Seismic Vulnerability, Behavior and Design of Underground Piping Systems

Quasi-Static Analysis Formulation For Straight Buried Piping Systems

by

Leon Ru-Liang Wang

Sponsored by National Science Foundation

Research Applied to National Needs (RANN)

Grant No. ENV76-14884

Technical Memorandum (SVBDUPS Project) No. 3

July 1978

Department of Civil Engineering
Rensselaer Polytechnic Institute
Troy, New York 12181

50272-101

REPORT DOCUMENTATION PAGE	1. REPORT NO. NSF/RA-780337	2.	PB 290100

4. Title and Subtitle
Quasi-Static Analysis Formulation for Straight Buried Piping Systems, (Seismic Vulnerability, Behavior, and Design of Underground Piping Systems, Technical Memorandum No. 3)

5. Report Date
July 1978

6.

7. Author(s)
L.R. Wang

8. Performing Organization Rept. No.
3

9. Performing Organization Name and Address
Rensselaer Polytechnic Institute
Department of Civil Engineering
Troy, New York 12181

10. Project/Task/Work Unit No.

11. Contract(C) or Grant(G) No.
(C)
(G) ENV7614884

12. Sponsoring Organization Name and Address
Applied Science and Research Applications (ASRA)
National Science Foundation
1800 G Street, N.W.
Washington, DC 20550

13. Type of Report & Period Covered
Technical Memorandum

14.

15. Supplementary Notes

16. Abstract (Limit: 200 words)

Overall aims of this research are to develop a systematic way of assessing the adequacy and vulnerability of water/sewer distribution systems subjected to seismic loads and also to develop future design methodologies for water/sewer systems. This technical memorandum develops the quasi-static governing equilibrium equations for the axial response of buried pipelines subjected to earthquakes. The formulation is general and includes such parameters as variable elasticity, segment lengths and cross sections of pipes, variable joint stiffnesses, variable soil resistant characteristics and end conditions. Variations of seismic wave form, propagation velocity and delay time can be incorporated into the numerical procedure.

17. Document Analysis a. Descriptors

Design	Water pipes	Piping systems
Earthquakes	Water supply	Equilibrium methods
Earthquake resistant structures	Sewer pipes	Axial strain
Dynamic structural analysis	Hazards	

b. Identifiers/Open-Ended Terms

Equilibrium equations
Earthquake engineering
Buried pipelines

c. COSATI Field/Group

18. Availability Statement
NTIS.

19. Security Class (This Report)

20. Security Class (This Page)

22. Price
PC-A03/MF-A01

(See ANSI-Z39.18) See Instructions on Reverse OPTIONAL FORM 272 (4-77) (Formerly NTIS-35) Department of Commerce

TABLE OF CONTENTS

Preceding page blank

Department of Civil Engineering
Telephone — (518) 270-6360

Rensselaer Polytechnic Institute Troy, New York 12181

List of NSF SVBDUPS (Seismic, Vulnerability, Behavior and Design of Piping Systems) Project Technical Memoranda

No. 1 Leon Ru-Liang Wang
Discussion on Soil Restraint Against
Horizontal Motion of Pipe
Jan. 1978

No. 2 Leon Ru-Liang Wang and Warrent T. Lavery
Engineering Profile of Latham Water District,
Albany, New York
April 1978

No. 3 Leon Ru-Liang Wang
Quasi-Static Analysis Formulation For
Buried Straight Piping Systems
July 1978

ACKNOWLEDGEMENT

This is the third in a series of Technical Memoranda under the general title of 'Seismic Vulnerability, Behavior and Design of Underground Piping Systems' (SVBDUPS). A technical memorandum is written with somewhat limited objectives and scope as compared to a technical report.

The research has been sponsored by the Earthquake Engineering Program of NSF-RANN under grant No. ENV76-14884 in which Drs. S.C. Liu and William Hakala are the Program Managers. Dr. Leon Ru-Liang Wang is the Principal Investigator of this project. The overall aims of this research are to develop a systematic way of assessing the adequacy and vulnerability of water/sewer distribution systems subjected to seismic loads and to develop future design methodologies.

The author wishes to express his appreciation for the input and discussions from Dr. Michael O'Rourke, Senior Investigator and Messrs. Kwong M. Cheng, Richard R. Pikul and Eric S.L. Fok, Research Assistants on the project.

Appreciation also goes to the Advisory Panel which consists of Mr. Holly A. Cornell, Chairman of Board, CH2M Hill, Inc., Corvallis, Oregon; Mr. Warren T. Lavery, Superintendent of Latham Water District, Latham, N.Y.; Dr. Richard Parmelee, Professor of Civil Engineering, Northwestern University and Drs. Jose Roesset and Robert Whitman, Professors of Civil Engineering, M.I.T., for their constructive comments and suggestions.

The typing and proofreading of this report by Mrs. Jo Ann Grega is also appreciated.

Please note that although the project is sponsored by the National Science Foundation, any opinions, findings and conclusions or recommendations expressed in this publication are those of the author and do not necessarily reflect the view of the National Science Foundation.

Abstract

This technical memorandum develops the quasi-static governing equilibrium equations for the axial response of buried pipelines subjected to earthquakes.

The formulation is general and includes such parameters as variable elasticity, segment lengths and cross sections of pipes, variable joint stiffnesses, variable soil resistant characteristics and end conditions. Variations of seismic wave form, propagation velocity and delay time can be incorporated into the numerical procedure.

Introduction

The response of buried pipelines during seismic shaking has been found to be predominant in the axial direction of the pipeline[6,8,9,12,15,17]. Pipeline damage caused by longitudinal earthquake excitation has been observed to be a major mode of failure[3,4,5]. State of the art papers[7,20,21] on buried lifeline earthquake engineering have been published recently. The fundamental behavior of underground piping systems has been studied by the investigators at Weidlinger Associates[2] and at Rensselaer Polytechnic Institute[13,18]. To aid the design of buried pipelines, both the static displacement approach[11,16] and the dynamic interference response spectra approach[10] have been proposed.

The purpose of this technical memorandum is to develop a rigorous quasi-static analysis model for the response of buried pipelines subjected to earthquake motion in the axial direction to supplement the simplified evaluation reported earlier[18]. Since the dynamic effects on the response behavior of buried pipelines[6,8,12,17] have been found to be negligible, the inertia and damping terms in the dynamic equations of motion will be dropped. Because the input ground motion is a function of time, the response will also be a function of time. Thus, the analysis is called quasi-static.

Description of the General Model

A long buried piping system consisting of n-segments is shown in Fig. 1 where K_1, K_2, .. K_i, .. K_{n-1} are spring constants at joints between pipe segments; K_0, K_n are spring constants at the end supports; X_1, X_2, .. X_{2i-1}, X_{2i}, .. X_{2n-1}, X_{2n} are longitudinal displacements at the ends of pipe segments; X_{G1}, X_{G2}, .. X_{Gn-1} are the corresponding ground displacements at the segment intersections in the same direction as the pipeline axis; X_{G0} and X_{Gn} are the ground movements at the ends; L_1, L_2, .. L_i, .. L_n are pipe lengths; and k_1,

k_2, .. k_i, .. k_n are soil resistant spring constants per unit length along the pipe segments.

(1) Strain Energy in A Pipe Segment

Referring to Fig. 2, for a pipe segment subjected to two end forces and a linearly distributed soil resistance force, the displacement function within a segment is a cubical variation as:

$$\phi_i(x) = [(1 - \frac{x^3}{L_i^3}) \quad \frac{x^3}{L_i^3}] \begin{bmatrix} X_{2i-1} \\ X_{2-i} \end{bmatrix} \tag{1}$$

The strain function is:

$$\varepsilon_i(x) = \frac{d\phi_i(x)}{dx} = [\frac{-3x^2}{L_i} \quad \frac{3x^2}{L_i}] \begin{bmatrix} X_{2i-1} \\ X_{2i} \end{bmatrix} \tag{2}$$

and the stress function is:

$$\sigma_i(x) = E_i \; \varepsilon(x) = E_i[\frac{-3x^2}{L_i} \quad \frac{3x^2}{L_i}] \begin{bmatrix} X_{2i-1} \\ X_{2i} \end{bmatrix} \tag{3}$$

The strain energy within a pipe segment i

$$U^i_{pipe} = \frac{1}{2} \int_v \sigma_i^{\,t} \, \varepsilon_i dv = \frac{1}{2} \int_0^{L_i} \sigma_i^{\,t} \, \varepsilon_i \, A_i \, dx \tag{4}$$

where $\sigma_i^{\,t}$ is transpose of σ_i.

Substituting Eqns. (2) and (3) into Eqn. (4) and integrating, one obtains

$$U^i_{pipe} = \frac{1}{2} [X_{2i-1} \; X_{2i}] \begin{bmatrix} \frac{9E_iA_i}{5L_i} & \frac{-9E_iA_i}{5L_i} \\ \frac{-9E_iA_i}{5L_i} & \frac{9E_iA_i}{5L_i} \end{bmatrix} \begin{bmatrix} X_{2i-1} \\ X_{2i} \end{bmatrix} \tag{5}$$

The sum of energies of all segments in the system is:

$$U_{pipe}^{Total} = \sum_{i=1}^{n} U_{pipe}^{i}$$

$$= \frac{1}{2} \{X\}^t \, [K_{pipe}] \, \{X\} \tag{6}$$

where

$$\{X\}^t = \underset{1x2n}{[X_1, X_2 \,..\, X_3\, X_4 \,..\, X_{2i-1}\, X_{2i} \,..\, X_{2n-1}\, x_{2n}]} \tag{7}$$

and $[K_{pipe}]$ is a symmetrically tridiagonal matrix as shown below

$$[K_{pipe}] = \frac{9}{5} \underset{2nx2n}{\begin{bmatrix}
\frac{E_1A_1}{L_1} & \frac{-EA_1}{L_1} & & & & & & \\
\frac{-E_1A_1}{L_1} & \frac{E_1A_1}{L_1} & 0 & & & & & \\
& 0 & \frac{E_2A_2}{L_2} & \frac{-E_2A_2}{L_2} & & & & \\
& & \frac{-E_2A_2}{L_2} & \frac{E_2A_2}{L_2} & 0 & & & \\
& & & \bullet & & & & \\
& & & 0 & \frac{E_iA_i}{L_i} & \frac{-E_iA_i}{L_i} & & \\
& & & & \frac{-E_iA_i}{L_i} & \frac{E_iA_i}{L_i} & 0 & \\
& & & & & \bullet & & \\
& & & & & 0 & \frac{E_nA_n}{L_n} & \frac{-E_nA_n}{L_n} \\
& & & & & & \frac{-E_nA_n}{L_n} & \frac{E_nA_n}{L_n}
\end{bmatrix}} \tag{8}$$

(2) Strain Energy in A Joint Spring

Referring to Fig. 3, the spring elongation and spring force can be expressed:

$$D_i = [-1 \quad 1] \begin{bmatrix} X_{2i} \\ X_{2i+1} \end{bmatrix} \tag{9}$$

$$F_i = [-K_i \quad K_i] \begin{bmatrix} X_{2i} \\ X_{2i+1} \end{bmatrix} \tag{10}$$

The strain energy within a spring is

$$U^i_{spring} = \frac{1}{2} F_i^{\,t} D_i$$

$$= \frac{1}{2} [X_{2i} \quad X_{2i+1}] \begin{bmatrix} K_i & -K_i \\ -K_i & K_i \end{bmatrix} \begin{bmatrix} X_{2i} \\ X_{2i+1} \end{bmatrix} \tag{11}$$

Two special cases are developed for the end supports.

For the beginning support, $i = 0$, it is assumed that $X_0 = X_{G0}$. Thus,

$$U^{start}_{spring} = \frac{1}{2} [X_{G0} \quad X_1] \begin{bmatrix} K_0 & -K_0 \\ -K_0 & K_0 \end{bmatrix} \begin{bmatrix} X_{G0} \\ X_1 \end{bmatrix} \tag{12}$$

For the end support, $i = n$, it is assumed that $X_{2n+1} = X_{Gn}$ and

$$U^{end}_{spring} = \frac{1}{2} [X_{2n} \quad X_{Gn}] \begin{bmatrix} K_n & -K_n \\ -K_n & K_n \end{bmatrix} \begin{bmatrix} X_{2n} \\ X_{Gn} \end{bmatrix} \tag{13}$$

The total strain energy contained in all joint springs is as follows:

$$U^{Total}_{spring} = \sum_{i=0}^{n} U^{i}_{spring}$$

$$= \frac{1}{2} \{\bar{X}\}^t [\bar{K}_{spring}] \{\bar{X}\} \tag{14}$$

where

$$\{\bar{X}\}^t = [X_{G0} \,\vdots\, X_1 \; X_2 \; .. \; X_{2i-1} \; X_{2i} \; .. \; X_{2n-1} \; X_{2n} \,\vdots\, X_{Gn}]$$

$$1 \times 2(n+1)$$

and $[\bar{K}_{spring}]$ is a symmetrical tridiagonal matrix as follows:

$$[\bar{K}_{spring}] = \left[\begin{array}{c|cccccccccccc|c}
K_0 & -K_0 & & & & & & & & & & & & \\
\hline
-K_0 & K_0 & 0 & & & & & & & & & & & \\
 & 0 & K_1 & -K_1 & & & & & & & & & & \\
 & & -K_1 & K_1 & 0 & & & & & & & & & \\
 & & & & \ddots & & & & & & & & & \\
 & & & & 0 & K_i & -K_i & & & & & & & \\
 & & & & & -K_i & K_i & 0 & & & & & & \\
 & & & & & & & \ddots & & & & & & \\
 & & & & & & & 0 & K_{n-1} & -K_{n-1} & & & & \\
 & & & & & & & & -K_{n-1} & K_{n-1} & 0 & & & \\
 & & & & & & & & & 0 & K_n & & & -K_n \\
\hline
 & & & & & & & & & & -K_n & & & K_n
\end{array}\right] \tag{15}$$

$$(2n+2)(2n+2)$$

Using partitioning matrix manipulation, the total strain energy of all joint springs may be expressed as follows:

$$U^{Total}_{spring} = \frac{1}{2} K_0 \, X^2_{G0} + \frac{1}{2} K_n \, X^2_{Gn}$$

$$+ \{X\}^t \{\bar{K}_{ba}\} \; X_{G0} + \{X\}^t \{\bar{K}_{bc}\} \, X_{Gn}$$

$$+ \frac{1}{2} \{X\}^t [K_{spring}] \{X\} \tag{16}$$

where

$$\{\bar{K}_{ba}\}^t = \underset{1x2n}{[-\ K_0\ 0\ 0\ .\ .\ 0]} \tag{17}$$

$$\{\bar{K}_{bc}\}^t = \underset{1x2n}{[0\ .\ .\ .\ .\ 0\ -K_n]} \tag{18}$$

and $[K_{spring}]$ is a symmetrically tridiagonal matrix as:

$$[K_{spring}] = \underset{2nx2n}{\begin{bmatrix} K_0 & 0 & & & & & & & & & \\ 0 & K_1 & -K_1 & & & & & & & & \\ & -K_1 & K_1 & 0 & & & & & & & \\ & & 0 & K_2 & -K_2 & & & & & & \\ & & & -K_2 & K_2 & 0 & & & & & \\ & & & & 0 & K_i & -K_i & & & & \\ & & & & & -K_i & K_i & 0 & & & \\ & & & & & & 0 & K_{n-1} & -K_{n-1} & & \\ & & & & & & & -K_{n-1} & K_{n-1} & 0 & \\ & & & & & & & & 0 & K_n \end{bmatrix}} \tag{19}$$

(3) <u>Strain Energy of Soil Springs Along A Pipe Segment</u>

Referring to Fig. 4 for the soil resistant distribution, the soil spring displacement and resistance per unit length can be expressed:

$$y(x) = [(1 - \frac{x}{L_i}) \quad \frac{x}{L_i}] \begin{bmatrix} (X_{2i-1} - X_{Gi-1}) \\ (X_{2i} - X_{Gi}) \end{bmatrix}$$

$$= [(1 - \frac{x}{L_i}) \quad \frac{x}{L_i} \quad -(1 - \frac{x}{L_i}) \quad \frac{-x}{L_i}] \begin{bmatrix} X_{2i-1} \\ X_{2i} \\ X_{Gi-1} \\ X_{Gi} \end{bmatrix} \tag{20}$$

and

$$R(x) = \left[k_i\left(1 - \frac{x}{L_i}\right) \quad \frac{k_i x}{L_i} \quad -k_i\left(1 - \frac{x}{L_i}\right) \quad \frac{-k_i x}{L_i}\right] \begin{bmatrix} X_{2i-1} \\ X_{2i} \\ X_{Gi-1} \\ X_{Gi} \end{bmatrix} \tag{21}$$

The strain energy produced by the soil resistance within a pipe segment is:

$$U^i_{soil} = \frac{1}{2} \int_0^{L_i} R^t(x)\ y(x)\ dx \tag{22}$$

Substituting Eqns. (20) and (21) into Eqn. (22) and integrating, one obtains:

$$U^i_{soil} = \frac{1}{2}[X_{2i-1}\ X_{2i} \mid X_{Gi-1}\ X_{Gi}] \left[\begin{array}{cc|cc} \frac{k_iL_i}{3} & \frac{k_iL_i}{6} & \frac{-k_iL_i}{3} & \frac{-k_iL_i}{6} \\ \frac{k_iL_i}{6} & \frac{k_iL_i}{3} & \frac{-k_iL_i}{6} & \frac{-k_iL_i}{3} \\ \hline \frac{-k_iL_i}{3} & \frac{-k_iL_i}{6} & \frac{k_iL_i}{3} & \frac{k_iL_i}{6} \\ \frac{-k_iL_i}{6} & \frac{-k_iL_i}{3} & \frac{k_iL_i}{6} & \frac{k_iL_i}{3} \end{array}\right] \left[\begin{array}{c} X_{2i-1} \\ X_{2i} \\ \hline X_{Gi-1} \\ X_{Gi} \end{array}\right] \tag{23}$$

Using 2x2 partitioning matrices, Eqn. (23) can be written as follows:

$$\begin{aligned} U^i_{soil} = {} & \frac{1}{2}[X_{2i-1}\ X_{2i}] \begin{bmatrix} \frac{k_iL_i}{3} & \frac{k_iL_i}{6} \\ \frac{k_iL_i}{6} & \frac{k_iL_i}{3} \end{bmatrix} \begin{bmatrix} X_{2i-1} \\ X_{2i} \end{bmatrix} \\ & + \frac{1}{2}[X_{Gi-1}\ X_{Gi}] \begin{bmatrix} \frac{k_iL_i}{3} & \frac{k_iL_i}{6} \\ \frac{k_iL_i}{6} & \frac{k_iL_i}{3} \end{bmatrix} \begin{bmatrix} X_{Gi-1} \\ X_{Gi} \end{bmatrix} \\ & - [X_{2i-1}\ X_{2i}] \begin{bmatrix} \frac{k_iL_i}{3} & \frac{k_iL_i}{6} \\ \frac{k_iL_i}{6} & \frac{k_iL_i}{3} \end{bmatrix} \begin{bmatrix} X_{Gi-1} \\ X_{Gi} \end{bmatrix} \end{aligned} \tag{24}$$

The sum of total strain energy of entire soil system

$$U_{Soil}^{Total} = \sum_{i=1}^{n} U_{soil}^{i}$$

$$= \frac{1}{2} \{X\}^t [K_{soil}] \{X\} + \frac{1}{2} \{X_G\}^t [K_{soil}] \{X_G\} - \{X\}^t [K_{soil}] \{X_G\} \quad (25)$$

where

$$\{X_G\}^t = \underset{1x2n}{[X_{G0}\ X_{G1} \mid X_{G1}\ \ldots\ X_{Gi} \mid X_{Gi}\ \ldots\ X_{Gn-1} \mid X_{Gn-1}\ X_{Gn}]} \quad (26)$$

and $[K_{soil}]$ is a symmetrically tridiagonal matrix as follows:

$$[K_{soil}] = \underset{2nx2n}{\begin{bmatrix} \frac{k_1L_1}{3} & \frac{k_1L_1}{6} & & & & & & \\ \frac{k_1L_1}{6} & \frac{k_1L_1}{3} & 0 & & & & & \\ & 0 & \frac{k_2L_2}{3} & \frac{k_2L_2}{6} & & & & \\ & & \frac{k_2L_2}{6} & \frac{k_2L_2}{3} & 0 & & & \\ & & & \bullet & & & & \\ & & & 0 & \frac{k_iL_i}{3} & \frac{k_iL_i}{6} & & \\ & & & & \frac{k_iL_i}{6} & \frac{k_iL_i}{3} & 0 & \\ & & & & & \bullet & & \\ & & & & & 0 & \frac{k_nL_n}{3} & \frac{k_nL_n}{6} \\ & & & & & & \frac{k_nL_n}{6} & \frac{k_nL_n}{3} \end{bmatrix}} \quad (27)$$

(4) <u>Total Potential Energy of the Soil-Structure Interaction System</u>

The total potential energy of the buried piping system is the sum of strain energies of pipe segments, joint and soil resistant springs. Mathematically, it is expressed as:

$$U_{system}^{Total} = U_{pipe}^{Total} + U_{spring}^{Total} + U_{soil}^{Total} \tag{28}$$

Substituting Eqns. (6), (16) and (25) into Eqn. (28), one obtains the potential energy of the entire system as

$$\begin{aligned} U_{system}^{Total} &= \frac{1}{2} \{X\}^t [K_{pipe}] \{X\} \\ &+ \frac{1}{2} K_0 X_{G0}^2 + \frac{1}{2} K_n X_{Gn}^2 \\ &+ \{X\}^t \{\bar{K}_{ba}\} X_{G0} + \{X\}^t \{\bar{K}_{bc}\} X_{Gn} \\ &+ \frac{1}{2} \{X\}^t [K_{spring}] \{X\} \\ &+ \frac{1}{2} \{X\}^t [K_{soil}]\{X\} + \frac{1}{2} \{X_G\}^t [K_{soil}] \{X_G\} - \{X\}^t [K_{soil}] \{X_G\} \end{aligned} \tag{29}$$

(5) <u>Governing Equilibrium Equation of Buried Pipelines</u>

According to variational principle[1], the variation of total potential energy of an equilibrium system is equal to zero, i.e.

$$\frac{\partial U_{system}^{Total}}{\partial \{X\}} = 0$$

or

$$\frac{\partial U_{pipe}^{Total}}{\partial \{X\}} + \frac{\partial U_{spring}^{Total}}{\partial \{X\}} + \frac{\partial U_{soil}^{Total}}{\partial \{X\}} = 0 \tag{30}$$

Carrying through the derivatives[14] of U_{system}^{Total} shown Eqn. (29), one obtains the following equation of equilibrium of the system:

$$\begin{aligned} &[K_{pipe}] \{X\} + [K_{spring}] \{X\} + [K_{soil}] \{X\} \\ &\quad = [K_{soil}] \{X_G\} - \{\bar{K}_{ba}\} X_{G0} - \{\bar{K}_{bc}\} X_{Gn} \end{aligned} \tag{31}$$

After grouping from Eqn. (31), one obtains the following simplified equation:

$$\underset{2n\times 2n}{[K_{system}]} \; \underset{2n\times 1}{\{X\}} = \underset{2n\times 2n}{[\bar{K}_{soil}]} \; \underset{2n\times 1}{\{X_G\}} \tag{32}$$

where both $[K_{system}]$ and $[\bar{K}_{soil}]$ matrices are symmetrical tridiagonal matrices as follows:

$$[K_{system}] = \begin{bmatrix} (\frac{9E_1A_1}{5\ L_1} + \frac{k_1L_1}{3} + K_0) & (\frac{-9E_1A_1}{5\ L_1} + \frac{k_1L_1}{6}) & & & \\ (\frac{-9E_1A_1}{5L_1} + \frac{k_1L_1}{6}) & (\frac{9E_1A_1}{5\ L_1} + \frac{k_1L_1}{3} + K_1) & -K_1 & & \\ & -K_1 & (\frac{9E_2A_2}{5\ L_2} + \frac{k_2L_2}{3} + K_1) & (\frac{-9E_2A_2}{5\ L_2} + \frac{k_2L_2}{6}) & \\ & & \bullet & & \\ & K_{2i-1,2i-2} & K_{2i-1,2i-1} & K_{2i-1,2i} & \\ & & K_{2i,2i-1} & K_{2i,2i} & K_{2i,2i+1} \\ & & & & \bullet \\ & & & & \quad \bullet \end{bmatrix} \tag{33}$$

$$K^{system}_{2i-1,2i-1} = \frac{9E_iA_i}{5\ L_i} + \frac{k_iL_i}{3} + K_{i-1}\ ; \quad 1 \leq i \leq n \tag{34a}$$

$$K^{system}_{2i-1,2i} = K^{system}_{2i,2i-1} = -\frac{9E_iA_i}{5\ L_i} + \frac{k_iL_i}{6}; \quad 1 \leq i \leq n \tag{34b}$$

$$K^{system}_{2i,2i} = \frac{9E_iA_i}{5\ L_i} + \frac{k_iL_i}{3} + K_i; \quad 1 \leq i \leq n \tag{34c}$$

$$K^{system}_{2i,2i+1} = K^{system}_{2i+1,2i} = -K_i \quad 1 \leq i \leq n-1 \tag{34d}$$

and

$$
[\bar{K}_{soil}] = \begin{bmatrix}
(\frac{k_1L_1}{3} + K_0) & \frac{k_1L_1}{6} & & & & & & & & \\
\frac{k_1L_1}{6} & \frac{k_1L_1}{3} & 0 & & & & & & & \\
& 0 & \frac{k_2L_2}{3} & \frac{k_2L_2}{6} & & & & & & \\
& & \frac{k_2L_2}{6} & \frac{k_2L_2}{3} & 0 & & & & & \\
& & & & \bullet & & & & & \\
& & & & 0 & \frac{k_iL_i}{3} & \frac{k_iL_i}{6} & & & \\
& & & & & \frac{k_iL_i}{6} & \frac{k_iL_i}{3} & 0 & & \\
& & & & & & & \bullet & & \\
& & & & & & & 0 & \frac{k_nL_n}{3} & \frac{k_nL_n}{6} \\
& & & & & & & & \frac{k_nL_n}{6} & (\frac{k_nL_n}{3} + K_n)
\end{bmatrix}_{2n\mathrm{x}2n} \tag{35}
$$

In general terms, the elements of $[\bar{K}_{soil}]$ are defined as

$$\bar{K}^{soil}_{2i-1,2i-1} = \bar{K}^{soil}_{2i,2i} = \frac{k_iL_i}{3} \; ; \quad 1 \leq i \leq n \tag{36a}$$

$$\bar{K}^{soil}_{2i,2i+1} = \bar{K}^{soil}_{2i+1,2i} = 0 \; ; \quad 1 \leq i \leq n-1 \tag{36b}$$

$$\bar{K}^{soil}_{2i-1,2i} = \bar{K}^{soil}_{2i,2i-1} = \frac{k_iL_i}{6} \; ; \quad 1 \leq i \leq n \tag{36c}$$

except for the two ends which are as follows:

$$\bar{K}^{soil}_{11} = \frac{k_1L_1}{3} + K_0 \tag{36d}$$

$$\bar{K}^{soil}_{2n,2n} = \frac{k_nL_n}{3} + K_n \tag{36e}$$

(6) Ground Motion Input

The solution of pipe motion $\{X\}$ shown in Eqn. (32) depends on the inputs of the ground motion $\{X_G\}$. Since $\{X_G\}$ is a function of time, the solution of $\{X\}$ is also a function of time. Thus, the method proposed is called the 'Quasi-static' model.

Assuming that the wave form of the traveling seismic excitation remains constant over the entire length of the pipeline which is divided into n-segments, the inputs of the time-space varying ground motions starting from the first support are:

$$X_{G0} = \begin{cases} 0 & t < 0 \\ \Delta_{max}\, h(t) & t \geq 0 \end{cases} \tag{37a}$$

$$X_{G1} = \begin{cases} 0 & t-\Delta T_1 < 0 \\ \Delta_{max}\, h(t-\Delta T_1) & t-\Delta T_1 \geq 0 \end{cases} \tag{37b}$$

$$X_{Gi} = \begin{cases} 0 & t-\Delta T_i < 0 \\ \Delta_{max}\, h(t-\Delta T_i) & t-\Delta T_i \geq 0 \end{cases} \tag{37c}$$

where Δ_{max} is maximum ground displacement input in a record; h(t) is the displacement time function; ΔT_i is the delay time of seismic wave traveling from the first support to the end face of i^{th} pipe segment considered, which can be expressed as

$$\Delta T_i = \sum_{j=1}^{i} L_j / C_j \tag{38}$$

and C_j is the traveling wave velocity of soil surrounding the pipe segment j.

Models For Special Cases

For wider applications of the quasi-static model to analyze buried lifeline systems subjected to seismic excitations, two special case models; one for a long buried pipeline system and the other for a rigid pipe segment system will be developed.

(1) A Buried Continuous Pipeline System

For a long buried continuous pipeline system, there will be no joint springs. In this case, $X_{2i} = X_{2i+1} = \chi_{i+1}$ is observed. In a n-segment pipeline system, there will be n+1 degrees of freedom as shown below:

$$\{X^t_{cont}\} = \underset{1x(n+1)}{[\chi_1, \chi_2, \cdot\cdot\ \chi_i\ \cdot\cdot\ \chi_n\ \chi_{n+1}]} \tag{39}$$

and

$$\{X^t_{G,\ cont}\} = \underset{1x(n+1)}{[X_{G0}, X_{G1}, \cdot\cdot\ X_{Gi}, \cdot\cdot\ X_{Gn-1}, X_{Gn}]} \tag{40}$$

Observing the $2i^{th}$ and $(2i+1)^{th}$ rows of Eqn. (32) as:

$$\left(\frac{-9E_iA_i}{5L_i} + \frac{k_iL_i}{6}\right) X_{2i-1} + \left(\frac{9E_iA_i}{5\ L_i} + \frac{k_iL_i}{3} + K_i\right) X_{2i} - K_i\ X_{2i+1}$$

$$= \frac{k_iL_i}{6} X_{Gi-1} + \frac{k_iL_i}{3} X_{Gi} \tag{41}$$

and

$$-K_i\ X_{2i} + \left(\frac{9E_{i+1}A_{i+1}}{5\ L_{i+1}} + \frac{i_{i+1}L_{i+1}}{3} + K_i\right)\ X_{2i+1} + \left(\frac{-9E_{i+1}A_{i+1}}{5\ \ L_{i+1}} + \frac{k_{i+1}L_{i+1}}{6}\right) X_{2i+2}$$

$$= \frac{k_{i+1}L_{i+1}}{3} X_{Gi} \qquad \frac{k_{i+1}L_{i+1}}{6} X_{Gi+1} \tag{42}$$

Letting $X_{2i} = X_{2i+1} = \chi_{i+1}$; $X_{2i-1} = \chi_i$; $X_{2i+2} = \chi_{i+2}$ and adding Eqns. (41) and (42), one obtains the governing equilibrium equation for a long buried continuous pipeline system as:

$$\left(\frac{-9E_iA_i}{5\,L_i} + \frac{k_iL_i}{6}\right) X_i + \left(\frac{9E_iA_i}{5\,L_i} + \frac{9E_{i+1}A_{i+1}}{5\,L_{i+1}} + \frac{k_iL_i}{3} + \frac{k_{i+1}L_{i+1}}{3}\right) X_{i+1}$$

$$+ \left(\frac{-9E_{i+1}A_{i+1}}{5\,L_{i+1}} + \frac{k_iL_{i+1}}{6}\right) X_{i+2}$$

$$= \frac{k_iL_i}{6} X_{Gi-1} + \frac{1}{3}(k_iL_i + k_{i+1}L_{i+1}) X_{Gi} + \frac{k_{i+1}L_{i+1}}{6} X_{Gi+1} \qquad (43)$$

Note that the above equation is good for any $1 \leq i \leq n-1$ values. The equilibrium of the two end supports are written as follows:

For the first row, it is

$$\left(\frac{9E_1A_1}{5\,L_1} + \frac{k_1L_1}{3} + K_0\right) X_1 + \left(\frac{-9E_1A_1}{5\,L_1} + \frac{k_1L_1}{6}\right) X_2$$

$$= \left(\frac{k_1L_1}{3} + K_0\right) X_{G0} + \frac{k_1L_1}{6} X_{G1} \qquad (44)$$

and the last row

$$\left(\frac{-9E_nA_n}{5L_n} + \frac{k_nL_n}{6}\right) X_n + \left(\frac{9E_nA_n}{5\,L_n} + \frac{k_nL_n}{3} + K_n\right) X_{n+1}$$

$$= \frac{k_nL_n}{6} X_{Gn-1} + \left(\frac{k_nL_n}{3} + K_n\right) X_{Gn} \qquad (45)$$

For free end supports, K_0 in Eqn. (44) and K_n in Eqn. (45) should be removed.

For fixed end support condition, $X_1 = X_{G0}$ and $X_{n+1} = X_{Gn}$, one can write (n-1), equations from Eqn. (43). Eqns. (44) and (45) become unnecessary in this case.

(2) A Rigid Pipe Segmented System

For a rigid pipe segment system, it is observed that $X_1 = X_2$, .. $X_{2i-1} = X_{2i}$. For an n-segment pipeline system, there will be n degrees of freedom as shown below:

$$\{\chi^t_{rigid}\} = [\chi_1, \chi_2, .. \chi_i .. \chi_n] \tag{46}$$

but there will be n+1 ground displacements as

$$\{X^t_{G,rigid}\} = [X_{G0}, X_{G1}, .. X_{GI}, .. X_{Gn-1}, X_{Gn}] \tag{47}$$

Observing $(2i-1)^{th}$ and $2i^{th}$ rows of Eqns. (32) as

$$-K_{i-1} X_{2i-2} + (\frac{9E_iA_i}{5\ L_i} + \frac{k_iL_i}{3} + K_{i-1}) X_{2i-1} + (\frac{-9E_iA_i}{5\ L_i} + \frac{k_iL_i}{6}) X_{2i}$$

$$= \frac{k_iL_i}{3} X_{Gi-1} + \frac{k_iL_i}{6} X_{Gi} \tag{48}$$

and

$$(\frac{-9E_iA_i}{5\ L_i} + \frac{k_iL_i}{6}) X_{2i-1} + (\frac{9E_iA_i}{5\ L_i} + \frac{k_iL_i}{3} + K_i) X_{2i} - K_i X_{2i+1}$$

$$= \frac{k_iL_i}{6} X_{Gi-1} + \frac{k_iL_i}{3} X_{Gi} \tag{49}$$

Letting $X_{2i-1} = X_{2i} = \chi_i$; $X_{2i-2} = \chi_{i-1}$; $X_{2i+1} = \chi_{i+1}$ and adding Eqns. (48) and (49), one obtains the equilibrium equation for a rigid pipe segment system as:

$$-K_{i-1} \chi_{i-1} + (k_iL_i + K_{i-1} + K_i) \chi_i - K_I \chi_{i+1}$$

$$= \frac{1}{2} k_iL_i X_{Gi-1} + \frac{1}{2} k_iL_i X_{Gi} \tag{50}$$

Note that the above equation is good for any $2 \leq i \leq n-1$ values.

For $i = 1$, $X_0 = X_{G0}$, the first row of the equilibrium equation is

$$(k_1L_1 + K_0 + K_1) \chi_1 - K_1 \chi_2$$

$$= (K_0 + \frac{1}{2} k_1L_1) X_{G0} + \frac{1}{2} k_1L_1X_{G1} \tag{51}$$

For $i = n$, $X_{n+1} = X_{Gn}$, the last row of the equilibrium equation is

$$-K_{n-1} X_{n-1} + (k_n L_n + K_{n-1} + K_n) X_n$$

$$= \frac{1}{2} k_n L_n X_{Gn-1} + (\frac{1}{2} k_n L_n + K_n) X_{Gn} \qquad (52)$$

Discussions

Note that the quasi-static analysis model shown in Eqn. (32) can be solved easily since the equation is basically a static one. If seismic inputs $X_{G1}(t_1)$, $X_{G2}(t_2)$ ... are given, the response of pipe segments $X_1(t)$, $X_2(t)$ ... are calculated numerically. With this numerical procedure, various wave forms of input and time lag can be incorporated without difficulty as long as we can define the ground displacement variation in time and in space. Furthermore, in order to evaluate the system including the possibility of soil resistance failure surrounding the pipe, an idealized elasto-plastic soil resistance curve as shown in Fig. 5 can also be easily incorporated.

After solving for displacements, the joint spring force F_i within the elastic soil resistance range can be found as:

$$F_i^{start} = K_{i-1}(X_{2i-1} - X_{2i-2}); \; F_i^{end} = K_i(X_{2i+1} - X_{2i}) \qquad (53)$$

The axial stress, σ_i, and strains, ε_i, in pipe i either in tension or compression are then computed:

$$\sigma_i^{start} = F_i^{start}/A_i; \; \sigma_i^{end} = F_i^{end}/A_i \qquad (54a)$$

$$\varepsilon_i^{start} = F_i^{start}/E_i A_i; \; \varepsilon_i^{end} = F_i^{end}/E_i A_i \qquad (54b)$$

These stresses and strains can be used to evaluate the vulnerability/serviceability of the pipeline segments during an earthquake using a strength, ductility or buckling criteria[19].

The failure possibility of joint i, D_i, by pull-out or crushing for instance is indicated by

$$D_i = F_i/K_i = X_{2i+1} - X_{2i} \tag{55}$$

Summary

This technical memorandum has developed all the necessary equations for the quasi-static analysis of buried pipelines in axial motion. The formulation is very general involving the following parameters:

- Pipe segments variable in either length or cross sectional area;
- Variable joint spring stiffnesses;
- Variable end conditions;
- Variable soil spring constants and with soil strength yield possibilities;
- Variable time delay of traveling waves

Note that a variation of soil properties may be taken into account by the soil stiffness and/or traveling wave delay time. Since the solution will be performed using a numerical procedure, the variation of wave form can be easily taken into account.

With the above development, a computer program can be written to include all parameters for the general quasi-static seismic analysis of buried pipelines.

References

1. Crandall, S.J.
   Engineering Analysis
   McGraw-Hill Book Co., 1956

2. Isenberg, J., Weidlinger, P., Wright, J.P., and Baron, M.L.
   Underground Pipelines in A Seismic Environment
   Proceedings, Specialty Conference on the Current State of Knowledge of Lifeline Earthquake Engineering, August 30-31, 1977, U.C.L.A., T.C.L.E.E., American Society of Civil Engineers, pp. 267-281

3. Kachadoorian, R.
   Earthquake: Correlation Between Pipeline Damage and Geologic Environment
   Journal of AWWA, March 1976, pp. 165-167

4. Katayama, T., Kubo, K. and Sato, N.
   Earthquake Damage to Water and Gas Distribution Systems
   Proc. of U.S. National Conf. on Earthquake Engr., Ann Arbor, Mich., EERI, 1975, pp. 396-405

5. King, P.V. and Betz, J.M.
   Earthquake Damage to a Sewer System
   Journ. of Water Pollution Control Federation (WPCF), May 1972, pp. 859-867

6. Kubo, K.
   Behavior of Underground Waterpipes During An Earthquake
   Proc. of 5th World Conference (1973) on Earthquake Engineering, IAEE, Rome, 1974, pp. 569-578

7. Kubo, K., Katayama, T. and Ohashi, A.
   Present State of Lifeline Earthquake Engineering in Japan
   Proc. of ASCE Current State of Knowledge of Lifeline Earthquake Engineering Conference, Los Angeles, Aug. 1977, pp. 118-133

8. Kuribayashi, E., Iwasaki, T. and Kawashima, K.
   Dynamic Behavior of A Subsurface Tubular Structure
   Bulletin of the New Zealand National Society for Earthquake Engineering, Vol. 7, No. 4, Dec. 1974, pp. 200-209

9. Nakayama, S., Kiyomiya, O. and Tsuchida, H.
   Observation of Dynamic Behavior of Kinuura Submerged Tunnel During Earthquakes
   Preprint, 9th Joint Meeting US-JAPAN on Wind and Seismic Effects, Tokyo, May 1977

10. Nelson, I. and Weidlinger, P.
    Development of Interference Response Spectra for Lifeline Seismic Analysis
    Report IR-2, Weidlinger Associates, July 1977

11. Newmark, N.M. and Hall, W.J.
    Pipeline Design to Resist Large Fault Displacement
    Proc. of U.S. National Conf. on Earthquake Engr., Ann Arbor, Mich., EERI, 1975, pp. 416-425

12. Okamoto, S. and Tamura, C.
Behavior of Subaqueous Tunnel During Earthquakes
Int. Journ. of Earthquake Engr. and Struct. Dynamics, John Wiley & Sons, Ltd., Vol. 1, No. 3, Jan.-Mar. 1973, pp. 253-266

13. O'Rourke, M. and Wang, L.R.L.
Earthquake Response of Buried Pipeline
Technical Report (SVBDUPS Project) No. 4, Dept. of Civil Engineering, R.P.I., March 1978
Also in Proceedings of ASCE Geotechnical Division Specialty Conference on Earthquake Engineering and Soil Dynamics, Pasadena, Calif., June 1978

14. Pipes, L.A.
<u>Matrix Methods for Engineering</u>
Prentice-Hall, 1963

15. Sakurai, A. and Takahashi, T.
Dynamic Stresses of Underground Pipelines During Earthquakes
Proc. of 4th World Conference on Earthquake Engineering, Chilean Asso. on Seismology and Earthquake Engineering, Santiago, 1969, pp. 811-895

16. Tamura, C.
Design of Underground Structures by Considering Ground Displacement During Earthquakes
Proc. of US-JAPAN Seminar on Earthquake Engineering Research With Emphasis on Lifeline Systems, Tokyo, Nov. 1976, pp. 417-434

17. Tamura, C., Okamoto, S. and Hamada, M.
Dynamic Behavior of A Submerged Tunnel During Earthquakes
Report of the Institute of Industrial Science, Univ. of Tokyo, Vol. 24, No. 5, Serial No. 154, March 1975

18. Wang, L.R.L. and Cheng, K.M.
Seismic Response Behavior of Buried Pipelines
Technical Report (SVBDUPS Project) No. 5, Dept. of Civil Engineering, R.P.I., June 1978
Also to be presented at ASME Winter Meeting, San Francisco, CA, December 1978

19 Wang, L.R.L. and Fung, R.C.Y.
Seismic Design Criteria for Buried Pipelines
Technical Report (SVBDUPS Project), Dept. of Civil Engineering, R.P.I., in progress
Accepted for presentation at ASCE Pipeline Division Specialty Conference, New Orleans, LA, Jan. 1979

20. Wang, L.R.L. and O'Rourke, M.J.
State of the Art of Buried Lifeline Earthquake Engineering
Technical Report (SVBDUPS Project) No. 1, Dept. of Civil
Engineering, R.P.I., Jan. 1977
Also presented and published in Proc. of Specialty Conference
on The Current State of Knowledge of Lifeline Engineering,
Los Angeles, Calif., Aug. 1977, pp. 252-266

21. Wang, L.R.L. and O'Rourke, M.J.
An Overview of Buried Lifeline Earthquake Engineering
Technical Report (SVBDUPS Project) No. 1A, Dept. of Civil
Engineering, R.P.I., Jan. 1978
Also to be discussed at ASCE Geotechnical Engineering Division
Specialty Conference on Earthquake Engineering and Soil Dynamics,
Pasadena, Calif., June 1978 and accepted for publication in ASCE
Journal of Technical Councils

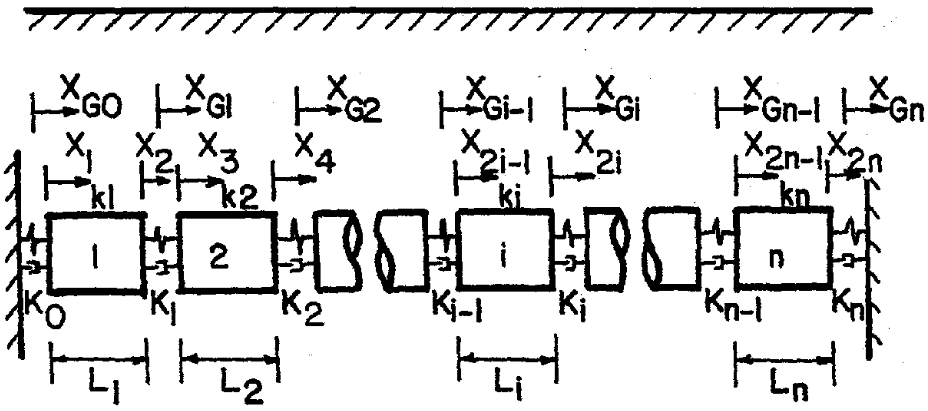

FIG. 1 A BURIED LONG PIPING SYSTEM

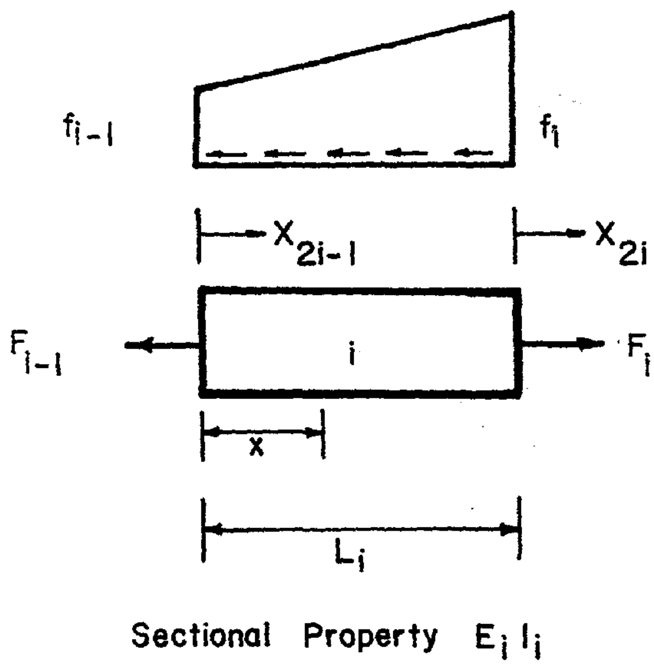

FIG. 2 A PIPE SEGMENT WITH LINEAR
SURFACE RESISTANCE FORCE

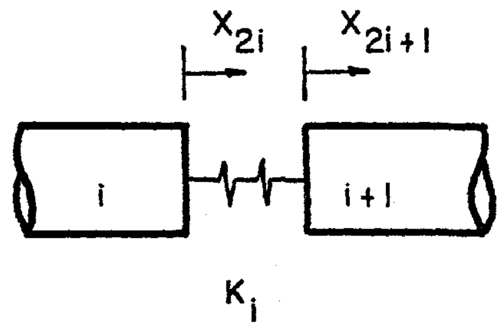

FIG. 3 A JOINT SPRING

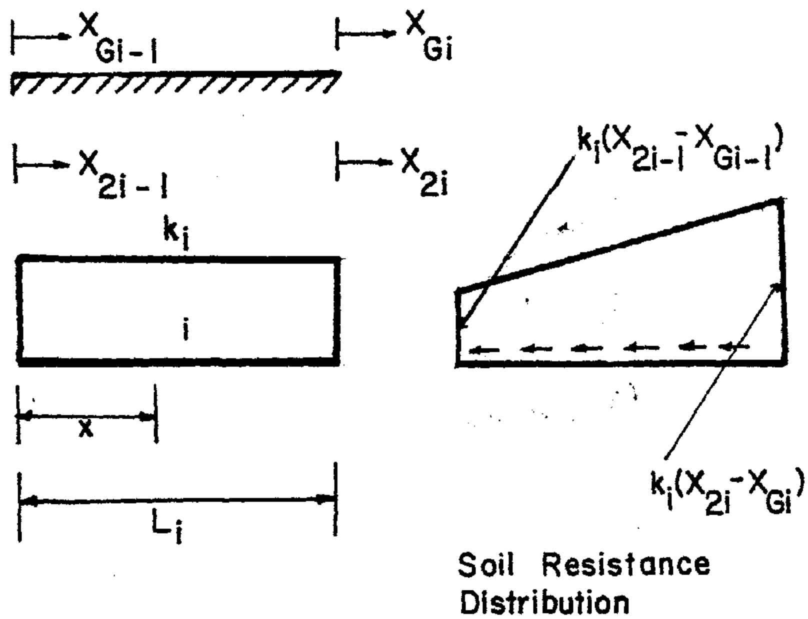

FIG. 4 SOIL RESISTANT DISTRIBUTION ALONG A PIPE SEGMENT

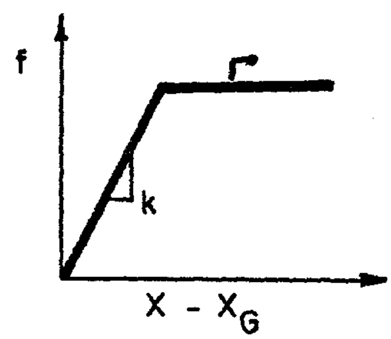

FIG. 5 SOIL RESISTANT CHARACTERISTICS