### Abstract

The frequency response characteristics of two shaking tables have been determined experimentally. The lighter table, weighing 2,000 lb (900 kg), was used primarily to determine the effects of a resonant structure on a shaking table's frequency response. The heavier table, weighing 100,000 lb (45,300 kg), was used primarily to determine the effects of foundation compliance on a shaking table's frequency response.

Mathematical models were formulated for both tables, and the models were refined by adjusting parameters to obtain the best correspondence between the computed and experimental frequency responses. The mathematical models were then used to study the effects of a resonant structure and of foundation compliance on the frequency responses of shaking tables and on the ability of shaking tables to reproduce earthquake-type motions.

It was found that the magnitudes of the peak and notch distortions in the frequency response of a shaking table are sensitive to the amount of force feedback employed by the control system. In addition, the magnitudes depend on the ratio of the mass of the structure to the mass of the shaking table and to the transmissibility function of the structure with respect to the table. Although the peak and notch effect may cause difficulties in determining the frequency response of structures by means of shaking tables, it has little effect on the accuracy to which a shaking table can reproduce earthquake-type motions.

It was found that foundation compliance affects the frequency response of a shaking table only at low frequencies, and the magnitude of the effect is limited to an amount which depends on the transmissibility function of the foundation with respect to the table.
ABSTRACT

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It was found that foundation compliance affects the frequency response of a shaking table only at low frequencies, and the magnitude
of the effect is limited to an amount which depends on the transmissibility function of the foundation with respect to the table.
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1. INTRODUCTION

Shaking tables are being employed with increasing frequency in earthquake engineering to study the effects of earthquake type excitations on models of structures and small structures constructed from realistic structural components (1). The objective of these tests is to determine the responses of the test specimens to earthquake type excitations, especially in cases where the intensity of the excitation is strong enough to cause nonlinear behavior in the specimen.

Shaking tables designed specifically for use in earthquake engineering are generally driven by high-performance hydraulic rams, or actuators, equipped with servo-valves. In these systems, the position of the actuator piston is controlled by means of an electronic closed-loop displacement feedback system. The basic displacement feedback is supplemented by velocity and force feedback signals to improve performance characteristics.

There are physical limitations to the intensity of motions shaking tables may undergo. Generally, at low frequencies the limitations are displacements imposed by the actuator strokes, at intermediate frequencies the limitations are velocities imposed by the maximum flow capacities of the servo-valves or pumps, and at higher frequencies the limitations are accelerations imposed by the force capacities of the actuators. These limitations for a particular shaking table are depicted in Fig. 1.1.

Within their physical limitations, shaking tables loaded only with rigid masses will exhibit frequency responses typical of second-order systems. If the control system were open loop, the corner
frequency would occur at the oil column resonant frequency which is a function of the total mass being driven and the compliance of the oil contained in the actuators' cylinders. However, in closed-loop systems the corner frequency may be significantly lower than the oil column resonant frequency. The form of the frequency response limitations is also depicted in Fig. 1.1.

A shaking table loaded with a structure may have a distortion in its frequency response at the resonant frequency of the structure\(^{(2)}\). In addition, compliance in the actuator supports, which is likely in the case of large shaking tables, may adversely affect the frequency response\(^{(3)}\). Since these two effects may impair the performance of a shaking table, the degree to which they can alter frequency response characteristics has been investigated by a combination of experimental and analytical procedures.

Experimentally, the effects of a resonant structure on the frequency response characteristics of a shaking table can be investigated most conveniently by means of a relatively light table. In contrast, the investigation of foundation compliance needs a relatively heavy table. Thus two tables, one weighing 2,000 lb (900 kg) and the other 100,000 lb (45,300 kg) were used in the experimental phase of the investigation.

In the analytical phase of the investigation mathematical models were formulated for both tables, and the models were refined by altering their parameters in order to match the computed frequency responses with the experimental ones. The effects of a resonant structure and foundation compliance on the frequency response of shaking tables and on their ability to reproduce earthquake type motions were then studied by means of the mathematical models.
2. EXPERIMENTAL FREQUENCY RESPONSES

Experimental frequency responses were determined for two shaking tables. The lighter shaking table, weighing 2,000 lb (900 kg), was used to determine the effects of load resonance on the frequency response since a large ratio of structure mass to table mass could be achieved easily. The effects of foundation compliance on frequency response were investigated experimentally by means of a 100,000 lb (45,300 kg) shaking table because then the table weight is large enough relative to the foundation weight that some significant effects might be expected.

2.1 2,000 lb Shaking Table

A shaking table with one horizontal direction of motion only, driven by a single hydraulic actuator, and loaded by a single degree of freedom test structure, is shown in Fig. 2.1. The shaking table is a rectangular steel platform with overall dimensions of 10 ft x 7 ft (3 m x 2 m) weighing 2,000 lb. The longer sides are formed by two 10 x 12 WF beams which are connected transversely by four 6 ft (1.8 m) lengths of rectangular tubing. Four 10 ft (3 m) lengths of 6 x 2 in. (150 x 50 mm) channel sections are welded across the bottom faces of the rectangular tubes and run the length of the platform. The table is supported on two V and two single Thompson linear bearings. Each bearing runs on an 18 in. (450 mm) length of 2 in. (50 mm) diameter bar which is grouted into a 3 x 1 1/2 in. (75 x 38 mm) channel bolted to a strong floor.

The single degree of freedom structure shown mounted on the table in Fig. 2.1 consists of a platform supported by four columns.
The platform, which is identical to the shaking table, weighs 2,000 lb (900 kg). The far two columns have pillow block bearings at both ends and thus do not contribute to the horizontal stiffness of the single degree of freedom structure. The near two columns also have pillow block connections at their tops but are bolted rigidly to the shaking table and provide the horizontal stiffness of the single degree of freedom structure by bending about their weak axes.

The table is driven by an hydraulic actuator having an effective area of 25.4 in.$^2$ (164 cm.$^2$) and a stroke of 12 (±6) in. (300(±150)mm). The actuator is equipped with a two stage servo-valve that can feed oil to the actuator at a rate up to 175 gallons (0.67 m$^3$) per minute. The position of the actuator's piston is controlled by means of a closed loop feedback system. In addition to the primary displacement feedback signal, secondary feedback signals consisting of actuator force and piston velocity are used for stabilizing the primary feedback loop. A schematic diagram of the control system is shown in Fig. 2.2. MTS Systems supplied the hydraulic actuator and its associated hydraulic and electronic components.

Frequency responses were determined for the 2,000 lb (900 kg) shaking table loaded with a rigid mass weighing 2,000 lb (900 kg) and for the shaking table loaded with the single degree of freedom structure shown in Fig. 2.1. Displacement frequency responses for the table loaded with the rigid mass and for amplitudes of 0.025, 0.05, and 0.1 in. (0.6, 1.3 and 2.5 mm) are shown in Fig. 2.3. The resonant or corner frequency decreases from 24 cps (24 Hz) for the smallest amplitude to 18 cps (18 Hz) for the largest amplitude.

An acceleration frequency response of the single degree of freedom structure shown on the table in Fig. 2.1 is shown in Fig. 2.4(a).
This frequency response was observed while the shaking table frequency response, shown in Fig. 2.4(b), was being observed. The table's frequency response contains a peak and notch at the same frequency as the resonant frequency of the structure. This effect is caused by the reaction loads of the structure on the table and is sensitive to the amount of force feedback used to stabilize the primary displacement feedback loop, see Fig. 2.2. The effect of varying amounts of force feedback on the table's frequency response is illustrated in Fig. 2.5.

2.2 100,000 lb Shaking Table

The 100,000 lb (45,300 kg) shaking table, shown in Fig. 2.6(a), is constructed from a combination of reinforced and prestressed concrete. Structurally, it may be considered as a 1 ft (300 mm) thick 20 ft (6 m) square plate. The plate is stiffened by heavy central transverse ribs that are 1 ft (300 mm) wide and extend 1 ft 9 in. (525 mm) below the bottom surface of the plate, and by lighter diagonal ribs that are also 1 ft (300 mm) wide and extend 4 in. (100 mm) below the bottom surface of the table. The hydraulic actuators that drive the table horizontally are attached to the table by means of one of the transverse ribs. The vertical actuators, as well as test structures, are attached to the table by means of prestressing rods located in 2 in. (50 mm) diameter pipes that run vertically through the table on a 3 ft (1 m) square grid.

The shaking table is driven horizontally by three 50 kip (220 kN) hydraulic actuators, one of which is shown in Fig. 2.6(b), and vertically by four 25 kip (110 kN) hydraulic actuators, one of which is shown in Fig. 2.6(c). The actuators have swivel joints at both ends so that they rotate about the foundation swivel joints as the table moves.
moves. The total length of each horizontal actuator, including swivel joints, is 10 ft 6 in. (3.2 m), and the total length of each vertical actuator is 8 ft 8 in. (2.7 m). The length of the actuators helps to decouple the vertical and horizontal motions of the table, and further decoupling is accomplished by electronic means. The actuators are located in a pit beneath the shaking table as shown in Fig. 2.7.

The horizontal actuators are equipped with 175 gpm (0.67 m³/min) servo-valves and the vertical actuators with 90 gpm (0.34 m³/min) servo-valves. The flow rate of the servo-valves limits the maximum velocities in the horizontal and vertical directions to 20 in/sec (500 mm/sec) and 15 in./sec (380 mm/sec), respectively. The strokes are 12 in. (±6) (300 mm(±150)) for the horizontal actuators and 4 in. (±2)(100 mm(±50)) for the vertical actuators. However, the horizontal actuators will be limited to displacements of ± 5 in. (126 mm) to improve the resolution of table motion in the horizontal direction.

In operation, the air in the pit within the foundation and beneath the shaking table is pressurized so that the total dead weight of the table and the test structure is balanced by the difference in air pressure between the air in the pit and the air above the shaking table. The pit entrance is sealed by two air-tight doors that provide a lock chamber and, thus, access to the pit while the air in the pit is pressurized. The 1 ft (300 mm) gap between the shaking table and the interior foundation walls is sealed by a 24 in. (600 mm) wide strip of vinyl covered nylon fabric. The fabric, in its inflated position, can be seen in Fig. 2.6(a). A differential air pressure of 1.55 psi (10.7 kN/m²) is required to balance the dead weight of the shaking table alone; the maximum air pressure is not expected to exceed 4 psi.
The actuator forces are counteracted by a massive foundation, which is a reinforced concrete structure in the form of an open box with 5 ft (1.5 m) thick sides. The outside dimensions of the box are 32 ft x 32 ft x 15 ft (10 m x 10 m x 4.5 m), and the inside dimensions are 22 ft x 22 ft x 10 ft (7 m x 7 m x 3 m). The shaking table forms a closure for the box; the top of the shaking table being flush with the top of the foundation walls which in turn are flush with the floor slab of the building housing the shaking table, see also Fig. 2.6(a). The foundation weighs 1,580 kips (6.6 MN).

The electronic control system for the shaking table, which was supplied by MTS Systems Corporation, Minneapolis, Minnesota, who also supplied the hydraulic actuators, is based on controlling five degrees of freedom of the shaking table\(^{(5)}\). The sixth degree of freedom, translation perpendicular to the direction of the horizontal translational degree of freedom, is controlled by a sliding mechanism. Transducers are installed in each actuator to measure displacements and forces. From the displacement signals, feedback signals representing the average horizontal and vertical displacement, the pitch, roll and yaw (or twist) are derived on the assumption that the table is a rigid body. Corresponding force signals are also derived that are used to supplement the primary displacement feedback signals. Normally the pitch, roll and yaw command signals are zero, and the horizontal and vertical command signals represent translational displacement time histories of an earthquake record.

Frequency response functions for vertical and horizontal motions of the shaking table are shown in Fig. 2.8. The gain factors exhibit varying degrees of flatness and peaking because the control settings
were different for each frequency response measurement. The control system is quite sensitive to gain settings of the primary loops in the translational degrees of freedom, and to the amount of force stabilization in the pitch degree of freedom. A particular frequency response could be improved slightly by searching for an optimum control setting. However, since such adjustments will be difficult to perform with a test structure on the shaking table, the curves should be regarded as typical.

The frequency response functions indicate closed loop resonant frequencies of about 8 cps (Hz). These resonant frequencies are about 50% of the open-loop or oil column resonant frequencies which have been determined to be 15 and 16 cps (Hz) for vertical and horizontal motions, respectively. Although the closed loop resonant frequencies may be increased by increasing the loop gain, the improvement is small before the system becomes unstable.

The foundation transmissibility functions have been established by operating the table under harmonic motion of constant acceleration amplitude and varying frequency. The transmissibility functions for vertical and horizontal motions are shown in Fig. 2.9(a) and 2.9(b) respectively. The gain factors show that at frequencies below 10 cps (10 Hz) the soil stiffness is predominant in counteracting actuator forces, while at frequencies above 20 cps (20 Hz) the inertia mass of the foundation becomes predominant in counteracting the actuator forces. At frequencies between 10 and 20 cps (10 and 20 Hz), there is a transition zone where soil stiffness and foundation inertia are combining to counteract the actuator forces. In the vertical direction of motion the ratio of foundation acceleration to table acceleration
reaches 4% at a frequency of 24 cps (24 Hz). At 24 cps (24 Hz) the ratio appears to be rapidly approaching its limiting value of 6.3%, which is the ratio of table weight to foundation weight. The ratio of foundation acceleration to table acceleration for horizontal motion reaches the limit of 6.3% at 25 cps (25 Hz) and will probably exceed this value because the actuator forces are applied in a plane above the center of gravity of the foundation, see Fig. 2. Thus the foundation pitches as well as translates under the action of the horizontal actuators. The foundation acceleration measurements for Fig. 2.9(b) were made at the level of the horizontal actuators.

Soil of greater stiffness would have improved the foundation transmissibility functions over the complete frequency range (6). Since there are resonances in the transmissibility functions at about 8 cps (8 Hz), a lighter foundation would have improved the transmissibility functions in the frequency region below 10 cps (10 Hz) while making them worse at higher frequencies.
3. ANALYSIS OF CONTROL SYSTEM FOR 2,000 lb SHAKING TABLE

Servovalves control the flow of fluid through orifices and therefore they are inherently nonlinear devices. The nonlinear differential equations governing the behavior of systems incorporating servovalves may be solved directly by means of digital computers. But such analyses are expensive, and they do not easily impart a physical understanding of how a system behaves. On the other hand linear analyses, although valid only for small excursions about some operating point, are easily interpreted in terms of physical behavior. Since linear analyses for excursions about the zero position have been found adequate to describe the behavior of many electrohydraulic systems, a mathematical model of the 2,000 lb (900 kg) shaking table was formulated for such analyses. Frequency responses for the mathematical model are then compared with experimental frequency responses of the 2,000 lb (900 kg) shaking table. Finally, the response of the shaking table to earthquake type excitations when it is loaded with single degree of freedom linear and nonlinear structures is discussed.

3.1 Frequency Response of 2,000 lb Shaking Table

A schematic diagram of a rigid mass shaking table driven by an hydraulic actuator and a two stage servovalve is shown in Fig. 3.1. The equations governing such a system incorporating a single state servovalve have been derived by Merritt (7). Following Merritt's assumptions the equations for the system shown in Fig. 3.1 may be written as follows:

\[ m \ddot{x} + c \dot{x} + k x = F_i \]  \hspace{1cm} (3.1)

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\[ q_{lp} = f(x_p, P_{ls}) \]  
\[ q_{ls} = f(x_s, P_{la}) \]  
\[ q_{lp} = x_{lp} \dot{x}_{lp} + \frac{V_s}{4\beta} \dot{P}_{ls} \]  
\[ q_{ls} = x_{ls} \dot{x}_{ls} + \frac{V_a}{4\beta} \dot{P}_{la} \]  
\[ m_s \ddot{x}_s + c_s \dot{x}_s + k_s x_s = A_s P_{ls} = F_s \]  
\[ m_t \ddot{x}_t + c_t \dot{x}_t = A_t P_{la} = F_a \]

where

\[ x_p, x_s, x_t = \text{pilot stage spool, slave stage spool, and} \]
\[ \text{actuator piston displacements, respectively, and} \]
\[ \text{the dot notation is used to signify differentiation} \]
\[ \text{with respect to time.} \]

\[ c_p, c_s, c_t = \text{viscous damping coefficients for the pilot spool,} \]
\[ \text{slave spool, and actuator piston, respectively.} \]

\[ k_p, k_s = \text{stiffness coefficients tending to center the pilot} \]
\[ \text{and slave spools, respectively. The pilot spool} \]
\[ \text{coefficient is derived partly from a mechanical} \]
\[ \text{spring and partly from fluid flow, whereas the} \]
\[ \text{slave spool coefficient is derived entirely from} \]
\[ \text{fluid flow.} \]

\[ q_{lp}, q_{ls} = \text{the pilot and slave stage load flows. The load} \]
\[ \text{flow is the average of the flows entering and} \]
\[ \text{leaving their respective stages, see ref. (7).} \]
\( f_p, f_s \) = nonlinear functions governing the pilot and slave stage load flows.

\( p_s, p_a \) = differential pressure across slave spool and actuator piston, respectively.

\( V_s, V_a \) = volume of oil undergoing differential pressure changes in the slave stage and actuator, respectively.

\( A_s, A_a \) = effective areas of pilot spool and actuator piston, respectively.

\( F_s, F_a \) = differential pressure force acting on the slave spool and actuator piston, respectively.

\( F_i \) = external force applied to move pilot spool.

\( \beta \) = bulk modulus of fluid.

Equations (3.1) through (3.7) are depicted in block diagram form in Fig. 3.2 where \( S \) denotes the Laplacian operator.

Normally, two stage servovalves have some form of feedback from the slave stage to the pilot stage, and the servovalve in the control system of the 2,000 lb (900 kg) shaking table feeds back a signal from a displacement transducer that is proportional to slave spool position \( x_s \). The effect of such feedback is to linearize the behavior of the servovalve so that the slave spool position is proportional to the applied force in the system's operating frequency range:

\[ x_s = k F_i \]  

(3.8)

In addition, for small slave spool excursions about its central position,
the nonlinear function \( f \) of equation (3.5) may be linearized by means of a Taylor series expansion of the form (see ref. (7))

\[
q_k = K_q x_s - K_c p_a
\]  

where

\[
K_q = \text{slave stage flow gain, and}
K_c = \text{slave stage flow-pressure coefficient.}
\]

Equations (3.1) through (3.7) can be replaced by equations (3.6) through (3.9) and the latter equations solved to determine the transfer function relating the position of the actuator piston to the force applied to the pilot spool:

\[
\frac{x_t}{F_i} = \frac{k K_A}{q a} \left( \frac{V_m}{4 \beta} S^2 + \left( \frac{V_a}{4 \beta} + K_c m \right) S + \left( A_a^2 + K_c c t \right) \right)
\]  

Since \( K_c c t \) is negligible in comparison to \( A_a^2 \), see ref. (7), and assuming

\[
F_i = k' x_i
\]  

where

\[
x_i = \text{command table displacement}
\]

\[
k' = \text{an electronic amplifier gain,}
\]

then the transfer function relating \( x_t \) and \( x_i \) is

\[
\frac{x_t}{x_i} = \frac{\omega_0^2 k' k K A_a}{q a} \left( S^2 + 2 \tau_0 \omega S + \omega_0^2 \right)
\]
where
\[ \omega_0^2 = \frac{4 \beta A_a^2}{V_m m_t}, \]  
(3.13)

and
\[ \zeta_0 = \frac{c_t}{2 m_t \omega_0} + \frac{4 \beta K_c}{2 V_a \omega_0}. \]  
(3.14)

The open loop natural frequency \( \omega_0 \) is commonly referred to as
the oil column resonant frequency. The oil column resonant frequency
for the 2,000 lb (900 kg) shaking table loaded with a 2,000 lb (900 kg)
weight (\( m_t = 4,000/386 \text{ lb - sec}^2/\text{in.} \times 1,800/9.81 \text{ kg - sec}^2/\text{m} \)),
\( A_a = 25.4 \text{ in}^2 (164 \text{ cm}^2) \), \( V_a = 25.4 \times 12.5 \text{ in}^3 (164 \times 31.8 \text{ cm}^3) \), and
assuming \( \beta = 2 \times 10^5 \text{ psi} (1.4 \times 10^6 \text{ kN/m}^2) \) is 396 rad/sec or 63
cps (63 Hz).

The damping factor for the open loop system cannot be evaluated
reliably from equation (3.13) because neither \( c_t \) nor \( K_c \) are known
accurately, and there are other sources of damping that have been
neglected in the analyses. However, the effect of \( K_c \), the slave stage
flow-pressure coefficient, is to increase the equivalent damping of
the open loop system; and the effects of \( c_t \) and \( K_c \) and other sources of
damping may be incorporated into an equivalent viscous damping co-
efficient \( \zeta_e \), and its associated equivalent damping coefficient is
given by
\[ c_e = 2 \zeta_e \omega_0 m_t. \]  
(3.15)

Thus equations (3.7) and (3.9) become, respectively,

\[ m_t \ddot{x}_t + c_e x_t = A_a p_{la} = F_a \]  
(3.7a)
and

$$q_s = K q s$$ \hspace{1cm} (3.9a)

and equations (3.6), (3.7a), (3.8), (3.9a), and (3.11) are the equivalent linearized equations for the open loop system operating about its central position. These equations are depicted in block diagram form in Fig. 3.3.

Electrohydraulic shaking tables employ closed loops for control in which the position of the actuator piston (which is the same as the table's position assuming they are rigidly coupled) is the primary feedback and the force the actuator is exerting and the velocity of the piston are supplementary feedback signals. These feedback signals are shown added to a modified linearized open loop system in Fig. 3.4. The transfer function relating actual table displacement and command displacement for the closed loop system is

$$\frac{x_t}{x_i} = \frac{K}{\frac{V_m a}{4B A_a} s^3 + \frac{V c}{4B A_a} s + (K k_{ff} m + 48 e + K k_{ff} + K k_{dd}) s + K k_{ff} + K k_{vd}} \hspace{1cm} (3.16)$$

where

- $K = k' k_{qq}$
- $k_{ff}$ = gain of force feedback
- $k_{vf}$ = gain of velocity feedback, and
- $k_{df}$ = gain of displacement feedback.

In practical shaking tables, the equivalent viscous damping coefficient is small and velocity feedback does not have much effect. Thus $c_e$ and $k_{vf}$ may both be assumed equal to zero in equation (3.16),
and since the table displacement is required to equal the command displacement $k_{df}$ is unity. Therefore equation (3.16) simplifies to

$$\frac{x_T}{x_i} = \frac{\frac{V}{4\beta} + k_{ff} S^2}{m_t + A S + K}$$

(3.17)

where the subscript $a$ has been dropped from both $V$ and $A$. The associated block diagram is depicted in Fig. 3.5.

The frequency response function $H(i\omega)$ of the closed loop system may be obtained by substituting $i\omega$ for $S$ in equation (3.17) so

$$H(i\omega) = \frac{K}{K \left(1 - k_{ff} \frac{m_t}{4\beta} \omega^2\right) + i \left(A \omega - \frac{V m_t}{4\beta} \omega^3\right)}$$

$$= |H(i\omega)| e^{-i\phi(\omega)}.$$  

(3.19)

The system depicted in Fig. 3.5 was simulated on a digital computer and the gain factor $|H(i\omega)|$ and the phase factor, $\phi(\omega)$, were determined. The values of $A$, $m_t$, $V$, and $\beta$ were made 25.4, 4,000/g, 318, and $2 \times 10^5$, respectively, so that the system would represent the 2,000 lb (900 kg) shaking table loaded with a 2,000 lb (900 kg) mass for which frequency responses were presented in Fig. 2.3. The parameters $K$ and $k_{ff}$ were varied in order to obtain values that would reproduce these experimental frequency responses.

The effects of varying $K$ while maintaining $k_{ff}$ constant and of varying $k_{ff}$ while maintaining $K$ constant are shown in Fig. 3.6(a) and (b), respectively. The effectiveness of force feedback in controlling the resonant response of the system can be seen in Fig. 3.6(b). Once the force feedback reaches an adequate level it effectively prevents
resonance at the oil-column resonant frequency. However, it also has
the adverse effect of reducing the corner frequency of the frequency
response function. It is apparent that values of $K$ in the range
2,000 to 3,000 and values of $k_{ff}$ in the range $4 \times 10^{-6}$ to $8 \times 10^{-6}$ will
produce frequency responses similar to those obtained experimentally.

3.2 Effects of Test Specimen Reaction on 2,000 lb Shaking Table

The mathematical model of the 2,000 lb (900 kg) shaking table
was modified to incorporate a single degree of freedom structure
attached to the shaking table, and the block diagram of the model is
shown in Fig. 3.7. The transfer function relating table displacement
to the input is

\[
\frac{X_t}{X_i} = \frac{K (\frac{V S^3}{4B A} + K k_{ff} S^2) m_t (1 + \frac{m s x}{m t x_t}) + A S + K}{(V S^3/4B A + K k_{ff} S^2) m_t (1 + \frac{m s x}{m t x_t}) + A S + K}
\] (3.20)

The system shown in Fig. 3.7 was simulated on a digital computer. The
parameters in the system were selected so that it would represent the
single degree of freedom structure and shaking table to which the
experimental frequency response of Fig. 2.4 pertains.

Frequency responses in the form of gain factors versus frequency
for the system are shown in Fig. 3.8, and peaks and notches similar to
those shown in Fig. 2.4(b) and Fig. 2.5 are evident. The effect of the
amount of force feedback on the magnitudes of the peak and notch is
shown in Fig. 3.8(a). Increasing the amount of force feedback increases
the magnitudes of the peak and notch. Similarly, as shown in Fig. 3.8(b),
the magnitudes of the peak and notch increase as the ratio of the mass
of the structure to the mass of the table ($m_s/m_t$) increases. The
magnitudes of the peak and notch also increase if the damping capacity of the structure decreases as shown in Fig. 3.8(c).

The cause of the peak and notch may be seen by examining equation 3.20. This transfer function is similar to the transfer function for the table alone, equation 3.17, except that the table mass $m_t$ has been replaced by an effective table mass $m_e$ where

$$m_e = m_t \left( 1 + \frac{m_s}{m_t} \frac{x_s}{x_t} \right). \quad (3.21)$$

The effective mass depends on the ratio of the masses of the structure and table, $m_s/m_t$, and on the ratio $x_s/x_t$, which is the transmissibility function relating the absolute displacements of the structure and table:

$$\frac{x_s}{x_t} = T(\omega) = \frac{1 + i \times \frac{\zeta \omega_n}{\omega}}{1 - \left(\frac{\omega}{\omega_n}\right)^2 + i \times \frac{\zeta \omega_n}{\omega}} \quad (3.22)$$

where

- $\omega_n = \text{circular natural frequency of the structure}$
- $\zeta = \text{damping factor of the structure}.$

The transmissibility function is approximately unity for frequencies up to 70% of the natural frequency of the structure, and it is approximately zero for frequencies greater than 1.5 times the natural frequency. Thus at the lower frequencies the effective table mass is

$$m_e = m_t \left( 1 + \frac{m_s}{m_t} \right) = m_t + m_s \quad (3.23)$$

and at the higher frequencies it becomes
Near the natural frequency of the structure the transmissibility function varies rapidly in magnitude and phase depending on the damping factor of the structure. At frequencies just less than the natural frequency the effective table mass is given approximately by

\[ m_e = m_t \left( 1 + \frac{m_s}{m_t} \frac{1}{2\zeta} \right), \tag{3.25} \]

and at frequencies just greater than the natural frequency it is given approximately by

\[ m_e = m_t \left( 1 - \frac{m_s}{m_t} \frac{1}{2\zeta} \right). \]

If \( m_s/2\zeta \) is large compared with \( m_t \), which will be the case when the mass of the structure is nearly as large as, or larger than, the mass of the table and the damping capacity of the structure is small, then the effective table mass just below and just above the natural frequency is \( m_s/2\zeta \) and \(- m_s/2\zeta\), respectively.

A shaking table loaded with a resonant structure appears to the control system as a mass which varies with frequency. However, as shown in Fig. 3.9, the gain factor for a shaking table without load is a function of the table mass. The curves in this figure were obtained by assuming different values, including a negative value, for the mass of the shaking table in the system shown in Fig. 3.5. At low frequencies the effective table mass for a table weighing 2,000 lb (900 kg) loaded with a structure weighing 2,000 lb (900 kg) is 4,000/g(1800 kg/g)
and the frequency response will be curve 2 in Fig. 3.9. Just below
the natural frequency of the structure the effective table mass may
increase due to resonance to 20,000/g (9,000 kg/g) and the frequency
response changes from curve 2 to curve 3, Fig. 3.9. Just above the
natural frequency, the effective table mass may become a negative
20,000/g (9,000 kg/g) and curve 4 is the appropriate frequency response.
Finally, at higher frequencies the effective mass becomes 2,000/g
(900 kg/g), and curve 1 is the appropriate frequency response. Thus a
peak and notch are formed in the frequency response function as
illustrated by the dotted line in Fig. 3.9.

In practice most shaking tables must use force feedback to
control the table's resonant response and thus, when loaded with a
resonant structure, their frequency response contains a peak and notch.
Because of this effect, caution must be exercised when determining the
frequency response characteristics of structures by means of shaking
tables.

In order to determine the effect of the peak and notch in the
frequency response on the ability of the shaking table to simulate
earthquake motions, the acceleration time history of the N-S component
of the El Centro (1940) earthquake was doubly integrated to obtain a
displacement command signal. The command signal was then fed to the
computer model of the shaking table loaded with a resonant structure
as shown in Fig. 3.7. The amount of force feedback in the model was
intentionally, made sufficient to produce a significant peak and notch
in the frequency response of the shaking table.

The commanded El Centro acceleration time history is compared
with the simulated earthquake motion in Fig. 3.10(a), and, although the
acceleration peaks tend to be slightly smaller in the simulated earthquake, the fidelity of the simulation is excellent. As a further check on the fidelity of the simulation the structure's response to the simulated shaking table motion and to the commanded motion are compared in Figs. 3.10(b) and (c). Figure 3.10(b) shows the response of a structure in which the spring characteristic remained linear throughout the acceleration time history. Figure 3.10(c) shows the response of a structure in which the spring characteristic was bi-linear hysteretic and the yield force was low enough so that the response of the structure was nonlinear during the simulation. In the case of linear behavior, Fig. 3.10(b), the response of the structure to the simulation is slightly smaller than the response of the structure to the commanded motion, but in the case of nonlinear behavior, Fig. 3.10(c), there are no discernible differences other than the inevitable phase shift.
4. ANALYSIS OF CONTROL SYSTEM FOR 100,000 lb SHAKING TABLE

The 100,000 lb (45,300 kg) shaking table is driven vertically by four 25 kip (110 kN) hydraulic actuators and horizontally by three 50 kip (220 kN) hydraulic actuators. The vertical actuators are controlled electronically to operate in phase, and the horizontal actuators are also synchronized independently of the vertical actuators. The actuators are sufficiently long so that horizontal and vertical motions are essentially uncoupled. Assuming the synchronization circuits have no effect on the main control loops, the horizontal and vertical actuators may each be treated as a single independent actuator, and the analytical model of Fig. 3.5 may be used for either vertical or horizontal motion.

The frequency response of the analytical model representing vertical motion of the 100,000 lb shaking table was determined by digital computer. The area of the equivalent vertical actuator was made equal to the sum of the areas of the four 25 kip (110 kN) actuators, 38.44 in² (248 cm²). The volume of oil in the actuators was made equal to the total volume of oil in the four 25 kip (110 kN) actuators, 192.20 in³ (3150 cm³). The bulk modulus of the oil was chosen so that the model would have the same oil column resonant frequency, 16 cps (16 Hz), as the 100,000 lb (45,300 kg) shaking table. The amounts of main loop gain and force feedback were varied in order to achieve optimum frequency responses.

An optimum frequency response for the analytical model of the 100,000 lb (45,300 kg) shaking table is shown in Fig. 3.11. The closed loop resonant frequency of this system is approximately 8 cps (8 Hz)
which is about half of the oil column or open loop resonant frequency. The computed frequency response is very similar to the one shown in Fig. 2.8(a) that was obtained for the actual table.

4.1 Effects of Flexibility in Couplings and Foundation

In the analytical model shown in Fig. 3.5 it has been assumed that the piston is coupled rigidly to a rigid shaking table, and that the cylinder of the actuator is attached rigidly to a rigid foundation. In practice, the couplings, shaking table, and foundation are not rigid. When the flexibilities associated with these components are introduced into the block diagram of Fig. 3.7, the analytical model shown in Fig. 3.12 is obtained. In the model shown in Fig. 3.12, the subscript $g$ refers to the foundation or ground, the subscript $c$ refers to the cylinder and its coupling to the foundation, the subscript $p$ refers to the piston, the subscript $t$ refers to the table and its coupling to the piston, and the subscript $s$ refers to the structure. The coupling between the piston and shaking table includes the flexibility of the shaking table. The primary feedback, for both the 2,000 lb (900 kg) and 100,000 lb (45,300 kg) shaking tables, is the displacement between the piston and cylinder of the actuator.

Since the primary function of the shaking table is to reproduce an absolute table motion, the transfer function of interest is that relating absolute table displacement to input, which is

$$\frac{x_t}{x_i} = \frac{K}{\left(V S^3 + K k_{ff} S^2\right) m_t \left(1 + m_p x_p + m_s x_s\right) + (A S + K) \left(\frac{x - x_c}{x_t}\right)}$$  \hspace{1cm} (4.1)

This transfer function is of the same basic form as the transfer...
function for the simplified analytical model used previously, with an effective table mass given by

\[ m_e = m_t \left( 1 + \frac{m_p}{m_t} \frac{x_p}{x_t} + \frac{m_s}{m_t} \frac{x_s}{x_t} \right) \]  

(4.2)

and with the last two terms of the numerator \((A S + K)\) multiplied by \((x_p - x_c)/x_t\).

In the case of the 100,000 lb (45,300 kg) table, the coupling between the piston and shaking table and the shaking table itself are sufficiently rigid that the transmissibility function \(x_p/x_t\) is close to unity within the operating frequency of the table. Then, since \(m_p/m_t\) is very small, the effective mass reduces to

\[ m_e = m_t \left( 1 + \frac{m_s}{m_t} \frac{x_s}{x_t} \right) \]  

(4.3)

which is the same as that discussed previously for the 2,000 lb table in section 3.2. Thus the 100,000 lb (45,300 kg) table when loaded with a resonant structure will also exhibit a peak and notch in its frequency response function.

Since the transmissibility function \(x_p/x_t\) is approximately unity, the multiplier of the last two terms of the numerator \((A S + K)\) in equation (4.1) becomes \((1 - x_c/x_t)\). Also, since the cylinders of the actuators in the 100,000 lb (45,300 kg) table are prestressed onto the foundation, flexibility in these couplings is negligible so that \(x_c = x_g\). Thus the multiplier becomes \((1 - x_g/x_t)\) and the transmissibility function \(x_g/x_t\) is shown for vertical motion of the 100,000 lb (45,300 kg) table in Fig. 2.9(a). The transmissibility function \(x_g/x_t\) ranges from zero to a maximum value of 0.06, and thus the multiplier \((1 - x_g/x_t)\)
can only range between 1 and 0.94. Thus foundation flexibility in the 100,000 lb (45,300 kg) shaking table can reduce the magnitude of the terms \((A S + K)\) in the numerator of equation 4.1, by up to 6%. However, since these terms are significant only at low frequencies, the effects of foundation flexibility on the performance of the 100,000 lb (45,300 kg) table are negligible.
5. CONCLUSION

Mathematical models have been formulated that describe adequately the small amplitude dynamic behavior of electro-hydraulic shaking tables. The accuracy of the models has been confirmed by comparing the computed frequency response functions for the models with experimental frequency responses for a 2,000 lb (900 kg) and a 100,000 lb (45,300 kg) shaking table. The models simulated the corner frequency characteristic and the peak-and-notch distortion caused by a resonant structure on the shaking table.

Force feedback from the actuator was found to have a significant influence on the frequency response characteristics of shaking tables. Once the force feedback reaches an adequate level it effectively prevents resonance at the oil-column resonant frequency. However, force feedback also reduces the corner frequency and leads to the peak-and-notch distortion in the frequency response.

In addition to the amount of force feedback, the magnitudes of the peak and of the notch are sensitive to the ratio of the mass of the structure to the mass of the shaking table and to the transmissibility function of the structure. A shaking table loaded with a resonant structure appears to the control system as a mass whose magnitude varies with frequency. The variation has the form of the transmissibility function of the structure which has a maximum amplitude and change of phase at the resonant frequency. The transmissibility function increases the effective mass just below the resonant frequency of the structure producing the peak in the frequency response. Because of the phase change the transmissibility function reduces the effective mass just above the resonant frequency response.
The peak and notch distortion in the frequency response results in difficulties in determining the frequency response of structures by means of shaking tables. Corrections for this distortion need to be made either during the experimental work or later in the analysis of the data. However, it was found that the peak and notch distortion had little effect on the accuracy to which a shaking table could reproduce earthquake type motions.

The effect of foundation compliance on the frequency response characteristics of shaking tables was also examined. It was found that foundation compliance can only affect the frequency response at low frequencies, and the magnitude of the effect is limited to an amount which depends on the transmissibility function of the foundation with respect to the table.
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