50272-101

REPORT DOCUMENTATION PAGE	1. REPORT NO. NSF/RA-760773	2.	PB 288372

4. Title and Subtitle
Identification of the Energy Absorption Characteristics of an Earthquake Resistant Structure, Description of the Identification Method, Proceedings *

5. Report Date
1976

6.

7. Author(s)
H.D. McNiven, and V.C. Matzen

8. Performing Organization Rept. No.

9. Performing Organization Name and Address
University of California at Berkeley
Earthquake Engineering Research Center
Berkeley, California 94720

10. Project/Task/Work Unit No.

11. Contract(C) or Grant(G) No.
(C)
(G) AEN7604262

12. Sponsoring Organization Name and Address
Applied Science and Research Applications (ASRA)
National Science Foundation
1800 G Street, N.W.
Washington, DC 20550

13. Type of Report & Period Covered

14.

15. Supplementary Notes
* Conference on Dynamic Response of Structures, University of California at Los Angeles, 1976

16. Abstract (Limit: 200 words)
The construction of a mathematical model of a single story steel frame, when subjected to earthquake inputs, using the system identification method, is the subject of this paper. The system identification method consists of three parts: the differential equation which reflects the physics of the system and which contains a set of parameters; the error function which reflects the difference in the structure's behavior and that predicted by the model; and the algorithm by which the parameter space is searched to find the set which minimizes the error function. The program for estimating the parameters was tested. It was first subjected to a numerical experiment using simulated data. The input is the acceleration time history of a recorded earthquake, but response data is generated by assigning values to each of the four parameters and obtaining the solution of the resulting equation. The program can be considered to be successful if the optimization algorithm, in a reasonable number of iterations, identifies the minimum point of the error function surface, as having the coordinates of the assigned values, when the search is begun at a variety of points in the remote terrain. The mathematical model was found to predict a response close to the physical response of the structure yet two major ways in which the estimation of the parameters could be improved are suggested.

17. Document Analysis a. Descriptors

Earthquake resistant structures
Structural analysis
Steel construction
Mathematical models
Dynamic structural analysis

b. Identifiers/Open-Ended Terms

Energy absorption

c. COSATI Field/Group

18. Availability Statement	19. Security Class (This Report)	21. No. of Pages
NTIS		13
	20. Security Class (This Page)	22. Price A02-A01

(See ANSI-Z39.18) See Instructions on Reverse OPTIONAL FORM 272 (4-77) (Formerly NTIS-35) Department of Commerce

CAPITAL SYSTEMS GROUP, INC,
6110 EXECUTIVE BOULEVARD
SUITE 250
ROCKVILLE, MARYLAND 20852

1. INTRODUCTION

For years engineers have been using differential equations to explain and predict the behavior of building frames when they are subjected to dynamic loads. Most of the attention has been directed to linear behavior and for these problems the form of the differential equations has been accepted and the parameters in the equations, representing physical quantities, have, by various means, been identified accurately. The equations, together with the parameters they contain, form a mathematical model of a physical structure.

When the model of a frame is linear, it implies that the frame will respond elastically to the imposed dynamic loads. Earthquake ground motions, however, can be very intense and it is generally accepted that a design of the frame that would ensure an elastic response would be economically unrealistic. When a more realistically designed frame responds to an earthquake, yielding will occur at several locations of the frame. To predict this kind of response, the mathematical model of the building must reflect this yielding, rendering at least some of the equations nonlinear.

When the mathematical model becomes nonlinear, it is much more difficult to construct. One is no longer so confident about the form of the differential equation: that is how the nonlinearity should be incorporated, and it is by no means certain what values the parameters should have. Accordingly, it is important, when constructing a mathematical model of a particular structure, to have a method for appraising the model in light of the behavior of the structure itself. Further, to modify the model sensibly it is necessary to be able to appraise its two parts, the parameters and the equation itself, separately.

To achieve this kind of appraisal, we feel that the method should have two ingredients. The first is an experimental capability by which a physical model of the structure can be subjected earthquake forcing functions. This capability introduces two distinct advantages. The experimental response can be used for the estimation of the parameters. The other has to do with the appraisal of the model. When the mathematical model of a particular frame is appraised, one is able, from physical experiments, to compare the response predicted by the mathematical model with the experimental response of the physical model, when both are

subjected to the same earthquake input. This is the ultimate test of a mathematical model, short of being able to test an actual structure.

We are fortunate at the Earthquake Engineering Research Center at the University of California at Berkeley, in having a shaking table capable of imposing on a large-scale model of a building, arbitrary base motions. It is possible to perform experiments under controlled conditions in which accurate records are made of the table input and frame responses at any number of chosen locations. Having this capability, we have been able to carry out this kind of appraisal of one model. In another paper at this conference such an appraisal is made for a single story steel frame.

The other quality the method should have is a program for estimating the parameters of the equation, that will allow for a separate appraisal of the form of the equation. This is best understood by discussing a method that does not have this ability. If, for example, the parameters are found by trial and error, with recourse to physical insight, a model is formed. When the response of this model is compared to a physical response it may be inadequate. For such a case, one is unable to decide whether to attribute the inadequacy to poor parameters or to a poor equation.

To overcome this dilemma we use System Identification to estimate the parameters in the equation. Using this method, the estimated set of parameters is, by definition, the set that will give the best mathematical model within the limitation of the differential equation. If the match of the responses using System Identification, is inadequate, the fault can be laid to the form of the equation.

The construction of a mathematical model of a single story steel frame, when subjected to earthquake inputs, using System Identification is the subject of this paper.

2. SYSTEM IDENTIFICATION

System identification consists of three parts. The first, and most important, is the mathematical model. This consists of the differential equation, (or equations), which here reflects the physics of the system and which contains a set of parameters. The second is the error function. The error reflects the difference in the behavior of the structure and that predicted by the model. In our case the error is the squared differences between the responses of the structure and the model at each time step, accumulated over the period of exposure of each to the same earthquake input. The third part is the algorithm by which the parameter space is searched to find the set which minimizes the error function.

Each of the three parts will be reviewed separately

(a) The Differential Equation

In many identification problems nothing is known about the system, so that it takes on the name of a "black box". In other systems, such as ours, various degrees of insight are available to help in construction of the model. In our case, we resort to physics and engineering and are able to exploit what is generally accepted to be the major response mechanisms of a single story steel frame. In

the formulation we account for linear viscous damping and hysteretic energy absorption. The equation that results is:

$$M\ddot{x} + C\dot{x} + P(x) = -M\ddot{x}_g \tag{1}$$

In Eq. (1),

x is the displacement

C is the viscous damping coefficient

$\ddot{x}_g$ is the ground (or table) acceleration.

$P(x)$ reflects the global inelastic behavior during response. As P does not accommodate rate dependent behavior, it is limited to modeling elastic and plastic behavior. Except for the special case of a linearly elastic material, $P(x)$ will render the equation nonlinear.

For mild steel, subjected to bending, there are two major forms for $P(x)$. The first is a bilinear model, the second is the Ramberg-Osgood model. Both characterizations are popular but the choice here is Ramberg-Osgood.

The choice is made on the bases of two considerations. The first is physical and is based on the accuracy with which the model is able to represent the behavior of structural steel assemblages, subjected to quasi-static loading that induces bending in the members. This accuracy has been reported by Popov and Bertero (1) in tests conducted on a variety of assemblages subjected to an array of cyclic loading histories.

The second reason is mathematical. The Ramberg-Osgood model has many properties that simplify the solution of the differential equation. The solution of the equation will not be discussed here, even though it is important to the success of the identification program, as a report exclusively devoted to the solution will be issured shortly.

In this mathematical context, it should be pointed out that the Ramberg-Osgood equations have the disadvantage that they do not express P as a function of x, which is desirable, but rather express x as a function of P. The equations are

$$x(P) = \frac{P}{K}\left[1 + A\left|\frac{P}{K}\right|^{R-1}\right] \tag{2}$$

for skeletal curves, and

$$x(P) - x_{RE}(P) = \frac{P-P_{RE}}{K}\left[1 + A\left|\frac{P-P_{RE}}{2K}\right|^{R-1}\right] \tag{3}$$

for branch curves.

Even though the equation is derived using physical knowledge of the system, preconceived ideas of the values of the parameters C, A, K and R are abandoned, and their values are left to the mercy of the identification process. There is one notable exception to this. The optimization algorithm for estimating parameters requires a first estimate, and the more realistic this first estimate is, the shorter the search for the best estimate will be. A method for gaining insight into this first estimate of the parameters from the physical response is described in another paper at this conference.

(b) The Error Function

The general form of the error function adopted here is that used in the large majority of identification problems. It is the accumulated squared error of the responses between the two systems, physical and mathematical.

In this study, as we measure the response of the physical model in terms of accelerations and displacements, the error function has the following form:

$$J(\bar{\beta},T) = \int_0^T \left\{[\ddot{x}(\bar{\beta},t) - \ddot{y}(t)]^2 + [x(\bar{\beta},t) - y(t)]^2\right\} dt \tag{4}$$

where

$\bar{\beta}$ is a vector in four dimensional parameter space

$x(\bar{\beta},t)$ describes the motion of the mathematical model

$y(t)$ describes the motion of the physical model, and

T is an interval of time during which both models are exposed to an earthquake input. It may represent the total exposure of the input or a chosen part of it.

(c) The Optimization Algorithm.

The final part of the identification problem is the estimation the parameters. It is a minimization problem in that it requires the determination of a four dimensional vector $\bar{\beta}$, $\bar{\beta} \in R^4$, which minimizes the error function $J(\bar{\beta})$. It is convenient to think of the error function $J(\bar{\beta})$ as a four-dimensional surface imbedded in five dimensional space and to note that we seek the point at which the surface has its global minimum.

The method used here is a "gradient method," that is a method in which the direction along which we search the terrain from a particular point is a function of the gradient at that point.

The method is generated by expanding the error function in a Taylor Series

$$J(\bar{\beta}_{i+1},T) = J(\bar{\beta}_i) + \nabla J(\bar{\beta}_i)^T (\bar{\beta}_{i+1} - \bar{\beta}_i) + \frac{1}{2}(\bar{\beta}_{i+1} - \bar{\beta}_i)^T \nabla^2 J(\bar{\beta}_i)(\bar{\beta}_{i+1} - \bar{\beta}_i) + O(3)+ \tag{5}$$

where $\bar{\beta}_i$ represents $\bar{\beta}$ at the i^{th} iteration and ∇ is the usual del operator, here used in four dimensional parameter space. We do in fact truncate the series to include only terms of order two or less. The resulting equation becomes increasingly accurate as the terms of order three and higher become increasingly less significant, or as the surface representing the error function becomes increasingly quadratic.

To minimize the error function, we set the gradient of $J(\bar{\beta})$, with respect to $\bar{\beta}_{i+1}$ equal to the zero vector. Carrying out this operation on Eq. (5) leads to the vector equation

$$\nabla J(\bar{\beta}_i) + \nabla^2 J(\bar{\beta}_i)(\bar{\beta}_{i+1} - \bar{\beta}_i) = 0 \tag{6}$$

or

$$\bar{\beta}_{i+1} = \bar{\beta}_i - \nabla^2 J(\bar{\beta}_i)^{-1} \nabla J(\bar{\beta}_i). \tag{7}$$

In Eq. (7) $\nabla J(\bar{\beta}_i)$ represents the gradient of the surface $J(\bar{\beta})$ at the point representing the i^{th} iteration. The term $\nabla^2 J(\beta_i)$ is the Hessian matrix calculated at the same point. Space does not permit the development of these expressions.

The gradient is

$$\bar{\nabla}_i J(\bar{\beta}) = 2 \sum_{n=1}^{N} \left\{ \left[\ddot{x}(\bar{\beta},t_n) - \ddot{y}(t_n) \right] \frac{\partial \ddot{x}}{\partial \beta_i}(\bar{\beta},t_n) + \left[x(\bar{\beta},t_n) - y(t_n) \right] \frac{\partial x}{\partial \beta_i}(\bar{\beta},t_n) \right\} \Delta t \tag{8}$$

The Hessian matrix can be written as the sum of two matrices

$$\nabla^2_{ij} J(\bar{\beta}) = A_{ij}(\bar{\beta}) + B_{ij}(\bar{\beta}) \tag{9}$$

where

$$A_{ij}(\bar{\beta}) = 2 \sum_{n=1}^{N} \left\{ \frac{\partial \ddot{x}}{\partial \beta_i}(\bar{\beta},t_n) \frac{\partial \ddot{x}}{\partial \beta_j}(\bar{\beta},t_n) + \frac{\partial x}{\partial \beta_i}(\bar{\beta},t_n) \frac{\partial x}{\partial \beta_j}(\bar{\beta},t_n) \right\} \Delta t \tag{10}$$

and

$$B_{ij}(\bar{\beta}) = 2\sum_{n=1}^{N} \left\{ \left[\ddot{x}(\bar{\beta},t_n) - \ddot{y}(t_n) \right] \frac{\partial^2 \ddot{x}}{\partial \beta_i \, \partial \beta_j} + \left[x(\bar{\beta},t_n) - y(t_n) \right] \frac{\partial^2 x}{\partial \beta_i \, \partial \beta_j} \right\} \Delta t \tag{11}$$

In Eqs.(8,10 and 11) integration has been replaced by summation over incremental time steps.

We adopt the usual method associated with the Gauss-Newton scheme and neglect the B matrix. We now return to Eq. (7) and rewrite it as

$$\bar{\beta}_{i+1} = \bar{\beta}_i + \Delta\bar{\beta} \tag{12}$$

where

$$\Delta\bar{\beta} = - \alpha A^{-1}(\bar{\beta}) \cdot \nabla J(\bar{\beta}) \tag{13}$$

The heart of the optimization method appears in these two equations. The step by which the parameter vector is improved requires both direction and step size. The $A(\bar{\beta})$ matrix affects both. The direction given by Eq. (13) is not in fact the direction of the gradient, but this direction modified by $A^{-1}(\bar{\beta})$. The $A(\bar{\beta})$ matrix also influences the step size, but it is expedient to introduce the factor α so that the step size can be chosen independently.

The search of the terrain representing the surface $J(\bar{\beta})$ begins at a point which is the first estimate. The gradient and the A matrix are found for this point and the step direction is established according to Eq. (13). Having the direction, a one dimensional search is made in this direction to find the minimum point, or a point close to the minimum, along this unknown profile. To this end, we set α equal to one and find the coordinates of the point which this step represents. If none of the coordinates violates a constraint (here C, A, $K > 0$, $R > 1$) the slope of the surface is found at this point. The slope at the initial point is by definition downward and, if the slope at the point $\alpha = 1$ is opposite, the minimum lies between them. A cubic polynomial is constructed along the search line which passes through the two end points with the correct slopes. The minimum point of this polynomial is found. At this point the values of the cost function and slope in the search direction are found.

If the slope in the search direction is less than a specified tolerance the one-dimensional search ends and the next iteration is started. If the slope is greater, the cubic interpolation is repeated, using two points on either side of the minimum, until the slope is tolerated establishing the minimum.

It is not clear how demanding to be in a one-dimensional search. There is a trade off between an exacting one-dimensional search with fewer steps to the minimum point, and a more casual one-dimensional search entailing more steps to the minimum. Our one-dimensional searches have started out with fairly large slope tolerances at points remote from the minimum, with the search becoming more exacting as the minimum point is approached.

When a step size of α equal to one violates a constraint, one gets back into the acceptable domain in the following way. We study the vector between the two end points of the interval and, using linear interpolation, scale back to find the intersection of this vector and the constraint. We establish the acceptable end point of the adjusted interval by moving an additional ten percent of the shortened vector length back from the constraint.

A comment is appropriate concerning the minimization process. Satisfaction of Eq. (6) could lead to an array of stationary points, but with the terrain

representing $J(\bar{\beta})$ with which we have had experience, and with the physical insight into the possible values of the parameters, the stationary point which we have located is always a minimum and represents a likely solution. For the simulated data, which we will discuss later, the stationary point identified is the one assigned a priori, which by definition is the global minimum, and for the point derived from the physical response the coordinates of this point identified as the minimum represent parameters for which the response is entirely satisfactory.

The search, as was pointed out, begins with the calculation of the elements of the gradient and the A matrix at the initial point on the surface. Examination of Eqs. (8) and (10) shows that these values depend on finding the derivatives $\partial x/\partial \beta_i$ etc. which are called the sensitivity coefficients.

The sensitivities, which are found at each time increment, are derived by returning to Eq. (1). If we differentiate Eq. (1) with respect to a particular parameter β_j we have

$$M \frac{\partial \ddot{x}_n}{\partial \beta_j} + C \frac{\partial \dot{x}_n}{\partial \beta_j} + \frac{\partial P_n}{\partial \beta_j} = \frac{-\partial C}{\partial \beta_j} \dot{x}_n \tag{14}$$

where the n indicates the time step and $\beta_j (j = 1\text{-}4) = C, K, A, R$.

As was pointed out earlier, the form of the Ramberg-Osgood equations is unfortunate, giving the term $\partial P_n/\partial \beta_j$ which is not a sensitivity. This minor problem is resolved by returning to Eqs. (2) and (3) and differentiating both sides with respect to β_j. $\partial P_n/\partial \beta_j$ is established from the resulting equations, and substituted in Eqs. (14). The resulting set of equations, from Eq.(2) is

$$M \frac{\partial \ddot{x}_n}{\partial C} + C \frac{\partial \dot{x}_n}{\partial C} + TS_n \frac{\partial x_n}{\partial C} = -\dot{x}_n$$

$$M \frac{\partial \ddot{x}_n}{\partial K} + C \frac{\partial \dot{x}_n}{\partial K} + TS_n \frac{\partial x_n}{\partial K} = -P_n/K.$$

$$M \frac{\partial \ddot{x}_n}{\partial A} + C \frac{\partial \dot{x}_n}{\partial A} + TS_n \frac{\partial x_n}{\partial A} = TS_n \frac{P_n|P_n|^{R-1}}{K^R}$$

$$M \frac{\partial \ddot{x}_n}{\partial R} + C \frac{\partial \dot{x}_n}{\partial R} + TS_n \frac{\partial x_n}{\partial R} = TS_n \frac{P_n|P_n|^{R-1}}{K^R} A \ln \left| \frac{P_n}{K} \right|, \tag{15}$$

governing the sensitivities.

In Eqs. (15) the P_n at the n^{th} time step is known from the solution of Eq. (1), the "tangent stiffness" at the P_n is known from

$$TS_n = \left[\frac{1}{K} + \frac{AR}{K^R} |P_n|^{R-1} \right]^{-1} \tag{16}$$

A set of equations, comparable to Eqs. (15), for branch curves is derived from Eq. (3). The sensitivities, which are the dependent variables in Eqs. (15), are those corresponding to a particular point on the surface $J(\bar{g})$ so that the values of the parameters appearing in Eqs. (15) and Eq. (16) are the corrdinates of that point. Eqs. (15) are therefore a set of uncoupled, nonhomogeneous, differential equations with constant coefficients for time step "n" whose solution offers no particular problem.

3. ESTIMATION OF THE PARAMETERS

The program for estimating the parameters must be tested. It is first subjected to a numerical experiment, that is one using simulated data. The input is the acceleration time history of a recorded earthquake, but response data is generated by assigning values to each of the four parameters and obtaining the solution of the resulting equation. The program can be considered to be successful if the optimization algorithm, in a reasonable number of iterations, identifies the minumum point of the error function surface, as having the coordinates of the assigned values, when the search is begun at a variety of points in the remote terrain.

The first search used four seconds for the error function of a full earthquake duration of twelve seconds. The path converged on the minimum point, from each beginning point, in very few iterations (see Figure 1). The search was repeated using only one second exposure. The same set of parameters was found (the assigned set), but surprisingly, in even fewer iterations.

Even though the numerical experiments were useful, certain problems arose, using physical data, that we had not anticipated. The surface representing the cost function was "badly behaved" particularly in the neighborhood of the minimum. The surface was not convex in much of the region explored and it did not begin to become quadratic until the immediate neighborhood of the minimum was reach. Further the minimum was very close to a constraint. As a result, a large number of iterations were needed to identify the minimum (see Figure 2).

4. FUTURE STUDY

Even though the mathematical model of the single story steel frame, obtained by the identification method described, predicts a response close to the physical response of the structure (revealed in another paper at this conference) we feel that there are two major ways in which the estimation of the parameters could be improved.

The error function would be improved if the influence of the displacement was increased relative to the acceleration. This can easily be done by associating a weighting factor with the term representing the difference in displacements.

The second area of modification is the method of choosing the direction from one point to the next on the error function surface in the optimization algorithm. Due to the unusual nature of the terrian, we feel that it might be well, at points remote from the minimum, to choose the direction of the steepest descent, and to increase the influence of the A matrix on this direction as the minimum is approached, until, in the neighborhood of the minimum, the matrix assumes its full influence.

REFERENCES

(1) Popov, E. P., and Bertero, V. V., "Cyclic Loading of Steel Beams and Connections," Journal of the Structural Division, ASCE, Vol. 99, No. ST6, June 1973, pp. 1189-1204.

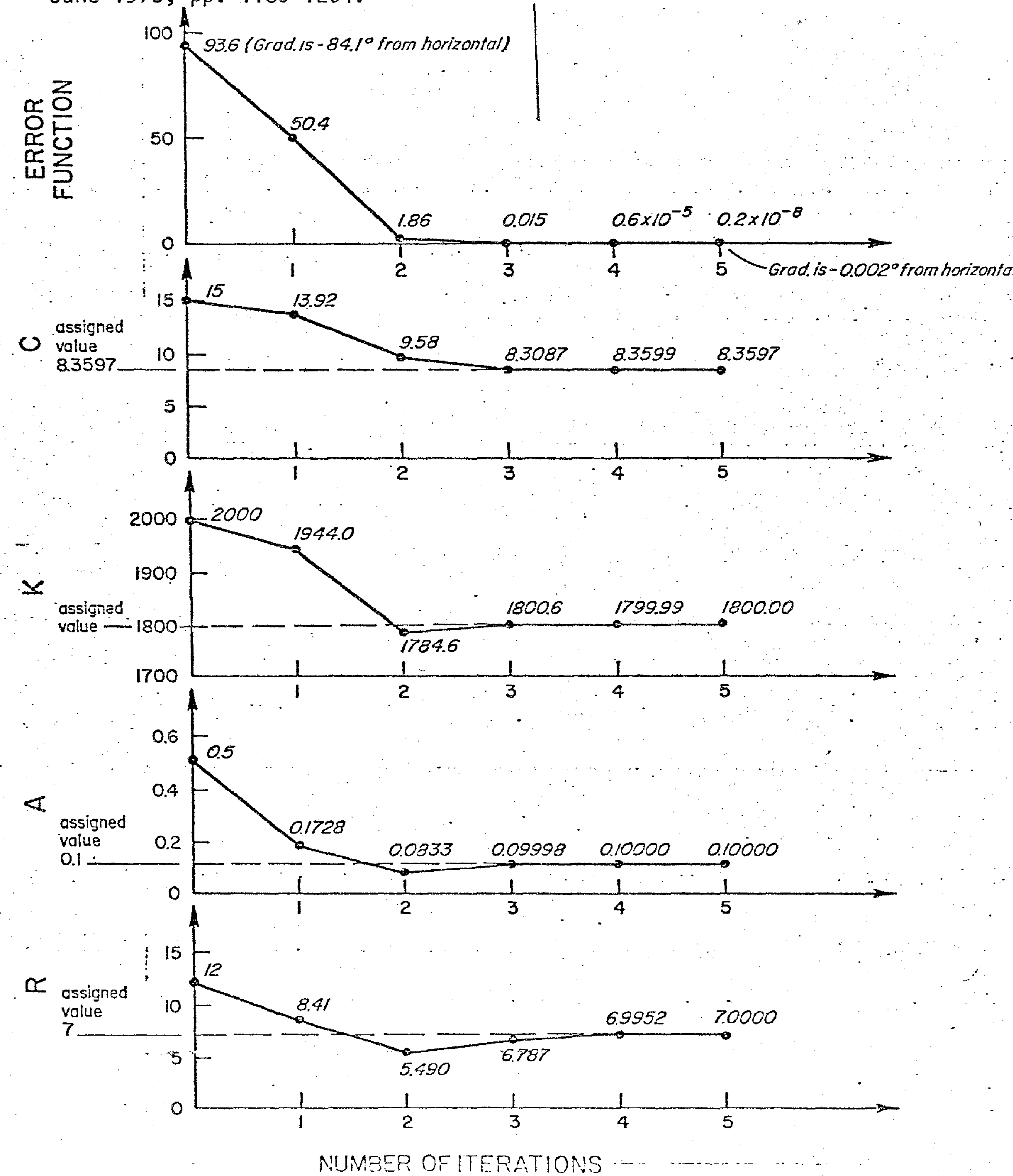

Figure 1. Simulated Data

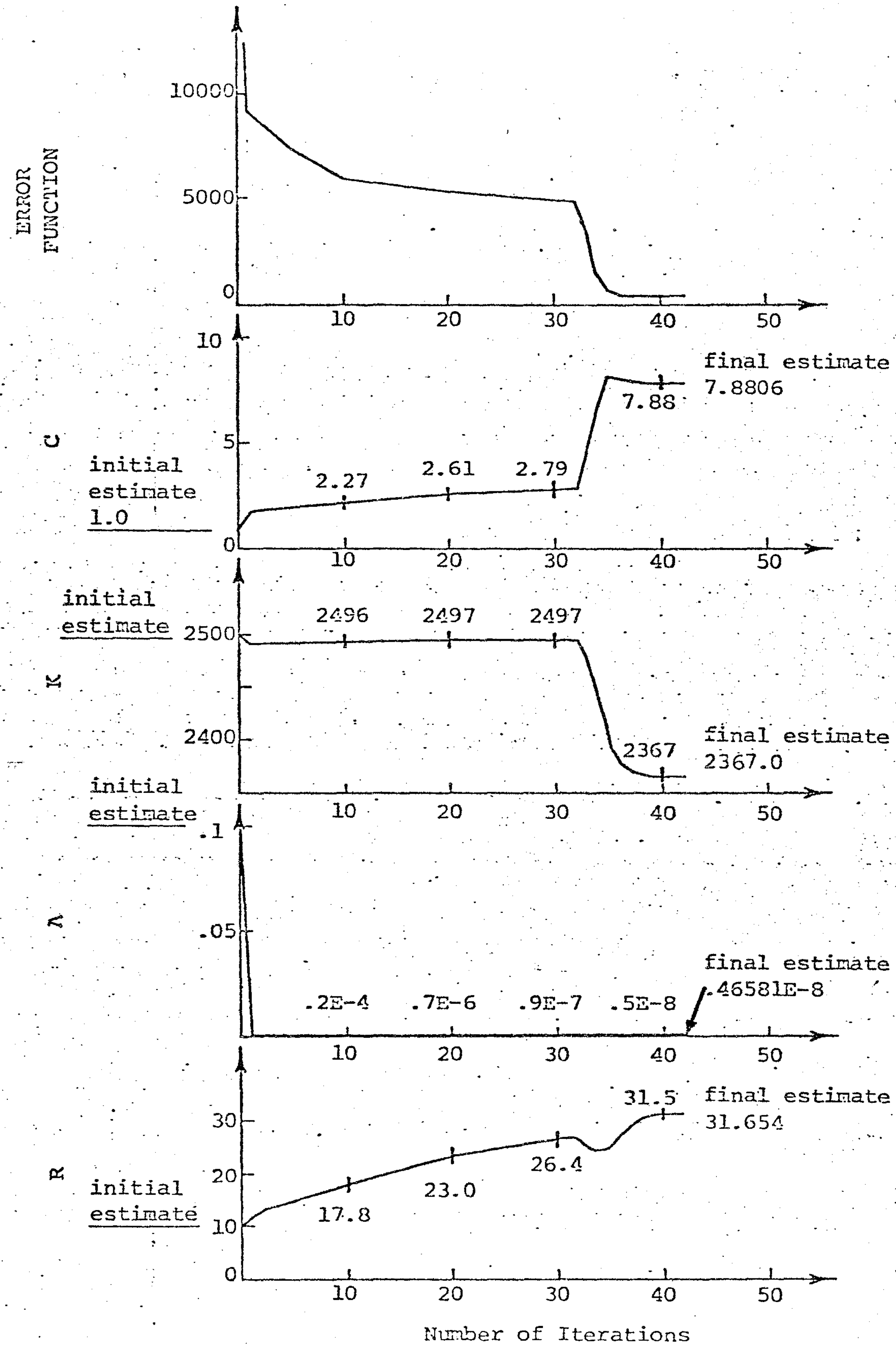

Figure 2. Actual Test Data