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Minimax Procedures for Specifying Earthquake Motion

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16. Abstracts
This research continued the development of a method which ultimately would enable earthquake engineers to make confident earthquake resistance guarantees. The method, applied to existing structures, leads to assessments of their earthquake resistance. The assessments appear to be somewhat conservative but consistent with the design practices among experienced engineering firms. This method, described in this report, may be relied on by civil engineers in the design, and in design reviews, of structures in seismic regions. Several modifications and extensions of the method are possible. Their application in practice is likely to be most appropriate for structures whose social or economic value invites conservative design, and some care and investment in the way it is achieved.

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Introduction and Summary

One of the aims of earthquake engineering is the design of structures whose survival at their locations can be guaranteed with a fairly definite degree of confidence, and whose resistance to damage on various levels can be similarly guaranteed (though possibly with lesser assurance). The object of the research under this grant was to continue the development of a method which would ultimately enable earthquake engineers to make such guarantees. The evidence, as of this writing, is that this object is being achieved. A method whose development was begun under an earlier grant has now been carried to the point at which it can be applied, and in fact has been applied, to existing structures. It leads to assessments of their earthquake resistance which appear to be somewhat conservative but quite consistent with the design practices among experienced engineering firms.

The evidence on which this remarks are based was developed under this grant, and it is summarized in this final report. Its implication is felt to be that perhaps the method to be described here or, more likely, some modifications and estimations of it will often be relied on by civil engineers in the design, and in design reviews, of structures in seismic region. Several such modifications and extensions are possible. In fact, the theory of some major ones (e. g. the generalization of the method from elastic to inelastic structures) were developed under this grant. Their application in practice is likely to be most appropriate for structures whose social or economic value invites conservative design, and some care and investment in the way it is achieved.

Comment

A proposal was submitted to NSF/Ranu in January 1976 for support for a continuation of this research.

The Critical Excitation Concept

In order to guarantee the earthquake resistance of a structure and to do so on a certain level of confidence, one should be able to demonstrate two things. First, one must show that the structure will survive all ground motions that can be expected at that location, on that level of confidence; and second, one should show with comparable confidence that it will escape damage beyond some acceptable limit. The approach taken under this grant (and under an earlier one, No. GK 14550) seeks to come up with such assurances by relying on a concept which has come to be called a "critical excitation" and which is new to civil engineering.

An excitation of a structure is called "critical" for it, among some designated class of excitations, if it drives one of the structural variables to a larger response peak than any other excitation in that class.

A rough idea of how this concept is used in the prediction of earthquake resistance is then the following. Suppose that the erection of a new structure is planned for some site in an earthquake-prone region, or that the safety of an already existing one is to be reviewed. Suppose further that it has been possible to isolate a class of excitations which consists of all ground motion that can realistically be expected at that location. Among the excitations in this class, there will then be some that generate the highest response peaks in variables, such as the forces, moments, and displacements of the structure. If these peaks are found to lie within the tolerances of the design, the structure can be guaranteed as correspondingly safe against damage and/or failure due to any of the excitations under consideration.

The procedure by which such a guarantee is arrived at, consists of three major steps. The question of how to take each of these represents a problem in its own right. The first step is the isolation of a class of excitations which are "realistic" potential ground notions at a given location. The second is the determination of those among them that are critical for a given structure. And the last is the computational problem of calculating the peaks of the critical responses, and the engineering problem of assessing their seriousness.

None of these problems have been precisely formulated here. In fact, each has many formulations and correspondingly many solutions. Some earthquake engineers may prefer one, and others another. The objective of the research under this grant was to formulate them in one particular and, it is hoped, realistic way; and to show that the approach leads to equally realistic solutions. This was in fact done. The formulation, as well as the three-step procedure to which it led, is described in the next section. It might be added here that other, and in some respects more attractive, formulations are possible. Indications are that they lead to similarly realistic results. They were not investigated under this grant however.

A Procedure for the Prediction of Earthquake Resistance

The prediction of the earthquake resistance of a structure, by way of its critical excitations, proceeds in three steps, as has just been explained, namely these.

(a) Isolation of a class of realistic ground motions

(b) Determination of the critical excitations in that class

(c) Calculation and engineering assessment of the critical response peaks.

The approach taken under this grant was the following.

The first of these problems was initially formulated as follows. It was assumed that, among the statistical information available regarding possible ground motions at the geographical location of a structure, only one item was known on the level confidence with which the prediction was to be made. This was the distribution of the ground motion intensities. A design engineer, in other words, who guaranteed a structure as 90% safe against ground motions up to a certain intensity could be quite certain that there was only one chance in ten of the actual ground motion exceeding that intensity limit. (The square-integral of the ground acceleration was used as intensity measure, though others could have been used as well.)

This assumption in effect constitutes a solution to the first problem listed above: The intensity limit set by the designer isolates a class of ground motions, namely, all those with intensities not exceeding that limit. One could therefore proceed directly to the next problem listed above, and this was in fact done under a previous grant. It was however found that this class is too large. It contains many excitations that are quite unlike any ground motions that have ever been recorded, and the critical excitations for structures unfortunately often are among those.

One can put this in other words by saying that the distribution of intensities is not in fact all that is known about all forseeable ground motions. Other information is available, and on a comparable level of confidence. This information must accordingly be utilized and those excitations eliminated from consideration which are as unlikely as those of very high intensities.

Such elimination is quite possible and can be effected in a number of ways. However, in the procedure that was used under this grant, computational simplicity suggested another solution. It consisted of two steps. First, the critical excitation was determined relative to the large class of ground motions that has just been described, i. e. the one defined only by an intensity limit. This determination is easy, at least for elastic structures, and has been described on several occasions, (e. g. [1]). The theory behind it was extended under the present grant also to inelastic structures. In either case, however, it often leads to critical excitations which are quite unrealistic, as has just been mentioned. To circumvent this difficulty, these excitations were replaced with others, called "subcritical," which were related to the critical ones but were more "realistic" in the following sense.

In defining what meaning to attach to the term "realistic" it was decided that any already recorded ground motion was patently realistic (though not necessarily for all locations). Accordingly, a set of such records was chosen as a representative basis. Beyond these, all linear superpositions among them were added as being presumably equally realistic. The superposition which differed least from a critical excitation, in the least-squares sense, was then used as its sub-critical replacement. In more mathematical language, the sub-critical excitation is the projection of the critical one into the space spanned by the chosen basis of recorded ground motions. The numerical execution of the projection is performed easily and quickly by computer, which is the main reason for the choice of this approach. It does, at any rate, represent a solution to problems (a) and (b).

The calculation of the response peaks generated by the sub-critical excitation is also straightforward, though somewhat time-consuming even for elastic structures. In fact, no calculations were performed for realistic inelastic structures under this grant. The risk to a structure, as a consequence of having one of its variables driven to a high peak, was finally assessed by strength calculations for typical structural numbers, and by determining the ductility ratios induced in them by the response peaks.

In this way, a solution was arrived at to the last of the three problems mentioned above.

The result obtained with this approach are briefly described and discussed in the next section.

Predictions of Earthquake Resistance

The procedure for the prediction of structural earthquake resistance which was sketched in the preceding section was applied to eight existing structures, four high-rise buildings, a hospital pavillon, two structures associated with nuclear reactors, and a tall, re-inforced concrete chimney. The results of the analyses are described in detail in the papers which form the appendices to this report, notably in

Appendices A and B. In summary, the following was found.

Two high rise buildings, both designed by a well-known and experienced engineering firm (H. J. Degenkolb and Ass.'s, San Francisco) were analyzed and are judged to be fully resistant to any kind of ground shaking with an intensity up to that recorded during the 1940 El Centro earthquake. By contrast, another high rise (building, namely Bldg. 180 of the Jet Propulsion laboratory in Pasadena, California) was judged to have inadequate earthquake resistance. This appraisal apparently is consistent with one arrived at independently by the Laboratory management which has contracted for a general strengthening of the frame. A fourth building was analyzed as well. In this case however the building was nonexistant and its design could be drawn up, based on the approach described here. This was done. It was found to lead to strength specifications that were consistent with good design practice.

The hospital facility was was studied was the psychiatric pavillon of the Olive View Hospital in Sylmar, California. The study showed that it was not survivable under ground motions of intensities of the 1940 El Centro earthquake. The ground shaking it experienced during the San Fernando earthquake of 1971 may have exceeded that intensity since it collapsed on that occasion. The tall chimney, on the other hand, was found to be safe under such excitation intensities. It had been designed by Ammann and Whitney, Consulting Engineers, with this in mind.

The two nuclear facilities were analyzed from dynamical equations which had been prepared elsewhere. In one case, they were obtained from the literature [2] and characterized a rigid reactor-soil combination.

In the second, they were supplied by a New York engineering consulting firm. This information was adequate for the derivation of the critical response peaks of the two structures but not for any strength calculations. The results of the analysis nevertheless indicated the earthquake resistance to be adequate for the first structure. The evidence regarding the second was not equally reassuring.

In summary, the result obtained under this grant indicate that the new approach leads to fairly reliable, if somewhat conservative, predictions of the earthquake resistance of structures. One can therefore conclude that this method is well suited to such predictions, especially when the economic or social value of the structure is high enough to invite some conservatism and to justify the investment in the necessary analyses.

The best prognosis as of this writing is that ultimately variations on this method, rather the method itself, will be adapted by earthquake engineers. The present transition from the critical to the sub-critical excitation seems to lead to some inconsistencies in the assessments of structural safety which are considered undesirable and which can probably be eliminated if the method is suitably modified. Moreover, the computational effort that is now needed before a comprehensive assessment can be made seems unnecessarily large, and it will no doubt become larger when ground motion in three or more dimensions, interaction with water, and inelasticities are allowed for. The theory which governs such allowances has been largely developed under this grant but no applications to existing structures were attempted. It is however already clear that these extensions entail greater computational effort which will discourage many practicing design engineers. It is therefore important to develop simplified procedures which avoid some of that effort.

Reports and Publications

As of this writing, the research supported by this grant has led to six papers, all but one accepted for publication, and two already published. Three others are in various stages of preparation. In addition, memoranda and interim reports were prepared for limited distribution, at the suggestion of NSF/RANN.

Reprints and pre-prints of these papers are added to this report as Appendices.

Appendix A: Critical Excitation and Response of Free Standing Chimneys, by P. C. Wang, W. Wang, R. F. Drenick and J. Velozzi; Proc. of the International Symposium on Earthquake Structural Engineering, St. Louis, Mo., August 1976.

Appendix B: On a Class of Non-Robust Problems in Stochastic Dynamics, by R. F. Drenick, Proc. of the Symposium on Stochastic Problems in Dynamics, sponsored by the International Union on Theoretical and Applied Mechanics, Southampton, UK, July 1976.

Appendix C: The Critical Excitation of Nonlinear Systems, by R. F. Drenick, accepted for publication in the Journal of Applied Mechanics.

Appendix D: Critical Excitation and Response of Structures, by P. C. Wang and R. F. Drenick, accepted for publication in the Proc. of the 6th World Conference on Earthquake Engineering, New Delhi, India, January 1977.

Appendix E: The Critical Excitation of Inelastic Structures, by R. F. Drenick and H. Kano, accepted for publication in the Proc. of the 6th World Conference on Earthquake Engineering, New Delhi, India, January 1977.

Appendix F: The Critical Excitation and Response of High-Rise Buildings by P. C. Wang and W. Y. L. Wang, submitted for publication in the ASCE Journal for Structures.

The three papers under preparation are:

(1) The integrity of nuclear reactor structures during earthquakes, by P. C. Wang and W. Y. L. Wang.

(2) The time interval of effective ground shaking, by P. C. Wang and R. F. Drenick.

(3) Critical response spectra of inelastic structures, by H. Kano and R. F. Drenick.

The limited-distribution memoranda and reports prepared under this grant were:

(a) Case study of Critical Excitation and Response of Structures, Preliminary Report, by R. F. Drenick, July 15, 1975.

(b) Case study of Critical Excitation and Response, Second Preliminary Report, by R. F. Drenick, August 5, 1975.

(c) Case study of Critical Excitation and Response of Structures, Interim Report, by P. C. Wang, W. Wang, R. F. Drenick, Nov. 1, 1975.

Scientific Collaborators

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1. R. F. Drenick, Model-Free Design of Aseismic Structures, Jour. Eng. Mech., ASCE, Vol. 96 (1970) p. 483.

2. C. A. Miller and C. J. Costantino, Structure-Foundation Interaction of a Nuclear Power Plant with a Seismic Disturbance, Nuclear Eng. and Design, Vol. 14 (1970) p. 332.

APPENDIX A

INTERNATIONAL SYMPOSIUM ON
EARTHQUAKE STRUCTURAL ENGINEERING
St. Louis, Missouri, USA, August, 1976

CRITICAL EXCITATION AND RESPONSE OF FREE STANDING CHIMNEYS

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U. S. A.

SUMMARY

This paper deals with the problem of the seismic design of free standing chimneys, of constant as well as tapered cross sections. It is more particularly shown that seismic deisgns based on the so-called critical excitations of these structures are conservative, but not overly conservative, and that they should be appropriate either for localities in which ground motion records are scarce or for structures whose loss would have serious consequences, economically or socially. This conclusion is based on computed "critical design factors" which are the ratios of the response peaks generated by a critical excitation to those produced by an actual ground acceleration of same intensity. These factors were found to be in the order of 0.93 to 1.3 for at least one structural design variable of each of the two structures, implying the conclusion that design based on the critical excitation method would be more, but not greatly more, conservative than one based on an already observed ground motion. Design calculations for the additional steel reinforcement implied by those factors confirm this conclusion.

INTRODUCTION

Free standing chimney are comparatively susceptible to seismic damages due to their inherent weak supporting condition and lack of structural redundancy. The most damaging (critical) ground excitation for an assigned design variable (moments, shears, or deflections) possesses characteristic frequency contents, duration, and energy level. The first two characteristics are dependent on the structural properties while the other depends on the nature and intensity of the ground motion.

Structural response is characterized by the frequencies of the modes of its free vibrations. Intuitively, one should expect the most damaging (i. e., the critical) excitation of a structure to have a frequency spectrum that matches that of the structure. This is actually the case, as experience indicated. It is known, for instance, that ground motion matching in frequency with the lower vibration modes of a structure is likely to cause severe damage in it. It is also well known that excitations at short distances from the epicenter which exhibit intense vibrations at high frequencies may induce damage in apparently strong but rigid structures, yet light or no damage to seemingly weaker but flexible structures. Mathematical confirmation [3] of these observations shows that the critical excitation of an elastic structure, for a given intensity and relative to one of the design variables, is the time-reversed impulse response of that variable.

It develops however, that the kind of precise frequency matching which is afforded by the time-reversed impulse response is not in general achieved by realistic ground motions. In other words, the response peaks to which it leads are typically much too large, and the designs that would escape damage, much too conservative to be useful. It has accordingly been necessary in this study to modify the time-reversed impulse response and to treat the modified excitation as the critical. To distinguish the original and its modification, they are called the "first-class" and the second-class" critical excitations in what follows.

This paper starts with a discussion of the first-class critical excitation for structures with a single-degree-of-freedom, as well as some assumptions and concepts that are pertinent to it, and then proceeds to the case of multi-degree-of-freedom systems. The idea of the second-class critical is introduced next. The succeeding sections present the methods and the results of the analyses of the two types of chimney, namely one with constant and the other with tapered cross sections. A critical discussion of the results is contained in the concluding section.

EFFECTIVE DURATION AND INTENSITY OF GROUND EXCITATION

The response $y(t)$ of a design variable of an elastic structure to a ground acceleration $\ddot{x}_g(t)$ is given by the Duhamel integral

$$y(t) = \int_0^t \ddot{x}_g(\tau)\, h(t-\tau)\, d\tau \tag{1}$$

in which $h(t-\tau)$ is the unit impulse response at a time $(t-\tau)$. For a structure with a single degree of freedom it is given by

$$h(t-\tau) = \frac{1}{\omega_D} e^{-\xi\omega(t-\tau)} \sin \omega_D(t-\tau) \tag{2}$$

where $\omega_D = \omega\sqrt{1-\xi^2}$ is the damped frequency, ω is the undamped frequency, and ξ is the damping ratio. Thus, if the maximum response of a structure occurs at time t_e, the duration of excitation needs not be taken longer than the value of (t_e-t_o) so that $h(t_e-t_o) = 0$, or more practically $h(t_e-t_o)$ decays to a certain percentage of the maximum of $h(t)$. The decay percentage can often be left to the judgment of the designers. For example, if the decay to a ten percent was assigned to a structure based on its fundamental period of vibration of 2 seconds, with a damping ratio of 5% then the duration of excitation need not be taken greater than

$$(t_e-t_o) = \frac{-\ln 0.1}{\xi\omega} = \frac{2.3}{0.05\,\frac{2\pi}{2}} = 14.6 \text{ seconds} \tag{2a}$$

The definition of the intensity of ground excitation has been the subject of extensive discussions. In this paper, following the derivation of reference [3], the intensity of an excitation was defined as

$$E = \left[\int_{t_o}^{t_e} \ddot{x}_g^2(t)\, dt\right]^{\frac{1}{2}} \tag{3}$$

Since the duration of excitation $(t_e - t_o)$ used for the critical excitations and the comparative recorded excitations, as will be seen in the later discussions, are the same, the intensity of excitation defined here is similar to that defined by Housner [6].

$$E = \frac{1}{t} \int_o^t \ddot{x}_g^2(t)\, dt \tag{4}$$

FIRST-CLASS CRITICAL EXCITATION

The maximum response of a multi-degree-of-freedom system represented by modal superposition is as follows:

$$y_k(t_e) = \sum_i \phi_{ki}\eta_i(t_e) = \int_{t_o}^{t_e} \ddot{x}_g(\tau) \sum_i \phi_{ki} P_i h_i(t_e - \tau)\, d\tau$$

$$= \int_{t_o}^{t_e} \ddot{x}_g(\tau)\, \overline{h}(t_e - \tau)\, d\tau \tag{5}$$

where $y_k(t_e)$ is the k^{th} response variable, ϕ_{ki} is the k^{th} element of the i^{th} mode shape, $\eta_i(t_e)$ is the normal coordinate of i^{th} mode, $P_i = \phi_i^T M \hat{I} / \phi_i^T M \phi_i$ is the i^{th} mode participation factor with M as the mass matrix and $\hat{I}$ is a vector with 1's or o's to indicate the existence or not of excitation in the vector elements of y. Squaring the response y_k and setting up the inequality, the following relation is obtained.

$$y_k^2(t_e) = \left[\int_{t_o}^{t_e} \ddot{x}_g(\tau)\, \overline{h}(t_e - \tau)\, d\tau\right]^2$$

$$\leq \left[\int_{t_o}^{t_e} \ddot{x}_g^2(\tau)\, d\tau\right] \left[\int_{t_o}^{t_e} \overline{h}(t_e - \tau)\, d\tau\right]$$

$$\leq E^2 N^2 \tag{6}$$

or $\qquad y_k(t_e) \leq EN$

where E is the intensity of excitation as defined in Eq. (3) and N^2 is the square integral of the unit impulse response. The maximum response is the product EN and can be obtained by applying a first-class critical excitation $\ddot{x}_{c1}(t)$, so that

$$\ddot{x}_{c1}(\tau) = \frac{E}{N}\, \overline{h}(t_e - \tau) \tag{7}$$

The shape of the unit impulse $\overline{h}(t)$ and $\ddot{x}_{c1}(t)$ are shown in Fig. 1.

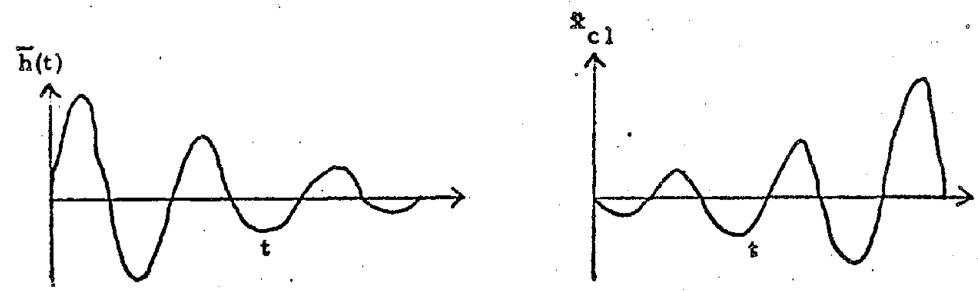

Fig. 1. SHAPE OF $\overline{h}(t)$ AND $\ddot{x}_{c1}$

The maximum response due to the first class critical excitation is

$$y_{c1} = \frac{E}{N} \int_{t_o}^{t_e} \overline{h}(t_e - \tau) \cdot \overline{h}(t_e - \tau)\, d\tau$$

$$= EN \tag{8}$$

The intuitive interpretation of this result was already mentioned in the Introduction. It indicates that the frequency content of the first class critical excitation matches exactly with that of the structural vibration and therefore that the corresponding critical response y_{c1} is the maximum peak among those produced by all the excitations with same intensity E.

SECOND-CLASS CRITICAL EXCITATION

It has been mentioned in the introduction, that the response peaks produced by the first-class critical excitations often are too large to be realistic, and the results reported below for two free-standing chimney will be seen to confirm this. It has therefore been found necessary to introduce a modification which is called the "second-class critical excitation" here.

The second-class critical excitation is obtained by superposition of a number of recorded groung excitations (or artificially generated excitations) and least-square fitted with the first class critical excitation as follows:

$$\ddot{x}_{c2}(t) = \sum_{i=1}^{n} c_i \ddot{x}_i(t) \tag{9}$$

and

$$\int_{t_o}^{t_e} [\ddot{x}_{c1}(t) - \ddot{x}_{c2}(t)]^2 dt = \text{minimum}$$

$$\int_{t_o}^{t_e} \ddot{x}_{c2}^{2}(t)\, dt = E^2$$

The response to the second-class critical excitation is

$$y_{c2} = \int_{t_o}^{t_e} \bar{h}\,(t_e-\tau)\, \ddot{x}_{c2}\,(\tau)\; d\tau \tag{10}$$

The second-class critical excitation $\ddot{x}_{c2}$ resembles the recorded excitations more closely than the first-class one and the peak of its resultant response y_{c2} is more reasonable. However it is still larger than that of any of the responses due to the component excitations used for the least-squares of it.

In order to find the first-class critical excitation $\ddot{x}_{c1}$ and the corresponding response y_{c1} for a particular structural design variable based on the time-reversed unit impulse response, the designer only needs the specification of a reference ground motion intensity E. However, in order to obtain the second-class critical excitation $\ddot{x}_{c2}$ which is a least-squares fit, a number of appropriate ground motions must be selected to make the combination as shown by Eq. (9). Finally, in order to have a basis of comparison, a few recorded accelerograms must be selected and structural responses calculated for them as well. This section describes the choices that were made for these purposes.

In regard to the first requirement of obtaining the least-squares fitted excitation x_{c2}, twelve accelerograms were selected including two of the three selected for comparative studies. These accelerograms were chosen with the following stipulations:

1. The ground excitations are characterized by relatively short epicentral distances, say 25 to 30 kilometers.

2. The shape of the accelerogram should have a gradual build-up period.

3. The site conditions of the selected earthquakes should resemble as much as possible the condition prevailing at the location of the structure.

The third stipulation may be difficult to satisfy unless a choice can be made from a rather large variety of accelograms, probably larger than now exists. At any rate, in the present study twelve ground motions recorded in Southern California were chosen and assumed to be representative for the locations of the chimneys to be analyzed below. Appendix 1 lists these twelve earthquakes and their intensities E.

Typical examples of second-class critical excitation obtained in this way are shown in Fig. 2.

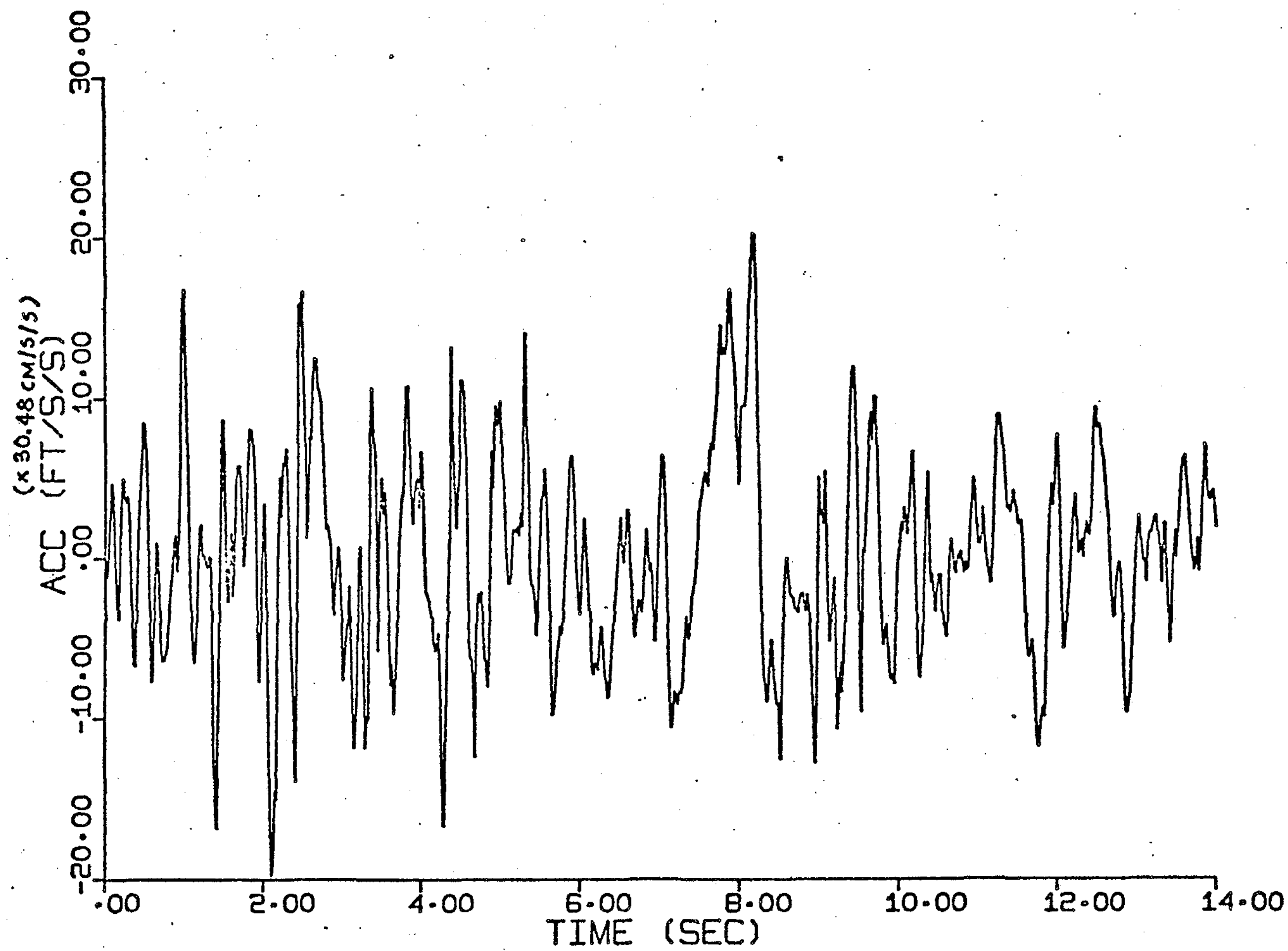

FIG. 2. TYPICAL 2ND CLASS CRITICAL EXCITATION WITH SAN FERNANDO, PACOIMA DAM INTENSITY

For comparative studies, three accelerograms were selected from the published results [2], namely, (1) 1971 San Fernando, Pacoima S14W, (2) 1940 Imperial Valley, El Centro S00E, (3) 1954 Eureka N79E. Each of these three accelerograms has certain special characteristics: the first is the strongest (1.17g) that has ever been recorded, the second one is strong and of relatively long duration, while the third one is moderately intense and of relatively short duration.

CHIMNEYS WITH CONSTANT CROSS SECTIONS

Chimneys with constant cross sections are simple prismatic cantilevers. Its natural frequency of vibration of the i^{th} mode is given by [1]:

$$\omega_i = \alpha_i^2 \sqrt{\frac{EI}{mL^4}} \tag{11}$$

where α_i is obtained from the transcendental equation

$$\cos \alpha_i \cosh \alpha_i = -1 \tag{12}$$

The mode shapes are given by

$$\phi_i(\frac{x}{L}) = \sin \alpha_i \frac{x}{L} - \sinh \alpha_i \frac{x}{L} + A_i(\cosh \alpha_i \frac{x}{L} - \cos \alpha_i \frac{x}{L})$$

with

$$A_i = \frac{\sin \alpha_i + \sinh \alpha_i}{\cos \alpha_i + \cosh \alpha_i} \tag{13}$$

In the above expressions, E is the modules of elasticity, I is the moment of inertia, m is the distributed mass per unit height, L is the height of the chimney, and x is the distance from the base of the chimney.

For a reinforced concrete chimney of 304.80 m in height, 18.288 m in outside diameter, and 0.4572m in thickness, the mass per unit height is 1910.677 Kg-sec^2/m^2. Based on modulus of elasticity 2.9489 x 10^9 kg/m^2 and moment of inertia 1018.5m^4, the period of vibration in seconds of the first six modes are 2.400, 0.383, 0.137, 0.070, 0.042 and 0.028. The participation factors $\int_o^L \phi_i \, dx / \int_o^L \phi_i^2 \, dx$ are 0.783, 0.434, 0.251, 0.001 for the first four modes. The design variables selected are top deflection Δ, base moment M, and base shear V. The results of the dynamic analysis for the three reference earthquakes are shown in Table 1. The entries in the table are more specifically the response peaks generated by these excitations shown in the left column. The peaks to which the first-class critical excitation leads are seen to be consistently much higher than those due to the actual ground motion.

Those produced by the second-class critical are however much more realistic. The ratios of those peaks to the ones generated by the actual ground motions are listed in Table 2, under the heading of "critical design factors." These factors are seen to range over values from 1 to 3.

TABLE 1. RESPONSE PEAKS OF CHIMNEY WITH CONSTANT CROSS SECTION

Excitations	Intensity E ($cm/sec^{3/2}$)	Response Variables: Top Deflection Δ (m)	Base Moment M (10^5 Kg-m)	Base Shear V (10^3 Kg)
Pacoima Dam		0.676	10288.8	1280.2
1st cl. critical	6.471	2.396	26151.1	1194.3
2nd cl. critical		5.101	59678.2	3565.7
El Centro		0.432	4967.4	349.9
1st cl. critical	2.572	0.953	10394.4	474.7
2nd cl. critical		2.028	23720.6	1417.3
Eureka		0.243	3423.3	286.6
1st cl. critical	2.008	0.744	8115.5	370.6
2nd cl. critical		1.583	18519.9	1106.5

TABLE 2. CRITICAL DESIGN FACTORS OF THE CHIMNEY WITH CONSTANT CROSS SECTIONS

Excitations	Top Deflection Δ	Base Moment M	Base Shear V
Pacoima Dam	3.54	2.54	0.93
El Centro	2.21	2.09	1.36
Eureka	3.06	2.37	1.29

TAPERED CHIMNEYS

Most chimneys have tapered shapes. Although expressions similar to (11) and (13) for frequency and mode shapes can be derived, it is simpler to use discrete lumped mass approach.

The chimney selected for this study is a 304.8 m free standing tapered reinforced concrete cylinder. The bottom outside diameter is 25.298 m with wall thickness of 0.889 m. The top outside diameter is 10.262 m with a thickness of 0.216 m. The 0.64 cm steel lining is not considered as the integrated structural element. The detailed vertical chimney wall cross section is shown in Figure 3.

A discrete finite element method was used to find the free vibration as well as dynamic analysis. The height of the chimney is divided into 17 sections with the respective horizontal cross sectional area and moment of inertia computed as shown in Table 3. The lumped masses at the nodal points are also shown in Table 3. The condensed stiffness matrix refers to the horizontal displacements at the nodal points corresponding to each mass point. The mode shapes and periods of vibration are shown in Figure 4. The design variables selected for study are again the top deflection Δ, the base moment M, and the base shear V. The dynamic

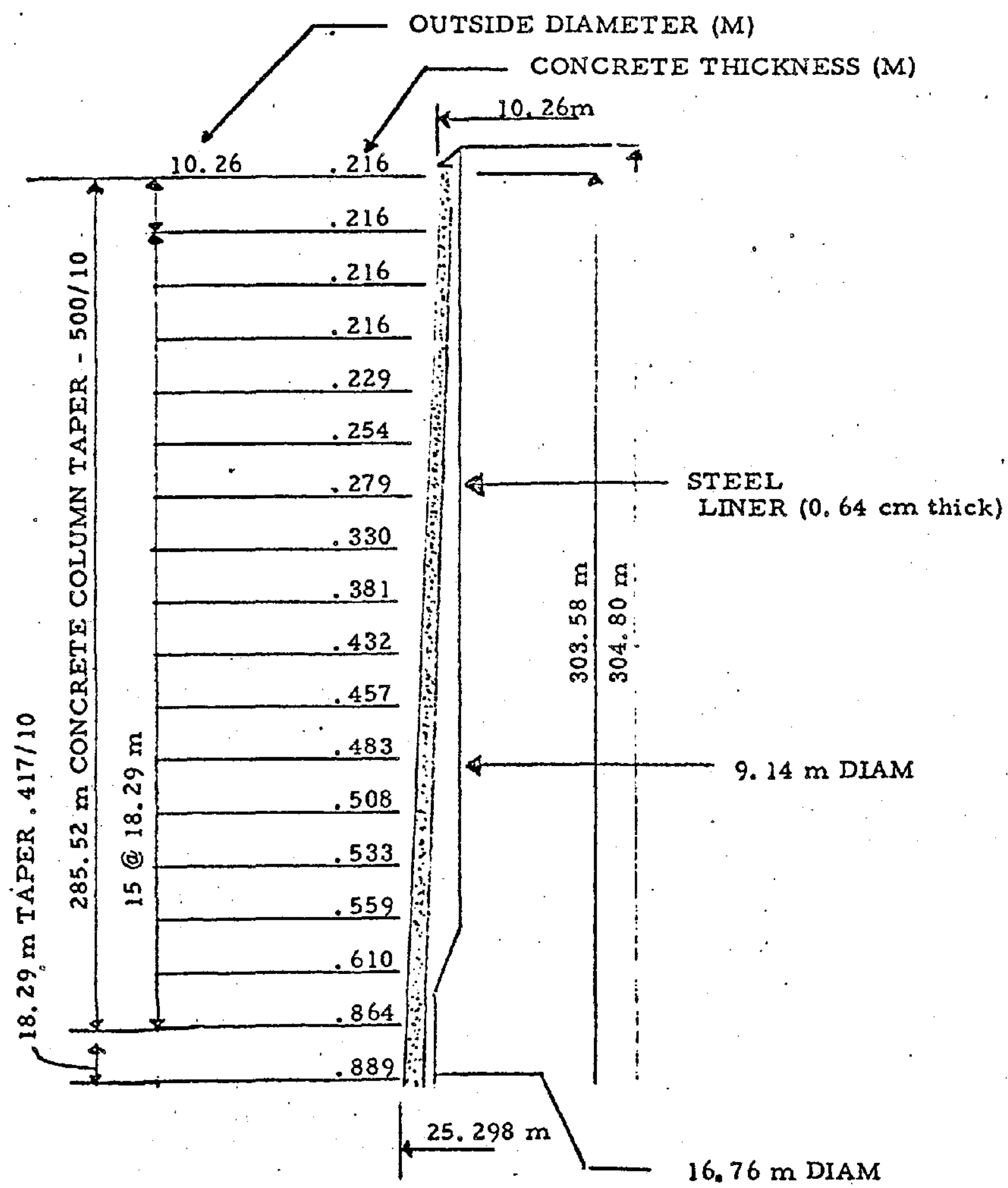

FIG. 3. VERTICAL CROSS SECTION OF CHIMNEY WITH TAPERED CROSS SECTIONS

TABLE 3. AREA, MOMENT OF INERTIA AND LUMPED MASSES OF TAPERED CHIMNEY

Element	Area (m^2)	Moment of Inertia (m^4)	Node	Lumped Mass ($Kg\text{-}sec^2/m$)
17	7.005	93.420	18	10119.08
16	7.505	114.90	17	27976.28
15	8.125	145.791	16	37648.93
14	8.999	186.856	15	40476.32
13	10.448	24.652	14	45535.86
12	12.293	330.866	13	53571.6
11	14.888	450.039	12	62053.77
10	18.334	617.540	11	76934.77
9	22.956	823.209	10	93006.25
8	25.348	1044.520	9	110268.21
7	28.108	1274.539	8	122321.82
6	31.010	1540.536	7	134970.67
5	34.056	1846.109	6	148363.57
4	37.242	2194.955	5	162351.71
3	41.448	2644.194	4	176935.09
2	54.024	3684.179	3	200149.45
1	66.189	4788.865	2	290477.12
			1	

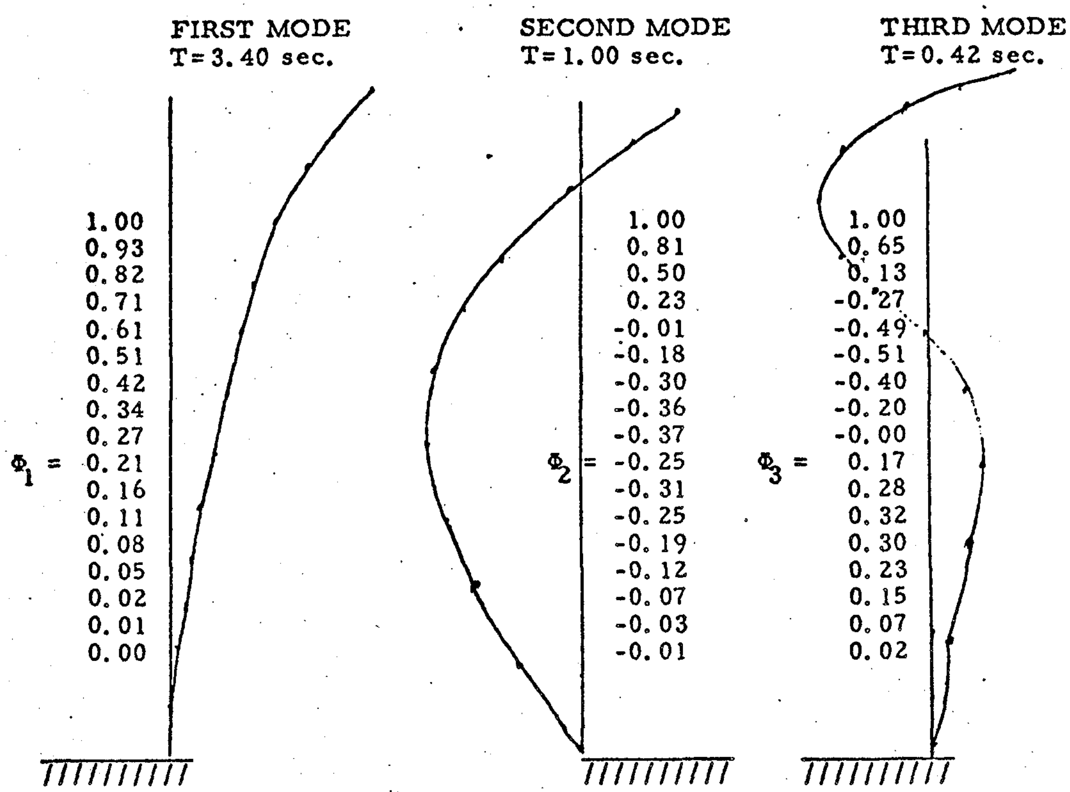

FIG. 4. MODE SHAPES AND PERIODS FOR THE TAPERED CHIMNEY

analyses of this chimney led to results which are summarized in Tables 4 and 5. Table 4 lists the response peaks that were generated by the actual ground motions, along with those due to the first-class and second-class critical excitations of the same intensities. Table 5 presents the critical design factors.

TABLE 4. RESPONSE PEAKS OF CHIMNEY WITH TAPERED CROSS SECTIONS

Excitation	Intensity E ($m/sec^{3/2}$)	Response Variables: Top Deflection Δ (m)	Base Moment M (10^5 Kg-m)	Base Shear V (10^4 Kg)
Pacoima Dam		1.383	5536.6	703.6
1st cl. critical	6.996	3.849	10930.4	1044.0
2nd cl. critical		21.924	58122.5	5191.0
El Centro		0.694	2134.7	313.3
1st cl. critical	2.895	1.594	4523.7	432.1
2nd cl. critical		9.074	22684.8	2148.4
Eureka		0.448	1580.3	2307.4
1st cl. critical	2.034	1.119	3177.9	3035.3
2nd cl. critical		6.373	15935.9	15092.2

TABLE 5. CRITICAL DESIGN FACTORS OF THE CHIMNEY WITH TAPERED CROSS SECTIONS

Excitations	Top Deflection Δ	Base Moment M	Base Shear V
Pacoima Dam	2.78	1.97	1.48
El Centro	2.51	2.12	1.33
Eureka	2.50	2.01	1.32

A design of the base cross section of the chimney was also made, based on the elastic design approach as well as on an inelastic one with ductility factor of $\mu = 4$. The results are shown in Table 6. The reinforcing that would be required for adequate strength against the second-class critical is considered to be rather high, but not beyond reason, when compared with that needed against the El Centro ground motion.

CONCLUSIONS

The proposed method of assessing seismic resistance of structures, based on the second-class critical excitation, was applied to uniform cross sectional and tapered chimneys. The conclusions from this study are as follows.

1. The method proposed here is an upper bound analysis in view of the fact that precise nature of earthquake, frequency of occurrence, interaction of structure and soil, and other earthquake related factors are not

TABLE 6. REINFORCEMENT FOR THE TAPERED CHIMNEY

	Excitations	Steel Ratio p%	$A_{s(req)}(cm^2)$	Reinforcement	$A_s(cm^2)$ provided
$\mu = 1$ (elastic)	El-Centro	0.60	4070.5	632-#9 bars, both sides, at 24.26 cm dist.	4077.4
	2nd cl. ctitical	5.00	32499.1	1260-#18S bars, both sides, at 12.19 cm dist.	32516.3
$\mu = 4$ (inelastic)	El-Centro	0.06	409.22	578-#3 bars, both sides, at 26.7 cm dist.	410.2
	2nd cl. critical	0.90	6087.5	944-#9 bars, both sides, at 16.3 cm dist.	6090.4

readily available.

2. In the structural design of the two chimneys, the method appears to be effective, though still somewhat conservative. If desired, further reduction of the bound can be achieved by the judgment of the design engineer in reducing the specified intensity E, or in eliminating some of the selected component earthquakes in the least-squares fitting process. By observation of the coefficient of the least-squares fitting process, it appears that the earthquakes which most resemble the shape of the time-reversed unit impulse response excitation are the ones which may cause larger response. If these earthquakes are not likely to occur at a given location, they can be profitably omitted.

3. Both the intensity of the earthquake E, and the square integral N depend on the effective duration t_e-t_o used in the integration process. In general, the duration depends on the fundamental period of vibration and the damping of the structure, being shorter for shorter period and larger damping. It is suggested that one may use the duration of decay of the unit impulse response to a judiciously selected percentage (say 20%) of the peak.

4. When plastic behavior is considered by using a ductility factor of 3 for a recorded earthquake, a ductility factor of roughly 6(=2x3) is required for the same structural strength against the least-squares fitted excitation. This ductility factor appears somewhat on the high side but not entirely out of proportion.

5. Based on the above discussions, it is suggested that the assessment of seismic resistance based on critical excitation be used for structures with major importance the destruction of which would cause severe human and economic losses. Another instance for adopting this approach is for those localities where seismicity is active but reliable ground motion data are scarce.

6. The practicality of the method is still undergoing examination by applying to various realistic structures at the time of this writing. Hopefully, consistent comprehensive recommendations can be drawn from these results in the near future.

ACKNOWLEDGMENT

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APPENDIX 1

12 EARTHQUAKES USED FOR LEAST-SQUARES FITTING

File #		Quake	Record	Comp	$E=\left(\int_0^{14} a_g^2\,dt\right)^{1/2}$ $(m/sec^{3/2})$
IIA	1	Imperial Valley	El Centro	S00E	2.895
IIA	10	San Jose	San Jose	N31W	0.660
IIA	13	San Francisco	San Francisco	N45E	0.340
IIA	14	San Francisco	San Francisco	N09W	0.294
IIA	15	San Francisco	San Francisco	N10E	0.397
IIA	16	San Francisco	San Francisco	S09E	0.543
IIA	17	San Francisco	San Francisco	N26E	0.234
IIA	18	Hollister	Hollister	S01W	0.838
IIA	19	Borrego Mt.	El Centro	S00W	0.875
IIC	41	San Fernando	Pacoima	S14W	6.996
IIC	48	San Fernando	Los Angeles	N00W	2.565
IID	56	San Fernando	Castaic	N21E	1.929

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APPENDIX B

ON A CLASS OF NON-ROBUST PROBLEMS IN STOCHASTIC DYNAMICS

BY

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SYNOPSIS

The stochastic treatment of dynamical systems frequently assumes that the excitations and responses form random processes whose probabilistic structure is completely known. This is rarely true in practice, but not really objectionable unless the quantity of interest is very sensitive to small changes in that structure. Evidence is presented that shows that the failure probability of a system is among those qualities. A method is suggested for the treatment of problems in which such quantities are of interest. It combines probabilistic and worst-case analyses to obtain bounds on the desired quantity. The estimation of the earthquake resistance of elastic and inelastic structures is presented as an example of the method.

INTRODUCTION

This paper deals with certain problems in the reliability or safety of mechanical systems which will fail if the magnitude of the response exceeds a certain limit. These problems are often treated by probability theory. It is then assumed that the system responses are sample functions from a random process, usually a Gaussian one, and an attempt is made at calculating or at least estimating the exceedance probability of that limit.

The first point of this paper is that this procedure is not robust, and that it can easily lead to very misleading results. The supporting evidence is presented in Sect. 2. The conclusion that is reached there should not be overly surprising. Failures are, or had better be, rare events in most instances. They are therefore events whose probabilities are strongly dependent on the shapes of the tails of the underlying distributions which are usually the least known portions, and those least accessible to statistical estimation. The failure probabilities that are calculated from them are subject to large errors.

Sect. 3 raises the question of what one should do in such problems. The answer that is reached there is, by what seems to be a fairly generally valid argument, that one should perform a combination of probabilistic and worst-case analyses. The probabilistic portion should more particularly utilize all information that is available regarding the statistics of the underlying random process and that possesses the desired level of assurance. The worse-case analysis is then used to obtain bounds on the failure probability that are consistent with that information.

Sect.'s 4 and 5 present an example taken from earthquake engineering, which, according to rather recent work, appears to be producing practical results. This is the assessment of the seismic resistance of structures. The ground motions during earthquakes probably form a very good example of a random process whose probabilistic structure is poorly known, expecially on the tails of its distributions. It is therefore natural to apply the general ideas presented in Sect. 3. This is done for elastic structures in Sect. 4 and for inelastic ones in Sect. 5.

ACKNOWLEDGMENT

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NON-ROBUSTNESS PROBLEMS

The kind of problem to be discussed in this paper is one in which a failure of a system is brought about by an excessively severe response. It will be convenient to assume that the "severity" of a response variable y is measured by the norm $\|y\|$. The norm that seems most useful in practice is

$$\|y\| = \sup_t |y(t)| \tag{2.1}$$

where t ranges over the interval T of interest. One can then define system failure as an event of the form $\{\|y\| > L\}$ where L is some failure limit.

In the stochastic treatment of the problem one seeks to determine the probability

$$P\{\|y\| > L\} = P\{\sup_t |y(t)| > L\} \tag{2.2}$$

of the event of failure. In order to do so one assumes that the response variable y forms a random process with a completely known probability measure. (For simplicity, the symbol y will be used in what follow to denote the random process as well as an individual sample function; the wording of the text will, it is hoped, avoid misunderstandings due to this imprecise notation.) It is in fact usually assumed that the probability measure is Gaussian, and the probability in (2.2) is then calculated, or at least estimated, on that assumption.

The point to be made in this section is that the value of the failure probability (2.2) is very sensitive to the assumption of Gaussianity ("ill-conditioned," in the language of numerical analysis): small changes away from it can produce very large changes in the value of the failure probability. The evidence to be presented indicates more particularly that the failure probability is most sensitive to those characteristics of the underlying random process y that are least likely to be well known, namely the behavior for large $|y(t)|$.

With a few exceptions, closed expressions for the distribution of $\|y\|$ in (2.1) are known only when the random process y is stationary and Gaussian. One that was derived fairly recently by Pickands [1] is typical of most others. It is of the familiar double exponential form

$$P\{\|y\| > L\} = 1 - \exp\{-\exp[-\tfrac{L}{\sigma}(2\log 2n)^{\frac{1}{2}} + \eta]\} \quad (2.3a)$$

in which n and η are constants. The first is more specifically a coefficient in the Maclaurin series for the autocovariance $R_y(\tau)$ of y, i.e.

$$R_y(\tau) = R_y(0)[1 - n|\tfrac{\tau}{T}| + 0\,(\tfrac{\tau}{T})], \quad (2.3b)$$

and the second is

$$\eta = 2\log 2n + \tfrac{1}{2}(\log \pi - \log\log 2n) \quad (2.3c)$$

Formulae (2.3) are asymptotically valid for large L and large n, in the sense that terms of order $O(L/\sigma)^{-1}$ and $O(\log n)^{-1}$ are neglected. It is of interest that the expression (2.3a) is essentially the same as for the exceedance probability of n independent Gaussian variables, each with the same density as $y(t)$. (The only discrepancy is in the factor of $\log \pi$ in (2.3c).

The derivation of (2.3) rests very heavily on the Gaussian nature of the process y. Accordingly no similar expressions are known for non-Gaussian processes, to the writer's knowledge. However, it may be at least plausible to expect an equivalence to exist, between non-Gaussian processes and a suitable number n of independent non-Gaussian variables, which is of the kind that has just been described for Gaussian ones. There exists no mathematical proof of the equivalence, but it is difficult to think of any reasons why it should fail to hold, at least if all conditions are satisfied under which (2.3) is valid and if the departure from Gaussianity is small.

If one can accept this equivalence, one can proceed further. This will be done here, at any rate. To be more specific, a non Gaussian random process will be considered whose one-dimensional probability density $p(y)$ has the following properties.

(1) Between two limits ($\pm y_o$), $p(y)$ is Gaussian

$$p(y) = \frac{a_1}{\sigma}\exp(-\frac{y^2}{2\sigma^2}), \quad (|y| < y_o). \quad (2.4a)$$

(2) Beyond these two limits, $p(y)$ is "of the exponential type" (in the terminology of Gumbel [2, p. 120]):

$$p(y) = \frac{a_2}{s}|\tfrac{y}{s}|^m \exp(-\tfrac{1}{r}|\tfrac{y}{s}|^r) \quad (|y| > y_o, r > 1). \quad (2.4b)$$

(3) At $|y| = y_o$, $p(y)$ is continuous.
(4) n is large.
(5) L is large.

The last two assumptions are fairly traditional in the theory of extremes [3, p. 374]. The first three are made here in order to be able to postulate a random process which is Gaussian in a region in which observational data are available (namely for $y(t)$ values which are not very large) but which may depart from Gaussianity where such data are scarce and where such a departure would be difficult to ascertain statistically. The case of no departure is included: one merely sets

$$r = 2, \; m = 0, \; a_1 = a_2 = (2\pi)^{-\frac{1}{2}}, \; s = \sigma. \quad (2.5)$$

Suppose now also, as suggested above, that the exceedance probability $P\{\|y\| > L\}$ in (2.2) for the random process y is the same as of n independent variables $y_1, y_2, \ldots y_n$, each with the density (2.4).

Under these assumptions, one can derive a formula for the exceedance probability $P\{\|y\| > L\}$ which is analogous to (2.3). The derivation is laborious but straightforward. It is simplified if one can made a sixth assumption, namely $L \geq y_o > s$, which is not unreasonable and which will be made here. One then finds

$$P\{\|y\| > L\} = 1 - \exp\{-\exp[-\tfrac{L}{s}(r\log 2n)^{\frac{r-1}{r}} + \eta']\} \quad (2.6a)$$

with

$$\eta' = \log a_2 + r\log 2n + \frac{m-r+1}{r}(\log r + \log\log 2n) \quad (2.6b)$$

This is again valid assymptotically for large (L/s) and large n, but in the sense that terms of orders $O(L/s)^{-r}$ and $O(\log n)^{-1}$ are negligibly small.

Expression (2.5) for the exceedance probability is of roughly the same double exponential form as its counterpart (2.5). Since all parameters of the underlying density $p(y)$ enter into the second exponent, and some even exponentially so, the probability is very sensitive to even small changes in them.

The changes that are of interest here are those in the parameters a_1, a_2, s, m, and r, away from the values (2.5) which they take if the random process y is Gaussian. Their effect on the failure probability $P\{\|y\| > L\}$ can be evaluated by a contentional perturbation calculation. If p_g is used to denote the value of this probability when y is Gaussian, and δp the change induced by small departure δ_s, δ_r, and δ_m, from Gaussianity one finds

$$\frac{\delta p}{p_g} = [M_1\frac{\delta s}{\sigma} + M_2\delta m + M_3\delta r]\log p_g \quad (2.7)$$

where

$$\begin{aligned} M_1 &= (\tfrac{y_o}{\sigma})^2 - 1 - \tfrac{L}{\sigma}(2\log 2n)^{\frac{1}{2}} \\ M_2 &= \log\tfrac{y_o}{\sigma} + \tfrac{1}{2}(\log 2 + \log\log 2n) \\ M_3 &= \tfrac{1}{4}(\tfrac{y_o}{\sigma})^2(1 - 2\log\tfrac{y_o}{\sigma}) + \log 2n \\ &\quad + \tfrac{1}{4}\tfrac{L}{\sigma}(2\log 2n)^{\frac{1}{2}}(1 - \log 2 - 4\log\log 2n) \\ &\quad - \tfrac{1}{4}(1 + \log 2 + 4\log\log 2n) \end{aligned} \quad (2.8)$$

The expressions are valid if, as before, terms of orders $O(L/\sigma)^{-1}$ and $O(\log n)^{-1}$ are considered negligible relative to 1, and if the same is true of terms of order $O(y_o/\sigma)^{-2}$.

A mere inspection of (2.7) and (2.8) shows that even small changes in the one-dimensional density of the process y are prone to produce large changes δ_p in the failure probability. Numerical work confirms this. Suppose, for example, that a system had been designed on the assumption that y is Gaussian, and for a failure probability of $p_g = .05$. This would mean that L/σ would have been set at $L/\sigma = 1.64$. In order to simplify the formulae (2.8), suppose further th. $y_o = L$ (i.e. that the departure from Gaussianity most pronounced beyond the failure limit L), and that $n = 20$ (i.e. that the random process is equivalent to 20 independent random variables). In that excitation process x. The transformation from one to the other can be difficult. By contrast, the question of whether the value of 0 applies in (3.3), or 1, seems relatively easy to settle in practice.

The example to be treated in the next two sections will, it is hoped, bear out these comments.

THE CRITICAL EXCITATIONS AND RESPONSES OF LINEAR SYSTEMS

An example which illustrates the general remarks just made will be discussed in this section and the next. It arose from a problem in earthquake engineering.

The ground motions during earthquakes form an almost ideal example of a random process whose precise statistics are very imperfectly known and unlikely to be well-known in the near future. It has been customary in recent years to make the assumption that the ground motions form a Gaussian random process. However, very little evidence in this direction has ever been presented and what evidence exists, apparently does not support the assumption [5]. This uncertainty, of course, is transmitted to the response. As a consequence, and as explained in Sect. 2, any statement regarding structural failure or survival is liable to be in serious error.

Based on the above remarks, one should next inquire what information concerning the statistics of ground motion during earthquakes is well enough established, to be used towards the prediction of structural failure. One can perhaps say that the distribution of ground motion intensities is based on a sample of sufficient size to qualify in this respect. Such information has been accumulated over many years, as pointed out by Housner [6, p. 97-99]. There are admittedly many possible definitions for the term "intensity." In this paper it will be convenient to define it as the L_2-norm $\|x\|$ of the ground acceleration x, i.e. by

$$\|x\|^2 = \int_{-\infty}^{\infty} x^2(t)\,dt\,.$$

(Other norms, in particular the maximum ground acceleration, could be used equally well and might even seem more natural here.) One can then perhaps assume that the distribution of $\|x\|$ can be equally well documented no matter which definition is adopted and in fact that it is of the roughly exponential form that has been pointed out by Housner [6, ibid.].

Suppose now that the distribution of $\|x\|$ is actually all that is reliably known regarding the stochastic nature of the ground motion. Suppose further that the response of a variable, such as the base shear or base moment is of interest in an elastic structure. This response is then related to the ground acceleration by the Duhamel integral

$$y(t) = \int_{-\infty}^{\infty} h(t-\tau)\,x(t)\,dt. \qquad (4.1)$$

in which h is the impulse response of the variable. The sets S introduced in the preceding section are of a special form in this example, namely

$$S = \{y: y(t) = \int_{-\infty}^{\infty} h(t-\tau)x(t)dt;\ \|x\| \leq M\}\,. \qquad (4.2)$$

Each consists of all responses that are generated by ground motions with intensities $\|x\| \leq M$. The probability measure of each S is

$$P(S) = P\{\|x\| \leq M\} \qquad (4.3)$$

which, as was just assumed, is all that is known regarding the statistics of the ground motions.

The question now, according to (3.3), is whether or not there are any responses in S whose peaks $\|y\|$ exceed the failure limit L. The answer can be given quite easily, by a straightforward use of the Schwarz inequality [7]. One finds that a response in S with the highest peak $\|y\|$ is

$$y^*(t) = \frac{M}{N} \int_{-\infty}^{\infty} h(t-\tau)\,h(-\tau)\,dt\,,$$

and that all others differ from y^* only by time shifts or by a change in sign. Here, N has been used for

$$N^2 = \int_{-\infty}^{\infty} h^2(t)\,dt < \infty.$$

The peak of y^*, namely

$$y^*(0) = MN\,, \qquad (4.4)$$

occurs at t=0. It is generated by the excitation

$$x^*(t) = \frac{M}{N}\,h(-t)$$

which is, except for the constant factor, (M/N), the time-reversed impulse response of the variable under consideration. The pair x^*, y^* have been called the "critical excitation" and the "critical response" of the strucutral variable; relative to the set S. Hence, the title of this section.

It should be added that the set S, as defined in (4.2) developed to be too large in many cases in practice: the critical response peaks (4.4) were often unrealistically large. However, the fact that the term "unrealistic" can be used here at all implies that some information is available regarding ground motions, other than merely the distribution (4.3) of their intensities, as has been assumed here. For, if that were really all that is known it would not be possible to disqualify some of the response peaks as being excessively large. There has been some speculation of what this additional information might be. Shinozuka [8] has suggested the envelope of the Fourier amplitude spectrum as one possible item, and Iyengar [9] the envelope of the time history of the ground acceleration as another. Either suggestion amounts to a restriction of the sets (4.2) or, equivalently, a refinement of the kind that has been advocated in Sect. 3 for Σ', as being helpful towards the reduction of excessive conservatism. The writer and his colleagues have experimented with yet another restriction, which seems to be successful in that the residual conservatism is quite well consistent with good structural design practice [10].

All of the restrictions mentioned here, however, suffer from the same defect, namely, that it is very difficult to say just what the probabilities P(S) of the resulting sets S are. Beyond that, the approach has been criticized on several counts, for instance, the fact that each structural variable has its own critical excitation and response and hence must in principle be analyzed individually, or the implicit assumption that all uncertainties in the response statistics are imputed to the ground motion and none to the structure. Work is under way which will, it is hoped, meet these and other objections.

case one finds

$$\frac{\delta p}{p_g} = 15.3 \frac{\delta s}{\sigma} - 22.4\, \delta r - 4.50\, \delta m.$$

This shows that merely a change in m alone from 0 to 1, produces a change in the failure probability by a factor of 4.5.

Such a change would be extremely difficult to detect statistically, on the level of confidence which one would often wish to attach to an estimate of the failure probability. The usual statistical tests in particular which aim at the estimation of certain mean values of the density p(y) of y, are known to yield no information regarding the behavior for large values of y [4].

The evidence presented here therefore indicates that a reliable estimation of the failure probability will often be very difficult, basically of course because it depends on the behavior of the underlying random process for large values of its sample functions. That, however, is the region that is the least accessible to robust statistical tests.

PROBABILISTIC AND WORST-CASE ANALYSIS COMBINED

The discussion in the preceding section has, it is hoped, made a reasonably persuasive case for the non-robust nature of the probability of a system failure which is induced by the magnitude of its response. Unless the stochastic characteristics of the latter are very well known precise pronouncements regarding the former will often be impossible. Under the circumstances, one may have to settle for weaker statements regarding this probability, especially upper bounds, and seek to make these as robust as possible. In order to do so, one may have to follow a line of reasoning which seems to be quite generally valid and which, the writer believes, will frequently be inevitable. It leads to a cross between probability theory and worst-case analysis.

In this procedure, one would first of all utilize any information which is known on the desired level of statistical confidence and which bears on the probabilistic structure of the random process y under study. It is possible in principle that this information characterizes the random process completely. This is unlikely however, for in that case it would have to specify the probability measure on the sigma algebra Σ of all (measurable) sets of sample functions of the process. More often, the reliable information will be incomplete, in the sense that it specifies with the desired assurance the probability measure only on the sets of a family Σ' within Σ (Σ' will in fact either be a coarser subsigma algebra of Σ, or else will have to be embedded in one.)

What matters here is that, so far as any statement regarding the random process y are concerned, they cannot be made on the sets in Σ but only on those in the coarser Σ'. They will be correspondingly weaker statements, and the best thing to do is to make Σ' as fine as it can be made, consistently with the available information.

The next question is how to arrive at those weaker statements. One can, and may even be forced to, proceed as follows.

Suppose that S is a set in Σ', and more particularly one that consists of, or at least contains, all sample functions that are of interest in a particular problem. The question that is considered in this paper is the probability that some among those sample functions induce failure. In other words, it is desired to know the probability of the intersection

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$$\{y : \|y\| > L\} \cap \{y \in S\} \quad , \qquad (3.1)$$

using the somewhat imprecise notation introduced above. This probability clearly obeys the inequality

$$0 \leq P\{\|y\| > L \mid y \in S\}\, P(S) \leq P(S) \qquad (3.2)$$

The upper bound is attained if almost all sample functions $y \in S$ of the process exceed the failure limit L, i.e., if the intersection (3.1) is essentially equal to S; the lower bound applies if almost none do, i.e., if the intersection is essentially empty.

One can now use these two bounds towards statements such as "the failure probability of a system will not exceed P(S) when $y \in S$." or "the system will not fail under this condition." Moreover, these statements will carry the same degree of assurance as the information that led to the definition of the set S in the first place. They may however, be rather extreme. The first one in particular may be extremely conservative, in fact, even pessimistic in many cases: the upper bound P(S), as just mentioned, is attained only if essentially all sample functions in S produce failure. The second one will, for similar reasons, be attained only rarely.

The point to be made here is that, pessimistic or not, it often is impossible to do much better. There will, of course, be the temptation of reducing the conservatism of these statements or, which is saying the same thing, of estimating the magnitude of the factor $P\{\|y\| > L \mid y \in S\}$ in (3.2). This is actually usually done. The tacit argument in such cases is that it is better to avoid excessive conservatism than to avoid unreliable information. Consequently, various assumptions are made which are thought to be reasonable and which allow a calculation, or at least an estimation, of $P\{\|y\| > L \mid y \in S\}$. The moral of the discussion of the proceding section, however, is that this is risky business: it will often be better to make only those statements that can be make on a level of confidence that is consistant with the one attached to the data, and to let the resulting conservatism fall where it may. These are then statements of the kind that have been suggested above. They amount to setting

$$P\{\|y\| > L \mid y \in S\} = 0 \text{ or } 1 \; , \qquad (3.3)$$

depending on whether the intersection (3.1) is, or is not empty.

The problem then becomes one of first making the family Σ' as fine as possible, i.e., of utilizing all information that is considered to be reliable enough to be used. In this way, the upper bound P(S) in (3.2) will be tightened as much as possible. Also, the achievement of the lower bound, namely zero, will be made more likely. Secondly, the intersection (3.1) must be studied: if it is found empty, the lower bound applies; if not, the upper.

This procedure is in effect a combination of probability theory with worst-case analysis: probability theory is used in setting the measures P(S) of the sets $S \in \Sigma'$; the worst-case analysis complements it, via (3.3), by allowing no probabilities other than 0 and 1.

In practice, the probabilistic part seems to be more difficult than the second. It is often doubtful just what information is available, and also reliable enough to be used in the determination of the sets S. A further complication often is that the information which is available does not pertain directly to the response process y of a system, but to the

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A recent generalization [11] of the result mentioned in the preceding section from linear to nonlinear systems may be of sufficient interest to be reported on here briefly.

What has been shown more specifically is this. The critical excitation x^* and response y^* of a nonlinear system obey to sets of simultaneous equations. One is, of course, the set which defined the system under consideration. The second set is obtained from the first by

(a) linearizing it about x^* and y^*,
(b) replacing x with $k\delta$ where δ is the unit impulse function and where k is so determined that $\|x\| = M$, and
(c) reversing time, i.e., replacing t with $(-t)$.

For example, if the system under consideration is given by a single differential equation of the form

$$g\left(\frac{d^n y}{dt^n}, \frac{d^{n-1}y}{dt^{n-1}}, \ldots y\right) = x \quad , \tag{5.1}$$

the critical excitation and response obey two differential equations, namely

$$g\left(\frac{d^n y^*}{dt^n}, \frac{d^{n-1}y^*}{dt^{n-1}}, \ldots y^*\right) = x^* \tag{5.2}$$

which merely expresses the fact that y^* is the response to x^*, and

$$a_n(t)\frac{d^n x^*}{dt^n} + a_{n-1}(t)\frac{d^{n-1}x^*}{dt^{n-1}} + \ldots + a_o(t) = k\delta \tag{5.3}$$

where

$$a_k(t) = (-1)^k \left.\frac{\partial^k g}{\partial y^{(k)}}\right|_{y=y^*}, \quad \left(y^{(k)} = \frac{d^k y}{dt^k}\right)$$

which is necessary for the criticality of x^* and y^*.

Another way of stating this result which brings out the parallel with linear systems is the following. The critical excitation x^* is again, except for the constant factor k, a time-reversed impulse response. However, by contrast to linear systems, it is not the impulse response of the given system but of a linearized version of it. The linearization must more particularly be around the critical excitation/response pair.

The result holds not only for systems that are, or can be, defined by a single differential equation, such as (5.1). On the contrary, substantially more general excitation/response relationships are admissible than nonlinear differential equations. In particular history-dependent failure mechanisms, such as material fatigue, are subsumed under it.

The result is derived in roughly the following way. The system is first assumed to be specified by its Volterra series [12], rather than by its differential equations. This is done partly for sake of greater generality and partly to preserve the analogy to the Duhamel integral in (4.1). The result then follows very quickly by a variational argument. Some attention must be paid to the fact that Volterra series frequently have small radii of convergence, and to the transition from those series to other system representations, such as (5.1).

The solution that is obtained in this way is valid under fairly general conditions. It has, however, certain drawbacks as well. Among those are, to begin with, all those mentioned in the preceding section in connection with linear systems. In addition, the solution for nonlinear ones need not be unique. There may, in other words, be more than one excitation/response pair that satisfies eq.'s (5.2) and (5.3), or others like these. Finally, and perhaps most importantly, these equations unfortunately cannot be solved simultaneously; the obstacle develops to be time-reversal in the second equation, as one recognizes quite easily. The solution can often be carried out by successive approximation, however. On the basis of some limited computational experience, there is in fact hope that the approximation will converge quite rapidly in many problems of practical interest.

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APPENDIX C

THE CRITICAL EXCITATION OF NONLINEAR SYSTEMS

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ABSTRACT

The critical excitation of a mechanical system, in the terminology of this paper, is one that drives the system to a larger response peak than any other in some class of allowed excitations. The critical excitation is of interest in questions related to the reliability and safety because the magnitude of the response peak is frequently an indicator of the survivability of the system. The problem of finding it has been solved for linear systems some time ago. This paper deals with the generalization of the problem to nonlinear systems. It is shown that its solution is in many ways analogous to its earlier counterpart.

1. Introduction

This paper deals with a problem that is encountered in questions of the reliability or safety of mechanical systems. It is then often important to know what the largest response peak is to which the system can be driven by any of some class of possible excitations. The idea is that the magnitude of that maximum response peak will indicate whether or not one should be prepared for a possible system failure. The excitation that achieves this peak has been called the "critical excitation" of the system, and the response which it generates the "critical response." The terminology is used also here.

The problem is patently undefined unless a class of excitations is specified among which the critical is to be found. In this paper, that class is assumed to consist of all excitations whose square-integral is limited to a certain value. This assumption is often appealing because the square-integral can be interpreted as representing the energy, or the intensity of the excitation.

As it happens, the assumption is appealing for another reason as well: for it develops that it leads to a particularly neat solution when the system under consideration is linear. The excitation that is critical under this assumption is, except for a constant factor, the impulse response of the system, reversed in time [2].

The restriction to linearity is, however, quite inappropriate to problems that touch on system failures. Nonlinearities are virtually inevitable as failure is approached, and failure itself is patently a nonlinear phenomenon as well. The generalization of the problem to nonlinear systems is therefore highly desirable. It is the subject of this paper.

The main result is derived in Section 3. It is shown there that an interesting parallel exists between the solutions for linear and for nonlinear systems. The critical excitation of a nonlinear system is found to be again, except for a constant factor, a time-reversed impulse response, and more particularly that of a certain linearization of the given system.

Section 4 discusses some supplementary questions. The non-uniqueness of the solution is emphasized first. Relations to earlier work are reviewed next. In particular, the possibility of arriving at a very similar result through optimal control-theory is pointed out. A computational procedure, its convergence and divergence, are discussed. The application is illustrated by a simple example.

The solution derived in this paper uses some concepts from functional analysis which are collected and briefly explained in Section 2.

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2. Preliminaries

This paper considers systems which transform excitations x, subject to

$$(2.1a) \qquad \int_{-\infty}^{\infty} x^2(t)dt < \infty,$$

into responses y which obey

$$(2.1b) \qquad \sup_b |y(t)| < \infty \quad (-\infty < t < \infty).$$

Such systems can be represented in many ways, and several will in fact be used below. One representation is in terms of an operator H which carries x into y, or its inverse $G = H^{-1}$, i.e.,

$$(2.2) \qquad y = H(x), \qquad x = G(y).$$

Most physical systems have the property that no excitation x by itself determines a response y, and that an additional variable (the "initial state" q) must be added to define y uniquely. H is then a function of x and q. In this paper, this complication will be avoided by the convention that the system is initially at rest and that the initial state is fixed accordingly. The same assumption, if needed, will be understood to apply to G.

Under certain further assumptions, operators such as H and G can be expanded into series which have properties similar to Taylor series in the elementary calculus. In fact, one writes them in the same form, namely

$$(2.3) \qquad y = H(x) = H(\bar{x}) + H'(\bar{x})\,(x-\bar{x}) + \frac{1}{2!}\,H''(\bar{x})\,(x-\bar{x})^2 + \ldots$$

$$(2.4) \qquad x = G(y) = G(\bar{y}) + G'(\bar{y})\,(y-\bar{y}) + \frac{1}{2!}\,G''(\bar{y})\,(y-\bar{y})^2 + \ldots .$$

The interpretation of the terms in such series, conditions for their existence, and regions of convergence are known (e.g., [4], p. 112). It is further known that these conditions in general insure the existence of similar series for the "derivatives" $H^{(k)}(x)$ and $G^{(k)}(x)$. For instance,

(2. 5) $$H'(x) = H'(\bar{x}) + H''(\bar{x})(x-\bar{x}) + \frac{1}{2!} H'''(\bar{x})(x-\bar{x})^2 + \dots \quad .$$

Furthermore, since H and G are each other's inverses, the derivatives in the series (2. 4) and (2. 5) are related. Thus, if

$$\bar{y} = H(\bar{x}), \qquad \bar{x} = G(\bar{y})$$

the first derivatives in (2. 4) and (2. 5) obey ([6], p. 36)

(2. 6) $$H'(\bar{x})\, G'(\bar{y}) = G'(\bar{y})\, H'(\bar{x}) = I$$

where I is the identity operator.

For many purposes, it is inconvenient to represent a system by an operator which characterizes the response y for all times t, in terms of the excitation x, or vice versa. It is often more appropriate to have a representation which defines the response $y(t)$ at only specified time t, or conversely. The restriction to such specified times leads to two functionals h and g, which are the counterparts to the operators H and G, and for which

(2. 7) $$y(t) = h(t;\, x), \qquad x(t) = g(t;\, y),$$

in place of (2. 2).

The Taylor series (2. 4) and (2. 5), too, have their analogs for the functionals f and g. One can show more specifically that, under the condition (2. 1), a series expansion is possible for the functional h. It is

$$y(t) = \bar{y}(t) + \int_{-\infty}^{\infty} d\tau h_1(\bar{x}; t-\tau_1)\,[x(\tau_1) - \bar{x}(\tau_1)] + \ldots +$$

(2.8)

$$+ \int_{-\infty}^{\infty} d\tau_1 \int_{-\infty}^{\infty} d\tau_2\, h_2(\bar{x}; t-\tau_1, t-\tau_2)\,[x(\tau_1) - \bar{x}(\tau_1)][x(\tau_2) - \bar{x}(\tau_2)]$$
$$+ \ldots$$

and it reduces to the well-known Volterra series ([8], p. 21) when the reference excitation and response $\bar{x}(t) = \bar{y}(t) \equiv 0$.

The series for the functional g is not in general of the same form as (2.8) under the condition (2.1). In fact, (2.7b) is frequently a differential equation in y, say

$$x = g(y^{(n)}, y^{(n-1)}, \ldots, y). \tag{2.9}$$

The Taylor series for g is then a conventional one, namely

$$x(t) = \bar{x}(t) + \sum_i \frac{\partial g}{\partial y^{(i)}}\,[y^{(i)}(t) - \bar{y}^{(i)}(t)]$$

(2.10)

$$+ \frac{1}{2!} \sum_i \sum_j \frac{\partial^2 g}{\partial y^{(i)} \partial y^{(j)}}\,[y^{(i)}(t) - \bar{y}^{(i)}(t)][y^{(j)}(t) - \bar{y}^{(j)}(t)] + \ldots$$

in which the derivatives of g are evaluated at $y(t) = \bar{y}(t)$. Mechanical systems are not usually defined by single differential equations of the form (2.10). Nevertheless, series expansions are typically possible. They are then often certain combinations of the forms (2.8) and (2.10)

The operator equations (2.5) and (2.6) have their counterpart for the functionals h and g, as well. Thus, the analog of (2.5) is

$$h_1(x;t) = h_1(\bar{x};t) + \frac{1}{1!} \int_{-\infty}^{\infty} d\tau_2\, h_2(\bar{x}; t, t-\tau_2)[x(\tau_2) - \bar{x}(\tau_2)] + \ldots \tag{2.11}$$

and the analog of (2.6), assuming g to be the simple differential equation (2.9),

$$\sum_i \frac{\partial g}{\partial y^{(i)}} h_1(\bar{x};t) = \delta(t), \tag{2.12}$$

with the derivatives evaluated at $y(t) = \bar{y}(t)$ and with $\delta(t)$ the unit impulse function. The latter is a linear differential equation with time-variable coefficients and certainly has a solution if $\partial g/\partial y^{(n)} \neq 0$ for $y = \bar{y}$ and all t. The solution, namely h_1, is therefore the unit impulse response. This last observation applies also if g is of a more complicated form than (2.9), as one easily recognizes.

3. Critical Excitations

The objective in this section is to derive the main result of this paper namely, a characterization of the critical excitation of a nonlinear system. This is the excitation which drives the system to a larger response peak than any other, of some given class of admissible excitations. Which excitation turns out to be critical evidently depends on the class that is considered admissible in the first place. In this paper, all excitations will be considered admissible which obey

$$\int_{-\infty}^{\infty} x^2(t)\, dt = M^2, \tag{3.1}$$

for some given "intensity" M^2. (A comment on the possibility of replacing (3.1) with the more appropriate inequality constraint is made below in Section 4.3.)

For a linear system, the characterization of the critical excitation x^* under the constraint (3.1) is known [2]: if the system has the impulse response h, the critical excitation is, up to a constant factor, the time-reversed version h(-t) of h(t). To be more specific,

$$x^*(t) = \pm \frac{M}{N} h(-t) \tag{3.2a}$$

where

$$N^2 = \int_{-\infty}^{\infty} h^2(\tau)\, d\tau , \tag{3.2b}$$

provided only this quantity is finite (as it is for any stable linear system).

It will be shown here that a very similar result holds also for a nonlinear system. Its critical excitation is again the time-reversed version of an impulse response, but it is not that of the given system but of a certain linearized version of it. A proviso similar to (3.2b) develops to be necessary as well.

The first result to be derived here is the following.

Theorem 3.1. Let a system of the type described in Section 2 be given. An excitation x^* that is critical under the constraint (3.1) is then the time-reversed impulse response of the impulse response of a linear system, namely of the linearization of the given system at $x = x^*$, provided this linearization is stable in the sense that

$$\int_{-\infty}^{\infty} h_1^2(x^*;\tau)\, d\tau < \infty . \tag{3.3}$$

Here, h_1 is the kernel in the Volterra-Taylor representation (2.8) of the system operator.

Proof. Since the given system is assumed to be time-invariant, it is no restriction to specify the time t_m at which the response peak is to occur, and to set that time $t_m = 0$. This will be done here. It is then desired to maximize, with respect to all excitations obeying (3.1) the magnitude of

$$\begin{aligned} y(0) = \bar{y}(0) &+ \int_{-\infty}^{\infty} d\tau_1\, h_1(\bar{x};-\tau_1)[x(\tau_1) - \bar{x}(\tau_1)] \\ &+ \frac{1}{2!}\int_{-\infty}^{\infty} d\tau_1 \int_{-\infty}^{\infty} d\tau_2\, h_2(\bar{x};-\tau_1,-\tau_2)[x(\tau_1) - \bar{x}(\tau_1)][x(\tau_2)-\bar{x}(\tau_2)] \\ &+ \dots \end{aligned} \tag{3.4}$$

and where $\bar{x}$ is a suitable reference input to be specified later on. For the moment, one need only require that $\bar{x}$ leads to a response $\bar{y}$ with $|\bar{y}(0)| < \infty$. The critical excitation will then be among those for which the first variation $\delta y(0)$ of $y(0)$ vanishes, with respect to all variations $\delta x(\tau)$ that are consistent with the constraint (3.1). It is, as always, helpful to introduce this constraint into the variation of $y(0)$ by way of a Lagrange multiplier λ. A necessary condition for x^* to be critical under the constraint (3.1) is then that

$$0 = \delta y(0) + \lambda \delta \int_{-\infty}^{\infty} x^2(\tau)\, d\tau$$
$$= \int_{-\infty}^{\infty} \delta x(\tau) d\tau \left[h_1(\bar{x};-\tau) + \frac{1}{1!}\int_{-\infty}^{\infty} h_2(\bar{x};-\tau,-\tau_2)x^*(\tau_2)d\tau_2 + \ldots + 2\lambda x^*(\tau)\right]$$

should hold identically for all $\delta x(\tau)$. In writing this expression it has been assumed that the kernels $h_2, h_3, \ldots$ are symmetric functions of their time-argument, which is alright, as is well known [8, p. 19]. The identity can however hold only if

$$2\lambda x^*(\tau) + h_1(\bar{x};-\tau) + \frac{1}{1!}\int_{-\infty}^{\infty} h_2(\bar{x};-\tau,-\tau_2)x^*(\tau_2)d\tau_2 + \ldots + = 0.$$

The terms beyond the first, as a comparison with (2.11) shows, represent the linearization of the system operator H at x^*. That is,

$$h_1(\bar{x};-\tau) + \frac{1}{1!}\int_{-\infty}^{\infty} h_2(\bar{x};-\tau,-\tau_2)x^*(\tau_2)d\tau_2 + \ldots = h_1(x^*;-\tau).$$

Equation (3.3) can therefore be written

(3.5) $$-2\lambda x^*(\tau) = h_1(x^*;-\tau),$$

and bears out the main assertion of the theorem. The reference excitation $\bar{x}$, as this result shows, is best chosen to coincide with x^*. In particular, convergence is then assured when $\|\delta x\|$ is small. The factor (-2λ) in (3.5) must now be so determined that (3.1) holds, i.e., so that

(3.6) $$\int_{-\infty}^{\infty} h_1^2(x^*;\tau)d\tau = 4\lambda^2 M^2$$

which is clearly possible if (3.3) holds. The proof of the theorem is thus complete.

The proof assumes the system under consideration to be specified by a Taylor series of the form (2.8), preferably around x^* as reference excitation. In practice, however, a system is typically specified not by such a series but by a set of differential or similar equations. It is of interest to inquire what form the result above takes in such a case. It will be convenient to start by assuming that the system is specified by a single differential equation of the form (2.9), i.e.,

$$g(y^{(n)}, y^{(n-1)}, \dots, y) = x \tag{3.7}$$

in which $y^{(n)}$ stands for the n-th derivative of y, and g is such that the equation is meaningful and has a unique solution y either for every admissible x or for at least an interesting subset of these. In such a case, the following can be said.

Corollary 3.1. Let a system be described by an equation of the form (3.7). Its critical excitation x^* then differs by only a constant from the time-reversed impulse response of the linear system

$$\frac{\partial g}{\partial y^{(n)}} v^{(n)} + \frac{\partial g}{\partial y^{(n-1)}} v^{(n-1)} + \dots + \frac{\partial g}{\partial y} v = u \tag{3.8}$$

in which the derivatives $\partial g / \partial y^{(i)}$ are evaluated for the critical response $y = y^*$.

Proof. It merely needs to be shown here that the impulse response of the system (3.8) is $h_1(x^*, \tau)$. The assertion of the corollary then follows from (3.5). Recall from the discussion in Section 2 that (3.7) can be considered the representation of an operator G which transforms the response y into the excitation x, as in (2.1b), and which is therefore the inverse of H. Their first derivatives are then related by (2.6) or, if written in terms of the functionals g and h by (2.12). Restated here, for convenience, the relation is

$$\text{(3.9)} \qquad \frac{\partial g}{\partial y^{(n)}} h_1^{(n)} + \frac{\partial g}{\partial y^{(n-1)}} h_1^{(n-1)} + \ldots + \frac{\partial g}{\partial y} h_1 = \delta$$

where h_1 has been written for $h_1(x^*;t)$ and where the derivatives of g are evaluated at $y = y^*$. According to this equation, $h_1(x^*;t)$ is indeed the impulse response of the system (3.8), and the corollary is proven.

It may be noticed that no essential use has been made in this proof of the fact that the system is described by a single differential equation, namely (3.7). The functional g in that equation could actually have been assumed to be of a more general kind. Thus, instead of depending merely on y and its derivatives at one and the same time t, as g does in (3.7), the dependence could be on y as well as its derivatives at times other than t. In particular, therefore, the single differential equation could be replaced with a set of simultaneous ones, involving along with the response y other auxiliary variables, for instance, the components of some state vector. Moreover, these variables need not enter only through their values at one and the same time t. They could also do so by way of the history of their values in the past (as they would, e.g., for a system with hysteresis effects). These observations are summarized as

<u>Corollary 3.2.</u> The preceding corollary remains valid if the differential equation (3.7) is replaced with a system of simultaneous differential or functional equations which relate the response y of the system to the excitation x in a one-to-one fashion.

4. Discussion

It may be useful to supplement the results derived in the preceding section with some remarks which relate them to those obtained by others in the same and in other fields, and indicate how the results might be used and interpreted. This is done below. Also, an example is presented.

4.1 Uniqueness Questions

As is well known, variational arguments of the kind used to prove Theorem 3.1 merely lead to necessary conditions: the critical excitation x^* satisfies the condition (3.5) but other excitations may do the same. Those others may generate responses whose response peaks y_o represent local maxima, rather than the global one that is of interest. For that matter, $y(0)$ may even be a minimum or some other stationary value. The condition (3.5), in other words, does not in general characterize the critical excitation uniquely.

It would be desirable to have criteria which distinguish maxima from other stationary solutions, and the global one from local ones, if any. Criteria which do the former are known ([1], p. 177). One can also set up sufficient conditions for the latter. Those, however, seem to be difficult to verify in practice.

4.2 Connection with Earlier Work

As mentioned in the Introduction, the problem of determining the critical excitation is a problem in optimal control theory. In fact, this theory is directly applicable if the system is represented by one or more differential equations (see, e.g., [1], p. 47). However, the straightforward use of the maximum principle of that theory leads to a characterization of the critical excitation, namely in terms of the so-called adjoint system (ibid., or

[7], p. 85) that differs from the one obtained in Section 3. The adjoint system is a system of linear differential equations which are analogous, but not identical, to (3. 5) and which are solved with a different set of initial conditions. Moreover, the parallel to the result (3. 2) for linear systems is partly lost. This reason, and its slightly greater generality, suggested the approach that has been used in this paper.

The solution obtained from the maximum principle is more general in one respect than the one that is obtained here: it allows the constraint (3. 1) to be changed from an equality to the more appropriate inequality

$$\int_{-\infty}^{\infty} x^2(t)\, dt \leq M^2. \tag{4.1}$$

It is not difficult to convince oneself, however, that this generalization can also be incorporated in the approach used in this paper, without major changes in the proof and without a change in the result. One merely needs to argue that if x^* lay in the interior of the region (4. 1), λ could be set equal to zero in the proof of Theorem 1. 1. In that case, however,

$$\delta x(\tau) = \epsilon\, h_1(x^*;\tau)$$

would lead to an increase $\delta y(0)$, and $y(0)$ could not have been a maximum. The critical excitation, in other words, always obeys the equality constraint (3. 1).

The problem of finding the critical excitation for a nonlinear system was also treated by Iyengar [5]. His solution unfortunately does not in general lead to an excitation that generates a maximum response peak or, for that matter, to a value with extremal properties of any kind, local or otherwise. This was pointed out later on in a discussion of that paper [5].

4.3 Computational Questions

The characterization of the critical excitation which is most likely to be of interest in practice is the one contained in the corollaries to Theorem 3.1. Considering the one in Corollary 3.1, for instance, the critical excitation x^* and the critical response y^* are jointly determined by two differential equations, namely (3.7) and (3.8). They could therefore be calculated by solving them simultaneously if it were not for the time-reversal and the constant factor that have to be applied to the solution of the latter before it can be used in the former. Under the circumstances, a simultaneous solution unfortunately is not possible. The situation is reminiscent of, and in fact equivalent to, the two-point boundary value problems with split boundary conditions to which one is led by a solution via the maximum principle of optimal control theory.

There are several computational procedures that can be considered for the determination of x^* and y^*. One that suggests itself, and which incidentally avoids the numerical instabilities of boundary value problems, is a successive approximation method which proceeds as follows. Start with an excitation $\bar{x}$, preferably with one that is suspected of being at least similar to x^*, and determine the response to it by solving the equations defining the system, for instance (3.7). It is of course possible that $\bar{x}$ in fact is critical, i.e., $\bar{x} = x^*$. In that case, the response $\bar{y}$ will be critical also, $\bar{y} = y^*$. Equation (3.9), with the derivatives $\partial g/\partial y^{(i)}$ evaluated at this response, will then have a solution h_1 which differs from $\bar{x} = x^*$ by only a time-reversal and a constant factor which insures (3.1). More often, however, the initial $\bar{x}$ will not be critical, and neither will be the response $\bar{y}$ to it that is obtained by solving (3.7). One can nevertheless evaluate the derivatives $\partial g/\partial y^{(i)}$ for that response and solve (3.9). The solution h_1

found in this way, reversed in time and modified by a constant factor to enforce (3.1), can now be used as an excitation in place of $\bar{x}$, and the same procedure started all over again.

One can expect that the successive approximations process which is generated in this way will often converge on the critical excitation x^*, and in practice this in fact frequently happens (see Sect. 4.5 below).

4.4 Divergence

The approximation procedure described above will often converge on a critical excitation that generates at least a local maximum in the magnitude of the response peak y(0). It can, however, happen that the succession of solutions h_1 that are obtained in this process from (3.9) have square integrals

$$(4.2) \qquad \int_{-\infty}^{\infty} h_1^2\,(x;t)\,dt = N^2(x)$$

which grow successively larger and diverge to infinity. In the limit, therefore, the proviso (3.3) of Theorem 3.1 and of its corollaries is violated and the result invalid. One can surmise that in such a case the physical system is driven to a failure by its critical excitation. As a partial mathematical confirmation one can mention that the Taylor series (3.4) for y(0) is considered to be divergent near the excitation x if $N^2(x)$ in (4.2) is not finite.

4.5 Example

As an illustration of the ideas developed above consider a system with a single degree of freedom,

$$(4.3a) \qquad \ddot{y} + 2\xi\,\omega_o\dot{y} + f(y) = x,$$

with a bi-linear spring

(4.3b) $$f(y) = \begin{cases} \omega_o^2 y & \text{for } |y| \le \beta, \\ \omega_1^2 y + (\omega_o^2 - \omega_1)^2 \beta & \text{for } |y| > \beta, \quad \omega_o > \omega_1 \ge 0. \end{cases}$$

The linearization of (4.3) in the neighborhood of an excitation/response pair $(\bar{x}, \bar{y})$ is then

(4.4a) $$\ddot{y} + 2\xi \omega_o \dot{y} + f_1(y) = x$$

where

(4.4b) $$f_1(y) = \begin{cases} \omega_o^2 y & \text{when } |\bar{y}| \le \beta, \\ \omega_1^2 y & \text{when } |\bar{y}| > \beta. \end{cases}$$

It is desired to find a pair $(\bar{x}, \bar{y})$ such that $\bar{x}$ is the time-reversed solution y of (4.4) with $x = k\delta$, δ being the unit impulse and k so chosen that (3.1) is satisfied.

Suppose that the successive approximation procedure which has been described (in Section 4.3) is used, and that

(4.5) $$\bar{y}_o(t) = \omega_n \exp(-\xi \omega_o t) \sin \omega_n t \quad (\omega_n^2 = \omega_o^2 (1-\xi^2), \ t > 0)$$

is used as starting solution. To obtain its companion $\bar{x}_o$, one determines the impulse response h_o of (4.4), for $\bar{y} = \bar{y}_o$, and the constant k so that kh_o obeys (3.1). Both are done by numerical integration. Then $\bar{x}_o$ is the time reversed version of kh_o. This excitation is next used in (4.3) and a response $y = \bar{y}_1$ is calculated. The companion $\bar{x}_1$ to $\bar{y}_1$ is now obtained by the same procedure as $\bar{x}_o$ from $\bar{y}_o$. The process is repeated, and a succession of pairs $(\bar{x}_o, \bar{y}_o)$, $(\bar{x}_1, \bar{y}_1)$, ... derived. The response peaks $\bar{y}_o(0), \bar{y}_1(0), \ldots,$ should increase monotonely, if the procedure works as desired. It is stopped, when the increase does.

Computational experience with this type of example has been quite good over a fairly wide range of the parameters ω_o, ω_1, and β. Some tact and thoughtfulness in the choice of $\bar{y}_o$, tended to produce large returns in speed of convergence. (4.5) in particular is not always a good choice. The procedure then converged in less than ten successive approximations in all cases.

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APPENDIX D

CRITICAL SEISMIC EXCITATION AND RESPONSE OF STRUCTURES

by

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SYNOPSIS

The practicality of the critical excitation method for the design of structures is investigated. In the terminology of this paper, an excitation is critical for a structure if it generates a larger response peak in one of the design variables than any other possible excitation in some given class. The basic idea of the method is to draw up the design in such a way that the structure has sufficient reserve strength to sustain its own critical excitations up to a certain maximum intensity. This paper investigates more specifically a modification of the idea. It is shown, by an analysis of several existing and planned structures, that the modification leads to fairly realistic, if somewhat conservative designs. The results encourage the conclusion that the modified mehtod, or a similar one, may become a useful design tool of structures whose impostance justifies conservative design.

INTRODUCTION

The design of structures against seismic excitation is a process of decision-making under uncertainty. In most seismically active sites in the world, few recorded accelerograms and little reliable geological information are available. Someone entrusted with the design of an important structure must nevertheless decide what kind of ground acceleration it is to withstand. Under the circumstances, he may study the records obtained elsewhere, at localities with similar geological features, and base his design on one of these records. He would do so in the hope that this accelerogram or response spectrum represents an excitation likely to happen at that site.

A somewhat more rational procedure was proposed sometime ago[(1)]. A designer who follows it would select not a single accelerogram but a certain class of excitations which he considers to be realistic for the locality in question, and would then determine those in that class which generate the largest response peaks in each structural variable, such as a joint displacement or a member force. These excitations have been called "critical", and so have the responses of the structure. The idea is that the designer would draw up the design in such a way that it would have sufficient reserve strength to sustain its own critical excitations.

This design procedure can be cast in many forms. The paper begins by describing two. The first is one that is intuitively and conceptually appealing but unfortunately often leads to overly conservative designs. The second is a modification of the first[(2)], intended to avoid excessive conservativeness without introducing excessive computation complexity.

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The main purpose of the paper is to present results that were obtained using the modified procedure. In order to verify its practicality, a number of realistic structures were analyzed. The response peaks were calculated in each case for the modified critical excitation, along with those generated by several recorded ground motions of the same intensity. The ratios of the peaks, clled the "critical design factors" in this paper, are indicators of the degree of conservativeness of the procedure. These factors are shown to fall into the range of 1.1 to 2.9. There are two reasons for believing that factors in that order represent a reasonable degree of conservativeness. For one, the modified critical excitations appear to be fairly realistic samples of possible ground motions that cannot be disregarded out of hand. Secondly, as a fairly broad sample of strength calculations shows, designs by experienced engineering firms frequently have sufficient reserve strength to sustain these excitations.

The results suggest the conclusion that the modification of the critical excitation method, or some similar procedure, may become a practical engineering design tool for structures whose importance justifies some conservativeness.

ACKNOWLEDGEMENT

The results reported here were obtained through research supported by the National Science Foundation, Washington, D.C. The firm D.J. Degenkolb Associates, San Francisco, California, in the person of Mr. G. Dean, supplied the drawings for two of the buildings, as well as many helpful comments and discussions. Dr. J. Vellozzi worked up the structural data on these two buildings, and supplied the designs for two others. Dr. W. Wang of the Polytechnic Institute carried out the computer work. These contributions are gratefully acknowledged.

CRITICAL EXCITATION

In the case of forced vibration, the most damaging excitation on a structure can be expected to have a frequency content that closely matches that of the structure. This is actually so, under certain assumptions. As has been demonstrated before(1), the critical excitation $x_c^*(t)$ for a linear system is the time-reversed impulse response, multiplied by an intensity modification factor,

$$X_c^*(t) = \frac{E}{N} h(-t), \tag{1}$$

if E, the reference intensity of the ground acceleration is defined as

$$E^2 = \int_0^{t_e} \ddot{x}_g^2 (t)dt \tag{2}$$

and N is the square integral of the unit impulse response

$$N^2 = \int_0^{t_e} h^2(t)dt, \tag{3}$$

In both Eqs. (2) and (3), the "effective duration" t_e of a ground excitation is the period over which the excitation contributes significantly to the maximum response of the structure. t_e depends on the modes as well as the damping of the structure. The response peak under the critical excitation always occurs at the end of that period. It is

$$y_c^*(t_e) = EN \tag{4}$$

The frequency content of $x_c^*(t)$ is the same as that of the structure, in the sense that its Fourier amplitude spectrum differs from that of the structure only by the factor (E/N). A plot of the typical critical excitation with El Centro intensity is shown in Fig. 1.

MODIFICATION OF CRITICAL EXCITATION

The response peak (4) is often found to be unrealistically high. The intuitive reason for this is quite plain. The frequency content of x_c^* often differs greatly from that of any realistic ground acceleration. It is therefore necessary to exclude from the class of excitations that are being considered, all those with frequency contents that are unlike those of realistic ground motions. This can be done in many ways. One that seems particularly simple is the following.

Among the ground motions that are considered realistic for a particular site, one should presumably include a number n of recorded ones, x_i (i = 1, 2,..n), preferably those that have occurred at locations with similar geological features. In addition, all linear superpositions of the x_i might be considered to be realistic as well, provided only that their combined intensity does not exceed a prescribed maximum. This, at any rate, was done in the study reported here. Moreover, in order to avoid computational complication, it was not the critical excitation among these superpositions that was determined. Rather, an excitation x_ℓ^* was calculated which differed least (in the least-squares sense) from x_c^*. In symbols

$$x_\ell^* = \sum_{i=1}^{n} a_i x_i \tag{5}$$

so that

$$\int_0^{t_e} (x_\ell^* - x_c^*)^2 dt = \text{Minimum} \tag{6}$$

and

$$\int_0^{t_e} x_\ell^{*2}(t)dt = E^2 \tag{7}$$

A plot of a typical modified critical excitation x^* with El Centro intensity is shown in Fig. 2. It is, by all appearances, a sample of a perfectly realistic ground motion during an earthquake. One cannot, in other words, ignore it in the process of a design on grounds of its being "unrealistic" or "unlikely".

APPLICATION TO REALISTIC STRUCTURES

Several realistic structures were analyzed and some of the structural members were investigated by the critical excitation and response approach. The essential results are summarized in Tables 1 and 2.

Table 1 shows the "critical design factors" of some of the design variables of seven structures. The "critical design factors" are based on the ratio of the response of the second class critical excitation with that of the reference excitation of same intensity. The reference excitations are 1971 Pacoima dam S14W, 1940 El Centro S00E, and 1954 Eureka N79E. These factors range from 1.14 to 2.88.

Table 2 shows strength requirement for a modified critical excitation of El Centro intensity. The approximate ductility requirements for some of the members as they were designed are shown in the last column.

CONCLUSIONS

1. A modification of the critical excitation method is applied to several realistic structures. From the "critical design factos" calculated for each, and from strength checking on already designed ones, it appears that the method leads to results which are on the safe side but not overly conservative. This conclusion is further supported by plots of many of the modified critical excitations which are, by all indications, quite realistic samples of possible ground motions during earthquakes.

2. The modified mehtod, or some similar procedure, seems to have promise as a practical and useful tool for the design of structures in cases in which conservative design is desirable. This is likely to apply to structures of major importance, the destruction of which would cause severe human or economic losses.

3. Its attraction in such cases may lie in its ability to spot weak points in a design, and the fact that it eliminates much of the arbitrariness from the choice of the excitation on which designs now often are based.

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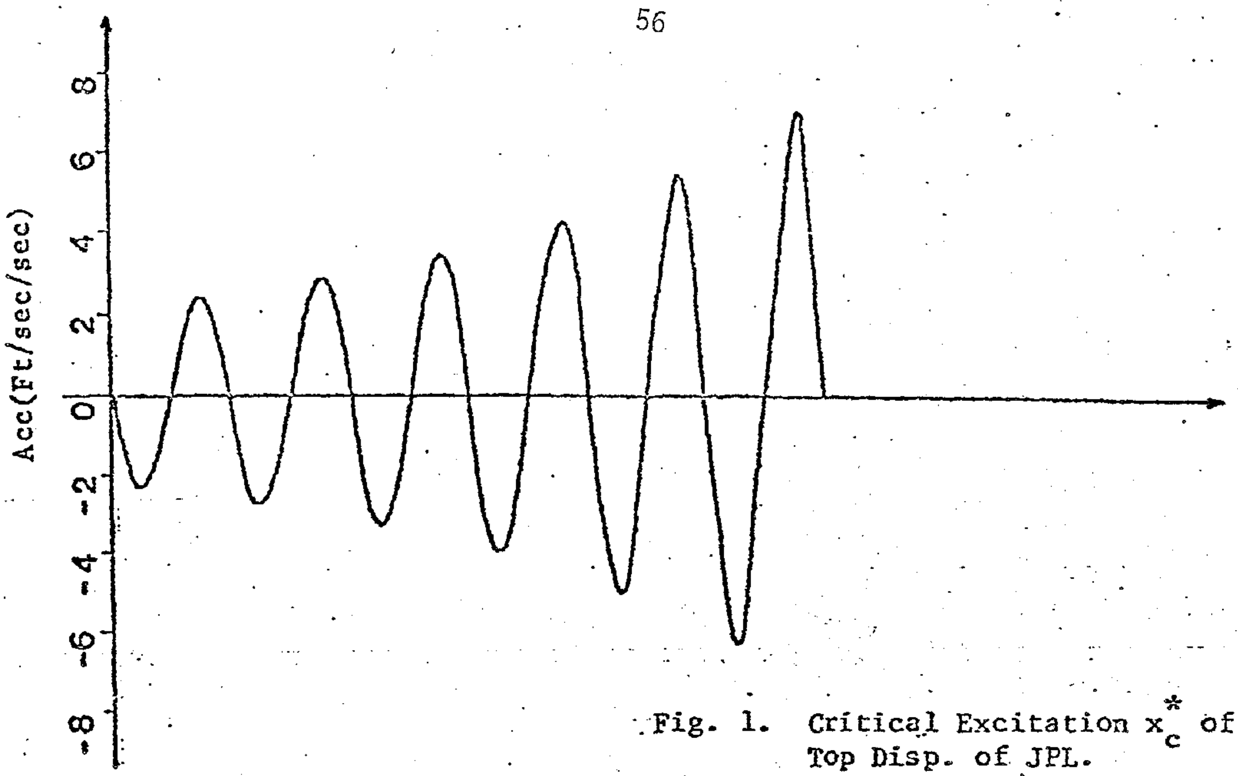

Fig. 1. Critical Excitation x_c^* of Top Disp. of JPL.

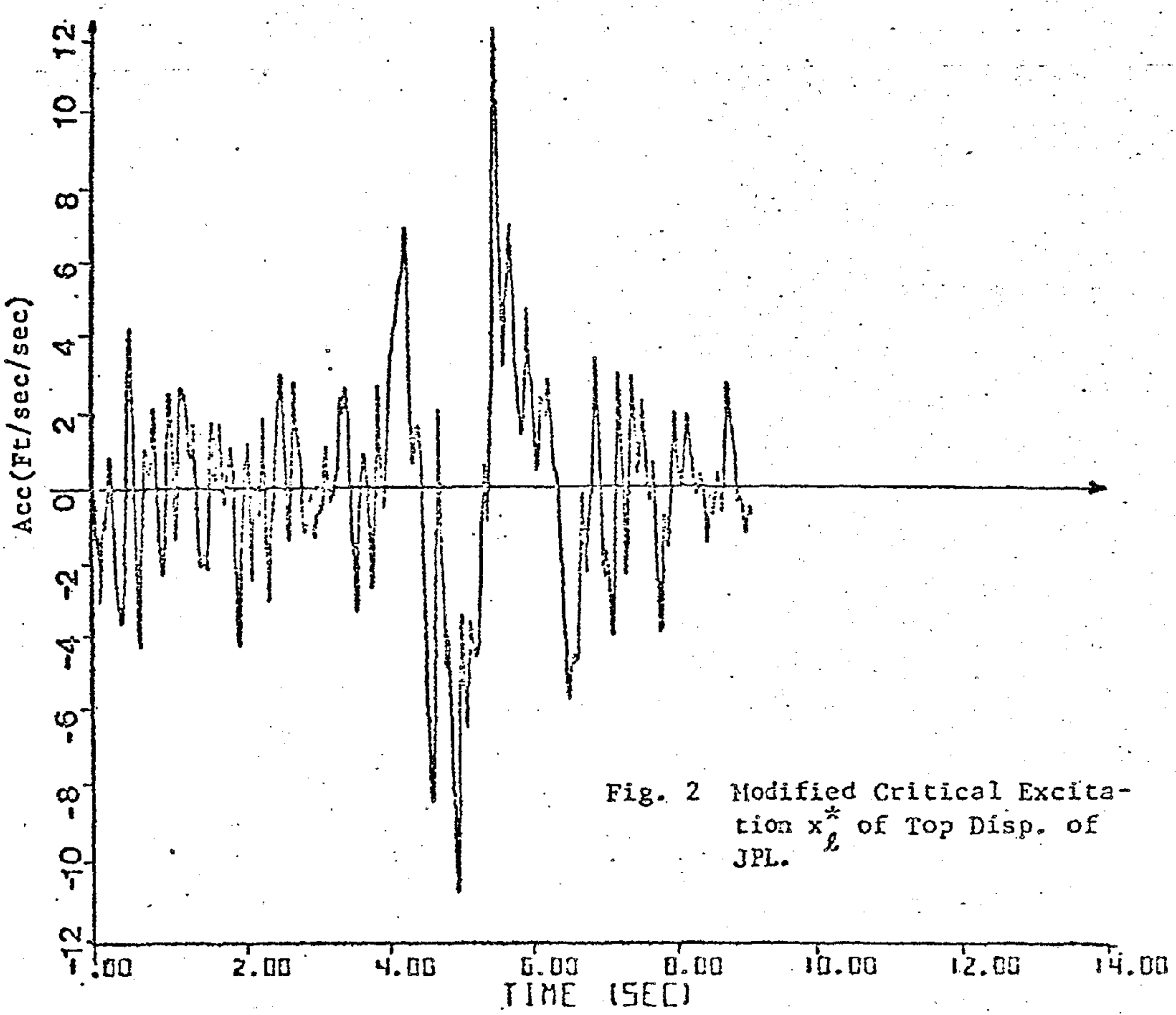

Fig. 2 Modified Critical Excitation x_ℓ^* of Top Disp. of JPL.

Table 1. CRITICAL DESIGN FACTORS

Structure	Fund. Period (sec)	Comparison Grd. Motion	Design Struct. Variable	Critical Design Factor	Source
R.C. Flat Plate Bldg. 16 stories	4.95	Pacoima Dam El Centro	Base M of Ext. Col.	1.59 1.24 1.50	J. Vellozzi Amman & Whitney
Tapered R.C. Chimney 1000 ft.	3.40	Pacoima Dam El Centro Eureka	Base V	1.48 1.33 1.32	J. Vellozzi Amman & Whitney
Reactor Shell	0.416	Pacoima Dam Eureka	Centroidal Defl.	1.39 1.14	Miller & Costantino(3)
Steel Struc. on Conc. Dike	0.214	Pacoima Dam Eureka	Top Defl.	2.29 2.50	Stone & Webster
Bank of Cal. (steel frame) 24 stories	3.664	Pacoima Dam El Centro Eureka	Base M of Col.	2.33 2.86 2.13	Degenkolb & Assoc.
JPL Bldg. No. 180	1.488	Pacoima Dam JPL Basement	Base M of Col.	1.97 2.88	Wood(4)
Int'l Bldg. (steel frame) 24 stories	1.456	Pacoima Dam El Centro Eureka	Top Defl.	1.23 2.61 2.69	Degenkolb & Assoc.

Table 2. STRENGTH REQUIREMENT FOR THE CRITICAL EXCITATION OF EL CENTRO INTENSITY

Structure	Structural Element	Requirements or Secs. Provided	Approximate Ductility Req'd
R.C. Flat Plate Bldg.	Bottom Story Ext. Col.	20"x20"col. f'_c=3ksi f_y=60ksi, 12-#14 #6 Ties @8"	1
R.C. Chimney	Bottom Sec.	f'_c~3ksi, f_y≈50ski, #9 Vert. Reinf.@6½" both faces	4
Bank of California	Ground Floor Ext. Col.	14WF456 A441 Steel	3.75
JPL Bldg. 180	2nd Story Col.	14WF158 A36 Steel	5
Int'l Building	Bottom Ext. Column	14WF320+2Pl.24x3½ A7 Steel	1.4

APPENDIX E

5B

THE CRITICAL EXCITATION OF INELASTIC STRUCTURES

by

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SYNOPSIS

A critical excitation of a structure drives one of its variables to a higher response peak than any other among some class of allowed excitations. This paper reports on the generalization of earlier results from elastic to inelastic structures.

PROBLEM AND SOLUTION

A question of importance in earthquake engineering which is rarely answered is this. Suppose that a structure is to be certified as resistant to earthquakes of some given intensity; what particular ground motions should it be able to withstand? In most current work, this question would probably be answered by saying that it should be able to withstand certain already recorded ground motions, or else certain artificially ones that are randomly generated. A third possible answer is an excitation which has been called "critical". It is an artificial ground motion also, but one which drives the structure to a higher response peak than any other of some designated class of allowed ground motions.

As has been shown previously, critical excitations are rather easily calculated if all ground motions up to a certain intensity are allowed, and if the structure is treated as elastic. If the intensity is measured by the square-integral of the ground acceleration, the critical excitation is more particularly found to differ by only a constant factor from the time-reversed impulse response of the structural variable of interest.

This result has recently been generalized to inelastic structures. It was found that the critical excitation is again a time-reversed impulse response. However, it is not the one of the structural variable itself but of one defined by a linearized set of equations. The linearization is more specifically the one that applies in the neighborhood of the critical excitation and response pair.

In order to determine such a pair, two sets of equations must be solved simultaneously: the nonlinear equations defining the inelastic structure and the linear ones defining the critical pair. It develops that, because of the time reversal, the solution can only be carried out by successive approximation. Moreover, the solution need not be unique: there may be more than one excitation/response pair that satisfies the equations.

Limited experience with numerical work indicates that the approximation process converges quite quickly.

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APPENDIX F

The Critical Excitation And Response Of High-Rise Buildings

by

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INTRODUCTION

High-rise building construction is the result of intensive land usage of urban area. As the population of a city grows, it is a natural trend to construct buildings upward to save space. These buildings represent heavy investment not only in economical sense but also in human life. Futhermore if these buildings house communication centers or military facilities, they become also important for national security. In view of these factors, it appears that large safe margins should be placed in the design of high-rise buildings especially when seismic excitation is a major consideration in regions of high seismic activities. This paper discusses a "critical excitation and response" approach to design important high-rise buildings. It will also demonstrate that the method is conservative but not overly so by the application of the method to several realistic or already built structures.

In the past, most practicing engineers approach to aseismic design of high-rise buildings is based on the applicable building codes. These codes in general treat ground excitation as pseudo static loads acting in the lateral directions of the building. The basis of this approach is originated from the response behavior of a single-degree-of-freedom elastic system with some consideration of ductility of the construction material (9), (2). The resultant elastic responses due to these pseudo static loads are in general smaller than those obtained from a dynamic analysis using the recorded

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ground excitation such as 1940 El centro N-S component. Recent proposed revisions of the building codes suggested using response spectra (8), (1) including the effects of higher modes of vibration and ductility of building materials. However, it is also well known that response spectrum analysis of seismic design is not an exact method and may be used at the preliminary design phase. For final analysis, it is desirable to perform an exact time-history analysis based on "properly" selected ground excitations. In the past, these "properly" selected ground excitations are either taken from the recorded seismographs or generated artificially based on the principles of random process. There is no assurance as to whether these excitations will cause the worst responses of the structural design variables such as moments, shears of the members, or story drifts.

The term critical excitation used herein is defined as the excitation among a certain class of excitations with an intensity limitation E(defined in a later section) that will drive a designated response variable to its maximum. To achieve this end, an unconstraind "critical" excitation is first obtain base on Drenick's (4) approach where the class of excitations are unlimited. This critical excitation is essentially the time-reversed unit impluse response of the design variable multiplied by an intensity modification factor. To make the shape of the excitation more realistic, a constrained "critical" excitation is next obtained by a least-squares fitting of a number of recorded excitations with the unconstrained one.

The proposed method was applied to several already designed or existing buildings. The required strength of some of the members based on this method were compared with that obtained by conventional approach (either by building code or dynamic analysis using recorded ground excitations). The results show that a safe but not overly conservative approach to design high-rise buildings against seismic excitation is indeed achieved.

THE UNCONSTRAINED CRITICAL EXCITATION

It is well known that the most damaging ground excitation on a structure among an unconstrained class of excitations with a limiting intensity (as defined in a later section) is the one that has the frequency spectrum closely matching that of the structural design variable. Such an unconstrained critical excitation was derived by Drenick (4) as

$$\ddot{x}_{c1}(t)=K\,h(-t) \tag{1}$$

wher K is an excitation intensity modification factor as described in reference 4 and h(-t) is the time-reversed unit inpuse response. For a single-degree-of-free system, h(t) is given by

$$h(t)=\frac{1}{\omega_D}\,e^{-\lambda\,\omega t}\,\sin\omega_D t \tag{2}$$

where ω and ω_D are undamped and damped frequencies of vibration and λ is the ratio to critical damping of the structure. For a multi-degree-of freedom system the unconstrained critical excitation can be obtained by modal superposition. First, consider the maximum response due to a general excitation $\ddot{x}_g$.

$$y_k(t)=\Sigma\ \phi_{ki}\,\eta_i(t)=\int_o^t \ddot{x}_g(\tau)\,\Sigma\ \phi_{ki}\,P_i\,h_i(t-\tau)d\tau$$

$$=\int_o^t \ddot{x}_g(\tau)\,\overline{h}\,(t-\tau)d\tau \tag{3}$$

where $y_k(t)$ is the k^{th} response variable, ϕ_{ki} is the k^{th} element of the i^{th} mode shape. $\eta_i(t)$ is the normal coordinate of the i^{th} mode, $P_i=\phi_i^T M\,\hat{I}/\phi_i^T M\,\phi_i$ is the i^{th} mode participation factor with M as the mass matrix and $\hat{\Gamma}$ is a vector with 1's or o's to indicate the existence or not of excitation of the vector elements of y. By squaring the response y_k and setting up the inequality, the following relationship is obtained

$$y_k^2(t)=[\int_o^t \ddot{x}_g(\tau)\bar{h}(t-\tau)d\tau]^2$$

$$\leq [\int_o^t \ddot{x}_g^2(\tau)d\tau]\ [\int_o^t \bar{h}^2(t-\tau)d\tau]$$

$$\leq E^2 N^2 \qquad (4)$$

or

$$y_k(t) \leq EN \qquad (5)$$

where E^2 and N^2 are square integrals of the ground excitation and the unit impulse response respectively in the time duration t.

The most damaging or critical ground excitation is

$$\ddot{x}_{cl}(\tau)= \frac{E}{N}\bar{h}(t-\tau) \qquad (6)$$

and is based on the fact that when it is substituted into (4), the response becomes

$$y_{cl}(t)=EN \qquad (7)$$

The quantity $E^2=\int_o^t \ddot{x}_g(\tau)d\tau$ is defined here as the excitation intensity measure. The upper limit of the integral was set at infinity in the mathematical derivation of reference(4). However, in real problems, the choice of the upper limit presents a difficult consideration and will be discussed in detail as an effective duration in a later section.

If this definition of excitation intensity is adopted, then the unconstrained critical excitation with this intensity for a designated design variable is precisely and conveniently given by Eq. (6) while the corresponding response is given by Eq. (7) with equal convenience.

CONSTRAINED CRITICAL EXCITATION AND RESPONSE

Although the unconstrained critical excitation and response as given in Eqs. (6) and (7) are simple and can be conveniently used to access seismic resistance of structures in an unconstrained or gross maximal sense (5), it often gives responses too high for practical application. The overconservativeness becomes even more pronounced as the structure or building increases in its flexibility. This departure from realistic response obviously lies in the fact that the unconstrained critical excitation matches in the frequency content with the response charater of the structural variable closely,and since most high-rise buildings have the foundamental period of 1 to 5 seconds, while earthquake excitations have predominant frequences less than 1 second (6), it can be concluded that the unconstrained critical excitation is not a realistic ground excitation. The question then is what kind of ground excitations are considered realistic and also critical. There are various approaches to modify the unconstrained critical one to be more realistic. The approach adopted in this paper is as follows:

The class of excitations can be constrained to those that have been recorded at sites with similar epicenterial distance, geological feature, etc., as that of the site of the structure to be built or already built. In addition,the linear superpositions of these recorded excitations can also be included into this class, provided that their combined intensity remains within the limitation. Furthermore, in order to avoid computational complication, it was not the critical excitation among these superpositions that was determined. Rather, an excitation $\ddot{x}_{c2}$ was calculated which differed least (in the least - squares sence) from $\ddot{x}_{c1}$. $\ddot{x}_{c2}$ is called the constrained critical excitation. In symbols

$$\ddot{x}_{c2} = \sum_{i=1}^{n} a_i \ddot{x}_i \tag{8}$$

so that

$$\int_0^t (\ddot{x}_{c2} - \ddot{x}_{c1})^2 dt = \text{minimum} \tag{9}$$

and

$$\int_0^t \ddot{x}_{c2}^2 (t)dt = E^2 \tag{10}$$

the constrained critical excitations thus obtained are more realistic and resembles observed ground excitations more closely as will be shown in the following practical examples. Once the constrained critical excitation of a design variable has been obtained, the corresponding response can be calculated by carrying out the Duhamel integral since the unit impulse response function has already been obtained in the process. In general, the maximum response always occurs at the end of the effective duration (defined in the next section) since partial frequency matching has been incorporated in the procedure.

$$y_{c2}(t) = \int_0^t \overline{h}\,(t-\tau)\, \ddot{x}_{c2}(\tau)d\tau \tag{11}$$

EFFECTIVE DURATION AND INTENSITY OF EARTHQUAKES

Recorded earthquakes have durations varies from few second to few minutes. However, the effective duration on a particular structure is controlled by the stiffness and the damping of the structure. It can be obtained by inspection of the Duhamal integral relating the response y(t) to the excitation $\ddot{x}(t)$ as follows:

$$y(t) = \int_0^t \ddot{x}(\tau)h(t-\tau)d\tau \tag{12}$$

where $h(t-\tau)$ is the response at time t due to a unit impulsive ground motion at time τ as discussed previously. Thus from Eq. (12)if the peak response occurs at t_e then the excitation occured at and before t_o so that $h(t_e-t_o)$ is zero need not be considered. In another words, (t_e-t_o) is the effective duration. In practice, the duration (t_e-t_o) can be judiciary chosen so that $h(t_e-t_o)$ decays to a certain percentage of the peak value of h(t). For example, if the decay to a ten percent was assigned, and the period of the system is 2 seconds with a damping ratio of 5%, the effective duration is $(t_e-t_o) = \dfrac{\ell n\ 0.1}{0.05 \times \frac{2\pi}{2}} = 14.6$ seconds. For multi-degree-of-freedom systems, the fundment mode can be used as the basis of competing the effective duration. The intensity of ground motion has been subjected to many discussions (7). Commonly used criteria are the Richter scale, modified Mercalli intensity number, the peak acceleration, etc. In this paper, the intensity of excitation is expressed as the square integral of the ground acceleration during the effective duration, i.e.

$$E^2 = \left[\int_{t_o}^{t_e} \ddot{x}^2(x)d\tau \right] \tag{13}$$

This expression of intensity is believed to be more meaningful since it represent a sort of energy and also it mathematical leads to the unconstrained critical excitation in Reference (4).

PRACTICAL APPLICATION

The critical excitation method has been applied to three existing office buildings and one conventionally designed appartment house. (see also reference(10). For each structure, a typical bent was selected for analysis. Based on the dimension of the structure and the conventionally designed member sizes, the unit impulse responses of selected design variables were computed by the computer program XTABS(11). The effective durations of ground excitation based on 66.7 per cent decay were computed and the intensity of 1940 El Centro N-S accelerogram based on the effective durations were used as reference. The time-reversed unit impulse response with the effective duration and adjusted to the reference intensity is used as the critical excitation $\ddot{x}_{c1}$. In the process of least-squares fitting, twelve recorded excitations were selected as the bases $\ddot{x}_i$'s, and they are listed in Appendix I (3). After the constrained critical excitation $\ddot{x}_{c2}$ was obtained by least-squares fitting of $\ddot{x}_{c1}$ and the superpositions of x_i's, the corresponding constrained critical response y_{c2} was computed by the Duhamal integral. The ratio of the constrained critical response and the response from the above mentioned El Centro excitation was listed as the critical design ratio. It is one of the indicators of the conservativeness of the method, being more conservative when this ratio is larger. The second indicator of conservativeness is obtained by carrying out the actual design of some of the numbers based on the results of the responses from the constrained critical excitation and comparing with those from the El Centro excitation. A brief discussion of each structure follows.

THE BANK OF CALIFORNIA BUILDING, SAN FRANCISCO

This building consists of steel frames and reinforced concrete core walls and was designed by Henry J. Degenkolb and Associates. Ashen and Allen are the architects. The building is twenty four stories high with bottom three stories below ground. Although the seismic resistance was provided by the combined action of the reinforced concrete core wall and the steel frames, a single frame in the north-south direction was isolated for analysis (Fig. 1). This departure from real structural system was adopted to simplify the analysis and since the comparative results are based on the same system, the conclusions drawn from the comparison may be considered valid. The members selected for strength checking are two columns above the ground floor and one beam on the second floor. The lateral deflection on the the top floor was also selected for comparison. Following are the essential results:

a) Periods in seconds of the first three modes of vibration: 3.364, 1.125 and 0.695.

b) Effective duration: 11.9 seconds

c) Damping ratio: 5%

d) The unconstrained critical excitations of the top floor displacement with El Centro intensity is shown in Fig. 2.

e) The constrained critical excitation of the top floor displacement with the same intensity is shown in Fig. 3.

f) The responses due to El Centro and the constrained critical excitations are summarized in Table 1.

Table 1 Summary of Responses of The Bank of California Building

Design Variable	El Centro	Constrained Critical	Critical Design Ratio
Top Floor Disp (ft)	1.362	3.414	2.51
Bottom Mom. (ft-k)			
Column 1	975	2785	2.86
Column 2	1046	3003	2.87
Axial Force (k)			
Column 1	952	2500	2.63
Column 2	168	421	2.51
Right End Mom. ft-k)			
Beam 1	1076	2979	2.77
Beam 2	636	1869	2.94
1 ft = 0.3048m, 1k = 4.45 kN, 1 ft-k = 1.356 m-kN			

g) The strength checking was made for the columns and beams as summarized in Table 2.

In Table 2, the strength checking is based on AISC 1963 specification. The ratio between yield point stress and allowable working stress is 1.67. For the columns, under El Centro excitations, the stresses are slightly higher than yielding while under constrained critical excitation they are 4.17/1.67 = 2.5 and 3.25/1.67 = 2.0 times the yielding. For the beam, the bending stress is 2.87/1.67 = 1.72 times the yield stress under El Centro and 7.14/1.67 = 4.28 times the yeild stress under the constrained critical excitation. Due to the fact that the frame is assumed to take its entire tributary earthquake forces without the participation of the core wall, it appears that the structure as it is designed will have enough ductility to resist the constrained critical excitation. Some interesting observations can be drawn from the study: First, base on the same excitation intensity E^2, the constrained critical excitation drives the structure to responses between 2.5 to 3 times those driven by the El Centro excitation, although the peak acceleration of the former is slightly less than that of the latter. It proves that peak acceleration is not a suitable measure of intensity of ground motion. Also, since the shape of the constrained critical excition shown in Fig. 3 appears realistic, certainly it should not be excluded from the design consideration. Second, for important structures such as the Bank of California building designed by experienced engineers, it appears that safe margins were already considered so that it can even withstand severe excitations such as the constrained critical one.

Table 2. Strength Checking of Bank of California Building

Member Forces And Stresses	Column 1 14 W=456(A 441)		Column 2 14 W=456 (A441)		Beam 1 27 W= 84 (A441)	
	El Centro	Constrained Critical	Const.	Critical	El Centro	Constrianed Critical
Axial Force (k) (D+L)	1613	1613	1806	1806		
Axial Force (k) (EQ)	951	2500	510	1049		
Combined P (k)	2564	4113	2316	2855		
Moment (ft-k) (D+L)	210^{1k}	210^{1k}	70^{1k}	70^{1k}	202	202
Moment (ft-k) (EQ)	975	2785	1046	3003	1076	2979
Combined M_b (ft-k)	1185	2995	1116	3073	1278	3181
A (in^2)	134.1	134.1	134.1	134.1		
S_x (in^3)	758.5	758.5	758.5	758.5	212	212
I_x (in^4)	7221	7221	7221	7221		
I_y (in^4)	2568	2568	2568	2568		
r_x (in)	7.34	7.34	7.34	7.34		
r_y (in)	4.39	4.39	4.39	4.39		
$f_a = P/A$ (ksi)	19.10	30.6	17.20	21.3		
$f_b = M_b/S_x$ (ksi)	18.11	47.4	17.66	48.6	72.3	180.0
F_a (ksi)	18.34	18.34	18.34	18.34		
F_b (ksi)	25.20	25.2	25.00	25.20	25.2	25.2
Fe'_x (ksi)	84.65	84.65	84.65	84.65		
$\frac{f_a}{F_a} + \frac{C_m f_b}{F_b(1 - \frac{f_a}{Fe'_x})}$	1.83	4.17	1.68	3.35	2.87	7.14

1 k = 4.45 kN, 1 ft-k = 1.356 m-kN, 1 in = 25.4 mm, 1 ksi = 6.9MN/M^2

THE INTERNATIONAL BUILDING, SAN FRANCISCO

This building consists also of reinforced concrete core walls and steel frames and was designed by the same team of architects and engineers. It has also twenty four stories but with only one floor under ground. A frame in the north-south direction is again isolated for analysis (Fig. 4). However, different from the first structure, the core wall was incorporated in the second, third and fourth bays of the frame. In modelling the structure, core walls are assumed as shear panels connected to the beams and columns on the four sides of the panel within each bay and between floors. Same dynamic analysis as described for the first structure was carried out and following are the essential results:

a) Periods in seconds of the first three modes: 1.456, 0.423, 0.206.

b) Effective duration: 5.6 seconds

c) Damping ratio: 5%

d) & e) The unconstrained critical and constrained critical excitation are in the same shape as Fig. 2 and 3 except with shorter duration.

f) The responses due to El Centro and constrained critical excitations are summarized in Table 3.

g) The strength checking was made for the columns, beams and the shear wall panels as summarized in Table 4.

Observations drawn from this study are: First, critical design factors a re smaller for this building than the first building and appears to orginated from the smaller fundamental periods. Second, well designed structures by experienced engineers can withstand critical excitation without relying on large ductilities.

Table 4. Strength Checking of International Building

Member Forces Or Stresses	Column 1 14 W F 320 + 2pl. 24x 3 1/2 (A7) El Centro	Constrained Critical	Beam 1 27 W F 94 (A7) El Centro	Constrained Critical	Shear Wall Panel 25" x 324" #6@ 12" Vert. E. F. El Centro	Constrained Critical
Axial Force (k) (D+L)	2607	2607			1573	1573
Axial Force (k) (EQ)	1096	2721			43	106
Combined P (k)	3703	5328			1616	1679
Moment (ft-k) (D+L)	52	52	23	23		
Moment (ft-k) (EQ)	721	1123	413	657	22,842	40,790
Combined M_b (ft-k)	773	1175	436	680		
A (in^2)	262	262				
S (in^3)	808	808	242.8	242.8	ACI ultimate strength design shows minimum steel will be satisfactory.	
I_x (in^4)	22566	22566				
I_y (in^4)	9699	9699				
r_x (in)	9.28	9.28				
r_y (in)	6.08	6.08				
$f_a = P/A$ (ksi)	14.14	20.34				
$f_b = M_b/S_y$ (ksi)	10.71	17.45	21.5	33.6		
F_a (ksi)	17.2	17.2				
F_b (ksi)	20.0	20.0				
$Fe'y$ (ksi)	67.51	67.51				
$\frac{f_a}{F_a} + \frac{C_m f_b}{F_b(1-f_a/Fe'y)}$	1.40	2.24				

1 k = 4.45 kN, 1 ft-k = 1.356 m-kN, 1 in = 25.4 mm, 1 ksi = 6.9 MN/M^2

Table 3. Summary of Responses of the International Building

Design Variable	El Centro	Constrained Critical	Critical Design Ratio
Top Floor Disp (ft)	0.458	1.196	2.61
Bottom Mom. (ft-k)			
Column 1	721	1123	1.56
Column 2	522	810	1.55
Axial Force (k)			
Column 1	1096	2073	1.89
Column 2	1463	2770	1.89
Right End Mon. (ft-k)			
Beam 1	413	657	1.59
Beam 2	843	1314	1.56
Shear Panel			
Axial Force (k)	43	106	2.45
Bott. Mom. (ft-k)	22842	40720	1.78
1 ft = 0.3048m. 1 k = 4.45 kN, 1 ft-k = 1.356 m-kN			

JET PROPULSION LABORATORY BUILDING 180

The JPL building 180 is a nine-story steel frame structure with 5 inches lighweight concrete slab. The twelve steel frames in the north-south direction are constructed with trusses at the floor levels and wide flange columns embedded in concrete protection. A typical north-south direction frame was isolated for analysis (Fig. 5). The equivalent beam stiffness of the truss and the composite stiffness of the columns were modelled according to Wood (12). The same dynamic analysis as discribed for the first structure was carried out and the essential results are as follows:

a) Periods in seconds of the first three mode: 1.488, 0.472, 0.259

b) Effective duration: 9.1 seconds

c) Damping ratio: 5%

d) The unconstrained critical excitation of the top floor displacement with El Centro intensity is shown in Fig. 6.

e) The constrained critical excitation of the top floor displacement with the same intensity is shown in Fig. 7.

f) The responses due to El Centro and constrained critical excitations are summarized in Table 5.

Table 5. Responses of the JPL Building 180

Design Variables	El Centro	Constrained Critical	Critical Design Ratio
Top Floor Disp (ft)	0.529	1.865	3.53
Bott. Mom. (ft-k) of Column	1021	3334	3.27
Bott. Shear (k) of Column	109	327	2.99
1 ft = 0.3048m, 1 k = 4.45 kN, 1 ft-k = 1.356 m-kN			

g) The strength checking was made for the bottom story column as summarized in Table 6.

The strength checking shows that the bottom story column will stressed to 3.06/1.67 = 1.83 times the yield stress under the El Centro excitation and 8.31/1.67 = 4.98 times the yield stress under the constrained critical excitation. It appears that the structure under the present designed condition needs strengthening if it is to sustain an ground excitation of EL Centro intensity, especially if constrained critical excitation is considered.

FLAT PLATE APARTMENT BUILDING

This building is a conventional flat plate reinforced concrete building without shear walls. It has 16 stories with 6 in. (15.24 cm) typical floor slab. The column sizes varies from 20" x 20" (0.51m x 0.51m) to 12" x 12"

Table 6. Strength Checking For JPL Building 180

Member Forces Or Stresses	Bottom Story Column 14 W = 158 (A36)	
	El Centro	Constrained Critical
Axial Load (k) (D+L)	483 k	483 k
Axial Load (k) (EQ)	369 k	1144 k
Combined P (k)	852	1627
Bottom Mom (ft-k) (D+L)	176	176
Bottom Mom (ft-k) (EQ)	1021	3334
Combined M_b (ft-k)	1197	3510
A (in^2)	46.5	46.5
S (in^3)	253	253
r_x (in)	10.32	10.32
r_y	7.01	7.01
$f_a = P/A$ (ksi)	18.32	34.99
$f_b = M_b$ S(ksi)	56.77	166.5
F_a (ksi)	20.15	20.15
F_b (ksi)	24.0	24.0
F_e (ksi)	338.63	338.63
$\frac{f_a}{F_a} + \frac{C_m f_b}{F_b(1-f_a/F_e)}$	3.06	8.31

1 k = 4.45 kN, 1 ft-k = 1.356 m-kN, 1 in = 25.4 mm. 1 ksi = 6.9 MN/ M^2

(.31m x .31m). A typical transverse frame consists of two bays was isolated for analysis (Fig. 8). The dynamic analysis similar to that described for the first structure was carried out and the essential results are summarized as follows:

a) Periods in second of the first three modes: 4.95, 1.743, 1.001.

b) Effective duration: 14.0 seconds

c) Damping ratio: 5%

d) The unconstrained critical excitations has the same shape as the others.

e) The constrained critical excitation of the top floor displacement with the same intensity as El Centro is shown in Fig. 9.

f) The responses of the design variables are summarized in Table 7.

Table 7. Responses of the Flat Plate Building

Design Variables	El Centro	Constrained Critical	Critical Design Ratio
Top Floor Disp (ft)	2.38	3.02	1.27
Bott. Mom. (ft-k) Ext. Column	640	796	1.24
Bott. Shear (k) Ext. Column	46.8	57.5	1.23
1 ft = 0.3048m, 1 k = 4.45 kN, 1 ft-k = 1.356 m-kN			

g) The strength checking of the bottom story exterior column was carried out and summarized in Table 8.

Table 8. Strength Checking of the Flat Plate Building

Member Forces	Ext. Column El Centro	20in x 20in $f_c = 5$ ksi, $f_y = 60$ ksi Constrained Critical
Axial Load (k) (D+L)	446	446
Bott. Mom. (ft-k) (D+L)	16	16
Bott. Mom. (ft-k) (EQ)	816	1014
Combined M_b (ft-k)	832	1030
Required Vert Reinf	12 - #10	12 - #14
1k = 4.45 kN, 1 ft-k = 1.356 m-kN, 1 in = 25.4 mm, 1 ksi = 6.9 MN/in^2		

In the strength checking, the loads have been multiplied by the ultimated load factor as specified by the ACI Building Code. The columns thus designed does not need any reserved ductility factor to sustain either the El Centro excitation or the constrained critical excitation. Properly designed ties acting as shear stirrups are required to resist shear. The slab reinforcing has also to be properly designed to resist ground excitations. But it appears that for a new design, the details can be worked out without overly excessive reinforcement to sustain the constrained critical excitation.

CONCLUSION

1. The method proposed here is an upper bound approach applicable to buildings of major importance, the destruction of which would cause severe human and economic losses.

2. The motivation behind the method is based on the fact that precise nature of earthquake, frequency of occurrence, interaction between structure and soil and other earthquake related factors are not readily available.

3. The practical application on existing buildings shows that the method is on the safe side but not overly conservative.

4. Although the constrained critical excitation derived here is base on elastic behavior but it may also be used as a first approximation in the piece wise linearization of non-linear responses of a structure.

5. Additional studies are required to evaluate the effect on the resultant constrained critical excitation by using different sets of recorded excitations as well as the numbers of excitations in the least-squares fitting procedure.

Acknowledgement

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APPENDIX - I

12 EARTHQUAKES USED FOR LEAST-SQUARES FITTING

File Number In Ref. B	Quake	Record	Comp	$E = (\int_0^{14} x_g^2 \, dt)^{1/2}$ $(cm/sec^{1/2})$	E $(ft/sec^{1/2})$
IIA 1	Imperial Valley	El Centro	S00E	289.5264	9.499
IIA 10	San Jose	San Jose	N31W	66.0062	2.166
IIA 13	San Francisco	San Francisco	N45E	34.0232	1.116
IIA 14	San Francisco	San Francisco	N09W	29.3481	0.963
IIA 15	San Francisco	San Francisco	N10E	39.6909	1.302
IIA 16	San Francisco	San Francisco	S09E	54.2891	1.781
IIA 17	San Francisco	San Francisco	N26E	23.4391	0.769
IIA 18	Hollister	Hollister	S01W	83.7635	2.748
IIA 19	Borrego Mt.	El Centro	S00W	87.5406	2.872
IIA 41	San Fernando	Pacoima	S14W	699.6152	22.952
IIA 48	San Fernando	Los Angeles	N00W	256.5239	8.416
IID 56	San Fernando	Castaic	N21E	192.8584	6.327

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APPENDIX III - NOTATIONS

The following symbols are used in this paper:

A	=	area of a member
C_m	=	coefficient applied to bending term in interaction formula of columns
D & L	=	dead plus live load
E^2	=	intensity measure of ground excitation
E_Q	=	earthquake load
f_a	=	axial compressive stress
f_b	=	bending stress
F_a	=	allowable axial compressive stress
F_b	=	allowable bending stress
F_e	=	Euler stress divided by a factor of safety
$h(t)$	=	unit impulse response
$\hat{I}$	=	a column vector of 1's and o's
I_x, I_y	=	moments of inertia about x and y axes
M	=	mass matrix
M_b	=	bending moment
N^2	=	square integral of the critical excitation
P	=	axial force
P_i	=	participation factor of the ith mode
r_x, r_y	=	radii of gyration about x and y axes
t	=	time
$t_e - t_o$	=	effective duration

$\ddot{x}_{c1}(t)$	=	unconstrained critical excitation
$\ddot{x}_{c2}(t)$	=	constrained critical excitation
$\ddot{x}_g(t)$ or $\ddot{x}_i(t)$	=	historic ground excitation
$y_{c1}(t)$	=	response to unconstrained critical excitation
$y_{c2}(x)$	=	response to constrained critical excitation
λ	=	ratio to the critical excitation
ω	=	undamped frequency of vibration
ω_D	=	damped frequency of vibration
η_i	=	i-th modal coordinate
ϕ_{ki}	=	i-th mode shape

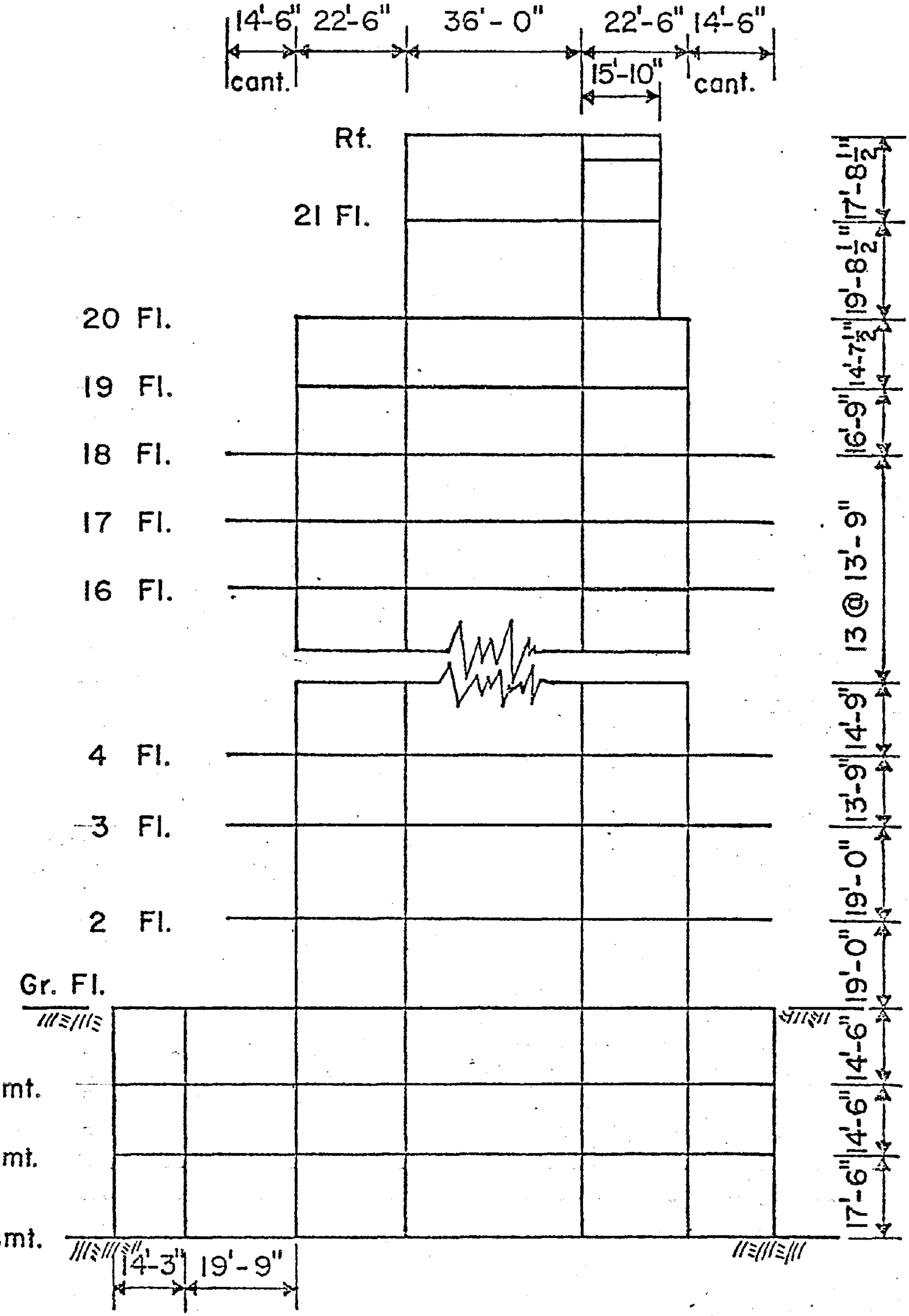

Fig I. BANK OF CALIFORNIA BLDG. (1ft = 0.3048 m)
SAN FRANCISCO

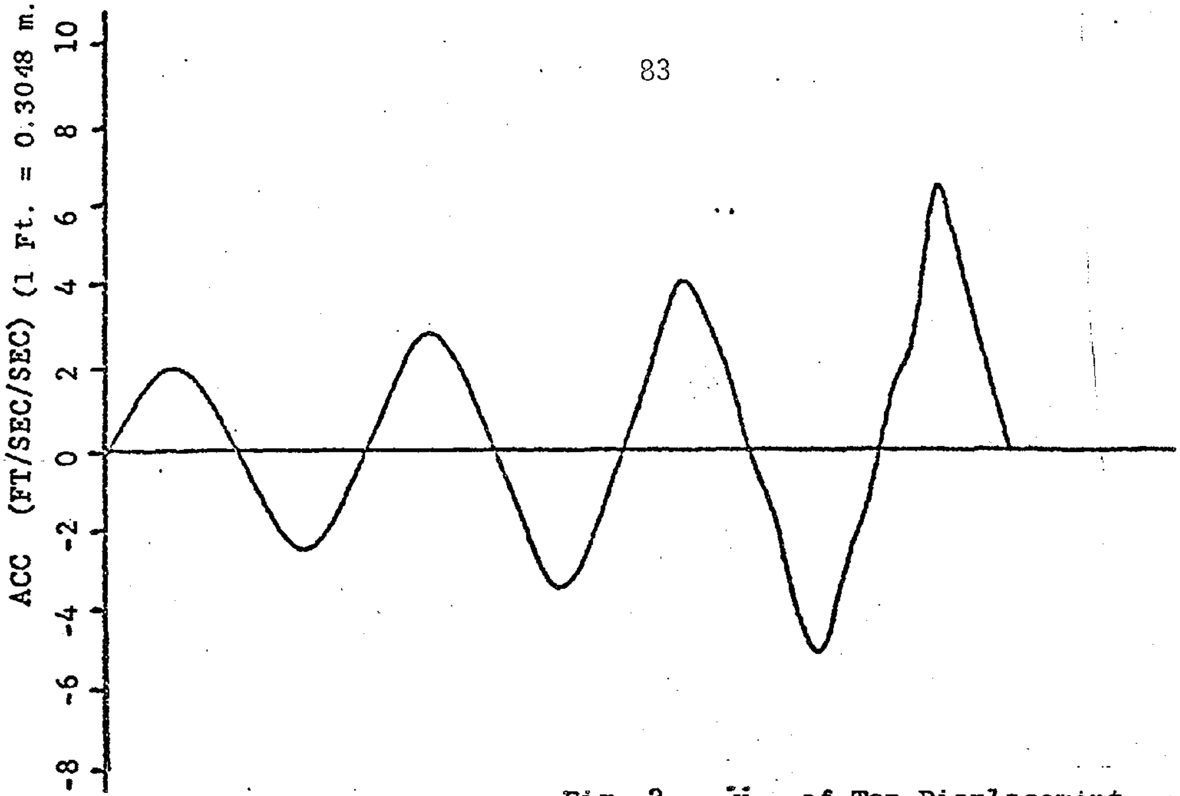

Fig. 2. - $\ddot{x}_{c1}$ of Top Displacement
Bank of Cal. Building

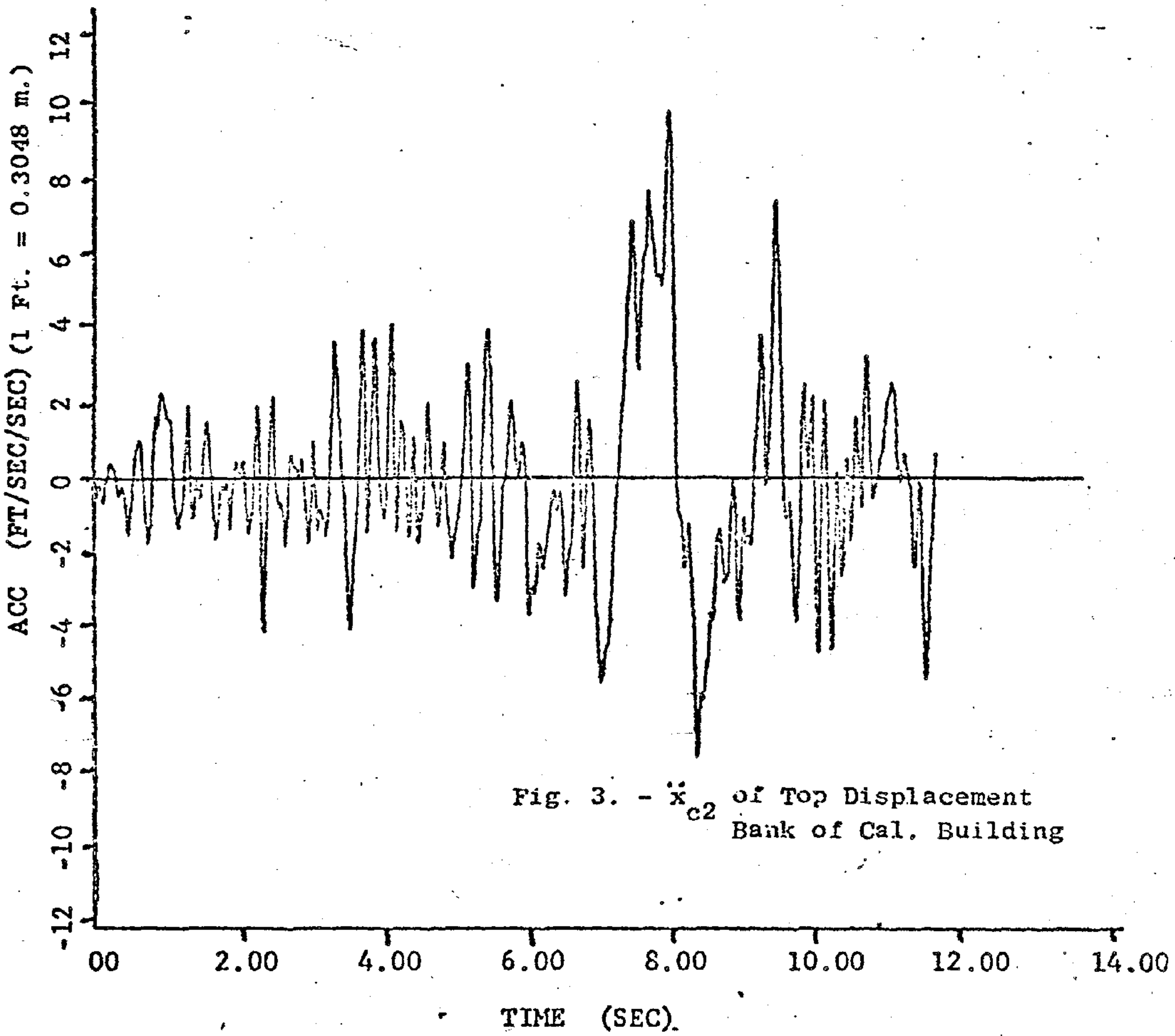

Fig. 3. - $\ddot{x}_{c2}$ of Top Displacement
Bank of Cal. Building

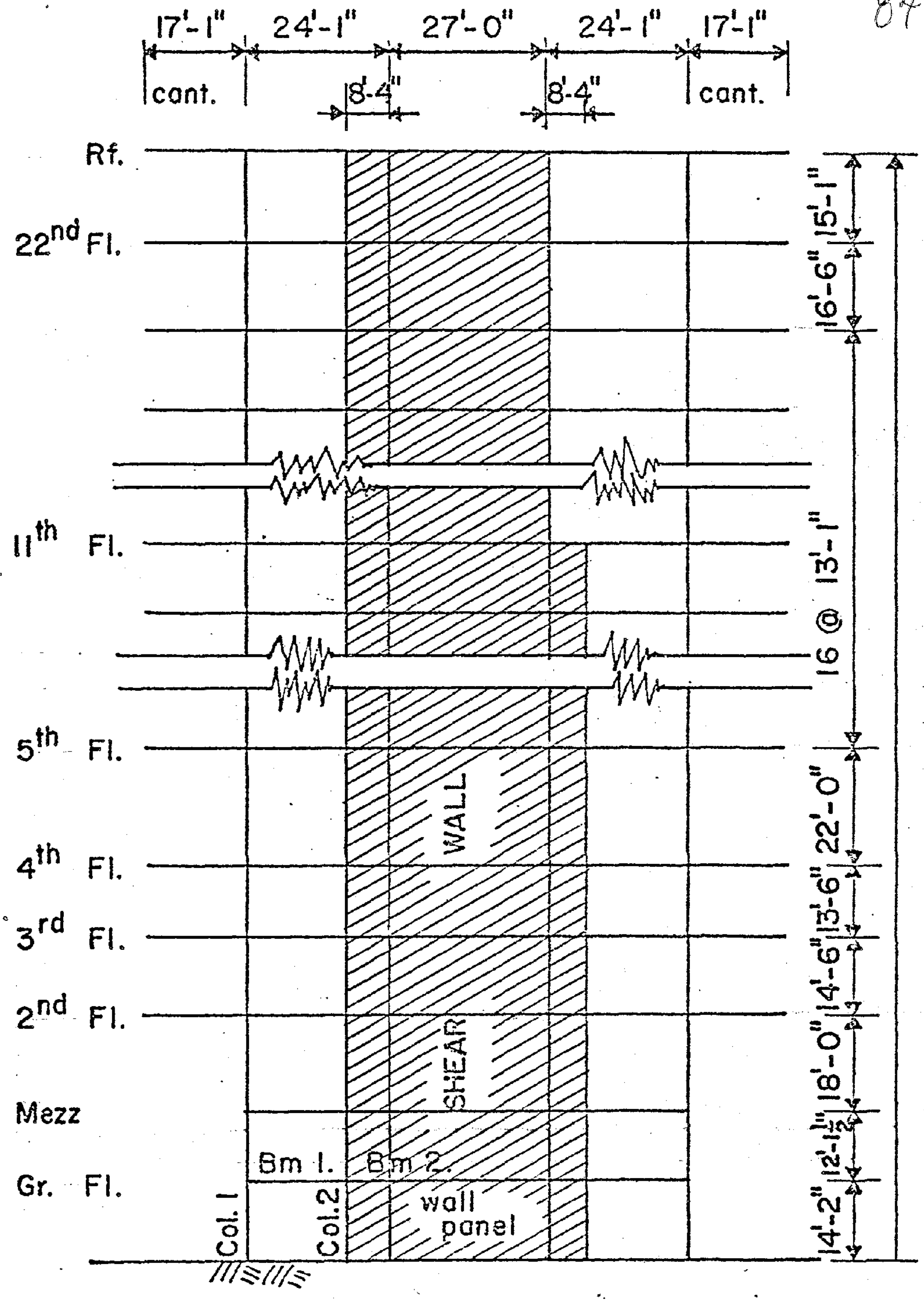

Fig 4. INTERNATIONAL BUILDING (1ft = 0.3048 m)
SAN FRANCISCO

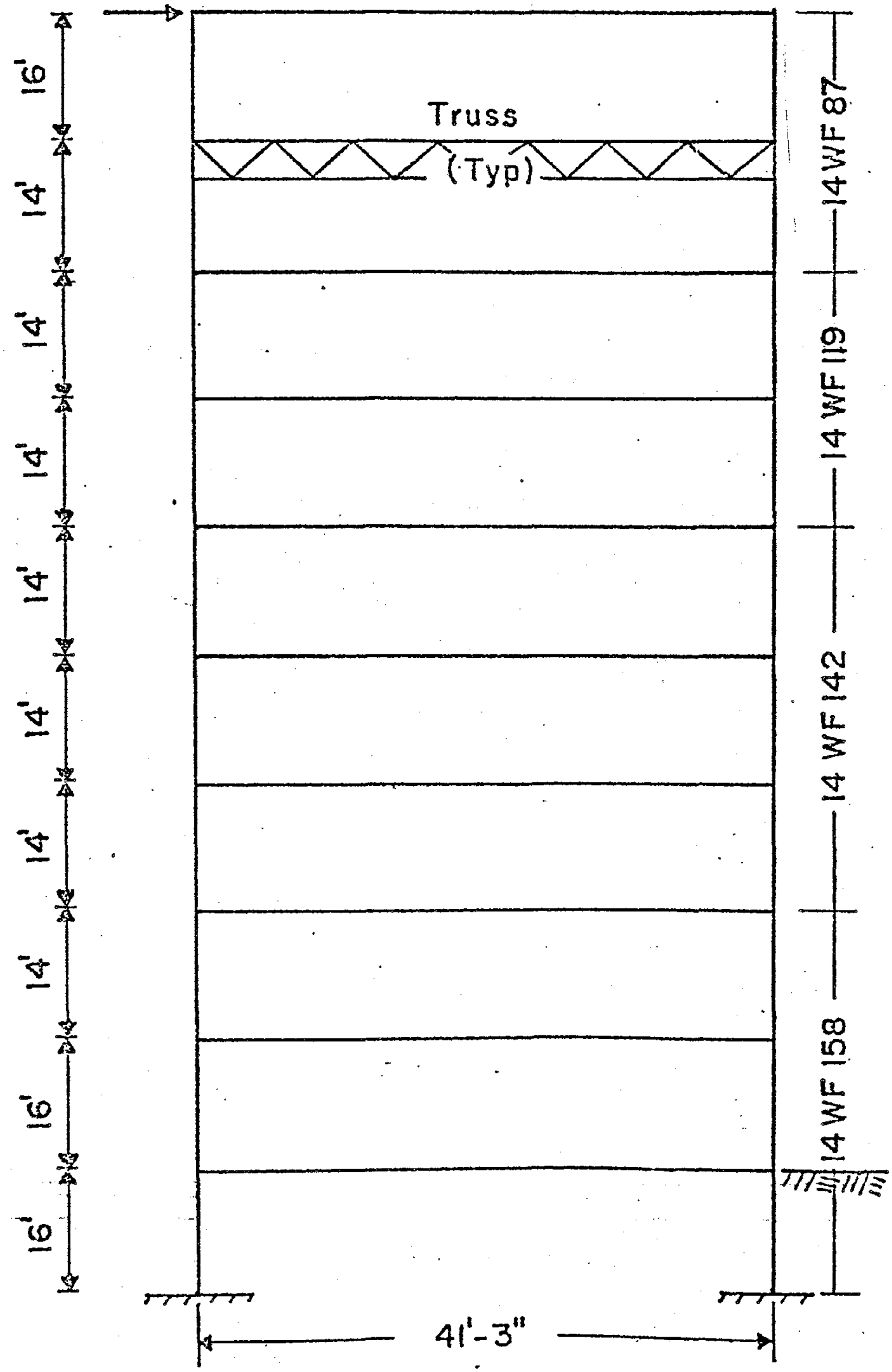

Fig.5 JET PROPULSION LAB (1ft=0.3048 m)

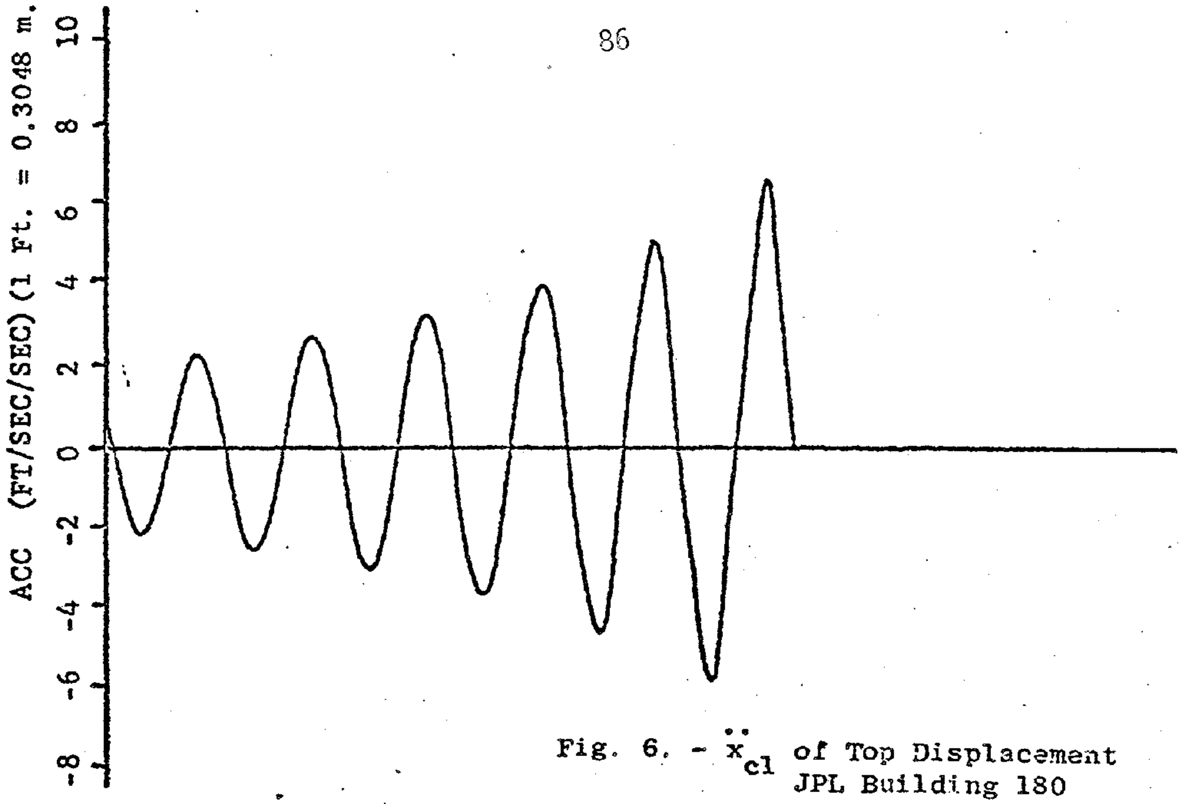

Fig. 6. - $\ddot{x}_{c1}$ of Top Displacement JPL Building 180

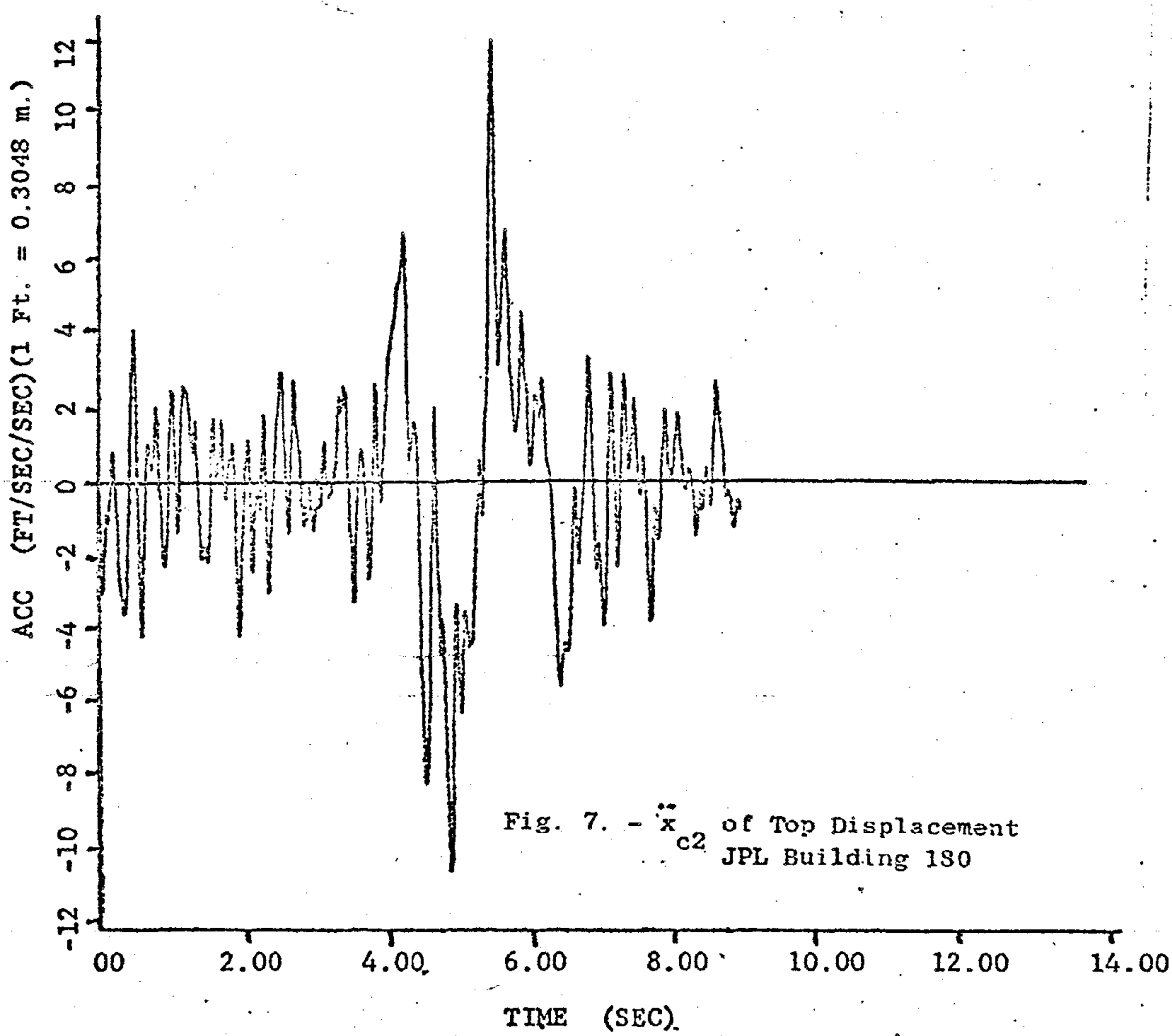

Fig. 7. - $\ddot{x}_{c2}$ of Top Displacement JPL Building 180

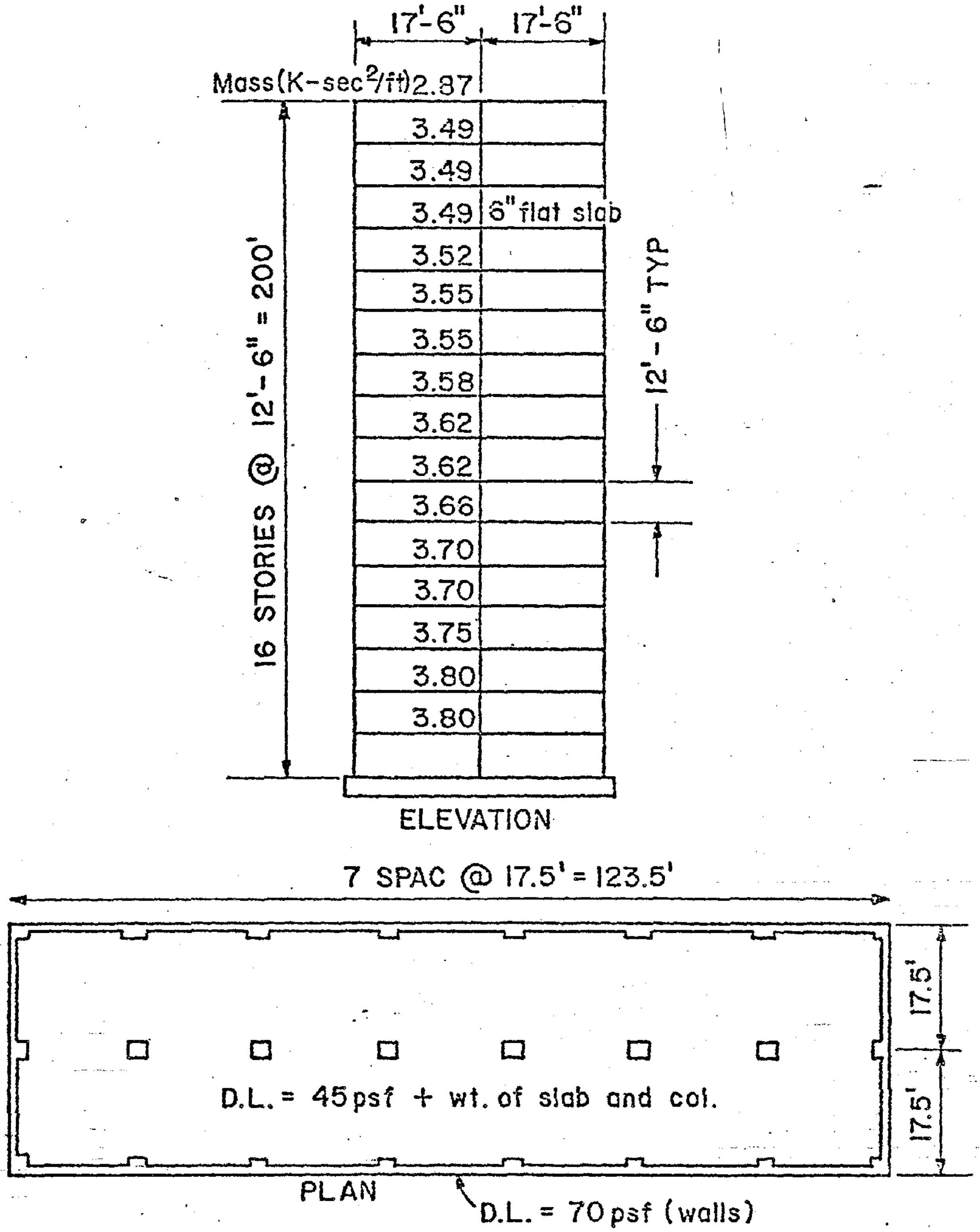

FIGURE 8. REINFORCED CONCRETE BUILDING
(1ft = 0.3408 m , 1 psf. = 0.0479 kN/m^2,
1K - sec^2/ft = 1.331 kg - sec^2/m)

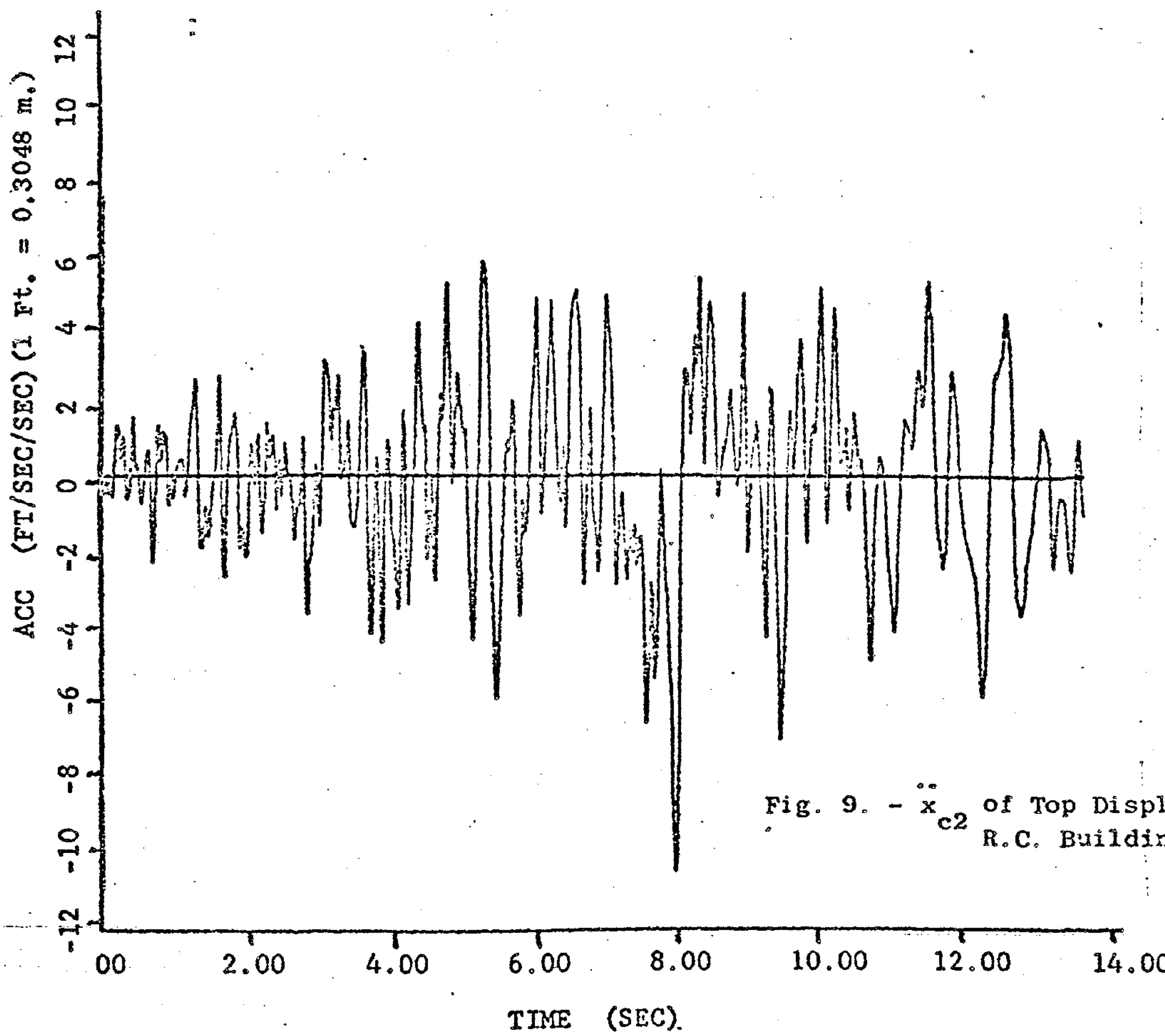

Fig. 9. - $\ddot{x}_{c2}$ of Top Displacement R.C. Building