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A TIMOSHENKO BEAM MODEL FOR VIBRATION OF PLANE FRAMES

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16. Abstracts A detailed dynamic analysis, reported in this series, was conducted on the seismic response and structural safety of key subsystems (steam generator, high pressure steam piping, coal handling equipment, cooling tower, chimney) of Unit #3 of TVA at Paradise, Kentucky in order to: (1) determine for the key components the natural frequencies below 50 Hz and the corresponding normal modes; (2) determine response of plant to seismic disturbances; (3) verify through full scale tests results obtained in (1) and determine estimates of damping needed in (2); (4) determine potential failure modes of major structural components; and (5) determine a spare parts policy for a power system so that outages due to damage from seismic disturbances are minimal. Analytical and experimental methods are used. This volume includes discussion of the use of the Timoshenko beam model with bending and shear rigidities evaluated by a procedure that takes the joint rotation into account in a quasi-static manner. Explicit formulas for bending and shear rigidities of the model are derived in terms of the member dimensions and material properties of the original frame structure. A shear beam model is derived from the Timoshenko beam model. Solutions for three evaluative examples are presented and compared with exact finite element solutions.

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A TIMOSHENKO BEAM MODEL FOR VIBRATION OF PLANE FRAMES

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Summary Report

Prior to 1974 there has been no detailed dynamic analysis of the seismic structural response and safety of large fossil-fuel steam generating plants. In March, 1974, under NSF Grant GI41897, a detailed dynamical analysis was begun on the seismic response and structural safety of key subsystems

(steam generator,
high pressure steam piping,
coal handling equipment,
cooling tower,
chimney)

of Unit #3 of TVA at Paradise, Kentucky to accomplish the following objectives:

a) Determine for the key components the natural frequencies below 50 Hz and the corresponding normal modes.

b) Determine response of plant to seismic disturbances.

c) Verify through full scale tests, where possible, results obtained in a), and determine estimates of damping needed in b).

d) Determine potential failure modes of major structural components.

e) Determine a spare parts policy for a power system so that outage due to damage from seismic disturbances are minimal.

Analytical and experimental methods are used.

The attached Reports present what has been accomplished to date.

Before making a few summarizing remarks on the individual Reports, some comments must be made in order to provide perspective on the study.

Paradise, Unit #3 of TVA was selected for study because near-by mine operations provide excitation (due to blasting) for the plant, and TVA was willing to cooperate in the conduct of the study. It should be pointed out that this plant was not designed to resist earthquakes. However, it was felt that this disadvantage was outweighed by the experimental possibilities.

The key components selected for study are critical for operation of the plant and would cause significant outage if damaged. All components can be studied using similar types of analyses. These are the basic reasons for including in this study only the steam generator, high pressure piping, coal handling equipment, cooling tower, and chimney.

Basic data for the analyses were obtained from drawings provided by TVA and Babcock-Wilcox. In addition to these data, a number of assumptions had to be introduced into the analyses. These assumptions refer in the main to the nature of the connections among elements of known properties, the fixity of columns, the properties of hanger elements, etc. Choices were made based on physical as well as computational reasons.

The analyses were confined to the linear range. After such a study, it is possible to assess at what level of excitation parts of the structure become nonlinear.

Structure-foundation interaction was neglected. Unit #3 of Paradise rests on excavations in limestone. It is assumed that there is little interaction. However, experimental studies will be made on this point.

It was decided at the start that all computations would be carried out with an existing computer program. SAP IV was chosen. Some program modifications have proved necessary, but these have been relatively minor. To obtain familiarity with the program it was necessary to study a number of special cases of the actual structure to ensure that it was functioning properly. For example, substructures within the steam generator support were considered seperately; assumed values of viscous damping coefficients were used in generating time histories*; etc. We found the program execution

* It should be noted that the magnitude of the response with zero damping must be interpreted with some caution as systems with slightly different frequencies can exhibit significantly different magnitudes of response.

time slow in some respects which indicates that some of its internal subroutines, such as eigen value solution, could be improved. It is beyond the scope of this project, however, to improve existing programs.

The experimental part of the study has proved much more difficult to conduct than anticipated. TVA has been most cooperative. However, the sheer physical size of the units, the weather, etc. have caused a number of difficulties that were not easy to foresee. Progress is gradually being achieved.

Interest in simple models stems from their possible use in design studies. It was decided to develop a methodology for constructing simple models. At present, our simple models are in the embryonic stage. It is hoped that after the study of two more plants a useful methodology can be obtained. Simple models developed could have been used for one component under study; however, timing made this impossible.

No recommendations will be made or conclusions drawn at this time, except in special situations. The partial examination of one plant does not provide a sufficient basis for such actions. At the completion of the study conclusions and recommendations will be presented.

A number of factors of some importance have not been considered so far. For example, the steam generator's internal elements can move with respect to it, the steam piping exerts dynamic forces on its supports, dynamic stresses in steam piping are just part of its stress system, many different seismic excitations are available, plus many more. Also a spare parts policy was not considered. As additional progress is made, we shall consider some of these problems. However, it must be recognized that it is possible to consider in this study only those factors of major importance. A spare parts policy involves economic considerations; it may not be possible to acquire the information needed to address this point.

Contact with industry in this country and Japan clearly indicates that the current detailed study is of great interest.

An Advisory Committee consisting of

Carl L. Canon	- Babcock & Wilcox Product Design Supervisor for Structural Steel and Design
William A. English	- Tennessee Valley Authority Head Civil Engineer
Clinton H. Gilkey	- Combustion Engineering, Inc. Manager, Engineering Science
Richard F. Hill	- Federal Power Commission Acting Director, Office of Energy Systems

R. Bruce Linderman	-	Bechtel Power Corporation Engineering Specialist
D. P. Money	-	Foster-Wheeler Corporation Supervisor of Stress Analysis
R. D. Sands	-	Burns & McDonnell Chief Mechanical Engineer
Erwin P. Wollak	-	Pacific Gas & Electric Company Supervisor, Civil Engineering Division

has been formed to provide a forum for an interchange of practical and conceptual views on various aspects of the study. The aim is to ensure that what is developed (in simple models) will be of practical use to industry. The Advisory Committee has met twice and reviewed plans and the progress of the investigation.

Contact is also maintained with the following firms:

Mitsubishi Heavy Industries
Babcock-Hitachi
Ishikawajima Harima Heavy Industries
Kawasaki Heavy Industries
Taiwan Power Company

The initial visit provided considerable information on the methods they have used in seismic response studies conducted by the research groups in each organization and plant experience under seismic disturbances.

Comments from the Advisory Committee and reviewers have been most helpful and encouraging. Many of the comments have been considered. However, it is not possible to take account in our studies of all points that have been brought to our attention.

Five professors, 8-10 graduate students, 2 technicians, and a secretary devoted part time to the study. A great deal of effort was devoted to acquiring information and equipment. The cooperation of TVA and Babcock-Wilcox was most helpful and deeply appreciated. Progress was excellent when it is remembered that education of students is a major function of a University.

This research project was sponsored by NSF through Grant No. GI41897.

The Reports in this series are as follows:

Dynamic Behavior of the Steam Generator and Support Structures of the 1200 MW Fossil Fuel Plant, Unit #3, Paradise, Kentucky, by T.Y. Yang, M.I. Baig, J.L. Bogdanoff.

The High Pressure Steam Pipe, by C.T. Sun, A.S. Ledger, H. Lo.

Coal Handling Equipment, by K.W. Kayser and J.A. Euler.

Theoretical Study of the Earthquake Response of the Paradise Cooling Tower, by T.Y. Yang, C.S. Gran, J.L. Bogdanoff.

Theoretical Study on Earthquake Response of a Reinforced Concrete Chimney, by T.Y. Yang, L.C. Shiau, H. Lo.

A Simple Continuum Model for Dynamic Analysis of Complex Plane Frame Structures, by C.T. Sun, H. Lo, N.C. Cheng, and J. L. Bogdanoff.

A Timoshenko Beam Model for Vibration of Plane Frames, by C.T. Sun, C.C. Chen, J.L. Bogdanoff, and H. Lo.

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1. Introduction

The conventional shear beam approximation for multistory buildings has long been recognized to be inadequate as it cannot account for the joint rotation and gross flexural deformation [1]. Extending the shear beam model to include the gross bending effect can be achieved by using the Timoshenko beam. However, there are no known methods for including an independent degree of freedom in the conventional Timoshenko beam theory. An alternative approach is to use the concept of couple stress as described in [2].

Because of its simplicity and popularity among the structural engineers, the conventional shear beam approach deserves additional attention in overcoming its disadvantages in its original form. The purpose of this research is to use the Timoshenko beam model with bending and shear rigidities evaluated by a procedure that takes the joint rotation into account in a quasi-static manner. Explicit formulas for bending and shear rigidities of the model are derived in terms of the member dimensions and material properties of the original frame structure. A shear beam model is derived from the Timoshenko beam model. Solutions for three evaluative examples are presented and compared with exact finite element solutions.

2. The Timoshenko Beam Theory

For convenience of reference, the basic formulation of the Timoshenko beam theory is reviewed first. The strain energy and kinetic energy per unit length of beam are given by

$$U = \frac{1}{2} EI(\frac{\partial \psi}{\partial x})^2 + \frac{1}{2} \kappa \; A \; G(\frac{\partial v}{\partial x} - \psi)^2 \qquad (1)$$

and

$$T = \frac{1}{2} m(\dot{v})^2 + \frac{1}{2} \rho \; I(\dot{\psi})^2 \tag{2}$$

respectively [3]. In Equations (1-2), v is the transverse displacement, ψ is the rotation of cross-section, m is the mass per unit length, ρI is the rotatory inertia, and κ is a shear correction coefficient. In this work, we will adopt the value $\kappa = 1$.

In the absence of lateral loadings, the equations of motion are expressed as

$$\begin{aligned} \frac{\partial}{\partial x}(EI \frac{\partial \psi}{\partial x}) + \kappa \, A \, G(\frac{\partial v}{\partial x} - \psi) &= \rho \; I \; \ddot{\psi} \\ \frac{\partial}{\partial x}[\kappa \, A \, G(\frac{\partial v}{\partial x} - \psi)] &= m \; \ddot{v} \end{aligned} \tag{3}$$

The shear force Q and the bending moment M are related to the displacements as

$$Q = \kappa \, A \, G(\frac{\partial v}{\partial x} - \psi) \tag{4}$$

and

$$M = EI \frac{\partial \psi}{\partial x} \tag{5}$$

respectively.

3. The Simple Model

The use of a Timoshenko beam to represent a frame structure requires determination of the effective bending rigidity EI and shear rigidity GA of the frame. Once these properties are established, the vibration analysis is performed in the usual manner. Evaluation of the overall shear and bending rigidities of a frame structure by considering all the members exactly can be done on the computer with the aid of the finite element method. However, such approach apparently defeats the purpose of a simple model. In view of this, we try to break down the frame structure into typical substructures and then evaluate the contribution of each substructure in the gross shear and bending rigidities.

3.1 Evaluation of the Shear Rigidity

The typical substructures are indicated in Fig. 1. Since the base of the structure is assumed fixed, the behavior of the structural members at the ground level is substantially different from the rest, and, hence, are considered separately. For a tall building where the boundary effect is relatively small, it might not be necessary to make such distinction.

It is due to make a note about substructures Type a and Type b for which only half length of the column is taken. In a lower mode vibration, it is assumed that, away from the base, the deformation is somewhat "smooth" in the longitudinal direction, and consequently the middle point of a column between two adjacent floors is a point of inflection. Similar argument leads to the conclusion that the midpoint of a girder is a point of inflection. For the present purpose, these points are then replaced by equivalent hinges and rollers as shown in Fig. 1. The substructures at the ground level for the shear rigidity analysis are depicted also in Fig. 1.

To illustrate the procedure for evaluating the effective shear rigidity contributed by the substructures, we consider substructure Type a. Applying a horizontal force P at the joint as shown in Fig. 2 we can easily obtain the corresponding displacement δ. In this analysis, the rigid frame assumption is taken. In other words, the axial deformation is neglected. The resulting displacement is obtained as

$$\delta = \frac{L_c^3(2 + \alpha\beta)}{24EI_c \ \alpha\beta} \ P \tag{6}$$

where E is the Young's modulus assumed to be identical for columns and girders, I_c is the moment of inertia of the column, L_c is the column length between two adjacent floors, and α and β are defined by

$$\alpha = I_g/I_c$$
$$\beta = L_c/L_g \tag{7}$$

in which a subscript g denotes girder.

The equivalent shear strain is

$$\gamma = \frac{\delta}{L_c/2} \tag{8}$$

The shear force associated with this amount of shear strain in an equivalent Timoshenko beam is

$$(GA)_a\gamma = (GA)_a \frac{2\delta}{L_c} \tag{9}$$

where $(GA)_a$ is the effective shear rigidity provided by the column under consideration. Substitution of Eq. (6) in (9) yields

$$(GA)_a = \frac{12\alpha\beta EI_c}{(2 + \alpha\beta)L_c^2} \tag{10}$$

for substructure Type a.

By similar procedures, we obtain

$$(GA)_b = \frac{12\alpha\beta EI_c}{(1 + \alpha\beta)L_c^2} \tag{11}$$

for substructure Type b,

$$(GA)_c = \frac{12(1 + 3\alpha\beta)EI_c}{(4 + 3\alpha\beta)L_c^2} \tag{12}$$

for substructure Type c, and

$$(GA)_d = \frac{6(1 + 6\alpha\beta)EI_c}{(2 + 3\alpha\beta)L_c^2} \tag{13}$$

for Type d. The total effective shear rigidity of the frame structure between two floors is the sum of the shear rigidities of the appropriate substructures.

3.2 Evaluation of the Bending Stiffness

The gross bending effect can be large for tall structures. In the present consideration, we will assume that the joints of the same floor level displace linearly in the vertical direction with respect to the centroidal axis of the cross-sectional areas of columns as shown in Fig. 3. This assumption is obviously valid only in lower modes of vibration. In addition, it is assumed that the restoring forces in the vertical direction are provided by the axial forces in columns. Denoting the incremental rotation at a floor level relative to the lower one by ϕ (see Fig. 3), the relative displacement at joint i is given by

$$\delta_i = d_i \; \phi \tag{14}$$

where d_i is the horizontal distance from the joint to the centroidal axis. If we assume that strain is constant in the column, then the corresponding axial force is

$$P_i = EA \frac{\delta_i}{L_c} = EA_i \; d_i \; \frac{\phi}{L_c} \tag{15}$$

where A_i is the cross-sectional area of the column. The moment about the centroid due to P_i is

$$M_i = P_i \; d_i = EA_i \; d_i^2 \; \phi/L_c \tag{16}$$

The total bending moment of the whole structure is obtained as

$$M = \sum_i M_i = E \frac{\phi}{L_c} \sum_i A_i \, d_i^2 \tag{17}$$

Equating Eq. (17) to Eq. (5) we obtain

$$EI \frac{\partial \psi}{\partial x} = E \frac{\phi}{L_c} \sum_i A_i \, d_i^2 \tag{18}$$

where EI represents the effective bending stiffness of the equivalent Timoshenko beam model. Since strain is assumed constant in columns, we have

$$\frac{\partial \psi}{\partial x} = \frac{\phi}{L_c} \tag{19}$$

between any two adjacent floors. As a result, the effective bending stiffness can be expressed in the form

$$EI = E \sum_i A_i \, d_i^2 \tag{20}$$

3.3 Diagonal Bracing

Diagonal bracing is often used in flexible frames to provide additional lateral resistance against earthquake motion. Since a brace is usually designed to take axial forces only, it can be considered as an axial member without bending rigidity. The additional shear and overall bending rigidities of the structure due to a brace will be calculated based upon this assumption. Furthermore, it is assumed that the braces and frames act independently, so that the bracing stiffness can be added to the frame stiffness. This approach was taken also by Clough and Jenschke [4] in a study on the effect of diagonal bracing on the earthquake performance of a steel frame building.

With the foregoing assumptions, the shear stiffness and the longitudinal stiffness of a brace can be obtained by analyzing the simple problems shown in Fig. 4a and Fig. 4b, respectively. The additional effective shear rigidity is obtained as

$$(GA)_{Br} = A_b\, E \cos^2\theta \sin\theta \tag{21}$$

where A_b is the cross-sectional area of the brace. The above expression can also be written as

$$(GA)_{Br} = \frac{\beta}{(1 + \beta^2)^{3/2}} A_b\, E \tag{22}$$

in which β is defined by Eq. (7).

Similarly, the additional effective gross bending rigidity due to the brace is obtained as

$$(EI)_{Br} = E\, A_b\, d^2 \sin^3\theta \tag{23}$$

or

$$(EI)_{Br} = \frac{\beta^3}{(1 + \beta^2)^{3/2}} A_b\, E\, d^2 \tag{24}$$

in which d denotes the distance between the upper joint of the brace to the centroidal axis of the cross-sectional areas of columns.

4. The Timoshenko Beam Finite Element

Since the resulting equivalent Timoshenko beam for the frame structure is, in general, nonhomogeneous, it will be more convenient to employ the finite element method for solution. Derivation of the stiffness and consistent mass matrices is quite straightforward. Hence, only the results will be presented here.

Consider a Timoshenko beam finite element of length L, bending stiffness EI, shear rigidity GA, mass per unit length m, and rotatory inertia ρI (ρ is the mass density and the moment of inertia I is obtained from the value of EI). By assuming the shape functions for the beam displacement variables as

$$\begin{aligned} v &= a_0 + a_1 x + a_2 x^2 + a_3 x^3 \\ \psi &= b_0 + b_1 x + b_2 x^2 + b_3 x^3 \end{aligned} \tag{25}$$

together with the energy functions given by Eqs. (1) and (2), the total strain energy and kinetic energy in the beam element can be computed. The matrix equation of motion for the discrete system is obtained by using Hamilton's principle. We have

$$\{F\} = [k]\{\Delta\} + [m]\{\ddot{\Delta}\} \tag{26}$$

where [k] and [m] are the element stiffness matrix and element mass matrix, respectively. In Eq. (26), the force vector {F} and the displacement vector $\{\Delta\}$ are defined by

$$\{F\} = \begin{Bmatrix} Q_1 \\ N_1 \\ M_1 \\ \mu_1 \\ Q_2 \\ N_2 \\ M_2 \\ \mu_2 \end{Bmatrix} \text{ and } \{\Delta\} = \begin{Bmatrix} v_1 \\ v_1' \\ \psi_1 \\ \psi_1' \\ v_2 \\ v_2' \\ \psi_2 \\ \psi_2' \end{Bmatrix} \tag{27}$$

respectively. The subscripts 1 and 2 denote node 1 and node 2, respectively, and a prime indicates the slope. It should be noted that with this higher order element, only Q and M can be realized in the boundary conditions; the generalized forces N and μ are set equal to zero at both clamped end and free end.

The stiffness matrix is obtained as

$$[k] = \left[\begin{array}{c|c} k_1 & k_2 \\ \hline k_2^T & k_4 \end{array}\right] \tag{28}$$

where the submatrices are given by

$$[k_1] = \begin{bmatrix} 504b & -42Lb & -210Lb & -42L^2b \\ & 56L^2b & -42L^2b & 0 \\ & & 36a + 156L^2b & 3La + 22L^3b \\ \text{sym.} & & & 4L^2a + 4L^4b \end{bmatrix} \tag{29}$$

$$[k_2] = \begin{bmatrix} -504b & -42Lb & -210Lb & -42L^2b \\ 42Lb & -14L^2b & 42L^2b & -7L^3b \\ 210Lb & 42L^2b & -36a + 54L^2b & 3La - 13L^3b \\ 42L^2b & 7L^3b & -3La + 13L^3b & -L^2a - 3L^4b \end{bmatrix} \tag{30}$$

$$[k_4] = \begin{bmatrix} 504b & 42Lb & 210Lb & -42L^2b \\ & 56L^2b & -42L^2b & 0 \\ & & 36a + 156L^2b & -3La + 22L^3b \\ \text{sym.} & & & 4L^2a + 4L^4b \end{bmatrix} \tag{31}$$

In Eqs. (29-31),

$$a = \frac{EI}{30L} \quad , \quad b = \frac{\kappa G A}{420L} \tag{32}$$

The mass matrix is expressed in the form

$$[m] = \left[\begin{array}{c|c} m_1 & m_2 \\ \hline m_2^T & m_4 \end{array}\right] \tag{33}$$

where

$$[m_1] = \begin{bmatrix} 156c & -22Lc & 0 & 0 \\ & 4L^2c & 0 & 0 \\ & & 156e & 22Le \\ \text{sym.} & & & 4L^2e \end{bmatrix} \tag{34}$$

$$[m_2] = \begin{bmatrix} 54c & 13Lc & 0 & 0 \\ -13Lc & -3L^2c & 0 & 0 \\ 0 & 0 & 54e & -13Le \\ 0 & 0 & 13Le & -3L^2e \end{bmatrix} \tag{35}$$

$$[m_4] = \begin{bmatrix} 156c & 22Lc & 0 & 0 \\ & 4L^2c & 0 & 0 \\ & & 156e & -22Le \\ \text{sym.} & & & 4L^2e \end{bmatrix} \tag{36}$$

and

$$c = \frac{\rho A L}{420} \quad , \quad e = \frac{\rho I L}{420} \tag{37}$$

5. Shear Beam Model

For shorter frame structures, the gross bending effect could be negligible. If the gross rotation ψ is set equal to zero in the Timoshenko beam model, then we obtain a shear beam model which is infinitely rigid in bending. The strain energy per unit length becomes

$$U_s = \frac{1}{2}\kappa A G\left(\frac{\partial v}{\partial x}\right)^2 \quad (38)$$

and the kinetic energy reduces to

$$T_s = \frac{1}{2} m(\dot{v})^2 \quad (39)$$

The corresponding equation of motion is

$$\frac{\partial}{\partial x}\left[\kappa A G \frac{\partial v}{\partial x}\right] = m \ddot{v} \quad (40)$$

It is important to note that the present shear beam model is different from the conventional shear beam model for which the floors are assumed to be rigid. Besides being able to account for the girder flexibility, the present shear beam model is also more elaborate in evaluating the effective shear rigidity as has been described in Section 3.

To retain the simplicity, the following displacement function is assumed for the shear beam finite element:

$$v = (1 - \frac{x}{L})v_1 + v_2 x \quad (41)$$

where v_1 and v_2 are the nodal displacements and L is the element length. The stiffness matrix is

$$[k] = \frac{\kappa G A}{L} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \tag{42}$$

A lump-mass matrix of the form

$$[m] = \begin{bmatrix} m_{11} & 0 \\ 0 & m_{22} \end{bmatrix} \tag{43}$$

will be used. The lump-mass method in general overestimates the inertia effect, and is thus more suitable for the shear beam as the suppression of bending also has a stiffening effect of the structure.

6. Evaluative Examples

Free vibrations of three plane frames of different characteristics are investigated by the present simple models and by the conventional finite element method (with detailed structures). The latter solution is regarded as the exact solution for comparison purposes.

Example 1: 10 Bay - 9 Story Plane Frame

The geometry and material constants of the plane frame are given in Fig. 5. The corresponding Timoshenko beam is also shown. Nine Timoshenko beam finite elements are used with the floor mass lumped at the nodes. The mass per unit length, m, of the Timoshenko beam is calculated according to

$$m = \sum \rho A_c \tag{44}$$

The effective shear rigidities of the four types of substructures are obtained from Eqs. (10-13) as

$$(GA)_a = 2.4 \frac{EI_c}{L_c^2}$$

$$(GA)_b = 4 \frac{EI_c}{L_c^2}$$

$$(GA)_c = \frac{60}{11} \frac{EI_c}{L_c^2} \qquad (45)$$

$$(GA)_d = \frac{48}{7} \frac{EI_c}{L_c^2}$$

The effective shear rigidity of the Timoshenko beam element at the ground level is obtained as

$$GA = (9 \times \frac{48}{7} + 2 \times \frac{60}{11}) \frac{EI_c}{L_c^2} = 3.137 \times 10^5 \text{ kips} \qquad (46)$$

The rest have the effective shear rigidity given by

$$GA = (9 \times 4 + 2 \times 2.4) \frac{EI_c}{L_c^2} = 1.762 \times 10^5 \text{ kips} \qquad (47)$$

Similarly, the effective bending rigidity is obtained as

$$EI = 5.7024 \times 10^{10} \text{ kips-ft}^2 \qquad (48)$$

The angular frequencies for the first three modes of vibration obtained by using nine Timoshenko beam elements as well as nine shear beam elements are shwon in Table 1. Although the Timoshenko beam model yields a better agreement with the "exact" solution, it should be noted that the shear beam model also proves adequate. In fact the difference between the two approximate solutions is negligible from the practical standpoint. The small discrepancies between the Timoshenko beam model and the shear beam model in this case should be expected as the structure is relatively short as compared to its lateral dimension, and, as a consequence, the bending effect is not pronounced.

Fig. 6 shows the mode shapes of the Timoshenko beam model, the shear beam model, and the mode shapes of the middle column based on the exact finite element solution. Excellent agreement is noted.

Example 2: 4 Bay - 15 Story Plane Frame

Fig. 7 shows the dimensions and material constants of the structure. The floor masses are again lumped at the respective levels as shown in the figure. Fifteen elements are used for both Timoshenko beam model and shear beam model. The angular frequencies for the first five modes are presented in Table 2. The Timoshenko beam solutions are again in close agreement with the exact solutions. The shear beam solutions are still quite acceptable as the maximum error stays within 10% from the exact solutions.

One would naturally expect that the frequencies obtained based on the shear beam model should be higher than those according to the Timoshenko beam model. This, however, is not reflected from the present results. The reason is that in the shear beam finite element solution, we employ the lump-mass matrix which overestimates the inertia effect especially in the higher modes.

In Fig. 8 the mode shapes for the first three modes are shown. Excellent agreement is evident.

Example 3: 4 Bay - 7 Story Plane Frame with Bracing

When bracing is present, the additional stiffnesses due to the braces are calculated according to Eqs. (22) and (24). For the braced frame structure shown in Fig. 9 the floor masses are lumped at each level; and the brace masses are divided equally between two adjacent floors. The frequencies are presented in Table 3, and the mode shapes are shown in Fig. 10. It should be pointed out that the third mode is a longitudinal mode and cannot be accounted for by either the Timoshenko beam model or the shear beam model. The result for mode 3 presented in Table 3 is obtained by considering the structure as an axial member with the effective axial rigidity

$$EA = E \sum A_c + E \sum \frac{\beta^3}{(1 + \beta^2)^{3/2}} A_b \tag{49}$$

The stiffness matrix of the axial member is the same in form as that for the shear beam except that κGA should now be replaced by EA as given by Eq. (49).

As revealed in this example, the longitudinal mode might appear as one of the lower modes in vibration if the structure is heavily braced. In this case, the axial (longitudinal) motion must be also investigated and compared with the results obtained from the Timoshenko beam model.

7. Conclusions

A Timoshenko beam model and its reduced shear beam model for analyzing vibration of plane frames are presented. Explicit formulas for evaluating the effective stiffnesses are derived. Finite elements based on these simple models are also formulated. Comparisons of the simple model solutions with the exact finite element solutions show that the present beam models are quite adequate in predicting the natural frequencies and the mode shapes for the lower modes. When the frame structures are heavily braced, it is found that longitudinal motion might appear in the lower modes of vibration. A model of axial member for the longitudinal motion is also derived.

The simple models proposed in this report could be useful in seismic dynamic analysis of frame structures where lower modes usually dominate the response. It could be of particular interest to the designer in the primary design stage using parametric study to measure structural safety based upon dynamic considerations.

References

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[3] Y. C. Fung, Foundations of Solid Mechanics, Prentice-Hall, 1965.

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MODE	EXACT FREQUENCY	TIMOSHENKO BEAM		SHEAR BEAM	
		FREQUENCY	ERROR %	FREQUENCY	ERROR %
1	0.768	0.768	0	0.769	0.1
2	2.351	2.299	-2.2	2.281	-3.0
3	4.073	3.813	-6.4	3.718	-8.7

Table 1 Angular frequencies for the first three modes in Example 1.

MODE	EXACT FREQUENCY	TIMOSHENKO BEAM		SHEAR BEAM	
		FREQUENCY	ERROR %	FREQUENCY	ERROR %
1	4.26	4.49	5.4	4.68	9.98
2	13.08	13.88	6.1	14.00	7.1
3	23.18	24.03	3.6	23.15	-0.1
4	33.03	33.16	0.4	32.04	-3.0
5	43.32	42.33	-2.3	40.55	-6.4

Table 2 Angular frequencies for the first five modes in Example 2.

MODE	EXACT FREQUENCY	TIMOSHENKO BEAM		SHEAR BEAM	
		FREQUENCY	ERROR %	FREQUENCY	ERROR %
1	22.18	20.86	-5.9	20.80	-6.2
2	62.73	66.54	6.1	66.45	5.9
3	105.13	105.32	0.2		
4	110.62	119.05	7.6	119.00	7.6
5	131.56	161.53	22.8	161.41	22.7

Table 3 Angular frequencies for the first five modes in Example 3.

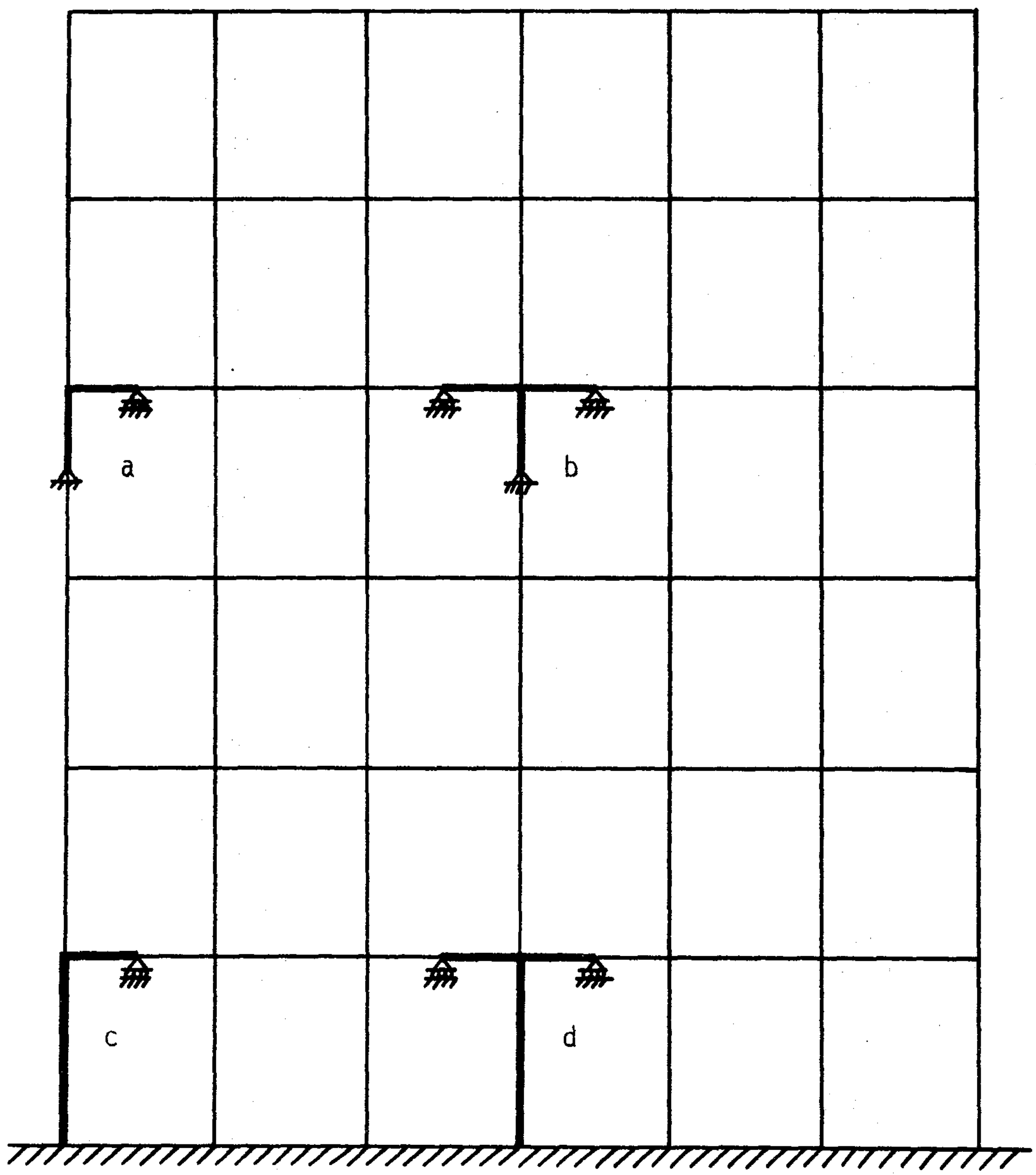


Fig. 1. Four types of substructures.

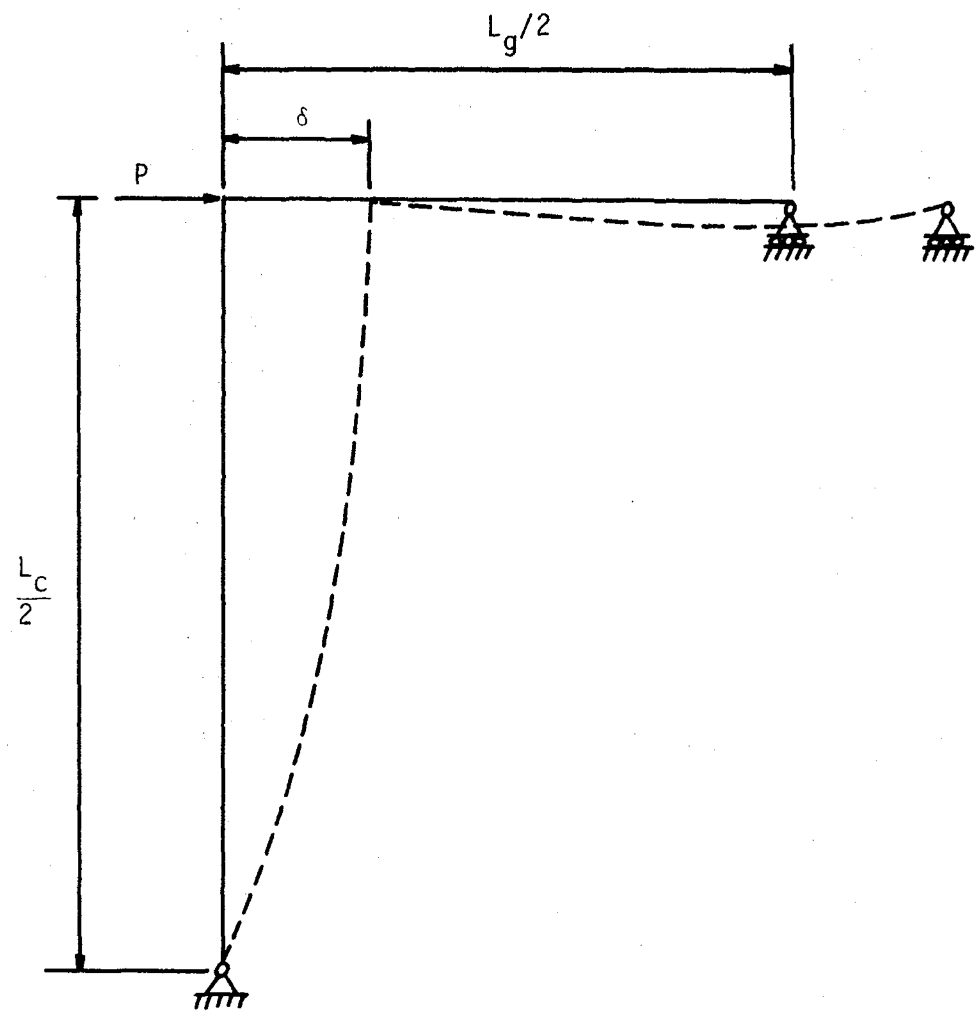


Fig. 2. Effective shear rigidity of Substructure Type a.

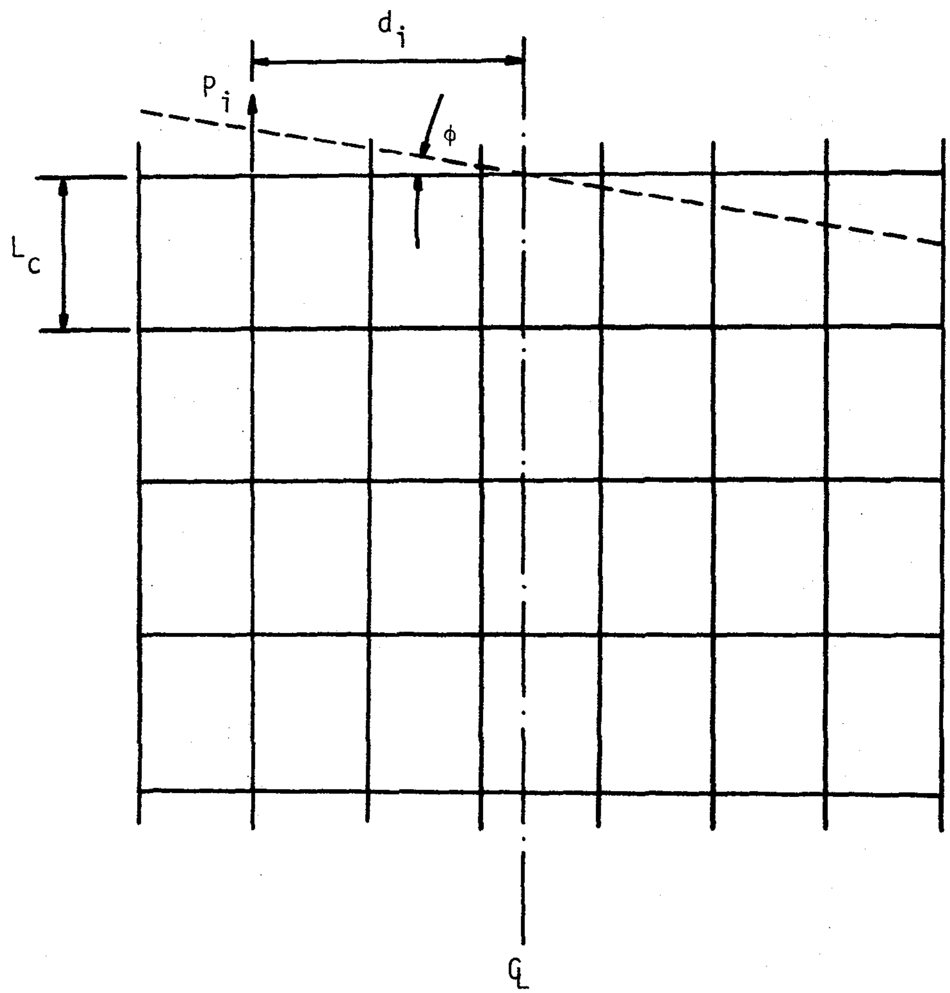


Fig. 3. Gross bending deformation.

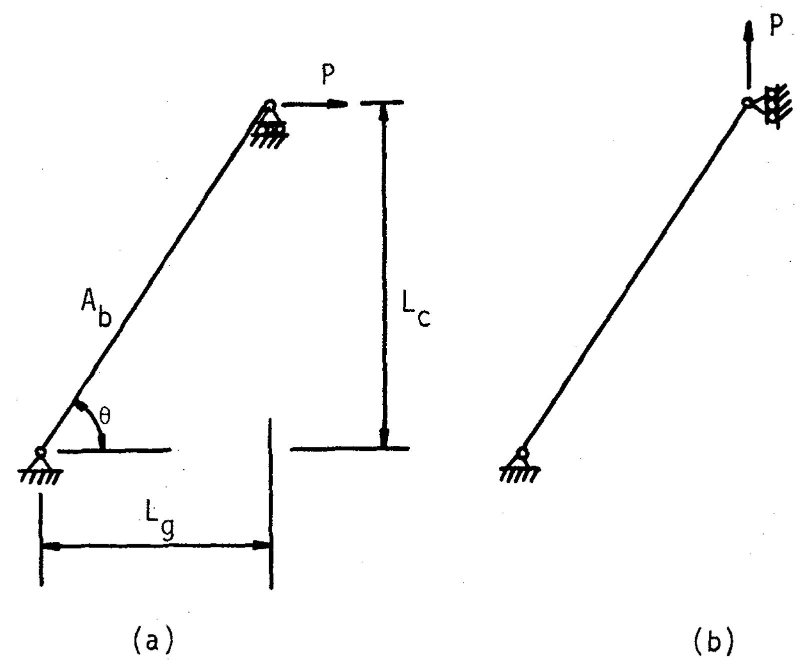
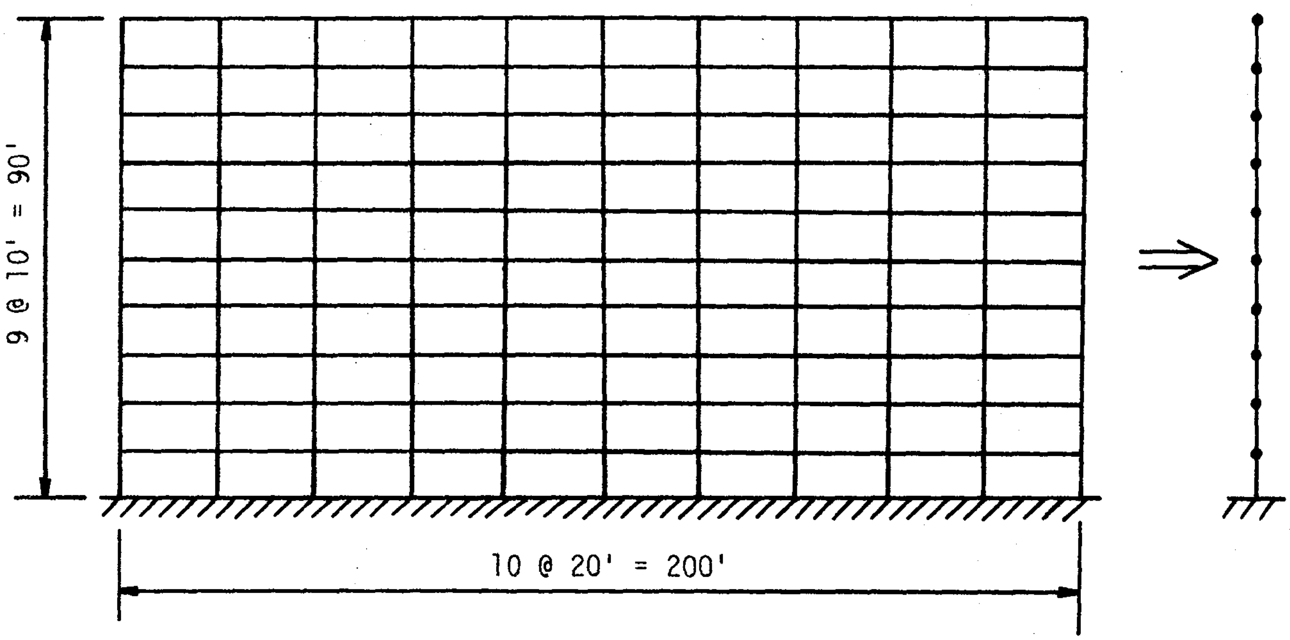


Fig. 4. Effective shear and bending stiffness of a brace.



E = 432000 , ρ = 1.0

$A_c = A_g = 3.0$, $I_c = I_g = 1.0$ for all members.

(units: FT, KIPS)

Fig. 5. Example 1.

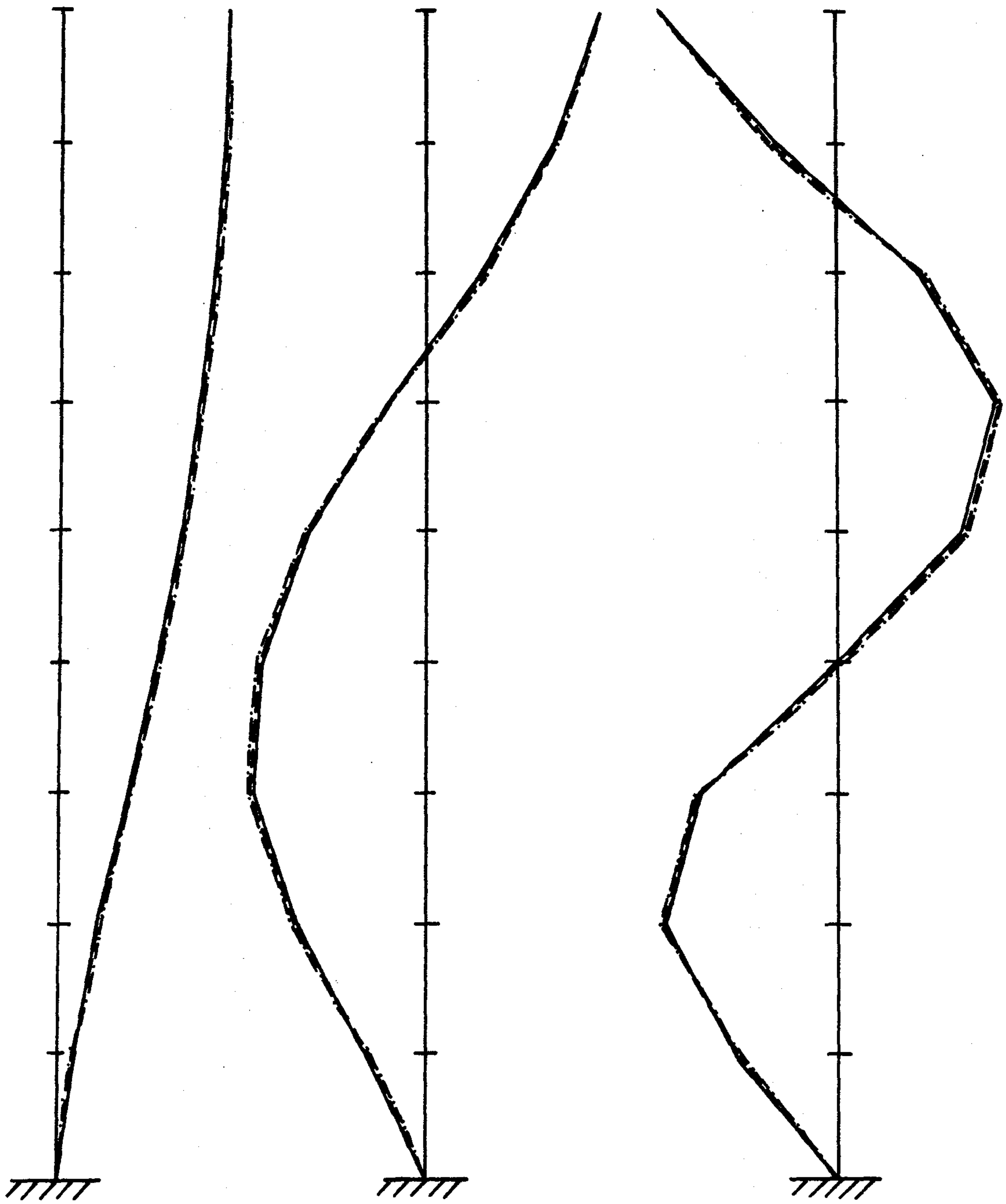
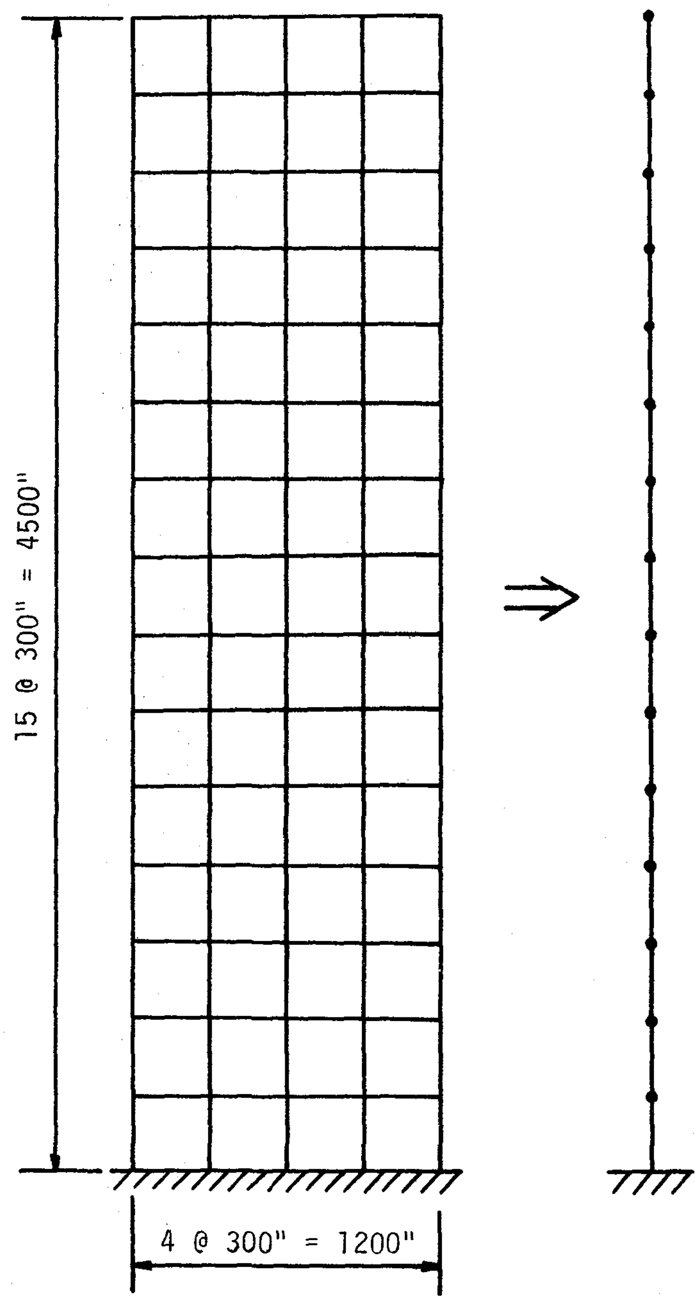


Fig. 6. Mode-shapes for Example 1 (——— exact, ---- Timoshenko beam, —·—·—· Shear beam).



$E = 3.0 \times 10^7$, $\rho = 7.45 \times 10^{-4}$

$A_c = A_g = 29.1$, $I_c = I_g = 4000$

(units: IN, LB)

Fig. 7. Example 2.

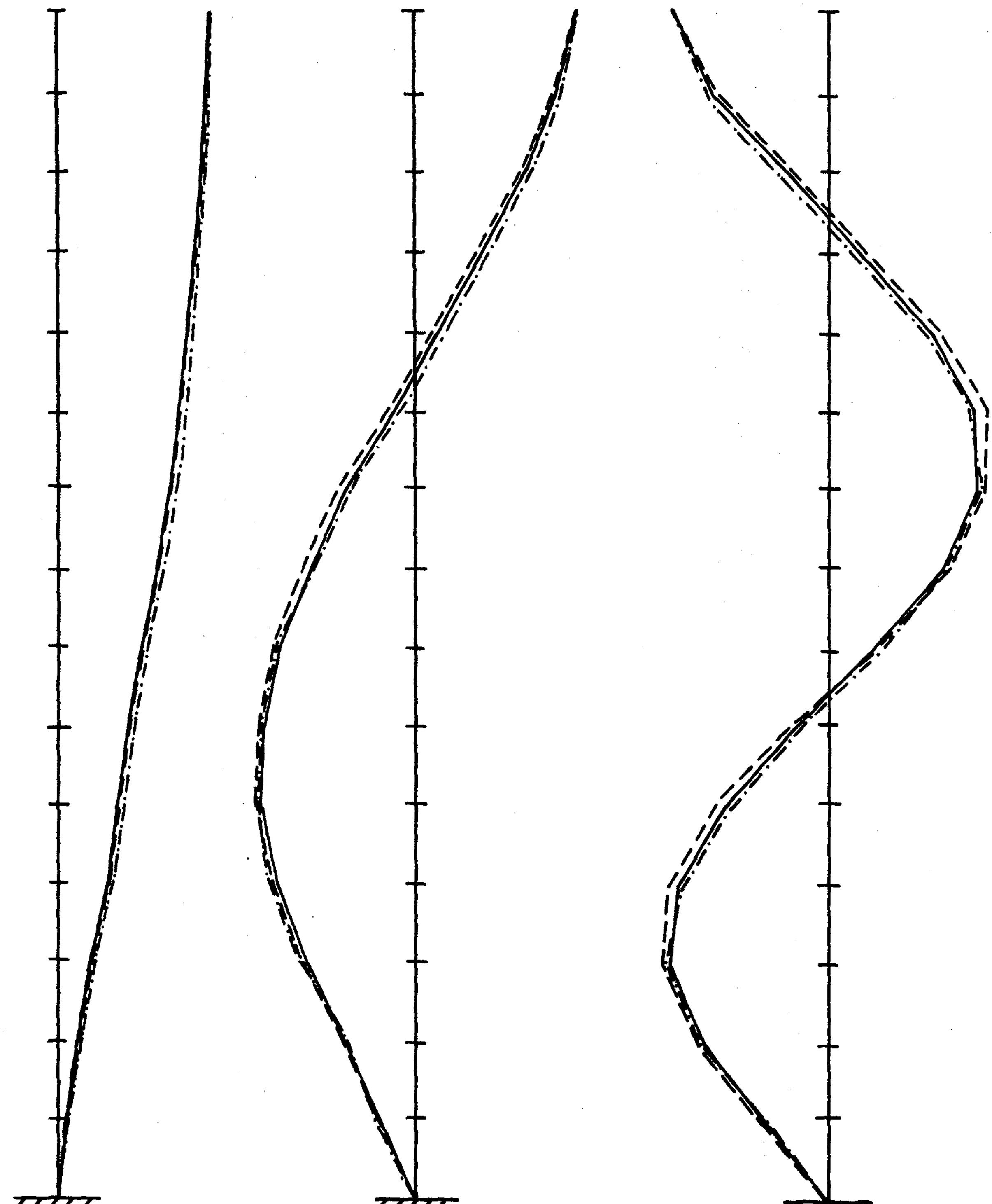
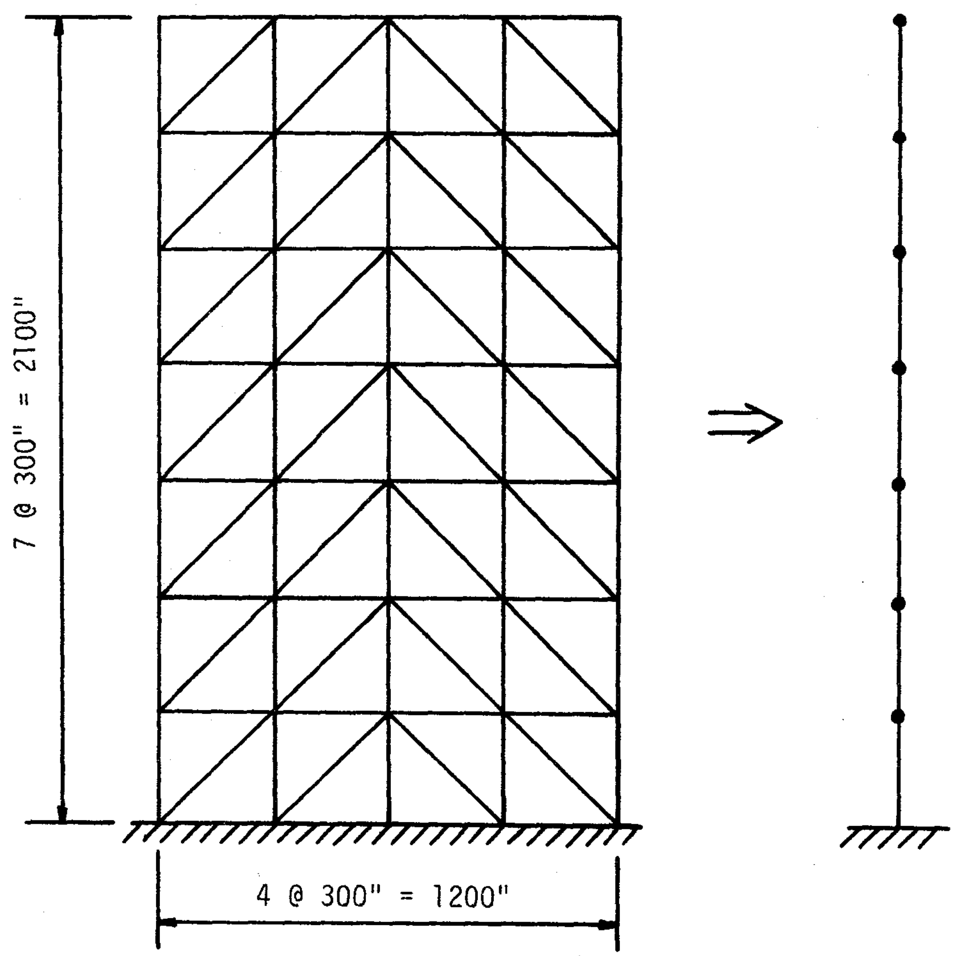


Fig. 8. Mode shapes for Example 2 (——— exact, ---- Timoshenko beam, —·—·— shear beam).



$E = 3.0 \times 10^7$, $\rho = 7.45 \times 10^{-4}$

$A_c = A_g = 29.1$, $I_c = I_g = 4000$

$A_b = 5$

(units: IN, LB)

Fig. 9. Example 3.

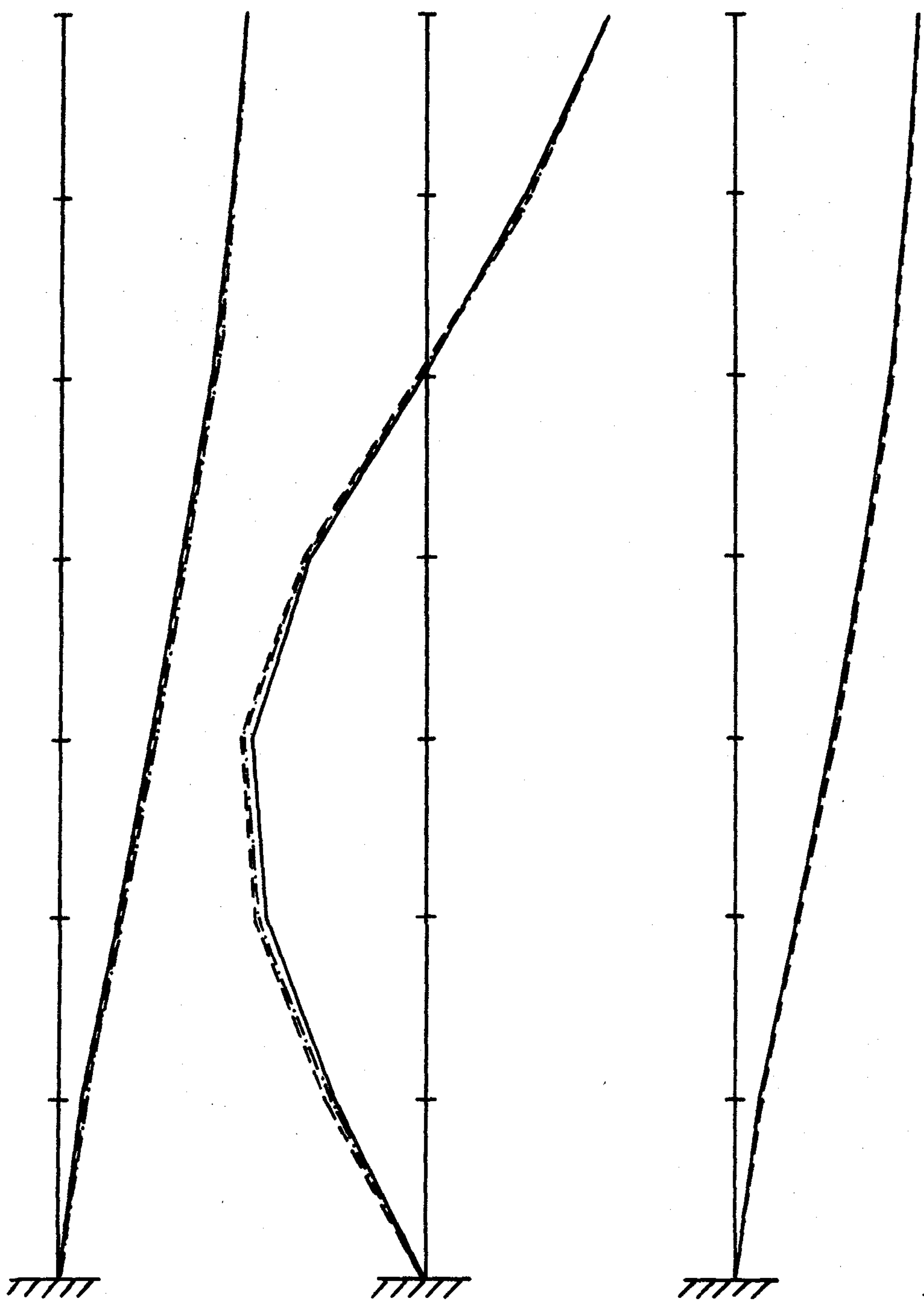


Fig. 10. Mode shapes for Example 3 (—— exact, ---- Timoshenko beam, —·—·— shear beam). The third mode is a longitudinal mode and the amplitude indicates vertical displacement.