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DYNAMIC INTERACTION BETWEEN ADJACENT STRUCTURES

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ABSTRACT

A finite element type formulation for the solution of truly three-dimensional soil-structure interaction problems is presented. The formulation is based on the use of the consistent boundary developed by Waas for two-dimensional problems and by Kausel for three-dimensional conditions (with cylindrical coordinates) to determine the displacement on the surface of a layered soil deposit due to a unit harmonic load. The method is applied first to the determination of the dynamic stiffnesses of a square foundation and the results are compared to those of an equivalent circular footing to check their validity. An excellent agreement is obtained.

The interaction, through the underlying soil, between two rigid masses (and two structures idealized as simple one-degree-of-freedom systems) is studied next for the case of a harmonic force applied at one of the masses (or structures) and for the case of a base motion representing an earthquake-type excitation. In both cases it is found that the interaction effects increase when the two structures have the same natural frequency (on a flexible foundation), when their masses increase and as the distance between them decreases. When applying a force to one of the structures, the presence of the other tends to increase the peak response for the cases considered. For a base motion, on the other hand, the peak response tends to decrease within the range of parameters studied. In all cases the most significant effect is a change in the natural frequencies of the soil-structures system.

PREFACE

The work described in this report represents a summary of the thesis of José J. Gonzalez, presented to the Civil Engineering Department at M.I.T. in partial fulfillment of the requirements for the degree of Master of Science. The research was supervised by Professor José M. Roesset and was made possible through Grant AEN-7417835 from the National Science Foundation, Division of Advanced Environmental Research and Technology.

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1. Research Report R76-8 by Mohammed M. Ettouney, "Transmitting Boundaries: A Comparison," January 1976.

2. Research Report R76-9 by Mohammed M. Ettouney, "Nonlinear Soil Behavior in Soil Structure Interaction Analysis," February 1976.

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INTRODUCTION

The vibrations of soils and soil deposits and the dynamic interaction between soils and structures have received considerable attention in recent years. There are a variety of practical engineering problems associated with this general area: the design of foundations for vibrating machinery and the response of buried structures to blast loading were among the first ones to be studied; the effect of local soil conditions on the characteristics of earthquake motions, the stability of slopes and earth dams under seismic excitation (including the possibility of liquefaction) and the assessment of vibrations caused by transit systems were all considered later. In the last few years and because of the need to perform a seismic analysis for all nuclear power plants, the earthquake response of structures accounting for the flexibility of the foundation (soil-structure interaction) has been a problem of considerable interest and research.

A substantial amount of work has been done in all phases of the soil-structure interaction problem, from the determination of the motions at the foundation level, before the structure is built, for different types of waves (of particular importance for embedded foundations) to the development of analytical or semi-analytical solutions for the foundation stiffnesses as a function of frequency (strip footings, circular foundations and rectangular foundations can now be studied on the surface of a horizontally stratified soil deposit), the derivation of simplified formulas from parametric studies, the consideration of nonlinear effects due to the soil behavior or to the possible separation of the mat from the soil, and the implementation of general computer programs to solve the complete problem for some idealized situations (normally with very simple models of the structure).

Two general approaches are used now for the solution of soil-structure interaction problems:

— A one-step or direct approach, in which the soil and the structure are analyzed together, modelling them through finite elements (or finite differences) and linear members. This procedure has a clear theoretical advantage if inelastic behavior, particularly in the soil, is to be accounted for through a step-by-step numerical integration of the equations of motion in the time domain. The advantage is hampered by the fact that the input motion must be specified at the base of the model, where it is not known a priori. When the design earthquake is specified at the free surface of the soil deposit, as is now normally the case, a deconvolution is necessary to obtain first a compatible motion at bedrock.

— A three-step approach also referred to as the substructure or spring method. In this case the first step is the determination of the seismic motion at the foundation level, considering a rigid but massless foundation (for an embedded structure the motion will have both translational and rotational components). The second step is the determination of the dynamic stiffnesses of the foundation, complex functions of the frequency. Finally the dynamic analysis of the structure resting on frequency dependent "springs" as obtained in the second step is carried out for the base motions computed in the first. This procedure implies the validity of linear superposition and is therefore restricted in rigor to linear analyses or studies in which nonlinearities are simulated through an equivalent linearization. It offers on the other hand considerably more flexibility in the way each step is handled and it is particularly suited to parametric studies.

The advantages and disadvantages of each one of these two approaches and the possibility of obtaining with either sensible results, when properly implemented, has been extensively discussed, although some controversy seems to persist, unfortunately, on the validity or adequacy of each method.

HISTORICAL REVIEW

It has long been recognized that no building stands alone, and that the presence of neighboring structures may affect its dynamic response, particularly under a seismic excitation, but very little work has been done to date to determine in a systematic way the importance of these interactions. The problem is complicated by the large number of parameters involved: the number of structures, the relative position in space, their size, mass and stiffness and the characteristics of the soil profile. The interaction of two structures through the underlying, or surrounding, soil is not only of interest in seismic analyses but also in the assessment of potential damage to very sensitive structures due to vibrations induced from adjacent buildings (vibrating machinery, wind loads, etc.).

In 1969 Richardson (5) studied the case of two rigid cylinders (with the same dimensions) resting on the surface of an elastic half-space when one of them was excited by external harmonic forces. This solution was based on an analytical formulation using an averaging procedure to determine the motions of each cylinder from the free field displacements. With the two cylinders at a distance of ten radii between centers, Richardson's results indicate that the direct effects in the excited mass (vertical displacement due to a vertical load) are only slightly affected by the presence of the second mass, but that new effects appear which would not have occurred had the mass been alone (horizontal displacements and rotations under a vertical load). Under this type of excitation the vertical displacement of the second, passive mass, is about 20% of that of the first with a similar variation with frequency. The horizontal displacement and the rotation of the second mass are, however, much larger than those of the first. This result is quite logical if one takes into account that these effects are induced first in the passive mass and then transmitted, through a feedback mechanism, to the active one.

In 1974 Chang Liang (1) presented the results of a more extensive set of parametric studies on the interaction effects between two rigid masses resting on the surface of a soil layer of finite depth (underlain by rigid rock). A two-dimensional (plane strain) problem was considered and the solution was obtained using a finite element type formulation with the consistent boundary originally developed by Waas (6). Both the case of a harmonic force (a horizontal force or a rocking moment) applied to one of the masses, and the case of a base motion affecting the two masses, and simulating a seismic type excitation, were studied.

For the first case and within the range of parameters studied (values of the masses, relative distances between their centers and stratum depths), Chang Liang concluded that the interaction effect between the two masses seemed to be less important for very shallow layers of soil (resting on rigid rock) than for layers of moderate thickness. In all cases the existence of a second mass at distances of 2.5 to 5 times the base width affected only slightly the direct response of the excited mass (although there were again vertical displacements that would not have occurred had the mass been alone). The significant part of this kind of study would thus be the determination of the vibrations induced in the second, unexcited mass. The horizontal displacement of this mass due to a horizontal force applied at the first is of the order of 50% that of the active mass when the stratum is relatively deep and the distance between the foundations is 2.5 times their base width; it reduces to about 20% if the stratum is very shallow or if the distance increases to five widths. This ratio is larger when the masses increase (particularly the second one). Similar results are obtained for the rotations induced by a rocking moment, but the ratios of the effects in the passive structure to those in the active one are slightly smaller (of the order of 30 to 40% for the deep stratum and the smaller distance). These observations are basically consistent with those of Richardson.

For the second case, when a base motion simulating an earthquake excitation (caused by a train of shear waves propagating vertically through the soil at a specified frequency) was imposed on both masses, the main effect of the adjoining mass seemed to be a change in the natural frequencies of the system. As a consequence, if the displacements at the base of one of the masses (including the effect of the other) were divided by the corresponding result if the mass were alone, amplifications and deamplifications would take place at different frequencies. The amplifications could be of the order of 50 to 100% when the masses were heavy and the deamplifications were similarly of the order of 40 to 60%. The net effect under an actual earthquake would depend, however, on the frequency content of the specified motion. For a narrow band process interaction effects between the two masses could be very important: if most of the energy of the earthquake is around one of the frequencies where amplifications take place, a large increase in response could occur (the range of amplification is very narrow for the swaying frequency of the masses, but broader for the rocking frequency); on the other hand, if the energy was centered in the frequency range where the results show deamplifications, the effect of the adjoining mass would be beneficial. For a white noise or a wide band process, the main effect would be the appearance of small shifts in the frequencies at which the peaks of the response spectra occur. Whether the amplitude of these peaks would increase or decrease is not obvious from the results presented, but it appears that small amplifications might occur. In particular the presence of a larger and heavier structure seemed to have a detrimental effect on the response of a lighter one.

More recently (1975) Lysmer et al. (4) have presented results for the seismic response of a nuclear power plant, including the effect of two auxiliary buildings. The formulation was based on a finite element solution by the one-step approach, using basically a two-dimensional model to which viscous dashpots are added on the lateral faces to increase the amount of damping and thus simulate radiation in the direction perpendicular to the plane. The model will have, however, the foundation stiffnesses and natural frequencies corresponding to a two-dimensional solution rather than those of the true three-dimensional problem. Furthermore, the buildings must be centered along a common axis, and their foundations must have the same width in the direction perpendicular to the plane (a serious limitation for the analysis of general three-dimensional situations).

In this study two equal auxiliary buildings were included, one on each side of the nuclear power plant (symmetrically placed). A layer of sand with a depth of 65 ft. resting on rigid rock was considered. The main building had a base width of about 2 times the stratum depth and was embedded to about one-third of this depth. The auxiliary buildings had base widths approximately equal to the stratum depth and were deeply embedded to about two-thirds of this depth. The control motion, with a maximum acceleration of 0.25g and a spectrum according to the AEC Regulatory Guide,seemed to be specified at the depth of the reactor foundation in the free field. The results presented were the acceleration response spectra for 2% oscillator damping at a point of the containment building at the level of the soil surface, considering only the main building (two-dimensional solution) and with the effect of the auxiliary buildings (for a two-dimensional and a pseudo three-dimensional solutions).

These results are particularly striking and show much stronger effects than those of previous studies. When the two-dimensional model is considered throughout, the peak of the response spectrum is amplified by a factor of almost 2.5, due to the presence of the auxiliary buildings. This would suggest that present practice, analyzing nuclear power plants under seismic excitation without due account for the presence of adjacent structures,could lead to very erroneous results and unsafe designs, a very disquieting thought. It would also indicate that the interaction of adjacent buildings is much more important than other effects which are now being studied.

The comparison of the two-dimensional solution without auxiliary buildings and the pseudo three-dimensional results shows an increase in the peak of the response spectrum of only 1.5 to 2 (still very significant), but it is not very meaningful because of the inconsistency in the models. The building alone should have been studied also with the pseudo three-dimensional model for a valid comparison.

Finally, the comparison between the normal two-dimensional solution and the one with additional viscous dashpots on the lateral faces (pseudo three-dimensional approach) shows a reduction in response which can be easily explained by the increase in damping provided in the latter.

An interpretation of these results is difficult without the complete data on the structures (stiffnesses and masses) and on the motion characteristics. It is possible on one hand that the interaction effects be so marked because the input at the base of the building has a large peak at a specific frequency, or because the parameters are not realistic for typical structures (they would increase with very large masses). On the other hand, the very deep embedment of the auxiliary buildings may cause a box-type effect and decrease the effective radiation damping (it would be necessary to assess first what the soil-structure interaction effect is for the containment building alone and what is the amount of effective damping due to radiation). This is a case where a solution using the three-step approach would have greatly facilitated the identification of the factors contributing to the increase in response.

It seems, in any case, that because of these results, more studies are needed to improve present knowledge on the nature and magnitude of structure-soil-structure interaction effects.

SCOPE

In this work a formulation is presented for the general three-dimensional solution of structure-soil-structure interaction problems with the three-step approach. Any number of structures with foundations of arbitrary shape can be considered. The main two limitations are that the structures are founded on the surface (the method could be extended to embedded foundations, but it would become much more expensive computationally) and that a horizontally stratified soil deposit of finite depth (resting on much harder, rock-like material) must be considered. The formulation is essentially an extension of the procedure used by Chang-Liang in his studies (1) to the three-dimensional case.

The formulation is described first and evaluated by comparing the free field displacements due to concentrated loads on the surface of a layer of finite depth to results of a semi-analytical solution suggested by Gazetas (2). The method proposed here furnishes results which are in good agreement with the analytical solution at considerably less computational expense. In addition, the dynamic stiffnesses of a square foundation are obtained and compared to those of an equivalent circular footing, and the convergence characteristics of the method are investigated as a function of the mesh size.

Results are then presented for the cases of two rigid masses and two simple structures resting on the surface of a homogeneous soil deposit with a depth equal to twice the foundation width. In order to reduce the number of parameters, both foundations are assumed to be square, of equal size, parallel and centered along a common horizontal axis. The values of the masses, the natural frequencies and the distance between the foundations are varied. Both the cases of a harmonic excitation applied to one of the masses or structures and of a base motion (simulating an earthquake) are considered. The results are presented in dimensionless form: the displacements are multiplied by the shear modulus of the soil and the side of the foundation, and divided by the value of the applied force (they correspond to unit values of these three quantities). They are plotted versus a dimensionless frequency which is the actual frequency in cycles per second (Hz) multiplied by the size of the foundation and divided by the shear wave velocity of the soil. (Notice that this is not exactly the same dimensionless frequency used in other studies where the frequency in radians per second is multiplied by the radius of a circular foundation, or half the size of a square one, and divided by the shear wave velocity. A factor of π should be applied to the values used here to obtain the other).

FORMULATION

The starting point for the proposed formulation is the determination of the displacements at any point on the surface of the soil deposit due to a harmonic unit horizontal force x, a unit horizontal force y, and a unit vertical force z applied at the origin, as a function of the frequency of excitation. A semi-analytical type solution for this problem was suggested by Gazetas (2) using a double Fourier expansion in the x-y plane. In this work the solution is based on a finite element type formulation using the consistent boundary developed by Waas (6) and extended by Kausel (3) to the three-dimensional case. It is important to notice that contrary to what has been sometimes reported (see 4, for instance) the use of this boundary matrix is not restricted to the solution of axisymmetric problems. When part of the soil profile or the structure are reproduced by axisymmetric (toroidal) finite elements, then the geometry of the problem must be indeed axisymmetric, although the loads or excitation can have any distribution expanding them in a Fourier series along the circumference (the approach normally used for the solution of shells of revolution under arbitrary loadings). This restriction becomes meaningful when the soil-structure interaction problem is to be solved in a single step but no longer applies if the three-step solution is used. All that is necessary is to express the coordinates of the points where displacements are desired in cylindrical coordinates and to refer the radial and tangential displacement to orthogonal cartesian components.

Figures 1 and 2 show the horizontal and vertical displacements at the free surface of a soil layer of finite depth due to a unit horizontal or vertical force at the origin, respectively. The abscissa is the distance to the point of application of the load divided by the stratum thickness H. To evaluate these results it must be taken into account that in the present solution a concentrated load is considered within the accuracy of the finite element method. In the semi-analytical solution, on the other hand, the load is expanded in a double Fourier series using a discrete Fourier transform. The solution corresponds then to a rectangular pulse, or a load uniformly distributed over a rectangle, with sides equal to 1/6 of the stratum depth for the case shown. One can expect therefore that in the immediate neighborhood of the point of application of the load, the results provided by the finite element model will be slightly larger (the actual condition of a point load is better modelled). A second parameter which affects the accuracy of Gazetas' solution is the number of points used for the double Fourier expansion. The results shown are for a mesh with 64 equally spaced points in each horizontal direction. Considering these differences the agreement in the results seems very good. The curves shown are for frequencies of 0.1 C_s/H and 0.4 C_s/H, where C_s is the shear wave velocity of the soil and H is the stratum depth. They are typical of those obtained for a number of frequencies studied.

For the problem at hand the foundation or foundations considered are discretized by a grid of equally spaced points in the x and y directions (Figure 3). (The method would accept of course unequal spacing of the points, but in this work only square foundations were treated). Assuming that each side of a foundation is divided into N equal segments, there are $(N+1)^2$ points for each foundation and a total of $2(N+1)^2$ points (if both foundations are equal). By considering a unit force applied at one of these points and determining the displacements u, v and w at all the others, one can form a column of a global flexibility matrix F of dimensions $6(N+1)^2$ by $6(N+1)^2$ (since there are three displacement components at each node), or in general $3N_1 + 3N_2$ if N_1 is the total number of points in the first foundation and N_2 the corresponding quantity in the second. It is important to notice that it is not necessary to repeat the computation for each point (or column of the flexibility matrix), since many of the coefficients can be derived from the others by a simple shift or by applying symmetry and antisymmetry conditions.

If P_1 represents a vector of forces (in the x, y and z directions) applied at the mesh points of the first foundation, U_1 is the vector of displacements at the same points, and P_2, U_2 are the corresponding variables for the second foundation.

$$U = \begin{Bmatrix} U_1 \\ U_2 \end{Bmatrix} = FP = F \begin{Bmatrix} P_1 \\ P_2 \end{Bmatrix}$$

Imposing now the condition of a rigid body motion for each foundation, the displacements of the mesh points can be related to those of the corresponding centroid by a transformation of the form

$$U_1 = T_1 \, u_1$$
$$U_2 = T_2 \, u_2$$

where U_1, U_2 have $3(N+1)^2$ components (or in general $3N_1$ and $3N_2$ respectively). T_1 and T_2 are matrices $3(N+1)^2$ by 6 (or in general $3N_1$ by 6 and $3N_2$ by 6), and u_1 and u_2 have six components each (three translational components along the x, y, and z axis and three rotations around each one of these axes).

The resultants of the forces applied at the mesh points of a foundation with respect to its centroid will consist of three forces and three components.

Denoting by p_1 and p_2 the vectors of these six force components,

$$p_1 = T_1^T P_1$$

and $$p_2 = T_2^T P_2 \quad .$$

It is then possible to write

$$p = \begin{Bmatrix} p_1 \\ p_2 \end{Bmatrix} = \begin{bmatrix} T_1^T & 0 \\ 0 & T_2^T \end{bmatrix} F^{-1} \begin{bmatrix} T_1 & 0 \\ 0 & T_2 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = K\, u$$

K is then the dynamic stiffness matrix (with terms complex functions of frequency) of the system of two foundations on the soil layer. It should be noticed that in order to obtain K it is not necessary to invert the flexibility matrix F. It is enough instead to form directly the product $F^{-1} \begin{bmatrix} T_1 & 0 \\ 0 & T_2 \end{bmatrix}$ by solving a system of equations with F as matrix of coefficients and the columns of the transformation matrix as right-hand side vectors. For the case of two foundations, matrix K will be of size 12 x 12. It can be partitioned into four submatrices of the form

$$K = \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix}$$

where K_{11} and K_{22} are symmetric and $K_{21} = K_{12}^T$.

It must be noticed that due to the interaction effects K_{11} and K_{22} are not equal to the stiffness matrices of the individual foundation considered alone. Furthermore, even if the two foundations are equal, K_{11} and K_{22} are not identical.

When the two foundations are parallel and centered along the x axis (as shown in fig. 3) in-plane and out-of-plane effects can be uncoupled. The in-plane effects are represented by forces and displacements in the x (horizontal) and z (vertical) directions, and moments and rotations around the y axis. The out-of-plane effects consist of forces and displacements in the y direction, and moments and rotations around the x axis (rocking) and around the z axis (torsion). For this situation the in-plane matrices are of the form

$$K_{11} = \begin{bmatrix} K_{xx} & K_{xz} & K_{x\phi} \\ K_{xz} & K_{zz} & K_{z\phi} \\ K_{x\phi} & K_{z\phi} & K_{\phi\phi} \end{bmatrix} \qquad K_{22} = \begin{bmatrix} K_{xx} & K_{xz} & -K_{x\phi} \\ K_{xz} & K_{zz} & -K_{z\phi} \\ -K_{x\phi} & -K_{z\phi} & K_{\phi\phi} \end{bmatrix}$$

and

$$K_{21} = \begin{bmatrix} K'_{xx} & K'_{xz} & K'_{x\phi} \\ K'_{xz} & K'_{zz} & K'_{z\phi} \\ -K'_{x\phi} & -K'_{z\phi} & K'_{\phi\phi} \end{bmatrix} \qquad K_{12} = K_{21}^T$$

Similar relationships take place for the out-of-plane effects.

In order to check the accuracy of the solution procedure, the case of a single foundation, rigid and square, was considered. Following the approach used by Kausel (3), the convergence of the solution with mesh size was investigated by considering different grids and plotting the values of the stiffnesses versus mesh size. In all cases it was considered that the thickness of each sublayer of soil for the determination of the consistent boundary matrix (and the computation of the surface displacements) was equal to the mesh size. As in Kausel's studies the values of the stiffnesses for different meshes fell almost exactly on straight lines. The extrapolated values of the static stiffnesses obtained from the study were

$$K_{xx} = K_{yy} \simeq \frac{9.2\ GB}{2-\nu}\left(1 + 0.6\,\frac{B}{H}\right) \qquad \text{(horizontal)}$$

$$K_{zz} \simeq \frac{4.6\ GB}{1-\nu}\left(1 + 1.6\,\frac{B}{H}\right) \qquad \text{(vertical)}$$

$$K_{\phi\phi} \simeq \frac{4GB^3}{1-\nu}\left(1 + 0.11\,\frac{B}{H}\right) \qquad \text{(rocking)}$$

$$K_{\theta\theta} \simeq 8.2\ GB^3\left(1 + 0.05\,\frac{B}{H}\right) \qquad \text{(torsion)}$$

where G is the shear modulus of the soil, B is half the side of the square foundation, and H is the stratum depth.

By comparison for a circular foundation, Kausel had suggested the formulas (3)

$$K_{xx} = \frac{8GR}{2-\nu}\left(1 + 0.5\,\frac{R}{H}\right)$$

$$K_{\phi\phi} = \frac{8GR^3}{3(1-\nu)}\left(1 + 0.17\,\frac{R}{H}\right)$$

Taking an equivalent radius so as to obtain the same area for the circular and the square foundation $R_e = \sqrt{4/\pi}\,B$ the first expression would become

$$K_{xx} = \frac{9.1GB}{2-\nu}\left(1 + 0.56\,\frac{B}{H}\right)$$

Taking $R_e = \sqrt[4]{16/\pi}\,B$ to obtain the same moment of inertia, the second expression yields

$$K_{\phi\phi} = \frac{11.9GB^3}{3(1-\nu)}\left(1 + 0.2\,\frac{B}{H}\right).$$

These results indicate that the usual procedure of adapting the stiffnesses of a circular foundation to the case of a square footing by defining an equivalent radius provides an excellent approximation. Furthermore, the variation of the stiffness coefficients with frequency was almost identical to the results of Kausel using again the equivalent radius and modifying accordingly the values of the dimensionless frequency.

It should be noted that the accuracy provided by a given mesh is not the same for all the terms of the stiffness matrix. Thus for instance for a coarse mesh with each side divided in three equal segments (a total of sixteen points under the foundation) the error in the terms K_{xx} K_{yy} or K_{zz} may be of the order of 15 to 20% (depending on Poisson's ratio) the error in the terms $K_{\phi\phi}$ or $K_{\theta\theta}$ is of the order of 50%. In all cases the results converge monotonically from above: that is to say, the computed values are larger than the actual ones. It is thus recommended, if the method is used for practical applications to use a mesh which is sufficiently fine, or even better, to compute results for two meshes (a coarse one and a medium one) and to obtain improved estimates using a linear extrapolation.

In order to study the interaction effects between two rigid masses, it is sufficient to form the inertia matrices of each mass. If M_i is the mass, I_{x_i} I_{y_i} and I_{z_i} are the mass moments of inertia with respect to axes parallel to the x, y and z directions passing through the center of gravity, and E_i is the height of this point with respect to the base, the inertia matrix referred to the base displacements is of the form

$$M_{b_i} = \begin{bmatrix} M_i & 0 & 0 & 0 & M_iE_i & 0 \\ 0 & M_i & 0 & -M_iE_i & 0 & 0 \\ 0 & 0 & M_i & 0 & 0 & 0 \\ 0 & -M_iE_i & 0 & I_{x_i} + M_iE_i^2 & 0 & 0 \\ M_iE_i & 0 & 0 & 0 & I_{y_i} + M_iE_i^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & I_{z_i} \end{bmatrix}$$

The dynamic equations of motion for a steady state response in the frequency Ω are then

$$(K - \Omega^2 M)\, U = F$$

where

$$M = \begin{bmatrix} M_{b_1} & 0 \\ 0 & M_{b_2} \end{bmatrix}$$

and F is the vector of applied harmonic forces at the base of the masses. For the case of a base motion, simulating an earthquake type excitation if $\ddot{U}$ are the absolute accelerations at the base of the masses (for a frequency Ω) and $\ddot{U}_G$ represents a vector of free field ground accelerations (with the specified amplitudes of acceleration in the appropriate degrees of freedom of each mass and zeroes for all other components), the corresponding equations are

$$(K - \Omega^2 M)\, \ddot{U} = K\, \ddot{U}_G$$

For two (or more) structures it is necessary to form first the dynamic stiffness matrix $D^i = K^i - \Omega^2 M^i$ of each structure. If this matrix is partitioned in the form

$$D^i = \begin{bmatrix} D^i_{11} & D^i_{12} \\ D^i_{21} & D^i_{22} \end{bmatrix}$$

where D_{11} corresponds to all the degrees of freedom above (and excluding) the foundation, D_{22} to the six degrees of freedom of the foundation (assumed to be rigid) and D_{12}, $D_{21} = D^T_{12}$ are the coupling terms, the equations of motion for a steady state condition are

$$A\,U = F$$

with

$$A = \begin{bmatrix} D^1_{11} & 0 & D^1_{12} \\ 0 & D^2_{11} & D^2_{12} \\ D^1_{21} & D^2_{21} & B+K \end{bmatrix}$$

$$B = \begin{bmatrix} D^1_{22} & 0 \\ 0 & D^2_{22} \end{bmatrix}$$

K is the stiffness matrix of the two foundations and the soil stratum as obtained from the proposed method of analysis. F is the vector of harmonic forces applied at any of the degrees of freedom of the structures and the two foundations. Partitioning it in a manner analogous to that used for matrix A,

$$F = \begin{Bmatrix} F_1 \\ F_2 \\ F_b \end{Bmatrix}$$

Finally, to determine the dynamic response of two (or more) structures to a base motion representing a seismic input at the free surface of the soil deposit (in the far field) obtaining the transfer functions of the absolute accelerations, the same equations can be used replacing the vector U by $\ddot{U}$ and the vector of applied forces F by a vector of the form

$$\left\{ \begin{array}{c} 0 \\ 0 \\ K\ddot{U}_G \end{array} \right\}$$

with the same partitioning used above and $\ddot{U}_G$ as previously defined.

INTERACTION BETWEEN TWO RIGID MASSES

In this section results are presented illustrating the interaction between two rigid masses on equal, square foundations and resting on the surface of a homogeneous soil deposit. The depth of the stratum (underlain by rigid rock) is equal to four times the side of each foundation ($H = 4B$); a Poisson's ratio of 1/3 and an internal hysteretic damping in the soil of 5% were assumed.

The masses are defined in terms of the dimensionless parameter $\bar{m} = M/8\rho B^3$ where M is the actual value of the mass and ρ is the mass density of the soil. Values of $\bar{m} = 1$ and 2 were studied; the first one may be typical of a heavy nuclear power plant; the second one would correspond to an extremely heavy building. These large values were selected in order to emphasize the interaction effects. It was assumed in all cases that the height of the center of gravity above the foundation was equal to 1/3 of the base size and that the radius of gyration (for the computation of the mass moments of inertia) was equal to half the base. Distances between the centers of the foundations of 3B (a clear spacing of half the foundation size), 4B, 5B, and 10B were considered. The results presented here are all for the case $D = 3B$ where the interaction effects are more marked. They are plotted versus the dimensionless frequency $f_o = f\,(2B/c_s)$ defined above.

Figure 4 shows the variation with frequency of the horizontal displacement at the base of the first mass when it is excited by a harmonic horizontal force (applied also at the base). The first, excited, mass has m = 1. It can be seen that the presence of the second mass originates shifts in the natural frequencies with amplification of the motion in certain ranges and deamplifications in others. Similar results are obtained when $m_1 = 2$ (figure 5), but it can be seen that the effect of interaction increases when the second mass is heavier (and also as both masses increase). To interpret these figures it is interesting to notice that the swaying rocking frequencies of the masses alone (frequencies at which the peaks occur) are approximately

for m = 1	0.125	0.21	0.375
for m = 2	0.125	0.16	0.34

The first peak is affected by the natural frequency of the soil deposit (at precisely 0.125). For the case of a half-space one would have expected to find only two peaks.

Figure 6 shows the horizontal displacements induced in the second mass. For a distance of 3B between the centers of the foundations, the displacement of the passive mass is about 0.6 times that of the excited one for $m_2 = 1$ and about 0.8 times for $m_2 = 2$. This ratio decreases obviously as the distance between the foundations increases, but it is still of the order of 0.25 to 0.30 for D = 10B. For the case (1,1) the variation with frequency of the motion of the second mass is similar to that of the first. For the other cases it tends to show only one peak. Notice that the results for (1,2) and (2,1) are almost identical (the difference in the scale of the figures should be taken into account).

Figure 7 shows the vertical displacements of the two masses for the two cases (1,1) and (2,2). It must be remembered that this displacement is entirely a result of the adjoining mass, since for a mass alone no vertical motion should take place (under a horizontal excitation). The displacement is almost zero below the fundamental frequency of the stratum, but it becomes significant in the neighborhood of the vertical frequencies of the masses (0.23 for m = 1 and 0.20 for m = 2). The effect ot the horizontal rocking frequency can still be seen when both masses are equal to 2, but is very small in the other instances.

When the two masses are very close to each other (D = 3B) the vertical displacement induced in the active mass by the presence of the second can have a peak amplitude of the order of 1/5 to 1/8 of the peak horizontal displacement. The maxima of both effects occur, however, at different frequencies: if the excitation were to take place in the neighborhood of the vertical frequency, the vertical displacements could be almost as large as the horizontal ones (from 0.5 to 1 times, depending on the mass ratios). These vertical effects decay, though, very fast as the distance between the foundations increases. They are about half to two-thirds of the values shown at D = 4B, and only 1/20 of these values at D = 10B.

The vertical displacements of the passive mass have a frequency variation very similar to those of the first mass. It is important to notice that their amplitude is much larger (about 1.7 times when D = 3B and up to 10 times for D = 10B). This illustrates the fact that this displacement is induced first in the adjoining mass and then transmitted as a feedback to the active one.

The effect of the adjoining mass on the rotations caused by a unit horizontal force or a unit rocking moment is very similar to that described above for the horizontal displacement. Analogous conclusions are reached comparing vertical displacements due to vertical loads. The corresponding quantities in the passive mass are again of the order of 50 to 60% those of the primary mass for D = 3B and decrease to 10 to 20% for D = 10B. Finally, if a horizontal force is applied in the y direction to the first mass, torsional rotations appear in both. The behavior of these torsional motions is of the same nature as that of the vertical displacements caused by a horizontal force.

Figure 8 shows the horizontal accelerations at the base of the masses due to a ground acceleration specified at the free surface of the soil deposit (in the far field). The main effect of the adjoining mass is again the appearance of shifts in the frequences of the peaks. The values of the peaks seem to increase slightly (of the order of 5 to 20%), but whether an amplification or a deamplification will occur depends on the specific frequency of interest, or generally on the frequency content of the earthquake.

Figure 11 shows the torsional accelerations induced by the presence of the second mass when the ground motion takes place in the y direction. The torsional frequencies of the masses on the soil stratum are 0.35 for m = 1 and 0.26 for m = 2. When both masses are equal to 1 the torsional acceleration shows two peaks: one at the second natural frequency in swaying rocking (0.21); the second one, larger, at the torsional frequency (0.35). The horizontal acceleration at the edge of the mass due to this torsional rotation would be about 10 to 20% of the acceleration of the centroid. When the second mass is heavier, the first peak increases slightly and shifts towards the frequency of the second mass; the second peak, on the other hand, increases by a factor of almost 2.5. In this case the lateral displacement of the edge of the mass due to torsion would be about 50% of the centroidal displacement (at that frequency). As the distance between the two foundations increases, the first peak changes very slowly; the second peak decreases faster and shifts to the left.

When the first mass is the heavy one ($m_1 = 2$) and the adjoining mass is smaller, peaks appear at frequencies of 0.125 (very small), 0.21 and 0.26 (torsional frequency of m = 2). The edge displacement caused by the torsion at 0.21 is about 50% of the centroidal displacement at that frequency, but is very small compared to the maximum horizontal displacement (at a different frequency). When both masses are equal to 2 the excited frequencies are 0.125, 0.16 (the two swaying-rocking frequencies) and 0.26 (the torsional frequency). The maximum response occurs again at the torsional frequency, but this peak decreases faster with distance.

INTERACTION BETWEEN TWO STRUCTURES

In order to study the interaction between two structures, introducing their own natural frequencies as parameters, two simple structures, idealized as single-degree-of-freedom systems, were used. Each structure was modelled as a single mass, lumped at a certain height, h, and a shear spring. The foundation was assumed to be rigid but massless. The structure was rigid in the vertical direction and had the same rotation at the base and at the top. The two foundations were still square, of equal size, and centered along the x axis so that in-plane and out-of-plane effects could be uncoupled. Considering only the in-plane effects, each structure alone, on the elastic foundation, had four degrees of freedom but only two dynamic ones.

In order to define the structural characteristics, the dimensionless frequency $k = K/2GB$ (K is the stiffness of the shear spring), the dimensionless mass $m = M/8\rho B^3$ (defined before), and the height h are used. In all cases it was assumed that h = B. A hysteretic type damping of 5% was employed. The soil profile was the same one of the previous studies with rigid masses. The results are shown again for a distance of 3B between the centers of the foundations.

The structures considered and their natural frequencies on a rigid base and on the elastic foundation were:

	m=1 k=4	m-1 k=2	m=0.5 k=2	m=0.5 k=1
for a rigid base	0.318	0.225	0.318	0.225
for an elastic foundation	0.170	0.154	0.220	0.175

It is interesting to notice that although structures 1 and 3 (and 2 and 4) have the same natural frequencies on a rigid base, the actual frequencies when considering the elastic foundation are different. For the soil structure system cases 1 and 4 have almost the same natural frequency. It is the frequency accounting for the flexibility of the foundation which is important in interpreting the shape of the response curves.

Figure 12 (a,b, and c) shows the displacements at the top of the two structures for various combinations of the structural models when a horizontal harmonic force is applied at the top of the first.

When the two structures are equal (m=1 k=4), the amplitude of the response at the resonant frequency (slightly shifted) increases by about 30 to 40% due to the presence of the second structure, but the peak is narrower. The displacement of the passive structure has a similar shape with a peak response about 50% of that of the first.

When $m_1=1$ $k_1=4$ and $m_2=1/2$ $k_2=2$ (same natural frequency on rigid base), the interaction effect is much smaller, with an increase in the peak value of about 10%. The response of the passive structure is still similar (controlled by the characteristics of the first) and of the order of 40%.

If $m_1=1$ $k_1=4$ and $m_2=1/2$ $k_2=2$ (same natural frequency approximately on elastic foundation), the peak displacement of the excited structure is increased by a factor of 1.2 to 1.3 over the value for the structure alone, a situation intermediate between the two previous ones. The response of the second structure becomes relatively larger, about 75% of that of the first. If the situation is reversed and the excited structure is the lighter one ($m_1=1/2$ $k_1=2$ $m_2=1$ $k_2=4$), the increase in the peak response of the excited structure is smaller (15 to 20%). The response of the passive structure is now much smaller (about 25% of that of the active one).

When the two structures have the same mass but different natural frequencies ($m_1=1$ $k_1=4$ $m_2=1$ $k_2=2$), the increase in the peak response is again of the order of 20 to 30%, and the response of the passive structure is half that of the active.

Finally, if both structures are equal but with smaller masses ($m=0.5$ $k=1$), the increase in the peak response of the excited structure is only of the order of 20%, and the response of the passive structure is less than half (about 40%).

It appears therefore that the interaction effects are more pronounced when both structures have the same natural frequency on the elastic foundation and when the masses are heavy. To illustrate better the reasons for the increase in the peak response due to the presence of the second structure, figure 13 shows the base displacements and rotations for the case when both structures are equal, with $m=1$ and $k=4$ (for the case of both structures and one structure alone). The main increase in response takes place in the rotation.

Figure 14 shows the horizontal accelerations at the top of the first structure when they are both excited by a ground acceleration in the x direction specified at the surface of the soil in the free field. In all cases the first structure has m=1 k=4; the second structure takes the values of each one of the four models. When the second structure is equal to the first, the interaction effect is more marked. Two peaks appear instead of one, and the peak response decreases by about 30%. When m_2=0.5 and k_2=2, the main effect is a shift in the frequency of the peak; the decrease in the amplitude of the peak is only of the order of 5%. If m_2=0.5 and k_2=1, the decrease is of the order of 15%. Finally, if m_2=1 and k_2=2, two peaks occur again and the amplitude of the larger is about 5 to 10% smaller than that of the structure alone.

Figure 15 shows the horizontal accelerations at the top of the second structure for the same situations. When m_2=0.5 and k_2=2, there is an increase in response at the frequency of the first structure and a slight decrease at its own frequency. In the second case, when m_2=0.5 and k_2=1 (both structures have approximately the same natural frequency on the elastic foundation), the reduction in the peak response is very marked, of the order of 40%. This implies that the fact that the other structure has a larger mass reinforces the interaction effect. In the last case (m_2=1 k_2=2), the interaction effect is relatively small.

The decrease in response at the peak is again primarily due to the base rotation. The results for a base excitation are in general terms, similar to those of the applied harmonic force, but the effect of the rotation is of opposite sign (it decreases the acceleration at the top for the case of a base motion and increases it in the other.

CONCLUSIONS

From the studies carried out and described here, it appears that the proposed formulation, based on a finite element type procedure with the consistent boundary developed by Kausel, can provide an excellent solution for truly three-dimensional problems and for the study of interaction effects between adjacent structures. In order to use the procedure in practice it would be necessary to use a finer mesh under each foundation or to obtain solutions for two different meshes and apply a linear extrapolation. (The results shown were obtained with a 3 x 3 mesh or a total of 16 points under each foundation for reasons of economy).

One of the main effects of an adjoining mass or structure is the excitation of modes of vibration which would not appear if the structure were alone (such as vertical vibrations under a horizontal force in the x direction and torsional rotations under a force in the y direction). When only one of the masses (or structures) is excited by an external force, these effects are due to a feedback from the passive structure, and they decay very rapidly with increasing distance between the foundations. When the two masses are excited (as in the case of a base motion), they become more significant and their rate of decay with distance is much slower.

A second important effect is a change in the natural frequencies of the combined soil structure system. This change is more pronounced when dealing with two rigid masses, because their frequencies are affected only by the foundation stiffnesses. For the case of two structures, their own stiffness is an additional factor.

For the case of two rigid masses, when one of them is excited by a horizontal force, the effect of the adjoining mass on its horizontal displacement is only small. The effect is a little more pronounced in the rotations induced by a horizontal force or a rocking moment. The response of the passive mass is of the same order of magnitude as that of the excited one: from 30 to 80% when the masses are close (D=3B) and 10 to 30% when they are at a distance D=10B.

Under a base motion the existence of two masses tends to increase slightly the peaks of the base translation, but it reduces more importantly the rotation. The effect is more marked when both masses have the same natural frequency on elastic foundation and as their masses increase.

For the case of two structures, with one of them excited by a horizontal force at the top (where the mass is applied), the top displacement tended to increase in all cases due to the presence of the other structure, the increase being more noticeable when both of them had the same natural frequency on elastic foundation and as their masses increased. The increase in the peak response varied between 5 and 40% for the cases considered with the two foundations at a distance of 3B. The main cause for the increase lies in the base rotation.

Under a base motion, simulating a seismic input, the presence of the second structure tended instead to decrease the peak response. The decreases are of the same order as the increases found in the case of an exciting force and are also caused by the base rotation.

The results obtained are in general agreement with those reported by Richardson (5) and by Chang Liang (1). Because of the large number of parameters involved in this problem and the relatively small number of cases considered, it is difficult yet to present more specific conclusions. It appears, however, from these studies that simple rules could be derived to estimate the magnitude of the vibrations caused in a passive structure as a function of distance and feedback effects. These rules could then be extended to the case where both structures are excited.

In order to understand better the interaction effects, it would be convenient to isolate further translational and rotational effects (notice that in this work even for the case of rigid masses the horizontal force creates rotations due to the elevation of the center of mass). The rotational effects seem to be the most important ones. It would also be more convenient to work with a half-space rather than with a finite layer of soil to eliminate the effect of the stratum frequencies.

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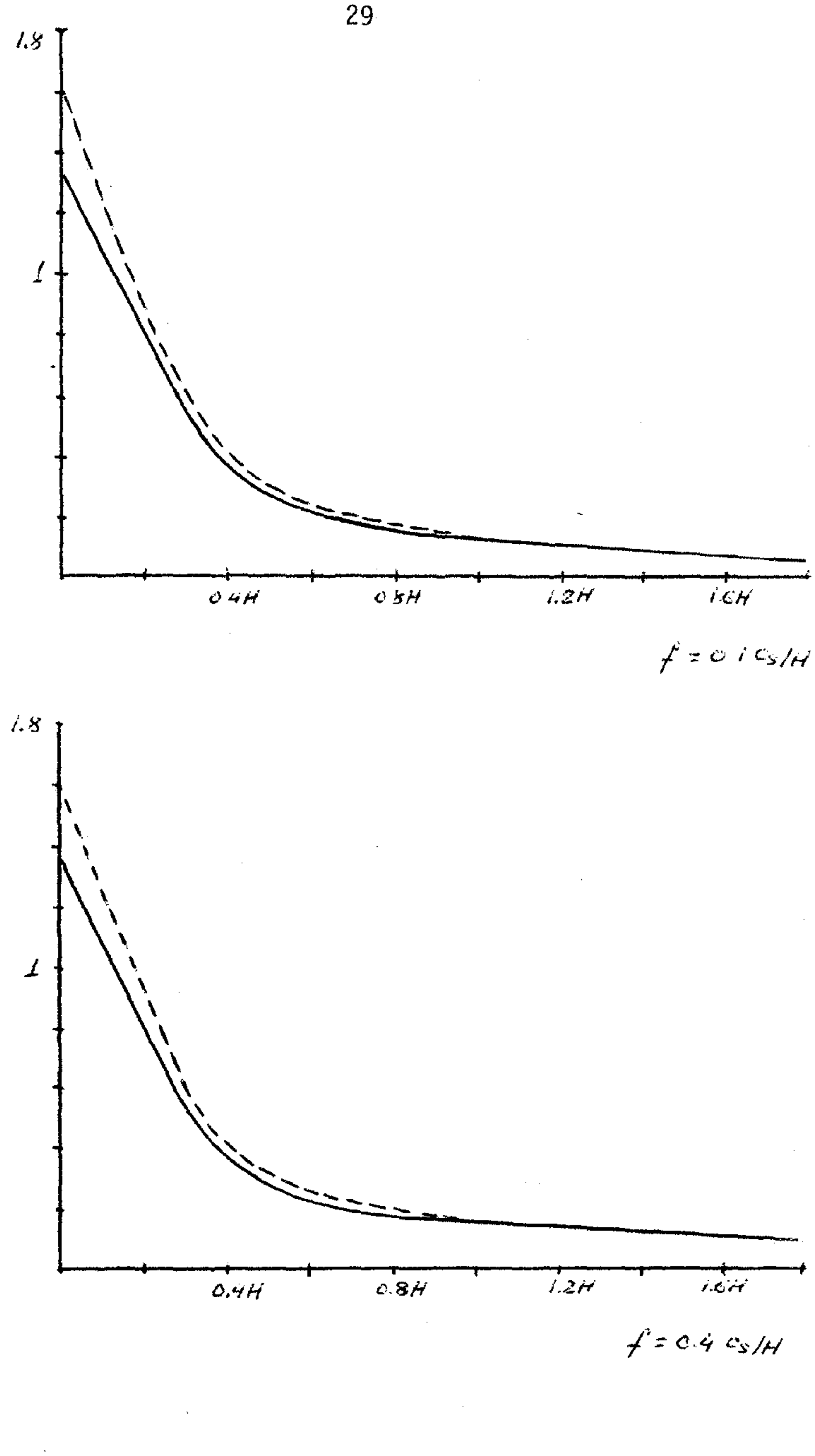
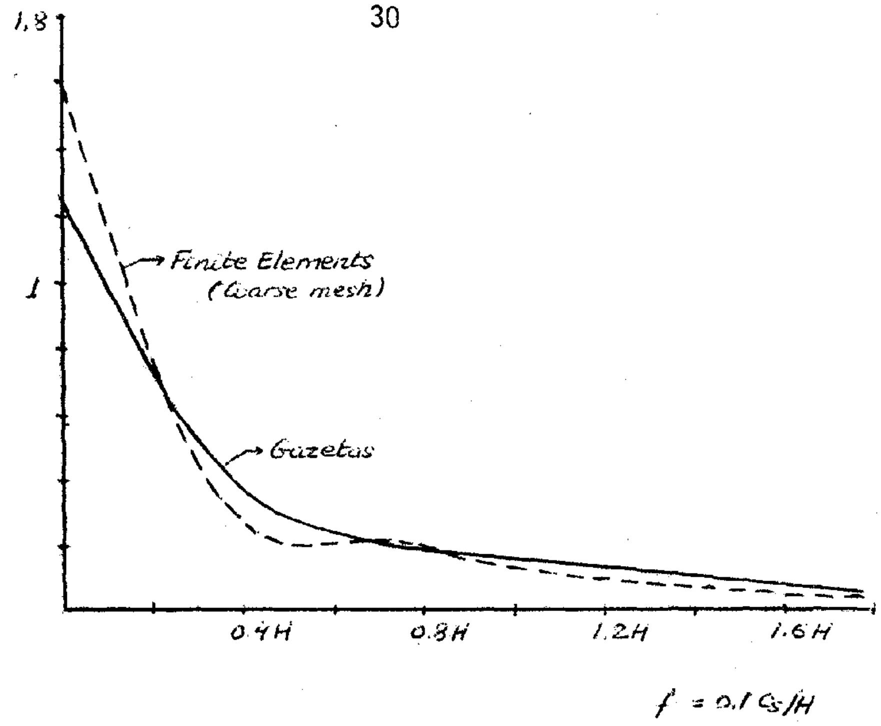


FIGURE 1. Horizontal Displacements due to a horizontal force.



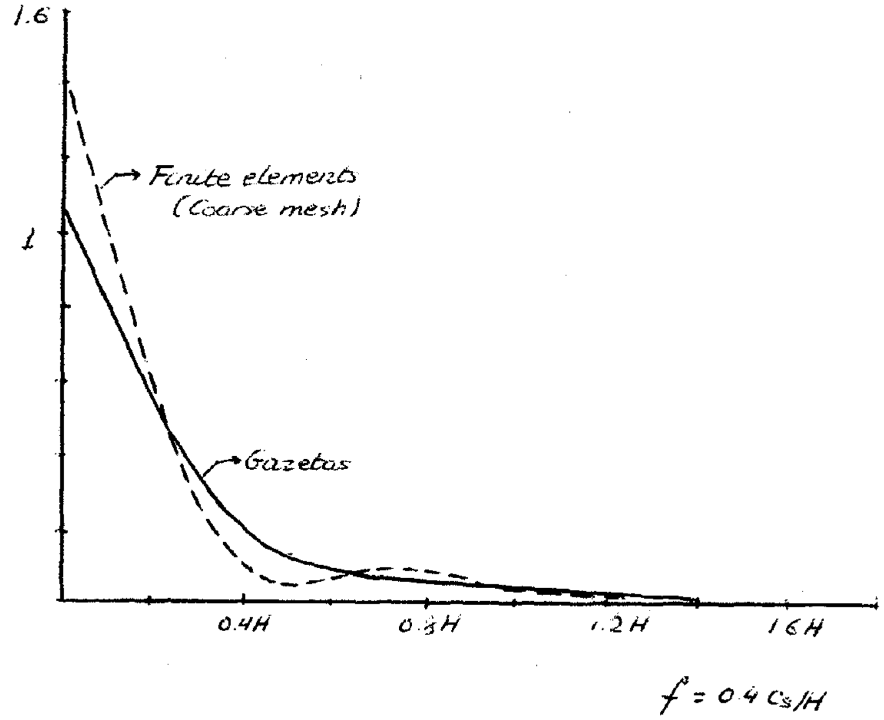


FIGURE 2 - Vertical displacements due to a vertical force

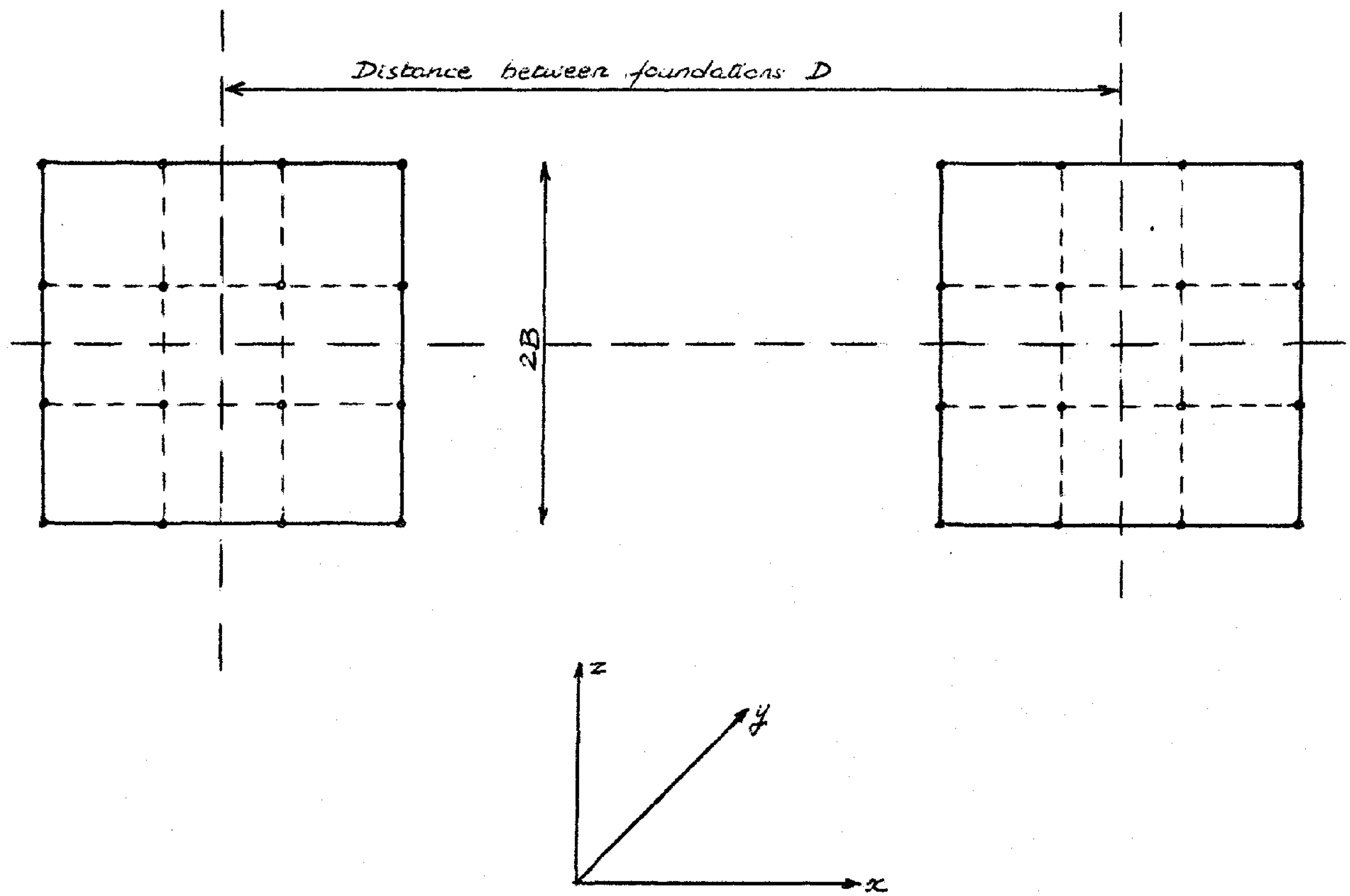


FIGURE 3 - BASIC GEOMETRY - PLAN VIEW

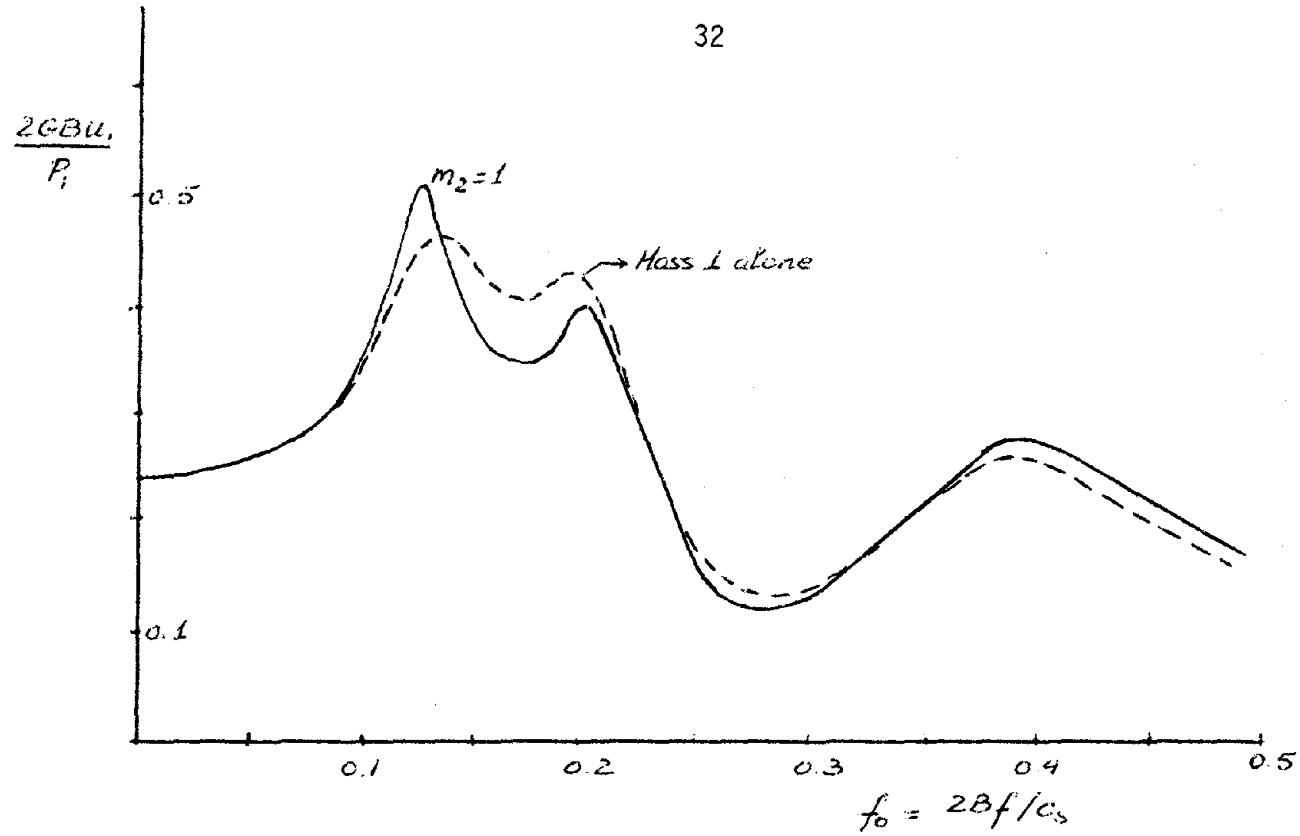
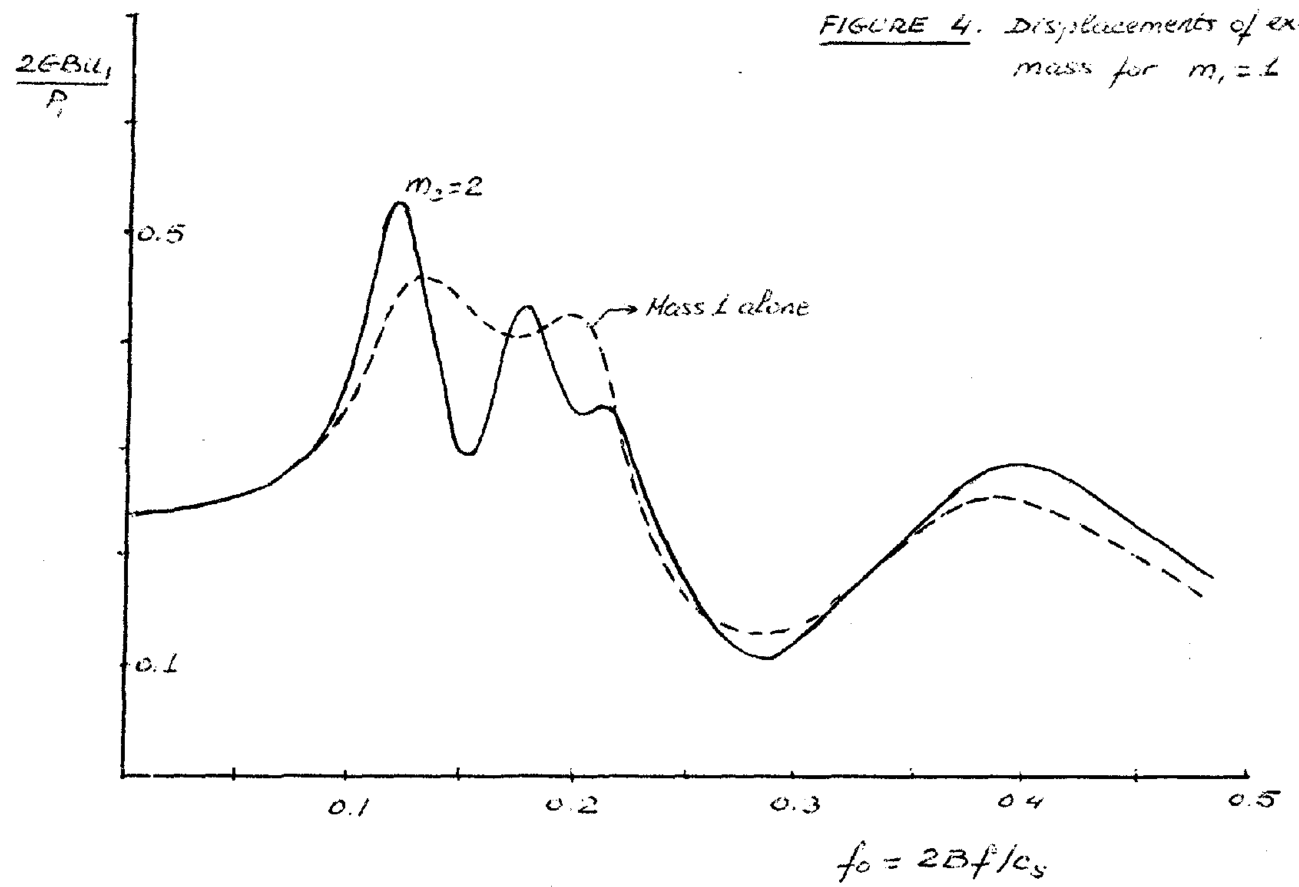


FIGURE 4. Displacements of excited mass for $m_1=1$



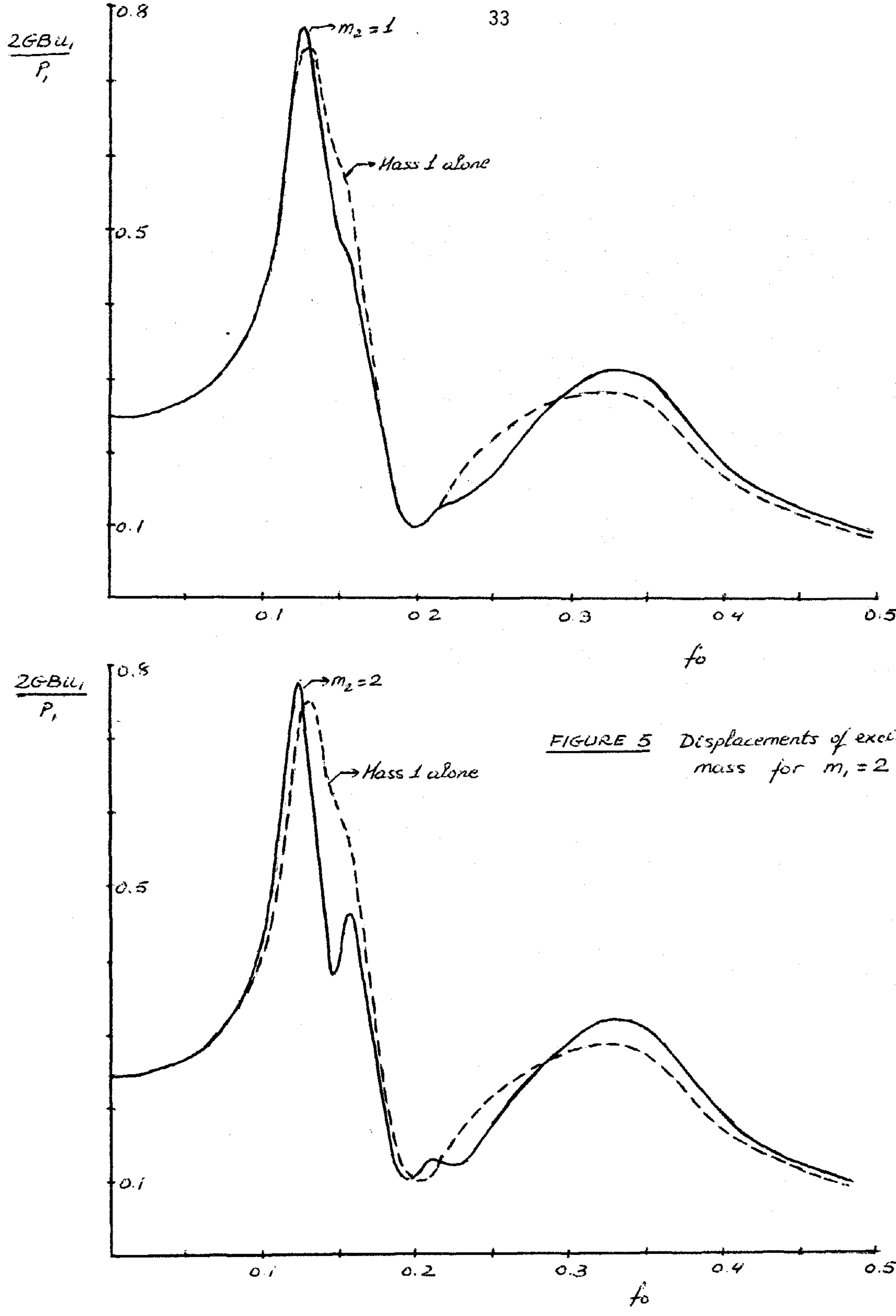


FIGURE 5 Displacements of excited mass for $m_1 = 2$

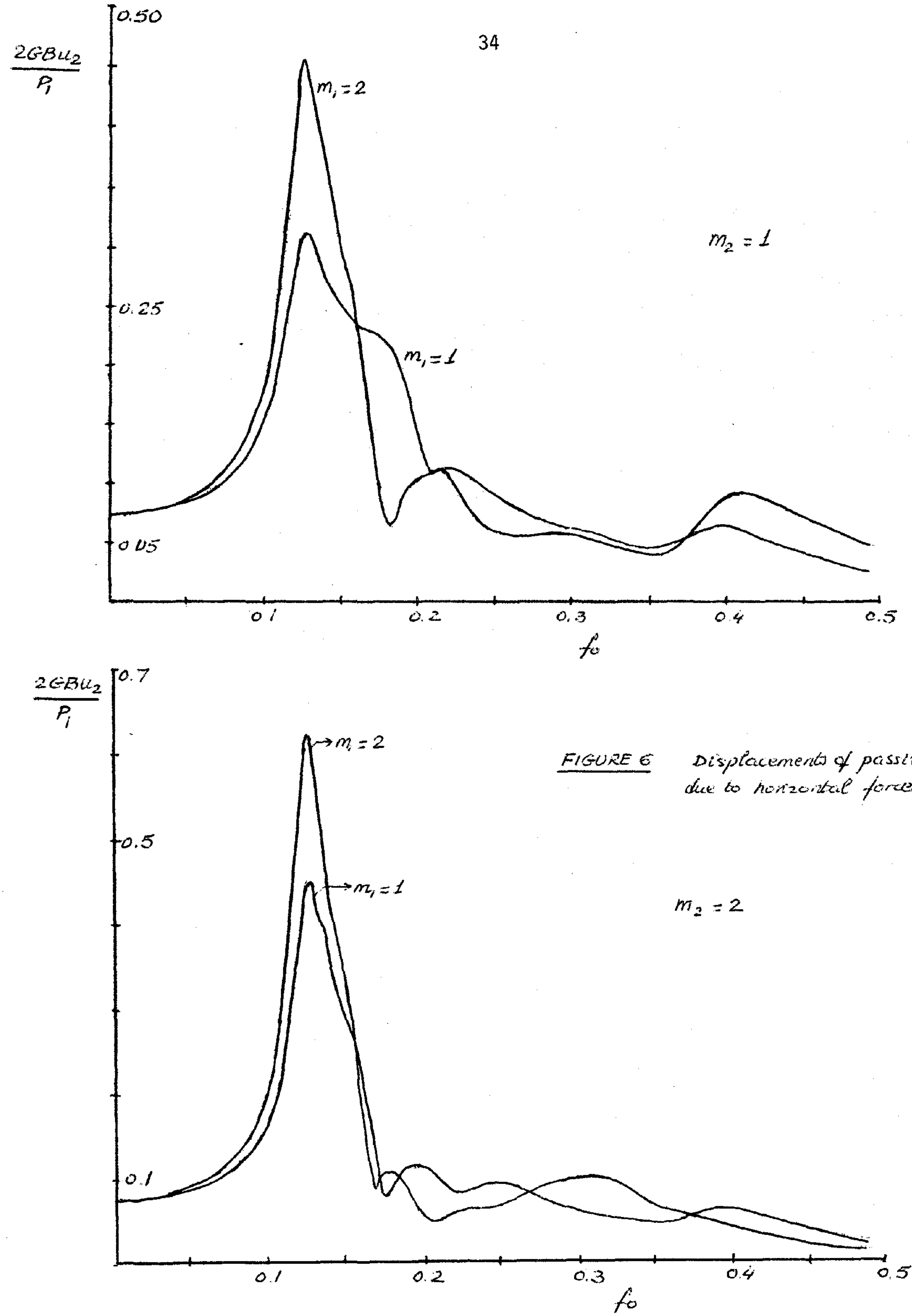


FIGURE 6 Displacements of passive mass due to horizontal force

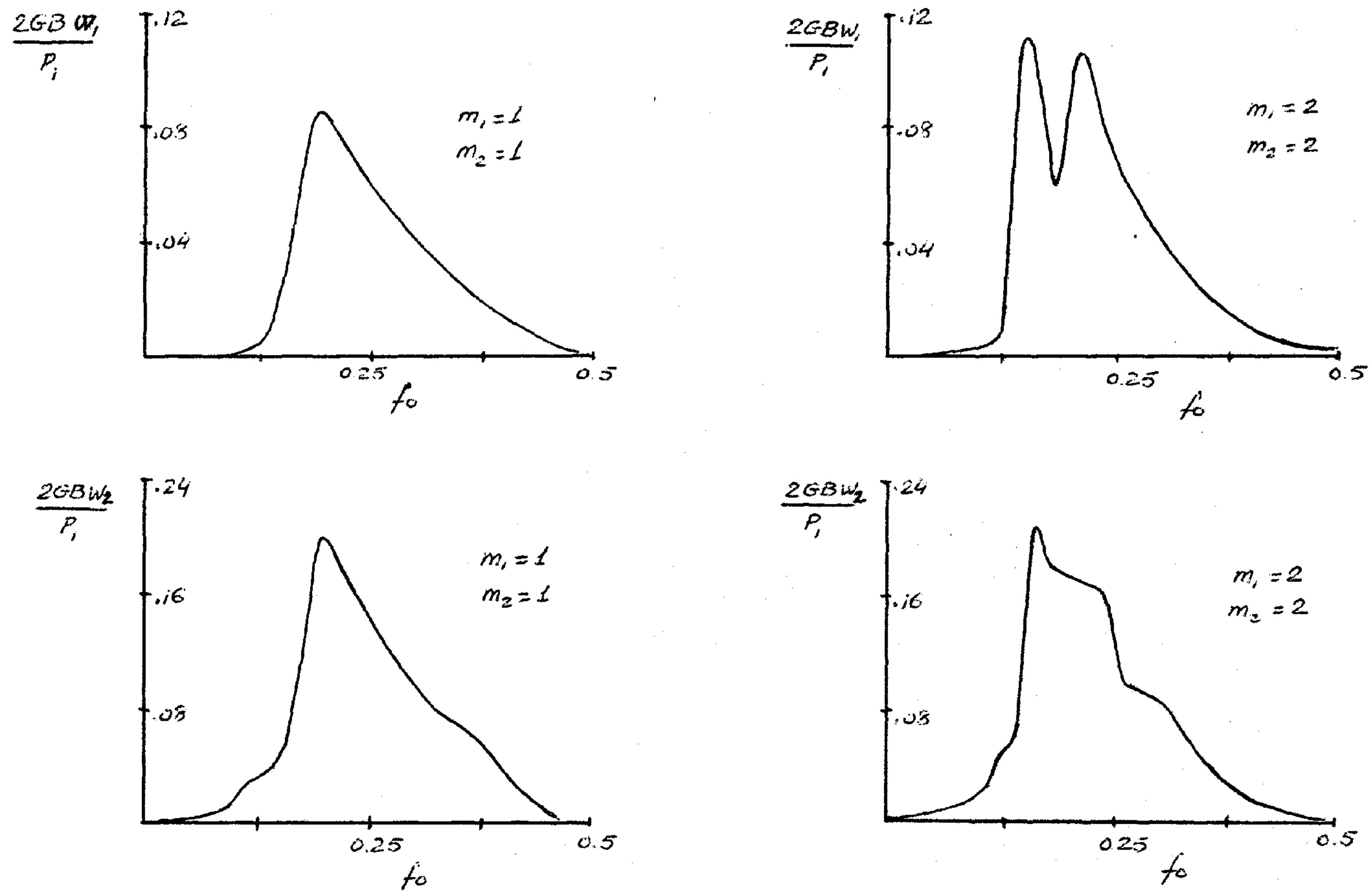


FIGURE 7. Vertical displacements of the masses due to a horizontal force at the base of the first one.

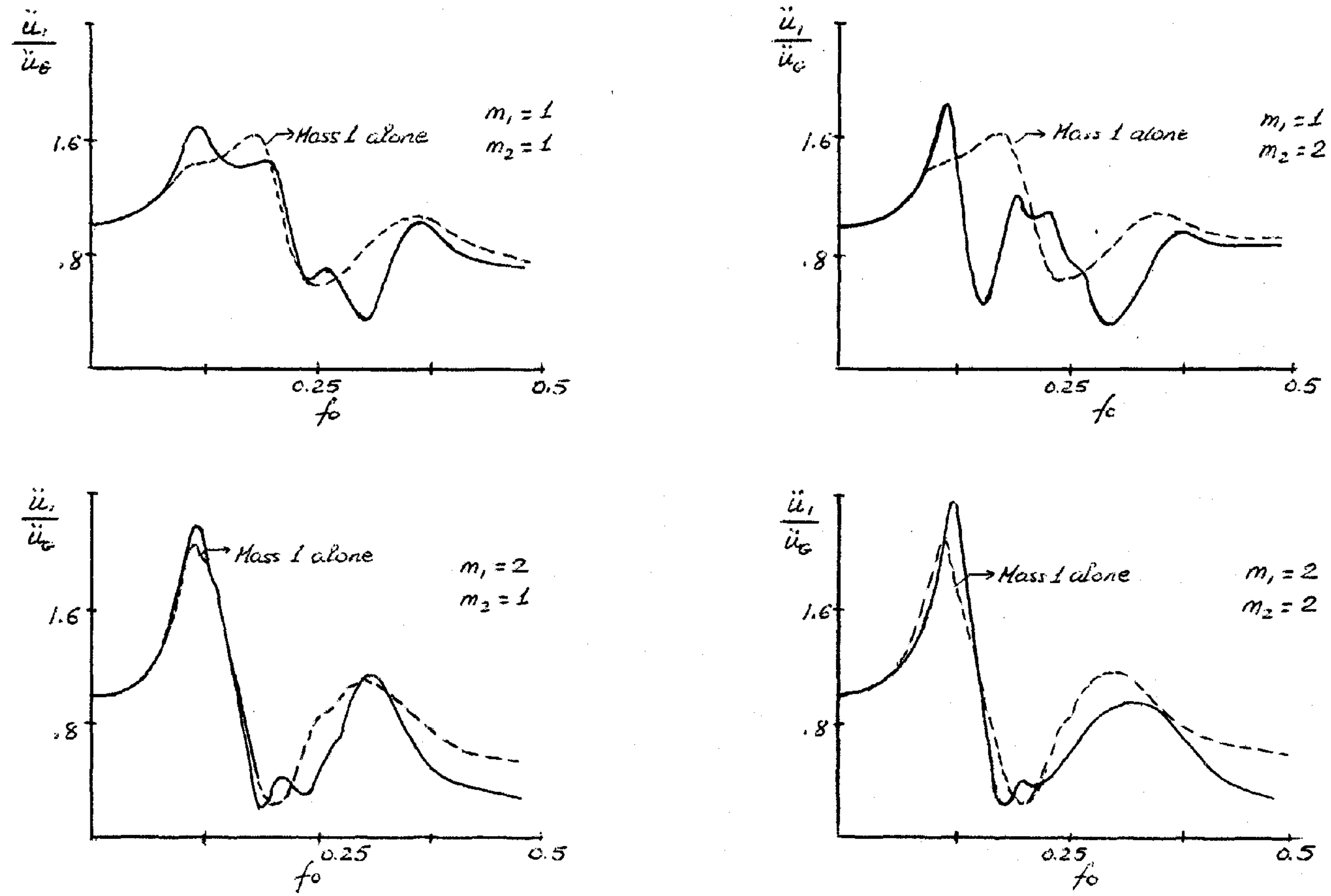


FIGURE 8. Horizontal accelerations at the base of the first mass due to a base motion.

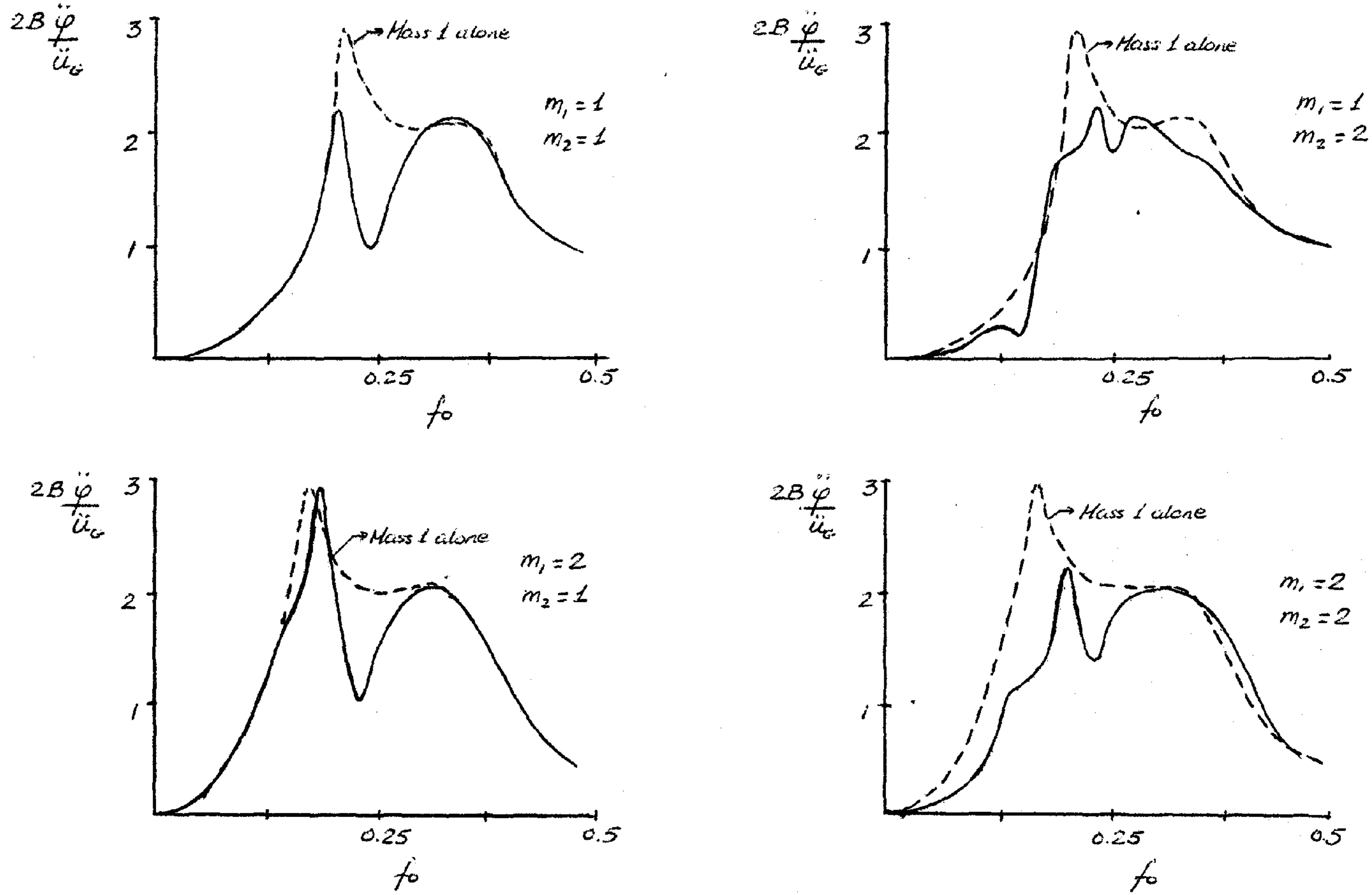


FIGURE 9. *Rotational accelerations at the base of the first mass due to a horizontal base motion.*

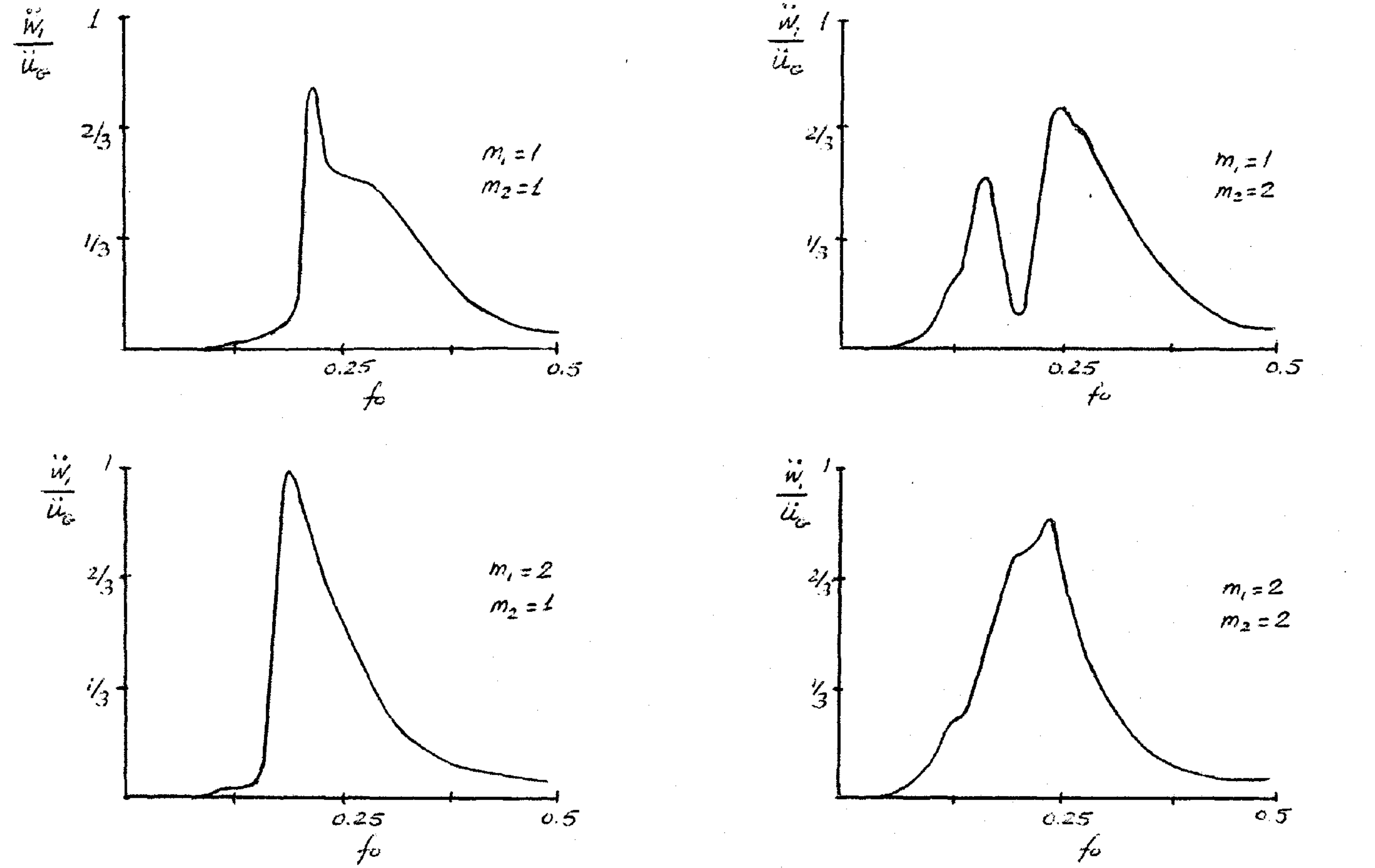


FIGURE 10 - Vertical accelerations of first mass due to a horizontal ground motion.

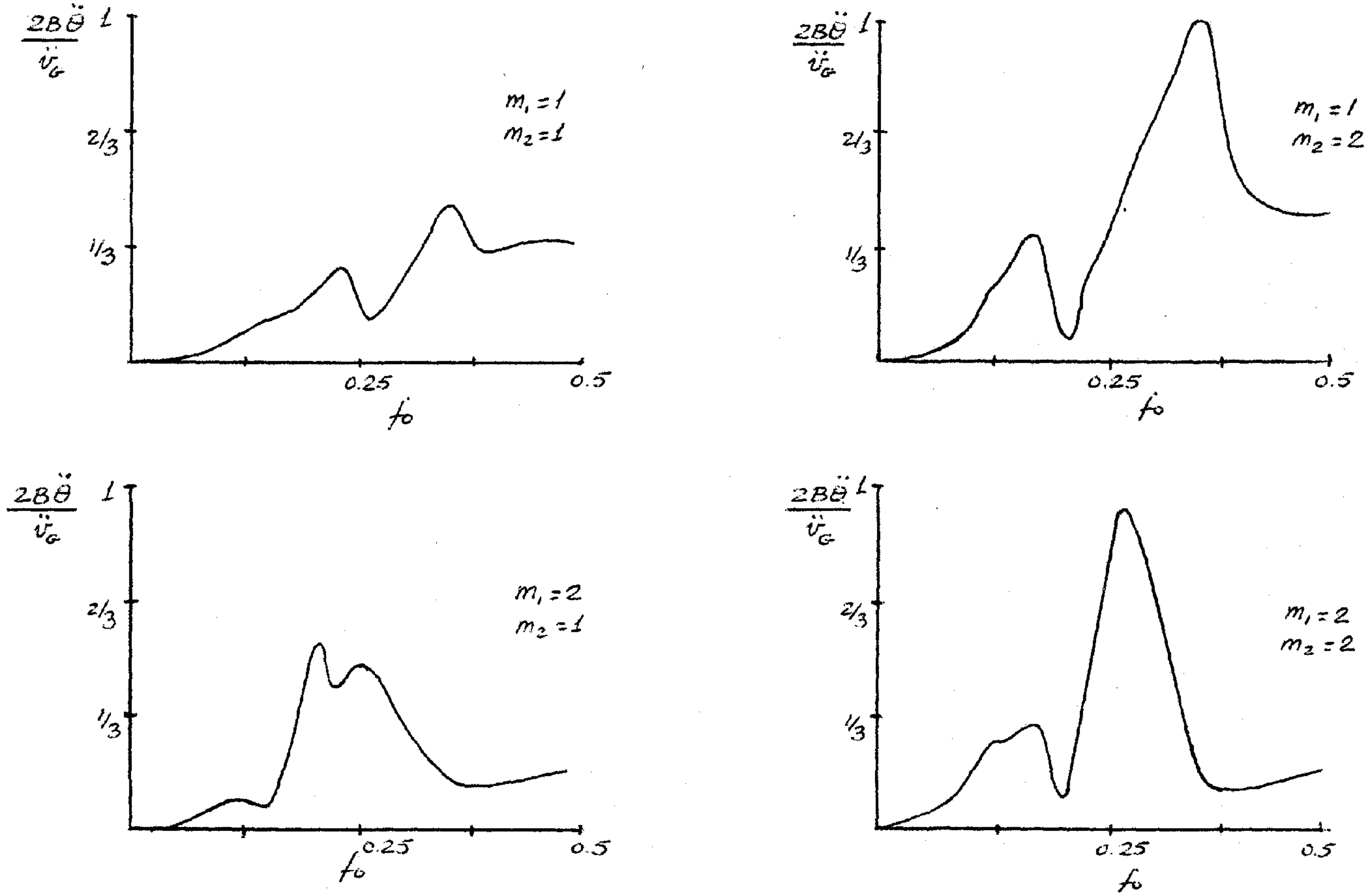


FIGURE 11 - Torsional accelerations of first mass due to a horizontal ground motion.

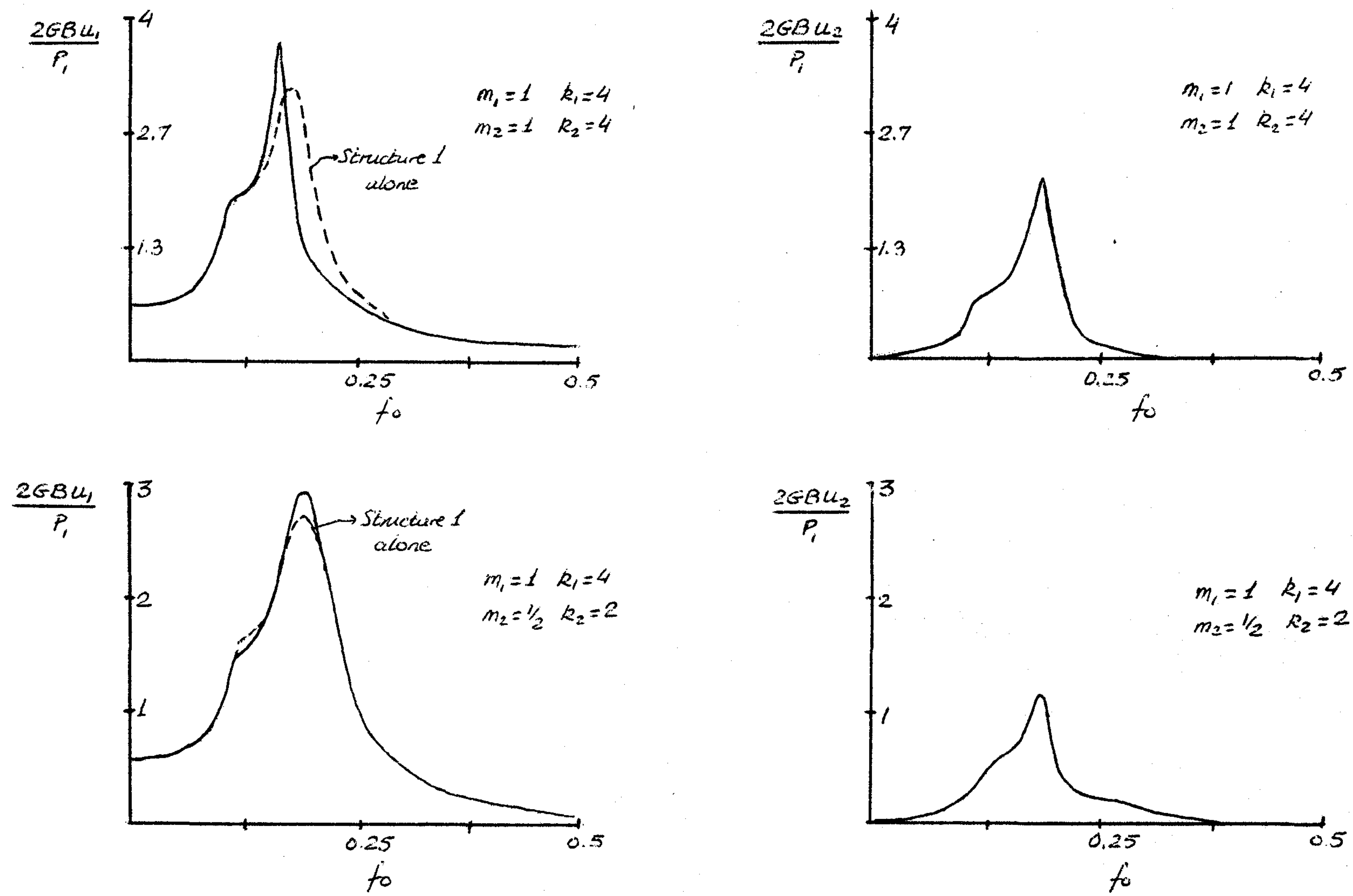


FIGURE 12a. Displacements at the top of the structures due to a horizontal force applied at the top of the first.

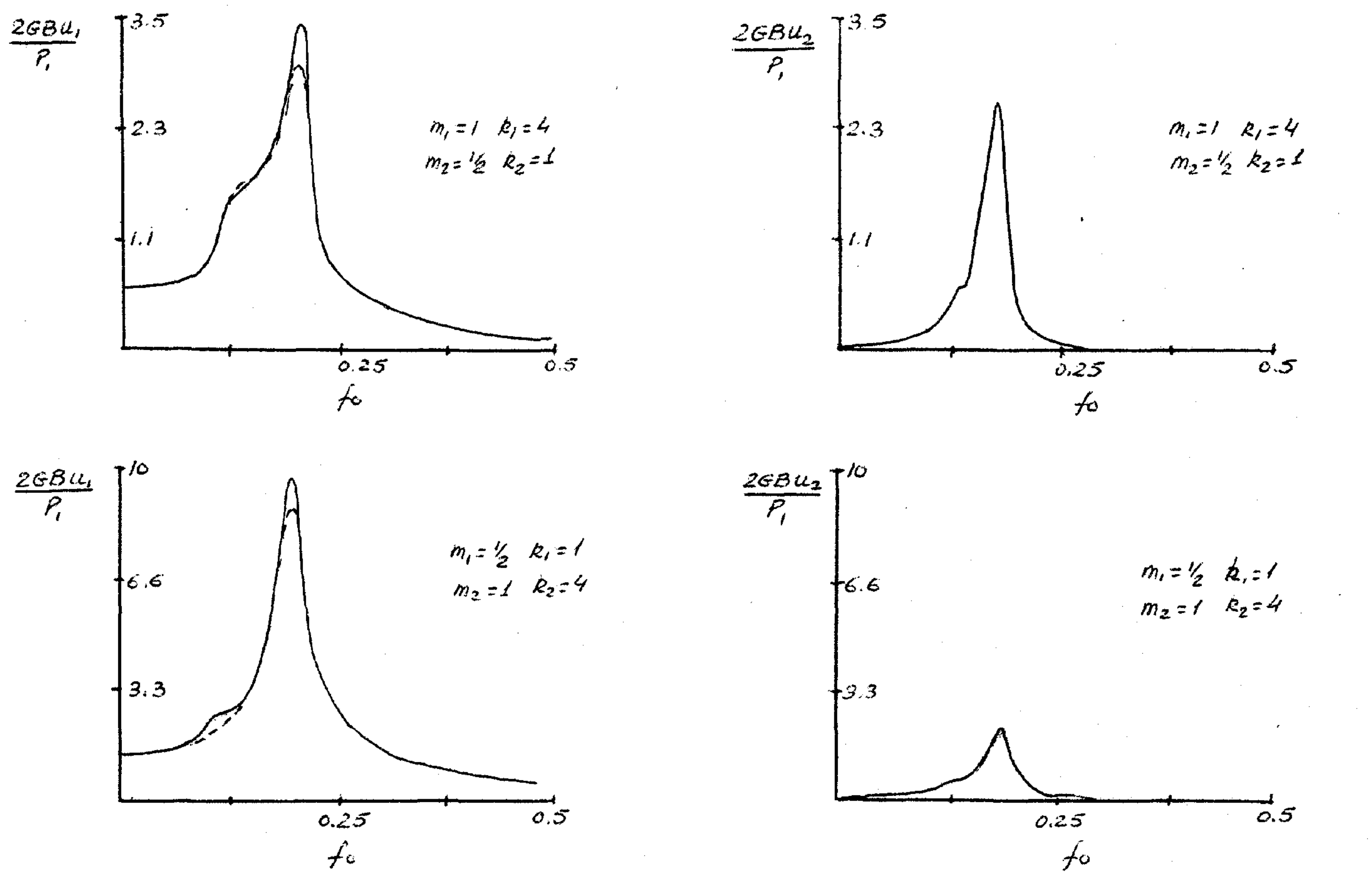


FIGURE 12b Displacements at the top of the structures due to a horizontal force applied at the top of the first.

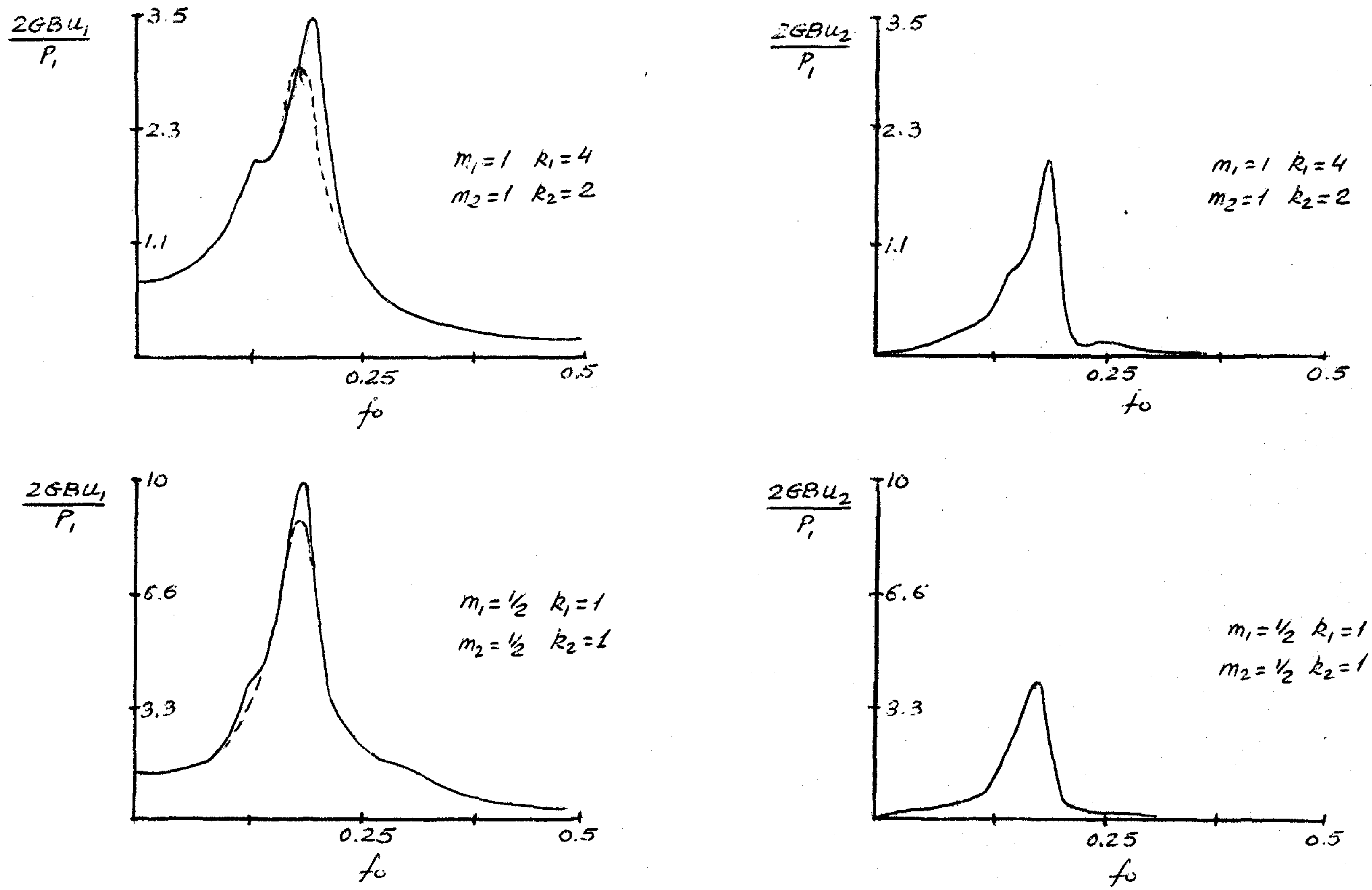


FIGURE 12C Displacements at the top of the structures due to a horizontal force applied at the top of the first.

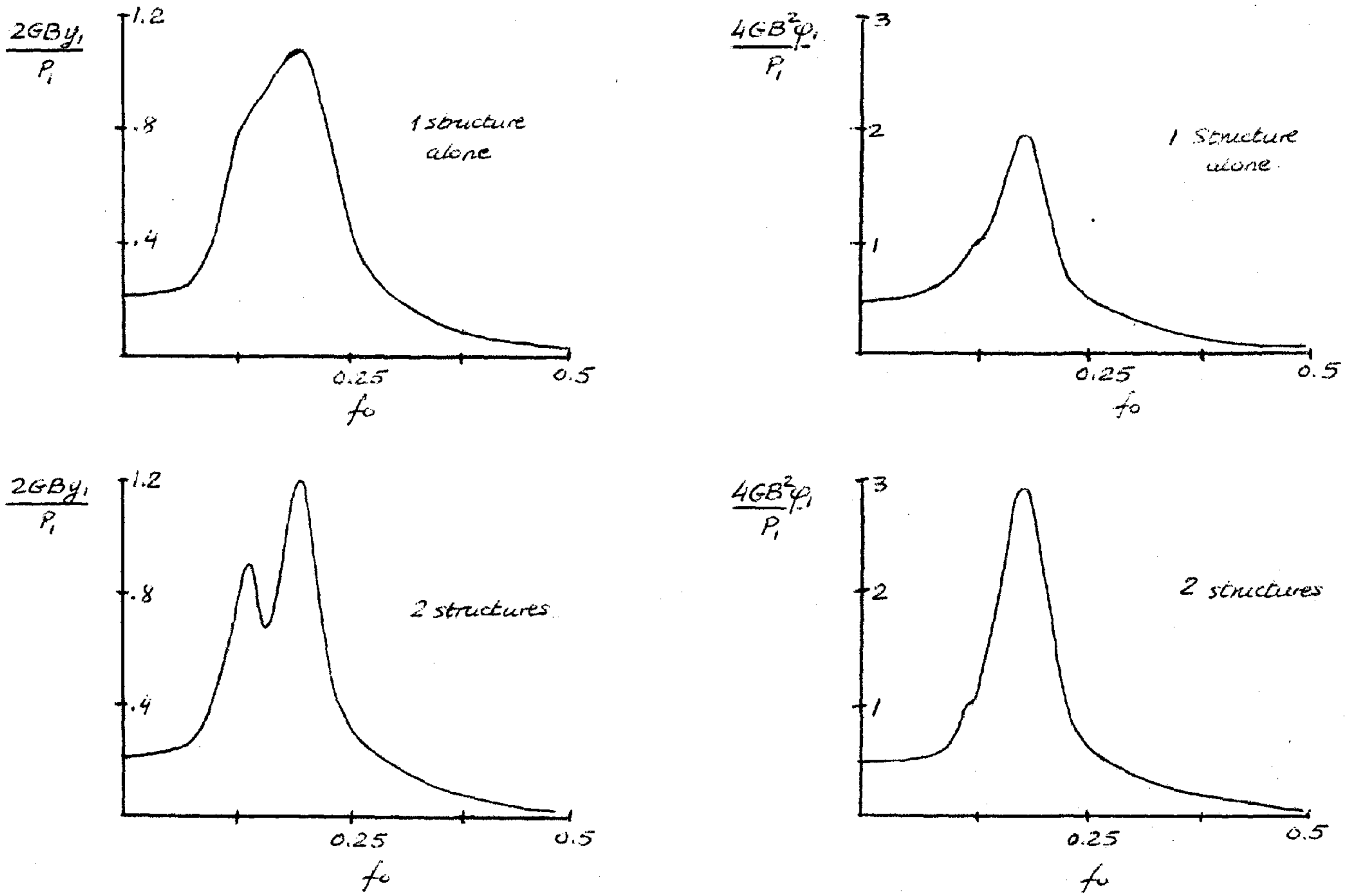
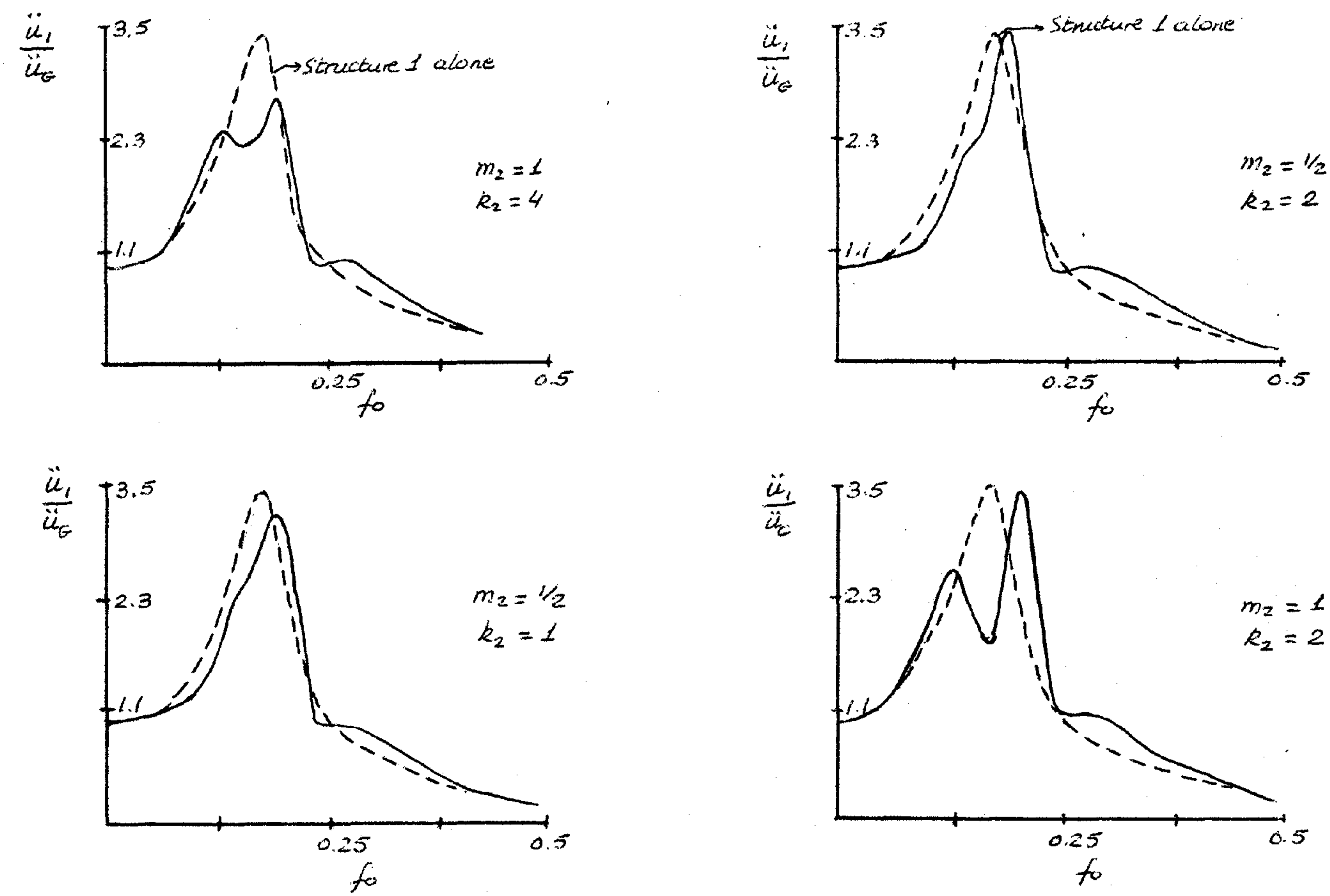


FIGURE 13. Displacement and rotation at the base of the excited structure
$m_1 = 1 \quad k_1 = 4 \quad m_2 = 1 \quad k_2 = 4$



<u>FIGURE 14.</u> Accelerations at top of first structure due to a horizontal ground motion. $m_1 = 1$ $k_1 = 4$

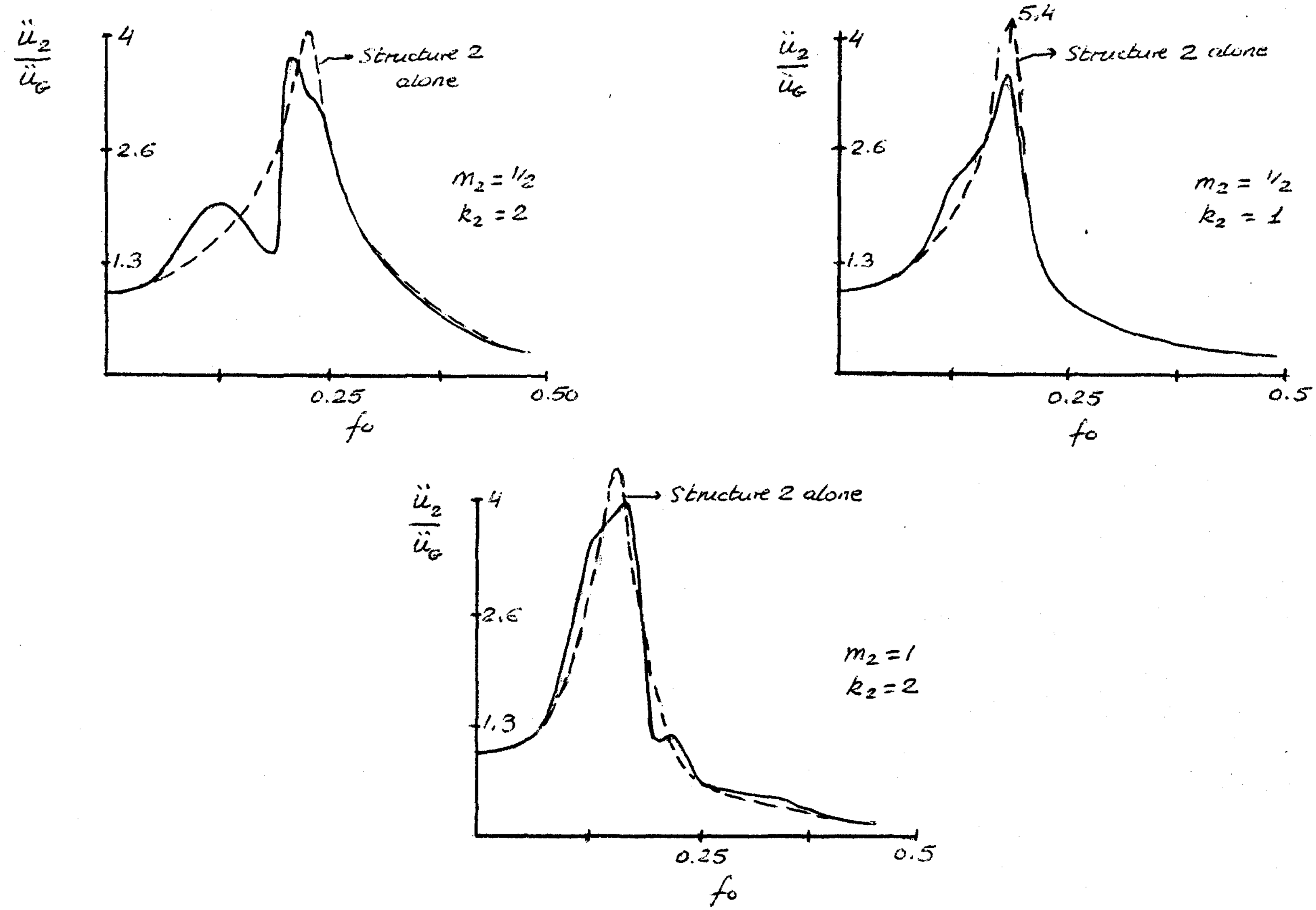


FIGURE 15 Accelerations at the top of the second structure due to a horizontal ground acceleration. $m_1 = 1$ $k_1 = 4$