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NONLINEAR STIFFNESS OF FOUNDATIONS

by

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The effects of nonlinear soil behavior on the dynamic stiffness of foundations are evaluated through a series of parametric studies. Approximate formulae for use in preliminary analyses are developed. A two-dimensional plane-strain model, corresponding to a strip footing resting on the surface of a layer of finite depth and an equivalent linearization technique were used. The dynamic excitation of a machine foundation and a building subjected to seismic motion were investigated.

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ABSTRACT

A series of parametric studies are conducted in order to assess the effect of nonlinear soil behavior on the dynamic stiffness of foundations. The results of these studies are translated into simple, approximate formulae and rules which can be used for preliminary soil-structure interaction analyses.

Two different situations are considered: the case of a machine foundation, where the dynamic excitation is caused exclusively by the vibrations induced in the structure, and the case of a building subject to an earthquake motion, where substantial nonlinearities would take place in the soil due to the seismic waves even if no structure were present. In both instances a two-dimensional, plane-strain model, corresponding to a strip footing resting on the surface of a layer of finite depth, and an equivalent linearization technique based on an iterative procedure were used for the analyses.

In the case of a vibrating machine, the static stiffnesses are obtained first as a function of the ratio of layer depth to foundation width and parameters representative of the nonlinearity of the soil. The frequency dependence of the dynamic stiffnesses and the effect of the level of excitation on this dependency are discussed next.

In the case of seismic excitation, the variation of effective soil properties with depth is determined first as a function of the original soil properties and the intensity of shaking. The stiffnesses of a foundation resting on a soil profile with these properties are determined next, and the characteristics of an "equivalent" uniform soil deposit are obtained.

Because of the limitations in the model itself (two-dimensional, iterative solution) and in the number of parametric studies conducted, the results obtained and the rules suggested are only of a crude approximate nature. It is believed, however, that they can be used for preliminary estimates of interaction effects. (It should be noticed that many of these limitations are also present in sophisticated computer programs used today).

PREFACE

The work described in this report represents a summary of the Thesis of Moshe Jakub, submitted to the Civil Engineering Department of M.I.T. in partial fulfillment of the requirements for the degree of Master of Science. The research was supervised by Professor José M. Roesset and was made possible through Grant AEN-7417835 from the National Science Foundation, Division of Advanced Environmental Research and Technology.

This is the fifth of a series of reports published under this grant. The other four were:

1. Research Report R76-8 by Mohammed M. Ettouney, "Transmitting Boundaries: A Comparison," January 1976.

2. Research Report R76-9 by Mohammed M. Ettouney, "Nonlinear Soil Behavior in Soil Structure Interaction Analysis," February, 1976.

3. Research Report R77-30 by José J. Gonzalez, "Dynamic Interaction between Adjacent Structures," September 1977.

4. Research Report R77-33 by F. Elsabee and J. P. Morray, "Dynamic Behavior of Embedded Foundations," September 1977.

INTRODUCTION

Most of the analytical or semi-analytical solutions for the dynamic stiffness of foundations are based on the assumption of linear elastic soil behavior. It has been long recognized, however, that if the results of a soil structure interaction analysis are to be realistic, the nonlinear behavior of the soil must be taken into account. Three procedures, with varying degree of sophistication, are used at present for this purpose:

a) To perform a linear analysis using soil properties consistent with the expected level of strains in the soil. These properties are estimated in the simplest case on the basis of approximate rules, applying reduction factors to the values of shear modulus corresponding to low levels of strain and adding some internal damping of a hysteretic nature. In other cases they are obtained from more complicated analyses on simpler models.

b) To perform a series of linear analyses using soil properties (modulus and damping) consistent with the level of strains resulting from the previous analysis. This iterative procedure has been extensively used and is implemented at present in such programs as SHAKE (8) (one-dimensional situations) and FLUSH (6) (two-dimensional geometries).

c) To perform a true nonlinear analysis in the time domain using a discrete model (finite elements or finite differences) with appropriate nonlinear constitutive equations for the soil. Two main categories of models are being used for this purpose:

— Empirical-type models based on the results of relatively simple tests, such as the hyperbolic or the Ramberg-Osgood model. These models are particularly appropriate for one-dimensional problems, but their extension to two- or three-dimensional states of stress requires some arbitrary decisions in the selection of the characteristic strain and the variation of a second elastic parameter (Young's modulus, the bulk modulus, the constrained modulus or Poisson's ratio).

— Models based on the theories of Plasticity or Elasto-Plasticity. While these models have the advantage of a much more rigorous and logical treatment of two- or three-dimensional states of stress, their parameters are often hard to adjust from the results of simple tests. In addition, most of the models suggested to date are applicable to the case of monotonic loading or to the study of failure conditions under loads of short duration and high intensity but will not reproduce properly the behavior under cyclic loading with large reversals of shear strains, the case of interest under seismic excitations.

Comparative studies carried out by Constantopoulos (2) for the one-dimensional case using the Ramberg-Osgood model for a true nonlinear analysis in the time domain and consistent laws of variation of shear modulus and hysteretic damping for the iterative solution indicated that the results from both types of solutions are in relatively good agreement (within 20%) for maximum accelerations or response spectra, although maximum displacements and strains are badly underestimated in the iterative approach (a point of concern if one is interested in the soil behavior or in the possibility of failure of the foundation rather than only the structural response). Similar comparisons by Ettouney (3) for the two-dimensional case (plane-strain problem) showed larger differences between the results provided by the two approaches. The models used were not, however, entirely consistent since complete memory, with perfect closing of the hysteretic loops (implicitly assumed in the iterative procedure) was not enforced in the time solution. Further studies enforcing this condition would seem to provide better agreement for the two-dimensional case, with conclusions similar in type to those reported by Constantopoulos.

In spite of its limitations the iterative procedure is still widely used for lack of a completely satisfactory nonlinear model which could be implemented in the time domain. A substantial amount of research is being done now in this area, and constitutive relations with both isotropic and kinematic hardening are being derived which can reproduce much better the cyclic shear behavior of certain classes of soils (undrained, saturated clays for instance). It can thus be expected that in a few years nonlinear analyses will become more popular; in the meantime, however, most practical solutions of soil structure interaction problems will be based on one of the first two approaches.

The purpose of this work is to determine, through parametric studies, simplified procedures which could be used with a type a) approach to account, at least in an approximate way, for nonlinear soil behavior. Two different situations are considered: the case of a foundation for vibrating machinery, and the case of a structure subjected to an earthquake motion. In the first case forces applied by the structure on the foundation are the only source of excitation, and the nonlinearity of the soil must be studied in two or ideally three dimensions. In the second case one can distinguish between primary and secondary nonlinearities. The former are the result of the seismic waves travelling through the soil and take place even when no structure is present; the latter are the result of the disturbances produced by the soil structure interaction effects and are generally restricted to the immediate neighborhood of the foundation. Studies by Kausel, Roesset and Christian (5) and by Ettouney (3) indicate that while the secondary nonlinearities play an important role in the strains and values of moduli around the foundation (particularly along the side boundaries of an embedded foundation and in the vicinity of the corners), their effect on the structural response is relatively small (if the seismic excitation is moderate to large). This is an important observation, because the primary nonlinearities can be estimated economically with an iterative scheme using either an exact analytical solution or a discrete model in each cycle of the iteration; on the other hand, application of the iterative procedure to the complete analysis of a discretized soil structure system is expensive since each cycle of the analysis with a large number of finite elements is time consuming.

DYNAMIC STIFFNESSES FOR SURFACE EXCITATION

Formulation. A linear, two-dimensional finite element model was used to derive the dynamic foundation stiffnesses for the case of a surface excitation. The iterative procedure was implemented to account for nonlinear soil behavior: at the beginning of each cycle of analysis, values of modulus and damping were selected for each finite element on the basis of a characteristic shear strain computed from the results of the previous analysis. The process was repeated until results from two consecutive cycles differed by less than a specified tolerance (5 percent).

The finite element model consisted of a core region, discretized with square finite elements (linear displacement expansion) and limited by two types of boundaries: a rigid boundary at the bottom, simulating a much stiffer rock-like material, and a consistent lateral boundary (1) to model the layered far field. Since the behavior of the soil in the layered outside region is assumed to be linearly elastic, these lateral boundaries were placed at a distance of 10B from the edge of the foundation (B is half the foundation width). It was felt (based on the results of previous studies such as those conducted by Ettouney) that at this distance the nonlinearities caused by the structural vibration would be negligible. The model would reproduce therefore the behavior of a strip footing on the surface of a layer of finite depth. The size of the elements in the core region was taken equal to 0.25B so that there were eight elements across the total width of the footing, and the stiffness functions could be reproduced with good accuracy up to values of the dimensionless frequency a_o of π. ($a_o = \Omega B/c_s$, where Ω is the frequency of the excitation in radians/second and c_s is the shear wave velocity of the soil for low levels of strain). Three different layer depths were considered: H = B (a very shallow stratum), H = 2B and H = 4B.

A Ramberg-Osgood model was used to simulate the nonlinear constitutive equations of the soil. This model was initially applied to the solution of soil dynamics problems by Constantopoulos (2) and is particularly appropriate for one-dimensional situations. Then the shear stress and the shear strain are related for the backbone curve (monotonic loading) by an equation of the form

$$\frac{\gamma}{\gamma_y} = \frac{\tau}{\tau_y}\left(1 + \alpha \left|\frac{\tau}{\tau_y}\right|^{r-1}\right)$$

where α and r are parameters controlling the position of the stress-strain curve and its curvature and γ_y τ_y define the tangent modulus at the origin (value of the shear modulus for low levels of strain).

The equation for unloading and reloading is, in order to satisfy Masing's law

$$\frac{\gamma-\gamma_0}{2\gamma_y} = \frac{\tau-\tau_0}{2\tau_y}\left(1 + \alpha \left|\frac{\tau-\tau_0}{2\tau_y}\right|^{r-1}\right)$$

where γ_0 τ_0 are the coordinates of the last reversal point.

When applying the model to a two-dimensional problem, some arbitrary decisions must be made. In this work it was assumed that the above equations apply to the maximum shear strain

$$\gamma = [(\varepsilon_x - \varepsilon_y)^2 + \gamma_{xy}^2]^{1/2}$$

and the maximum shear stress

$$\tau = [(\sigma_x - \sigma_y)^2 + \tau_{xy}^2]^{1/2}$$

This is the assumption commonly made with the iterative procedure and implemented in such widely accepted and used programs as LUSH (7) or FLUSH (6).

In order to define the variation of a second elastic parameter, it was assumed that the bulk modulus of the material remained constant. This assumption, while not entirely correct, would appear to be more appropriate than maintaining a constant Poisson's ratio and forcing Young's modulus to change with shear strain proportionally to the shear modulus (as done in LUSH or FLUSH).

Calling $G_o = \tau_y/\gamma_y$ the initial shear modulus for very low levels of strain, the secant modulus corresponding to a strain amplitude γ or a shear stress amplitude τ (for a harmonic excitation) is given by

$$\frac{G}{G_o} = \frac{1}{1 + \alpha \left|\frac{\tau}{\tau_y}\right|^{r-1}}$$

The effective damping ratio D (of a hysteretic nature) corresponding to this level of deformation is then

$$D = \frac{2}{\pi}\alpha\frac{r-1}{r+1} \frac{\left|\frac{\tau}{\tau_y}\right|^{r-1}}{1 + \alpha \left|\frac{\tau}{\tau_y}\right|^{r-1}}$$

or

$$D = \frac{2}{\pi}\alpha\frac{r-1}{r+1} \frac{G}{G_o} \left|\frac{\tau}{\tau_y}\right|^{r-1}$$

Finally the value of the effective Poisson's ratio ν can be obtained from the condition of a constant bulk modulus

$$K = \frac{2G_o(1+\nu_o)}{3(1-2\nu_o)} \quad \text{and} \quad \nu = \frac{1}{2}\frac{3K-2G}{3K+G} = \frac{(1-\frac{G}{G_o})+\nu_o(1+2\frac{G}{G_o})}{(2+\frac{G}{G_o})+2\nu_o(1-\frac{G}{G_o})}$$

In order to use a Ramberg-Osgood model for a specific soil, it would be necessary to adjust the parameters α, r, γ_y and τ_y to fit experimental data either on the backbone curve or better on the variation of modulus and damping with shear strain (notice that r need not be an odd integer the way the model is defined here). The studies by Constantopoulos (2) indicated that values of r between 2 and 3, values of α from 0.05 to 0.1 and values of γ_y from 10^{-6} to 10^{-5} seemed to match reasonably well the curves provided by Seed and Idriss (9). In this work, since no particular soil deposit was considered, a value of r=2 was assumed for simplicity (various values of α were used). The above expressions simplify then to

$$\frac{G}{G_o} = \frac{1}{1 + \alpha|\frac{\tau}{\tau_y}|}$$

and $$D = \frac{2\alpha}{3\pi}\frac{G}{G_o}|\frac{\tau}{\tau_y}|$$

In order to determine the dynamic stiffnesses of the foundation, assumed to be massless and rigid, harmonic horizontal displacements and rotations were imposed and the forces necessary to preserve equilibrium were computed. The resultant of these forces due to a horizontal displacement is the term K_{xx} of the stiffness matrix and the corresponding moment is $K_{\phi x}$. In the same way the resultant due to a base rotation is $K_{x\phi}$ and the moment is $K_{\phi\phi}$. The equations of motion are then written as

$$\begin{Bmatrix} F \\ M \end{Bmatrix} = \begin{bmatrix} K_{xx} & K_{x\phi} \\ K_{\phi x} & K_{\phi\phi} \end{bmatrix} \begin{Bmatrix} u \\ \phi \end{Bmatrix}$$

for a given frequency of vibration Ω and a specified amplitude of the displacement and rotation.

To make a distinction between the internal dissipation of energy in the soil due to hysteretic (nonlinear) behavior and the loss of energy by radiation of waves away from the foundation (radiation damping) the stiffness terms were written in the form

$$K_{xx} = K_x(k_1 + ia_oc_1)(1 + 2iD_1)$$

$$K_{\phi\phi} = K_\phi(k_2 + ia_oc_2)(1 + 2iD_2)$$

where K_x, K_ϕ are the static values of the horizontal and rocking stiffnesses

k_1c_1 k_2c_2 are frequency dependent stiffness coefficients

D_1 and D_2 are the effective values of hysteretic damping in swaying and rocking

a_o is the dimensionless frequency $\Omega B/c_s$.

The variation of the static stiffnesses K_x K_ϕ with amplitude of motion was investigated first. K_x was obtained by applying a horizontal translation to the rigid foundation and K_ϕ by applying a rotation. The terms $K_{x\phi}$ and $K_{\phi x}$ were also computed, but they were found to be small, as in the case of a surface foundation on an elastic stratum. Clearly when both a horizontal force and a moment are applied to the foundation (giving raise to both a horizontal translation and a rotation), due to the inelastic action, K_x and K_ϕ will be different from the values computed here, considering each type of excitation independently. It was felt, however, that these coupling effects could be ignored as a first approximation for the purposes of this study.

The variation with frequency of the stiffness coefficients k_1c_1 k_2c_2 and the effective damping ratios D_1D_2 were determined next, considering again a horizontal motion of varying amplitude for k_1c_1 and D_1 and a rotation for k_2c_2 and D_2.

Static Stiffnesses

By analogy to the expressions for the secant modulus using the Ramberg-Osgood model, the static stiffnesses of the foundation were written in the form

$$K_x = K_{xo} \frac{1}{1 + a\alpha \frac{F}{2B\tau_y}}$$

$$K_\phi = K_{\phi o} \frac{1}{1 + b\alpha \frac{M}{4B^2\tau_y}}$$

where F is the shear force on the foundation, M the rocking moment, B the half-width of the foundation, α and τ_y characteristics of the soil model and K_{xo} and $K_{\phi o}$ the elastic stiffnesses for very low levels of excitation.

$$\tau_y = G_o \gamma_y$$

Values of α of 0.025, 0.05 and 0.1 were considered and layer depths of H=B, 2B and 4B. Alternatively the expressions above can be rewritten as

$$\frac{K_{xo}}{K_x} = 1 + a\,\alpha\frac{F}{2B\tau_y} \qquad \frac{K_{\phi o}}{K_\phi} = 1 + b\,\alpha\frac{M}{4B^2\tau_y}$$

Figures 1 and 2 show plots of $\frac{K_{xo}}{K_x}$ and $\frac{K\phi_o}{K_\phi}$ versus $\frac{F}{2B\tau_y}$ and $\frac{M}{4B^2\tau_y}$ for the two extreme cases H=B and H=4B and a value of $\alpha = 0.05$. These plots are representative of those obtained for all the cases. It can be seen from these figures that a straight line as suggested by the intended formulae does provide a reasonable fit. The slopes of these lines represent the terms $a\alpha$ and $b\alpha$; a and b could be functions of both α and B/H, but the results obtained seem to indicate that:

a is almost independent of α and can be approximated by $a = \frac{1}{4}(1 + \frac{B}{H})$

b is almost independent of α and $\frac{B}{H}$ and approximately equal to $\frac{1}{2}$.

It is thus possible to write for the range of parameters studied

$$K_x \simeq K_{xo}\frac{1}{1 + 0.25(1 + \frac{B}{H})\alpha\frac{F}{2B\tau_y}}$$

$$K_\phi \simeq K_{\phi o}\frac{1}{1 + 0.5\alpha\frac{M}{4B^2\tau_y}}$$

For each one of the cases studied the amplitudes of the maximum shear stresses τ were computed in all the finite elements. As an alternative to the above formulae the stiffnesses were written in the form

$$K_x = \frac{K_{xo}}{1 + \alpha\frac{\tau_s}{\tau_y}}$$

$$K_\phi = \frac{K\phi_o}{1 + \alpha\frac{\tau_r}{\tau_y}}$$

where τ_s and τ_r would be characteristic shear stresses for swaying (horizontal excitations) and rocking. For each case the values of τ_s and τ_r were then computed from the known values of K_x K_{xo} K_ϕ $K_{\phi o}$ α and τ_y and the location of these stresses within the soil mass was identified. It was found that for the very shallow profile (H=B) τ_s occurred approximately at a depth of 0.33B under the edge of the footing, while for the two other cases (H=2B or 4B) it occurred at a depth of 0.5B (again under the edge). For rocking τ_r was in all cases the stress at a depth of 0.75B under the edge of the footing.

While additional and more refined studies would be necessary to determine better the location of these characteristic stresses, it would seem that within the level of approximation sought in this study (with the soil model used and the iterative procedure), one could for preliminary purposes determine the stiffnesses K_x and K_ϕ from the previous expressions using as a characteristic strain:

- for swaying the maximum shear stress from an elastic (static) analysis occurring at a depth of 0.5B under the edge of the footing (if $H \geq 2B$);
- for rocking the maximum shear stress from an elastic (static) analysis at a depth of 0.75B under the edge of the footing.

In order to use in practice any of the above formulae, it is necessary to estimate appropriate values of the soil parameters G_o (needed to compute K_{xo} and $K_{\phi o}$), α and τ_y, from experimental data (such as a monotonic loading shear stress - shear strain curve or better curves showing the variation of modulus and damping with shear strain). If these curves are not available (as may be the case for preliminary studies), values of α = 0.05 and $\gamma_y = 10^{-5}$ can be taken. τ_y can be computed then as $G_o \gamma_y$.

It must be emphasized again that the expressions suggested in this work are only crude approximations. To obtain more reliable ones it would be necessary to repeat the analyses for other values of the parameter r, other soil models, combinations of horizontal forces and rocking moments and ideally three-dimensional discretizations of the soil and the foundation.

Considering, however, the uncertainties often present in the estimation of the actual soil properties and the lack of accurate nonlinear constitutive equations, it is felt that they can provide a reasonable feeling for the effect of the magnitude of the applied forces on the foundation stiffnesses and that they can be used at least for preliminary analyses. It must be noticed, in addition, that most of the limitations just mentioned are also present in the pseudo nonlinear analyses performed in practice today.

Dynamic Stiffness Coefficients

Writing the dynamic stiffnesses first in the form

$$K_{xx} = K_x \left(k_x + i\, a_o\, c_x\right)$$

$$K_{\phi\phi} = K_\phi \left(k_\phi + i\, a_o\, c_\phi\right)$$

figures 3 to 6 show the variation of k_x and k_ϕ with frequency for various levels of excitation and the two extreme cases H=B, H=4B. It can be seen from these figures that as the level of excitation and the nonlinearity in the soil increase, all the curves are shifted to the left. This shift reflects a reduction in the natural frequencies of the soil and can be accounted for approximately by using in the expression for $a_o = \Omega B/c_s$ the shear wave velocity corresponding to the reduced shear modulus

$$G = \frac{G_o}{1 + \alpha \dfrac{\tau_s}{\tau_y}} \quad \text{or} \quad \frac{G_o}{1 + \alpha \dfrac{\tau_r}{\tau_y}}$$

For the horizontal stiffness k_x and the case of the deeper stratum (H=4B), a clear reduction in the amplitude of the oscillations is also apparent due to the increased internal damping. When dealing with a circular or rectangular foundation on a stratum of moderate depth ($H \geq 2B$), it can be expected that the variation of k_x with a_o will tend to approach the solution for a half space as the excitation increases.

For the rocking stiffness k_ϕ the shift in the curves is caused not only by the softening of the soil (reduction in modulus and increase in damping), but also by the increase in Poisson's ratio resulting from the assumption of a constant bulk modulus (see for instance ref. 10 for the variation of k_ϕ with ν in the case of a circular foundation on an elastic half space). This effect can also be reproduced by taking the value of ν corresponding to the effective modulus G.

Figures 7 to 10 show the variation of $a_o\ c_x$ and $a_o\ c_\phi$ with frequency for the same two cases H=B and H=4B. These terms start with a constant value, function of the level of excitation. Until the fundamental frequency of the stratum is reached (the fundamental shear frequency for the term c_x and the dilatational frequency for c_ϕ). Beyond these frequencies they increase almost linearly with a_o, showing again oscillations. Their values in the low frequency range, and particularly for a_o=0, can be interpreted as a hysteretic damping ratio $2D_s$ or $2D_r$. Tables 1 and 2 show the values resulting for the case H=4B and compare them to those obtained from the formulae

$$D_s = \frac{2\alpha}{3\pi}\ \frac{G}{G_o}\ \frac{\tau_s}{\tau_y} \qquad\qquad D_r = \frac{2\alpha}{3\pi}\ \frac{G}{G_o}\ \frac{\tau_r}{\tau_y}$$

It can be seen that the agreement is very good, particularly for moderate to large levels of excitation.

Table 1 - Values of effective internal soil damping, D_s

Applied Force F x 1000/G	τ_s x 1000 / G	D_s from τ_s x 100	D_s measured x 100
0.19	0.006	0.6	1.6
0.89	0.028	2.6	3.3
1.66	0.052	4.4	4.9
6.01	0.188	10.3	9.8
9.85	0.308	12.9	12.1

Table 2 - Values of effective internal soil damping D_r

Applied Moment $M \times 1000/G$	$\frac{\tau_r \times 1000}{G}$	D_r from τ_r x 100	D_r measured x 100
5.90	0.030	2.8	3.7
22.42	0.112	7.6	8.7
37.51	0.188	10.3	11.1
110.96	0.555	15.6	15.7
171.29	0.856	17.2	17.1

Using these values of D_s D_r, the dynamic stiffnesses can be written in the alternate form

$$K_x = K_{xo} \, (k_1 + i \, a_o \, c_1)(1 + 2i \, D_s)$$

$$K_\phi = K_{\phi o} \, (k_2 + i \, a_o \, c_2)(1 + 2i \, D_r)$$

The terms c_1 c_2 can then be interpreted as representing the effect of radiation damping. Figures 11 to 14 show the variation of these coefficients with frequency. For the horizontal excitation the main effect of increasing the level of motion is to diminish the oscillations and to reduce the frequency at which radiation starts to take place (this effect can be again accounted for by using in the expression of the dimensionless frequency a_o a reduced shear wave velocity of the soil). It should be noticed, however, that the average value of c_1 over the high frequency range is almost independent of the applied force. Since these values were computed using for a_o the initial shear wave velocity c_{so} (corresponding to low levels of strain), the expression K_x should be written as

$$K_x = K_{xo} \, (k_1 + i \, \frac{\Omega B}{c_{so}} \, c_1)(1 + 2i \, D_s)$$

or alternatively

$$K_x = K_{xo} \, (k_1 + i \, a_o \, \sqrt{G/G_o} \, c_1)(1 + 2i \, D_s)$$

k_1 and c_1 can then be taken the same as for the foundation in elastic soil using for $a_o = \Omega B/c_s$, with c_s the reduced shear wave velocity corresponding to G.

For rocking, on the other hand, there is not only a shift in the frequency variation of c_2, but also an increase in its value with increasing magnitude of moment. This increase is consistent with the reduction in the shear wave velocity of the multiplier a_o, and therefore the expression for K_ϕ can be used directly with $a_o = \Omega B/c_s$ both for the computation of c_2 and for the product $a_o\ c_2$.

Summary. It appears from the studies conducted that for the case of a machine foundation the dynamic stiffnesses including the effect of nonlinear soil behavior can be approximated by estimating first characteristic stresses τ_s and τ_r. These stresses may be computed as the values from an elastic static analysis under the edge of the footing at depths of 0.5B and 0.75B when applying a shear force or a rocking moment respectively. Alternatively for the soil model used here, they can be obtained from the formulae

$$\tau_s \simeq 0.25\ (1 + \frac{B}{H})\ \frac{F}{2B}$$

$$\tau_r \simeq 0.5\ \frac{M}{4B^2}$$

When both a horizontal force and a moment are applied on the foundation, as would be normally the case, within the degree of approximation sought here, a unique characteristic stress τ_c may be used. τ_c would be the maximum shear stress from an elastic static analysis (with both the shear force and the rocking moment) at an average depth under the edge of the footing, of say 2/3 B (or 2/3 R for a circular foundation). All the parametric studies conducted in this part of the work corresponded, however, to a soil stratum which had initially uniform properties. If the modulus increased with depth, a smaller value, of say 0.5B might be more appropriate.

Once the characteristic stresses τ_s and τ_r or τ_c are known, equivalent values of the shear modulus G, the shear wave velocity c_s, Poisson's ratio ν and internal damping D may be computed. The dynamic stiffnesses can then be written as

$$K_x = K_{xo}\ (k_x + i\ a_o \sqrt{G/G_o}\ \ c_x)$$

$$K_\phi = K_{\phi o}\ (k_\phi + i\ a_o\ c_\phi)$$

where K_{xo} $K_{\phi o}$ k_x k_ϕ c_x and c_ϕ are the solutions for a linear elastic soil deposit with the above properties and $a_o = \omega B/c_s$ (or $\omega R/c_s$ for a circular foundation).

DYNAMIC STIFFNESSES FOR SEISMIC EXCITATION

Formulation. For this part of the study a horizontally stratified soil deposit, without any foundation, was considered first, in order to determine the effect of the so-called "primary nonlinearties" (5). Three different profiles were studied: two uniform profiles with initial shear wave velocities of 750 and 1500 ft/sec and a variable soil profile with the shear modulus increasing as a function of the square root of the depth. In this last case the shear wave velocity of the first layer (at a depth of 2.5 ft) was taken equal to 500 ft/sec. The profiles were divided into layers with 5-ft. thickness, and artificial earthquakes, generated to match the Newmark-Blume-Kapur response spectra, were applied at the surface. These earthquakes were scaled to maximum accelerations of 0.05, 0.15, 0.25 and 0.50g. A deconvolution process was then applied, assuming a one-dimensional condition (shear waves propagating vertically through the soil) and using the equivalent linearization technique (iterative procedure) to simulate nonlinear soil behavior. A Ramberg-Osgood model with r=2 was again selected to obtain the variation of shear modulus and damping with shear strain. Maximum values of accelerations and shear stresses and strain compatible values of modulus and damping were then computed at various depths.

The deconvolution process is normally performed in the frequency domain. It should be noticed, however, that it is not necessary to consider the whole soil profile simultaneously, but that the computations can proceed one layer at a time, starting from the top.

If $F(\Omega)$ represents the Fourier transform of the acceleration at the free surface of the soil deposit, the Fourier transform of the displacements in the first layer is given by

$$U_1(\Omega) = E_1(e^{ip_1 z} + e^{-ip_1 z})$$

with

$$p_1^2 = \frac{\rho_1 \Omega^2}{G_1(1 + 2iD_1)}$$

ρ_1 is the mass density of the layer, G_1 its effective shear modulus and D_1 the effective hysteretic damping.

$$E_1 = -\frac{1}{2\Omega^2} F(\Omega)$$

The shear strains are then given by

$$\Gamma_1(\Omega) = ip_1 E_1 \, (e^{ip_1 z} - e^{-ip_1 z})$$

For the second layer

$$U_2(\Omega) = E_2 \, e^{ip_2 z} + F_2 \, e^{-ip_2 z}$$

with

$$2E_2 = E_1 \, [e^{ip_1 h_1}(1+\mu_2) + e^{-ip_1 h_1}(1-\mu_2)]$$

$$2F_2 = E_1 \, [e^{ip_1 h_1}(1-\mu_2) + e^{-ip_1 h_1}(1+\mu_2)]$$

$$\mu_2 = \frac{p_1 G_1(1 + 2iD_1)}{p_2 G_2(1 + 2iD_2)}$$

and

$$\Gamma_2(\Omega) = ip_2 \, [E_2 \, e^{ip_2 z} - F_2 \, e^{-ip_2 z}]$$

In general for the n^{th} layer, calling

$$p_n^2 = \frac{\rho_n \, \Omega^2}{G_n(1 + 2iD_n)}$$

and

$$\mu_n = \frac{p_{n-1} G_{n-1}(1 + 2iD_{n-1})}{p_n G_n(1 + 2iD_n)}$$

$$2E_n = E_{n-1}(1 + \mu_n)e^{ip_{n-1}h_{n-1}} + F_{n-1}(1 - \mu_n)e^{-ip_{n-1}h_{n-1}}$$

$$2F_n = E_{n-1}(1 - \mu_n)e^{ip_{n-1}h_{n-1}} + F_{n-1}(1 + \mu_n)e^{-ip_{n-1}h_{n-1}}$$

$$\Gamma_n(\Omega) = ip_n[E_n e^{ip_n z} - F_n e^{-ip_n z}]$$

In these expressions z is always measured from the top of each individual layer and h_n is the thickness of the n^{th} layer.

Since the values of E_n F_n are functions of the unknown properties $G_n D_n$ (through the term μ_n), an iterative procedure must be established for each layer. In order to perform the iteration it is common practice, in such programs as SHAKE (8), to compute the inverse Fourier transform of $\Gamma(\Omega)$ and scan the time history $\gamma(t)$ for the maximum value γ_{max} in time. A characteristic strain of $\gamma_{ch} = \frac{2}{3} \gamma_{max}$ is then selected to obtain consistent values of G and D from the appropriate curves.

In this work a much simpler and more economical procedure suggested in (5) was adopted, defining

$$\gamma_{ch} = \frac{2}{3} \frac{\text{Maximum Input Acceleration}}{\text{RMS Input Acceleration}} \times \text{RMS strain}$$

It should be noticed that the maximum input acceleration and the rms input acceleration are computed only once, at the beginning of the process, and that the rms strain for each layer is easily obtained from $\Gamma(\Omega)$ without the need to compute the inverse Fourier transform. $\Gamma(\Omega)$ was evaluated in all cases at the middepth of the layer for $z = \frac{1}{2} h_n$.

An alternative to this approach is to model the soil deposit by an equivalent discrete system with lumped masses and interconnecting springs. The first mass is then $M_1 = \frac{1}{2} \rho_1 h_1$, and the remaining masses are $M_n = \frac{1}{2} \rho_{n-1} h_{n-1} + \frac{1}{2} \rho_n h_n$. Each spring has a constant of the form $k_n = \frac{G_n}{h_n}$. The equation of motion for the first mass yields then

$$M_1 \ddot{u}_1 + \tau_1 = 0$$

or in the frequency domain $M_1 \ddot{U}_1(\Omega) + T_1(\Omega) = 0$, where $\ddot{U}_1(\Omega) = F(\Omega)$ and $T_1(\Omega)$ is the Fourier transform of the shear stress τ_1.

Taking then $\tau_{ch} = \frac{\text{Maximum Input Acceleration}}{\text{RMS Input Acceleration}} \times \text{RMS stress}$,

the consistent values of shear modulus and damping for the first layer can be computed, without any iteration, from the corresponding curves, or in the present case from the formulae

$$G = \frac{G_o}{1 + \alpha \frac{\tau_{ch}}{\tau_y}} \qquad D = \frac{2\alpha}{3\pi} \frac{G}{G_o} \frac{\tau_{ch}}{\tau_y}$$

Then $\ddot{U}_2(\Omega) = \ddot{U}_1(\Omega) + \frac{\Omega^2}{k_1(1+2iD_1)} T_1(\Omega) = \ddot{U}_1(\Omega) - \frac{M_1\Omega^2}{k_1(1+2iD_1)} \ddot{U}_1(\Omega)$

For the n^{th} layer the equation of motion is

$$\sum_{i=1}^{n} M_i\ddot{U}_i(\Omega) + T_n(\Omega) = 0$$

or

$$T_n(\Omega) = - \sum_{i=1}^{n} M_i\ddot{U}_i(\Omega)$$

and once G_n and D_n are known

$$\ddot{U}_{n+1}(\Omega) = \ddot{U}_n(\Omega) + \frac{\Omega^2}{k_n(1+2iD_n)} T_n(\Omega) = \ddot{U}_n(\Omega) - \frac{\Omega^2}{k_n(1+2iD_n)} \sum_{i=1}^{n} M_i\ddot{U}_i(\Omega)$$

It can be noticed that in this procedure it is necessary to differentiate twice the shear forces in order to compute the acceleration of each new mass. In the frequency domain this represents multiplying $T_n(\Omega)$ by the factor $-\Omega^2$. As a result the amplitudes in the high frequency range will continuously increase as the calculations proceed downward and eventually the solution will blow up. The same problem is encountered with the first approach, although it may not be as apparent from the form of the equations. It is a result of the assumption that the seismic motions are caused exclusively by vertically polarized shear waves. In order to be able to carry out the analysis until an appropriate depth is reached, it is common practice to truncate the Fourier transform of the input earthquake above a threshold frequency, of the order of 15 or 20 cps. This same approach was used in this work.

Both procedures were used here, yielding essentially identical results.

If the effect of secondary nonlinearities in the structural response is not important, as suggested by previous studies (3,5), once the equivalent values of modulus and damping are obtained, a single linear analysis could be conducted to estimate the foundation stiffnesses (or to do the analysis of the complete soil-structure system in a single step). A further simplification can be introduced, at least for preliminary analyses, by considering a uniform profile with some equivalent properties and using available solutions as those reported by Veletsos and Wei (10) or Veletsos and Verbic (11). In order to determine these equivalent properties, additional studies were conducted determining the foundation stiffnesses for the soil profiles with the strain compatible properties. Effective values of modulus and damping were then obtained so as to match the static stiffnesses, and the depth at which these values occurred was identified. The frequency variation of the dynamic stiffnesses for the actual profiles and the equivalent uniform deposit were then compared. These studies were conducted using both an analytical solution developed by Gazetas (4) and a finite element model with the consistent transmitting boundary placed at the edge of the footing (1) with almost identical results. Ratios of layer depth H to half the foundation width B of 2 and 4 were considered, and the bottom was assumed to be rigid.

Strain Compatible Soil Properties. Figures 15 to 17 show the variation of the effective shear modulus with depth for the three soil profiles considered and various levels of acceleration at the free surface. The corresponding values of hysteretic damping are shown in figures 18 to 20.

Figures 21 to 23 show the variation of the dimensionless parameter $\tau_m/\rho a z$ with depth, where τ_m is the maximum shear stress at depth z, a is the maximum surface acceleration in consistent units and ρ is the mass density of the soil (one can replace ρa by γa with γ the unit weight of the soil and a the acceleration in g's). This parameter provides an indication of the variation of the maximum acceleration: if the acceleration were uniform τ_m would increase linearly with depth and the above ratio would be constant, equal to 1.

It can be seen that $\tau_m/\rho az$ is much less dependent on the value of the maximum acceleration a than the soil properties. It is still a function, however, of the original values of the shear wave velocity c_{so}. In an effort to find a relationship which included the soil characteristics and the acceleration level, a function of the form

$$\frac{\tau_m}{\rho az} = f(\frac{z}{H})$$

was tried, with

$$H = (\frac{c_{so}}{4f_o})^{5/6} (\frac{\tau_y}{\alpha\rho a})^{1/5}$$

c_{so} is the initial shear wave velocity of the soil, for low levels of strain, at depth z; f_o is the fundamental frequency of the earthquake, of the order of 2.5 cps for the motions considered; a is again the maximum surface acceleration, ρ is the mass density of the soil and τ_y and α are characteristics of the Ramberg-Osgood model (values of $\alpha = 0.05$ and $\tau_y = G_o \times 10^{-5}$ were used in this part of the study).

Figure 24 shows the variation of $\tau_m/\rho az$ versus z/H for the two uniform soil profiles and all the levels of acceleration. It can be seen that the scatter in the results is extremely small and that a unique curve can be adopted as valid for all cases.

Figure 25 shows the results for the variable soil profile. The scatter due to the acceleration level is again very small.

From these curves it is thus possible to determine for any specific soil deposit and a desired level of acceleration at the surface, the values of the maximum shear stresses τ_m at various depths. Corresponding values of shear modulus G and hysteretic damping D can then be computed using a characteristic stress $\tau_{ch} = \frac{2}{3}\tau_m$ and the appropriate formulae for the desired soil model (the Ramberg-Osgood equations, for instance). The effect of the paramters f_o and α was not studied, and further verification of the expression for H is therefore necessary. For seismic motions based on the Newmark-Blume-Kapur response spectra (as those selected for the study) the curves can be used with some degree of confidence with $f_o = 2.5$ cps. A value of $\alpha = 0.05$ is recommended.

Static Stiffnesses. For the soil profile which had originally uniform properties with c_{so} = 1500 fps, and for the variable soil profile, parametric studies were conducted using the strain-compatible moduli and damping obtained in the previous step and applying a unit horizontal displacement and a unit rotation to a massless rigid foundation. Two ratios of layer depth to foundation half width (H/B = 2 and 4) were considered.

From the resulting values of the horizontal and rocking stiffnesses an equivalent shear wave velocity c_s and an equivalent damping D corresponding to a uniform soil profile were determined (so as to provide the same foundation stiffnesses). The depth H_{eq} at which these properties occurred was then evaluated. Table 3 shows these results for the originally uniform soil profile. It can be seen that in swaying H_{eq} varies from 0.5B to B, increasing with layer depth and decreasing with increasing level of acceleration. In rocking, on the other hand, the variation of H_{eq} is less pronounced (from 0.5B to 0.6B). Table 4 shows the corresponding results for the variable soil profile: in swaying H_{eq} varies from 0.3B to 0.5B and in rocking from 0.3B to 0.45B.

By conducting a more extensive set of parametric studies, it might be possible to obtain approximate formulae expressing the effective shear modulus and damping as a function of the level of acceleration, the characteristics of the soil profile, the Ramberg-Osgood parameters, the ratio of layer depth to foundation width and the total foundation width. Due to the cost of computation, however, only a small number of cases would be covered here, and the amount of data available was not sufficient to fit approximate expressions with any reliability. Until more studies of this nature are conducted, it is believed that a reasonable approximation can be obtained by:

- using for a soil profile which has originally uniform properties the modulus and damping consistent with the strains caused by the seismic motion at a depth of 0.5 to 0.7B (say 2/3 B).

- using for a soil profile which has originally a modulus increasing with the square root of depth the modulus and damping consistent with the strains caused by the seismic motions at a depth of 0.3 to 0.5B (say 0.4B).

Table 3 - Uniform Soil Profile

a		Swaying c_s	H_{eq}/B	D%	H_{eq}/B	Rocking c_s	H_{eq}/B	D%	H_{eq}/B
0.15g	H=2B	1370	0.6	3.3	0.5	1370	0.6	3.3	0.5
	H=4B	1310	1	4.4	0.8	1360	0.65	3.5	0.55
0.25g	H=2B	1310	0.6	5	0.6	1310	0.6	5	0.6
	H=4B	1240	0.8	6.2	0.8	1290	0.65	5.4	0.65
0.50g	H=2B	1190	0.5	7.5	0.5	1190	0.5	7.5	0.5
	H=4B	1120	0.7	9	0.7	1160	0.6	8.2	0.6

Table 4 - Variable Soil Profile

a		Swaying c_s	H_{eq}/B	D%	H_{eq}/B	Rocking c_s	H_{eq}/B	D%	H_{eq}/B
0.05g	H=2B	760	0.4	2.3	0.3	760	0.4	2.3	0.3
	H=4B	810	0.5	2.5	0.4	780	0.45	2.4	0.35
0.15g	H=2B	690	0.4	5.7	0.3	690	0.4	5.7	0.3
	H=4B	730	0.5	5.9	0.4	710	0.45	5.7	0.3
0.25g	H=2B	650	0.4	7.9	0.3	640	0.4	7.9	0.3
	H=4B	675	0.5	8.2	0.35	650	0.4	8	0.3

It should be noticed that for a circular foundation the variation of the static horizontal stiffness with the H/B ratio is much less pronounced than for a strip footing. The increase in H_{eq} to values of almost B for deep profiles in the swaying case is thus probably characteristic of the two-dimensional problem and may not occur in a three-dimensional geometry.

Dynamic Stiffness Coefficients. The selection of the modulus and damping of an equivalent uniform soil deposit was based on the values of the static stiffnesses. To investigate the effect of this approximation on the variation with frequency of the dynamic stiffnesses, the soil profile which had originally a uniform shear wave velocity of 1500 ft/sec, subject to an earthquake with a maximum surface acceleration of 0.5g, was again considered. The dynamic stiffnesses were obtained for a ratio H/B = 2, using the variable soil properties consistent with the earthquake excitation and an equivalent profile with c_s = 1190 ft/sec and D = 7.5%. Writing the dynamic stiffnesses again in the form

$$K_{xx} = K_{xo} \ (k_x + i \ a_o \ c_x)$$
$$K_{\phi\phi} = K_{\phi o} \ (k_\phi + i \ a_o \ c_\phi)$$

figure 26 shows the frequency variation of k_x and k_ϕ for the two cases. Figure 27 shows the corresponding results for $a_o\ c_x$ and $a_o\ c_\phi$. It can be seen that in this case the natural frequencies of the soil deposit are slightly overestimated by the equivalent c_s. If c_s had been selected so as to match these natural frequencies, the agreement between the dynamic stiffness coefficients would have improved, but a larger error would have been introduced in K_{xo} and $K_{\phi o}$. Considering on the other hand the uncertainty existing in practice in the actual soil properties and the degree of approximation sought in this study, the agreement seems reasonable.

Summary. It appears from these studies that for the case of a seismic excitation the effect of nonlinear soil behavior on the foundation stiffnesses can be estimated by:

- computing first from figures 24 or 25 the maximum shear stresses at various depths, defining a characteristic stress $\tau_{ch} = \frac{2}{3} \tau_m$, and obtaining appropriate values of G and D from the soil curves or the Ramberg-Osgood model.

— making a single linear analysis using for the soil profile the strain compatible properties determined above or using the available solutions for the foundation stiffnesses (10,11) with the values of G and D at a depth of 0.6B (or 0.6R) if the soil deposit were originally uniform or at a depth of 0.4B (0.4R) if the original modulus increased with depth.

CONCLUSIONS AND RECOMMENDATIONS

The purpose of this work was to determine, through parametric studies, simplified rules which could be used with a single linear analysis to account, at least in an approximate way, for nonlinear soil behavior. Within the limitations of this study (two-dimensional problem, nonlinear soil model, etc.), it appears that the procedures suggested should provide reasonable results at least for preliminary analyses. It should be noticed that many of the limitations of this work are also present in the iterative procedure used in practice with complete and expensive two-dimensional finite element models (or pseudo three-dimensional solutions).

In order to improve the reliability of the approximate procedures, more parametric studies should be conducted both for the case of a surface excitation (vibrating machinery) and for the case of seismic loading: for the former in order to consider combinations of forces and moments acting simultaneously on the foundation; for the latter in order to investigate the effect of the frequency content of the earthquake and variations in the soil parameters. More soil profiles and geometries should be considered (including embedded foundations).

Similar studies for circular or rectangular foundations would be desirable, but these cases would be considerably more expensive from the point of view of computation. In all instances the use of better nonlinear constitutive equations for the soil, when they become available, and a solution in the time domain would greatly increase the reliability of the results.

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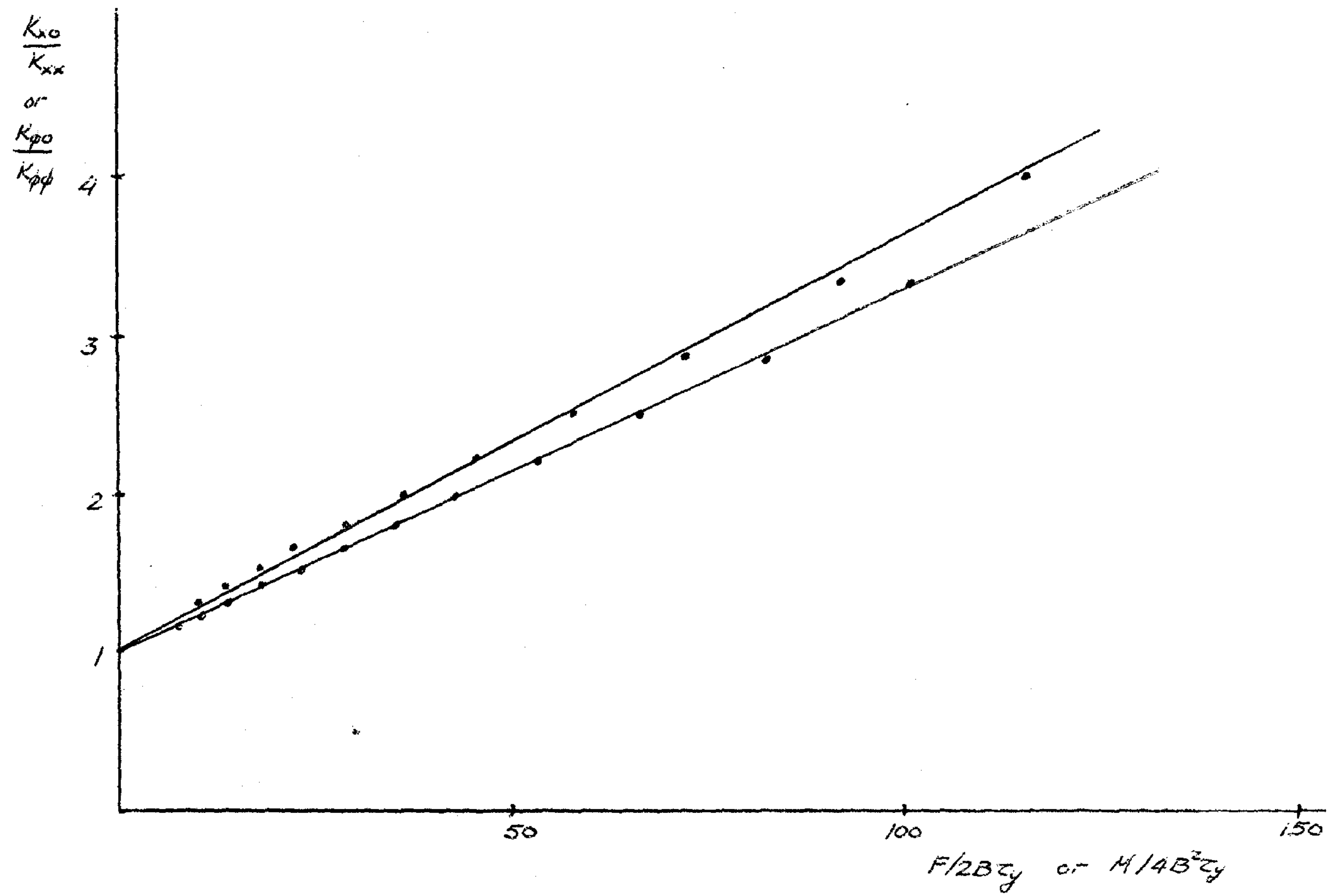


FIGURE 1 - REDUCED STIFFNESSES VS. LEVEL OF EXCITATION

$H = B$

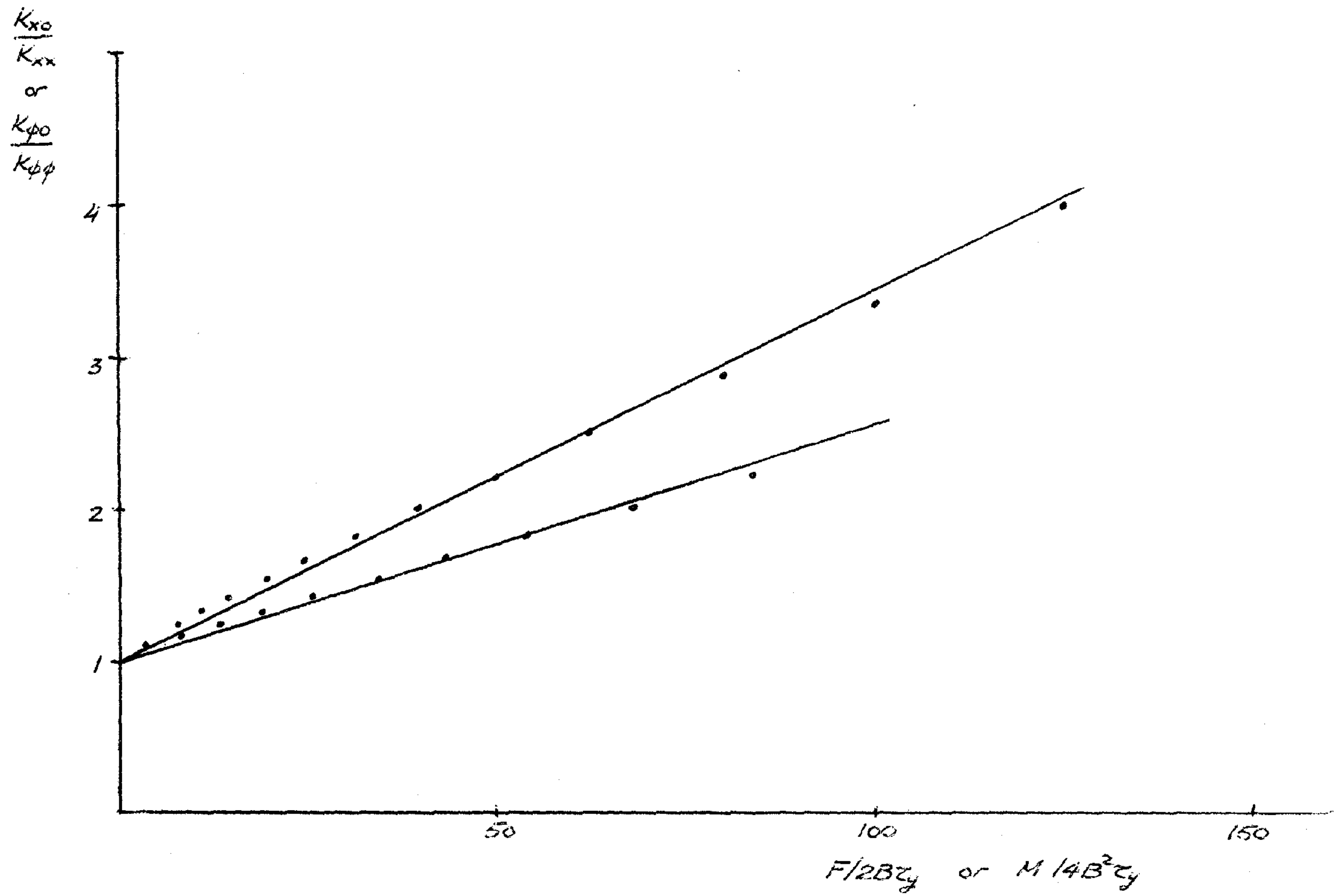
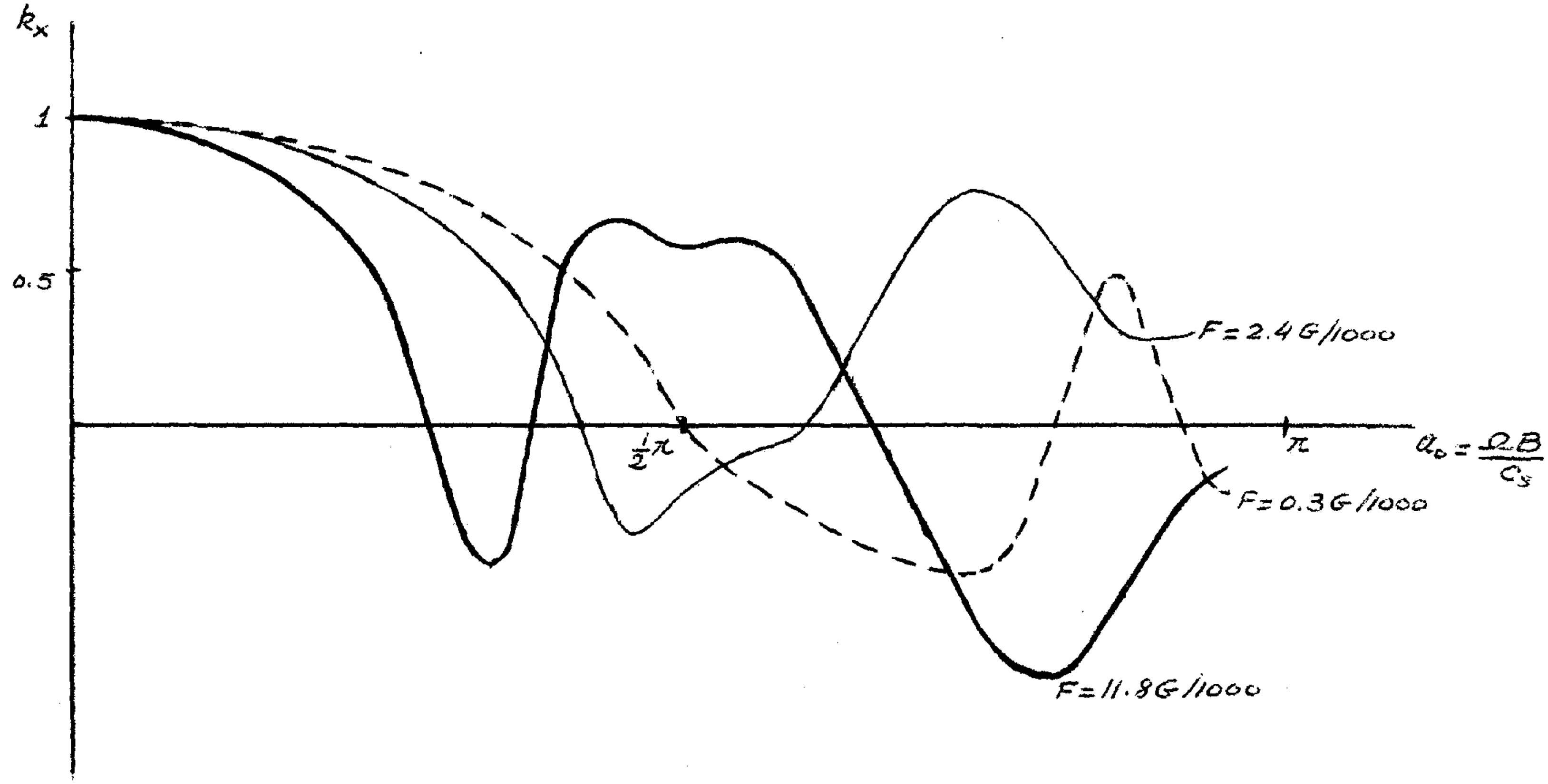


FIGURE 2 - REDUCED STIFFNESSES VS. LEVEL OF EXCITATION
$H = 4B$



<u>FIGURE 3</u> - Horizontal Stiffness Coefficients $H = B$

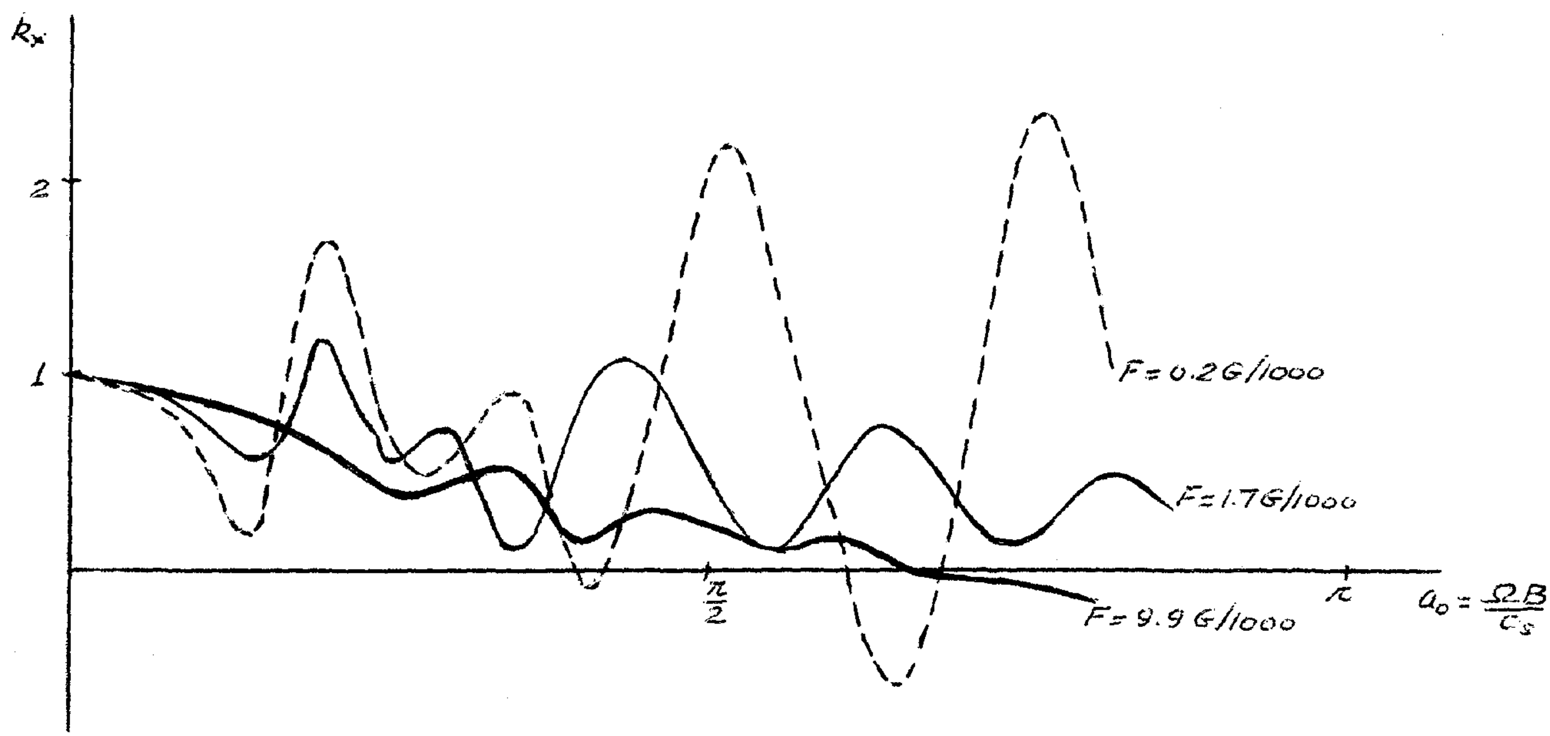


FIGURE 4. Horizontal Stiffness Coefficients - $H = 4B$

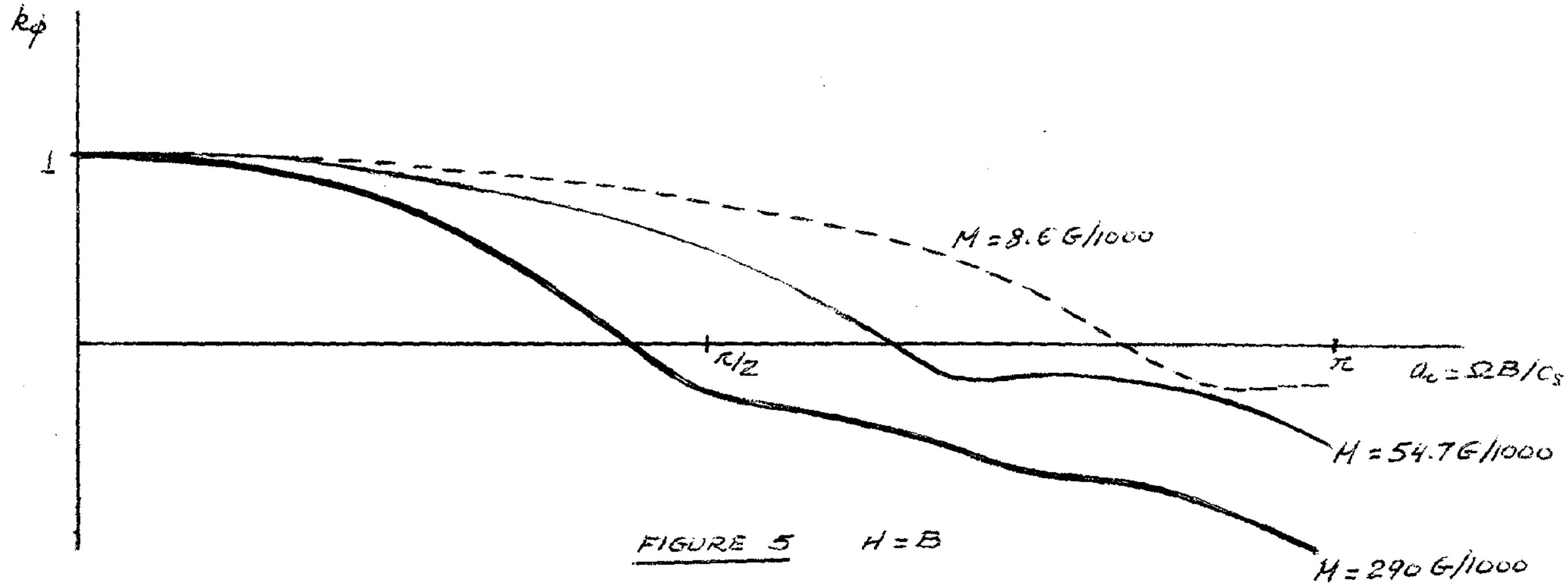


FIGURE 5 H = B

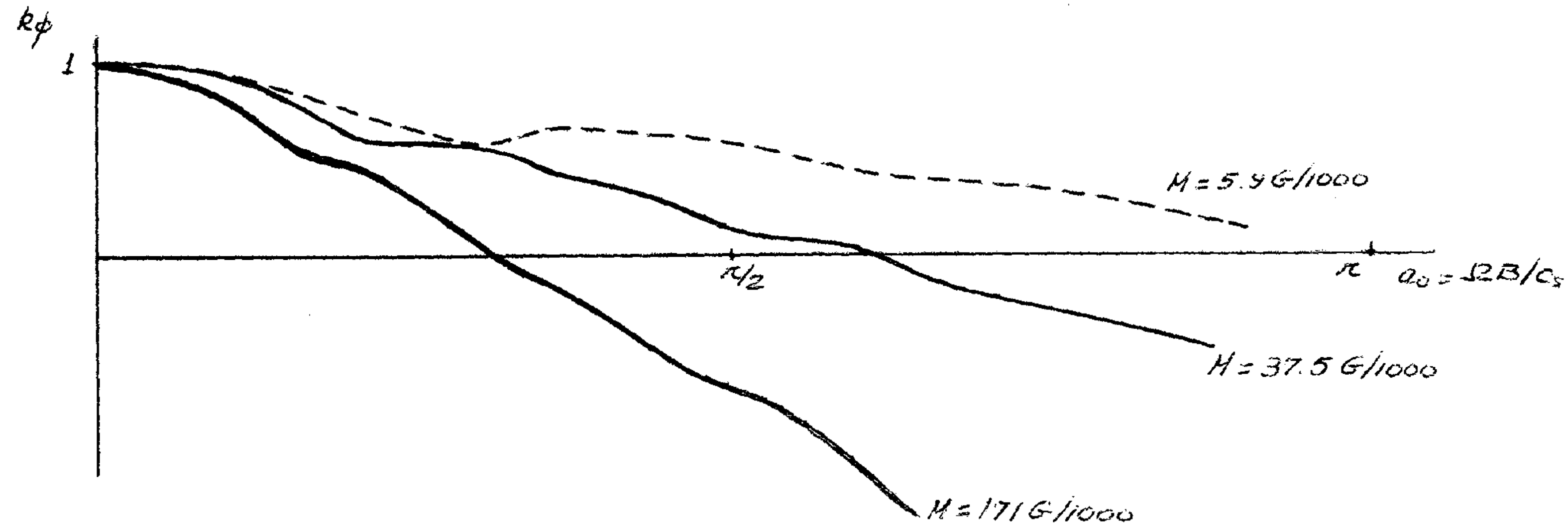


FIGURE 6 H = 4B

ROCKING STIFFNESS COEFFICIENTS

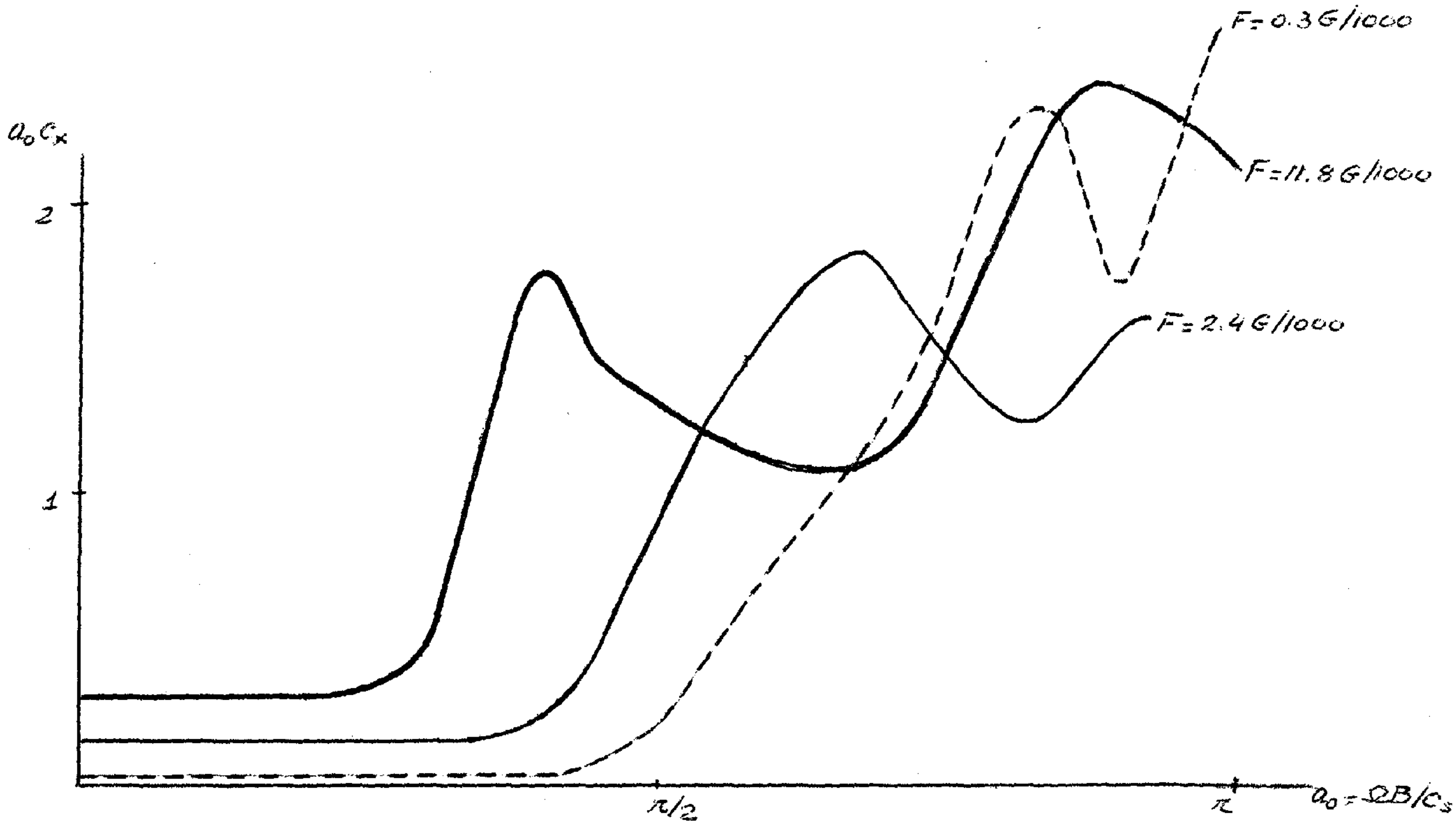


FIGURE 7 HORIZONTAL DAMPING COEFFICIENTS H = B

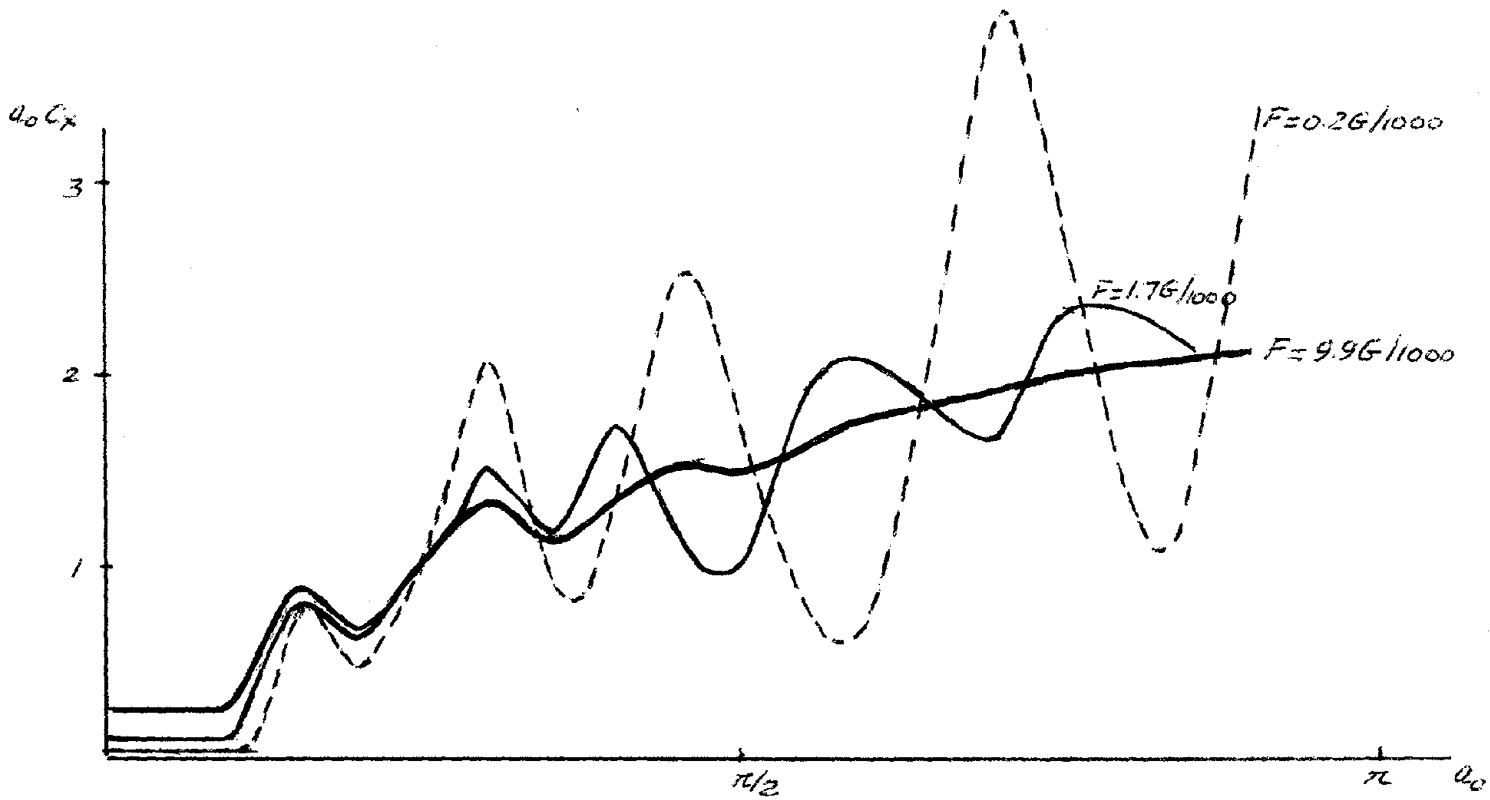


FIGURE 8. HORIZONTAL DAMPING COEFFICIENTS. H=4B

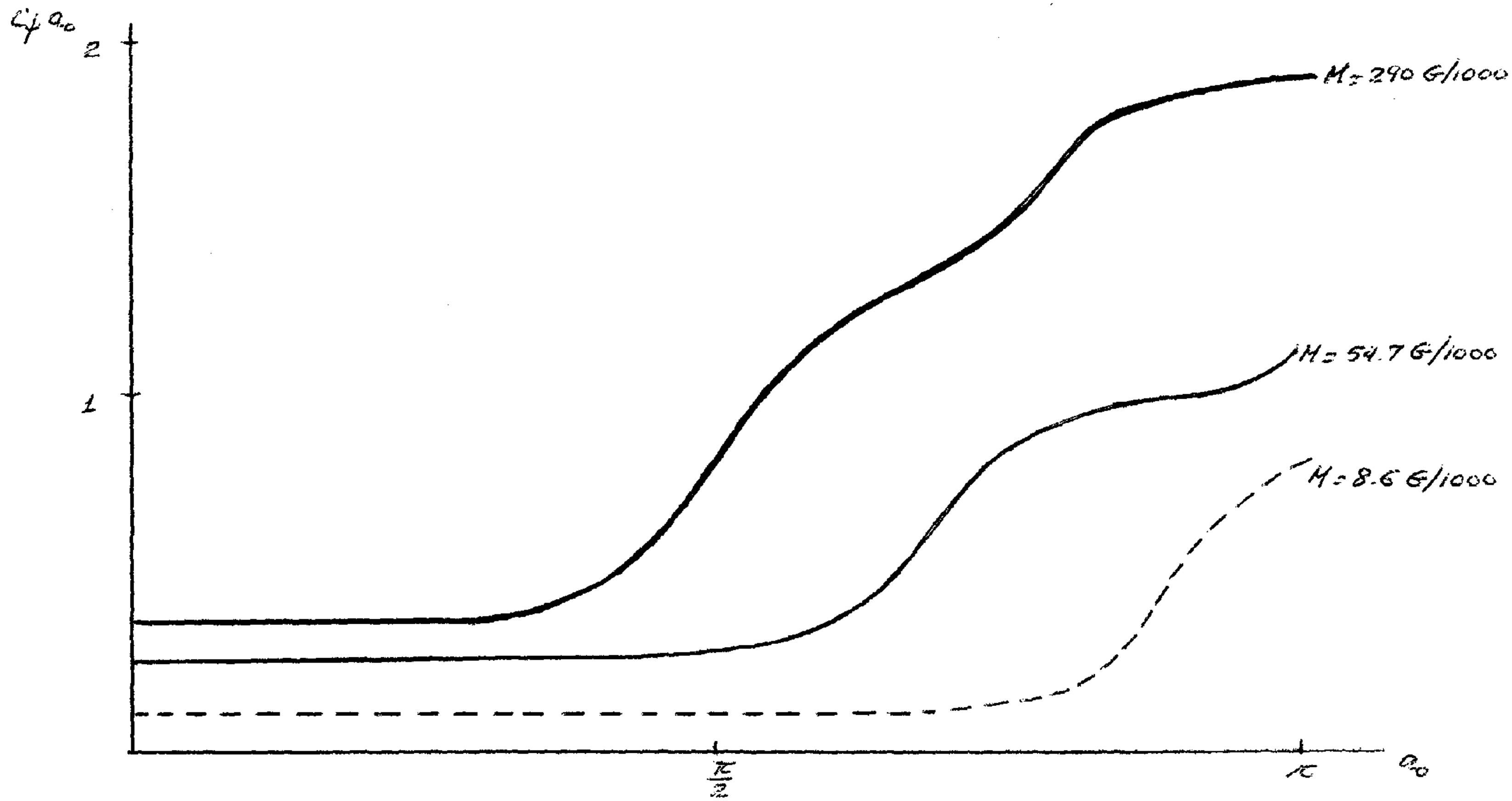


FIGURE 9 . ROCKING DAMPING COEFFICIENTS. H = B

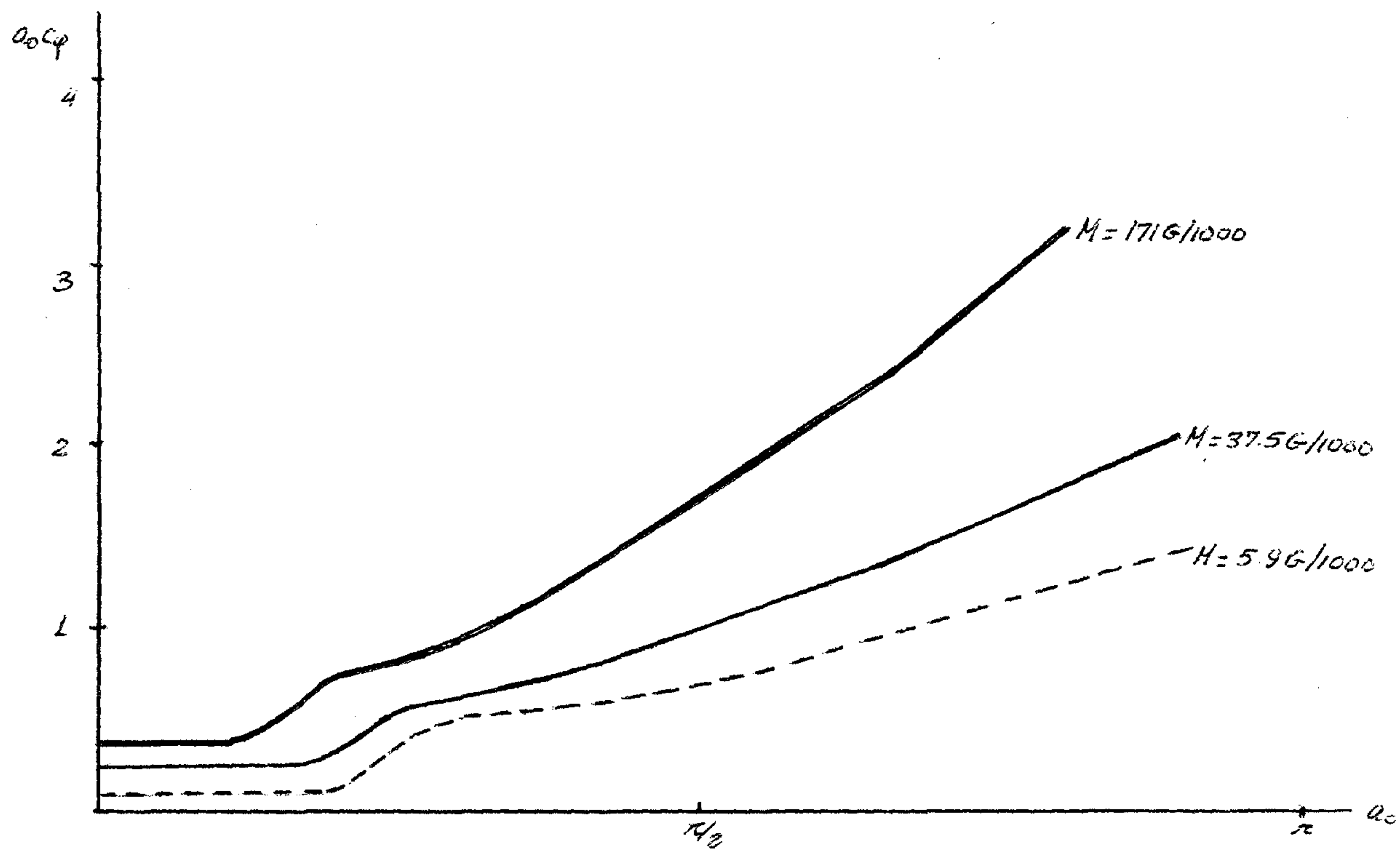


FIGURE 10. ROCKING DAMPING COEFFICIENTS. H = 4B

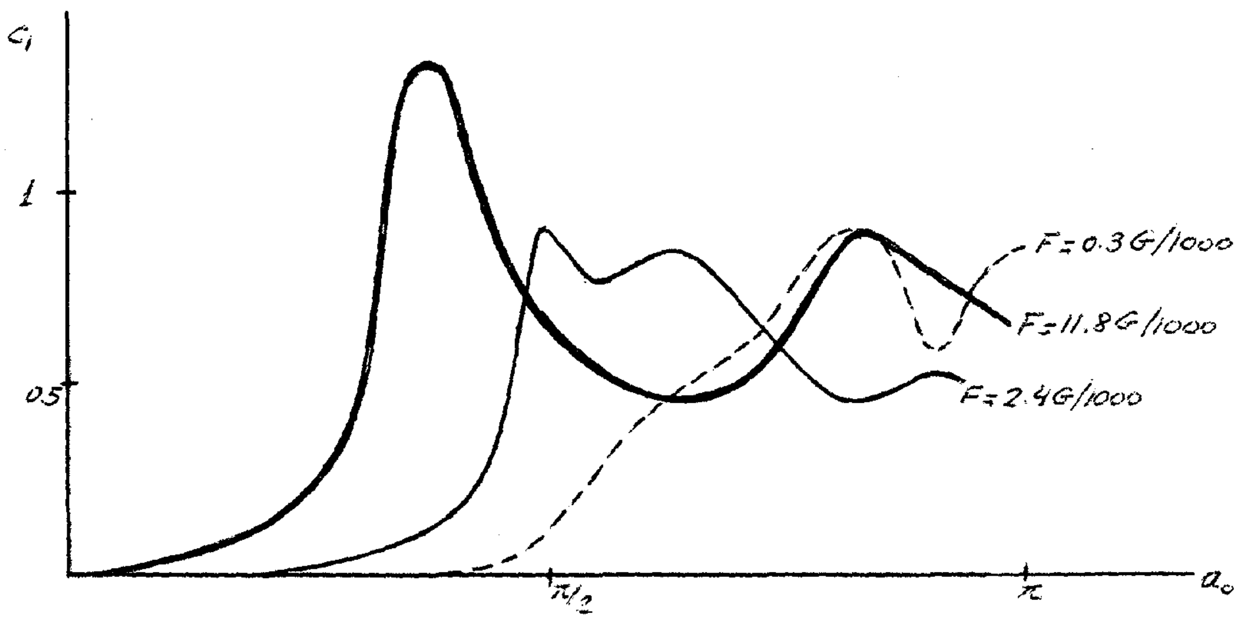


FIGURE 11 HORIZONTAL RADIATION DAMPING H=B

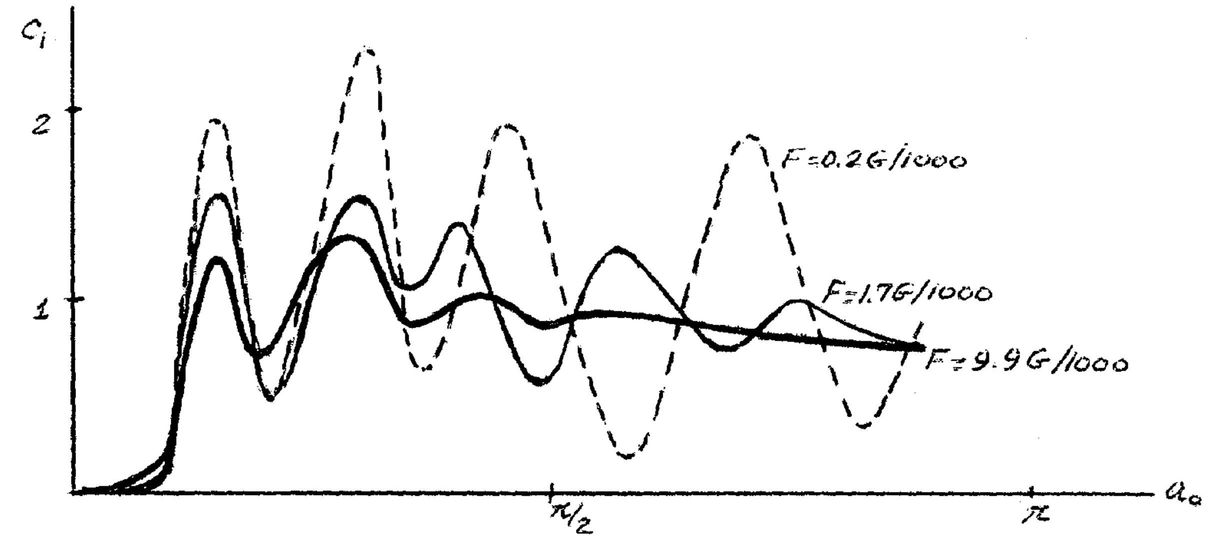


FIGURE 12. HORIZONTAL RADIATION DAMPING H=4B

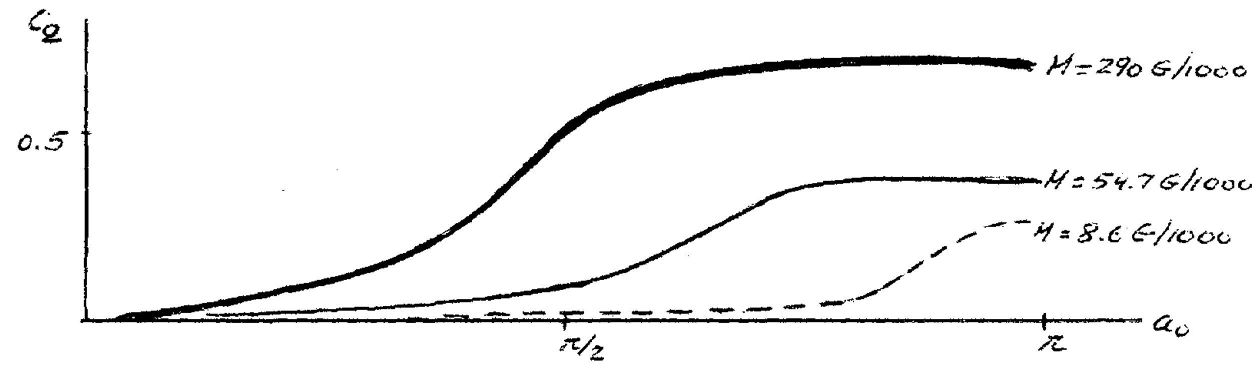


FIGURE 13 H = B

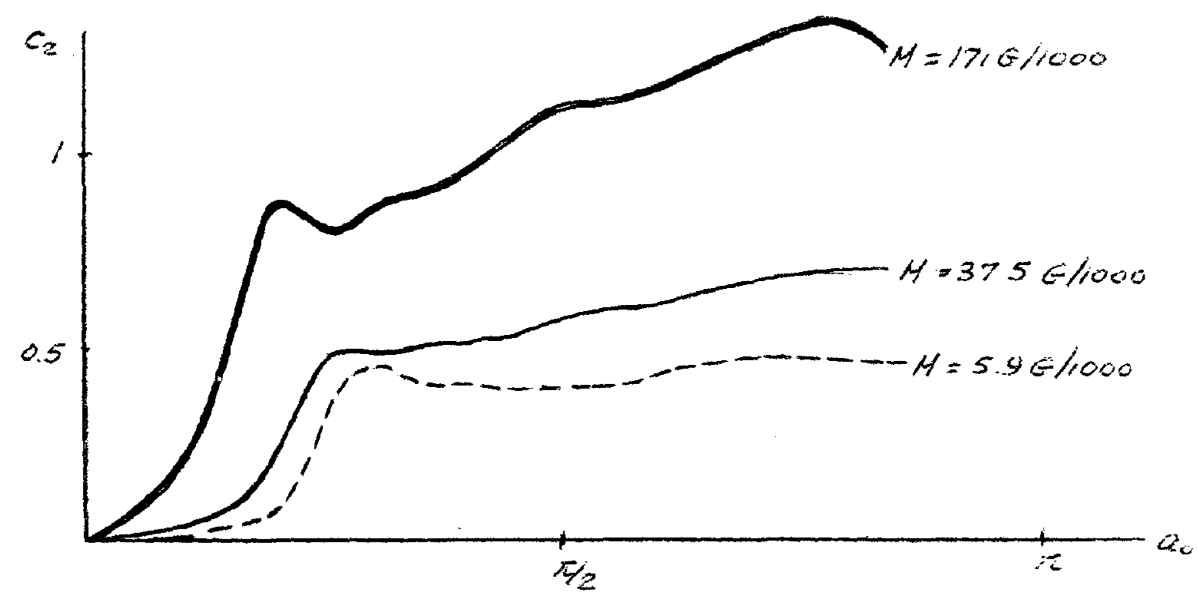


FIGURE 14 H = 4B

RADIATION DAMPING IN ROCKING

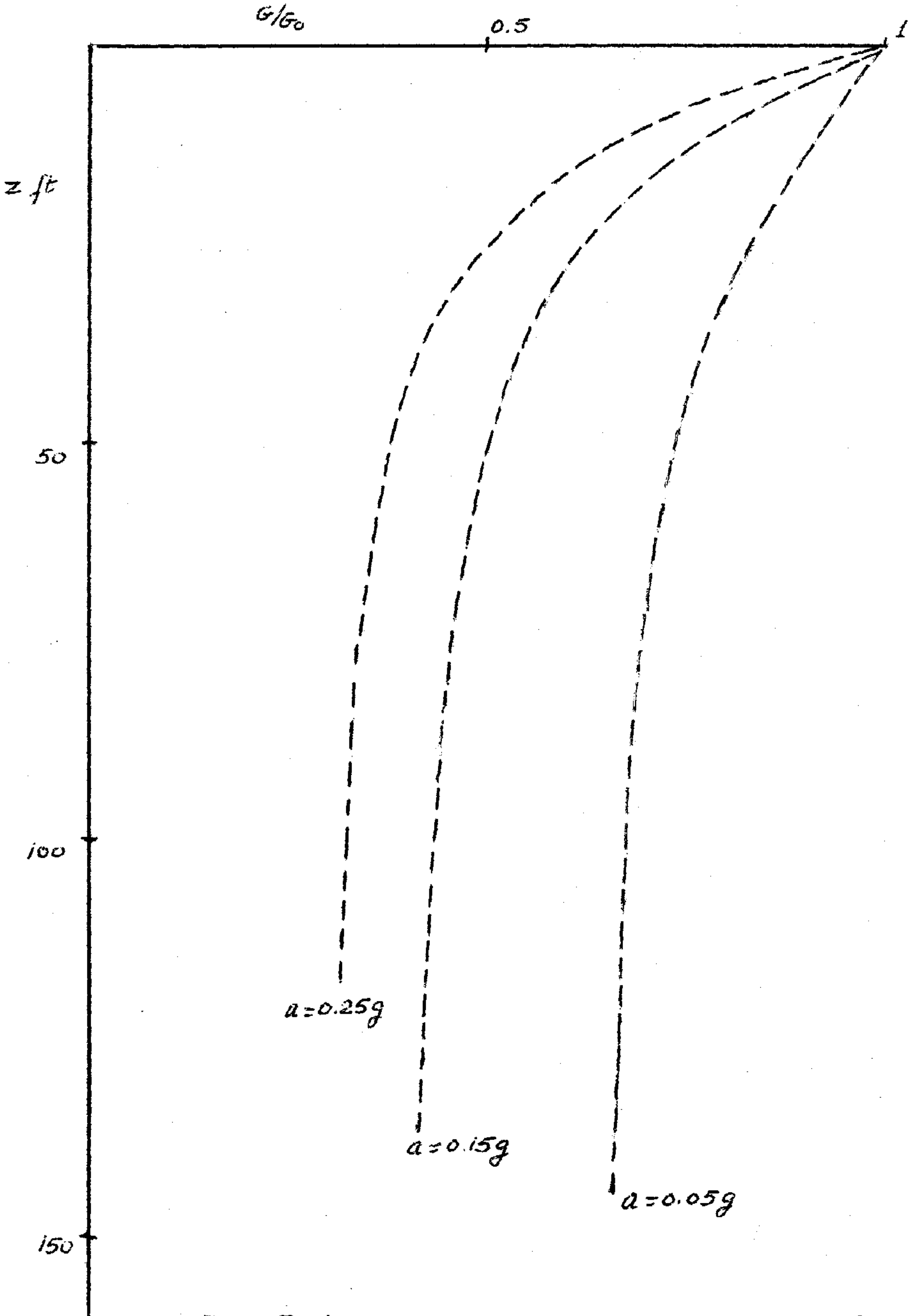


FIGURE 15 VARIATION OF MODULUS WITH DEPTH
UNIFORM SOIL PROFILE C_{s0} = 750 ft/sec.

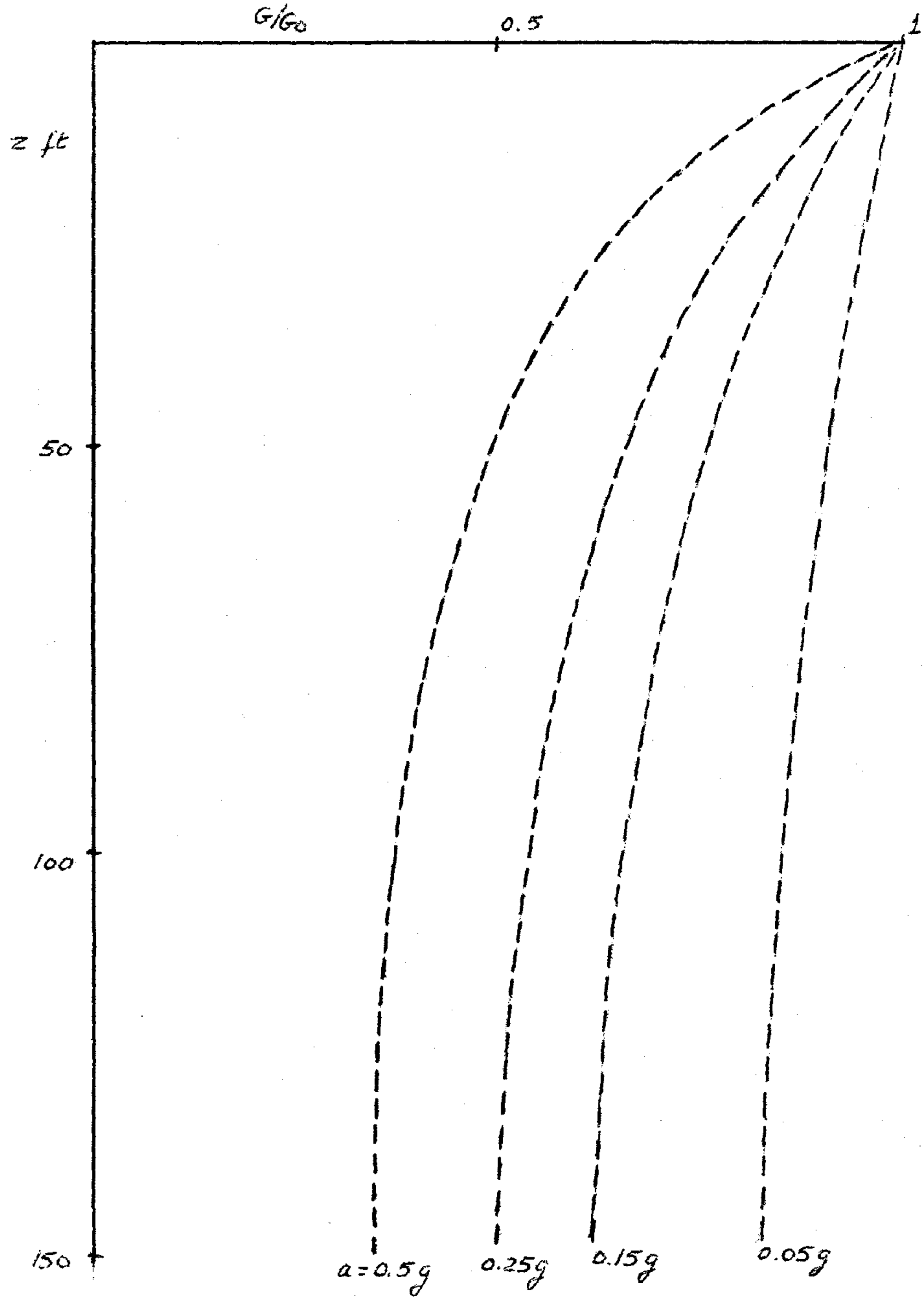


FIGURE 16 VARIATION OF MODULUS WITH DEPTH
UNIFORM SOIL PROFILE C_{So} = 1500 ft/sec.

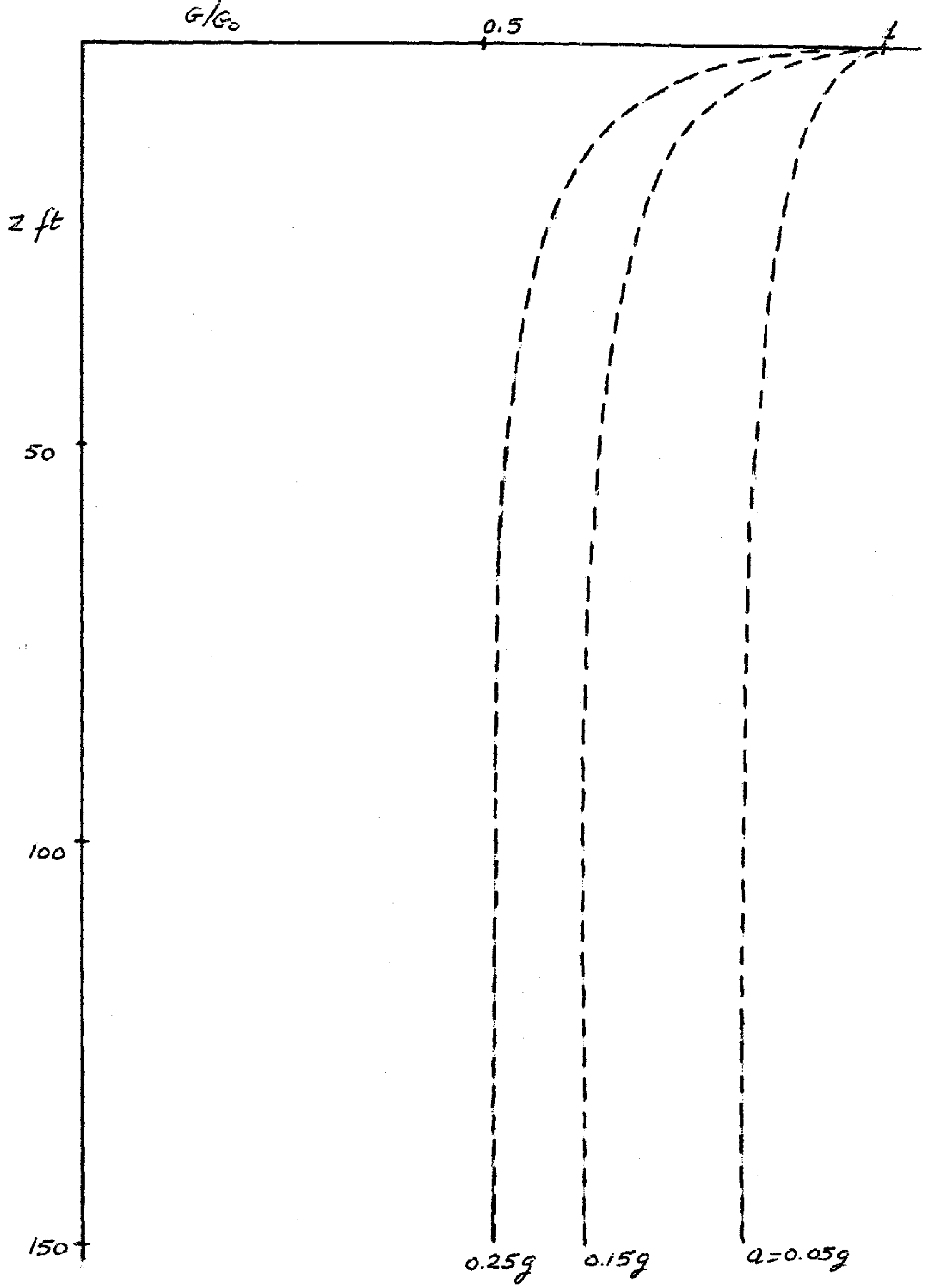


FIGURE 17 VARIATION OF MODULUS WITH DEPTH
VARIABLE SOIL PROFILE

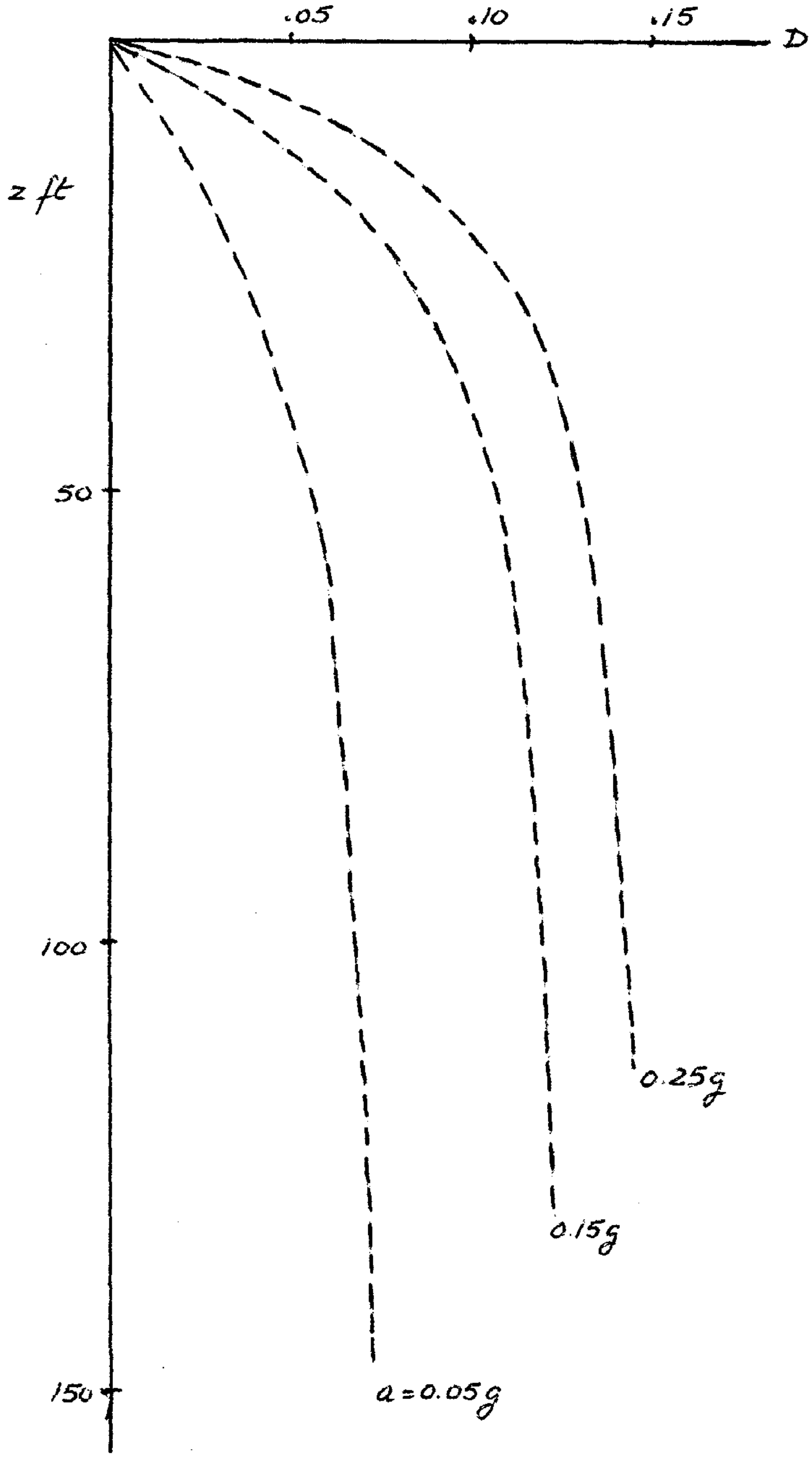


FIGURE 18 - VARIATION OF DAMPING WITH DEPTH
UNIFORM SOIL PROFILE c_{so} = 750 ft/sec.

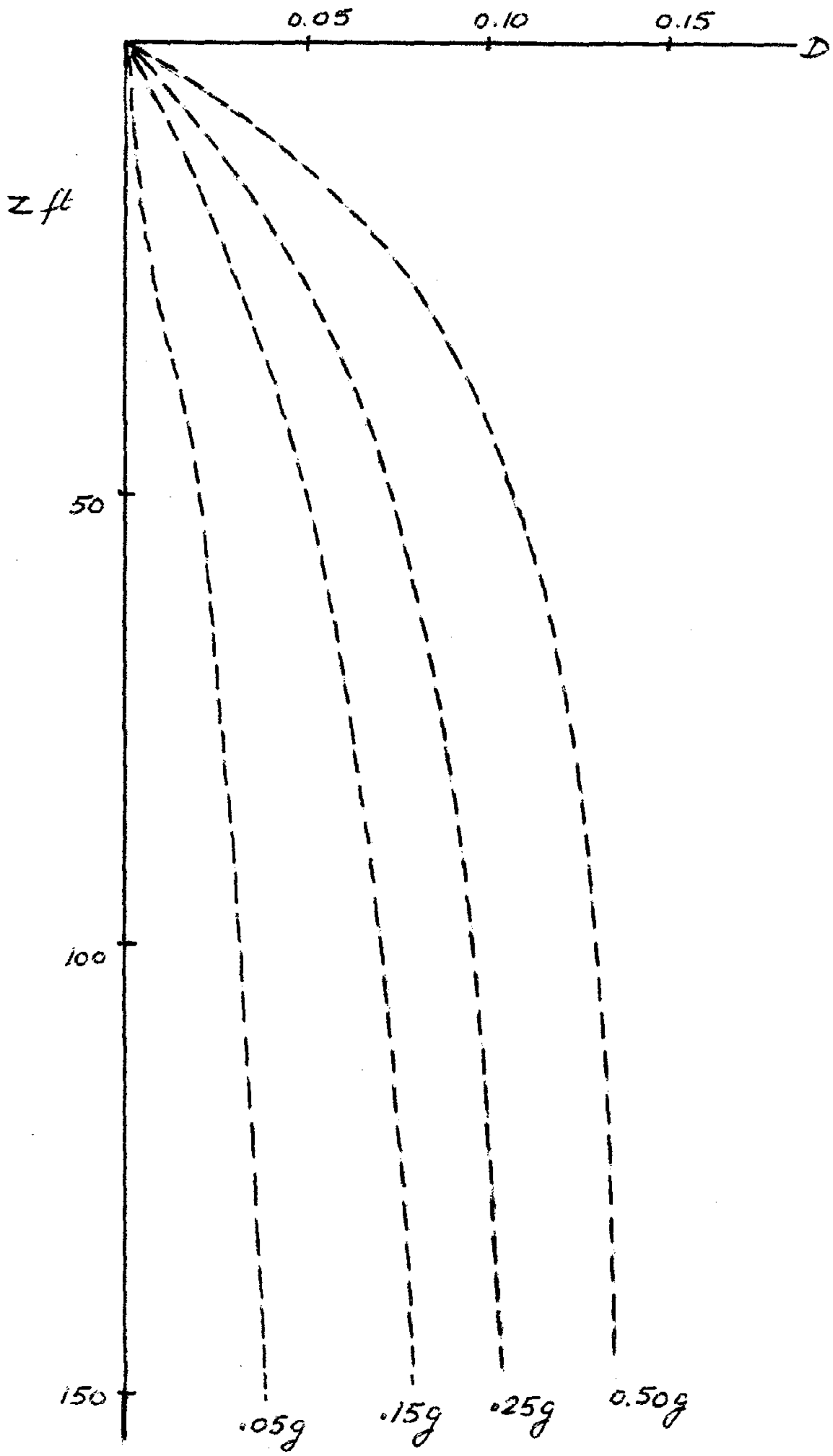


FIGURE 19 VARIATION OF DAMPING WITH DEPTH
UNIFORM SOIL PROFILE C_{so} = 1500 ft/sec

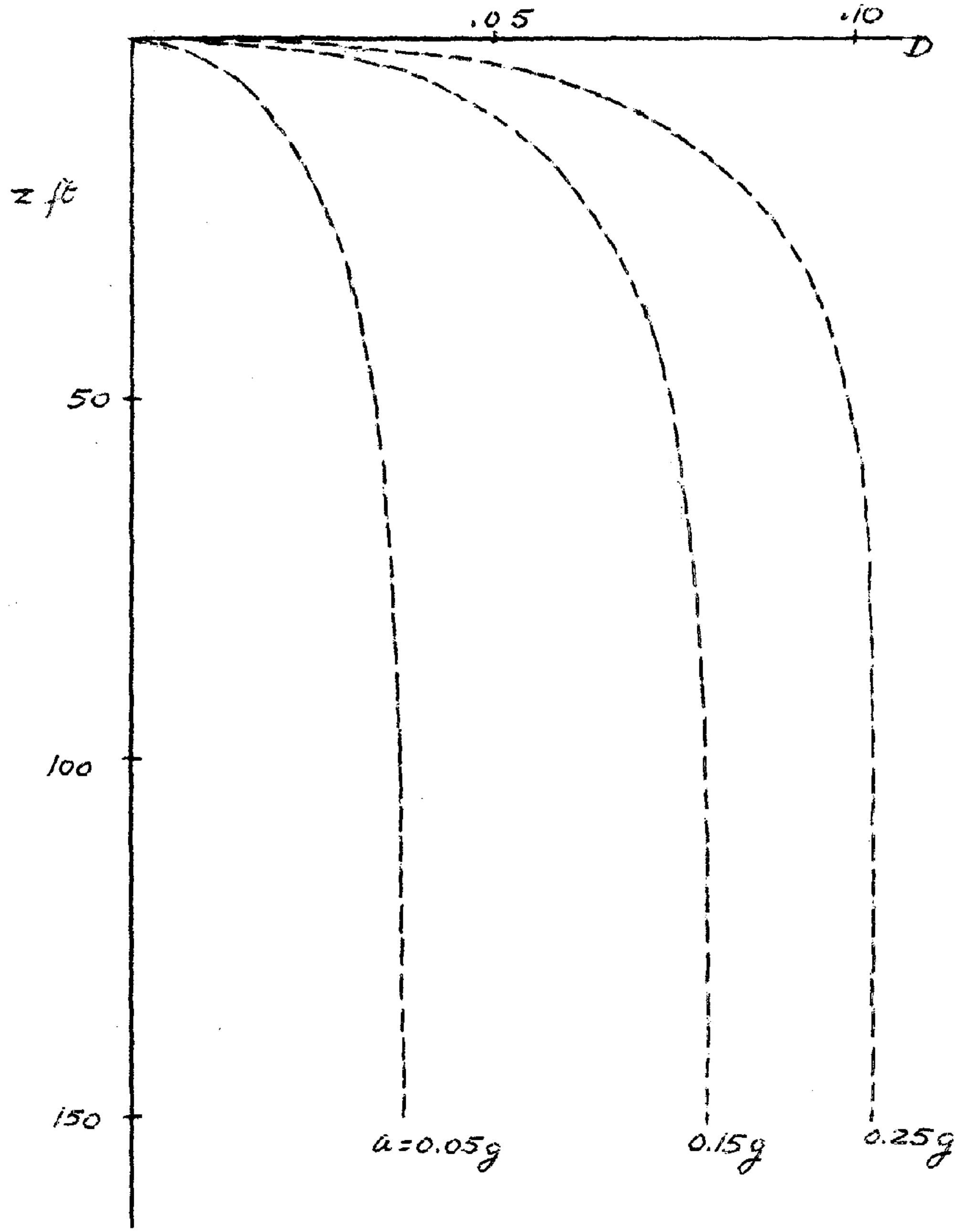


FIGURE 20 VARIATION OF DAMPING WITH DEPTH
VARIABLE SOIL PROFILE

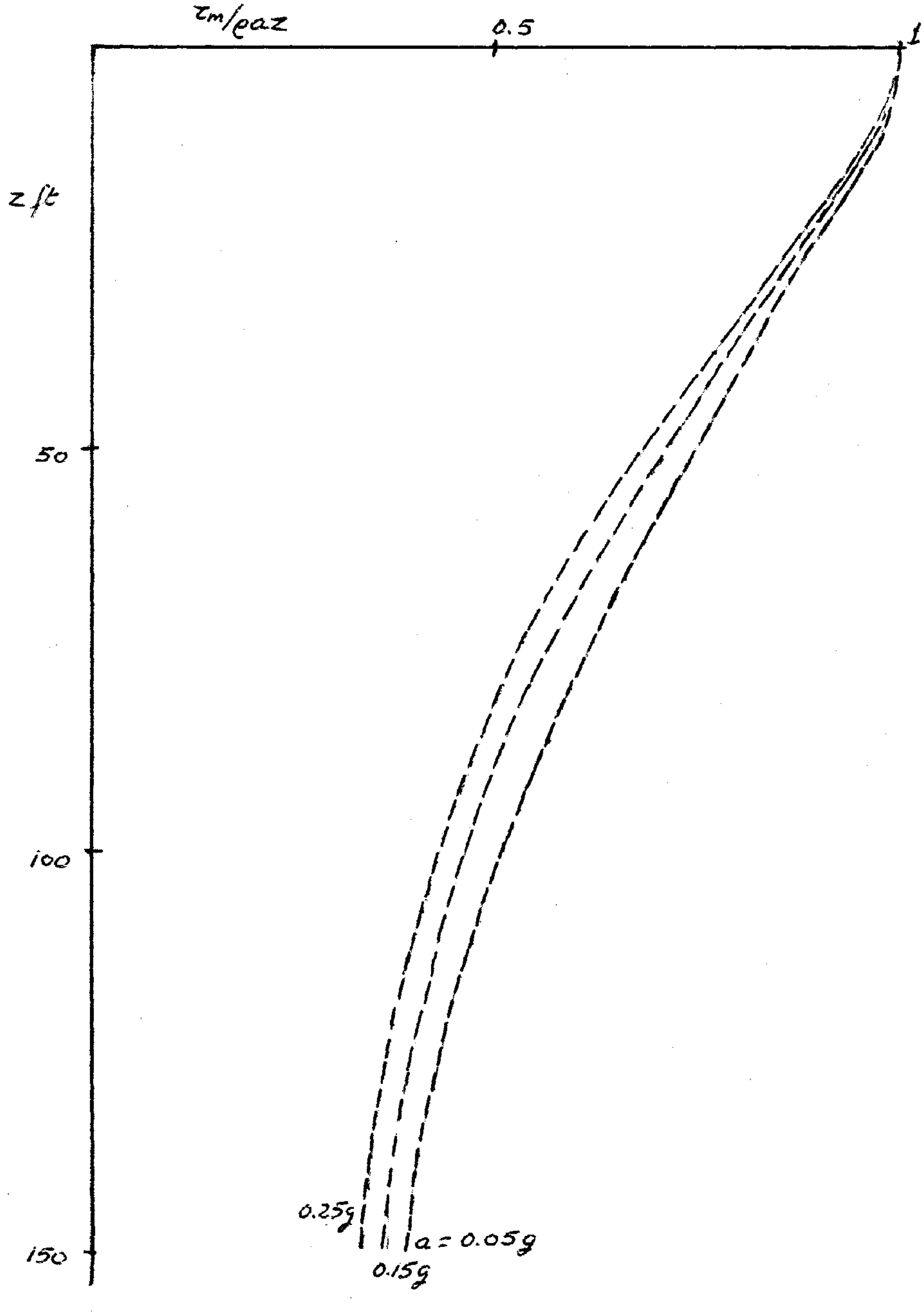


FIGURE 21 VARIATION OF SHEAR STRESS WITH DEPTH
UNIFORM SOIL PROFILE C_{S0} = 750 ft/sec.

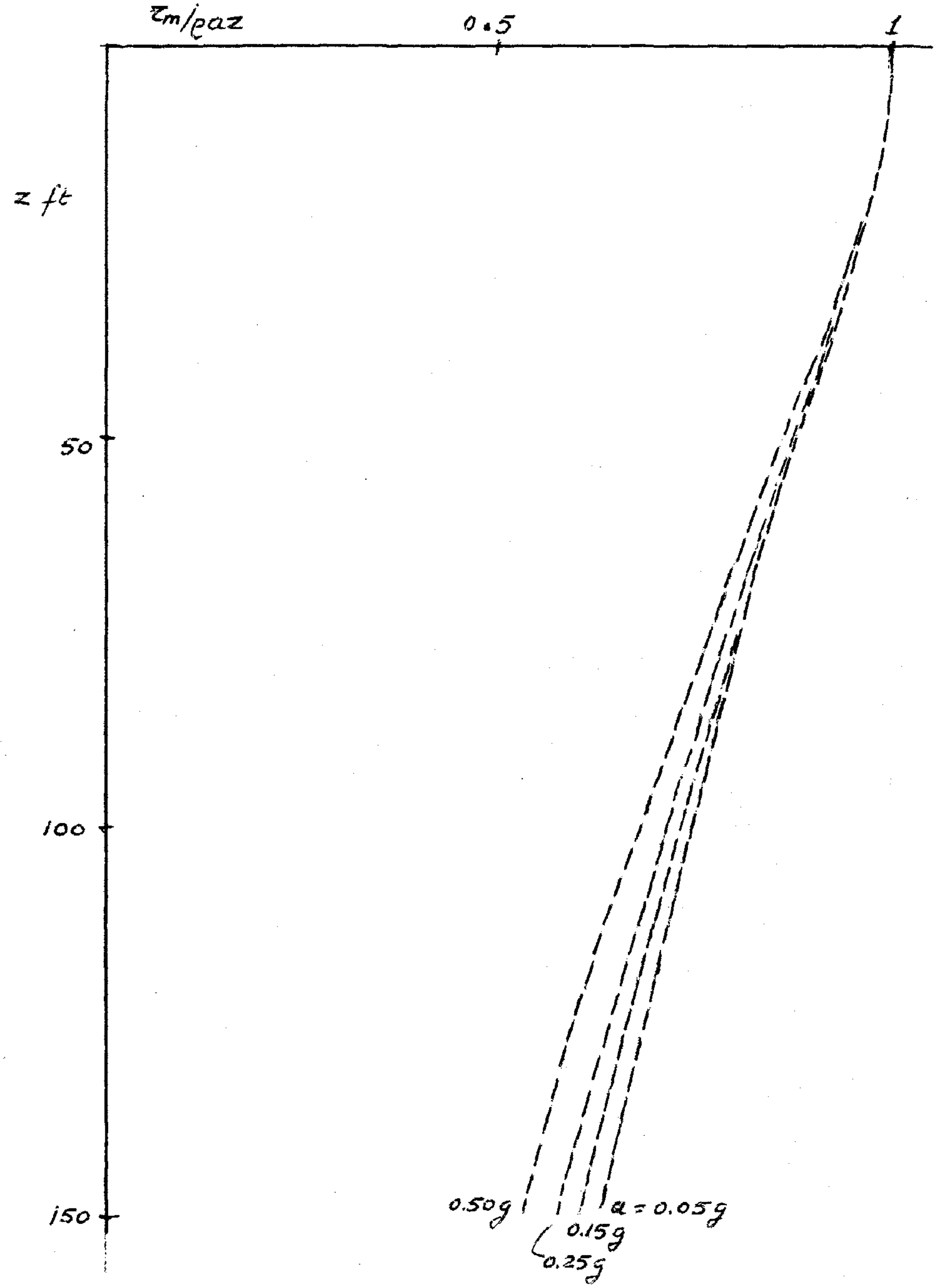


FIGURE 22. VARIATION OF SHEAR STRESS WITH DEPTH
UNIFORM SOIL PROFILE C_{S0} = 1500 ft/sec

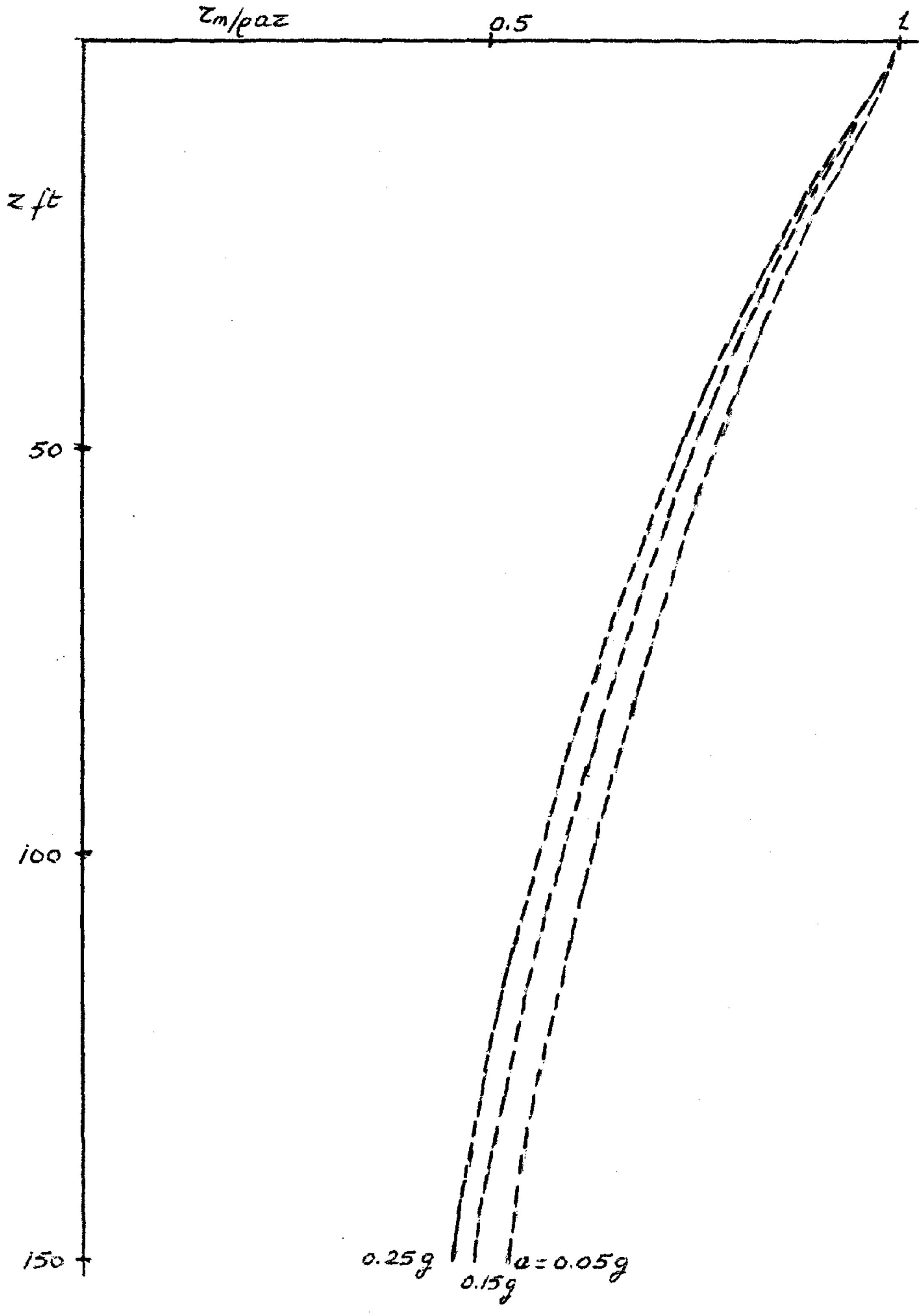


FIGURE 23 VARIATION OF SHEAR STRESS WITH DEPTH
VARIABLE SOIL PROFILE

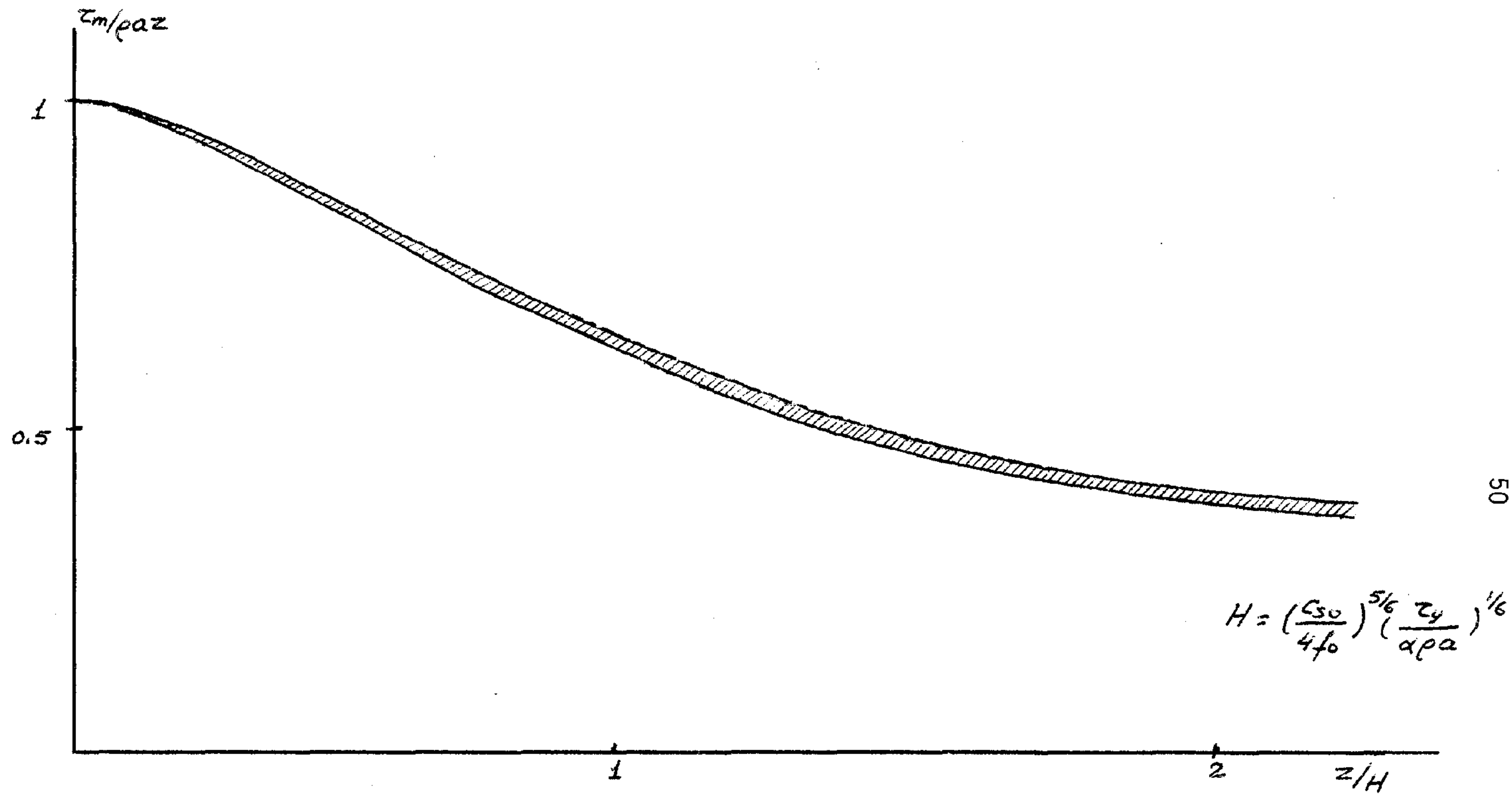


FIGURE 24 - VARIATION OF $\tau_m/\rho a z$ VS z/H - UNIFORM SOIL PROFILES

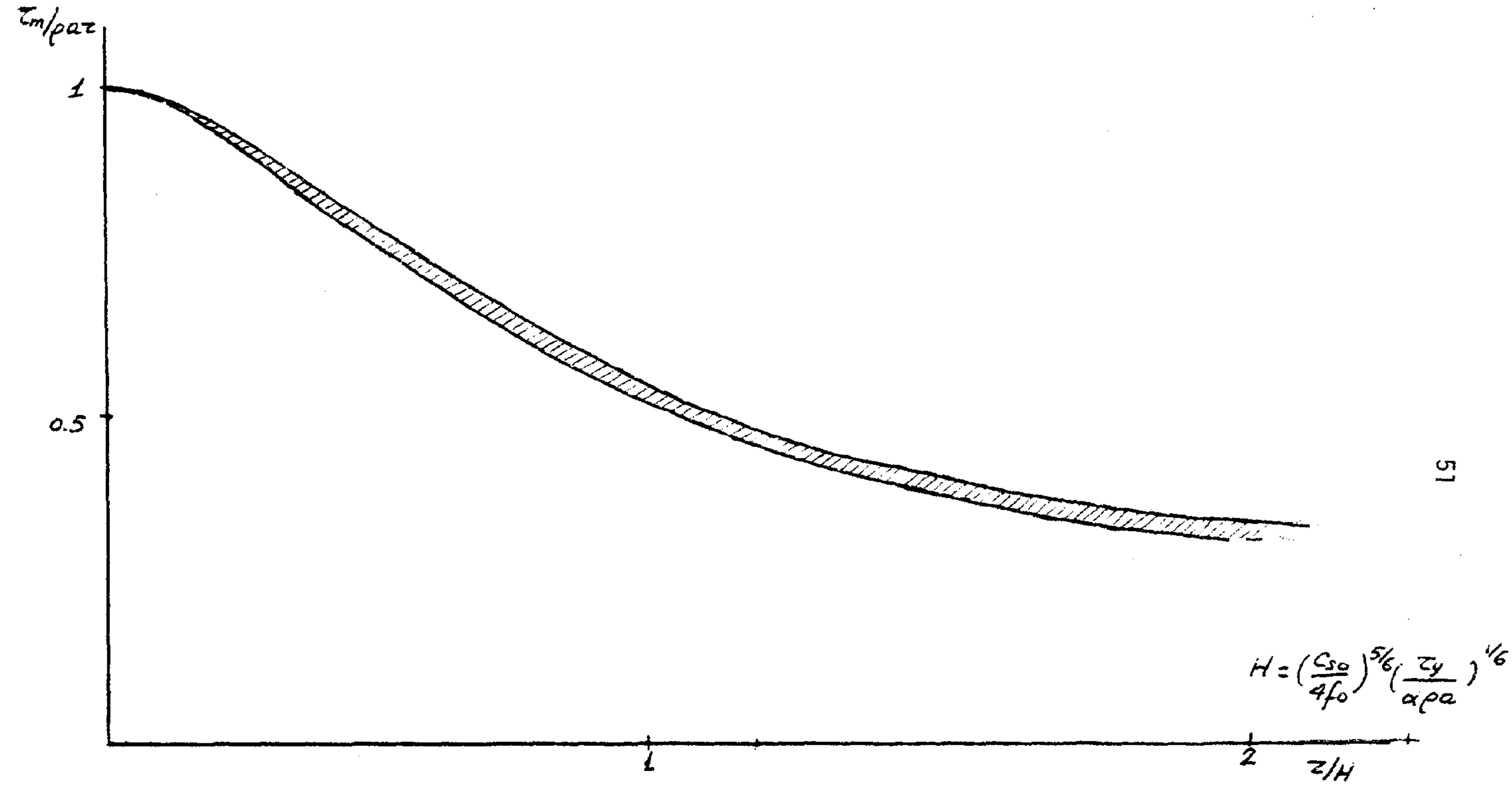
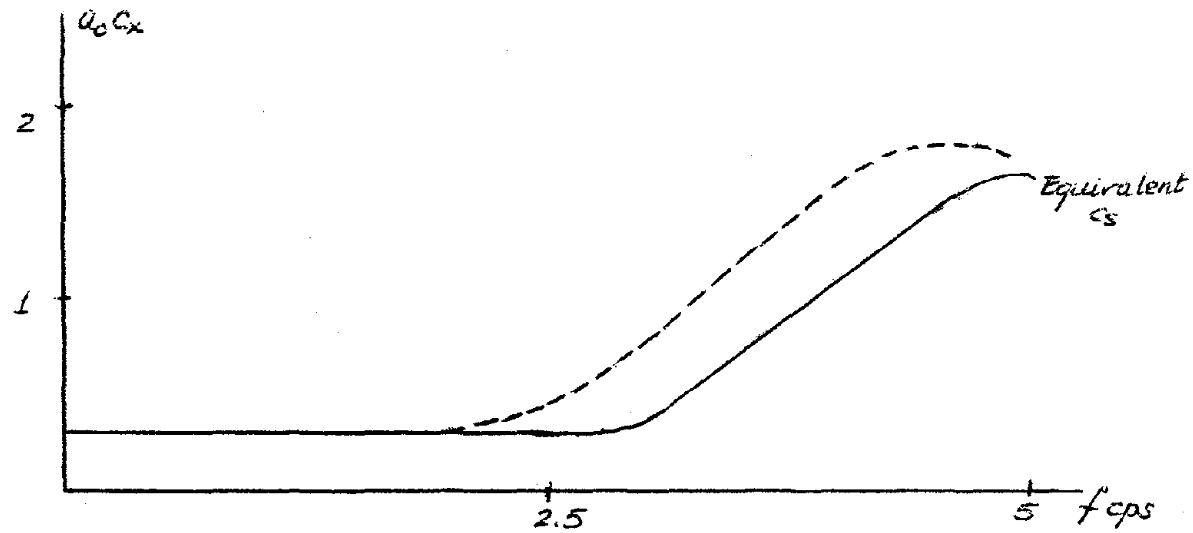


FIGURE 25 - VARIATION OF $\tau_m/\rho a z$ VS z/H - VARIABLE SOIL PROFILE



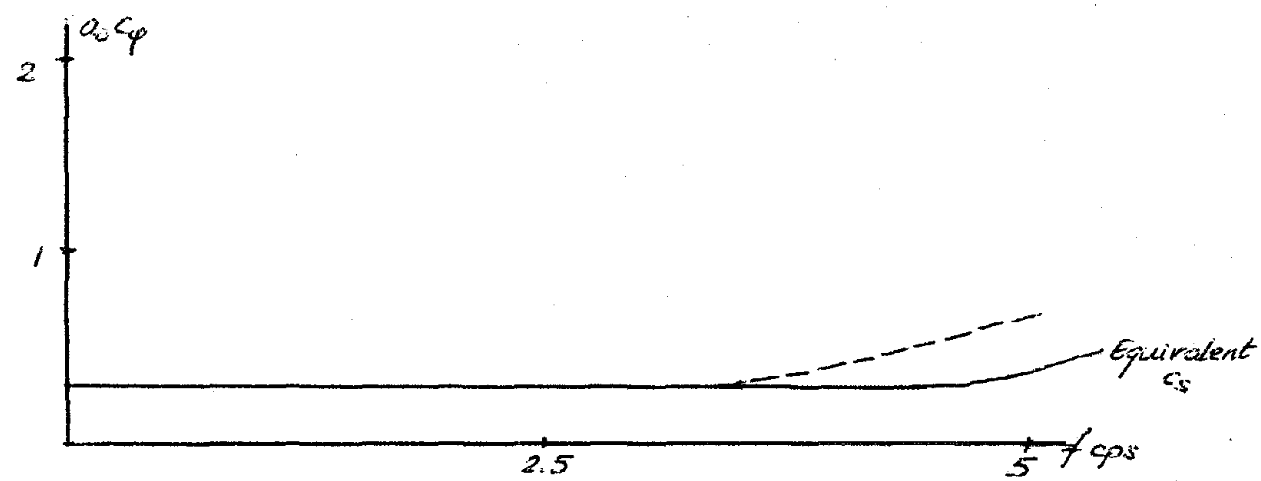


FIGURE 27 DYNAMIC DAMPING COEFFICIENTS