

PB 273 279

REPORT NO.  
EERC 76-3  
OCTOBER 1976

EARTHQUAKE ENGINEERING RESEARCH CENTER

# DYNAMIC BEHAVIOR OF A MULTISTORY TRIANGULAR-SHAPED BUILDING

by

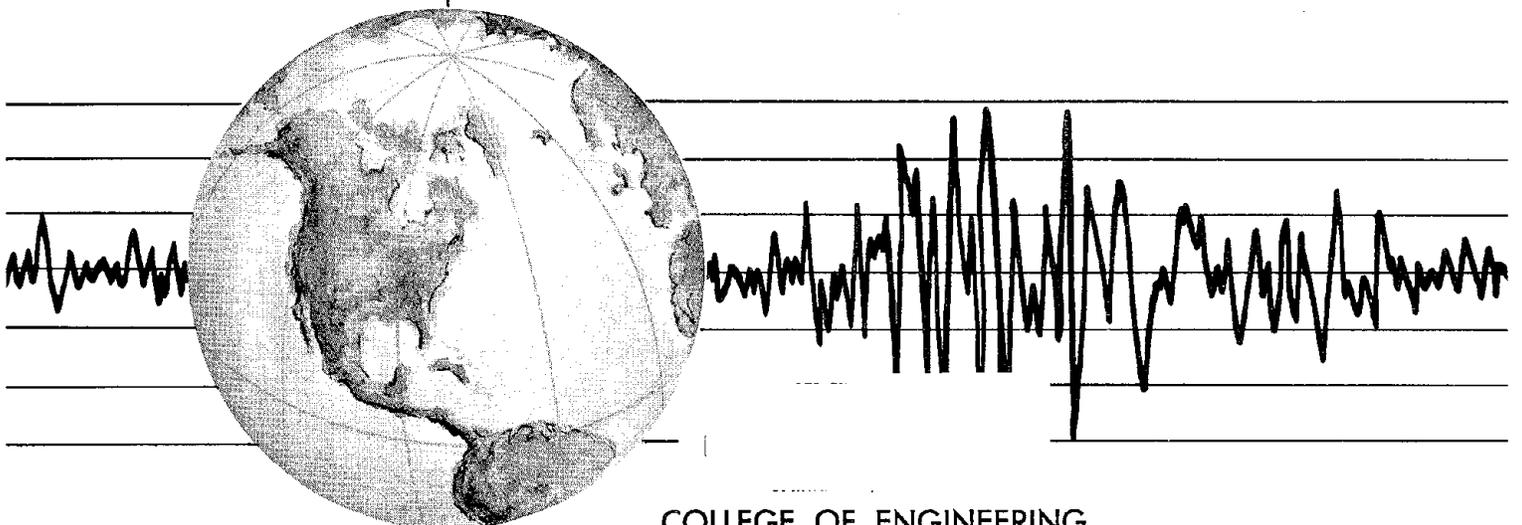
JAKIM PETROVSKI

R. M. STEPHEN

E. GARTENBAUM

J. G. BOUWKAMP

Report to the National Science Foundation



COLLEGE OF ENGINEERING

UNIVERSITY OF CALIFORNIA • Berkeley, California



BIBLIOGRAPHIC DATA SHEET	1. Report No. EERC 76-3	2.	3. Recipient's Accession No. 10273219
	4. Title and Subtitle "Dynamic Behavior of a Multistory Triangular-Shaped Building"		5. Report Date October 1976
7. Author(s) J. Petrovski, R.M. Stephen, E. Gartenbaum, J.G. Bouwkamp		8. Performing Organization Rept. No. 76-3	
9. Performing Organization Name and Address Earthquake Engineering Research Center University of California, Berkeley 47th Street and Hoffman Blvd. Richmond, California 94804		10. Project/Task/Work Unit No.	11. Contract/Grant No. AEN73-07732 A0Z
12. Sponsoring Organization Name and Address National Science Foundation 1800 G Street, N.W. Washington, D.C. 20550		13. Type of Report & Period Covered	
15. Supplementary Notes		14.	
16. Abstracts <p>As a part of a continuing program to evaluate the dynamic response of actual structures and to accumulate a body of information on the dynamic properties of structures, especially when these structures have novel design features, a dynamic test program was conducted on the forty-story Century City Theme Tower building.</p> <p>The dynamic tests of the building included both a forced vibration study and an ambient vibration study. These results are compared and in general show very good correlation. A mathematical computer model of the Theme Tower was formulated and the results of the analysis are presented and compared to the experimental results. Again in general the results compare very favorably.</p>			
17. Key Words and Document Analysis. 17a. Descriptors			
17b. Identifiers/Open-Ended Terms			
17c. COSATI Field/Group			
18. Availability Statement Release Unlimited		19. Security Class (This Report) UNCLASSIFIED	21. No. of Pages 138
		20. Security Class (This Page) UNCLASSIFIED	22. Price PCAO7 MFA01



EARTHQUAKE ENGINEERING RESEARCH CENTER

DYNAMIC BEHAVIOR OF A MULTISTORY  
TRIANGULAR-SHAPED BUILDING

A Report to the  
National Science Foundation

by

Jakim Petrovski  
R. M. Stephen  
E. Gartenbaum  
J. G. Bouwkamp

Report No. EERC 76-3  
College of Engineering  
Department of Civil Engineering  
University of California  
Berkeley, California  
October 1976



# TABLE OF CONTENTS

	Page
TABLE OF CONTENTS . . . . .	i
LIST OF FIGURES . . . . .	iii
LIST OF TABLES . . . . .	vi
ABSTRACT . . . . .	vii
1. INTRODUCTION . . . . .	1
1.1 General . . . . .	1
1.2 Acknowledgement . . . . .	2
2. THE CENTURY CITY THEME TOWERS . . . . .	3
2.1 General . . . . .	3
2.2 Structural System and Elements . . . . .	3
3. FORCED VIBRATION STUDY . . . . .	14
3.1 General . . . . .	14
3.2 Experimental Apparatus . . . . .	14
3.2.1 Vibration Generators . . . . .	14
3.2.2 Accelerometers . . . . .	16
3.2.3 Equipment for Measurement of Frequency . . . . .	16
3.2.4 Recording Equipment . . . . .	16
3.3 Experimental Procedure and Data Reduction . . . . .	17
3.3.1 Resonant Frequencies . . . . .	17
3.3.2 Mode Shapes . . . . .	18
3.3.3 Damping Capacities . . . . .	19
3.4 Experimental Results . . . . .	20
3.5 Discussion of Experimental Results . . . . .	24

	Page
4. AMBIENT VIBRATION STUDY . . . . .	62
4.1 General . . . . .	62
4.2 Field Measurements . . . . .	63
4.2.1 Measuring Equipment . . . . .	63
4.2.2 Measurement Procedure . . . . .	64
4.3 Data Analysis . . . . .	68
4.3.1 Fourier Analysis . . . . .	68
4.3.2 Data Processing . . . . .	71
4.3.3 Frequencies and Mode Shapes . . . . .	72
4.3.4 Damping . . . . .	73
5. COMPARISON OF FORCED AND AMBIENT VIBRATION STUDIES . . . . .	89
6. FORMULATION OF MATHEMATICAL MODEL . . . . .	105
6.1 General . . . . .	105
6.2 Computer Program . . . . .	105
6.3 Formulation of Mathematical Model . . . . .	106
6.4 Analytical Dynamic Properties . . . . .	107
7. COMPARISON OF EXPERIMENTAL AND ANALYTICAL RESULTS . . . . .	110
8. GENERAL CONCLUSIONS . . . . .	113
REFERENCES . . . . .	114

## LIST OF FIGURES

FIG. NO.	Page
2.1 General View of the Theme Tower Buildings from North-West . . .	6
2.2 South Theme Tower . . . . .	7
2.3 South Elevation of the Building . . . . .	8
2.4 Cross-Section of the Building . . . . .	9
2.5 Typical Floor Plan and Location of Vibration Generators (42nd Floor). . . . .	10
2.6 Floor Plan on Plaza Level . . . . .	11
2.7 Floor Plan on F Level . . . . .	12
2.8 Corner Column Section - Plaza Level . . . . .	13
3.1 Vibration Generator on the 42nd Floor of South Theme Tower . .	26
3.2 Vibration Force Output vs. Speed-non-Counter Balanced (after Hudson) . . . . .	27
3.3 Frequency Response, First Mode E-W . . . . .	28
3.4 Frequency Response, Second Mode E-W . . . . .	29
3.5 Frequency Response, Third Mode E-W . . . . .	30
3.6 Frequency Response, Fourth Mode E-W . . . . .	31
3.7 Frequency Response, Fifth Mode E-W . . . . .	32
3.8 Frequency Response, Sixth Mode E-W . . . . .	33
3.9 Frequency Response, First Mode N-S . . . . .	34
3.10 Frequency Response, Second Mode N-S . . . . .	35
3.11 Frequency Response, Third Mode N-S . . . . .	36
3.12 Frequency Response, Fourth Mode N-S . . . . .	37
3.13 Frequency Response, Fifth Mode N-S . . . . .	38
3.14 Frequency Response, First Torsional Mode . . . . .	39
3.15 Frequency Response, Second Torsional Mode . . . . .	40
3.16 Frequency Response, Third Torsional Mode . . . . .	41
3.17 Frequency Response, Fourth Torsional Mode . . . . .	42

	Page	
3.18	Frequency Response, Fifth Torsional Mode . . . . .	43
3.19	Frequency Response, Sixth Torsional Mode . . . . .	44
3.20	Mode Shapes, First Translational Mode E-W . . . . .	45
3.21	Mode Shapes, Second Translational Mode E-W . . . . .	46
3.22	Mode Shapes, Third Translational Mode E-W . . . . .	47
3.23	Mode Shapes, Fourth Translational Mode E-W . . . . .	48
3.24	Mode Shapes, Fifth Translational Mode E-W . . . . .	49
3.25	Mode Shapes, Sixth Translational Mode E-W . . . . .	50
3.26	Mode Shapes, First Translational Mode N-S . . . . .	51
3.27	Mode Shapes, Second Translational Mode N-S . . . . .	52
3.28	Mode Shapes, Third Translational Mode N-S . . . . .	53
3.29	Mode Shapes, Fourth Translational Mode N-S . . . . .	54
3.30	Mode Shapes, Fifth Translational Mode N-S . . . . .	55
3.31	Mode Shapes, First Torsional Mode . . . . .	56
3.32	Mode Shapes, Second Torsional Mode . . . . .	57
3.33	Mode Shapes, Third Torsional Mode . . . . .	58
3.34	Mode Shapes, Fourth Torsional Mode . . . . .	59
3.35	Mode Shapes, Fifth Torsional Mode . . . . .	60
3.36	Mode Shapes, Sixth Torsional Mode . . . . .	61
4.1	Ambient Vibration Equipment . . . . .	75
4.2	Location of Ranger Seismometers on the 42nd Floor for Resonant Frequency Response . . . . .	75
4.3	Location of Ranger Seismometers for the Mode Shapes . . . . .	76
4.4	First Translational Mode Shape, E-W . . . . .	77
4.5	Second Translational Mode Shape, E-W . . . . .	78
4.6	Third Translational Mode Shape, E-W . . . . .	79
4.7	Fourth Translational Mode Shape, E-W . . . . .	80
4.8	Fifth Translational Mode Shape, E-W . . . . .	81
4.9	First Translational Mode Shape, N-S . . . . .	82

	Page
4.10 Second Translational Mode Shape, N-S . . . . .	83
4.11 Third Translational Mode Shape, N-S . . . . .	84
4.12 Fourth Translational Mode Shape, N-S . . . . .	85
4.13 First Torsional Mode Shape . . . . .	86
4.14 Second Torsional Mode Shape . . . . .	87
4.15 Third Torsional Mode Shape . . . . .	88
5.1 Ratio of Resonant Frequencies . . . . .	92
5.2 First Translational Mode Shape, E-W . . . . .	93
5.3 Second Translational Mode Shape, E-W . . . . .	94
5.4 Third Translational Mode Shape, E-W . . . . .	95
5.5 Fourth Translational Mode Shape, E-W . . . . .	96
5.6 Fifth Translational Mode Shape, E-W . . . . .	97
5.7 First Translational Mode Shape, N-S . . . . .	98
5.8 Second Translational Mode Shape, N-S . . . . .	99
5.9 Third Translational Mode Shape, N-S . . . . .	100
5.10 Fourth Translational Mode Shape, N-S . . . . .	101
5.11 First Torsional Mode Shape . . . . .	102
5.12 Second Torsional Mode Shape . . . . .	103
5.13 Third Torsional Mode Shape . . . . .	104
6.1 Marcoelement Simulation of Structure . . . . .	108
6.2 Mode Shapes . . . . .	109
7.1 Typical Mode Shape . . . . .	112



## LIST OF TABLES

Table No.	Page
3.1 Resonant Frequencies . . . . .	21
3.2 Damping Factors (%) from Resonance Curve . . . . .	22
3.3 Resonant Force Amplitudes . . . . .	22
3.4 Resonant Displacement Amplitude . . . . .	22
3.5 Resonant Rotation Amplitude, 42nd Floor . . . . .	23
3.6 Resonant Rotation Amplitude, 2nd Floor . . . . .	23
3.7 Ratio of Resonant Frequencies . . . . .	24
4.1 Wind Direction and Velocity . . . . .	66
4.2 Location of Seismometers . . . . .	67
4.3 Resonant Frequencies . . . . .	72
4.4 Ratio of Resonant Frequencies . . . . .	73
4.5 Damping Ratios . . . . .	74
5.1 Comparison of Resonant Frequencies and Damping Factors . . . . .	90
6.1 Results of Mathematical Model Resonant Frequencies . . . . .	107
7.1 Comparison of Resonant Frequencies and Damping Factors . . . . .	111



## ABSTRACT

As a part of a continuing program to evaluate the dynamic response of actual structures and to accumulate a body of information on the dynamic properties of structures, especially when these structures have novel design features, a dynamic test program was conducted on the forty-story Century City Theme Tower building.

The dynamic tests of the building included both a forced vibration study and an ambient vibration study. These results are compared and in general show very good correlation. A mathematical computer model of the Theme Tower was formulated and the results of the analysis are presented and compared to the experimental results. Again in general the results compare very favorably.



# 1. INTRODUCTION

## 1.1 General

The design of multistory structures subjected to dynamic forces resulting from foundation motions requires a consideration of both the characteristics of the ground motion and the dynamic properties of the structure. Ground motions as caused by an earthquake are random and although not prescriptible for aseismic design have been fairly well studied for certain well known past earthquake. The engineer is therefore mainly interested in the dynamic properties of the structure when designing for earthquake forces and is only indirectly concerned with the ground motion characteristics.

The availability of high speed digital computers and the sophistication of the idealization of structures and the computer model formulation of the structure have made available the elastic, and in certain structural systems, the inelastic response of structures when subjected to earthquakes. However, the accuracy of the results in large measure depend upon the computer model formulation of the structure and its foundation. In order to determine the accuracy of the calculated results and to accumulate a body of information on the dynamic properties of structures, especially when these structures have novel design features, a number of dynamic tests have been conducted on full-scale structures (1).

For the above reasons dynamic tests, using forced and ambient methods, were performed on the Century City South-Theme Tower in Los Angeles. Because of the favorable advantage of the ambient vibration method in dynamic testing of full-scale structures it was desirable to compare both methods in order to assess the accuracy of each method in evaluating the dynamic properties of the structural systems.

The building is described in Chapter 2 and the results of the dynamic tests, from forced as well as ambient vibration study, are given in Chapters 3 and 4, respectively. Comparison of the dynamic properties of the building from both studies is presented in Chapter 5. A mathematical model of the structural system was formulated, and the calculated and experimental dynamic properties were compared. The formulation of the mathematical model and the analytical dynamic properties obtained are described in Chapter 6.

## 1.2 Acknowledgement

The authors gratefully acknowledge the financial support provided by the National Science Foundation under Grant AEN73-07732 A0Z. They also wish to thank the owner, Delta Towers Joint Venture; the managing agent, Century City, Inc., especially Mr. Robert D. Burford and Mr. John F. O'Laughlin; the general contractor, Tishman Realty Construction Co. Inc.; and the Structural Engineers, Skilling, Helle, Christiansen, Robertson, for their help and cooperation in coordinating and carrying out the test program.

## 2. THE CENTURY CITY THEME TOWERS

### 2.1 General

The Century City Theme Towers, located in Los Angeles, California, are built as twin South and North Towers. Both buildings from the structural and architectural point of view are practically identical. The dynamic tests were performed on the South Tower during November 1974 and March, 1975. The building is a multi-story structure, forty-four stories in height above the plaza level, and six under ground parking levels. The height of the building above the plaza level is about 575 feet, with the equilateral triangular floor plan having sides of 254 feet. Figure 2.1 shows both buildings looking from North-West and Fig. 2.2 shows South Theme Tower.

### 2.2 Structural Systems and Structural Elements

The steel frame structure extends from B level to the roof and consists of core triangular frames with three corner columns connected with deep beams at the second floor of the building. From the second floor to the roof, the core steel framing continue like in the lower floors whereas the exterior walls are constructed as three identical moment resistant frames with twenty-three bays of 10 feet 2 inches (Fig. 2.5). A deep steel girder covering the top two floors is rigidly connected to all three exterior frames. Thus, the structural system consists of the equilateral triangular core and exterior moment resistant frames (Figs. 2.3, 2.4 and 2.5) connected at each floor with shear end connected beams. The floor slabs in the core part are 4 1/2 inches deep without steel decking and the exterior slabs are 4 1/2 inches over 18 gage steel deck.

A typical floor plan of the building from the second to forty-fourth floor is given in Fig. 2.5 showing the basic structural elements. The floor plan on the plaza level is given in Fig. 2.6 showing the core structural elements and corner columns. A typical cross-section of the corner columns from level B to second floor is shown in Fig. 2.8 as well as architectural finish line forming the final shape of the corner column at the plaza level.

The core frame columns are rolled sections of W14 shape, and the core beams are in general rolled sections, mostly wide flange shape varying from W12 to W36. Exterior columns consist of standard wide flange sections and built-up sections. Exterior frame spaced beams are built-up girders with a constant depth of four feet and changeable plate thickness. Deep exterior beam on the top of the building and the second floor are also built-up sections with a depth of twenty-eight feet, and seven feet, respectively.

The structural steel used in the building for both the beams and the columns is A36 and A50, the latter high strength steel being used in general in the lower floors.

From the B level down to the foundation (F level) the structural system consists of reinforced concrete elements, forming periferial core walls with a thickness of 20 inches, and core reinforced concrete columns connected with rigid slabs on each parking level. The corner columns are also reinforced concrete with the dimensions at the F level of twenty by twenty feet. A partial plan of the building on the F level is given by Fig. 2.7.

The core part of the building rests on a triangular shaped mat eight feet thick about sixty-six feet below the plaza level.

The corner columns are resting on individual foundation mats with thicknesses of fourteen feet and plan dimensions of about 40x45 feet for the north and 40x49 feet for the south columns. The foundation mat is placed in a silty sand layer.

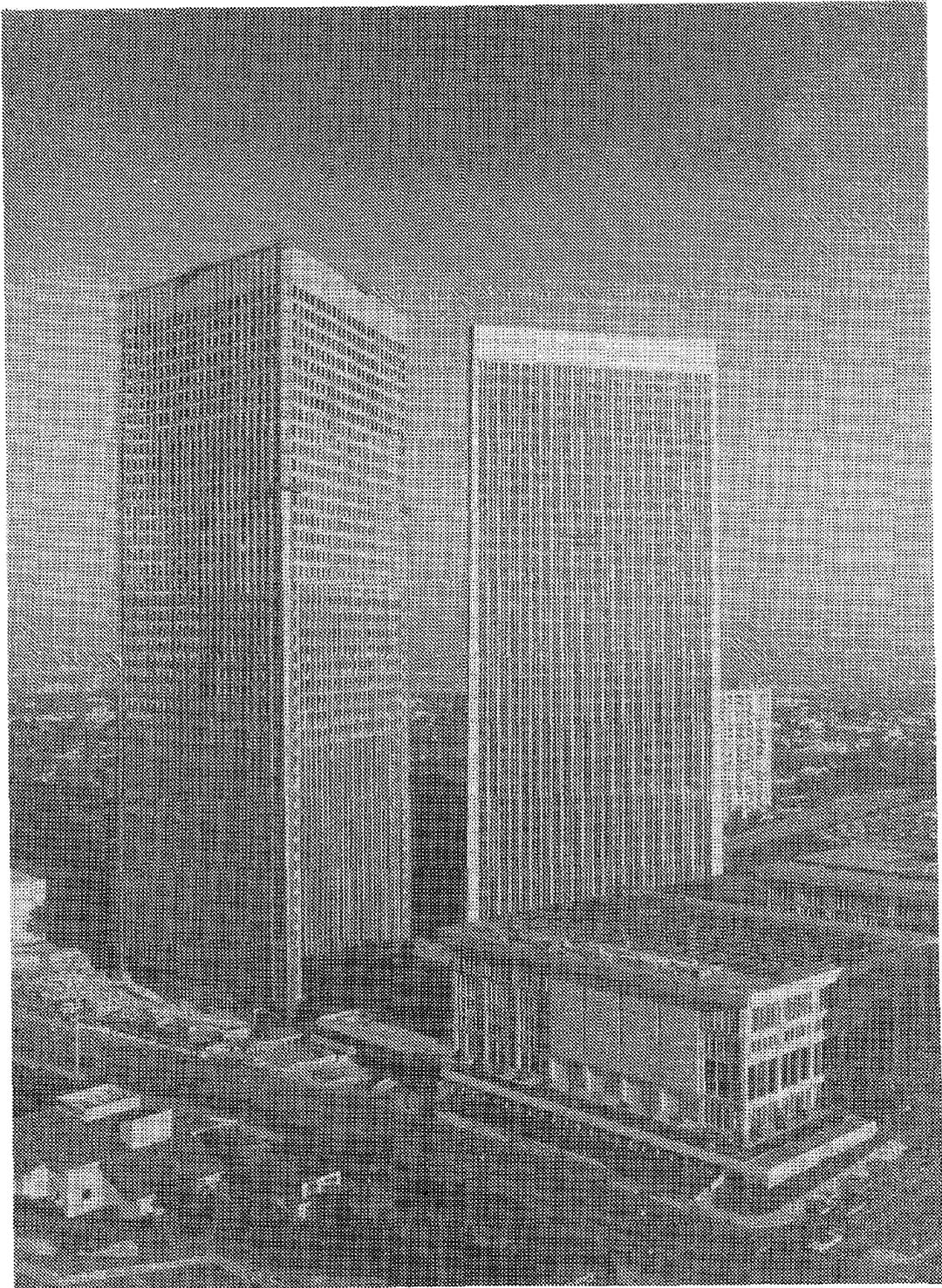


FIG. 2.1 GENERAL VIEW OF THE THEME TOWER BUILDINGS FROM NORTH-WEST

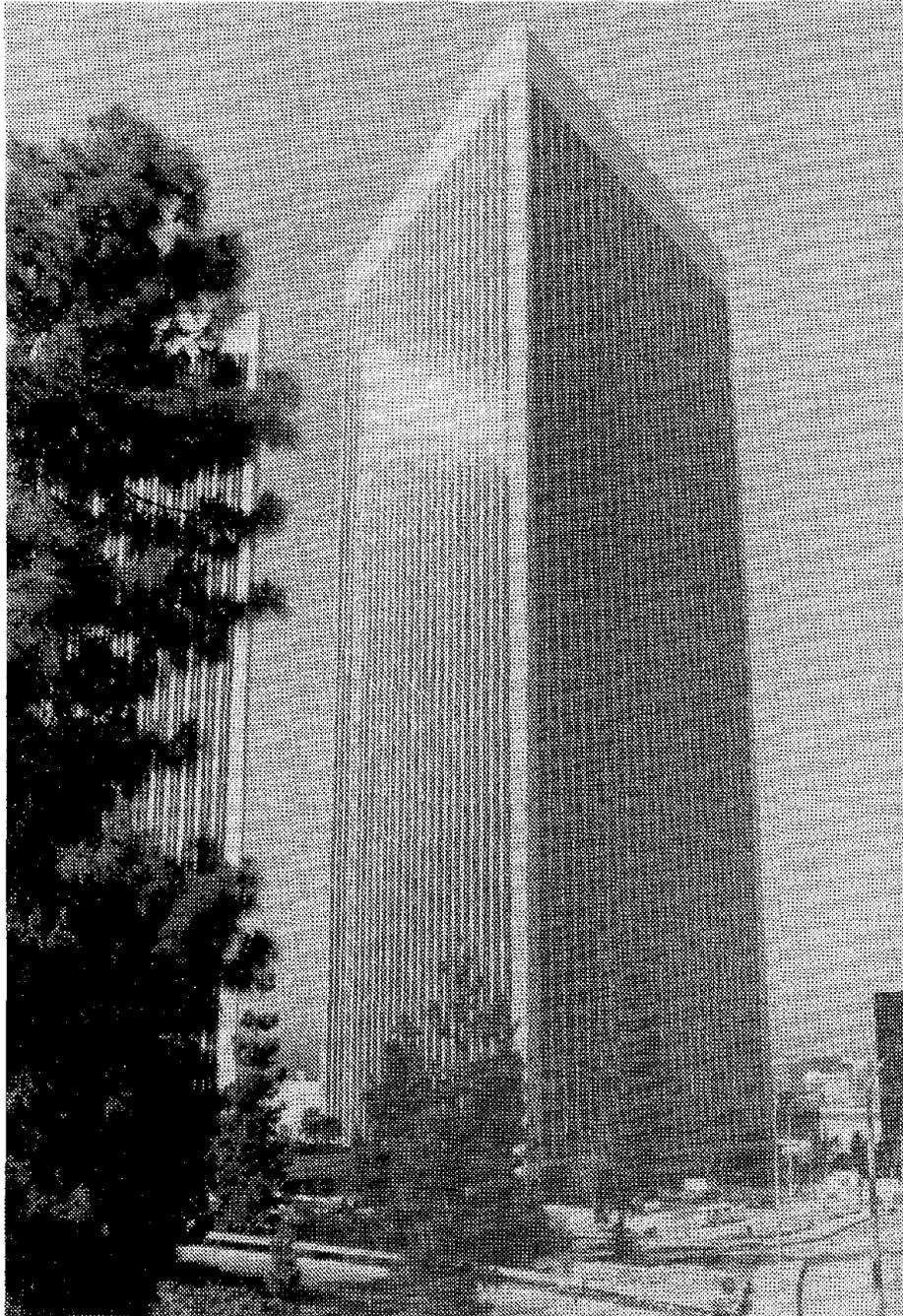


FIG. 2.2 SOUTH THEME TOWER

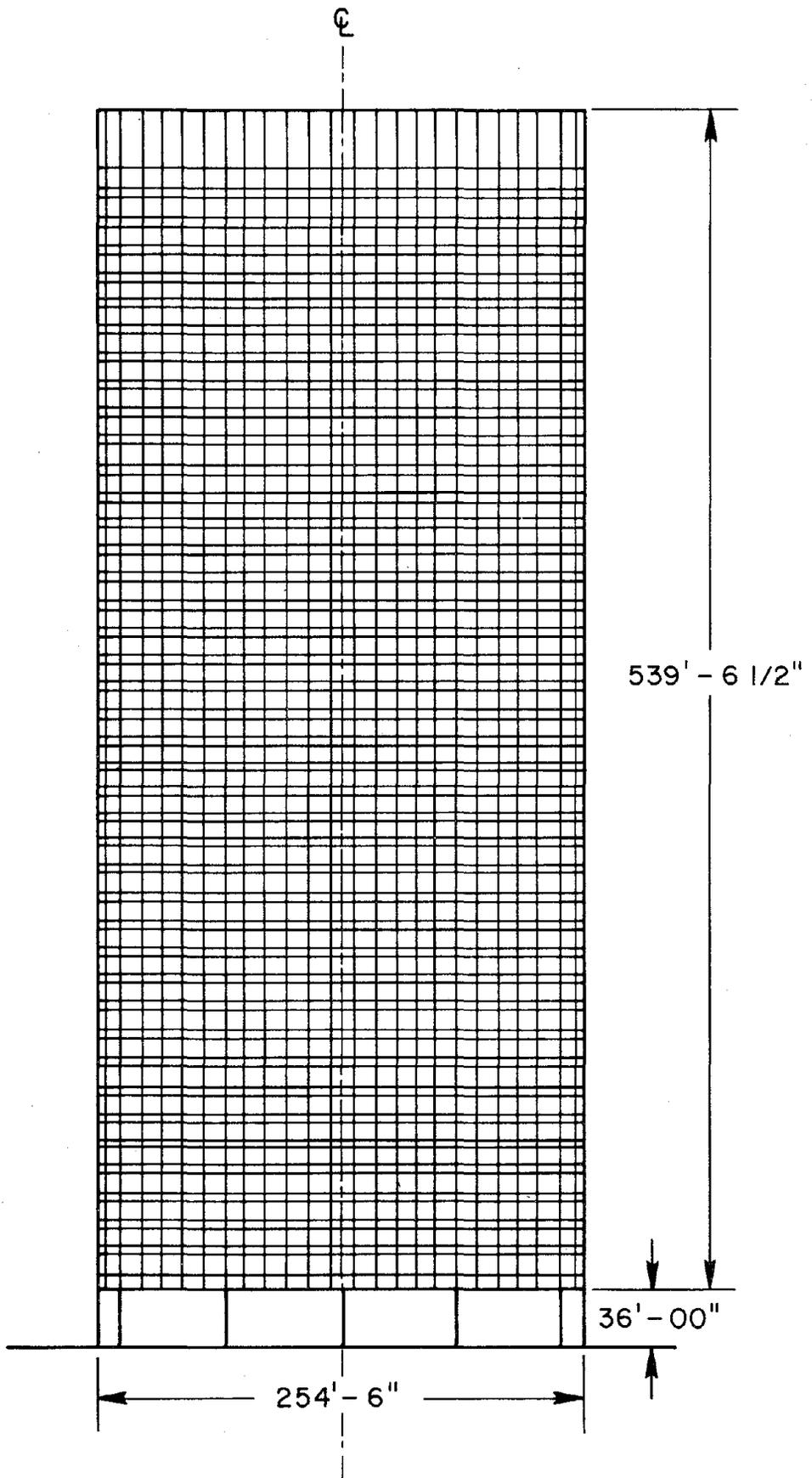


FIG. 2.3 SOUTH ELEVATION OF THE BUILDING

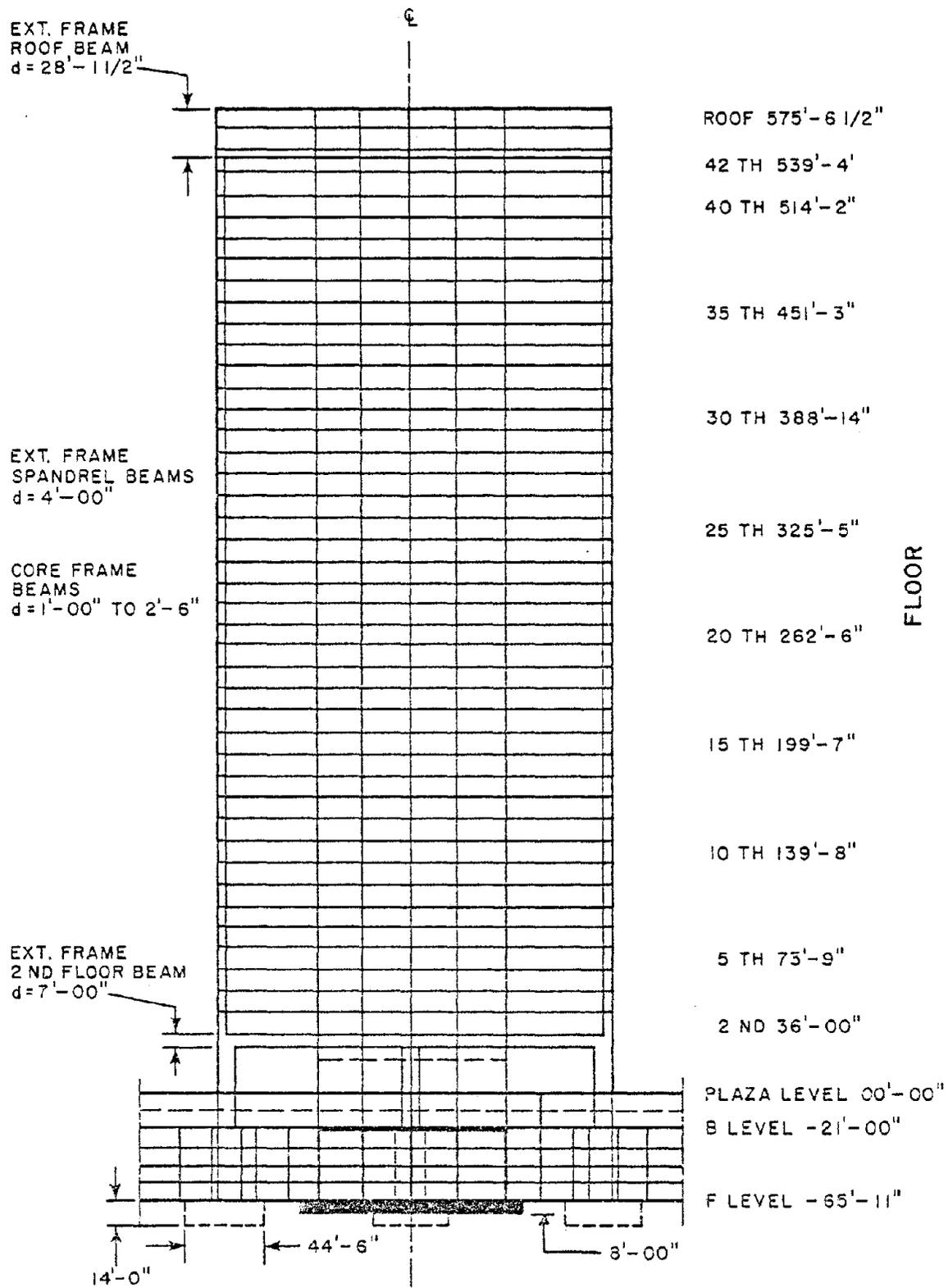


FIG. 2.4 CROSS SECTION OF THE BUILDING

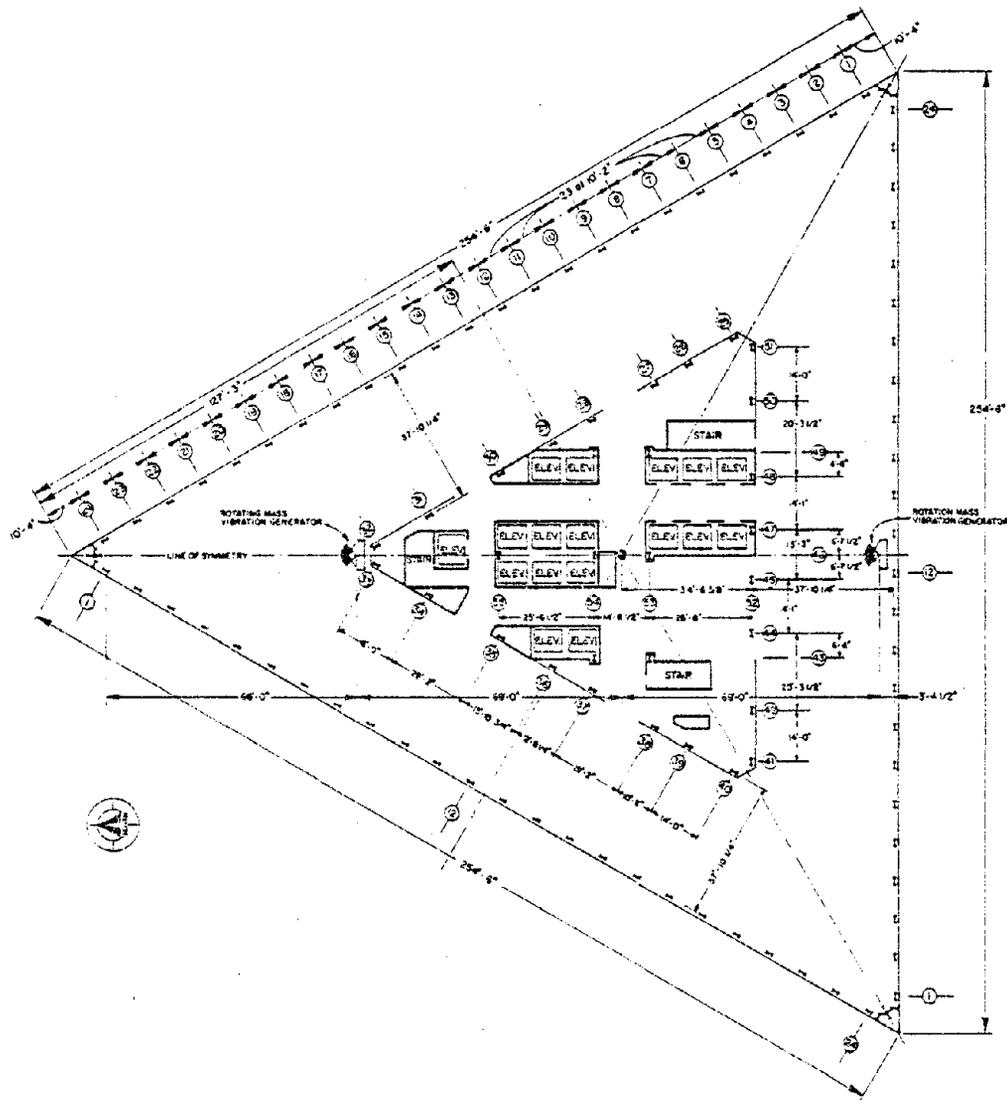


FIG. 2.5 TYPICAL FLOOR PLAN AND LOCATION OF VIBRATION GENERATORS (42 ND FLOOR)

Reproduced from  
best available copy.

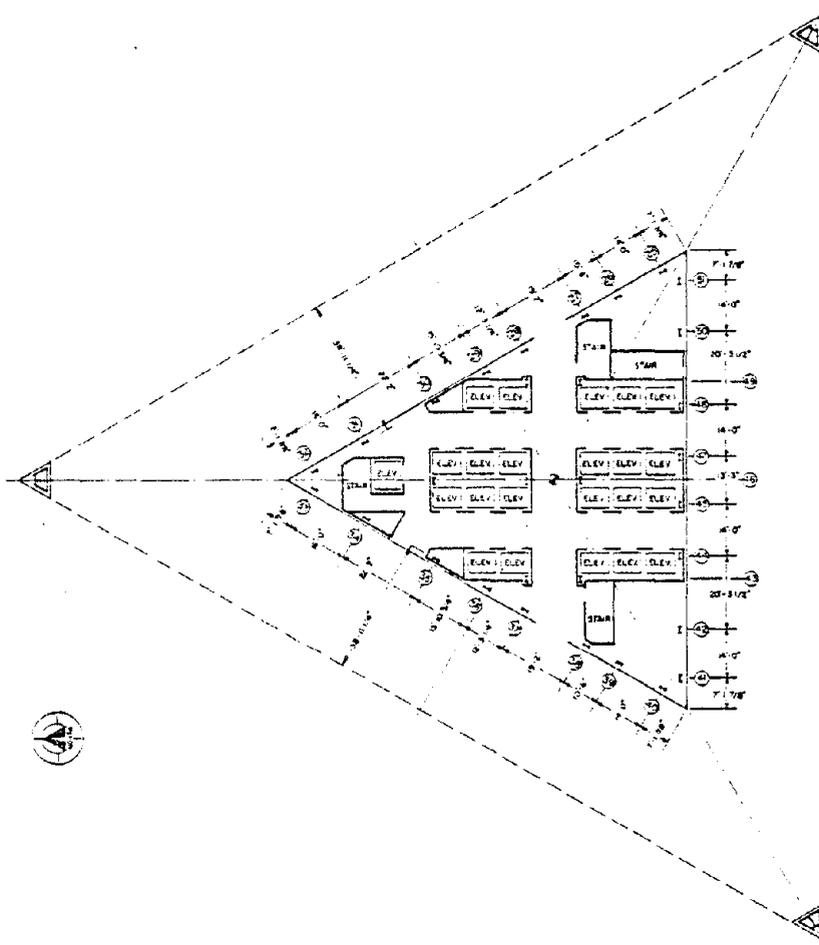


FIG. 2.6 FLOOR PLAN ON PLAZA LEVEL

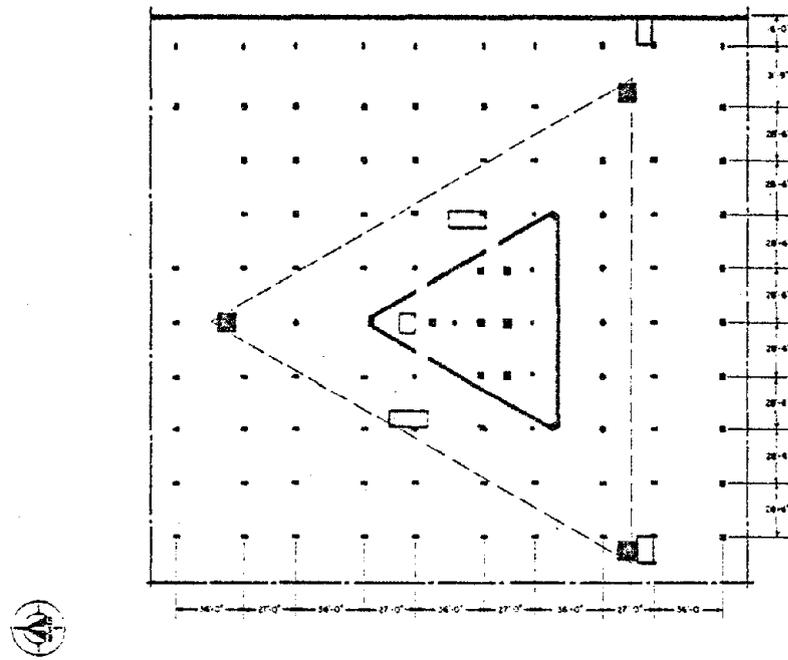


FIG. 2.7 FLOOR PLAN ON F LEVEL

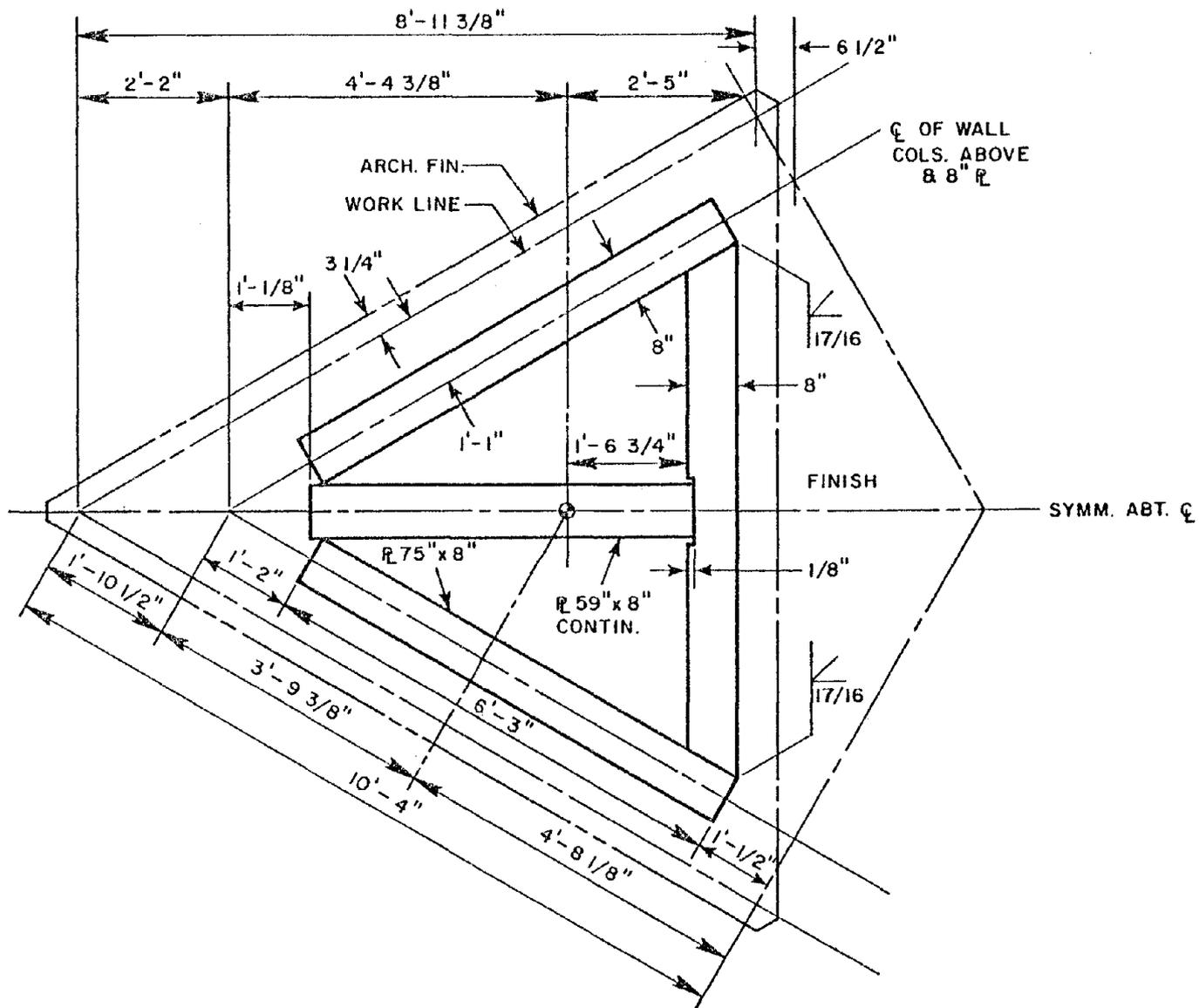


FIG. 2.8 CORNER COLUMN SECTION-PLAZA LEVEL

### 3. FORCED VIBRATION STUDY

#### 3.1 General

The forced vibration study was carried out and completed during November 1974. The building was structurally completed prior to the experimental work, and all of the facing cover with glass and aluminum, as well as, partition walls and installations in the core part of the building were in place. The experimental apparatus employed in the dynamic test is described below. The general experimental procedures, equipment used, and procedures for data reduction applied, for forced vibration study conducted are also described. Finally, the experimental results are presented and discussed.

#### 3.2 Experimental Apparatus

The experimental apparatus employed in the tests were two vibration generators, twelve accelerometers and equipment for the measurement and recording of the frequency responses. The apparatus is described in the following sections.

##### 3.2.1 Vibration Generators

Forced vibrations were produced by two rotating-mass vibration generators or shaking machines, one of which is shown in Fig. 3.1. These machines were developed at the California Institute of Technology under the supervision of the Earthquake Engineering Research Institute for the Office of Architecture and Construction, State of California. Each machine consists of an electric motor driving two pie-shaped baskets or rotors, each of which produces a centrifugal force as a result of the rotation. The two rotors are mounted on a common vertical shaft and rotate in opposite directions so that the

resultant of their centrifugal forces is a sinusoidal rectilinear force. When the baskets are lined up, a peak value of the sinusoidal force will be exerted. The structural design of the machines limits the peak value of force to 5,000 lbs. This maximum force may be attained at a number of combinations of eccentric mass and rotational speed, since the output force is proportional to the square of the rotational speed as well as the mass of the baskets and the lead plates inserted in the baskets. The maximum force of 5,000 lbs. can be reached for a minimum rotational speed of 2.5 cps when all the lead plates are placed in the baskets. At higher speeds the eccentric mass must be reduced in order not to surpass the maximum force of 5,000 lbs. The maximum operating speed is 10 cps, and the minimum practical speed is approximately 0.5 cps. At 0.5 cps with all lead plates in the baskets, a force of 200 lbs. can be generated. The relationship between output force and frequency of rotation of the baskets for different basket loads is shown in Fig. 3.2. Although, the rotating mass vibration generators are very difficult to accurately generate at frequencies lower than 0.5 cps and at the same time develop sufficiently large forces to record the motion of the building, in this dynamic test with extremely careful performance, it was possible to obtain frequency response for the first modes. The frequencies excited were in the range of 0.2 to 0.4 cps and the exciting resonant force was within 115 and 204 lbs.

The speed of rotation of each motor driving the baskets is controlled by an Electronic Amplidyne housed in a control unit. The control unit allows the machines to be synchronized or operated 180° out-of-phase. This makes it convenient to excite, in structures with a line of symmetry, either torsional or pure translational vibrations

without changing the position of either machine. A complete description of the vibration generators is given in (7).

The vibration generators were mounted on the 42nd floor at the south end and north core corner on an equal distance (69 ft) from the centroid, along the north-south angle symmetry line of the building. Associated vibration control and recording equipment was also placed on the 42nd floor (Fig. 2.5 and 3.1).

### 3.2.2 Accelerometers

The transducers used to detect horizontal floor accelerations of the building were Statham Model A4 linear accelerometers, with a maximum rating of  $\pm 0.25g$ .

### 3.2.3 Equipment for Measurement of Frequency

For the vibration generators the vibration excitation frequencies were determined by measurement of the speed of rotation of the electric motor driving the baskets. A tachometer, attached to a rotating shaft driven by a transmission belt from the motor, generated a sinusoidal signal of frequency 300 times the frequency of rotation of the baskets. Hence, the maximum accuracy of frequency measurements was  $\pm 1$  count in the total number of counts in a period of 1 second (the gating period), i.e.,  $\pm 1/3$  of 1% at 1 cps and  $\pm 1/9$  of 1% at 3 cps.

### 3.2.4 Recording Equipment

The electrical signals for all accelerometers were fed to amplifiers and then to a Honeywell Model 1858 Graphic Data Acquisition System with 8" wide chart. In frequency-response tests, the digital counter reading was observed and recorded manually on the chart alongside the associated traces.

### 3.3 Experimental Procedure and Data Reduction

The quantities normally determined by a dynamic test of a structure are: resonant frequencies, mode shapes, and damping capacities. The experimental procedures and reduction of data involved in determining these quantities are described in the next section.

#### 3.3.1 Resonant Frequencies

With the equipment described on the previous page resonant frequencies are determined by sweeping the frequency range of the vibration generators from 0.2 to 10 cps.

In the case of the vibration generators the exciting frequency is increased slowly until acceleration traces on the recording chart are large enough for measurement. Above this level the frequency is increased in steps until the upper speed limit of the machine is reached. Near resonance, where the slope of the frequency-response curve is changing rapidly, the frequency-interval steps are as small as the speed control permits. These steps are relatively large in regions away from resonance. Each time the frequency is set to a particular value, the vibration response is given sufficient time to become steady-state, before the acceleration traces are recorded. At the same time, the frequency of vibration as recorded on a digital counter, is observed and written on the chart with its corresponding traces. Plotting the vibration response at each frequency step results in a frequency-response curve.

Frequency-response curves, in the form of acceleration amplitude versus exciting frequency, may be plotted directly from the data on the recording chart. However, the curves are for a force which

increases with the square of the exciting frequency, and each acceleration amplitude should be divided by the corresponding square of its exciting frequency to obtain so-called normalized curves equivalent to those for a constant force (assuming linear stiffness and damping for the structural system). If the original acceleration amplitudes are divided by the frequency to the fourth power, displacement frequency-response curves for constant exciting forces are obtained. In cases of fairly low damping (under 5%), there is little difference between results obtained for resonant frequencies and damping capacities measured from the different curves.

### 3.3.2 Mode Shapes

Once the resonant frequencies of a structure have been found, the mode shapes at each of these frequencies may be determined. Generally, there are insufficient accelerometers, or insufficient recorder channels, to measure the vibration amplitude of all the required points simultaneously. Thus, it is necessary, after recording the amplitudes of a number of points, to stop the vibration, shift the accelerometers to new positions, and then vibrate the structure at resonance once more. This procedure is repeated until the vibration amplitude of all required points has been recorded.

The structure may not vibrate at exactly the same amplitude in each test run because it is impossible to vibrate the structure at precisely the same frequency each run. Therefore, it is necessary to maintain one reference accelerometer (preferably at a point of maximum displacement) during all the mode shape measurements for a particular mode. Subsequently, all vibration amplitudes can be adjusted to a constant modal amplitude.

In addition it is necessary to make corrections to the recorded amplitudes to compensate for differences between calibration factors. Absolute calibration is not required for mode shapes and cross-calibration is sufficient. The accelerometers and all equipment associated with them in their respective recording channels are cross-calibrated simply by placing them all together so that they measure the same vibration. Cross-calibration is generally carried out at the beginning and end of each day. The average calibration factors as derived from the pre- and post-test cross-calibration runs are used to adjust the recorded amplitudes.

The number of points required to define a mode shape accurately depends on the mode and the number of degrees of freedom in the system. For example, in a dynamic test on a 15-story building (12) four points were sufficient to define the first mode, whereas it required measurements of the vibration of all 14 floors and the roof to define the 5th mode shape accurately.

### 3.3.3 Damping Capacities

Damping capacities may be found from resonance curves in the normalized frequency-response curves by the formula:

$$\xi = \frac{\Delta f}{2f}$$

where

- $\xi$  = damping factor,
- $f$  = resonant frequency,
- $\Delta f$  = difference in frequency of the two points on the resonance curve with amplitudes of  $1/\sqrt{2}$  times the resonant amplitude.

Strictly, the expression for  $\xi$  is only applicable to the displacement resonance curve of a linear, single degree of freedom system with a small amount of viscous damping. However, it has been used widely for systems differing appreciable from that for which the formula was derived, and it has become accepted as a reasonable measure of damping. In this respect it should be remembered that in the case of full-size civil engineering structures it is not necessary to measure damping accurately in a percentage sense. It is sufficient if the range in which an equivalent viscous damping coefficient lies is known. Meaningful ranges might be defined as: under 1%, 1-2%, 2-5%, 5-10%, over 10% (1,10).

The bandwidth method described above is extremely useful when damping factor lies in the range 1-10% of critical. However, if the damping lies below 1%, difficulties may be encountered in observing sufficient points on the resonance curve. Also the small frequency difference between two relatively large frequencies becomes difficult to measure accurately. Above 10% of critical damping, resonance curves often become poorly defined due to interference between modes and the results from the bandwidth method have little meaning.

### 3.4 Experimental Results

The vibration equipment was bolted to the 42nd floor through the test program as shown in Fig. 2.5 and with the appropriate adjustments to the vibration generator equipment it was possible to produce translational or torsional vibrations of the building. The first six and five translational modes, respectively in E-W and N-S direction were excited, as well as, the first six torsional modes. Frequency response curves, in the region of the resonant frequencies

for all excited translational and torsional modes, are shown successively in Figs. 3.3 through 3.19. The curves are plotted in the form of normalized displacement amplitude versus exciting frequency. The ordinates were obtained by dividing the recorded acceleration amplitude by the square of the exciting frequency to obtain acceleration amplitudes for a constant equivalent force amplitude. The values thus obtained are divided by the square of the circular frequency (rad/sec) to obtain normalized displacement amplitudes. For convenience the actual exciting force ( $F_p$ ) and displacement amplitude ( $u_p$ ) for each of the excited resonancies are given in Figs. 3.3 through 3.19, as well as, calculated damping factors.

The resonant frequencies and damping factors evaluated from the response curves are summarized in Tables 3.1 and 3.2, respectively. The generated exciting force by both shaking machines and corresponding resonant displacement amplitude for each resonant frequency are given in Tables 3.3 and 3.4, respectively.

TABLE 3.1 RESONANT FREQUENCIES (c.p.s.)

Excitation	Mode					
	1	2	3	4	5	6
E-W	0.267	0.76	1.317	1.89	2.74	3.133
N-S	0.267	0.783	1.373	1.973	2.85	-
Torsional	0.357	0.99	1.69	2.47	3.317	4.133

TABLE 3.2 DAMPING FACTORS (%) FROM RESONANCE CURVES

Excitation	Mode					
	1	2	3	4	5	6
E-W	2.62	1.57	1.18	1.27	1.73	2.39
N-S	4.34	1.60	1.64	1.72	1.49	-
Torsional	1.34	0.81	0.95	1.01	1.36	1.51

TABLE 3.3 RESONANT FORCE AMPLITUDES (1b)

Excitation	Mode					
	1	2	3	4	5	6
E-W	115	930	2790	5748	7298	6708
N-S	115	986	3033	6264	7896	-
Torsional	204	1577	4596	7692	4342	1817

TABLE 3.4 RESONANT DISPLACEMENT AMPLITUDES ( $\times 10^{-2}$  in.) 42nd FLOOR

Excitation	Mode					
	1	2	3	4	5	6
E-W	1.65	1.03	0.588	0.311	0.057	0.023
N-S	1.51	1.03	0.584	0.304	0.064	-
Torsional	2.883	3.028	0.909	0.465	0.064	0.014

The mode shapes for the translational and torsional modes that were excited are shown in Figs. 3.20 through 3.36. Particular attention has been given to observe in plane deformations on the 42nd and 2nd floor for each of the excited resonances. The horizontal

in-plane floor vibrational mode associated with each of the translational and torsional modes of vibration, as well as the resonant displacement amplitudes of the floor center and the rotational amplitudes about the center, are given in Figs. 3.20 through 3.36. Resonant rotation amplitudes for the 42nd and 2nd floor are summarized in Tables 3.5 and 3.6, respectively.

TABLE 3.5 RESONANCE ROTATION AMPLITUDES  
42nd FLOOR ( $\times 10^{-7}$  rad.)

Excitation	Mode					
	1	2	3	4	5	6
E-W	4.991	2.536	1.047	0.282	2.375	1.811
N-S	0.036	0.107	0.143	0.178	0.003	-
Torsional	113.2	185.8	92.4	24.4	17.3	0.76

TABLE 3.6 RESONANCE ROTATION AMPLITUDES  
2nd FLOOR ( $\times 10^{-7}$  rad.)

Excitation	Mode					
	1	2	3	4	5	6
E-W	-	0.123	1.543	1.296	0.488	0.426
N-S	-	0.819	0.891	1.211	0.317	-
Torsional	9.20	34.3	47.7	20.5	20.5	1.22

### 3.5 Discussion of Experimental Results

The forced-vibration study of the building was conducive to obtaining accurate resonant frequency for the first five translational modes of vibration in N-S direction, along one of the symmetry lines of the floor plane, and the first six translation modes in orthogonal E-W direction, as well as the first six torsional modes of vibration. The resonant frequencies were well separated, and it was of interest to note the ratios of the observed higher mode frequencies with respect to the fundamental one. These ratios are given in Table 3.7 for all three directions of excited vibrations, and they indicate a type of over-all structural response.

TABLE 3.7 RATIO OF RESONANT FREQUENCIES

Mode	Translational E-W		Translational N-S		Torsional	
	$f_i$ (cps)	$f_i/f_1$	$f_i$ (cps)	$f_i/f_1$	$f_i$ (cps)	$f_i/f_1$
1	0.267	1.0	0.267	1.0	0.357	1.0
2	0.76	2.85	0.783	2.93	0.99	2.77
3	1.317	4.93	1.373	5.14	1.69	4.73
4	1.89	7.08	1.973	7.39	2.47	6.92
5	2.74	10.26	2.85	10.67	3.317	9.29
6	3.133	11.73	-	-	4.133	11.58

From these results it may be concluded that the building vibration in both translational directions as well as torsional vibration are predominantly of the shear type, because the determined frequency ratios follow closely the ratios 1, 3, 5, 7, 9, 11, ..., which apply for the uniform shear beam.

For equilateral triangular floor plan of the building it should be expected that the translational modes will exist in the direction of the lines of symmetry (Fig. 2.5). One of the translational direction of vibration excitation was selected along the N-S line of symmetry and the second orthogonal to this one in E-W direction. It has been found that amplitudes of rotation on the 42nd and 2nd floor for both directions are of the same order of magnitude (Table 3.5 and 3.6, and Figs. 3.20 through 3.31) and comparing them with the amplitudes of rotation in the torsional modes they are about two order of magnitude smaller. Thus, it could be concluded that the modes of vibrations excited along the line of symmetry (N-S) and orthogonal to it (E-W) are actual translational modes, with practically the same resonant frequencies and mode shapes (Table 3.1 and Figs. 3.20 through 3.31). In the torsional modes of vibration it appears that for all excited modes the line of rotation is crossing the floor plane centroid (Figs. 3.31 through 3.36). From these results it could be concluded that the mass and stiffness center coincide with the geometric center, and that the floor slabs are practically rigid in their plane.

Damping coefficients varied within 1 and 2% of critical damping in all modes, except for the first mode in N-S and E-W direction. It should be mentioned that the damping coefficients for the first mode in both the translational directions are probably higher due to difficulties in properly controlling the building vibration at such low frequencies (0.2 and 0.4 cps) and at a very low exciting force amplitude (115 lb). Similar damping values have been reported from the other full-scale forced vibration studies of steel frame high-rise buildings (13, 14, 15).

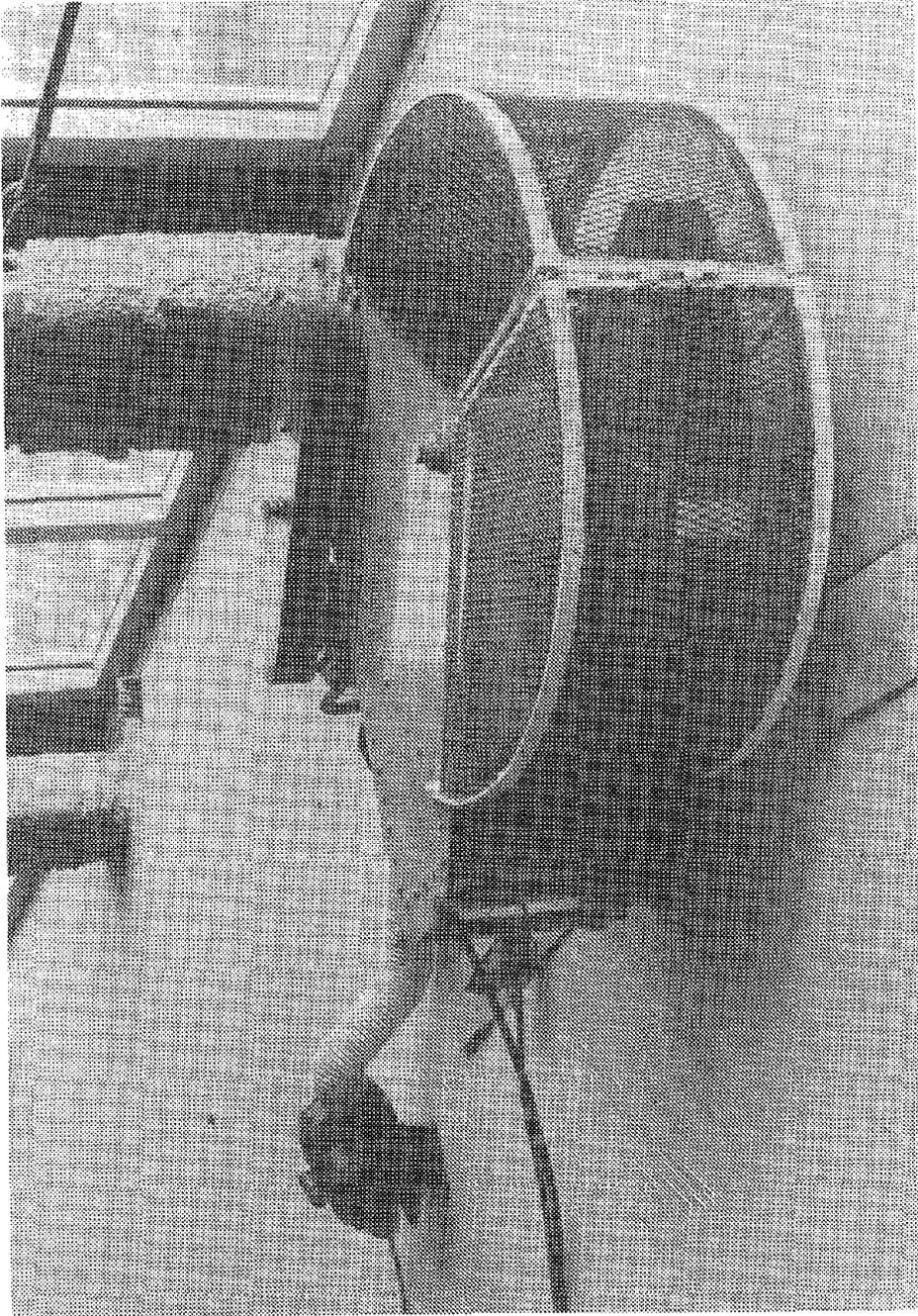


FIG. 3.1 VIBRATION GENERATOR ON THE 42 ND  
FLOOR OF SOUTH THEME TOWER

MAXIMUM LOAD LIMIT = 5000 LBS.

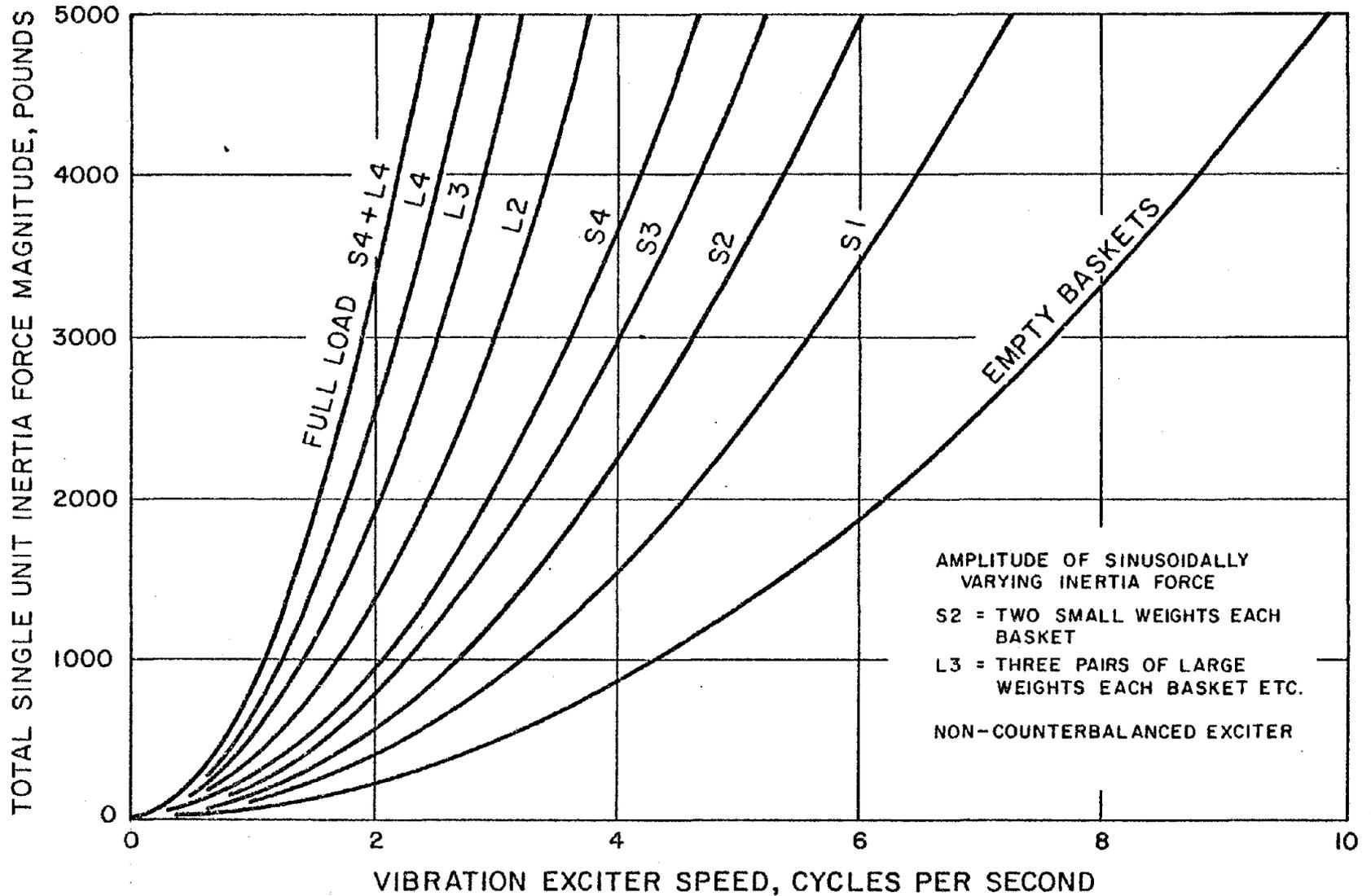


FIG. 3.2 VIBRATION FORCE OUTPUT VS. SPEED—NON-COUNTERBALANCED (AFTER HUDSON)

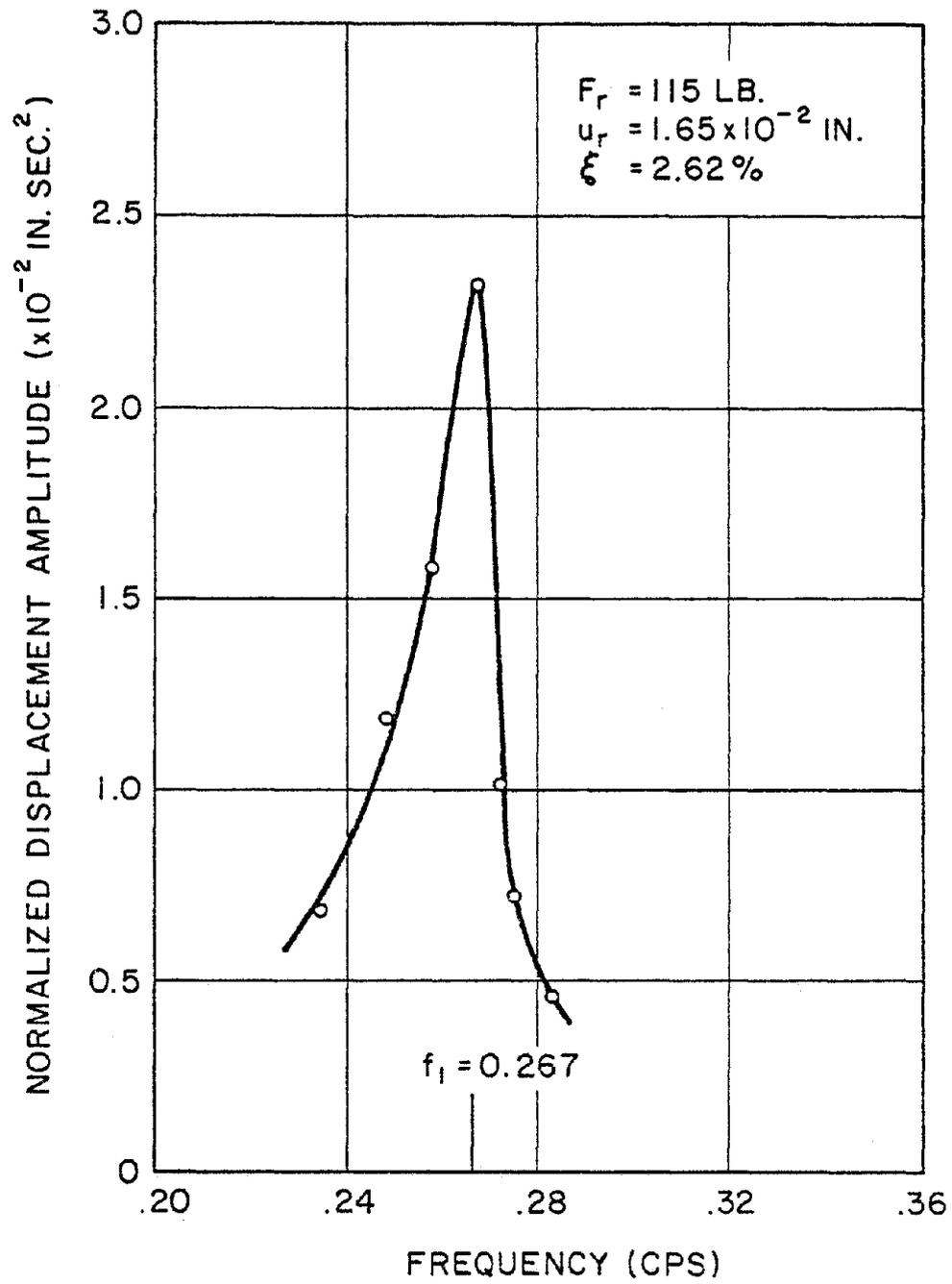


FIG. 3.3 FREQUENCY RESPONSE; FIRST MODE E-W

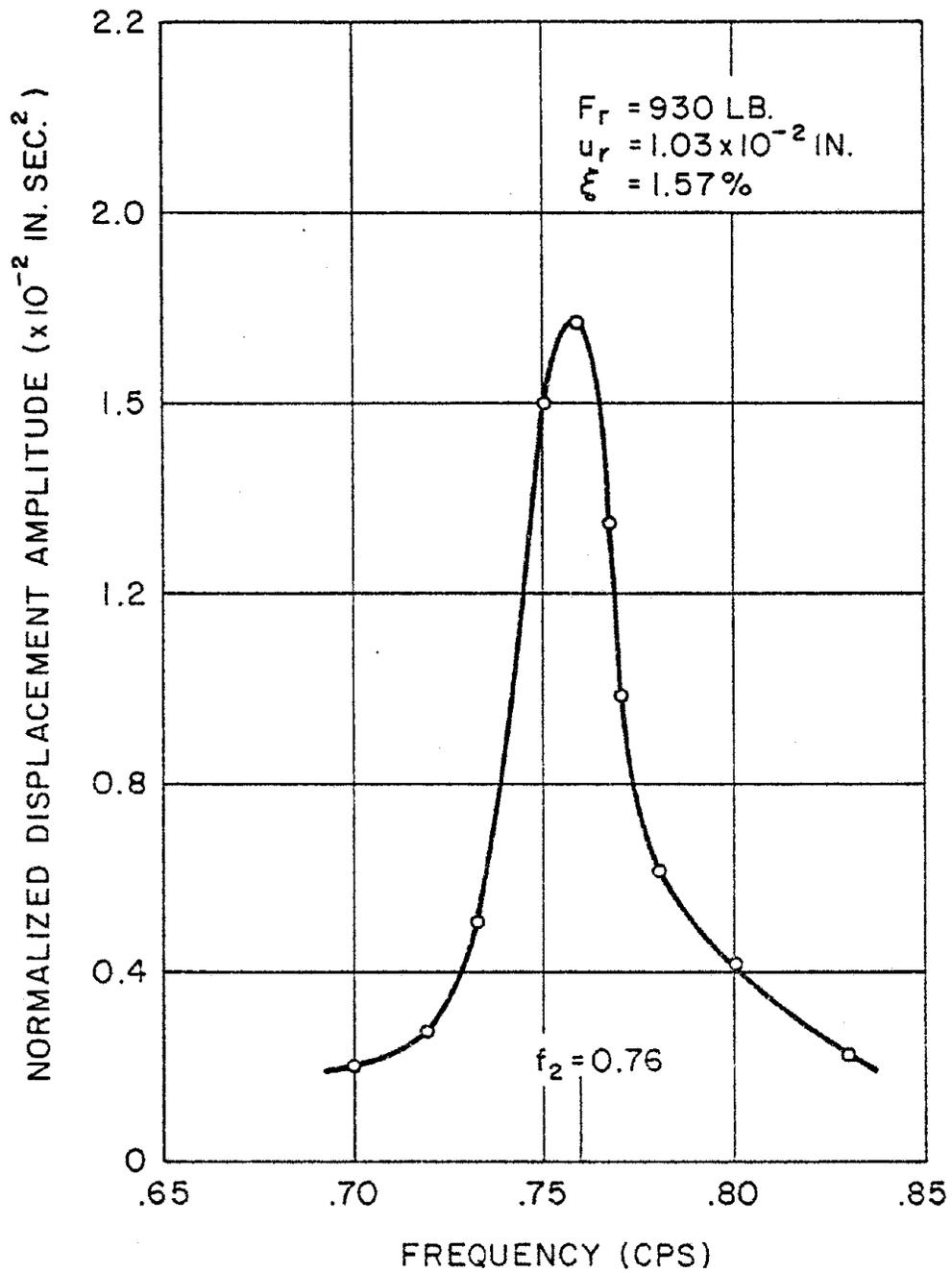


FIG. 3.4 FREQUENCY RESPONSE; SECOND MODE E-W

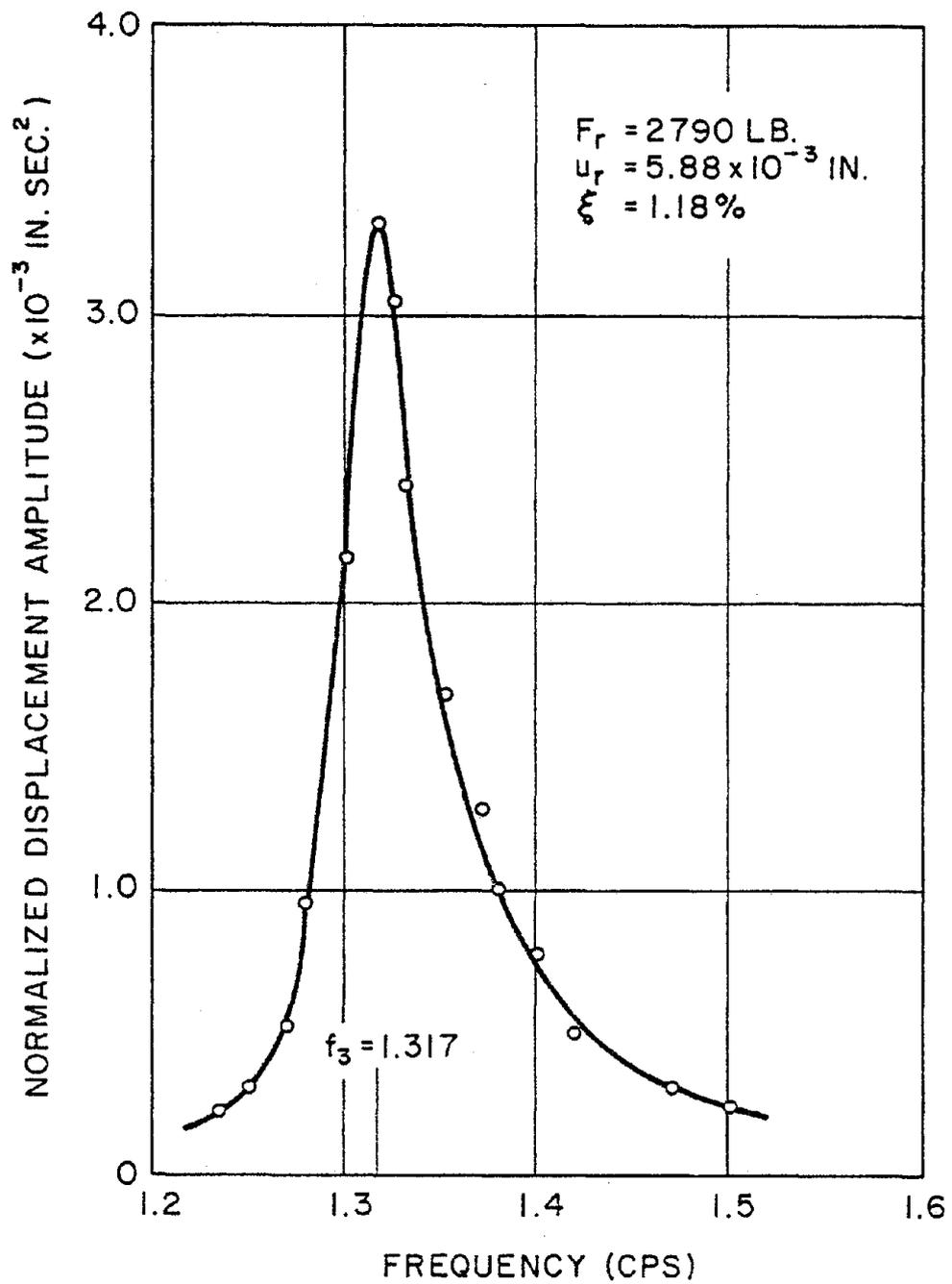


FIG. 3.5 FREQUENCY RESPONSE; THIRD MODE E-W

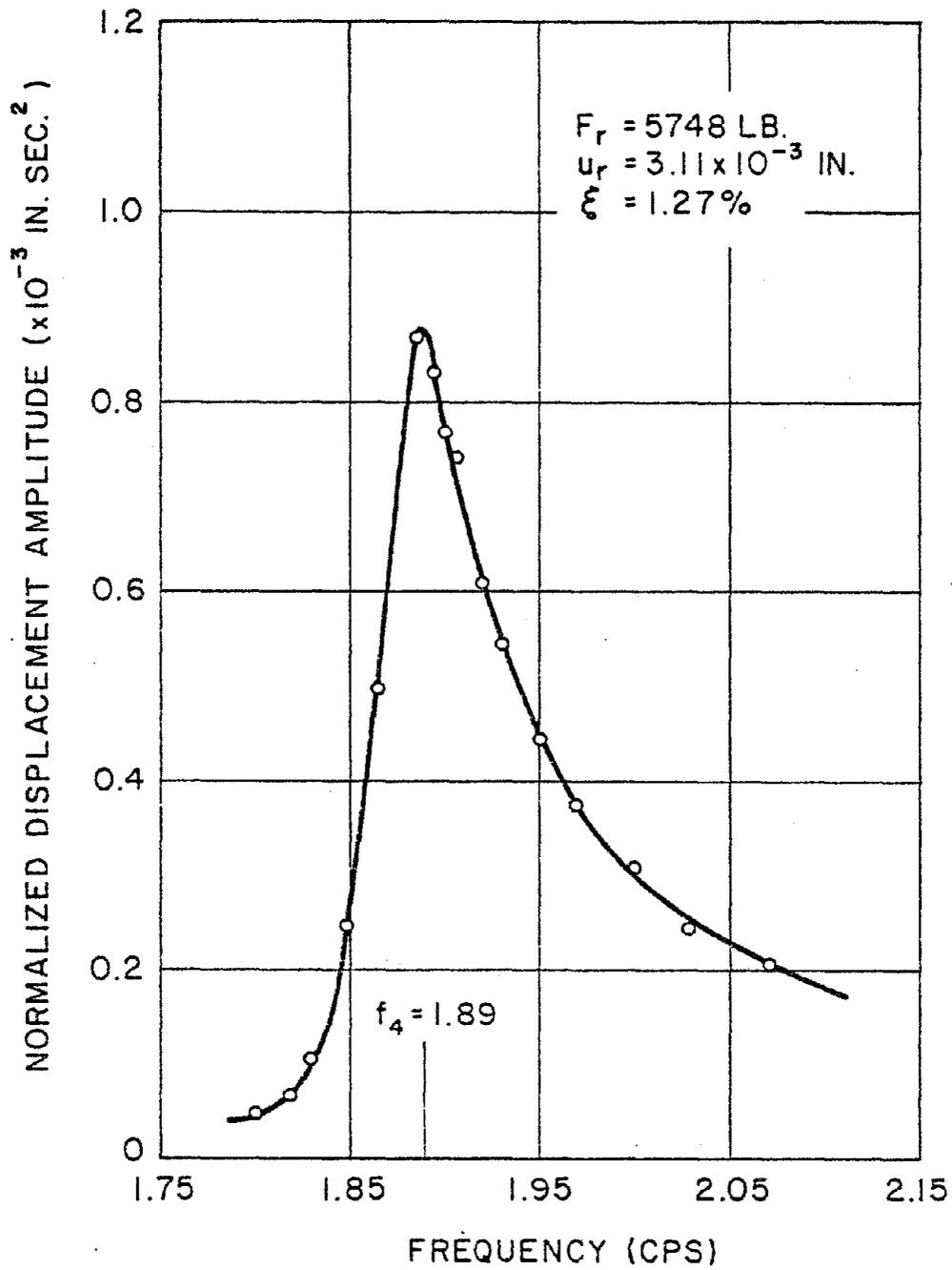


FIG. 3.6 FREQUENCY RESPONSE; FORTH MODE E-W

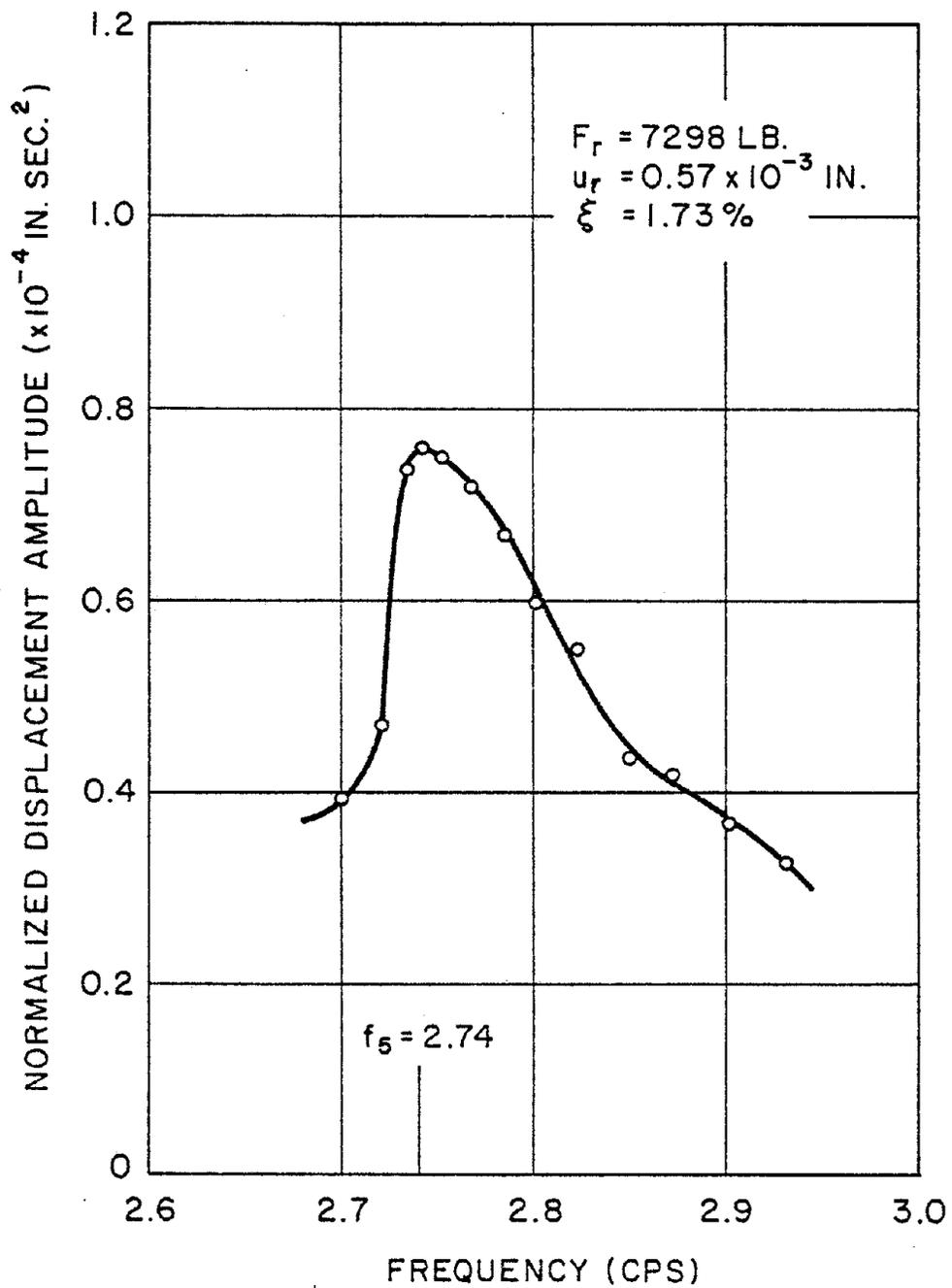


FIG. 3.7 FREQUENCY RESPONSE; FIFTH MODE E-W

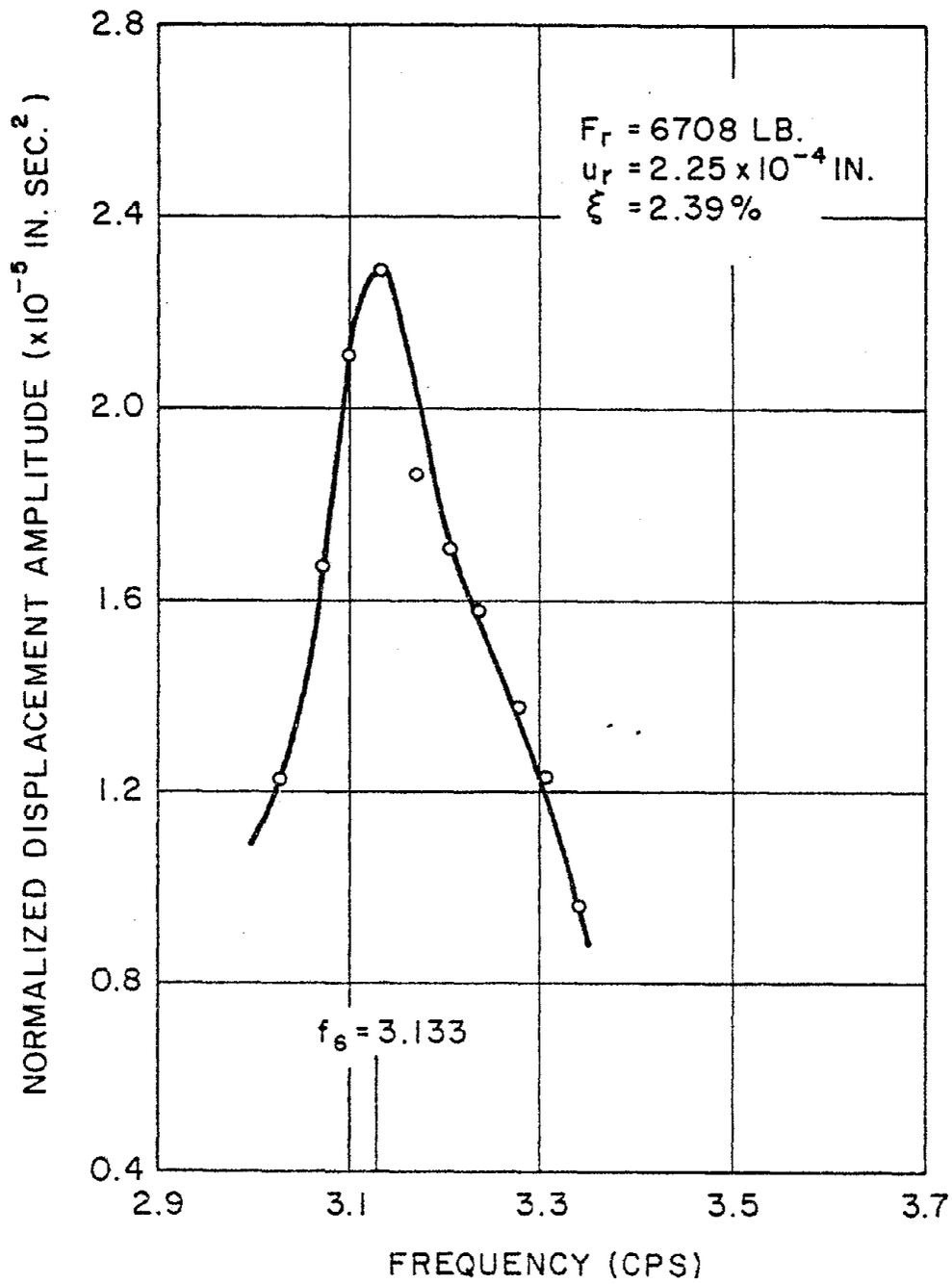


FIG. 3.8 FREQUENCY RESPONSE; SIXTH MODE E-W

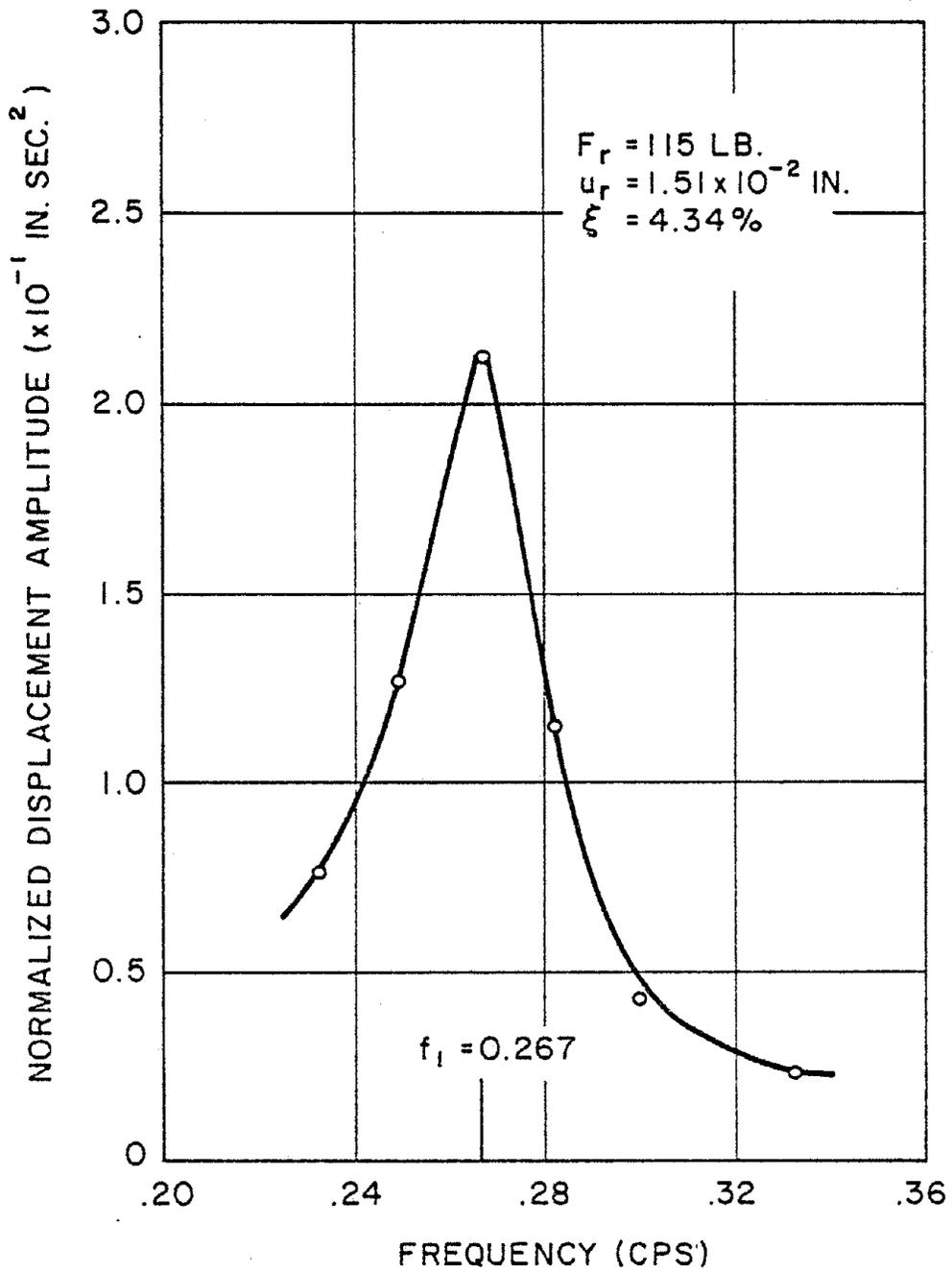


FIG. 3.9 FREQUENCY RESPONSE; FIRST MODE N-S

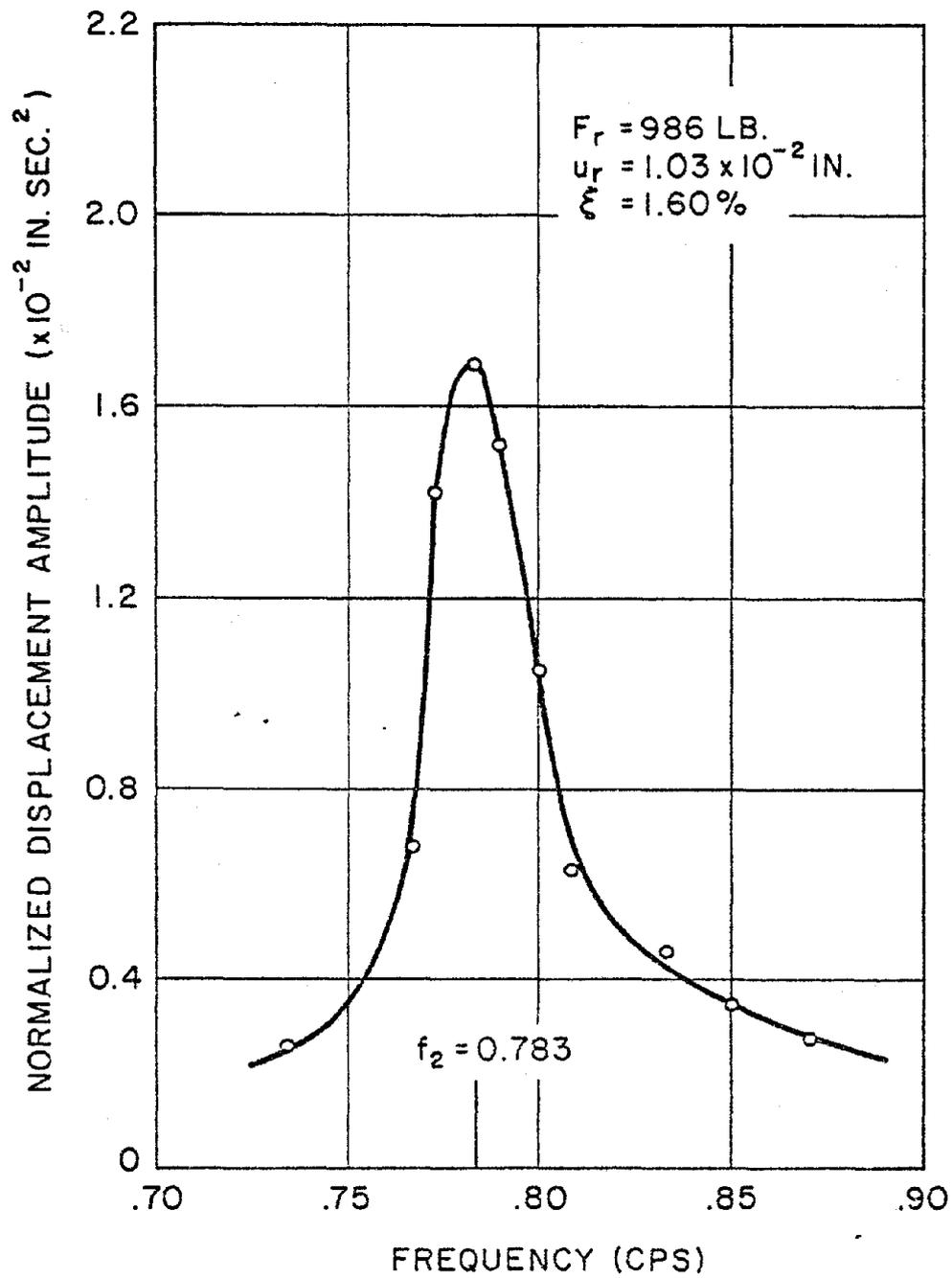


FIG. 3.10 FREQUENCY RESPONSE; SECOND MODE N-S

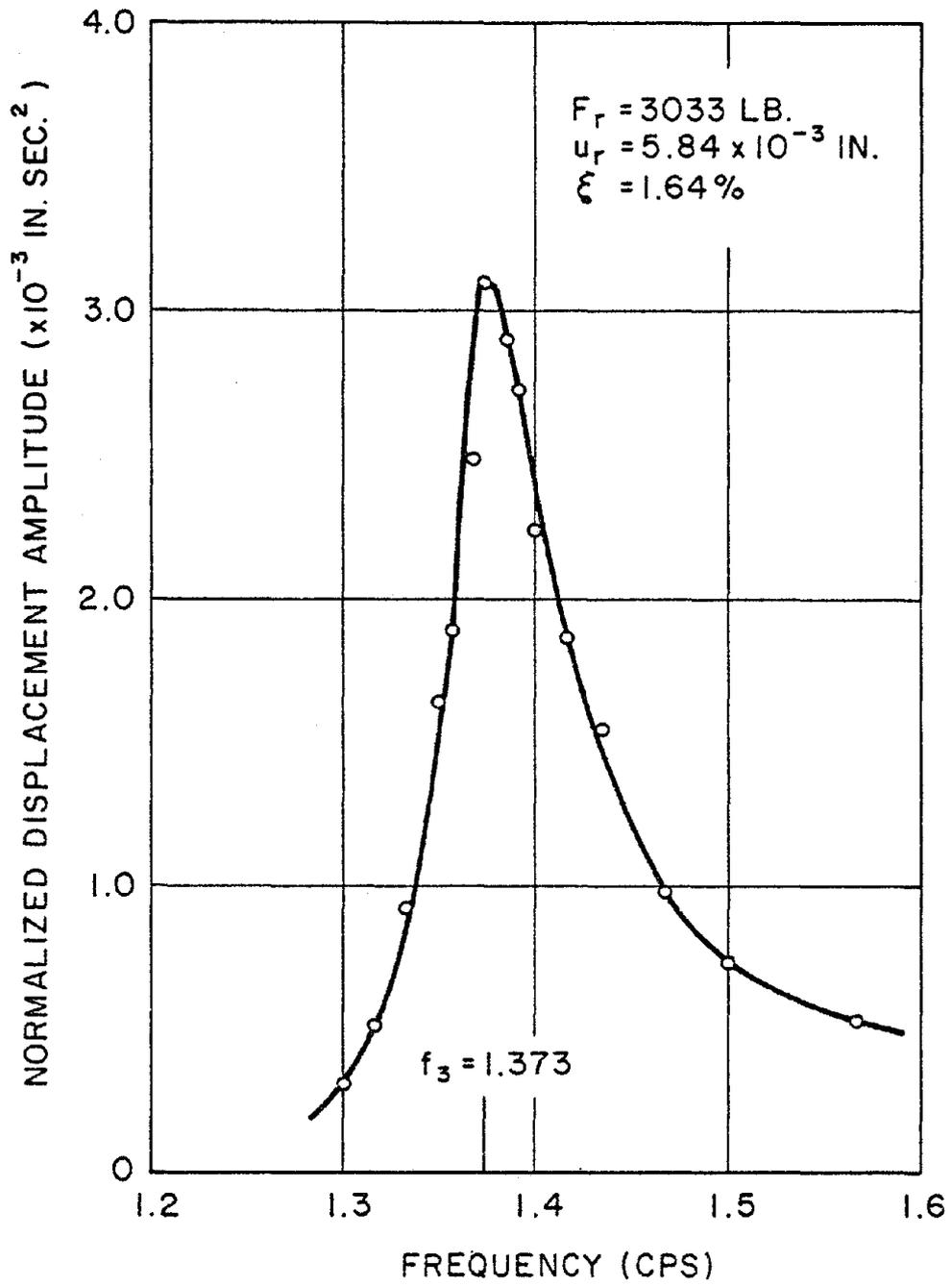


FIG. 3.11 FREQUENCY RESPONSE; THIRD MODE N-S

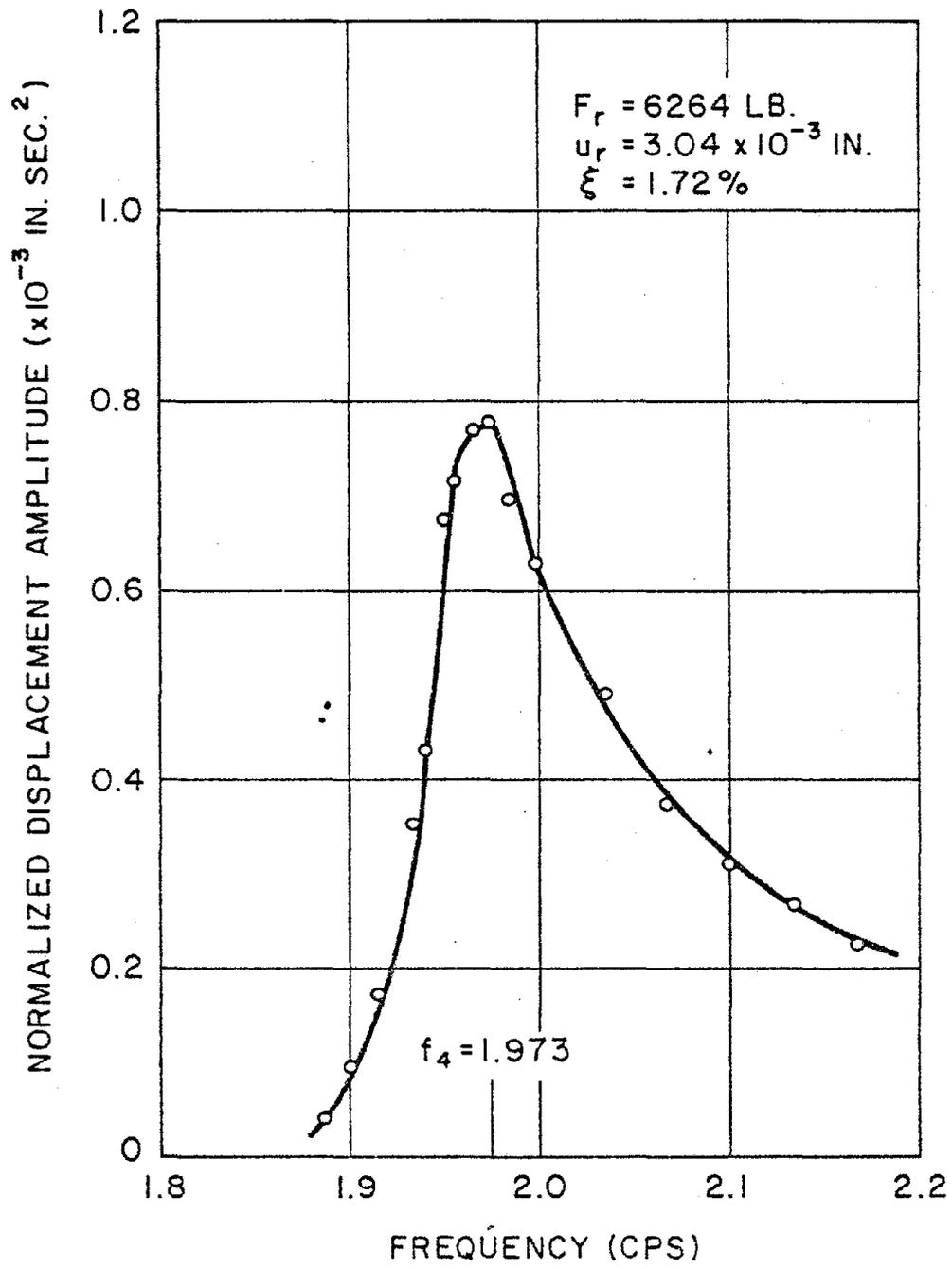


FIG. 3.12 FREQUENCY RESPONSE; FORTH MODE N-S

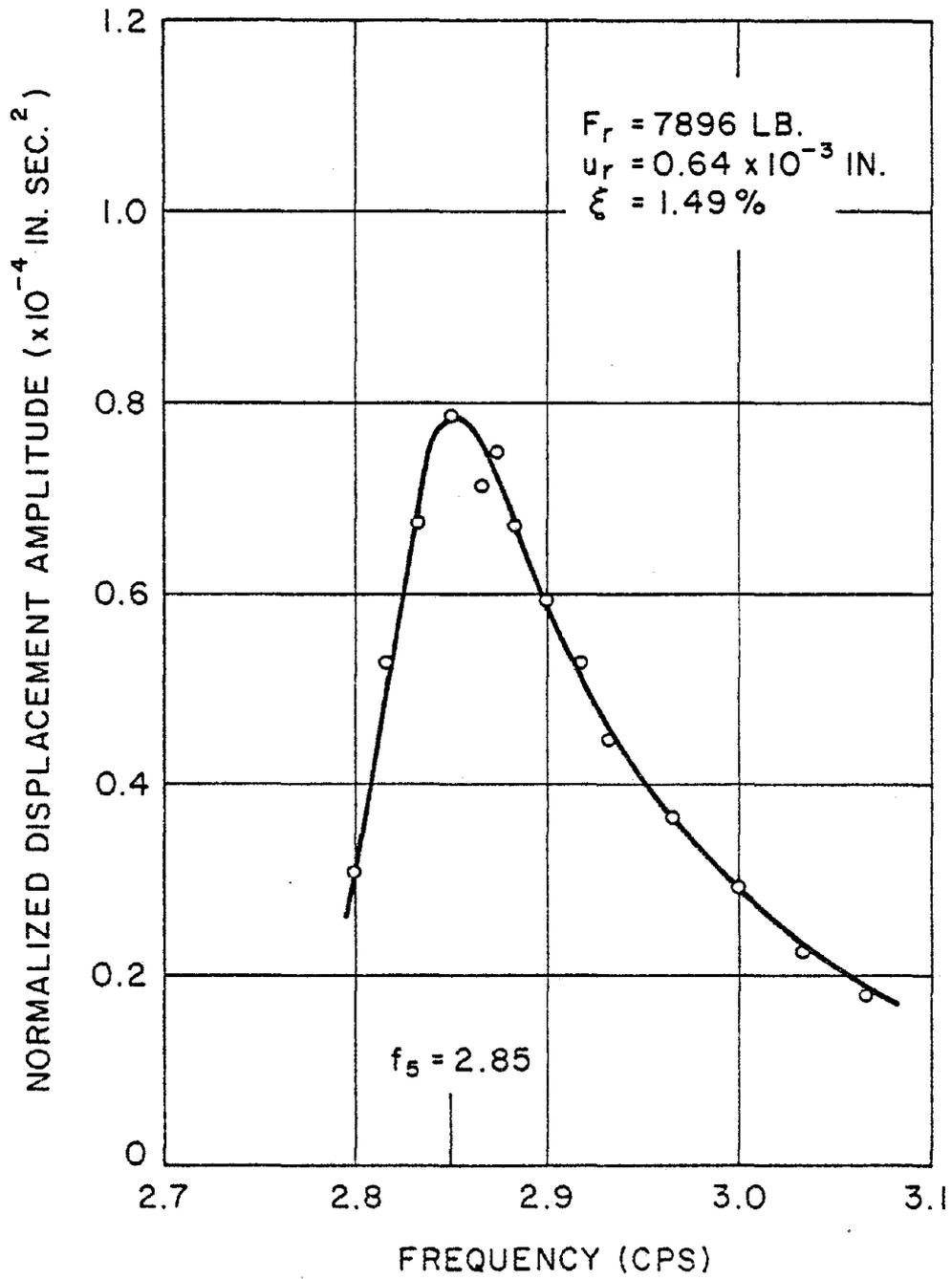


FIG. 3.13 FREQUENCY RESPONSE; FIFTH MODE N-S

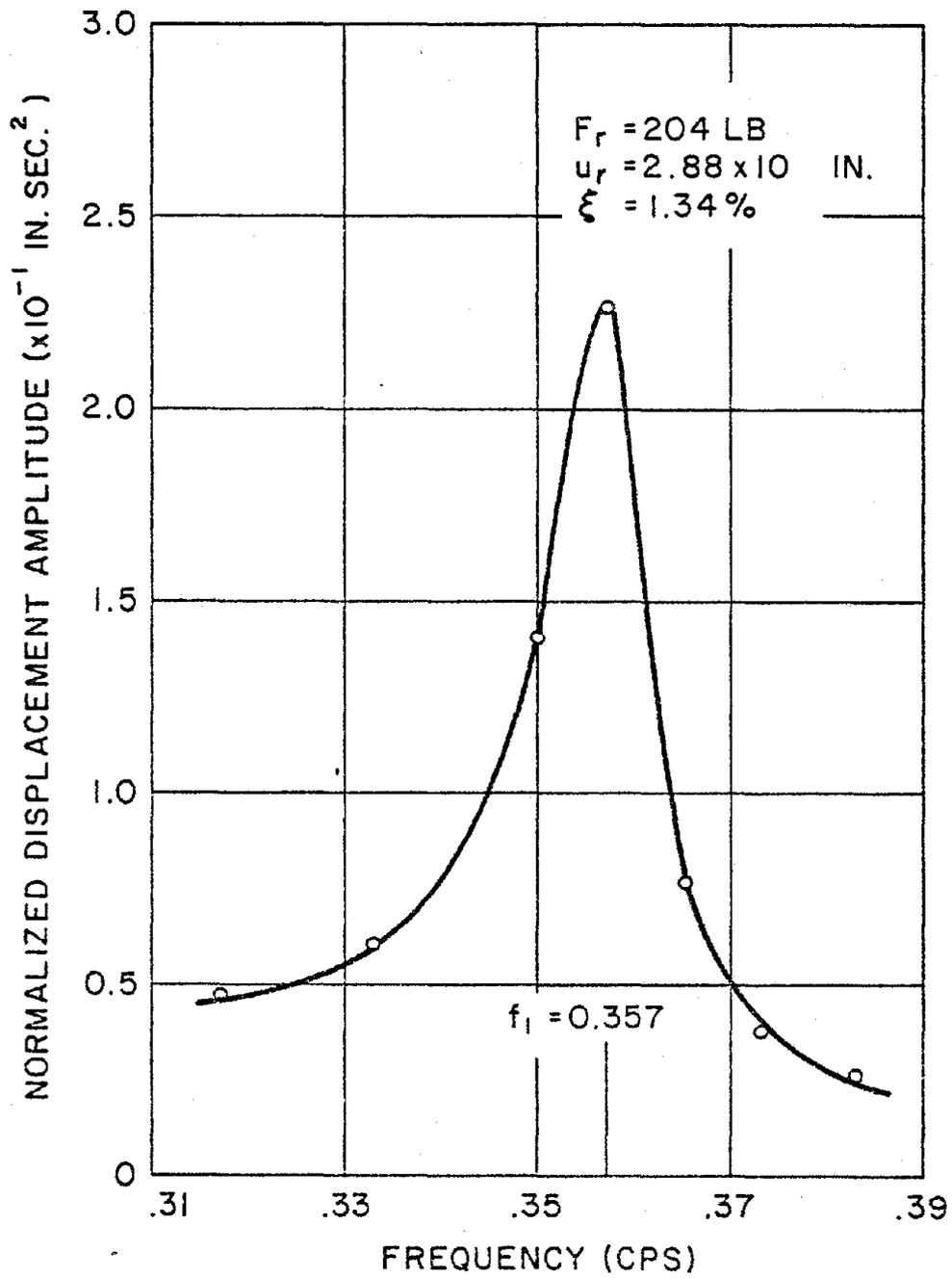


FIG. 3.14 FREQUENCY RESPONSE; FIRST TORSIONAL MODE

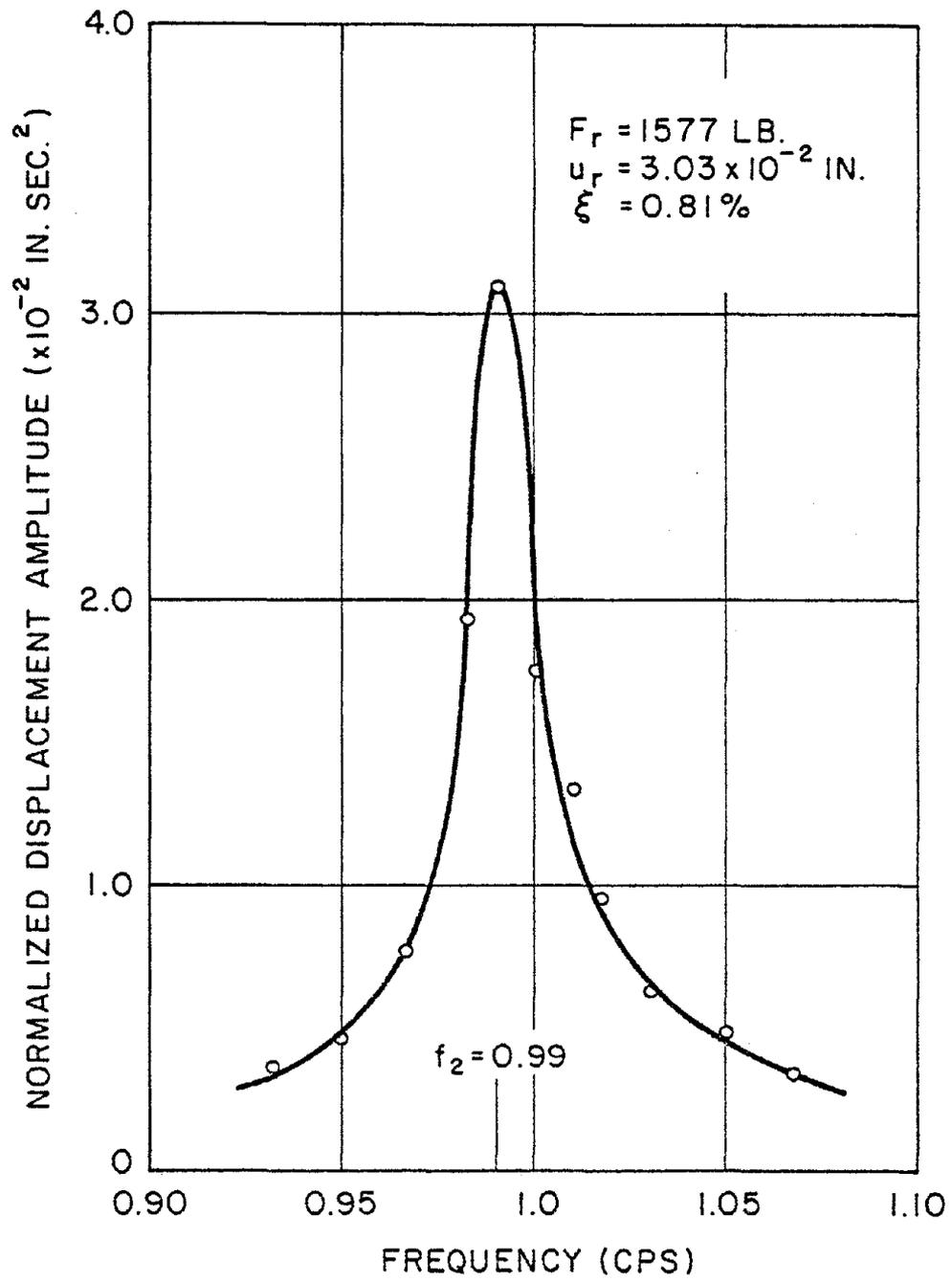


FIG. 3.15 FREQUENCY RESPONSE ; SECOND TORSIONAL MODE

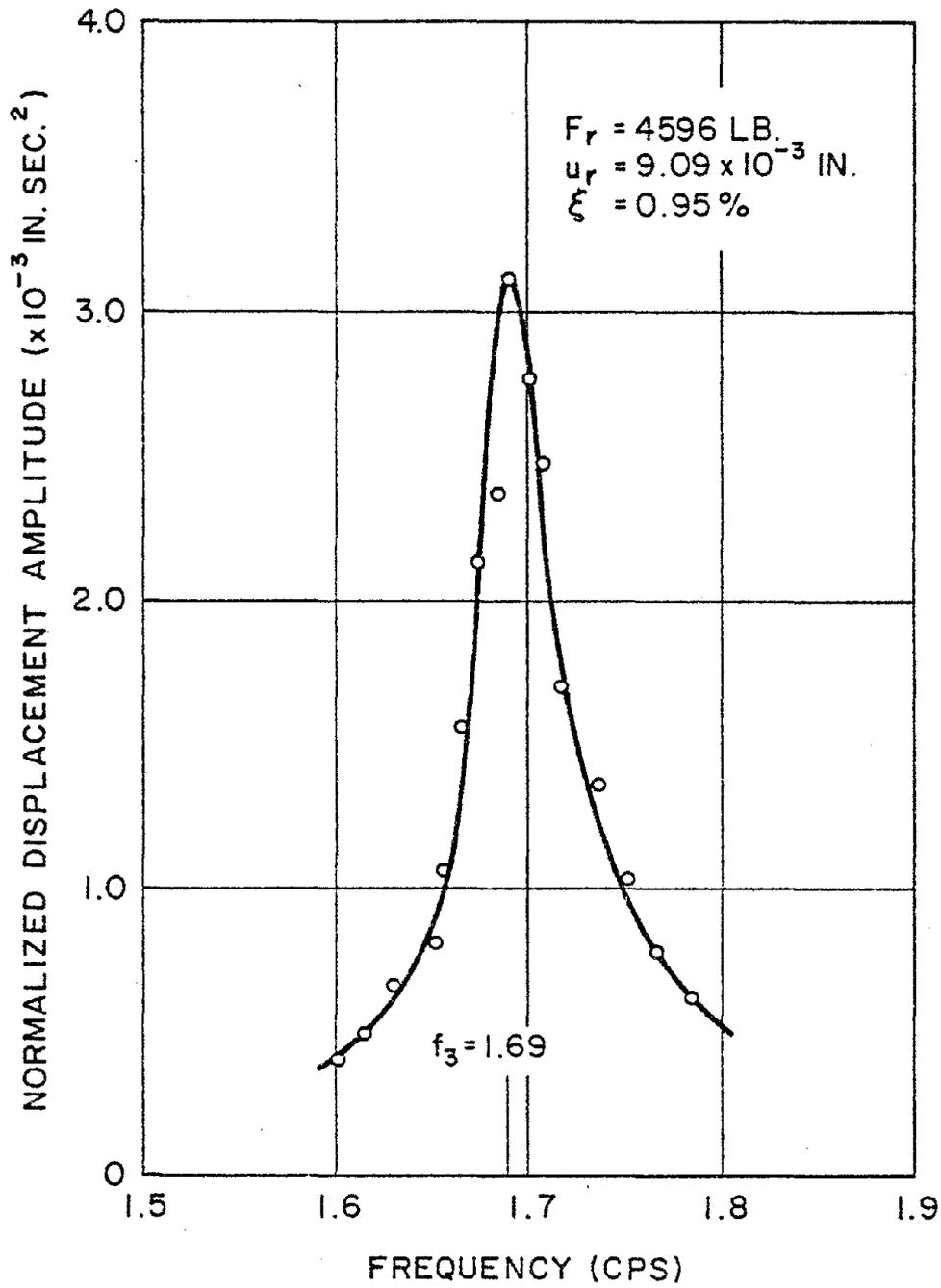


FIG. 3.16 FREQUENCY RESPONSE; THIRD TORSIONAL MODE

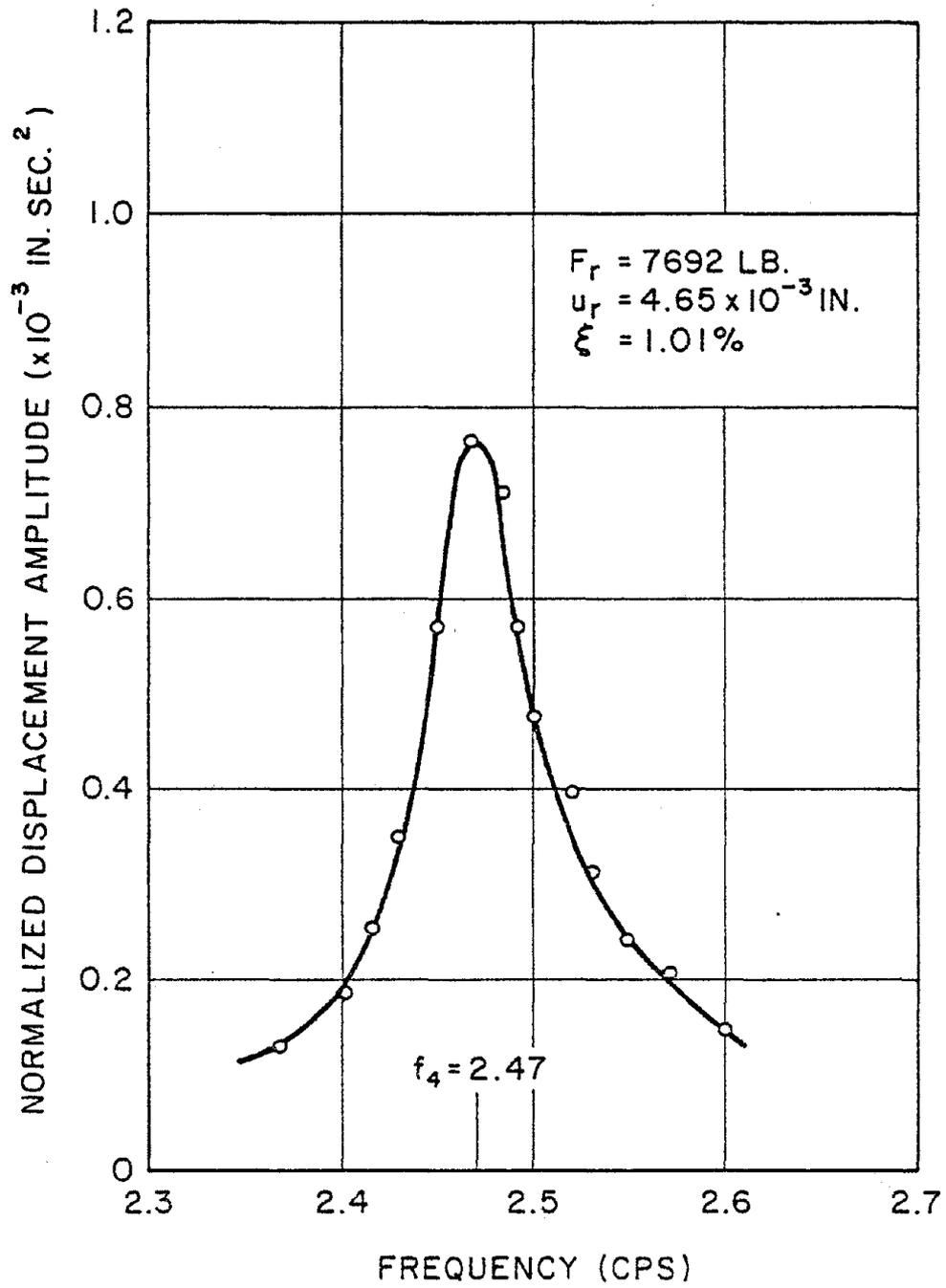


FIG. 3.17 FREQUENCY RESPONSE; FORTH TORSIONAL MODE

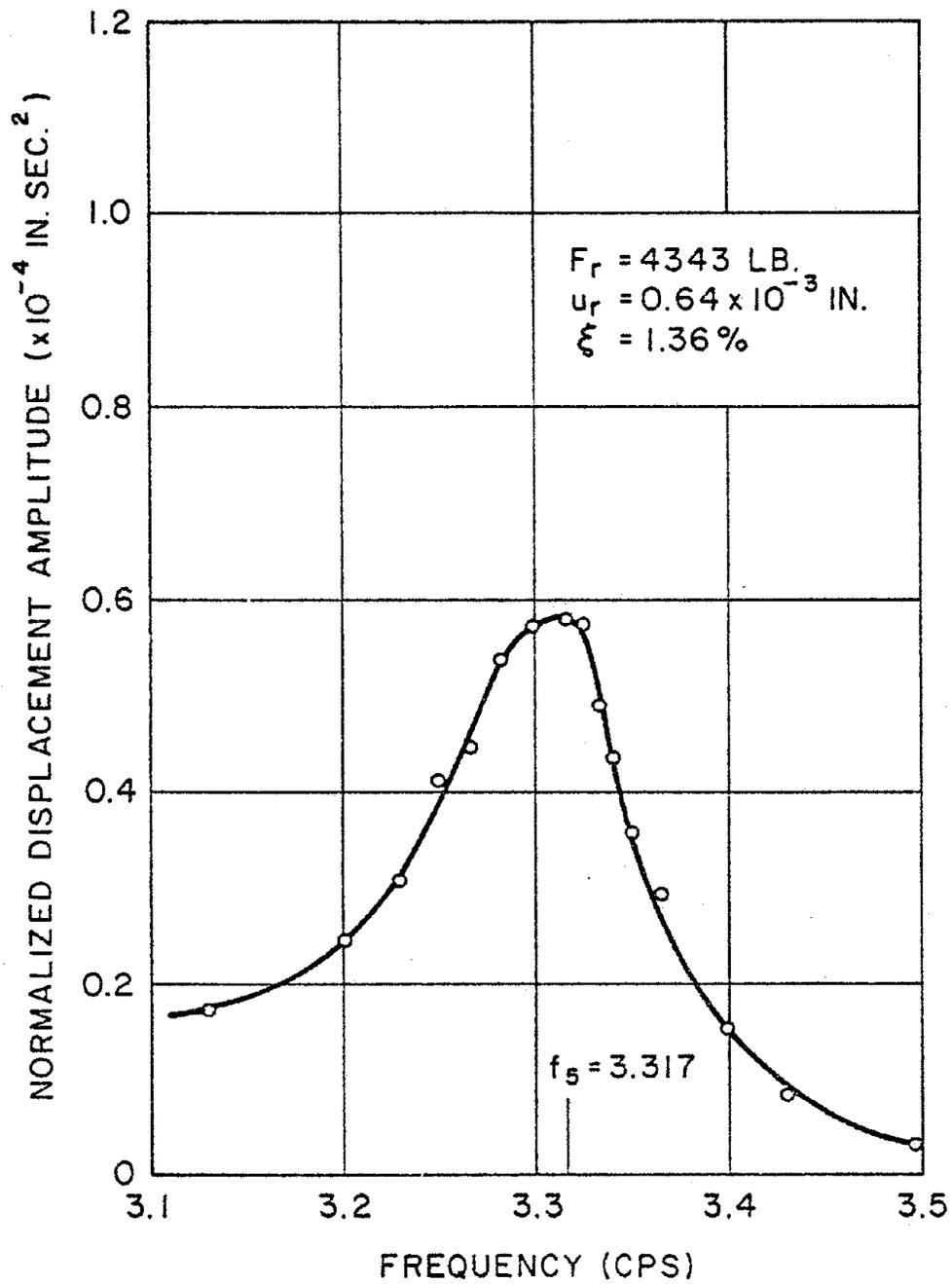


FIG. 3.18 FREQUENCY RESPONSE; FIFTH TORSIONAL MODE

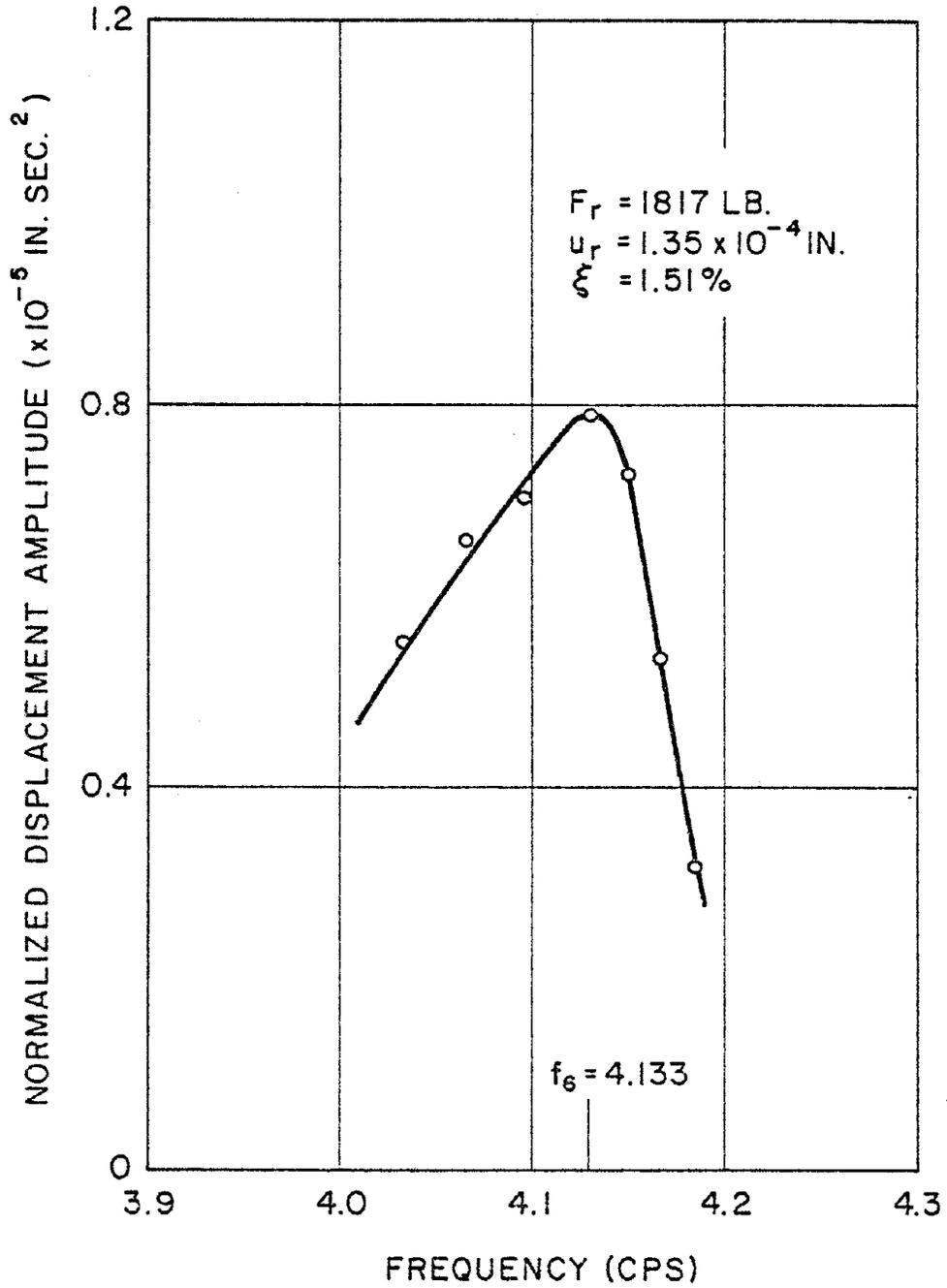


FIG. 3.19 FREQUENCY RESPONSE; SIXTH TORSIONAL MODE

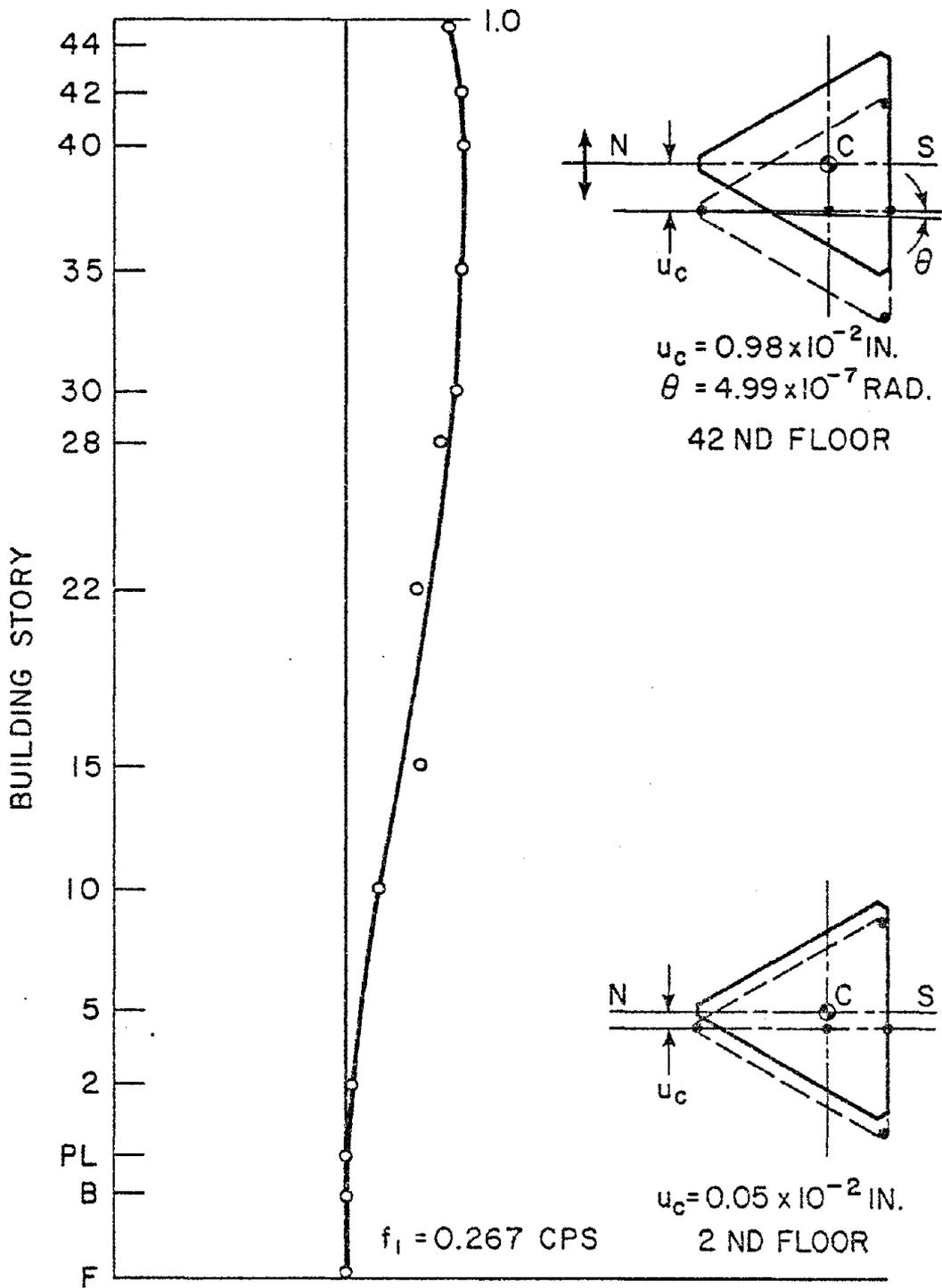


FIG. 3.20 MODE SHAPES, FIRST TRANSLATIONAL MODE E-W

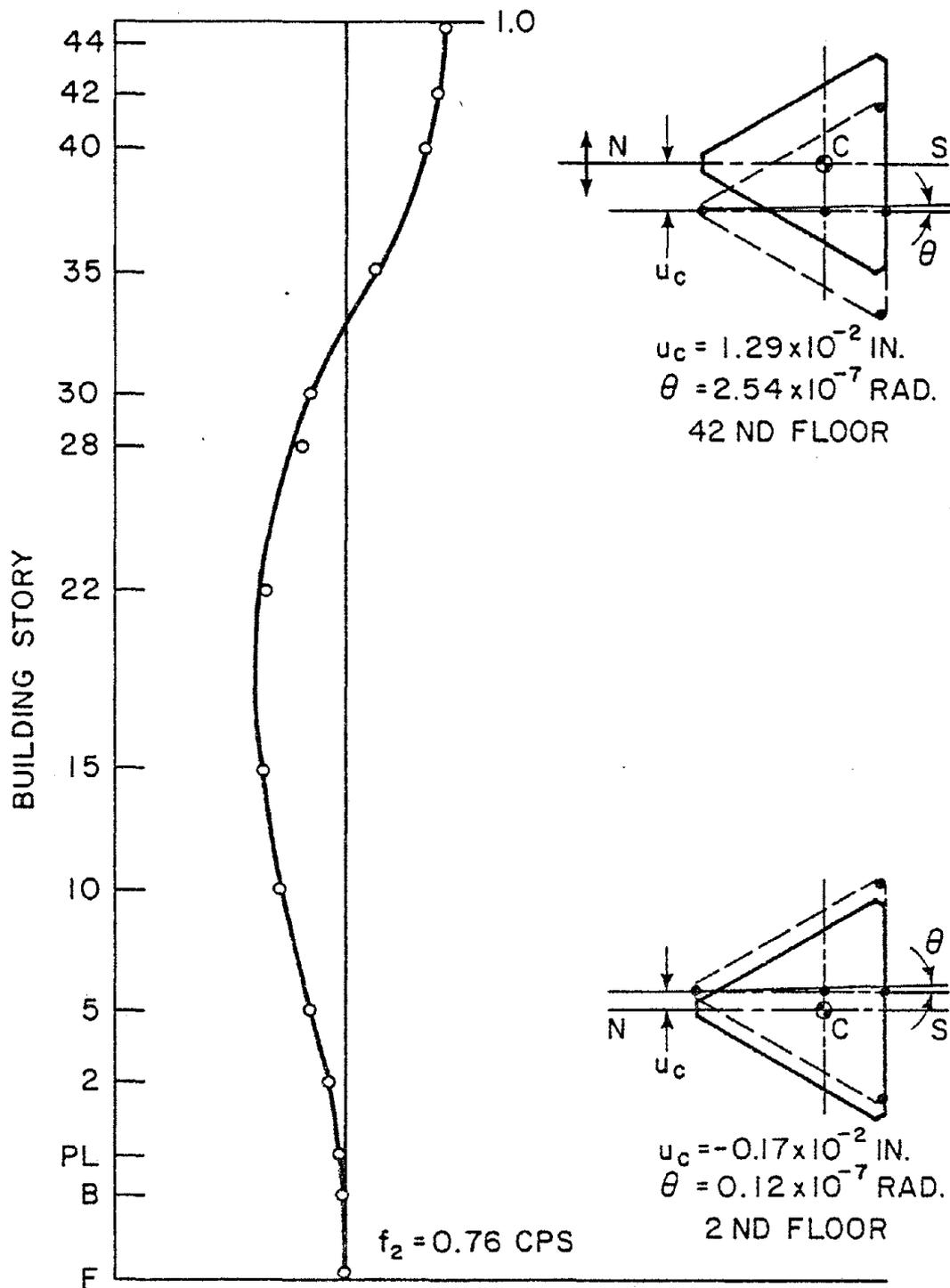


FIG. 3.21 MODE SHAPES, SECOND TRANSLATIONAL MODE E-W

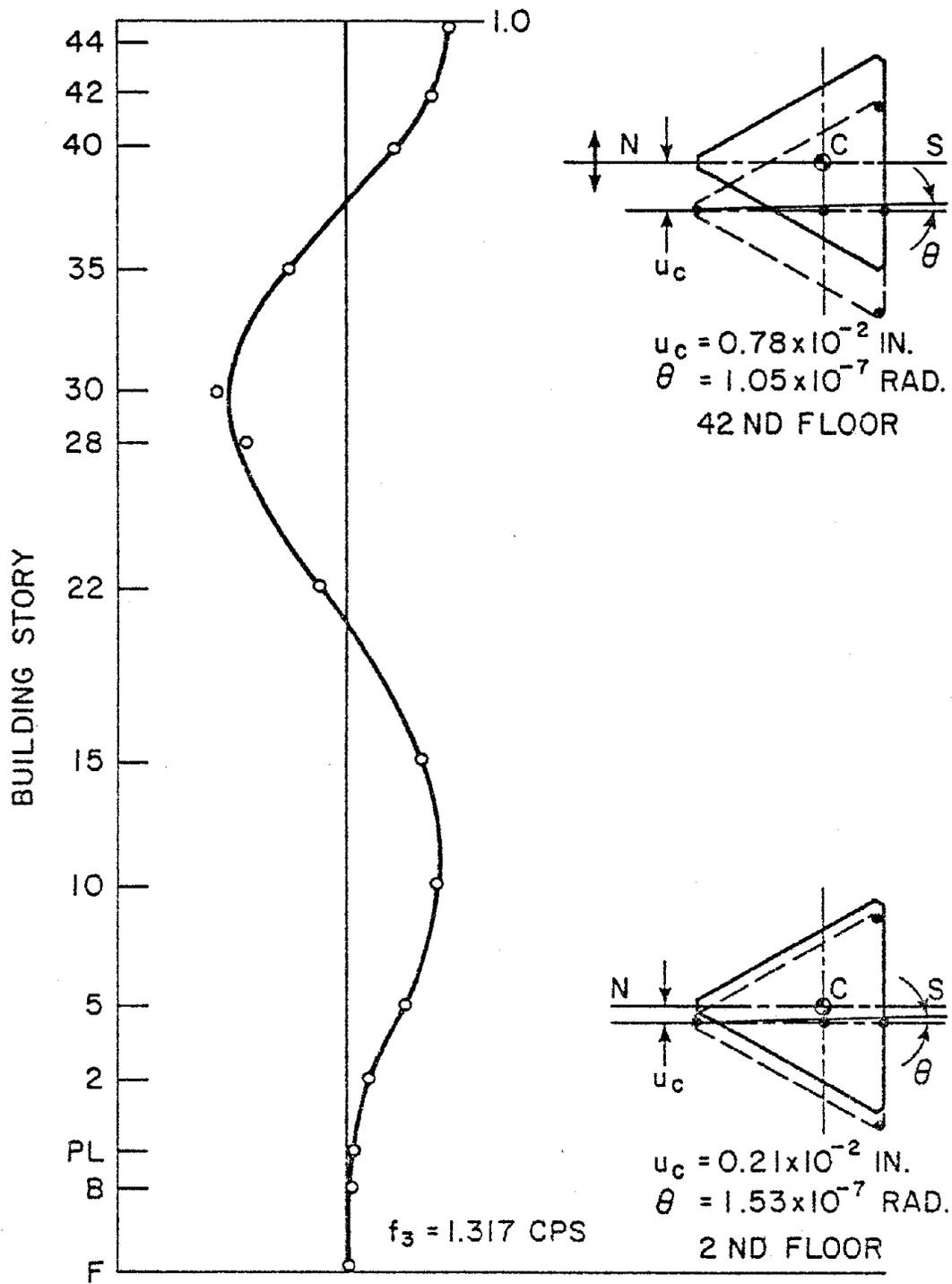


FIG. 3.22 MODE SHAPES, THIRD TRANSLATIONAL MODE E-W

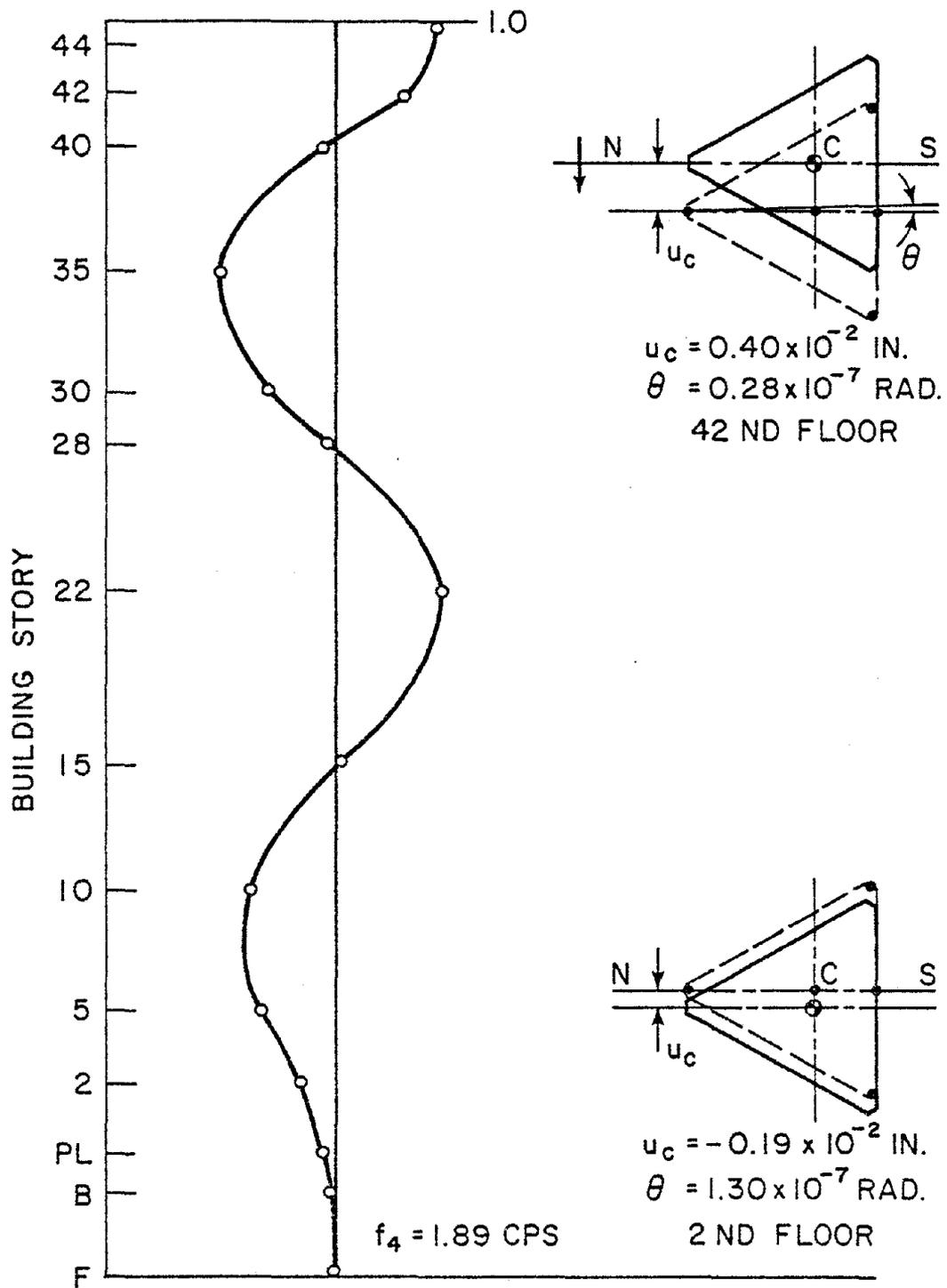


FIG. 3.23 MODE SHAPES, FOURTH TRANSLATIONAL MODE E-W

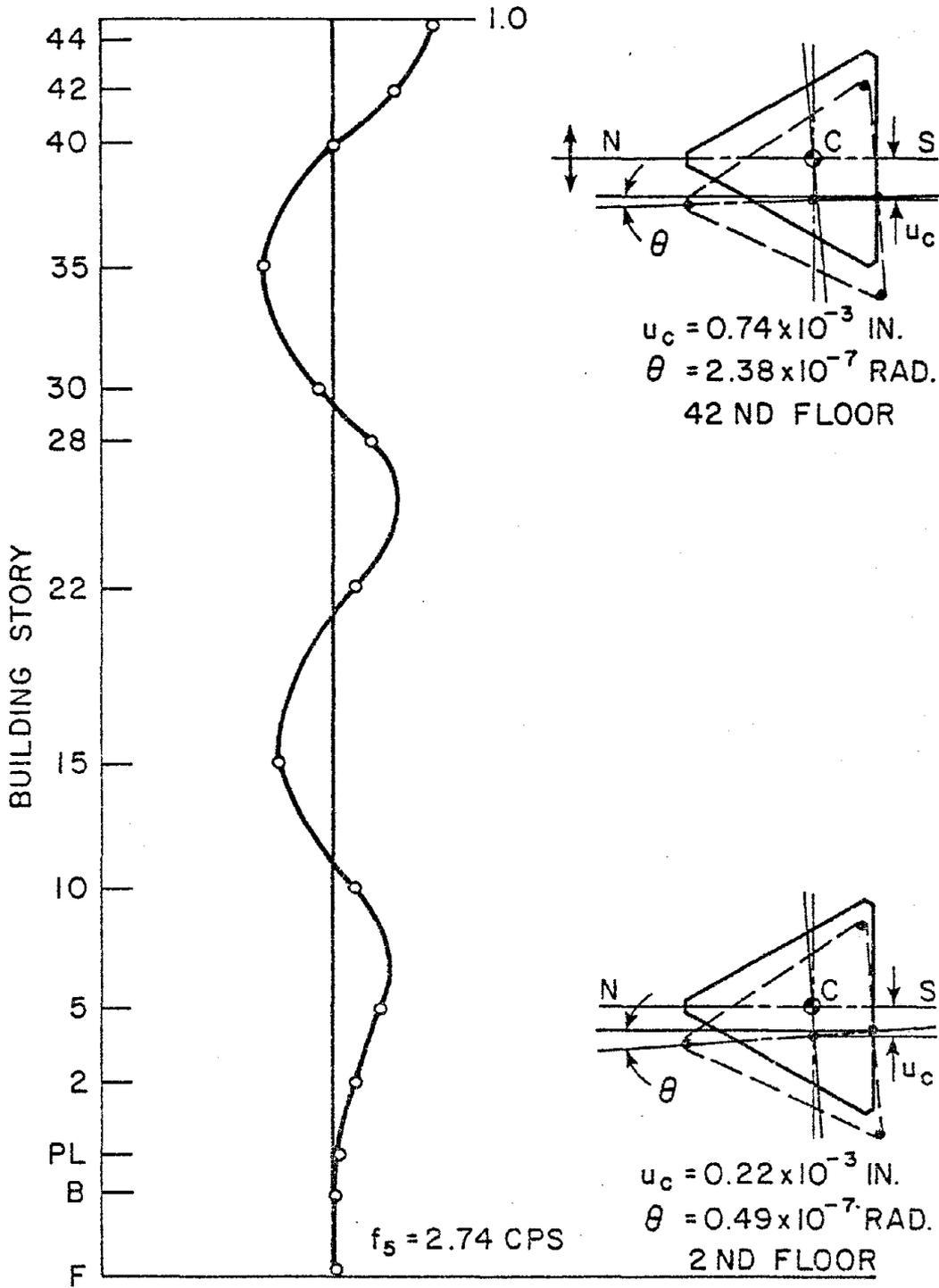


FIG. 3.24 MODE SHAPES, FIFTH TRANSLATIONAL MODE E-W

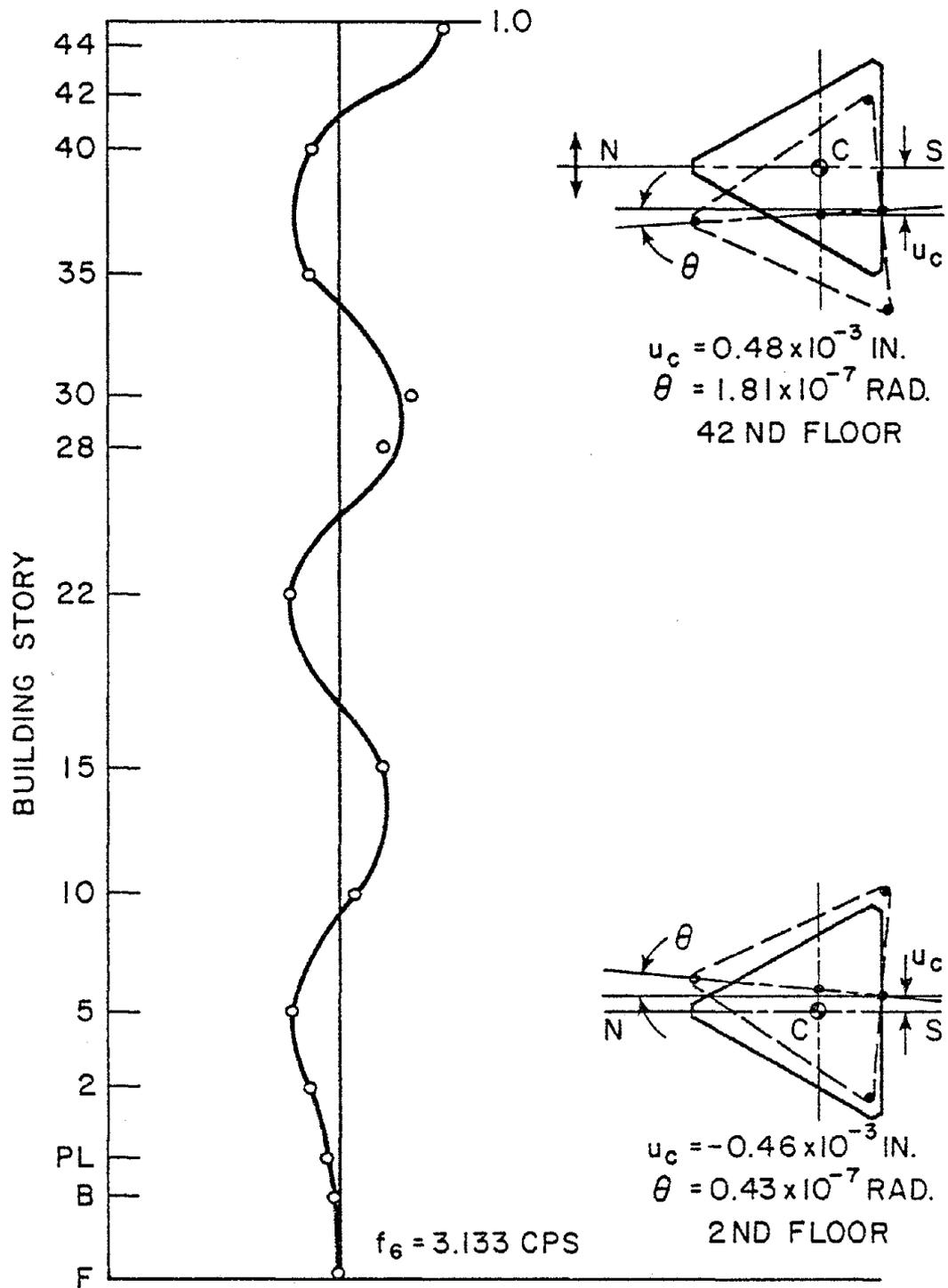


FIG. 3.25 MODE SHAPES, SIXTH TRANSLATIONAL MODE E-W

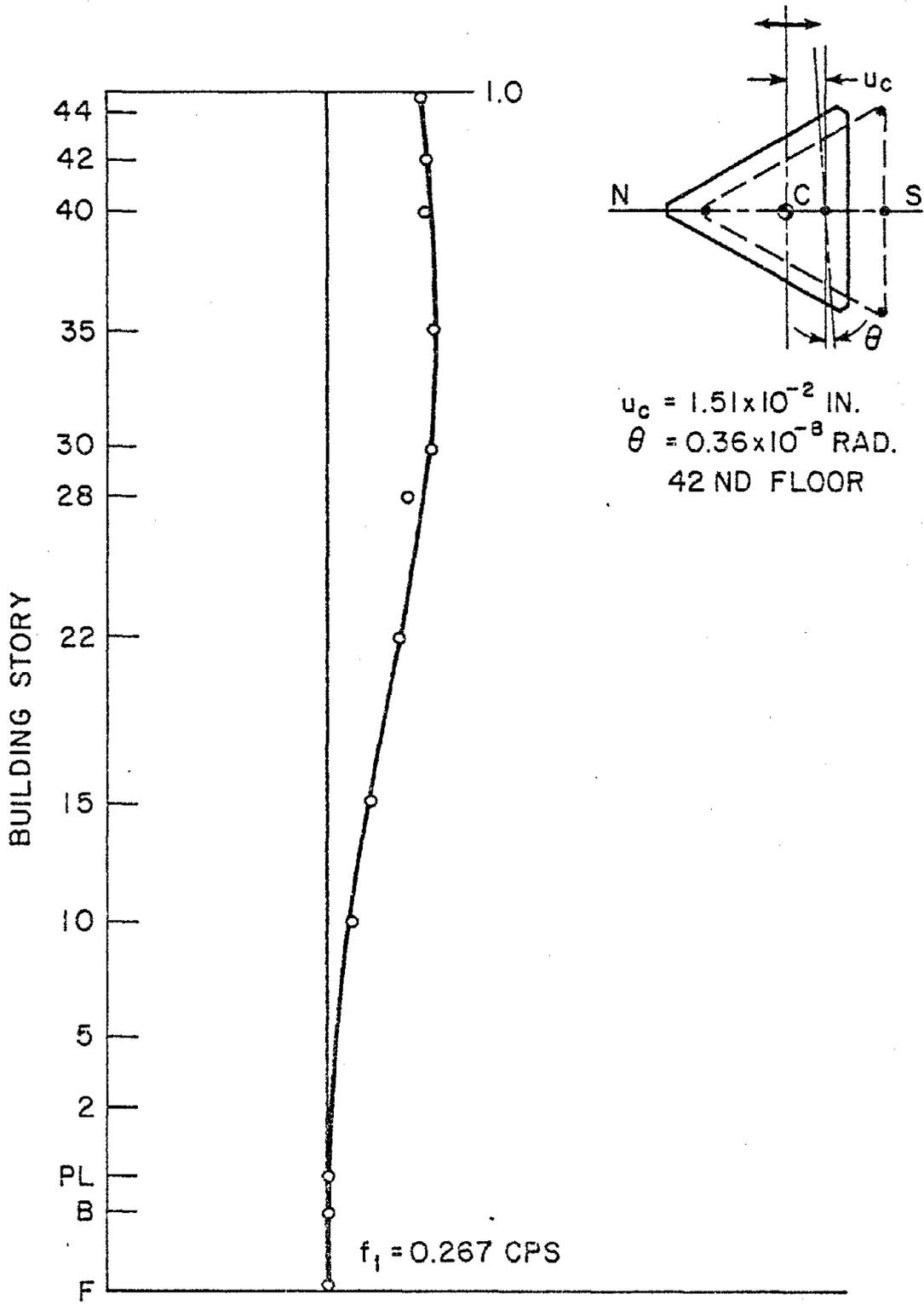


FIG. 3.26 MODE SHAPES, FIRST TRANSLATIONAL MODE N-S

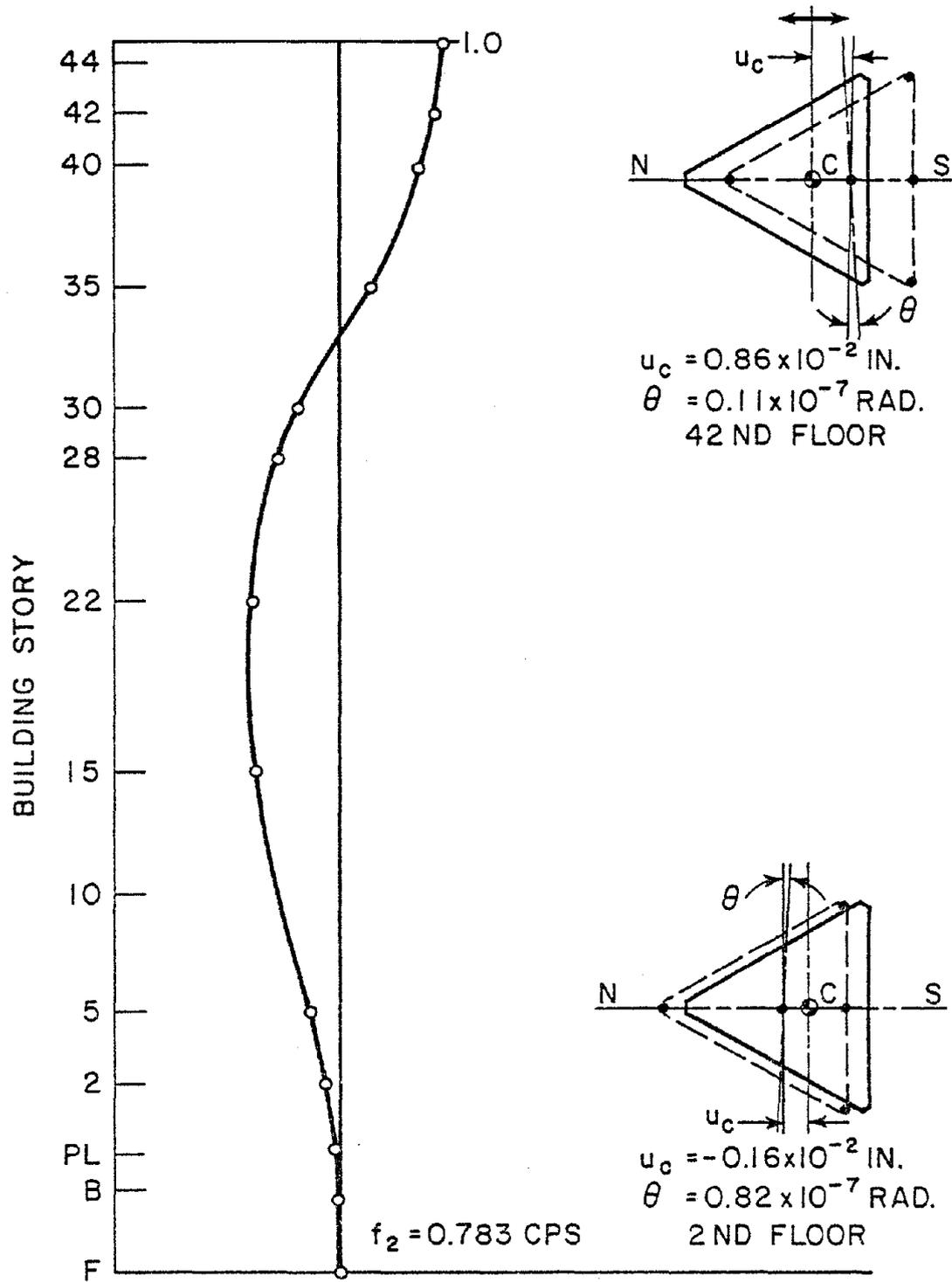


FIG. 3.27 MODE SHAPES, SECOND TRANSLATIONAL MODE N-S

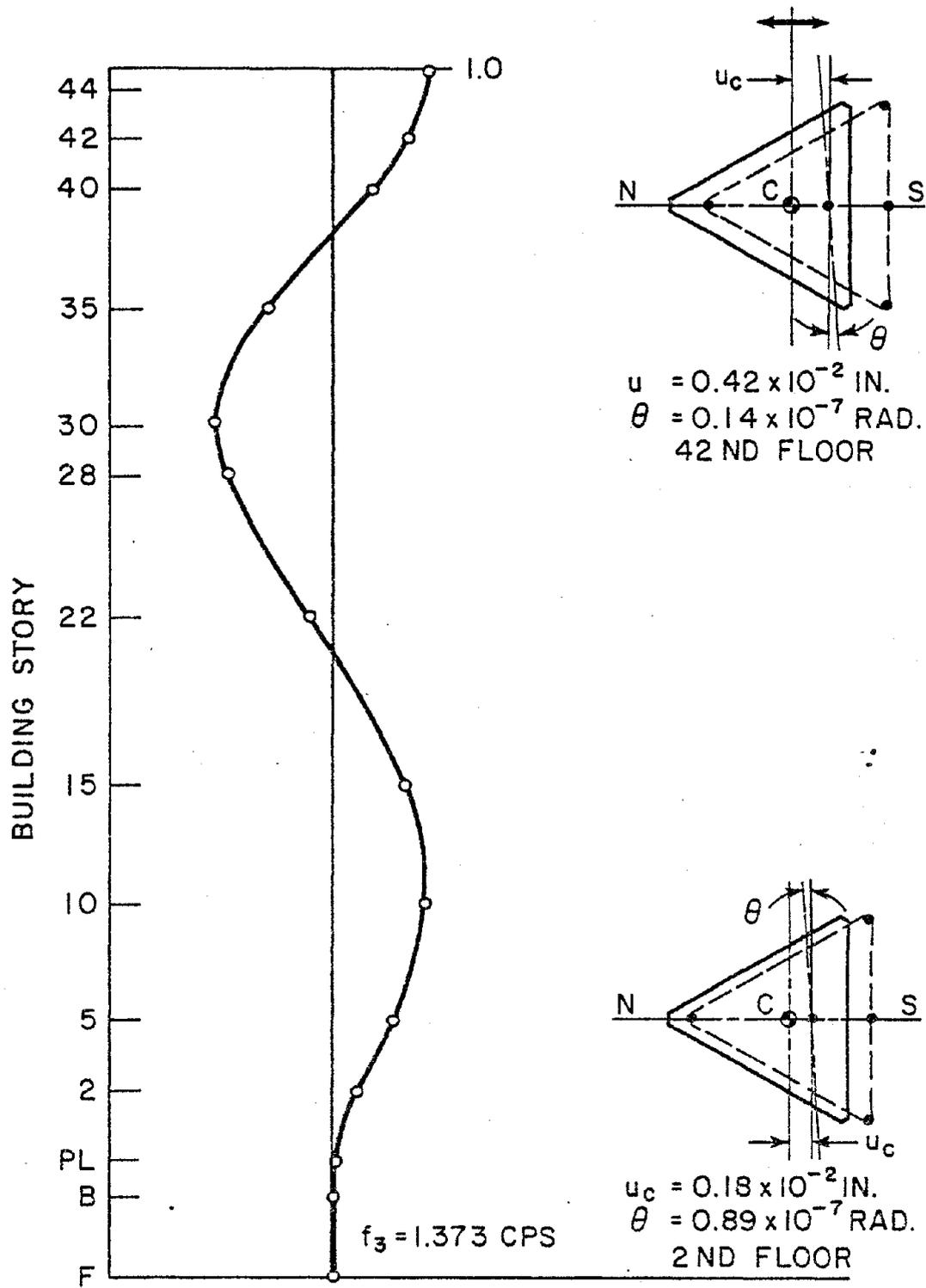


FIG. 3.28 MODE SHAPES, THIRD TRANSLATIONAL MODE N-S

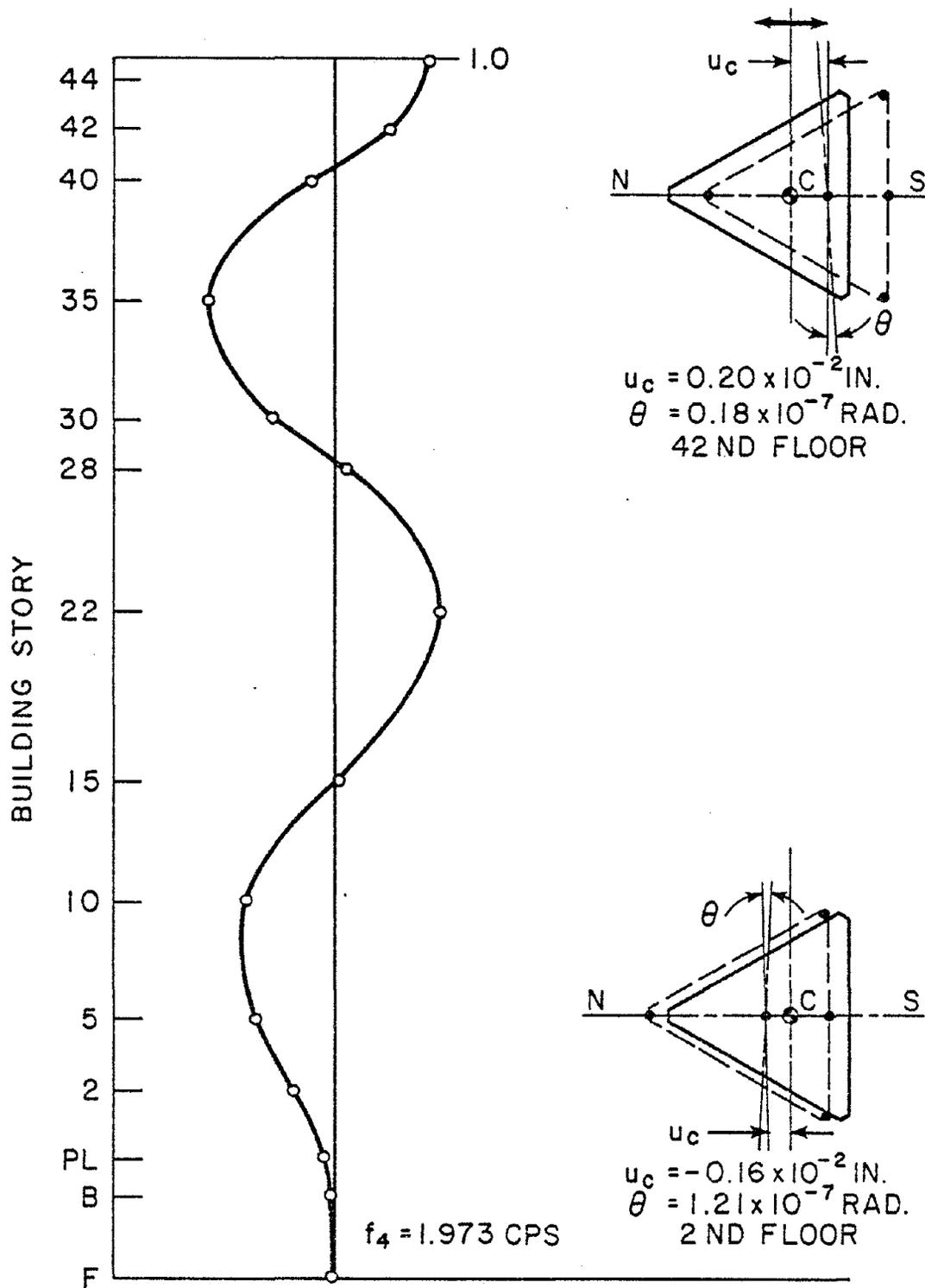


FIG. 3.29 MODE SHAPES, FOURTH TRANSLATIONAL MODE N-S

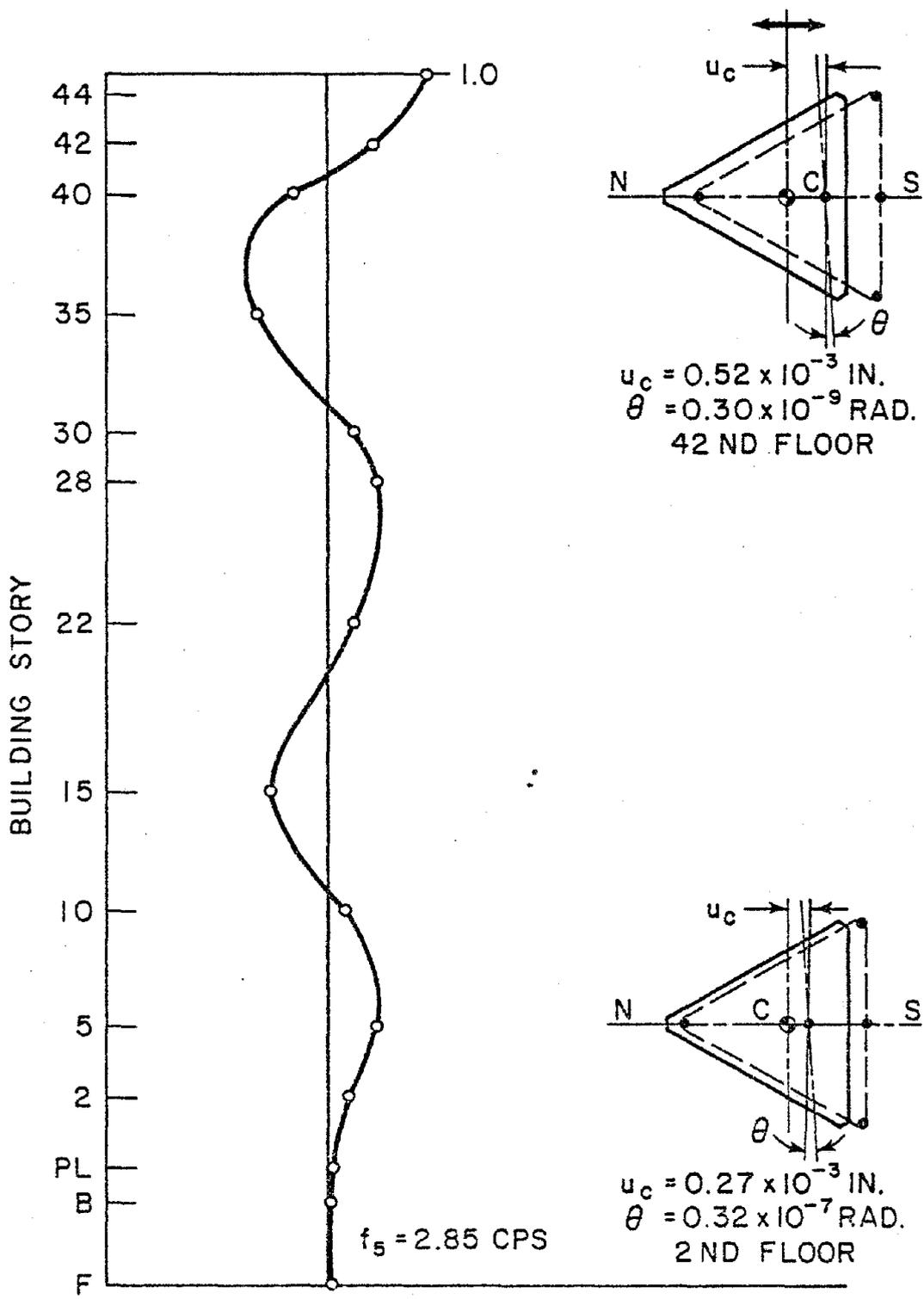


FIG. 3.30 MODE SHAPES, FIFTH TRANSLATIONAL MODE N-S

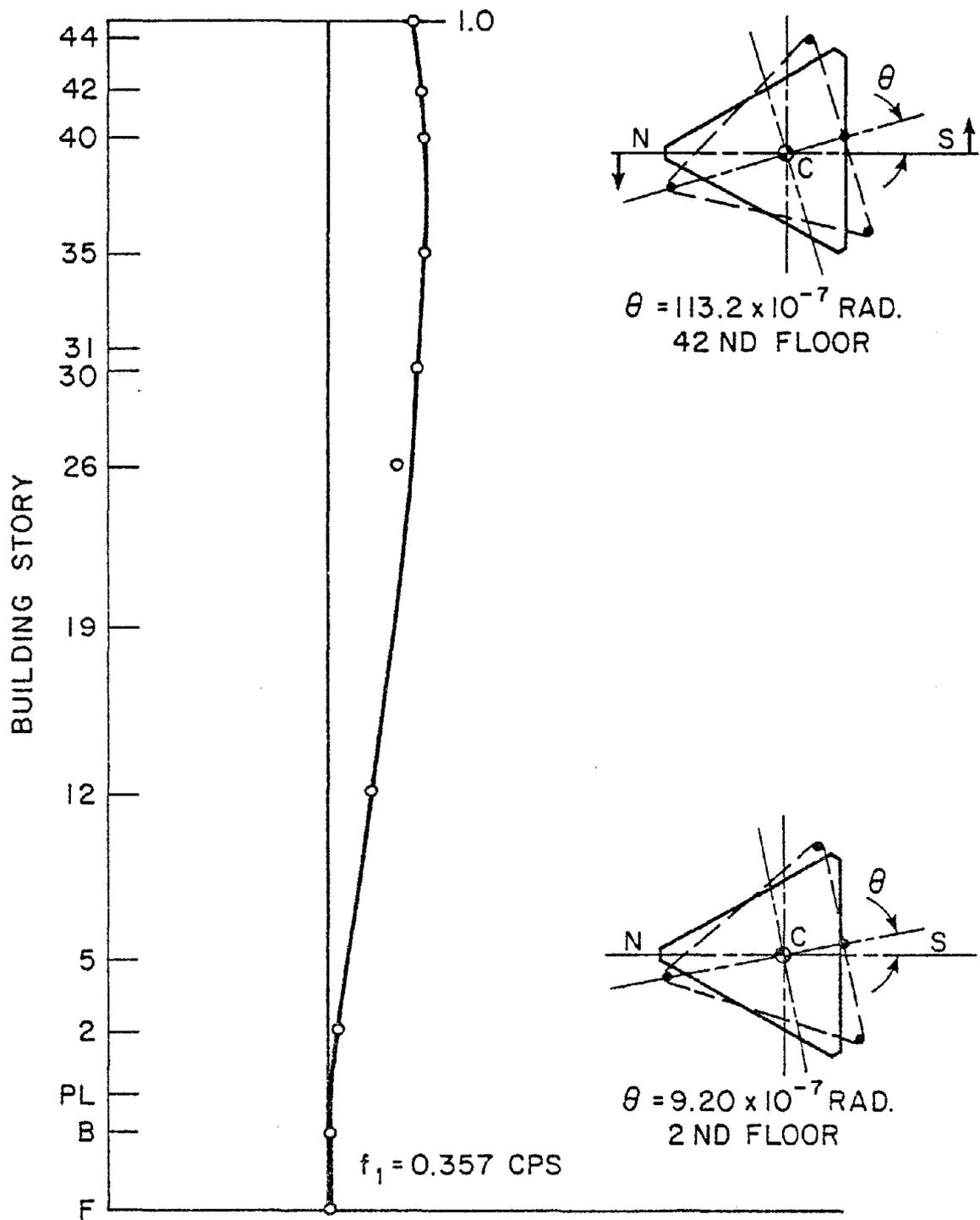


FIG. 3.31 MODE SHAPES, FIRST TORSIONAL MODE

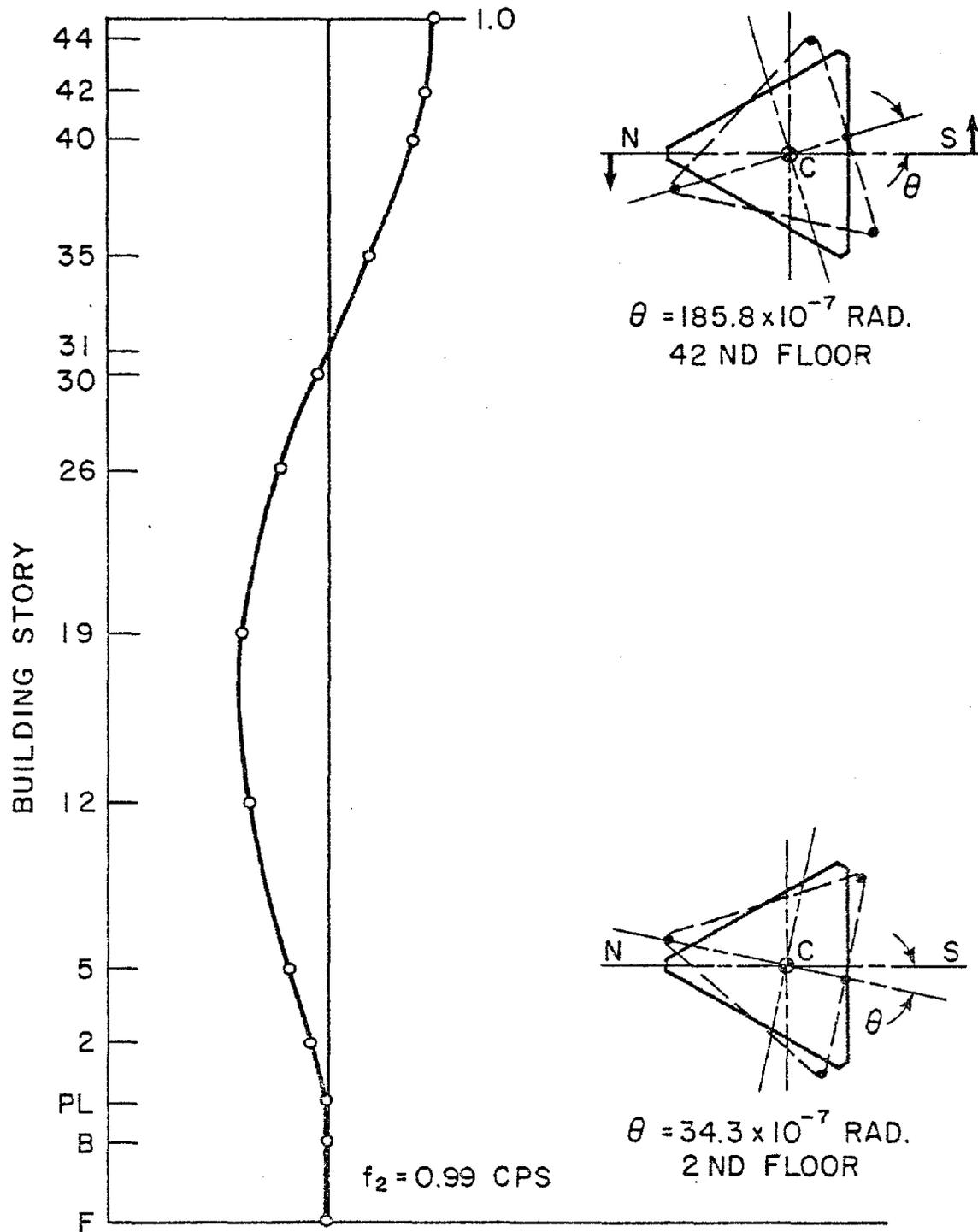


FIG. 3.32 MODE SHAPES, SECOND TORSIONAL MODE

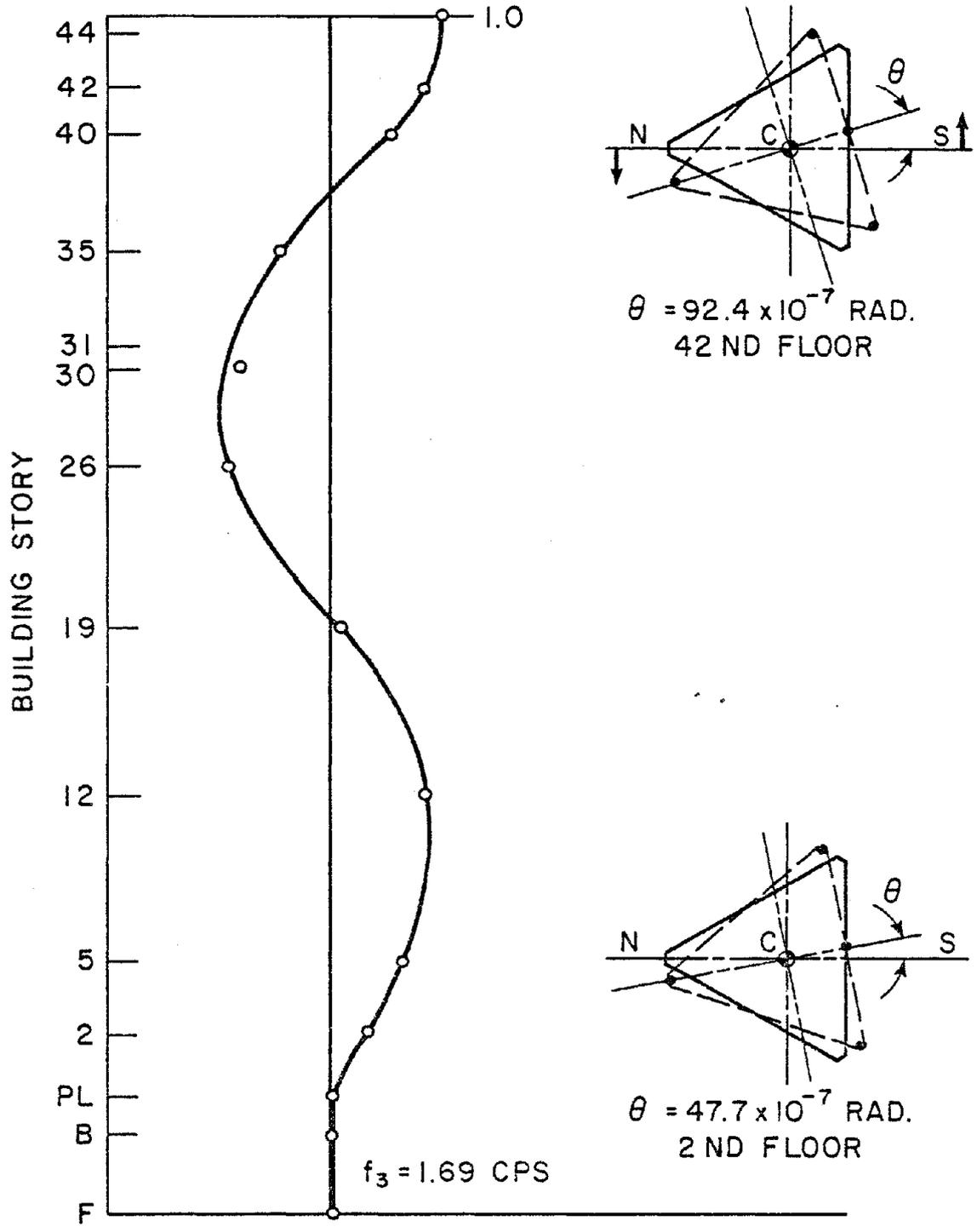


FIG. 3.33 MODE SHAPES, THIRD TORSIONAL MODE

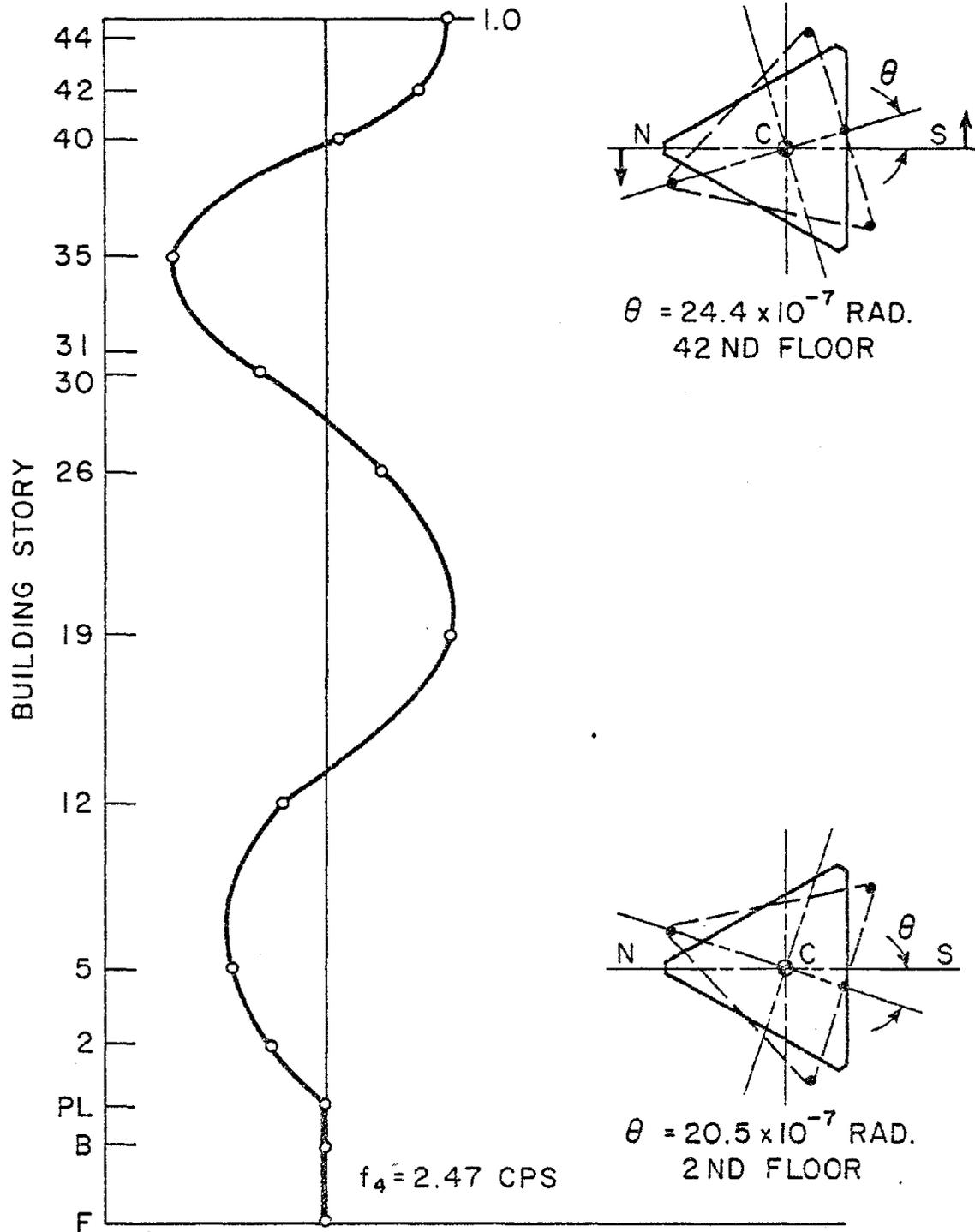


FIG. 3.34 MODE SHAPES, FOURTH TORSIONAL MODE

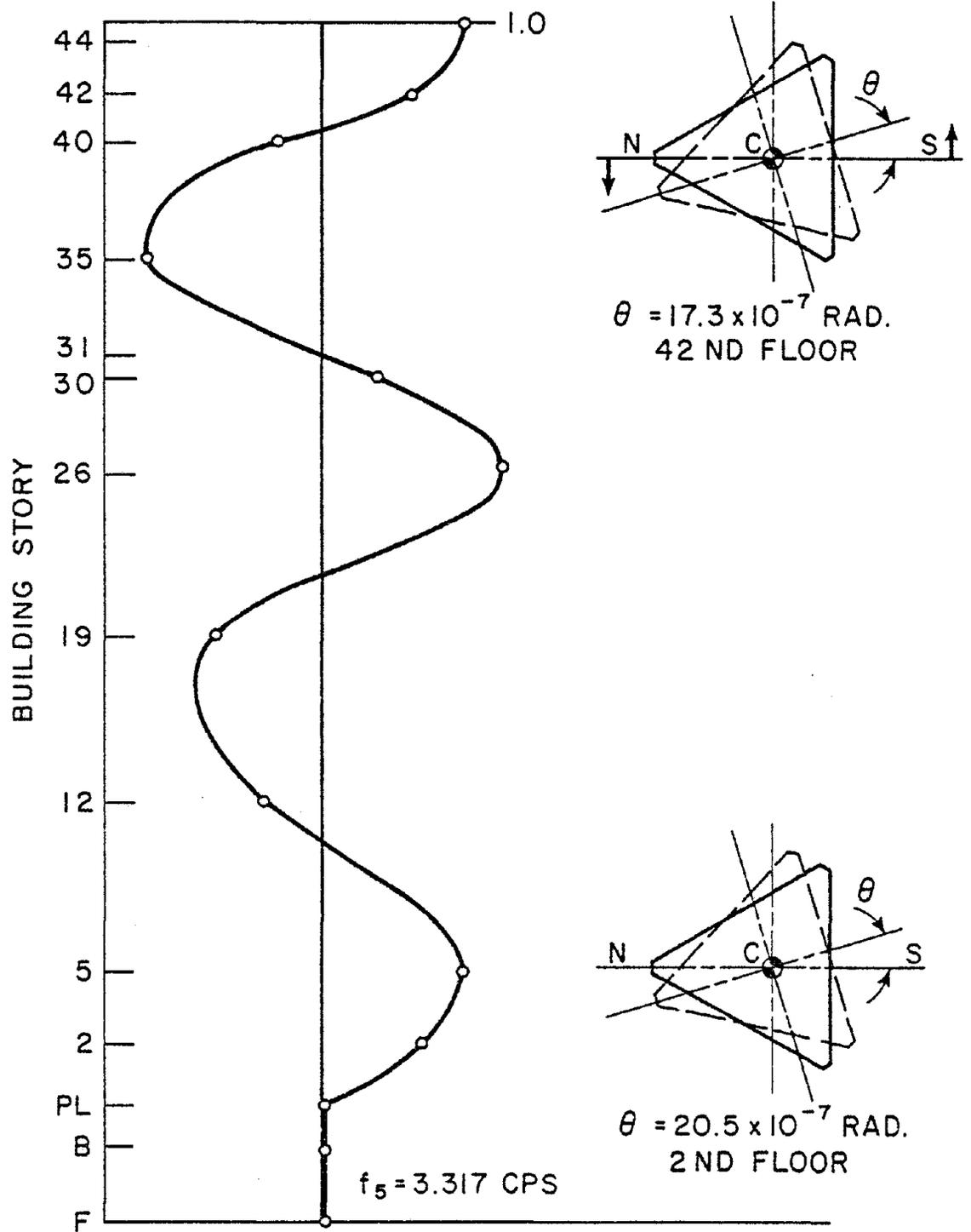


FIG. 3.35 MODE SHAPES, FIFTH TORSIONAL MODE

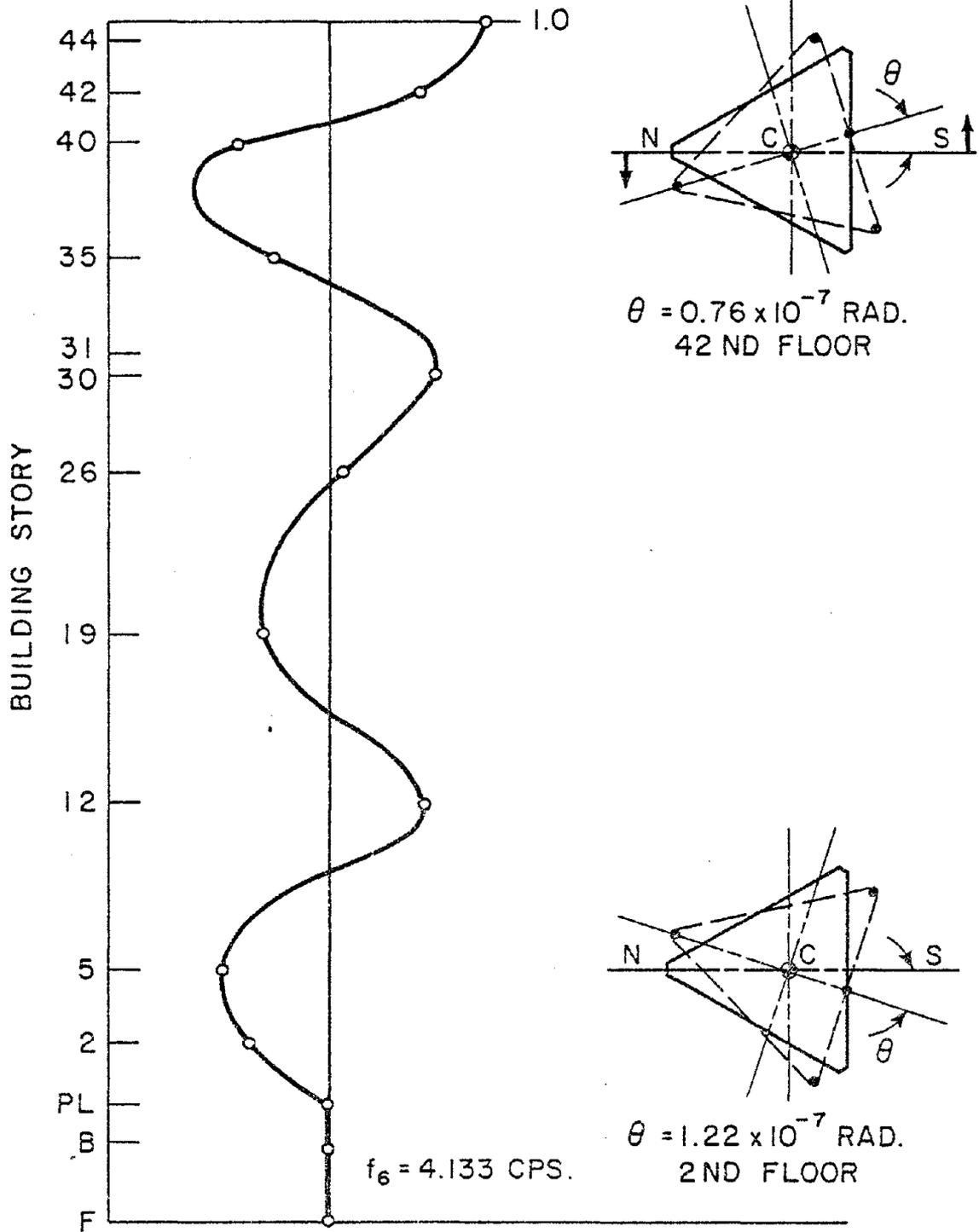


FIG. 3.36 MODE SHAPES, SIXTH TORSIONAL MODE

## 4. AMBIENT VIBRATION STUDY

### 4.1 General

In recent years a method for testing of full-scale structures based on wind and microtremor-induced vibrations has been developed. Although the method has been in use for almost 40 years by the United States Coast and Geodetic Survey (16) to measure fundamental periods of the buildings, it was not until recently that this was extended to higher modes (5, 8, 9, 13, 14, 17).

The ambient vibration study of the dynamic properties of the structures is a fast and relatively simple method of field measurements. It does not interfere with normal building function, and the measuring instruments and equipment can be installed and operated by a small crew.

The objective of performing the ambient vibration study was to obtain dynamic properties of the building, and then compare these results with those obtained from the forced vibration study to assess efficiency of both techniques.

The ambient vibration, experimental and analytical procedures were first suggested by Crawford and Ward (5,17). An assumption in the analysis technique is that the exciting forces are stationary random process, possessing reasonably flat frequency spectrum. For multistory buildings and other large above ground structures the largest ambient vibrations are produced by wind. If the frequency spectrum of the vibrational exciting forces is reasonably flat, a structure subjected to this input will respond in all its normal modes.

The ambient vibration study of the Century City South-Theme Tower was carried out on March 19, 1975. The building was practically in the same condition as it was during the performance of forced vibration

study in November 1974. Wind direction and velocity on the day of dynamic test measured at nearby Santa Monica airport are given in Table 4.1. The direction of the wind was almost constant at azimuth 200-230°, and velocity of 9-14 mph.

TABLE 4.1 WIND DIRECTION AND VELOCITY  
(at Santa Monica Airport, March 19, 1975)

Time	Wind Blowing	Direction (°)	Velocity (mph)
10:40		200	11.50
11:42		230	10.36
12:40		210	9.21
1:40		230	13.82

The vibration measuring equipment employed in the ambient vibration-dynamic test is described below. The general experimental procedures and procedures for data analyses applied are also described. Finally, the experimental results are presented and discussed.

## 4.2 Field Measurements

### 4.2.1 Measuring Equipment

The wind induced vibrations were measured using Kinometrics Ranger Seismometers, Model SS-1. The seismometer has a strong permanent magnet as the seismic inertial mass, moving within a stationary coil attached to the seismometer case. Small rod magnets at the periphery of the coil produce a reversed field which provides a destabilizing force to extend the natural period of the mass and its suspension.

The resulting seismometer frequency was 1 Hz. Damping was set at 0.7 critical. The output for a given velocity is a constant voltage at all frequencies greater than 1 Hz and falls off at 12 dB/octave for frequencies less than 1 Hz.

The Kinematics Signal Conditioner, Model SC-1 (Fig. 4.1) was used to amplify and control simultaneously four seismometer signals. The four input channels have isolated circuitry to integrate and to differentiate the amplified input signal. All outputs are simultaneously or independently available for recording. Each channel provides a nominal maximum gain of 100,000. An 18 dB/octave low pass filter is available with a cut-off frequency continuously selectable between 1 Hz and 100 Hz, for each channel.

The amplified analogue signals were recorded and directly converted to digital format using the Kinematics Digital Data System, Model DDS-1103. A direct recording oscillograph was provided to display and monitor the four signal levels during tape recordings (Fig. 4.1). The data was digitized at 10 samples per second. The DDS-1103's rate of scan across multiple input channels is 40,000 Hz. This rapid scan rate is sufficient to retain the phase relationship between channels.

#### 4.2.2 Measurement Procedures

When measuring ambient and forced vibrations of the buildings, it is usually assumed that the structure can be approximated by a one-dimensional, damped discrete or continuous system. In most of the cases (11, 13, 14) measurements indicate that for the level of excitations applied, floor structures are sufficiently stiff so that the above assumption is acceptable. In the case of Century City South-Theme Tower, it is assumed that the structural behavior may be approximated

by a linear one-dimensional model.

In the experimental study of building vibration which is based on the linear model, it is assumed that the resulting motions can be expressed as the superposition of modes associated with the discrete frequencies (2,3). This approach then requires a simultaneous measurement of motion in a given direction at at least two different floors to obtain their relative amplitude and phase, the two quantities needed to determine mode shapes. During the measurements of wind induced vibrations, it is not necessary to find the actual amplitudes that are recorded, because all that is ever used in determining mode shapes is the relative amplitude of the same two instruments.

The first field measurements was a calibration run at the 42nd floor. All four seismometers were at the center of the floor oriented South to record in parallel the identical structure motion. This measurement provides a relative amplitude and phase calibration between channels which includes the entire seismometer, amplifier, filter, tape recorder and analogue-to-digital conversion system. To obtain translational mode frequencies two pair of seismometers were located at the floor center oriented in South and West direction, respectively, and recorded the vibrations for 840 seconds (Fig. 4.2a and Table 4.2). To obtain information about the torsional frequencies the experiment was conducted at the 42nd floor with two pairs of seismometers each of them located at the south end and the floor center, oriented parallel in South direction. The recording in this test lasted 300 seconds (Fig. 4.2 b and Table 4.2).

TABLE 4.2 LOCATION OF SEISMOMETERS

Run No.	Excitation Measured	Duration (Sec)	Floor, Direction			
			Seis. No.1	Seis. No.2	Seis. No.3	Seis. No.4
1	CAL	20	42S	42S	42S	42S
2	T*	840	42S	42W	42S	42W
3	$\theta$ *	300	42W	42W	42W	42W
4	T	60	42S	42W	RS	RW
5	$\theta$	60	42W	42W	RW	RW
6	T	60	42S	42W	40S	40W
7	$\theta$	60	42W	42W	40W	40W
8	T	60	42S	42W	35S	35W
9	$\theta$	60	42W	42W	35W	35W
10	T	60	42S	42W	30S	30W
11	$\theta$	60	42W	42W	30W	30W
12	T	60	42S	42W	28S	28W
13	$\theta$	60	42W	42W	28W	28W
14	T	60	42S	42W	25S	25W
15	$\theta$	60	42W	42W	25W	25W

\* T = translation

$\theta$  = torsion

TABLE 4.2 LOCATION OF SEISMOMETERS  
(Continued)

Run No.	Excitation Measured	Duration (Sec)	Floor, Direction			
			Seis. No.1	Seis. No.2	Seis. No.3	Seis. No.4
16	T	60	42S	42W	22S	22W
17	⊖	60	42W	42W	22W	22W
18	T	60	42S	42W	20S	20W
19	⊖	60	42W	42W	20W	20W
20	T	60	42S	42W	15S	15W
21	⊖	60	42W	42W	15W	15W
22	T	60	42S	42W	10S	10W
23	⊖	60	42W	42W	10W	10W
24	T	60	42S	42W	5S	5W
25	⊖	60	42W	42W	5W	5W
26	T	60	42S	42W	2S	2W
27	⊖	60	42W	42W	2W	2W
28	T	60	42S	42W	PS	PW
29	⊖	60	42W	42W	PW	PW
30	CAL	120	42S	42S	42S	42S

For measurement of the translational modes it would be ideal to place the seismometers in the center of torsion. This was available because the center of stiffnesses and masses coincide with the floor center, which was already established from the previous forced vibration test. Thus, for measurement of the translational modes two of the seismometers remained at the 42nd floor placed in the floor center and oriented South and West, respectively. The other two seismometers were oriented in the same way and relocated in approximately five floor increments for simultaneous measurements of motion along the height of the building (Fig. 4.3 a and Table 4.2). For the torsional modes one of the seismometers on the 42nd floor remained at the floor center, and the other one was moved at the South end position, both oriented parallel to the West direction. The other two seismometers were oriented in the same way and relocated along the height of the building like in the translational modes (Fig. 4.3 b and Table 4.2). Each mode shape run was recorded continuously for 60 seconds. The low pass filter was set on each channel at 5Hz to attenuate all higher frequencies, thus completely removing electrical noise and other possible high frequency vibrations. The maximum system output was at 0.6 Hz frequency, and is normalized to a relative gain of 1.0 at this frequency. The voltage output to the recorder was adjusted to not exceed about  $\pm 1.4$  volts. This resulted in a system sensitivity of 12 volts/in/sec<sup>2</sup> for most of the measurements. Corresponding first mode acceleration and displacement were about  $\pm .03 \times 10^{-5} g$  and  $\pm 4 \times 10^{-5}$  inches respectively.

#### 4.3 Data Analysis

##### 4.3.1 Fourier Analysis

It is convenient to use Fourier transforms to analyze low

level structural vibrations. They may be used to exhibit the frequency content of the recorded vibration, thus identifying modal frequencies when the input force frequency spectrum is reasonably flat. Comparing measured amplitude and phase between various points on the structure provides an estimate of the mode shape.

A measured time-series signal  $x(t)$ , can be transformed to the frequency domain, with certain restrictions using the integral

$$X(f) = \int_{-\infty}^{\infty} x(t)e^{-2\pi ift} dt \quad (4-1)$$

where  $X(f)$  represents the frequency domain function,  $f$  is frequency and  $i = \sqrt{-1}$ .

The time-series  $x(t)$  can be recovered by the inverse transformation

$$x(t) = \int_{-\infty}^{\infty} X(f)e^{2\pi ift} df \quad (4-2)$$

Equations 4-1 and 4-2 may be expressed in functional notation as

$$X(f) = F [x(t)] \quad (4-3)$$

$$x(t) = F^{-1} [X(f)] \quad (4-4)$$

Equation 4-3 is the direct transform and Equation 4-4 is the inverse transform. Together they are called a Fourier Transform Pair. The direct transform maps a time-series (time domain) into a function of  $f$  (frequency domain). The inverse transform reverses the process.  $X(f)$  is a complex number with both amplitude and phase.

$|X(f)|$  is known as the amplitude spectrum of  $x(t)$ . The function  $|X(f)|^2$  is known as the power spectrum of  $f(t)$ .

Consider the elastic structure representing a multistory building. The set of time-series  $x_1(t), x_2(t), \dots, x_i(t) \dots, x_n(t)$  recorded for corresponding floor level are transformed to the frequency domain,

$$\begin{aligned}
 X_1(f) &= F [x_1(t)] \\
 X_2(f) &= F [x_2(t)] \\
 &\cdot \quad \cdot \\
 &\cdot \quad \cdot \\
 &\cdot \quad \cdot \\
 X_i(f) &= F [x_i(t)] \\
 &\cdot \quad \cdot \\
 &\cdot \quad \cdot \\
 &\cdot \quad \cdot \\
 X_n(f) &= F [x_n(t)]
 \end{aligned}
 \tag{4-5}$$

Modal frequencies of the structure appear as peaks in the plots of amplitude spectra  $|X_n(f)|$ . The  $i$ 'th mode shape coefficient  $a_{ij}$  at each natural frequency  $f_j$ , normalized to the value at coordinate 1, is simply:

$$a_{ij} = \frac{|X_i(f_j)|}{|X_1(f_j)|}
 \tag{4-6}$$

The relative phase of the complex product  $X_1(f) X_i(f)$  gives the mode shape direction.

Actual calculations are based on a limited time measurement of  $X(t)$ . In the time interval  $T$ , the Fourier transform (4-1) becomes

$$X(f) = \int_{-T/2}^{T/2} x(t)e^{-2\pi ift} dt \quad (4-7)$$

The Hanning time window is one of the simplest methods used to minimize the spectral spreading effect caused by the finite record length. It is used for the routine Fourier amplitude spectrum calculations in this report. The standard Fourier amplitude spectrum is smoothed by 1/4, 1/2, 1/4 weights as follows

$$|X_i(f)|_{sm} = 1/2 |X_i(f)| + 1/4 \{ |X_{i+1}(f)| + |X_{i-1}(f)| \} \quad (4-8)$$

Estimates of equivalent viscous damping are obtained from the width of the peak corresponding to the modal frequency of interest

$$\xi = \frac{\Delta f}{2f} \quad (4-9)$$

where  $\xi$  is the critical damping ratio and  $\Delta f$  is the peak width (bandwidth in Hz) measured at  $1/\sqrt{2}$  of the amplitude spectrum value  $|X(f_i)|$ .

#### 4.3.2 Data Processing

Four simultaneous outputs were recorded on magnetic tape during each of the 30 runs listed in Table 4.2. Each run was on a digital system and converted to 10 discrete points per second. Because of the high frequency filtering present in the field instrumentation, no significant frequencies above 5 Hz were found in the recordings. This gave a final sample rate of 10 samples per second corresponding to a Nyquist frequency,  $f_n = 5.0$  Hz. For the resonant frequencies runs 8,192 and 2048 data points were selected for the translational and torsional modes, respectively.

For each mode shape run 1024 data points were selected corresponding to about 60 seconds of record. The Fourier amplitude spectrum was computed giving 1024 spectral estimates and an equal number of phase angles.

The spectral estimates were smoothed by 1/4, 1/2, 1/4 weights. The 1024 spectral estimates are uniformly distributed between 0 Hz and 5 Hz giving a frequency resolution of 5/1024, or about 0.0049 Hz.

Relative phase angles were used to determine the positive (in phase) or negative (180 degrees out of phase) sign of the mode amplitude. The phase angle calculated for each component at each modal frequency were taken from a computer output listing. Subtracting the phase angle in degrees at each floor from the corresponding phase angle at the reference instrument gave relative phase.

#### 4.3.3 Frequencies and Modes of Vibration

The natural frequencies of vibration for six E-W and N-S translational modes and for four torsional modes are given in Table 4.3.

TABLE 4.3 RESONANT FREQUENCIES (cps)

Excitation	Mode					
	1	2	3	4	5	6
E-W	.263	.76	1.338	1.914	2.695	3.096
N-S	.273	.791	1.397	2.015	2.852	3.15
Torsional	.347	1.00	1.70	2.49	-	-

The natural frequencies were determined considering the distribution of all peaks in the Fourier spectra for all 30 runs.

The ratios of the higher mode frequencies with respect to fundamental ones are given in Table 4.4. The values obtained are closely

to the ratios 1, 3, 5, 7, 9, 11 ..., indicating that the building vibration in all studied directions are predominantly of shear type.

TABLE 4.4 RATIO OF RESONANT FREQUENCIES

Mode	Translational, E-W		Translational, N-S		Torsional	
	$f_i$ (cps)	$f_i / f_1$	$f_i$ (cps)	$f_i / f_1$	$f_i$ (cps)	$f_i / f_1$
1	.263	1.00	.273	1.00	.347	1.00
2	.76	2.89	.791	2.90	1.00	2.88
3	1.338	5.09	1.397	5.11	1.70	4.90
4	1.914	7.28	2.015	7.38	2.49	7.18
5	2.695	10.25	2.852	10.45	-	-
6	3.096	11.77	3.15	11.54	-	-

Mode shapes were calculated for five and four modes in E-W and N-S translational direction, respectively, as well as, three torsional modes. All determined modes of vibration are given in Figs. 4.4 through 4.15. From the calculated Fourier response spectra it was very difficult to separate structural spectral values from the noise above the frequency of 2.0 cps. This is mainly due to low density of the recorded data and the instrument response characteristic used to provide maximum signal for the lowest frequency modes. It appeared that 10 data points per second are insufficient to obtain clearly higher frequency modes. It would require a minimum of 50 data points per second in order to obtain results as good as during the forced vibration study

#### 4.3.4 Damping

In the case of force vibration study damping in the

structure can be determined in several ways; by band-width method, by measuring relative peak amplitudes, or when there is no wind by measuring a free vibration response.

During the ambient vibrations, strictly speaking all these methods fail unless measurements can be taken during the period when wind excitations are random and stationary in time (17). According to the criteria described in (17), during the ambient vibration study of Century City South-Theme Tower, wind excitation could be considered in most of the runs as random and nearly stationary in time. There was also reasonably good separation of the translational and torsional modes and no overlapping in the peak areas was noticed.

Estimation of the equivalent viscous damping factors from this study are given in Table 4.5. The damping for the translational and torsional modes were calculated from the average spectra of all records for corresponding direction on the 42nd floor.

TABLE 4.5 DAMPING FACTORS (%)

Excitation	Mode					
	1	2	3	4	5	6
E-W	2.37	1.33	.35	.26	.17	-
N-S	3.30	1.28	1.17	.74	.55	.15
Torsional	.90	.43	.16	.13	-	-

It appears that estimated damping values are too low, except for the first translational and torsional modes, most probably due to insufficient density of the recorded data.

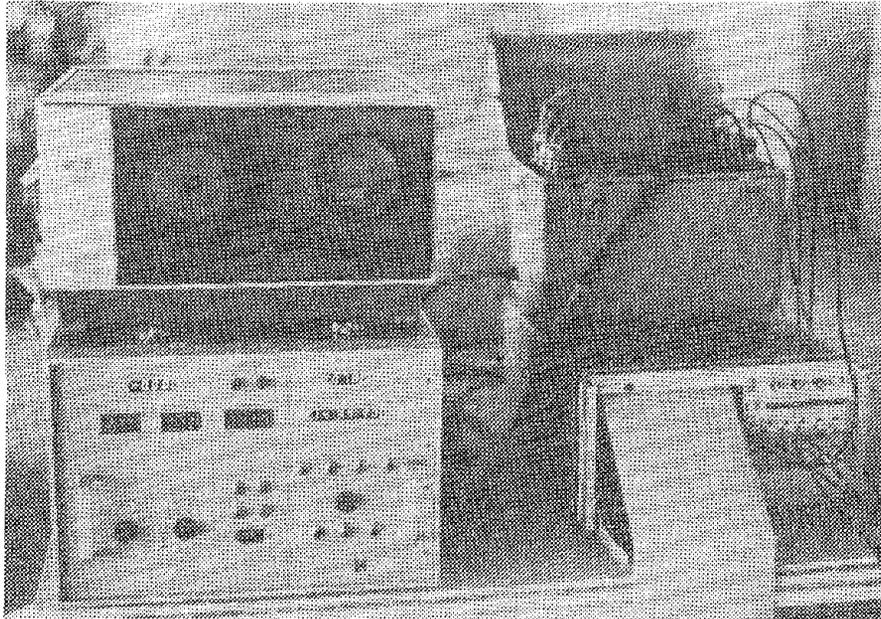


FIG. 4.1 AMBIENT VIBRATION EQUIPMENT

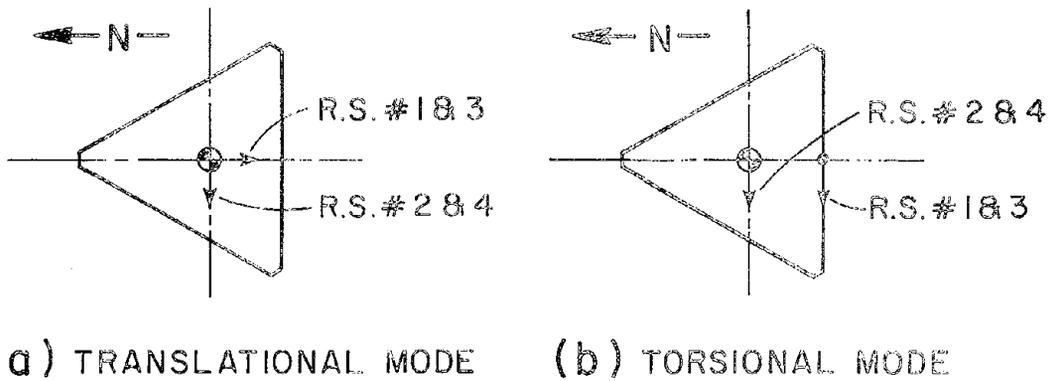


FIG. 4.2 LOCATION OF RANGE SEISMOMETERS ON THE 42 TH FLOOR FOR RESONANT FREQUENCY RESPONSE

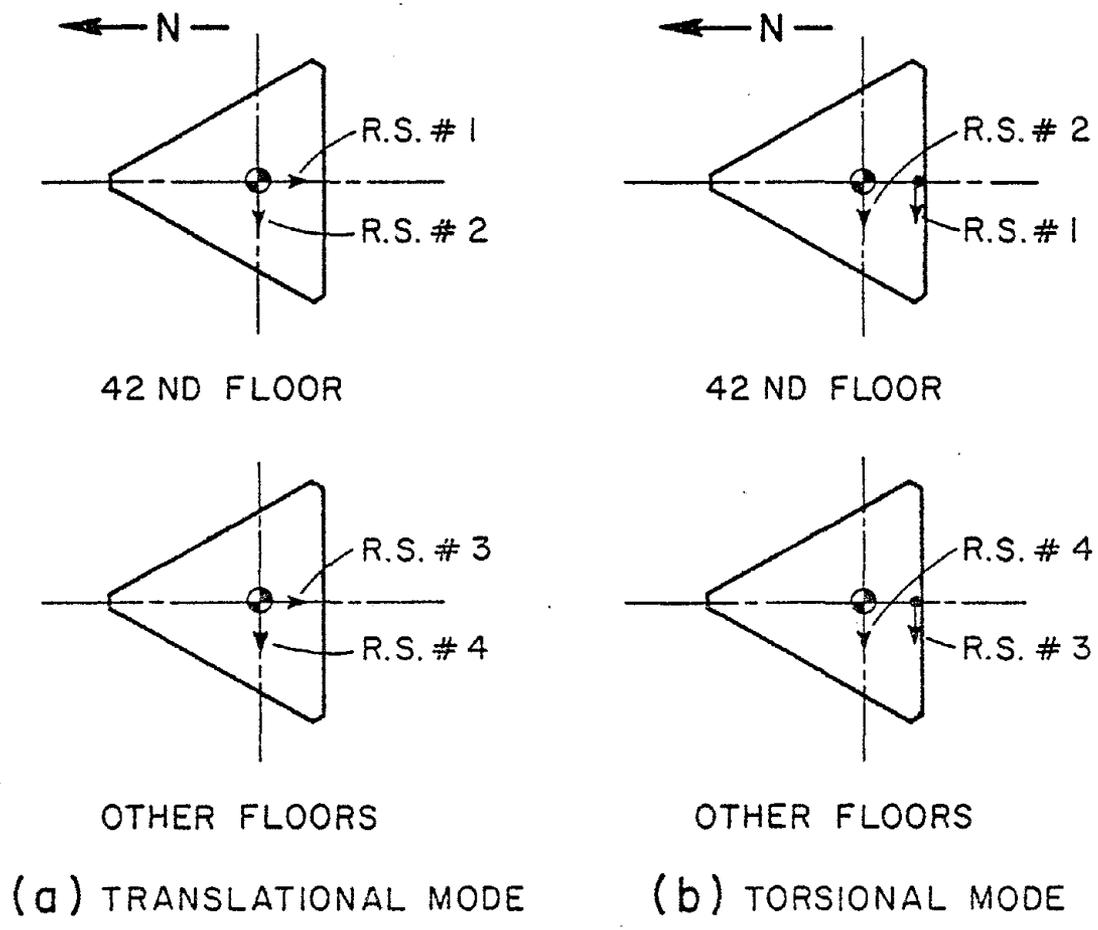


FIG. 4.3 LOCATION OF RANGE SEISMOMETERS FOR THE MODE SHAPES

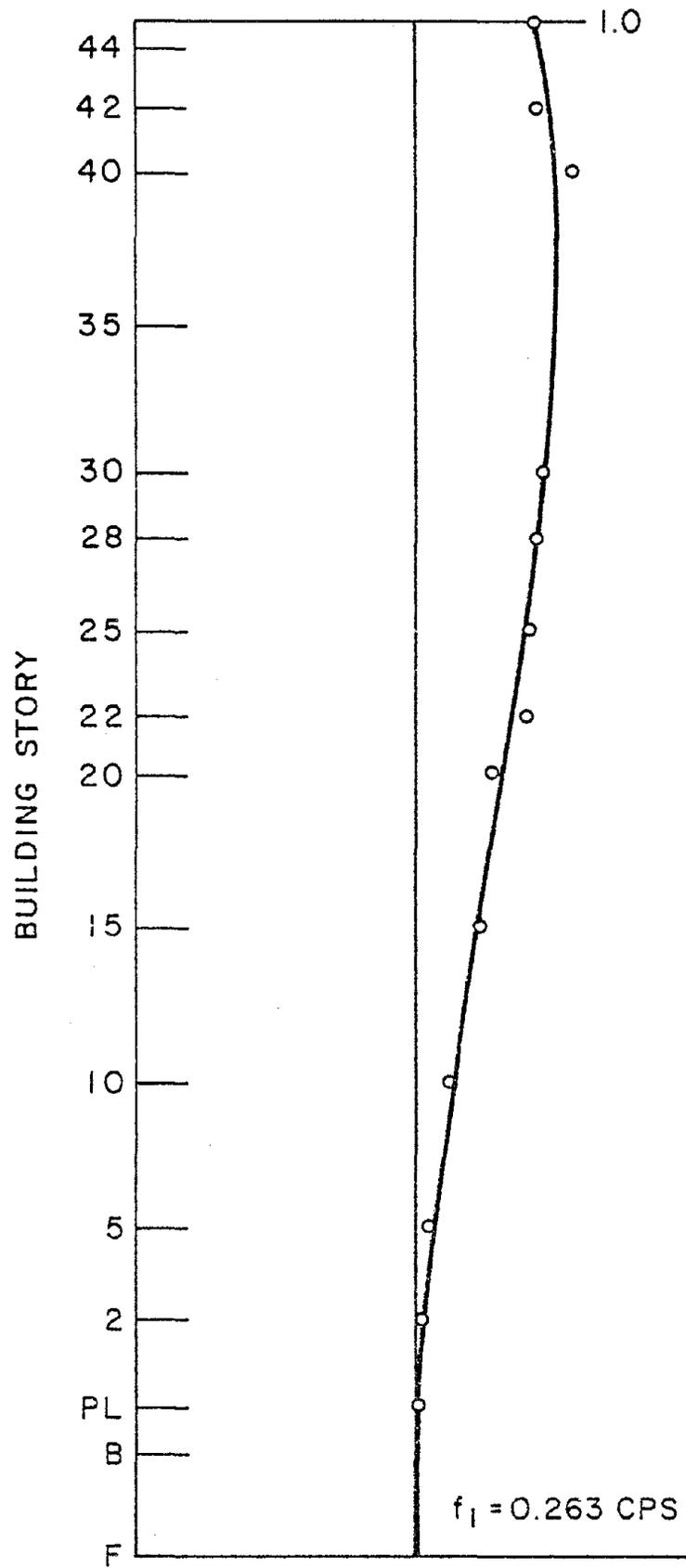


FIG. 4.4 FIRST TRANSLATIONAL MODE SHAPE, E-W

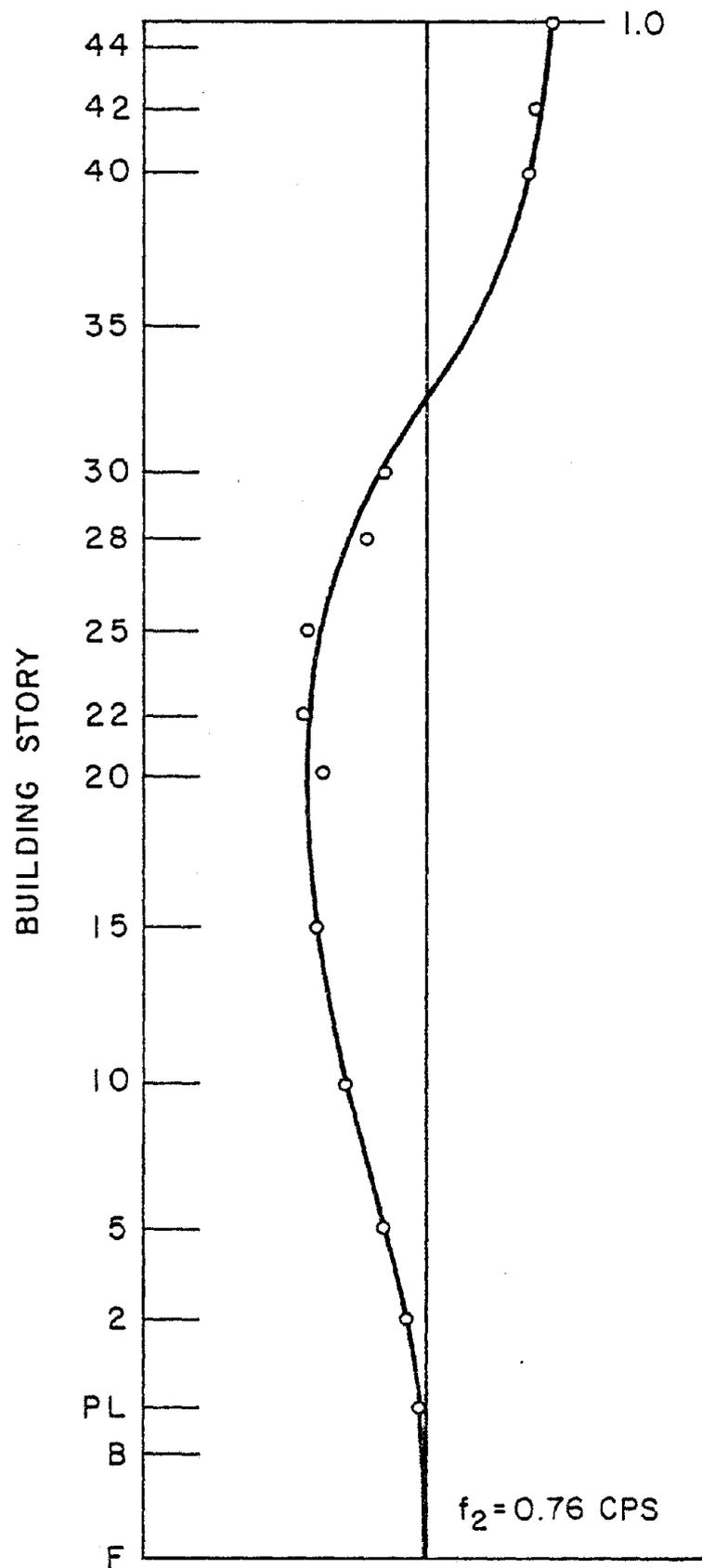


FIG. 4.5 SECOND TRANSLATIONAL MODE SHAPE, E-W

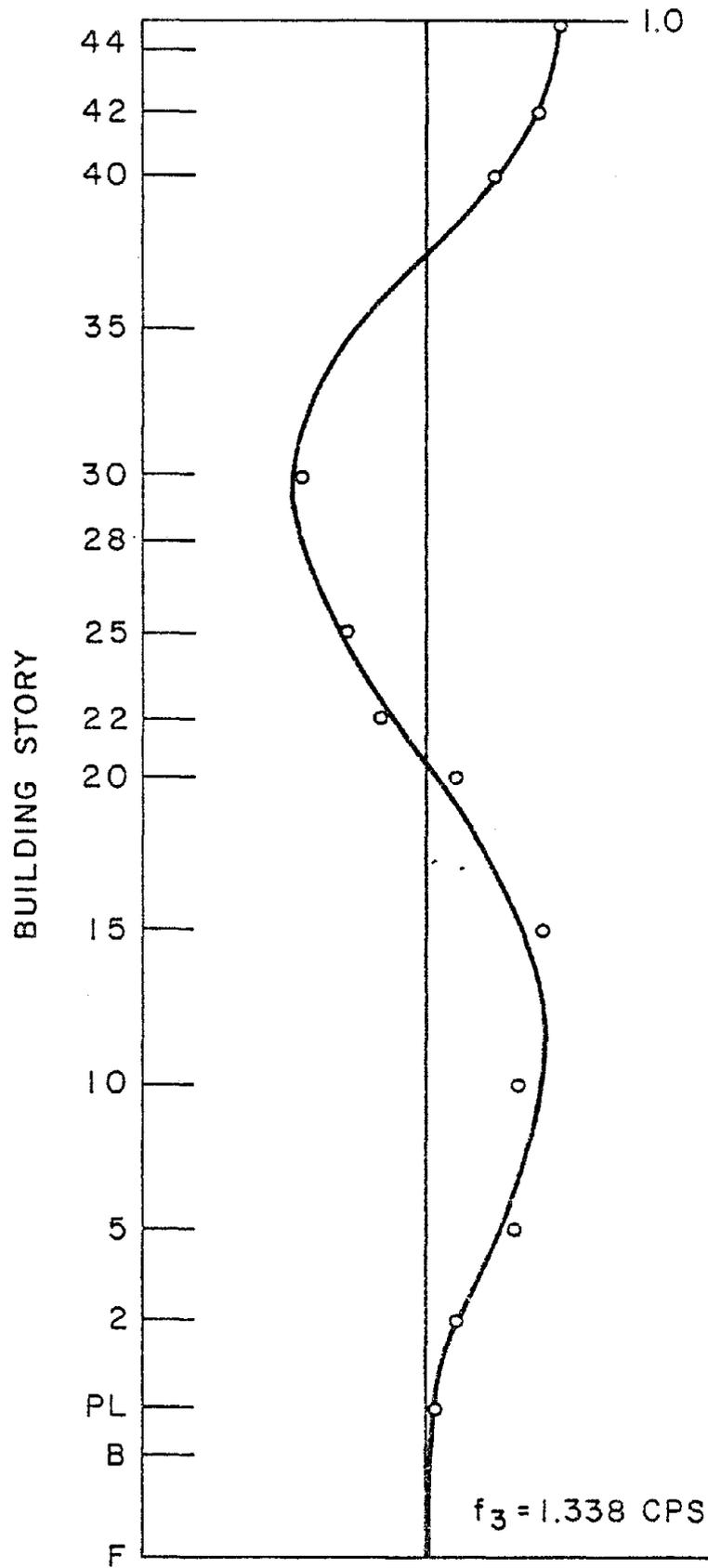


FIG. 4.6 THIRD TRANSLATIONAL MODE SHAPE, E-W

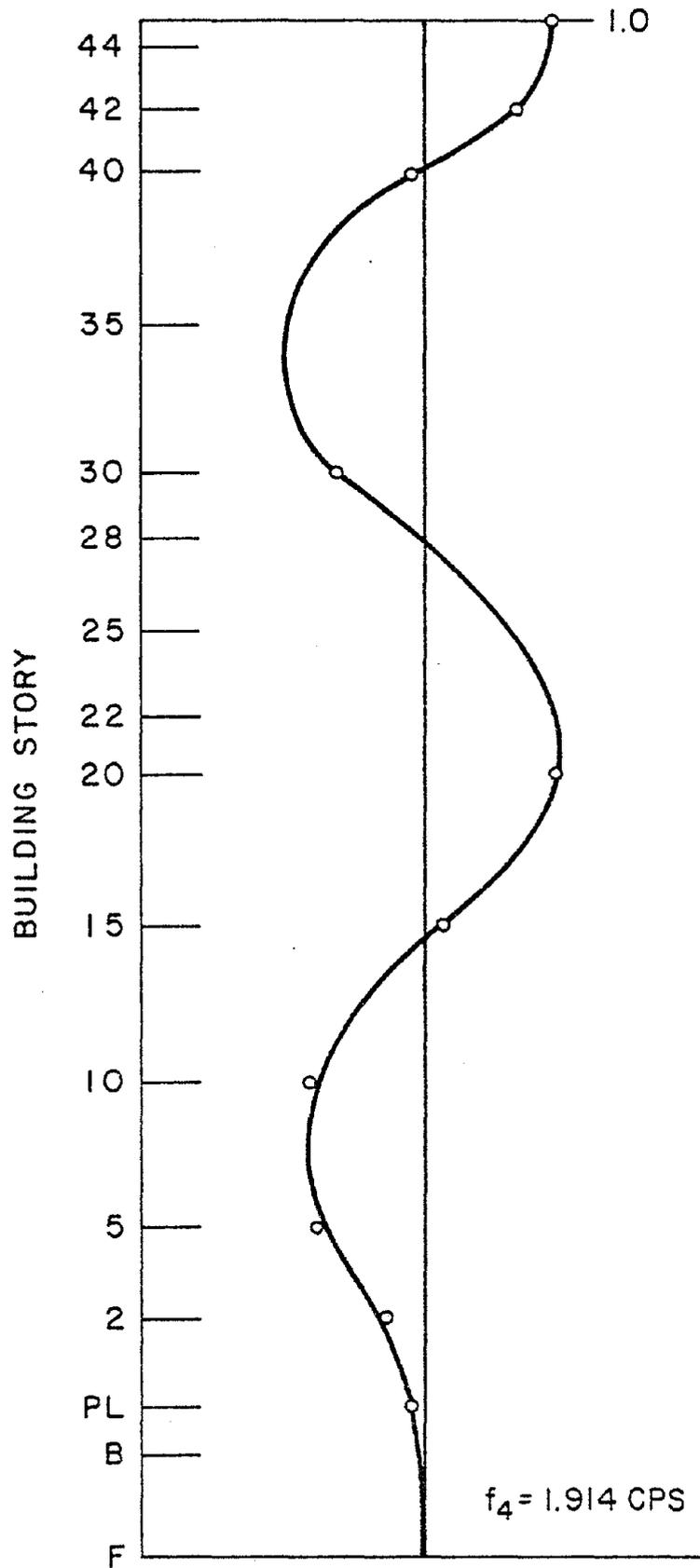


FIG. 4.7 FOURTH TRANSLATIONAL MODE SHAPE, E-W

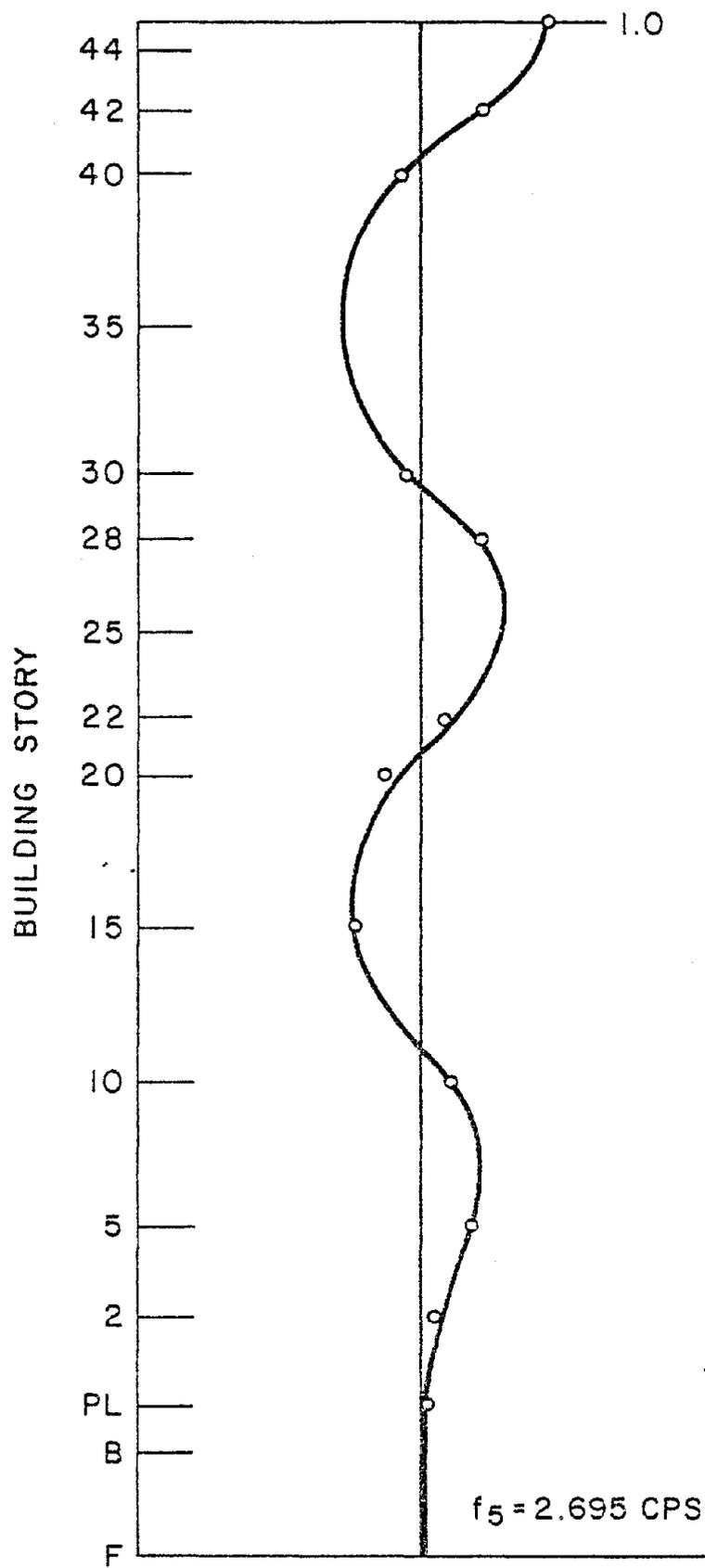


FIG. 4.8 FIFTH TRANSLATIONAL MODE SHAPE, E-W

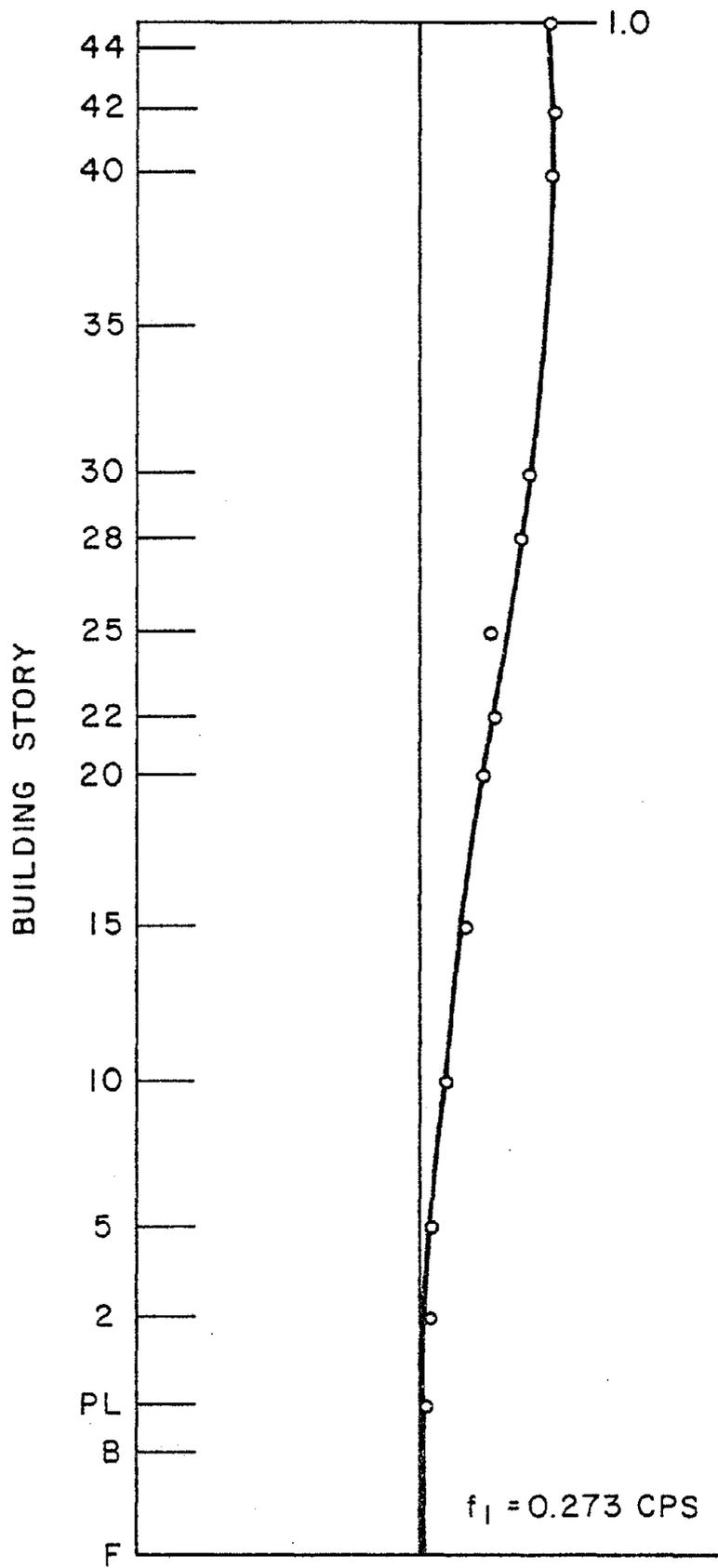


FIG. 4.9 FIRST TRANSLATIONAL MODE SHAPE, N-S

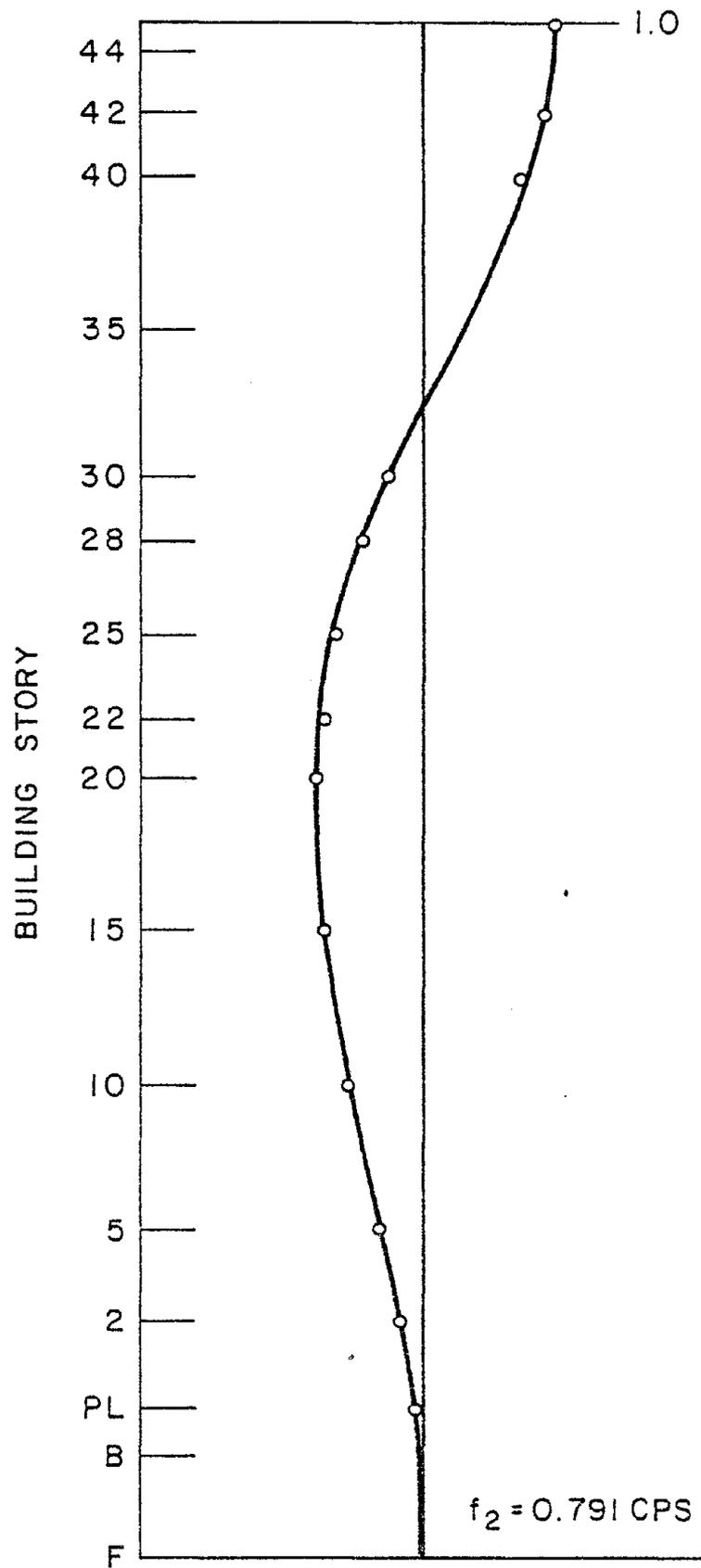


FIG. 4.10 SECOND TRANSLATIONAL MODE SHAPE, N-S

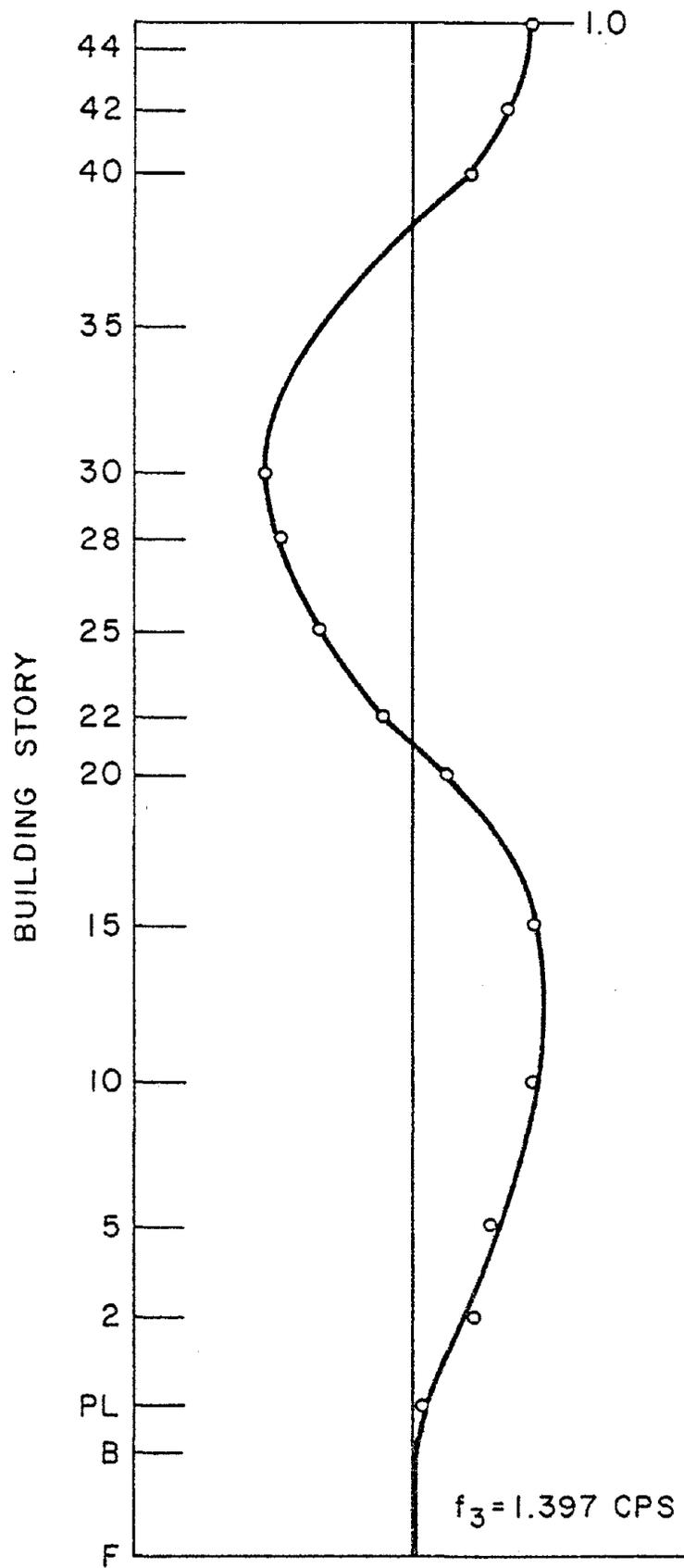


FIG. 4.11 THIRD TRANSLATIONAL MODE SHAPE, N-S

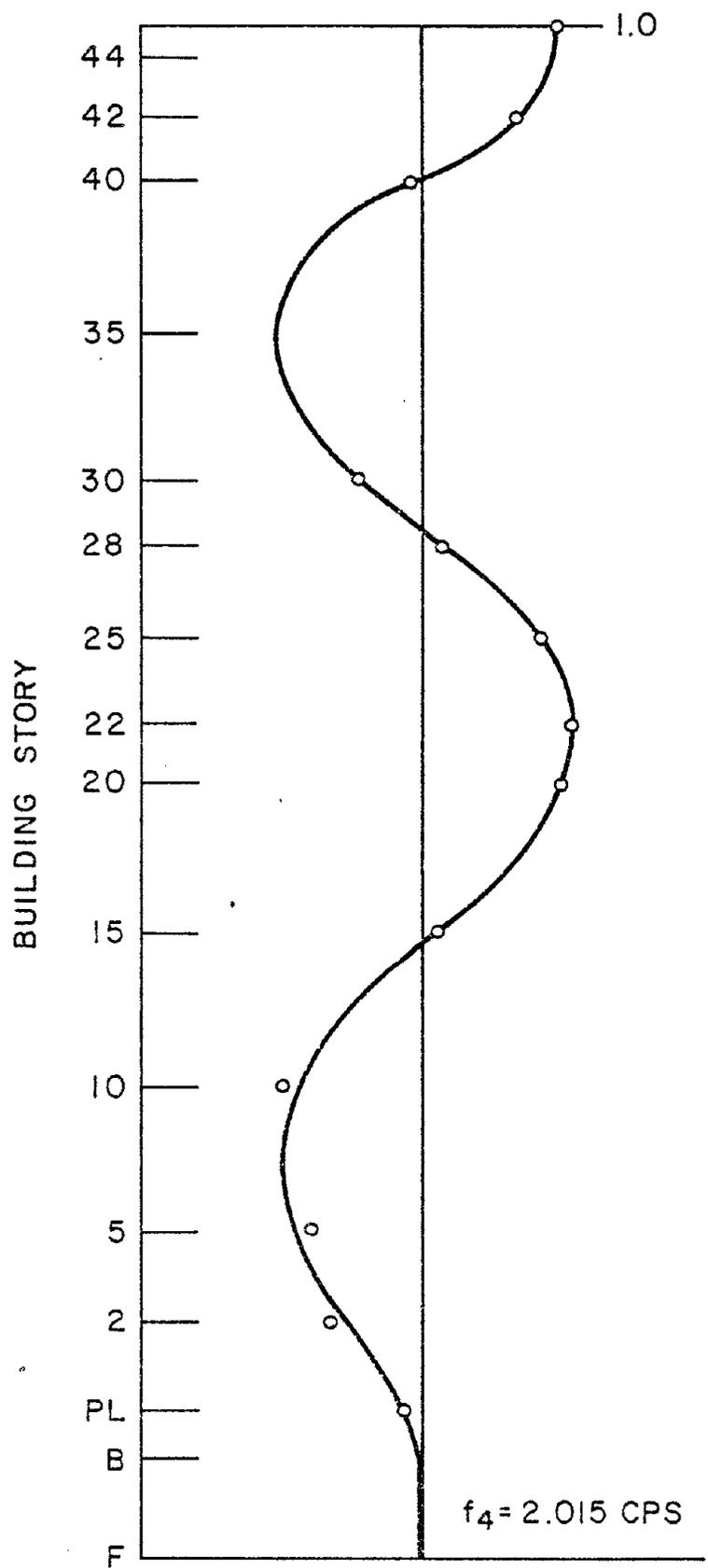


FIG. 4.12 FOURTH TRANSLATIONAL MODE SHAPE, N-S

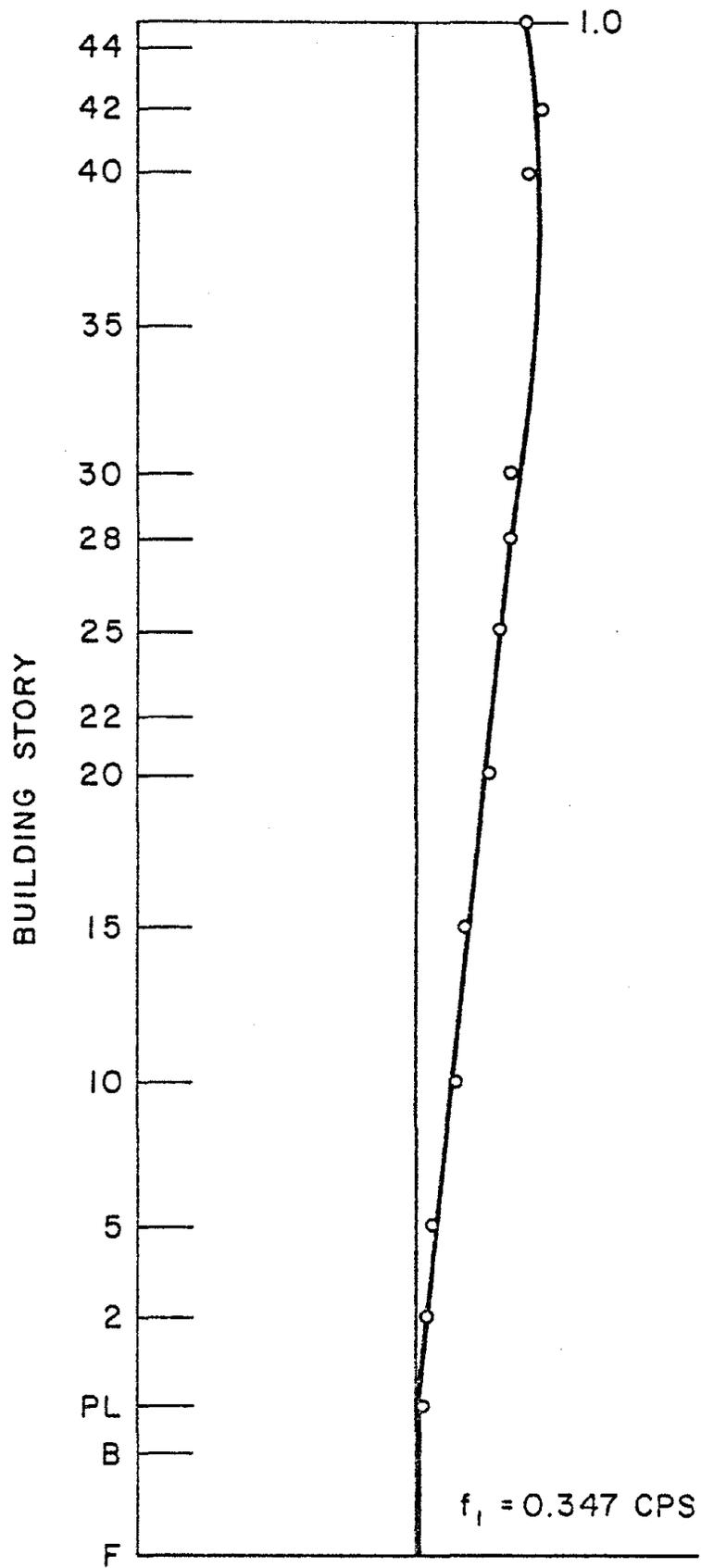


FIG. 4.13 FIRST TORSIONAL MODE SHAPE

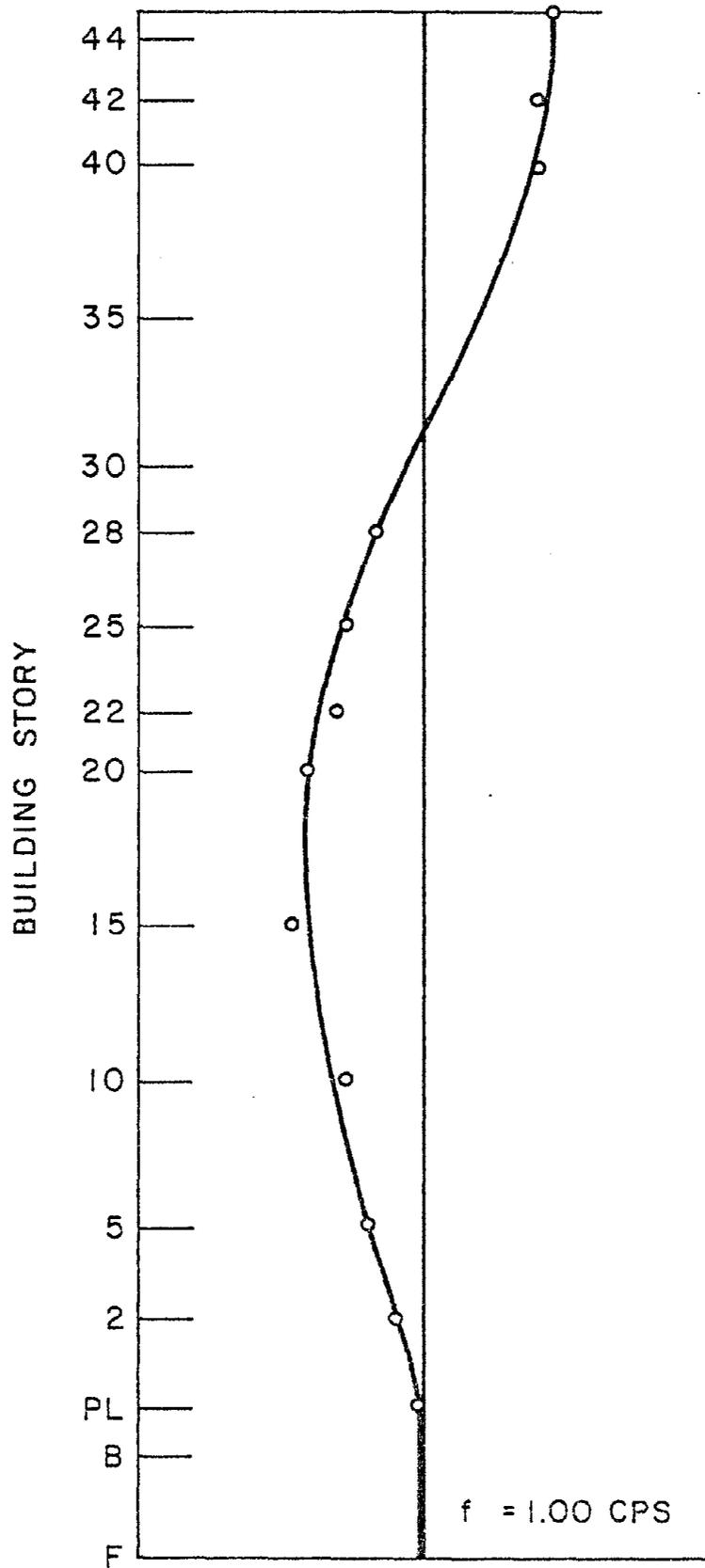


FIG. 4.14 SECOND TORSIONAL MODE SHAPE

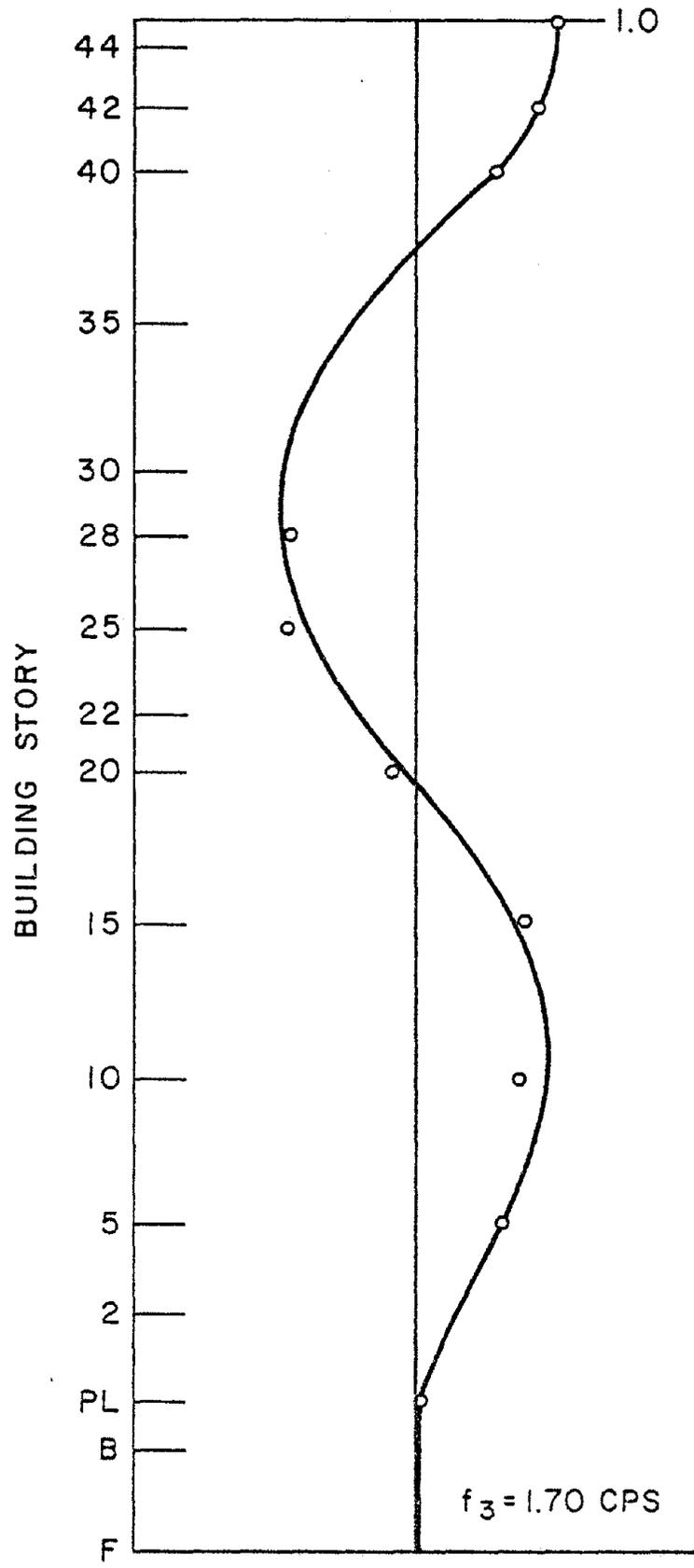


FIG. 4.15 THIRD TORSIONAL MODE SHAPE

## 5. COMPARISON OF FORCED AND AMBIENT VIBRATION STUDIES

The dynamic properties: resonant frequencies, modes of vibration and damping values, were determined by full-scale dynamic test using forced and ambient vibration methods. Resonant frequencies and damping factors from both studies are summarized and compared in Table 5.1.

The resonant frequencies from both studies are in very good agreement in all separated modes of vibration with the maximum difference smaller than 2%. The ratios of the observed higher mode frequencies with respect to the fundamental one from both dynamic studies of the building are plotted in Fig. 5.1. These ratios for both translational directions as well as the torsional direction indicate that over-all structural response is predominately of the shear type. Equivalent viscous damping factors for the reasons discussed in Chapter 4 show significant difference. It appears that it is rather difficult to obtain appropriate damping values from the ambient vibration study, particularly in cases when equivalent viscous damping is expected to be rather low.

Mode shapes for the translational (E-W and N-S) directions as well as torsional ones, are compared in Figs. 5.2 through 5.13. All presented mode shapes are in good agreement from both studies.

Comparison of the forced and ambient vibration experiments of Century City South-Theme Tower demonstrate the consistency of the two method in determining with adequate accuracy the natural frequencies and mode shapes of a typical modern building. Difficulties in the evaluation of equivalent viscous damping factors from ambient vibrations studies are present and probably it will be more realistic from this type of study to expect assessment of the range of damping factors,

TABLE 5.1 COMPARISON OF RESONANT FREQUENCIES AND DAMPING FACTORS

Mode No.	Translational E-W				Translational N-S				Torsional			
	Forced		Ambient		Forced		Ambient		Forced		Ambient	
	f (cps)	$\xi$ (%)	f (cps)	$\xi$ (%)	f (cps)	$\xi$ (%)	f (cps)	$\xi$ (%)	f (cps)	$\xi$ (%)	f (cps)	$\xi$ (%)
1	0.267	2.62	0.263	2.37	0.267	4.34	0.273	3.30	0.357	1.34	0.347	0.90
2	0.76	1.51	0.76	1.33	0.783	1.60	0.791	1.28	0.99	0.81	1.0	0.43
3	1.317	1.18	1.338	0.35	1.373	1.64	1.397	1.17	1.69	0.95	1.70	0.16
4	1.89	1.27	1.914	0.26	1.973	1.72	2.015	0.74	2.47	1.01	2.49	0.13
5	2.74	1.73	2.695	0.17	2.85	1.49	2.852	0.55	3.317	1.36	-	-
6	3.133	2.39	3.096	-	-	-	3.15	0.15	4.133	1.51	-	-

rather than damping values associated with each mode of vibration.

The field effort involved in the ambient vibration study was significantly smaller than for the forced vibration experiment because the measuring equipment used for ambient vibration test is much lighter and has fewer components. A group of three people required for both ambient and forced vibration experiments can perform necessary measurements for the ambient test in 1 to 2 days. The time necessary to complete forced vibration test was about two weeks. The total number of necessary measurements in ambient test is significantly smaller, and also each individual measurement requires a shorter time interval. On the other hand, data analysis is slightly more complicated because it requires Fourier analysis using digital computers.

Both ambient and forced vibration studies may lead to the determination of up to six and more modes of vibration. The number of mode shapes resolved depends mainly on the level of the high-frequency noise and the number of measuring stations in the building. Although both methods of dynamic testing of full-scale structures are based on small levels of excitation, compared to strong earthquake ground motion, the derived dynamic properties of the structural systems are invaluable since they offer a sound basis for rational improvements of the formulation of the mathematical models in the elastic range of behavior of the structural systems.

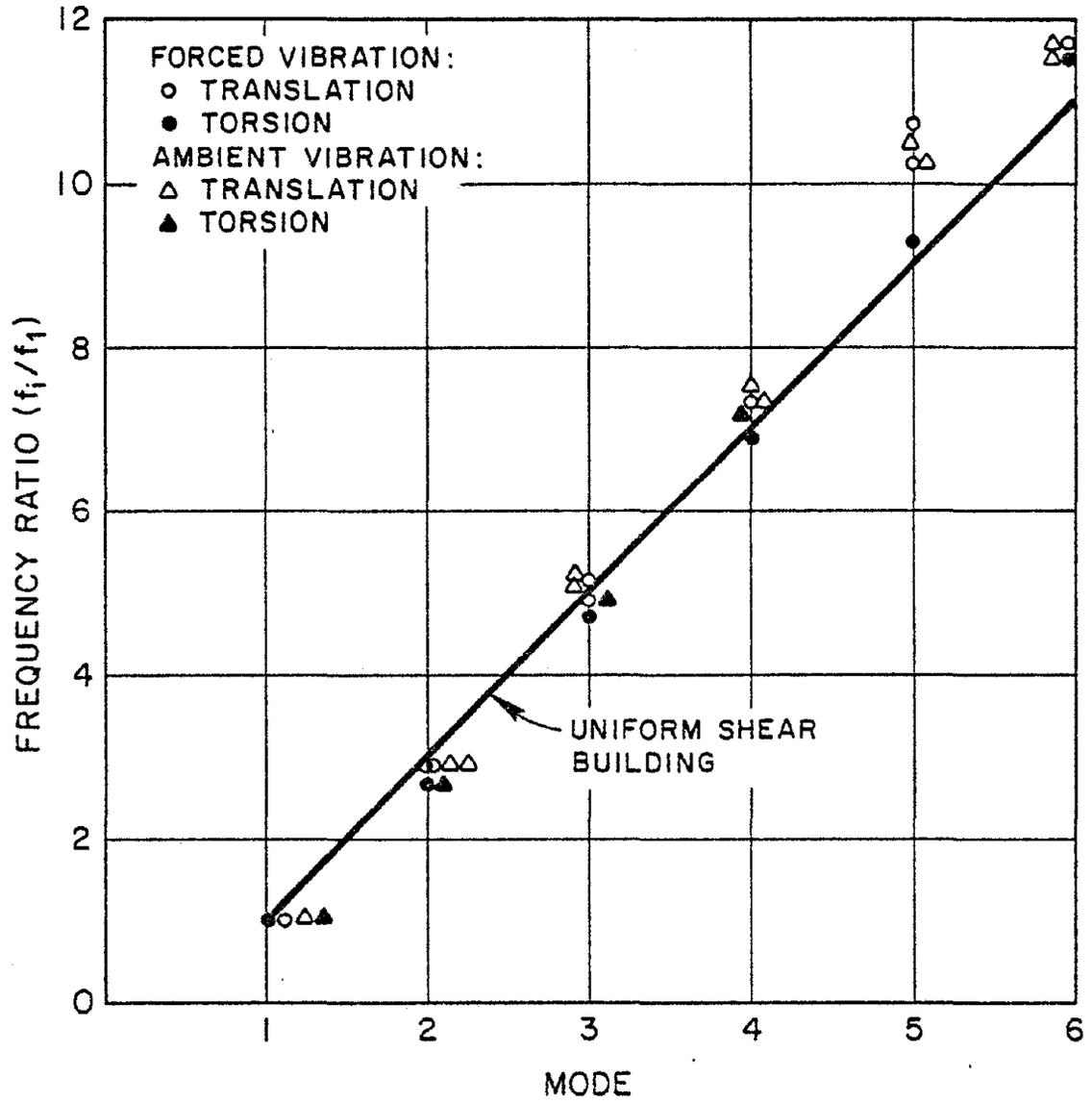


FIG. 5.1 RATIO OF RESONANT FREQUENCIES

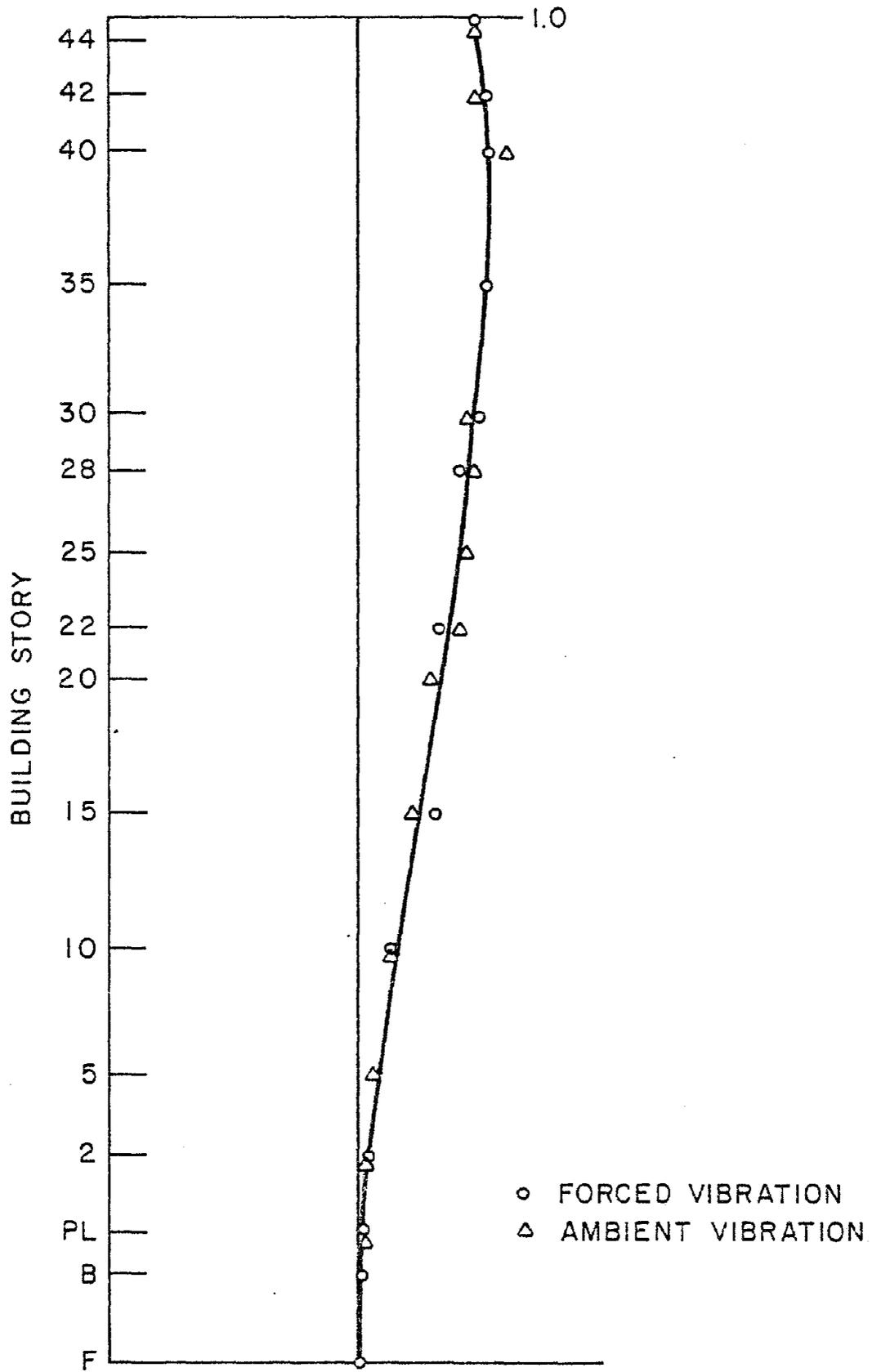


FIG. 5.2 FIRST TRANSLATIONAL MODE SHAPE, E-W

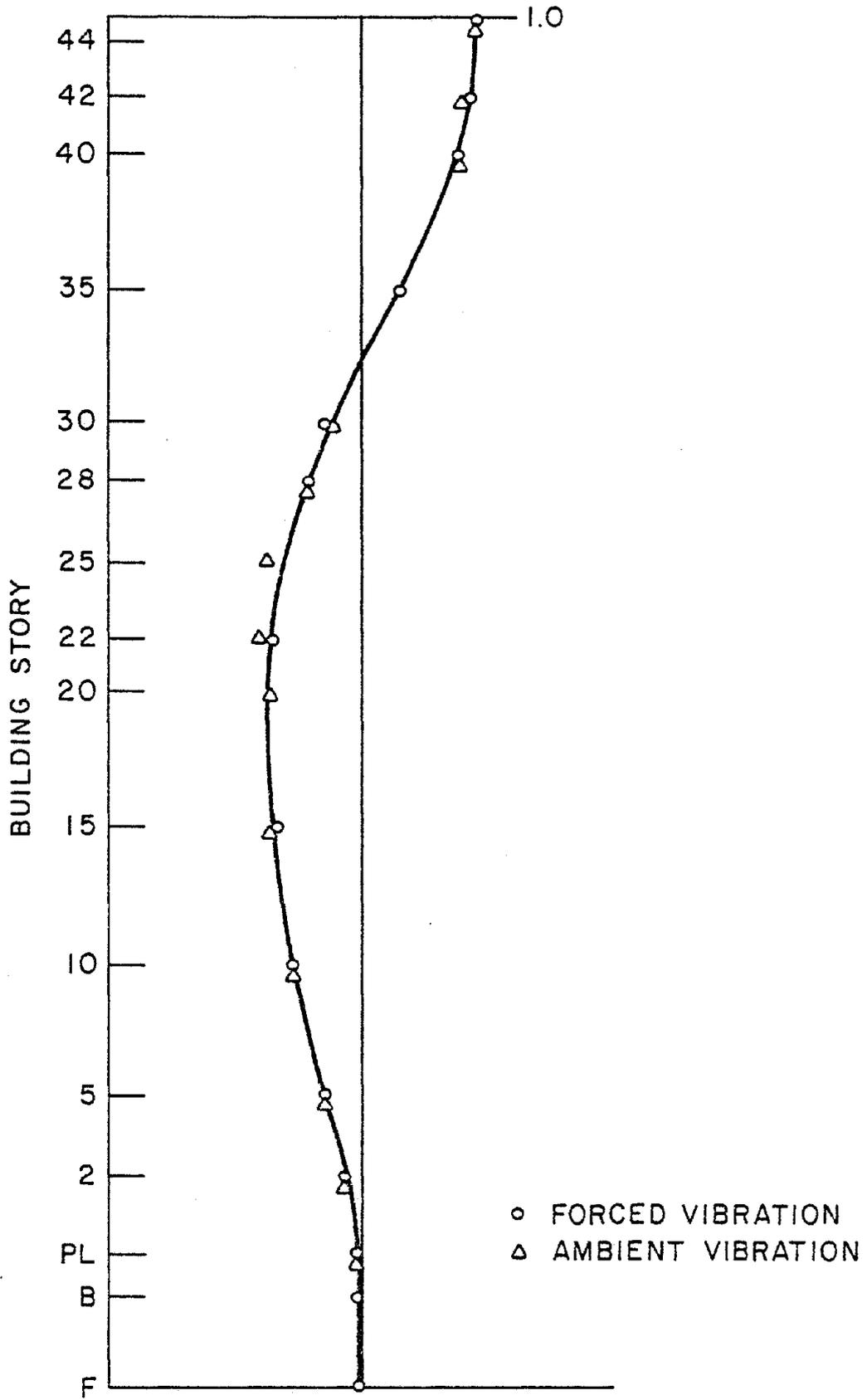


FIG. 5.3 SECOND TRANSLATIONAL MODE SHAPE, E-W

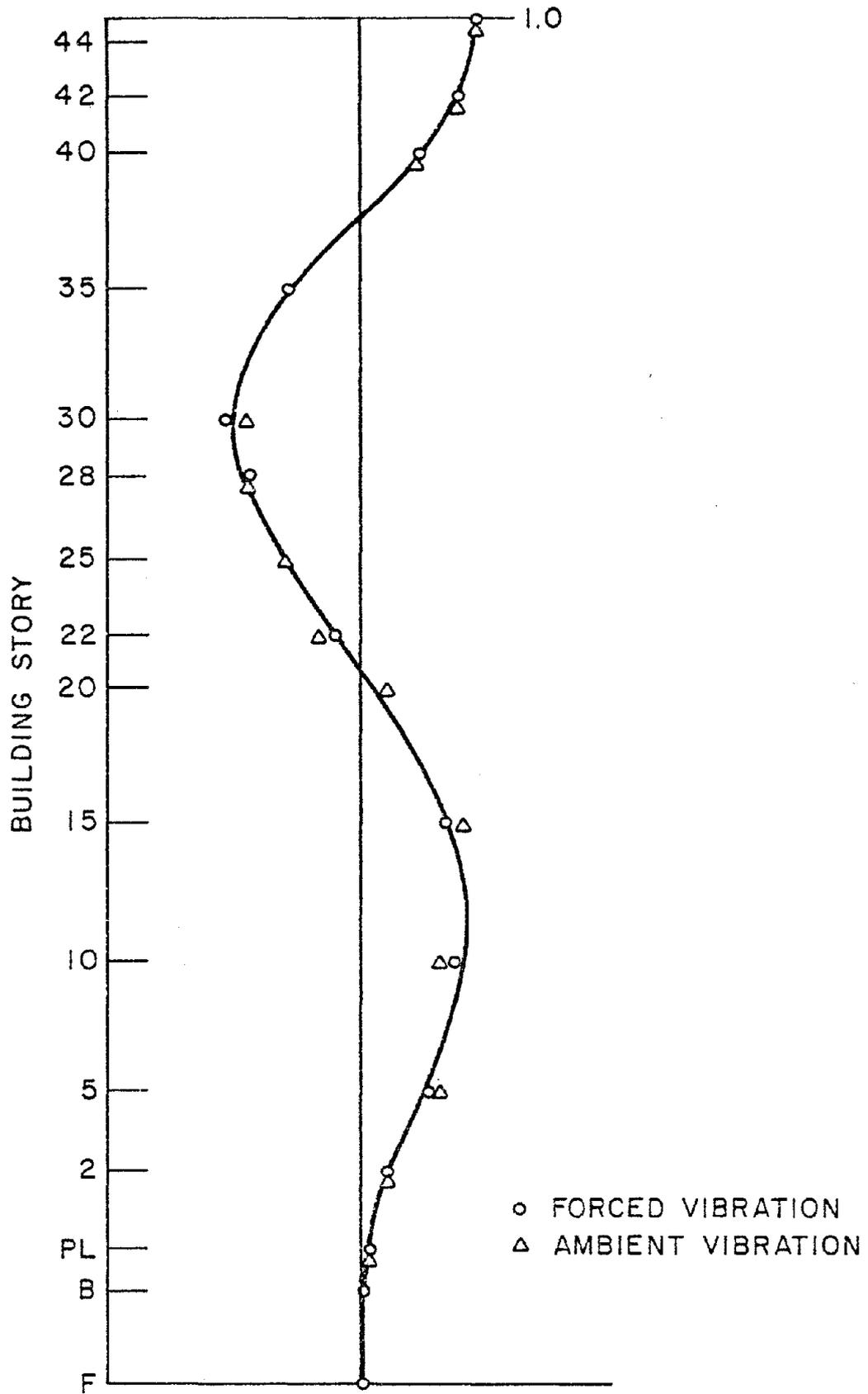


FIG. 5.4 THIRD TRANSLATIONAL MODE SHAPE, E-W

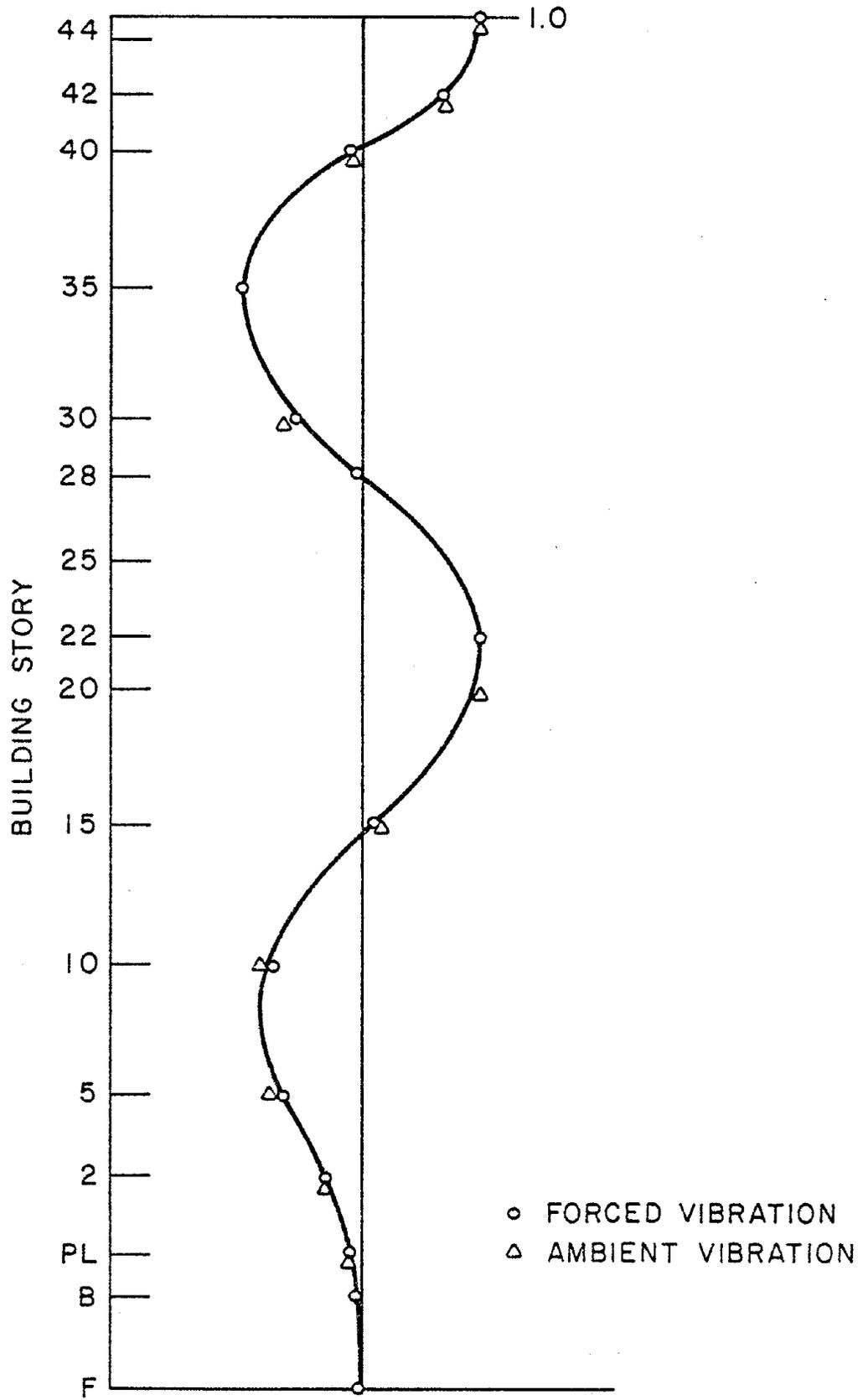


FIG. 5.5 FOURTH TRANSLATIONAL MODE SHAPE, E-W

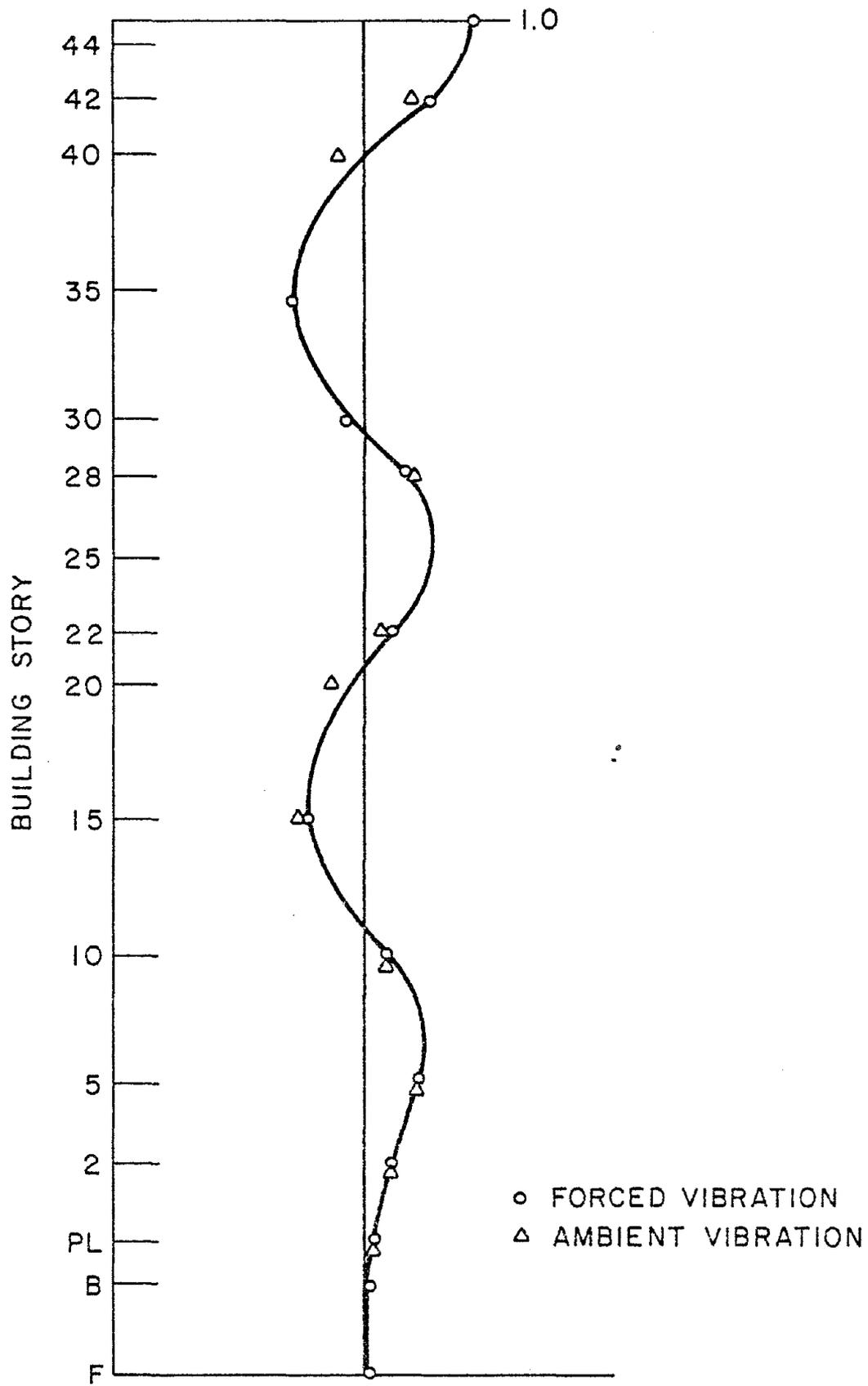


FIG. 5.6 FIFTH TRANSLATIONAL MODE SHAPE, E-W

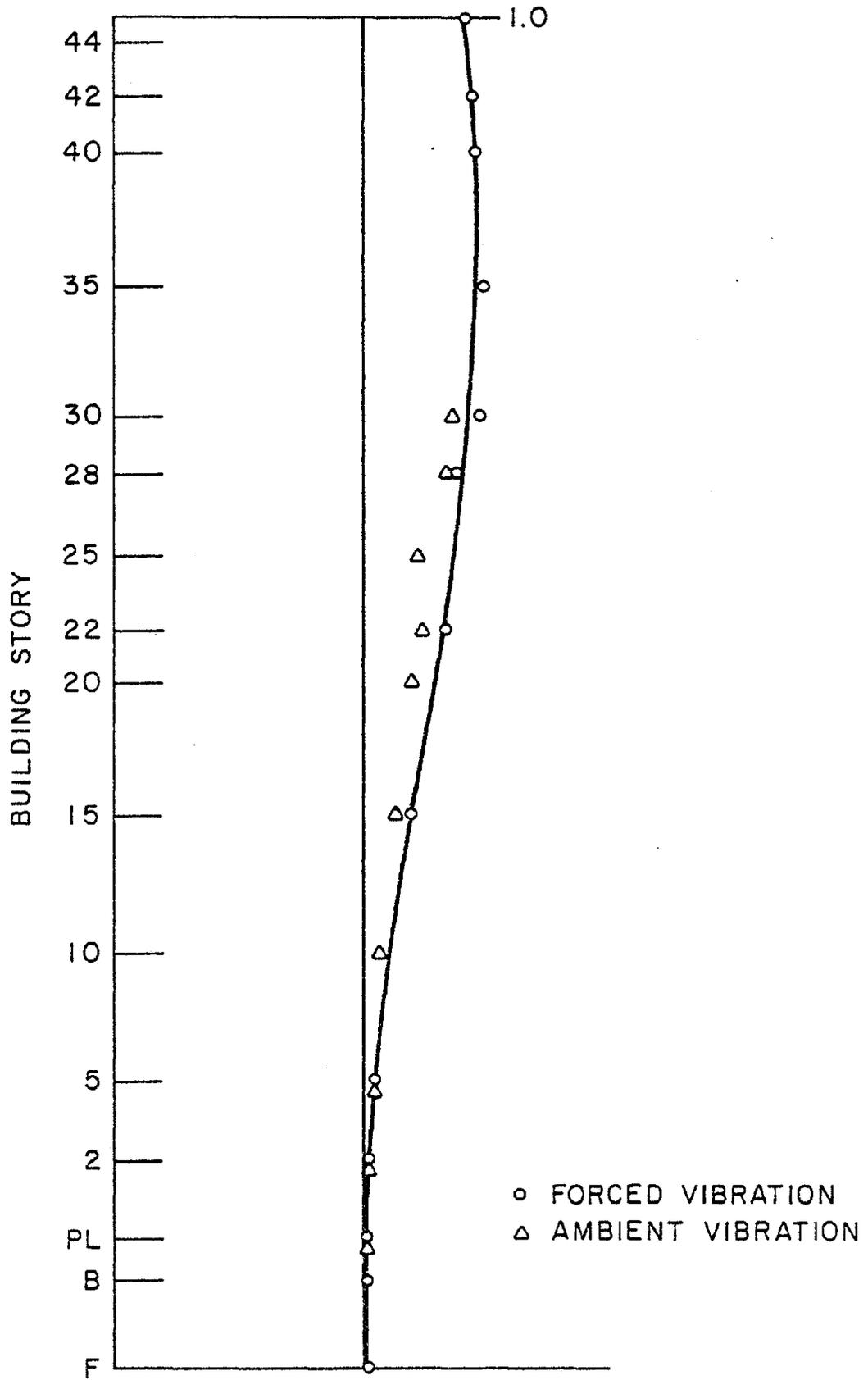


FIG. 5.7 FIRST TRANSLATIONAL MODE SHAPE, N-S

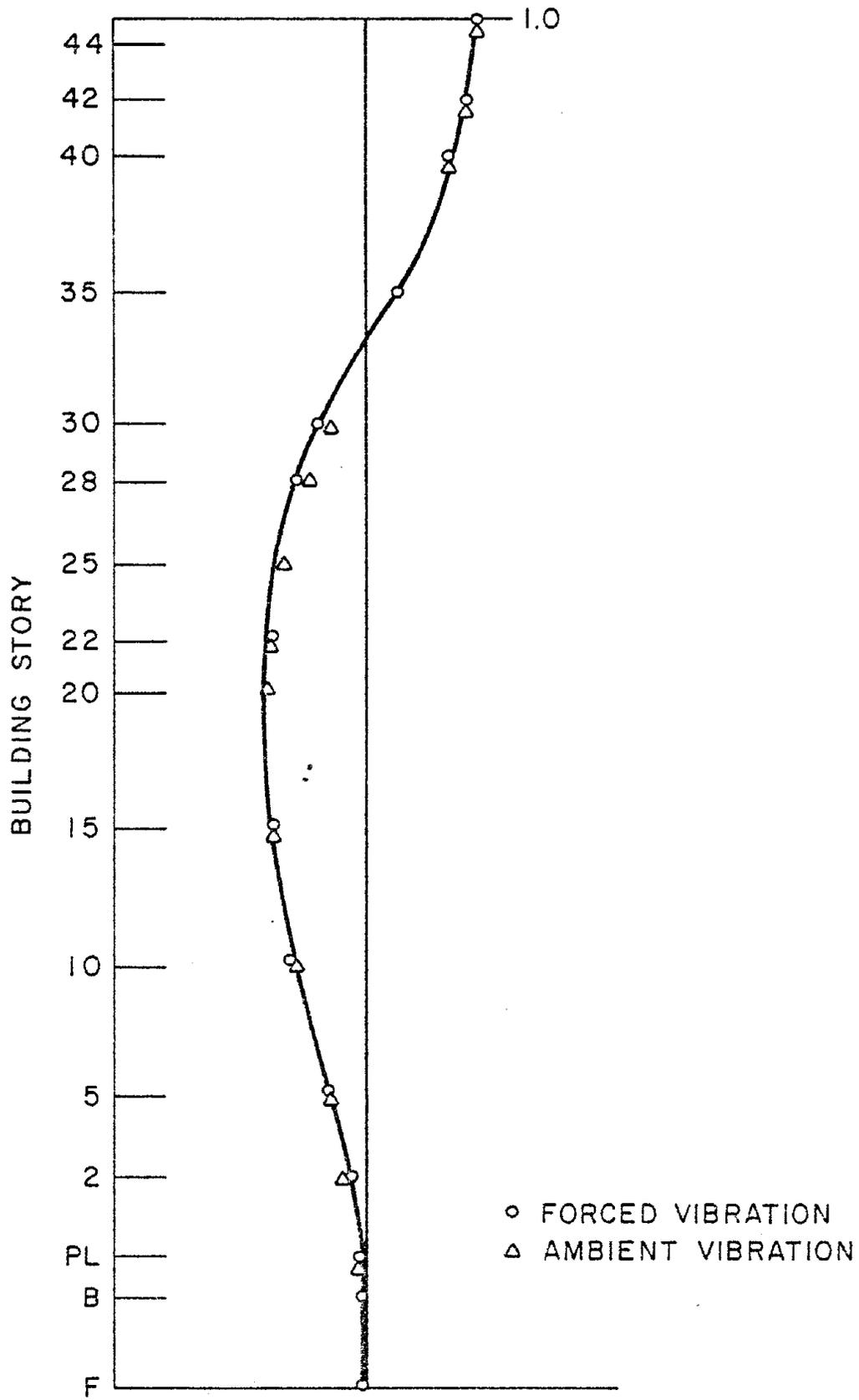


FIG. 5.8 SECOND TRANSLATIONAL MODE SHAPE, N-S

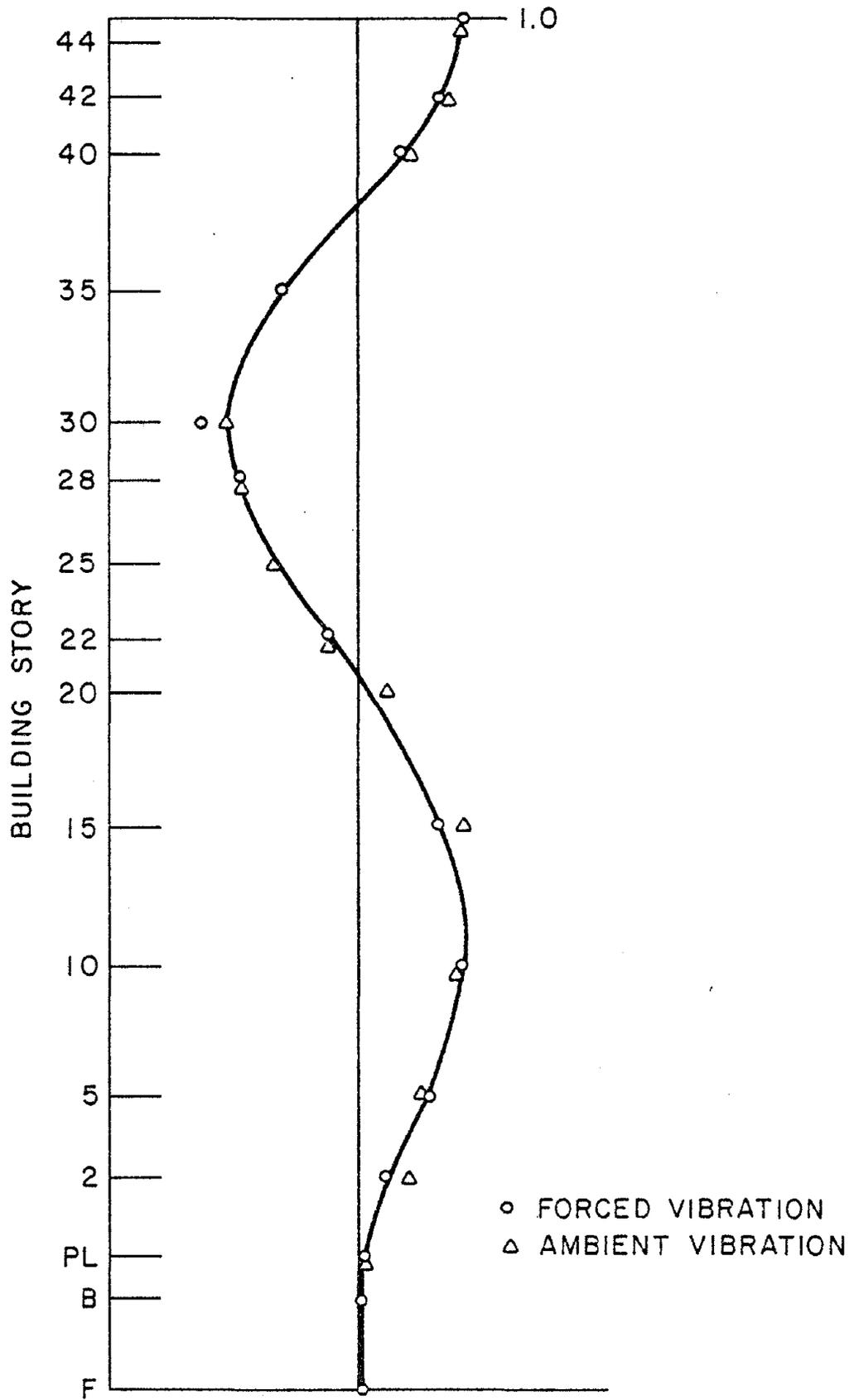


FIG. 5.9 THIRD TRANSLATIONAL MODE SHAPE, N-S

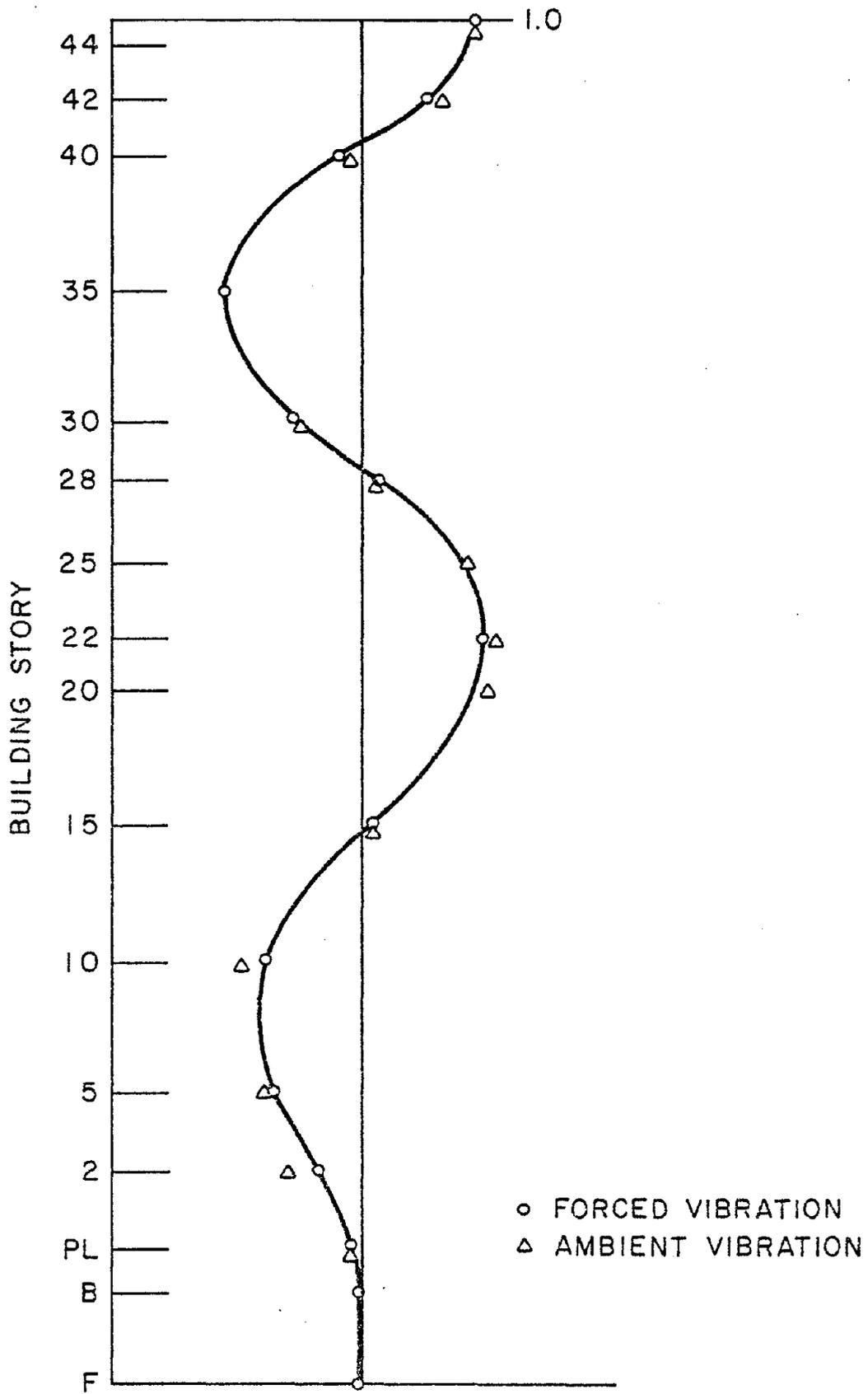


FIG. 5.10 FORTH TRANSLATIONAL MODE SHAPE, N-S

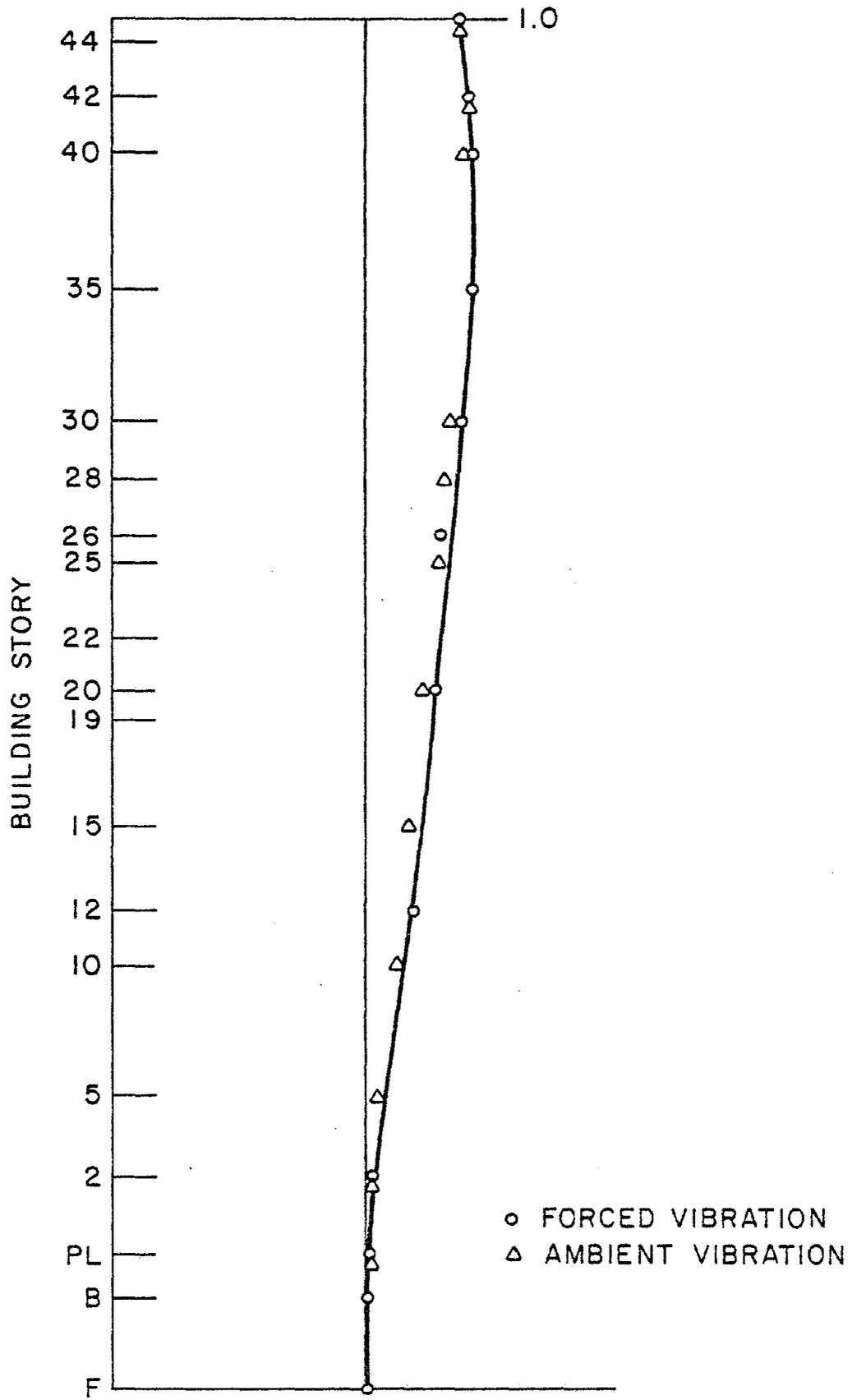


FIG. 5.11 FIRST TORSIONAL MODE SHAPE

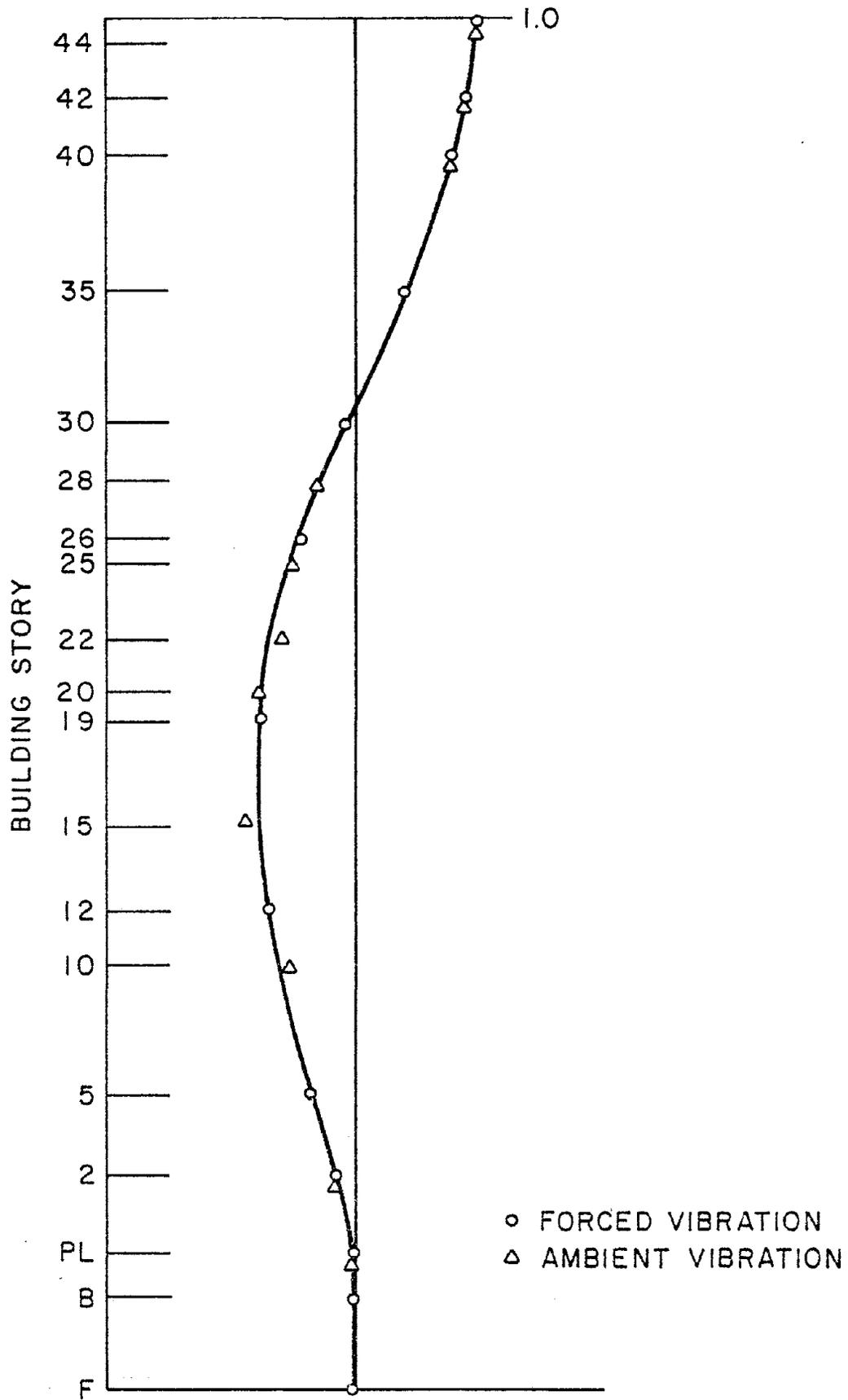


FIG. 5.12 SECOND TORSIONAL MODE SHAPE

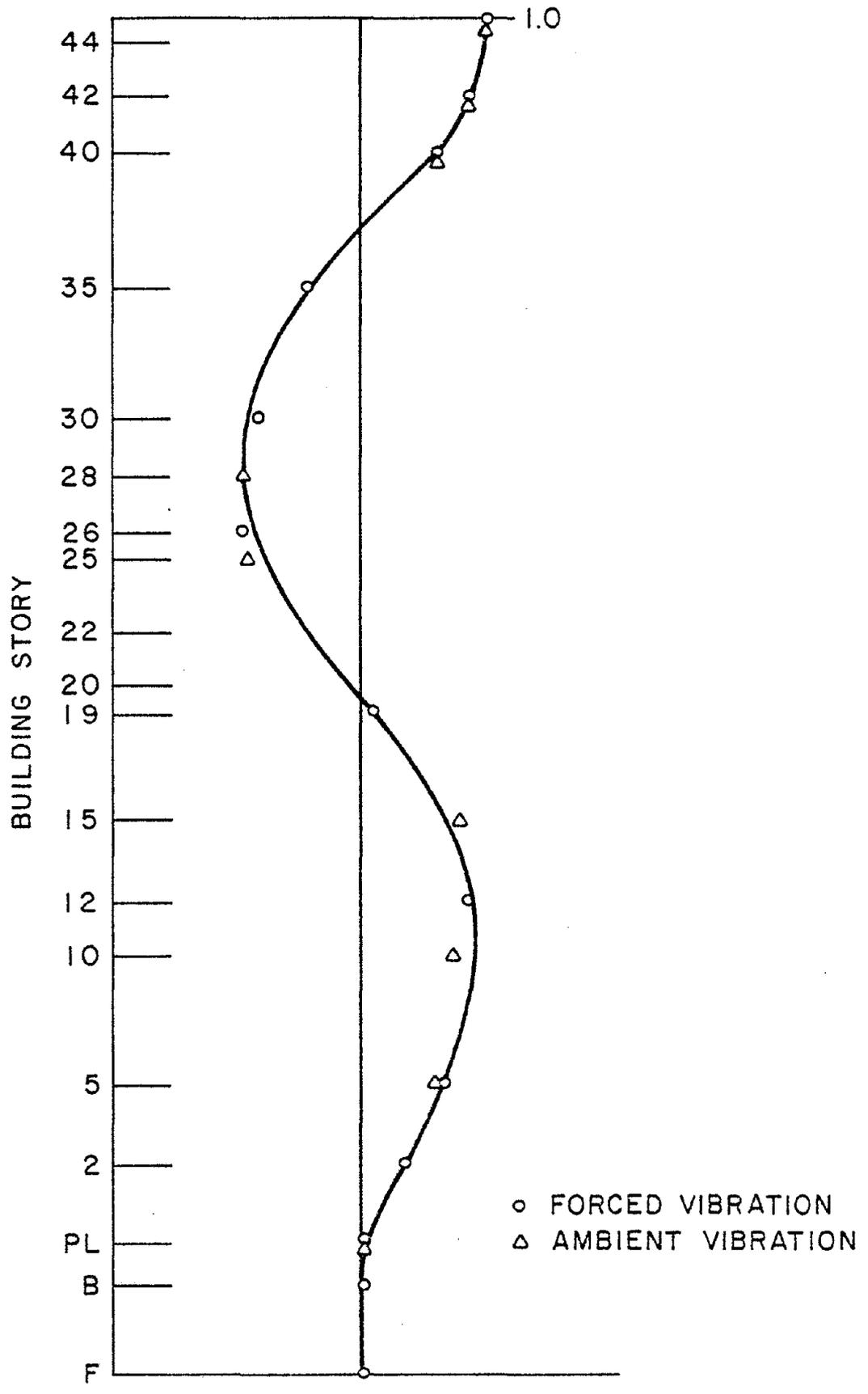


FIG. 5.13 THIRD TORSIONAL MODE SHAPE

## 6. FORMULATION OF MATHEMATICAL MODEL

### 6.1 General

A mathematical computer model of the South theme tower was formulated to assess the dynamic properties for the N-S, E-W and rotational characteristics. In this particular formulation the motion of each component was studied individually by restraining all other degrees of freedom. The model and the computer program employed for the analysis are described below.

### 6.2 Computer Program

MACTUB a special purpose computer program developed in the Division of Structural Engineering and Structural Mechanics of the Department of Civil Engineering at the University of California, Berkeley was used to compute the mode shapes and frequencies of the building. A complete description of the program is given in reference (19).

Employing an extension of the finite element concept, MACTUB was created for the analysis of multistory tube structures consisting of an assembly of plane frames. Each element incorporates a rectangular portion of a frame consisting of a number of columns and a number of beams. Elements are interconnected at nodes along their sides. Both static and dynamic analysis can be performed, assuming linear material behavior.

The analysis was performed by a CDC 6400 digital computer using the aforementioned program. The program is based on the direct stiffness method which first generates the element stiffness matrices in their local coordinates and then performs the transformation into the global coordinates of the complete structural system giving by summation an overall structural stiffness. Since the total number of joints modeled are less

than the actual number of beam column joints, the computer solution time required is greatly reduced. In addition each floor is assumed to displace as a rigid system.

The computer time required for a typical modal extraction (5 modes) was in the order of 2.5 minutes. The basic computer performance for the solution portion of the problem is shown below.

FORM ELEMENT STIFFNESS	8.466 sec.
ASSEMBLE & REDUCE GLOBAL STIFFNESS	68.789 sec.
DYNAMIC ANALYSIS	20.418 sec.
TOTAL SOLUTION TIME	97.653 sec.

### 6.3 Formulation of Mathematical Model

The building was modeled as a structural system composed of three planar frames forming the exterior face of the building. A typical frame shown in Fig. 6.1 consists of a series of macroelements connected at the nodal points. Each element represents an average stiffness of the beams and columns in the actual structure over a segment of floors and bays. In addition corner elements were employed to simulate the stiffness properties of the corner columns. The core was represented by a column with equivalent bending and axial stiffness placed at the geometrical center of the building. In the modeling all secondary framing existing between the exterior face and the core have been neglected since their effect on the overall structural stiffness is minimal.

The floor dead load was distributed by a lumped mass system. These masses were assumed to be located at the geometric center of the building, and at the corner and mid-side points of the interior and exterior frames.

The analysis was carried out by treating the modal response of each degree of freedom independently from the other. The assumption seems reasonable due to the symmetry of the building.

#### 6.4 Results of Mathematical Model

Under the idealizations and assumptions described in the previous section, a dynamic analysis of the mathematical model was performed. The results of the resonant frequencies for the translational and torsional motions for the first five modes are outlined in Table 6.1. Because of the building symmetry the N-S and E-W resonant frequencies and modes are the same. The first five translational modes and first three torsional modes are shown in Figs. 6.2.

Table 6.1 RESULTS OF MATHEMATICAL MODEL  
RESONANT FREQUENCIES (cps)

EXCITATION	MODE				
	1	2	3	4	5
E-W	.276	.725	1.267	1.821	2.394
N-S	.276	.725	1.267	1.821	2.394
TORSIONAL	.403	1.089	1.840	2.625	3.465

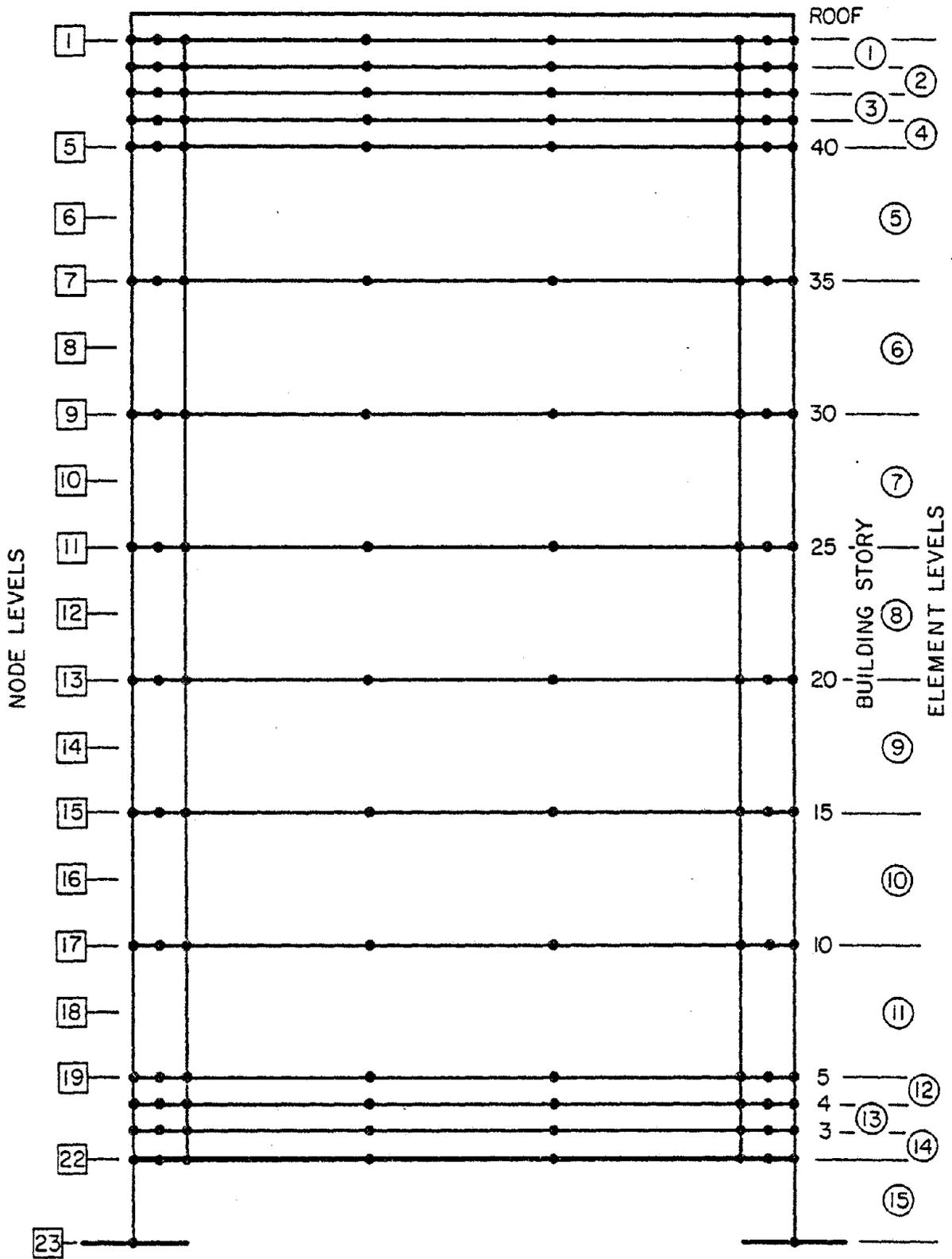


FIG. 6.1 MACROELEMENT SIMULATION OF STRUCTURE

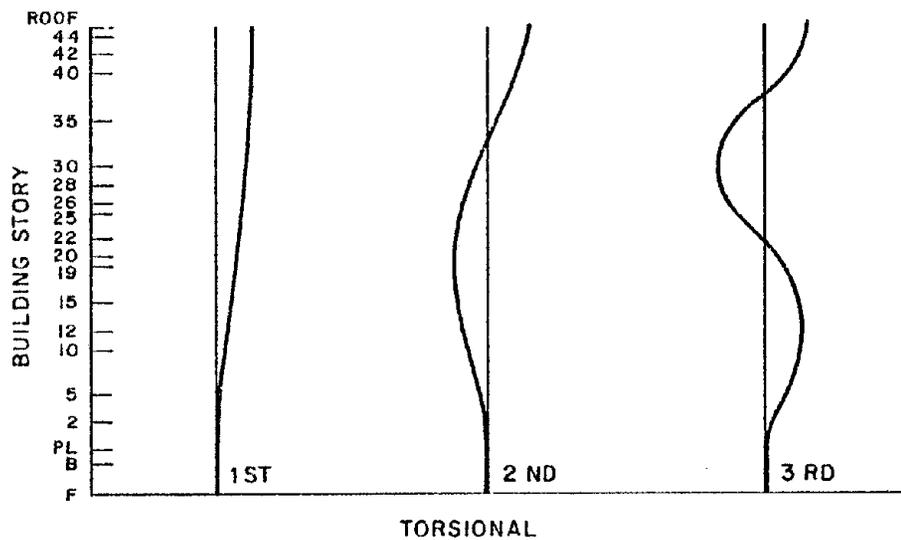
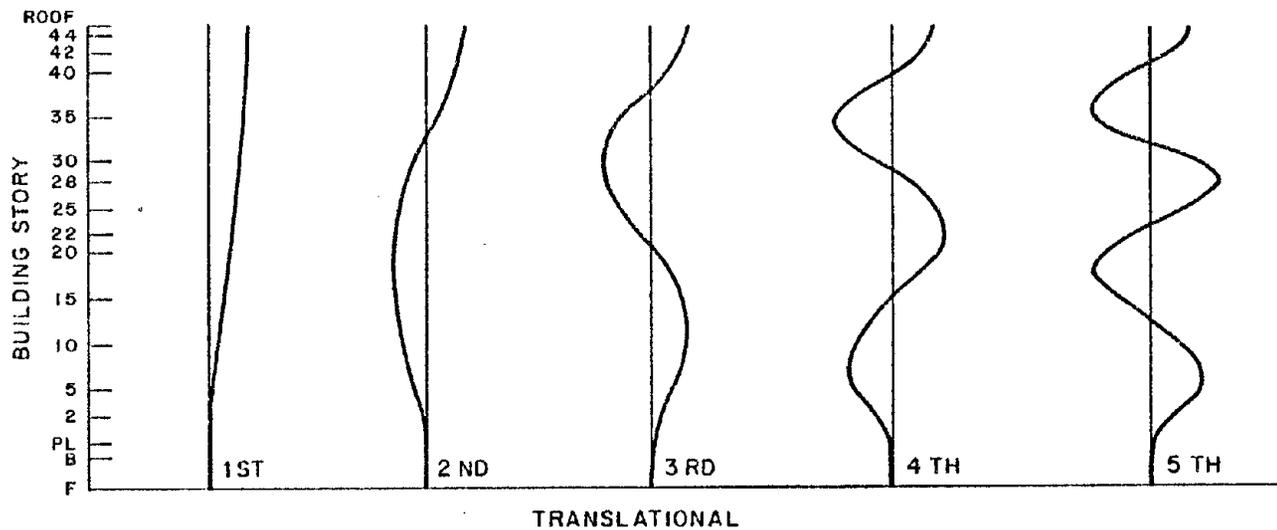


FIG. 6.2 MODE SHAPES

## 7. COMPARISON OF EXPERIMENTAL AND ANALYTICAL RESULTS

The resonant frequencies and damping factors obtained from the full-scale tests are summarized and compared in Table 7.1. The analytical results for the E-W translational were the same as the N-S translational and are only listed once. The mode shapes for the translational as well as the torsional motions are compared in Fig. 7.1.

A comparison of the translational analytical results show very good agreement with the experimental studies, the maximum difference ranging from about 3% at the first mode to about 14% at the higher modes. It would appear from the first translational mode shape that the actual building is slightly more flexible than what the analysis indicates. In comparing the torsional analytical results with the experimental the differences are in the range of 10%.

TABLE 7.1 COMPARISON OF RESONANT FREQUENCIES AND DAMPING FACTORS

Node No.	Translational E-W				Translational N-S					Torsional				
	Forced		Ambient		Forced		Ambient		Anal.	Forced		Ambient		Anal.
	f (cps)	$\xi$ (%)	f (cps)	$\xi$ (%)	f (cps)	$\xi$ (%)	f (cps)	$\xi$ (%)	f (cps)	f (cps)	$\xi$ (%)	f (cps)	$\xi$ (%)	f (cps)
1	0.267	2.62	0.263	2.37	0.267	4.34	0.273	3.30	.276	0.357	1.34	0.347	0.90	.403
2	0.76	1.51	0.76	1.33	0.783	1.60	0.791	1.28	.725	0.99	0.81	1.0	0.43	1.089
3	1.317	1.18	1.338	0.35	1.373	1.64	1.397	1.17	1.267	1.69	0.95	1.70	0.16	1.840
4	1.89	1.27	1.914	0.26	1.973	1.72	2.015	0.74	1.821	2.47	1.01	2.49	0.13	2.625
5	2.74	1.73	2.695	0.17	2.85	1.49	2.852	0.55	2.394	3.317	1.36	-	-	3.465
6	3.133	2.39	3.096	-	-	-	3.15	0.15	-	4.133	1.51	-	-	-

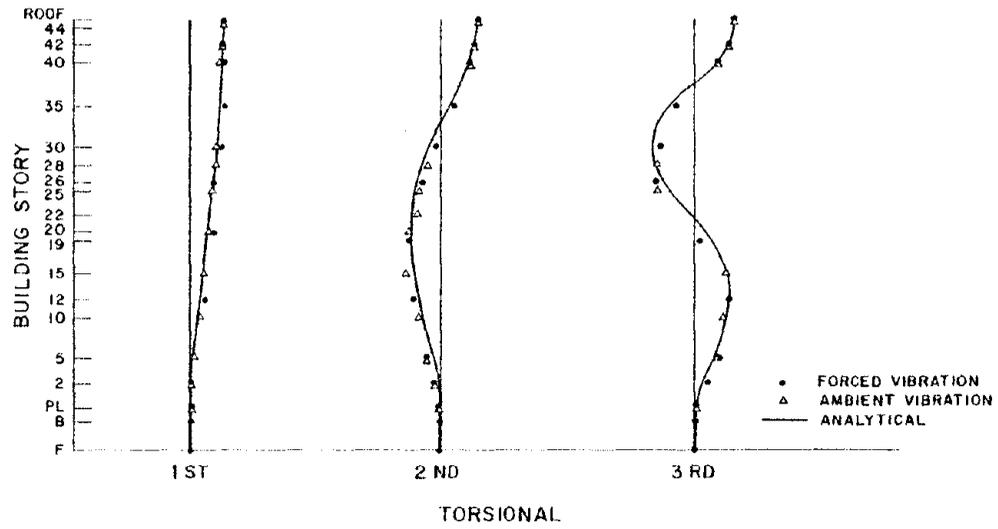
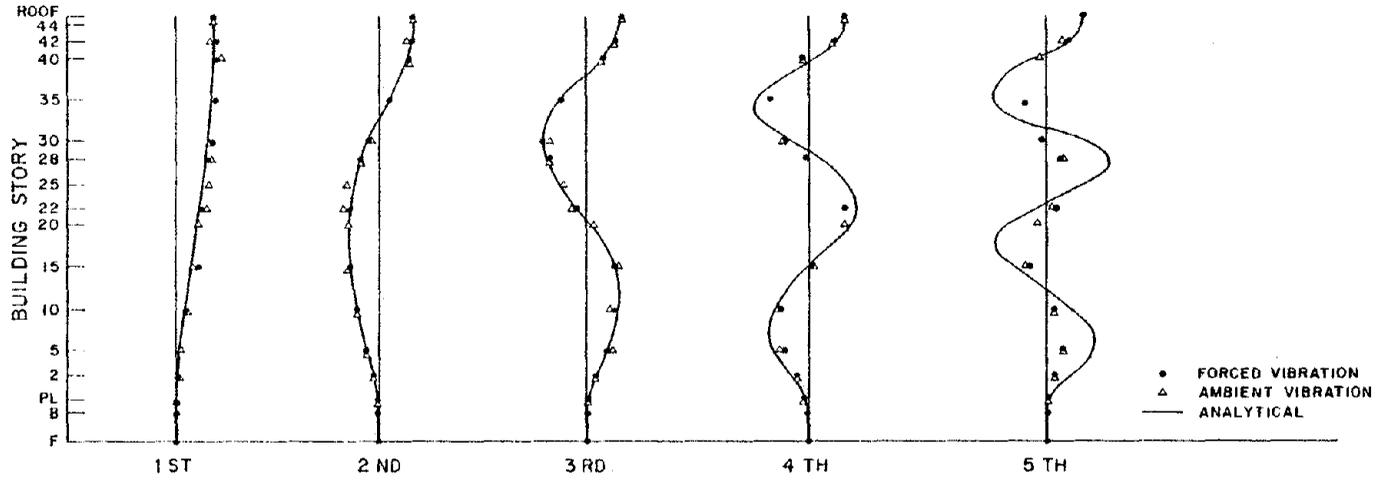


FIG. 7.1 TYPICAL MODE SHAPES

## 8. GENERAL CONCLUSIONS

The dynamic properties of the translational modes in the N-S and E-W directions, as well as the torsional modes of the Century City South-Theme Tower, were determined by forced vibration and ambient vibration studies.

The resonant frequencies from both studies are in very good agreement in all separated modes of vibration with the maximum difference smaller than 2%. The ratios of the observed higher mode frequencies with respect to the fundamental one from both dynamic studies of the building indicate that the overall structural response is predominantly of the shear type.

Comparison of the forced and ambient vibration experiments demonstrate that it is possible to determine with adequate accuracy the natural frequencies and mode shapes of typical modern buildings using the ambient vibration method. Difficulties in evaluation of equivalent viscous damping factors from ambient vibrations studies are present and probably it will be more realistic from this type of study to expect assessment of the range of damping factors, rather than damping values associated with each mode of vibration.

A comparison of the analytical results with the experimental results show very good agreement in the translational motion especially in the lower modes and fair agreement in the torsional modes.

## REFERENCES

1. Bouwkamp, J.G. and Rea, Dixon, "Dynamic Testing and Formulation of Mathematical Models", Chapter VIII in Earthquake Engineering, editor Wiegel, R.L., Prentice-Hall, 1970.
2. Caughey, T.K., "Classical Normal Modes in Damped Linear Systems", J. Appl. Mech., 27, 1960, p. 269-271.
3. Clough, R.W. and Penzien, J., Dynamics of Structures, McGraw-Hill, 1975.
4. Cooley, J.W. and Tukey, J.W., "An Algorithm for the Machine Calculation of Complex Fourier Series", Math. of Comp. 19, p. 297-301.
5. Crawford, R. and Ward, H.S., "Determination of the Natural Periods of Buildings", Bull. Seis. Soc. Am., 54, 1964, p. 1743-1756.
6. Horner, J.B. and Jennings, P.C., "Modal Interference in Vibration Tests", J. Engrg. Mech. Div., Proc. ASCE 95, EM4, 1965, p. 827, p. 827-839.
7. Hudson, D.E., "Synchronized Vibration Generators for Dynamic Tests of Full Scale Structures", Earthquake Engineering Research Laboratory Report, California Institute of Technology, Pasadena, 1962.
8. Kawasumi, K. and Kanai, K., "Small Amplitude Vibration of Actual Buildings", Proc. 1st World Conf. Earthquake Eng., Berkeley, 1956.
9. McLamore, V.R., "Ambient Vibration Survey of Chesapeake Bay Bridge", Teledyne Geostronics, Rep. No. 0370-2150, 1970.
10. Petrovski, J. and Jurukovski, D., "Dynamic Properties of Structures from Full-Scale Forced Vibration Studies and Formulation of Mathematical Models", UNESCO Interregional Seminar on Low Cost Construction Resistant to Earthquake and Hurricanes, Skopje, November 1971.
11. Petrovski, J. Jurukovski, D. and Percinkov, S., "Dynamic Properties of Multistory Trade Building and Formulation of the Mathematical Model", Publ. No. 24, Institute of Earthquake Engineering and Engineering Seismology, University of Skopje, 1971.
12. Rea, D., Bouwkamp, J.G. and Clough, R.W., "The Dynamic Behavior of Still Frame and Truss Buildings", AISI, Bulletin No. 9, April 1968.
13. Stephen, R.M., Hollings, J.P. and Bouwkamp, J.G., "Dynamic Behavior of a Multistory Pyramid-Shaped Building", Report No. EERC 73-17, Earthquake Engineering Research Center, University of California, Berkeley, 1973.
14. Trifunac, M.D., "Wind and Microtremor Induced Vibrations of a Twenty-Two Story Steel Frame Building", Earthquake Engineering Research Laboratory, EERL 70-10, California Institute of Technology, Pasadena, 1970.

15. Trifunac, M.D., "Comparisons Between Ambient and Forced Vibration Experiments", Earthquake Engineering and Structural Dynamics, 1, 1972, p. 133-150.
16. U.S. Coast and Geodetic Survey, "Earthquake Investigations in California, 1934-1935", Special Publication No. 201, U.S. Dept. of Commerce, Washington, D.C., 1936.
17. Ward, H.S. and Crawford, R., "Wind Induced Vibrations and Building Modes", Bull. Seism. Soc. Am. 56, 1966, p. 793-813.
18. Wilson, E.L., Bathe, K.J., Petersons, F.E. and Dovey, H.H., "Computer Program for Static and Dynamic Analysis of Linear Structural Systems", Earthquake Engineering Research Center, Report No. EERC 72-10, November 1972.
19. DeClercq, H., Powell, G.H., "Analysis and Design of Tube-Type Tall Building Structures", Earthquake Engineering Research Center, Report No. EERC 76-5, February 1976.

EARTHQUAKE ENGINEERING RESEARCH CENTER REPORTS

- EERC 67-1 "Feasibility Study Large-Scale Earthquake Simulator Facility," by J. Penzien, J. G. Bouwkamp, R. W. Clough and D. Rea - 1967 (PB 187 905)
- EERC 68-1 Unassigned
- EERC 68-2 "Inelastic Behavior of Beam-to-Column Subassemblages Under Repeated Loading," by V. V. Bertero - 1968 (PB 184 888)
- EERC 68-3 "A Graphical Method for Solving the Wave Reflection-Refraction Problem," by H. D. McNiven and Y. Mengi 1968 (PB 187 943)
- EERC 68-4 "Dynamic Properties of McKinley School Buildings," by D. Rea, J. G. Bouwkamp and R. W. Clough - 1968 (PB 187 902)
- EERC 68-5 "Characteristics of Rock Motions During Earthquakes," by H. B. Seed, I. M. Idriss and F. W. Kiefer - 1968 (PB 188 338)
- EERC 69-1 "Earthquake Engineering Research at Berkeley," - 1969 (PB 187 906)
- EERC 69-2 "Nonlinear Seismic Response of Earth Structures," by M. Dibaj and J. Penzien - 1969 (PB 187 904)
- EERC 69-3 "Probabilistic Study of the Behavior of Structures During Earthquakes," by P. Ruiz and J. Penzien - 1969 (PB 187 886)
- EERC 69-4 "Numerical Solution of Boundary Value Problems in Structural Mechanics by Reduction to an Initial Value Formulation," by N. Distefano and J. Schujman - 1969 (PB 187 942)
- EERC 69-5 "Dynamic Programming and the Solution of the Biharmonic Equation," by N. Distefano - 1969 (PB 187 941)

---

Note: Numbers in parenthesis are Accession Numbers assigned by the National Technical Information Service. Copies of these reports may be ordered from the National Technical Information Service, 5285 Port Royal Road, Springfield, Virginia, 22161. Accession Numbers should be quoted on orders for the reports (PB --- ---) and remittance must accompany each order. (Foreign orders, add \$2.50 extra for mailing charges.) Those reports without this information listed are not yet available from NTIS. Upon request, EERC will mail inquirers this information when it becomes available to us.

- EERC 69-6 "Stochastic Analysis of Offshore Tower Structures,"  
by A. K. Malhotra and J. Penzien - 1969 (PB 187 903)
- EERC 69-7 "Rock Motion Accelerograms for High Magnitude  
Earthquakes," by H. B. Seed and I. M. Idriss - 1969  
(PB 187 940)
- EERC 69-8 "Structural Dynamics Testing Facilities at the  
University of California, Berkeley," by R. M. Stephen,  
J. G. Bouwkamp, R. W. Clough and J. Penzien - 1969  
(PB 189 111)
- EERC 69-9 "Seismic Response of Soil Deposits Underlain by  
Sloping Rock Boundaries," by H. Dezfulian and  
H. B. Seed - 1969 (PB 189 114)
- EERC 69-10 "Dynamic Stress Analysis of Axisymmetric Structures  
under Arbitrary Loading," by S. Ghosh and E. L.  
Wilson - 1969 (PB 189 026)
- EERC 69-11 "Seismic Behavior of Multistory Frames Designed by  
Different Philosophies," by J. C. Anderson and  
V. V. Bertero - 1969 (PB 190 662)
- EERC 69-12 "Stiffness Degradation of Reinforcing Concrete  
Structures Subjected to Reversed Actions," by  
V. V. Bertero, B. Bresler and H. Ming Liao - 1969  
(PB 202 942)
- EERC 69-13 "Response of Non-Uniform Soil Deposits to Travel  
Seismic Waves," by H. Dezfulian and H. B. Seed - 1969  
(PB 191 023)
- EERC 69-14 "Damping Capacity of a Model Steel Structure," by  
D. Rea, R. W. Clough and J. G. Bouwkamp - 1969  
(PB 190 663)
- EERC 69-15 "Influence of Local Soil Conditions on Building  
Damage Potential during Earthquakes," by H. B. Seed  
and I. M. Idriss - 1969 (PB 191 036)
- EERC 69-16 "The Behavior of Sands under Seismic Loading  
Conditions," by M. L. Silver and H. B. Seed - 1969  
(AD 714 982)
- EERC 70-1 "Earthquake Response of Concrete Gravity Dams," by  
A. K. Chopra - 1970 (AD 709 640)
- EERC 70-2 "Relationships between Soil Conditions and Building  
Damage in the Caracas Earthquake of July 29, 1967," by  
H. B. Seed, I. M. Idriss and H. Dezfulian - 1970  
(PB 195 762)

- EERC 70-3 "Cyclic Loading of Full Size Steel Connections," by E. P. Popov and R. M. Stephen - 1970 (PB 213 545)
- EERC 70-4 "Seismic Analysis of the Charaima Building, Caraballeda, Venezuela," by Subcommittee of the SEAONC Research Committee: V. V. Bertero, P. F. Fratessa, S. A. Mahin, J. H. Sexton, A. C. Scordelis, E. L. Wilson, L. A. Wyllie, H. B. Seed and J. Penzien, Chairman - 1970 (PB 201 455)
- EERC 70-5 "A Computer Program for Earthquake Analysis of Dams," by A. K. Chopra and P. Chakrabarti - 1970 (AD 723 994)
- EERC 70-6 "The Propagation of Love Waves across Non-Horizontally Layered Structures," by J. Lysmer and L. A. Drake - 1970 (PB 197 896)
- EERC 70-7 "Influence of Base Rock Characteristics on Ground Response," by J. Lysmer, H. B. Seed and P. B. Schnabel - 1970 (PB 197 897)
- EERC 70-8 "Applicability of Laboratory Test Procedures for Measuring Soil Liquefaction Characteristics under Cyclic Loading," by H. B. Seed and W. H. Peacock - 1970 (PB 198 016)
- EERC 70-9 "A Simplified Procedure for Evaluating Soil Liquefaction Potential," by H. B. Seed and I. M. Idriss - 1970 (PB 198 009)
- EERC 70-10 "Soil Moduli and Damping Factors for Dynamic Response Analysis," by H. B. Seed and I. M. Idriss - 1970 (PB 197 869)
- EERC 71-1 "Koyna Earthquake and the Performance of Koyna Dam," by A. K. Chopra and P. Chakrabarti - 1971 (AD 731 496)
- EERC 71-2 "Preliminary In-Situ Measurements of Anelastic Absorption in Soils Using a Prototype Earthquake Simulator," by R. D. Borcherdt and P. W. Rodgers - 1971 (PB 201 454)
- EERC 71-3 "Static and Dynamic Analysis of Inelastic Frame Structures," by F. L. Porter and G. H. Powell - 1971 (PB 210 135)
- EERC 71-4 "Research Needs in Limit Design of Reinforced Concrete Structures," by V. V. Bertero - 1971 (PB 202 943)
- EERC 71-5 "Dynamic Behavior of a High-Rise Diagonally Braced Steel Building," by D. Rea, A. A. Shah and J. G. Bouwkamp - 1971 (PB 203 584)

- EERC 71-6 "Dynamic Stress Analysis of Porous Elastic Solids Saturated with Compressible Fluids," by J. Ghaboussi and E. L. Wilson - 1971 (PB 211 396)
- EERC 71-7 "Inelastic Behavior of Steel Beam-to-Column Subassemblages," by H. Krawinkler, V. V. Bertero and E. P. Popov - 1971 (PB 211 335)
- EERC 71-8 "Modification of Seismograph Records for Effects of Local Soil Conditions," by P. Schnabel, H. B. Seed and J. Lysmer - 1971 (PB 214 450)
- EERC 72-1 "Static and Earthquake Analysis of Three Dimensional Frame and Shear Wall Buildings," by E. L. Wilson and H. H. Dovey - 1972 (PB 212 904)
- EERC 72-2 "Accelerations in Rock for Earthquakes in the Western United States," by P. B. Schnabel and H. B. Seed - 1972 (PB 213 100)
- EERC 72-3 "Elastic-Plastic Earthquake Response of Soil-Building Systems," by T. Minami - 1972 (PB 214 868)
- EERC 72-4 "Stochastic Inelastic Response of Offshore Towers to Strong Motion Earthquakes," by M. K. Kaul - 1972 (PB 215 713)
- EERC 72-5 "Cyclic Behavior of Three Reinforced Concrete Flexural Members with High Shear," by E. P. Popov, V. V. Bertero and H. Krawinkler - 1972 (PB 214 555)
- EERC 72-6 "Earthquake Response of Gravity Dams Including Reservoir Interaction Effects," by P. Chakrabarti and A. K. Chopra - 1972 (AD 762 330)
- EERC 72-7 "Dynamic Properties on Pine Flat Dam," by D. Rea, C. Y. Liaw and A. K. Chopra - 1972 (AD 763 928)
- EERC 72-8 "Three Dimensional Analysis of Building Systems," by E. L. Wilson and H. H. Dovey - 1972 (PB 222 438)
- EERC 72-9 "Rate of Loading Effects on Uncracked and Repaired Reinforced Concrete Members," by S. Mahin, V. V. Bertero, D. Rea and M. Atalay - 1972 (PB 224 520)
- EERC 72-10 "Computer Program for Static and Dynamic Analysis of Linear Structural Systems," by E. L. Wilson, K.-J. Bathe, J. E. Peterson and H. H. Dovey - 1972 (PB 220 437)

- EERC 72-11 "Literature Survey - Seismic Effects on Highway Bridges," by T. Iwasaki, J. Penzien and R. W. Clough - 1972 (PB 215 613)
- EERC 72-12 "SHAKE-A Computer Program for Earthquake Response Analysis of Horizontally Layered Sites," by P. B. Schnabel and J. Lysmer - 1972 (PB 220 207)
- EERC 73-1 "Optimal Seismic Design of Multistory Frames," by V. V. Bertero and H. Kamil - 1973
- EERC 73-2 "Analysis of the Slides in the San Fernando Dams during the Earthquake of February 9, 1971," by H. B. Seed, K. L. Lee, I. M. Idriss and F. Makdisi - 1973 (PB 223 402)
- EERC 73-3 "Computer Aided Ultimate Load Design of Unbraced Multistory Steel Frames," by M. B. El-Hafez and G. H. Powell - 1973
- EERC 73-4 "Experimental Investigation into the Seismic Behavior of Critical Regions of Reinforced Concrete Components as Influenced by Moment and Shear," by M. Celebi and J. Penzien - 1973 (PB 215 884)
- EERC 73-5 "Hysteretic Behavior of Epoxy-Repaired Reinforced Concrete Beams," by M. Celebi and J. Penzien - 1973
- EERC 73-6 "General Purpose Computer Program for Inelastic Dynamic Response of Plane Structures," by A. Kanaan and G. H. Powell - 1973 (PB 221 260)
- EERC 73-7 "A Computer Program for Earthquake Analysis of Gravity Dams Including Reservoir Interaction," by P. Chakrabarti and A. K. Chopra - 1973 (AD 766 271)
- EERC 73-8 "Behavior of Reinforced Concrete Deep Beam-Column Subassemblages under Cyclic Loads," by O. Kustu and J. G. Bouwkamp - 1973
- EERC 73-9 "Earthquake Analysis of Structure-Foundation Systems," by A. K. Vaish and A. K. Chopra - 1973 (AD 766 272)
- EERC 73-10 "Deconvolution of Seismic Response for Linear Systems," by R. B. Reimer - 1973 (PB 227 179)
- EERC 73-11 "SAP IV: A Structural Analysis Program for Static and Dynamic Response of Linear Systems," by K.-J. Bathe, E. L. Wilson and F. E. Peterson - 1973 (PB 221 967)
- EERC 73-12 "Analytical Investigations of the Seismic Response of Long, Multiple Span Highway Bridges," by W. S. Tseng and J. Penzien - 1973 (PB 227 816)

- EERC 73-13 "Earthquake Analysis of Multi-Story Buildings Including Foundation Interaction," by A. K. Chopra and J. A. Gutierrez - 1973 (PB 222 970)
- EERC 73-14 "ADAP: A Computer Program for Static and Dynamic Analysis of Arch Dams," by R. W. Clough, J. M. Raphael and S. Majtahedi - 1973 (PB 223 763)
- EERC 73-15 "Cyclic Plastic Analysis of Structural Steel Joints," by R. B. Pinkney and R. W. Clough - 1973 (PB 226 843)
- EERC 73-16 "QUAD-4: A Computer Program for Evaluating the Seismic Response of Soil Structures by Variable Damping Finite Element Procedures," by I. M. Idriss, J. Lysmer, R. Hwang and H. B. Seed - 1973 (PB 229 424)
- EERC 73-17 "Dynamic Behavior of a Multi-Story Pyramid Shaped Building," by R. M. Stephen and J. G. Bouwkamp - 1973
- EERC 73-18 "Effect of Different Types of Reinforcing on Seismic Behavior of Short Concrete Columns," by V. V. Bertero, J. Hollings, O. Kustu, R. M. Stephen and J. G. Bouwkamp - 1973
- EERC 73-19 "Olive View Medical Center Material Studies, Phase I," by B. Bresler and V. V. Bertero - 1973 (PB 235 986)
- EERC 73-20 "Linear and Nonlinear Seismic Analysis Computer Programs for Long Multiple-Span Highway Bridges," by W. S. Tseng and J. Penzien - 1973
- EERC 73-21 "Constitutive Models for Cyclic Plastic Deformation of Engineering Materials," by J. M. Kelly and P. P. Gillis - 1973 (PB 226 024)
- EERC 73-22 "DRAIN - 2D User's Guide," by G. H. Powell - 1973 (PB 227 016)
- EERC 73-23 "Earthquake Engineering at Berkeley - 1973" - 1973 (PB 226 033)
- EERC 73-24 Unassigned
- EERC 73-25 "Earthquake Response of Axisymmetric Tower Structures Surrounded by Water," by C. Y. Liaw and A. K. Chopra - 1973 (AD 773 052)
- EERC 73-26 "Investigation of the Failures of the Olive View Stairtowers during the San Fernando Earthquake and Their Implications in Seismic Design," by V. V. Bertero and R. G. Collins - 1973 (PB 235 106)

- EERC 73-27 "Further Studies on Seismic Behavior of Steel Beam-Column Subassemblages," by V. V. Bertero, H. Krawinkler and E. P. Popov - 1973 (PB 234 172)
- EERC 74-1 "Seismic Risk Analysis," by C. S. Oliveira - 1974 (PB 235 920)
- EERC 74-2 "Settlement and Liquefaction of Sands under Multi-Directional Shaking," by R. Pyke, C. K. Chan and H. B. Seed - 1974
- EERC 74-3 "Optimum Design of Earthquake Resistant Shear Buildings," by D. Ray, K. S. Pister and A. K. Chopra - 1974 (PB 231 172)
- EERC 74-4 "LUSH - A Computer Program for Complex Response Analysis of Soil-Structure Systems," by J. Lysmer, T. Udaka, H. B. Seed and R. Hwang - 1974 (PB 236 796)
- EERC 74-5 "Sensitivity Analysis for Hysteretic Dynamic Systems: Applications to Earthquake Engineering," by D. Ray - 1974 (PB 233 213)
- EERC 74-6 "Soil-Structure Interaction Analyses for Evaluating Seismic Response," by H. B. Seed, J. Lysmer and R. Hwang - 1974 (PB 236 519)
- EERC 74-7 Unassigned
- EERC 74-8 "Shaking Table Tests of a Steel Frame - A Progress Report," by R. W. Clough and D. Tang - 1974
- EERC 74-9 "Hysteretic Behavior of Reinforced Concrete Flexural Members with Special Web Reinforcement," by V. V. Bertero, E. P. Popov and T. Y. Wang - 1974 (PB 236 797)
- EERC 74-10 "Applications of Reliability-Based, Global Cost Optimization to Design of Earthquake Resistant Structures," by E. Vitiello and K. S. Pister - 1974 (PB 237 231)
- EERC 74-11 "Liquefaction of Gravelly Soils under Cyclic Loading Conditions," by R. T. Wong, H. B. Seed and C. K. Chan - 1974
- EERC 74-12 "Site-Dependent Spectra for Earthquake-Resistant Design," by H. B. Seed, C. Ugas and J. Lysmer - 1974

- EERC 74-13 "Earthquake Simulator Study of a Reinforced Concrete Frame," by P. Hidalgo and R. W. Clough - 1974 (PB 241 944)
- EERC 74-14 "Nonlinear Earthquake Response of Concrete Gravity Dams," by N. Pal - 1974 (AD/A006583)
- EERC 74-15 "Modeling and Identification in Nonlinear Structural Dynamics, I - One Degree of Freedom Models," by N. Distefano and A. Rath - 1974 (PB 241 548)
- EERC 75-1 "Determination of Seismic Design Criteria for the Dumbarton Bridge Replacement Structure, Vol. I: Description, Theory and Analytical Modeling of Bridge and Parameters," by F. Baron and S.-H. Pang - 1975
- EERC 75-2 "Determination of Seismic Design Criteria for the Dumbarton Bridge Replacement Structure, Vol. 2: Numerical Studies and Establishment of Seismic Design Criteria," by F. Baron and S.-H. Pang - 1975
- EERC 75-3 "Seismic Risk Analysis for a Site and a Metropolitan Area," by C. S. Oliveira - 1975
- EERC 75-4 "Analytical Investigations of Seismic Response of Short, Single or Multiple-Span Highway Bridges," by Ma-chi Chen and J. Penzien - 1975 (PB 241 454)
- EERC 75-5 "An Evaluation of Some Methods for Predicting Seismic Behavior of Reinforced Concrete Buildings," by Stephen A. Mahin and V. V. Bertero - 1975
- EERC 75-6 "Earthquake Simulator Study of a Steel Frame Structure, Vol. I: Experimental Results," by R. W. Clough and David T. Tang - 1975 (PB 243 981)
- EERC 75-7 "Dynamic Properties of San Bernardino Intake Tower," by Dixon Rea, C.-Y. Liaw, and Anil K. Chopra - 1975 (AD/A008406)
- EERC 75-8 "Seismic Studies of the Articulation for the Dumbarton Bridge Replacement Structure, Vol. I: Description, Theory and Analytical Modeling of Bridge Components," by F. Baron and R. E. Hamati - 1975
- EERC 75-9 "Seismic Studies of the Articulation for the Dumbarton Bridge Replacement Structure, Vol. 2: Numerical Studies of Steel and Concrete Girder Alternates," by F. Baron and R. E. Hamati - 1975

- EERC 75-10 "Static and Dynamic Analysis of Nonlinear Structures,"  
by Digambar P. Mondkar and Graham H. Powell - 1975  
(PB 242 434)
- EERC 75-11 "Hysteretic Behavior of Steel Columns," by E. P. Popov,  
V. V. Bertero and S. Chandramouli - 1975
- EERC 75-12 "Earthquake Engineering Research Center Library Printed  
Catalog" - 1975 (PB 243 711)
- EERC 75-13 "Three Dimensional Analysis of Building Systems,"  
Extended Version, by E. L. Wilson, J. P. Hollings and  
H. H. Dovey - 1975 (PB 243 989)
- EERC 75-14 "Determination of Soil Liquefaction Characteristics by  
Large-Scale Laboratory Tests," by Pedro De Alba, Clarence  
K. Chan and H. Bolton Seed - 1975
- EERC 75-15 "A Literature Survey - Compressive, Tensile, Bond and  
Shear Strength of Masonry," by Ronald L. Mayes and  
Ray W. Clough - 1975
- EERC 75-16 "Hysteretic Behavior of Ductile Moment Resisting Reinforced  
Concrete Frame Components," by V. V. Bertero and  
E. P. Popov - 1975
- EERC 75-17 "Relationships Between Maximum Acceleration, Maximum  
Velocity, Distance from Source, Local Site Conditions  
for Moderately Strong Earthquakes," by H. Bolton Seed,  
Ramesh Murarka, John Lysmer and I. M. Idriss - 1975
- EERC 75-18 "The Effects of Method of Sample Preparation on the Cyclic  
Stress-Strain Behavior of Sands," by J. Paul Mulilis,  
Clarence K. Chan and H. Bolton Seed - 1975
- EERC 75-19 "The Seismic Behavior of Critical Regions of Reinforced  
Concrete Components as Influenced by Moment, Shear and  
Axial Force," by B. Atalay and J. Penzien - 1975
- EERC 75-20 "Dynamic Properties of an Eleven Story Masonry Building,"  
by R. M. Stephen, J. P. Hollings, J. G. Bouwkamp and  
D. Jurukovski - 1975
- EERC 75-21 "State-of-the-Art in Seismic Shear Strength of Masonry -  
An Evaluation and Review," by Ronald L. Mayes and  
Ray W. Clough - 1975
- EERC 75-22 "Frequency Dependencies Stiffness Matrices for Viscoelastic  
Half-Plane Foundations," by Anil K. Chopra, P. Chakrabarti  
and Gautam Dasgupta - 1975
- EERC 75-23 "Hysteretic Behavior of Reinforced Concrete Framed Walls,"  
by T. Y. Wong, V. V. Bertero and E. P. Popov - 1975

- EERC 75-24 "Testing Facility for Subassemblages of Frame-Wall Structural Systems," by V. V. Bertero, E. P. Popov and T. Endo - 1975
- EERC 75-25 "Influence of Seismic History of the Liquefaction Characteristics of Sands," by H. Bolton Seed, Kenji Mori and Clarence K. Chan - 1975
- EERC 75-26 "The Generation and Dissipation of Pore Water Pressures during Soil Liquefaction," by H. Bolton Seed, Phillippe P. Martin and John Lysmer - 1975
- EERC 75-27 "Identification of Research Needs for Improving a Seismic Design of Building Structures," by V. V. Bertero - 1975
- EERC 75-28 "Evaluation of Soil Liquefaction Potential during Earthquakes," by H. Bolton Seed, I. Arango and Clarence K. Chan 1975
- EERC 75-29 "Representation of Irregular Stress Time Histories by Equivalent Uniform Stress Series in Liquefaction Analyses," by H. Bolton Seed, I. M. Idriss, F. Makdisi and N. Banerjee 1975
- EERC 75-30 "FLUSH - A Computer Program for Approximate 3-D Analysis of Soil-Structure Interaction Problems," by J. Lysmer, T. Udaka, C.-F. Tsai and H. B. Seed - 1975
- EERC 75-31 "ALUSH - A Computer Program for Seismic Response Analysis of Axisymmetric Soil-Structure Systems," by E. Berger, J. Lysmer and H. B. Seed - 1975
- EERC 75-32 "TRIP and TRAVEL - Computer Programs for Soil-Structure Interaction Analysis with Horizontally Travelling Waves," by T. Udaka, J. Lysmer and H. B. Seed - 1975
- EERC 75-33 "Predicting the Performance of Structures in Regions of High Seismicity," by Joseph Penzien - 1975
- EERC 75-34 "Efficient Finite Element Analysis of Seismic Structure - Soil - Direction," by J. Lysmer, H. Bolton Seed, T. Udaka, R. N. Hwang and C.-F. Tsai - 1975
- EERC 75-35 "The Dynamic Behavior of a First Story Girder of a Three-Story Steel Frame Subjected to Earthquake Loading," by Ray W. Clough and Lap-Yan Li - 1975
- EERC 75-36 "Earthquake Simulator Study of a Steel Frame Structure, Volume II - Analytical Results," by David T. Tang - 1975
- EERC 75-37 "ANSR-I General Purpose Computer Program for Analysis of Non-Linear Structure Response," by Digambar P. Mondkar and Graham H. Powell - 1975

- EERC 75-38 "Nonlinear Response Spectra for Probabilistic Seismic Design and Damage Assessment of Reinforced Concrete Structures," by Masaya Murakami and Joseph Penzien - 1975
- EERC 75-39 "Study of a Method of Feasible Directions for Optimal Elastic Design of Framed Structures Subjected to Earthquake Loading," by N. D. Walker and K. S. Pister - 1975
- EERC 75-40 "An Alternative Representation of the Elastic-Viscoelastic Analogy," by Gautam Dasgupta and Jerome L. Sackman - 1975
- EERC 75-41 "Effect of Multi-Directional Shaking on Liquefaction of Sands," by H. Bolton Seed, Robert Pyke and Geoffrey R. Martin - 1975
- EERC 76-1 "Strength and Ductility Evaluation of Existing Low-Rise Reinforced Concrete Buildings - Screening Method," by Tsuneo Okada and Boris Bresler - 1976
- EERC 76-2 "Experimental and Analytical Studies on the Hysteretic Behavior of Reinforced Concrete Rectangular and T-Beams," by Shao-Yeh Marshall Ma, Egor P. Popov and Vitelmo V. Bertero - 1976
- EERC 76-3 "Dynamic Behavior of a Multistory Triangular-Shaped Building," by J. Petrovski, R. M. Stephen, E. Gartenbaum and J. G. Bouwkamp - 1976