UILU-ENG-77-2016

EFFECT OF BEAM STRENGTH AND STIFFNESS ON DYNAMIC BEHAVIOR OF REINFORCED CONCRETE COUPLED WALLS

Volume 1: Text

By

John M. Lybas

and

Mete A. Sozen

A Report to the
NATIONAL SCIENCE FOUNDATION
Research Grant ENV74-22962

University of Illinois
at Urbana-Champaign
Urbana, Illinois
July 1977

BIBLIOGRAPHIC DATA SHEET	1. Report No. NSF/RA-770274	2.	3. Recipient's Accession No. PB273876
4. Title and Subtitle Effect of Beam Strength and Stiffness on Dynamic Behavior of Reinforced Concrete Coupled Walls, Volume 1: Text (Civil Engineering Studies, Structural Research Series No. 444)			5. Report Date July 1977
			6.
7. Author(s) J.M. Lybas, M.A. Sozen			8. Performing Organization Rept. No. UILU-ENG-77-2016
9. Performing Organization Name and Address University of Illinois at Urbana-Champaign Urbana, Illinois 61801			10. Project/Task/Work Unit No.
			11. Contract/Grant No. ENV7422962
12. Sponsoring Organization Name and Address Research Applied to National Needs (RANN) National Science Foundation Washington, D.C. 20550			13. Type of Report & Period Covered
			14.

15. Supplementary Notes

Volume 2 is also available from NTIS

16. Abstracts

This report attempted to develop an understanding of the response of reinforced concrete coupled wall systems to seismic loading. Five test structures (approximately one-twelfth scale) were subjected to one component of the earthquake base motion measured at El Centro, California (1940). The base motions were strong enough to cause yielding of the test structures. A sixth test structure was subjected to slowly applied cyclic lateral loading. The experimental program is outlined in Chapter 2, while the results are presented in Chapter 3. The details of experimental procedures, along with the characteristics of the test specimens and materials, are given in Appendix A. An analytical study of the static hysteretic response of the test structures was undertaken. Equivalent viscous damping factors, consistent with the calculated overall structure hysteresis relation, were determined. The variation of damping factor with response mode and response amplitude was studied. The study of static hysteretic response is presented in Chapter 5. The feasibility of simulating the observed dynamic responses with a linear viscously dampled analytical model was investigated. Both response-spectrum analyses and response-history analyses were performed. The study is presented in Chapter 7. Finally, the experimental results were compared with the results of the analytical studies. The comparison is described in Chapter 8.

17. Key Words and Document Analysis. 17a. Descriptors

Earthquakes
Earthquake resistant structures
Earthquake resistant construction
Concrete construction
Earth pressure
Loads (forces)

17b. Identifiers/Open-Ended Terms

Seismic loading

17c. COSATI Field/Group

18. Availability Statement NTIS	19. Security Class (This Report) UNCLASSIFIED	21. No. of Pages 256
	20. Security Class (This Page) UNCLASSIFIED	22. Price A12-A01

FORM NTIS-35 (REV. 10-73) ENDORSED BY ANSI AND UNESCO. THIS FORM MAY BE REPRODUCED USCOMM-DC 8265-P74

CAPITAL SYSTEMS GROUP, INC.
6110 EXECUTIVE BOULEVARD
SUITE 250
ROCKVILLE, MARYLAND 20852

ACKNOWLEDGMENT

The work described in this report was part of a continuing study of the response of reinforced concrete buildings to earthquake loading, being carried out in the Structural Research Laboratory of the Department of Civil Engineering of the University of Illinois. The work is sponsored by the Research Applied to National Needs Program of the National Science Foundation under grant ENV74-22962.

The writers acknowledge the helpful suggestions of S. Otani, former Research Associate in the Department of Civil Engineering, and W. C. Schnobrich, Professor of Civil Engineering. Appreciation is also due to A. R. Robinson, Professor of Civil Engineering, for his numerical method which simplified the work for determining eigenvalues. J. Bryja, S. Staffier, M. Killion and N. Gavlin, research assistants in the Department of Civil Engineering, participated in the experimental portion of the study.

Acknowledgment is due to Professor V. J. MacDonald and Mr. G. Lafenhagen for their invaluable assistance in instrumentation and data reduction. Mr. W. McKenzie helped in the casting of the test specimens. Mr. O. H. Ray and his staff helped in preparing the test specimens and setup.

The IBM 360/75 and DEC System 10 computer systems of the Digital Computation Laboratory of the University of Illinois were used for the data reduction and analytical studies.

This report was based on a doctoral dissertation by J. M. Lybas.

TABLE OF CONTENTS

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

Figure | Page

Figure | Page

Figure | Page

CHAPTER 1

INTRODUCTION

1.1 Object and Scope

This report describes a study aimed at developing an understanding of the response of reinforced concrete coupled wall systems to seismic loading. The study had analytical and experimental phases as described below.

Five test structures (approximately one-twelfth scale) were subjected to one component of the earthquake base motion measured at El Centro, California (1940). The base motions were strong enough to cause yielding of the test structures. A sixth test structure was subjected to slowly applied cyclic lateral loading. The experimental program is outlined in chapter 2, while the results are presented in chapter 3. The details of experimental procedures, along with the characteristics of the test specimens and materials, are given in appendix A.

An analytical study of the static hysteretic response of the test structures was undertaken. The effect of the hysteresis relations of the members on the overall hysteresis relation of the structure was studied. Equivalent viscous damping factors, consistent with the calculated overall structure hysteresis relation, were determined. The variation of damping factor with response mode and response amplitude was studied. The study of static hysteretic response is presented in chapter 5.

The feasibility of simulating the observed dynamic responses with a linear viscously damped analytical model was investigated. Both response-spectrum analyses and response-history analyses were performed. The study is presented in chapter 7.

Finally, the experimental results were compared with the results of the analytical studies. The comparison is described in chapter 8.

1.2 Previous Research

Most previous research in the response of reinforced concrete coupled wall systems to lateral loading has been analytical in nature. Recently, several experimental studies have been undertaken.

One class of analytical models for the response of coupled wall systems to lateral loading considers the connecting beams to be replaced by a continuous lamina. Several papers discussing the application of this model to planar structures are Beck (1962), Coull and Choudhury (Feb. 1967), Coull and Choudhury (Sept. 1967), Coull (1971) and Rosman (1964). The concept was extended to three dimensional buildings by Rosman (1970). Several limitations of the laminar models are discussed by MacLeod (1970). The laminar concept of analysis is modified to account for piers of grossly unequal width by Arvidsson (1974).

A method for calculating the strength of coupled wall systems is presented by Winokur and Gluck (1968). Paulay (1970) applies the laminar approach in a step-by-step manner, considering elasto-plastic member behavior, to determine the failure load and mechanism for a coupled wall system. Consideration of ductility requirements are emphasized. Gluck (1973) also applies the laminar method to determine a failure mechanism, and also considers ductility requirements.

Mahin and Bertero (1976) report an analytical study of the nonlinear behavior of an 18-story coupled wall structure under earthquake base motion. The importance of the strength and stiffness of the coupling beams on dynamic behavior is emphasized.

A number of experimental studies are described in the literature.

A study is described by Aristizabal and Sozen (1976), in which ten story coupled wall systems were tested under earthquake base motion and the results compared to a linear dynamic response model.

Paulay (1971) reports tests of isolated coupling beams with various amounts of longitudinal and shear reinforcement. Tests of small-scale coupling beams are reported by Irwin and Ord (1976), in which variables include depth and reinforcement ratio. Paulay and Binney (1974) report further tests of coupling beams in which the concept of diagonally placed reinforcement is presented as a means for avoiding shear failure.

Test of coupled wall systems, with diagonally reinforced coupling beams, are reported by Paulay and Santhakunar (1976).

CHAPTER 2

OUTLINE OF EXPERIMENTAL PROGRAM

A series of five reinforced concrete coupled wall systems were subjected to earthquake base motions on the University of Illinois Earthquake Simulator. An additional specimen was subjected to statically applied lateral loading. Each test structure consisted of two walls fastened to the earthquake simulator parallel to each other, such that earthquake motion would result in bending of the walls in their strong direction. Each wall consisted of two piers coupled at six levels by deep beams. Each pier had a nominal depth of seven inches and a nominal thickness of one inch, and was of uniform section throughout the height of the wall (Fig. A.17). The reinforcing steel was uniformly distributed over the cross-section for a steel ratio of one percent (Fig. A.18-20). The beams were spaced uniformly along the height of the wall nominally at nine in. center to center and had a nominal span of four in. and a nominal thickness of one in. Hence, the test structure had a total height of approximately 60 in. The beams were doubly reinforced, with equal steel areas at the top and at the bottom. The longitudinal steel ratio (each layer) varied from 2.2% to 0.59%, while the depth varied from 2.25 to 1.5 in. In a given test structure all beams were identical. Dead load was simulated by 2000 lb. of steel placed at the levels of the second, fourth and sixth connecting beams. This provided a total of 6000 lb. of dead load on a test structure. The weights were connected at four points, such that the vertical load was applied through the centerline of the four piers, so as not to induce moments in the piers or beams. The connection was also such that rotation of the piers about their strong axis was not restrained. Failure of the test structure about the weak axis of the piers was prevented by steel diaphragms bolted at top and bottom to the steel weights (Fig. A.28). The specimen-to-simulator connection was designed to simulate a fixed base condition for the test structure.

The principal variable in the series was the strength and stiffness of the connecting beams. The specimens were grouped into three classes according to their beam cross-section. For Specimen Type A, the depth was 2.25 in. with a longitudinal steel ratio of 2.2%; for type B, 1.5 in. and 1.02%; and for type C, 1.5 in. and 0.59%. Designations of test structures of different types are recorded in Fig. 2.1. Dynamic and static tests have the prefix D and S.

All dynamic tests used the north-south component of the base motion measured at El Centro, California in the 1940 Imperial Valley Earthquake. The time scale of the earthquake was compressed by a factor of 5.0 to be compatible with the test structure. The acceleration level was magnified to suit the needs of the particular test run. Each dynamic test consisted of several test runs. In essence, the test structure was subjected to the earthquake motion several times, the acceleration levels of the base motion being increased in successive runs for a given structure. Each specimen was tested to failure. During each test run, continuous measurements of the lateral deflection and acceleration at the level of each weight, in the direction of simulator motion, were recorded. A continuous recording was also made of the acceleration at the base of the test structure (Fig. A.29).

The static test was carried out with the specimen mounted on the earthquake-simulator platform and the simulator restrained from motion. This was done to provide base conditions similar to that in the dynamic tests. The loading was applied to the test structure, along the axis of dynamic test simulator motion by two-way hydraulic rams at the levels of the three weights. Several times the test structure was loaded into the inelastic range, unloaded, loaded into the inelastic range in the opposite direction and then unloaded again. The loads in the three rams were maintained in a constant ratio given by the shape of the computed first mode of the test structure. Continuous measurements were recorded of the lateral deflections at the levels of the weights and of the loads in the rams.

Detailed information on the test structures, testing procedures, instrumentation and data reduction is provided in Appendix A. The test results are presented in Chapter 3. Figure 2.1 illustrates the organization of the experimental program.

CHAPTER 3

OBSERVED RESPONSE

3.1 General Comments

(a) Organization of Presentation

The results of the experimental program outlined in the previous chapter are presented here. The organization of the presentation is such that the results are first grouped according to specimen type. Several important classes of results are described for each specimen type, all test runs for the particular specimen type being included under each class.

(a) The condition of the specimen at the start of the initial test run of each test is discussed. Comments are made concerning whether it was damaged in removing the forms after casting, in transporting the specimen from forms to simulator, or in placing the test weights and completing the test setup.

(b) The earthquake base motions are discussed. As described in Appendix A, the north-south component of the 1940 Imperial Valley Earthquake measured at El Centro, California is used for all test runs. However, the maximum acceleration was varied from test run to test run. Elastic response spectra computed from the observed base motion for a single degree of freedom system are provided for several values of viscous damping coefficient. The usual tripartite plot format is first provided, showing all relations plotted together in a compact manner. This format, however, has its disadvantages for qualitatively observing or quantitatively measuring the variation of acceleration or displacement with frequency. The logarithmic scale of the tripartite plot format make variations in response with frequency less obvious and considerably more difficult to measure. Hence, the response spectra for each test run are also plotted with the acceleration and displacement on a linear scale. In general, response spectra are provided for the north wall only. However, for one run in each test, linear response spectra are provided for both walls.

(c) The natural frequencies of the specimen measured in free vibration tests before each test run are discussed. The natural frequency observed during the final two seconds of specimen response is also provided. In most cases, it was possible to excite and measure the frequencies of both the first and second modes.

(d) The observed horizontal displacements and accelerations are discussed. At this point, some clarification of the plot format is in order. The accelerometers and differential transformers were placed two to each test weight, along the axes of the two walls of the test structure. Therefore, for each type of instrument, it is possible to think of two groups of three instruments each, a group along the axis of the south wall and a group along the axis of the north wall. The response histories are plotted three to a page, each page representing the response measured from either the south group of instruments or the north group of instruments. The lowest plot on the page is associated with the instrument attached to the lower test weight, the middle plot with the instrument attached to the middle weight, and the top plot with the instrument attached to the top weight. The plots of observed horizontal displacements and accelerations are presented for the north wall only.

(e) The observed base shear and base moment are discussed. The observed accelerations were used to calculate the response histories for base shear and base moment on a point-by-point basis, resulting in a response history for the north wall and a response history for the south wall for each of the two functions. These were plotted along with observed base acceleration, each page of plots consisting of base acceleration, base shear, and base moment for a wall. In most cases these plots are provided for the north wall only. For one run in each test, they are provided for both walls.

(f) The distribution and development of the cracks are illustrated in figures and described. Along, with this the failure mechanism is described. The yielding and other alterations in specimen behavior with successive test runs is illustrated by comparing maximum observed responses to spectrum intensity of observed base motions.

(g) The deflected shape is illustrated by plotting the observed deflections at the levels of the three weights one above the other, at several predetermined times. The times were chosen to correspond to either positive relative maxima or negative relative maxima in the response history.

Results of dynamic tests are summarized in Tables 3.1 through 3.7. As mentioned in the introduction, the response histories and response spectra are included in Volume II of this report.

(b) Terminology

Additional comments need to be made concerning certain terminology on the figures and in the text. In several places, for example, the figures depicting variation of response with spectrum intensity, reference is made to average maximum response. This refers to the average of the maximum response observed for the two instruments attached to a particular test weight. This is reasonable in many cases because the two response histories measured at a given test weight are almost identical.

Another qualification made in several places is maximum double-amplitude displacement, as opposed to maximum single-amplitude displacement. Maximum double-amplitude displacement is the largest total of a positive relative maximum and a negative relative maximum which are parts of the same cycle of response. In cases of significant residual plastic deformation, or permanent set, this is a more useful measure of displacement than single-amplitude maxima.

Reference is made to response in a given mode. By "first mode" it is meant that the responses at the three levels at a given time are phased and occur at a frequency that would be compatible with the first mode of the structure of which dynamic characteristics change during a given test.

(c) Spectrum Intensity and Maximum Base Acceleration

In describing the behavior of the system to increasingly intense base motions, it is necessary to choose some function or parameter to represent this base motion intensity. Two parameters often used are maximum base acceleration and Housner's spectrum intensity (Ref. 17). Figure 3.1 compares these two parameters for each test run in the dynamic test program.

Points are reported for records observed at the bases of both the north and south walls. Where results are the same for both north and south walls, only one point is plotted. The two parameters are proportional for tests D1, D4, and D5, although D1-5 deviates somewhat from the pattern set by earlier runs in the same dynamic test. Similar results would be obtained comparing either parameter to the maximum observed responses. This proportionality is not present, however, for the tests D2 and D3. It was decided to use spectrum intensity for the response comparisons. Maximum base acceleration was judged to be more sensitive to high frequency components or narrow, isolated peaks in the base acceleration response history. These isolated peaks would have little effect upon an integrated quantity such as spectrum intensity.

3.2 Dynamic Tests of Specimen Type A

(a) State Before Test

The only cracks observed in the test structure were those due to shrinkage. The pattern of shrinkage cracks is depicted in Fig. 3.7.

(b) Loading

The maximum base acceleration ranged from 0.12 G for test run D1-1 to 2.2 G for test run D1-5, the intention being to double the maximum base acceleration successively for each test run, as listed in Table 3.5. The measured response histories for base accelerations are plotted in Fig. 3.6. There are some high-frequency noise components in the response for test runs D1-1 and D1-2. This was due to the low amplitude of the base motion. The level of the base acceleration was rather close to the level of accuracy of the accelerometer. Linear response spectra are provided in Fig. 3.2 and 3.3.

(c) Frequencies

The observed first-mode frequency (very small amplitude, free vibration) varied from 12 Hz before test run D1-1 to 3.3 Hz at end of test run D1-5. The second mode of the test structure was not excited in its undamaged state. It was possible, however, to obtain an observation before test run D1-2. At this stage, the frequency was 32 Hz and decreased to 20 Hz after test run D1-5. The observed frequencies are listed in Table 3.6.

Because of the amplitude difference, the frequency at the end of a test run should not be compared directly with the frequency measured before the following test run.

(d) Accelerations

The response histories for horizontal accelerations are shown in Fig. 3.4. The maximum observed horizontal accelerations are listed in Table 3.1 During runs D1-1 through D1-3, the acceleration response was primarily in the first mode. Despite the high-frequency content of the floor 2 record, the phasing and variation over the height of the acceleration amplitudes were consistent with the first mode.

The character of the lower level acceleration for test runs D1-4 and D1-5 is due to the fact that the base accelerations constitute a visibly large portion of the absolute acceleration.

In general the acceleration response histories exhibit very little noise. There is some noise in test run D1-1, but this is not surprising considering the low amplitude level of the test run.

Finally, it should be noted that the accelerations of the north and south walls were almost identical. Torsional response does not appear to have been significant.

(e) Displacements

The response histories for displacement are shown in Fig. 3.5. The maximum single-amplitude displacements at the level of the top weight range from 0.059 in. in test run D1-1 to 1.05 in. for test run D1-5. The maximum observed responses are listed in Tables 3.2 and 3.3.

Again, due to the low amplitude of response, high frequency noise is present in the records for test runs D1-1 and D1-2. To some extent, this is also true for test run D1-3.

The records exhibit first-mode phasing for all test runs. There is no evidence of higher mode components in the response histories, not even in the final test run.

The residual displacements in the test structure are listed for each test run in Table 3.4. No significant permanent displacement developed during test runs D1-1 through D1-3. During test run D1-4, however, permanent inelastic displacement did begin to develop and eventually attained a value of 0.21 in. at the top level at the close of test run D1-5. It should be noted that residual displacements for the north and south walls differed in the last two runs. However, the observed maximum responses (Table 3.2 and 3.3) indicated negligible torsional component.

(f) Base Shear

The response histories for base shear are provided in Fig. 3.6. The response varies from 0.5 kip in test run D1-1 to 3.5 kips in test run D1-5. The maxima are listed in Table 3.5.

The base shear response, although dominated by the first mode, does appear to contain a higher mode component that becomes stronger with succeeding test runs. This component had a frequency of approximately 20 Hz in test run D1-5, and is most likely associated with the second mode. There is no evidence of torsion in the base shears calculated for the two walls.

(g) Base Moment

The response histories for base moment are provided in Fig. 3.6. The maxima vary from 20 k-in. in test run D1-1 to 105 k-in. in test run D1-5 (Table 3.5).

The base moment also exhibits a higher mode component that becomes increasingly obvious in successive test runs. The component does not, however, become nearly as strong as in the base shear response. Again its frequency during test run D1-5 appears to be approximately 20 Hz. There is no torsion apparent in the response histories for base moment.

(h) Failure Mechanism

The crack patterns are depicted for each wall, at the end of each test run, in Fig. 3.7. Several of the sketches include two successive runs. The crack pattern at the end of the earlier run of the set is shown by solid lines. The additional cracks due to the later run of the set is shown by dashed lines.

The failure mechanism for the test structure was characterized by the bases of the piers attaining their maximum axial tension capacity. None of the connecting beams appears to have yielded.

The variation of observed response with spectrum intensity is depicted in Fig. 3.8 through 3.10. Fig. 3.11 illustrates the variation of base shear and base moment with displacement. The responses plotted are the average of the maxima measured for the north and south walls. The variation of top level acceleration with spectrum intensity indicates a decrease in slope with increasing spectrum intensity, until the slope becomes quite small. Similar trends are observed in the variation of base moment with spectrum intensity and base moment with top level displacement. This indicates the yielding experienced by the test structure in later test runs. This effect may also be observed in the variation of deflection with spectrum intensity (Fig. 3.9). The increase in deflection with increasing spectrum intensity becomes more rapid after test run D1-4. It may also be observed that the lower level acceleration (Fig. 3.8) does not exhibit a decrease in slope with increasing spectrum intensity. For the middle level acceleration, the decrease in slope is much less dramatic than for top level acceleration. This is related to the change in the relative strengths of the first and second modes that occurs with successive test runs. A similar comment may be made concerning the observation that neither the increase in base shear with spectrum intensity nor the increase in base shear with top level displacement is decreased for high values of spectrum intensity.

(i) Deflected Shape

The deflected shape of the test structure was observed at several predetermined times corresponding to positive or negative peaks in the deflection response histories. For each particular time, the deflection was taken off the observed response history for each of the three levels and plotted in Fig. 3.12. Measurements were taken at six different times for each test run for the south wall only. An examination of Fig. 3.12 shows the results to be quite consistent. The deflected shape is almost linear, with a concentration of rotation near the base of the test structure.

3.3 Dynamic Tests of Specimen Type B

(a) State Before Test

As with the type A specimen, the only cracks observed were those due to shrinkage. These are shown in Fig. 3.23a for structure D2 and Fig. 3.24a for structure D3.

(b) Loading

The maximum base acceleration ranged from 1.1 G to 4.1 G., as listed in Table 3.5. The measured response histories for base acceleration are plotted in Fig. 3.17 for test D2 and Fig. 3.22 for test D3. The high frequency noise components observed in test D1 did not occur in tests D2 and D3. There were no runs of such low amplitude that the response level was close to the level of accuracy of the instrumentation. Linear response spectra are provided in Fig. 3.13 and 3.14 for test D2 and in Fig. 3.18 and 3.19 for test D3. The spectrum intensities are listed in Table 3.7.

(c) Frequencies

The observed first-mode frequency, measured in the same manner as for the type A specimen, varied from 7.8 Hz before test run D2-1 and 7.6 Hz before test run D3-1 to 2.2 Hz at the end of test run D2-2 and 2.1 Hz at the end of test run D3-2. The observed second mode frequency varied from 39 Hz before test run D2-1 and 35 Hz before test run D3-1 to 16 Hz at the end of test run D2-2 and 12 Hz at the end of test run D3-2. The observed frequencies are listed in Table 3.6. As for specimen type A, the frequency measured at the end of a test run should not be compared directly with the free-vibration frequency measured before the following test run because of the difference in amplitude.

(d) Accelerations

The response histories for horizontal accelerations are shown in Fig. 3.15 for test D2 and Fig. 3.20 for test D3. The maximum observed horizontal accelerations for both tests are listed in Table 3.1

For all test runs, the horizontal accleration appears to have a very strong higher mode component. The frequency of this component is consistent with the second mode. The phasing of the horizontal accelerations is also consistent with the second mode.

Torsional response does not appear to have been significant for the horizontal accelerations in either test.

(e) Displacements

The response histories for displacement are shown in Fig. 3.16 for test D2 and Fig. 3.21 for test D3. Maximum single-amplitude displacements for type B specimens at the level of the top weight ranged from 0.43 in. in test run D2-1 to 1.36 in. in test run D2-2. The maximum observed responses are listed in Tables 3.2 and 3.3.

For all test runs, the phasing and variation over the height of the displacement amplitudes is consistent with the first mode. There is evidence, however, of a small, but visible higher mode component. The frequency of this component is consistent with the second mode.

Residual displacements developed during the second run of both tests D2 and D3. The residual displacements of the north and south walls differed significantly, however, the observed maximum displacements (Table 3.2 and 3.3) indicated negligible torsional component.

(f) Base Shear

The response histories for base shear are provided in Fig. 3.17 for test D2 and Fig. 3.22 for test D3. The maximum response varies from 1.54 kips in test run D2-1 to 2.5 kips in test runs D2-2 and and D3-2. The maxima are listed in Table 3.5.

The base shear response exhibits a strong higher mode component, the frequency of which is consistent with the second mode.

As for specimen type A, there is no evidence of torsion in the base shears calculated for the two walls.

(g) <u>Base Moment</u>

The response histories for base moment are provided in Fig. 3.17 for test D2 and Fig. 3.22 for test D3. The maxima vary from 56 kip-in. in test run D3-1 to 65 kip-in. in test run D3-2 (Table 3.5).

The base moment response also exhibits a higher mode component, although not so strongly as the base shear. Again, the frequency of the component is consistent with the second mode.

As for specimen type A, there is no torsion apparent in the response histories for base moment.

(h) <u>Failure Mechanism</u>

The crack patterns are depicted in Fig. 3.23 for structure D2 and in Fig. 3.24 for structure D3. Each figure shows the crack patterns for both the north and south walls. One illustration shows the shrinkage cracks before the first test run. The other illustration uses solid lines to denote the crack pattern at the end of the first test run. The dashed lines denote additional cracks that appear during the second test run.

The failure mechanism in both tests consisted of flexural yielding of the beams at their ends, followed by flexural yielding of the piers at their bases.

After the first test run all connecting beams had very fine (approx. 0.002 in.) cracks at their ends and there were no visible residual cracks at the base-pier interfaces (Fig. 3.23 and 3.24). However, there were very fine cracks in the piers between the base and the first-level beam. These cracks could be seen only with the help of the detection ink and were smaller than 0.001 in. The cracks in the connecting beams had enlarged almost uniformly to widths of approximately 0.03 in. after test run 2. The cracks in the pier bases had residual widths of approximately 0.02 in. No spalling of the concrete was observed in any part of the structure.

The variation of observed response with spectrum intensity is depicted in Fig. 3.25 through 3.28. As for specimen type A, the yielding of the test structure is apparent in the variations of displacement and base moment with spectrum intensity. The variation of base moment with top level displacement also suggests the yielding of the test structure (Fig. 3.29). The variation of horizontal acceleration and base shear with spectrum intensity, by not exhibiting a decrease in slope at higher spectrum intensities, show the increasing effect of the second mode.

(i) Deflected Shapes

The deflected shape of the test structure was observed at six predetermined times for each test run in a manner identical to the method used for specimen type A. The deflected shapes are plotted in Fig. 3.29 for test D2 and in Fig. 3.30 for test D3. In a manner similar to that for specimen type A, rotation appears to be concentrated below the lower level weight.

3.4 Dynamic Tests of Specimen Type C

(a) State Before Test

As with both the type A and type B specimens, the only cracks observed were those due to shrinkage. These are shown in Fig. 3.41a for structure D4 and in Fig. 3.42a for structure D5.

(b) Loading

The maximum base acceleration ranged from 1.1 G to 2.4 G, as listed in Table 3.5. The measured response histories for base acceleration are plotted in Fig. 3.35 for test D4 and in Fig. 3.40 for test D5. As for tests D2 and D3, high frequency noise components were not present in any response histories. Again, there are no extremely low amplitude test runs. Linear response spectra are provided in Fig. 3.31 and 3.32 for test D4 and in Fig. 3.36 and 3.37 for test D5. The spectrum intensities are listed in Table 3.7.

(c) Frequencies

The observed first mode frequency, measured in the same manner as for specimen types A and B, varied from 6.9 Hz before test run D4-1 and 8.4 Hz before test run D5-1 to 2.2 Hz at the end of test run D4-2 and 2.1 Hz at the end of test run D5-2. The observed second mode frequency varied from 31 Hz before test runs D4-1 and D5-1 to 13 Hz at the end of test runs D4-2 and D5-2. The observed frequencies are listed in Table 3.6. As for specimen types A and B, the frequency measured at the end of a test run should not be compared directly with the free-vibration frequency measured before the following test run because of the difference in amplitude.

(d) Accelerations

The response histories for horizontal accelerations are shown in Fig. 3.33 for test D4 and in Fig. 3.38 for test D5. The maximum observed horizontal accelerations for both tests are listed in Table 3.1.

For all test runs, a higher mode component is quite visible in the horizontal accelerations. The frequency of this component is consistent with the second mode. Again, the phasing of the horizontal accelerations is consistent with the second mode.

Torsional response does not appear to have been significant for the horizontal accelerations in either test.

(e) Displacements

The response histories for displacement are shown in Fig. 3.34 for test D4 and in Fig. 3.39 for test D5. Maximum single amplitude displacements at the level of the top weight ranged from 0.48 in. in test runs D4-1 and D5-1 to 1.23 in. in test run D5-2. The maximum observed responses are listed in Tables 3.2 and 3.3.

For all test runs, the phasing and variation over the height of the displacement amplitudes is consistent with the first mode. A higher mode component is barely visible. The frequency of this component is consistent with the second mode.

Residual displacements developed during the first run of both tests and increased during the second run. In contrast to specimen types A and B, the residual displacements of the north and south walls did not differ significantly. Similarly, the observed maximum displacements (Table 3.2 and 3.3) indicated negligible torsional component.

(f) Base Shear

The response histories for base shear are provided in Fig. 3.35 for test D4 and in Fig. 3.40 for test D5. The maximum base shear response varies from 1.35 kips in test run D4-1 to 2.6 kips in test run D4-2. The maxima are listed in Table 3.5.

The base shear response exhibits a strong higher mode component, the frequency of which is consistent with the second mode.

As for specimen types A and B, there is no evidence of torsion in the base shears calculated for the two walls.

(g) Base Moment

The response histories for base moment are provided in Fig. 3.35 for test D4 and in Fig. 3.40 for test D5. The maximum response varies from 51 kip-in. in test run D5-1 to 63 kip-in. in test run D5-2 (Table 3.5).

The base moment response also exhibits a higher mode component, however, in general, this component is not so strong as in the base shear. Again, the frequency of the component is consistent with the second mode.

There is no torsion apparent in the response histories for base moment.

(h) Failure Mechanism

The crack patterns are depicted in Fig. 3.41 for structure D4 and in Fig. 3.42 for structure D5. The format of the illustrations is the same as for the type B specimens.

Similarly to specimen type B, the failure mechanism in both tests consisted of flexural yielding of the beams at their ends, followed by flexural yielding of the piers at their bases. The cracking pattern for structures D4 and D5 were also fairly similar to that for the type B structures. After the first test run, all connecting beams had very fine (approx. 0.002 in.) cracks at their ends and for structure D4 there were no visible residual cracks at the base-pier interfaces. Structure D5, however, did exhibit some visible cracking in this area. (Fig. 3.41 and 3.42). For structure D4 there were extremely fine cracks in the piers between the base and the first-level beam. These could be seen only with the help of detection ink and were smaller than 0.001 in. For structure D5, however, these cracks were considerably larger (approx. 0.004 in.) and were visible with the unaided eye. At the end of test run 2, the cracks at the ends of the beams had enlarged to approximately 0.03 in. The cracks at the bases of the piers had residual widths of approximately 0.02 in. These test structures had some spalling at the end of test run 2. This was present at the ends of the upper three beams and at the outside edges of the piers (edges farthest from the connecting beams).

The variation of observed response with spectrum intensity is depicted in Fig. 3.43 through 3.45. As for previous specimens, the yielding of the test structure is apparent in variation of displacement and base moment with spectrum intensity. The variation of base moment with top level displacement also suggests the yielding of the test structure (Fig. 3.46). Again the variation of horizontal acceleration and base shear with spectrum intensity do not exhibit a decrease in slope at higher spectrum intensities, showing the increasing effect of the second mode.

(i) Deflected Shapes

The deflected shape of the test structure was again observed at six predetermined times during each test run in a manner identical to that for specimen types A and B. The deflected shapes are plotted in Fig. 3.47 for test D4 and in Fig. 3.48 for test D5. In a manner similar to that for specimen types A and B, rotation appears to be concentrated below the lower level weight.

3.5 Static Test of Specimen Type B

(a) General Comments

It was mentioned in Chapter 2 that a type B specimen was tested under statically applied lateral loading as part of the experimental program. The results of that test are presented in this section.

The loads were applied to the test structure by three hydraulic rams, one at the level of each test weight. The rams were positioned such that the loads were applied along an axis parallel to and midway between the axes of the two walls that comprised the test structure, causing the test structure to bend about its strong axis. The test setup is shown in Fig. A.33, A.34 and A.35. Using mechanical dial gages, horizontal deflections of each of the two walls were observed at the levels of the three test weights. Differential transformers, built into each of the three hydraulic rams, measured horizontal deflections along the loading axes. Dial gages were also used to measure horizontal and vertical deflections of the bases of the test structure. The differential transformers operated throughout the test, while the dial gages were operative only during a portion of the first one-quarter cycle of loading. The instrumentation scheme is illustrated in Fig. A.34. Appendix A describes the test setup and test procedure in detail.

(b) Loading

The hydraulic rams were programmed to maintain a predetermined ratio among the three lateral loads. This ratio is shown in Fig. 3.49. The load ratio corresponds to the shape of the first mode of the test structure, computed as described in Chapter 4. The test was conducted by applying certain predetermined increments of top level deflection. The bottom and middle rams would simultaneously load to the appropriate ratio of the load in the top ram. The schedule of top level deflections is shown in Fig. 3.49.

(c) Deflections Measured by Mechanical Dial Gages

The observed horizontal deflections are shown for each of the north and south walls at the levels of the bottom, middle, and top weights in Fig. 3.50. In the figure, each dot corresponds to a point at which the test was stopped and the dial gages were read. These measurements include rotation and sliding of the base of the wall. Vertical and horizontal deflections are shown in Fig. 3.51. The labelling of Fig. 3.51 may be explained in relation to Fig. A.34. The labels N-horiz. and S-horiz. refer to the horizontal deflection of the bases of the north and south walls. The labels NE and NW refer to vertical deflections measured at the east and west edges of the north wall. The labels SE and SW refer to similar gage locations for the south wall. The horizontal measurements are shown as positive in Fig. 3.51 for deflection to the west. The ram loads were also being applied in a westward direction. The base moved in the direction of load application. The NE and SE deflections are positive upward while the NW and SW deflections are positive downward. The bases tend to rotate in a sense consistent with the direction of wall bending. Fig. 3.52 illustrates the method of correcting the observed deflections for these base motions. The corrected deflections for each of the north and south walls at the bottom, middle, and top levels are shown in Fig. 3.53. It should also be noted that the torsional motion of the test structure was negligible.

(d) Deflections Measured by Differential Transformers

The deflections observed at the bottom, middle, and top levels are shown in Fig. 3.54. Note that after the first one-eighth cycle, these are the result of a continuous recording. The test was halted only when it was desired to reverse the direction of loading. It should also be noted that since these observations were taken midway between the north and south walls, they may be thought of as an average deflection of the two walls. Finally, the deflections are not corrected for base deflections. However, considering the magnitude of the correction applied to the dial gage readings (its effect on the initial slope was less than one percent for the top-level deflections), this is not critical.

The salient feature of the observed hysteresis is its low stiffness at low loads. As the load increases, the load-deflection relation stiffens and eventually reaches the same maximum load attained in the first one-quarter cycle. The result, however, is that with each successive cycle of loading, the test structure must reach a higher and higher deflection to attain its maximum load capacity. The small loops were intentional.

CHAPTER 4

STRENGTH AND DEFORMATION PROPERTIES

4.1 Transformed Sections

(a) Uncracked

The section stiffnesses of the beams and piers based on linearly elastic behavior (no cracking in concrete) were computed using a transformed section, in which the reinforcement was transformed into concrete through the modular ratio, $n = E_s/E_c$. The sections, for the beams and piers, are shown in Fig. 4.1. The transformed moments of inertia and areas were computed for each test structure using the Dec System 10 computer of the Digital Computation Laboratory of the University of Illinois. The mean measured dimensions of the test structures (Tables A.6 through A.11), along with reinforcement areas obtained from measured diameters (Table A.3) and the mean secant modulus of concrete (Table A.1) were used in the computations. Young's modulus for reinforcement was assumed equal to 29000 ksi.

Referring to Fig. 4.1 (a), the transformed area of the beams is given by,

$$A_{tr} = bd_w + 2\ (n-1)\ A_s \tag{4.1}$$

The transformed moment of inertia is given by.

$$I_{tr} = \frac{1}{12}\ bd_w^3 + (n-1)\ A_s\ [(\frac{d_w}{2} - d')^2 + (d - \frac{d_w}{2})^2] + A_{tr}\ (\bar{c} - \frac{d_w}{2})^2 \tag{4.2}$$

The results, for each test structure, are listed in Table 4.1.

Referring to Fig. 4.1 (b), the transformed area of the piers was given by,

$$A_{tr} = bd_w + 6\,(n-1)\,A_s \tag{4.3}$$

The transformed moment of inertia is given by,

$$I_{tr} = \frac{1}{12}\,bd_w^3 + (n-1)\,A_s \sum_{i=1}^{6} \left(\frac{d_w}{2} - d_i\right)^2 + A_{tr}\,\left(\bar{c} - \frac{d_w}{2}\right)^2 \tag{4.4}$$

The results for each test structure are listed in Table 4.2.

(b) <u>Cracked</u>

For each test structure, the section stiffnesses were also computed for a fully cracked state. The concrete was assumed to be linearly elastic in compression and to have no tensile strength. A linear strain distribution was assumed. Again measured section dimensions (Tables A.6 through A.11) along with measured steel area (Table A.3) and measured concrete modulus (Table A.1) were used. Young's Modulus of steel was assumed equal to 29000 ksi. As for the uncracked sections, steel was transformed into concrete through the modular ratio and calculations were performed on the Dec System 10 computer.

The approach for the connecting beams is illustrated in Fig. 4.2. Since the section is linearly elastic and there is no axial load, the neutral axis corresponds to the centroid of the section. Assuming that only one steel layer is subjected to tensile force (Fig. 4.2 (a)), the transformed area is given by,

$$A_{cr} = b\bar{c} + (2n-1)\,A_s \tag{4.5}$$

From the definition of a centroid,

$$\bar{c}\, A_{cr} = \frac{1}{2}\, b\bar{c}^2 + (n-1)\, A_s\, d' + n\, A_s d \tag{4.6}$$

After combining with equation 4.5 and algebraic manipulation, a quadratic equation in $\bar{c}$ was obtained,

$$\bar{c}^2 + \frac{2}{b}\,(2n-1)\, A_s\, \bar{c} - \frac{2}{b}\,(n-1)\, A_s d' - \frac{2}{b}\, n\, A_s d = 0 \tag{4.7}$$

The above quadratic equation was solved for $\bar{c}$, and the transformed area was computed from equation 4.5. The transformed moment of inertia was then obtained from,

$$I_{cr} = \frac{1}{3}\, b\bar{c}^3 + (n-1)\, A_s\, (\bar{c}-d')^2 + nA_s\, (d-\bar{c})^2 \tag{4.8}$$

The fully cracked section may also be characterized by both reinforcement layers being in tension. This is illustrated in Fig. 4.2 (b). The transformed area is given by,

$$A_{cr} = b\bar{c} + 2n\, A_s \tag{4.9}$$

The centroid of the section is given by,

$$\bar{c}\, A_{cr} = \frac{1}{2}\, b\bar{c}^2 + nA_s\, (d'+d) \tag{4.10}$$

The resulting quadratic equation is,

$$\bar{c}^2 + \frac{4n}{b}\, A_s\, \bar{c} - \frac{2n}{b}\, A_s\, (d'+d) = 0 \tag{4.11}$$

The transformed moment of inertia is given by,

$$I_{cr} = \frac{1}{3}\, b\bar{c}^3 + nA_s\, [(d'-\bar{c})^2 + (d-\bar{c})^2]$$

For each test structure four calculations were needed. Both states, discussed above, were investigated. However, the measured dimensions did not characterize a symmetrical section. The upper and lower reinforcement layers were not symmetric about the midheight of the section. Calculations were performed for compression at the top edge of the beam section and for compression at the bottom edge of the section. The two results were averaged.

There was some variation among test structures concerning the number of steel layers subjected to tension. In several test structures, this characteristic was even altered by reversing the sense of the applied moment on the section. For test structures D2 and D5, both reinforcement layers were in tension for both directions of loading. For test structure D1, for both directions of loading, only one reinforcement layer was subjected to tension. For test structures D3, D4 and S1, the number of steel layers in tension was dependent upon the direction of loading. The results are presented in Table 4.1.

The calculation of fully cracked section stiffnesses for the piers involved assumptions similar to those for the beams. The approach was complicated, however, by the presence of axial load. Both the cracked transformed area and cracked transformed moment of inertia are functions of the axial load. The presence of axial load further causes the two above parameters to become functions of the moment applied to the section. Hence, computations were performed at several values of axial load and applied moment, both senses for the applied moment being considered.

The basis for the calculations is illustrated in Fig. 4.3. The neutral axis does not correspond to the centroid of the section, hence, equations are derived directly from considerations of axial load equilibrium. Also, the derivation is general with respect to the number of reinforcement layers in compression. This quantity is denoted by the integer, k. Referring to Fig. 4.3, the cracked transformed area is given by,

$$A_{cr} = c_o b + k\,(n-1)\,A_s + (6-k)\,nA_s \tag{4.13}$$

The cracked transformed moment of inertia is given by,

$$I_{cr} = \frac{1}{12} bc_o^3 + \left(\frac{c_o}{2} - \bar{c}\right)^2 bc_o + (n-1)\,A_s \sum_{i=1}^{k} (\bar{c}-d_i)^2 + nA_s \sum_{i=k}^{6} (\bar{c}-d_i)^2 \tag{4.14}$$

Rearranging,

$$I_{cr} = \frac{1}{12} bc_o^3 + \left(\frac{c_o}{2} - \bar{c}\right)^2 bc_o + (6n-k)\,A_s \bar{c}^2 - 2A_s \bar{c}\,\left[n \sum_{i=1}^{6} d_i - \sum_{i=1}^{k} d_i\right]$$

$$+ A_s \left[n \sum_{i=1}^{6} d_i^2 - \sum_{i=1}^{k} d_i^2\right] \tag{4.15}$$

The centroid of the section is given by,

$$\bar{c} = \frac{\frac{1}{2}\, bc_o^2 + nA_s \sum_{i=1}^{6} d_i - A_s \sum_{i=1}^{k} d_i}{A_{cr}} \tag{4.16}$$

Also,

$$\sigma_a = \frac{P}{A_{cr}} \tag{4.17}$$

From simple bending theory, the applied moment may be expressed as,

$$M = \frac{\sigma_a I_{cr}}{\bar{c} - c_o} \tag{4.18}$$

Solving equation 4.18 for c_o,

$$c_o = \frac{\bar{c}M - \sigma_a I_{cr}}{M} \tag{4.19}$$

The position of the neutral axis, c_o, is determined iteratively using equations 4.13 through 4.19. First, a value of k is guessed. An initial guess for c_o is also made. The values for the variables A_{cr}, I_{cr}, $\bar{c}$ and σ_a are then computed from equations 4.13, 4.15, 4.16 and 4.17. Equation 4.19 is used to compute a new value for c_o. The difference between the new value and the initial guess is compared with a predetermined tolerance, indicative of the desired level of accuracy. If the difference is too great the calculation procedure is repeated, using the new value of c_o. When the difference between two successive values of c_o is acceptable, the value is compared with the assumed value of k. If c_o is not consistent with k, a new value of k is assumed and the process is repeated. If they are consistent, the most recent values of A_{cr} and I_{cr} are taken as the section properties.

Calculations were performed for each test structure for axial loads of 0.0, 0.5 kip, 1.5 kips, and 3.0 kips compression, in addition to 0.5 kip tension. For each test structure these calculations were performed for the cracking moment of the pier section of the particular structure, the ultimate moment of the pier section for test structure S1, and the average of the two. For each moment-load combination, bending in both senses was considered. Hence, 30 calculations were performed for each test structure. The cracked transformed areas and moments of inertia are presented in Table 4.2. The values tabulated represent the averages obtained for the two directions of moment application. The variations of section stiffness with axial load and applied moment, although considerable for the transformed area, is insignificant for the transformed moment of inertia.

4.2 Structure Deformation Properties

(a) Frequencies and Mode Shapes

The natural frequencies and mode shapes for each test structure were computed from the model illustrated in Fig. 4.4. The structure has been cut in half at the midspan of the beams. A roller is idealized at this point. The implicit assumption is that there is a point of inflection at the midspan of the beams, hence, a bending pattern in the beams anti-symmetric about the midspan, with the two piers experiencing identical bending patterns. In addition to flexural deformation, axial deformation in the piers is considered. The finite joint sizes are modelled by the infinitely rigid blocks, shown hatched in the figure. The mass is concentrated along the centerline of the pier, at the centerlines of the second, fourth and sixth level beams, as shown at the right side of Fig. 4.4. The section properties used are those computed in Section 4.1 and listed in Tables 4.1 and 4.2. The secant modulus of concrete for each test structure is taken as the mean from Table A.1. The calculations were performed using a computer program written in the Fortran IV Language for the 360/75 computer of Digital Computer Laboratory of the University of Illinois. The program used is described in Appendix D.

Computations were performed for test structures D1 through D5 and S1. For each test structure, calculations were performed for five cases.

The symbols used in the following expressions are illustrated in Fig. 4.4.

(1) Uncracked: $I_{pi} = I_{tr}, A_{pi} = A_{tr}, i = 1$ through 6

$$I_{bi} = I_{tr}, i = 1 \text{ through } 6$$

(2) Beams cracked: $I_{pi} = I_{tr}, A_{pi} = A_{tr}, i = 1$ through 6

$$I_{bi} = I_{cr}, i = 1 \text{ through } 6$$

(3) Beams and lower pier cracked:

$$I_{pi} = I_{cr}, A_{pi} = A_{cr}, \; i = 1$$

$$I_{pi} = I_{tr}, A_{pi} = A_{tr}, \; i = 2 \text{ through } 6$$

$$I_{bi} = I_{cr}, \; i = 1 \text{ through } 6$$

(4) Uncoupled piers, uncracked:

$$I_{pi} = I_{tr}, A_{pi} = A_{tr}, \; i = 1 \text{ through } 6$$

$$I_{bi} = 0, \; i = 1 \text{ through } 6$$

(5) Uncoupled piers, lower pier cracked:

$$I_{pi} = I_{cr}, A_{pi} = A_{cr}, \; i = 1$$

$$I_{pi} = I_{tr}, A_{pi} = A_{tr}, \; i = 2 \text{ through } 6$$

$$I_{bi} = 0, \; i = 1 \text{ through } 6$$

The first and second mode natural frequencies for each test structure for each of the above cases are listed in Table 4.4. The shapes of the first and second modes are given in Table 4.5. The organization of Table 4.5 is not by test structure. One value of first mode shape is provided for each calculation case. The shape of the first mode was identical for all test structures. The shape of the second mode is provided for each case for the type A test structure, then for the types B and C test structures. Although the statistical variation in dimensions and material properties did not affect the mode shapes, the difference in beam depth between the type A structure and the types B and C structures did affect the shape of the second mode (Fig. A.17).

(b) Initial Stiffness

The stiffness of each test structure was computed using the same model, with the same assumptions, as for modal analysis. The calculations were performed, however, using the computer program STRUDL-II of the ICES System developed at Massachusetts Institute of Technology. The model was subjected to lateral loads applied to the joints as shown in the inset of Fig. 4.4. The ratios between the lateral loads were chosen to correspond to the computed first mode shape of the test structures, and are identical to the ratios used in the static test (Chapter 3). The rationale for this choice, as described in Chapter 3, was that the structure responds primarily in flexure and that the first mode is dominant in the response history for base moment during the interval of highest amplitude response.

Stiffnesses were calculated for each test structure for the same cases as in the modal analysis. However, direct analyses of the model of Fig. 4.4 were done only for the completely uncracked state (Case (1)). The stiffnesses for the other four cases were computed using the uncracked stiffness as a reference stiffness and using the ratios of the first mode frequencies calculated previously in this section. The implicit assumption is that the structure responded as a single degree of freedom system. Considering two cases, Case (a) and Case (b) for a single-degree-of-freedom system,

$$f_a = \frac{1}{2\pi}\sqrt{\frac{k_a}{m}} \qquad f_b = \frac{1}{2\pi}\sqrt{\frac{k_b}{m}} \tag{4.20}$$

Hence,

$$\frac{f_b}{f_a} = \sqrt{\frac{k_b}{k_a}} \tag{4.21}$$

Rearranging,

$$k_b = k_a \left[\frac{f_b}{f_a}\right]^2 \tag{4.22}$$

where,

k_a = system stiffness for Case (a).

k_b = system stiffness for Case (b).

f_a = system frequency for Case (a).

f_b = system frequency for Case (b).

Knowing the stiffness for the case of the uncracked structure and knowing the first mode frequencies for all five cases, the stiffnesses for the remaining four cases were calculated.

The stiffnesses for each test structure, for each of five cases, were expressed as the ratio of base moment to top level deflection, and are presented in Table 4.6.

4.3. Moment-Load-Curvature Relations

(a) Cracking Moment

Cracking moments were computed for the beams and piers of each test structure. The moments were computed using simple bending theory referring to Fig. 4.1,

$$M_{cr} = \frac{I_{tr}}{(d_w - \bar{c})} \left(f_t + \frac{P}{A_{tr}}\right) \tag{4.23}$$

where,

P = the axial load on the section (positive for compression)

The cracking moment for the pier section, a function of axial load, was computed for several axial loads. The values for the uncracked transormed moments of inertia were those from Table 4.2. The tensile strength of concrete was taken as the mean splitting stress from Table A.2. The cracking moment varied, depending upon the direction in which bending was assumed to occur. This was due, again, to the fact that the measured sections were not symmetric. The distance to the center of gravity depended upon which edge was assumed to be in compression. The results are presented for each test structure, at several values of axial load, in Table 4.3.

The cracking moment for the beam section was computed for each test structure using the uncracked transformed moments of inertia from Table 4.1. As for the pier sections the tensile strength for the concrete for each test structure was taken as the mean splitting stress from Table A.2. Again the value of the cracking moment was dependent upon the direction in which bending was assumed to occur. The results for the two directions were averaged, and are presented, for each test structure, in Table 4.1.

The calculations, for both the beams and the piers, were performed on the Dec System 10 computer of the Digital Computer Laboratory of the University of Illinois.

(b) Stress-Strain Idealization

In order to compute the moment-curvature relations and moment-axial load interaction for the cross-sections of the members in the test structures, it was necessary to idealize the measured stress-strain relations for the concrete and for the reinforcement.

The idealized stress-strain relation for the concrete is shown in Fig. 4.5 (a). The ascending portion of the compressive region of the relation is the parabola used by Hognestad (16) and applied in several previous studies in the laboratory (15, 25). This is given by,

$$f_c = f'_c \left[2\left(\frac{\varepsilon_c}{\varepsilon_o}\right) - \left(\frac{\varepsilon_c}{\varepsilon_o}\right)^2 \right] \qquad 0 \leq \varepsilon_c \leq \varepsilon_o \qquad (4.24)$$

For strains greater than ε_o, the stress is taken equal to the maximum stress, in essence,

$$f_c = f'_c \qquad \varepsilon_c \geq \varepsilon_0 \tag{4.25}$$

The flat portion of the relation is assumed to extend to infinity. In essence, the concrete is considered to be well confined by the helical reinforcement of the piers and the closely spaced stirrups of the beams. For each test structure, the values of f'_c and ε_0, were the means presented in Table A.1. The tensile region of the relation was given by,

$$f_c = 2f'_c \left(\frac{\varepsilon_c}{\varepsilon_0}\right) \qquad \varepsilon_t \leq \varepsilon_c \leq 0 \tag{4.26}$$

$$f_c = 0 \qquad \varepsilon_c < \varepsilon_t \tag{4.27}$$

This is a linear relation with a slope equal to the initial slope of the compressive portion of the relation. The tensile strength of the concrete for each test structure, f_t, was taken as the mean splitting stress presented in Table A.2. The value of ε_t was derived from the tensile strength and the initial slope of the parabola.

The idealized stress-strain relation for the reinforcement is shown in Fig. 4.5 (b). The idealization is of the same form all three sizes of reinforcement and is assumed to be identical for tension and compression. The value for Young's modulus is assumed to be 29000 ksi, it being judged that the measured values, from Appendix A, exhibited too much scatter for use in analyses. The steel is assumed to maintain its maximum stress to an infinite strain.

In summary,

$$f_s = E_s\, \varepsilon_s \qquad -\varepsilon_{sy} \leq \varepsilon_s \leq \varepsilon_{sy} \tag{4.28}$$

$$f_s = f_{sy} \qquad -\varepsilon_{sh} \leq \varepsilon_s \leq -\varepsilon_{sy} \text{ or } \varepsilon_{sy} \leq \varepsilon_s \leq \varepsilon_{sh} \tag{4.29}$$

$$f_s = f_{sy} + E_{sh}(\varepsilon_s - \varepsilon_{sh}) \qquad -\varepsilon_{su} \leq \varepsilon_s \leq -\varepsilon_{sh} \text{ or } \varepsilon_{sh} \leq \varepsilon_s \leq \varepsilon_{su} \tag{4.30}$$

$$f_s = f_{su} \qquad \varepsilon_s \geq \varepsilon_{su} \text{ or } \varepsilon_s \geq -\varepsilon_{su} \tag{4.31}$$

where,

$$E_{sh} = \frac{f_{su}-f_{sy}}{\varepsilon_{su}-\varepsilon_{sh}}$$

The values of ε_{sh} and ε_{su} for each size of reinforcement are taken from the means in Table A.4. The value of f_{sy} for each size of reinforcement in each test structure was taken equal to the mean in Table A.5. The value for f_{su} for each size of reinforcement in each test structure was obtained by multiplying the mean ratios of ultimate stress to yield stress for each size of reinforcement (Table A.4) by the mean yield stresses for the reinforcement in each test structure (Table A.5).

(c) General Calculation Method

The main calculation procedure was identical for both the moment-curvature relations and the moment-axial load interaction relations. The calculation procedure was also generalized to accomodate both the beams and the piers. The method of calculation is illustrated in Fig. 4.7. Given a rectangular section, consisting of a specific number of piers, q, with openings, with the positions of several concentrated steel layers defined, the compressive strain in concrete at the compression edge of the section, ε_{cm}, defined, and the axial load on the section, P, defined, the problem was to compute the bending moment about the plastic centroid. By performing the calculations for various combinations of

ε_{cm} and P, both the moment-curvature relations and the moment-load interaction relations were constructed.

A linear strain distribution was assumed, in essence, the individual piers were perfectly coupled, to behave as a single section. The stress-strain idealizations were those presented in the previous section. The explanation will be general with respect to the number of reinforcement layers, m, and the number of distinct piers, q. Each pier constitutes a rectangular section.

It is desired that the moments computed be with respect to the plastic centroid, defined as the point of application of load when the section is subjected to its maximum axial compressive load and no moment. This is illustrated in Fig. 4.6. The first step was to compute the distance $\bar{d}_p$ of this point from the edge of the section in maximum compression (Fig. 4.6). The maximum compressive load, in the absence of moment, is given by,

$$P_m = \sum_{i=1}^{q} f'_c bh_i + \sum_{i=1}^{m} A_s(f_{su}-f'_c) \tag{4.32}$$

The position of the plastic centroid is described by equating the moments of the distributed loads and equivalent axial force in Fig. 4.6,

$$\bar{d}_p P_m = \sum_{i=1}^{q} f'_c\, bh_i \bar{d}_i + \sum_{i=1}^{m} A_s(f_{su}-f'_c)d_i \tag{4.33}$$

Rearranging,

$$\bar{d}_p = \frac{f'_c b \sum_{i=1}^{q} h_i \bar{d}_i + A_s(f_{su} - f'_c) \sum_{i=1}^{m} d_i}{f'_c b \sum_{i=1}^{q} h_i + mA_s(f_{su} - f'_c)} \tag{4.34}$$

The next step was to compute the position of the neutral axis (Fig. 4.7). From axial load equilibrium,

$$\sum_{i=1}^{q} \int_{c_{1i}}^{c_{2i}} f_c b dc + \sum_{i=1}^{k} A_s(f_{si} - f_{ci}) + \sum_{i=k+1}^{m} A_s f_{si} = P \tag{4.35}$$

where c is zero at the neutral axis and positive in the region of the section subjected to compression. Stresses, strains, and loads are positive for compression. Since the strain distribution is linear,

$$dc = \frac{c_o}{\varepsilon_{cm}} d\varepsilon_c \tag{4.36}$$

Finally,

$$b \frac{c_o}{\varepsilon_{cm}} \sum_{i=1}^{q} \int_{\varepsilon_{1i}}^{\varepsilon_{2i}} f_c d\varepsilon_c + A_s \sum_{i=1}^{k} (f_{si} - f_{ci}) + A_s \sum_{i=k+1}^{m} f_{si} = P \tag{4.37}$$

An initial value for the neutral axis distance, c_o, is guessed. The left hand side of equation 4.37 is evaluated and compared to the given axial load, P. An algorithm is applied to adjust c_o in successive repetitions until equation 4.37 is satisfied to within a specified tolerance. The moment, referenced to the plastic centroid is then computed from,

$$M = \sum_{i=1}^{q} \int_{c_{1i}}^{c_{2i}} f_c b(c-\bar{c}_p)\ dc + \sum_{i=1}^{k} A_s(f_{si}-f_{ci})(c_{si}-\bar{c}_p) + \sum_{i=k+1}^{m} A_s f_{si}(c_{si}-\bar{c}_p) \tag{4.38}$$

Since the strain distribution is linear,

$$c = \frac{c_o}{\varepsilon_{cm}}\ \varepsilon_c \qquad\qquad dc = \frac{c_o}{\varepsilon_{cm}}\ d\varepsilon_c \tag{4.39}$$

Hence,

$$M = \sum_{i=1}^{q} \left(b\left(\frac{c_o}{\varepsilon_{cm}}\right)^2 \int_{c_{1i}}^{c_{2i}} f_c \varepsilon_c d\varepsilon_c - b\bar{c}_p\left(\frac{c_o}{\varepsilon_{cm}}\right) \int_{c_{1i}}^{c_{2i}} f_c d\varepsilon_c \right)$$

$$+ \sum_{i=1}^{k} A_s(f_{si}-f_{ci})(c_{si}-\bar{c}_p) + \sum_{i=k+1}^{m} A_s f_{si}(c_{si}-\bar{c}_p) \tag{4.40}$$

The curvature was computed from,

$$\phi = \frac{\varepsilon_{cm}}{c_o} \tag{4.41}$$

The calculations were performed on the IBM 360/75 computer of the Digital Computer Laboratory of the University of Illinois. The computer programs are described in Appendix B.

(d) Cases for Calculations

The general calculated shape of the moment-curvature relationships for the doubly reinforced connecting beams (equal reinforcement top and bottom) is shown in Fig. 4.8. Numerical studies showed that the influence of the observed dimensional scatter on the three points indicated in Fig. 4.8 was very small. It did not matter whether the average of the moment-curvature curves or the moment-curvature curve based on mean dimensions was used. Moment-curvature relationships based on mean dimensions for the three types of connecting beams are shown in Fig. 4.9a through c. The differences between positive- and negative-moment strengths are due to differences in effective depth of the reinforcement.

Moment-curvature relations and moment axial load interaction relations were computed for the pier cross-section of structure S1 only. The mean depth and width of the section (Table A.11) were used. The reinforcement layers were considered to be in their nominal positions, in essence, uniformly distributed over the depth of the section. The variations of section and material properties among test structures was not considered sufficient to produce a significant variation in strength properties for the piers. The relations are shown in Fig. 4.10 and 4.11. The results are listed in Table 4.3.

4.4 Failure Mechanism

(a) General Comments

The failure mechanism for each test structure was investigated using beam strengths and pier strengths computed as discussed in section 4.3. For the calculations, the story heights were assumed to be equal to their nominal values. For these larger distances, the variation of measured distance from the nominal was not considered to be significant.

The loading for the calculation and the resulting reactions are shown in Fig. 4.12. Concentrated loads at three levels, at the centerline of each pier, corresponded to the vertical dead load of the test weights.

Lateral loads were applied at the levels of the second, fourth and sixth beams. The loads were considered as one, three and five times a constant Q_m. These values were chosen to correspond to the computed shape of the first mode for the test structure (Table 4.5). Depending upon the governing mechanism, the values of T_b and C_b were determined either from maximum pier section strength or from maximum beam shears and vertical equilibrium of the pier. Next, M_1 and M_2 were obtained from the computed interaction diagram (Fig. 4.11) at axial loads of T_b and C_b, respectively. Moment equilibrium about point 0 was then considered, obtaining Q_m. The base shear, V_b, was then computed considering horizontal equilibrium of the structure.

(b) Type A Test Structure

The failure mechanism is depicted in Fig. 4.13. The mechanism is characterized by the development of the maximum tensile capacity at the base of one pier and a combination of compression and flexure at the base of the other pier. The beams do not yield. The maximum forces for the mechanism are listed in Table 4.7.

This mechanism can also be described as failure of the entire structure as a cantelever, with the maximum load being computed for flexural failure at the base of the cantelever. Hence, the maximum base moment may be computed by considering each wall as a single section, as in Fig. 4.14 and computing the section strength at the appropriate axial load. The reinforcement layers were considered to be in their nominal positions. Other dimensions were mean values from Table A.6. Reinforcement areas were consistent with Table A.3. The two piers act as a completely coupled unit, as in Fig. 4.7. Moment-Axial Load interaction diagrams at several values of ε_{cm}, were computed for the above section using the computer program of Section 4.3 and Appendix B. For the compressive axial load corresponding to the dead load on one wall (3.0 kips), the maximum moment capacity was calculated to be 81 kip-in. As expected, this result was equal to that obtained from the mechanism analysis.

(c) Types B and C Test Structures

Again, the failure pattern is shown in Fig. 4.13. The pattern consists of flexural hinges at the ends of the beams and at the bases of the piers. The mechanism forces (Table 4.7) are provided for two cases, Mechanism 1 and Mechanism 2. Mechanism 1 considers the beams to have developed their yield moments, while Mechanism 2 considers them to have developed their ultimate moments.

CHAPTER 5

STUDY OF STATIC HYSTERESIS

5.1 Analytical Model

(a) General Comments

An analytical model was developed to study the static response of the test structure subjected to reversals of lateral loading. The model considered the cyclic structural response of the test structure for deformations into the inelastic range. In essence, the model enabled the hysteresis properties of the entire structure to be studied given the moment-rotation responses of the individual elements. This section describes the model itself, while subsequent sections describe several studies performed using the model. These studies were oriented toward studying the overall mechanism of energy dissipation, along with the effect of response amplitude and mode of response on energy dissipation.

(b) Structural Idealization

The analytical model is depicted in Fig. 5.1. The analysis considered one-quarter of a test structure or one-half of a wall. The forces resulting from the analysis were doubled to correspond to forces for one wall. This idealization assumed that a point of inflection existed at the midspan of each beam and that there be no axial loads in the beams. The existence of such a point of inflection depended upon the existence of identical deformation patterns in each of the two piers of a given wall. This required that the two piers carry the same load and possess identical distributions of stiffness. In early stages of loading the piers may have possessed nearly identical properties, however, variations in axial load between the piers would cause the stiffness of one pier to be different from the other, leading to different shears in the two piers. In a prototype structure, a difference in the shears carried by the two piers would cause the generation of axial thrusts in the connecting beams, altering the mechanical properties of those members. For the test structure, however, this was not a major consideration. The lateral load was applied directly to each of the two piers through a very stiff steel weight. As the stiffnesses of the two piers deviated, redistribution of the loads could occur through the steel weights themselves, rather than through the beams. This behavior was further encouraged by the fact that the steel weights were approximately 800 times as stiff, with respect to axial deformation, as the two beams at the same level in the test structure. Furthermore, the response of an entire wall was approximated by using the pier hysteresis relations corresponding to an axial load equal to the applied dead load. The applied dead load was an average of the axial loads in the two piers of a wall. This condition was required for vertical equilibrium of the connecting beams. The axial force induced in one pier by the connecting beams had to be of equal magnitude and opposite sense to that induced in the other pier. These forces induced by the connecting beams represented the entire deviation of the axial load in the piers from that axial load due to vertical dead load. The rationale in using a hysteresis for this "average" axial load was that an "average" load for the two piers of a wall would be computed for the pier of the analytical model. The nearly linear nature of the moment-axial load interaction relation for the pier section for the range of axial loads encountered in the study (Fig. 4.11) lends credence to this approach. When the forces computed for the analytical model were doubled, the result was a reasonable approximation for an entire wall.

The analytical model considered inelastic action through the approach of piecewise linear response. A piecewise linear hysteresis relation, composed of moment and curvature, was idealized for each member, or for each beam and each story of the pier. Each member was considered to behave in a linearly elastic manner during each of several steps of loading and unloading, a step being terminated when any member attained a load corresponding to a discontinuity of stiffness in its idealized hysteresis relation. The altered stiffness of the member was then applied in the next step of loading or unloading, this step being terminated when another point of stiffness discontinuity was reached, either in the same member or a different member. The dead load of the test structure was simulated by concentrated vertical loads of 500 lb. each, along the centerline of the pier at the levels of the second, fourth, and sixth connecting beams. These corresponded to the load of the steel weights in the test structures. The lateral loading was also applied at the levels of the second, fourth, and sixth connecting beams. The ratios of the lateral loads were assumed to remain constant through all stages of loading and unloading, as in the static test. Referring to Fig. 5.1, the factors a_1, a_2, and a_3 remained constant throughout loading and unloading, only the value of Q varied.

The model ignored axial and shear deformations in the members. The finite sizes of the joints were considered using the infinitely stiff blocks, depicted in Fig. 5.1. The test structure was considered to be fixed at its base.

For purposes of calculation, the model considered the individual members to be not only linearly elastic, but of uniform section stiffness throughout their lengths. The calculations were performed considering the same piecewise linear moment-curvature hysteresis relation to apply for the entire length of a given member. This facilitated standard, linearly elastic structural analysis. This, however, was not a realistic assumption for higher amplitude stages of loading, when each member would experience yielding over a portion of its length. Hence, the uniform section stiffnesses applied for each member during each loading step was an equivalent or pseudo-uniform section stiffness derived from a more realistic relation between member end moment and member end rotation.

The moment-rotation relations, considering partial member yielding, are described in parts (d) and (e); while the moment-curvature relations used to obtain those moment rotation relations are presented in part (c). The method of deriving equivalent uniform section stiffnesses from these moment rotation relations is described in part (f). The calculation procedure for the structural analysis is further clarified in part (g).

(c) Idealized Moment-Curvature Relations

The moment-curvature relation for the beam section was idealized tetra-linearly as depicted in Fig. 5.2. The first discontinuity of slope corresponded to yield of the reinforcement layer subjected to tension, the second to the attainment of the maximum compressive stress in the concrete at the edge of the beam section, and the third to the attainment of the strength of the reinforcement layer subjected to tension. The moments and curvatures corresponding to these three events were the averages of the values calculated for the beam cross-sections of test structure S1 (chapter 4). Hence, the section stiffnesses for the first three segments of the relation were defined. The plateau of the relation was nominally of zero slope, but to facilitate analysis was assigned the small slope shown in

Fig. 5.2. The limiting curvature, ϕ_ℓ, corresponded to a tensile strain of 0.20 in the reinforcement.

For the cross-section of the pier, the moment curvature relation was also idealized from that described in chapter 4 (Fig. 4.10). The tri-linear idealization was performed for an axial load of 1.5 kips and is shown in Fig. 5.3, superimposed on the calculated relation. The limiting curvature, ϕ_ℓ, was chosen to correspond to a maximum tensile reinforcement strain of 0.20 in the section.

The parameters M_{y1}, M_{y2}, M_{y3}, M_ℓ, ϕ_{y1}, ϕ_{y2}, ϕ_{y3}, and ϕ_ℓ for both beam and pier sections are listed in Table 5.1.

(d) Moment-Rotation Relations for Beams

The computation of the end moment-end rotation relation for the connecting beams is illustrated in Fig. 5.4. The geometry of the beam is shown in Fig. 5.4(a). The distribution of moment along the beam is obtained directly from statics, and is illustrated in Fig. 5.4(b). The idealized moment-curvature relation, presented in part (c) of this section was used to obtain a curvature distribution along the beam. The end rotation was then computed as follows.

$$\delta = \int_0^{\ell_e} [\phi(x)]\ x\ dx \tag{5.1}$$

$$\theta_E = \frac{\delta}{\ell_E} \tag{5.2}$$

where the symbols refer to Fig. 5.4 and $\phi(x)$ is the curvature as a function of the distance along the beam.

Because the moment distribution along the beam was linear and the moment-curvature relation was idealized as piecewise linear, the variation of curvature along the beam for any end moment, was piecewise linear, greatly simplifying the evaluation of the integral of Equation 5.1. It was necessary only to compute the moments of several trapezoidal areas about the hinged end of the beam. Three distinct classes of curvature distributions were delineated. These are shown in Fig. 5.4(c) through (f) and were based upon the relation of the end moment, M_e, to the moments M_{y1}, M_{y2}, and M_{y3} in Fig. 5.2. For Fig. 5.4(c) through (f), the end moment M_e was less than or equal to the moment M_{y1}, greater than the moment M_{y1} but less than or equal to the moment M_{y2}, greater than the moment M_{y2} but less than or equal to the moment M_{y3}, and greater than the moment M_{y3}, respectively. For a given value of M_e, the value of δ (Equation 5.1, Fig. 5.4(a)) was computed from Equation 5.3, 5.4, 5.5, or 5.6, depending upon the magnitude of M_e.

$$\delta = \frac{1}{3} \phi_e \ell_e^2 \qquad M_e \le M_{y1} \tag{5.3}$$

$$\begin{aligned}\delta &= \frac{1}{3} \phi_{y1} (\Delta x_2)^2 + \phi_{y1} (\Delta x_1)[\frac{\Delta x_1}{2} + \Delta x_2] \\ &+ \frac{1}{2} (\phi_e - \phi_{y1}) (\Delta x_1)[\frac{2}{3} (\Delta x_1) + \Delta x_2] \\ &\qquad M_{y1} < M_e \le M_{y2}\end{aligned} \tag{5.4}$$

$$\begin{aligned}\delta &= \frac{1}{3} \phi_{y1} (\Delta x_3)^2 + \phi_{y1} (\Delta x_2)[\frac{\Delta x_2}{2} + \Delta x_3] + \phi_{y2} (\Delta x_1)[\frac{\Delta x_1}{2} + \Delta x_2 + \Delta x_3] \\ &+ \frac{1}{2}(\phi_{y2} - \phi_{y1}) (\Delta x_2) [\frac{2}{3}(\Delta x_2) + \Delta x_3] \\ &+ \frac{1}{2} (\phi_e - \phi_{y2}) (\Delta x_1)[\frac{2}{3}(\Delta x_1) + \Delta x_2 + \Delta x_3] \\ &\qquad M_{y2} < M_e \le M_{y3}\end{aligned} \tag{5.5}$$

$$\delta = \frac{1}{3}\,\phi_{y1}\,(\Delta x_4)^2 + \phi_{y1}\,(\Delta x_3)\,[\frac{\Delta x_3}{2} + \Delta x_4]$$

$$+ \phi_{y2}\,(\Delta x_2)[\frac{\Delta x_2}{2} + \Delta x_3 + \Delta x_4] + \phi_{y3}\,(\Delta x_1)[\frac{\Delta x_1}{2} + \Delta x_2 + \Delta x_3 + \Delta x_4]$$

$$+ \frac{1}{2}\,(\phi_{y2} - \phi_{y1})\,(\Delta x_3)\,[\frac{2}{3}(\Delta x_3) + \Delta x_4]$$

$$+ \frac{1}{2}\,(\phi_{y3} - \phi_{y2})\,(\Delta x_2)\,[\frac{2}{3}(\Delta x_2) + \Delta x_3 + \Delta x_4]$$

$$+ \frac{1}{2}\,(\phi_e - \phi_{y3})\,(\Delta x_1)[\frac{2}{3}\,(\Delta x_1) + \Delta x_2 + \Delta x_3 + \Delta x_4]$$

$$M_e > M_{y3} \qquad (5.6)$$

Calculations were performed for several values of M_e in each of the four above ranges. For each case, the values of ϕ_e, ϕ_{y1}, ϕ_{y2} and ϕ_{y3} were obtained from the idealized moment-curvature relation. The values of Δx_1, Δx_2, Δx_3 and Δx_4 were obtained from the moment distribution (Fig. 5.4a) and the magnitudes of M_{y1}, M_{y2} and M_{y3} (Table 5.1).

The end rotation, θ_E, was then determined from Equation 5.2. The computed relation between end moment, M_e, and end rotation, θ_E, is presented in Fig. 5.5. The point of maximum rotation on the moment-rotation relation corresponds to the rotation obtained from the case where the maximum curvature along the beam, ϕ_e, is equal to the maximum curvature on the idealized moment-curvature relation. Finally, the moment-rotation relation was idealized into the tri-linear form also depicted in Fig. 5.5. The values of moment and rotation corresponding to slope discontinuity in the idealization are listed in Table 5.2.

(e) Moment-Rotation Relation for Pier

The computation of the moment rotation relation for each story of the pier is illustrated in Fig. 5.7. Each member consisted of two infinitely rigid end portions and the deformable portion of length, ℓ_p. Referring to Fig. 5.7(a), the moment rotation relation was composed of the sum of the end moments, M_e, where,

$$M_e = M_{eb} + M_{et} \tag{5.7}$$

The linear distribution of moment was obtained directly from statics (Fig. 5.7b).

The idealized moment-curvature relation, presented in part (c) of this section, was used to obtain the curvature distribution for a given pair of end moments, M_{eb} and M_{et} (Fig. 5.7c through e). The total rotation was obtained from

$$\theta_E = \int_0^{\ell_p} \phi(x)dx \tag{5.8}$$

Again, the curvature distribution was piecewise linear, enabling the integral of Equation 5.8 to be evaluated as the sum of several trapezoidal areas.

Considering M_{et} to be less than or equal to M_{y1} for the pier, θ_E was computed from Equation 5.9, 5.10 or 5.11, depending upon the magnitude of M_{eb} (Fig. 5.7c, d and e).

$$\theta_E = \frac{1}{2} (\phi_{et} + \phi_{eb}) \ell_p \qquad M_{eb} \leq M_{y1} \tag{5.9}$$

$$\theta_E = \frac{1}{2}[(\phi_{eb} + \phi_{y1})(\Delta x_1) + (\phi_{y1} + \phi_{et})(\Delta x_2)]$$

$$M_{y1} < M_{eb} \leq M_{y2} \qquad (5.10)$$

$$\theta_E = \frac{1}{2}[(\phi_{eb} + \phi_{y2})(\Delta x_1) + (\phi_{y1} + \phi_{y2})(\Delta x_2) + (\phi_{y1} + \phi_{et})(\Delta x_3)]$$

$$M_{eb} > M_{y2} \qquad (5.11)$$

For given values of M_{eb} and M_{et}, the curvatures ϕ_{eb} and ϕ_{et} were obtained from the idealized moment-curvature relation (Fig. 5.3). Knowing the values for M_{y1} and M_{y2}, the values of Δx_1, and Δx_2 and Δx_3 were computed from the linear distribution of moment (Fig. 5.7b).

In relating θ_E to M_e there was some question concerning how the individual end moments M_{eb} and M_{et} vary as the total, M_e, is varied. In computing a moment-rotation relation, M_{et} was considered constant, while only M_{eb} varied. The relation was then computed from Equations 5.9 through 5.11. However, the moment-rotation relation was different for different values of M_{et}, necessitating computation of the relation for several values of M_{et}.

The calculated moment rotation relations are presented in Fig. 5.6. The point on each relation corresponding to maximum rotation corresponded to a value of ϕ_{eb} equal to the maximum curvature consistent with the idealized moment-curvature relation (Fig. 5.3). The calculated curves were finally idealized into the trilinear form shown in Fig. 5.6. The values of moment and rotation corresponding to discontinuity of slope in the idealized relations are listed in Table 5.2.

(f) Equivalent Section Stiffness

As mentioned in part (b), the analytical model assumed a prismatic section along the length of any given member. Due to local yielding, the section stiffness does vary along the length of both beams and pier members. In order to account for the variation of section stiffness, an equivalent section stiffness was used. This was obtained by setting the ratio of the end moment to the end rotation for a member with uniform section stiffness equal to the slope of the calculated moment-rotation relation (Fig. 5.5 and 5.6). The uniform section stiffness satisfying this criterion was then used in the analysis. The procedure will first be illustrated for the beams. The geometry and distribution of moment for the equivalent beam would be that depicted in Fig. 5.4a and b. The distribution of curvature is shown in Fig. 5.8, where EI_{eq} represents the uniform section stiffness. The end rotation of the member was computed by applying Equations 5.2 and 5.3.

$$\theta_E = \frac{1}{3}\left(\frac{M_e}{EI_{eq}}\right)\frac{\ell_e^2}{\ell_E} \tag{5.12}$$

Denoting the ratio of end moment to end rotation for the equivalent member by $\left(\frac{M_e}{\theta_E}\right)_{eq}$,

$$\left(\frac{M_e}{\theta_E}\right)_{eq} = 3(EI_{eq})\frac{\ell_E}{\ell_e^2} \tag{5.13}$$

Denoting the slope of the inelastic moment rotation relation by $\left(\frac{M_e}{\theta_E}\right)_y$, the criterion to be satisfied was,

$$\left(\frac{M_e}{\theta_E}\right)_{eq} = \left(\frac{M_e}{\theta_E}\right)_y \tag{5.14}$$

Combining Equations 5.13 and 5.14,

$$EI_{eq} = \frac{1}{3} \left(\frac{M_e}{\theta_E}\right)_y \left(\frac{\ell_e^2}{\ell_E}\right) \qquad (5.15)$$

In general, during a given load step, each beam was at a different stage in its hysteretic response, hence, $\left(\frac{M_e}{\theta_E}\right)_y$, and therefore EI_{eq}, was a different numerical value for each beam. By determining a new value of EI_{eq} for each step of loading or unloading, a condition was maintained in which the equivalent member had the same moment rotation stiffness (overall stiffness) as the more realistically modeled inelastic member.

The same fundamental concept as for the beams was used to obtain an equivalent uniform section stiffness for the pier members. The appropriate geometry and moment distributions for the equivalent member were those depicted in Fig. 5.7(a) and (b). The distribution of curvature is depicted in Fig. 5.9. The computation of the total rotation, θ_E, was accomplished by applying Equation 5.9. Hence,

$$\theta_E = \frac{1}{2} \left(\frac{M_{et} + M_{eb}}{EI_{eq}}\right) \ell_p \qquad (5.16)$$

As was required for the equivalent beam member,

$$\left(\frac{M_e}{\theta_E}\right)_{eq} = \left(\frac{M_e}{\theta_E}\right)_y \qquad (5.17)$$

Combining Equations 5.7, 5.16 and 5.17,

$$EI_{eq} = \frac{\ell_p}{2} \left(\frac{M_e}{\theta_E}\right)_y \qquad (5.18)$$

As for the beam a new value of EI_{eq} was computed whenever the slope of the inelastic moment-rotation relation changed. In this manner the overall stiffness of the equivalent member was maintained equal to that of a realistic yielding member.

(g) Calculation Procedure

The calculation procedure applied to the model of Fig. 5.1 is outlined in Fig. 5.10. Cyclic loading is modelled by applying various hysteresis schemes to the primary moment-rotation relations of Fig. 5.5 and 5.6. Hence, in addition to the structural idealization of Fig. 5.1, the input for the analysis consisted of a piecewise linear hysteretic moment-rotation relation for each member in the structure. An example would be that of Fig. 5.13. As discussed previously, the analysis was performed in a series of steps of loading or unloading, members being linearly elastic in each step. For each step the uniform section stiffness, EI_{eq}, to be applied to a member was determined from the slope of the applicable portion of the moment-rotation hysteresis relation, using either Equation 5.15 or 5.18. The resulting set of uniform section stiffnesses was then used to assemble the equivalent structure stiffness matrix, $[K_{eq}]$, a 12 by 12 matrix. This represented a tangent stiffness for the non-linear hysteretic structure. The degrees of freedom considered for the structure were the lateral displacement and rotation for each of the six beam-column joints (Fig. 5.1). However, the fact that the externally applied moment at each joint was zero was used to condense the stiffness matrix into a six by six format, where the six degrees of freedom were the lateral displacement of each joint. Explicit consideration of the rotations of the hinged ends of the beams was avoided by modelling each beam as a rotational spring of stiffness given by,

$$k_{sp} = \frac{\ell_E}{\ell_e} \left(\frac{M_e}{\theta_E}\right)_{eq} \qquad (5.19)$$

where $\left(\frac{M_e}{\theta_E}\right)_{eq}$ was given by Equation 5.13.

Again, in general, k_{sp} had a different numerical value for each beam. The degrees of freedom are depicted in Fig. 5.11, which is a representation of the structure of Fig. 5.1. The degrees of freedom U_i and θ_i represent lateral displacement and rotation of the i^{th} joint. The load P_i was the lateral applied load at the i^{th} joint. The six member vectors, $\{U\}$, $\{\theta\}$ and $\{P\}$ were composed of the values of U_i, θ_i and P_i, respectively.

The incremental load vector $\{\Delta P\}$ consistent with the loading pattern depicted in Fig. 5.1 and 5.11 was given by,

$$\{\Delta P\} = \{R_\ell\} \quad (\Delta Q) \tag{5.20}$$

where ΔQ was the increment of the load Q (Fig. 5.1). The value of ΔQ was guessed at this stage of the analysis. The vector $\{R_\ell\}$ denoted the predetermined ratio of the lateral loads, which remained constant throughout the analysis.

$$\{R_\ell\} = \begin{Bmatrix} a_3 \\ 0 \\ a_2 \\ 0 \\ a_1 \\ 0 \end{Bmatrix} \tag{5.21}$$

where a_1, a_2 and a_3 are defined in Fig. 5.1. What resulted was a straightforward problem in linearly elastic structural analysis. The equilibrium equation was given by,

$$[K_{eq}] \{\Delta U\} = \{\Delta P\} \tag{5.22}$$

where the six member vector, $\{\Delta U\}$, contained the incremental lateral displacement at each joint (Fig. 5.11). Solving,

$$\{\Delta U\} = [K_{eq}]^{-1} \{\Delta P\} \tag{5.23}$$

The six joint rotations (θ_1 through θ_6 in Fig. 5.11) were derived from $\{\Delta U\}$ via a six by six transformation matrix, [T].

$$\{\Delta\theta\} = [T]\ \{\Delta U\} \tag{5.24}$$

Combining equations 5.20, 5.23, and 5.24,

$$\{\Delta\theta\} = [T]\ [K_{eq}]^{-1}\ \{R_\ell\}\ (\Delta Q) \tag{5.25}$$

The structure geometry was such that the six joint rotations were identical to the end rotations, θ_E, of the beams. The total end rotations, θ_E, for the pier members were obtained by summing the two appropriate joint rotations. Hence, the incremental joint rotations, $\{\Delta\theta\}$, were directly translatable into increments of the member end rotations, $\Delta\theta_E$, for which the piecewise linear moment-rotation hysteresis relations were developed. Using the moment-rotation relations, incremental end moments, ΔM_E, were defined. For each member there existed a factor, f, such that,

$$(M_E)_{lim} = (M_E)_0 + f(\Delta M_E) \tag{5.26}$$

where,

(ΔM_E) = incremental end moment for a member implied by the vector, $\{\Delta\theta\}$, as calculated in Equation 5.25.

$(M_E)_0$ = end moment of a member at beginning of the loading step in question.

$(M_E)_{lim}$ = end moment of a member corresponding to a change in slope of the moment-rotation relation.

Rearranging,

$$f = \frac{(M_E)_{lim} - (M_E)_0}{(\Delta M_E)} \tag{5.27}$$

Equation 5.27 was evaluated for each member in the structure. The smallest resulting value was designated f_{min}. A vector, $\{\Delta M_E'\}$ of member end moments was then defined by,

$$\{M_E'\} = f_{min} \{\Delta M_E\} \tag{5.28}$$

This represented the vector of incremental member end moments at which the stiffness distribution of the structure needed to be altered. Because the structure was assumed to respond linearly during each load step,

$$\{\Delta P'\} = f_{min} \{\Delta P\} \tag{5.29}$$

and

$$\{\Delta U'\} = f_{min} \{\Delta U\} \tag{5.30}$$

where $\{\Delta P'\}$ and $\{\Delta U'\}$ represented the incremental lateral loads and joint deflections corresponding to $\{\Delta M_E'\}$. The lateral loads and lateral joint deflections at the onset of the loading step in question were denoted by $\{P_o\}$ and $\{U_o\}$, respectively. The lateral deflections and lateral loads for the level of each beam at the end of the step were then obtained from,

$$\{U\} = \{\Delta U'\} + \{U_o\} \tag{5.31}$$

and

$$\{P\} = \{\Delta P'\} + \{P_o\} \tag{5.32}$$

The values of $\{U\}$ and $\{P\}$ from Equations 5.31 and 5.32, then became the new values of $\{U_o\}$ and $\{P_o\}$ for the next step of loading or unloading.

By repeating the preceding sequence for load step after load step, the lateral load-lateral deflection hysteresis for the structure was computed. The result was, of course, a piecewise linear relation. The calculated lateral loads were then used to compute the base moment. The structure hysteresis was then illustrated by the relation between top level deflection and base moment.

5.2 Study of Hysteresis Shape

(a) General Comments

The analytical model described in Section 5.1 was used to study the effect upon the overall structure hysteresis of various hysteresis models applied to the beams. This section describes that study. The first hysteresis model investigated was that devised by Takeda (ref. 36). This was a general model for reinforced concrete used in previous studies (ref. 15,25). Subsequent investigations applied modifications of the Takeda model to the beams. These were designed to simulate phenomena such as total loss of concrete for the beam section adjacent to the pier edge, slip of beam longitudinal reinforcement in the joint, and yielding of beam longitudinal reinforcement in compression as cracks close on the beam adjacent to the pier edge. For all cases, the pier was assigned an unaltered Takeda model.

The imposed deflection schedule was identical for all models and is depicted in Fig. 5.12. The limiting top level deflection for each quarter cycle was chosen to be equal to the limiting top level deflection for the corresponding quarter cycle of Test S1. The loading history considered a total of one and one quarter cycles.

As described in the preceding section, the model utilized a predetermined ratio among the lateral loads which remained constant throughout a given analysis. For this study, this ratio was chosen to correspond to that for Test S1. The loads were constrained to be in a ratio given by the shape of the first mode for the test structure. Referring to Fig. 5.1, the ratio $a_1:a_2:a_3$ was equal to 1:3:5.

The following parts will describe each of the five models studied and present the resulting overall structure hysteresis, in terms of base moment and top level deflection.

(b) Hysteresis Model 1

Model 1 (Fig. 5.13) was an unaltered Takeda model. As for all cases, in the first quarter-cycle the relation corresponded to the calculated moment-rotation relation (Fig. 5.5). The points in Fig. 5.13 corresponding to the first and second yield levels are denoted by Y1 and Y2, respectively. The primary curve was also defined in the opposite direction of loading (shown as a broken line in Fig. 5.13), the points corresponding to the first and second yield levels being denoted by -Y1 and -Y2, respectively.

The hysteresis rules were defined as follows. The maximum rotation experienced by the beam during the first one-quarter cycle was denoted θ_{m1} with the corresponding point on the moment-rotation relation being denoted M1. The maximum rotation was a result of the analysis and was the rotation consistent with the predetermined limiting top level deflection for the first one-quarter cycle. The slope of the unloading segment was determined from,

$$s_{r1} = s_1\left(\frac{\theta_{y1}}{\theta_{m1}}\right)^{0.5} \tag{5.33}$$

where s_1 was the slope of the first segment of the primary curve. The point on the hysteresis relation corresponding to zero moment, point R1, was defined as the intersection of the unloading segment with the rotation axis. The first reloading segment was defined as a straight line connecting points R1 and -Y1. The path of reloading during the third quarter cycle then followed the primary curve "breaking" at point -Y2 and reversing at point M2. The slope of the second unloading segment was computed in a manner similar to that of the first unloading segment.

$$s_{r2} = s_1 \left(\frac{\theta_{y1}}{\theta_{m2}}\right)^{0.5} \tag{5.34}$$

The intersection of the unloading segment with the rotation axis defined point R2. The next reloading segment was defined as a straight line between points R2 and M1. Further reloading, with rotations greater than θ_{m1} occurred along the primary moment-rotation curve, terminating at a rotation θ_{m3} (point M3). All six beams were rotated beyond θ_{y2} during all of the first, third and fifth quarter cycles. The calculated values of θ_{m1}, θ_{r1}, θ_{m2}, θ_{r2} and θ_{m3} for each of the six beams are listed in Table 5.3.

The six pier members (one member for each story) also followed the Takeda model, but they did not experience such extensive yielding as did the beams. The rotations for the pier members of stories two through six did not exceed θ_{y1} at any time (Fig. 5.6). The first story pier member did experience limited yielding. During the first quarter cycle, the rotation did not exceed θ_{y1}, hence, unloading and reloading occurred along the original loading path. During the third quarter cycle, the maximum rotation exceeded θ_{y1}, but not θ_{y2}. The point corresponding to the maximum rotation, θ_{m2}, was denoted M2. In a manner similar to that for the beams, the slope of the unloading segment was computed from Equation 5.34. Again the intersection of the unloading segment with the rotation axis defined point R2 and the reloading segment was defined as a straight line connecting points R2 and M1, even though θ_{m1} was less than θ_{y1}. The reloading then followed the primary curve to the maximum fifth quarter cycle rotation θ_{m3}. The values of θ_{m1}, θ_{r1}, θ_{m2}, θ_{r2} and θ_{m3} are listed in Table 5.3.

The calculated overall structure hysteresis is shown in Fig. 5.14 as the relation between base moment and top level lateral deflection. The regions of the moment-deflection hysteresis corresponding to significant events in the moment rotation hysteresis of the beams (yielding, attainment of zero moment) are indicated in the figure. The numbers in parentheses after the type of the event indicate the order in which the beams at the various levels experienced the event.

(c) Hysteresis Model 2

The second hysteresis model considered the beams to lose all concrete in the region adjacent to the edge of the pier after the first quarter cycle of loading. In this region the section consisted only of the two layers of reinforcement. This is illustrated in Fig. 5.15.

To modify the Takeda model for the above condition, it was necessary to compute the moment-rotation relation for the beam of Fig. 5.15(d). It was considered that the yield of the reinforcement adjacent to the pier edge would lead to the development of an indefinite concentrated rotation of the beam in the region adjacent to the pier. In essence, it was assumed that the reinforcement did not strain harden. The resulting moment-rotation relation is illustrated in Fig. 5.16(c). The yield moment, M_{ys}, was computed from,

$$M_{ys} = f_{sy} A_s (d-d') \tag{5.35}$$

as illustrated in Fig. 5.16(a), where f_{sy} was obtained from Table A.5 for the #11 size reinforcement of test structure S1, A_s was obtained from the measured diameter for the #11 wire (Table A.3), and the depths, d and d', were equal to the nominal values for the test structure. The computation of the beam rotation corresponding to the development of the yield strength of the reinforcement adjacent to the pier, θ_{ys}, (Fig. 5.16(c)) involved considerable judgment. Fortunately, as will be illustrated later, the final structure hysteresis relation was not sensitive to the value of θ_{ys}. The reinforcement was assumed to experience a uniform curvature over the width of the crack, ℓ_{cr} (Fig. 5.15). Due to the moment gradient along the beam this assumption was not strictly correct. Due to the insensitivity of the results to θ_{ys}, this was deemed an acceptable assumption. Hence, θ_{ys} was computed, as illustrated in Fig. 5.16(a). From geometry,

$$\phi_{ys} = \frac{2\varepsilon_{sy}}{d-d'} \tag{5.36}$$

where ϕ_{ys} was the localized curvature at yield and ε_{sy} was the strain in the reinforcement at yield. Since the curvature was assumed uniform over the length of the crack,

$$\theta_{ys} = \phi_{ys}\ \ell_{cr} \tag{5.37}$$

Assuming bond between steel and concrete to be destroyed for some distance beyond the actual separation in the concrete, a value of 0.25 in. was considered for ℓ_{cr}, resulting in a value of 0.001 radians for θ_{ys}.

The resulting modifications to the Takeda hysteresis model are illustrated in Fig. 5.17. The primary moment-rotation relation calculated for the beam was applied as for the unaltered Takeda model. The moment-rotation relation for the damaged beam (Fig. 5.16) is also depicted in the figure, the points corresponding to yield for positive and negative directions of loading being denoted by (S) and (-S), respectively. Loading in the first quarter cycle to point (M1) occured along the primary curve as for the Takeda model. Similarly, the unloading slope, s_{r1} was defined as for the Takeda model. The value of s_{r1} defined the location of point (R1). At this stage of the loading, the crack at the end of the beam was assumed to be partially developed (Fig. 5.15(c)). The reloading segment was defined as a straight line connecting points (R1) and (-S). This represented complete opening of the crack. Further reloading occurred along the moment-rotation relation for the damaged beam, the maximum rotation, θ_{r2}, being attained at point (M2). The slope of the unloading segment was determined as for the Takeda model (Equation 5.34), determining the location along the rotation axis of point (R2). Next, the point (M1') was defined as that point on the moment-rotation relation for the damaged beam characterized by a rotation equal to θ_{m1}. The reloading segment was then defined as a straight line connecting points (R2) and (M1'). Further reloading, beyond a rotation of θ_{m1}, occurred along the moment-rotation relation for the damaged beam, the maximum rotation attained being denoted θ_{m3} and the corresponding point being (M3).

All six beams experienced the complete sequence of events depicted in Fig. 5.17. In essence, they all experienced the complete yielding process in each direction of loading. The calculated values of θ_{m1}, θ_{r1}, θ_{m2}, θ_{r2}, and θ_{m3} for each beam are listed in Table 5.3.

At this point the insensitivity of the hysteresis to the magnitude of θ_{ys} should become apparent. The only effect of this rotation upon the entire hysteresis relation is its effect upon the slope of the first reloading segment (third quarter cycle). Referring to the values for θ_{r1}, listed in Table 5.3, a reduction of θ_{ys} by 50% to a value of 0.0005 radian, would increase the slope of the reloading segment by 12%, for the first level beam. The effect for other beams would be much smaller. Hence, the uncertainty in the choice of θ_{ys} is not a factor for serious concern.

The Takeda hysteresis model was applied to the pier members, with the pattern of behavior paralleling that for Hysteresis Model 1. The members for stories two through six remained elastic throughout the analysis while the first story member followed the sequence of loading and unloading described for Model 1. The calculated values for θ_{m1}, θ_{r1}, θ_{m2}, θ_{r2} and θ_{m3} are listed in Table 5.3.

The computed overall structure hysteresis relation, in terms of base moment and top level deflection is shown in Fig. 5.18. As for Model 1 (Fig. 5.14) the regions of the moment-deflection relation corresponding to the various significant events in the hysteretic response of the beams are indicated, along with the sequence in which the various beams experienced each event.

(d) Hysteresis Model 3

The third hysteresis model studied considered the effect of slip of the longitudinal beam reinforcement in the beam-pier joint. Although the construction of the test structure makes the bond problem somewhat different from that of a prototype reinforced concrete structure, a case can be made for analogous behavior.

The mechanism of slip of reinforcement in the beam-pier joint of a prototype reinforced concrete structure is illustrated in Fig. 5.19. Deformed reinforcement would be present and the slip would be a manifestation of the elastic deformation of the bar in the joint. In Fig. 5.19(a), the top layer of reinforcement is loaded to its yield stress, f_{sy}, at the face of the joint. The force in the bar associated with the yield stress must be in equilibrium with the total force developed by the bond stresses along the length, ℓ_d, the development length for the bar. The stress in the bar at a distance ℓ_d into the joint would be zero. There would be an elastic deformation, $\Delta\ell$, for the bar, associated with this change in stress over the length ℓ_d. A manifestation of this deformation would be a deflection of the lugs on the reinforcement, accompanied by localized crushing of concrete adjacent to the lugs. This would cause the development of the voids illustrated in Fig. 5.19(a). When the direction of loading in the beam is reversed, the reinforcement layer, after unloading, must slip the distance $\Delta\ell$ before the lugs can bear on their opposite faces, allowing the reinforcement to develop compressive stress, and the beam to develop load in the opposite direction. The corresponding rotational slip in the beam would be given by,

$$\Delta\psi_1 = \frac{\Delta\ell}{d-d'} \tag{5.38}$$

As reloading occurs, the development of tensile stress in the bottom reinforcement layer will cause damage to concrete similar to that for the top layer in the first quarter cycle (Fig. 5.19(b)). When the direction of loading is again reversed, slippage must occur twice. The bottom reinforcement layer must slip a distance, $\Delta\ell$, to develop compressive force, while the top layer must also slip a distance, $\Delta\ell$, to develop tensile stress (Fig. 5.19(c)). The corresponding rotational slip would be given by,

$$\Delta\psi_2 = \frac{2(\Delta\ell)}{d-d'} \tag{5.39}$$

As tensile stress is again developed in the top reinforcement layer, further crushing will occur adjacent to the lugs. The slippage of the reinforcement layer will become $2(\Delta\ell)$. Hence, the rotational slippage of the beam at zero load will be increased by the increment $\Delta\psi_1$ for each successive reversal in loading.

The situation was somewhat different for the beam-pier joint of the test structure. The reinforcement was underformed wire. Positive anchorage was obtained by spot welding the beam longitudinal reinforcement to the vertical reinforcement of the pier. It can be argued that tensile forces in the longitudinal beam reinforcement transferred to the pier reinforcement through welds, are then resisted by compression in the concrete adjacent to the welds. Crushing of concrete may occur, creating a slip mechanism analogous to that described in the previous paragraph (Fig. 5.20).

As discussed previously, the incremental slip for each cycle, $\Delta\ell$, will be equal to the elastic deformation of the longitudinal beam reinforcement over its development length in the joint. The computation of this value is illustrated in Fig. 5.21. The stress in the reinforcement at the face of the joint was assumed to be equal to the yield strength of the steel. The variation of the tensile stress in the reinforcement along its length was assumed linear, the stress equalling zero at a distance, ℓ_d, into the joint. The implication was that the bond stresses between steel and concrete along the length of the reinforcement were uniform. The differential deformation, $d\ell$, is given by,

$$d\ell = \varepsilon_s \, dx \qquad (5.40)$$

where ε_s is the strain in the steel at a distance, x, from the point of zero steel stress (Fig. 5.21). The total deformation is given by,

$$\Delta\ell = \int_{o}^{\ell_d} d\ell \tag{5.41}$$

Combining Equations 5.40 and 5.41 and expressing the results in terms of stress, results in,

$$\Delta\ell = \int_{o}^{\ell_d} \frac{f_s}{E_s}\, dx \tag{5.42}$$

where f_s is the stress in the steel at a distance, x, from the point of zero steel stress and E_s is Young's modulus. The linear variation of stress with, x, may be expressed as,

$$f_s = f_{sy} \frac{x}{\ell_d} \tag{5.43}$$

where f_{sy} is the yield strength of the reinforcement. Combining Equations 5.42 and 5.43 and evaluating the resulting integral resulted in,

$$\Delta\ell = \frac{1}{2} \frac{f_{sy}\ell_d}{E_s} \tag{5.44}$$

The corresponding rotational slip for the end of the beam was then expressed as,

$$\Delta\psi_1 = \frac{1}{2} \frac{f_{sy}\ell_d}{E_s(d-d')} \tag{5.45}$$

The value for f_{sy} was taken from Table A.5 for test structure S1. Young's modulus was assumed equal to 29000 ksi. The depths, d and d', were assumed equal to their nominal values for the type B test structure. The development length, ℓ_d, was assumed equal to three inches. This implied that three of the vertical wires in the pier resisted the entire force in the longitudinal reinforcement of the beam. The result, computed from Equation 5.45, was a rotational slip, $\Delta\psi_1$, equal to 0.003 radian.

The modification of the Takeda hysteresis model to account for the slip mechanism is shown in Fig. 5.22. The model was identical to the Takeda model during the first quarter cycle of loading to a rotation equal to θ_{m1} (point M1). Similarly the slope of the unloading segment, defining the location of point R1 was consistent with the Takeda model. However, before reloading could occur, a slip equal to $\Delta\psi_1$ (0.003 radian) was assumed to occur. This slip corresponded to the effect of the voids in the concrete depicted in Fig. 5.19(a). In this manner, point (R1') was located. The first reloading segment was defined as a straight line connecting points (R1') and (-Y1). Further reloading occurred, as for the Takeda model, along the primary curve, to a maximum rotation equal to θ_{m2} (point M2). The unloading segment was consistent with the Takeda model (Equation 5.34), defining the location of point (R2). Before reloading occurred, a slip equal to 2 ($\Delta\psi_1$) was assumed to occur, locating point (R2'). This slip corresponded to the slippage of the reinforcement through the voids illustrated in Fig. 5.19(b). For reloading, the slip of the top reinforcement layer in Fig. 5.19(b) must also manifest itself in the translation of the primary moment-rotation relation a distance $\Delta\psi_1$, along the rotation axis. The translated moment-rotation relation is shown dashed in Fig. 5.22. The point (M1') on the translated relation, at a rotation equal to θ_{m1}, was defined. The first reloading segment was then defined as a straight line connecting points (R2') and (M1'). Further reloading was assumed to occur along the translated moment-rotation relation, terminating at a rotation equal to θ_{m3} (point M3). All six beams experienced the entire sequence of loading and unloading depicted in Fig. 5.22. The calculated values for θ_{m1}, θ_{r1}, θ_{m2}, θ_{r2}, and θ_{m3} are listed in Table 5.3.

The pier members were assumed to follow an unaltered Takeda hysteresis. The sequence of loading and unloading was similar to that for Models 1 and 2. The second through sixth story members remained elastic. The calculated values for θ_{m1}, θ_{r1}, θ_{m2}, θ_{r2}, and θ_{m3} for the first story member are listed in Table 5.3.

The calculated structure hysteresis in terms of base moment and top level deflection, is shown in Fig. 5.23. As for Models 1 and 2, the regions of the moment-rotation relation corresponding to the various events in the hysteretic response of the beams are indicated, along with the sequence in which the beams experienced each event.

(e) Hysteresis Model 4

The fourth beam hysteresis model was a modification of the second model. As for Model 2, the section of the beam immediately adjacent pier was assumed to be characterized by total loss of the concrete. Only the reinforcing steel remained. This state is depicted in Fig. 5.15, 5.16, and 5.24(a). For Model 2, the beam in this region was assumed to experience indefinite concentrated rotation, performing as a section composed only of two yielded reinforcement layers. For Model 4, however, the beam was assumed to experience only a specific amount of rotation before closure of the crack or gap adjacent to the pier occurred (Fig. 5.24(b)). This transformed the section into one composed of both concrete and steel, enabling further reloading to occur.

Fig. 5.24 also illustrates the computation of the rotation necessary to cause closure of the crack. The rotation, $\theta_{c\ell}$, consistent with closure, was given by,

$$\theta_{c\ell} = \frac{2\ell_{cr}}{d_w} \tag{5.46}$$

assuming the mid-height of the section to be the center of rotation. For this purpose, the crack width, ℓ_{cr}, was assumed equal to 0.005 in., resulting in a rotation of 0.0067 radian. Consistent with the crudeness of the assumptions, a rotation to closure of 0.006 radian was applied in the analysis.

The modification applied to the Takeda hysteresis model is illustrated in Fig. 5.25. The primary moment-rotation relation and the moment-rotation relation for the damaged beam (Fig. 5.15 and 5.24) are shown as for Model 2 (Fig. 5.17). The moment-rotation relation for the damaged beam is identical to that for Model 2. The loading relation in the first quarter cycle, the first unloading segment, and the first reloading segment, terminating at point (-S), are identical to those for Model 2. Further reloading follows the moment-rotation relation for the damaged beam for rotations less than $\theta_{c\ell}$ (point C1). At this point closure of the crack (Fig. 5.24(b)) was assumed to occur, and further reloading occurred along a segment with a slope equal to s_1; the damaged concrete and steel section was assumed to have a stiffness equal to that of the intact beam section. The reloading segment was assumed to terminate along the primary moment-rotation relation at point (C1'). At this moment level, yielding of the member was assumed to occur, and further reloading was consistent with the primary relation, terminating at a rotation equal to θ_{m2} (point M2). The unloading segment was defined as for Model 2, defining the location of point (R2). The reloading segment was identical to that for Model 2, until a rotation equal to $\theta_{c\ell}$ in the opposite direction was attained (point C2). The member was then assumed to stiffen, and reload along a segment of slope s_1. This segment was assumed to terminate at point (C2'), a point on the reloading segment for the unaltered Takeda hysteresis model. In essence, the beam was assumed to yield at a moment level consistent with the strength of the unaltered Takeda model. Further reloading was consistent with the unaltered Takeda model, achieving a maximum rotation equal to θ_{m3} (point M3). With one exception, the beams experienced the entire sequence of loading and unloading depicted in Fig. 5.25. The exception was that, for the first level beam, θ_{m1} was less than $\theta_{c\ell}$. The result was that, for the second reloading phase (fifth quarter cycle), an interpretation of the hysteresis rule, specifically for low amplitude response was required. The interpretation is illustrated in Fig. 5.27. The figure shows the loading paths of the first and fifth quarter cycles along with the unloading paths of the second and sixth quarter cycles. For the fifth quarter cycle the system was assumed to follow the standard loading path connecting points R2 and M1' until it attained the intersection point, M1", with the second quarter cycle unloading path (line connecting points M1 and R1). The system was then assumed to reload along the second quarter cycle unloading path, yielding at a moment M1 and following the primary moment-rotation relation for larger rotations. The rationale for this procedure was that since the beam had never experienced rotations larger than θ_{m1}, it should not be modeled to respond as a section devoid of concrete for rotations larger than θ_{m1}. In essence, the crack must close when the beam attains the largest rotation it had previously experienced. If the direction of loading were reversed while the beam was loading along the path connecting points

M1" and M1, it would unload along this same path. If the load were reversed at a rotation less than θ''_{m1}, the unloading slope would be computed from the usual relation (analogous to Equations 5.33 and 5.34). This final point will be significant in a later study. The calculated values of θ_{m1}, θ_{r1}, θ_{m2}, θ_{r2} and θ_{m3} for each beam are listed in Table 5.3.

The pattern of loading and unloading for the pier members paralleled that for the other models. The pier members of stories two through six remained elastic. The values of θ_{m1}, θ_{r1}, θ_{m2}, θ_{r2} and θ_{m3} for the first story pier are listed in Table 5.3.

The calculated overall structure hysteresis relation, in terms of base moment and top level deflection is shown in Fig. 5.26. As for Models 1, 2 and 3 the regions of the moment-rotation relation corresponding to the various events in the hysteretic response of the beams are indicated, along with the sequence in which the beams experienced each event.

(f) Hysteresis Model 5

The fifth hysteresis model was a modification of the fourth model. The only modification occurred in the fifth quarter cycle. After the beam had been cycled once in each direction, it was assumed that the faces of the crack, or separation, at the beam-pier interface were deteriorated sufficiently to preclude reloading after crack closure with a stiffness equal to that for the intact beam member. Hence, in the fifth quarter cycle, this stiffness was reduced. The modification is illustrated in Fig. 5.25. After the crack closed at a rotation equal to $\theta_{c\ell}$, reloading was assumed to occur along the straight line connecting points (C2) and (M1), the strength of the section again being consistent with Takeda model. As for Model 4, further reloading was consistent with the Takeda model, attaining a maximum rotation, θ_{m3} (point M3).

The application of this modification to the first level beam again, presented a special problem. As for Model 4, this was necessitated by the fact that, for this member, θ_{m1} was less than $\theta_{c\ell}$. The path of reloading was identical to that for Model 4. The beam was assumed to reload along the line connecting points (R2) and (M1') (Fig. 5.27). Reloading then occurred along the line connecting points (M1") and (M1), the unloading segment for the second quarter cycle. Further reloading occurred along the unaltered Takeda relation to a maximum rotation of θ_{m3} (point M3). The calculated values of θ_{m3} for the beams are listed in Table 5.3. Other rotational values were the same as for Model 4.

The alteration in the structure hysteresis relation for Model 4, due to the modification characterizing Model 5, is denoted in Fig. 5.26. Again, the significant events in the hysteretic response of the beams are indicated.

(g) Calculation of Base Moment for Observed Response

The following paragraphs describe the calculation of the base moment top level deflection hysteresis relation corresponding to the response observed during test S1. This step was preparatory to comparing the results of the analytical study of hysteresis shape (sections 5.2(a) through (f)) with the observed response (section 3.5).

Figure 3.54 presents the observed hysteresis relations, in terms of load in a ram and structure deflection at the point of application of that same load, for each of the three levels along the height of the structure at which load was applied. The general shape of the hysteresis relation was the same for all three levels. After the first two quarter cycles of response, the reloading paths exhibit some distinctive characteristics. As reloading commences, the structure begins to exhibit a steady decrease in stiffness. This progresses until the stiffness becomes quite low. After a certain amount of deformation the structure regains its lost stiffness and eventually reaches the maximum load observed in previous cycles. The loss of stiffness, deformation at a low stiffness level, and regaining of stiffness becomes increasingly pronounced with successive reloading cycles. It is the purpose of this section to interpret the above phenomena in terms of the behavior mechanism for the connecting beams.

In preparation for comparison with the analytical results, the observed responses were expressed in the same terms as the analytical results, a relation between base moment and top level deflection. This was accomplished using the observed relation between top level load and top level deflection (Fig. 3.54c) and considering the ratios among the three applied lateral loads to be those intended for test S1 (Fig. 3.49a). The base moment was expressed directly as a constant times the top level load recorded in Fig. 3.54(c). A major consideration in the validity of this approach was how closely the actual applied loads conformed to the intended ratios. If the loading equipment did not closely maintain the intended load ratios, base moments computed as a multiple of the top level ram load might be inaccurate. Using the continuously recorded hysteresis relations (Fig. 3.54), however, this was the only computation method applicable. Base moments could not be calculated using the ram load at each of the three levels because there was no direct way to choose values of ram load occurring at the same instant of time. However, during the first quarter cycle of testing, ram loads were recorded at discrete times, with the application of load temporarily halted (Fig. 3.53). For this data, ram loads corresponding to the same times could be used in the calculation of base moment. Using the discrete loads, a comparison was made between base moment calculated considering all three ram loads and their appropriate moment arms (true base moment) and base moment calculated as a multiple of top level load (load-multiple base moment) (Fig. 5.28). The results of this comparison are presented in Fig. 5.29. The figure shows the variation with true base moment of the deviation of the "load-multiple base moment" from the true base moment as a per cent of true base moment. For base moments greater than four kip-in., the error was insignificant. With this result in mind, the base moment for the first one and one quarter cycles of test S1 was calculated as shown in Fig. 5.28 for various values of top level deflection using Fig. 3.54(c). The results are depicted by the broken curve in Fig. 5.30.

(h) Comparison of Analytical Results and Observed Response

The following paragraphs compare the results of the study of hysteresis shape with the hysteresis relation observed in test S1. The objective of this comparison was to relate the various mechanisms of beam behavior, or energy dissipation, with the observed response.

The comparison is illustrated in Fig. 5.30, as variations of base moment with top level deflection. The observed relation is represented by the broken line. There were characteristics of this relation that required careful interpretation. During early stages of the reloading portions of the hysteresis relation (third and fifth quarter cycles), the stiffness of reloading became progressively lower, until a rather low level of stiffness prevailed. In later stages of reloading, an apparent restiffening occurred, followed by an apparent decrease in stiffness as the maximum moment was approached. Figure 5.30 illustrates that this phenomenon was more noticeable for the fifth quarter cycle than for the third quarter cycle. In fact, Fig. 3.54 (broken curve) illustrates that the behavior became increasingly pronounced with each succeeding quarter cycle of loading throughout the test.

The moment-deflection relation for beam hysteresis model 1 (section 5.2(b)) did not exhibit the successive loss of stiffness and restiffening characterizing the observed response. This is apparent in Fig. 5.30. This beam hysteresis model followed the general rules given by Takeda (ref. 36).

Beam hysteresis model 2 (section 5.2(c)) considered gross cracking of the concrete at the ends of the beams. This model did exhibit a marked decrease in stiffness upon reloading. It did not, however, exhibit the restiffening. Even the initial decrease in stiffness during reloading was probably not so pronounced as for the observed response. Finally, the model did not exhibit the maximum moment capacity apparent in the observed response. The apparent moment capacity of the observed response indicates that, at maximum deflection, the section of the beam consisted of more than merely the two steel layers considered in model 2. A certain amount of concrete was apparently acting in compression.

Beam hysteresis model 3 (section 5.2(d)) considered the slip of the longitudinal reinforcement of the beams in the beam-pier joints. The model exhibits the apparent moment capacity of the observed response, after the reinforcement had slipped, the concrete could act in compression, and the moment capacity of model 1 was available. Although the model did exhibit an initial loss of stiffness upon reloading, followed by restiffening, the restiffening occurred much sooner, during reloading, than for the observed reloading. Evidently, the phenomenon determining the shape of the observed moment-deflection relation, was capable of causing greater incremental deflections (e.g. greater incremental rotations of the beams) at low stiffness levels than was the slip of reinforcement in the joints. A mechanism was required that would allow the beams to rotate through a greater angle before reloading.

Beam hysteresis model 4 (section 5.2(d)) was an attempt to provide for the greater rotations referred to in the preceding paragraph. This model was actually a modification of model 2, allowing for closure of the wide cracks at the ends of the beams. This model would allow for larger beam rotations than the reinforcement slip model, yet allow the concrete at the ends of the beams to act in compression in the late stages of reloading. The results from model 4 shown in Fig. 5.30 indicate that the restiffening does occur during later stages of reloading, as it did for the observed response. A reasonable magnitude of beam rotation at low stiffness appears to have been attained. The restiffening was, however, somewhat more abrupt than indicated by the observed response. This model assumed that the beams restiffened at their initial, first-quarter-cycle stiffness. Apparently, the beam was not so stiff as the crack closed. This was possibly due to reseating of the edges of the crack as closure took place.

Beam hysteresis model 5 was a modification of model 4, allowing for a more gradual closure of the crack at the end of the beam (Fig. 5.25). This appears to improve correlation of the analytical model with observed response.

Beam hysteresis model 5 exhibited the general characteristics of the observed response. Fine tuning the analytical model to correspond to observed response was probably not warranted within the degree of refinement of the study. The beam rotation necessary to initiate closure of the crack, along with the stiffness of the beam while closure is taking place, are difficult variables to quantify. Similarly, crack closure may not even terminate at point M1 (Fig. 5.25). The reseating of the two edges of the crack, along with localized crushing or loss of concrete, may be such that the full moment capacity of the beam section is attained only at some rotation greater than θ_{m1} (Fig. 5.25). This could well cause the apparent discrepancy between beam hysteresis model 5 and the observed response during the final stage of reloading for the fifth quarter cycle (Fig. 5.30).

In conclusion, the mechanism of energy dissipation for the beams appears to entail the development of wide cracks, accompanied by loss of concrete, at the ends of the beams. As the beam rotates, as lateral loading is applied to the structure, these wide cracks repeatedly open and close.

It should also be mentioned that, apparently, the six beams did not share equally in the dissipation of energy, some beams attained considerably higher maximum rotations than others (Table 5.3). The fourth and fifth level beams exhibited the greatest degree of inelastic action, the first level beam, the least degree of inelastic action. Similarly, significant events in the hysteretic response of the beams (yielding, stiffening, attainment of zero moment) occurred first in the "middle" beams (levels 2,3,4), and occurred later in the bottom and top beams. This is shown in Fig. 5.14, 5.18, 5.23 and 5.26 for overall structure response corresponding to each beam hysteresis model.

A final comment, concerning the piers, is in order. Only a small degree of inelastic behavior occurred during the response of these members, and that was confined to the base. This is mentioned in sections 5.2(b) through (f) and may be verified in Tables 5.2 and 5.3. Furthermore, the overall hysteresis relation for the entire structure (base moment related to top level deflection) was quite sensitive to changes in the moment rotation hysteresis of the beam members. This was consistent with the basic linearity of response for the piers. This point also established the beams as the significant source of energy dissipation in the structure as a whole.

5.3 Study of Equivalent Damping

(a) General Comments

The analytical model for static loading was also used to compare the damping capacity of the test structure when responding in the first mode to its damping capacity when responding in the second mode. Lateral loads, with the ratios between the second, fourth and sixth level loads corresponding to the first and second mode shapes, were applied to the structure. As in the hysteresis shape study, the ratios were maintained as constant throughout each analysis. Additionally, for the first mode loading, the structure was analyzed for two widely different maximum response amplitudes. This enabled the effect of response amplitude upon damping capacity to be studied. For the beams, hysteresis model 5 (section 5.2) was applied. An unaltered Takeda hysteresis was applied to the piers. The structure hysteresis, in terms of base moment and top level deflection, was then calculated for each of the two load ratios (two modes). Viscous damping coefficients, consistent with each of the two overall structure hysteresis relations, were derived using a concept developed by Jacobsen (ref. 19). Subsequent parts of this section describe the study in detail.

(b) First Mode Load Case

The ratio of the lateral loads for the first mode was that applied in test S1 and used for the hysteresis shape study (section 5.2). The ratio is illustrated in Fig. 5.31(a).

The deflection schedule is shown in Fig. 5.32(a). The maximum top level deflections were chosen to be consistent with observed values from the dynamic and static tests. The maximum response level for the lower amplitude cycles was also chosen with the observed hysteresis from test S1 in mind (Fig. 3.54). The observed hysteresis exhibited a lower stiffness at lower deflection values. The limiting amplitude for the low amplitude portion of the analysis was chosen to be consistent with the deflections corresponding to the low stiffness region of the observed hysteresis. Also note the bifurcation point, B, in the deflection schedule (Fig. 5.32a). After the structure had been cycled to the state corresponding to point B, two cases were investigated, represented by the two paths in the figure. In one case, loading continued to the upper deflection limit. In the other case, the cycles were limited to low amplitudes.

The application of hysteresis model 5 to the study of equivalent damping is illustrated in Fig. 5.33. The first five quarter cycles were identical to the relation shown in Fig. 5.25. The rule for the sixth quarter cycle was the same as that for the second and fourth quarter cycles. The rule for the seventh quarter cycle was the same as that for the fifth quarter cycle. Point B in the seventh quarter cycle corresponds to the bifurcation point, B, in Fig. 5.32(a). For the high amplitude response, the seventh quarter cycle was the final quarter cycle, terminating at point M4, following the path described by points R3, B, C3, M2 and M4. For the low amplitude portion of the study the seventh quarter cycle terminated at point B and was followed by four additional quarter cycles, terminating at point M6. Note that the rotation, θ_{m4}, occurred twice, once in the high amplitude portion of the study and once in the low amplitude portion.

The values of θ_{m1}, θ_{r1}, θ_{m2}, θ_{r2}, θ_{m3}, θ_{r3}, θ_{m4}, θ_{r4}, θ_{m5}, θ_{r5}, θ_{m6} for each beam are listed in Table 5.4. Note that for the second through sixth level beams, during the high amplitude cycles, each of θ_{m1}, θ_{m2}, θ_{m3} and θ_{m4} exceeded the beam rotation for crack closure, $\theta_{c\ell}$. These beam cycled through the complete sequence of events in Fig. 5.33. Only for the first level beam was this not the case. The discussion in section 5.2(e) relating to Fig. 5.27 would apply for the first level beam. During the low amplitude response, closure of the cracks did not occur. Reversal of load, for all beams, occurred without stiffening of the section. This behavior is shown in Fig. 5.33, and corresponds to the dashed unloading segment of Fig. 5.27.

Figure 5.34 depicts the resulting overall structure hysteresis relation in terms of base moment and top level deflection. The bifurcation, corresponding to the bifurcation in the deflection schedule (Fig. 5.32(a)) is labeled point B.

(c) Second Mode Load Case

The ratios of the lateral loads for the second mode loading were obtained from the linear response history study (chapter 8). The calculated mode shape amplitudes for the second mode, at each of three appropriate levels, were averaged over all 26 cases studied in chapter 8. This operation resulted in the load ratios depicted in Fig. 5.31(b).

The deflection schedule is shown in Fig. 5.32(b). For the viscous damping factors for the two modes to be comparable, the maximum deflections used in the analysis for the first and second modes were required to represent similar levels of overall structure response, again, the results of the response history analysis, described in chapter 8, were used to accomplish this. Cases having first mode components of response most closely describing the test results were chosen. (Analyses 6, 16, 20, and 24 in chapter 8). The maximum top level deflection (sum of two modes), averaged over the four analyses in question, was approximately 0.35 in. Note that the observed maximum deflections for test runs D2-1, D3-1, D4-1, and D5-1, which the response analysis in question simulated, was approximately 0.5 in. The top level deflection for second mode response, averaged over the four applicable cases was 0.016 in. This result was then adjusted for the variation of the maximum observed top level deflection during the dynamic tests from that calculated in the four response history analyses considered. The calculated second-mode deflection was multiplied by the ratio of the observed first-mode deflection (0.50 in.) to the calculated first-mode deflection (0.35 in.). The result was 0.023 in. A deflection of 0.03 in. was chosen as the maximum for the static analysis. This is the magnitude shown in Fig. 5.32(b).

The rules for the moment-rotation response of the beams were those depicted in Fig. 5.25, 5.27 and 5.33.

As would be expected, for second-mode response, the pattern of maximum end rotations for the beams varied radically from that calculated for first-mode response. The results are presented in Table 5.4. The fourth level beam remained elastic. None of the beams experienced rotation of a magnitude sufficient to cause closure of the cracks at the end of the beams as simulated by the hysteretic model. The load reversal in each cycle was analogous to that in Fig. 5.27 and 5.33 when reversal occurs at a rotation less than θ''_{m1}.

The calculated overall structure hysteresis relation, in terms of base moment and top level deflection, is shown in Fig. 5.35.

(d) Damping Factors

The dynamic hysteretic response of a structure may be modeled as that of a linearly elastic substitute structure with reduced overall stiffness and an array of viscous dashpots to account for the energy dissipated by hysteretic response. Fig. 5.36(a) depicts a sample overall structure hysteresis. Consider the path of loading and unloading to be identical for cycle after cycle. Response of the substitute structure would be linearly elastic with stiffness k_r. The area enclosed by the hysteresis loop, ABCD, of Fig. 5.36(a) is directly proportional to the energy dissipated by the structure in one cycle of response. This dissipation of energy is modeled in the substitute structure by the viscous dashpot depicted in Fig. 5.36(b), where the force in the dashpot is proportional to the velocity of the mass, m_s. The work performed by the force in the dashpot models the energy dissipated by the inelastic hysteresis (Fig. 5.36a). A single degree of freedom system of the type shown in Fig. 5.36(b) was defined for each of the two modes of response described in parts (b) and (c) of this section. The equation of motion of each single degree of freedom system was of the form,

$$m_s \ddot{x}_s + c_s \dot{x}_s + k_r x_s = -m_s a_b \tag{5.47}$$

where a_b was the acceleration of the base.

Hence,

$$\ddot{x}_s + \frac{c_s}{m_s} \dot{x} + \frac{k_r}{m_s} x_s = -a_b \tag{5.48}$$

Let,

$$\frac{k_r}{m_s} = \omega_s^2 \tag{5.49}$$

and,

$$\frac{c_s}{m_s} = 2\beta_s\omega_s \tag{5.50}$$

The factor, c_s, controls the magnitude of force in the viscous dashpot, and therefore the capacity for energy dissipation. Hence, the energy dissipation capacity of the single degree of freedom system may be expressed in terms of a viscous damping factor, β_s, where,

$$\beta_s = \frac{c_s}{2m_s\omega_s} \tag{5.51}$$

The system desired was characterized by a value of β_s corresponding to a viscous dashpot (Fig. 5.36b) that would dissipate the same quantity of energy per cycle of response as the hysteretic system. To realize this goal, it was necessary to solve the equation of motion (Equation 5.47) and use the result to express β_s as a function of the energy dissipated by the viscous dashpot. Consider the base acceleration to be a sinusoidal function of time with a circular frequency denoted by ω_b. Equation 5.48 becomes,

$$m_s\ddot{x}_s + c_s\dot{x}_s + k_r x_s = -p \cos \omega_b t \tag{5.52}$$

where p is a constant. The closed form solution for such a system is a response with time given by,

$$x_s = \Omega \cos (\omega_b t + \eta) \tag{5.53}$$

where Ω and η are constants. The energy dissipated by the viscous dashpot acting through an infinitesimal deflection, dx_s, is given by

$$dE_v = c_s\dot{x}_s dx_s \tag{5.54}$$

The deflection, x_s, however, is a function of time and,

$$dx_s = \dot{x}_s dt \tag{5.55}$$

where dt is an infinitesimal time interval. Hence, the energy dissipated during an entire cycle is given by,

$$\Delta E_v = \int_0^{T_s} c_s (\dot{x}_s)^2 \, dt \tag{5.56}$$

where T_s is the period. Differentiating Equation 5.53 to obtain the variation of velocity with time produces,

$$\dot{x}_s = -\Omega\omega_b \sin (\omega_b t + \eta) \tag{5.57}$$

substituting Equation 5.57 into Equation 5.56 and performing the integration, the energy dissipated per cycle of response by the viscous dashpot is expressed as,

$$\Delta E_v = \Pi c_s \omega_b \Omega^2 \tag{5.58}$$

The maximum strain energy for the single degree of freedom system was expressed as,

$$E_k = \frac{1}{2} k_r \Omega^2 \tag{5.59}$$

Combining Equations 5.58 and 5.59, an energy ratio was derived,

$$\frac{\Delta E_v}{E_k} = \frac{2\Pi c_s \omega_b}{k_r} \tag{5.60}$$

Combining Equations 5.60, 5.51 and 5.49, the following formula for the viscous damping factor was obtained,

$$\beta_s = \frac{1}{4\Pi} \frac{\Delta E_v}{E_k} \frac{\omega_s}{\omega_b} \tag{5.61}$$

By assuming that the frequency of the base motion was close to the natural frequency of the single degree of freedom system, in essence,

$$\frac{\omega_b}{\omega_s} \approx 1 \qquad (5.62)$$

the result was simplified to,

$$\beta_s = \frac{1}{4\Pi} \quad \frac{\Delta E_v}{E_k} \qquad (5.63)$$

It was desired that the energy dissipated by the hysteretic system per cycle be equal to that dissipated by the viscous dashpot system, in essence,

$$\Delta E_h = \Delta E_v \qquad (5.64)$$

where E_h was the energy dissipated for hysteretic response (Fig. 5.36a). The magnitude of ΔE_h was proportional to the area, ΔA_h, enclosed by the hysteresis loop, (in Fig. 5.36, the area enclosed by parallelogram ABCD), while the magnitude of E_k was proportional to A_k, the area under the line representing the linearly elastic response of the substitute system (in Fig. 5.36, the area enclosed by triangle OAE).
Hence,

$$\frac{\Delta A_h}{A_k} = \frac{\Delta E_v}{E_k} \qquad (5.65)$$

Equations 5.63 and 5.65 were then used to compute the equivalent viscous damping factor. The operation was performed for the structure hystereses, in terms of base moment and top level deflection, for response in each of the first and second modes (Fig. 5.34 and 5.35). The calculation for the first mode was performed for both high and low amplitude response levels.

For high-amplitude first mode response, the area enclosed by the load paths of fourth through seventh quarter cycles was utilized in the computation. For the low amplitude response the area enclosed by the loading paths of the eighth through eleventh quarter cycles was utilized. For the second mode response, the area enclosed by the loading paths of the fourth through seventh quarter cycles was utilized. The results are summarized in Table 5.5.

(e) Discussion of Results

The following paragraphs make some pertinent comments relative to the results of the study of equivalent damping.

Referring to Table 5.5, the calculated viscous damping factor for the first mode for high amplitude response was reasonable for a reinforced concrete structure undergoing significant inelastic response.

The magnitude of the viscous damping factor consistent with the low amplitude response was somewhat surprising, however. As listed in Table 5.5, the viscous damping factor was considerably higher than for high amplitude response. Apparently, this was due to the "fatness" of the low amplitude hysteresis relation compared to the high amplitude relation (Fig. 5.34). Certainly the beam dissipated more energy per cycle of high amplitude response than it did per cycle of low amplitude response, the area enclosed by the hysteresis realtion is larger for high amplitude response. This result illustrates the meaning of the viscous damping factor in a substitute structure. The viscous damping factor is not a direct measure of the energy dissipated by the structure per cycle of response. Referring to Fig. 5.36(a), the area of triangle OAE represents the potential energy of the system when it is at point A in its response. Since the total system energy is given by the sum of the kinetic and potential energy, and the system is motionless at point A, the triangular area represents the total system energy at that stage of response. Because the viscous damping factor is given by Equation 5.63, it represents the fraction of system energy, corresponding to a given mode of response, dissipated during one cycle of response.

Finally, it was interesting that the viscous damping factor for the second mode of response was comparable to that for the first mode. This point will be further considered in the study of the dynamic linear response of the test structure (chapter 8).

CHAPTER 6

FOURIER ANALYSIS OF OBSERVED RESPONSE

6.1 General Comments

This chapter is concerned with the analyses performed to determine the relative importance of various modes of response in the response history records of the dynamic tests. This was done using Fourier analysis, performing the numerical integration with the Fast Fourier Transform. The analysis, which was performed for one run of each dynamic test, separated the observed response into two portions, that attributable to all frequency components above 10 Hz. and that attributable to all frequency components below 10 Hz. The lower range would be associated with the first mode, the higher range with the sum of all higher modes. Comparison of the two portions provided a measure of the importance of the first mode relative to higher modes.

The next section describes the results of the Fourier analysis. The steps used in the analysis are provided for reference in appendix E.

6.2 Results of Fourier Analysis

(a) Cases for Analysis

The Fourier Analysis was performed for one test run from each dynamic test, including test runs D1-4, D2-1, D3-1, D4-1 and D5-1. Analyses were performed only upon response histories for the north wall. For tests D2, D3, D4 and D5, this was accomplished for horizontal acceleration at the bottom, middle and top levels, for base shear, and for base moment. For test D1, the horizontal deflections at the three levels were analyzed, in addition to the above.

For each dynamic test, the test run chosen for analysis was the one in which the maximum base acceleration was approximately 1.0g. This run was also the one modelled by the static test (S1), and was the test run analyzed in subsequent linearly elastic response analyses (chapter 8). In this manner, the results of the Fourier analysis, the static-hysteresis analysis, and the linearly elastic response analysis were made comparable. This run was chosen, as opposed to other acceleration levels, because, for all but one dynamic test, it represented a "virgin" test structure. Furthermore the data indicated that the test structures had general yielding at a base acceleration of 1.0g.

The response histories computed in the Fourier analysis are provided in Fig. 6.1 through 6.16. Two sets of response histories are provided, side by side, on each page. Each set of three curves corresponds to one response-history curve as reported in chapter 3. The top plot represents the response due to all frequency components less than or equal to 10 Hz., the middle plot represents the response due to all frequency components greater than 10 Hz., and the bottom plot represents the total response. The bottom plot is identical to the observed response presented in chapter 3. The maximum responses computed in the analysis are listed in Tables 6.1 and 6.2. Table 6.3 lists frequencies measured from the response histories. The following paragraphs describe the analysis results for deflection, acceleration, base shear, and base moment.

(b) Horizontal Displacements

Fourier analyses were performed for test D1-4 for observed displacements at the lower, middle, and top levels (Fig. A.29). The response histories are shown in Fig. 6.1. The maximum responses, and frequencies measured from response histories are provided in Tables 6.1 and 6.3, respectively.

Both the listed maxima and the response histories imply that the observed displacement was dominated by the first mode. The measured frequency for the lower frequency levels (Table 6.3) compared well with the first mode frequency of the test structure, as determined in a free vibration test (Table 3.6). The dominant frequency of the higher mode portion of the response was approximately 11 Hz. This is much too low a frequency to be attributable to the second mode of response of the test structure (Table 3.6). The base motion (Fig. 3.6) is rich in frequencies in this intermediate range.

Because the observed displacements for later dynamic tests were similar to those for test D1, in terms of apparent first mode dominance, the Fourier analysis was not performed for the displacements observed during tests D2 through D5.

(c) Horizontal Acceleration

Fourier analyses were performed on the observed horizontal accelerations measured at the lower, middle, and top levels. The response histories are shown in Fig. 6.2 through 6.6. The maximum responses and frequencies measured from the response histories are listed in Tables 6.1 and 6.3, respectively.

The response histories indicate that the higher mode response was quite significant at all three levels. This was consistent with general observations made concerning the dynamic test results (chapter 3).

For the lower level acceleration, for all five test runs, the dominant frequency for the low frequency portion of the response (Table 6.3) was too high to correspond to a first-mode frequency, as measured in free-vibration tests (Table 3.6). The measured base accelerations (Fig. 3.6, 3.17, 3.22, 3.35, 3.40) appear to contain frequency components in this intermediate range, between the first and second modes of the test structures. This is the likely source of this dominant component in the lower frequency response. For the middle and top level accelerations, the dominant frequency component of the lower frequency response was consistent with the first-mode frequencies of the test structures (Table 3.6).

For the higher frequency portion of the response, the measured frequencies for tests D2 through D5 are consistent with the second mode frequencies as measured in a free vibration test (Table 3.6). Test D1-4, for the lower level acceleration, exhibits a dominant frequency component in the higher frequency response considerably lower than that measured in free vibration tests. Again, this is due to the influence of the base acceleration. The frequencies exhibited by the middle and top level accelerations, for all five test runs, are reasonably close to those measured in free vibration tests.

(d) <u>Base Shear and Moment</u>

The response histories obtained in the Fourier Analysis of the base shears are shown in Fig. 6.8 through 6.11. The maximum responses scaled from the response histories are listed in Table 6.2.

The results confirm that the higher frequency portion of the response is quite significant for the base shear. The higher frequency components are slightly less visible in test D1 than for other tests.

The response histories resulting from the Fourier analysis of the base moments are shown in Fig. 6.12 through 6.16. The response maxima scaled from the response histories are listed in Table 6.2.

As would be expected, the waveform for base moment is dominated by the first-mode response.

It is interesting to note, though as an observation of narrow scope, that the total shear and moments were always less than the absolute sum of the modal components and, in general, comparable to the "root-sum-square" value.

CHAPTER 7

STUDY OF DYNAMIC RESPONSE

7.1 General Comments

This chapter describes an analytical study of the inelastic, dynamic response of the test structures to seismic base motion. The nonlinear, hysteretically responding test structure was replaced by a substitute structure with reduced stiffness, and viscous damping to account for hysteretic energy dissipation. The concept is illustrated in idealized form, for a simple system, in Fig. 7.1. The assumed paths of loading and unloading for the actual structure are shown by the solid lines with arrowheads in Fig. 7.1(b). A stable hysteresis loop that develops after the first cycle of loading and unloading is depicted by the path ABCD in the figure. The path of loading and unloading for the substitute structure is the line COA. The substitute structure, itself, is depicted in Fig. 7.1(c). The substitute structure has the same maximum response as the actual structure. The reduction in stiffness from $k_{e\ell}$ to k_r is referred to as the damage ratio. In essence, the damage ratio, μ_{dr}, is given by,

$$\mu_{dr} = \frac{k_{e\ell}}{k_r} \tag{7.1}$$

This parameter is, in general, not equal to the response deflection ductility, given by,

$$\mu_{dc} = \frac{x_{sm}}{x_{sy}} \tag{7.2}$$

where the variables x_{sm} and x_{sy} are the maximum deflection and yield deflection, respectively, for the actual structure. Both variables do, however, quantify the same concept. As the substitute structure goes through a cycle of response, loading from point 0 to point A, unloading to point 0 and reloading to point C, then unloading to point 0, the viscous dashpot, with velocity coefficient c_s, dissipates the same quantity of energy as that indicated by the interior of the hysteretic response path, ABCD, of the actual structure. This concept is similar to that applied in section 5.3(c).

A series of substitute structures were used, applying various viscous damping factors and various damage ratios to the actual structure. The response histories of the substitute structures were calculated, using as input, the observed base acceleration records from the various dynamic tests. An investigation was also performed on the maximum response of the substitute structures, using the linear response spectra computed from the base acceleration records observed in the dynamic tests. A large body of analytical results on the maximum response of substitute structures with various damping factors and stiffness levels was amassed in this portion of the study.

The next section of this chapter discusses the structural idealizations associated with the analytical model. The analysis procedure is described in detail in appendix F.

The third section of this chapter explains the study of maximum responses in more detail and presents the results. The final section of the chapter performs a similar function for the study of response history. The computer program written to perform the calculations is describe in appendix D.

7.2 Structural Idealization

The analytical model is depicted in Fig. 7.2. As for the static hysteretic model, one quarter of a test structure was modelled. The forces resulting from the analysis were then doubled to correspond to forces for one wall. As for the static hysteretic model, a point of inflection was assumed to occur at the midspan of the connecting beams of a wall. The comments made about this assumption in chapter 5 also apply here. The pier was considered to be fixed against rotation at its base. The base motion was assumed to involve horizontal translation only.

For the actual tests on the earthquake simulator, mass was simulated by three steel weights as described in chapter 3 and appendix A. In the analytical model, these weights were simulated by concentrations of mass at the points where the weights were connected to the pier. The connections were at the centerline of the pier, at the levels of the centerlines of the second, fourth and sixth connecting beams. The mass, m_h, associated with horizontal acceleration of a given point of mass concentration was equal to one quarter of the mass of a complete 2000 lb. test weight (appendix A). The mass associated with the vertical acceleration of a point of mass concentration was that mass consistent with the rotational inertia of the complete test weight for rotation about an axis perpendicular to the plane of a wall and passing through the midspan of the connecting beam. The concept is illustrated in Fig. 7.3, where I_{rot} represents the rotational inertia of the 2000 lb. weight and m_v represents the mass for vertical accelerations in the analytical model. The variables a_θ and a_v represent rotational and vertical acceleration, respectively. To obtain equivalent force in the pier, equality of applied moment was desired, in essence,

$$m_v a_v \bar{\ell} = \frac{1}{2} I_{rot} a_\theta \tag{7.3}$$

where the factor of one half accounts for the presence of two walls in a test structure.

From geometry,

$$a_v = \frac{\bar{\ell}}{2} a_\theta \tag{7.4}$$

The appropriate mass for vertical accelerations was obtained by combining Equations 7.3 and 7.4,

$$m_v = \frac{I_{rot}}{\bar{\ell}^2} \tag{7.5}$$

There was no mass associated with the rotational acceleration of the point of concentration of mass. This assumption was consistent with the hinge connection between the piers of the test structure and the test weights (appendix A).

The individual members were idealized as prismatic and completely linearly elastic. As for the static hysteretic model the pier was considered to be six individual members, one for each story.

A damage ratio was applied to the uniform section stiffness for each beam and pier member.

The model considered only flexure in the beams. Both flexural and axial deformations were considered in the pier members. Shear deformation was not considered at all. The idealization, therefore, considered three degrees of freedom at each beam-pier joint, a horizontal displacement, a vertical displacement, and a rotation, for a total of 18 degrees of freedom in the model.

The response history of the idealized model was calculated using modal superposition and the linear-acceleration method as described in appendix F.

7.3 Study of Maximum Structure Response

(a) Introductory Remarks

As mentioned in the first section of this chapter, one means by which the analytical model was used to interpret the observed structure responses during the dynamic tests was by using the linear model to calculate the maximum responses of the various test structures, assuming a range of damage ratios and damping factors. The analysis was performed using the modal-analysis portion of the computer program written for the dynamic response analytical model (appendix D), in conjunction with the linearly elastic response spectra for the observed base accelerations of the dynamic tests. This section describes the analyses and presents the analytical results.

(b) Initialization of Study

Modal analyses were performed for two different structures. One was the type A test structure of test D1, the other was an "average" structure representing the types B and C structures of tests D2 through D5. To consider inelastic response, the member section stiffnesses were reduced by various damage ratios, as described in sections 7.1 and 7.2. Figure 7.4 depicts the reduction in stiffness for the linearly elastic moment-curvature relation of a section of a member in the substitute structure. The damage ratio was given by,

$$\mu_{dr} = \frac{EI_{ref}}{EI_{sub}} \tag{7.6}$$

where EI_{sub} was the stiffness of the section in the substitute structure and EI_{ref} was a reference stiffness corresponding to the estimated section stiffness at the beginning of the dynamic test in question. Hence, the damage ratio directly corresponded to damage incurred during the dynamic test. Damage incurred during casting and handling was included in the reduction in stiffness from EI_{unc} to EI_{ref}. Where axial section stiffness was considered (in essence, the pier members) an analogous concept was applied to obtain EA_{sub}, the axial section stiffness in the substitute structure. The reference stiffness was obtained by considering the section stiffnesses for all members to be reduced by identical factors relative to the stiffness of uncracked sections. In essence, any damage sustained by the structure prior to the start of testing was assumed to be distributed uniformly over the structure. Furthermore, for purposes of this initial stiffness reduction, the structure was idealized as a single degree of freedom system, characterized in its uncracked (undamaged) state by the computed first-mode frequency for that state (Table 4.4), and characterized at the start of the dynamic test in question by the appropriate measured first-mode frequency (Table 3.6). The reference section stiffness for each beam and each pier was then obtained from the relation between stiffness and natural frequency for a single degree of freedom system, in essence,

$$EI_{ref} = E_c I_{tr} \left(\frac{f_{ref}}{f_{unc}}\right)^2 \qquad (7.7)$$

Similarly, the reference axial stiffness for the pier members was obtained from,

$$EA_{ref} = E_c A_{tr} \left(\frac{f_{ref}}{f_{unc}}\right)^2 \qquad (7.8)$$

The implicit assumption was made that flexural stiffness and axial stiffness reduced by the same factor. For each test structure, the values for E_c were those listed in Table A.1. The average values of A_{tr} for the piers, and I_{tr} for the beams and piers, for each test structure, were the properties of the uncracked sections described in chapter 4 and listed in Tables 4.1 and 4.2. The values of f_{unc} for each test structure were the first-mode frequencies described in chapter 4 and listed in Table 4.4. The values of f_{ref} were those first-mode frequencies obtained in a low amplitude free vibration test prior to the start of the appropriate dynamic test (prior to the first test run) and are listed in Table 3.6. For the type A test structure (test D1), the values of EI_{ref} and EA_{ref} for each member, computed in Equations 7.7 and 7.8, were used directly as the reference to which damage ratios were applied. For the types B and C test structures (tests D2 through D5), Equations 7.7 and 7.8 were evaluated for each member in each test structure, and, for each member, the resulting reference stiffnesses were then averaged over the four structures to obtain an "average" reference structure. These values are listed in Table 7.1.

(c) Cases for Calculation

Three distinct distributions of member damage ratios were used in the analysis. For all three distributions, the same damage ratio, μ_{bm}, was applied to all six beams. For the first distribution, a damage ratio equal to one was applied to the pier members (μ_{pr} = 1.0). For the second distribution, the damage ratio for the first story pier member, applied to both flexural and axial stiffness, was always equal to that applied to the beams ($\mu_{pr} = \mu_{bm}$). For the third distribution, the damage ratio for the first story pier member, again applied to both flexural and axial stiffness, was always equal to one-half that applied to the beams ($\mu_{pr} = \frac{1}{2}\mu_{bm}$). For both the second and the third distributions, the stiffness of the pier members for the second through sixth stories was not reduced below the reference value (damage ratio equal to one). For each of the three distributions of stiffness, a wide range of values of beam damage ratio, μ_{bm}, were applied. Hence, the analysis considered three distributions of response ductilities, each representing a specific relation between beam damage ratio and pier damage ratio, and within each distribution, several overall levels of response ductility were considered.

For each test, the linearly elastic response spectra, computed for the observed base acceleration record, were used in the analysis. Hence, separate results were obtained for each test, in spite of the use of an "average" test structure for tests D2 through D5. For all tests, the test run analyzed was that exhibiting a maximum base acceleration approximating 1.0g. This justified comparisons between the results of the study of static hysteresis, the Fourier analysis, and the study of dynamic response. The test runs considered were D1-4, D2-1, D3-1, D4-1 and D5-1. For each test run, analyses were performed for two different viscous damping factors, two percent of critical damping and ten percent of critical damping. The same viscous damping factor was applied to both the first and second modes of response. Responses calculated were the top level deflection, the base shear for one wall and the base moment for one wall. For the computation of maximum top level deflection, only the first mode of response was considered.

Hence,

$$\Delta_{mt} = b_1 \phi_{13} S_1 \tag{7.9}$$

where Δ_{mt} was the maximum top level deflection, b_1 was the modal participation factor for the first mode, ϕ_{13} was the element of $\{\phi_1\}$ corresponding to the top level deflection, and S_1 was the spectral displacement corresponding to the first-mode frequency.

For the maximum base shear and maximum base moment, both first and second response modes were considered. For the first mode,

$$V_{m1} = 2\omega_1^2 \; S_1 \; m_h b_1 \; (\phi_{11} + \phi_{12} + \phi_{13}) \tag{7.10}$$

$$M_{m1} = 2\omega_1^2 \; S_1 \; m_h b_1 \; (H_1\phi_{11} + H_2\phi_{12} + H_3\phi_{13}) \tag{7.11}$$

where M_{m1} and V_{m1} were the maximum base moment and base shear, respectively, for the first mode, m_h was the mass at each level associated with horizontal acceleration, ω_1 was the circular frequency for the first mode, H_1, H_2 and H_3 were the distances from the base to the bottom, middle, and top concentrated masses, respectively, and ϕ_{11}, ϕ_{12}, ϕ_{13} were the elements of the mode shape, $\{\phi_1\}$, corresponding to the bottom, middle and top masses, respectively. The factor of two appeared in the equations because the shear and moment were computed for one wall (one-half of a test structure), while the mass, m_h, was for one-quarter of a test structure. This was done so that the results would be comparable with the observed responses and with the results of the analytical study of response hysteresis. Similarly, for the second mode of response,

$$V_{m2} = 2\omega_2^2 \; S_2 \; m_h b_2 \; (\phi_{21} + \phi_{22} + \phi_{23}) \tag{7.12}$$

$$M_{m2} = 2\omega_2^2 \; S_2 \; m_h b_2 \; (H_1\phi_{21} + H_2\phi_{22} + H_3\phi_{23}) \tag{7.13}$$

where the variables are defined in a manner analogous to those for the first mode. Finally, the maximum responses for the two modes were added directly. Hence,

$$V_{mtot} = V_{m1} + V_{m2} \tag{7.14}$$

$$M_{mtot} = M_{m1} + M_{m2} \tag{7.15}$$

where V_{mtot} and M_{mtot} were the maximum total base shear and base moment, respectively.

(d) Variation of Second-Mode Frequency with First-Mode Frequency

As discussed in section 7.3(c), a number of structures were considered in the analysis, representing a range of damage ratios for the beams and three overall distributions of response ductility. Each structure was, of course, characterized by its own particular first and second-mode frequencies. The calculated variation of the second-mode frequency with the first-mode frequency, for each of the three distributions of response ductility, is depicted in Fig. 7.5, first for the type A structure, then for the types B and C structures. For the type A structure, the ratio of second mode frequency to first-mode frequency varied from approximately 8 at a first-mode frequency equal to 3 Hz to approximately 4.5 at a first mode frequency equal to 10 Hz. For the types B and C structures, the same ratio varied from approximately 5.5 for a first-mode frequency equal to 4 Hz to approximately 4.0 for a first mode frequency equal to 7.5 Hz. The implication of the reduction of the above ratio with increasing first-mode frequency (decreasing beam damage ratio) was that as the beams became stiffer, the structure more strongly assumed the characteristics of a frame; a reasonable result.

(e) Variation of Frequency with Damage Ratio

One aspect of comparing the results of the modal analysis-response spectrum analysis with observed responses was evaluating what damage ratios in the structure could be expected to accompany a given response frequency. To make such an evaluation possible, the relation between damage ratio and first-mode frequency, as obtained in the results of the response spectrum-model analysis was considered. For each set of damage ratios considered for the beams and pier, a first-mode frequency was calculated in the modal analysis. These results are shown as relations between beam damage ratio and calculated first-mode frequency in Fig. 7.6. There is a separate set of relations for each of the two structures considered, the type A test structure, and the "average" structure representing the types B and C test structures. Each set of relations consists of three separate curves, one for each damage distribution, as noted on the figure.

A primary characteristic of the relations was that, for a given response frequency, the corresponding damage ratio in the beams decreased sharply when damage was introduced into the first level pier member. This is a significant trend that will be used in interpreting observed responses in later chapters.

(f) Variation of Maximum Responses with Frequency

To aid further in reconciling the dynamic response analysis with the observed response, the results of the modal spectral analysis were expressed in terms of two parameters directly observed during the dynamic tests, maximum response level and first-mode frequency. This approach facilitated the comparison of analytical and experimental results which will be discussed in chapter 8.

The results of the spectral analysis in terms of maximum structure response and first-mode frequency is shown in Fig. 7.7 through 7.11. Each page contains the results for one dynamic test for one viscous damping factor. For each page, the maximum base moment, maximum base shear and maximum top level deflection, are plotted as functions of first-mode frequency. The results are for one wall (one-half test structure), and were computed as described in part (c) of this section (Equations 7.9 through 7.15). For the top level deflection, only the first mode is plotted. The second-mode component was considered to be insignificant. For the base shear and base moment, the maximum response obtained considering only the first mode is depicted by the broken lines (Equations 7.10 and 7.11). The solid lines indicate the results obtained considering the direct sum of the maximum responses for the first and second modes (Equations 7.14 and 7.15). It will be noted that there are several solid lines and several broken lines for each parameter. This is because the results for all three distributions of damage considered in the analysis (in essence, $\mu_{pr} = \mu_{bm}$, $\mu_{pr} = 1.0$, and $\mu_{pr} = \frac{1}{2}\,\mu_{bm}$) are plotted together on the same set of axes. There are not three distinct solid lines and three distinct broken lines, because in several cases the maximum responses for the three damage distributions did not differ sufficiently, in relation to the scale of the plots, to constitute distinctly separate relations. In no case did the results for the three damage distributions differ by a significant amount. For this reason, the relations are not labelled with respect to which damage distribution to which they correspond. For each response parameter, the maximum observed response during the appropriate dynamic test is denoted by a horizontal solid line.

Although the detailed interpretation of the results of the spectral study, in terms of other analyses and experimental results, will be presented in a later chapter, some general observations relative to the results of the study are appropriate for this chapter.

One of the most striking characteristics of the results displayed in Fig. 7.7 through 7.11 is that, at a given first-mode frequency for the structure, the response was virtually independent of the distribution of damage ratios between the beams and the pier. This result, however, is really not highly surprising, considering the nature of the analytical model. The calculated response in a particular mode is a function of two parameters: spectral displacement or acceleration, and mode shape multiplied by the appropriate modal participation factor. Consider the first-mode response. The spectral response must be the same for all three damage distributions, since the frequency is the same. Only the effect of the distribution of damage upon the shape of the first mode could cause variations in base moment and base shear. Table 7.4 illustrates that such variations in the shape of the first mode are minor. The first-mode maximum base shear and base moment should not be expected to vary significantly with distribution of damage. For the second mode of response, the frequency, and hence, the spectral response, will vary somewhat among the three damage distributions. These frequency variations are shown in Fig. 7.5 for the type A and types B and C structures. Although the frequency variations are significant for the type A structure, they are not so large for the types B and C structures. However, for the range of frequencies being considered, none of the linear response spectra for the test runs considered (Fig. 3.3, 3.14, 3.19, 3.32 and 3.37) exhibit a high rate of variation of spectral response with frequency. One would not expect the variation of frequency with damage distribution to affect strongly the structural response. As for the first mode, the variation of mode shape with damage distribution is not large. One would not expect the maximum base shear and base moment, attributable to the second mode of response, to vary significantly with the distribution of damage ratios among the beams and pier.

For tests D1-4 and D3-1, the calculated deflections tended to decrease with increasing first-mode frequency, while the calculated deflections for tests D2-1, D4-1, and D5-1 showed no overall trend in magnitude over the first-mode frequency range investigated (approximately three to seven Hz). For all tests, the deflection exhibited a localized peak at approximately five Hz. The peak was more pronounced for two percent damping than for ten percent damping. For the calculated base shears and base moments, a similar localized peak, stronger for two percent damping than for ten percent, occurred at approximately 5.7 Hz. All of the above observations are consistent with the characteristics of the linear response spectra for the appropriate tests. Finally, all calculated responses were reduced in magnitude and rendered less erratic in their variation with first mode frequency by the increase in damping factor from two percent to ten percent.

Comments can also be made concerning the relative contributions of the first and second modes of response to the base shears and base moments. These results are presented in Fig. 7.12 through 7.16. The figures depict, for each viscous damping factor considered in the study, the variation with first-mode frequency of the ratio of the second-mode response to the first-mode response. Because the variation of maximum responses with frequency was independent of the relation between damage ratio in the beams and damage ratio in the pier, the ratio of the responses for the two modes was calculated for only one such case, that case corresponding to a damage ratio of one in the pier. Referring to Fig. 7.12 through 7.16, the second mode of response was stronger, relative to the first mode, for the base shear, than for the base moment. This was true for all five tests at both damping factors. The analysis for test D1, a test of the type A structure, was characterized by a much weaker second-mode component than the analyses for tests of types B and C structures. Finally, for all cases, the contribution of the second mode relative to that of the first mode increased as the first-mode frequency decreased. These observations will be discussed further when the results of the spectral study are reconciled with the observed responses and with the results of the analytical study of response history.

Finally, a significant characteristic of Fig. 7.7 through 7.11 is the frequency at which the calculated maximum response was equal to the observed response, as shown by the horizontal lines in the figures. This information will be used later in reconciling the results of the spectral study with the experimental results and the results of other analyses.

7.4 Study of Response History

(a) Introductory Remarks

This section describes the study of response history, as introduced in section 7.1, for a number of substitute structures. Response histories were computed for several of the substitute structures having different combinations of natural frequency and viscous damping. The goal was to correlate the response of various substitute structures with the observed response from the dynamic tests, with the waveform separated into frequency components. The calculations were performed using the analytical model described in section 7.2. Hence, the modal analysis procedure was the same as for the study of maximum response (section 7.3) and the variation of response with time was calculated by the numerical analysis described in section F.5.

The next part of this section (section 7.4(b)) will describe the choice of the substitute structures for investigation. Section 7.4(c) will present the results of the study of response history.

(b) Cases for Study

The study of response history included each of the five test runs (D1-4, D2-1, D3-1, D4-1, D5-1) considered in the analytical study of maximum response (section 7.3), and subjected to the Fourier analysis of observed responses (chapter 6). This promoted comparability of the analytical studies with each other and with the observed responses. For each test run, the base motion input for the study of response history was the corresponding observed base acceleration record.

As for the study of maximum response, a given substitute structure was characterized by a particular first-mode response frequency (overall damage level or stiffness reduction), a distribution of damage ratios, or stiffness reductions, throughout the structure, and a set of viscous damping factors for the first and second modes of response. A major question concerned what combinations of the above parameters to consider. Of the three distributions of damage ratio between the connecting beams and the lower level pier member considered in the study of maximum response, only one distribution was considered for each test structure in the study of response history. For the type A structure, only that distribution characterized by equal damage ratios for the beams and lower story pier

($\mu_{pr} = \mu_{bm}$) was considered. For the types B and C structures, only that distribution characterized by a damage ratio of one for the pier ($\mu_{pr} = 1$) was considered in the study of response history. The decision to use only one distribution of damage was based upon the similarity of calculated response for the three distributions of damage at any given first-mode frequency as obtained in the study of maximum response (Fig. 7.7 through 7.11).

Having set the distribution of damage to be considered, the next consideration was the first mode frequencies (overall damage levels) to be considered in the study. The main objective of the study was, of course, to correlate the results of the analytical study of response history with observed response histories. One would want to consider, for the study, structure damage levels consistent with those existing immediately before, during, or immediately after the test runs being considered. The use of frequencies measured in the pre-test free vibration tests was first considered. These are listed in Table 3.6. However, the results of the study of maximum response (section 7.3) indicated that this would not be a promising choice for analysis. This is illustrated by the variation of maximum base moment and maximum base shear with first mode frequency (Fig. 7.7 through 7.11). For test D1-4, if the viscous damping factor is taken to be ten percent of critical damping, the calculated and observed maximum responses become equal to each other at a frequency only slightly less than that measured in a pre-test free vibration test. For the other four tests, however, the calculated response becomes equal to observed response only for frequency levels much lower than those consistent with the pre-test free vibration tests. Hence, for the study of response history, it was deemed more reasonable to consider frequencies, or structure damage levels, occurring at various times during the observed response of the structures. The observed responses, however, exhibited a continuous variation of frequency over the duration of response. Practical considerations limited the number of discrete frequency levels that could be investigated. Two frequencies were considered. The first was termed the early frequency and was the average observed response frequency considering the first 1.5 sec. of response. This represented the interval of highest amplitude response for the observed records. The second frequency level considered was termed the late frequency and was the average response frequency considering the final 2.0 sec. of response. The early and late frequencies, as calculated for each of the five test runs considered in the analysis, are listed in Table 7.2.

First-mode frequencies were related to damage ratios in the same manner as for the study of maximum response. Section stiffnesses throughout the structure were reduced uniformly from the value for an uncracked section, such that the first-mode frequency would be equal to the frequency measured in the pre-test free vibration test. This represented a reference state of the structure, for which all members were assumed to have a damage ratio equal to one. A uniform damage ratio was then applied to the beams, reducing the structure's first-mode frequency from the reference value to that value being investigated. A reference structure was defined for the type A structure and an average reference structure was defined for the four structures of types B and C. In practice, the damage ratios necessary to produce the desired first-mode frequencies were obtained from the results of the study of maximum response.

For each test run, for the early and late frequencies, the corresponding damage ratios in the beams, for input into the computer program, were obtained from Fig. 7.6.

The third consideration involved what viscous damping factors to consider in the investigation. There were two aspects to this consideration, the value of the viscous damping factor for the first mode response and the relative values of the viscous damping factors for the first- and second-mode responses. For test D4-1, the effect of the relative values of the two viscous damping factors was investigated. For all other tests, the viscous damping factors used for the two modes of response were considered to be equal to each other. Viscous damping factors of two percent and ten percent of critical damping were considered. In addition, for test D4-1, a case with both damping factors equal to fifteen percent of critical damping was investigated.

A summary of the variables considered in the study is provided in Table 7.3. The table lists, for various combinations of first and second mode damping factors, which test runs were analyzed. Each combination was performed at both early and late frequencies, providing a total of 26 analysis cases, considering all test runs. Table 7.4 lists, for each analysis case, the various structural parameters.

(c) Results of Study

The maximum calculated responses are presented in Table 7.5. Results are included for the top level deflection, base shear, and base moment, for the response histories corresponding to the first-mode response, second-mode response, and sum of the first- and second-mode responses.

The calculated response histories are presented in Fig. 7.17 through 7.42. Each figure presents the results of one analysis, for analyses 1 through 26 (Tables 7.4 and 7.5). Response histories of top level deflection, base shear, and base moment are presented for each analysis, as three sets of three response histories each. Within each set of response histories, the top relation corresponds to the response obtained by considering only the first mode response of the structure, the middle relation to that response obtained by considering only the second mode response, and the bottom relation to that response obtained by considering the sum, at each point in time, of the first and second mode responses. Each figure spans one and one-half pages.

The following paragraphs will describe the general characteristics of the calculated results, considering the effects of varying the quantity of viscous damping and various first,mode frequencies. Reconciliation of the results with observed responses and other analyses will be presented in chapter 8. As would be expected, for a given first-mode frequency level and test run, increasing the viscous damping factor from two percent of critical damping to ten percent of critical damping decreased all responses, top level deflection, base shear and base moment. Consistent with the implications of response spectra, comparison of the results of the various analyses for test D4-1 indicated that the effect of increasing the viscous damping from ten percent of critical damping to fifteen percent of critical damping has much less effect than increasing the damping from two percent of critical damping to ten percent of critical damping.

For structures characterized by the early frequency, the high amplitude response occurred early in the response history, as for the observed response. For structures characterized by the late frequency, high amplitude response occurred later in the response history. This was a consistent result among the various tests and characterized top level deflections, base shears and base moments. It represented the difference in the overall nature of the response histories obtained for structures at the two frequency levels. These gross differences in the two classes of response histories will be important in reconciling the results of the study of dynamic response with the observed responses (chapter 8).

The trends in the maximum responses, comparing the responses of structures characterized by the two frequency levels, are consistent with the linearly elastic response spectra for the base acceleration records used as input for the analyses. Table 7.6 lists the trends in calculated maximum response, in terms of per cent increase or decrease in response, as the frequency considered in the analysis changed from early frequency to late frequency.

The trends in the top level deflection varied among the damping factors and test runs. Comparison with the response spectra for relative deflection (plotted on a linear scale) showed the trends to be consistent with the variation of the response spectra over the appropriate frequency range for each test run. The response spectra are shown in Fig. 3.3, 3.14, 3.19, 3.32 and 3.37 for test runs D1-4, D2-1, D3-1, D4-1 and D5-1, respectively. The trends for base shear and base moment shown in Table 7.6, are provided for both first and second mode responses, as are the response histories. For all test runs, except D3-1, the first-mode base shear and moment decreased when the first-mode frequency of the structure decreased from early frequency to late frequency. For test D3-1 and a damping factor of ten percent, the response increased. These trends were consistent with the response spectra for each test run. For tests D1-4, D2-1, D4-1 and D5-1 the spectral response decreased throughout the interval of 5.0 Hz to 3.3 Hz, the range of interest. For test D3-1, spectral response decreased at low damping factors but increased slightly for higher damping factors, as the frequency decreased. For all cases, except the base shear for test run D2-1 at ten percent damping, the maximum second-mode response for base shear and base moment increased when the first mode frequency of the structure decreased from the early frequency to the late frequency. This was consistent with the variations in the response spectra for all cases, except the base moment for test D2-1 at ten percent damping. Except for test D2-1, all response spectra increased as the frequency decreased from 30 Hz to 27 Hz for test D1-4, and decreased from 23 Hz to 22 Hz for test runs D3-1, D4-1 and D5-1. For test run D2-1, the spectral acceleration decreased as the frequency varied from 23 Hz to 22 Hz. The maximum base shear followed this same pattern, while the maximum base moment increased slightly (Tables 7.4 and 7.5). This anomaly was not disturbing, however, as the shape of the second-mode changed, over the frequency of interest, in a manner that would favor an increase in base moment (Table 7.5).

In sum, the results of the study of response history, in terms of the effect of various viscous damping factors and first mode frequencies upon the response, were reasonable, and were consistent with the characteristics of the linear response spectra for the test runs being analyzed.

The results of the study of dynamic response are reconciled with the observed responses and the study of static hysteretic response in Chapter 8.

CHAPTER 8

COMPARISON OF OBSERVED AND CALCULATED RESPONSES

8.1 General Comments

This chapter compares the results of the analytical and experimental results described in previous chapters, including the observed responses presented in chapter 3, the calculated strength and deformation properties presented in chapter 4, the calculated hysteretic properties presented in chapter 5, the Fourier analyzed observed responses presented in chapter 6, and the calculated response histories presented in chapter 7. Several chapters discussed certain implications of the results of individual experimental or analytical studies relative to the behavior of the test structures. Much interpretation of the behavior of the test structures, however, requires comparison of the results of several of the studies listed above. This chapter provides such a unification.

The reconciliation is made in four parts. The first part, presented in section 8.2, compares the initial stiffnesses, as calculated in the strength and deformation study, and observed in the free-vibration tests and in the static test. The second part, presented in section 8.3, compares the strength of the test structures, as calculated in the strength and deformation study, as observed in the static test and calculated in the static hysteretic analysis, and as implied by the observed dynamic responses. The third part, presented in section 8.4, interprets what level of viscous damping factor, for a linear substitute structure, was required to simulate the observed responses. Results from the study of linear dynamic response, the study of static hysteretic response and the Fourier analysis of the observed responses are compared. The fourth part, presented in section 8.5, considers the level of structural damage exhibited by the test structures, using results from the study of linear dynamic response, the strength and deformation study, the study of static hysteretic response, the static test and the Fourier analysis of observed response histories. Section 8.6 summarizes the results.

8.2 Reconciliation of Initial Stiffnesses

(a) Introductory Remarks

This section will compare and interpret the low load level stiffness of the test structures as measured in test S1, as calculated in analyses considering linearly elastic response, and as measured in low-amplitude free-vibration tests. The comparison is for type B test structures only, because this was the only type for which a test under statically applied loads, directly measuring initial stiffness, was performed. The free-vibration tests considered were also those for type B structures, tests previous to test runs D2-1 and D3-1.

(b) Summary of Results

The initial stiffness properties of the type B test structures, obtained in various manners are summarized in Table 8.1.

The first group of results, the calculated properties, were obtained as described in section 4.2 and listed in Tables 4.4 and 4.6. Three classes of calculation results are included. The first considers the structure to be completely uncracked, the second considers fully cracked section for every beam, while the piers are considered to be completely uncracked. The third considers fully cracked section for every beam and for the portion of the pier below the first level beam. Results are provided for both the stiffness, itself, in terms of the ratio of base moment for one wall to top level deflection, and in terms of first-mode frequency.

The next group of results pertain to low-amplitude free-vibration tests performed prior to test runs D2-1 and D3-1. The stiffness values, in terms of base moment and top-level deflection, listed for the free vibration test, were obtained by comparison with the calculated stiffness and frequency for the appropriate test structure, considering the structure completely uncracked. The calculations followed the method described in Section 4.2(b) (Equations 4.20 through 4.22).

The last three sets of results in Table 8.1 represent attempts, during the static test, to measure the initial stiffness of the test structure. As discussed in Sections 3.5 and A.6, the deflections of the test structure were measured using both mechanical dial gages and differential transformers. The initial stiffnesses listed in the table, in terms of base moment and top-level deflection, were obtained directly from Fig. 3.50 and 3.53. Those values obtained from dial gage measurements are provided uncorrected for base movement (Fig. 3.50) and corrected for base movement (Fig. 3.53). The results obtained using differential transformers are, of course, uncorrected for base movement. The corresponding first-mode frequencies were obtained by comparison of the initial stiffnesses with calculated stiffness and frequency for structure S1, considering a completely uncracked structure. The calculation method was analogous to that in Section 4.2(b).

The initial stiffnesses and corresponding first-mode frequencies for the various cases listed in Table 8.1 vary over a considerable range. The variations will be discussed in subsequent parts of this section.

(c) Comparison of Stiffnesses Measured During the Static Test

As shown in Table 8.1 there is some variation in the three initial stiffness measurements pertaining to test S1. The following paragraphs will discuss those variations.

The stiffness obtained from mechanical dial gage readings corrected for base movement, as illustrated in Fig. 3.52, was greater than that consistent with the dial gage readings uncorrected for base movement. This is a reasonable result, base movement increases flexibility of the test structure for low-amplitude response.

An additional comparison may be made between the initial stiffness as determined from dial gage readings uncorrected for base movement, and the initial stiffness as determined from differential transformer readings, also, of course, uncorrected for base movement. The differential transformer readings implied a significantly lower stiffness. This discrepancy may be attributed to the fact that the dial gages bore directly against small plates on the edge of the pier of the test specimen, while the differential transformers bore against the steel weights. This point is described in section A.6. The connection between the steel weights and the test specimen itself, may have been the source of some relative movement. Micrometer measurements indicated an allowance of 0.025 in. between the bolt diameter and the inside diameter of the hole through the specimen. To investigate the plausibility of such an origin for the observed stiffness variation, the difference between the deflections at each of three levels as measured by differential transformers and as measured by dial gages are compared with base moment in Fig. 8.1. The differential deflections for the middle and top weights increased almost linearly, until a certain magnitude of deflection was attained. The variation in deflection for the middle weight became constant with moment at a deflection difference of approximately 0.03 in., relatively close to the estimated allowance in the weight-to-specimen connections. The difference in deflections for the top level weight attained somewhat higher values (0.04 in.), but also appeared to approach an asymtote to the vertical axis in the figure. The difference in deflections for the lower level weights increased continuously with base moment, but did not exceed the estimated allowance (0.025 in.). It appeared that the slip in the weight-to-specimen connections could account for the variation between initial stiffness measured by differential transformers and initial stiffness measured by mechanical dial gages (uncorrected for base movement).

Overall, the dial gage readings, corrected for base movement, would appear to be the most reliable of the three measures of the initial stiffness of test structure S1, eliminating both base movement and slip in the weight-to-specimen connections.

(d) Comparison of Stiffness from Dial Gage Readings and Free Vibration Tests

The initial stiffness of test structure S1 implied by the mechanical dial gage readings, corrected for base movement, may be compared to the initial stiffnesses implied by the results of the pre-test free vibration tests for test structures D2 and D3. Referring to Table 8.1, the results were quite comparable, considering that they represent different test specimens, cast on different days, exhibiting somewhat different material properties (Table A.1). It appears that the low-amplitude free-vibration tests provided a reasonably accurate measure of initial stiffness.

(e) Comparison of Measured Initial Stiffnesses and Calculated Stiffnesses

It is instructive to compare the measured initial stiffnesses (corrected dial gages readings and free vibration test results) with various calculated stiffnesses (Table 8.1). The calculated stiffness for each of three cases listed in Table 8.1, along with the measured initial stiffness from dial gages or a free-vibration test, are provided in Table 8.2, as fractions of the stiffness of the uncracked structure, for each of the three appropriate test structures.

The reduction in measured structure stiffness below that for an uncracked structure is apparently due to shrinkage cracks and other cracks incurred during casting and handling of the test specimens. Although the calculated stiffnesses listed in Table 8.2 do not include the effect of shear deformations, it was determined that this effect could not account for the discrepancy between the stiffnesses implied by the measurements and those consistent with the uncracked state. For test structure S1, the shear deformations were found to reduce the stiffness for the uncracked state by 13 percent. The extent of the reduction in stiffness between the uncracked state and measured values is emphasized in Table 8.2. The measured stiffness was comparable to the calculated stiffness for the structure, considering all beams cracked and the piers below the first level beam cracked. This result may seem unreasonable. It was noted in chapter 3 that none of the specimens suffered apparent damage in casting, or handling prior to testing. However, the result can be explained without admitting visible cracking. If microcracking is considered to have occurred at locations with abrupt changes in geometry, stiffness reduction could have been attained by reducing all section stiffnesses to approximately 0.35 of the uncracked section stiffness (Table 8.2). The section stiffnesses for completely cracked beams and piers ranged from 0.19 to 0.26 of the uncracked section stiffnesses (Table 8.2). Hence, the observed stiffness reduction is plausible, although it does illustrate the effect that microcracking can have on the initial stiffness of the structures.

8.3 Comparison of Observed and Calculated Strengths

(a) Introductory Remarks

This section compares the measured strengths of the test structures with the calculated strengths. The observed responses considered included the Fourier analysis results for the critical test runs (D1-4, D2-1, D3-1, D4-1, D5-1) and the results for the final run of each test (test runs D1-5, D2-2, D3-2, D4-2, and D5-2).

Assuming that the test structures were loaded well into the nonlinear range of response (Sections 3.2, 3.3 and 3.4), the observed maximum base shear and moment responses can be considered to provide an indication of the strengths of the structures.

The calculated strengths were those for the failure mechanisms discussed in Section 4.4.

Section 8.3(b) summarizes the calculated strengths and observed responses presented in previous chapters. The calculated and observed values are compared in Sections 8.3(c) through 8.3(e).

(b) Presentation of Results

Table 8.3 summarizes the observed maximum base shear and base moment for each test run for each test structure, along with the calculated maximum base shear and base moment consistent with the failure mechanisms for each test structure.

The failure mechanisms are described in Section 4.4 and the base shears and base moments consistent with each mechanism are taken from Table 4.7.

The observed responses include the Fourier analysis results for the critical run of each dynamic test (test runs D1-4, D2-1, D3-1, D4-1 and D5-1). The maximum base shears and base moments for these test runs are taken from Table 6.2. The shears and moments considering only first-mode response are noted, in addition to the total shear and moment. The shears and moments considering only the first response mode were significant in that they were more directly comparable with the shears and moments consistent with the failure mechanisms than were the total observed shears and moments. This was due to the fact that the calculated strengths considered purely first-mode loading (section 4.4).

The maximum observed base shear and base moment for the final run of each dynamic test (test runs D1-5, D2-2, D3-2, D4-2 and D5-2) are also provided in Table 8.3. Only the total observed responses are listed, the values being taken from Table 3.5.

The maximum base shear and base moment, for each direction of loading, for the static test (test S1), are also listed in Table 8.3. The maximum shears and moments were obtained directly from Fig. 3.54, considering the maximum load at each of three levels, along with the heights of the points of load application above the base of the structure.

(c) Discussion of Results from the Test Runs with $A_{max} \cong 1.0g$

For convenience, the measured maximum base moments for test runs with $A_{max} \cong 1.0g$ are summarized below, along with strengths calculated for mechanisms 1 and 2, as defined in section 4.4. It should be noted that mechanisms 1 and 2 refer to the same pattern of flexural yield hinges, the only difference being the strengths of the hinges. For mechanism 1, the yield strength of the beams is considered, for mechanism 2, the maximum moment capacity is considered. The mechanism for structure type A refers to yielding at the base only.

	Calculated		Measured	
Type	Mech. 1 k-in.	Mech. 2 k-in.	First-Mode k-in.	Total k-in.
A(D1)	81	-	78	86
B(D2)	47	56	49	58
B(D3)	47	56	51	56
C(D4)	40	46	50	54
C(D5)	40	46	49	51

The ratio of the maximum first-mode component to the maximum total measured value is approximately 0.9, which is consistent with the results of the linear-response analyses. Because the magnitudes indicated by first-mode components are more reliable measurements of the base moment, the observed ratio, which agrees with the calculated ratio, tends to provide confidence in the observed maximum total values.

In general, the measured moments agreed reasonably well with the calculated ones. The measured values for type C structures, especially the first-mode components, were almost the same as the measured values for type B structures, contrary to the trend indicated by the calculated values. The calculated strengths reflected the influence of the reduction of the reinforcement ratio of the beams. If the measured first-mode base moments are considered to be reliable, it would appear that the calculations underestimated either the relative contribution of the piers to structure strength or the effect of strain hardening on the strength of the beams. The mean total moment for type B structures (57 k-in.) was higher than that for type C structures (53 k-in.), but not high enough to confirm the difference implied by the calculations.

(d) Discussion of Maximum Observed Moments

Evaluation of the maximum observed moments is of interest because the strengths implied by these data can be compared directly with strengths calculated from physical characteristics of the test structures. However, before the quantities themselves are considered, one feature of the measured quantities must be discussed.

The "measured" moment is a quantity calculated from accelerations measured at three levels in the test structure. Consequently, if the acceleration data contain "spikes" and if two of those "spikes" are recorded as having occurred at the same time, the influence on the calculated moment of these "spikes," which may or may not be real, can be quite large. Furthermore, the superposition of such "spikes" is highly sensitive to small variations in the phase relations among the accelerations at the three levels in the test structure. Therefore, "spikes" in the waveform of the moment-response plot must be considered very carefully before associating the magnitude of such spikes with structure strength.

Referring to Table 8.3, the maximum total base moments for the final test runs, in general, appear quite high, relative to the calculated strengths. However, for all cases, except test structure D2, for which the observed maximum compared to the calculated strength, the observed maxima were associated with "spikes" or isolated peaks in a "jagged" response history. This is apparent in Fig. 3.6, 3.17, 3.22, 3.35 and 3.40, where the maximum base moment for structure D1 occurred at 0.4 sec. after the start of response and the maximum base moments for other structures occurred at 1.1 sec. after the start of response. The situation is especially noticeable for test structure D1. The isolated "spike" that produced this high maximum moment corresponded to "spikes" in the acceleration records (Fig. 3.4). For all cases, except structure D2, the physically significant observed maximum moments were probably somewhat lower than the apparent maxima.

8.4 Interpretation of Observed Response Using Linear-Response Models

(a) Introductory Remarks

This section compares the observed responses with the results of the study of linear dynamic response and the study of static hysteretic response, to determine the overall magnitudes of viscous damping factors consistent with the observed responses. The objective was to study the feasibility of using a viscously damped substitute structure to simulate the response of the test structures.

The spectral study (section 7.3) was first evaluated to determine what viscous damping factors would be required to make the calculated responses equal to the observed responses at reasonable frequency levels. This was done for base shear, base moment and top level deflection and is described in part (b) of this section.

The basis for determining the feasibility of using a viscously damped, linear, substitute structure to simulate the observed responses was a comparison of the results of the study of response history (section 7.4) and the results of the Fourier analysis of observed responses (chapter 6). The comparison was based upon both the overall shape, or character, of the response histories and the magnitude of viscous damping factors needed, in the analytical model, to simulate the observed responses. The comparison is described in parts (c) through (f) of this section.

The final portion of the section (part g) provides a general discussion of the results derived from the comparisons.

(b) Comparison of Observed Response and Calculated Spectral Response

Figure 7.7-11, described in Section 7.3, contain plots of three calculated response quantities (deflection, base shear, and base moment), as a function of the first-mode frequency at two damping factors (0.02 and 0.10), for each test structure ($A_{max} \cong 1.0g$). The magnitude of the maximum observed response is indicated in each plot.

Before considering the comparison of the calculated response histories with the measured response histories, it is helpful to review the overall implications of these plots.

In all cases, the observed response can be reconciled with the response calculated for a particular combination of first-mode frequency (between approximately 4 and 10 Hz.) and viscous damping factor (between 0.02 and 0.10).

Because of the necessity of invoking unreasonably low first-mode frequencies to effect reconciliation at low damping factors, reconciliation at damping factors approaching 0.10 appears more plausible.

Using the "early frequency" (section 7.4) and a damping factor of approximately 0.10, it is possible to match the observed and calculated responses for base shear and moment, but not for top-story displacement.

(c) Scheme for Reconciliation

Sections 8.4(c) through (f) compare calculated response-histories with observed response histories, in order to study the possibility of using a linear analytical model to simulate the test results. The comparisons considered the first-mode response, second-mode response and total response for the base shear and base moment, and the total response for the top-level deflection. The results of the analytical study of response history are presented in Fig. 7.17 through 7.42. The results of the Fourier analysis are presented in Fig. 6.1 through 6.16.

The results for the analytical model are compared with the observed responses, separately, for two particular intervals during the test duration. The first interval refers to the first 1.5 sec. and the final interval refers to the final 2.0 sec. of the total test duration of 6.0 sec. The first interval was significant because maximum response was registered during this interval. The consideration of the second interval provided information on whether one substitute structure could be used to simulate an entire response history, or whether a response history had to be simulated in pieces, by several substitute structures.

Several overall approaches were considered. One approach involved comparing observed responses during both the first and final intervals with the response histories for a substitute structure characterized by a first-mode frequency equal to the early frequency (chapter 7). Although such a reconciliation could be made for the type A test structure (test run D1-4), for types B and C test structures, the response during the final interval could not be reconciled in such a manner. A first-mode viscous damping factor much greater than 0.10 would be required of a substitute structure to obtain the first-mode observed response in this interval. This conclusion was derived from the response histories for base shear and base moment.

Because of the poor correlation between calculated and measured forces, the approach of using a substitute structure with a first-mode frequency equal to the early frequency to simulate response during both first and final intervals was discarded.

A second possible approach involved calculating the observed response using a substitute structure characterized by a first-mode frequency equal to the late frequency. However, the response in the first interval could not be simulated plausibly using models with their stiffnesses based on the late frequency. This was true of base shear, base moment and top level deflection and is shown in Fig. 7.19, 7.20, 7.23, 7.24, 7.26, 7.29, 7.30 7.33, 7.34, 7.37, 7.38, 7.41 and 7.42.

Reconciliation of observed and calculated responses was obtained using a substitute structure characterized by the early frequency to calculate response in the first interval and a substitute structure characterized by the late frequency to calculate response in the final interval.

(d) Type A Structure

The observed response histories for the type A test structure (test run D1-4) are shown in Fig. 6.2, 6.7 and 6.12. The calculated response histories are represented by analyses 1 through 4 (tables 7.4 and 7.5) and are shown in Fig. 7.17 through 7.20. The substitute structures with viscous damping factors of 0.02 resulted in responses well in excess of the first-mode response for both intervals of response. However, the substitute structures with viscous damping factors of 0.10 led to overall first-mode magnitudes comparable to those measured in both intervals. The waveforms of the calculated response histories also compared well to the results of the Fourier analysis of observed response.

The overall magnitude of the calculated second-mode response for base shear was apparently not highly sensitive to the viscous damping factor (see, for example, Fig. 7.17 and 7.18). The observed response could be matched using a viscous damping factor for the second mode of either 0.02 or 0.10. For base moment, however, the second-mode response was underestimated, even using substitute structures with a second-mode damping factor of 0.02.

The total responses calculated in analyses 2 and 4, using viscous damping factors of 0.10 for both first-and second-mode responses matched the total magnitudes of response for both first and final intervals well, due to the dominant influence of the first mode. However, the second-mode-contributions were vastly underestimated by these analyses. The result was that the calculated response histories for base shear and base moment for these analyses lacked much of the "jaggedness" produced by the second mode in the observed response histories.

(e) Type B Structures

The Fourier-analysis results for the observed response histories of the type B test structures (test runs D2-1 and D3-1)are shown in Fig. 6.8, 6.9, 6.13 and 6.14.

The calculated response histories are represented by analyses 19 through 22 (table 7.4). All of the above analyses were characterized by identical viscous damping factors for the two response modes (either 0.02 or 0.10).

For both tests, the overall magnitude of response during the first interval, for base shear, base moment and top level deflection, was grossly over-estimated by the analyses for damping factors of 0.02. The magnitude of these first-interval responses were matched closely for both first and second modes of response by substitute structures characterized by damping factors of 0.10. The total response for base shear and moment, for the first interval, was well matched, even to the degree of "jaggedness" in the response.

The overall magnitude of the first-mode response in the final interval was slightly overestimated for base shear, base moment and top level deflection, by substitute structures characterized by damping factors equal to 0.10. The implied damping factor for the observed final-interval response was only slightly greater than 0.10. This was true for both tests.

The overall magnitude of the second-mode response in the final interval for base shear, base moment and top level deflection, was overestimated by substitute structures characterized by damping factors equal to 0.02, for both tests, well matched by substitute structures characterized by damping factors equal to 0.10, for test run D2-1, and underestimated by substitute structures characterized by damping factors equal to 0.10, for test run D3-1. Evidently, the second-mode equivalent damping factor, for late stages of response, was on the order of 0.10 for test run D2-1 and between 0.02 and 0.10 for test run D3-1.

The substitute structures characterized by damping factors equal to 0.10 modelled the general shape and "jaggedness" of the response well for test run D2-1, while for test run D3-1, such analytical models did not lead to waveforms with the degree of "jaggedness" observed in the measured response, reflecting the lower apparent viscous damping factor for the second mode.

(f) Type C Structures

The observed response histories for the type C test structures (test runs D4-1 and D5-1) are shown in Fig. 6.10, 6.11, 6.15 and 6.16. The calculated response histories are represented by analyses 5 through 14, for test run D4-1, and by analyses 23 through 26, for test run D5-1 (Table 7.4). The calculated response histories are shown in Fig. 8.21 through 8.30, for test run D4-1 and Fig. 8.39 through 8.42, for test run D5-1. Analyses 5 through 10 and 23 through 26 were for substitute structures characterized by equal damping factors for the first and second modes of response. Analyses 11 through 14 represented a study of the effect of dissimilar damping factors for the first and second modes of response. The first-mode viscous damping factor was equal to 0.10 for all of analyses 11 through 14, while the second-mode viscous damping factor was either 0.02 or 0.05.

Considering first the substitute structures characterized by equal damping factors for the first and second modes of response, the results were very similar to those for the type B structures. The responses in the first interval were grossly overestimated by substitute structures characterized by viscous damping factors equal to 0.02. Substitute structures with viscous damping factors equal to 0.10 matched the observed response in the first interval for base shear and base moment, quite well, for both first and second modes of response. The overall shape of the calculated relations was also consistent with the observed response. The total first-interval response was also well matched by results based on these structures. Even the overall degree of "jaggedness" of the total response was well simulated. Hence, calculations using the substitute structures matched reasonably well the manner in which the two response modes combined.

The matching of the observed first-interval deflections, was somewhat more of a problem. The substitute structures with viscous damping factors for both response modes equal to 0.10 could be used to match the magnitude of the first two or three peaks in the observed deflection. However, later peaks in the first interval were underestimated. It was noted that the observed base shear and moment did not exhibit such behavior. The maximum response was attained during the first two or three excursions. Response increased little for subsequent peaks during the first time interval. The success in matching of the maximum forces, but not deflections, was thought of as a manifestation of yielding of the test structures during the interval of early response. This point will be discussed further in section 8.5.

The final-interval responses, for base shear, base moment, and top level deflection, for both tests, were slightly overestimated using substitute structures with viscous damping factors for both response modes equal to 0.10. Damping factors of 0.10 or somewhat greater, for both modes, were apparently consistent with the observed responses. These substitute structures also led to results which matched the total responses for the final interval, including the manner in which the two modes of response combined.

Analyses 11 through 14 helped to provide additional support for an interesting conclusion, described in the preceding paragraphs. This was the conclusion that, for reconciliation of analytical and observed responses, for type B and C structures, the viscous damping factor for the second mode of response needed to be of the same order as that for the first mode of response. Analyses 11 through 14 considered substitute structures characterized by first-mode damping factors equal to 0.10. The second-mode damping factor was either 0.02 or 0.05. The results for analyses 11 and 13 (second-mode damping factor equal to 0.02) grossly overestimated the second-mode early response for base shear and base moment. This result also manifested itself in the total first-interval responses. Only with a second-mode damping factor equal to 0.10 (analysis 6) did the modal contributions to first-interval response become reasonable.

The final-interval second-mode response for base shear and base moment were not as sensitive to the magnitude of viscous damping factor as were the early responses. A second-mode damping factor equal to 0.02, however, did result in overestimation of the second-mode contribution, while a second-mode damping factor of 0.05 slightly overestimated the second-mode contribution. The change in the overall magnitude of final-interval second-mode response as the viscous damping factor increased from 0.05 to 0.10 was noticeable, but not drastic. The viscous damping factor for reconciliation of calculated and observed response must be on the order of 0.10.

(g) Discussion of Results

The damping factor required to match the results from linear models with the observed results was generally the same for all types of test structures.

The required first-mode damping factor for first-interval and high-amplitude response appeared to be on the order of 0.10. This is a reasonable value for a reinforced concrete structure undergoing extensive yielding.

The required first-mode damping factor for final-interval and low amplitude response was equal to, or slightly greater than, 0.10, for all cases. The calculated first-mode damping factor for low-amplitude response was greater than that for high-amplitude response, as discussed in section 5.5 (e), in relation to the shape of the hysteresis relation and the definition of an equivalent viscous damping factor. Certainly, the numerical values of the calculated damping factors in Table 5.5 do not correlate well with those values obtained from the response history study. However, considering the crude manner in which the hysteresis is defined in chapter 5, close numerical correlation may not be expected. The results of chapter 5 enhance the understanding of the results of the response history study and help support the concept that low-response amplitude does not necessarily imply low damping factor.

Another interesting result was that, with the exception of the type A structure (test run D1-4), the second-mode viscous damping factors required for the substitute structures to predict the observed responses were of the same order as the first-mode damping factors (equal to or only slightly less than 0.10). This result was also anticipated by the equivalent damping study of section 5.5, where, again, this problem is described in terms of the shape of the hysteresis relation and the definition of an equivalent viscous damping factor. The calculated damping factors for the first-and second-modes of response are shown in Table 5.5. The second-mode value was meant to correspond to high amplitude (first-interval) response. The hysteresis model used to obtain the values in Table 5.5 was, of course, very crude, and the results should be thought of only as providing support for the general concept that the second-mode damping factor may be of a magnitude similar to that of the first-mode damping factor. Sources of error for the numerical values of the damping factors of Table 5.5 included the use of an idealized, or approximate, mode shape, or loading pattern, uncertainty concerning the relative amplitudes of the two modes of response, and idealization of the moment-rotation hysteresis relations for the members.

For the type A structure (test run D1-4) the second-mode viscous damping factor for reconciliation of calculated and observed response histories was on the order of 0.02, quite different from those for the type B and C structures.

Finally, the reconciliation of the analytical study of response history with the results of the Fourier analysis of the observed response histories produced reasonable and interesting results, in terms of the magnitudes of viscous damping factors required for the linear substitute structures to estimate the observed responses. The damping factors obtained were of reasonable overall magnitude for reinforced concrete structures undergoing significant yielding. The results indicated, however, that the general concept that higher modes are less heavily damped than the first mode and that viscous damping factor decreases as response amplitude decreases may not be universally correct.

8.5 Interpretation of Damage to the Test Structures

(a) Introductory Remarks

This section will interpret the degree of structural damage implied by the maximum first quarter cycle response during test S1 and by the major peaks in the observed response histories for the critical test runs (D1-4, D2-1, D3-1, D4-1, D5-1).

For the static test (S1), the degree of structural damage was assessed by considering the calculated maximum member deformations from the analytical study of hysteretic response. For the dynamic tests, the structural damage level was assessed through a series of linear substitute structures, with stiffness levels compatible with (a) the maximum base moment and maximum top level deflection for the various peaks in the observed response histories, and (b) the frequencies in the first and final intervals. In a similar manner, a linear substitute structure was defined to exhibit the maximum base moment and maximum top level deflection observed during the first quarter cycle of test S1.

In the first stage of the consideration of structural damage, the stiffness and frequencies for the substitute structures were compared with each other and with the overall structure stiffnesses calculated in chapter 4 for various combinations of cracked and uncracked beams and piers and completely missing beams. In the second stage of the interpretation, using the methods of chapter 7, damage ratios for the beams and piers were associated with the various substitute structures, based upon their first-mode frequencies. These were compared with each other and with member damage ratios implied by the results of the study of static hysteresis. All damage ratios were based on the stiffness for a cracked section.

Section 8.5 (b) presents a summary of the results to be considered in the comparison. Section 8.5 (c) discusses the stiffnesses and first-mode frequencies for the various substitute structures. Section 8.5 (d) discusses the member damage ratios associated with various substitute structures and with the study of static hysteretic response.

(b) Summary of Results

The overall stiffnesses of the substitute structures, in terms of the ratio of base moment for one wall to top level deflection, are listed in table 8.4. The stiffnesses for various combinations of cracked and uncracked beams and piers and missing beams (first five cases in the table) were taken directly from table 4.6. The stiffnesses for substitute structures characterized by the frequencies in the first and final intervals were taken from table 7.2.

The stiffnesses for the substitute structures representing the maximum first quarter cycle response for test S1 and the study of static hysteretic response were obtained from Fig. 3.54 and 5.30. The substitute structures for the major peaks in the response histories were defined for purely first-mode response. This promoted comparability with those substitute structures considering the static test and static hysteresis analysis and with the analyses considering various combinations of cracked and uncracked section stiffnesses for the members. The overall stiffnesses for substitute structures corresponding to the pre-test free vibration tests were calculated from the corresponding observed frequencies (Table 3.6), following the method of section 4.2(b).

The first-mode frequencies corresponding to the stiffnesses listed in Table 8.4 are listed in Table 8.5. The results were taken from Tables 4.4, 3.6, 7.2 and 8.4. The method of section 4.2(b) was used to convert stiffnesses to frequencies. Through the results of section 7.3, member damage ratios were associated with the substitute structures corresponding to the early frequency and also with those corresponding to the peaks in the response histories. Using the first-mode frequencies listed in Table 8.5, the uniform damage ratios for the beams and for the lower level piers were obtained from Fig. 7.6, for the three distributions of structural damage provided in the figure. The resulting ranges of member damage ratios for each substitute structure are listed in Table 8.7. These damage ratios were based upon the reference structures used in the study of dynamic response (section 7.3). The member section stiffness for these reference structures (reference section stiffnesses) are listed in Table 8.6. To promote overall comparability in the study and to make the member damage ratios more physically meaningful, the member damage ratios of Table 8.7 were factored by a stiffness ratio, so as to be based upon the cracked section stiffnesses (Table 8.6). The results are listed in Table 8.8. Note that when damage ratios are based on cracked section stiffnesses, the identifications for the damage distributions ($\mu_{pr} = 1$, $\mu_{bm} = 2\mu_{pr}$, $\mu_{pr} = \mu_{bm}$) no longer reflect the numerical relations between beam and pier damage ratios.

Member damage ratios were also computed for the maximum first quarter cycle response during the study of static hysteretic response. The maximum member end rotations, as listed in Table 5.3, were considered. The damage ratios were expressed in terms of cracked section stiffnesses for the members and are listed in Table 8.9.

(c) Discussion of Stiffnesses and Frequencies for Substitute Structures

The stiffnesses corresponding to pre-test free vibration tests, for test structures D2, D3 and D4, were very close to the stiffness of the test structure with all beams and the lower level piers fully cracked. The free vibration test for D1 represented only a slightly lower stiffness, and that for D5 a somewhat higher stiffness. The comments made relative to the initial stiffness comparison of section 8.2 apply here, however. The analyses being considered allowed only the lower level pier to be fully cracked. The remainder of the pier was assumed completely uncracked. The results of Tables 8.4 and 8.5 do not require that one assume the beams and lower level pier to be fully cracked at the start of the dynamic tests. Finally, the free vibration test results for the types A and C structures represent damage levels fairly similar to those for the type B structures, as discussed in section 8.2.

The substitute structures corresponding to the final-interval frequency exhibited stiffnesses very similar to those obtained considering uncoupled piers and the lower level pier fully cracked. This result was consistent with the structural damage observed at the conclusions of the critical test runs (D1-4, D2-1, D3-1, D4-1 and D5-1).

The stiffnesses associated with peaks in the response histories (Table 8.4) were significant relative to the comparison of observed and calculated response histories, as discussed in section 8.4. It was mentioned in section 8.4 that, for types C test structures, substitute structures exhibiting a first-mode frequency equal to the early frequency and a viscous damping factor of 0.10 could be used to match the maximum base moment, but not the maximum deflection. The top-level deflection calculated was close to that for the peak at a time 0.4 sec. into the observed response. The substitute structure characterized by the early frequency modelled the peak for a time of 0.4 seconds. It also predicted the base moment and top level deflection for this peak reasonably well. For subsequent peaks, the observed base moment increased only slightly, while the observed deflection increased more significantly. The structure was yielding. The substitute structure corresponding to the early frequency was simply too stiff to predict the maximum deflection excursion.

In effect, for a more faithful simulation of the response, it would have been preferable to subdivide the first interval, using models with different stiffnesses in the two subdivisions.

For the type A structure (test D1), the results were somewhat different. The maximum response peak, occurring at 0.7 sec., was associated with a stiffness only slightly less than that associated with the early frequency, and well in excess of the stiffness associated with the late frequency.

Finally, the stiffnesses for the substitute structures associated with the maximum first quarter cycle response for the static test and for the static hysteresis analysis (Table 8.4) correlated well with stiffnesses for those substitute structures associated with the late frequency of the dynamic test structures, and hence, with those substitute structures associated with the highest amplitude peak (1.2 sec) in the observed response histories. The static test was designed to simulate the maximum observed response, and, if anything, represented a higher deformation level than the dynamic tests. It is also to be noted that the first quarter cycle response for the static test represented considerable overall yielding (Fig. 3.54). This was consistent with the yielding implied by the observed response histories, as discussed in preceding paragraphs.

(d) Discussion of Member Damage Ratios

The damage ratios, based upon fully cracked section stiffnesses, for the beams and pier, are listed in Table 8.8 for various substitute structures. As mentioned previously, the overall stiffness level for each substitute structure may be satisfied by an infinite number of combinations of beam damage ratio and pier damage ratio. The table provides results for three damage distributions, $\mu_{pr} = 1$, $\mu_{pr} = \mu_{bm}$, and $\mu_{bm} = 2\mu_{pr}$.

It should first be mentioned that the beam and pier damage ratios are for equivalent prismatic members. The values may be thought of as related to the average damage ratios over the length of the actual members. For the beams, especially, the concentrated damage ratios at the beam-pier interface would be larger. Furthermore, the values in the table assume that all beams exhibited identical average damage ratios, even though some beams must exhibit larger average damage ratios over their length. The two preceding observations suggest that in judging the damage ratios computed for the substitute structures, the quantities listed should be interpreted as indicating trends rather than precise values.

For the assumed damage distribution $\mu_{pr} = 1$, the results show pier damage ratios less than one. This merely indicates the section stiffness for the lower level pier was greater than that for a fully cracked section. The section stiffness was equal to the reference section stiffness for the study of dynamic response (chapter 7). For the early frequency and for the first major peak in the response history (0.4 sec), the damage distribution with $\mu_{pr} = 1$ requires very high beam damage ratios. For structure D3, a beam damage ratio of 17.9 is required. Even at this early loading stage, a pier section stiffness closer to that for a fully cracked section was likely. For the maximum excursion of the dynamic tests (1.2 sec), the required damage ratios for the beams would be absurd, exceeding a value of 30.

For test structure D1 (type A) at maximum response (0.7 sec), the damage ratio for the beams could be made small only by assuming enormous damage at the base of the pier. It should be noted that the observed and calculated failure mechanism for this structure (sections 3.4(h) and 4.4(b)) consisted of failure of the bases of the piers in axial tension and no apparent beam damage. The implication from the substitute structure was that the lower level pier damage was very large, which was consistent with the observed result.

Among the substitute structures for type B and C structures it is interesting to consider the structures for the final major peak in the response history (1.2 sec). A distribution of damage characterized by $\mu_{bm} = \mu_{pr}$ resulted in damage ratios for the lower level pier which were too high (in the range 2.6 to 3.0). The member damage ratios for the distribution, $\mu_{bm} = 2\mu_{pr}$, could be used to explain the structure response. Damage ratios for the pier, for the various structures, would be in the range 1.8 to 2.1, while those for the beams would be in the range 3.2 to 5.0. The damage ratio for the pier may also be thought of as being somewhat less than 1.8, while the beam damage ratios could be thought of as somewhat higher than 3.2 to 5.0. In sum, the maximum responses observed in tests D2-1, D3-1, D4-1 and D5-1 can be associated with plausible damage ratios for the members.

Damage ratios, based upon fully cracked section stiffnesses, are listed, in Table 8.9, for the maximum response during the first quarter cycle of the analytical study of static hysteretic response. These results should be compared with those for the substitute structures for tests D2-1 and D3-1 at a time of 1.2 seconds. The beam damage ratios implied by the hysteretic study appear high (7.5 to 14.2) compared with those for the dynamic tests for $\mu_{bm} = 2\mu_{pr}$ (4.4 to 5.0). It should first be remembered that the results for the dynamic tests represent average damage ratios over all six beams. Some beams could be thought of as having larger damage ratios. Furthermore, as discussed in the previous paragraph, the damage distribution of $\mu_{bm} = 2\mu_{pr}$ may not reflect the behavior of the test structures precisely. The pier damage ratio may be thought of as somewhat lower than the 1.8 to 2.0 for the substitute structures. As indicated by Fig. 7.6, for the frequency range under consideration (3.6 to 3.7 Hz.), the damage ratios for the beams may be quite sensitive to a decrease in the damage ratio for the lower level pier. Furthermore, the static test, and the hysteretic analysis, as described in section 8.3, apparently represented a slightly higher level of loading than did tests D2-1 and D3-1. Slightly higher damage ratios for the beams in the static test structure, than for those in test structures D2 and D3, may be reasonable. The magnitudes of the beam damage ratios are not only reconcilable with those for the substitute structures for test D2-1 and D3-1, but are plausible, in terms of general magnitude.

8.6 Summary of Results

The results of low-amplitude free-vibration tests, measurements of initial stiffness during the static test, and calculations of structure stiffness indicated that shrinkage cracks and other microcracks reduced the initial stiffness of the test structures significantly below that indicated by a calculation based on completely uncracked sections for the members.

Reasonable correlation was obtained between the observed structure strengths, considering first-mode response only, and the structure strengths consistent with the failure mechanisms. Type C structures developed slightly higher displacements during the first test run than did the type B structures.

For all critical test runs, it was found that a linear substitute structure characterized by the early frequency could be used to match the level of force response during the first interval in the response history, while a linear substitute structure chracterized by the late frequency could be used to match the level of force response during the final interval in the response history. Furthermore, the linear response models could be used successfully in estimating the general shape of the response histories.

Calculations based on substitute structures characterized by a damping factor of 0.10 matched the overall magnitude of the first-mode base shear and base moment for first and final intervals in the response histories. For the type A structure, the overall magnitude of the second-mode base shear and base moment would be matched by calculations based on a substitute structure with a damping factor less than 0.02. For types B and C structures, with the exception of test structure D3, the overall magnitude of the second-mode base shear and base moment was matched by substitute structures with damping factors on the order of 0.10. For structure D3, a damping factor slightly less than 0.10 would produce reconciliation for the second-mode base shear and base moment. When both the first- and second- mode base shear or base moment were well simulated, the total base shear or moment was well matched. In essence, the linear dynamic model could be used to model the manner in which the observed first and second modes of response combined, to form the total response.

For types B and C structures, a linear substitute structure with a first-mode damping factor of 0.10 simulated the overall magnitude of top-level deflection late in the response histories and matched the first major deflection peak (0.4 sec) early in response. Subsequent peaks in the observed deflection histories were characterized by higher magnitudes than those calculated using the substitute structure. This was a manifestation of yielding of the test structure and consequent reduction in apparent structure stiffness, which could not be modelled by the linear substitute structure characterized by the early frequency.

An interesting finding of the study of linear dynamic response was that the second-mode damping factor for a linear substitute structure was not necessarily significantly less than the first-mode damping factor. Furthermore, low-amplitude response was not necessarily consistent with a lower damping factor than higher amplitude response. These results were reinforced by those of the study of equivalent damping using the analytical model for static hysteretic response. The shape of the structure hysteresis can alter vastly the relations among the various damping factors.

Comparison of the early and late frequencies with the calculated structure deformation properties, as presented in chapter 4, indicated that, in addition to significant structural damage to the beams, major damage to the lower level piers accompanied the maximum response.

Associating various substitute structures with damage ratios, for the type A structure, implied that for the maximum observed response for test D1-4, the damage ratios for the lower level piers, based upon a fully cracked section, had to be very high (significantly greater than five) if the beam damage ratios were to conform to the observation of no heavy beam damage.

Associating the stiffnesses of various substitute structures with damage ratios (Table 8.8) for the types B and C structures, implied that for the maximum responses in the critical runs (1.2 sec after the start of response), the average damage ratios for the beams had to be on the order of five and those for the lower level pier on the order of 1.5. These values are based upon the stiffness of a fully cracked section. Several beams had to exhibit damage ratios significantly greater than the average. The results of the static hysteretic analysis, representing a somewhat higher level of deformation than tests D2-1 and D3-1, gave an upper limit to what these ratios might be. The damage ratio for the most severely deformed beam could be on the order of 14. Furthermore, the various combinations of beam and pier damage ratios calculated for the substitute structures indicated that a certain amount of lower level pier damage, relative to a cracked section, was required, if the maximum responses during the dynamic tests were to be explained. For a damage ratio of one in the lower level piers, the necessary beam damage ratios became unreasonable.

CHAPTER 9

SUMMARY AND CONCLUSIONS

The object of this study was to develop information toward a better understanding of the dynamic response of reinforced concrete coupled-wall systems subjected to strong earthquake motions. The experimental work included tests of six small-scale structures as described in Fig. A.17 through A.20 and Tables A.6 through A.11. The main experimental variables were the strength and stiffness of the connecting beams as shown below.

Type	Mark	Beam Depth/Span	Beam Reinf. Ratio
A	D1	0.6	0.022
B	D2,D3,S1	0.4	0.010
C	D4,D5	0.4	0.006

Five of the test structures were subjected to base motions simulating one component of the record obtained at El Centro, California in 1940. One test structure (S1) was loaded with slowly applied cyclic lateral forces. Both the base motions and the static loading produced yielding of the structures.

Material properties and test procedures are described in appendix A. The target concrete strength was 4500 psi. The nominal yield stress of the reinforcement was 43,000 psi. The walls were reinforced uniformly, the longitudinal and transverse reinforcement ratios being 0.01. For the beams, the reinforcement ratio varied as indicated above.

Instrumentation for the dynamic tests measured accelerations and displacements. The data from all test runs were reduced to obtain base shear and moment. The modal components in the data from test runs with A_{max} = 1.0g were separated using standard Fourier Analysis techniques.

The data from the static test was studied to provide information on the actual initial stiffness of the test structures, the limiting capacity of the system for a particular distribution of lateral loading, and hysteretic response.

The influence of member hysteretic response on the overall hysteresis of the structure was studied analytically, using the results from the static test as a check for the results obtained using the analytical model.

A series of studies were made to investigate the possibility of using linear-dynamic response models to obtain calculated values comparable to the observed base shear, base moment and displacement responses from the dynamic tests. These studies were made for each test structure for the test run with $A_{max} \cong 1.0$g.

The following general conclusions were drawn from the experimental results:

*The initial measured frequency for the test structures could be closely matched using the initial stiffness measured during the static test. The measured initial stiffness was much lower than that computed considering uncracked sections.

*The apparent natural frequency of the test structures decreased continuously as the structures deteriorated under successive and increasingly severe applications of the base motion. Table 3.6 shows this trend.

*The maximum top-level deflection observed during the test runs with A_{max} = 1.0g was from 2.7 to 4.6 times the deflection calculated using response spectrum-modal analysis and a completely uncracked test structure. The deflections are listed in Table 9.1.

*The relative contribution to base shears and moments of higher modes increased with decrease in strength and stiffness of the connecting beams.

The following conclusion was drawn from the results of the static hysteretic analysis:

*The hysteresis relations for the connecting beams had a major effect on the overall hysteresis relation for the structure and, therefore, on the energy dissipation capacity of the structure.

Several conclusions follow from comparison of the results of the linear dynamic response analysis with the results of the dynamic tests with $A_{max} \cong 1.0g$:

*A linear dynamic response model could be used to simulate the base shear and base moment responses observed during the dynamic tests. Better results were obtained using a model having different stiffness levels for the initial and final portions of the response duration.

*The equivalent damping factors required for the linear response model to simulate the observed base shears and moments were virtually constant, at approximately 0.10, for all levels of response amplitude.

*For test structures with shallow beams (types B and C described in Fig. A.20), the equivalent damping factors required for the linear response model to simulate the observed base shear and base moment responses were the same for mode 2 as for mode 1.

*For test structures with shallow beams, the deflections obtained from the analytical model (with its natural frequency set equal to the observed apparent mean frequency) were less than those observed during the dynamic tests.

*For the test structure with deep beams, observed deflections were also well simulated by the linear model (with its natural frequency set equal to the observed apparent mean frequency).

APPENDIX A

DESCRIPTION OF EXPERIMENTAL PROGRAM

A.1 Concrete Properties

The concrete used throughout this study is small-aggregate concrete similar to that used in previous studies in the Structural Research Laboratory of the Department of Civil Engineering at the University of Illinois. The proportions by dry weight for the mix were 1.00:3.83:0.96 (cement:coarse aggregate:fine aggregate). The cement used was high early strength (Type III), the coarse aggregate was Wabash River sand, and the fine aggregate was fine lake sand. The aggregates were kept "bone-dry." The water-cement ratio was 0.8, chosen on the basis of attaining a desired compressive strength. The water content by volume was 0.27. This was chosen to obtain maximum possible workability of the mix.

Mechanical properties were determined from tests performed on the same day that each wall specimen was tested. Cylinders were tested in compression and by splitting, and modulus of rupture tests were performed. Results for each test are summarized in Tables A.1 and A.2.

Compressive properties were determined by testing 4 x 8 cylinders using a 120-kip universal testing machine. Strains were determined from a 0.001-in. mechanical dial gage with a 5-in. gage length. A representative stress-strain relation is shown in Fig. A.1. Due to limitations of the equipment, it was not always possible to obtain the descending portion of the stress-strain relation. The data in that range were very erratic. The mean compressive strength, obtained from these tests, for each pair of specimens, along with their respective standard deviations and ranges are compared with age at testing in Fig. A.2. This data compares well with that obtained for concrete in previous studies at the University of Illinois (Table A.1 in Ref. 25).

The initial modulus of the concrete, taken as the slope of the secant drawn from zero to 1000 psi, is compared with the square root of compressive strength in Fig. A.3. All points fall between two lines described by $40\sqrt{f'_c}$ and $50\sqrt{f'_c}$.

The tensile properties of the concrete were determined by splitting tests on 4 x 8-in. cylinders and from the modulus of rupture determined from prisms with a 2 x 2-in. cross section loaded at the center of a span of 6 in. For each pair of specimens, the mean tensile parameters are compared with the square root of the mean compressive strength in Fig. A.4. The mean modulus of rupture is compared with the mean splitting strength for each pair of specimens in Fig. A.5. These, again, compare well with results from previous studies in the laboratory (Ref. 25).

A.2 Reinforcement Properties

(a) General Comments

The steel used for flexural and shear reinforcement throughout the study was black annealed wire. The supplier cut the wire into 6-ft lengths and covered it with heavy oil for protection from weather during shipping. To help insure proper bond between steel and concrete in the tests, the wire was soaked in a petroleum-based solvent to remove the oil and then in acetone to remove any residual film. Three gauges of wire were used; #8 gauge, #11 gauge, and #13 gauge with nominal diameters of 0.162 in., 0.125 in., and 0.0915 in. respectively. The cross-sectional dimensions of the wire were checked by micrometer readings. The nominal area was within 2% of the actual area in all cases. Measured dimensions are shown in Table A.3.

Tension tests of the steel were performed on a 60-kip universal testing machine. Strains were measured by a clip-on electrical resistance strain gauge with a 0.5 in. gauge length.

Preliminary tensile tests of the #11 and #13 gauge wire indicated that a portion of the bars had yield stresses less than 40 ksi. Yield stresses this low were considered unacceptable. Hence, it was necessary to select bars, to be used in the specimen, by individual coupon tests. For the #11 wire, 307 bars were used in specimens. Of these, stress-strain curves were obtained for 33 and yield stresses for the remainder. For the #13 wire, samples were tested from 72 bars. Of these, stress-strain curves were obtained for nine wires and yield stresses for the remainder. From this sample of 72 bars, seven were selected for use in the specimens. Of the six #8 gauge wires used in the specimens, stress-strain tests were run for three. Stress-strain results are also available for five additional bars not used in the specimens. These results are discussed in part (b) of this section.

The cages for the specimens were assembled by welding lightly with a 2.5 KVA Taylor-Winfield spot welder. For this reason, an additional stress-strain study was performed on #11 wire to investigate the welding effect. This is described in part (c) of this section.

Tests were also performed on 0.046-in. diameter wire. This wire was wound into coils and placed as a spiral around the vertical #11 wire in the piers. These tests are discussed in part (d) of this section.

(b) <u>Properties of Black Annealed Wire Before Welding</u>

As listed in Table A.5, the mean yield stress for the #11 gauge wire ranged from 42.2 to 45.3 ksi for all test structures and the coefficient of variation for #11 wires in a given test structure did not exceed 0.07. Statistical information on parameters delineating the measured stress-strain curves for #11 wires is tabulated in Table A.4. As would be expected, strain parameters are subject to considerably greater scatter than stress parameters. This is also true for Young's Modulus. The fracture strain was not measured for all specimens. It is recorded for all those cases in which it was measured. Tables A.4 and A.5 also list the stress-strain parameters of the #8 and the #13 wires.

A measured stress-strain curve is presented in Fig. A.6. An upper yield stress was observed in several specimens, although this could not be accurately measured with the equipment used. A rounding of the curve in the region of the yield stress was observed in five samples.

Variation of the various stress-strain parameters with bar size is depicted in Figs. A.7 through A.10. Fig. A.7 shows the yield stress and ultimate stress for the stress strain sample. Fig. A.8 shows the ratio of the ultimate stress to the yield stress. Fig. A.9 depicts the strain at strain hardening and the strain when the ultimate stress is reached. The measured Young's moduli are shown in Fig. A.10. The erratic variation with bar size and the scatter of these values becomes apparent in the figure.

Measured yield stresses, based on measured cross section, are summarized in Fig. A.11 for the #11 wire. This shows the yield stress statistics of the particular #11 and #13 wires that were used in the specimens for each static or dynamic test.

(c) Effect of Welding

To study the effect of welding on the stress strain properties of the black annealed wire, four #11 wires were selected at random. Each of these was cut into eight 9-in. samples. The first, third, fifth, and seventh samples were tested directly. The second, fourth, sixth and eighth sample each had a #11 gauge cross-bar welded to it. This resulted in conditions similar to those encountered in the fabrication of the cages. Conducting the study in this manner should make it possible to separate the effects of welding upon the yield stress and ultimate stress from variations from wire to wire and variations along a given wire.

The results of the study are illustrated in Fig. A.12 through A.16. Fig. A.12 through A.15 show the measured ultimate stress and the yield stress measured at 0.2 percent offset for each sample. Fig. A.16 consists of composite stress-strain curves for the welded and unwelded samples in each of the four groups of samples. Each relation was obtained by averaging the stresses from the appropriate curves at a number of values of strain.

It is apparent that the stress-strain relation was affected by welding. The effect, however, was quite erratic. Although the proportional limit was reduced in all cases, the stress at a strain of 2% was relatively insensitive to the welding process. For two of the specimen groups even, the yield stress measured to a 0.2% offset strain was not significantly affected. The ultimate stress is also quite insensitive. It should also be noted that the most severe welding effects occurred with the higher strength specimens. The steel of specimen group four, lacking an abrupt slope discontinuity in its stress-strain relation at yield, would not have been used in a shear wall test specimen. After consideration of these observations, it was decided that the effect of welding upon the yield and ultimate stresses of the steel would be ignored.

(d) Helical Reinforcement

Determining the properties of the steel used for "spirals" was complicated by the mechanical deformation that the material had been subjected to. The steel was received by the laboratory in a roll. The wire was unrolled and then deformed by machine into a helix with a nominal outside diameter of 0.875 in. and longitudinal spacing of 0.25 in. To obtain a measure of its mechanical properties, coupons from this batch of steel were tested as received and also after it was made into a helix (coupons were straightened in both cases before testing). For both cases the proportional limit was approximately

20,000 psi. Five samples of the wire as received indicated a mean yield stress of 41,400 psi (stress at approx. 0.04 strain). The corresponding value was 41,500 psi for four samples of the wire straightened from the helix.

A.3 Specimen Details

(a) Overall Configuration

Each test structure comprised two walls (Fig. A.17). Each wall comprised two piers interconnected by beams at six levels (A.17). The total beam depth was nominally 2.25 in. for test D1 and 1.5 in. for all other tests. All other nominal dimensions were identical for each test.

Each specimen was cast monolithically with a heavy base, as shown in the figures.

Holes were provided along the centerline of each pier at the levels of the second, fourth and sixth level beams. These facilitated connection of the weights as described in section A.4.

The overall placement scheme of the reinforcement in the specimens is shown in Fig. A.18. The reinforcing pattern will be described in detail in the next two parts for the piers (or the structural walls) and the connecting beams.

(b) Pier Reinforcement

The reinforcement of the pier was common to all specimens and was unchanged throughout the height of any given specimen. The nominal cross-sectional geometry is shown in Fig. A.19. The reinforcement consisted of six #11 wires uniformly spaced throughout the depth of the pier placed along the centerline of the small dimension of the pier.

This provided a steel ratio of 0.98%. Horizontal reinforcement was spaced at uniform intervals of one in. along the height of the piers, providing a steel ratio of 1.11%.

It was necessary that the piers be capable of developing their maximum flexural capacity at the base of the frame. The vertical pier steel had to be able to develop its ultimate stress at this location. To insure this, the vertical steel was welded to a steel plate imbedded in the base of the specimen (Fig. A.21).

(c) Beam Reinforcement

The cross-sectional geometry of the connecting beams was a major variable in the experimental study. The nominal cross-sectional dimensions for various tests are shown in Fig. A.20. Type A beams were used for the specimen for Test D1, Type B beams for the specimens for Tests D2 and D3, and Type C beams for the specimens for tests D4 and D5. Both the total beam depth and the reinforcement ratio were varied from test to test. The nominal reinforcement ratios, based on the total steel area and the gross area of the section, were 3.7% for Type A, 1.52% for Type B, and 0.88% for Type C. It should be noted, however, that within any given test specimen, all beams had identical nominal dimensions. It was desired that the connecting beams be capable of developing their maximum moment capacity in flexure. Hence, a major problem in designing the test specimen was to provide sufficient anchorage length for the longitudinal (flexural) steel in the beams to enable the beam steel to develop its ultimate stress at the face of the piers. Therefore, the beam longitudinal steel was spot welded to the vertical wall steel as shown in Fig. A.22.

The connecting beams were also provided with #13 closed stirrups with 14-diameter laps.

Initially, the transverse reinforcement ratio was computed to provide the shear strength necessary to resist the shear force corresponding to the attainment of the maximum moment capacity (based on strain hardening of longitudinal reinforcement) of the beams at the face of the piers. The contribution of concrete to shear strength was ignored. In this way it was intended to suppress a shear failure in the beam. The beams were designed to fail in flexure. However, the number of stirrups necessary to provide this condition would have left most of the length of the beam entirely unreinforced for shear. Therefore, additional stirrups were placed at a reasonable uniform interval as shown in Fig. A.20.

(d) Base Detail

The reinforcement details of the base of the specimen are shown in Fig. A.21. The longitudinal reinforcement was provided such that the base could resist, without cracking, the maximum overturning moment capacity of the frame of the specimen. The vertical steel of the piers was welded to the steel plate in the base. Steel tubing (Fig. A.21) provided vertical holes in the base to bolt the specimen to the platform of the earthquake simulator.

(e) Casting and Curing

The two walls for each test structure were cast simultaneously.

The concrete for both walls and for the cylinders and prisms was mixed in one batch in the laboratory. Proper placement of the concrete, including elimination of voids, was insured through the use of a mechanical stud vibrator. The vibrator was used inside the concrete for the base of the specimen and against the formwork for the frame of the specimen. Approximately one half hour after placement, the concrete was struck off and then finished with a metal trowel.

The walls were covered with plastic and allowed to cure overnight in the laboratory. Approximately 24 hours after casting, they were uncovered and the side forms were removed. The walls were then covered with wet burlap and plastic was placed over the burlap. Seven days after casting, the burlap and the plastic were removed. The walls were stored in the laboratory. The cylinders and prisms received the same treatment.

The details of the forms and the placement of the completed cages in the forms is shown in Fig. 23.

(f) Measured Specimen Dimensions

The measured dimensions of the specimens varied slightly from the nominal dimensions. This was due to the general level of accuracy inherent in fabrication and casting. Hence, actual specimen dimensions were recorded, including effective depths for the steel after each test.

Fig. A.24 shows the positions on the specimens for which measurements were taken. The pier and opening widths (dimensions A and B) were measured at two levels between each pair of connecting beams, for a total of 96 measurements of pier width and 48 measurements of opening width in a test structure. Results were also computed for the portion of the above sample taken below the level of the lowest level connecting beams (dimensions A1 and B1); a sample of 16 pier-width measurements and eight opening-width measurements. The steel placement in the pier (dimensions F1 thru F7) was measured at one section near the base of each pier, for a total of four samples per test structure for each of the appropriate dimensions. The opening height (dimension D) was measured at each end of each opening on each face of a wall, for a total of four samples per opening and 48 samples in a test structure. The pier thickness (dimension T) is measured at two positions across the width of each pier at the level of each connecting beam and at midheight between each pair of connecting beams, for a total of 96 samples per test structure. Results were also computed for the portion of this sample taken at the midheight between the base and the first level beam (dimension T1), for a total of eight samples per test structure. The beam section geometry (dimensions E,G,HT and HB) was measured at each end of each connecting beam, for a total (in each test structure) of 48 samples each of E and G and 24 samples each of HT and HB. Tables A.6 thru A.11 summarize the results.

A.4 Dynamic Tests

(a) The Earthquake Simulator

The dynamic test specimens were tested on the University of Illinois Earthquake Simulator. The overall test setup is shown in Figure A.25. A hydraulic ram of 75 kips capacity drives a 12 foot x 12 foot platform, providing one component of horizontal motion. The test specimen is attached to the platform. Both the ram and the platform are attached to the structural test floor of the laboratory.

The connection of the platform to the floor is such that no restraint is provided by the floor in the direction of ram motion. The frequency range of the simulator response is from zero to 100 Hz. The maximum single amplitude platform displacement is 2.5 in. The desired acceleration record for the test is input from magnetic tape. The record is integrated twice to produce a displacement record. A servomechanism then controls the hydraulic ram to reproduce the displacement record.

Further details about the earthquake simulator are given by Otani (1972); Sozen, Otani, Gulkan and Nielsen (1969); and Sozen and Otani (1970).

(b) Weights and Connections

The platform of the Earthquake Simulator is equipped with a rectangular pattern of 1/2" nominal diameter threaded holes 12 inches on centers. This pattern is used to fasten the test structure to the earthquake simulator. This is accomplished through bolts that pass with a loose fit through vertical holes in the base of the specimen and provide a vertical force in a steel plate which bears against the top surface of the specimen base. There were four such connections for each base. Sliding of the specimen bases with respect to the simulator platform in the direction of excitation is further prevented by large steel angles bolted to the platform hole pattern at each end of the bases. This can be seen in Fig. A.26.

The dead load of the structure is provided by 2000-pound steel weights placed at the levels of the second, fourth and sixth level connecting beams. Each weight transfers its load to the specimen at four points; one point along the vertical centerline of each pier. The connection is such that the weights offer no restraint to bending of the piers about the strong axis of the pier. Eight steel angles, two for each connection point, were bolted to the weight through their horizontal legs. A ball-bearing assembly was press fitted into each of the vertical legs. A bolt was passed through the center of the ball bearing fixtures and through a hole in the pier. Washers were placed between the inner ring of the ball bearing assembly and the surface of the pier, preventing the specimen from touching the connecting angle. The centerline of the hole in the pier corresponded to the centerline of the beam at that level. The detail of the connection is shown in Fig. A.27. The configuration of the weights is shown in Fig. A.26.

A major problem with the test structure for this study was that it possessed very little strength about its weak axis of bending. Without restraint of some nature, failure of the specimen might occur about this weak axis, aborting the experiment. To avoid this, steel diaphragms were provided in the direction perpendicular to the direction of motion of the simulator platform. The diaphrams were equipped with light hinges to prevent them from providing restraint in the direction of motion of the simulator platform. The placement of the diaphragms is depicted in Fig. A.28.

(c) Instrumentation

The instrumentation of the test set up consisted of differential transformers (LVDT) to measure deflections and accelerometers to measure accelerations. Accelerometers were attached to the east edge of the weights along the axis of the centerline of each of the two coupled shear wall frames to measure accelerations in the direction of motion of the simulator platform. AC-type differential transformers were attached in a similar orientation to the west edge of the weights. The differential transformers were mounted on a steel A-frame with a natural frequency of approximately 60 Hertz. Hence, the differential transformers measured deflections in the direction of motion of the simulator platform relative to the deflection of the simulator platform. An accelerometer was attached to each base to measure the base acceleration experienced by the specimen. Four DC type differential transformers, two for each base, monitored any vertical uplift of the bases. These were mounted on heavy steel fixtures which were bolted to the simulator platform. Each weight was also equipped with two accelerometers to measure vertical accelerations; one at the west edge of the weight and one at the east edge. For Test D1, these were attached at the centerline of the weights, an equal distance from each of the two test specimens. However, there was concern that vertical vibration of the weight as a beam supported at the two specimens might influence the acceleration at this point. Hence, in subsequent tests, these accelerometers were placed along the axis of the south specimen. The placement of the instruments is illustrated in Fig. A.29. Results obtained from accelerometers along the axis of the specimen were comparable to those obtained from the accelerometers located at the centerline of the weight.

(d) Data Recording

The voltage output of the differential transformers and the accelerometers was continuously recorded in an analog format on magnetic tape. This required a total of 24 channels on analog magnetic tape; a channel for each instrument. Three tape recorders were needed to accomplish the recording.

Since the test data as recorded on tape was purely in terms of voltages, a calibration was needed to facilitate conversion of the data to units that would be pertinent to a structural study; in essence, deflection and acceleration units. Before each test, calibrations were performed on both the accelerometers and the differential transformers. Differential transformers were calibrated by metal gage blocks machined to either 0.25 in. or 1.0 in. The accelerometers were calibrated against the Earth's gravitational field by placing them first vertically, then horizontally. The voltage outputs corresponding to these known instrument response levels were then recorded on the tape upon which the test data was to be recorded. This provided a comparison with the test data.

The data recording scheme is illustrated in Fig. A.31.

(e) Test Procedure

Immediately after the test specimen had been bolted to the platform of the earthquake simulator and the weights placed on the specimen any cracks in the specimen were recorded by marking on the specimen in colored pencil along the crack. These could have been incurred through either shrinkage or handling. The specimen was soaked with "Partek" P1-A Fluorescent (Magnaflux Corporation, Chicago, Illinois). The fluid was allowed to dry, and a black light was applied to the specimen. The fluid contained fluorescent particles and glowed when subjected to the black light. The greater fluid concentration in cracks caused the cracks to show as bright lines under the black light.

Next, the tightness of all bolts on the test setup were checked. This included the weights, the specimen base, and the instrumentation fixtures. The mounting and alignment of all differential transformers and accelerometers was rechecked. Finally, the mechanical calibrations were performed on the accelerometers and differential transformers. The direction of the calibration step and its magnitude in inches or G was recorded in a notebook.

The following sequence of operations was performed for each run of each dynamic test:

1) The tightness of the bolts fixing the specimen to the platform of the earthquake simulator was checked.
2) The simulator platform was displaced very gently to induce a low amplitude free vibration in the specimen.
3) The specimen was subjected to the desired earthquake base motion at the desired acceleration level.

4) During the run, the motion of the specimen was recorded by a video tape machine, and by several movie cameras
5) Immediately after the test, still pictures were taken of the condition of the specimen. Special attention was given to any wide, visible cracks and to any spalling of the concrete.
6) Notes were made of the nature and general distribution of the damage sustained in the run.
7) The specimen was soaked with "Partek" P-1A Fluorescent and any new cracks were marked.

After conducting the entire dynamic test, as described above, the weights were removed from the specimen. The crack pattern was sketched on paper in colored pencil, different colors denoting the results of different runs. The crack pattern on each specimen was then darkened in stages with a magic marker, allowing the crack pattern at the end of each test run to be photographed.

A.5 Reduction of Dynamic Test Data

The data, as obtained in the test, consisted of a series of instrument responses in voltage units for various times. These were recorded on magnetic tape in analog format. For purposes of reporting and interpretation the data was needed in the form of plots of acceleration-time relations or displacement-time relations. The variation of base shear and base overturning moment with time was also required. Finally, the elastic response spectra for the measured base motions were needed.

The analog records were converted into digital records using the Spiras-65 computer of the Department of Civil Engineering. These were also placed on magnetic tape. The digitization rate was 1000 points per second. These tapes were then copied on the Burroughs 6700 computer of the Department of Civil Engineering to enable them to be used on the IBM 360-75 computer of the Digital Computer Laboratory of the University of Illinois.

The next step involved the determination of the calibration factors and zero levels for the data. The calibration steps recorded on tape were read by a computer program in terms of voltage units. By knowing the instrument response in terms of acceleration or displacement that these calibration steps corresponded to, the appropriate calibration factors for the data were computed. By reading the portion of the data record immediately before the onset of the earthquake, the same computer program obtained the zero levels for each gage response in voltage units.

A second computer program was used to process the data into its final form for permanent storage on magnetic tape. The organization of the data was altered, to place it into the form of a series of response-time relations. The previously obtained zero levels and calibration factors were also applied to the data. The data was then in the form of a series of time histories in the units of either inches or G.

A computer program was also written to compute the base shear-time relations and the base overturning moment-time relations. The base shear and base overturning moment were computed directly at each time from the measured acceleration response at the appropriate time at the level of each of the three weights and the measured mass of each weight.

Two computer programs were written for the purpose of plotting response-time relations. One routine plotted the relations three curves to a page and was used to plot large quantities of data for purposes of comparison. Another routine could plot any portion of one curve to any time scale and response scale desired. This was useful for close examination of a specific relation and taking measurements from a plot.

A computer program was also directed toward computing the response spectra for the base acceleration-time relations measured in the tests. The program used a numerical approach to compute the response of a single degree of freedom system to the measured acceleration record, considering linearly elastic response. The spectra were plotted in tripartite form and in a linear form.

A final program was written to integrate any response-time relation either once or twice. For example, it would be possible to compute the displacement-time relations kinematically consistent with the measured acceleration-time relations.

The data reduction process is illustrated by a chart in Fig. A.32.

A.6 Static Tests

(a) Loading Method

A drawing of the static test setup is shown in Fig. A.33. The specimen is mounted on the platform of the earthquake simulator. This is to insure that the static test includes the same base conditions as the dynamic tests. Loading is accomplished by three hydraulic two-way servorams each of 20 kips capacity. The rams were bolted to a steel A-frame and applied their loads to the steel weights, placed at the levels of the second, fourth, and sixth beams. The rams can deflect the specimen to the east through a direct connecting rod. Four 0.5 in. nominal diameter rods passing through the entire weight system and bearing on the east edge of the weights provide for westward deflection.

Some comments should be made concerning the manner in which the application of load in the experiment was controlled. Built into each ram assembly was a load cell and a differential transformer. The top ram was operated by controlling deflection. The ram would continue to apply load until its differential transformer sensed a certain preset limiting deflection, at which time the ram would stop and maintain its deflection. The two lower rams were operated by controlling load. The rams would continue to apply load until their load cells sensed a certain preset fraction of the load in the top ram. In this way, a certain predetermined ratio was maintained among the three ram loads and the test was conducted by applying predetermined increments of top-story deflection.

(b) <u>Loading Pattern</u>

Since a major objective of the static test was to measure hysteresis relations that would be applicable to the results of the dynamic tests, it was considered appropriate that the ratios among the applied lateral loads reflect the ratios present in the dynamic tests. Since the failure of the specimens was a flexural type failure, it was reasonable that the load or acceleration ratios predominant during periods of high base overturning moment should be used. Examination of base overturning moment-time relations for the dynamic tests indicated that during these periods of large amplitude response, the first mode of the specimen was predominant. Hence, the ratio among the applied loads was chosen to correspond to the shape of the first mode of the specimen. The loading pattern is depicted in Fig. A.33.

(c) Weights and Connections

The dead load for the static test specimen was simulated using the same weights that were used in the dynamic tests. The features of the weights and the weight-to-specimen connections were the same as described for the dynamic tests.

(d) Instrumentation

The instrumentation for the test is shown in Fig. A.34. Deflections were measured by mechanical dial gages of 0.0001-in. accuracy and by differential transformers. A differential transformer was built into each hydraulic ram, measuring deflection of the weight at the point of load application. Six AC-type differential transformers measured lateral deflections of the weights. There were also two dial gages measuring lateral deflections at the same levels. The dial gages, however, beared directly upon the east edge of the specimens, rather than upon the weights. Two dial gages were mounted on steel fixtures bolted to the platform of the earthquake simulator and measured east-west sliding of each of the two specimen bases. Four dial gages were mounted on wood fixtures bolted to the platform of the simulator and monitored uplift of the specimen bases. The load in each of the three hydraulic rams was measured by load cell built into the ram assembly.

(e) Data Recording

All load cell and differential transformer responses were recorded in analog format on magnetic tape during the test. Automatic plotting instruments provided a continuous plot, in ink, of ram load cell reading and ram differential transformer reading for each of the three hydraulic rams. Mechanical dial gage readings were recorded manually on paper. Fig. A.36 illustrates the data recording scheme.

(f) Test Procedure

Before the beginning of the test, calibrations were performed for the differential transformers in a manner identical to that described for the dynamic tests. The crack pattern for the specimen was also marked before the test; again in a manner identical to that for the dynamic tests.

In the earliest stage of the test the top-story deflection was applied in a step by step manner in small increments. The increments were initially in the range of 0.002 in. and were gradually increased into the range of 0.07 in. After each increment of deflection, all instrument readings were recorded, as described in part (e) of this chapter.

When a top-story deflection of approximately 0.3 in. was attained, the mechanical dial gages were removed from the test setup. The top-story deflection was increased continuously until a deflection of 0.56 in. was attained. The loading rate was 200 seconds/cycle. At this point the direction of deflection application was reversed and the loading rate for the test was increased to 100 seconds/cycle. The specimen was then subjected to several cycles of loading, the maximum deflection being increased for each successive cycle. After removal of the dial gages, load cell and differential transformer readings were continuously recorded as described in part (e). Notes were taken during the test concerning the onset of large cracks, spalling of concrete and other major behavior phenomena. Cracks were marked at the conclusion of the test.

After the test the weights were removed and the crack pattern was recorded on a sketch. Then the cracks on the specimen were marked over with felt-tip pen and photographed.

APPENDIX B

COMPUTER PROGRAMS FOR MOMENT, AXIAL LOAD AND CURVATURE

This appendix describes the computer programs written to calculate the moment-curvature relations and the moment-axial load interaction relation. The program was written in the Fortran IV language for the IBM 360/75 computer of the Digital Computer Laboratory of the University of Illinois.

The programs were written for a rectangular section consisting of several district piers with the reinforcement concentrated in any number of layers. The various assumptions are as described in Chapter 4 (Fig. 4.7). The procedure for the primary calculations was that outlined in equations 4.32 through 4.40.

The input data for the moment-curvature program consisted of section dimensions, steel area for each layer, and stress-strain parameters for both concrete and steel, along with a set of axial loads, P, and maximum concrete compressive strains, ε_{cm}. The program then computed a moment-curvature relation for each axial load, P. To define each curve, a calculation was performed at each value of ε_{cm}. The output, for each point of moment-curvature relation, consisted of moment about the plastic centroid, M, curvature, ϕ, corresponding neutral axis depth, c_o, maximum concrete strain, ε_{cm}, and strain at the level of each reinforcement layer. The input data for the moment-axial load interaction program consisted of section dimensions, steel area for each layer, stress-strain parameters, and a set of maximum compressive concrete strains, ε_{cm}. An interaction diagram was then computed for each value of ε_{cm} provided. The program was equipped with an algorithm to determine the axial loads at which points on the moment-axial load interaction diagram were to be computed. The output, for each point on each interaction diagram, consisted of moment about the plastic centroid, position of the neutral axis, maximum compressive strain in the concrete, and strain at the level of each reinforcement layer.

Fig. B.1 and B.2 provide flowcharts for the moment-curvature relation program and the moment-axial load interaction program, respectively. The two programs contained an identical "core" routine which, provided with values for maximum compressive concrete strain, ε_{cm}, and axial load, P, computed the neutral axis location, c_o, and the moment, M, about the plastic centroid. This routine included, with the exception of the calculation of the plastic centroid, the calculation routine described by equations 4.35 through 4.40. A flowchart is provided in Fig. B.3.

APPENDIX C

COMPUTER PROGRAM FOR STATIC ANALYTICAL MODEL

C.1 General Comments

The purpose of this appendix is to describe the computer program that was developed to perform the calculations for the static analytical model presented in Chapter 5. The program was named STAT and was written in the version of the BASIC language used on the DEC System 10 computer of the Digital Computer Laboratory at the University of Illinois.

In a general sense, the input consisted of the applied lateral loading and the distribution of member stiffnesses throughout the system (Fig. 5.1). As described in Chapter 5, a piecewise linear analysis was performed, the stiffnesses remaining constant during any given step. In this manner a lateral load-lateral deflection relation for the structure was developed. The program was designed to operate in an interactive fashion: the program stopped and asked the user for input data at the beginning of each step. Hence, the program was independent of any specific hysteresis relation and the user could begin or conclude analysis at any stage of loading.

In practice, the lateral loading was applied to the structure in stages, the system responding linearly in each stage. Consider a typical loading increment. At the beginning of the increment, the structure was under some set of external forces, joint deflections, and member forces. These were the initial responses. To apply the increment, the user first input the values of the lateral loads. The direction of the loading increment was determined by the sign of the loads. The magnitudes did not matter, it was important only that the loads be in the proper ratio.

A uniform section stiffness was input for each member. The program analyzed the structure with this data to obtain a set of joint deflections and member end forces. These were referred to as the unfactored incremental responses. Also input was a set of critical responses. These were values of member forces or joint deflection which the user did not want to exceed in the increment. The program multiplied the unfactored responses by a modification factor chosen such that when the above product was added to the initial responses, none of the critical responses was exceeded. The resulting set of responses was the new set of total responses, and became the set of initial responses for the next loading increment. A series of such loading increments would constitute an analysis.

By applying the proper sequence of member stiffnesses and critical responses, and by reversing the signs of the lateral loads at the proper loading increments, the user could subject the structure to virtually any hysteresis relation he desired.

C.2 Programming Scheme

A flowchart for STAT is shown in Fig. C.1. The program consisted of the following:

(1) A series of input statements which received the data for a given step in the analysis. This included the uniform section stiffness for each member, the ratio of the applied lateral loads, and a set of critical responses, the attainment of any one of which caused the termination of the loading step.

(2) A computation routine which assumed linear response throughout the system and analyzed the system for the stiffnesses

and loads input in part (1) above. Note that the loading used was <u>numerically equal</u> to that input in part (1). Hence, the resulting member forces and joint displacements needed to be multiplied by a factor to satisfy the constraints of the critical responses.

(3) A routine to compute a modification factor for the results of part (2). The factor was computed such that none of the critical responses input in part (1) was exceeded. What was obtained after multiplying the factor times the results from part (2) was the largest load step which would exceed none of the critical responses input.

(4) A data file on magnetic disk (DATA5) which was used to store the total response between loading steps. At the conclusion of a step, the file was erased and, the new total responses were written in the file. The next step read the file and used the contents for its initial responses.

(5) A second data file on magnetic disk (DATA4), which contained the values of certain responses at the end of the most recent step. For each loading step, the program erased the content of this file and wrote the new total responses onto it.

(6) An output data file (DATA1), again on magnetic disk onto which the program wrote the final responses for each loading stage. Results were accumulated in this file as load increment after load increment was applied. This was the permanent record of the results of the analysis.

(7) A series of output statements that displayed on the screen of the cathode ray tube (CRT) the input data as understood

by the program and told the user, for each step, which critical response constrained the step.

(8) Two decision points at which the user first decided whether or not to record a given step and then decided whether or not to terminate the program.

In addition to the main program, there was a small auxiliary program called ZERO5. This program wrote data onto the file DATA5 described above. It had to be run at the beginning of each analysis, and was used to initialize this data file. This also enabled the user to begin an analysis from any intermediate point in a hysteresis relation, he could write any set of responses he wished onto DATA5. These would then be the initial responses for the first step of the analysis.

C.3 Operation of STAT

The input data for each step was entered in response to a series of questions displayed on the CRT. The following is a list of the questions posed by the program for each loading step and the format in which the user answered them. Reference to Fig. 5.1 will clarify the explanations.

(1) "THE LATERAL LOAD INCREMENTS IN KIPS ARE?"

Six joint loads were typed in, starting with the first story level and proceeding upward to the top story. (Zero load input is permitted). If the values entered were positive, the loads were applied toward the right as shown in Fig. 5.1. If the signs were negative, the loading was in the opposite direction. The absolute values of the loads were not important. It was important only that they be in the proper ratios to each other.

(2) "THE VALUES OF EI FOR EACH MEMBER IN KIP-IN2 ARE?"

The uniform section stiffnesses for the step for both beams and piers, were entered.

(3) The program now echoed the data as input in parts (1) and (2).

(4) "THE CRITICAL MOMENTS ARE?"

The critical responses used to limit the step size were entered through the CRT at this time. The parameters considered were the end rotation for each beam and the top level deflection. Hence, a total of seven critical responses were entered.

(5) The program echoed the critical responses. At this point the program performed the structural analysis, and computed and applied the modification factor, f_{min} (Fig. C.1).

(6) The program echoed back the number of the step and which critical parameter constrained the step size.

(7) "DO YOU WANT TO RECORD THE STEP?"

Typing "YES" on the CRT caused the results to be written in Files DATA1, DATA4, and DATA5. The program then proceeded to part (8). Typing "NO" caused the program to discard the step; it branched back to part (1) of this section to redo the step.

(8) "DO YOU WANT TO TERMINATE?"

Typing "NO" caused the program to branch back to part (1). The user would then proceed to enter the next load step. Typing "YES" caused the program to terminate.

APPENDIX D

COMPUTER PROGRAMS FOR STUDY OF DYNAMIC RESPONSE

D.1 General Comments

This appendix describes the computer programs written to perform the calculations for the analysis presented in chapter 7. This section will give an introduction to the format of the programs; the next section will describe the various parts of the main calculation program, and the final section will describe a second computer program, written to compute the response histories of base shear and base moment from the results of the first program.

Both programs were written in the FORTRAN IV language for the IBM 360/75 computer of the Digital Computer Laboratory at the University of Illinois. The programs received input from punched cards and magnetic tape and produced output on line printer and magnetic tape. The main calculation program could operate in two basic capacities. In one capacity, it performed a modal analysis for a structure with a given set of section stiffnesses for the members (Fig. 7.2). The program was designed to handle several sets of section stiffnesses (several distinct analysis cases) successively, in a single run of the program. The program calculated the natural frequencies, mode shapes, and modal participation factors for the first three response modes of the system. In its second capacity, in addition to calculating the modal parameters, the program calculated the response histories for the structure, in terms of horizontal acceleration and horizontal displacement at the levels of the three masses (Fig. 7.2).

A base acceleration history was provided as input. As for the first operational capacity, results could be computed successively for several sets of section stiffnesses for the various members and several sets of viscous damping factors for the various models. The response histories were output to magnetic tape, where they were stored in the same format as the observed responses. The analytical results could then be plotted using the same plotting routines as for the observed response.

The program was written as a series of subroutines, each performing a specific subtask in the analysis. Section D.2 will present the flow of calculations in the program and briefly describe each subroutine.

Section D.3 describes an auxiliary program which read the displacement response histories computed by the main calculation program, from magnetic tape, and used them to compute the response histories for base shear and base moment. These results were stored on magnetic tape, as for the other calculated response histories.

D.2 Main Calculation Program

(a) Introductory Remarks

The flowchart for the main calculation program for the study of dynamic response is provided in Fig. D.1. The names of the subroutines performing the various operations in the flowchart are denoted either at the upper left corner of the block for an operation, or at the upper left corner of a dashed block, enclosed several operations in the same subroutine.

The following paragraphs will briefly describe what each subroutine did along with explaining the flowchart.

(b) Control Routine

This was the main or core routine. It called other routines and received punched card input containing control information, such as whether a full response history analysis, or only a modal analysis, was to be performed, and how many analyses (sets of structural properties), $\overline{N}$, were to be performed. When all cases were analyzed it terminated execution of the program.

(c) Input Routine (INCRD)

This routine was called by the core routine and received input on punched cards. The input included a uniform flexural section stiffness for each beam, uniform axial and flexural section stiffnesses for each story of the pier, story heights, total depth of beams and piers, the number of response modes to be considered in the analysis, and, if a response history analysis was to be performed, the viscous damping factors to be used. The data input was output to line printer.

(d) Assembly of Stiffness Matrix for Structure (STIFF)

Because the program was to be used for only one general structural configuration, the program did not synthesize the stiffness matrix for the structure from stiffness matrices for the members. The coefficients for the structure stiffness (18 x 18 matrix) were directly derived in terms of member section stiffnesses, member lengths and member depths. The depths were necessary due to the consideration of finite joint sizes, as discussed in section 7.2. The information received by subroutine INCRD was then used by subroutine STIFF to compute the stiffness matrix for the structure.

The degrees of freedom considered in the matrix were the horizontal displacement, vertical displacement, and rotation for each of the six beam-pier joints, as described in chapter 7.

(e) Condensation of Structure Stiffness Matrix (CNDNSE)

The stiffness matrix for the structure was condensed from the 18 x 18 format of subroutine STIFF to a 6 x 6 format, as described in appendix F (Equations F.2 through F.5). The degrees of freedom in the 18 x 18 matrix, which did not correspond to mass in the test structures, were eliminated. The degrees of freedom for the 6 x 6 matrix included the horizontal and vertical displacements at the beam-pier joints of the second, fourth and sixth level beams.

(f) Assemble Mass Matrix for Structure (MASS)

The mass matrix for the structure was assembled directly from the lumped mass considered for each of the six degrees of freedom of the condensed structure stiffness matrix. This involved horizontal and vertical inertia for each of the three appropriate joints.

(g) Modal Analysis (MODAL)

Using the 6 x 6 stiffness and mass matrices for the structure, a modal analysis was performed. A first approximation to the mode shapes and natural frequencies was obtained using the routine EIGENZ of the IBM Scientific Subroutine Package. As described in section F.4, the results provided by this routine were of insufficient accuracy, due to poor matrix conditioning induced by the axial deformations considered for the pier members. An iterative improvement technique (ref. 30) was employed, which used each approximation for mode shapes and frequencies to obtain a better approximation. The resulting loop is shown in the flowchart (Fig. D.1). After each iteration through the improvement technique, the approximation to the first mode frequency after the iteration, $\overline{f}_j$, was compared with that before the iteration, $\overline{f}_{j-1}$. When the percent change in the frequency was within a certain tolerance, the results for the frequencies and mode shapes were accepted and transferred to the control routine.

(h) Printed Output (OUTPRT)

This routine sent the mode shapes, frequencies and modal participation factors, for the number of response modes desired, to line printer for output.

(i) Decision Point

If the data input to the control routine at the beginning of the program indicated that only a modal analysis was to be performed, a check was made to ascertain if all cases for modal analysis had been executed, in essence, if the counter, $\overline{I}$, was equal to the number of sets of structural parameters to be processed, $\overline{N}$. If not, the program branched back to perform the analysis for the next set of structural properties, beginning with subroutine INCRD. If all cases had been processed, execution terminated.

(j) Numerical Integration (PRPG)

If the data input at the start of the program indicated that a complete response history analysis was to be performed, the numerical integration of the equations of motion was performed at this stage.

The base acceleration record to be used as loading for the analysis was read from magnetic tape. This was the function a_b, in Equation 7.19, defined at a number of discrete times.

Step-by-step numerical integration was performed, to solve the equation of motion for the single degree of freedom system corresponding to each response mode to be considered in the analysis. This class of equations is represented by Equation F.14 and the solution procedure was that described in section F.5. The results of the analysis were the values of $\ddot{\xi}(t)$ and $\xi(t)$ (Equation F.14) for each response mode considered, at each of the discrete times at which the base acceleration was provided.

(k) Structure Response Histories (OUTTP)

The mode shapes, modal participation factors, and frequencies computed in subroutine MODAL were transmitted from the core routine to subroutine OUTTP, along with the single degree of freedom responses ($\xi(t)$ and $\ddot{\xi}(t)$) at the various discrete points in time. For each call to subroutine OUTTP, several response histories were computed considering the various discrete points in time, using the appropriate modal participation factors, mode shape values, and single degree of freedom response histories (Equations F.7 and F.9). There were four calls to subroutine OUTTP for each analysis case. During the first call, the response histories for horizontal acceleration at each of three levels, for response in the first mode and response in the second mode, a total of six response histories, were computed and stored on magnetic tape. During the second call, the same was done for the response histories for horizontal displacement, again, a total of six response histories. During the third call to OUTTP, the response histories for horizontal acceleration at each of three levels, for the sum of the first and second response modes, was computed and stored on magnetic tape. During the fourth call to OUTTP, the same was done for the horizontal displacements. The calculation of base shears and moments will be discussed in section D.3.

(l) Decision Point

The counter, $\overline{I}$, was compared to the number of sets of structural parameters, $\overline{N}$, for which response history analyses were to be performed. If all sets of parameters had been processed ($\overline{I} = \overline{N}$), execution of the program terminated. If not, flow branched back, to perform the analysis for the next set of structural parameters, beginning with subroutine INCRD.

D.3 Program for Base Shear and Base Moment

(a) Introductory Remarks

As mentioned in section D.1, a second computer program was written to compute the response histories for base shear and base moment, using, as input, the response histories for displacement computed in the main calculation program. The following paragraphs describe the flow of the program.

(b) Description of Program

The flowchart for the program is given in Fig. D.2. The program could, in one run, compute the base shear and base moment response histories for several sets of member section shiftnesses and viscous damping factors, as could the main response history program (section D.2). The number of cases to be considered was input on punched cards as an integer, $\overline{N}$.

In the next step, additional punched card input was received. This included the first and second mode frequencies, the story heights, and the mass matrix (3 x 3) for the structure. The degrees of freedom for the mass matrix were the horizontal displacement at each of the three levels in the structure corresponding to lumped mass. The response histories for displacements at each of the three levels, for the first and second response modes, were input from the magnetic tape on which they were stored by the main calculation program (Section D.2).

The response histories for base shear were computed in a point-by-point manner from the response histories for displacement, as described in section F.6. The calculations were performed, first, considering only the first response mode, then considering only the second response mode. Finally, the responses for the two modes were added on a point-by-point basis, obtaining the results for the sum of the two modes. Each response history was stored on magnetic tape. A similar procedure was followed for the base moment.

The counter, $\overline{I}$, was incremented by one, and the result compared with the number analysis cases, $\overline{N}$, to be considered. If all cases had been analyzed, execution of the program terminated. If there were cases yet to be analyzed, the flow branched back to receive data from punched cards and magnetic tape for the next analysis case.

APPENDIX E
FOURIER ANALYSIS THEORY

E.1 General Comments

This appendix presents a description of the Fourier analysis method used to identify the relative contributions of the various modes of response present in the observed response histories from the dynamic tests. The material may be found in greater detail in Clough (ref. 6). The results obtained in the analysis are presented in chapter 6.

E.2 Fourier Analysis

(a) General Concept

The objective of the analysis was to consider a given response history and to determine the portion of that response history attributable to various frequency domains. This was accomplished by deriving a function $\overline{w}(\omega)$, which, for the response history, w(t), described the relative importance of various frequency levels as a continuous function of the circular frequency, ω. The transformation necessary to obtain $\overline{w}(\omega)$ from w(t) was such that the same transformation could be used to obtain the function w(t), from the function $\overline{w}(\omega)$. By applying the transformation to $\overline{w}(\omega)$ over the interval ω_0 to ω_f, the portion of the response history, w(t), associated with frequencies in the interval ω_0 to ω_f was obtained.

(b) Formulation for Periodic Functions

An arbitrary response history, if it is assumed harmonic, may be expressed as a summation of sine and cosine functions of time. For a response history w(t),

$$w(t) = Y + \sum_{i=1}^{\infty} Y' \cos \frac{2\pi i}{T_p} t + \sum_{i=1}^{\infty} Y'' \sin \frac{2\pi i}{T_p} t \tag{E.1}$$

where T_p is the period of the lowest mode of harmonic response. From the orthogonality properties of the sine and cosine functions, the constants Y, Y' and Y" may be evaluated as,

$$Y = \frac{1}{T_p} \int_0^{T_p} w(t)\, dt \tag{E.2}$$

$$Y' = \frac{2}{T_p} \int_0^{T_p} w(t) \cos \frac{2\pi i}{T_p} t\, dt \tag{E.3}$$

$$Y'' = \frac{2}{T_p} \int_0^{T_p} w(t) \sin \frac{2\pi i}{T_p} t\, dt \tag{E.4}$$

The above relations, however, are somewhat complicated. A more concise form can be established through the use of complex numbers. The trignometric functions may be expressed in complex form through the relations,

$$\sin \theta = - \frac{z}{2} (e^{z\theta} - e^{-z\theta}) \tag{E.5}$$

$$\cos \theta = \frac{1}{2} (e^{z\theta} + e^{-z\theta}) \tag{E.6}$$

where e is the base of the natural logarithm and z is the complex variable,

$$z = \sqrt{-1} \tag{E.7}$$

Equations E.5 and E.6 are obtained from the power series expansions for the sine, and cosine, and exponential functions (Kaplan pp. 359, 368).

By applying Equations E.5 and E.6 to Equation E.1, w(t) is obtained in complex form as,

$$w(t) = \sum_{i=-\infty}^{\infty} Z_i e^{zi\omega_1 t} \tag{E.8}$$

where ω_1 is the circular frequency for the first mode. In essence,

$$\omega_1 = \frac{2\pi}{T_p} \tag{E.9}$$

Also,

$$Z_i = \frac{1}{T_p} \int_0^{T_p} w(t) e^{-zi\omega_1 t} \, dt \tag{E.10}$$

The details of the development of Equation E.8 and E.10 from Equations E.1, E.5 and E.6 is described in greater detail by Kaplan (pp. 433-435).

(c) Extension to Nonperiodic Functions

As they appear above, Equations E.8 and E.10, apply only to a periodic function, w(t). It is desired to extend these relations to nonperiodic functions, w(t). In Equations E.1 and E.8, a summation over various discrete frequency components is being taken. A factor Z_i is defined for each frequency level. Consider a function $\overline{w}(\omega)$, given by,

$$\overline{w}(\omega_i) = T_p \, Z_i \tag{E.11}$$

Assume, the frequencies, ω_i, used in the summation to occur at increments of frequency, $\Delta\omega$, such that,

$$\Delta\omega = \omega_1 \tag{E.12}$$

Hence,

$$\omega_i = i\omega_1 \tag{E.13}$$

From Equation E.9,

$$T_p = \frac{2\pi}{\Delta\omega} \tag{E.14}$$

Equations E.8 and E.10 become,

$$w(t) = \frac{\Delta\omega}{2\pi} \sum_{i=-\infty}^{\infty} \bar{w}(\omega_i)e^{z\omega_i t} \tag{E.15}$$

and,

$$\bar{w}(\omega_i) = \int_{\frac{-T_p}{2}}^{\frac{T_p}{2}} w(t)e^{-z\omega_i t}\, dt \tag{E.16}$$

Nonperiodic response is accounted for by allowing the period of the periodic response to approach infinity ($T_p \rightarrow \infty$). In addition the frequency increment, $\Delta\omega$, for the summation was assumed to become infinitesimally small ($\Delta\omega \rightarrow d\omega$). In essence, $\bar{w}(\omega)$ becomes a continuous function, rather than one defined only at several discrete frequencies, ω_i. Hence Equations E.15 and E.16 become,

$$w(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \bar{w}(\omega)\; e^{z\omega t}\; d\omega \tag{E.17}$$

$$\bar{w}(\omega) = \int_{-\infty}^{\infty} w(t)e^{-z\omega t}\; dt \tag{E.18}$$

Equations E.17 and E.18 are referred to as a Fourier Transform Pair. The reciprocal nature of the functions $w(t)$ and $\bar{w}(\omega)$ is to be noted. By applying the Fourier Transformation of Equation E.18 to $w(t)$, the function $\bar{w}(\omega)$ is obtained. By applying the Fourier Transformation of Equation E.17 to the function $\bar{w}(\omega)$, the original function, $w(t)$ is obtained again. It is also important to note that the magnitude of the function $\bar{w}(\omega)$ for a specific frequency, ω', represents a weighting factor for the contribution of the frequency ω' to the function $w(t)$. In other terms, the variation of the integral of Equation E.17 for an infinitesimal increment of frequency is given by,

$$dw(t') = \frac{1}{2\pi}\,\bar{w}(\omega)\;e^{z\omega t'}\;d\omega \tag{E.19}$$

where t' is a specific time for which $w(t)$ is being computed. The result, $dw(t')$, represents the increment of response at time, t', due to frequencies within the frequency increment, $d\omega$. Hence, if one needed to compute the portion of the response, $w(t)$, due to frequencies in the domain ω_0 through ω_f, one need only change the limits of integration in Equation E.17 to ω_0 and ω_f, rather than $-\infty$ and $+\infty$. Similarly, a practical function, $w(t)$, is nonzero over a finite time interval, $t = 0$ through $t = t_f$. Hence, given a response history, $w(t)$, of duration, t_f, the function $\bar{w}(\omega)$ would be computed from,

$$\bar{w}(\omega) = \int_0^{t_f} w(t)e^{-z\omega t}\,dt \tag{E.20}$$

To determine the portion of $w(t)$, $w^*(t)$, at a given time, t', associated with frequencies in the domain ω_0 through ω_f, the following transformation would be performed,

$$w^*(t') = \frac{1}{2\pi}\int_{\omega_0}^{\omega_f} \bar{w}(\omega)\;e^{z\omega t'}\;d\omega \tag{E.21}$$

(d) Application to Test Data

Equations E.20 and E.21 represent the basis for the Fourier Analysis of the dynamic test results. In practice, the integrals of Equations E.20 and E.21 are evaluated numerically. The equations are discretized into summations and the functions $\overline{w}(\omega)$ and $w^*(t)$ are computed for an array of discrete values of circular frequency and time. Referring to Equation E.20, the interval of response, t_f, is divided into N time increments of magnitude, Δt. The i^{th} discrete time is given by,

$$t_i = i(\Delta t) \tag{E.22}$$

Similarly, the frequency is discretized by intervals, $\Delta\omega$, such that,

$$\omega_j = j(\Delta\omega) \tag{E.23}$$

Hence, the value of $\overline{w}(\omega)$ at the circular frequency, ω_j, is computed from,

$$\overline{w}(\omega_j) = \sum_{i=0}^{N} w(t_i)\, e^{-z\omega_j t_i}\, \Delta t \tag{E.24}$$

After Equation E.24 is evaluated for each value of the index j, the function $w^*(t)$ is computed at several values of time, t_i, as follows,

$$w^*(t_i) = \frac{1}{2\pi} \sum_{j=j_0}^{j_f} \overline{w}(\omega_j)\, e^{z\omega_j t_i}\, \Delta\omega \tag{E.25}$$

where the index values j_0 and j_f are those corresponding to the circular frequencies ω_0 and ω_f, respectively.

In executing the Fourier analysis of the test results a computer program was utilized that arranged the numerical integration computations in a highly efficient form known as the Fast Fourier Transform. The details of this arrangement of the computations will not be discussed here. A brief introduction to the Fast Fourier Transform is given in the text by Clough and Penzien (pp. 114-115).

In performing the analysis, it was deemed appropriate to ascertain the accuracy of the numerical integration procedure used (Fast Fourier Transform). The integration to obtain $\overline{w}(\omega)$ was performed as in Equation E.20. The integration of Equation E.21 was, then performed, but over a wide frequency range, rather than only from ω_0 through ω_f. The result was compared to the original response history, w(t), and correlation was satisfactory.

The calculations, for the entire Fourier analysis were performed on the IBM 360/75 computer of the Digital Computer Laboratory of the University of Illinois.

APPENDIX F

ANALYTICAL MODEL FOR STUDY OF DYNAMIC RESPONSE

F.1 General Comments

This appendix describes the calculation methods used for the study of dynamic response, as presented in chapter 7. The analysis was performed for the structure shown in Fig. 7.2. The structure was completely linearly elastic, with various other idealizations, as described in section 7.2. The analysis procedure, to obtain response histories, was one of modal analysis, with a response history being computed for the resulting single degree of freedom system for each response mode. Modal superposition was then applied, to obtain the response histories for deflections, base shears and base moments for the structure.

Subsequent sections of this appendix describe various portions of the analytical procedure.

F.2 Stiffness Matrix

In its most general form, the equation of motion for the structure, in matrix format, could be expressed by,

$$[\bar{K}] \{\bar{U}\} = \{\bar{P}\} \tag{F.1}$$

where $[\bar{K}]$ represents the 18-degree-of-freedom stiffness matrix (three degrees of freedom at each of six beam-pier joints), $\{\bar{U}\}$ represents displacements relative to the base for all 18 degrees of freedom and $\{\bar{P}\}$ represents a load vector consisting of external loads at the joints. For seismic response, these loads were solely the inertial loads due to the idealized mass distribution for the structure. However, the structure idealization, as described in section 7.2, was such that there was mass associated with only six of the 18 degrees of freedom. These six degrees of freedom were the horizontal and vertical displacements at the beam-pier joints for the second, fourth and sixth level beams. The members of $\{\overline{P}\}$ corresponding to the other 12 degrees of freedom were zero. This observation was used to reduce the analysis problem to one with six degrees of freedom.

By partitioning $[\overline{K}]$, Equation F.1 became,

$$\left[\begin{array}{c|c} \overline{K}_1 & \overline{K}_2 \\ \hline \overline{K}_3 & \overline{K}_4 \end{array}\right] \left\{\begin{array}{c} \overline{U}_1 \\ \hline \overline{U}_2 \end{array}\right\} = \left\{\begin{array}{c} \overline{P}_1 \\ \hline \overline{P}_2 \end{array}\right\} \tag{F.2}$$

where $\{\overline{P}_1\}$ contained the six nonzero inertial load terms, $\{\overline{P}_2\}$ was a vector of zeroes, and the dimensions of $[\overline{K}_1]$ $[\overline{K}_2]$, $[\overline{K}_3]$, $[\overline{K}_4]$, $\{\overline{U}_1\}$, $\{\overline{U}_2\}$,$\{\overline{P}_1\}$ and $\{P_2\}$were 6 x 6, 6 x 12, 12 x 6, 12 x 12, 6 x 1, 12 x 1, 6 x 1, and 12 x 1, respectively. From Equation F.2,

$$[\overline{K}_3]\ \{\overline{U}_1\} + [\overline{K}_4]\ \{\overline{U}_2\} = \{0\} \tag{F.3}$$

Combining Equations F.2 and F.3, the relation,

$$\{[\overline{K}_1] - [\overline{K}_2]\ [\overline{K}_4]^{-1}\ [\overline{K}_3]\}\ \{\overline{U}_1\} = \{\overline{P}_1\} \tag{F.4}$$

was obtained. Equation F.4 represented the equation of motion in terms of a condensed (6 x 6) stiffness matrix, corresponding to a six degree of freedom system. Hence, the stiffness matrix for the six degree of freedom substitute structure was obtained from,

$$[K] = [\overline{K}_1] - [\overline{K}_2]\,[\overline{K}_4]^{-1}\,[\overline{K}_3] \tag{F.5}$$

F.3 Equations of Motion

The equation of motion for the six degree of freedom substitute structure was expressed in matrix form as,

$$[M]\,\{\ddot{\Delta}\} + [C]\,\{\dot{\Delta}\} + [K]\,\{\Delta\} = -[M]\,\{A_b\} \tag{F.6}$$

where [M], [C], and [K] were the mass, viscous damping and stiffness matrices, respectively. The six member vector, $\{\Delta\}$, represented the horizontal and vertical displacements, relative to the base, at the locations of the concentrated masses used in the structure idealization. The vectors, $\{\dot{\Delta}\}$ and $\{\ddot{\Delta}\}$ were the corresponding velocities and accelerations. The vector, $\{A_b\}$, was a six member vector with each member equal to the base acceleration. All four of the above vectors were functions of time. The object of the analysis was to determine the magnitude of the vectors $\{\Delta\}$ and $\{\ddot{\Delta}\}$ at a number of discrete times during the response of the structure, given the magnitude of the vector $\{A_b\}$ at those same discrete times during the response.

Consistent with a modal analysis approach, the vectors $\{\Delta\}$, $\{\dot{\Delta}\}$, and $\{\ddot{\Delta}\}$ were idealized by the summations,

$$\{\Delta\} = \sum_{i=1}^{N_s} b_i\{\phi_i\}\xi_i(t) \tag{F.7}$$

$$\{\Delta\} = \sum_{i=1}^{N_s} b_i\{\phi_i\}\dot{\xi}_i(t) \tag{F.8}$$

$$\{\Delta\} = \sum_{i=1}^{N_s} b_i\{\phi_i\}\ddot{\xi}_i(t) \tag{F.9}$$

where b_i was the modal participation factor for the i^{th} mode of response, $\{\phi_i\}$ was the mode shape corresponding to the i^{th} mode of response and $\xi_i(t)$, $\dot{\xi}_i(t)$, and $\ddot{\xi}_i(t)$ were the displacement, velocity and acceleration, respectively, relative to the base, as functions of time, for a single degree of freedom system corresponding to the i^{th} mode of response. Equations F.7, F.8, and F.9 were substituted into Equation F.6. The result was premultiplied by $\{\phi_j\}^T$, the transpose of the mode shape for a specific mode, mode j. The orthogonality relations,

$$\{\phi_j\}^T [M] \{\phi_i\} = 0 \text{ for } i \neq j \tag{F.10}$$

$$\{\phi_j\}^T [K] \{\phi_i\} = 0 \text{ for } i \neq j \tag{F.11}$$

were applied. Additionally, the viscous damping coefficients were considered to be linear functions of mass and stiffness.

$$[C] = \alpha_1 [M] + \alpha_2 [K] \tag{F.12}$$

Combining Equations F.10, F.11 and F.12,

$$\{\phi_j\}^T [C] \{\phi_i\} = 0 \text{ for } i \neq j \tag{F.13}$$

Considering all of the above operations applied to Equation F.6, and rearranging, the result was,

$$b_j \ddot{\xi}_j(t) + 2\beta_{sj}\omega_{sj} b_j \dot{\xi}_j(t) + \omega_{sj}^2 b_j \xi(t)$$

$$= - \frac{\{\phi_j\}^T [M]\{I\}}{\{\phi_j\}^T [M]\{\phi_j\}} a_b(t) \tag{F.14}$$

where,

$$2\beta_{sj}\,\omega_{sj} = \frac{\{\phi_j\}^T[C]\{\phi_j\}}{\{\phi_j\}^T[M]\{\phi_j\}} \tag{F.15}$$

$$\omega_{sj}^2 = \frac{\{\phi_j\}^T[K]\{\phi_j\}}{\{\phi_j\}^T[M]\{\phi_j\}} \tag{F.16}$$

Allowing,

$$b_j = \frac{\{\phi_j\}^T[M]\{I\}}{\{\phi_j\}^T[M]\{\phi_j\}} \tag{F.17}$$

The result was a single degree of freedom equation of motion for each mode of response,

$$\ddot{\xi}_j(t) + 2\beta_{sj}\omega_{sj}\dot{\xi}_j(t) + \omega_{sj}^2\xi_j(t) = -a_b(t) \tag{F.18}$$

In this manner, the problem of defining the vectors $\{\ddot{\Delta}\}$ and $\{\Delta\}$ (equations F.7 and F.9) at several discrete times was reduced to two major operations. One was the modal analysis of the substitute structure to determine the mode shape, $\{\phi_i\}$, and circular frequency, ω_{si}, for each mode, i, of response. That operation is discussed in section F.4. The second operation was to determine the acceleration and displacement, $\ddot{\xi}_i$ and ξ_i, respectively, for each mode of response, i, at several discrete times during the response of the structure. That operation is described in section F.5.

F.4 Eigenanalysis

The eigenvalue problem defined by the relation,

$$[M]\{\ddot{\Delta}\} + [K]\{\Delta\} = \{0\} \tag{F.19}$$

was solved to obtain the mode shpae, $\{\phi_i\}$, and the participation factor, b_i, for each mode, i. Note that the effect of the viscous damping upon the mode shapes and frequencies was not considered. For reasonable damping factors, this effect is generally small.

The eigenanalysis was performed using the subroutine EIGENZ of the FORTUOI Library of the Digital Computer Laboratory of the University of Illinois. The resulting mode shapes and natural frequencies were inaccurate due to the effect of the axial deformation of the pier, which was considered in the analytical model. The routine, EIGENZ, was not reliable at handling the poor matrix conditioning associated with the inclusion of the axial effects. The results obtained using EIGENZ were applied as the initial guess in an iterative eigenanalysis improvement technique developed by Robinson and Harris (ref. 30). In this manner, refined results for the natural frequencies and mode shapes were obtained.

F.5 Response History of Single Degree of Freedom System

This part describes the computation of the response history corresponding to Equation F.18. A numerical approach was used, obtaining the response of the single degree of freedom system at a number of discrete points in time. The duration of structure response was divided into a series of uniform intervals, each of duration Δt. The endpoints of the time intervals became the discrete times for which structure response was to be computed. For the following discussion, two such discrete times, t' and t'' (where $t'' = t' + \Delta t$) are considered.

The single degree of freedom system must, of course, conform to its equation of motion (Equation F.18) for each of the discrete times. For t = t", after rearranging the equation,

$$\ddot{\xi}(t'') = -a_b(t'') - 2\beta_{sj}\omega_{sj}\dot{\xi}(t'') - \omega_{sj}^2\xi(t'') \qquad (F.20)$$

In this manner, the acceleration at a discrete time was expressed in terms of the displacement, velocity and base acceleration for that same discrete time. Kinematics and an assumption concerning the variation of acceleration over an interval were used to express the velocity and displacement at t = t" in terms of velocity, displacement and acceleration at t = t'. The assumption for the acceleration was that between any two points in time, such as t' and t", for which the response was to be computed, the acceleration varied linearly with time. Referring to Fig. F.1, the acceleration at any time between t' and t" was described as,

$$\ddot{\xi}(t) = \ddot{\xi}(t') + \frac{\ddot{\xi}(t'') - \ddot{\xi}(t')}{\Delta t}(t-t') \qquad t' \leq t \leq t'' \qquad (F.21)$$

The expressions for $\xi(t)$ and $\dot{\xi}(t)$ in the interval where then obtained by integration,

$$\dot{\xi}(t) = \dot{\xi}(t') + \int_{t'}^{t} \ddot{\xi}(t)\, dt \qquad t \leq t \leq t'' \qquad (F.22)$$

$$\xi(t) = \xi(t') + \int_{t'}^{t} \dot{\xi}(t)\, dt \qquad t' \leq t \leq t'' \qquad (F.23)$$

Evaluating Equations F.22 and F.23 for t = t" resulted in,

$$\dot{\xi}(t'') = \dot{\xi}(t') + (\frac{\Delta t}{2})[\ddot{\xi}(t) + \ddot{\xi}(t')] \qquad (F.24)$$

$$\xi(t'') = \xi(t') + (\Delta t)\,[\dot{\xi}(t')] + \frac{(\Delta t)^2}{6}[2\ddot{\xi}(t') + \ddot{\xi}(t'')] \qquad (F.25)$$

Substituting Equations F.24 and F.25 into Equation F.20, a direct relation between the acceleration at t = t" and the acceleration, velocity, and displacement at t = t' was obtained,

$$\ddot{\xi}(t'') = W_1\ a_b(t'') + W_2\ (t') + W_3\ (t') + W_4\ (t') \qquad (F.26)$$

where,

$$W_1 = - \frac{1}{\left| 1 + \beta_{sj}\omega_{sj}(\Delta t) + \frac{\omega_{sj}^2(\Delta t)^2}{6} \right|} \qquad (F.27)$$

$$W_2 = - \frac{\left| \beta_{sj}\omega_{sj}(\Delta t) + \frac{\omega_{sj}^2(\Delta t)^2}{3} \right|}{\left| 1 + \beta_{sj}\omega_{sj}(\Delta t) + \frac{{}_{sj}(\Delta t)^2}{6} \right|} \qquad (F.28)$$

$$W_3 = - \frac{[2\beta_{sj}\omega_{sj} + \omega_{sj}^2(\Delta t)]}{\left| 1 + \beta_{sj}\omega_{sj}(\Delta t) + \frac{\omega_{sj}^2(\Delta t)^2}{6} \right|} \qquad (F.29)$$

$$W_4 = - \frac{\omega_{sj}^2}{\left| 1 + \beta_{sj}\omega_{sj}(\Delta t) + \frac{\omega_{sj}^2(\Delta t)^2}{6} \right|} \qquad (F.30)$$

The formulation of the initial value problem was complete at this point. Knowing the acceleration, velocity, and displacement relative to the base at t = t', and the base acceleration at t = t' and t = t", the acceleration relative to the base at t = t" was computed from Equation F.26. The velocity and displacement for t = t" were then computed from Equations F.24 and F.25. The results for t = t" were then used to compute the responses for $t = t'''$. Knowing the response at one discrete time, the response at the next discrete time was computed. Only a set of initial responses was needed to begin the calculation process. This was accomplished through the condition of zero acceleration, velocity and displacement for $t = 0$.

F.6 Deflections, Shears and Moments

Modal superposition was used to compute the deflections, base shear and base moment for the structure, using the results of sections F.4 and F.5.

The response history for the structure, for horizontal deflections, was computed from Equation F.7, where the horizontal deflections are represented by three members of the six member vector, $\{\Delta\}$. The modal participation factors and mode shapes, b_i and $\{\phi_i\}$, respectively, for each mode, were obtained as described in section F.4. The single degree of freedom displacement, $\xi(t)$, for each mode, was calculated for a number of discrete points in time, as described in section F.5.

The response histories for base shear and moment were computed from the response histories for deflection. The six member vector of inertial loads, $\{\overline{P}_{1i}\}$, for the i^{th} mode, was calculated from,

$$\{\overline{P}_{1i}\} = -\omega_{si}^{2}[M]\{\Delta_i\} \tag{F.31}$$

where $\{\Delta_i\}$ represented the vector of displacements for the i^{th} mode, calculated at a number of discrete points in time. Modal superposition was applied to the inertial loads, obtaining,

$$\{P_1\} = \sum_{i=1}^{N_s} \{\overline{P}_{1i}\} \tag{F.32}$$

for each of the times for which the vectors, $\{\Delta_i\}$, were calculated. The base shear and base moment for each time were then calculated directly from the three members of $\{\bar{P}_1\}$ which represented horizontal loads.

APPENDIX G

NOTATION

The following symbols are used in this report:

a_1 = multiplication factor for the applied lateral load for the structure at the centerline of the second level beam.

a_2 = multiplication factor for the applied lateral load for the structure at the centerline of the fourth level beam.

a_3 = multiplication factor for the applied lateral load for the structure at the centerline of the sixth level beam.

$a_b(t)$ = acceleration of the base for a single degree of freedom system, a function of time.

a_v = vertical acceleration of a pier, associated with rotational acceleration of a test weight, for the analytical model for the study of dynamic response.

a_θ = rotational acceleration of the steel weights of the test specimen.

b = width of section.

b_i = modal participation factor for the i^{th} response mode.

c = distance from the neutral axis of the section, positive in the region of compressive strains.

$\bar{c}$ = depth to centroid of section, measured from edge characterized by maximum compressive strain.

c_{1i} = distance from the neutral axis of the section to the edge of the i^{th} pier farthest from the level of maximum compressive strain, where $i = 1$ corresponds to the pier experiencing the maximum compressive strain. Positive in the region of compressive strains.

c_{2i} = distance from the neutral axis of the section to the edge of the i^{th} pier closest to the level of maximum compressive strain, where $i = 1$ corresponds to the pier experiencing the maximum compressive strain. Positive in the region of compressive strains.

c_o = depth to the neutral axis of the section.

$\bar{c}_p$ = distance from the neutral axis to the plastic centroid of the section, positive in the region of compressive strains.

c_s = velocity coefficient associated with a viscous dashpot in a single degree of freedom system.

d = depth, measured from edge of section characterized by maximum compressive strain, to farthest layer of reinforcement in a doubly reinforced beam.

d' = depth, measured from edge of section characterized by maximum compressive strain, to closest layer of reinforcement in a doubly reinforced beam.

d_i = depth, measured from edge of the section characterized by maximum compressive strain, to i^{th} layer of reinforcement, where i = 1 corresponds to the closest layer.

$\overline{d}_i$ = depth, measured from the edge of the section characterized by maximum compressive strain, to centroid (mid-height) of i^{th} pier, where i = 1 corresponds to the closest pier.

$\overline{d}_p$ = depth to the plastic centroid of the section, measured from the edge characterized by the maximum compressive strain.

d_w = total section depth.

e = the base of the natural logarithm.

f = multiplication factor, in the static analytical model, for incremental joint rotations.

f_a = frequency of the single degree-of-freedom system corresponding to Case (a).

f_b = frequency of the single degree-of-freedom system corresponding to Case (b).

f_c = stress in concrete. Positive in compression

f'_c = compressive strength of concrete, obtained from tests of 4 x 8-in. cylinders.

f_{ci} = stress in concrete at the level of the i^{th} reinforcement layer, where i = 1 corresponds to the layer closest to the edge characterized by maximum compressive strain. Positive in compression.

$\overline{f}_J$ = for the computer program for the study of dynamic response, the J^{th} approximation to the frequency, in the eigenanalysis improvement procedure.

f_{min} = for the static hysteretic model, minimum value of the multiplication factor, f, considering all members in the structure.

f_{ref} = for the study of dynamic response, the first-mode frequency, for the structure to which stiffness reductions for the substitute structures were referenced.

f_s = stress in the reinforcement. Positive in compression.

f_{si} = stress in the i^{th} layer of reinforcement, where i = 1 corresponds to the layer closest to the edge of the section characterized by maximum compressive strain. Positive in compression.

f_{su} = ultimate strength of reinforcement.

f_{sy} = yield strength of reinforcement.

f_t = tensile strength of concrete, corresponding to splitting stress of 4 x 8 in. cylinders.

f_{unc} = for the study of dynamic response, the calculated first-mode frequency for the completely uncracked test structure.

f_y = yield strength of reinforcement.

h_i = depth of i^{th} pier, where i = 1 corresponds to pier experiencing maximum compressive strain.

i = an index variable.

j = an index variable.

j_f = in the Fourier analysis, the value of the index for discrete circular frequencies, corresponding to the circular frequency, ω_f.

j_o = in the Fourier analysis, the value of the index for discrete circular frequencies, corresponding to the circular frequency, ω_o.

k = number of reinforcement layers subjected to compression for a section.

k_a = stiffness of the single degree-of-freedom system corresponding to Case (a).

k_b = stiffness of the single degree-of-freedom system corresponding to Case (b).

$k_{e\ell}$ = spring stiffness for a linearly elastic single degree of freedom system, which has not yet yielded.

k_r = spring stiffness, reduced for equivalent linear response, of a single degree of freedom system.

k_{sp} = rotational stiffness of a linearly elastic spring used to model the bending stiffness of a beam in the static analytical model.

ℓ_{cr} = the length of a beam, adjacent to the beam-pier joint, characterized by total loss of concrete.

ℓ_d = length of reinforcement imbeddment in a joint necessary for the development of the yield stress of the reinforcement at the face of the joint.

ℓ_e = length of a beam in the static analytical model measured from the face of the rigid joint to the pinned end.

ℓ_E = length of a beam in the static analytical model measured from the centerline of the pier to the pinned end.

ℓ_p = length of a pier member measured from face of joint to face of joint.

$\Delta\ell$ = Increment of slippage per cycle of loading reversals for a reinforcing bar imbedded in a joint.

$\overline{\ell}$ = distance between the vertical centerlines of the two piers in a wall.

m = total number of reinforcement layers in the section, considering all piers.

m_h = lumped mass associated with horizontal acceleration, for the analytical model for the study of dynamic response.

m_s = concentrated mass associated with a single degree of freedom system.

m_v = lumped mass associated with rotational acceleration of the test weights, for the analytical model for the study of dynamic response.

m_w = mass corresponding to the applied dead load at one level, for one quarter of a test structure.

n = ratio of Young's modulus of reinforcement to secant modulus of concrete.

p = a constant determining the amplitude of the harmonic base motion for a single degree of freedom system.

q = number of distinct piers comprising the section.

s_1 = slope of the initial segment of the piecewise linear moment rotation relation of the static analytical model.

s_{r1} = slope of the first unloading segment (second quarter cycle) for a piecewise linear hysteretic relation between member end moment and end rotation.

s_{r2} = slope of the second unloading segment (fourth quarter cycle) for a piecewise linear hysteretic relation between member end moment and end rotation.

t = variable to denote time in a system response.

t' = a specific time during the response of a structural system.

t'' = a specific time during the response of a structural system.

t''' = a specific time during the response of a structural system.

Δt = an interval, or increment, of time.

t_i = in the Fourier analysis, the i^{th} discrete value of time, when the interval of response is divided into uniform increments, Δt.

$w(t)$ = a function of time describing a response of a structure (displacement, acceleration, shear, moment).

$\overline{w}(\omega)$ = in Fourier analysis, a function of frequency describing the relative importance of various frequency components in a response history.

$w^*(t)$= portion of a response history, $w(t)$, attributable to frequency components in the range ω_0 through ω_f.

x = variable used to measure distance along a member.

Δx_1 = distance along a member, in the static analytical model, from the section of maximum curvature, to the nearest section corresponding to slope discontinuity in the piecewise linear curvature distribution.

Δx_2 = distance along a member, in the static analytical model, between the first and second discontinuities in slope of the piecewise linear curvature distribution, counting discontinuities from the section of maximum curvature.

Δx_3 = distance along a member, in the analytical model, between the second and third discontinuities of slope in the piecewise linear curvature distribution, counting discontinuities from the section of maximum curvature.

Δx_4 = distance along a member, in the analytical model, between the third and fourth discontinuities of slope in the piecewise linear curvature distribution, counting discontinuities from the section of maximum curvature.

x_s = displacement of the concentrated mass of a single degree of freedom system.

$\dot{x}_s$ = velocity of the concentrated mass associated with a single degree of freedom system.

$\ddot{x}_s$ = acceleration of the concentrated mass associated with a single degree of freedom system.

x_{sm} = maximum response deflection for a single degree of freedom system.

x_{sy} = deflection of a single degree of freedom system, corresponding to yield.

z = the complex number $(\sqrt{-1})$.

A_{cr} = transformed area of a section, considering a fully cracked condition for the concrete.

ΔA_h = area enclosed, on a relation between base moment and deflection, by one complete cycle of hysteretic response.

A_k = on a relation between base moment and deflection, the area enclosed by a line defining a linearly elastic response, the deflection axis (horizontal axis), and a vertical line corresponding to a deflection equal to the maximum linearly elastic response.

A_{pi} = area of the i^{th} level pier, where i = 1 corresponds to the lowest level.

A_s = area of a reinforcement layer.

A_{tr} = transformed area of a section, considering an uncracked condition for the concrete.

C = compressive force in a reinforcing bar.

C_b = maximum compression at the base of a pier, corresponding to a failure mechanism. Positive in compression.

E_c = secant modulus for concrete measured between 0 and 1000 psi.

ΔE_h = energy dissipated, per cycle of response, by a nonlinear hysteretic system.

E_k = strain energy stored in a linear spring.

E_s = Young's modulus for the reinforcement.

E_{sh} = strain-hardening modulus for the reinforcement.

E_v = energy dissipated by a viscous dashpot in a linearly elastic single degree of freedom system.

ΔE_v = energy dissipated, per cycle of response, by a viscous dashpot in a linearly elastic single degree of freedom system.

EA_{ref} = for the study of dynamic response, the axial section stiffness for a member, to which the damage ratio for the member was referenced.

EI_{bi} = uniform section stiffness for the i^{th} level beam in the static analytical model.

EI_{ci} = uniform section stiffness for the i^{th} story pier member in the static analytical model.

EI_{eq} = for the static analytical model, the uniform section stiffness of the equivalent member.

EI_{ref} = for the study of dynamic response, the flexural section stiffness for a member, to which the damage ratio for the member was referenced.

EI_{sub} = for the study of dynamic response, the flexural section stiffness for a member in the substitute structure.

$\bar{I}$ = in the computer program for the study of dynamic response, a counter.

I_{bi} = moment of inertia of the i^{th} level beam, where i = 1 corresponds to the lowest level.

I_{cr} = transformed moment of inertia of a section, considering a fully cracked condition for the concrete.

I_{pi} = moment of inertia of the i^{th} level pier, where i = 1 corresponds to the lowest level.

I_{rot} = moment of inertia, associated with rotation, for the steel weights of the test specimen.

I_{tr} = transformed moment of inertia of a section, considering an uncracked condition for the concrete.

$\bar{J}$ = in the computer program for the study of dynamic response, a counter.

M = applied moment for a section.

M_1 = base moment, corresponding to a failure mechanism, in the pier in which lateral loads induce tensile force.

M_2 = base moment, corresponding to a failure mechanism, in the pier in which lateral loads induce compressive force.

M_3 = base moment, corresponding to a failure mechanism, associated with the couple comprised of the axial forces in the piers.

M_b = total base moment for one wall (one-half test structure).

M_{crn} = for dynamic test runs subjected to Fourier analysis, the total observed maximum base moment.

M_e = end moment for a member in the static analytical model. For a beam, M_e was the moment at the face of the rigid joint. For a pier, M_e was the sum of the moments at the faces of the upper and lower joints.

M_{eb} = for the static analytical model, the moment in a pier member at the face of the lower joint.

M_{et} = for the static analytical model, the moment in a pier member at the face of the upper joint.

M_E = moment in a beam in the static analytical model measured at the centerline of the pier.

$(\frac{M_e}{\theta_E})_{eq}$ = slope of the relation between member end moment and member end rotation for the equivalent uniform member of the static analytical model.

$(\frac{M_e}{\theta_E})_y$ = slope of the relation between member end moment and member end rotation considering progressive yielding along the member.

$(M_E)_o$ = for the static analytical model, end moment for a member at the beginning of a loading step.

(ΔM_E) = for the static analytical model, incremental end moment for a member, implied by the vector $\{\Delta\theta\}$ as calculated in Equation 5.25.

$(M_E)_{lim}$ = for the static analytical model, end moment for a member corresponding to a change in slope of a piecewise linear moment-rotation relation.

M_{frn} = for the final run of each dynamic test, the observed maximum base moment.

M_ℓ = moment level, in terms of either a section moment or member end moment, corresponding to a tensile strain of 0.20 in the reinforcement.

M_{m1} = for the spectral study, maximum base moment for the first mode of response.

M_{m2} = for the spectral study, maximum base moment for the second mode of response.

M_{mch1} = base moment corresponding to the failure mechanism for the type A structure and to the failure mechanism characterized by the attainment of the yield moments at the ends of the beams for types B and C structures.

M_{mch2} = base moment corresponding to the failure mechanism characterized by the attainment of the maximum moment capacity at the ends of the connecting beams, for types B and C structures.

M_{md1} = for dynamic test runs subjected to Fourier analysis, the maximum base moment obtained considering only that portion of the response attributable to frequency components below 10 Hz.

M_{mtot} = for the spectral study, the total maximum base moment, considering both first and second mode components.

M_{sneg} = observed maximum base moment, in the negative direction (third quarter cycle), for the static test.

M_{spos} = observed maximum base moment, in the positive direction (first quarter cycle), for the static test.

M_{y1} = moment at first slope discontinuity in a piecewise linear moment-curvature or moment-rotation relation.

M_{y2} = moment at second slope discontinuity in a piecewise linear moment-curvature or moment-rotation relation.

M_{y3} = moment at third slope discontinuity in a piecewise linear moment-curvature or moment-rotation relation.

M_{ys} = for the static analytical model, for the beam characterized by complete loss of concrete adjacent to the face of the beam-pier joint, the member end moment, measured at the face of the joint, corresponding to yield.

N = in the Fourier analysis, the number of increments, Δt, into which the interval of system response is divided.

$\overline{N}$ = for the computer program for the study of dynamic response, number of substitute structures (analysis cases) processed in one run of the program.

N_s = number of response modes considered for the calculation of a response history for the structure.

P = axial load on a section. Positive for compression.

ΔP_i = increment of applied lateral load at the level of the i^{th} beam.

P_m = maximum axial load capacity of a section with no moment applied to the section.

Q = factor determining overall magnitude of applied lateral loading.

ΔQ = increment of the factor determining the overall magnitude of statically applied lateral loads.

Q_m = factor determining overall magnitude of applied lateral loading, corresponding to a failure mechanism.

S_i = spectral displacement for the i^{th} response mode.

T = tensile force in a reinforcing bar.

T_b = maximum tension at the base of a pier, corresponding to a failure mechanism. Positive in tension.

T_p = period of response for the first response mode of a structure

T_s = natural period of a single degree of freedom system.

U_i = lateral deflection in the analytical model at the level of the centerline of the i^{th} beam.

V = shear force at a section in a member.

V_1 = base shear, corresponding to a failure mechanism, in the pier in which lateral loads induce tensile force.

V_2 = base shear, corresponding to a failure mechanism, in the pier in which lateral loads induce compressive force.

V_b = total base shear for one wall (one-half test structure), corresponding to a failure mechanism.

V_{crn} = for dynamic test runs subjected to Fourier analysis, the total observed maximum base shear.

V_{frn} = for the final run of each dynamic test, the observed maximum base shear.

V_{m1} = for the spectral study, maximum base shear for the first mode of response.

V_{m2} = for the spectral study, maximum base shear for the second mode of response.

V_{mch1} = base shear corresponding to the failure mechanism for the type A structure, and to the failure mechanism characterized by the attainment of the yield moment at the ends of the beams for the types B and C structures.

V_{mch2} = base shear corresponding to the failure mechanism characterized by the attainment of the maximum moment capacity at the ends of the connecting beams, for types B and C structures.

V_{md1} = for dynamic test runs subjected to Fourier analysis, the maximum base shear obtained considering only that portion of the response attributable to frequency components below 10 Hz.

V_{mtot} = for the spectral study, the total maximum base shear, considering both first and second mode components.

V_{sneg} = observed maximum base shear, in the negative direction (third quarter cycle), for the static test.

V_{spos} = observed maximum base shear, in the positive direction (first quarter cycle), for the static test.

W_1 = for the study of dynamic response, a constant in the numerical integration procedure.

W_2 = for the study of dynamic response, a constant in the numerical integration procedure.

W_3 = for the study of dynamic response, a constant in the numerical integration procedure.

W_4 = for the study of dynamic response, a constant in the numerical integration procedure.

Y = constant in a Fourier series.

Y' = constant coefficient of the cosine term in a Fourier series.

Y'' = constant coefficient of the sine term in a Fourier series.

Z_i = the coefficient, a complex function of time and frequency, in the complex, exponential form of the Fourier series.

α_1 = for the study of dynamic response, the coefficient for the mass in the viscous damping expression.

α_2 = for the study of dynamic response, the coefficient for stiffness in the viscous damping expression.

β_s = viscous damping factor, as a fraction of critical damping, for a single degree of freedom system.

β_{sj} = viscous damping factor for the single degree of freedom system corresponding to the j^{th} response mode.

δ = deflection of a line tangent to a beam at the face of a beam-pier joint from the undeflected beam. Measured at the pinned end.

ε_{1i} = strain at the edge of the i^{th} pier farthest from the level of maximum compressive strain, where $i = 1$ corresponds to the pier experiencing the maximum compressive strain. Positive for compression.

ε_{2i} = strain at the edge of the i^{th} pier closest to the level of maximum compressive strain, where $i = 1$ corresponds to the pier experiencing the maximum compressive strain. Positive for compression.

ε_c = strain in concrete. Positive in compression.

ε_{cm} = strain at the edge of the section characterized by the greatest compressive strain.

ε_o = compressive strain at which concrete attains its compressive strength.

ε_s = strain in the reinforcement. Positive in compression.

ε_s' = strain at the level of the top level of reinforcement in a doubly reinforced beam. Positive in compression.

ε_s'' = strain at the level of the lower level of reinforcement in a doubly reinforced beam. Positive in compression.

ε_{sh} = strain in reinforcement at the onset of strain-hardening.

ε_{si} = strain at the level of the i^{th} reinforcement layer, where i = 1 corresponds to the top layer. Positive in compression.

ε_{su} = strain in the reinforcement at the attainment of the ultimate strength.

ε_{sy} = yield strain of reinforcement.

ε_t = strain corresponding to tensile strength of concrete.

η = phase shift for the harmonic response of a single degree of freedom system.

θ = an angle, in radians.

$\theta_{c\ell}$ = beam end rotation to accomplish closure of the crack in the beam adjacent to the face of the beam-pier joint.

θ_E = end rotation for a member in the static analytical model. For a beam, θ_E was the rotation of the rigid beam-pier joint. For a pier member, θ_E was the difference of the rotations of the upper and lower beam-pier joints.

$\Delta\theta_E$ = incremental member end rotation.

$(\theta_E)_{lim}$ = end rotation of a member corresponding to a discontinuity of slope in a piecewise linear hysteretic relation between member end moment and end rotation.

$(\theta_E)_o$ = member end rotation at the beginning of a given loading step.

θ_i = rotation of the i^{th} level beam-pier joint, in the static analytical model.

θ_ℓ = member end rotation, θ_E, corresponding to a tensile strain of 0.20 in the reinforcement.

θ_{m1} = maximum member end rotation, θ_E, for the first quarter cycle of hysteretic response.

θ''_{m1} = member end rotation corresponding to the intersection of the reloading segment of a hysteretic moment-rotation relation with the unloading segment of the previous cycle.

θ_{m2} = maximum member end rotation, θ_E, for the third quarter cycle of hysteretic response.

θ_{m3} = maximum member end rotation, θ_E, for the fifth quarter cycle of hysteretic response.

θ_{m4} = maximum member end rotation, θ_E, for the seventh quarter cycle of hysteretic response.

θ_{m5} = maximum member end rotation, θ_E, for the ninth quarter cycle of hysteretic response.

θ_{m6} = maximum member end rotation, θ_E, for the eleventh quarter cycle of hysteretic response.

θ_{r1} = member end rotation, θ_E, corresponding to zero end moment for the second quarter cycle of hysteretic response.

θ_{r2} = member end rotation, θ_E, corresponding to zero end moment for the fourth quarter cycle of hysteretic response.

θ_{r3} = member end rotation, θ_E, corresponding to zero end moment for the sixth quarter cycle of hysteretic response.

θ_{r4} = member end rotation, θ_E, corresponding to zero end moment for the eighth quarter cycle of hysteretic response.

θ_{r5} = member end rotation, θ_E, corresponding to zero end moment for the tenth quarter cycle of hysteretic response.

θ_{y1} = member end rotation, θ_E, corresponding to the first slope discontinuity in a piecewise linear moment-rotation relation.

θ_{y2} = member and rotation, θ_E, corresponding to the second slope discontinuity in a piecewise linear moment-rotation relation.

θ_{ys} = for the static analytical model for the beam characterized by complete loss of concrete adjacent to the face of the beam-pier joint, the member end rotation corresponding to yield.

μ_{bm} = for the study of dynamic response, the damage ratio applied to the connecting beams.

μ_{dc} = response deflection ductility for a structure or structural element.

μ_{dr} = damage ratio for an element of a linearly elastic substitute structure.

μ_{pr} = for the study of dynamic response, the damage ratio applied to the first story pier.

$\xi_i(t)$ = displacement for the single degree of freedom system corresponding to the i^{th} response mode, a function of time.

$\dot{\xi}_i(t)$ = velocity for the single degree of freedom system corresponding to the i^{th} response mode, a function of time.

$\ddot{\xi}_i(t)$ = acceleration for the single degree of freedom system corresponding to the i^{th} response mode, a function of time.

σ_a = uniform axial stress in a reinforced concrete section, corresponding to the applied axial load. Positive in compression.

ϕ = curvature applied to a section.

ϕ_e = curvature of a beam in the static analytical model at the face of the beam-pier joint.

ϕ_{eb} = for the static analytical model, the curvature in a pier member at the face of the lower joint.

ϕ_{et} = for the static analytical model, the curvature in a pier member at the face of the upper joint.

ϕ_{ij} = member of the mode shape vector for the i^{th} response mode, corresponding to the level of the j^{th} weight in the test structure.

ϕ_ℓ = curvature of a section corresponding to a tensile strain of 0.20 in the reinforcement.

ϕ_{y1} = curvature corresponding to first slope discontinuity in a piecewise linear moment-curvature relation.

ϕ_{y2} = curvature corresponding to second slope discontinuity in a piecewise linear moment-curvature relation.

ϕ_{y3} = curvature corresponding to third slope discontinuity in a piecewise linear moment-curvature relation.

ϕ_{ys} = for the static analytical model for the beam section characterized by complete loss of concrete, the curvature at the face of the joint, corresponding to yield.

$\Delta\psi_1$ = rotational slip at the end of a beam for the first half-cycle of response, due to slip of reinforcement in the beam-pier joint.

$\Delta\psi_2$ = rotational slip at the end of a beam for the second half-cycle of response, due to slip of reinforcement in the beam-pier joint.

ω = circular frequency of a periodic waveform.

$\Delta\omega$ = in the Fourier analysis, the increment between uniformly spaced discrete values of circular frequency.

ω' = in the Fourier analysis, a specific value of circular frequency for analysis.

ω_1 = first mode circular frequency for a structure.

ω_b = circular frequency of the harmonic base motion for a single degree of freedom system.

ω_f = in the Fourier analysis, the upper limit for a range of circular frequencies.

ω_i = in the Fourier analysis, the i^{th} discrete value of circular frequency.

ω_j = in the Fourier analysis, the j^{th} discrete value of circular frequency.

ω_o = in the Fourier analysis, the lower limit for a range of circular frequencies.

ω_s = natural circular frequency for a single degree of freedom system.

ω_{sj} = circular frequency for the single degree of freedom system corresponding to the j^{th} response mode.

Δ_{mt} = for the spectral study, maximum deflection for the test structure at the top level weight.

Ω = constant determining the amplitude of harmonic response of a single degree of freedom system.

$\{A_b\}$ = for the study of dynamic response, the six by one vector representing the base acceleration, a function of time.

$[C]$ = for the study of dynamic response, six by six viscous damping matrix for the structure.

$\{I\}$ = a vector, whose every member is equal to one.

$[K]$ = for the study of dynamic response, the condensed six by six stiffness matrix for the structure.

$[\overline{K}]$ = for the study of dynamic response, the 18 x 18 stiffness matrix for the structure.

$[\overline{K}_1]$ = for study of dynamic response, upper left portion of partitioned structure stiffness matrix, $[\overline{K}]$.

$[\overline{K}_2]$ = for study of dynamic response, upper right portion of partitioned structure stiffness matrix, $[\overline{K}]$.

$[\overline{K}_3]$ = for the study of dynamic response, lower left portion of partitioned structure stiffness matrix, $[\overline{K}]$.

$[\overline{K}_4]$ = for the study of dynamic response, lower right portion of partitioned structure stiffness matrix, $[\overline{K}]$.

$[K_{eq}]$ = stiffness matrix for the structure of the static analytical model, considering equivalent uniform members.

$[M]$ = for the study of dynamic response, six by six mass matrix for the structure.

$\{\overline{P}\}$ = for the study of dynamic response, the 18 x 1 vector of external joint loads.

$\{\Delta P\}$ = vector of incremental, statically applied lateral loads for the structure.

$\{\Delta P'\}$ = vector of incremental lateral loads modified by the factor, f.

$\{\overline{P}_1\}$ = for the study of dynamic response, the upper portion of the partitioned vector of external joint loads, $\{\overline{P}\}$. The vector $\{\overline{P}_1\}$ represented the nonzero joint loads.

$\{\overline{P}_{1i}\}$ = for the study of dynamic response, the six by one vector of nonzero inertial joint loads corresponding to the i^{th} mode of response.

$\{\overline{P}_2\}$ = for the study of dynamic response, the lower portion of the partitioned vector of external joint loads, $\{\overline{P}\}$.

$\{P_o\}$ = vector of lateral loads at the beginning of a given load step.

$\{R_c\}$ = for the static hysteretic model, the vector of critical structure responses for a load step.

$\{R_F\}$ = for the static hysteretic model, vector of structure responses at the end of a load step.

$\{R_I\}$ = for the static hysteretic model, vector of structure responses at the beginning of a load step.

$\{R_\ell\}$ = vector determining the ratios between the statically applied lateral loads at the levels of the various beams.

$\{R_R\}$ = for the static hysteretic model, vector of unfactored incremental structure responses for a load step.

$[T]$ = matrix to transform lateral deflections of beam-pier joints into joint rotations.

$\{\overline{U}\}$ = for the study of dynamic response, the 18 x 1 vector of structure displacements.

$\{\Delta U\}$ = vector of the incremental lateral deflections of the beam pier joints.

$\{\Delta U'\}$ = vector of incremental lateral displacements of the beam-pier joints, modified by the factor, f.

$\{\overline{U}_1\}$ = for the study of dynamic response, upper portion of the partitioned vector of structure joint displacements, $\{\overline{U}\}$.

$\{\overline{U}_2\}$ = for the study of dynamic response, lower portion of the partitioned vector of structure joint displacements, $\{\overline{U}\}$.

$\{U_o\}$ = vector of lateral displacements of the beam-pier joints at the beginning of a given load step.

$\{\Delta\theta\}$ = vector of incremental rotations of the beam-pier joints.

$\{\phi_i\}$ = mode shape for the i^{th} response mode.

$\{\Delta\}$ = for the study of dynamic response, the six by one vector of structure joint displacements, a function of time.

$\{\dot{\Delta}\}$ = for the study of dynamic response, the six by one vector of structure joint velocities, a function of time.

$\{\ddot{\Delta}\}$ = for the study of dynamic response, the six by one vector of structure joint accelerations, a function of time.

REFERENCES

1. "Response of Buildings to Lateral Forces," Reported by ACI Committee 442, Journal of the American Concrete Institute, Proceedings, Vol. 68, No. 2, February 1971.

2. Adams, P. F., and J. G. MacGregor, "Plastic Design of Coupled Frame-Shear Wall Structures," Journal of the Structural Division, ASCE, Vol. 96, No. ST9, September 1970.

3. Arvidsson, K., "Shear Walls With Door Openings Near the Edge of the Wall," Journal of the American Concrete Institute, Proceedings, Vol. 71, No. 7, July 1974.

4. Beck, Hubert, "Contribution to the Analysis of Coupled Shear Walls," Journal of the American Concrete Institute, Proceedings, Vol. 59, No. 8, August 1962.

5. Biggs, J. M., Introduction to Structural Dynamics, McGraw-Hill, Inc., 1964.

6. Clough, R. W., and J. Penzien, Dynamics of Structures, McGraw-Hill Inc., 1975.

7. Coull, A., and J. R. Choudhury, "Stresses and Deflections in Coupled Shear Walls," Journal of the American Concrete Institute, Proceedings, Vol. 64, No. 2, February 1967.

8. Coull, A., and J. R. Choudhury, "Analysis of Coupled Shear Walls," Journal of the American Concrete Institute, Proceedings, Vol. 64, No. 9, September 1967.

9. Coull, A., "Interaction of Coupled Shear Walls with Elastic Foundations," Journal of the American Concrete Institute, Proceedings, Vol. 68, No. 6, June 1971.

10. Crandall, S. H., Engineering Analysis, Engineering Societies Monographs, McGraw-Hill, Inc., 1956.

11. Deschappeles, B. J., "Analytical Model for Lateral Load Effect on Buildings," Journal of the Structural Division, ASCE, Vol. 96, No. ST6, June 1970.

12. Gluck, J., "Elasto-Plastic Analysis of Coupled Shear Walls," Journal of the Structural Division, ASCE, Vol. 99, No. ST8, August 1973.

13. Gould, P. L., "Interaction of Shear Wall-Frame Systems in Multistory Buildings," Journal of the American Concrete Institute, Proceedings, Vol. 62, No. 1, January 1965.

14. Green, N. B., "Factors in the Aseismic Design of Reinforced Concrete Shear Walls Without Openings," Journal of the American Concrete Institute, Proceedings, Vol. 65, No. 8, August 1968.

15. Gulkan, P. and M. A. Sozen, "Response and Energy-Dissipation of Reinforced Concrete Frames Subjected to Strong Base Motions," Civil Engineering Studies, Structural Research Series No. 377, University of Illinois, Urbana, May 1971.

16. Hognestad, E., "Study of Combined Bending and Axial Load in Reinforced Concrete Members," Engineering Experiment Station Bulletin Series No. 399, University of Illinois, Urbana, November, 1951.

17. Housner, G. W., "Behavior of Structures During Earthquakes," Journal of the Engineering Mechanics Division, ASCE, Vol. 85, No. EM4, October 1959.

18. Irwin, A. W. and A.E.C. Ord, "Cyclic Load Tests on Shear Wall Coupling Beams," Proceedings of the Institution of Civil Engineers, Part 2, Vol. 61, June 1976.

19. Jacobsen, L. S., "Steady Forced Vibration as Influenced by Damping," Transactions, ASME, Vol. 52, Part 1, 1930.

20. Jennings, P. C., "Equivalent Viscous Damping for Yielding Structures," Journal of the Engineering Mechanics Division, ASCE, Vol. 94, No. EM1, February 1968.

21. Kaplan, W., Advanced Calculus, Addison-Wesley Publishing Co., Inc., 1952.

22. Khan, F. R., and J. A. Sbarounis, "Interaction of Shear Wall with Frames in Concrete Structures Under Lateral Loads," Journal of the Structural Division, ASCE, Vol. 90, No. ST3, June 1964.

23. Mahin, S. A. and V. V. Bertero, "Nonlinear Seismic Response of a Coupled Wall System," Journal of the Structural Divsion, ASCE, Vol. 102, No. ST9, September 1976.

24. MacLeod, I. A., "Connected Shear Walls of Unequal Width," Journal of the American Concrete Institute, Proceedings, Vol. 67, No. 5, May 1960.

25. Otani, S., M. A. Sozen, "Behavior of Multistory Reinforced Concrete Frames During Earthquakes," Civil Engineering Studies, Structural Research Series No. 392, University of Illinois, Urbana, November 1972.

26. Paulay, T., "An Elasto-Plastic Analysis of Coupled Shear Walls," Journal of the American Concrete Institute, Proceedings, Vol. 67, No. 11, November 1970.

27. Paulay, T., "Coupling Beams of Reinforced Concrete Shear Walls," Journal of the Structural Division, ASCE, Vol. 97, No. ST3, March 1971.

28. Paulay, T., and Binney, J. R., Diagonally Reinforced Coupling Beams of Shear Walls," Shear in Reinforced Concrete, Special Publication SP-42, Vol. 2, American Concrete Institute, 1974.

29. Paulay, T. and A. R. Santhakunar, "Ductile Behavior of Coupled Shear Walls, "Journal of the Structural Division, ASCE, Vol. 102, No. ST1, January 1976.

30. Robinson, A. R. and J. F. Harris, "Improving Approximate Eigenvalues and Eigenvectors," Journal of the Engineering Mechanics Division, ASCE, Vol. 97, No. EM2, April 1971.

31. Rosman, R., "Approximate Analysis of Shear Walls Subject to Lateral Loads," Journal of the American Concrete Institute, Proceedings, Vol. 61, No. 6, June 1964.

32. Rosman, R., "Analysis of Spatial Concrete Shear Wall Systems," The Institution of Civil Engineers, Supplement Vol. 45, February 1970.

33. Roy, H.E.H., "Designing for Lateral Loads," Structural Concrete Symposium, Toronto, Canada, May 1971.

34. Schwaighofer, J., "Shear Wall Structures," Proceedings, Structural Concrete Symposium, Toronto Canada, May 1971.

35. Shibata, A., and M. A. Sozen, "The Substitute-Structure Method for Earthquake-Resistant Design of Reinforced Concrete Frames," Civil Engineering Studies, Structural Research Series No. 412, University of Illinois, Urbana, October 1974.

36. Takeda, T., M. A. Sozen, and N. N. Nielsen, "Reinforced Concrete Response to Simulated Earthquakes," Journal of the Structural Division, ASCE, Vol. 96, No. ST12, December 1970.

37. Wight, J. K., and M. A. Sozen, "Shear Strength Decay in Reinforced Concrete Columns Subjected to Large Deflection Reversals," Civil Engineering Studies, Structural Research Series No. 403, University of Illinois, Urbana, August 1973.

38. Winokur, A., and J. Gluck, "Ultimate Strength Analysis of Coupled Shear Walls," Journal of the American Concrete Institute, Proceedings, Vol. 65, No. 12, December 1968.