EARTHQUAKE ENGINEERING RESEARCH CENTER

EARTHQUAKE INDUCED
DEFORMATIONS OF EARTH DAMS

by
NORMAN SERFF
H. BOLTON SEED
F. I. MAKDISI
C.-Y. CHANG

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the National Science Foundation

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UNIVERSITY OF CALIFORNIA
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The calculated deformations are in reasonable agreement with the displacements measured after the earthquake, and the calculated stresses and strains explain some of the observed effects of the earthquake on the dam, such as cracking in the outlet conduit and the development of slide scarps on the upstream slope.

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College of Engineering
University of California
Berkeley, California
Abstract

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Chapter 1

Earthquake Induced Deformations of Earth Dams

Introduction

A possible failure mechanism for an earth dam during an earthquake is the loss of freeboard, and the subsequent overtopping of the dam, caused either by slope failure or settlement of the crest due to strains induced in the dam by seismic inertia forces. Sophisticated methods for assessing the stability of embankment slopes are now available, having progressed from the pseudo-static approach to the currently available comprehensive dynamic analysis which incorporates the varying bedrock acceleration during the earthquake, the change in material properties with the level of strain induced by the earthquake inertia forces, and laboratory tests to determine the resistance of the embankment soils to the cyclic stresses induced by the earthquake motions (Seed et al., 1969, 1973). However, the information gained from such analyses to date has provided only an approximate assessment of the deformations occurring due to seismic force applications (Seed et al., 1973), and while this is all that is required in many cases, it would seem desirable that a general method to predict earthquake-induced deformations be made available to the designer of earth dams.

A comprehensive approach to the determination of earthquake induced deformation of an earth dam must include an analysis of both the initial stresses acting throughout the dam and the superimposed cyclic stresses due to the earthquake, together with a comprehensive program of laboratory testing to determine the strains induced in the soil by the superimposed
cyclic stresses. Finally, a system of integrating the strains induced in
individual elements to predict the deformed shape of the dam is required.

The initial and simplest approach to this problem was that used by
Seed et al. (1973) which simply involved determining an overall average shear
strain likely to develop in an extensive zone of an embankment. A second
approach was presented by Lee (1974) and applied to the analysis of the
deformations of five dams. It involved the concept that seismic deformation
of a dam is due to softening of the soil by seismic shaking and the
resultant settling of the dam to a new condition compatible with the changed
stress-strain properties of the embankment soils.

A different approach is presented in this report. The method
integrates the element strains, calculated by the approach advocated by
Seed et al. (1973), in a finite element formulation to calculate the
deforption of an earth dam due to seismic forces. The method is used to
calculate the deformation of the Upper San Fernando dam during the earth­
quake of February 9, 1971, and shown to give results in reasonable agree­
ment with the measured displacements.

Previous Studies

The earliest analytical technique used to study the dynamic
response of earth dams to earthquake ground motion was the vertical shear
beam method (Mononobe, Takata & Matamura, 1936; Ambraseys, 1960).
Limitations associated with the technique restrict the analysis to dams
which can be idealized as homogeneous structures; furthermore only shear
mode response is determined, and the effect of the vertical component of
the ground motion cannot be included. However, as this analysis considers
the dam to be a deformable body, it makes possible the determination of the
manner in which the seismic coefficient (representing the earthquake-induced inertia forces) varies with height within the dam as well as with time.

The most versatile analytical tool for dynamic response analysis currently available is the finite element method. It was first applied to the study of the dynamic response of earth dams by Clough & Chopra (1966) and of earth banks by Idriss & Seed (1967). These first studies incorporated a linearly elastic medium with uniform viscous damping. Later modifications of this approach incorporated an equivalent linear analysis, where the shear modulus of the soil and the damping are strain-dependent, and used an iterative procedure to achieve a strain-compatible result (Seed et al., 1973).

Methods of analysis of the deformation of a dam embankment under a combination of sustained and cyclic stresses can be divided into two categories—effective stress analyses or total stress analyses.

In an effective stress analysis the deformation-controlling characteristics of the soil are determined by a suitable program of laboratory tests. These characteristics include the pore-water pressure generation coefficients, the yield stress and the rate of plastic deformation at stresses above the yield stress, and their variation during the earthquake. Incorporating these results in appropriate analytical analysis, the magnitude of the deformation is assessed.

In a total stress analysis, the initial and superimposed cyclic stresses are determined analytically. In the laboratory, samples of the soil in each zone of the dam are consolidated under the initial stress conditions of corresponding elements in the dam, then subjected to uniform cyclic stress histories equivalent to those determined from the response analysis, and the resulting deformations noted.
With either an effective or a total stress analysis, it is necessary to integrate the individual element deformations to obtain the overall deformation of the dam. The results should be the same, whichever method is used, provided the method is correctly applied.

Until recently, methods used by most designers to determine the seismic stability of earth dams employed a limiting equilibrium concept, where the result of the analysis was expressed as a single number, such as a factor of safety against total failure. The effect of the earthquake was simulated by a static force acting on a potential sliding block to represent inertia forces in the dam, and a conventional stability analysis, such as the method of slices, was used to determine the stability of the dam. The procedure was similar to the static stability analysis, though the critical sliding surface was usually different for the two cases. The dam was determined to be stable under the specified loading conditions if the factor of safety was greater than unity. Otherwise, failure was considered total, and no estimate of the actual displacement could be made. The force on the sliding block, which represented the inertia forces, was expressed as a percentage of the weight of the block, and was characterized by a seismic coefficient. The seismic coefficient, which was related to the maximum ground acceleration which could be expected at the site, varied between about 0.05 and 0.15.

This method of analysis assumes that the seismic force on the dam acts in one direction for an infinite time, whereas the actual inertia forces reverse after a short period, approximately 0.25 to 0.5 seconds for most earthquakes, with possibly only minor slumping occurring before the forces reverse, even though the factor of safety may fall below unity for part of the cycle. However, this slumping, cumulative over a number of
strong ground motion cycles, could culminate in sufficient loss of
freeboard resulting in overtopping of the dam, even though no general slope
failure occurs. Thus the deformation of the dam will depend on the duration,
as well as the severity, of the ground motion. The time factor cannot be
incorporated into limiting equilibrium methods of analysis.

An early attempt to rationalize the pseudo-static approach of applying
a static force to simulate earthquake loading was made by Seed & Martin (1966).
As a knowledge of the variation of the inertia forces during the earthquake,
and the frequency of this variation, would be of more significance to the
designer than an equivalent static force, a method to determine these for a
given design earthquake was presented.

It was assumed that the random variation of the seismic coefficient
could be expressed as a number of uniform cycles at a given frequency.
For dams of uniform construction, design curves were presented which gave
the characteristics of this uniformly varying seismic coefficient, as a
function of the fundamental period of the dam, using the El Centro record
as design earthquake.

The technique used to calculate the dynamic response of the dam was
the vertical shear beam approach, where the dam is considered as a
triangular wedge with linear visco-elastic response characteristics, i.e.,
the response is controlled by shearing between horizontal slices and the
shear stress along any horizontal surface is uniform. This assumption
produces shear stresses along a horizontal plane that are an average value,
the actual values being higher near the center of the dam and lower near
the slopes. The material properties of the dam—-the shear modulus, damping
and density—were assumed constant throughout. Although the method does not
allow for absorption of energy due to plastic deformation, this could be
simulated by an increase in the damping ratio of the materials.

A theoretical solution for the fully recoverable horizontal displacement with time was programmed and used to determine the displacement of each element in the dam; from the displacements the shear strain and, subsequently, the shearing force records are calculated.

The calculation of the varying seismic coefficient is simplified by assuming that the sliding mass is a triangular wedge with a horizontal base. The time-varying record of the average value of the seismic coefficient acting over the height of the wedge, for the applied earthquake, is calculated by determining the varying shear force acting on the base of the wedge, then dividing by the mass of the wedge to give the average horizontal acceleration which would produce the same shearing force.

Since laboratory tests are usually performed using uniform stress cycles, the random variation with time of the seismic coefficient thus calculated was represented by an equivalent number of constant amplitude cycles, an equivalent maximum seismic coefficient, and a corresponding predominant period.

It was found that the equivalent seismic coefficient increases as potential sliding wedges at higher elevations within the dam are considered, and decreases, for any given section of the dam, as the height of the dam increases. In the earlier pseudo-static approach, the seismic coefficient was usually taken to be constant for all dams in a given area, and throughout the height of any embankment. Only the horizontal component of the ground acceleration is used in the analysis, a restriction of the shear beam approach, but the effect of vertical acceleration could be included by inclining the direction of the resultant force acting on the sliding mass. This method of approach was subsequently extended to potential sliding masses with other shapes than triangular
wedges by Ambraseys and Sarma (1967).

The first method of analysis aimed at calculating the permanent deformations in dams, was proposed by Newmark (1965) and successfully applied by Goodman and Seed (1966) to the analysis of the dynamic behavior of model sand embankments on a shaking table.

Newmark's approach is based on the concept of a yield acceleration, in which no movement takes place along a potential sliding surface until the acceleration of the sliding mass exceeds some limiting value. Using a procedure analogous to that of analyzing the movement of a sliding block on an inclined plane, the time record of acceleration of the sliding mass, which is calculated using a method such as the elastic shear wedge theory, is compared with the yield acceleration. Whenever the acceleration of the mass exceeds the yield acceleration, a process of double integration is used to calculate the progressive down-slope displacement (Fig. 1.1).

Difficulties in applying this procedure may arise in the determination of the yield acceleration for saturated soils. The yield acceleration is a function of the soil strength, which in turn is a function of the effective stress and thus dependent on the transient pore-water pressures generated during the earthquake. At the present time, it is not possible to predict, in general, the variation of the pore-water pressure during an earthquake. However, Martin et. al. (1975) have developed a procedure which allows the calculation of the pore water pressure history for the case where no initial shear stress acts, and this method might ultimately be extended to embankment problems. Furthermore, data from undrained cyclic load tests may be interpreted to determine an effective yield stress for use in this type of analysis.

The concept of deformations caused by accelerations in excess of the yield acceleration (Newmark's method) is primarily applicable to cases where
FIG. 1.1 INTEGRATION OF ACCELEROMETERS TO DETERMINE DOWNSLOPE DISPLACEMENTS
the movement of slopes occurs along well-defined failure zones. Such a failure mechanism occurs in dense, cohesionless soils and in cohesive soils. Experimental evidence has shown (Seed & Goodman, 1964) that in dry, dense, uniform slopes, subjected to a fairly uniform acceleration, failure occurs by mass sliding of a thin surface zone, making it a problem amenable to solution by Newmark's method. Movement downslope occurs during each cycle of acceleration where inertia forces are large enough to cause a temporary instability. Movement stops when the force is reversed, and the overall effect is a progressive downstream movement, causing a flattening of the slope at the toe and settlement of the crest.

Shaking table model tests (of dry sand embankments) were conducted by Goodman and Seed (1966) and deformation was analyzed by Newmark's approach. The mechanism of failure observed was a slide in a thin surface layer, similar to that reported previously. A formula, based on the strength of the soil, was developed to calculate the yield acceleration. By allowing for the decrease of soil strength, and hence yield acceleration, with increasing slope displacement, reasonable agreement was found between measured and calculated displacements.

However, in other soils and particularly saturated cohesionless soils, the shear stress may exceed the yield stress over a large part of the dam, and hence extensive shear zones will exist. In such cases, an analytical approach is required which can determine deformations over a wide zone and integrate them to give the overall deformation of the embankment.

An alternative method for the seismic design of earth dams, which followed the concept first proposed by Newmark (1963) that the stability of an embankment during an earthquake should be assessed on the basis of the deformations produced, was presented by Seed (1966), and has been used to
analyze the behavior of the Sheffield Dam in the 1925 Santa Barbara earthquake (Seed et al., 1969) and the Dry Canyon Dam during the 1952 Kern County earthquake (Lee and Walters, 1973). This method incorporated the consideration of the history of the stresses developed throughout the embankment, and the behavior of samples tested in the laboratory under similar stress conditions. The method is primarily applicable to conditions where drainage cannot occur during the earthquake, such as in saturated fine-grained soils, conditions usually found in the upstream slope of an earth dam.

As a preliminary step, the initial stress conditions acting along the assumed failure surface are determined. The method of slices, using the procedure of Lowe and Karafiath (1959), was used to determine the normal and shear stresses at the base of each slice, using drained soil strength data. From a Mohr circle, the corresponding principal stresses are found. In the laboratory, specimens of soil, compacted at field water content to field density, are consolidated under a range of confining pressures and stress ratios which encompass the range found in the dam.

Cyclic triaxial tests are then performed on the consolidated specimens to find the cyclic deviator stress causing failure. In the tests, only the major principal stress is varied. This test limitation is of minor importance as the confining pressure has no effect on the stress-strain relationship of a saturated soil.

The data from the laboratory tests is presented as a relationship between the cyclic shear stress along the failure plane causing failure and the normal stress on the failure plane before the earthquake. Strength curves are presented for a range of consolidation ratios.

The consolidation ratio and the normal stress on the assumed slip surface before the earthquake are known from the static analysis. Hence, the
maximum cyclic shear stress that can be developed at the base of each slice without causing excessive deformation is found from the laboratory data. By a method of analysis similar to that used to determine the initial static stresses, but including the maximum equivalent cyclic inertia force, as determined by an analysis such as that of Seed and Martin (1966), and acting in the direction to cause an increase in shear stress, trial values of a factor of safety are tried until the force system acting on the slices is in equilibrium. Undrained soil strength parameters are used.

The analysis is repeated for a range of potential sliding surfaces until the surface with the minimum factor of safety is found.

For some soils no well-defined failure takes place as strain increases. Strain, and consequently deformation of the dam, increase throughout the earthquake. As no analytical procedure existed at the time to relate axial strain of laboratory samples to deformations in the dam, empirical relationships were required. A discussion of the deformations of the Otterbrook dam, under static loading conditions, and of the corresponding axial strains of laboratory samples was presented (Seed, 1966). Recommendations on the relationship between axial strain of laboratory specimens and maximum tolerable field deformations are presented as an aid to the design engineer.

Subsequently this method of analysis was modified to provide a means for evaluating the strains developed throughout the cross-section of a dam (Seed, Lee and Idriss, 1969; Seed, Lee, Idriss and Makdisi, 1973). The laboratory procedure is similar to that proposed by Seed (1966), but the calculation of stresses is made by the finite element approach, using strain-dependent soil properties. As it is the basis upon which the deformations analysis presented in this report is founded, a full description of the method is given in the following chapter.
Chapter 2
Static and Dynamic Analyses of Earth Dams

Introduction

The design of an earth dam to withstand safely the effects of earthquake ground motions is an important engineering problem in seismically active areas of the world. The ground accelerations during a moderate to strong earthquake can cause large inertia forces throughout the dam. These forces, which reverse in direction with each cycle of ground motion, induce cyclic stresses, and consequently strains, throughout the embankment. The resulting permanent deformation is evidenced by slumping and sometimes cracking of the dam. If the strains are severe enough, or the slumping large enough, the dam may fail due to slope instability or overtopping.

The analysis of the stresses and strains induced in the dam by the earthquake, and the determination of the resulting deformations, is a complex and difficult problem. Before the development of the finite element method (Turner et al., 1956), its subsequent application to the dynamic response of earth dams (Clough and Chopra, 1966), and a means of interpreting the induced stresses in the light of the results of laboratory tests on the soil to give an overall picture of the behavior of the dam (Seed, 1966), no satisfactory solution to the problem existed.

To render the analysis more tractable, a number of simplifying assumptions are generally made. Foremost among these is the representation of the dam, a three-dimensional structure, by a two-dimensional transverse
cross-section in which a state of plane-strain exists. For the dynamic analysis of an embankment this assumption is usually necessary for reasons of analysis cost and computer capacity. However, as coupling between transverse and longitudinal motions in the dam is believed to be small, the response calculated by the plane-strain analysis will usually be sufficiently accurate for practical purposes.

In the finite element technique, the dam, a continuous structure, is represented by an assemblage of elements, usually either triangular or quadrilateral, connected only at their nodes. A relationship between nodal displacement and distribution of strain within the element is assumed, and based on this relationship and the stress-strain characteristics of the soil within the element, a stiffness is calculated for each element and then summed to give an overall stiffness for the dam which relates the forces acting within and on the dam to the nodal displacements. These forces, due to gravity, seepage, or inertia, are distributed to act only at the element nodes. From the nodal displacements, element strains and stresses are easily calculated. In a dynamic analysis, the ground acceleration at specific time intervals is used to calculate inertia forces and an analysis is run for each time step to produce stress histories for each node.

For a comprehensive analysis of the response of an earth dam to an earthquake, it is necessary to perform first a static stress analysis to determine the stress distribution throughout the dam before the earthquake; then a dynamic analysis to determine the history of the varying stresses during the earthquake; and finally a laboratory study to determine the behavior of the material of the dam under conditions of cyclic stress. A knowledge of the initial static stresses is required since the behavior of soil under dynamic stresses depends on the stresses under which the soil was
consolidated. Cyclic stresses applied in the laboratory are uniform in frequency and amplitude and so the stress histories for the elements of the dam must be converted to an equivalent number of uniform stress cycles in order to estimate behavior of the various elements of the dam from the laboratory tests. The following sections of this chapter discuss more fully the steps of the analysis outlined above, as first proposed by Seed et al, 1969.

Static Stress Analysis

Although it is possible to estimate the static stress distribution in a dam by approximate means (Lee and Idriss, 1975) the ultimate value of the complete stability analysis of the dam will depend on the accuracy with which each step is performed. The best method to date for calculating stresses is the finite element technique, and the most comprehensive approach using the finite element technique is that presented by Ozawa and Duncan (1973) and Wong and Duncan (1974) in which the method of construction of the dam is simulated by a progressive analysis where an additional layer of elements is added at each step. The nonlinear stress-strain properties of soils are also included in the analysis.

The number of layers used in the analysis need not be equal to the actual number of layers used in the construction of the dam. Approximately eight to ten layers are sufficient for a large dam. Though this type of analysis is necessary if an accurate measure of deformations during construction is required, the stresses calculated are not very different from an analysis using only one layer, i.e., a gravity turn-on analysis (Lee and Idriss, 1975). Displacements calculated from an analysis in which the dam is built up in layers are maximum near mid-height on the center line and smaller near the top and base of the dam, similar to the displacement
pattern measured in the field. The displacements calculated by a gravity
turn-on analysis are a maximum at the top or the dam and smallest at the
base (Clough and Woodward, 1966).

The method used (Ozawa and Duncan, 1973) to represent nonlinear
stress-strain curves is that proposed by Kondner (1963). The stress-strain
curve is assumed to be a hyperbola, expressed as:

\[ \frac{\varepsilon_a}{(\sigma_1 - \sigma_3)} = \frac{a}{a + b\varepsilon_a} \]  

(2.1)

where \( a, b \) are empirical constants;
\( \varepsilon_a \) is the axial strain; and
\( \sigma_1, \sigma_3 \) are principal stresses.

The formulation in Equation (2.1) represents the nonlinear stress-
strain curve for the confining pressure of \( \sigma_3 \). In order to determine the
values of the parameters 'a' and 'b', the equation is written in the
following linear form:

\[ \frac{\varepsilon_a}{(\sigma_1 - \sigma_3)} = a + b\varepsilon_a \]  

(2.2)

The graphical representation of both equations is shown in Fig. 2.1.
It can be seen from Fig. 2.1b that the parameters 'a' and 'b' are respectively
the intercept and slope of the straight line. It can be seen from Fig. 2.1a
that the value of the asymptotic stress difference \( (\sigma_1 - \sigma_3)_{\text{ult}} \) is always
greater than the stress difference at failure, \( (\sigma_1 - \sigma_3)_f \). These two values
are related as follows:

\[ (\sigma_1 - \sigma_3)_f = R_f (\sigma_1 - \sigma_3)_{\text{ult}} \]  

(2.3)

where \( R_f \) is a factor called the failure ratio. The failure ratio is always
less than unity, and is a measure of how well the stress-strain curve for a
\begin{align*}
\sigma_3 &= \text{Constant} \\
E_i &= \frac{1}{a} \\
(b = \frac{1}{(\sigma_1 - \sigma_3)_{ult}}) \\
\text{Axial Strain, } \varepsilon_a &\quad \text{Axial Strain, } \varepsilon_a \\
\text{FIG. 2.1a HYPERBOLIC REPRESENTATION OF STRESS-STRAIN CURVE} &\quad \text{FIG. 2.1b LINEAR TRANSFORMATION OF HYPERBOLIC STRESS-STRAIN CURVE}
\end{align*}
soil approaches a true hyperbola; a value of $R_f$ equal to unity corresponds exactly with a hyperbola.

As the stress-strain characteristics of a soil depend on the confining pressure ($\sigma_3$), the value of the initial tangent modulus, $E_i$, as shown in Fig. 2.1a, must be related to the confining pressures. The relationship used by Kulhawy et al. (1969) is that proposed by Janbu (1963):

$$E_i = K \frac{\sigma_3}{P_a}$$  \hspace{1cm} (2.4)

where $K$ is a modulus number;

$n$ is an exponent determining the rate of change of $E_i$ with $\sigma_3$ (both $K$ and $n$ are pure numbers); and

$P_a$ is the atmospheric pressure in the same units as $\sigma_3$.

The values of $K$ and $n$ are found by plotting the values of $E_i$, determined from a range of tests at differing confining pressure, against the confining pressure, on a log-log scale (Fig. 2.2) and fitting a straight line to the data.

To calculate the value of the failure ratio, $R_f$, the strength at failure is expressed as follows (Mohr-Coulomb relationship):

$$\left(\sigma_1 - \sigma_3\right)_f = \frac{2c \cos\phi + 2\sigma_3 \sin\phi}{1 - \sin\phi}$$  \hspace{1cm} (2.5)

and

$$R_f = \frac{\left(\sigma_1 - \sigma_3\right)_f}{\left(\sigma_1 - \sigma_3\right)_{ult}}$$  \hspace{1cm} (2.6)

Thus we have a method of relating stress to strain for a given value of confining pressure, using the hyperbolic relationship discussed, by means of the parameters $K$, $n$, $R_f$, $c$ and $\phi$. 

\[ K = E_i \text{ at } \sigma_3 = 1 \text{ atm.} \]
\[ n = \log \left( \frac{E_i}{K} \right) \text{ at } \sigma_3 = 10 \text{ atm.} \]

**FIG. 2.2 RELATIONSHIP BETWEEN INITIAL TANGENT MODULUS AND CONFINING PRESSURE**
It is possible to perform a nonlinear analysis by applying loads in increments, using the relationships derived above, using an expression to determine the tangent modulus for any point on the stress-strain curve:

$$E_t = \left[ 1 - \frac{R_f(1-\sin\phi)(\sigma_1-\sigma_3)}{2c \cos\phi + 2\sigma_3 \sin\phi} \right]^2 \frac{K \rho \left( \frac{\sigma_3}{P_a} \right)^n}{P_a}$$

(2.7)

The above expression for $E_t$ follows from the previous equations. Since $E_t$ is expressed only in terms of stress, and not strain, an analysis may be performed with any arbitrary initial state of stress.

The parameters are found from appropriate laboratory triaxial tests. For the conditions existing in the dam before the earthquake, consolidated drained tests are used. If no tests results are available, typical values of the nonlinear parameters are given in tabular form by Kulhawy et al. (1969).

In a finite element analysis, it is necessary to relate stress to strain by a generalized Hooke's Law. For isotropic materials, two independent parameters, usually Young's modulus ($E$) and Poisson's ratio ($\nu$) are used. Hence a nonlinear, stress dependent value of Poisson's ratio is required. The formulation used by Kulhawy is as follows:

$$\nu_t = \frac{G - F \log(\sigma_3/P_a)}{d(\sigma_1-\sigma_3)} \left[ \frac{K \rho \left( \frac{\sigma_3}{P_a} \right)^n}{P_a} \right]^2 \left\{ 1 - \frac{R_f(\sigma_1-\sigma_3)(1-\sin\phi)}{2c \cos\phi + 2\sigma_3 \sin\phi} \right\}$$

(2.8)

where $\nu_t$ is the tangent Poisson's ratio, and $G, F, d$ are nonlinear Poisson's ratio parameters which are found from a series of triaxial tests.

The Hooke's law relationship used in the analysis is written in terms of the bulk modulus ($K$) and shear modulus ($G$), as proposed by Clough.
and Woodward (1967). This makes it possible during the analysis to more closely model the behavior of actual soils after failure by reducing the value of $G$ to zero, but maintaining the value of $K$ at its pre-failure value. $K$ and $G$ are calculated from the relationships

$$K = \frac{E_t}{2(1+\nu)(1-2\nu)} \quad (2.9)$$

$$G = \frac{E_t}{2(1+\nu)} \quad (2.10)$$

A program to perform this static stress analysis has been written by Kulhawy, titled LSBUILD, and later modified by Ozawa (1973) and renamed ISBILD.

**Dynamic Stress Analysis**

As a comprehensive analysis of a dam involves a knowledge of the stresses induced in the embankment by the earthquake, a method of calculating these stresses is required. To perform such an analysis, both the dynamic soil properties of the dam and foundation and the acceleration history of the bedrock during the earthquake must be known or estimated. Dynamic soil properties are obtained from field tests, laboratory tests, or from a knowledge of the properties of similar soils. The acceleration record for the bedrock can be an artificially generated record or an accelerogram recorded at a similar site. Analysis of the dynamic stresses in a dam have, in the past, ranged from the shear beam approach in which only shear stresses are calculated, to the finite element modal analysis approach in which the modulus of the soil varies throughout the dam but the damping is constant, to the current variable damping finite element method.
The remainder of this section is divided into two parts: a discussion of the design earthquake and a discussion of the dynamic finite element analysis.

**Design Earthquake**

Although the records of earthquakes exhibit many features, for engineering design purposes three of these may be considered to characterize an earthquake record:

1. Maximum peak acceleration, \( a_{\text{max}} \);
2. Predominant period, \( T_p \); and
3. Number of significant cycles, \( N \).

The maximum peak acceleration is related to both the Richter magnitude \( M \) of the earthquake and the distance from the causative fault. The predominant period, \( T_p \), is the period of the most commonly occurring frequency in the record, and is usually taken as the period of the maximum peak of the acceleration spectrum. The predominant period is also related to magnitude and distance and increases with distance from the source of energy release. The number of significant cycles, \( N \), is the number of uniform average cycles which is equivalent to the actual acceleration record. A study by Seed, Idriss and Kiefer (1969) describes a procedure for selecting a design earthquake. Data on the variation of \( a_{\text{max}} \) and \( T_p \) with magnitude and distance, and the variation of \( N \) with magnitude, are presented. Before a design earthquake can be selected for a particular site, a seismicity study of the area must be made to estimate the distance to nearby faults and the maximum magnitude earthquake that could occur on each fault. The next step is to select rock motion records for earthquakes of similar magnitudes (in order that the significant number of cycles is appropriately represented) and modify the peak acceleration and predominant period. These records are
digitized for use with computer programs, and modification of the predominant period is simply a matter of changing the time step. After modification of a record, an appropriate base line correction such as that proposed by Berg and Housner (1961) should be made.

During an earthquake, the motion of the underlying bedrock is transferred to the dam by means of seismic waves propagating upward through the foundation and embankment. In most dynamic analyses of dams, the bedrock below the dam is considered rigid, although the earthquake motion is due to traveling seismic waves propagating outward from the hypocenter. In a study of the response of earth dams to traveling seismic waves, Dibaj and Penzien (1967) showed that if the ratio of base width of the dam, or dam and foundation system, to the shear wave velocity in the bedrock is less than 0.2 seconds, i.e.,

$$\frac{B}{V_S} < 0.2 \text{ seconds} \quad (2.11)$$

then the dynamic response of the dam to traveling waves will be similar to the response to rigid base motions. Assuming a shear wave velocity in the bedrock of 8000 fps, a rigid base motion analysis will be sufficiently accurate if the dam is no more than about 300 feet high.

**Finite Element Analysis**

The two requirements for a dynamic analysis are the base rock motion and the material dynamic properties. Selection of the design base rock motion is discussed in the previous section. The dynamic soil properties are characterized by the shear modulus and damping characteristics. By using strain-dependent values of the modulus and damping, and analyzing the dam in successive iterations until a strain compatible result is obtained,
the nonlinear characteristics of the soils are incorporated into the analysis. The variation of shear modulus and damping with shear strain for sands and saturated clays is shown in Fig. 2.3. These curves are from a study of the dynamic properties of soils by Seed and Idriss (1970). For soils which are a combination of materials, some judgment is required in selecting the curve to be used. Alternatively, a series of tests may be performed on the soil to produce a curve which can then be incorporated into the analysis.

The modulus of cohesionless soils also varies with mean effective confining pressure:

\[ G = 1000 \, K_2 (\sigma_m')^{1/2} \]  \hspace{1cm} (2.12)

where:

- \( G \) is the shear modulus in psf;
- \( K_2 \) is a parameter, and is a function of the soil type, relative density and shear strain;
- \( \sigma_m' \) is the mean effective pressure in psf.

At very low strains, in the order of \( 10^{-4} \) percent, the value of \( K_2 \) is a maximum. Once the value of \( K_2 \) is known, the curve in Fig. 2.3a may be used to calculate the value of \( K_2 \), and hence \( G \), for any level of strain. Typical values of \( K_2 \) for various cohesionless materials have been published. Alternatively, the value can be calculated from the results of shear wave velocity tests:

\[ V_s = \sqrt{\frac{G_{\max}}{\rho}} \]  \hspace{1cm} (2.13)

where:

- \( V_s \) is the shear wave velocity in fps;
- \( \rho \) is the mass density (density/g);
- \( G_{\max} \) is the shear modulus at low strains.
FIG. 2.3 AVERAGE SHEAR MODULI AND DAMPING CHARACTERISTICS OF SOILS
(After Seed and Idriss, 1970)
Once the mean effective stress at the position of the measurement of $V_s$ is calculated, the value of $(K_2)_{\text{max}}$ is easily found by the relationship in equation (2.12). The value of $(K_2)_{\text{max}}$ can also be found in the laboratory, e.g., by a resonant column test.

The shear modulus of a saturated cohesive soil varies with the undrained shear strength ($s_u$) and the shear strain, as shown in Fig. 2.3b.

The finite element analysis starts with the idealization of the cross-section. The finite element mesh should extend in the foundation sufficiently far upstream and downstream so that waves reflected from the boundaries are damped out and do not influence the solution. The size of the elements in the mesh is also important as this influences the maximum frequency which can be transmitted (Lysmer et al., 1974). If the dynamic analysis is by the step-by-step method, the element size also affects the stability of the solution, since if small elements are used a small time step is also necessary to insure stability. In general, the time step of the analysis should be less than the minimum shear wave travel time across any element of the mesh. It is convenient to use the same mesh for the static and dynamic analysis since the initial stresses acting in an element are used to determine the effect of the dynamic stresses on the behavior of the element. However, interpolation can be used to determine the static stresses in the elements of the dynamic mesh if a different mesh has been used in the static stress analysis.

It is usual in a dynamic analysis of an earth dam to apply only the horizontal component of the earthquake record. Laboratory tests have shown that volume changes, and associated pore pressure increases, caused by vertical motions are small compared to those caused by shear deformations. Also, the shear stresses induced in the dam by vertical motions are much less than those due to horizontal motions.
The maximum value of $K_2$ or the modulus, $G$, for a cohesionless soil, or the undrained shear strength for a cohesive soil, together with soil properties such as density, are supplied as input to the analysis. As the seismic response analysis is an iterative procedure which produces a strain-compatible result, a first approximation of the modulus and damping is also required.

Some areas of the dam may fail during the earthquake, thus causing the stress in surrounding areas to increase. These areas may in turn fail, redistributing the stress to neighboring zones. This type of progressive failure can be simulated in the analysis by running the analysis for a section of the earthquake, obtaining a strain-compatible result, and setting the modulus of any failed elements to a very low value. The analysis is then continued for following sections of the earthquake record until the analysis is completed. An element is said to have failed if a sample tested in the laboratory under similar consolidation stresses and cyclic stress fails, either through liquefaction or excessive strain. The method of laboratory testing is described in the following section.

The response of the dam found by the dynamic analysis consists of the acceleration history at each node of the mesh, and the stress histories for each element. To make a comparison between the behavior of laboratory samples and elements in the dam, these stress histories are expressed as an equivalent series of uniform stress cycles by an appropriate weighting of the ordinates of the stress history based on the results of laboratory cyclic test data, as proposed by Seed and described by Seed et al. (1975) and by Lee and Chan (1972). The uniform cyclic stress history is then used to determine the behavior of the dam.
Two types of programs are currently available to perform the dynamic analysis. The step-by-step direct integration procedure of Wilson and Clough (1962), incorporating Rayleigh-type damping, is employed by the program QUAD4 (Idriss et al., 1973). This form of damping damps out all but the lowest frequencies (in general there is little response above approximately 4Hz), but as most of the response of earth dams is in the lower modes this is not a serious drawback. The method also has stability problems. However, QUAD4 in general gives good results when applied to the analysis of earth dams, and has been used successfully on many analyses. The second method is an analysis in the frequency domain, and is employed by the program LUSH (Lysmer et al., 1974). This program can handle higher frequencies than QUAD4, being limited only by the time step of the digitized input acceleration and the element size in the finite element mesh. In practice, the full range of frequencies for which a solution is possible is not of interest in engineering problems, and the analysis is run only up to a selected maximum frequency.

**Cyclic Load Tests**

Cyclic load tests are usually performed by the triaxial method, although the cyclic simple shear test may simulate more accurately the field conditions in a dam during an earthquake. In the triaxial test, failure usually takes place along planes oriented at an angle of $45 + \phi/2$ degrees to the direction of the major principal plane, which in the finite element analysis of a dam deformation is usually considered to be along horizontal planes (Seed et al., 1973). The cyclic strength data from the triaxial test can be readily manipulated so that a direct comparison can be made between triaxial and simple shear or assumed field conditions (Peacock and Seed, 1968).
The data recorded during the cyclic triaxial test are the axial strain and the pore-water pressure vs. the number of stress cycles. Samples are consolidated under a range of consolidation ratios ($K_3$) and tests are run for a number of values of cyclic deviator stress ($\sigma_{dp}$) for each value of confining pressure ($\sigma_{3c}$). From the curves of axial strain ($\varepsilon_a$) vs. number of stress cycles (Fig. 2.4a), curves of $\sigma_{dp}$ vs. $N$ are drawn for a range of values of $\varepsilon_a$ (Fig. 2.4b). Curves are produced from these showing the relationship between $\sigma_{dp}$ and $N$ for a range of $K_3$ values, one set for each combination of $\varepsilon_a$ and $\sigma_{3c}$ (Fig. 2.5a). Each curve is for a specific value of $K_3$ and $\sigma_{3c}$.

The result of the finite element dynamic analysis is to determine the stress history of each element, expressed as an equivalent number of uniform stress cycles. This number of cycles, $N_{eq}$, is assumed to be the same for all elements. Using a value of $N$ equal to $N_{eq}$, curves are produced showing the relationship between $\sigma_{dp}$ and $\sigma_{3c}$ for a range of $K_3$ values. These curves are derived from Fig. 2.5a. Sets of curves are produced for different values of $\varepsilon_a$ (Fig. 2.5b).

To make it possible to apply the results of the laboratory testing to the analysis of the dam, it is necessary to know the value of the cyclic shear stress along the failure plane of the triaxial sample. Assuming that the failure plane is horizontal in the field, and is oriented at an angle of $45 + \phi/2$ degrees to the direction of the minor principal stress in the laboratory, the Mohr circle construction shown in Fig. 2.6 can be used to find this shear stress, termed $\tau_{cyclic}$. In cases of isotropic consolidation ($K_C = 1$) it has been shown that the cyclic shear stress on the failure plane is about 60 percent of the maximum shear stress in the triaxial test (De Alba et al., 1975). For values of $K_C > 1.5$, $\tau_{cyclic}$ it has been found
FIG. 2.4 RESULTS OF CYCLIC LOAD TESTS (1)
FIG. 2.5 RESULTS OF CYCLIC LOAD TESTS (2)
\[ \tau_{fc} = \text{Initial shear stress on potential failure surface} \]
\[ \sigma_{fc} = \text{Initial normal stress on potential failure surface} \]
\[ \alpha = \frac{\tau_{fc}}{\sigma_{fc}} \]
\[ \tau_{cyclic} = \text{Cyclic shear stress developed on potential failure surface} \]

**FIG. 2.6** PROCEDURE FOR INTERPRETING CYCLIC LOAD TRIAXIAL TEST DATA TO DETERMINE CYCLIC SHEAR STRESS ON POTENTIAL FAILURE SURFACE (after Seed et al)

**FIG. 2.7** RESULTS OF CYCLIC LOAD TESTS (3)
to be approximately equal to the maximum shear stress on the potential failure plane in the test specimen.

The final step in the reduction of the laboratory data is to produce a family of curves relating $\tau_{\text{cyclic}}$ to $\sigma_{\text{fc}}$ for different levels of strain. These are derived from the curves in Fig. 2.5 and the Mohr circle construction in Fig. 2.6. The curves are shown in Fig. 2.7, and a range of curves for varying values of $\alpha$ are produced, where

$$\alpha = \frac{\tau_{\text{fc}}}{\sigma_{\text{fc}}}$$

and $\tau_{\text{fc}}$ is the shear stress on the failure plane during consolidation; $\sigma_{\text{fc}}$ is the normal stress on the failure plane during consolidation.

Sets of these curves are produced for a range of values of $\varepsilon_a$.

**Analysis of Dam Behavior**

The final step in the analysis of the dam is to use the curves obtained from the laboratory tests in conjunction with the results of the static and dynamic finite element analyses, to determine the effects of the earthquake induced stresses on the individual elements of the embankment. For each element of the dam, assuming the horizontal plane is the critical plane, $\sigma_{\text{fc}}$ and $\tau_{\text{fc}}$ are known. From the dynamic analysis the superimposed cyclic shear stress in each element is known. By using the curves presented in Fig. 2.7, interpolating where necessary, the value of $\varepsilon_a$ produced by the appropriate combination of initial and cyclic stress conditions for each element is determined. This value of strain is for an element unrestrained by surrounding elements, and is termed the "strain potential." The actual strain induced by the earthquake will depend on the effects of interaction...
between elements.

To make the calculations of strain potential more direct, the curves of Fig. 2.7 can be replotted as $T_{\text{cyclic}}$ vs. $\varepsilon_a$, with one curve for each value of $\sigma_{fc}'$, and one set of curves for each value of $\alpha$.

It is usually convenient to examine the behavior of the dam by considering horizontal layers of the dam separately, and calculating strain potentials for all the elements of a layer together. A curve of the induced shear stress in the elements of the layer is plotted (Fig. 2.8). Superimposed on this are curves of the cyclic shear stress to cause different values of strain ($\varepsilon_a$). The strain potential for each element on the plane is then found directly from the plot.

If a progressive analysis is run, where the response of the dam is examined after progressively longer earthquake durations, and material properties adjusted accordingly, sets of curves similar to those of Figs. 2.4, 2.5 and 2.7 are derived for values of $N$ less than $N_{eq}$. These values of $N$ correspond to the number of uniform cycles associated with the shorter duration of shaking applied at each step.

**Summary**

The steps involved in the earthquake analysis of a dam as proposed by Seed and his coworkers are as follows:

1. Determine the initial stress in the embankment before the earthquake by performing a static finite element analysis.

2. Select the design earthquake(s) and determine the characteristics of the motions developed in the rock underlying the embankment and its soil foundation during the earthquake.
FIG. 2.8 CALCULATION OF POTENTIAL STRAINS
3. Determine the response of the embankment to the base rock motion and compute the dynamic stresses induced in representative elements of the embankment, using a dynamic finite element computer program.

4. Convert the dynamic stress histories for representative elements to a uniform stress level for a specified number of cycles, using curves developed from laboratory testing as a basis.

5. Perform laboratory tests to determine the dynamic response of the dam embankment and foundation materials (in terms of pore-water pressures and deformations produced) under cyclic loading conditions and various initial stress conditions.

6. Compare the computed dynamic stresses with the laboratory-determined response curves and estimate the potential strains in representative elements of the embankment.

7. From a knowledge of the strain potentials for representative elements in the embankment, evaluate the overall deformation and stability of the cross-section.

Using the results of this analysis, the stability of the dam after the earthquake can be checked (using the calculated values of strain potential to modify the strength of elements along a potential slip surface), and the factor of safety against sliding computed. For elements which have liquefied, for example, zero strength may be assigned in the stability analysis.

It is also possible to assess the overall stability of the dam by calculating the factor of safety against some level of strain (typically 5%) on an element by element basis. Such a study can be very helpful to the designer if the results show a clear trend; for example, if every element shows a factor of safety greater than unity against 5% strain then the
overall movements are likely to be tolerable. Alternatively, if all elements show a factor of safety less than unity against 20% strain, then the overall movements are likely to be excessive. For intermediate conditions judgement is required to interpret the significance of the results.

This method was first used by Seed (1970) in a study of Perris Dam, and has subsequently been used in numerous other studies. However, the strain potential assessment procedure developed in 1973 seems to offer greatly improved guidance in assessing embankment performance.

The last step in a comprehensive analysis of an earth dam is to estimate the permanent deformations induced by the earthquake. This problem is discussed in the following chapters, and a technique to calculate permanent deformations is developed.
Chapter 3
Analysis of Permanent Deformations

Introduction

The permanent deformations induced in an earth dam by the action of an earthquake are a function of both the static stress condition acting in the dam before the earthquake and the level and duration of the shaking due to the earthquake.

The steps taken in any analysis of the deformations must proceed from a basis of the knowledge of these criteria. Additionally, some technique to determine in the laboratory the effect of these initial stresses and applied inertia forces on the material of the dam is required.

Methods of determining the initial static stresses acting on the dam vary from an estimate based on previous experience to the most comprehensive technique currently available: a finite element approach which models the method of construction by building up the dam successively with layers of elements, while also accounting for the nonlinear behavior of the materials used.

A dynamic analysis which calculates the histories of the varying stresses throughout the dam, followed by use of these stress histories to determine the behavior in the laboratory of samples of the dam material, form the preliminary stages of a comprehensive deformation analysis. Analysis of these stress histories has developed from the original shear beam approach, which assumed the material of the dam to be uniform and that only shear forces acted, to the present comprehensive approach using the finite element method.
The static and dynamic stress analysis and the subsequent measurement in the laboratory of the behavior of specimens of the dam material subjected to these stresses, as used in the present research, are described fully in the previous chapter.

The behavior of soil samples consolidated under the initial stress conditions, due to gravity loading and seepage forces, and subjected to a cyclic stress history, is recorded in the form of the axial strain of the laboratory specimen. This strain, the major principal strain, is referred to as the strain "potential," since a corresponding element in the dam, unlike the laboratory sample, is restrained by surrounding material from undergoing the same degree of deformation. The final result of the dynamic stability analysis of the dam is the determination of the strain potential for each element in the dam, and use of these values both to modify the strength of elements along potential failure surfaces and to assess the overall deformations of the embankment. A stability analysis, using zero strength for the regions where the slip surface passes through "failure zones," i.e., areas where the strain potential exceeds the failure strain, and appropriately reduced strength where the strain potential is less than the failure strain, can be used to estimate the overall stability of the embankment after the earthquake. However, as with all limit analyses, the result indicates only whether or not total failure of the dam will occur. In the cases where failure does not occur, this approach gives no assessment of the deformations that may result as the soil undergoes strain in order to develop the strength necessary to resist failure.

A method of expressing the strain potential in terms of the overall earthquake-induced deformation of the dam is required in order to perform a complete analysis. A knowledge of these deformations, together with
calculated post-earthquake stress and strain distribution which is derived from such an analysis, would help the designer of the dam to recognize areas of potential weakness in the event of an earthquake.

Deformation Analyses

Analyses of the seismically induced deformations of an earth dam by four different approaches are presented in this chapter. The first approach is a rough approximation and follows directly from a knowledge of the strain potentials of the elements. It was used by Seed et al. (1973) in the analysis of the Upper San Fernando Dam. The second and third methods are based on the concept that the effect of the strain potential of an element is a reduction in the modulus of the material, followed by settlement of the dam to a new position of equilibrium under gravity loads. The second method is a linear gravity turn-on analysis, as proposed by Lee (1974), while the third method is a nonlinear version of the second, with the loads applied in increments, thus making it possible to approximate the stress-strain curve of the soil by a series of straight line segments.

The fourth method, which seems to offer the most rational approach, and for which a computer program has been developed (listed in the Appendix), is a true pseudo-static approach. The effect of the earthquake is represented by a set of nodal point forces which are derived from the element strain potentials. The method incorporates the nonlinear characteristics of the soils and loading is incremental.

The results of these analyses, when applied to the Upper San Fernando Dam to calculate the deformations due to the earthquake of February 9, 1971, are presented in the following chapter. The four methods are briefly described in the following paragraphs.
First Approximation

An approach for assessing embankment deformations, applied by Seed et al. (1973), is to average the strain potentials, expressed as shear strains, along a vertical section through the dam, and to calculate the movement at the top of the section from the product of the average shear strain and the height of the section. This method determines only horizontal displacements, and it assumes that the maximum shear stresses act along horizontal planes.

Modified Modulus Approach--Linear

An alternative approach involves the use of linear, static finite element analyses, in which the effect of the earthquake on the soil properties is simulated by a reduction in the modulus, as proposed independently by Lee (1974). The analysis is run in three stages.

In the first stage, a linear gravity turn-on analysis, which includes gravity loads and seepage forces, is run. Initial values of the modulus, $E_i$, are determined from static triaxial tests, or estimated from published data for similar soils. The calculated deformation of each node in the dam is recorded and stored for later use. The deformations are merely reference deformations and are not representative of any actual pre-earthquake deformation.

The effect of cyclic loading on a soil is represented as a softening of the soil, resulting in a reduced modulus. The dam will then deform under both its own weight and the hydrostatic forces acting on the upstream slope. Thus, the second stage of the analysis is to perform another linear gravity turn-on analysis, using reduced values of the modulus in the elements.

The third and final step calculates the difference between the deformations obtained from the second and first steps of the analysis to give
the displacement of each node in the dam due to the softening caused by the earthquake.

The initial values of the modulus, $E_i$, used in the first stage of the analysis, are determined from triaxial tests or published data for similar soils. To determine the final value of the modulus, $E_f$, it is assumed that the stresses developed during the earthquake return to the initial value and that the net change in stress is zero (Fig. 3.1). During the earthquake, the strain changes from the initial value $\varepsilon_i$ to the final value $\varepsilon_f$, an increase equal to the strain potential $\varepsilon_p$. The final value of the modulus is then:

$$E_f = \frac{C_i}{\varepsilon_i + \varepsilon_p} = \frac{C_i}{\varepsilon_f}$$  \hspace{1cm} (3.1)

Because the critical zones of the dam are saturated, and most of the rest of the dam is nearly saturated, the bulk modulus of the soil in the dam cannot change significantly. Hence, all of the softening is assumed to be due to a reduction of the shear modulus. To incorporate this in a finite element analysis, the stress-strain relationship is formulated in terms of the bulk modulus and shear modulus, rather than in the usual manner in terms of Young's modulus and Poisson's ratio. This formulation is that originally proposed by Clough and Woodward (1967):

$$C = \begin{bmatrix} K+G & K-G & 0 \\ K-G & K+G & 0 \\ 0 & 0 & G \end{bmatrix}$$  \hspace{1cm} (3.2)

where $C$ is the elastic stress-strain relationship (Hooke's Law) for plane
$E_i = \sigma_i / \varepsilon_i$  
Modulus before earthquake

$E_f = \sigma_i / (\varepsilon_i + \varepsilon_p)$  
Modulus after earthquake

FIG. 3.1 MODIFIED MODULUS METHOD - LINEAR
strain conditions, defined by

\[ \sigma = C \varepsilon \]

and expressed in terms of the bulk modulus (K) and the shear modulus (G) of the soil:

\[ K = \frac{E}{2(1+\nu)(1-2\nu)} \]  \hspace{1cm} (3.3)

\[ G = \frac{E}{2(1+\nu)} \]  \hspace{1cm} (3.4)

where \( E \) is the appropriate value of Young's modulus (\( E_i \) or \( E_f \)) and \( \nu \) is Poisson's ratio, which is usually estimated. For elements below the phreatic line, \( \nu \) is assumed to be 0.49; for other elements the value will vary with the material type and can be estimated from published values. The bulk modulus is calculated using \( E = E_i \) for both first and second stages of the analysis; the shear modulus is calculated using \( E = E_i \) in the first stage, and \( E = E_f \) in the second stage.

During the earthquake, drainage of excess pore water pressure does not have time to occur since the dam is effectively impermeable for the relatively short duration of shaking. Thus, in the second stage of the analysis, water forces may more appropriately be applied to the upstream slope of the dam, rather than as seepage forces throughout the dam.

A similar analysis has been published independently by Lee (1974). In this approach, a pseudo-secant modulus for each element is first calculated from the results of the finite element dynamic analysis. The modulus is defined as:

\[ E_p = \frac{\sigma_i}{\varepsilon_p} \]  \hspace{1cm} (3.5)
where \( \sigma_i \) is the initial stress in the element (i.e., before the earthquake) and \( \varepsilon_p \) is the element strain potential resulting from the earthquake shaking.

A final secant modulus is then defined by:

\[
\frac{1}{E_{ip}} = \frac{1}{E_i} + \frac{1}{E_p}
\]

(3.6)

where \( E_i \) is the initial (pre-earthquake) secant modulus.

Using the initial and final values of the secant moduli, two linear analyses are run, the difference in displacements giving the earthquake-induced deformations. Only the shear modulus is changed in the second stage of the analysis, as described previously, and hydrostatic forces are assumed to act on the upstream slope during the earthquake. The deformations calculated by Lee are compared with those determined in the present study in the following chapter.

**Modified Modulus Approach—Nonlinear**

This approach is an improvement on the preceding analysis and takes into account the nonlinear stress-strain behavior of soils. The analysis is run in three stages as before. The first stage consists of a finite element analysis to determine the conditions in the dam before the earthquake. The stress-strain relationship for the pre-earthquake condition is determined from the nonlinear parameters for the soil which are calculated by the method of Kulhawy et al. (1969) as explained in the previous chapter, or some similar approach. Fig. 3.2 shows representative curves for the stress-strain relationship in an element for the conditions before and after the earthquake. Deviator stress is plotted against principal strain.
\[(E_i)_{after} = E_i \left[ \frac{\varepsilon_i}{\varepsilon_i + \varepsilon_p} \right] \]

FIG. 3.2 MODIFIED MODULUS METHOD - NONLINEAR
In performing a nonlinear static analysis for the post-earthquake conditions, some additional assumptions must be made about the stress-strain relationships for the soil. Accordingly, it is assumed that the static shear stress in an element of the dam, represented by the static deviator stress, applied to a representative sample in the laboratory, will be unchanged due to the earthquake. During the earthquake, or simulated earthquake loading in the laboratory, the cyclic shear or deviator stress will pulsate about the pre-earthquake static shear stress condition. In the laboratory test, this effect will cause an accumulative strain, $\varepsilon_p'$, in the sample at the end of the simulated loading. Thus, the final total strain, $\varepsilon_f'$, after the earthquake will be $\varepsilon_i + \varepsilon_p'$, as shown in Fig. 3.2.

It is further assumed that the shape of the post-earthquake stress-strain curve between the origin and the final post-earthquake condition can be represented by a hyperbola similar to that of the pre-earthquake curve. Finally, it is assumed that the factors $n$ and $R_f$, as defined in Chapter 2, used to define this hyperbolic curve are the same as those used for the pre-earthquake curve and that the initial moduli before and after the earthquake are in direct proportion to the strain in the laboratory sample simulating the element in the dam:

$$\left(\frac{E_i}{E_i}\right)_{\text{after}} = \left(\frac{\varepsilon_i}{\varepsilon_i + \varepsilon_p'}\right) \quad (3.7)$$

In this way, a nonlinear stress-strain curve for an element for the post-earthquake condition can be fully defined.

The first step of the method consists of an incremental loading analysis using the initial stress-strain curve for each element. The stress-strain matrix used in the computer program is defined in terms of $G$ and $K$. The load vector for the finite element analysis is formed of
the gravity loads and the seepage forces and is applied in a number of steps. The stiffness properties are recalculated at each load step and the stiffness matrix reformed, thus following the stress-strain curve by a number of straight segments. In the second step, the analysis is repeated using the post-earthquake soil properties. The deformations calculated in the first two steps are subtracted in the third step to give the deformations of the dam due to the earthquake.

In the second step of the analysis, only the shear modulus is changed, the reasoning for this being as presented in the previous section on the linear approach.

The results of the preceding methods of analysis applied to the Upper San Fernando Dam are presented in the next chapter. However, since neither of the above methods includes consideration of the deformations produced by the inertia stresses induced by the earthquake shaking, another approach giving consideration to this aspect of the problem was developed, as described below.

**Equivalent Nodal Point Force Approach**

The ultimate effect of the earthquake, assuming the dam does not fail, is to cause each element of the dam to undergo some degree of strain; this may be expressed by the strain potential for each element, although the actual strain will necessarily be different from the strain potential values in order to ensure compatibility of deformations.

It is possible to postulate an array of static forces at the nodes of the finite element mesh which would result in the same deformations of the elements as those produced by the computed strain potentials. Thus, the effects of the earthquake can be represented by a series of equivalent static
forces producing a truly pseudo-static analysis.

To estimate the equivalent static nodal point forces for an element in the embankment, it is necessary to determine the change in stress for that element corresponding to the computed strain potential induced by the earthquake loading. This can be achieved once a stress-strain relationship for the material during the earthquake has been defined. In the present analysis two assumptions have been made: (1) the stress-strain behavior of the material during the earthquake may be represented by a nonlinear hyperbolic relationship determined from consolidated-undrained laboratory tests; and (2) the nonlinear stress-strain relationship for a particular soil during the earthquake is dependent only on the initial effective confining pressure existing in the embankment before the earthquake. The first assumption is a reasonable one in most cases, especially for low permeability soils, given the short duration of the earthquake loading during which the material can be assumed for all practical purposes to exhibit undrained behavior. The second assumption can reasonably be made since the approach followed in the analysis is a total stress approach where the properties of the material depend on the initial stress conditions before loading.

Accordingly, the increment in stress, \( \Delta \sigma_{p} \), corresponding to a specified strain potential, \( \varepsilon_{p} \), for an element of soil can be determined as shown in Fig. 3.3. The initial pre-earthquake stress condition \( (\sigma_{di}, \varepsilon_{i-D}) \) may be conveniently determined by an incremental finite element procedure (as described in Chapter 2) using nonlinear parameters obtained from drained triaxial test results. For the same initial stress conditions \( (\sigma_{3i}, \sigma_{di}) \), the undrained stress-strain relationship used during the earthquake is also shown in Fig. 3.3, and the corresponding strain on this curve for the same stress, \( \sigma_{di} \), is \( \varepsilon_{i-UD} \). It is assumed that deformations during the earthquake
FIG. 3.3 EQUIVALENT FORCE METHOD - DETERMINATION OF EQUIVALENT STRESS
start from this point on the undrained curve. The differences between
$\varepsilon_{i-UD}$ and $\varepsilon_{i-D}$ are very small compared to the strain potential, $\varepsilon_p'$, and
since the interest of the analysis is in the earthquake-induced deformations rather than the pre-earthquake displacements, such differences may
be neglected for practical purposes without affecting the results of the
analysis. The change in stress, $\Delta \sigma_d$, corresponding to the strain potential, $\varepsilon_p'$, can then readily be determined using the undrained stress-strain
relationship, as shown in Fig. 3.3.

Dynamic analyses have indicated that the response of earth dams to
horizontal base motions is, in general, predominantly a shear deformation
and that the maximum induced shear stresses occur along directions within
about $\pm 10^\circ$ of the horizontal. It may, therefore, be assumed conserva-
tively that the maximum induced dynamic shear stress, $\Delta \tau_{max}'$, acts along
the horizontal plane and is equal to $\Delta \tau_{xy}'$. The maximum dynamic shear
stress, $\Delta \tau_{max}'$, can readily be determined since it is equal to half the
deviator stress increment, $\Delta \sigma_d'$, determined above.

To estimate the equivalent nodal point forces, it is further assumed
that the distribution of shear stress over the area of an element is uni-
form and constant. Thus, the nodal point forces for an element in plane
strain can be estimated from $\Delta \tau_{xy}'$ (which is equal to $\Delta \tau_{max}'$) by multiply-
ing it by the width and the height of the element to determine the hori-
zontal and vertical node forces, respectively, as shown in Fig. 3.4.
These nodal forces are applied in the direction of the initial horizontal
shear stress acting on the element, since a soil element under simple
shear cyclic loading conditions will show a residual deformation in the
direction of the initial shear stress.

A nonlinear finite element analysis is then run in which only these
FIG. 3.4 EQUIVALENT FORCE METHOD - DETERMINATION OF NODAL POINT FORCES

\[ F_h = \Delta \tau_{\text{max}} x \frac{L}{2} \]
\[ F_v = \Delta \tau_{\text{max}} x \frac{H}{2} \]
forces are applied, and the resulting deformations in the dam are those
due to the earthquake. The loads due to gravity and seepage forces
are not included in the analysis since the effects of these forces have
already been included in the evaluation of the strain potential. Since
the soil was consolidated before dynamic testing under the stress condi­
tions existing in the embankment prior to the earthquake, these initial
stresses are due to the gravity loads and seepage forces.

A nonlinear finite element program (DEFORM) was written incorporating
the method presented above. A description and full listing of the program
is given in the Appendix. The nonlinear analysis procedure of Kulhawy et
al. (1969) is used. As a first stage, a nonlinear step-by-step loading
analysis, including seepage forces, is run to determine the initial stresses
in the dam. The pseudo-static nodal point forces representing the
earthquake are then calculated for each node based on the undrained stress–
strain relationship of the material during the earthquake and the strain
potentials for the elements.

With the initial stresses as a starting point, a second analysis is
run in which only the pseudo-static nodal point forces calculated in the
first stage are applied. The resulting displacements are the deformations
induced in the dam by the earthquake.

In the first stage of the analysis, soil properties for the drained
condition are used; in the second stage, undrained soil properties are
used, as these are more representative of the conditions existing in the
dam during the earthquake. The Hooke's Law relationship used throughout
the analysis is that previously described under Modified Modular Approach–
Linear. During the second stage of the analysis, only the shear modulus
of the soil is changed, the bulk modulus remaining constant, to model as
accurately as possible the behavior of saturated soil during the earthquake.

The analysis cannot incorporate the effect of time, although the deformations are not necessarily immediate and may continue after the earthquake has stopped. However, if it were possible to incorporate this time effect and allow the changing stresses to affect the soil properties, the effect would be for the soil modulus to decrease, causing a smaller stress change to be required to produce the potential strain. This would result in lower equivalent nodal point forces acting on a softer material. While it cannot be claimed that the end result would be the same, the indication is that the effect would tend to give similar displacements.

Due to the nature of the nonlinear stress-strain formulation, tensile stresses cannot be handled. Soils generally used in earth dams cannot support tension, resulting in the opening of cracks near the surface of the dam. If, during the analysis, large tensile stresses develop in any element, the modulus should be reduced to a very small positive value.

An alternative approach to the above, which is described below, may be more applicable in cases where the specified strain potential values for a considerable number of elements are of such large magnitude as to exceed the failure strain of the material, since the use of the hyperbolic stress-strain relationship shown in Fig. 3.3 may cause some mathematical difficulties in the computational procedure for the permanent deformations. For such conditions, an equivalent linear modulus approach may be used where the equivalent modulus $E_f$ is estimated from the nonlinear relationship on the basis of the strain potential, as shown in Fig. 3.5. The deviator stress increment, $\Delta\sigma_d$, used to establish the value of $E_f$ will be the same as that determined in Fig. 3.3.

The equivalent nodal point forces in this approach are determined in
FIG. 3.5 EQUIVALENT FORCE METHOD - ALTERNATIVE METHOD OF DETERMINING EQUIVALENT MODULUS
the same manner as that described in the nonlinear approach (see Fig. 3.4); however, in this case, the second stage displacement computations are carried out using a one-step linear analysis instead of the incremental nonlinear approach described above.

As with any similar analysis, the results produced are limited in usefulness by the accuracy with which the soil properties are known.

In the next chapter, the method is applied to evaluate the deformations of the Upper San Fernando Dam during the earthquake of February 1971.
Chapter 4

Analysis of Permanent Deformations of the
Upper San Fernando Dam

The Upper San Fernando Dam

Introduction

The Upper San Fernando Dam is a part of the Van Norman Lake Complex, a system of dams, reservoirs, dikes, and storm and diversion structures, which forms the terminal storage area for two aqueducts and was the main water distribution center for the surrounding area before the earthquake of February 9, 1971.

Construction of the Dam

Construction of the Upper San Fernando Dam was begun in 1921. Half a million cubic yards of material were placed that year, bringing the crest elevation to 1200 feet. It was originally planned to raise the embankment to a final crest elevation of 1238 feet in 1922. However, the dam was completed by the addition of a rolled fill section at the upstream side of the dam, bringing the final crest elevation to 1218 feet. The completed dam has a crest width of 20 feet and a 100-foot berm on the downstream side at elevation 1200 feet. The slopes are 2.5:1, and the upstream face is protected by concrete paving. The maximum cross-section height is approximately 82 feet (Fig. 4.1).

Construction details for the dam do not exist, but the general technique employed was the "semihydraulic" fill method, a variation on the hydraulic fill construction method. In the hydraulic fill method of
FIG. 4.1 IDEALIZED CROSS-SECTION THROUGH UPPER SAN FERNANDO DAM
construction, material for the dam is sluiced at the borrow area and conveyed to the site, through pipes by pumping or gravity. It is then discharged on beaches at the upstream and downstream edges of the dam and flows toward the center, coarser material being deposited quickly to form the shell of the dam and finer material settling out more slowly from the pool which forms at the center to form the "impermeable" core. Ideally, the end result is an overall gradation from coarse material at the face, forming a strong shell, to the finest material at the center, forming an impermeable core. In practice, this is rarely realized, and lenses of sand and silt sometimes penetrate the core partially negating the effect of the impermeable barrier and making the dam more prone to failure from piping. However, it was felt at the time by the designers that if adequate care was taken during construction, and if the rate of progress was controlled so that local failures did not occur on the slopes during construction, a sound embankment could be built.

Possibly due to lack of an adequate water supply, the hydraulic fill method was not used to build the Upper San Fernando Dam, though the Lower San Fernando Dam was built by this technique in 1912. In the semihydraulic fill method used, borrow material was loaded by Fresno scrapers or steam shovels at the borrow area and transported by horse-drawn carts to the dam site. Here the material was dumped on the beaches at the upstream and downstream toes and was spread by sluicing it with a jet of water pumped from a barge floating on the pool between the beaches. As with the hydraulic fill method, the finer material was transported down into the pool to form the core, while the coarser material stayed near the beaches to form the shell.

The volume of material placed by this method in 1921 was 500,000 cubic
yards. The following year the rolled fill section was added at the upstream face. This added a further 50,000 cubic yards to the dam. The material for the rolled fill section was brought to the dam by horse-drawn cart, spread in thin lifts by Fresno scrapers, sprinkled and then compacted by routing the hauling equipment over the filled area. Total cost of the dam was approximately $280,000.

Despite the difference in construction methods for the Upper and Lower San Fernando Dams, investigation by bore holes and trenches, made after the earthquake of February 1971, showed no appreciable difference in the material of the dams.

**Foundation of Dam**

The Upper San Fernando Dam is founded on alluvium, consisting of alternating layers of stiff clays and clayey gravels, varying from 50 feet to 60 feet deep. The alluvium is underlain by a poorly cemented conglomeritic sandstone and coarse-grained sandstone of the Saugus formation (Lower Pleistocene), which also forms the abutments.

The dam is constructed directly on the alluvium with a cut-off trench approximately 4 feet deep and 30 feet wide, under the axis of the dam, as the only site preparation (see Fig. 4.2).

**Reservoir and Auxiliary Structures**

The dam contains a reservoir of 1,850 acre-feet capacity. Originally, the reservoir capacity was 1,977 acre-feet, but this was reduced by alluvium washed down by the flood of 1938 and by later construction of dikes along the western side of the reservoir. The spillway, at elevation 1,212.5 feet, is located near the left abutment of the dam.

An outlet tower is located near the upstream toe of the midpoint of the dam. The tower is founded at elevation 1,149 feet and rises to
FIG. 4.2 CROSS-SECTION THROUGH UPPER SAN FERNANDO DAM
(after L.A. Dept. of Water and Power)
elevation 1,239 feet, approximately 90 feet high. The tower has an outside diameter of 20 feet, with stepped internal diameters. It connects to an 8-foot diameter cast-in-place concrete outlet conduit lined with a 62-inch inside diameter, concrete-lined steel pipe, which passes through the embankment. A second outlet pipe, 99 inches in diameter, and passing through the right abutment at an inlet elevation of 1,185 feet, was constructed in 1968.

Instrumentation

Observation wells on the berm and the downstream slope were used to locate the phreatic line. Monuments embedded in the embankment were used to measure deformations. Seepage losses were measured at drains at the abutments and at the downstream toe of the dam.

Earthquake of February 9, 1971

An earthquake, measuring $M = 6.6$ on the Richter Scale, occurred at 6:00 a.m. on February 9, 1971 in the San Gabriel Mountains north of the City of Los Angeles. The epicenter was approximately 6 miles NE of the San Fernando Dam complex. Fault movement was of the thrust type, the north block moving up and over the south block at an angle of about 45 degrees. The focal depth of the onset of rupture was approximately 8 miles. The fault break apparently propagated upward to the south, intersecting the ground surface in the San Fernando area. The fault scarp formed by the earthquake reached a maximum height of about 4 feet at its eastern end, diminishing in height toward the west. However, features resembling a fault break were traced nearly to the eastern edge of the Lower Van Norman reservoir.
Seismoscope records obtained on the crest and the abutment of the lower dam were obtained during the earthquake, and the record for the abutment has been interpreted by R. F. Scott to provide a record of the time history of accelerations at the bedrock underlying the dam. Based on a study of this record, and of the maximum accelerations recorded at sites with varying epicentral distances, Seed et al. (1973) concluded that the maximum rock acceleration at the site of the dams was in the range of 0.55 to 0.69.

Effects of the Earthquake on the Dam

When the water level in the reservoir of the Upper San Fernando Dam was drawn down after the earthquake, several longitudinal cracks were observed running nearly the full length of the dam. The cracks appeared to be multiple shear scarps resulting from the downstream movement of the main body of the dam, the crest showing a settlement of nearly 3 feet and a downstream movement of nearly 5 feet at the center-line of the dam (Fig. 4.2). The downstream movement was evident from the bowing of the parapet wall and the scarps on the upstream face.

A vertical longitudinal crack opened on the downstream slope of the rolled fill section (indicated in Fig. 4.2) and a 2-foot high pressure ridge formed at the downstream toe. Sand boils also formed below the downstream toe.

Damage to the outlet conduit was revealed on inspection after the earthquake. Several cracks, up to 3/4-inch, opened in the section of the conduit in the upstream and central areas of the dam. Compression failure occurred near the downstream toe (Fig. 4.2). However, the magnitude of the movements at the conduit was relatively small, indicating that most of
the movement occurred in the embankment above the conduit. A sinkhole, extending to the surface of the dam downstream from the berm, appears to have been formed by seepage and erosion through a crack in the conduit.

Although the transverse movement of the dam was the major movement, relative longitudinal movement of the abutments, probably less than 2 feet, caused cracks in the spillway and in the roadway across the crest of the dam.

Variations in the water level in three piezometers during and after the earthquake are shown in Fig. 4.3. The shear strains induced in the embankment during the earthquake caused increases in the pore water pressure, which slowly dissipated after the earthquake. These increases are evidenced by the changes in water level in the piezometers. During the earthquake, the water overflowed from piezometers #1 and #2, so the true increase is not known.

Judging from the field observations, movements appear to have been general throughout the dam, and not confined to a unique slip surface. The movements were probably due to a weakening of the soil due to the rise in pore water pressure. The sand boils indicate that liquefaction did take place in some areas, and part of the movements may have been due to strength loss due to liquefaction.

Measured Displacements

A survey of the monuments embedded in the dam was made shortly after the earthquake. Vertical and horizontal displacements were measured at several points on the maximum section of the dam (Fig. 4.4) including: the upstream parapet wall, the midpoint of the downstream slope of the rolled fill section, the upstream and downstream ends of the berm, the midpoint of the downstream slope and the downstream toe. The displacement
FIG. 4.3 CHANGES IN WATER LEVEL IN PIEZOMETERS FOLLOWING THE EARTHQUAKE - UPPER SAN FERNANDO DAM (after Seed et al)
FIG. 4.4 DISPLACEMENTS MEASURED AFTER EARTHQUAKE
of the crest was 4.9 feet downstream and 2.5 feet vertically downward. The horizontal movements were progressively larger toward the downstream end of the berm where the displacement was 7.2 feet. Settlement was 2.5 feet at the crest and 1.4 feet at the downstream edge of the berm, but only 0.2 feet at the toe of the rolled fill section. Unfortunately, this is the only measured movement of the central core section of the dam and so the maximum amount of heaving is not known. The measured displacement at the middle of the slope was 5.8 feet downstream with 1.7 feet vertical settlement, and the measured displacements at the downstream toe were 3.6 feet horizontal downstream movement and 0.2 feet vertical heaving. A 2-foot high pressure ridge was reported to have developed at the downstream toe of the dam -- though this ridge may have occurred in the hydraulic fill blanket somewhere below the toe of the dam. No measurements were made on the upstream slope.

Static and Dynamic Analysis of the Dam

Soil Properties

The field investigation carried out as a preliminary to the stability analysis of the Upper San Fernando Dam consisted of borings, trenches, and seismic surveys (Seed et al., 1973). Trenching showed the hydraulic fill to be made up of alternating layers of fairly clean sand and silty to clayey sands, with occasional layers of clay. Layering was most pronounced at the outer edge of the embankment where the material was generally coarser, while at the center of the embankment layering was not evident and the material was a fine sandy silt with some clay. Gradation curves of the material taken from the bottom of a transverse trench cut into the downstream berm are shown in Fig. 4.5. A cross-section through the middle of the embankment
FIG. 4.5 DISTRIBUTION OF GRAIN SIZES IN HYDRAULIC FILL FROM OUTER SHELL TOWARDS CENTER OF EMBANKMENT - UPPER SAN FERNANDO DAM (after Seed et al)
showing the locations of some of the borings and the materials encountered is shown in Fig. 4.5.

**Static Analysis**

A static finite element analysis was performed by Seed et al. (1973) as a preliminary step in the stability analysis of the Upper San Fernando Dam. The analysis, using nonlinear soil properties and in which construction conditions are simulated by progressively adding layers of elements to the model, is described fully in Chapter 2 and by Kulhawy, Duncan, and Seed (1969). This analysis, which includes seepage forces, gives the stress conditions acting in the dam prior to the earthquake. The initial stresses determined for each element are later used in conjunction with the dynamic analysis and laboratory testing program to calculate the strain potential for the element. The finite element mesh used is shown in Fig. 4.6. The nonlinear soil properties used are shown in Table 4.1. The contours of horizontal stress and strain ($\sigma_x$ & $\varepsilon_x$) and shear stress and shear strain ($\tau_{xy}$ & $\gamma_{xy}$) are shown in Fig. 4.7 and 4.8.

**Dynamic Analysis**

The dynamic finite element analysis performed on the Upper San Fernando Dam is described by Seed et al. (1973). The earthquake acceleration history used as input to the dynamic analysis was a modification of the record obtained at Pacoima Dam, with a peak acceleration of 0.6 $g$ (Fig. 4.13). Only the horizontal component of the Pacoima record was used in the analysis since shear stresses caused by vertical motions are insignificant compared to those caused by the horizontal motions. Also, the change in pore water pressure caused by vertical motions is small compared to that caused by horizontal motions. Since it is the weakening of the soil due to pore-water pressure
FIG. 4.6  FINITE ELEMENT MESH USED IN DEFORMATION ANALYSIS
### Table 4-1

**Soil Parameters Used in Nonlinear Static Analysis**

**Upper San Fernando Dam**

<table>
<thead>
<tr>
<th>Soil Parameter</th>
<th>Symbol</th>
<th>Values used in Analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Rolled Fill</td>
</tr>
<tr>
<td>Dry Unit Weight</td>
<td>$\gamma_d$ (pcf)</td>
<td>125</td>
</tr>
<tr>
<td>Buoyant Unit Weight</td>
<td>$\gamma_b$ (pcf)</td>
<td>78</td>
</tr>
<tr>
<td>Cohesion</td>
<td>$c$ (psf)</td>
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</tr>
<tr>
<td>Friction Angle</td>
<td>$\phi$</td>
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</tr>
<tr>
<td>Modulus Number</td>
<td>$K$</td>
<td>300</td>
</tr>
<tr>
<td>Modulus Exponent</td>
<td>$n$</td>
<td>0.76</td>
</tr>
<tr>
<td>Failure Ratio</td>
<td>$R_F$</td>
<td>0.90</td>
</tr>
<tr>
<td>Poisson's Ratio Parameters</td>
<td></td>
<td>G</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$F$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$d$</td>
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FIG. 4.7 STRESS AND STRAIN IN EMBANKMENT BEFORE EARTHQUAKE
FIG. 4.8 STRESS AND STRAIN IN EMBANKMENT BEFORE EARTHQUAKE
increases which permits significant deformations to occur during the earthquake, the neglect of the vertical component of ground motion is felt to have little effect on the results.

The method used is described fully in Chapter 2 of this report. The mesh was the same as that used for the static analysis. In conjunction with this analysis, cyclic triaxial tests were performed on all materials of the dam, under a range of consolidation conditions and cyclic loads. By comparing an element of the dam with a sample in the laboratory under the same initial stress conditions and cyclic stress history, interpolating as necessary, it is possible to determine a strain potential for each element of the dam. The results of the dynamic analysis, expressed as strain potential values, are shown in Figs. 4.10 and 4.11.

Analysis of Permanent Deformations

Approximate Method

The first attempt to approximate the deformations of the Upper San Fernando Dam was made by Seed et al. (1973). The average shear strain potential for a vertical section at the center of the dam was multiplied by the section height to obtain a first estimate of the downstream movement of the crest, as shown in Fig. 4.12. The value calculated for the horizontal downstream movement was approximately 6 feet. This method of analysis does not permit computation of vertical movements.

Linear Modified Modulus Analysis

Permanent deformations of the Upper San Fernando Dam may also be computed by the linear gravity turn-on analysis procedure. In this procedure it is assumed that the effect of the earthquake on the dam is a
FIG. 4.9 TIME HISTORY OF ACCELERATION IN BASE ROCK
FIG. 4.10 STRAIN POTENTIAL IN HYDRAULIC FILL - UPPER SAN FERNANDO DAM
(after Seed et al)
FIG. 4.11 CONTOURS OF SHEAR STRAIN POTENTIAL (PERCENT) IN HYDRAULIC FILL
Average shear strain for elements shown = 17%.
Average depth of shear zone = 38 ft.
Deformation of surface of embankment relative to alluvium = 0.17 x 38 = 6.5 ft.
Near crest: Average shear strain = 14%.
Average depth of shear zone = 40 ft.
Deformation of crest relative to base = 0.14 x 40 = 5.6 ft.

FIG. 4.12 DISTRIBUTION OF SHEAR STRAIN POTENTIAL VALUES FOR UPPER SAN FERNANDO DAM
weakening of the material due to the cyclic shear strains caused by the earthquake motions. The calculated deformations are the difference in displacements between an analysis made with the initial modulus and one made with modified modulus values. The initial modulus is easily found from the initial stress conditions \((E_i = \sigma_i / \epsilon_i)\). The modified modulus is calculated for each element using the initial stresses and strains and the value of the strain potential for the element. For the present study, the reservoir forces were considered to act on the upstream slope, which was assumed to be impermeable for the duration of the earthquake. The method is more fully described in the previous chapter. The results of the analysis applied to the Upper San Fernando Dam are shown in Fig. 4.13(a). With the exception of points near the crest, the horizontal movement at any point is greater than the vertical movement. However, values are considerably less than the measured displacements, by a factor of approximately 6 for the horizontal displacements and a factor of about 3 for the vertical. Vertical settlement over the core area is less than that in the areas upstream and downstream, indicating that a slight amount of heaving is taking place in the core.

The results of a similar analysis performed independently by K. L. Lee are presented in Fig. 4.13(b). Agreement with the method described above is good, with the exception of the central zone of the dam. This is due to a different technique used by Lee to calculate the strain potentials, which gave considerably higher values in the central core zone.

**Nonlinear Modified Modulus Analysis**

This analysis is similar to that discussed above, with the exception that nonlinear stress-strain properties are assumed for the soil. Water
FIG. 4.13(a) CALCULATED DISPLACEMENTS - LINEAR MODIFIED MODULUS METHOD

FIG. 4.13(b) CALCULATED DISPLACEMENTS - LINEAR ANALYSIS (after K. L. Lee)
forces were applied to the upstream slope as before, and only the shear modulus of the soil was considered to be affected by the cyclic shear strains. The method is described in detail in the previous chapter. The deformations calculated by this method are shown in Fig. 4.14. Although horizontal deformations are in general about twice as large as those calculated with the linear analysis approach, the overall deformed shape of the embankment is similar. However, computed displacements are still significantly below those measured after the earthquake.

Nonlinear Analysis Using Equivalent Nodal Point Forces

In this method, a set of nodal point forces was applied to the nodes of the finite element mesh to simulate the deforming effect of the earthquake. [In all the finite element analyses, the same mesh was used (Fig. 4.6).] Only one deformation analysis is required in this approach, but it is necessary to know the stress distribution throughout the dam before the earthquake. In the nonlinear analysis used, the strength properties of the soil are a function of the stress, and hence the first step of the deformations analysis is a determination of the initial stress conditions in the dam. A nonlinear incremental load analysis was used for this purpose; however, a study by Lee and Idriss (1975) has shown that a linear analysis can give good results for the initial stress conditions in a dam. The pre-earthquake stress analysis requires a knowledge of the seepage forces acting in the dam before the earthquake; these forces are easily calculated from a flow net, and finite element programs exist for this purpose (Finn, 1967). However, these analyses require the horizontal and vertical components of permeability as input, and no permeability tests were run on the samples obtained from the field investigation at the Upper San Fernando
FIG. 4.14  CALCULATED DISPLACEMENTS - NONLINEAR MODIFIED MODULUS METHOD
Dem. Fortunately, water level readings before the earthquake in three piezometers along the middle of the dam are known (Fig. 4.6) and from these an approximation to the phreatic line could be made. Equipotential lines were drawn by dividing the head loss through the dam into equal increments, and from these the magnitude and direction of the seepage forces at each element node below the phreatic line were evaluated. One-quarter the volume of the four surrounding elements is associated with each node. The average gradient was found across this area and the seepage force calculated. The seepage force can be assigned to act at the node in the direction of flow, which was estimated, and then resolved into its horizontal and vertical components to give an estimate of the seepage forces acting in the dam. It can be seen from Fig. 4.6 that the slope of the phreatic line is less in the central part of the embankment, the reverse of what would be expected if the material of the core has a lower permeability than the shell. This may be due to lenses or layers of silt or coarser material traversing the core, a possibility with hydraulic fill type of construction.

As the deformation analysis performed is nonlinear, a knowledge of the nonlinear soil parameters is required. These parameters for the drained condition have already been derived for the static analysis performed on the Upper San Fernando Dam by Seed et al. (1973). These parameters are appropriate to the conditions in the dam before the earthquake. For the saturated soils during the earthquake, excess pore pressure built up by the cyclic shear strains will not have time to dissipate, and hence, undrained conditions exist. Nonlinear soil parameters for these soils were therefore calculated from the results of consolidated-undrained tests. Very few such tests were run on the core material and thus considerable
uncertainty exists for the parameters for the clayey core material. The parameters used in this analysis are shown in Table 4.2. In the analysis, undrained parameters were used for material below the phreatic line and also for core material above the phreatic line, as capillary action in the fine core material makes use of these parameters more meaningful. Drained parameters were used elsewhere in the embankment.

A full description of the deformation analysis procedure is given in the previous chapter. Using the nonlinear, undrained stress-strain parameters shown in Table 4.2 and nodal point forces representative of the computed earthquake-induced strain potentials presented in Fig. 4.10, the embankment deformations were computed by the incremental non-linear approach using the program DEFORM-2 and the deformed shape together with the original section are presented in Fig. 4.15. In this computation a zone which is located downstream of the toe of the embankment (see Fig. 4.15) and which was found to have liquefied during the earthquake (as evidenced by sand boils at the ground surface) was assigned very low modulus values to simulate the loss of shearing resistance due to liquefaction. As can be seen from Fig. 4.15, the computed deformations are again much lower than those observed during the earthquake (Fig. 4.15), although they are qualitatively in reasonable agreement in terms of the direction of horizontal movement and vertical settlement at most locations in the embankment. The computed deformations were of the order of 1/4 to 1/5 of those measured after the earthquake. The results of a similar analysis using the procedure illustrated schematically in Fig. 3.5 and embodied in the program DEFORM-1 are shown in Fig. 4.16. Again the computed movements are considerably less than those observed.

However, it was noted in these analyses that significant tension zones
Table 4-2

Soil Parameters Used in Nonlinear Deformation Analysis (Equivalent Force Method)

<table>
<thead>
<tr>
<th>Soil Parameter</th>
<th>Symbol</th>
<th>Values Used in Deformation Analysis</th>
<th>Foundation Layer</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Rolled fill</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>above WT</td>
<td>below WT</td>
</tr>
<tr>
<td>Unit Weight (pcf)</td>
<td>$\gamma$</td>
<td>125</td>
<td>72</td>
</tr>
<tr>
<td>Cohesion (psf)</td>
<td>$c$</td>
<td>2600</td>
<td>1300</td>
</tr>
<tr>
<td>Friction Angle</td>
<td>$\phi$</td>
<td>25</td>
<td>20</td>
</tr>
<tr>
<td>Modulus Number</td>
<td>$K$</td>
<td>300</td>
<td>100</td>
</tr>
<tr>
<td>Modulus Exponent</td>
<td>$n$</td>
<td>0.76</td>
<td>0.76</td>
</tr>
<tr>
<td>Failure Ratio</td>
<td>$R_F$</td>
<td>0.90</td>
<td>0.90</td>
</tr>
<tr>
<td>Poisson's Ratio</td>
<td>$G$</td>
<td>0.30</td>
<td>0.49</td>
</tr>
<tr>
<td>Parameters</td>
<td>$F$</td>
<td>0.10</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>$d$</td>
<td>3.80</td>
<td>0</td>
</tr>
</tbody>
</table>

Notes:  
(1) buoyant weight used for soils below the water table

(2) data from consolidated-undrained tests (CU) used to determine parameters for soils below the water table (soil assumed impermeable during the earthquake)
FIG. 4.15 COMPUTED DISPLACEMENTS USING NODAL POINT FORCE ANALYSIS (DEFORM-2)

FIG. 4.16 COMPUTED DISPLACEMENTS USING NODAL POINT FORCE ANALYSIS (DEFORM-1)
developed in the dam, requiring the use of some computational modifications to simulate the cracking which would develop under these conditions. In fact, in making such analyses a special problem arises with elements near the center-line of the embankment where the initial shear stresses are zero. On either side of this zone, nodal point forces are applied, acting in the direction of the initial horizontal static shear stresses. Thus, elements upstream of the center-line are subjected to nodal point forces acting upstream, while elements downstream of the center-line are subjected to nodal point forces acting downstream. As a result, the central section of the core tends to hold the two sides together and is placed in a condition of tension. This is a fictitious condition since the soil would tend to fail in tension rather than hold the two sides of the embankment together. It may be noted that there was physical evidence of tension cracks in this zone following the earthquake.

As it is difficult in the present finite element analyses to correctly simulate the formation of tension cracks without the use of special joint elements, it was assumed that the above mechanism could be simulated approximately by a softening of the column of elements located in the central portion of the embankment where the tension cracks are expected to form. Accordingly, the computations were repeated with very low modulus values assigned to a column of elements located near the center-line of the embankment as shown in Fig. 4.17. The results for both the nonlinear and the equivalent linear modulus procedures are presented in Figs. 4.17 and 4.18. Again the displacements computed using the equivalent linear modulus procedure were about 30 to 40 percent higher than those using the non-linear procedure. However, the computed deformations in both procedures were found to be about twice to three times the values estimated earlier for the case where no simulation of the formation of tension cracks was attempted. A
FIG. 4.17 COMPUTED DISPLACEMENTS USING NODAL POINT FORCE ANALYSIS (DEFORM-2) WITH SOFT CENTRAL ZONE

FIG. 4.18 COMPUTED DISPLACEMENTS USING NODAL POINT FORCE ANALYSIS (DEFORM-1) WITH SOFT CENTRAL ZONE
comparison with the observed deformations in Fig. 4.4 shows that the computed results for the equivalent linear modulus procedure (Fig. 4.18) ranged from about 60 to 70 percent of the observed values near the crest and central portions of the embankment to about 30 percent at the downstream edge of the berm and the toe of the embankment. Considering the relative horizontal movement between the two points at the crest located on either side of the centerline of the embankment, Fig. 4.18 shows a total relative displacement of about 5 feet. This is analogous to the formation of an open longitudinal tension crack along the crest of the dam. Any tendency for a crack of such dimension to form would undoubtedly allow wedges on the upstream portion to slip into a configuration similar to that shown schematically in Fig. 4.19 providing a distorted section very similar to that observed during the earthquake.

In fact, from an examination of the multiple shear scarps on the upstream slope of the embankment after the earthquake (Fig. 4.2), it seems reasonable to believe that such a deformation pattern may have resulted from a failure mechanism similar to that proposed above. Thus, due to the inertia forces induced by the earthquake, longitudinal tension cracks might be expected to tend to open up along the crest of the embankment due to the tendency of the main body of the dam to move downstream; these cracks may extend to various depths depending on the magnitude of the inertia forces. As a result of this condition several wedges of the upstream portion of the embankment would then tend to slip in the downstream direction to fill up the resulting gaps. Such a mechanism, shown schematically in Fig. 4.19 would explain the formation of the observed shear scarps on the upstream face of the embankment after the earthquake.

Although the analytical approach described above is based on a number
FIG. 4.19 SCHEMATIC CONCEPT OF DEFORMATIONS OF UPPER SAN FERNANDO DAM
of simplifying assumptions and approximations and the exercise of some degree of judgment, considering the uncertainties in the material properties used in the analysis, it appears to provide a reasonable assessments of the deformations of the Upper San Fernando Dam during the earthquake and a basis for evaluating deformations in other dams where major movements are likely to occur. When movements are relatively small, the use of the program DEFORM, in either of the two formulations presented, would seem to provide a reasonable basis for assessing the overall deformed shape of an earth dam due to earthquake shaking.
Chapter 5
Summary and Conclusions

Summary

The deformations induced in the Upper San Fernando Dam by the earthquake of February 9, 1971, were studied by four different methods and the results compared with the deformations measured shortly after the earthquake.

The first method was a direct use of the strain potentials to obtain an approximation of the downstream movement of the crest, and was first used by Seed et al. (1973). Horizontal movements only can be calculated.

The second and third methods were linear and nonlinear analyses respectively, and used a reduced value of the modulus for the soil in conjunction with gravity loading to simulate the earthquake effects. Deformations calculated by the linear method (Lee, 1974) were too low; those calculated by the nonlinear method were higher, but still well below the measured values.

The fourth method, a pseudo-static approach using nonlinear soil properties, gave results in reasonable agreement with the measured deformations, where the upstream and downstream parts of the embankment were allowed to move relative to each other through the introduction of a softened core. This method is basically a pseudo-static technique in which the potential shear strains induced in elements of the dam are used with the nonlinear stress-strain properties to determine the corresponding shear stress changes. Forces are calculated which, if acting at the element nodes, would produce the same changes in shear stress. These forces are then
applied to the nodes of the mesh and the resulting deformations are calculated.

A number of assumptions were made in formulating the method. To calculate the stress change due to the induced strain it was necessary to assume that the confining pressure remains constant during the earthquake and that equivalent nodal point forces act in the direction of the pre-earthquake horizontal shear stresses. The horizontal plane is considered the most critical in the dam, and the shear stresses acting on this plane are considered to have the major effect on the behavior of the dam during the earthquake. These shear stresses are assumed to be the maximum shear stresses.

The applied nodal point forces represent all the forces acting on the dam during the earthquake, as the strain potential calculated in the dynamic analysis is a function of all the forces. Gravity and seepage forces are included in the stresses under which laboratory samples are consolidated. The inertia forces during the earthquake are represented by the cyclic stress applied to the sample.

In the nonlinear stress-strain formulation used in the analysis, the modulus of the soil is a function of the confining pressure. If tensile stresses occur, the modulus of the soil is reduced to a very low value in the analysis, and the surrounding elements are forced to assumed the extra load. Soils in general cannot take tension, requiring the use of special elements or a softened zone near the axis of the embankment to allow for this effect.

It should be noted that during the analysis of the Upper San Fernando Dam using the softened soil concept and gravity load procedure, large vertical settlements were calculated in the soft clay core under the down-
stream toe of the rolled fill section. The resulting deformation created tension in the rolled fill causing instability in the analysis. It appears to be a limitation of such analyses that adjacent zones of very different strength cannot be handled. In the analysis of the Upper San Fernando Dam the modulus number of the rolled fill had to be reduced to overcome this problem. As the major deformation occurred in the hydraulic fill, the softening of the rolled fill would not appreciably affect the calculated displacements of the dam.

In embankments of the type studied the potential strains may be very large. The theory of the finite element method used is based on small strains, and hence some error is introduced into the analysis due to this cause. Nevertheless the method appears to provide a reasonable basis for assessing the deformed shape of an embankment dam due to earthquake shaking with a sufficient degree of accuracy for most practical purposes.
References


APPENDIX

Computer Program "DEFORM"

DEFORM is a finite element program used to calculate the permanent deformations induced in an earth dam by an earthquake. The method of analysis is presented in Chapter 3 of this report; the organization of the program, subroutine by subroutine, is described in this Appendix. Instructions for preparing input for the program are also presented, followed by a listing of the program.

The analysis is run in two stages: in the first, the stresses in the dam before the earthquake are calculated, using a nonlinear step-by-step loading technique. The forces acting on the dam, including gravity and seepage forces, are applied gradually in a preselected number of steps, enabling the nonlinear stress-strain curve of the soil to be approximated by a number of straight segments. The stresses calculated in such an analysis are essentially the same as those determined by the more sophisticated technique where the dam is built up in layers to simulate the actual construction conditions (Kulhawy et al., 1969). The displacements calculated by the first stage of the analysis are not used.

The stress-strain curve for each element is a function of the confining pressure and, therefore, changes with each load step. When the stresses at the end of a load increment are determined, the tangent modulus can be calculated and, consequently, the new stiffness matrix can be formed. The stresses at the end of the next load increment are then calculated, but a process of iteration is required to ensure that the curved stress-strain relationship is followed. Two iterations per loadstep were found to give sufficient accuracy and are used in this
analysis. Thus, the tangent modulus calculated from the previous load step is used in the first iteration to obtain an intermediate value of the stress increase. A mean value of stress is calculated, a new tangent modulus formed, and the second iteration gives the final stress conditions at the end of the load step.

After the second iteration of the final load step, the final values of modulus and Poisson's ratio are calculated. The stresses will be the stresses acting throughout the dam before the earthquake. By assuming that the confining pressure acting on each element does not change due to the earthquake, and thus the strength curve is constant, the strain potential for an element can be used with the stress-strain curve to find the corresponding change in stress. From the change in element shear stress, a set of equivalent horizontal and vertical nodal point forces are calculated.

The second stage of the analysis is to apply the equivalent forces to the dam. The stresses and strains calculated in the first stage are the starting point for the second stage, but the displacement vector is first initialized to zero. Thus, the displacements calculated are those due to the earthquake, and the stresses and strains at the end of the stage are the conditions in the dam after the earthquake.

Two options are available to compute the displacements due to the application of the equivalent nodal point forces. The first option (referred to as DEFORM-1 in Chapter 4) is to use an equivalent linear modulus estimated from the specified strain potential. The second (referred to as DEFORM-2 in Chapter 4) is to use an incremental nonlinear approach where the load is applied in steps similar to that described for the first stage calculations. Both approaches have been described in detail earlier (Chapter 3).
Material properties used in the first stage will generally be determined from consolidated drained tests. Saturated zones of the embankment may be considered impermeable during the earthquake, and hence material properties derived from consolidated-undrained tests are more appropriate for the second stage of the analysis.

The program DEFORM prints the input data, the stresses, strains, moduli, etc. at the end of each load step, and the equivalent nodal point forces at the end of the first stage. At the end of each load step of the second stage, the displacements, both incremental and cumulative, are also printed.

DEFORM uses a dynamic storage technique where all variable arrays are stored in blank common. This technique uses core storage space most economically. After reading the input data, calculating the band width and allocating storage in blank common, the program transfers control to the subroutine EXEC. From EXEC, subroutines are called which form the load vector for the current load step and also the structure stiffness matrix, taking into account the nonlinear behavior of the soil. Boundary conditions are applied and an equation solver, USOL, is used to calculate the displacements. From these, the strains and stresses are calculated by STRESS and the process is repeated for the second iteration. This procedure is repeated for each load step. After the last load step of the first stage, the equivalent nodal point forces are calculated and substituted for the gravity and seepage forces in the load vector, and the analysis is repeated, resulting in the calculation of the deformations due to the earthquake.

Because in a nonlinear analysis as described in this report the modulus of the soil is a function of the stresses, it is necessary to
make an estimate of the stresses corresponding to the initial load step in order to start the analysis. The process of iteration that takes place ensures that the stresses estimated have been corrected by the end of the first load step. The initial estimate of the first load step stresses is made by the subroutine APPROX.
ORGANIZATION OF PROGRAM DEFORM
Organization of Program DEFORM

(1) DEFORM
(main program)
(a) Reads initial data required to set dynamic storage
(b) Dynamically assigns core storage to arrays
(c) Calls INPT - input data reader
(d) Calls BAND - band width calculator
(e) Calculates block size and number of blocks
(f) Calls EXEC - the control subroutine

(2) INPT
(subroutine)
Reads:
(a) Material properties
(b) Modal point data
(c) Element data
(d) Coordinates of boundary nodes (used by APPROX)
(e) Strain potentials of elements

(3) BAND
(subroutine)
Calculates band width of stiffness matrix

(4) EXEC
(subroutine)
Control subroutine
(a) Initializes stress, strain, displacement, and load vectors
(b) Calls BLOCK
(c) Calls USOL to solve simultaneous equations formed by BLOCK
(d) Reads displacements (calculated by USOL) from tape; prints displacement at each iteration punches displacements at end of analysis
(e) Calls STRESS
(f) Repeats steps "b" through "e" for each load step
(5) BLOCK
Forms the structure stiffness matrix in blocks and stores the blocks sequentially on tape. Forms the load vector in blocks and stores it on tape with the stiffness matrix:
(a) Calls MATRIX for each element
(b) Adds element stiffness matrices to form structure stiffness matrix
(c) Forms structure load vector from gravity loads and seepage forces (first stage) and from equivalent nodal point forces (second stage)
(d) Calls BCOND to apply boundary conditions

(6) MATRIX
(subroutine)
(a) Calls NONLIN
(b) Forms Hooke's law relationship for elements
(c) Calls ELMAT
(d) Calculates gravity load vector for elements (first stage only)

(7) NONLIN
(subroutine)
Uses the nonlinear material parameters and the stress in each element (estimated initially by the subroutine APPROX; determined by the subroutine STRESS for subsequent load steps) to determine the current value of the tangent and Poisson's ratio; from these the bulk modulus and shear modulus are calculated for use by subroutine MATRIX

(8) APPROX
(subroutine)
Estimates the initial stress in each element for the first iteration of the first load step (first stage only). The vertical stress is assumed equal to the weight of soil above the center of the element; the horizontal stress is calculated from the stress and the Poisson's ratio

(9) ELMAT
(subroutine)
Forms the isoparametric stiffness matrix for each element using 3x3 Gaussian quadrature; writes the strain-displacement relationship for elements on tape.

(10) BCOND
(subroutine)
Called by BLOCK when the structure stiffness matrix has been formed to apply boundary conditions. If a nodal point is fixed in either direction, the equation corresponding to this degree-of-freedom is
zeroed out and the diagonal term is set to unity. The corresponding value in the solution vector is set to zero, or the fixed displacement if the boundary condition is a specified displacement.

(11) **USOL**  
(subroutine)  
Solves the simultaneous equations block by block and stores the calculated displacements on tape. The technique used is an efficient Gaussian elimination algorithm, with no operations on zero terms in the stiffness matrix. The subroutine was coded by E. Wilson, U.C., Berkeley. Called by EXEC.

(12) **STRESS**  
(subroutine)  
(a) Reads strain-displacement relationship for each element from tape (written by ELMAT) and calculates the strain.

(b) Calculates the stresses; on the first iteration, the stresses calculated are used to determine the mean stress increase in the element; this is used to determine an intermediate value of the modulus. This modulus is used in the second iteration to arrive at a final value of stress increase for the load step. The stress increase is then added to the cumulative stress.

(c) Calls NONLIN at the second iteration of the last load step to calculate the final values of modulus and Poisson's ratio.

(d) Calls EQUIV on the second iteration for the last load step (first stage only).

(e) Transfers equivalent nodal point forces to the load vector at the end of the first stage.

(13) **EQUIV**  
(subroutine)  
Calculates the equivalent nodal point forces due to the change in element shear stress corresponding to the shear strain for the element.
List of Variables and Arrays Used in DEFORM

A array containing the current two blocks of the stiffness matrix. [The stiffness matrix is formed in square blocks - the number of equations equals the band width - and stored with the load vector on TAPE2.] Only two adjacent blocks are held in core storage at any given time for processing by the equation solver, USOL.

\(A_1, A_2, A_3\) working arrays used by equation solver, USOL.

B array containing current blocks of load vector (see under A above).

BINT working array containing intermediate values of load vector

BMOD array containing the bulk moduli of elements

C matrix contains Hooke's law relationship

CM array containing value of soil cohesion for each material

CODE array containing fixity code for nodes -
  1 fixed in X-direction
  2 fixed in Y-direction
  3 fixed in X & Y-direction

COEF array - nonlinear modulus parameter K

DISP matrix containing nodal displacements

DM array - nonlinear Poisson's ratio parameter \(d\)

DPHI array - change in \(\phi\) over 1 log cycle of pressure (\(\Delta\phi\))

ENF matrix - equivalent nodal point forces

EP array - strain potentials of elements

ETAN array - tangential Young's modulus of elements

EXP array - nonlinear modulus exponent \(n\)

FM array - nonlinear Poisson's ratio parameter \(F\)

GAM array - density of soil for each material

GAMW variable - density of water in units of analysis

GM array - nonlinear Poisson's ratio parameter G
ITER current iteration number - 2 iterations are made at each load step to determine the value of the tangent modulus

IX matrix - contains the 4 nodal points associated with each element and the material type of the element

MAT variable - number of different materials

MTYPE material number of element under consideration

NANA variable - defines one of the two options in Stage 2 analysis

NDP variable - number of nodal points in mesh

NEL variable - number of elements in mesh

NLC variable - number of nodes along upper boundary of mesh required to define the geometry of the dam and foundation

NLD variable - number of load steps to be taken in analysis

NSTEP variable - current value of load step

NUMBLK variable - number of blocks into which the stiffness matrix and load vector are divided (computed by program)

P array - element gravity loads

PATM variable - standard atmospheric pressure in units of analysis

PHI array - friction angle of each material ($\phi$)

Q array in blank common which holds all the variably dimensioned arrays used in the program

RF array - nonlinear parameter - failure ratio $R_f$

S matrix - element stiffness matrix

SIGMA matrix - element stresses ($\sigma_x, \sigma_y, \tau_{xy}$)

SIGIT matrix - intermediate element stresses used in iterating

SIGI3 array - minor principal stress for each element before earthquake. These values are used to compute modulus and Poisson's ratio for each element in Stage 2 analysis.

SL array - stress level (ratio of element deviator stress to deviator stress at failure - $SL = 1$ at failure)

SMOD array - shear moduli of elements

ST matrix - element strain-displacement relationship
STAGE variable - contains current stage of analysis
(STAGE=1, analysis to compute initial stresses before earthquake;
STAGE=2, analysis to compute permanent deformations due to
strain potential)

STRN matrix - element strains ($\varepsilon_x, \varepsilon_y, \gamma_{xy}$)

TNU array - tangent Poisson's ratio for elements

VOL variable - volume (area) of element

X array - horizontal nodal coordinates

XL array - horizontal nodal coordinates of boundary nodes

XLD array - horizontal nodal forces (seepage forces in Stage 1,
equivalent nodal point forces in Stage 2)

Y array - vertical nodal coordinates

YL array - vertical nodal coordinates of boundary nodes

YLD array - vertical nodal forces (as in XLD above)
### Files Used By DEFORM

<table>
<thead>
<tr>
<th>Tape</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>TAPE1</td>
<td>scratch tape (file) used by USOL</td>
</tr>
<tr>
<td>TAPE2</td>
<td>stores equations in block form (stiffness matrix and load vector). Written by BLOCK, read by USOL.</td>
</tr>
<tr>
<td>TAPE3</td>
<td>displacements written on TAPE3 by USOL, read by EXEC.</td>
</tr>
<tr>
<td>TAPE4</td>
<td>stores strain-displacement relationship (array ST) for each element. Stored by ELMAT, read by STRESS.</td>
</tr>
<tr>
<td>TAPE5</td>
<td>standard card input file</td>
</tr>
<tr>
<td>TAPE6</td>
<td>standard printer file</td>
</tr>
<tr>
<td>TAPE7</td>
<td>standard card punch file</td>
</tr>
<tr>
<td>TAPE8</td>
<td>scratch tape (file) used by USOL</td>
</tr>
<tr>
<td>TAPE9</td>
<td>scratch tape (file) used by USOL</td>
</tr>
<tr>
<td>TAPE10</td>
<td>physical tape (file) to store element stresses before earthquake used by STRESS, and read by INPT.</td>
</tr>
</tbody>
</table>
Input Data for DEFORM

(1) Control Cards

(a) Title Card (8A10)

Columns 1-80 Job Identification (any characters)

(b) Finite Element Mesh and Analysis Control Parameters (815)

Columns 1-5 NDP - No. of nodes in mesh
Columns 6-10 NEL - No. of elements
Columns 11-15 MAT - No. of different materials in dam
Columns 16-20 NLD - No. of steps in which loads are applied (can be 1 through 5) for both Stages 1 and 2 analyses
Columns 21-25 NLC - No. of nodes which are necessary to define the geometry of the upper boundary of the mesh (used to calculate the weight of soil above each element in order to estimate stresses for the first load step)

Columns 26-30 KPNCH - Key for punching element stresses at end of Stage 1 analysis and nodal displacements at end of Stage 2 analysis onto cards if KPNCH = 1

Columns 31-35 KSTART - Key for restarting Stage 2 analysis after initial stresses before earthquake has been computed (KSTART ≠ 0) and no Stage 1 analysis is made in current computer run.

If KSTART = 0, initial stresses are to be computed from Stage 1 analysis;
If KSTART = 1, initial stresses are to be read from cards;
If KSTART = 2, initial stresses are to be read from TAPE10.

Columns 36-40 NANA - Option to conduct Stage 2 analysis as a one-step linear analysis (NANA = 1) or an incremental nonlinear analysis (NANA = 2).
For example:

```
  43  54
   5  25
     61  80
```

NLC = 6

(c) Constants (2F10.0)

Columns 1-10   PATM - Standard atmospheric pressure (in units of analysis).
Columns 11-20  GAMW - Density of water (in units of analysis).

(2) Material Property Cards

Card 1 (I10,6F10.0)

Columns 1-10   N  - material number
Columns 11-20  GAM(N) - density of material γ
Columns 21-30  COEF(N) - modulus coefficient K
Columns 31-40  EXP(N) - modulus exponent n
Columns 41-50  DM(N) - Poisson's ratio parameter d
Columns 51-60  GM(N) - Poisson's ratio parameter G
Columns 61-70  FM(N) - Poisson's ratio parameter F

Card 2 (4F10.0)

Columns 1-10   CM(N) - cohesion c
Columns 11-20  PHI(N) - friction angle φ
Columns 21-30  DPHI(N) - change in φ per log cycle of pressure
Columns 31-40  RF(N) - failure ratio R_f

Note: Above sequence of two cards per material in the dam repeated for each different material, specifying pre-earthquake properties. Sequence is then repeated specifying the properties during the earthquake.
(3) Nodal Point Cards (2I5,4F10.0)

Columns 1-5 \( N \) - nodal point number
Columns 6-10 \( \text{CODE}(N) \) - fixity code for node (see below)
Columns 11-20 \( X(N) \) - \( x \) coordinate of node
Columns 21-30 \( H(N) \) - \( y \) coordinate of node
Columns 31-40 \( XLD(N) \) - horizontal component of nodal force
Columns 41-50 \( YLD(N) \) - vertical component of nodal force

Note 1:
\( \text{CODE}(N) = 0 \) Node free in \( x \)-direction, Node free in \( y \)-direction.
\( \text{CODE}(N) = 1 \) Node fixed in \( x \)-direction, Node free in \( y \)-direction.
\( \text{CODE}(N) = 2 \) Node free in \( x \)-direction, Node fixed in \( y \)-direction.
\( \text{CODE}(N) = 3 \) Node fixed in \( x \)-direction, Node fixed in \( y \)-direction.

Note 2: If nodal point cards are missing, coordinates of missing nodes will be calculated by linear interpolation; \( \text{CODE} \) will be set to 0.

Note 3: Coordinates are positive to the right and upward.

Note 4: Applied forces are seepage forces or any other forces applied to the dam.

(4) Element Cards (6I5)

Columns 1-5 \( M \) - element number
Columns 6-10 \( IX(M,1) \) - lower left node number
Columns 11-15 \( IX(M,2) \) - lower right node number
Columns 16-20 \( IX(M,3) \) - upper right node number
Columns 21-25 \( IX(M,4) \) - upper left node number
Columns 26-30 \( IX(M,5) \) - element material number

Note 1: For triangular elements, third node number is repeated.

Note 2: If elements are missing, nodal numbers will be generated for the missing elements by incrementing the nodal point numbers of the previous element by unity (NB - this interpolation can be used only if nodes are numbered in the same direction as the elements in a system which may be inconsistent with numbering for minimum band width) and the material number is set equal to that of the previous element.
Note 3: The sequence of numbering element nodes MUST be as listed above, otherwise the nodal forces calculated to simulate the earthquake will be applied incorrectly.

(5) Upper Boundary Nodal Point Cards (2F10.0)

Columns 1-10 \( x \) - coordinate of node

Columns 11-20 \( y \) - coordinate of node

One card per boundary node (i.e., NLC cards). Nodes in sequence from left to right (i.e., nodes 5, 25, 43, 54, 61, and 80) as shown in the example under the control cards input.

(6) Element Strain Potential Cards (I5,F10.0)

Columns 1-5 \( N \) - element number

Columns 6-15 \( EP(N) \) - element strain potential

One card per element; only elements with a non-zero strain potential need be supplied. However, a card MUST be supplied for the last element even if the strain potential is zero. Strain will be assumed zero for missing elements.

(7) Initial Stress Cards (I5,1X,2F7.1,6E10.3) Skip if KSTART = 0 on Card (1b)

Columns 1-5 \( N \) - element number

Columns 7-13 \( XC \)

Columns 14-20 \( YC \)

Columns 21-30 \( \Sigma(1,N) \) - element normal stress \( (\sigma_x) \)

Columns 31-40 \( \Sigma(2,N) \) - element normal stress \( (\sigma_y) \)

Columns 41-50 \( \Sigma(3,N) \) - element shear stress \( (\tau_{xy}) \)

Columns 51-60 \( STRN(1,N) \) - element strain \( (\varepsilon_x) \)

Columns 61-70 \( STRN(2,N) \) - element strain \( (\varepsilon_y) \)

Columns 71-80 \( STRN(3,N) \) - element shear strain \( (\gamma_{xy}) \)

END OF INPUT DATA
FORTRAN LISTING OF PROGRAM "DEFORM"
PROGRAM DEFORD (INPUT, OUTPUT, TAPE5=INPUT, TAPE6=OUTPUT, TAPE1=TAPE2, TAPE3, TAPE4, TAPE8=OUTPUT, TAPE10, PUNCH, TAPE1=PUNCH)

COMMON/CONST1/ NLD, VOL, NTYPE, NSTEP, PAH, GAMM, MNN, TIME(3)
COMMON/CONST2/ ITER, NBLD, NUMBLK, TITLE(8), STAGE, XL(50), YL(50), NLC
COMMON/CONST3/KPUNCH, KSTART, NANA

INTEGER CODE, STAGE

THE DIMENSION OF ARRAY Q AND THE CONSTANT NCOV SET FOR EACH PROBLEM
ACCORDING TO THE FORMULA BELOW:

MINIMUM VALUE OF NCOV AND DIMENSION OF Q

= 2
+ 21 TIMES NUMBER OF ELEMENTS (NEL)
+ 13 TIMES NUMBER OF NODAL POINTS (NJP)
+ 22 TIMES NUMBER OF DIFFERENT MATERIALS (MAT)
+ 2 TIMES BANDWIDTH SQUARED (KBAND**2)
+ 3 TIMES BANDWIDTH (KBAND)

COMMON Q(12000)
NCOV=12000

READ(5,1200) TITLE
WRITE(6,2200) TITLE
READ(5,1001) NDP, NEL, MAT, NLD, NLC, KPUNCH, KSTART, NANA
WRITE(6,2001) NDP, NEL, MAT, NLD, KPUNCH, KSTART, NANA
READ(5,1002) PAT, GAMW
WRITE(6,2002) PAT, GAMW

ASSIGN STORAGE IN BLANK COMMON

L01 = 2
L02 = L01 + NEL#*#
L03 = L02 + NEL
L04 = L03 + NEL#*#
L05 = L04 + NEL#*#
L06 = L05 + NDP
L07 = L06 + NDP#*#
L08 = L07 + NDP
L09 = L08 + NDP
L10 = L09 + NDP
L11 = L10 + NDP#*#
L12 = L11 + MAT#*#
L13 = L12 + MAT#*#
L14 = L13 + MAT#*#
L15 = L14 + MAT#*#
L16 = L15 + MAT#*#
L17 = L16 + MAT#*#
L18 = L17 + MAT#*#
L19 = L18 + MAT#*#
CALL INPT(Q(L01)*Q(L05)*Q(L06)*Q(L07)*Q(L08)*Q(L09)*Q(L11)*Q(L12)*
Q(L13)*Q(L14)*Q(L15)*Q(L16)*Q(L17)*Q(L18)*Q(L19)*Q(L20)*
Q(L21)*Q(L23)*Q(L22),NEL,NCP,NAT)

CALL BAND(Q(L01)*NEL)

M2 = MBAND*2
M3 = MBAND
N2 = NDP*2
N3 = MBAND/2
MP = MBAND
IF(MP .EQ. 0) GO TO 10
NBLK = (NDP/MB) + 1
GO TO 20
10 NBLK = NDP/MB
20 CONTINUE
WRITE(6,2005) NBLK

MB = (MBAND+1)*MBAND
L23 = L22 + NEL*2
L24 = L23
L25 = L24 + NBLK
L26 = L25 + NSB
L27 = L26 + MBAND
L28 = L27 + NDP*2
L29 = L28 + NDP*2
L30 = L29 + NDP*2
L31 = L30 + NEL
L32 = L31 + NEL
L33 = L32 + NEL
L34 = L33 + NEL
LEND = L34 + NEL

END OF STORAGE ALLOCATION

CHECK IF DIMENSION OF ARRAY IS SUFFICIENT

NDIF = NLEV - LEND
WRITE(6,2003) LEND
IF(NDIF .GE. 0) GO TO 30
NDIF = IABS(NDIF)
WRITE(6,2004) NDIF
STOP
30 CONTINUE
STAGE = 1
40 CONTINUE
IF(STAGE .EQ. 1) WRITE(6,2006)
IF(STAGE .EQ. 2) WRITE(6,2007)
CALL EXEC(Q(L21) + Q(L2) + Q(L3) + Q(L4) + Q(L5) + Q(L6) + Q(L7) + Q(L8)),
1 Q(L9) + Q(L10) + Q(L11) + Q(L12) + Q(L13) + Q(L14) + Q(L15) + Q(L16),
2 Q(L17) + Q(L18) + Q(L19) + Q(L20) + Q(L21) + Q(L22) + Q(L23) + Q(L24),
3 Q(L25) + Q(L26) + Q(L27) + Q(L28) + Q(L29) + Q(L30) + Q(L31) + Q(L32),
4 Q(L33), Q(L34),
5 NEL, NOP, MAT, N3, N5, N7, N9)

C STAGE = STAGE + 1
C IF(NANA = EQ(1)) NLD=1
IF(STAGE = EQ(2)) GO TO 40
C
100 FORMAT (9A17)
101 FORMAT (9I5)
102 FORMAT (9F14.7)
200 FORMAT (1H1, 8A10/)
201 FORMAT (20H NUMBER OF NODAL POINTS-------- I10 /
1 30H NUMBER OF ELEMENTS-------------------- I10 /
2 30H NUMBER OF DIFF. MATERIALS----------- I10 /
3 30H NUMBER OF LOAD STEPS--------------- I10 /
4 30H KEY FOR PUNCH OUTPUT----------------- I10/
5 30H KEY FOR RESTARTING RUN------------ I10, 29H(IF.MF,0, START WITH 6H STAGE 2) / 30H OPTION FOR TYPE OF ANALYSIS - - I10,
75H(1 FOR 1-STEP LINEAR, 2 FOR INCREMENTAL NONLINEAR))
202 FORMAT (1X, 29H ATMOSPHERIC PRESSURE-------- F17.3//
1 1X, 29H UNIT WEIGHT OF WATER-------- F10.3)
203 FORMAT (////10X, 24H STORAGE REQUIRED BY BLANK COMMON =, 18, 2X,
1 5H WORDS//)
204 FORMAT (1X, 49H SPECIFIED DIMENSION FOR ARRAY Q IN BLANK COMMON IS
* TOO SMALL BY - - 18, 2X, 49H WORDS)
205 FORMAT (////10X, 18H NUMBER OF BLOCKS =, 15)
206 FORMAT (1H1, 5X, 73HFIRST STAGE - CALCULATION OF INITIAL STRESSES AND
* EQUIVALENT NODAL FORCES)
207 FORMAT (1H1, 5X, 71HSECOND STAGE - CALCULATION OF PERMANENT DEFORMATIONS AND FINAL STRESSES)
C END

SUBROUTINE INPT(IX, CODE, X, Y, XLD, YLD, GAM, COEF, EXP, DX, X, F, Y, CH, PHI,
1 DPHI, RE, EP, SIGMA, STRN, VEL, NOP, VAP)
COMMON/CONST1/ NLD, VOL, NTYPE, NSTEP, PATM, GAMM, VAP, TIME(2)
COMMON/CONST2/ ITERM, MBA, MUL, T, TITLE(8), STAGE, XL(50), YL(50), NLC
COMMON/CONST3/ KPH, CH, START, NANA

DIMENSION GAM(WAT), COEF(WAT), EXP(WAT), DX(WAT), X(WAT), CH(WAT),
1 DPHI(WAT), RE(WAT), EP(WAT)
DIMENSION X(NOP), Y(NOP), XLD(NOP), YLD(NOP), CODE(NOP),
DIMENSION IX(NEL), EP(NEL), SIGMA(NEL, VEL), STRN(NEL)

INTEGER CODE, STAGE
WRITE (6, 214)
WRITE (6, 2003)
READ MATERIAL PROPERTIES
DO 40 J = 1, ?
IF (J * EQ. 1) GO TO 10
WRITE (6, 2004)
WRITE (6, 2003)
10 DO 30 I = 1, N, MAT
K = (J-1) * MAT
READ (5, 1023) N, GAM(N+K), COEF(N+K), EXP(N+K), CM(N+K), CV(N+K), FM(N+K)
1 + CM(N+K) * PH1(N+K) * PH2 + CM(N+K) * FM(N+K)
WRITE (6, 2004) N, GAM(N+K), COEF(N+K), EXP(N+K), CM(N+K), CV(N+K), FM(N+K)
30 CONTINUE
40 CONTINUE

READ NODAL POINT COORDINATES AND FORCES (INTERPOLATE FOR MISSING V
WRITE (6, 2006)
70 = 3
60 READ (5, 1025) N, CODE(N), X(N), Y(N), XLD(N), YLD(N)
NL = L + 1
ZX = N - L
IF (L * EQ. 0) GO TO 70
DX = (X(N) - X(L)) / ZX
DY = (Y(N) - Y(L)) / ZX
70 L = L + 1
IF (N-L) 100, 90, 80
80 CODE(L) = 0
X(L) = X(L-1) + DX
Y(L) = Y(L-1) + DY
XLD(L) = 0.0
YLD(L) = 0.0
GO TO 70
90 WRITE (6, 2005) (K, CODE(K), X(K), Y(K), XLD(K), YLD(K), K = NL, N
IF (NDP-N) 100, 110, 60
100 WRITE (6, 2009) N
STOP
110 CONTINUE

READ ELEMENT NODAL NUMBERS AND MATERIAL NUMBERS
WRITE (6, 2010)
N = 0
130 READ (5, 1001) M, (IX (M, I), I = 1, 5)
140 N = N + 1
IF (N * LE. N) GO TO 170
GO TO 150
150 IX (M, K) = IX (M-1, K) + 1
IX (N, 1) = IX (N-1, 1)
170 WRITE (6, 2012) N, (IX (N, I), I = 1, 5)
IF (M * GT. N) GO TO 140
IF (NEL * GT. N) GO TO 130
READ AND PRINT BOUNDARY COORDINATES
READ(5,1036) (XL(I),YL(I),I=1,NLC)
WRITE(6,2003)
WRITE(6,2012) (XL(I),YL(I),I=1,NLC)

READ STRAIN POTENTIAL
WRITE(6,2010)

M = 0
200 READ(5,1036) N,EP(N)
2-1 M = M + 1
IF(N.LE.M) GO TO 202
EP(M) = 0.0
202 WRITE(6,2011) Y,EP(M)
IF(N.GT.M) GO TO 201
IF(N.EQ.0) GO TO 200
NT=5
IF(KSTART .EQ. 1) NT=10
WRITE(6,3001)
DO 250 I=1,NEL
READ(INT,3000) NXC,YC,(SIGMA(J,N),J=1,3),(STRN(J,N),J=1,3)
WRITE(6,3103) NXC,YC,(SIGMA(J,N),J=1,3),(STRN(J,N),J=1,3)
250 CONTINUE
260 CONTINUE
C 300 RETURN
C 1001 FORMAT(6I5)
1003 FORMAT(1I3,6F10.0/4F10.0)
1004 FORMAT(15,F10.0)
1005 FORMAT(2I5,4F10.0)
1006 FORMAT(2F10.0)
2000 FORMAT(1H1,15X,7HELEMENT,13X,7HSIGMA-X,8X,7HSIGMA-Y,9X,6HTAU-YY,1 
10X,5HEPS-X,10X,5HEPS-Y,9X,6HGAM-XY/)
2002 FORMAT(113,3X,41X,15),5X,15)
2003 FORMAT(1HC,27X,18HMODULUS PARAMETERS,4X,24HP0ISSON RATIO PARAMETER,1 
1S,10X,19HSFRENGTH PARAMETERS/, 
2 1H3,6X,3HMAT. NO.,3X,7HDENSITY,5X,1HK,11X,1HN,9X,1HD,10X,3 
1HG,9X,1HF,9X,1HC,8X,3HPHI,5X,9DELTA PHI,5X,2HFX/)
2004 FORMAT(18X,13X,4X,15),10X,)
2005 FORMAT(1I2+1I0,2F12,26X,1F12,5X,1F12,3)
2006 FORMAT(1H1,8X,4HNODE,5X,4HTYPE,4X,7HX-COORD,5X,7HY-COORD,11X, 
1 7HX-FORCE,10X,7HY-FORCE/)
2008 FORMAT(1H1,8X,16HBOUNDARY NODES/6X,14H----------------- //20X, 
1 5HX-ORD,10X,5HY-ORD/)
2009 FORMAT (2H2NODAL POINT CARD ERROR M= 15)
2-17 FORMAT(1H1,8X,7HELEMENT,1IX,16HSTRAIN POTENTIAL/)
2-11 FORMAT(1I1,14X,41)
2-12 FORMAT(15X,F10.2,3X,F10.2)
2-13 FORMAT(1H1,5X,7HELEMENT,5X,1HI,5X,1HJ,5X,1HK,5X,1HL,5X, 
1 8HMATERIAl/)
2014 FORMAT(1H1,8X,37HMATERIAL PROPERTIES BEFORE EARTHQUAKE/)
2015 FORMAT(1H1,8X,37HMATERIAL PROPERTIES DURING EARTHQUAKE/)
SUBROUTINE EXEC (IX, S, SIG, SIGT, CODE, X, Y, XL, YLD, DISP, GAM, 1
   COEF, EXP, DM, CM, FM, CM, PHI, RF, EP, STRN, A, A1, A2, 2
   A3, B, ENF, EINT, SMOD, ETAN, TNU, SL, SIGI3, 3
   NEL, NDP, MAT, M2, M3, NS9, N2)

COMMON/CONST1/ NLD, VOL, MTYPE, NSTEP, PATM, GAMMA, NNN, TIME(3)
COMMON/CONST2/ ITER, MBAND, NUMBLK, TITLE(8), STAGE, XL(50), YL(50), NLC
COMMON/CONST3/ KPNCH, KSTART, NANA

DIMENSION GAM(MAT), COEF(MAT), EXP(MAT), DM(MAT), CM(MAT), FM(MAT), 1
   CM(MAT), PHI(MAT), RF(MAT), DM(MAT) 2
DIMENSION IX(NEL, 51), EP(NEL), SIGMA(3, NEL), SIGIT(3, NEL), STRN(3, NEL), 1
   BMOD(NEL), SMOD(NEL), ETAN(NEL), TNU(NEL), SL(NEL), SIGI3(NEL) 2
DIMENSION X(NDP), Y(NDP), XL(NDP), YLD(NDP), CODE(NDP), 1
   DISP(NDP, 2), ENF(NDP, 2), 3(N2), EINT(N2) 2
DIMENSION A(M2, M3), A1(NS9), A2(NS2), A3(M3)
DIMENSION P(8), ST(3, 8), C(3, 3), S(8, 8)

INTEGER CODE, STAGE

INITIALIZE DISPLACEMENTS, STRESSES AND STRAINS

DO 10 N = 1, NDP
DO 10 I = 1, 2
10 DISP(N, I) = 0.0
DO 20 J = 1, NEL
DO 20 I = 1, 3
20 SIGT(I, J) = 0.0
CONTINUE

IF (KSTART .NE. 0) GO TO 60
IF (STAGE .EQ. 2) GO TO 40

INITIALIZE STRESSES AND STRAINS (FIRST STAGE ONLY)

DO 30 J = 1, NEL
DO 30 I = 1, 3
30 SIGMA(I, J) = 0.0
STRTN(I, J) = 0.0
CONTINUE

GO TO 60
CONTINUE

CHANGE MATERIAL PROPERTIES FROM PRE-EARTHQUAKE TO EARTHQUAKE VALUE

DO 50 I = 1, MAT
J = I + MAT
GAM(I) = GAM(J)
COEF(I) = COEF(J)
EXP(I) = EXP(J)
DM(I) = DM(J)
GM(I) = GM(J)
FM(I) = FM(J)
CM(I) = CM(J)
PHI(I) = PHI(J)
DPHI(I) = DPHI(J)
RF(I) = RF(J)

CONTINUE

DO 500 NAM = 1, NLD
ITER=0
IF(STAGE.EQ.2.AND.NANA.EQ.1) ITER=1
IF (KSTART .NE. 0) ITER=1

CONTINUE

CALL SECOND(T1)
IF (KSTART .NE. 0) GO TO 100

GENERATE SIMULTANEOUS EQUATIONS

CALL BLOCK(A1,A2,A3,BINT,P,ST,IX,Y,CODE,YLD,YLD,GAM,CM,PHI,DPHI,COEF,
EXP,DM,GM,FM,RF,ETAN,SMOD,SMOD,TNU,SIGMA,SIGIT,SL,
SIGI3,NEL,NLD,MBAND,NUMB,NUMBLK,NS3,2,1,8,9,3)

CALL SECOND(T2)
TIME(1) = T2 - T1

SOLVE SIMULTANEOUS EQUATIONS

CALL USOL(A1,A2,A3,MBAND,MBAND,1,NUMBLK,NS3,2,1,8,9,3)

CALL SECOND(T3)
TIME(2) = T3 - T2

READ DISPLACEMETNS FROM TAPE

REWIND 3
NO=M-band*NUMB
DC BS I=1,NUMBLK
JN1=NO-M-band+1
SUBROUTINE STRESS(BMOD,SMOD,ETAN,TNNU,SL,SIGI,SIGA,STRN,EP,
1     GAM,COEF,EXP,DM,GM,FN,CM,PHI,CPHI,RF,SLG13,
2     IX,XY,CODE,XLD,YLD,BS,ENF,C,ST,SP,
3     NEL,NDP,MAT,M2,M2)

COMMON/CONST1/NLD,VOL,MTYP,NSTP,PATM,GAM,NAM,TIME(3)
COMMON/CONST2/ITER,VBAND,NUMLK,TITLE(8),STAGE,XY(50),YL(50),NLC
COMMON/CONST3/SPCH,START,NANA

DIMENSION GAM(MAT),COEF(MAT),EXP(MAT),DM(MAT),GM(MAT),FN(MAT),
*     CM(MAT),PHI(MAT),CPHI(MAT),RF(MAT)
DIMENSION IX(NEL,5),FP(NEL),SIGA(3,NEL),SIGI(3,NEL),STRN(3,NEL),
1     BMOD(NEL),SMOD(NEL),ETAN(NEL),TNNU(NEL),SL(NEL),SIG13(NEL)
DIMENSION X(NDP),Y(NDP),CODE(NDP),ENF(NDP+2),B(N2)
DIMENSION XLD(NDP),YLD(NDP)
DIMENSION P(B),ST(B,B),C(B,B),S(B,B),D(B,B),SIG(6)

INTEGER STAGE,CODE

REWIND 4

*PRINT=*:

INITIALIZE EQUIVALENT NODAL FORCES

IF(STAGE.NE.1) GO TO 20
DO 10 I = 1,NDP
DO 10 J = 1,2
ENF(I,J) = 0.0
10 CONTINUE

20 CONTINUE

START ELEMENT LOOP

DO 30 N = 1,NEL
IX(N,5) = IAPS(IX(N,5))
"TYPE = IX(N,5)
M = MTYPE

DO 30 I = 1,6
SIG(I) = 0.0
30 CONTINUE
DO 40 I = 1, 3
DO 40 J = 1, 3
ST(I, J) = 0.0
40 CONTINUE

IF (ITER.EQ.11) GO TO 60

DO 50 I = 1, 3
SIGIT(I, N) = SIGMA(I, N)
50 CONTINUE

60 CONTINUE
IF (KSTART .NE. 3) GO TO 90

DO 70 I = 1, 4
II = 2*I
JJ = 2*IX(N, I)
P(II-1) = B(JJ-1)
P(II) = B(JJ)
70 CONTINUE

READ ELEMENT STRAIN-DISPLACEMENT RELATIONSHIP FROM TAPE 4

READ(4) ST

DO 80 I = 1, 3
D(I+1) = 0.0
DO 80 K = 1, 8
D(I+1) = D(I+1) + ST(I, K)*P(K)
80 CONTINUE

C(1, 1) = BMOD(N) + SMOD(N)
C(1, 2) = BMOD(N) - SMOD(N)
C(1, 3) = C.0
C(2, 1) = C(1, 2)
C(2, 2) = C(1, 1)
C(2, 3) = C.0
C(3, 1) = C.0
C(3, 2) = C.0
C(3, 3) = SMOD(N)

DO 90 I = 1, 3
DO 90 K = 1, 3
SIGIT(I) = (SIGIT(I) + C(I, K)*D(K, I))
90 CONTINUE

IF (ITER.EQ.1) GO TO 110

AVERAGE STRESSES

110 DO 120 I = 1, 3
SIGIT(I, N) = SIGMA(I, N) - 0.5*SIG(I)
SIG(I) = SIGIT(I, N)
120 CONTINUE
GO TO 140
ADD STRESS INCREMENTS TO PREVIOUS STRESSES

DO 120 I = 1,3
   SIGA(I, N) = SIGA(I, N) + SIGA(IN, N) - SIGA(IN, N)
120 CONTINUE

OUTPUT STRESSES

IF (ITER.EQ.0) GO TO 300

CALCULATE PRINCIPAL STRESSES

CC = (SIGA(2) + SIGA(1)) / 2.0
BB = (SIGA(2) - SIGA(1)) / 2.0
CR = SQRT(5*BB**2 + SIGA(3)**2)

SIG(4) = 0.0
IF (SIGA(3).EQ.0.0.AND.BB.EQ.0.0) GO TO 150
SIG(4) = 28.648*ATAN2(SIGA(3), BB)
150 CONTINUE

SIG(5) = CC + CR
SIG(6) = CC - CR
IF (KSTART.EQ.0) OR STAGE.EQ.1. AND NSTEP.EQ.NLD. AND ITER.EQ.1)
   SIGIT(N) = SIG(6)

IN = IX(N+1)
JN = IX(N+2)
KN = IX(N+3)
LN = IX(N+4)
IF (KSTART.EQ.0) GO TO 175
IF (NSTEP.EQ.NLD) GO TO 160

DETERMINE FINAL VALUE OF MODULUS AND POISSON'S RATIO

SIGIT(1, N) = SIGA(1)
SIGIT(2, N) = SIGA(2)
SIGIT(3, N) = SIGA(3)

CALL NONLIN(IN, JN, KN, LN, N, ETAN, TNU, BMOD, SMOD, EI, HCS, GAM, COEF, 1
EXP, DM, 3X, FM, CV, PHI, DPHI, RF, SIGIT, SIGA, SL, X, Y, IX, 2
SIGI3, NEL, NDP, MAT)

160 CONTINUE

CALCULATE STRAINS (X, Y, XY)

DO 170 I = 1,3
STRN(I,N) = STRN(I,N) - D(I,1)*102.0
172 CONTINUE

175 CONTINUE
EPSX = STRN(1,N)
EPSY = STRN(2,N)
GAMXY = STRN(3,N)

IF (MPRINT) 190,190,190
190 WRITE (6,190)
191 FORMAT (1HI)
WRITE (6,191) NSTEP
192 MPRINT = MPRINT - 1
IF (KSTART .NE. 0) GO TO 230

IF (KN.EQ.LN) GO TO 230
XC = (X(IN)+X(JN)+X(KN)+X(LN))/4.0
YR TO 210
210 XC = (X(IN)+X(JN)+X(KN))/3.0
YC = (Y(IN)+Y(JN)+Y(KN))/3.0
213 CONTINUE

PRINT "MODULES, STRESSES AND STRAINS"
WRITE (6,2303) N*ETAN(N),BMOD(N),SMOD(N),TNJ(N),SIG(1),SIG(2),
1 SIG(3),EPSX, EPSY,GAMXY,SIG(4),SL(N),N

IF (NSTEP.EQ.NLD.AND.ITER.EQ.1.AND.STAGE.EQ.1)
1 WRITE (13,2303) N,XC,YC,SIG(1),SIG(2),SIG(3),EPSX,EPSY,GAMXY
IF (KPNCH.NE.1) GO TO 220
IF (NSTEP.EQ.NLD.AND.ITER.EQ.1)
1 WRITE (7,2303) N,XC,YC,SIG(1),SIG(2),SIG(3),EPSX,EPSY,GAMXY

220 CONTINUE
IF (STAGE.NE.1) GO TO 300
IF (NSTEP.NE.NLD) GO TO 300
IF (ITER.NE.1) GO TO 303
230 CONTINUE

NY=M+MAT
CALL NONLIN(N,IN,JN,KN,LN,NH,ETAN,TNU,BMOD,SMOD,EF,HCS,GAM,COEF,
1 EXP,DM,GM,FM,GM,PHI,DPHI,RF,SIGIT,SIGMA,SL,X,Y,IX,
2 SIGI2,NEL,NDP,MAT)

CALCULATE EQUIVALENT NODAL POINT FORCES
IF (EP(N) .LE. 7.0001) GO TO 250
CALL EQIV(N,EPSX,EPSY,GAMXY,EF,HCS,EPS,SIG,IX,ENFX,Y,
1 Y,NEL,NDP,ETAN,TNU,BMOD)
250 CONTINUE
WRITE (6,1001) N,ETAN(N),BMOD(N),SMOD(N),TNJ(N),SL(N)
1001 FORMAT (15*I,1P3E11.3,1O12.1,1*6T,73X,F7.3)
TRANSFER EQUIVALENT NODAL POINT FORCES INTO LOAD VECTOR

IF (KSTART .NE. 2) GO TO 305
IF (STAGE .NE. 1) GO TO 410
IF (NSTEP .NE. NLD) GO TO 410
IF (ITER .NE. 1) GO TO 410

305 DO 400 I = 1, NDP
   XLD(I) = ENF(I, 1)
   YLD(I) = ENF(I, 2)
   IF (CODE(I) .EQ. 1) GO TO 350
   IF (CODE(I) .EQ. 2) GO TO 310
   IF (CODE(I) .EQ. 3) GO TO 350

310 XLD(I) = .FALSE.
   IF (CODE(I) .EQ. 3) GO TO 320
   GO TO 360

320 YLD(I) = .FALSE.

350 CONTINUE

400 CONTINUE

PRINT EQUIVALENT NODAL POINT FORCES
WRITE(6, 2005)
WRITE(6, 2006) (N, CODE(N), X(N), Y(N), (ENF(N, L), L = 1, 2), N = 1, NDP)

410 ITER = ITER + 1
RETURN

2001 FORMAT (1X, 6HYOUNGS6, 6X, 443UKN6, 6X, 58HEAR6, 5X, 7HPOISS6, 11X, 14HELEME6T STRESS, 14X, 24HELEME6T STRAIN (PERCENT), 9X, 6HSTRESS/1H, 4HELE6YM, 4X, 7HMODUL6US, 4X, 7HMODUL6US, 4X, 7HMODUL6US, 5X, 5HRAT61, 5X, 7HSIG6MA, 4X, 7HSIG6MA, 3X, 6HTAU, XY, 6X, 5HEPS, 1X, 5HEPS, 1X, 5HEPS, 1X, 6HSAM, XY, 3X, 5HAN6LE, 3X, 5HLE6VEL, 2X, 4HE6LEM/1H)
2002 FORMAT (16, 2X, 1P3E11, 3, 1P3E11, 3, 2X, 1P3E11, 3, 2X, 1P3E11, 3, 2X, 1P3E11, 3, 2X, 1P3E11, 3)
2003 FORMAT (15, 1X, 1E13, 3)
2004 FORMAT (215, 1X, 1E13, 3)
2005 FORMAT (1H1, 5X, 23EQUIVALENT NODAL FORCES/6X, 23H------------ 1------------ 9X, 4HNODE, 4X, 4HTYPE, 4X, 7HX-COORD, 6X, 7HX-COORD, 9X, 7HX-FORCE, 11X, 7HX-FORCE/) 2006 FORMAT (1I12, 1I12, 2F12, 2, 6X, 1F12, 3, 5X, 1F12, 3)
2014 FORMAT (1X, 11HSTEP NUMBER, 13/)

END
COMMON/CONST3,KPNC,H,KSTART,NAMA

DIMENSION GAM(MAT),COEF(MAT),EXP(MAT),DM(MAT),GM(MAT),FM(MAT),
   CM(MAT),PHI(MAT),DPHI(MAT),RF(MAT)
DIMENSION IX(NEL),SIGMA(3,NEL),SIGIT(3,NEL),SIGIT(NEL),
   BND(NEL),M(OV(nel),ETAN(NEL),TNU(NEL),SL(NEL)
DIMENSION X(NDP),Y(NDP)

INTEGER STAGE

IF(ITER.EQ.0) GO TO 10
CC=(SIGIT(2,N)+SIGIT(1,N))/2.
BB=(SIGIT(2,N)-SIGIT(1,N))/2.
CR=SQRT(SIGIT(3,N)*SIGIT(3,N)+BB*BB)
GO TO 40
10 CONTINUE

IF(STAGE.EQ.2) GO TO 20

ESTIMATE INITIAL STRESSES FOR FIRST LOAD STEP

IF(NSTEP.EQ.1.AND.ITER.EQ.0)
   *CALL APPROX(N,1,J,K,L,M,SIGMA,X,Y,GM,GM,NEL,NDP,MAT)

20 CONTINUE
   CC=(SIGMA(2,N)+SIGMA(1,N))/2.
   BB=(SIGMA(2,N)-SIGMA(1,N))/2.
   CR=SQRT(SIGMA(3,N)*SIGMA(3,N)+BB*BB)
   IF(NSTEP.NE.1.AND.(ITER.NE.0)) GO TO 40
   IF(STAGE.EQ.2) GO TO 40
   DO 30 KJ=1,N
   DO 30 KJ=1,KJ
   SIGMA(KJ,J)=CC
   30 CONTINUE
   SIG1=CC+CR
   SIG3=CC-CR
   DEV=SIG1-SIG3
   IF(SIG1.LT.0.0) SIG1=0.0
   IF(SIG3.LT.0.0) SIG3=0.0
   IF(DEV.LT.0.0) DEV=0.0
   IF(STAGE.EQ.2) GO TO 45
   SIG13(N)=SIG3

45 CONTINUE

COMPUTE MODULI

SIG3=SIG13(N)
IF(SIG3.LT.0.0) GO TO 50
FI=(PHI(M)-PHI(M)* ALOG10(SIG3/PM))/37.28
GO TO 60
50 FI = PHI(M)/57.28
60 CONTINUE
   RCS=2.*(CM(M)*COS(FI)+SIG3*SIN(FI))/(1.0-SIN(FI))
   HCS = RCS/RF(M)
SUBROUTINE BLOCK(A,B,BINT,P,ST,IX,X,Y,CODE,XLD,YLD,GAM,CM,PHI, 
  DPHI,COEF,EXP,DM,GM,FM,RF,ETAN,BMOD,SMOD,TNU, 
  SIGMA,SIGIT,SL,SIG13,NEL,NDP,MAT,M2,M3,N2) 
  COMMON/CONST1/ NLD,VOL,*TYPE,NSTEP,PATM,GAMMN,NNN,TIME(3) 
  COMMON/CONST2/ ITER,MRAND,NUMELK,TITLE(8),STAGE,XY(50),YL(50),NLC 
  COMMON/CONST3/ XPYCH,KSTART,WANA 
  DIMENSION GAM(*,MAT),COEF(*,MAT),EXP(*,MAT),DM(*,MAT),GM(*,MAT),FM(*,MAT), 
  * DPHI(MAT),PHI(MAT),PHI(MAT),RF(MAT) 
  DIMENSION X(NDP),Y(NDP),XLD(NDP),YLD(NDP),CODE(NDP), 
  B(N2),BINT(N2) 
  DIMENSION IX(NEL,5),SIGMA(3,NEL),SIG13(3,NEL),SIGIT(3,NEL),SIG13(NEL),
* RMOD(NEL), SMOD(NEL), ETAN(NFL), TNU(NEL), SL(NEL)
  DIMENSION A(M,3), P(8), ST(3,3), C(3,3), S(9,3), LM(4)

INTEGER CODE, STAGE

REWIN 2  
REWIN 4  
NB = W3AND/I?  
ND = 2*NB  
ND2 = 2*ND  
NUMBLK = 0  

DO 10 N = 1, ND  
  B(N) = 0.0  
  DO 10 M = 1, ND  
  A(N,M) = C*T

FORM STIFFNESS MATRIX IN BLOCKS

30 NUMBLK = NUMBLK + 1  
  NH = NB*(NUMBLK+1)  
  NV = NH-NB  
  NL = NV-NB + 1  
  KSHIFT = 2*NL - 2

START ELEMENT LOOP

DO 210 N = 1, NEL
  DO 40 I = 1, 3
  DO 40 J = 1, 8
  40 ST(I, J) = C*T

  IF (IX(N, I)) 210, 210, 50
  50 DO 90 I = 1, 4
  60 IF (IX(N, I) = NL) 80, 70, 70
  70 IF (IX(N, I) = NH) 90, 90, 90
  80 CONTINUE
  GO TO 210

90 CONTINUE

CALL MATRIX(N, S, GAM, CY, PHI, NPHI, COEF, EXP, DM, GM, FM, RF, ETAN, RMOD, SMOD,  
  TNU, SL, SIGMA, SIGIT, IX, XY, P, C, S, ST, SIGI3, NEL, ND, MAT)

IF (VOL) 100, 100, 110
100 WRITE(6, 2003) N
ADD ELEMENT STIFFNESS TO TOTAL STIFFNESS

110 IX(N,5) = -IX(N,5)
DO 120 I = 1,4
LM(I) = 2*IX(N,I) - 2
120 CONTINUE

DO 130 I = 1,4
DO 130 K = 1,2
II = LM(I) + K - SHIFT
KK = 2*I - 2 + K
B(I) = B(I) + P(KK)
DO 130 J = 1,4
DO 130 L = 1,2
JJ = LM(J) + L - II + 1 - SHIFT
LL = 2*J - 2 + L
IF(JJ.LE.0) GO TO 130
130 CONTINUE

210 CONTINUE

FORM LOAD VECTOR FOR CURRENT STEP AND BLOCK

DO 240 N=NL*NM
IF(N.GT.NDP) GO TO 240
K=N+N-SHIFT
IF(ITER.EQ.1.AND.MINA.EQ.2) GO TO 230
IF(ITER.EQ.1.AND.STAGE.EQ.1) GO TO 230
BINT(N+N-1) = (S(K-1)+XLD(N))/NLD
BINT(N+N) = (2(K)+XLD(N))/NLD
BINT(K-1) = BINT(N+N-1)
230 CONTINUE

APPLY BOUNDARY CONDITIONS

DO 300 M = NL*NH
IF(M.GT.NDP) GO TO 300
MC = CODE(M) + 1
GO TO (320,250,250,250), MC
250 R=XLD(M)
N = 2*M - 1 - SHIFT
GO TO 280
250 R=YLD(M)
N = 2*M - SHIFT
MC = 1
280 CALL ECOND(A3N,D2,M,BAND,N+R,M2,W3*W2)
IF(MC.EQ.4) GO TO 260
300 CONTINUE
WRITE BLOCK OF EQUATIONS ON TAPE AND SHIFT UP LOWER BLOCK

WRITE(2) ((A(N,M),N=1,ND),M=1,MBAND),((B(N),N=1,ND)

DO 330 N = 1,ND
  K = N + ND
  A(K) = B(K)
  DO 330 M = 1,ND
  A(K,M) = A(K,M)
  IF(NM-NDP) 30,340,340
340 CONTINUE

RETURN

2001 FORMAT (2SHONEGATIVE AREA ELEMENT NO. 14)

END

SUBROUTINE MTRIX(N,GAM,CH,PHI,DPHI,COEF,EXP,DM,GM,FM,RF,ETAN,STAGE,XL(50),YL(50),NLC

COMMON/CONST1/ NLD, VOL, MTYPE, NSTEP, PATH, GAWW, NNN, TIME(3)

COMMON/CONST2/ ITER, MBAND, NUMSLK, TITLE(8), STAGE, XL(50), YL(50), NLC

COMMON/CONST3/ KPNCH, KSTART, NANA

DIMENSION GAM(MAT), COEF(MAT), EXP(MAT), DM(MAT), GM(MAT), FM(MAT)

DIMENSION X(NDP), Y(NDP)

DIMENSION IX(NEL,5), SIGMA(3,NEL), SIGO(3,NEL), SIG13(NEL)

DIMENSION P1(3), ST(3,8), C(3,3), S(3,3)

INTEGER STAGE

I = IX(N,1)
J = IX(N,2)
K = IX(N,3)
L = IX(N,4)

M = MTYPE

IF( STAGE = 20, 2, AND, NLD, Eq 1, AND, NANA = Eq 1 ) GO TO 50

FORY STRESS-STRAIN RELATIONSHIP

CALL NONLIN(N,I,J,K,L,M,STAN,TNU,BMOD,SMOD,ETAN,HCG,GAM,COEF,EXP,DM,
C FORM QUADRILATERAL STIFFNESS MATRIX FOR EACH ELEMENT

DO 100 II = 1,8
  P(II) = 0
  DO 100 JJ = 1,9
    S(II, JJ) = 0
  100 CONTINUE

VOL = 0.0

CALL ELMAT(I,J,K,L,C,S,ST,VOL,NDP)

CALCULATE GRAVITY LOADS

IF(STAGE.EQ.2) GO TO 300
IF(ITER.NE.2) GO TO 300

W = GAM(M)

FG = -W*VOL
IF (IX(N,3) .EQ. IX(N,4)) GO TO 150
FG = FG/4.
KL = 4
GO TO 160
150 FG = FG/3.0
KL = 3
160 CONTINUE
DO 200 IJ = 1, KL
  II = IJ + IJ
  P(II) = P(II) + FG
200 CONTINUE

300 CONTINUE

RETURN

END
SUBROUTINE USOL (A,B,MAXP,NEQG,MLL,NBLK,NSB,NORG,NMAX,NT1,NT2,NT3)

**THIS SUBROUTINE (CODED BY WILSON) USES AN EFFICIENT GAUSSIAN ELIMINATION TECHNIQUE TO SOLVE THE SIMULTANEOUS EQUATIONS**

DIMENSION A(NSB),B(NSB),MAXP,NEQG

NC=MLL+1
NSB=(MLL-1)/NEQG+1
INC=NEQG+1
NBR=NEQG+4
N2="TJ
N1="T'
REWIND NORG
REWIND N3K8

REDUCE EQUATIONS BLOCK BY BLOCK

DO 900 N=1,NBLK
   IF (N.GT.1.AND.NBR.EQ.1) GO TO 910
   IF (NBR.EQ.1) GO TO 905
   REWIND N1
   REWIND N2
905 N1=N1
   IF(N.EQ.1) N1=NORG
   READ (N1) A
100 DO 999 I=1,NEQG
   D=A(I)
   IF(D) 115,300,120
115 M=NEQG*(N-1)+I
   WRITE (6,116) M,D
116 FORMAT (33HSET OF EQUATIONS MAY BE SINGULAR /
      * 26H DIAGONAL TERM OF EQUATION 18, 84 EQUALS 1PE12.4)
   D=0
120 D=SGRT(D)
   A(1)=D
200 II=I
   DO 125 J=2,NC
      II=II+NEQG
125 A(II)=A(II)/D
999 CONTINUE

DO 130 J=1,MLL,NEQG
   IF (K(J).LT.2) MAXE(J)=J
130 CONTINUE

JJ=I+1
   IF (JL.GT.300) GO TO 300
   II=I
   DO 300 J=JL,NEQG
      II=II+NEQG
   300 C=A(II)
   IF (C.EQ.0.0) GO TO 200
   KK=J
   MAX=MAXE(J)
   DO 150 J=II,MEX,NEQG


```
160    KK=KK+NEQB
       C
       KK=J+NMB
       JJ=I+NMB
       DO 175 L=1,LL
       A(KK)=A(KK)-C*A(JJ)
       KK=KK+NEQB
175    JJ=JJ+NEQB
200    CONTINUE
300    CONTINUE
       WRITE (NBKS) A,MAXB

SUBSTITUTE INTO REMAINING EQUATIONS

       DO 800 NN=1,NBR
       IF(N+NN.GT.NBLOCK) GO TO 800
       NI=N1
       IF(V.EQ.1) NI=NORG
       IF(VN.EQ.NBR) NI=NORG
       READ (NI) B
       IL=I+NN*NEQB*NEQ8
       DO 700 I=1,NEQ8
       II=IL
       DO 690 K=I,NEQ8
       IF (II.GT.NMB) GO TO 690
       C=AWII)
       IF (C.EQ.0.0) GO TO 690
       MAX=MAXB(K)

       KK=I
       DO 640 JJ=II,MAX,NEQ8
       B(KK)=B(KK)-C*A(JJ)
640    KK=KK+NEQB
       C
       KK=I+NMB
       JJ=K+NMB
       DO 650 L=1,LL
       B(KK)=B(KK)-C*A(JJ)
       KK=KK+NEQB
650    JJ=JJ+NEQB

690    II=II-INC
700    IL=IL+NEQ8
       IF(NBR-NE.1) GO TO 750
       DO 740 I=1,NMB
740    A(I)=A(I)
       GO TO 900
750    WRITE (N2) B
800    CONTINUE
       N=N1
       N1=N2
900    N2=N
       C
BACKSUBSTITUTION - RESULTS ON TAPE NBRST
```
C
LS=LL*NEQ3
NEB=NEQ3=(NBR+1)
NBR=NBR*NEQ3
MAX=NEB*LL
DO 895 I=1,MAX
870 I=I-1,
REWRITE NEST
C
DO 1000 N=1,NBLOCK
BACKSPACE NBR
READ (NBR) A,MAXB
BACKSPACE NBR
DO 920 L=1,LL
K=L*NEB
DO 910 J=1,NUM
I=K-NEQ3
B(K)=B(I)
910 K=K-1
C
I=NEQ3
DO 920 L=1,LL
K=(L-1)*NEB
DO 920 J=1,NEQ3
I=I+1
K=K+1
920 B(K)=A(I)
C
DO 955 I=1,NEQ3
J=NEQ3+1-I
MAX=MAXP(J)
IF (A(J)*EQ.,0.) GO TO 940
DO 955 L=1,LL
KK=J+(L-1)*NEB
JJ=KK-1
IL=J+NEQ3
C=B(KK)
DO 940 II=IL,MAX,NEQ3
C=C-A(II)*B(JJ)
940 JJ=JJ+1
950 B(KK)=C/A(JJ)
955 CONTINUE
C
I=J
DO 960 L=1,LL
K=(L-1)*NEB
DO 960 J=1,NEQ3
K=K+1
I=I+1
960 A(I)=B(K)
C
WRITE (NBR) (A(I),I=1,LL)
1000 CONTINUE
C
RETURN
END
SUBROUTINE BAND (IX,NEL)

COMMON/CONST1/ NLD,VOL,TYPE,STEP,PATH,GAMMA,MNX,TMF(9)
COMMON/CONST2/ ITER,MHAN,NUMILK,TITLE(8),STAGE,YL(50),YL(50),NLC

DIMENSION IX(NEL+5)

DEFINE BAND WIDTH

JDIF = 0

DO 300 N = 1,NEL
DO 200 I = 1,3
DO 100 J = 1,3
L = I + J
IF (L. GE. 4) GO TO 200
NDIF = IABS (IX(N,I) - IX(N,L))
IF (NDIF .GT. JDIF) JDIF = NDIF
100 CONTINUE
200 CONTINUE
300 CONTINUE

MBAND = 2*JDIF + 2
WRITE (6,200) MBAND
RETURN
200 FORMAT (IH1,1X,1I1H,BANDWIDTH = ,6)
END

SUBROUTINE ELMAT (IA,JA,K,A,L,C,S,ST,X,Y,VOL,NDP)

THIS SUBROUTINE FORMS THE STIFFNESS MATRIX FOR A RECTANGULAR ELEMENT
SUBROUTINE IS A MODIFICATION OF GLST WRITTEN BY A. ALCAIDE (1971)

VS,VT LOCAL COORDINATES OF ELEMENT
VH COEFFICIENTS FOR GAUSSIAN INTEGRATION
ST STRAIN-DISPLACEMENT MATRIX AT ELEMENT CENTER
AB STRAIN-DISPLACEMENT MATRIX AT POINT VS,VT
S STIFFNESS MATRIX OF ELEMENT
AS FIRST DIFFERENTIAL OF S AT VS,VT
DJ MATRIX PROPORTIONAL TO INVERTED JACOBIAN MATRIX
CVM VARIABLE PROPORTIONAL TO JACOBIAN DETERMINANT

THE STRAIN-DISPLACEMENT RELATIONSHIP (ST) FOR EACH ELEMENT IS STORED ON TAPE 4 FOR LATER USE IN SUBROUTINE STRESS
DIMENSION X(NPD), Y(NPD)
DIMENSION C (2, 2), ST(2, 2), S(2, 2)
DIMENSION VH(6), VS(6), VT(6), AO(2, 4), AB(2, 2), AS(1, 9)

NP1=4
VH(1)=0.88888888
VH(2)=0.88888888
VH(3)=VH(1)
VH(4)=VH(1)
VH(5)=VH(1)
VH(6)=VH(1)

VS(1)=0.77458669
VS(2)=0.77458669
VS(3)=VS(1)
VS(4)=VS(1)
VS(5)=VS(1)
VS(6)=VS(1)

VT(1)=0.
VT(2)=0.
VT(3)=VS(1)

VT(NP1)=0.

VCL=(Y(JA)-Y(LA))*(X(JA)+X(JA)-Y(KA)-Y(LA))
VOL=VCL-(X(JA)-X(LA))*(Y(JA)+Y(JA)-Y(KA)-Y(LA))
VOL=1.*V*L
DO 1 I=1,NP1
DO 1 J=1,NP1
IF (I.EQ.NP1.AND.J.EQ.NP1) GO TO 10
IF (I.EQ.NP1) GO TO 1
IF (J.EQ.NP1) GO TO 1
1 AP(1,1)=-1.*JT(J)
AP(2,1)=-1.*VS(I)
AP(1,2)=-1.*JT(J)
AP(2,2)=1.*VS(I)
AP(1,3)=-1.*JT(J)
AP(2,3)=1.*VS(I)
AP(1,4)=-1.*JT(J)
AP(2,4)=1.*VS(I)
AJ1=AP(2,1)*Y(JA)+AP(2,2)*Y(JA)+AP(2,3)*Y(KA)+AP(2,4)*Y(LA)
AJ2=AP(1,1)*Y(JA)+AP(1,2)*Y(JA)+AP(1,3)*Y(KA)+AP(1,4)*Y(LA)
AJ3=AP(2,1)*X(JA)+AP(2,2)*X(JA)+AP(2,3)*X(KA)+AP(2,4)*X(LA)
AJ4=AP(1,1)*X(JA)+AP(1,2)*X(JA)+AP(1,3)*X(KA)+AP(1,4)*X(LA)
CWW=AJ1*A3J4-AJ2*AJ3
DO 2 K=1,4
   X1=K*X-1
   K2=2*K
   AB(1,K1)=AJ1+AP(1,K)+AJ2*AP(2,K)
   AB(2,K2)=AJ2
   AB(1,K2)=0.
   AB(2,K1)=AJ3+AP(1,K)+AJ4*AP(2,K)
   AB(3,K1)=AB(2,K2)
   AB(3,K2)=0.
   IF (I.EQ.NP1.AND.J.EQ.NP1) GO TO 20
   DO 40 K=1,8
   DO 4 L=1,3
   ST(L,K)=0.
   40 V=L*X
   ST(L,K)=ST(L,K)+C(L,V)*ST(V,X)
   DO 41 K=1,8
   DO 41 L=1,3
   AS(K,L)=0.
   AS(K,L)=AS(K,L)+AB(K,X)*ST(H,L)
SUBROUTINE BCOND (A,B,NEQ,M,BAND,N,U,M2,M3,N2)

THIS SUBROUTINE APPLIES THE BOUNDARY CONDITIONS TO THE STRUCTURE S
MATRIX

DIMENSION A(M2,M3),B(N2)

DO 40 M = 2,M,BAND
K = N-M+1
IF(K) 20,20,10
10 B(K) = B(K) - A(K,M)*U
A(K,M) = 0.0
20 K = N+M-1
IF(NEQ-K) 40,30,30
30 B(K) = B(K) - A(N,M)*U
A(N,M) = 0.0
40 CONTINUE
A(N+1) = 1.0
B(N) = U
RETURN
END

SUBROUTINE APPROX(N,J,K,L,M,SIGMA,X,Y,GAM,GAM,NEL,NDP,MAT)

COMMON/CONST1/ NLD,VOL,TYPE,NSTEP,PATH,GAMN,INN,TIME(3)
COMMON/CONST2/ ITER,M,BAND,NUMB,L,TITLE(8),STAGE,XL(50),YL(50),NLC
DIMENSION SIGMA(3,NEL)*X(NDP),Y(NDP),GAM(MAT),GAM(MAT)

COORDINATES OF CENTROID OF ELEMENT

IF (K.EQ.L) GO TO 10
\[ XC = \frac{(X(I)+X(J)+X(K)+X(L))}{4} \]
\[ YC = \frac{(Y(I)+Y(J)+Y(K)+Y(L))}{4} \]

GO TO 20

10 \[ XC = \frac{(X(I)+X(J)+X(K))}{3} \]
\[ YC = \frac{(Y(I)+Y(J)+Y(K))}{3} \]

20 CONTINUE

DETERMINE O/B PRESSURE

DO 30 IL = 1, NLC
  IF(XC.LE.XL(IL)) GO TO 40
30 CONTINUE

40 \[ XD = YL(IL) - YL(IL-1) \]
\[ YD = YL(IL) - YL(IL-1) \]
\[ YY = YD*(XC-XL(IL-1))/XD \]
\[ YH = YL(IL-1) + YM - YC \]
\[ ALPHA = \tan(2*YD*XD) \]
\[ SIGMA(2,N) = YH*GAM(M)/NLD \]
\[ AKO = GM(M)/(1.0-GM(M)) \]
\[ SIGMA(1,N) = SIGMA(2,N)*AKO \]
\[ SIGMA(3,N) = 0.5*SIGMA(2,N)*\sin(ALPHA) \]

RETURN

END

SUBROUTINE EQUIV(N,EPSX,EPSY,GAMXY,EI,HCS,EP,SIG,IX,ENF,X,
1  Y,NEL,NDP,ETAN,TNU,SMOD)

COMMON/CONST1/ NLD,VOL,MTYPE,NSTEP,PATH,GAMW,NNN,TIME(3)
COMMON/CONST2/ ITER,MBAND,NUM3LK,TITLE(8),STAGE,XL(50),YL(50),NLC
COMMON/CONST3/KPNCH,KSTART,NANA

DIMENSION EP(NEL),IX(NEL,5),X(NEL,5),Y(NEL,5),ENF(NEL,2),SIG(6)
DIMENSION ETAN(NEL),TNU(NEL),SMOD(NEL)

CALCULATE STRAIN CORRESPONDING TO INITIAL STRESSES

\[ DEV1 = SIG(N+1)-SIG(6) \]
\[ EE = EI*(1.-DEV1/HCS) \]
\[ EPS1 = DEV1/EE \]
\[ EPS1F = EPS1 + EP(M) \]
\[ DEV2 = EPS1F/(1.0/EI + EPS1F/HCS) \]

CALCULATE INCREASE IN SHEAR STRESS DUE TO STRAIN POTENTIAL
DTAU = (DEV2-DEV1)/2.0
IF(INAME.NE.11) GO TO 100
ETAN(N)=DTAU*2./EP(N)
SMOD(N)=ETAN(N)/(2.0*(1.0+TNU(N)))
100 CONTINUE

CALCULATE EQUIVALENT NODAL LOADS FOR ELEMENT

I=IX(N,1)
J=IX(N,2)
K=IX(N,3)
L=IX(N,4)
D=(ABS(X(I)-X(J)) + ABS(Y(K)-Y(L)))*C.5
DV=(ABS(Y(L)-Y(I)) + ABS(Y(K)-Y(J)))*C.5
TXY=SIG(3)
F=-SIGN(DTAU,TXY)*D/2.0
FV=-SIGN(DTAU,TXY)*DV*C.5
PM=TXY/SIG(2)
IF(ABS(PM) .LE. 0.001) F=0.
IF(ABS(PM) .LE. 0.001) FV=0.

ADD ELEMENT NODAL FORCES TO STRUCTURE NODAL FORCES

ENF(K,1) = ENF(K,1) + F
ENF(L,1) = ENF(L,1) + F
ENF(I,1) = ENF(I,1) - F
ENF(J,1) = ENF(J,1) - F
ENF(I,2) = ENF(I,2) - FV
ENF(L,2) = ENF(L,2) - FV
ENF(J,2) = ENF(J,2) + FV
ENF(K,2) = ENF(K,2) + FV

RETURN

END
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