><</u> −b	$\frac{\partial \phi}{\partial N} = -q$	

Otherwise

$$\frac{\partial \mathbf{N}}{\partial \phi} = \mathbf{0}$$

where

$$d = \left(\frac{\partial N}{\partial \phi}\right)_{\varepsilon=0}^{\varepsilon=0}$$



Fig. A.1 Primary Curve for Hysteresis Loops of a Wall Member Section



Fig. A.2 Evaluation of $\frac{\partial M}{\partial n}$ for Hysteresis Loops of a Wall Member Section







Fig. A.4 Idealized Axial Force-Axial Strain Relationship of a Wall Member Section



Fig. A.5 Relationship of Slope of Axial Force-Axial Strain Line and Curvature for a Wall Member Section

APPENDIX B

COMPUTER PROGRAM FOR NONLINEAR RESPONSE ANALYSIS OF COUPLED SHEAR WALLS

The computer program was developed to calculate nonlinear response of coupled shear walls under dynamic loads as well as static loads. The method of analysis was described in Chapter 4. The program is limited to the analysis of a symmetric coupled shear wall structure with less than 11 stories. A wall member can be divided into subelements in any arrangement up to 7 elements.

The total core space required to run the program is approximately 220 kilo-bites in IBM 360/75 computer, including temporary disk space for calculated response values. It took approximately 15 minutes of computing time to run the program for the nonlinear response analysis of a ten-story coupled shear wall structure subjected to 3 seconds of base motion at a .00035 second time interval. Calculated response values were temporarily stored in disk space and were plotted at the end of analysis on a CALCOMP plotter.

The flow diagram of the computer program for nonlinear response analysis of coupled shear walls is shown in Fig. B.l.



Fig. B.1 Flow Diagram of Computer Program for Nonlinear Response Analysis of Coupled Shear Walls



Fig. B.1 (continued)



Fig. B.1 (continued)



Fig. B.1 (continued)