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GADFLEA
A COMPUTER PROGRAM FOR THE ANALYSIS OF PORE PRESSURE GENERATION AND DISSIPATION DURING CYCLIC OR EARTHQUAKE LOADING

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A report on research sponsored by
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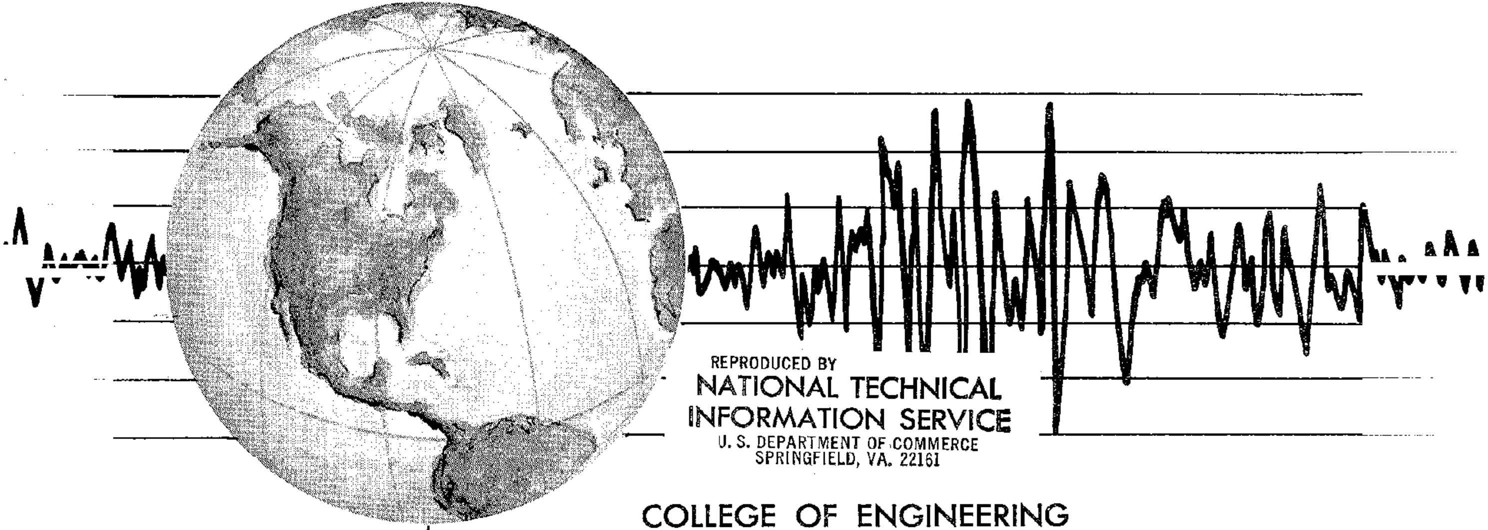

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16. Abstracts

During earthquakes, pore water pressures are induced in soils by the cyclic stress applications induced by the ground motions. At the same time, pore water pressures may dissipate from the soil due to drainage. Until very recently it had not been possible to couple the two effects. However, it has recently been shown that under one-dimensional conditions the pore-pressure generating effects of cyclic loading could be incorporated into a dissipation analysis by the introduction of a source term. This approach has also been extended to situations involving radial flow in studies of the feasibility of using gravel drains to stabilize potentially liquefiable soil deposits. An interesting aspect of this approach is that although it was initially developed to analyze cyclic loads induced by earthquakes it is equally applicable to many other forms of cyclic loading and may thus, for example, be used to analyze pore pressure generation and dissipation induced by wave action on off-shore structures. In this report a method of analysis of the equations governing pore pressure generation and dissipation based on the finite element method is developed and illustrated by application to a variety of problems.

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INTRODUCTION

There have been considerable advances in the understanding of the phenomenon of liquefaction in recent years. It is now widely accepted that a saturated granular material, subjected to cyclic loading involving shear stress reversal, will exhibit a tendency to compact and that if the material is unable to drain this tendency to decrease in volume will be accompanied by an increase in pore water pressure. Ultimately, if the cyclic loading is maintained the soil will reach a condition of zero effective stress and, depending on its relative density, will undergo essentially a complete loss of strength (liquefaction) or undergo excessively large strains (liquefaction with limited strain potential).

There has also been considerable progress in the development of test and test procedures to determine quantitative measures of the stress conditions which lead to these types of soil liquefaction. This development has been accompanied by development of methods of analysis (Seed and Idriss, 1967, 1971) which use these test results to evaluate the liquefaction potential of soil deposits in the field. These methods provide a useful basis for assessing probable site performance for prescribed earthquake performance.

A large proportion of the work described above assumes that undrained conditions prevail and is strictly only applicable to situations in which the redistribution and dissipation of pore pressures has no significant effect on the liquefaction potential of the soil mass. It has long been appreciated, however, that such effects may be quite significant. For example, if pore water pressures generated in a soil mass are to some extent dissipated, then liquefaction may be averted; conversely, the dissipation of pore water pressures generated deep within a soil mass may lead to upward seepage and consequent liquefaction of surface layers (Seed and Lee, 1966; Ambraseys and Sarma, 1969; Yoshimi and Kuwabara, 1973; and Seed, Martin and Lysmer, 1975).

Until very recently it had not been possible to couple the two effects. However, Seed et al (1975) showed that under one-dimensional conditions the pore pressure generating effects of cyclic loading could be incorporated into a dissipation analysis by the introduction of a source term. This approach was extended to situations involving radial flow by Seed and Booker (1976) in their examination of the feasibility of using gravel drains to stabilize potentially liquefiable soil deposits. An interesting aspect of this approach is that although it was initially developed to analyze cyclic loads induced by earthquakes it is equally applicable to many other forms of cyclic loading and may thus, for example, be used to analyze pore pressure generation and dissipation induced by wave action on off-shore structures, (Rahman, Seed and Booker, 1976).

In this report a method of analysis of the equations governing pore pressure generation and dissipation, based on the finite element method is developed and illustrated by application to a variety of problems.

BASIC EQUATIONS

In this section the equations governing the generation and dissipation of pore pressure for a horizontally stratified soil will be developed. It will be assumed that the flow of pore water is governed by Darcy's law so that the usual considerations of continuity lead to the equation

$$\{\nabla\}[k]\{\nabla \frac{u}{\gamma_\omega}\} = \frac{\partial \varepsilon}{\partial t} \tag{1}$$

where u is the excess pore pressure.

$$[k] = \begin{bmatrix} k_H & o & o \\ o & k_H & o \\ o & o & k_v \end{bmatrix}$$

is the matrix of permeability coefficients

$$\{\nabla\cdot\} = \left(\frac{\partial\cdot}{\partial x}, \frac{\partial\cdot}{\partial y}, \frac{\partial\cdot}{\partial z}\right)^T$$

γ_ω is the unit weight of water

ε is the volume strain, with volumetric reduction being considered positive

and where the z axis has been chosen to coincide with the vertical axis and the x and y axes lie in a horizontal plane.

Suppose that during an interval of time dt, the pore water pressure undergoes a change du, the element will also be subjected to dN cycles of alternating shear stress which will cause an additional pore pressure increase $\frac{\partial u_g}{\partial N} \cdot dN$, where u_g is the pore pressure generated by the alternating shear stresses for the appropriate prior strain history. It therefore follows that in situations where the material is isotropic with respect to deformation behavior and where the change in bulk stress is negligible, the volume change of the element during the time dt is just

$$d\varepsilon = m_{v_3}(du - \frac{\partial u_g}{\partial N} dN) \qquad (2a)$$

or

$$\frac{\partial\varepsilon}{\partial t} = m_{v_3}\left(\frac{\partial u}{\partial t} - \frac{\partial u_g}{\partial N}\frac{\partial N}{\partial t}\right) \qquad (2b)$$

where m_{v_3} is the coefficient of volume compressibility.

Combining equations (1,2) it is found that

$$\{\nabla\}^T[k]\{\nabla \frac{u}{\gamma_\omega}\} = m_{v_3}\left(\frac{\partial u}{\partial t} - \psi\right) \qquad (3)$$

where $$\psi = \frac{\partial u_g}{\partial N} \frac{\partial N}{\partial t}$$

Equation (3) is a diffusion equation with a source term $\psi = \frac{\partial u_g}{\partial N} \frac{\partial N}{\partial t}$ corresponding to the pore pressure generated by the alternating shear stresses. The numerical solution of equation (3) can be found by the finite element method; details of the solution method will be given in a later section.

In order to evaluate solutions to equation (3) it is first necessary to evaluate the two components of the source team $\frac{\partial u_g}{\partial N}$ and $\frac{\partial N}{\partial t}$ as well as the soil compressibility and permeability.

The value of $\frac{\partial u_g}{\partial N}$ can be found from undrained tests as described by Seed et al (1975). For many soils the relationship between u_g and N may be expressed in the form

$$\frac{u_g}{\sigma'_o} = \frac{2}{\pi} \text{ arc } \sin\left(\frac{N}{N_\ell}\right)^{\frac{1}{2\theta}} \tag{4}$$

where N is the number of cycles necessary to cause liquefaction.

σ'_o is the initial mean bulk effective stress for triaxial conditions or the initial vertical effective stress for simple shear conditions,

and θ is an empirical constant which depends on the soil type and test conditions.

The relationship given in equation (4) plotted in Fig. 1 for different values of θ. A value of $\theta = 0.7$ has been found to represent the average curve for many soils. Recent laboratory studies however, show that for some soils other values of θ may be required. It follows from equation (4) that

$$\frac{\partial u_g}{\partial N} = \frac{\sigma'_o}{\theta \pi N_\ell} \frac{1}{\sin^{2\theta-1} \chi \cos \chi}$$

where $\chi = \frac{\pi}{2} r_u$

and $r_u = u/\sigma'_o$ the pore pressure ratio.

For practical purposes it is found that irregular cyclic loading may be converted to an equivalent number of uniform cycles N_{eq} occurring during

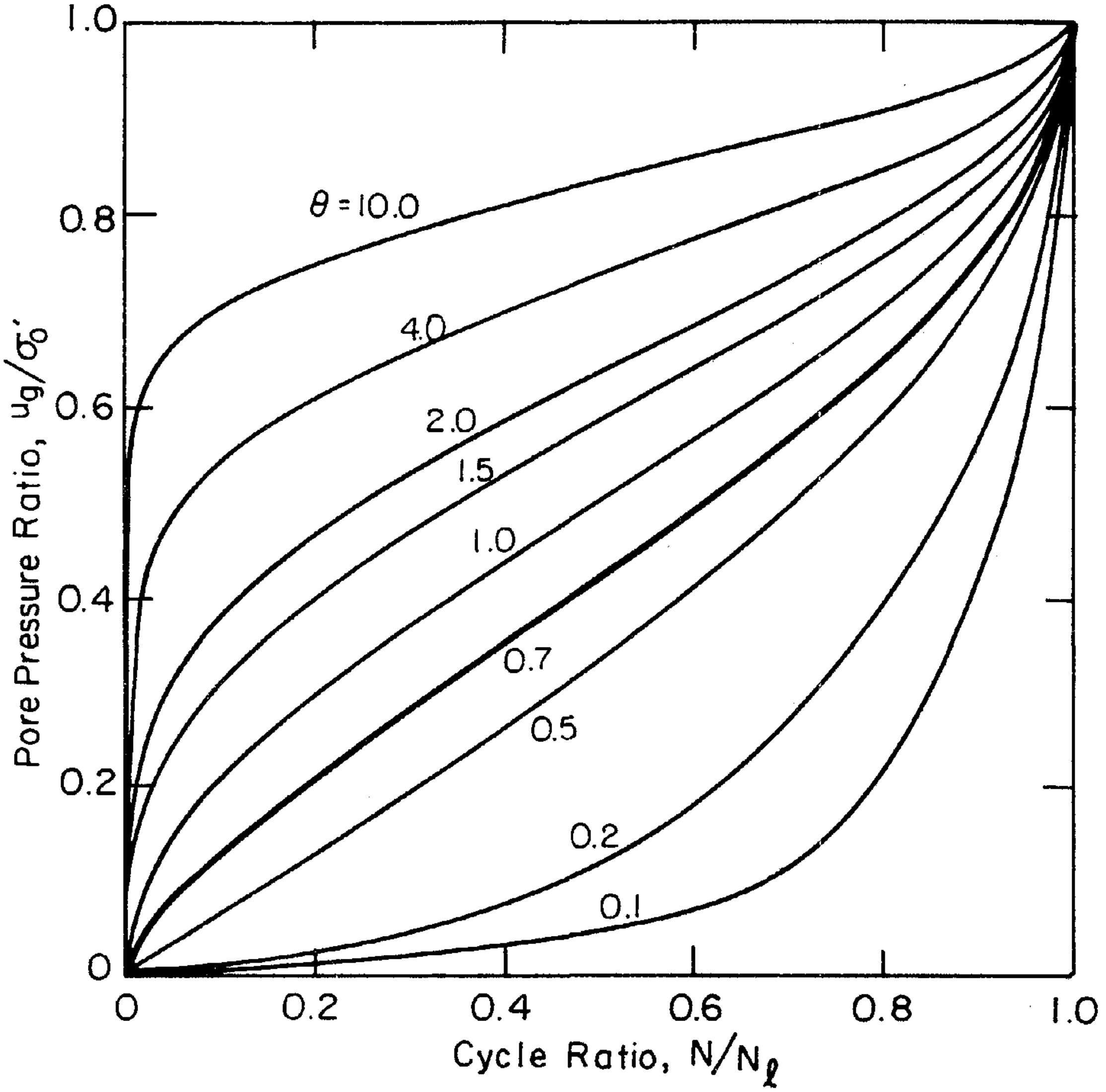

Fig. 1 RATE OF PORE PRESSURE GENERATION

a duration of time t_d. Details of this conversion are discussed by (Seed, Idriss, Makdisi and Banerjee, 1975). For such cases

$$\frac{\partial N}{\partial t} = \frac{N_{eq}}{t_d} \qquad 0 < t < t_d \tag{5}$$

$$= 0 \quad \text{otherwise}$$

The conversion of irregular cyclic loading to an equivalent number of cycles was developed primarily for earthquake induced loading. For some other forms of loading, such as that induced by a storm on an offshore structure, the amplitude of the cyclic stress changes may increase and decrease significantly during the period of application. For such cases, it may be advisable to apply a variation of equation(s) and break the loading into several distinct periods and approximate each of these by an equivalent number of uniform stress cycles.

For low values of pore pressure ratio it is found that the coefficient of volume compressibility m_{v_3} is fairly constant. However, for pore pressure ratios larger than about 60% the values of compressibility have been found to be influenced by both relative density and pore pressure ratio. Seed et al (1975) found that the variation of m_{v_3} with these two variables could be approximated by the relationship

$$\frac{m_{v_3}}{m_{v_{3o}}} = \frac{e^y}{1 + y + \frac{1}{2} y^2} \tag{6}$$

where $m_{v_{3o}}$ is the compressibility for zero pore pressure ratio

$y = A(r_u)^B$

$A = 5(1.5 - D_r)$

$B = 3(2)^{-2D_r}$

and D_r is the relative density. In any analyses, the value of m_{v3} does not decrease from the highest value attained.

The final parameters necessary to perform the analysis are the coefficients of permeability; these may be determined either from field or laboratory measurements or from various empirical relationships which relate grain size characteristics and the coefficients of permeability, Seed et al (1975). It is found that the coefficients of permeability do not vary to a marked extent during pore pressure generation or dissipation and therefore can be considered constant throughout the analysis.

FINITE ELEMENT ANALYSIS

Consider a volume of soil V with a surface S. Suppose that a portion of this surface S_D is free to drain, so that on S_D the excess pore pressure u will be zero. Suppose also that the remainder of the surface S_I is impermeable so that the component of the pore water velocity vector normal to S_I will vanish. It is well known that finding a pore pressure field u, which satisfies equation (3) and these boundary conditions is equivalent to finding a pore pressure field which satisfies the boundary conditions on S_p such that

$$\int \frac{1}{\gamma_\omega} \{\nabla \delta u\}^T \ [k] \ \{\nabla u\} + m_{v_3} \delta u \left(\frac{\partial u}{\partial t} - \psi \right) d_V = 0 \qquad (7)$$

for any virtual pore pressure field, i.e. any pore pressure field δu which is zero or S_D.

A numerical solution of equation (7) can be found by the finite element method, (Zienkiewicz (1971) and it is found that equation (7) may be approximated by a set of equations having the form

$$\{\delta u\}^T \left([A]\{u\} + [D](\{\frac{du}{dt}\} - \{\Psi\}) \right) = 0 \qquad (8)$$

where $\{\delta u\}$, $\{u\}$, $\{\Psi\}$ denote the vectors of nodal values of δu, u, ψ respectively. Additional details of the finite element solution are given in Appendix A.

Equation (8) is true for arbitrary variations $\{\delta u\}$ so that

$$[A]\{u\} + [D](\{\frac{du}{dt}\} - \{\Psi\}) = \{0\} \qquad (9)$$

It can be seen from Appendix A that the matrix [A] is constant and that the matrix [D] which depends upon the element compressibilities m_{v_3} will vary with pore pressure ratio.

Equation (9) can be regarded as a set of ordinary differential equations and may be approximately integrated over the interval t, t + Δt as follows

$$[A](\beta\{U_{t+\Delta t}\} + \alpha\{U_t\})\Delta t + \bar{D}(\{U_{t+\Delta t}\} - \{U_t\} - \{\bar{\Psi}\}\Delta t) = \{0\} \qquad (10)$$

where $\alpha + \beta = 1$

and the subscripts t, t + Δt indicate the values of a variable at times, t, t + Δt respectively and the bar denotes an average value over the interval t, t + Δt (usually calculated from the average pore pressure ratio over that interval). Different values of α correspond to different approximations; if $\alpha \leq 0.5$ the integration procedure is always stable. In the computer programs described in this report it will usually be assumed that = 0.5 (Crank Nicholson method). Equation (10) may be rewritten in the form

$$(\beta\Delta t[A] + [\bar{D}])\{u_{t+\Delta t}\} = \{b\} \qquad (n)$$

where $\{b\} = [\bar{D}](\{u_t\} + \{\bar{\Psi}\}\Delta t) - \alpha\Delta t[A]\{u_t\}$.

If it is assumed that the compressibility of the soil does not vary with pore pressure ratio the matrix $[\bar{D}]$ is constant and equation (11) can be used to march the solution forward in time. If the matrix $[\bar{D}]$ varies, equations (11) may be solved iteratively by using the current best estimate of pore pressure ratio to calculate $[\bar{D}]$ and repeating this procedure until the process converges.

It should be noted that the value of θ which determines the pore pressure generation function Ψ can be assigned different values at each node, corresponding to the appropriate rate of pore pressure development for the soil type and loading conditions involved.

DESCRIPTION OF COMPUTER PROGRAM

A computer program GADFLEA (Generation And Dissipation For cycling Loads and Earthquake Analysis) has been written to perform the finite element calculations described earlier in this report. The program consists of a main program GADFLEA and sixteen subroutines. The action of these subroutines is described in Table 1 and the logic of the program is shown in Fig. 2. A listing of the program is included in Appendix C.

Convergence Criterion

Equation (10) is nonlinear and must therefore in general be solved iteratively. In this program convergence is said to occur when successive estimates of all nodal values of the pore pressure ratios u/σ_o' are less than some specified quantity DIFFA. In order to prevent the formation of an infinite loop the number of iterations that can be performed is limited to ITA; if more than this number of iterations is needed for convergence a warning is printed out and the calculation is terminated.

Details of Storage

This program is set up so that it can have

- a maximum number of nodes NA
- a maximum semi-bandwidth MA
- a maximum number of elements NEA
- a maximum number of boundary conditions NBCA
- a maximum number of time increments NTIA

Values of these variables are set in the first five statements of the program. These statements must always be consistent with the dimension statements and the labelled common statements. Thus if statement 1 reads

$$NA = \overline{NA} \quad \text{etc.}$$

then it is necessary that the labelled common statements should have the

TABLE 1. SUBROUTINES IN PROGRAM GADFLEA

SUBROUTINE	DESCRPITION	DETAILS
OPTION	Read the problem title and option data and convergence criteria	Read problem title (TITLE). Read program (IOPT plane and radial flow). JOPT constant or variable compressibility. Read convergence criteria (ITA, DIFA).
NDATA	Reads nodal data and stores it in labelled common block/NDATAB/	Reads the number of nodes (NODE) then reads the node number (J) the nodal coordinates [X(J),Y(J)], the initial effective stress [ESV(J)], the number of cycles necessary to cause liquefaction under undrained conditions [ELIQ(J)] and pore pressure generation constants (THETA(J)}.
BDATA	Reads boundary data and stores it in the labeled common block / BDATAB /	Reads the number of boundary conditions (NBC) and the nodes at which the excess pore pressure is zero IBC(I).
QDATA	Reads the earthquake data and stores it in the labeled common block /QDATA/.	Reads the number of equivalent cycles (ENQ) and the duration of shaking (TD)
TDATA	Reads the time step data and stores it in the labeled common block /TDATAB/.	Reads the number of time increments (NTI), reads the number of steps of a given time increment [ITI(I)] the value of the time increment [PELT(I)], and the interval at which printout is required [DTPR(I)].
STIFF	Calculates the element matrices $[A_{ijkl}]$ and the global matrix [A]	Since permeability does not change this need only be called once.
TRISTF	Calculates the "stiffness" matrix for a triangular element	
DGNL	Calculates the element matrices $[D_{ijkl}]$ and the global matrix $[DIAG] = [\bar{D}]$	If the compressibility changes this needs to be called each for iteration in the solution process.

SUBROUTINE	DESCRIPTION	DETAILS
SETMTX	Calculates the matrix $[AAQ] = \beta \Delta t \; [A] + [\bar{D}]$	This portion needs to be calculated for each iteration in the solution process
SETLVI	Calculates the position of the load $\{BQ1\}$ = vector $(-\alpha \Delta t [A] + [\bar{D}]) \; \{u\}_t$	
SETLV2	Calculates the portion of the load vector $\{BQ2\} = [\bar{D}] \; \{\bar{\psi}\} \; \Delta t$	
DUG	Calculates the first estimate of $\{\Psi\}\Delta t$	
SLOPE	Calculates the value of ψ	
FUNCTION FAC(X, DR)	Calculates the current value of compressibility	
SYMSOL	Triangulises and solves equations	

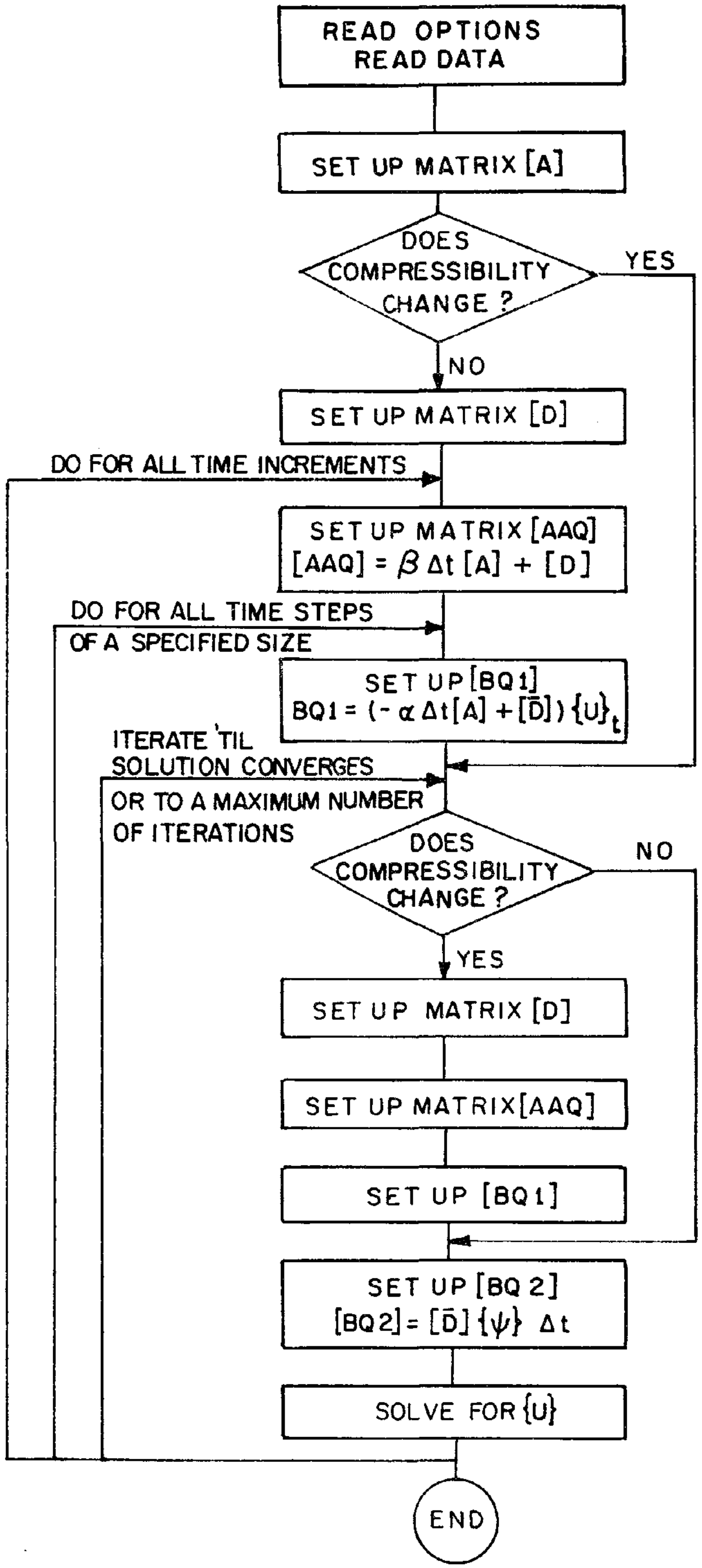

Fig. 2 FLOW CHART FOR PROGRAM

form

COMMON / NDATAB / X($\overline{\text{NA}}$), Y($\overline{\text{NA}}$), ESV($\overline{\text{NA}}$), ELIQ($\overline{\text{NA}}$), THETA($\overline{\text{NA}}$)
...
COMMON / EDATAB / NN(4,$\overline{\text{NEA}}$), PERM(2,$\overline{\text{NEA}}$) RMV($\overline{\text{NEA}}$), RMVO($\overline{\text{NEA}}$), RMVN($\overline{\text{NEA}}$),
DR($\overline{\text{NEA}}$)
...
COMMON / BDATAB / NBC, IBC($\overline{\text{NBCA}}$)
COMMON / TDATAB / NTI, ITT($\overline{\text{NTIA}}$), DELT($\overline{\text{NTIA}}$) DTPR($\overline{\text{NTIA}}$)

in the main program and all the subroutines and that the dimension statements should have the form

DIMENSION AAQ($\overline{\text{NA}}$,$\overline{\text{MA}}$), BQ($\overline{\text{NA}}$), BQ1($\overline{\text{NA}}$) BQ2($\overline{\text{NA}}$)
DIMENSION U($\overline{\text{NA}}$), UQ($\overline{\text{NA}}$), UN($\overline{\text{NA}}$), UGT($\overline{\text{NA}}$), UR($\overline{\text{NA}}$)

Input Details

Data for program GADFLEA must be input as shown in Table 2. An illustrative example is given later. The data may be in any set of consistent units.

Example Problem

The program outlined in this report has the capacity to analyse a wide range of problems and has to date been used to analyse problems as diverse as

(i) The generation and dissipation of pore pressure in a horizontally stratified soil deposit subjected to earthquake loading.

(ii) The use of gravel drains to stabilize potentially liquefiable soil deposits.

(iii) The generation and dissipation of pore pressure in earth and rock fill dams subjected to earthquake loading.

(iv) The generation and dissipation of pore pressure in marine deposits subjected to wave action.

TABLE 2. SEQUENCE FOR READING DATA

Number of Data Cards	Description	Format
	OPTION DATA	
1	Read problem title	12A6
	Read program options	3I5, 5X, F10.5
	IOPT, JOPT, ITA, DIFFA IOPT = 1 for plane flow IOPT = 2 for axisymmetric flow JOPT = 1 for a material with constant compressibility JOPT = 2 for a material with variable compressibility ITA = Maximum number of iterations in value solution DIFFA = Maximum difference between successive estimates of the base pressure ratio	
	NODAL DATA	
1	Read number of nodes (NODE)	I5
NODE	Read nodal data, read nodal number (J), nodal coordinates (X(J), Y(J)), nodal effective stress (ESV(J)), number of cycles to liquefaction (ELIQ(J)), and pore pressure generation constants (THETA(J)).	I5, 5X, 5F10.3
Sufficient cards for NBC boundary conditions	Read nodes at which the excess pore water pressure is zero (IBC(I))	16I5
	EARTHQUAKE DATA	
1	Read number of equivalent uniform stress cycles (ENQ) and duration of earthquake (TD)	2F10.3

Number of Data Cards	Description	Format
	TIME STEP DATA	
1	Read number of different time increments (NT1)	I5
NT1	Read number of time steps of a given size, (ITI(I)), size of these steps (DELT(I)), interval at which results should be printed out DTPR(I)	I5, 5X, 2F10.4

(v) The potential for liquefaction under off-shore structures subjected to storm loading.

In order to illustrate the use of the program it will be used to analyse the generation and dissipation of pore pressures in the horizontally stratified soil deposit shown in Fig. 3a.

The site consists of 20' of material with a relative density of 50% overlying 30' of a material with relative density 70% overlying 30' of material with relative density of 85% overlying 170' of material with relative density 90%. The initial vertical effective stresses can be calculated from a static analysis and are shown in Fig. 2b. Notice that the vertical stress at the surface has been set equal 0.001 to avoid difficulties in calculating the pore pressure ratio at the surface.

The site is assumed to be subjected to an earthquake of magnitude 7-1/2 and a duration of 30 seconds. The time history of shear stresses at different depths can be found from a site response analysis (Schnabel et al, 1972). Using well established methods the irregular time history of stresses may be converted to an equivalent history of uniform stress cycles and the average shear stress levels at different depths may be established and an equivalent number of cycles of stress application calculated. Then using the average shear stress levels and the cyclic strength curves for the corresponding soil layer, the number of cycles required to cause liquefaction are established; these are shown in Fig. 3c.

The finite element mesh shown in Fig. 2d was used to obtain a numerical solution to this problem. The problem was one of plane shear and so IOPT = 1, the compressibility was assumed to vary so that JOPT = 2, the maximum number of iterations to obtain convergence chosen as 10 and convergence was assumed to occur when successive estimates of the pore pressure ratio were all less than 0.005. There were 14 nodes and the coordinates, initial effective stresses and number of cycles required to cause liquefaction are shown in Fig. 3. Units of length, weight and time were feet, lbs. wt., and seconds respectively so that the unit weight of water was 62.4 lbs/ft^3. Six quadrilateral elements were used as shown in Fig. 2d and it was assumed that the material in each of these had a compressibility of 10^{-6} ft^2/lb and equal horizontal and vertical permeabilities of 3.3×10^{-4} ft/sec. The relative density for each of the elements is shown in Fig. 2a. The base and two sides were assumed impermeable and so the only two nodes (1,2) had a prescribed excess pore water pressure of zero. For this earthquake the number of equivalent cycles was assumed to be 30 and the duration of the earthquake was 30 seconds. Two time increments were used in the analysis and the first stage of the earthquake was analyzed using 30 one-second steps with values printed every 6 seconds; subsequently the earthquake was analyzed for an additional 5 time steps of 6 seconds and again results were printed after every 6 seconds. The data for these analyses is shown in Appendix B. It should be noted that the above problem is merely illustrative. In any practical situation, the error due to the choice of time step and spatial discretization should be the subject of an independent study.

The values of the pore pressure ratios obtained from the program is presented in Appendix B. It can be observed that the material at nodes 3, 4 has essentially liquefied after 24 seconds and remains in that state until the end of the earthquake. After the earthquake is over the pore pressures dissipate quite slowly and the pore pressure ratio at nodes 3 and 4 has only dropped to .9 after a further 30 seconds.

It is interesting to observe the effect of the time step used on the accuracy of the analysis. The problem described in this section was analyzed for the duration of the earthquake using the finite element discretization shown in Fig. 2 using time steps of 1, 2, 3, 5 and 10 seconds. The

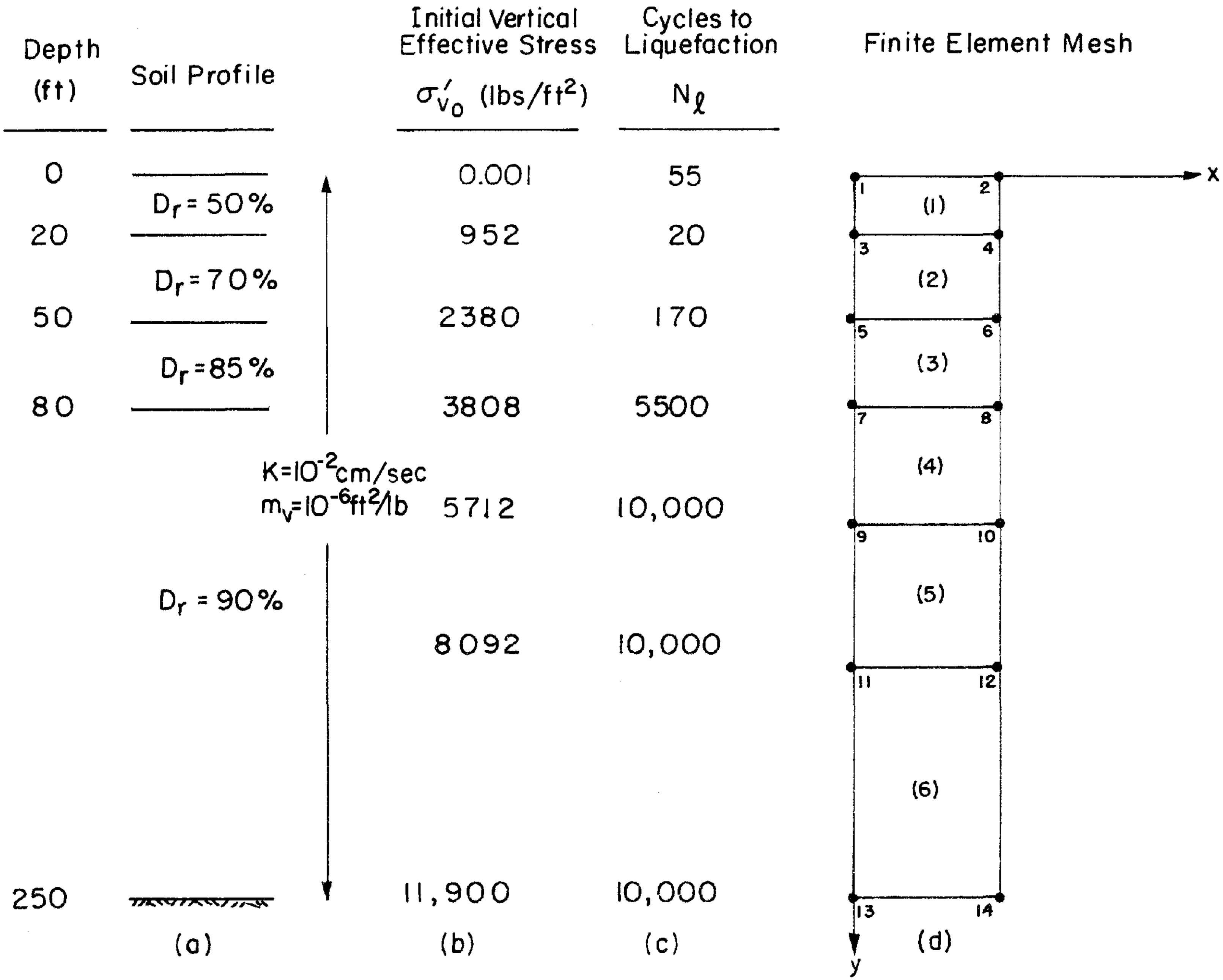

Fig. 3 DETAILS FOR ILLUSTRATIVE EXAMPLE

pore pressure ratio at node 3 for each of these analyses is shown in Fig. 4. It can be seen that for DT = 1, 2, 3 seconds the results are undistinguishable, the results for a time step of 5 seconds is extremely close to these results and even a time step of 10 seconds seems to give adequate accuracy.

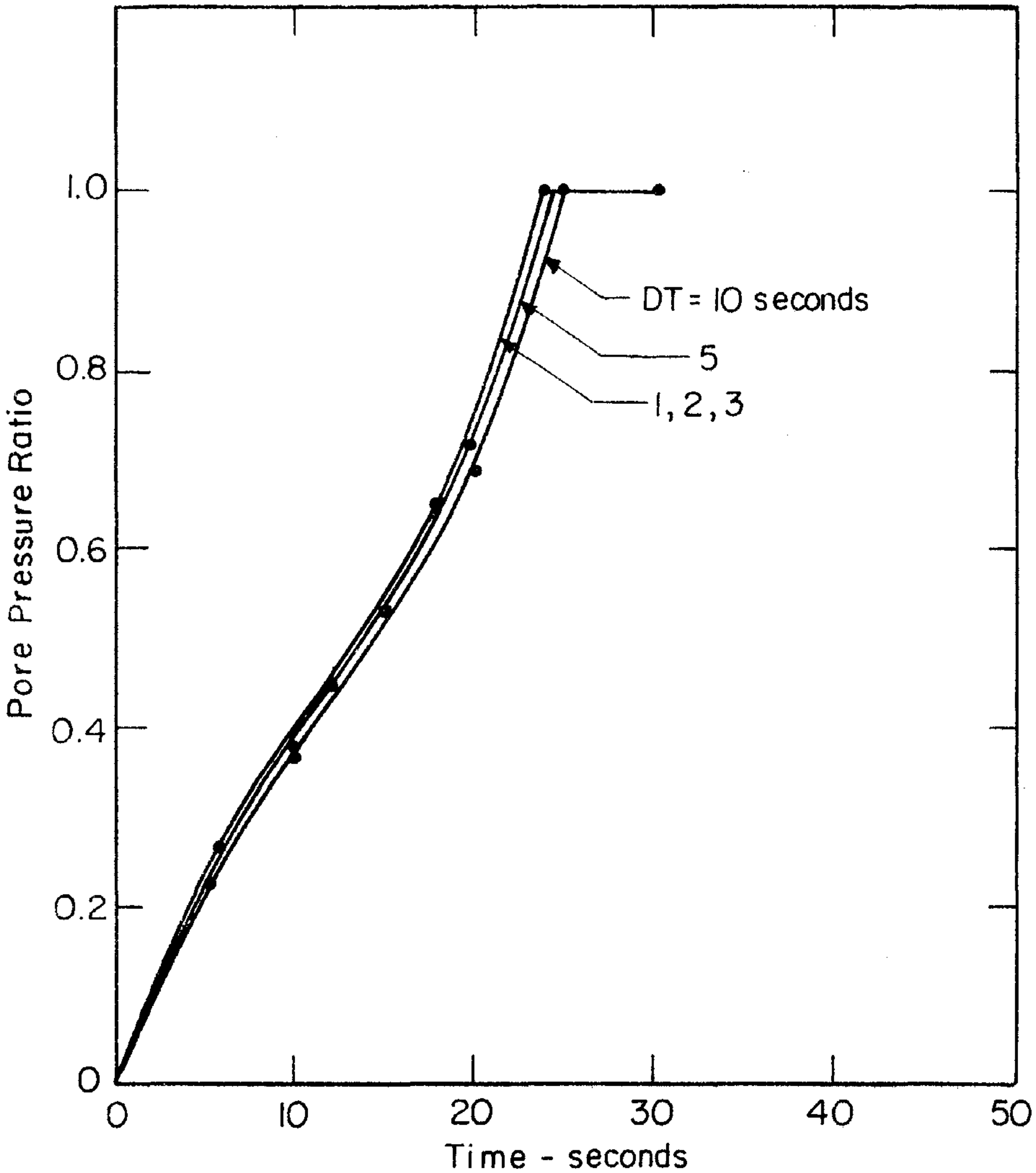

Fig. 4 EFFECT OF TIME STEP SIZE

REFERENCES

Ambrasys, N. and Sarma, S. (1969) "Liquefaction of Soil Induced by Earthquakes Bull. Seis. Soc. of America, Vol. 59, No. 2, 1969, pp. 651-664.

Rahman, M. S., Seed, H. B. and Booker, J. R. (1976), "Liquefaction Analysis for Offshore Oil Tanks", Geotechnical Engineering Report, University of California, Berkeley, September, 1976.

Schnabel et al. (1972) "SHAKE: A Computer Program for Earthquake Response Analysis of Horizontally Layered Sites," Earthquake Engineering Research Center, Report No. EERC 72-12, University of California, Berkeley, December.

Seed, H. B. and Booker, J. R. (1976) "Stabilization of Potentially Liquefiable Sand Deposits Using Gravel Drain Systems," Earthquake Research Center, Report No. EERC 76-10, University of California, Berkeley, April 1976.

Seed, H. B. and Idriss, I. M. (1967) "Analysis of Soil Liquefaction: Niigata Earthquake," Journal of the Soil Mechanics and Foundations Division, ASCE Vol. 93, No. SM3, Proc Paper 5233, May 1967, pp. 83-108.

Seed, H. B. and Idriss, I. M. (1971) "Simplified Procedure for Evaluating Soil Liquefaction Potential," Journal of the Soil Mechanics and Foundations Division, ASCE, Vol. 97, No. SM9, Proc. Paper 8371, September 1971, pp. 1249-1273.

Seed, H. B., Idriss, I. M., Makdisi, F., and Banerjee, N. (1975) "Representation of Irregular Stress Time Histories by Equivalent Uniform Stress Series in Liquefaction Analyses," Earthquake Engineering Research Center, Report No. EERC 75-29, University of California, Berkeley, October.

Seed, H. B. and Lee, K. L. (1966) "Liquefaction of Saturated Sands during Cyclic Loading," Journal of the Soil Mechanics and Foundations Division, ASCE, Vol. 92, No. SM6, Proc. Paper 4973, November, 1966, pp. 105-134.

Seed, H. B., Martin, P. P. and Lysmer, J. (1975) "The Generation and Dissipation of Pore Water Pressures During Soil Liquefaction," Earthquake Engineering Research Center, Report No. EERC 75-26, University of California, Berkeley, August 1975.

Wong, Robert T., Seed, H. Bolton and Chan, Clarence K. (1975) "Cyclic Loading Liquefaction of Gravelly Soils," Journal of the Geotechnical Engineering Division, ASCE, Vol. 101, No. GT6, Proc. Paper 11396, June.

Zienkiewicz, O. C. (1971) "The Finite Element Method in Engineering Science," McGraw-Hill.

Appendix A

This report will be restricted to problems of plane flow or flow with radial symmetry. It was found most convenient to divide the soil mass into triangular or quadrilateral elements of a simple type although there is no difficulty in extending the treatment to more complicated elements.

Plane Flow

Consider first the problem of plane flow in the x, y plane. If in the triangular element shown in Fig. 5, it is assumed that the excess pore pressure values linearly, then

$$u = u_i + b(x-x_i) + c(y-y_i) \tag{12}$$

where $b = b_i u_i + b_j u_j + b_R u_R$

$c = c_i u_i + c_j u_j + c_R u_R$

and $b_i = \dfrac{y_j - y_R}{2\Delta_{ijk}}$ etc.

$c_i = \dfrac{x_j - x_k}{2\Delta_{ijk}}$ etc.

where Δ_{ijk} = Area of the triangle i, j, k

$= 1/2(x_k - x_i)x(y_i - y_j) - (y_k - y_i)(x_i - x_j)$

Thus for a unit slice of the material

$$\int_{\substack{\text{element}\\ ijk}} \frac{1}{\gamma_\omega} \{\nabla\delta u\}[k]\{\nabla u\} \; dV \approx \{\delta u_{ijk}\}^T [A_{ijk}] \; \{u_{ijk}\} \tag{13}$$

where $\{u_{ijk}\} = (u_i, u_j, u_k)^T$ is the vector of nodal pore pressures

and $[A_{ijk}] = \dfrac{1}{\gamma_\omega} [E]^T [k] [E] \times \Delta_{ijk}$ where $[E] = \begin{bmatrix} b_i & b_j & b_k \\ c_i & c_j & c_k \end{bmatrix}$

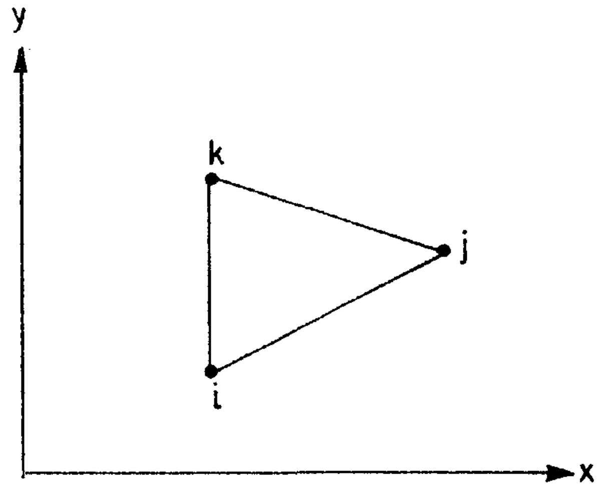

Fig. 5 TRIANGULAR ELEMENT
(Nodes numbered anti-clockwise)

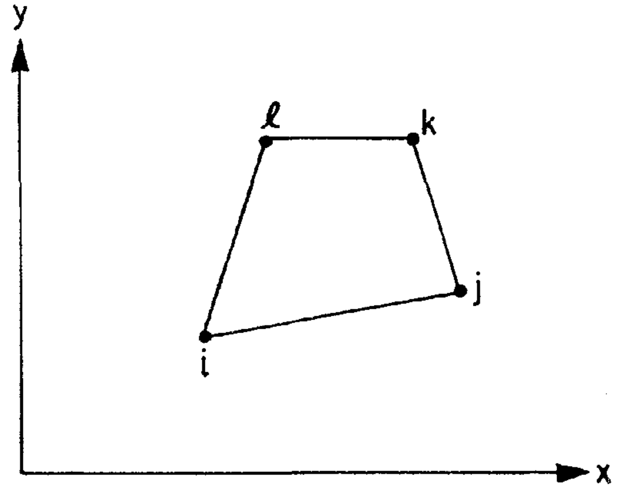

Fig. 6 QUADRILATERAL ELEMENT
(Nodes numbered anti-clockwise)

It is possible to obtain an approximation for the remaining portion of equation (7) similarly. However, it was found to be simpler to assume an approximation of the form

$$\int_{\substack{\text{element}\\ ijk}} m_{v3}\delta u \frac{\partial u}{\partial t} - \psi \, dV \approx \{\delta u_{ijk}\}^T[D_{ijk}](\{\frac{du_{ijk}}{dt}\} - \{\Psi_{ijk}\}) \qquad (14)$$

where $\{\delta U_{ijk}\}$ was defined previously $\{\Psi_{ijk}\} = (\psi_i, \psi_j, \psi_R)$

$$[D_{ijk}] = \frac{m_{v3}\Delta ijk}{3} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

and where m_{v3} denotes the average value of compressibility over the element.

So far the development of the theory has been restricted to triangular elements. A quadrilateral element of the type shown in Fig. 6 may also be used. It can be seen that such a quadrilateral may be broken into two triangles in two ways and thus it seems reasonable to make the approximation

$$\int \frac{1}{\gamma_\omega} \{\nabla\delta u\}[k]\{u\} \, dV \approx \{\delta u_{ijkl}\}^T[A_{ijkl}]\{u_{ijkl}\} \qquad (15)$$

where $\{u_{ijkl}\} = (u_i, u_j, u_k, u_l)^T$

and $[A_{ijkl}] = \frac{1}{2}([A_{ijk}] + [A_{jkl}] + [A_{kli}] + [A_{lji}])$

Similarly it seems reasonable to make the approximation

$$m_{v_3} \; \delta u\left(\frac{\partial u}{\partial t} - \psi\right)dV = \{\delta u_{ijkl}\}[D_{ijkl}] \; \frac{\{du_{ijkl}\}}{dt} - \{\Psi_{ijkl}\} \qquad (16)$$

where $[D_{ijkl}] = \frac{m_{v3}\Delta_{ijkl}}{4} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$\{\Psi_{ijkl}\} = (\psi_i, \psi_j, \psi_k, \Psi_l)$

$$\Delta_{ijkl} = \text{Area of quadrilateral } ijkl$$

and m_{v_3} denotes the average value of compressibility over the element.

The approximation of equation (7) for the entire soil body can be found by summing the approximations for all the individual elements and thus obtaining equation (8).

Flow with Radial Symmetry

It is sometimes of interest to analyze problems which exhibit radial symmetry. These problems are clearly related to those of plane flow and thus if it is assumed that in Figs (5 and 6) x denotes the radial coordinate, it can be shown that if a slice of angular span, one radian is considered then

$$\{\nabla\delta u\}[k]\{u\}\ dV \approx \{\delta U\}^T[D_{radial}](\frac{\{dU\}}{dt} - \{\Psi\}$$

where $[A_{radial}] = \bar{r}[A_{plane}]$

$[D_{radial}] = \bar{r}[D_{plane}]$

and $\bar{r} = \frac{1}{3}(x_i + x_j + x_k)$ for a triangular element

$= \frac{1}{4}(x_i + x_j + x_k + x_l)$ for a quadrilateral element.

APPENDIX B

INPUT AND OUTPUT DETAILS FOR THE EXAMPLE PROBLEM

INPUT DATA FOR EXAMPLE PROBLEM

DATA TITLE

EXAMPLE PROBLEM

OPTION DATA

JOPT	JOPT	ITA	DIFFA
2	1	10	.005

NODAL DATA

NODE

14

J	K(J)	Y(J)	ESV(J)	ELIQ(J)	THETA(J)
1	0.	0.	.001	55.	.7
2	1.	0.	.001	55.	.7
3	0.	20.	952.	20.	.7
4	1.	20.	952.	20.	.7
5	0.	50.	2380.	170.	.7
6	1.	50.	2380.	170.	.7
7	0.	80.	3808.	5500.	.7
8	1.	80.	3808.	5500.	.7
9	0.	120.	5712.	10000.	.7
10	1.	120.	5712.	10000.	.7
11	0.	170.	8092.	10000.	.7
12	1.	170.	8092.	10000.	.7
13	0.	250.	11900.	10000.	.7
14	1.	250.	11900.	10000.	.7

UNIT WEIGHT OF WATER

GAMAW

62.4

ELEMENT DATA

NUMEL

6

IE	NN(IIE)				PERM(IIE)		RMV(IE)	DR(IE)
1	1	2	4	3	.000001	.000328	.000328	.50
2	3	4	6	5	.000001	.000328	.000328	.70
3	5	6	8	7	.000001	.000328	.000328	.85
4	7	8	10	9	.000001	.000328	.000328	.90
5	9	10	12	11	.000001	.000328	.000328	.90
6	11	12	14	13	.000001	.000328	.000328	.90

BOUNDARY CONDITION DATA

NBC
2

IBC(I)	
1	2

EARTHQUAKE DATA

ENQ	TD
30.	30.

TIME STEP DATA

NTI
2

ITT(I)	DELT(I)	DTPR(I)
30	1.	6.
5	6.	6.

OUTPUT FOR EXAMPLE PROBLEM

```
    EXAMPLE PROBLEM

IOPT   =  1,-----PLANE PROBLEM

JOPT   =  2,-----VARYING COMPRESSIBILITY

ITA    =  10-----MAXIMUM NO. OF ITERATIONS

DIFFA =.005-----CONVERGENCE TOLERANCE

--------------------------------------------------------------------------


NUMBER OF NODES =    14


                                                      CYCLES TO
NODE  X-COORDINATE  Y-COORDINATE  VERTICAL STRESS   LIQUEFACTION      THETA

   1          0.           0.              .001        55.000          .700
   2      1.000            0.              .001        55.000          .700
   3          0.         20.000          952.000       20.000          .700
   4      1.000          20.000          952.000       20.000          .700
   5          0.         50.000         2380.000      170.000          .700
   6      1.000          50.000         2380.000      170.000          .700
   7          0.         80.000         3808.000     5500.000          .700
   8      1.000          80.000         3808.000     5500.000          .700
   9          0.        120.000         5712.000    10000.000          .700
  10      1.000         120.000         5712.000    10000.000          .700
  11          0.        170.000         8092.000    10000.000          .700
  12      1.000         170.000         8092.000    10000.000          .700
  13          0.        250.000        11900.000    10000.000          .700
  14      1.000         250.000        11900.000    10000.000          .700

--------------------------------------------------------------------------
```

```
UNIT WEIGHT OF WATER =     62.4000



NUMBER OF ELEMENTS =     6



ELEMENT  NODE   NODE   NODE   NODE   COMPRESSIBILITY  PERMEABILITY  PERMEABILITY    REL. DENSITY
            1      2      3      4                         XX            YY


      1     1      2      4      3       .10000E-05    .32800E-03    .32800E-03     .50000E+00
      2     3      4      6      5       .10000E-05    .32800E-03    .32800E-03     .70000E+00
      3     5      6      8      7       .10000E-05    .32800E-03    .32800E-03     .85000E+00
      4     7      8     10      9       .10000E-05    .32800E-03    .32800E-03     .90000E+00
      5     9     10     12     11       .10000E-05    .32800E-03    .32800E-03     .90000E+00
      6    11     12     14     13       .10000E-05    .32800E-03    .32800E-03     .90000E+00
------------------------------------------------------------------------------------------------
```

```
NUMBER OF BOUNDARY CONDITIONS =     2



NODES WITH AN EXCESS PORE WATER PRESSURE OF ZERO


   1   2
--------------------------------------------------------------------------------

NO. OF EQUIVT. CYCLES      =    30.000

DURATION OF CYCLIC ACTION =    30.000

--------------------------------------------------------------------------------


NUMBER OF TIME INCREMENTS =    2


NUMBER OF STEPS  STEP LENGTH   PRINT INTERVAL

       30          1.000          6.000
        5          6.000          6.000

--------------------------------------------------------------------------------
```

```
          -----PORE PRESSURE RATIO-----

----------------------------------------------------------------------------------------------------

AT TIME , T=      6.00  SECONDS

(  1)      0.      (  2)      0.      (  3)      .264  (  4)      .264  (  5)      .058
(  6)       .058   (  7)       .005   (  8)      .005  (  9)      .003  ( 10)      .003
( 11)       .003   ( 12)       .003   ( 13)      .003  ( 14)      .003  (
----------------------------------------------------------------------------------------------------

AT TIME , T=     12.00  SECONDS

(  1)      0.      (  2)      0.      (  3)      .446  (  4)      .446  (  5)      .096
(  6)       .096   (  7)       .009   (  8)      .009  (  9)      .005  ( 10)      .005
( 11)       .005   ( 12)       .005   ( 13)      .005  ( 14)      .005  (
----------------------------------------------------------------------------------------------------

AT TIME , T=     18.00  SECONDS

(  1)      0.      (  2)      0.      (  3)      .644  (  4)      .644  (  5)      .129
(  6)       .129   (  7)       .014   (  8)      .014  (  9)      .007  ( 10)      .007
( 11)       .007   ( 12)       .007   ( 13)      .007  ( 14)      .007  (
```

```
------------------------------------------------------------------------------------------------------------

 AT TIME , T=     24.00  SECONDS

 (  1)      0.      (  2)      0.      (  3)      .996  (  4)      .996  (  5)      .161
 (  6)       .161   (  7)       .018   (  8)      .018  (  9)      .009  ( 10)      .009
 ( 11)       .009   ( 12)       .009   ( 13)      .008  ( 14)      .008  (
------------------------------------------------------------------------------------------------------------

 AT TIME , T=     30.00  SECONDS

 (  1)      0.      (  2)      0.      (  3)      .996  (  4)      .996  (  5)      .203
 (  6)       .203   (  7)       .023   (  8)      .023  (  9)      .010  ( 10)      .010
 ( 11)       .010   ( 12)       .010   ( 13)      .010  ( 14)      .010  (




------------------------------------------------------------------------------------------------------------

 AT TIME , T=     36.00  SECONDS

 (  1)      0.      (  2)      0.      (  3)      .976  (  4)      .976  (  5)      .204
 (  6)       .204   (  7)       .026   (  8)      .026  (  9)      .010  ( 10)      .010
 ( 11)       .010   ( 12)       .010   ( 13)      .010  ( 14)      .010  (
------------------------------------------------------------------------------------------------------------
```

```
AT TIME , T=      42.00  SECONDS


(  1)      0.       (  2)      0.       (  3)      .955  (  4)      .955  (  5)      .204
(  6)       .204    (  7)       .028    (  8)      .028  (  9)      .011  ( 10)      .011
( 11)       .010    ( 12)       .010    ( 13)      .010  ( 14)      .010  (

----------------------------------------------------------------------------------------------


AT TIME , T=      48.00  SECONDS


(  1)      0.       (  2)      0.       (  3)      .935  (  4)      .935  (  5)      .205
(  6)       .205    (  7)       .031    (  8)      .031  (  9)      .011  ( 10)      .011
( 11)       .010    ( 12)       .010    ( 13)      .010  ( 14)      .010  (

----------------------------------------------------------------------------------------------


AT TIME , T=      54.00  SECONDS


(  1)      0.       (  2)      0.       (  3)      .916  (  4)      .916  (  5)      .205
(  6)       .205    (  7)       .033    (  8)      .033  (  9)      .011  ( 10)      .011
( 11)       .010    ( 12)       .010    ( 13)      .010  ( 14)      .010  (

----------------------------------------------------------------------------------------------


AT TIME , T=      60.00  SECONDS


(  1)      0.       (  2)      0.       (  3)      .897  (  4)      .897  (  5)      .206
(  6)       .206    (  7)       .036    (  8)      .036  (  9)      .011  ( 10)      .011
( 11)       .010    ( 12)       .010    ( 13)      .010  ( 14)      .010  (

----------------------------------------------------------------------------------------------
```

APPENDIX C

LISTING OF PROGRAM GADFLEA

```
      PROGRAM GADFLEA(INPUT,OUTPUT)
C
C
C
C        A PROGRAM FOR THE ANALYSIS OF PORE PRESSURE GENERATION
C
C        AND DISSIPATION DURING EARTHQUAKE OR CYCLIC LOADING
C
C
C
      COMMON/OPTIONB/IOPT,JOPT,ITA,DIFFA
      COMMON/NDATAB/X(30),Y(30),ESV(30),ELIQ(30),THETA(30),NODE
      COMMON/EDATAB/NN(4,20),PERM(2,20),RMV(20),RMVO(20),RMVN(20),DR(20)
     1,GAMAW,NUMEL
      COMMON/BDATAB/NBC,IBC(15)
      COMMON/TDATAB/NTI,ITI(10),DELT(10),DTPR(10)
      COMMON/QDATAB/ENQ,TD
      COMMON/LOVE/TIME
      DIMENSION AAQ(30,15),BQ(30),BQ1(30),BQ2(30)
      DIMENSION U(30),UO(30),UN(30),UGT(30),UR(30)
      DIMENSION A(30,15),DIAG(30,15)
C
C     THE FOLLOWING STATEMENTS ESTABLISH ALLOWABLE LIMITS OF STORAGE
C     ALL COMMON AND DIMENSION STATEMENTS SHOULD BE CONSISTENT
C     WITH THESE VALUES
C     -----------------------------------------------------------------
      NA=30
      MA=15
      NEA=20
      NBCA=15
      NTIA=10
C
C     DEFINE PROGRAM OPTIONS
C     -----------------------------------------------------------------
      PRINT 88
      PRINT 211
      CALL OPTION
C
C     READ NODAL DATA
C     -----------------------------------------------------------------
      PRINT 88
      CALL NDATA(NA)
C
C     READ ELEMENT DATA
C     -----------------------------------------------------------------
      PRINT 88
      PRINT 211
      CALL EDATA(NEA)
C
C     -----------------------------------------------------------------
      PRINT 88
      PRINT 211
      CALL BDATA(NBCA)
C
```

```
C     READ NUMBER OF EQUIVALENT CYCLES AND DURATION
C     -----------------------------------------------------------------
      PRINT 88
      CALL QDATA
C
C     READ TIME STEP DATA
C     -----------------------------------------------------------------
      PRINT 88
      CALL TDATA(NTIA)
      PRINT 88
      PRINT 211
      PRINT 62
      PRINT 88
      N=NODE
C
C     INITIALISE VARIABLES
C     -----------------------------------------------------------------
      DO 50 I=1,N
      U(I)=0.
      UN(I)=0.
      BQ(I)=0.
      UGT(I)=0.
 50   CONTINUE
C
C     SET UP STIFFNESS MATRIX, A
C     -----------------------------------------------------------------
      CALL STIFF(A,N,M,NA,MA)
      DO 20 IE=1,NUMEL
 20   RMVO(IE)=RMV(IE)
      IF(JOPT.EQ.1)CALL DGNL(DIAG,U,UN,N,M,NA,MA)
      TIME=0.0
      TTPR=0.
C
C     **CALCULATION LOOP FOR ALL TIME INCREMENTS
C     -----------------------------------------------------------------
      DO 100 L=1,NTI
      DT=DELT(L)
      TPR=DTPR(L)
      IF(JOPT.EQ.1)CALL SETMTX(A,DIAG,AAQ,N,M,NA,MA,DT,ALPHA)
      NSTEP=ITI(L)
      KKK=1
      IF(JOPT.EQ.1)CALL SYMSOL(AAQ,BQ,N,M,NA,MA,KKK)
C
C     ********CALCULATION LOOP FOR ALL TIME STEPS OF A GIVEN SIZE
C     -----------------------------------------------------------------
      DO 500 ISTEP=1,NSTEP
      TIME=TIME+DT
      IF(JOPT.EQ.1)CALL SETLV1(U,DIAG,A,BQ1,ALPHA,DT,N,NA,M,MA)
      ITR=1
C
C     **************ITERATION LOOP FOR SOLUTION OF EQUATIONS
C      ----------------------------------------------------------------
 111  CONTINUE
      ITR=ITR+1
```

```
      DO 222 I=1,N
 222  UO(I)=UN(I)
      IF(JOPT.EQ.1)GO TO 39
      CALL DGNL(DIAG,U,UN,N,M,NA,MA)
      CALL SETMTX(A,DIAG,AAQ,N,M,NA,MA,DT,ALPHA)
      KKK=1
C
C     TRIANGULISE MATRIX, AAQ
C      ---------------------------------------------------------------
      CALL SYMSOL(AAQ,BQ,N,M,NA,MA,KKK)
C
C     SET UP LOAD VECTOR
C      ---------------------------------------------------------------
      CALL SETLV1(J,DIAG,A,BQ1,ALPHA,DT,N,NA,M,MA)
 39   CONTINUE
      CALL SETLV2(DIAG,U,UN,UGT,DT,BQ2,N,NA,M,MA,ITR)
      DO 333 I=1,N
 333  BQ(I)=BQ1(I)+BQ2(I)
      KKK=2
C
C     SOLVE EQUATIONS
C      ---------------------------------------------------------------
      CALL SYMSOL(AAQ,BQ,N,M,NA,MA,KKK)
      DO 444 I=1,N
      IF(BQ(I).GT.ESV(I))BQ(I)=ESV(I)
      UN(I)=BQ(I)
 444  CONTINUE
      IF(ITR.GT.ITA)GO TO 555
      IF(ITR.EQ.2)GO TO 111
      DIFM=0.
      DO 666 I=1,N
      DIFF=ABS(UN(I)-UO(I))/ESV(I)
C
C     CHECK CONVERGENCE OF ITERATIVE SOLUTION
C      ---------------------------------------------------------------
      IF(DIFF.GT.DIFM) GO TO 777
      GO TO 888
 777  CONTINUE
      DIFM=DIFF
      KDIFM=I
 888  CONTINUE
      IF(DIFF.GT.DIFFA)GO TO 111
C
C     **************END OF ITERATION LOOP
C     ----------------------------------------------------------------
 666  CONTINUE
 555  CONTINUE
      DO 79 I=1,N
      U(I)=BQ(I)
      UR(I)=U(I)/ESV(I)
 79   CONTINUE
      TPRIN=ABS(TIME-TTPR-TPR)
      IF(TPRIN.LT.0.001)GO TO 155
      GO TO 255
```

```
 155  CONTINUE
      ITS=ITR-1
      PRINT 63,TIME
      PRINT 101,((I,UR(I)),I=1,N)
      IF(DIFM.LE.DIFFA)GO TO 198
      PRINT 412,KDIFM,ITS,DIFM
 198  CONTINUE
      PRINT 88
      TTPR=TTPR+TPR
 255  CONTINUE
      DO 981 IE=1,NUMEL
      IF(RMVO(IE).GT.RMVN(IE))RMVN(IE)=RMVO(IE)
 981  RMVO(IE)=RMVN(IE)
C
C     ********END OF CALCULATION LOOP FOR TIME STEPS
C     -------------------------------------------------------------------
  500 CONTINUE
C
C     **END OF CALCULATION LOOP FOR TIME INCREMENTS
C     -------------------------------------------------------------------
  100 CONTINUE
C
 211  FORMAT(1H1)
 412  FORMAT(/5X,*ERROR INFORMATION.....*,
     1/5X,*AT NODE =*,I5,/5X,*AFTER ITERATION =*,I5,/5X,*ERROR =*,F10.5)
 62   FORMAT(//15X,*-----PORE PRESSURE RATIO-----*//)
 63   FORMAT(//2X,*AT TIME , T=*,F10.2,2X,*SECONDS*//)
 88   FORMAT(/X,130(1H-))
  101 FORMAT(5(2X,1H(,I3,1H),F10.3))
      END
```

```
      SUBROUTINE OPTION
      COMMON/OPTION3/IOPT,JOPT,ITA,DIFFA
      DIMENSION TITLE(12)
      READ 11,TITLE
      PRINT 22,TITLE
      READ 1,IOPT,JOPT,ITA,DIFFA
      IF(IOPT.EQ.1)PRINT 33
      IF(IOPT.EQ.2)PRINT 44
      IF(JOPT.EQ.1)PRINT 55
      IF(JOPT.EQ.2)PRINT 66
      PRINT 77,ITA
      PRINT 88,DIFFA
1     FORMAT(3I5,5X,F10.5)
11    FORMAT(12A6)
22    FORMAT(//2X,12A6)
33    FORMAT(/2X,*IOPT   =  1,-----PLANE PROBLEM*)
44    FORMAT(/2X,*IOPT   =  2,-----AXISYMMETRIC PROBLEM*)
55    FORMAT(/2X,*JOPT   =  1,-----CONSTANT COMPRESSIBILITY*)
66    FORMAT(/2X,*JOPT   =  2,-----VARYING COMPRESSIBILITY*)
77    FORMAT(/2X,*ITA    =*,I4,*-----MAXIMUM NO. OF ITERATIONS*)
88    FORMAT(/2X,*DIFFA =*,F4.3,*-----CONVERGENCE TOLERANCE*)
      RETURN
      END
```

```
      SUBROUTINE NDATA(NA)
      COMMON/NDATAB/X(30),Y(30),ESV(30),ELIQ(30),THETA(30),NODE
      READ 1,NODE
      PRINT 2,NODE
      PRINT 15
      PRINT 5
      DO 10 IN=1,NODE
      READ 3,J,X(J),Y(J),ESV(J),ELIQ(J),THETA(J)
      PRINT 4,J,X(J),Y(J),ESV(J),ELIQ(J),THETA(J)
   10 CONTINUE
      ICH=1
      IF(NODE.GT.NA) GOTO 19
      RETURN
   19 CONTINUE
      PRINT 29
1     FORMAT(I5)
    2 FORMAT(//*  NUMBER OF NODES = *,I4//)
 3    FORMAT(I5,5X,5F10.3)
 4    FORMAT(2X,I5,5(4X,F10.3))
15    FORMAT(56X,*CYCLES TO*)
 5    FORMAT(2X,*NODE*,2X,*X-COORDINATE*,2X,*Y-COORDINATE*,2X,
     1*VERTICAL STRESS*,3X,*LIQUEFACTION*,6X,*THETA*/)
   29 FORMAT(*  DIAGNOSTICS FOR NDATA ICH = *,I4)
      END
```

```
      SUBROUTINE EDATA(NEA)
      COMMON/EDATAB/NN(4,20),PERM(2,20),RMV(20),RMVO(20),RMVN(20),DR(20)
     1,GAMAW,NUMEL
      READ 1,GAMAW
      PRINT 2,GAMAW
      READ 3,NUMEL
      PRINT 4,NUMEL
      ICH=1
      IF(NUMEL.GT.NEA) GOTO 19
      PRINT 8
      PRINT 9
      DO 10 JE=1,NUMEL
      READ 6,IE,(NN(I,IE),I=1,4),RMV(IE),(PERM(I,IE),I=1,2),DR(IE)
      PRINT 7,IE,(NN(I,IE),I=1,4),RMV(IE),(PERM(I,IE),I=1,2),DR(IE)
   10 CONTINUE
      RETURN
   19 CONTINUE
      PRINT 29,ICH
    1 FORMAT(F10.4)
    2 FORMAT(//,2X,*UNIT WEIGHT OF WATER = *,F10.4//)
3     FORMAT(I5)
    4 FORMAT(//,*  NUMBER OF ELEMENTS = *,I4//)
6     FORMAT(5I5,5X,4F10.8)
7     FORMAT(3X,5(2X,I4),3X,4(2X,E12.5))
    8 FORMAT(//2X,*ELEMENT*,2X,*NODE*,2X,*NODE*,2X,*NODE*,2X,*NODE*
     12X,*COMPRESSIBILITY*,2X,*PERMEABILITY*,2X,*PERMEABILITY*,
     23X,*REL. DENSITY*)
    9 FORMAT(5X,3X,6X,*1*,5X,*2*,5X,*3*,5X,*4*,25X,*XX*,12X,*YY*//)
   29 FORMAT(*  DIAGNOSTICS FOR EDATA ICH = *,I4)
      END
```

```
      SUBROUTINE BDATA(NBCA)
      COMMON/BDATAB/NBC,IBC(15)
      READ 1,NBC
      PRINT 2,NBC
      ICH=1
      IF(NBC.GT.NBCA) GOTO 19
      PRINT 5
      READ 3,(IBC(I),I=1,NBC)
      PRINT 4,(IBC(I),I=1,NBC)
      RETURN
   19 CONTINUE
      PRINT 29,ICH
1     FORMAT(I5)
    2 FORMAT(//*  NUMBER OF BOUNDARY CONDITIONS = *,I4//)
3     FORMAT(16I5)
    4 FORMAT(2X,20I4)
    5 FORMAT(//*  NODES WITH AN EXCESS PORE WATER PRESSURE OF ZERO*//)
   29 FORMAT(*  DIAGNOSTICS FOR BDATA ICH = *)
      END
```

```
      SUBROUTINE QDATA
      COMMON/QDATAB/ENQ,TD
      READ 2,ENQ,TD
      PRINT 3,ENQ,TD
2     FORMAT(2F10.3)
3     FORMAT(/2X,*NO. OF EQUIVT. CYCLES      =*,F10.3,
     1//2X,*DURATION OF CYCLIC ACTION =*,F10.3)
      RETURN
      END
```

```
      SUBROUTINE TDATA(NTIA)
      COMMON/TDATAB/NTI,ITI(10),DELT(10),DTPR(10)
      READ 1,NTI
      PRINT 2,NTI
      ICH=1
      IF(NTI.GT.NTIA) GOTO 19
      PRINT 3
      DO 10 I=1,NTI
      READ 5,ITI(I),DELT(I),DTPR(I)
      PRINT 4,ITI(I),DELT(I),DTPR(I)
   10 CONTINUE
      RETURN
   19 CONTINUE
      PRINT 29,ICH
1     FORMAT(I5)
    2 FORMAT(//*  NUMBER OF TIME INCREMENTS = *,I4)
3     FORMAT(//2X,*NUMBER OF STEPS*,2X,*STEP LENGTH*,
     13X,*PRINT INTERVAL*/)
4     FORMAT(8X,I4,4X,F10.3,5X,F10.3)
5     FORMAT(I5,5X,2F10.4)
   29 FORMAT(*  DIAGNOSTICS FOR TDATA ICH = *,I4)
      END
```

```
      SUBROUTINE STIFF(A,N,M,NA,MA)
      COMMON/NDATAB/X(30),Y(30),ESV(30),ELIQ(30),THETA(30),NODE
      COMMON/EDATAB/NN(4,20),PERM(2,20),RMV(20),RMVD(20),RMVN(20),DR(20)
     1,GAMAW,NUMEL
      COMMON/OPTIONB/IOPT,JOPT,ITA,DIFFA
      DIMENSION A(NA,MA)
      DIMENSION AUX(4,4)
      M=0
      N=NODE
      ICH=1
      RBAR=1.
      IF(N.GT.NA) GOTO 19
      DO 9 I=1,NA
      DO 9 J=1,MA
      A(I,J)=0.0
    9 CONTINUE
      DO 11 IE=1,NUMEL
      AREA=0.0
      DO 10 I=1,4
      DO 10 J=1,4
   10 AUX(I,J)=0.0
      N1=NN(1,IE)
      N2=NN(2,IE)
      N3=NN(3,IE)
      N4=NN(4,IE)
      FAC=1.0
      IF(IOPT.EQ.2)RBAR=(X(N1)+X(N2)+X(N3))/3.
      LIM=3
      RIM=3.0
      CALL TRISTF(N1,N2,N3,IE,AUX,AREA,0)
      IF(N4.EQ.0) GOTO 39
      FAC=0.5
      IF(IOPT.EQ.2)RBAR=(X(N1)+X(N2)+X(N3)+X(N4))/4.
      LIM=4
      RIM=8.0
      CALL TRISTF(N2,N3,N4,IE,AUX,AREA,1)
      CALL TRISTF(N4,N1,N2,IE,AUX,AREA,3)
      CALL TRISTF(N3,N4,N1,IE,AUX,AREA,2)
   39 CONTINUE
      DO 130 I=1,LIM
      II=NN(I,IE)
      DO 125 J=1,LIM
      JJ=NN(J,IE)
      KK=JJ-II+1
      IF(KK.GT.M) M=KK
      ICH=3
      IF(M.GT.MA) GOTO 19
      IF(KK.GT.0) A(II,KK)=A(II,KK)+AUX(I,J)*FAC*RBAR
  125 CONTINUE
  130 CONTINUE
   11 CONTINUE
      RETURN
   19 CONTINUE
      PRINT 29,ICH
```

```
      DO 130 I=1,LIM
      II=NN(I,IE)
      DIAG(II,1)=DIAG(II,1)+RMVN(IE)/RIM*AREA*RBAR
130   CONTINUE
11    CONTINUE
      RETURN
19    CONTINUE
      PRINT 29,ICH
29    FORMAT(2X,*DIAGNOSTIC FOR DGNL,   ICH=*,I5)
      END
```

```
      FUNCTION FACM(X,DR)
      IF(X.LT.0.05)GO TO 13
      IF(X.GT.1.)X=1.
      A=5.*(1.5-DR)
      B=3./(2.**(2.*DR))
      COM=A*(X**B)
      UP=EXP(COM)
      BEL=1.+COM+COM*COM/2.
      FACM=UP/BEL
      GO TO 14
13    CONTINUE
      FACM=1.
14    CONTINUE
      RETURN
      END
```

```
      SUBROUTINE TRISTF(N1,N2,N3,IE,AUX,AREA,KE)
      COMMON/NDATAB/X(30),Y(30),ESV(30),ELIQ(30),THETA(30),NODE
      COMMON/EDATAB/NN(4,20),PERM(2,20),RMV(20),RMVO(20),RMVN(20),DR(20)
     1,GAMAW,NUMEL
      DIMENSION AUX(4,4),EE(2,3)
      AREA1=AREA
      X1=X(N2)-X(N3)
      X2=X(N3)-X(N1)
      X3=-X1-X2
      Y1=Y(N2)-Y(N3)
      Y2=Y(N3)-Y(N1)
      Y3=-Y2-Y1
      AREA=0.5*(X2*Y3-Y2*X3)
      ICH=2
      IF(AREA.LT.0.0) GOTO 19
      DIV=0.5/AREA
      EE(1,1)=Y1*DIV
      EE(1,2)=Y2*DIV
      EE(1,3)=Y3*DIV
      EE(2,1)=X1*DIV
      EE(2,2)=X2*DIV
      EE(2,3)=X3*DIV
      DO 63 I=1,3
      II=I+KE
      IF(II.GT.4) II=II-4
      DO 63 J=I,3
      SUM=0.0
      JJ=J+KE
      IF(JJ.GT.4) JJ=JJ-4
      DO 64 K=1,2
   64 SUM=SUM+EE(K,I)*PERM(K,IE)*EE(K,J)
      AUX(II,JJ)=AUX(II,JJ)+SUM*AREA/GAMAW
      AUX(JJ,II)=AUX(II,JJ)
   63 CONTINUE
      AREA=AREA+AREA1
      RETURN
   19 CONTINUE
   29 FORMAT(*  DIAGNOSTICS FOR TRISTF ICH= *,I4)
      PRINT 29,ICH
      END
```

```
      SUBROUTINE DGNL(DIAG,U,UN,N,M,NA,MA)
      COMMON/NDATAB/X(30),Y(30),ESV(30),ELIQ(30),THETA(30),NODE
      COMMON/EDATAB/NN(4,20),PERM(2,20),RMV(20),RMVO(20),RMVN(20),DR(20)
     1,GAMAW,NUMEL
      COMMON/OPTIONB/IOPT,JOPT,ITA,DIFFA
      DIMENSION DIAG(NA,MA),U(NA),UN(NA)
      ICH=1
      RBAR=1.
      IF(N.GT.NA)GO TO 19
      DO 9 I=1,NA
      DO 9 J=1,MA
      DIAG(I,J)=0.
9     CONTINUE
      DO 11 IE=1,NUMEL
      N1=NN(1,IE)
      N2=NN(2,IE)
      N3=NN(3,IE)
      N4=NN(4,IE)
      IF(IOPT.EQ.2)RBAR=(X(N1)+X(N2)+X(N3))/3.
      LIM=3
      RIM=3.
      UAV=(U(N1)+U(N2)+U(N3))/3.
      UNAV=(UN(N1)+UN(N2)+UN(N3))/3.
      SAV=(ESV(N1)+ESV(N2)+ESV(N3))/3.
      UAV=(UAV+UNAV)/2.
      X1=X(N2)-X(N3)
      X2=X(N3)-X(N1)
      X3=-X1-X2
      Y1=Y(N2)-Y(N3)
      Y2=Y(N3)-Y(N1)
      Y3=-Y2-Y1
      AREA=0.5*(X2*Y3-Y2*X3)
      IF(N4.EQ.0)GO TO 39
      X1=X(N3)-X(N4)
      X2=X(N4)-X(N1)
      X3=-X1-X2
      Y1=Y(N3)-Y(N4)
      Y2=Y(N4)-Y(N1)
      Y3=-Y2-Y1
      AREA2=0.5*(X2*Y3-Y2*X3)
      AREA=AREA+AREA2
      ICH=2
      IF(AREA.LT.0)GO TO 19
      IF(IOPT.EQ.2)RBAR=(X(N1)+X(N2)+X(N3)+X(N4))/4.
      LIM=4
      RIM=4.
      UAV=(U(N1)+U(N2)+U(N3)+U(N4))/4.
      UNAV=(UN(N1)+UN(N2)+UN(N3)+UN(N4))/4.
      SAV=(ESV(N1)+ESV(N2)+ESV(N3)+ESV(N4))/4.
      UAV=(UAV+UNAV)/2.
39    CONTINUE
      RAT=UAV/SAV
      RMVN(IE)=RMV(IE)*FACM(RAT,DR(IE))
      IF(RMVO(IE).GT.RMVN(IE))RMVN(IE)=RMVO(IE)
```

```
      SUBROUTINE SETMTX(A,DIAG,AAQ,N,M,NA,MA,DT,ALPHA)
      COMMON/BDATAB/NBC,IBC(15)
      DIMENSION A(NA,MA),AAQ(NA,MA),DIAG(NA,MA)
      ALPHA=0.5
      BETA=1.0-ALPHA
      DO 11 I=1,N
      DO 10 J=1,M
      AAQ(I,J)=A(I,J)*DT*BETA+DIAG(I,J)
   10 CONTINUE
   11 CONTINUE
      DO 440 I=1,NBC
      K=IBC(I)
      DO 430 J=1,M
      L=K-J+1
      IF(L.GT.0) AAQ(L,J)=0.0
  430 AAQ(K,J)=0.0
  440 AAQ(K,1)=1.0
      RETURN
      END
```

```
      SUBROUTINE SETLV1(U,DIAG,A,BQ1,ALPHA,DT,N,NA,M,MA)
      COMMON/BDATAB/NBC,IBC(15)
      DIMENSION A(NA,MA),DIAG(NA,MA),U(NA),BQ1(NA)
      DO 300 I=1,N
      IM=I-1
      SUN=0.
      DUN=0.
      LIM=I+1-M
      IF(LIM.LT.1)LIM=1
      IF(LIM.GT.IM)GO TO 777
      DO 330 K=LIM,IM
      DUN=DUN+DIAG(K,I-K+1)*U(K)
      SUN=SUN+A(K,I-K+1)*U(K)
  330 CONTINUE
  777 CONTINUE
      LIM=M+I-1
      IF(LIM.GT.N)LIM=N
      DO 320 K=I,LIM
      DUN=DUN+DIAG(I,K-I+1)*U(K)
      SUN=SUN+A(I,K-I+1)*U(K)
  320 CONTINUE
      BQ1(I)=DUN-SUN*DT*ALPHA
  300 CONTINUE
      DO 331 I=1,NBC
      K=IBC(I)
  331 BQ1(K)=0.
      RETURN
      END
```

```
      SUBROUTINE SETLV2(DIAG,U,UN,UGT,DT,BQ2,N,NA,M,MA,ITR)
      COMMON/BDATAB/NBC,IBC(15)
      COMMON/NDATAB/X(30),Y(30),ESV(30),ELIQ(30),THETA(30),NODE
      COMMON/LOVE/TIME
      COMMON/QDATAB/ENQ,TD
      DIMENSION DIAG(NA,MA),BQ2(NA)
      DIMENSION U(NA),UGT(NA),UN(NA)
      EPS=.01
      DO 240 K=1,N
      ESO=ESV(K)
      ES=ESO-U(K)
      IF(ES.LT.EPS)U(K)=ESO
      EEE=ELIQ(K)
      IF(ITR.GT.2)GO TO 220
      CALL DUG(EEE,U(K),THETA(K),DT,ESO,UGH)
      GO TO 230
220   CONTINUE
      CALL SLOPE(EEE,U(K),UN(K),THETA(K),DT,ESO,UGH)
230   CONTINUE
      UGT(K)=UGH
      IF(ES.LT.EPS)UGT(K)=0.
240   CONTINUE
      DO 300 I=1,N
      IM=I-1
      BUN=0.
      LIM=I+1-M
      IF(LIM.LT.1)LIM=1
      IF(LIM.GT.IM)GO TO 777
      DO 330 K=LIM,IM
      BUN=BUN+DIAG(K,I-K+1)*UGT(K)
330   CONTINUE
777   CONTINUE
      LIM=M+I-1
      IF(LIM.GT.N)LIM=N
      DO 320 K=I,LIM
      BUN=BUN+DIAG(I,K-I+1)*UGT(K)
320   CONTINUE
      BQ2(I)=BUN
300   CONTINUE
      DO 331 I=1,NBC
      K=IBC(I)
331   BQ2(K)=0.
      RETURN
      END
```

```
      SUBROUTINE DUG(ENL,U,ALPHA,DT,ESV,UGH)
      COMMON/LOVE/TIME
      COMMON/QDATAB/ENQ,TD
      UGH=0.
      IF(TIME.GT.TD)GO TO 25
      BETA=1./ALPHA
      PI=3.14159265
      XX=1.
      RU=U/ESV
      ARG=0.5*(1.-COS(PI*RU))
      RN0=ARG**ALPHA
      DR=DT/TD*ENQ/ENL
      RN1=RN0+DR
      ARG=2.*RN1**BETA-1.
      IF(ARG.GT.XX)ARG=XX
      RU=0.5+ASIN(ARG)/PI
      UGH=RU*ESV-U
25    CONTINUE
      RETURN
      END
```

```
      SUBROUTINE SLOPE(EEE,U,UN,ALPHA,DT,ESV,UGH)
      COMMON/LOVE/TIME
      COMMON/QDATAB/ENQ,TD
      UGH=0.
      IF(TIME.GT.TD)GO TO 25
      BETA=1./ALPHA
      PI=3.14159265
      UM=(U+UN)/2.
      RU=UM/ESV
      IF(RU.LT.0.0001)RU=0.0001
      ARG=.5*(1.-COS(PI*RU))
      X=ARG**ALPHA
      XX=X**BETA
      UP=ESV*XX*ENQ*BETA
      BEL=PI*ABS(SQRT(XX-XX*XX))*X*TD*EEE
      DUT=UP/BEL
      UGH=DUT*DT
25    CONTINUE
      RETURN
      END
```

```
      SUBROUTINE SYMSOL(A,B,NN,MM,NA,MA,KKK)
      DIMENSION A(NA,MA),B(NA)
      EPS=0.000001
      ICH=1
      IF(NN.GT.NA) GOTO 19
      ICH=2
      IF(MM.GT.MA) GOTO 19
      IF(KKK.GT.1) GOTO 2000
 1000 DO 280 N=1,NN
      DO 260 L=2,MM
      ICH=3
      IF(ABS(A(N,1)).LT.EPS) GOTO 19
      C=A(N,L)/A(N,1)
      I=N+L-1
      IF(NN-I) 260,240,240
  240 J=0
      DO 250 K=L,MM
      J=J+1
  250 A(I,J)=A(I,J)-C*A(N,K)
  260 A(N,L)=C
  280 CONTINUE
 2000 DO 290 N=1,NN
      DO 285 L=2,MM
      I=N+L-1
      IF(NN-I) 290,285,285
  285 B(I)=B(I)-A(N,L)*B(N)
  290 B(N)=B(N)/A(N,1)
      N=NN
  300 N=N-1
      IF(N) 350,500,350
  350 DO 400 K=2,MM
      L=N+K-1
      IF(NN-L) 400,370,370
  370 B(N)=B(N)-A(N,K)*B(L)
  400 CONTINUE
      GOTO 300
  500 RETURN
   19 CONTINUE
      PRINT 29,ICH
   29 FORMAT(*DIAGNOSTICS FOR SYMSOL*,I4)
      END
```

EARTHQUAKE ENGINEERING RESEARCH CENTER REPORTS

EERC 67-1	"Feasibility Study Large-Scale Earthquake Simulator Facility," by J. Penzien, J. G. Bouwkamp, R. W. Clough and D. Rea - 1967 (PB 187 905)
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EERC 70-2 "Relationships Between Soil Conditions and Building Damage in the Caracas Earthquake of July 29, 1967," by H. B. Seed, I. M. Idriss, and H. Dezfulian - 1970 (PB 195 762)

EERC 70-3 "Cyclic Loading of Full Size Steel Connections," by E. P. Popov and R. M. Stephen - 1970 (Not available from NTIS)

EERC 70-4 "Seismic Analysis of the Charaima Building, Caraballeda, Venezuela," by Subcommittee of the SEAONC Research Committee, V. V. Bertero, P. F. Fratessa, S. A. Mahin, J. H. Sexton, A. C. Scordelis, E. L. Wilson, L. A. Wyllie, H. B. Seed, and J. Penzien, Chairman - 1970 (PB 201 455)

EERC 70-5 "A Computer Program for Earthquake Analysis of Dams," by A. K. Chopra - 1970 (AD 723 994)

EERC 70-6 "The Propagation of Love Waves Across Non-Horizontally Layered Structures," by J. Lysmer and L. A. Drake - 1970 (PB 197 896)

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EERC 71-7	"Inelastic Behavior of Steel Beam-to-Column Subassemblages," by H. Krawinkler, V. V. Bertero and E. P. Popov - 1971 (PB 211 335)
EERC 71-8	"Modification of Seismograph Records for Effects of Local Soil Conditions," by P. Schnabel, H. B. Seed and J. Lysmer - 1971 (PB 214 450)
EERC 72-1	"Static and Earthquake Analysis of Three Dimensional Frame and Shear Wall Buildings," by E. L. Wilson and H. H. Dovey - 1972 (PB 212 904)
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EERC 72-5	"Cyclic Behavior of Three Reinforced Concrete Flexural Members with High Shear," by E. P. Popov, V. V. Bertero and H. Krawinkler - 1972 (PB 214 555)
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EERC 73-7	"A Computer Program for Earthquake Analysis of Gravity Dams Including Reservoir Interaction," by P. Chakrabarti and A. K. Chopra - 1973 (AD 766 271)

EERC 73-8	"Seismic Behavior of Spandrel Frames - A Review and Outline for Future Research," by R. Razani and J. G. Bouwkamp - 1973 (PB 246 117)
EERC 73-9	"Earthquake Analysis of Structure-Foundation Systems," by A. K. Vaish and A. K. Chopra - 1973 (AD 766 272)
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