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A SIMPLE CONTINUUM MODEL FOR DYNAMIC ANALYSIS OF COMPLEX PLANE FRAME STRUCTURES

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16. Abstracts A detailed dynamic analysis, reported in this series, was conducted on the seismic response and structural safety of key subsystems (steam generator, high pressure steam piping, coal handling equipment, cooling tower, chimney) of Unit #3 of TVA at Paradise, Kentucky in order to: (1) determine for the key components the natural frequencies below 50 Hz and the corresponding normal modes; (2) determine response of plant to seismic disturbances; (3) verify through full scale tests results obtained in (1) and determine estimates of damping needed in (2); (4) determine potential failure modes of major structural components; and (5) determine a spare parts policy for a power system so that outages due to damage from seismic disturbances are minimal. Analytical and experimental methods are used. This volume includes discussion of a simple model developed for dynamic analysis of large frame structures that can account both for joint rotation and axial deformation. The governing equations are derived from a continuum theory for gridworks. Natural frequencies are computed by using the simple model and by finite element method. Comparisons of the solutions show that the simple model yields reasonable solutions for the first five modes.

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A SIMPLE CONTINUUM MODEL FOR DYNAMIC ANALYSIS OF COMPLEX PLANE FRAME STRUCTURES

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Summary Report

Prior to 1974 there has been no detailed dynamic analysis of the seismic structural response and safety of large fossil-fuel steam generating plants. In March, 1974, under NSF Grant GI41897, a detailed dynamical analysis was begun on the seismic response and structural safety of key subsystems

(steam generator,
high pressure steam piping,
coal handling equipment,
cooling tower,
chimney)

of Unit #3 of TVA at Paradise, Kentucky to accomplish the following objectives:

a) Determine for the key components the natural frequencies below 50 Hz and the corresponding normal modes.

b) Determine response of plant to seismic disturbances.

c) Verify through full scale tests, where possible, results obtained in a), and determine estimates of damping needed in b).

d) Determine potential failure modes of major structural components.

e) Determine a spare parts policy for a power system so that outage due to damage from seismic disturbances are minimal.

Analytical and experimental methods are used.

The attached Reports present what has been accomplished to date.

Before making a few summarizing remarks on the individual Reports, some comments must be made in order to provide perspective on the study.

Paradise, Unit #3 of TVA was selected for study because near-by mine operations provide excitation (due to blasting) for the plant, and TVA was willing to cooperate in the conduct of the study. It should be pointed out that this plant was not designed to resist earthquakes. However, it was felt that this disadvantage was outweighed by the experimental possibilities.

The key components selected for study are critical for operation of the plant and would cause significant outage if damaged. All components can be studied using similar types of analyses. These are the basic reasons for including in this study only the steam generator, high pressure piping, coal handling equipment, cooling tower, and chimney.

Basic data for the analyses were obtained from drawings provided by TVA and Babcock-Wilcox. In addition to these data, a number of assumptions had to be introduced into the analyses. These assumptions refer in the main to the nature of the connections among elements of known properties, the fixity of columns, the properties of hanger elements, etc. Choices were made based on physical as well as computational reasons.

The analyses were confined to the linear range. After such a study, it is possible to assess at what level of excitation parts of the structure become nonlinear.

Structure-foundation interaction was neglected. Unit #3 of Paradise rests on excavations in limestone. It is assumed that there is little interaction. However, experimental studies will be made on this point.

It was decided at the start that all computations would be carried out with an existing computer program. SAP IV was chosen. Some program modifications have proved necessary, but these have been relatively minor. To obtain familiarity with the program it was necessary to study a number of special cases of the actual structure to ensure that it was functioning properly. For example, substructures within the steam generator support were considered seperately; assumed values of viscous damping coefficients were used in generating time histories*; etc. We found the program execution

* It should be noted that the magnitude of the response with zero damping must be interpreted with some caution as systems with slightly different frequencies can exhibit significantly different magnitudes of response.

time slow in some respects which indicates that some of its internal subroutines, such as eigen value solution, could be improved. It is beyond the scope of this project, however, to improve existing programs.

The experimental part of the study has proved much more difficult to conduct than anticipated. TVA has been most cooperative. However, the sheer physical size of the units, the weather, etc. have caused a number of difficulties that were not easy to foresee. Progress is gradually being achieved.

Interest in simple models stems from their possible use in design studies. It was decided to develop a methodology for constructing simple models. At present, our simple models are in the embryonic stage. It is hoped that after the study of two more plants a useful methodology can be obtained. Simple models developed could have been used for one component under study; however, timing made this impossible.

No recommendations will be made or conclusions drawn at this time, except in special situations. The partial examination of one plant does not provide a sufficient basis for such actions. At the completion of the study conclusions and recommendations will be presented.

A number of factors of some importance have not been considered so far. For example, the steam generator's internal elements can move with respect to it, the steam piping exerts dynamic forces on its supports, dynamic stresses in steam piping are just part of its stress system, many different seismic excitations are available, plus many more. Also a spare parts policy was not considered. As additional progress is made, we shall consider some of these problems. However, it must be recognized that it is possible to consider in this study only those factors of major importance. A spare parts policy involves economic considerations; it may not be possible to acquire the information needed to address this point.

Contact with industry in this country and Japan clearly indicates that the current detailed study is of great interest.

An Advisory Committee consisting of

Carl L. Canon	- Babcock & Wilcox Product Design Supervisor for Structural Steel and Design
William A. English	- Tennessee Valley Authority Head Civil Engineer
Clinton H. Gilkey	- Combustion Engineering, Inc. Manager, Engineering Science
Richard F. Hill	- Federal Power Commission Acting Director, Office of Energy Systems

R. Bruce Linderman	-	Bechtel Power Corporation Engineering Specialist
D. P. Money	-	Foster-Wheeler Corporation Supervisor of Stress Analysis
R. D. Sands	-	Burns & McDonnell Chief Mechanical Engineer
Erwin P. Wollak	-	Pacific Gas & Electric Company Supervisor, Civil Engineering Division

has been formed to provide a forum for an interchange of practical and conceptual views on various aspects of the study. The aim is to ensure that what is developed (in simple models) will be of practical use to industry. The Advisory Committee has met twice and reviewed plans and the progress of the investigation.

Contact is also maintained with the following firms:

Mitsubishi Heavy Industries
Babcock-Hitachi
Ishikawajima Harima Heavy Industries
Kawasaki Heavy Industries
Taiwan Power Company

The initial visit provided considerable information on the methods they have used in seismic response studies conducted by the research groups in each organization and plant experience under seismic disturbances.

Comments from the Advisory Committee and reviewers have been most helpful and encouraging. Many of the comments have been considered. However, it is not possible to take account in our studies of all points that have been brought to our attention.

Five professors, 8-10 graduate students, 2 technicians, and a secretary devoted part time to the study. A great deal of effort was devoted to acquiring information and equipment. The cooperation of TVA and Babcock-Wilcox was most helpful and deeply appreciated. Progress was excellent when it is remembered that education of students is a major function of a University.

This research project was sponsored by NSF through Grant No. GI41897.

The Reports in this series are as follows:

Dynamic Behavior of the Steam Generator and Support Structures of the 1200 MW Fossil Fuel Plant, Unit #3, Paradise, Kentucky, by T.Y. Yang, M.I. Baig, J.L. Bogdanoff.

The High Pressure Steam Pipe, by C.T. Sun, A.S. Ledger, H. Lo.

Coal Handling Equipment, by K.W. Kayser and J.A. Euler.

Theoretical Study of the Earthquake Response of the Paradise Cooling Tower, by T.Y. Yang, C.S. Gran, J.L. Bogdanoff.

Theoretical Study on Earthquake Response of a Reinforced Concrete Chimney, by T.Y. Yang, L.C. Shiau, H. Lo.

A Simple Continuum Model for Dynamic Analysis of Complex Plane Frame Structures, by C.T. Sun, H. Lo, N.C. Cheng, and J. L. Bogdanoff.

A Timoshenko Beam Model for Vibration of Plane Frames, by C.T. Sun, C.C. Chen, J.L. Bogdanoff, and H. Lo.

1 Introduction

Dynamic analysis of complex frame structures can be very laborious if all the members are to be accounted for. Furthermore, in the analysis of seismic structural response, only the first few modes are used. Therefore, it is desirable to construct simple models that can yield accurate results for lower modes that usually represent the gross deformation of the structure. The simplest model for this purpose is the shear beam approach where floor beams are rigid and consequently, the joint rotations are suppressed. Ignoring joint rotations may lead to a substantial error in computing the fundamental frequencies of certain frame structure [1]. It has also been found that girder flexibilities can have significant effect on the dynamic behavior of tall buildings [2]. In addition, the axial deformation in columns is neglected in the shear beam model; hence, the gross flexural deformation is not included. Effect of axial deformation on natural frequencies can also be appreciable [3].

The present work aims to develop a simple model for dynamic analysis of large frame structures that can account for both joint rotation and axial deformation. The governing equations are derived from a continuum theory for gridworks [4]. Natural frequencies are computed by using the simple model and by finite element method. Comparisons of the solutions show that the simple model, in general, yields reasonable solutions for the first five modes.

2. Review of the Continuum Model

In [4], a two-dimensional continuous medium with couple stress is used to represent a plane gridwork that consists of orthogonally intersecting beams rigidly connected at the joints. The strain energy and kinetic energy densities for the continuum are given by

$$U = \frac{1}{2} a_1 (\frac{\partial u}{\partial x})^2 + \frac{1}{2} a_2 (\frac{\partial u}{\partial y})^2 + \frac{1}{2} a_3 (\frac{\partial u}{\partial y} + \theta)^2$$

$$+ \frac{1}{2} a_4 (\frac{\partial u}{\partial x}) - \theta)^2 + \frac{1}{2} a_5 (\frac{\partial \theta}{\partial x})^2 + \frac{1}{2} a_6 (\frac{\partial \theta}{\partial y})^2 \tag{1}$$

and

$$T = \frac{1}{2} b_1 (\dot{u})^2 + \frac{1}{2} b_2 (\frac{\partial \dot{u}}{\partial x})^2 + \frac{1}{2} b_3 (\frac{\partial \dot{u}}{\partial y})^2$$

$$+ \frac{1}{2} b_4 (\dot{v})^2 + \frac{1}{2} b_5 (\frac{\partial \dot{v}}{\partial x})^2 + \frac{1}{2} b_6 (\frac{\partial \dot{v}}{\partial y})^2$$

$$+ \frac{1}{2} b_7 (\dot{\theta})^2 + \frac{1}{2} b_8 (\frac{\partial \dot{\theta}}{\partial x}) + \frac{1}{2} b_9 (\frac{\partial \dot{\theta}}{\partial y})^2$$

$$+ b_{10} \dot{u} \frac{\partial \dot{\theta}}{\partial y} + b_{11} \frac{\partial \dot{u}}{\partial y} \dot{\theta} - b_{12} \dot{v} \frac{\partial \dot{\theta}}{\partial x} - b_{13} \frac{\partial \dot{v}}{\partial x} \dot{\theta} \tag{2}$$

respectively. In Equations (1) and (2), u is the displacement in x-direction, v is the displacement in y-direction, θ may be regarded as the "smoothed" joint rotation, and the coefficients are given by

$$a_1 = \frac{A_1 E_1}{L_2}, \qquad a_2 = \frac{A_2 E_2}{L_1}, \qquad a_3 = \frac{12 E_2 I_2}{L_1 L_2^2}$$

$$a_4 = \frac{12 E_1 I_1}{L_1^2 L_2}, \qquad a_5 = \frac{4 E_1 I_1}{L_2}, \qquad a_6 = \frac{4 E_2 I_2}{L_1}$$

$$b_1 = \frac{\rho_1 A_1}{L_2} + \frac{\rho_2 A_2}{L_1}, \qquad b_2 = \frac{\rho_1 A_1 L_1^2}{3 L_2}, \qquad b_3 = \frac{13 \rho_2 A_2 L_2^2}{35\ L_1}$$

$$b_4 = \frac{\rho_2 A_2}{L_1} + \frac{\rho_1 A_1}{L_2}, \qquad b_5 = \frac{13 \rho_1 A_1 L_1^2}{35\ L_2}, \qquad b_6 = \frac{\rho_2 A_2 L_2^2}{3 L_1}$$

$$b_7 = \frac{\rho_1 A_1 L_1^2}{210 L_2} + \frac{\rho_2 A_2 L_2^2}{210 L_1}, \qquad b_8 = \frac{\rho_1 A_1 L_1^4}{105 L_2}, \qquad b_9 = \frac{\rho_2 A_2 L_2^4}{105 L_1}$$

$$b_{10} = \frac{\rho_2 A_2 L_2^2}{12 L_1}, \qquad b_{11} = \frac{9\rho_2 A_2 L_2^2}{420 L_1}, \qquad b_{12} = \frac{\rho_1 A_1 L_1^2}{12 L_2}$$

$$b_{13} = \frac{9\rho_1 A_1 L_1^2}{420 L_2} \tag{3}$$

where A is the cross-sectional area, L is the length, ρ is the mass density, and the subscripts 1 and 2 indicate the beams parallel to the x-direction and y-direction, respectively.

3. The Simple Model

For simplicity and evaluative purposes, we assume that the framework is composed of identical columns and identical girders. Consequently, the representative continuum is homogeneous. The nonhomogeneous case can be easily extended.

The simple model will be derived following basically the same path in the derivation of Timoshenko beam theory. Denoting the longitudinal direction by x-axis, (see Fig. 1), the displacement field in the continuum representing the framework is approximated by

$$\begin{aligned} u &= u_o(x) + y\psi(x) \\ v &= v_o(x) \\ \theta &= \theta_o(x) \end{aligned} \tag{4}$$

where u_o is the displacement at the central axis, v_o is the lateral displacement, and ψ represents the gross flexural rotation of the structure. It is noted that both the lateral displacement and joint rotation are assumed to be uniform in the y-direction. Such assumption is obviously valid only for lower modes of vibration. A more accurate account of the displacement field is obtained by adding more terms in the approximate displacements. However, retaining more terms would result in more complicated governing equations, and thus lose the desired simplicity of the model.

Substituting the approximate displacement components, Equation (4), in Equations (1) and (2) and integrating over the width h we obtain the beam energy densities (in-lb/in) as

$$U_B = \int_{-h/2}^{h/2} U\, dy = \frac{1}{2} a_1\, h\left(\frac{\partial u_o}{\partial x}\right)^2 + \frac{1}{24} a_1 h^3 \left(\frac{\partial \psi}{\partial x}\right)^2 + \frac{1}{2} a_3\, h(\psi + \theta_o)^2 + \frac{1}{2} a_4\, h\left(\frac{\partial v_o}{\partial x} - \theta_o\right)^2 + \frac{1}{2} a_5\, h\left(\frac{\partial \theta_o}{\partial x}\right)^2 \quad (5)$$

$$T_B = \int_{-h/2}^{h/2} T\, dy = \frac{1}{2} b_1\, h(\dot{u}_o)^2 + \frac{1}{24} b_1\, h^3(\dot{\psi})^2 + \frac{1}{2} b_4\, h(\dot{v}_o)^2 + \frac{1}{2} b_7\, h(\dot{\theta}_o)^2 \quad (6)$$

It should be noted that the expression given by Equation (6) is the result of neglecting the strain rate terms which are of secondary importance in this application.

Corresponding to the independent variations δu_o, δv_o, $\delta \psi$ and $\delta \theta_o$, the variation of the work done by the external forces and moments is given by

$$\delta W_e = Q_{xL}\ \delta u_o(L) - Q_{xo}\delta u_o(o) + Q_{yL}\delta v_o(L)$$

$$- Q_{yo}\ \delta v_o(o) + M_L\delta\psi(L) - M_o\delta\psi(o)$$

$$+ m_L\delta\theta_o(L) - m_o\delta\theta_o(o) \tag{7}$$

where Q_x and Q_y are the end forces in the x-direction and y-direction, respectively; M can be regarded as the gross bending moment; m is the local moment or joint moment; the subscripts L and o on the forces and moments indicate the positions x=L and o, respectively.

From the Hamilton's principle

$$\delta \int_{t_o}^{t_1} dt \int_o^L (T_B - U_B)dx + \int_{t_o}^{t_1} \delta W_e\ dt = o \tag{8}$$

we obtain the equations of motion

$$\begin{aligned}
&a_1\ \frac{\partial^2 u}{\partial x^2} = b_1\ddot{u}\\
&a_4(\frac{\partial^2 v}{\partial x^2} - \frac{\partial\theta}{\partial x}) = b_4\ddot{v}\\
&a_5\frac{\partial^2\theta}{\partial x^2} + a_4(\frac{\partial v}{\partial x} - \theta) - a_3(\psi+\theta) = b_7\ \ddot{\theta}\\
&\frac{h^2}{12}\ a_1\ \frac{\partial^2\psi}{\partial x^2} - a_3(\psi+\theta) = \frac{h^2}{12}\ b_1\ \ddot{\psi}
\end{aligned} \tag{9}$$

and the natural boundary conditions

$$a_1\ h\ \frac{\partial u}{\partial x} = Q_x$$

$$a_4\ h(\frac{\partial v}{\partial x} - \theta) = Q_y$$

$$a_5 \, h \, \frac{\partial \theta}{\partial x} = m \tag{10}$$

$$\frac{h^3}{12} \, a_1 \, \frac{\partial \psi}{\partial x} = M$$

In Equations (9-10), the subscript o on the displacements is, and henceforth will be, deleted for the sake of brevity.

It can be easily noted that the longitudinal motion is governed by the first equation in Equation (9) and is uncoupled from the other motions.

4. Vibration of Plane Frame

We now consider free vibration of "homogeneous" plane frames. Since the lower modes are essentially lateral (horizontal) motions, we assume that u=0 and

$$\begin{aligned} v &= V \, e^{nx} \, e^{i\omega t} \\ \theta &= \Theta \, e^{nx} \, e^{i\omega t} \\ \psi &= \Psi \, e^{nx} \, e^{i\omega t} \end{aligned} \tag{11}$$

where V, Θ and Ψ are constant amphitudes, ω is the angular frequency, and n will be determined by the equations of motion as well as the boundary conditions.

Substitution of Equation (11) in Equation (9) yields

$$\begin{aligned} &(a_4 n^2 + b_4 \, \omega^2) \, V - a_4 n \, \Theta = 0 \\ &a_4 n V + [a_5 n^2 + b_7 \omega^2 - (a_3 + a_4)]\Theta - a_3 \Psi = 0 \\ &a_3 \, \Theta - (a_6 n^2 + a_7 \omega^2 - a_3) \, \Psi = 0 \end{aligned} \tag{12}$$

A nontrivial solution requires that the determinant of the coefficient matrix be vanishing, i.e.,

$$\begin{vmatrix} a_4n^2 + b_4\omega^2 & -a_4n & 0 \\ a_4n & a_5n^2 + b_7\omega^2 - (a_3 + a_4) & -a_3 \\ 0 & a_3 & -a_6n^2 - a_7\omega^2 + a_3 \end{vmatrix} = 0 \tag{13}$$

By expanding the determinant, a sextic equation with real coefficients in n results. For a given value of ω there are six roots for n; each root yields a partial solution. In order to satisfy the boundary conditions, all the six roots must be included. The general form of the solution is given by

$$\begin{aligned} v &= \sum_{i=1}^{6} V_i\, e^{n_i x}\, e^{i\omega t} \\ \theta &= \sum_{i=1}^{6} \alpha_i\, V_i\, e^{n_i x}\, e^{i\omega t} \\ \psi &= \sum_{i=1}^{6} \beta_i\, V_i\, e^{n_i x}\, e^{i\omega t} \end{aligned} \tag{14}$$

where n_i (i=1-6) are the six roots of n for an ω, and

$$\alpha_i = \frac{\theta_i}{V_i} \quad , \qquad \beta_i = \frac{\psi_i}{V_i} \tag{15}$$

The boundary conditions are

$$\begin{aligned} v = \theta = \psi = 0 \quad &, \qquad x = 0 \\ Qy = M = m = 0 \quad &, \qquad x = L \end{aligned} \tag{16}$$

Substituting the general solution, Equation (14), in Equation (16) we obtain six homogeneous equations:

$$\sum_{i=1}^{6} V_i = 0$$

$$\sum_{i=1}^{6} \alpha_i V_i = 0$$

$$\sum_{i=1}^{6} \beta_i V_i = 0 \qquad (17)$$

$$\sum_{i=1}^{6} (n_i-\alpha_i)e^{n_i L} V_i = 0$$

$$\sum_{i=1}^{6} \alpha_i n_i e^{n_i L} V_i = 0$$

$$\sum_{i=1}^{6} \beta_i n_i e^{n_i L} V_i = 0$$

The system of homogeneous equations for $V_i (i=1-6)$ has a nontrivial solution if the coefficient matrix $[D_{ij}]$ vanishes, i.e.,

$$|D_{ij}| = 0 \qquad (18)$$

Equation (18) is the frequency equation from which the natural frequencies are determined.

5. Further Approximation and Shear Correction

For some structures with wider base, the lower modes do not have significant gross flexural deformation. The equations of motion can be further simplified by suppressing the variable ψ. The result is a reduction in number of differential equations. We have

$$a_4 \frac{\partial^2 v}{\partial x^2} - a_4 \frac{\partial \theta}{\partial x} = b_4 \ddot{v}$$

$$a_5 \frac{\partial^2 \theta}{\partial x^2} + a_4 \frac{\partial v}{\partial x} - (a_3 + a_4)\,\theta = b_7 \ddot{\theta} \tag{19}$$

The natural boundary conditions become

$$Q_y = a_4\, h\left(\frac{\partial v}{\partial x} - \theta\right)$$

$$m = a_5\, h \frac{\partial \theta}{\partial x} \tag{20}$$

The frequency equation according to Equations (19-20) can be derived in the same manner as described in the previous section.

A shear correction factor is introduced to compensate the inaccurate assumed transverse shear deformation in the Timoshenko type beam and plate theories [5]. For plates and beams with a rectangular cross-section the shear correction factor is found to be $\pi^2/12$. In the present work, we will adopt this factor for "shear correction". With shear correction, the equations remain the same except that the coefficient a_4 should be replaced by $a_4\pi^2/12$. In the later numerical examples, it proves that the use of the shear correction factor actually improve the simple model solution.

6. Numerical Example and Discussion

A 15 story frame structure with 4 bays is first considered for evaluative purpose. The dimensions and material properties are given as follows (subscript 1 denotes column and 2 denotes girder):

Length: $L_1 = L_2 = 300$ in

Young's modulus: $E_1 = E_2 = 30 \times 10^6$ psi

Moment of inertia: $I_1 = I_2 = 4 \times 10^3$ in^4

Mass density: $\rho_1 = \rho_2 = 7.45 \times 10^{-4}$slug/in^3

Cross-sectional area: $A_1 = A_2 = 29.1$ in^2

The first five natural frequencies are calculated by using the simple model, the approximate equations, and the finite element method. The finite element solution that takes all members into consideration will be regarded as the "exact solution" for the sake of comparison. Both the solutions with and without shear correction are obtained. The results are presented in Table 1. In general, the simple model with or without shear correction yields excellent agreement with the solution obtained by finite element method (F.E.M.). On the other hand, the solutions according to the approximate equations, Equation (19), are acceptable from the engineering practice view point, although the discrepancies are in general larger. It is noted that the shear correction improves the approximate solution substantially. It is interesting to note that the discrepancy between the approximate solution and the finite element solution is larger for lower modes, as the gross flexural deformation is more pronounced in the lower modes.

In Figs. 2-4, the mode shapes of the first four modes for the lateral displacement v, the gross rotation ψ (flexural deformation), and the joint rotation θ are presented, respectively. The simple model solutions (without shear correction) are compared with the finite element solutions. The gross rotation with the finite element solution is computed by dividing the relative axial displacement of the outer columns by the width h, and the joint rotations are taken from the central columns. From the comparisons, it is easy to see that the agreement is excellent for the transverse displacement and gross rotation. As for the joint rotation, the simple model is not as accurate as in predicting the transverse displacement. However, the discrepancy seems to lie within practical error tolerance.

Two additional cases are investigated with the results shown by Table 2 and Table 3, respectively. The member dimensions and material properties are identical to those used in the first example, except that the number of story and the number of bays are now different. From the comparison with the finite element solution, it is found that accuracy of the simple model depends on the height of the frame structure. For shorter and wider structures, the error in natural frequency of the fifth mode (see Table 3) is 60 percent. This discrepancy is, of course, expected as such short structures under higher mode vibration can hardly be represented by a one-dimensional beam model.

The application of the present simple model to more realistic structures with varying member dimensions is conceptually straightforward. The result is a system of partial differential equations with variable coefficients rather than constant coefficients. Solving this type of differential equations is much more difficult. It seems more desirable to develop a finite element based upon the simple continuum model to handle the nonhomogeneous properties as well as other more general types of boundary conditions.

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Mode	F.E.M.	Simple Model		Approximate Equations	
		No Shear Correction	Shear Correction	No Shear Correction	Shear Correction
1	4.26	4.24	4.08	4.84	4.58
2	13.08	13.2	12.60	14.60	13.82
3	23.18	23.98	22.74	24.62	23.30
4	33.03	34.34	32.5	35.00	33.06
5	43.32	45.32	45.82	42.78	43.20

Table 1. Natural Frequencies ω for a 15 Story - 4 Bay Structure with $L_1 = L_2 = 300$ in.

Mode	F.E.M.	Simple Model		Approximate Equations	
		No Shear Correction	Shear Correction	No Shear Correction	Shear Correction
1	9.86	10.28	9.76	10.63	10.04
2	30.14	31.64	29.92	32.52	30.68
3	52.22	55.64	52.36	56.00	52.64
4	72.68	80.84	75.72	81.34	76.12
5	96.86	107.32	100.44	108.32	100.9

Table 2. Natural Frequencies ω for a 7 Story - 4 Bay Structure with $L_1 = L_2 = 300$ in.

Mode	F.E.M.	Simple Model		Approximate Equations	
		No Shear Correction	Shear Correction	No Shear Correction	Shear Correction
1	17.94	19.24	18.12	19.24	18.14
2	54.51	59.74	56.02	59.78	56.06
3	90.51	104.94	97.78	104.96	97.78
4	101.03	153.90	142.52	153.94	142.54
5	115.76	186.04	185.90	204.90	188.80

Table 3. Natural Frequencies ω for a 4 story - 15 Bay Structure with $L_1 = L_2 = 300$ in.

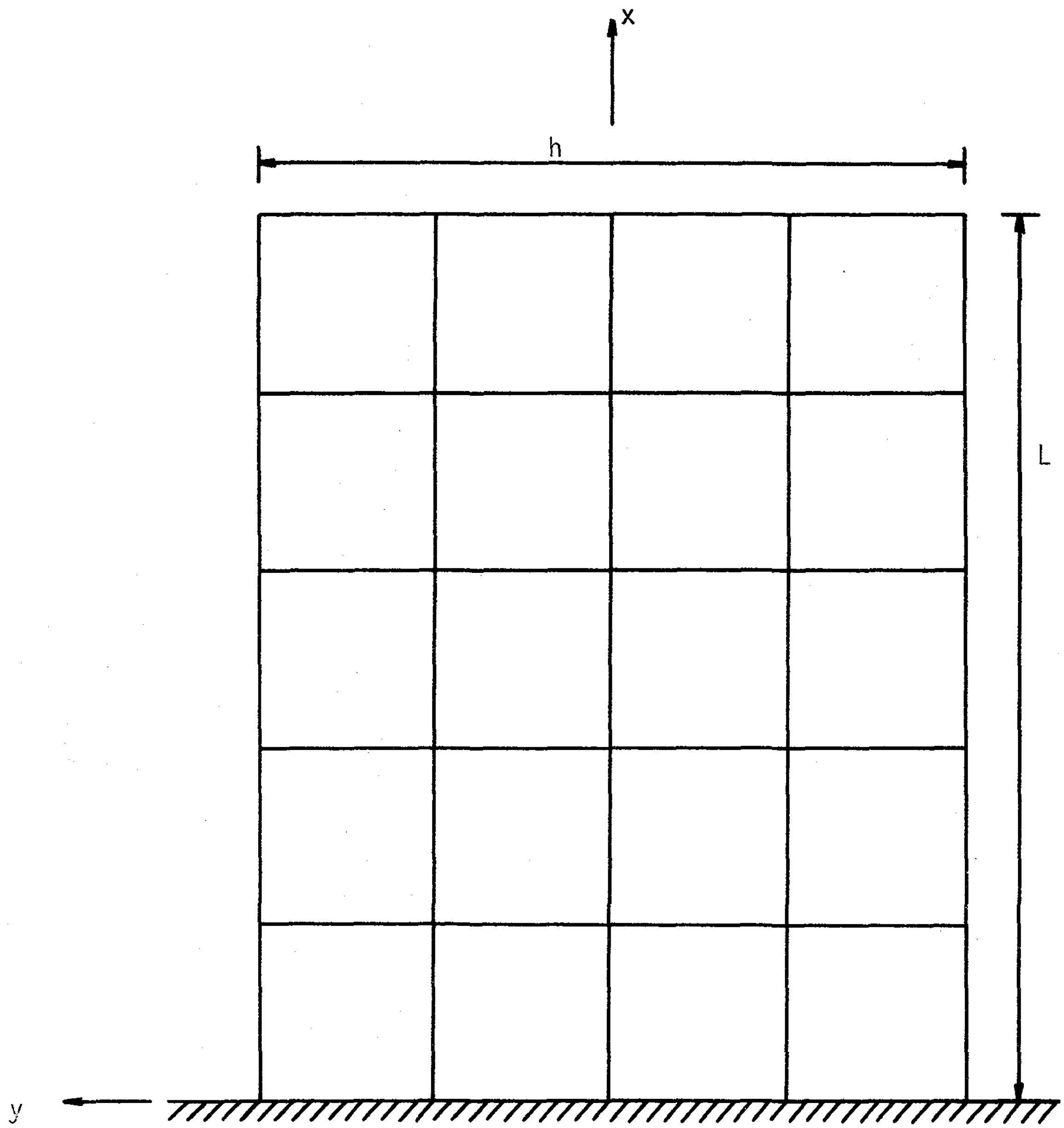

Fig. 1. Geometry and Coordinate System.

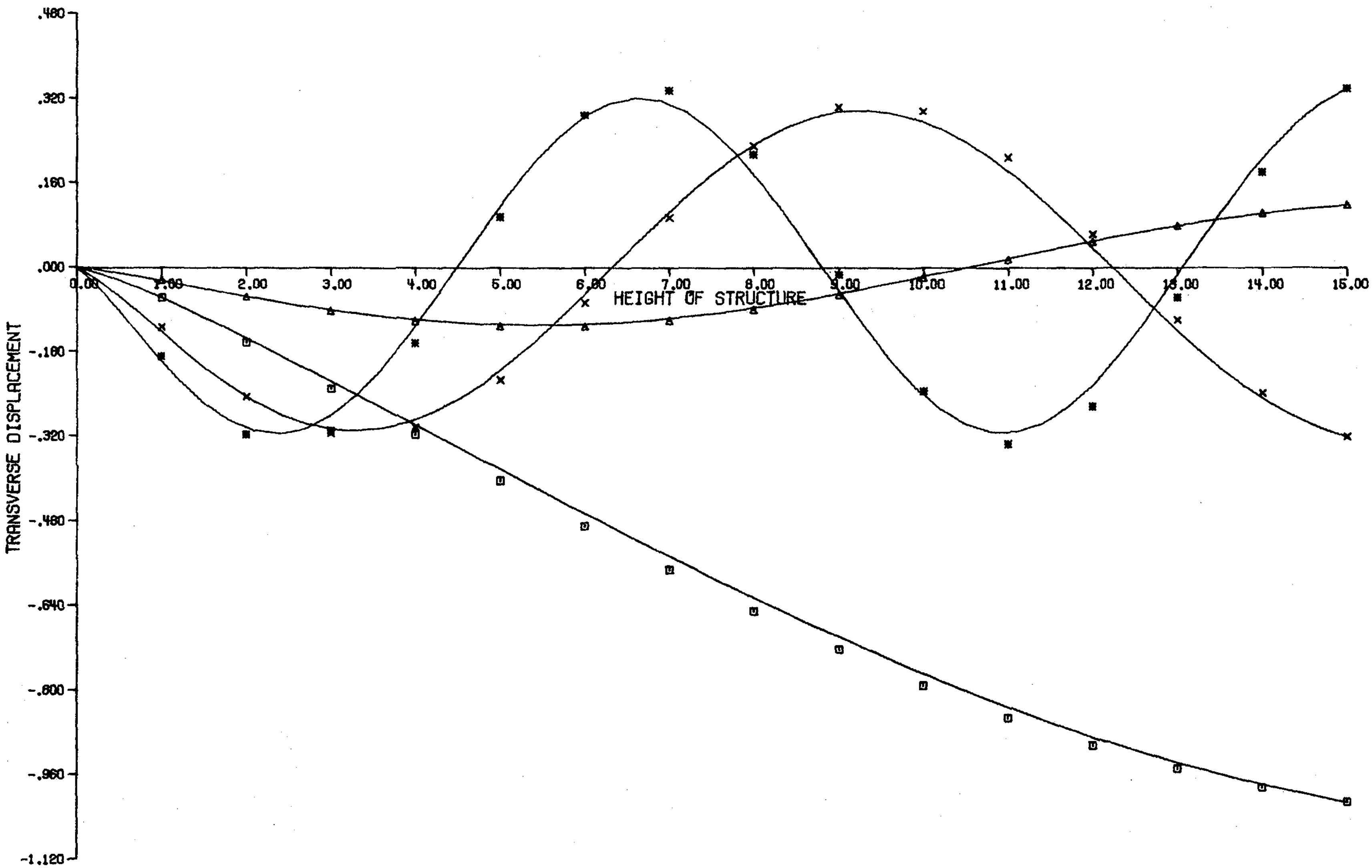

Fig. 2. The First Four Modes for Transverse Displacement. The Solid Lines Indicate Simple Model Solutions. Finite Element Solutions are indicated by ⊡ : First Mode, Δ : Second Mode, x : Third Mode, and * : Fourth Mode.

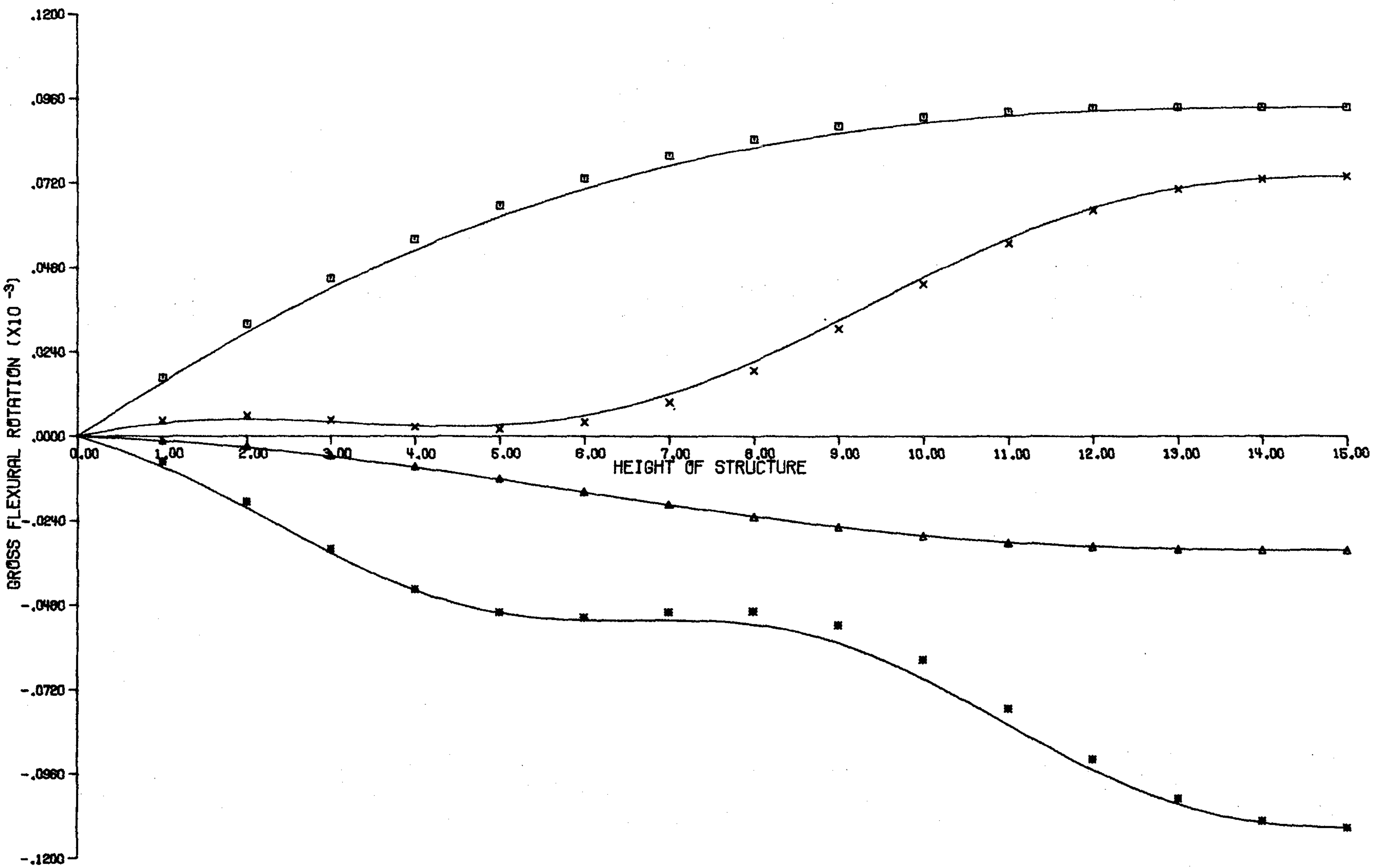

Fig. 3. Mode Shapes for the Gross Flexural Rotation ψ.

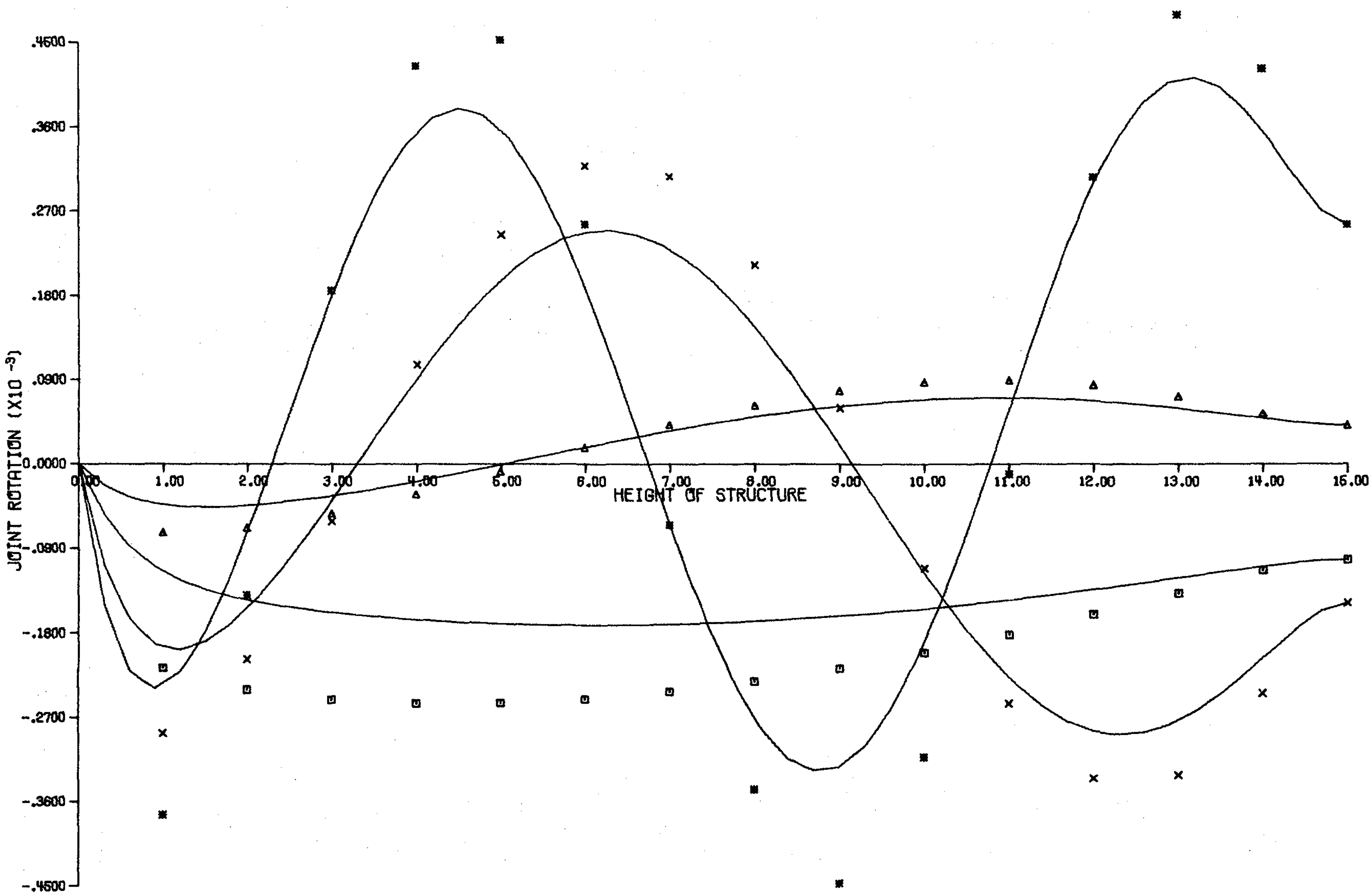

Fig. 4. Mode Shapes for the Joint Rotation θ.