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RESPONSE OF EMBEDDED FOUNDATIONS TO TRAVELLING WAVES

by

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ABSTRACT

The motion of rigid massless square foundations under various types of seismic waves is studied. Surface foundations are considered first and the results, obtained with the boundary element method, compared to those published by other researchers. Embedded foundations under vertically travelling shear waves are studied next and the results compared to those reported by Elsabee and Morray for circular foundations. Finally, the case of an embedded foundation under a combination of SV and P waves that will produce a free field motion compatible with the Newmark-Blume-Kapur spectra is investigated.

PREFACE

The work described in this report represents part of a research effort on Dynamic Soil Structure Interaction carried out at the Civil Engineering Department of M.I.T. under the sponsorship of the National Science Foundation, Division of Advanced Environmental Research and Technology, through Grant ENV 77-18339.

The work was conducted by Dr. José Dominguez, Assistant Professor at the Escuela Técnica Superior de Ingenieros Industriales, Madrid, Spain, who spent a year at M.I.T. under the auspices of a Fulbright grant. It was supervised by Professor José M. Roesset.

This is the second of a series of research reports published under this grant. The previous one was:

> R78-20, "Dynamic Stiffness of Rectangular Foundations," by José Dominguez, August 1978.

INTRODUCTION

It is often assumed in seismic analysis of structures that the base motion is applied uniformly at all points of the foundation. This assumption would be correct only for the case of shear or dilatational waves propagating vertically through the soil and a surface foundation. The effect of inclined (or travelling) waves on the foundation motion, giving rise to a difference in phase between the motions of the points under the foundation and as a result torsional and rocking components, has been discussed and investigated in some detail in recent years for surface foundations. The motion of embedded foundations under vertically propagating waves has also been studied, and simplified rules have been suggested to estimate both the translational and rotational components from the free field motion as a function of the embedment depth.

In this work the response of a massless, rigid foundation to non-vertically incident waves is studied as the first step of a seismic analysis using the substructure method. The magnitude of the effect produced by the angle of incidence of the waves will depend on the relation between the dimensions of the foundation and the wavelength of the waves projected on the surface. The existence of the foundation will produce a filtering of the waves, reducing the amplitude of the motion as a function of frequency.

Prior to this work several authors have treated the problem of obliquely incident waves for surface foundations. Newmark [7] studied the torsional effects produced by waves travelling horizontally and obtained an approximation to the value of this torsion by estimating the rotation around the vertical axis of the free field displacement below the foundation. The computed torsion was equal to the ratio of the velocity of the free field motion to the wave velocity projected on the surface. Yamahara [12] obtained an approximate value of the filtering effect of the horizontal motion for non-vertically incident SH waves. The motion of the foundation was made equal to the average value of the free field motion under it. Soil-structure interaction was not taken into account in the aforementioned works. Scanlan [9] considered a distributed set of springs placed between the soil and the foundation, but soil-structure interaction effects were not truly considered in the computation of the input motion. The input displacement was approximated by the average of the free field motion along the foundation, and the input torque, for the case of SH waves, was computed by integration of the torque produced by the distributed springs when excited by the free field motion.

Kobori, Minai and Shinozaki [4] and Luco [5] studied the torsional response of axisymmetric structures resting on the surface to obliquely incident SH waves, using similar analytical procedures that include soil-structure interaction effects and evaluate the input motion by solving numerically a Fredholm integral equation. Iguchi [3] proposed an approximation of the input torsion by an average over the foundation area of the tangential component of the free field motion.

Wong and Luco [11] studied the response of rectangular surface foundations to non-vertically incident waves including soil-structure interaction by means of a numerical procedure. The method is based on a subdivision of the surface of the foundation into elements with constant displacements and tractions. They established a relation between displacements and stresses by integrating the Green's functions for an elastic half-space. The integration for each element is not straightforward, because the Green's functions are not given in an explicit form, and they include a singularity.

In the present work the response of rectangular foundations (on the surface of or embedded in an isotropic elastic half-space) to non-vertically incident SH, SV and P waves is analyzed. The modeling of the system has been done by means of the Boundary Element Method. This numerical method is based on the discretization of the soil-foundation interface and the free soil around the foundation into elements throughout which the displacements and tractions are assumed to be constant or interpolated between nodal values. The method makes use of a fundamental solution which corresponds to the response of the complete space to a unit concentrated harmonic load. This fundamental solution has an explicit expression and its integration throughout the elements does not present any particular difficulty. The method has also been used to compute the stiffnesses of surface and embedded foundations [1].

FORMULATION

The starting point of this formulation will be to establish the equations for the Boundary Element solution of any elastodynamic problem and in particular of the soil-structure interaction problem for an elastic half-space. In our case the boundary will be: first the soil-foundation interface that will be considered divided into rectangular elements with constant displacements and tractions within each element (associated with a central node), and second, the free surface for which only a small zone around the foundation must be discretized, with the same kind of elements, in order to get satisfactory results. In reference [1] the formulation of the method was established in more detail, and results for foundation stiffnesses obtained with different meshes were compared. The formulation produces a system of linear algebraic equations that relate displacements and tractions over the boundary elements

$$[G]\ \{t\} = [H]\ \{u\}$$

where $\{t\}$ and $\{u\}$ are vectors formed by the three components of the element tractions and displacements, and $[G]$ and $[H]$ are two square matrices whose elements depend on the properties of the soil and the geometrical characteristics of the model. These coefficients are obtained by integration over the elements of the displacements and tractions of the fundamental solution that corresponds to a unit harmonic load concentrated in a point of the entire region.

For an embedded foundation of arbitrary shape subjected to a general train of waves, the previous system of equations can be established for the relative displacements and tractions of the elements with respect to the free field values.

$$[G]\ \{t-t^{\circ}\} = [H]\ \{u-u^{\circ}\}$$

where the super-index "o" indicates the free field value. Denoting with sub-index "I" the nodes in the soil-foundation interface and with "F" those of the free surface, one can partition the matrices in the form

$$\begin{bmatrix} G_{II} & G_{IF} \\ G_{FI} & G_{FF} \end{bmatrix} \begin{Bmatrix} t_I - t_I^o \\ 0 \end{Bmatrix} = \begin{bmatrix} H_{II} & H_{IF} \\ H_{FI} & H_{FF} \end{bmatrix} \begin{Bmatrix} u_I - u_I^o \\ u_F - u_F^o \end{Bmatrix}$$

which can be rearranged in the form:

$$\begin{bmatrix} G_{II} & -H_{IF} \\ G_{FI} & -H_{FF} \end{bmatrix} \begin{Bmatrix} t_I - t_I^o \\ u_F - u_F^o \end{Bmatrix} = \begin{bmatrix} H_{II} \\ H_{FI} \end{bmatrix} \begin{Bmatrix} u_I - u_I^o \end{Bmatrix}$$

Due to the fact that all the points in contact with the foundation have motions compatible with the rigid body motion of the foundation, the displacements $\{u_I\}$ will be

$$\{u_I\} = [L]\,\{u\}$$

where $\{u\}$ is the vector formed by the six components of the foundation motion and $[L]$ is the transformation matrix that depends only on the nodal coordinates.

The above equation can be written in the form:

$$\begin{bmatrix} G_{II} & -H_{IF} \\ G_{FI} & -H_{FF} \end{bmatrix} \begin{Bmatrix} t_I \\ u_F - u_F^o \end{Bmatrix} = \begin{bmatrix} H_{II} \\ H_{FI} \end{bmatrix} [L]\{u\} - \begin{bmatrix} H_{II} \\ H_{FI} \end{bmatrix} \{u_I^o\} + \begin{bmatrix} G_{II} \\ G_{FI} \end{bmatrix} \{t_I^o\}$$

and calling $[F]$ the first matrix on the left-hand side

$$\begin{Bmatrix} t_I \\ u_F - u_F^o \end{Bmatrix} = [F]^{-1} \begin{bmatrix} H_{II} \\ H_{FI} \end{bmatrix} [L]\,\{u\} - [F]^{-1} \left(\begin{bmatrix} H_{II} \\ H_{FI} \end{bmatrix} \{u_I^o\} - \begin{bmatrix} G_{II} \\ G_{FI} \end{bmatrix} \{t_I^o\} \right)$$

or $$\begin{Bmatrix} t_I \\ u_F - u_F^o \end{Bmatrix} = \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} \{u\} - \begin{Bmatrix} Y_1 \\ Y_2 \end{Bmatrix}$$

where the matrices [X] and {Y} are known, being the dimensions of [X] $3N \times 6$ and those of {Y} $3N \times 1$ (N is the number of elements in the model). $[X_1]$ and $\{Y_1\}$ are formed by the $3N_1$ first rows of [X] and {Y} where N_1 is the number of elements in the soil-foundation interface.

We can consider now only the upper part of the previous equation:

$$\{t_I\} = [X_1]\{u\} - \{Y_1\}$$

The six components of the resultant force over the foundation will be obtained from the tractions $\{t_I\}$ multiplied by a transformation matrix [M] of the same form as $[L]^T$ but with the terms multiplied by the area of the corresponding elements. For the determination of the input motion the foundation is considered massless and separated from the structure and consequently the resultant of the tractions applied on it will be zero.

$$[M]\{t_I\} = 0 = [M][X_1]\{u\} - [M]\{Y_1\}$$

$$\{u\} = ([M][X_1])^{-1}[M]\{Y_1\} .$$

The motion of the foundation is therefore obtained from the free field motions and tractions with some manipulations of the matrices [G] and [H]. In the cases of surface foundations the free field tractions are zero for all the elements and the formulation becomes somewhat simpler.

<u>INCIDENT WAVES</u>: Consider the half-space $z \geq 0$ and a train of plane waves propagating in directions parallel to the x-z plane. The motions are therefore independent of the coordinate y and the overall problem can be studied in two uncoupled parts, one corresponding to SH waves with a motion in the y direction, and the other to SV and P waves with coupled motions in the x and z directions. The solution adopted for the SH, SV and P waves is the one obtained by direct integration of the differential equations of motion in terms of amplitudes used in reference [6], as

opposed to solutions in terms of potentials proposed by other authors. The motion for an elastic medium with one-dimensional geometry in the case of SH waves has the form

$$u_y = [A_{SH} \exp\left(\frac{i\omega}{C_s} nz\right) + A_{SH}' \exp\left(-\frac{i\omega}{C_s} nz\right)] \cdot \exp\left(-\frac{i\omega}{C_s} \ell x\right) \cdot \exp(i\omega t)$$

where

ω is the circular frequency.

A_{SH} and A_{SH}' are amplitudes of the incident and reflected waves.

C_s is the shear wave velocity of the soil.

ℓ and n are the direction cosines of the direction of propagation.

For the half space $A_{SH} = A_{SH}'$, and for a unit amplitude of the motion on the surface,

$$u_y = 0.5 \left[\exp\left(\frac{i\omega}{C_s} nz\right) + \exp\left(-\frac{i\omega}{C_s} nz\right)\right] \cdot f(x,t)$$

or

$$u_y = \cos\left(\frac{\omega}{C_s} nz\right) \cdot f(x,t)$$

with $$f(x,t) = \exp\left(-\frac{i\omega}{C_s} \ell x\right) \cdot \exp(i\omega t)$$

The shear stresses are obtained from the displacement by differentiation.

$$\tau_{yx} = G\left(-\frac{i\omega}{C_s} \ell\right) \cos\left(\frac{\omega}{C_s} nz\right) \cdot f(x,t)$$

$$\tau_{yz} = -G\left(\frac{\omega}{C_s} n\right) \sin\left(\frac{\omega}{C_s} nz\right) \cdot f(x,t)$$

where G is the shear modulus.

In the case of SV and P waves the horizontal displacement u_x and the vertical displacement u_z depend on both waves, but the formulation can be simplified assuming the same variation in the x direction of all the displacement components. That means that

$$\frac{\ell}{C_p} = \frac{\ell'}{C_s}$$

and the displacement can be written:

$$u_x = [\ell A_p \exp(\frac{i\omega}{C_p} nz) + \ell A_p' \exp(\frac{i\omega}{C_p} nz) - n' A_{SV} \exp(\frac{i\omega}{C_s} n'z)$$

$$+ n' A_{SV}' \exp(-\frac{i\omega}{V_s} n'z)]\cdot f(x,t).$$

$$u_z = [-n A_p \exp(\frac{i\omega}{C_p} nz) + n A_p' \exp(-\frac{i\omega}{C_p} nz) - \ell' A_{SV} \exp(\frac{i\omega}{C_s} n'z)$$

$$- \ell' A_{SV}' \exp(-\frac{i\omega}{C_s} n'z)]\cdot f(x,t).$$

where

A_p and A_p' are the amplitudes of the P waves travelling in the negative z-direction and the reflected waves travelling in the positive z direction, respectively.

A_{SV} and A_{SV}' are the corresponding amplitudes for the SV waves.

ℓ, n and ℓ', n' are the direction cosines of the P and SV waves respectively.

C_p and C_s are the P and S wave velocity of the soil.

$f(x,t) = \exp(-\frac{i\omega}{C_s} \ell' x) \exp(i\omega t) = \exp(-\frac{i\omega}{V_p} \ell x) \exp(i\omega t).$

The stresses are obtained by differentiation of the previous expressions for the displacements.

$$\sigma_x = -\frac{i\omega}{C_p}[(\lambda+2G\ell^2) A_p \exp(\frac{i\omega}{C_p} nz) + (\lambda+2G\ell^2) A_p' \exp(-\frac{i\omega}{C_p} nz)$$

$$- 2G\ell n' A_{SV} \exp(\frac{i\omega}{C_s} n'z) + 2G\ell n' A'_{SV} \exp(-\frac{i\omega}{C_s} n'z)]\cdot f(x,t)$$

$$\sigma_z = -\frac{i\omega}{C_p}[(\lambda+2Gn^2) A_p \exp(\frac{i\omega}{C_p} nz) + (\lambda+2Bn^2) A_p' \exp(-\frac{i\omega}{C_p} nz)$$

$$+ 2G\ell n' A_{SV} \exp \quad \frac{i\omega}{C_s} n'z) - 2G\ell n' A_{SV}' \exp(-\frac{i\omega}{C_s} n'z)]\cdot f(x,t)$$

and

$$\tau_{xz} = \frac{i\omega}{C_s}\,[2G\ell' n\, A_p \exp\left(\frac{i\omega}{C_p} nz\right) - 2G\ell' n\, A_p' \exp\left(-\frac{i\omega}{C_p} nz\right)$$
$$+ G(\ell'^2 - n'^2)\, A_{SV} \exp\left(\frac{i\omega}{C_s} n'z\right) + G(\ell'^2 - n'^2) A_{SV}' \exp\left(-\frac{i\omega}{C_s} n'z\right)]\cdot f(x,t)$$

The above equations give the displacements and stresses in terms of the amplitudes of the SV and P waves. These amplitudes will be determined by the boundary conditions which in our case, for an elastic half-space, will be the stresses σ_z and τ_{xz} equal to zero on the surface z=0 and prescribed values for the displacements u_x and u_z on the same surface. Using these conditions on the equations of the displacements and stresses when z=0 and inverting the equations, one can obtain the following relations for the amplitudes

$$\begin{bmatrix} A_p \\ A_{SV} \\ A_p' \\ A_{SV}' \end{bmatrix} = \frac{1}{2} \begin{bmatrix} \dfrac{2G\ell}{\lambda+2G} & \dfrac{\ell'^2 - n'^2}{n} \\ -\dfrac{\lambda+2Gn^2}{n'(\lambda+2G)} & -2\ell' \\ \dfrac{2G\ell}{\lambda+2G} & -\dfrac{\ell'^2 - n'^2}{n} \\ \dfrac{\lambda+2Gn^2}{n'(\lambda+2G)} & -2\ell' \end{bmatrix} \begin{Bmatrix} u_x \\ u_z \end{Bmatrix}_{z=0}$$

FOUNDATION RESPONSE

<u>SH WAVES:</u> The motion of square surface and embedded foundations under the effects of SH waves with different angles will be considered first, using the previous formulation. Waves were assumed to produce a unit displacement on the free field. The Poisson's modulus ν of the soil was set equal to 1/3. The square surface foundation was modeled as shown in figure 1, using 16 square elements on the soil foundation interface and 16

rectangular elements to cover a band of free field 0.8B wide around the foundation. It was shown in reference [1] that these free field elements are sufficient to obtain satisfactory results.

A computer program was implemented which, given the wave characteristics and the soil model, computes the free field conditions and the elements of the [G] and [H] matrices, and then, following the transformations described in the previous section, obtains the response of the foundation.

Figure 3 shows the amplitude of the horizontal motion obtained for the square surface foundation when the SH waves have a direction of propagation corresponding to an angle ϕ=0, 45 and 90 degrees with the x axis. The results are compared with those reported by Wong and Luco [11] for the same angles using a more time-consuming numerical procedure. There is good agreement between both solutions, and it can be noticed that for the range of frequencies presented, the amplitude of the motion decreases with the frequency of the waves projected over the x axis (either when the frequency increases for a given angle or when the angle decreases for a certain frequency). The torsional motion of the massless square foundation is shown in figure 4 for the same angles of incidence and the same range of frequencies. The results are compared again with Wong and Luco's and the agreement is good. The amplitude of the torsion depends again mostly on the frequency of the waves projected over the x axis. There is also a third motion of the foundation under the influence of non-vertically propagating SH waves, a rocking around the x axis. This motion is very small in comparison with the other two. It is represented in figure 5 at a scale ten times larger than the one previously used. The variation with frequency is similar in shape to that of the torsional motion, but about two orders of magnitude smaller.

Figure 6 shows an approximation to the horizontal and torsional motion used by Unemori [10]. The solution for the horizontal displacement was obtained as an average of the free field motion along the foundation, as suggested by Yamahara [12], and the torsion was obtained in a way similar to Newmark's, [7], but taking again the average along the foundation.

The results for the horizontal motion for ϕ=0 or 45 degrees, compare well with those obtained in this work. However, the torsional response of the foundation for the same values of ϕ are overestimated by about 50 percent, using the approximate procedure.

To start the study of embedded foundations and in order to compare the solution with existing results for circular foundations, the response of a square foundation for two different levels of embedment was obtained for the case of vertically propagating S waves. The foundations had levels of embedment E/B = 4/3 and 2. The model used had 16 rectangular elements on the free field, the same as for the surface foundation shown in figure 3, and 33 or 45 equal square elements on the soil foundation interface, 9 on the bottom and two or three rows of 12 elements along the side walls. In figure 7 results are plotted versus the natural frequency of the layer of soil corresponding to the embedment, $f_0 = C_s/4E$. The values of the horizontal and rocking motions are compared with the approximate formulas proposed by Morray [2] for circular foundations embedded in a soil stratum. An equivalent radius $R_e = \sqrt[4]{16/3\pi}\,B$ was taken for the comparison, although this value is not necessarily the best approximation for an embedded foundation and the two types of motion. The equivalent radius would produce the same moment of inertia for a surface foundation. The comparison of both kinds of results is good, and the oscillation of the values around the approximate solution was also obtained by Elsabee and Morray in the case of a soil stratum.

Figures 8 to 10 show the results obtained for an embedded foundation with E/B = 2 and for the same angles of incidence used for the surface foundation. The values are plotted versus the natural frequency of the soil layer corresponding to the embedment (in this case $\omega_0 B/C_s = \pi/4$) and were obtained using the same model with 61 elements. It should be noticed in figure 8 that for waves travelling towards the foundation along the x axis (ϕ=0) the horizontal motion is similar to the motion of the surface foundation. On the other hand, when ϕ increases the motion does not tend towards unity as for the surface foundation, but the variation with frequency becomes less regular and stays below the ϕ=0 curve for most of the frequencies considered.

In figure 9 the torsional response is presented. This motion decreases with ϕ as could be expected and takes smaller values than for the case of a surface foundation. The rocking motion is presented in figure 10. This motion, which for the surface foundation was zero when $\phi = 90°$ and was negligible for other values of ϕ, has considerable importance when the foundation is embedded. For the range of frequencies studied, its value increases with frequency and with the angle ϕ, becoming the main rotational component for high values of this angle, while for low values of ϕ the main rotation of the foundation is the torsion.

SV and P WAVES: The present section studies the response of surface and embedded square foundations under the influence of a combination of SV and P waves. As in previous sections the results will be presented versus the dimensionless parameter a_0 and the natural frequency of the embedment layer, but in order to use the relation between free field vertical and horizontal motions commonly accepted in seismic design, the foundation dimensions and the soil properties were assumed to have typical values. The soil had a Poisson's ratio $\nu = 1/3$ and a shear wave velocity $C_s = 900$ ft/sec. The foundation was a square with dimensions 30' x 30' for the surface foundation case and had an embedment of also 30 feet in the other case. The boundary element discretizations were the same as for the SH waves. The amplitudes of the SV and P waves were determined by the ratio of horizontal to vertical free field motion resulting from the response spectra proposed by Newmark, Blume and Kapur [8], and by the angle of incidence ϕ of the SV waves that was set equal to 70, 80 and 90 degrees. The angle of incidence ψ of the P waves is determined by the relation $\ell/C_p = \ell'/C_s$. In table I the ratio of free field vertical to horizontal motion for values of frequencies up to 15 cps is presented: the relation is plotted in figure 11.

Figure 12 shows the filtering effect produced for the horizontal motion where the three angles of incidence of the SV and P waves are considered for a surface foundation. The effect is of the same type as for SH waves and can be again approximated with the solution by Unemori in figure 6. The variation with frequency and angle of incidence of the

TABLE I

f (cps)	$\lvert u_z \rvert_{ff} / \lvert u_x \rvert_{ff}$
1.5	0.676
3	0.723
4.5	0.838
6	0.939
7.5	1.035
9	1.128
10.5	1.158
12	1.199
13.5	0.125
15	1.316

filtered vertical motion have a similar shape (figure 13), but the effect is more pronounced, and Unemori's approximate solution cannot be used in these cases.

Figures 14 and 15 show the rocking motion produced by the non-vertically incident SV and P waves on a square surface foundation. In figure 14 the rocking motion is shown normalized with respect to the vertical motion of the free field and the variation with frequency for the range studied is essentially a straight line. On the other hand, if the motion is normalized with respect to the free field horizontal motion (figure 15), the variation with frequency reflects the factor of $\lvert u_z \rvert_{ff} / \lvert u_x \rvert_{ff}$, which indicates that the rocking motion depends basically only on the vertical motion. This result is in agreement with the negligible rocking motion taking place for SH waves that produces only horizontal motion. The rocking motion for the angles studied (figure 14) can be well predicted by Unemori's approximate solution (figure 6).

Figure 16 shows the filtering of the horizontal motion for the square embedded foundation. The filtering effect in the low range of frequencies increases with the angle ϕ but this is not true for all frequencies due to the oscillations of the results. For vertically incident waves (ϕ=90°) this variation is equal to that obtained when only an S wave was considered (SV and SH waves are the same for ϕ=90°).

The filtering of the vertical motion for an embedded foundation is plotted in figure 17. There can be noticed in this case a much smaller effect of the angle of incidence than for the horizontal motion. The filtering effect of both horizontal and vertical motions is an important factor for embedded foundations when the angles of incidence of the waves are close to 90°, while for surface foundations and the same angles of incidence the effect is small.

Figure 18 shows the rocking motion of the embedded square foundation for the three angles ϕ of the SV waves considered. It can be noticed that the variation with frequency does not change much for the angles considered. The curve for the vertically incident waves is the same as that obtained for an SV or SH wave in figure 10. Figure 19 shows the same motion but normalized with respect to the vertical free field motion.

CONCLUSIONS

A comparison of the results obtained in this work using the boundary element method with those reported by Wong and Luco for surface foundations indicates that the procedure can be used with confidence and more economically than previous solutions to study the motion of rigid (or flexible) foundations due to any train of waves. The method is particularly convenient when dealing with an elastic or viscoelastic halfspace.

The results for an embedded square foundation and vertically propagating shear waves are in good agreement in general terms with those reported by Morray corresponding to a circular footing embedded in a layer of finite depth.

While the number of cases studied is not sufficiently large to derive approximate formulae or general conclusions, it would appear that the effect of the angle of the waves, in the filtering of the translational components of motion and in the appearance of rotational components (rocking and torsion) are less pronounced for embedded foundations than for footings resting on the surface.

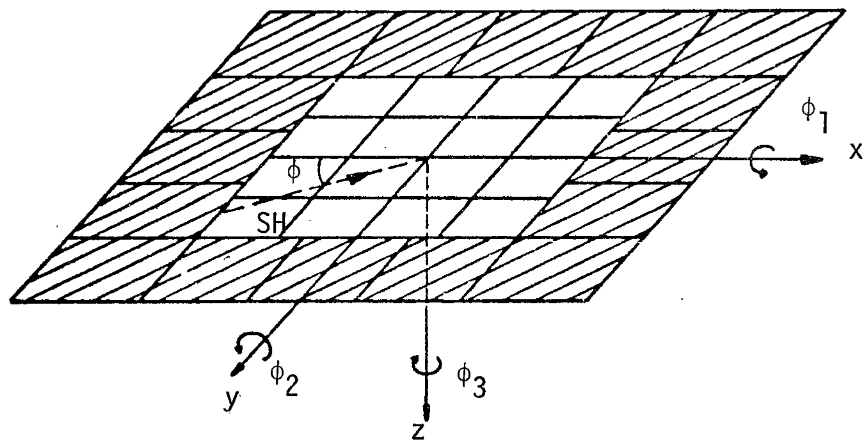

Figure 1 - Model for Surface Foundation and System of Coordinates

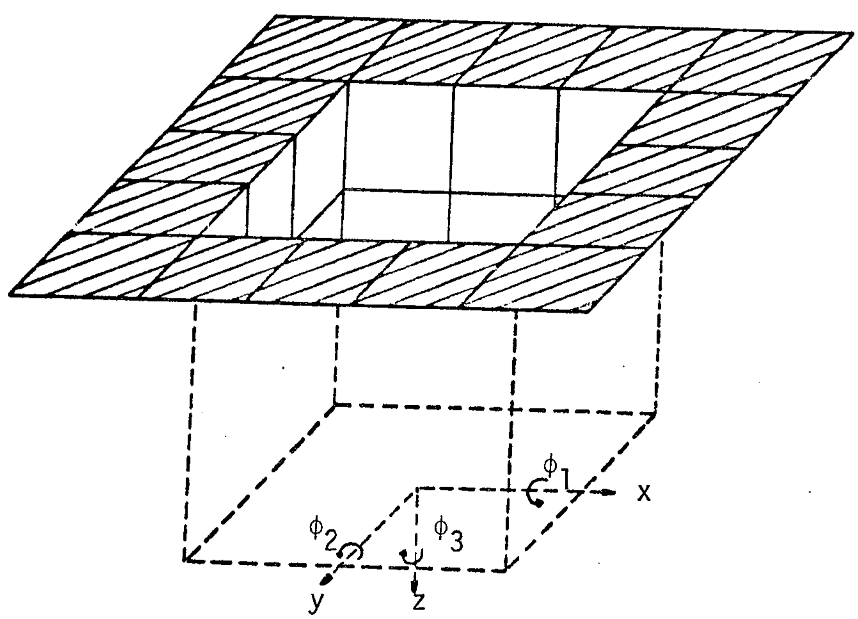

Figure 2 - Embedded Foundation

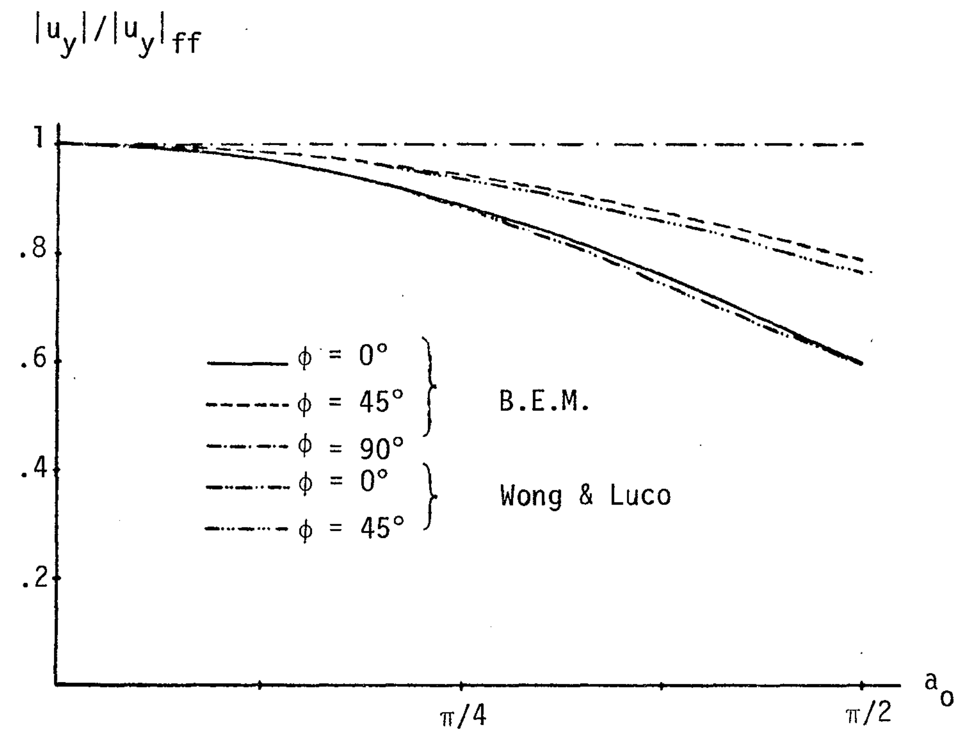

Figure 3 - Filtering of the Horizontal Motion for SH Waves. Surface Foundation.

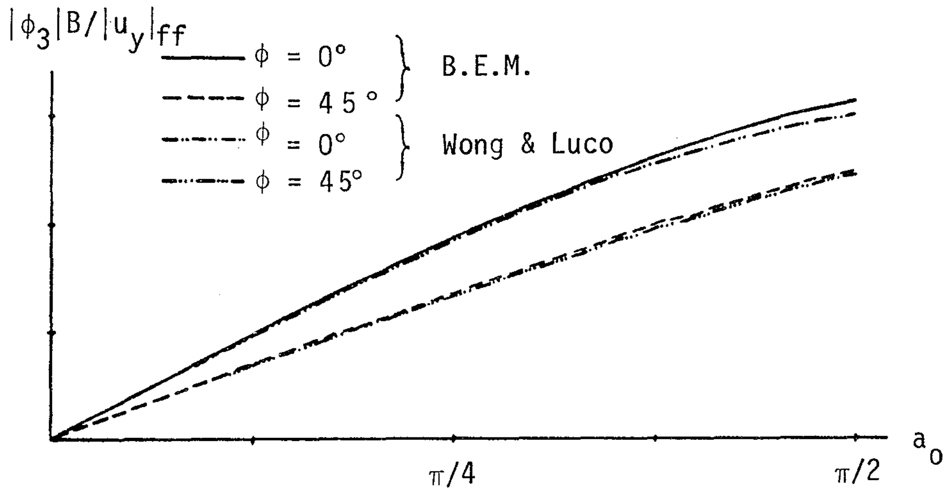

Figure 4 - Torsional Motion for SH Waves. Surface Foundation.

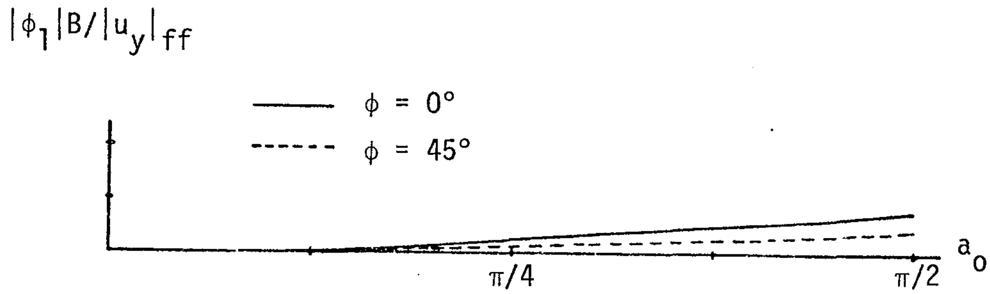

Figure 5 - Rocking Motion for SH Waves. Surface Foundation.

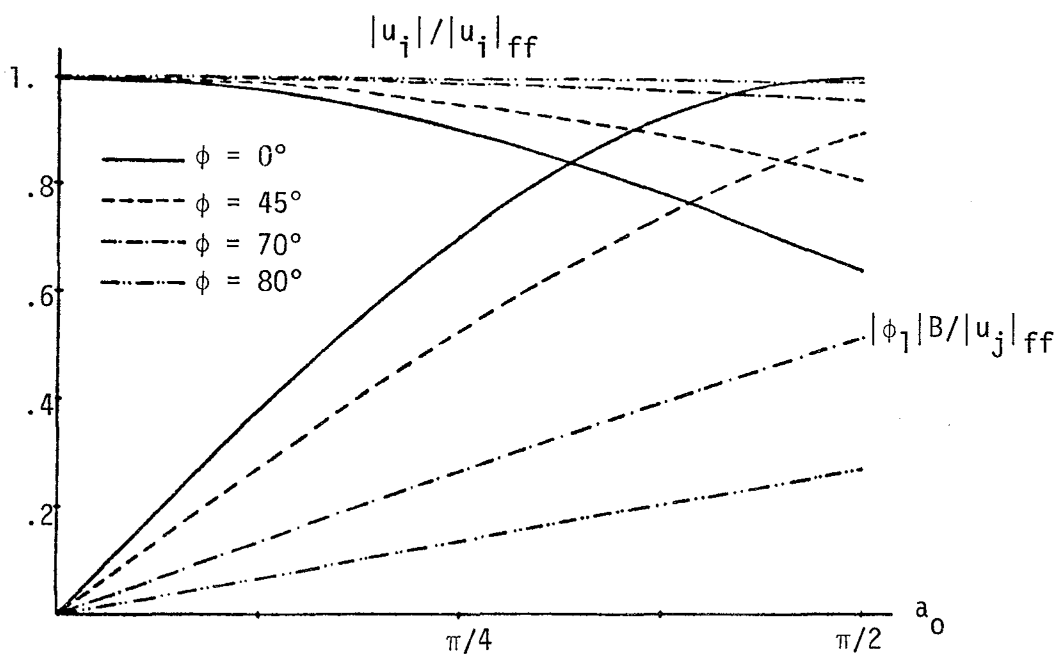

Figure 6 - Approximate Solution for the Filtering and Rotation by Unemori.

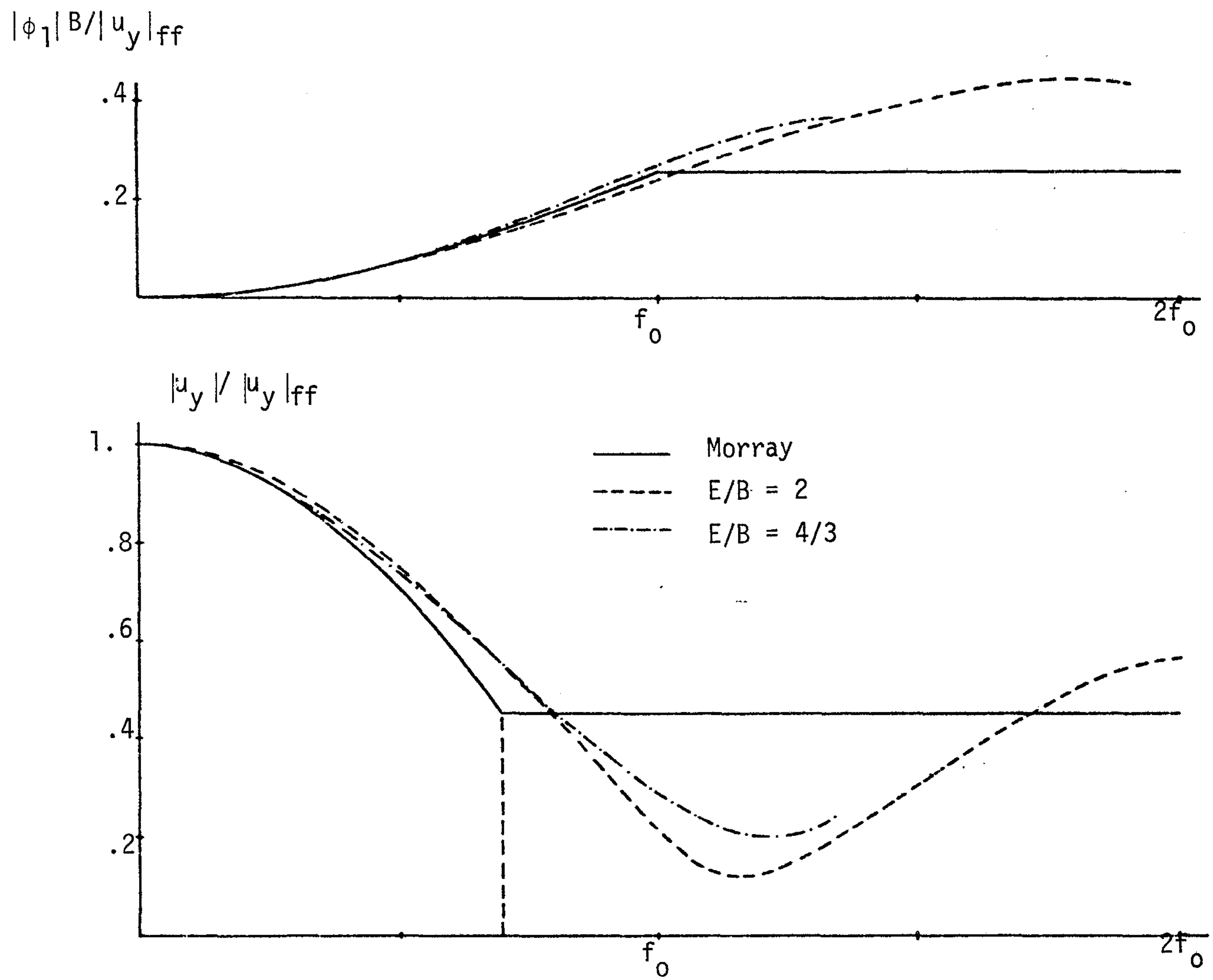

Figure 7 - Vertically Incident S Waves on Embedded Foundations.

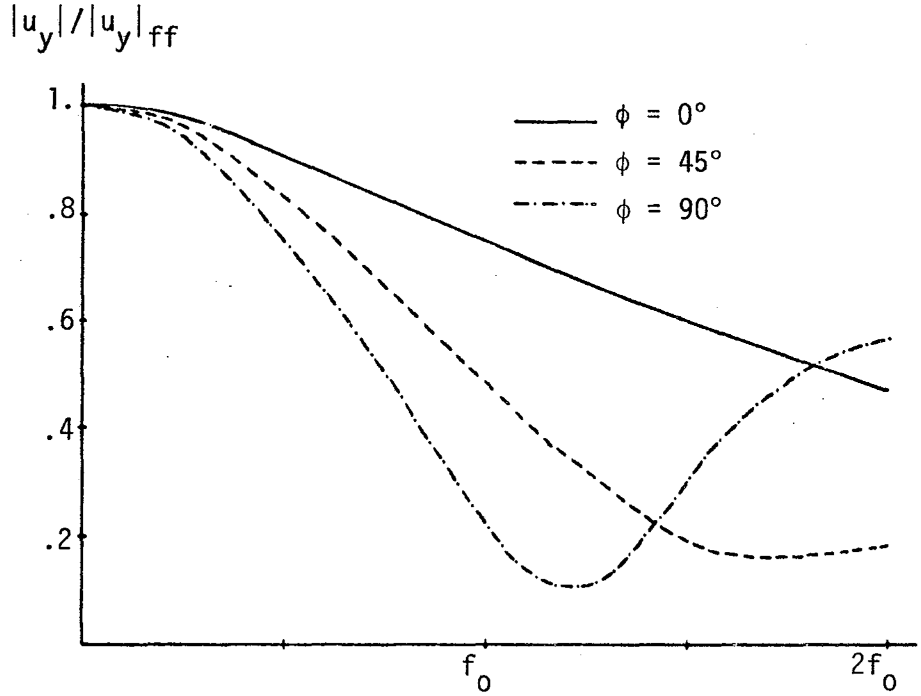

Figure 8 - Filtering of the Horizontal Motion for SH Waves. Embedded Foundation.

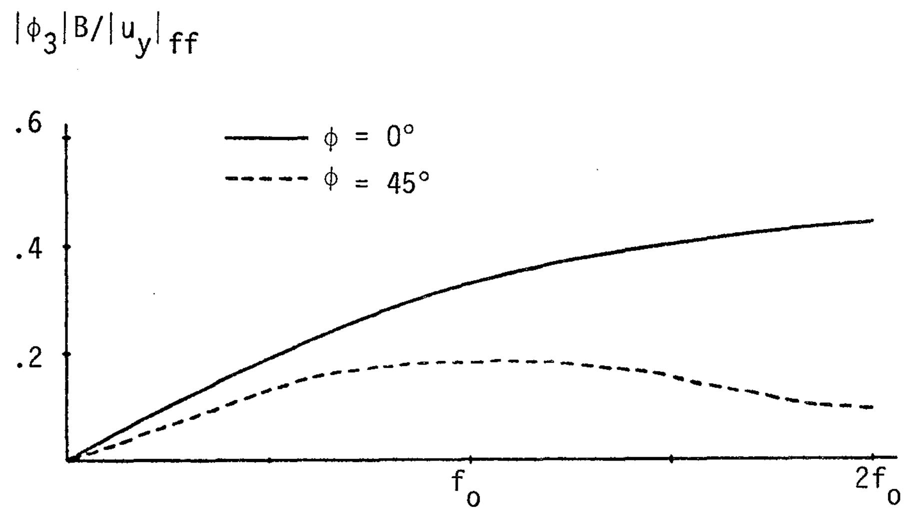

Figure 9 - Torsional Motion for SH Waves. Embedded Foundation.

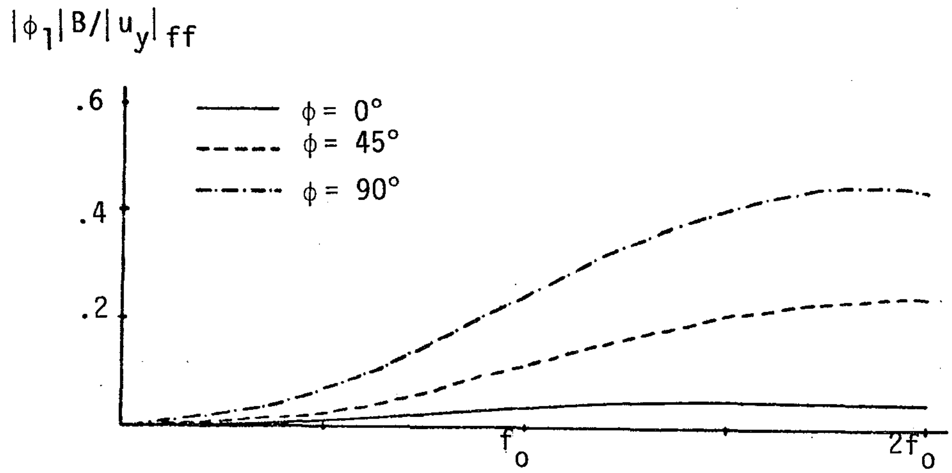

Figure 10 - Rocking Motion for SH Waves. Embedded Foundation.

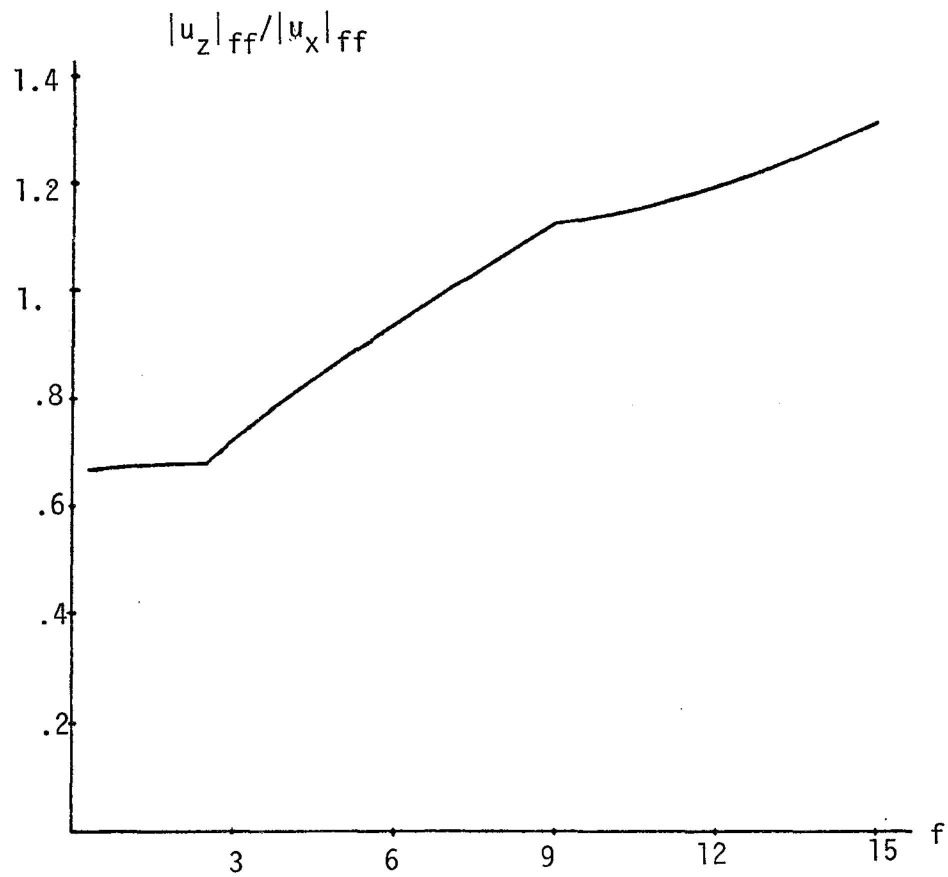

Figure 11 - Ratio of Amplitudes of Surface Motion for SV-P Waves.

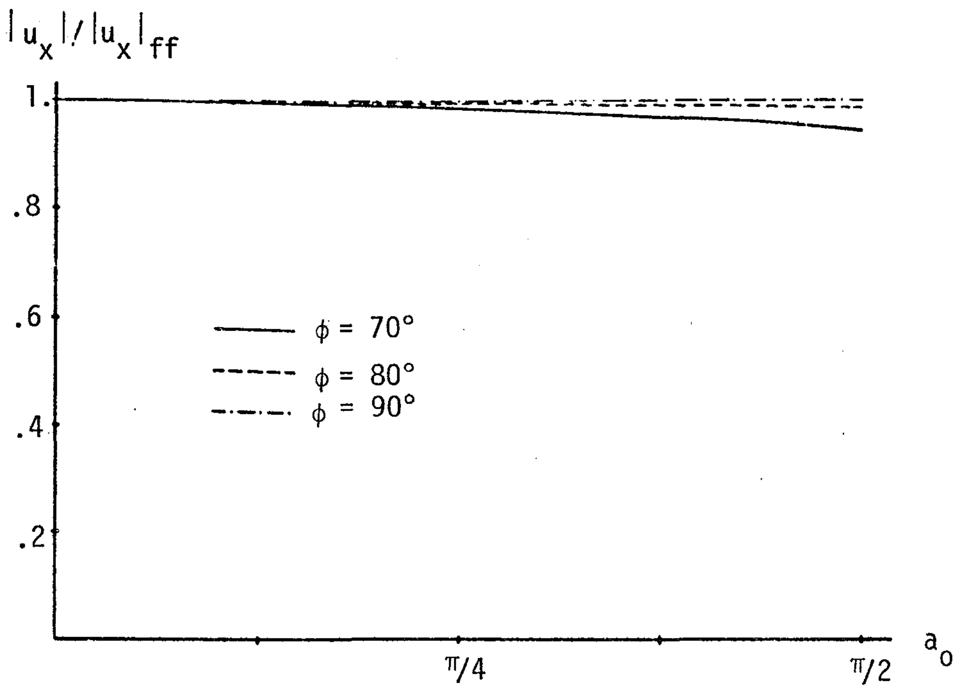

Figure 12 - Filtering of the Horizontal Motion for SV-P Waves.
Surface Foundation.

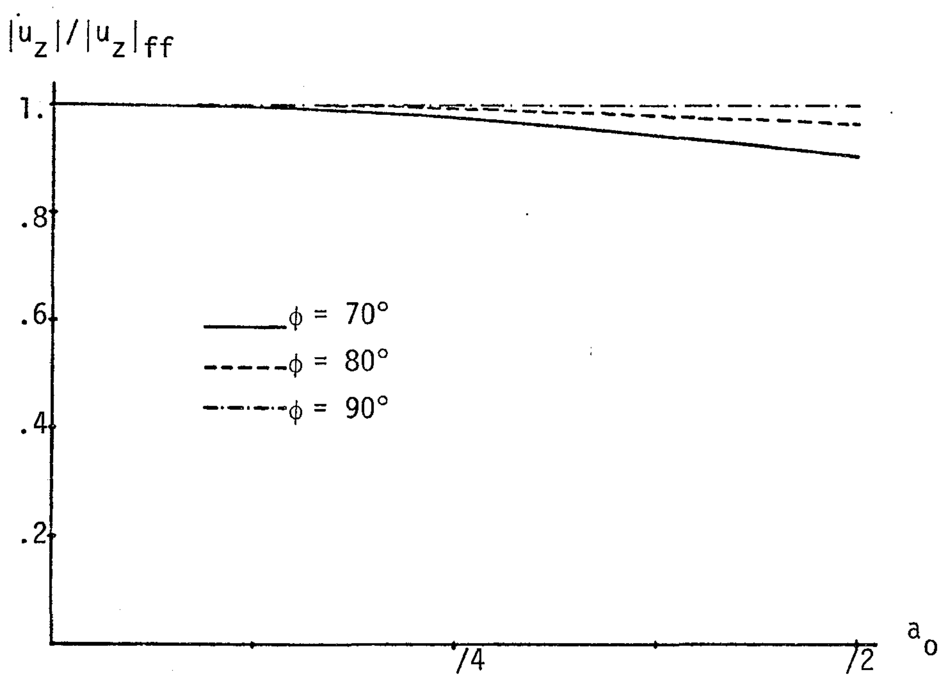

Figure 13 - Filtering of the Vertical Motion for SV-P Waves.
Surface Foundation.

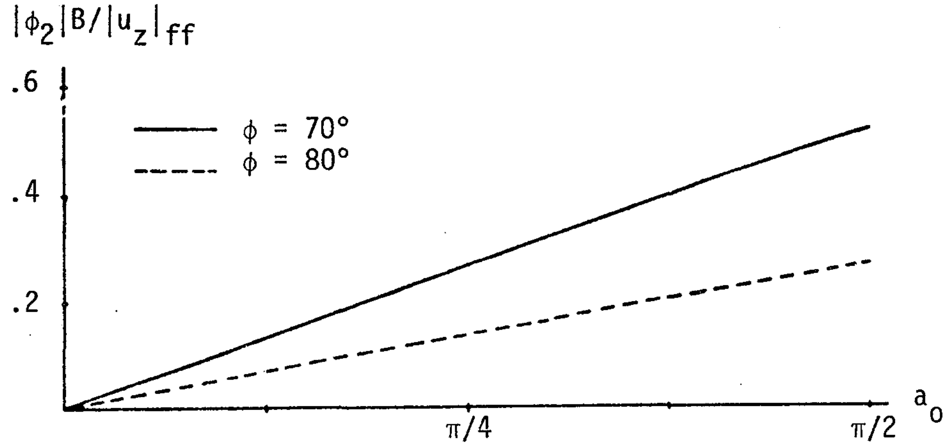

Figure 14 - Rocking Motion for SV-P Waves. Surface Foundations.

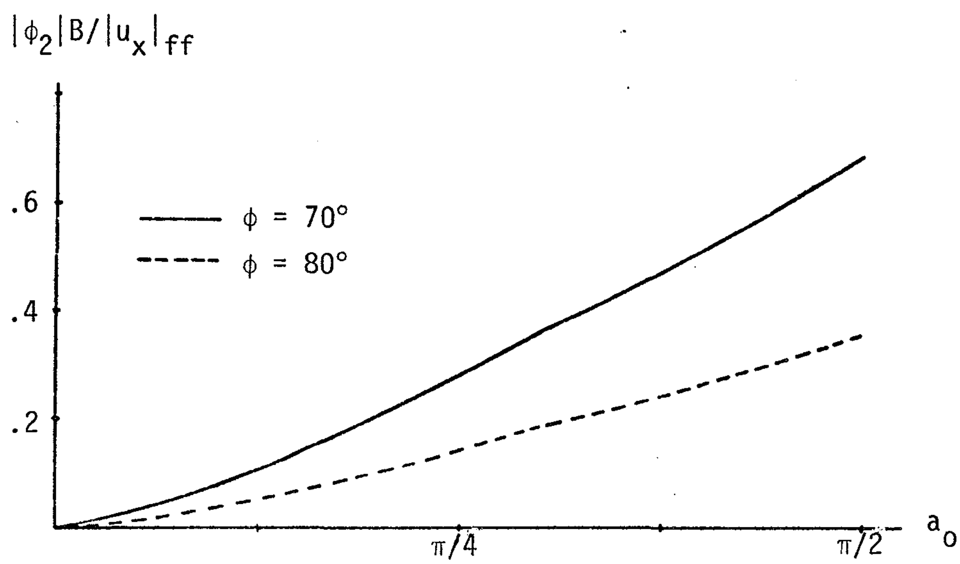

Figure 15 - Rocking Motion for SV-P Waves. Surface Foundations.

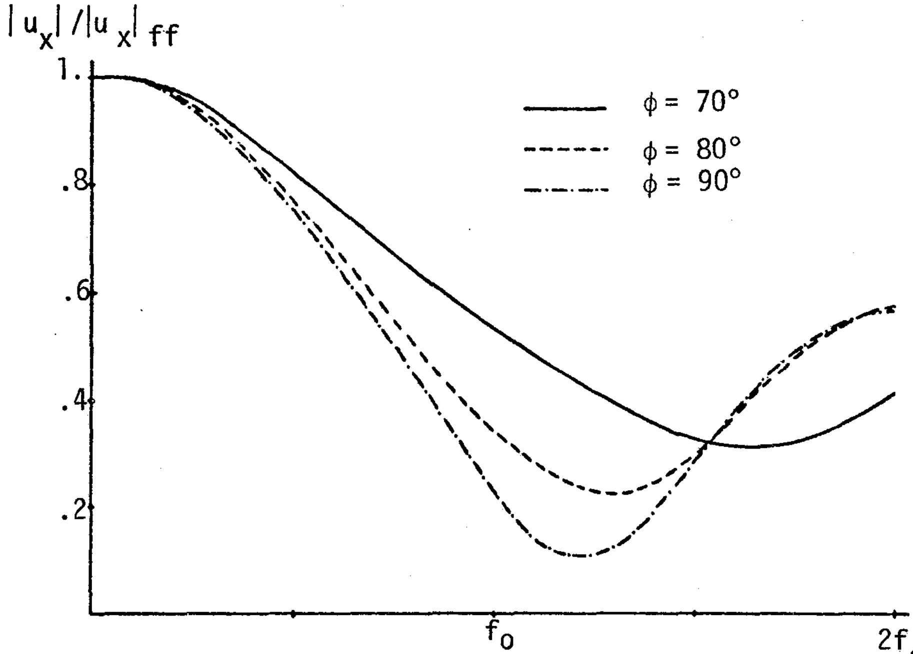

Figure 16 - Filtering of the Horizontal Motion for SV-P Waves. Embedded Foundation.

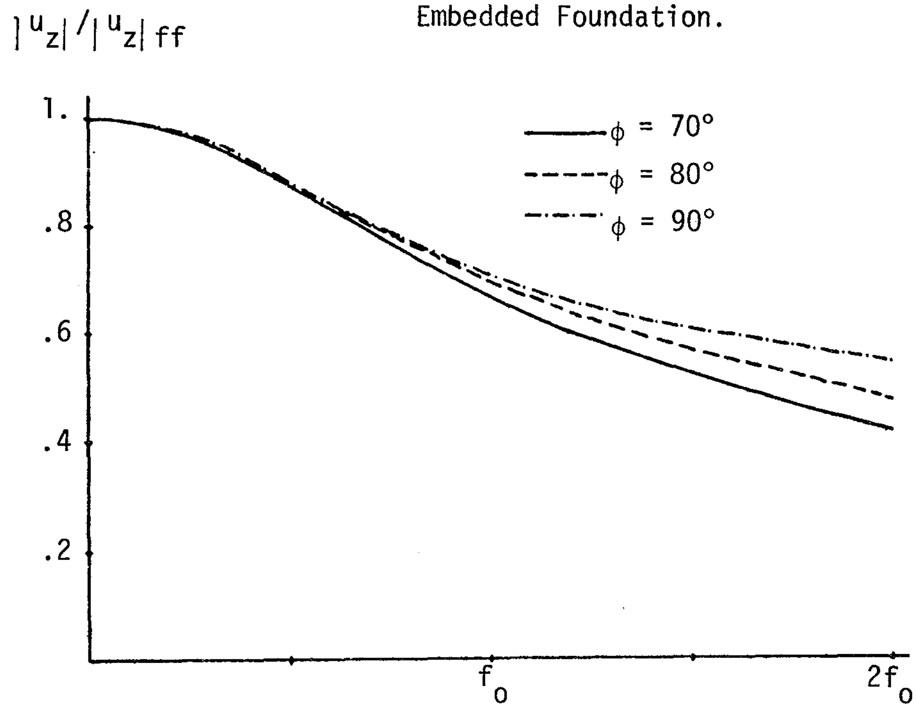

Figure 17 - Filtering of the Vertical Motion for SV-P Waves. Embedded Foundation.

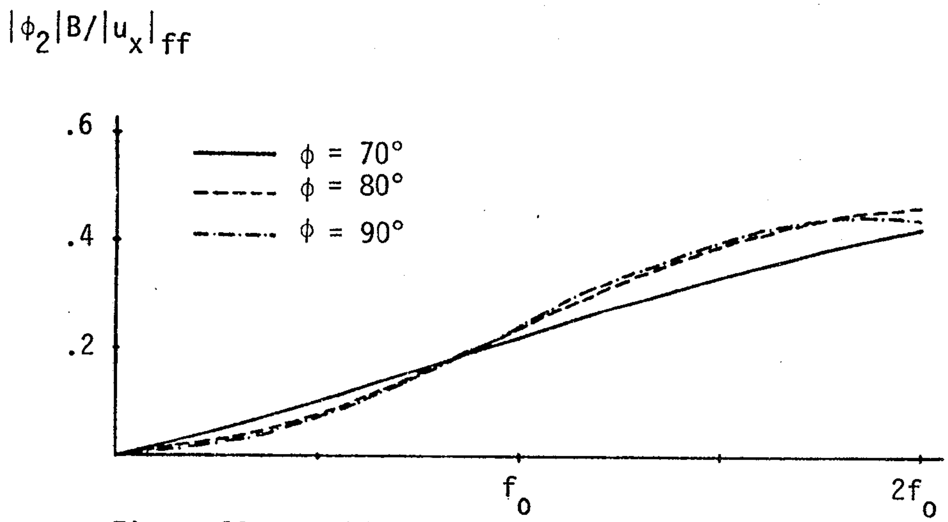

Figure 18 - Rocking Motion for SV-P Waves. Embedded Foundation.

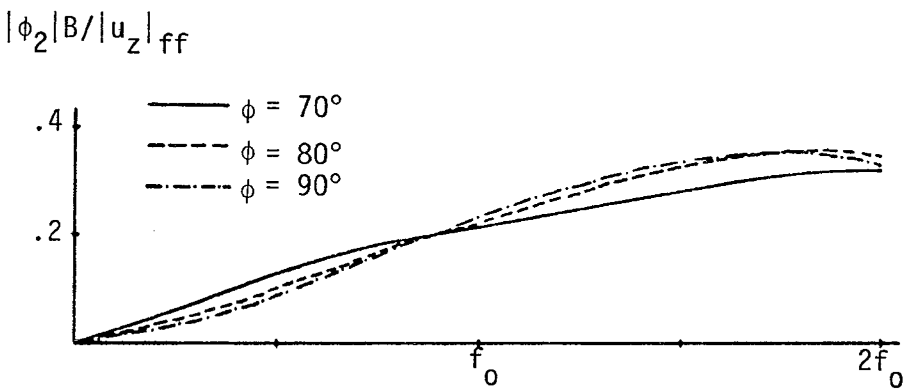

Figure 19 - Rocking Motion for SV-P Waves. Embedded Foundation.

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