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RATIONAL DESIGN METHODS FOR LIGHT EQUIPMENT IN STRUCTURES SUBJECTED TO GROUND MOTION

by

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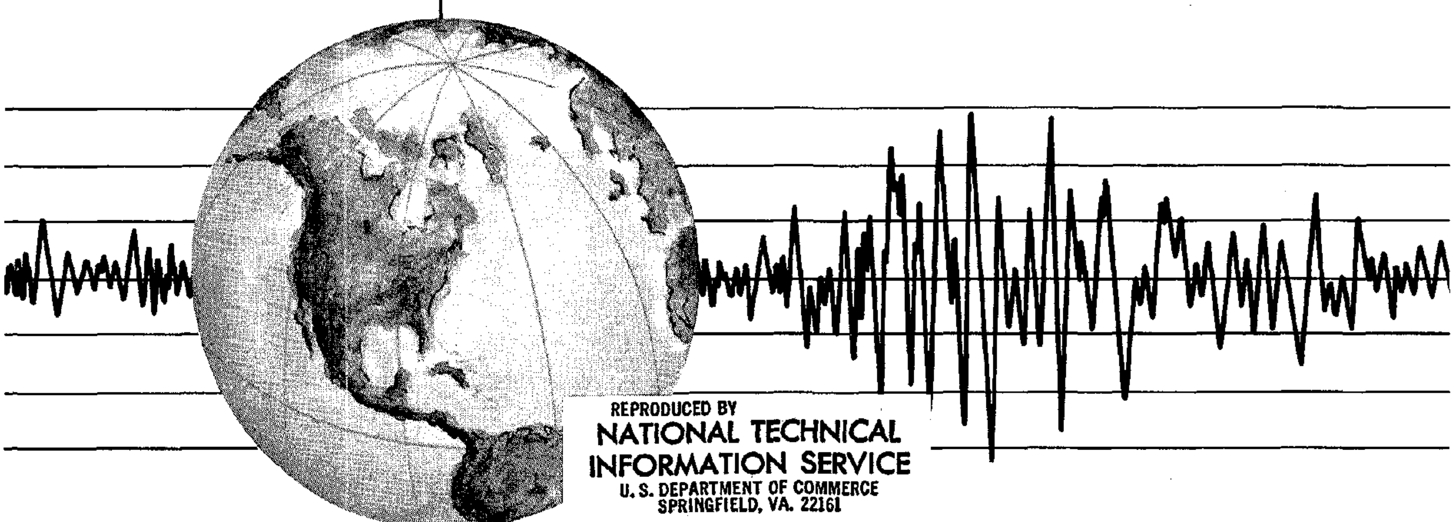

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The approach is based on the transient analysis of lightly damped tuned or slightly detuned equipment-structure systems in which the mass of the equipment is much smaller than that of the structure. It is assumed that the information available to the designer is a design spectrum for the ground motion, fixed-base modal properties of the structure, and fixed-base properties of the equipment. The results obtained are simple estimates of the maximum acceleration and displacement of the equipment. The method can also be used to treat closely spaced modes in structural systems, where the square root of the sum of squares procedure is known to be invalid.

This analytical method has also been applied to untuned equipment-structure systems for which the conventional floor spectrum method is mathematically valid. A closed-form solution is obtained which permits an estimate of the maximum equipment response to be obtained without the necessity of computing time histories, as required by the conventional floor spectrum method.

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ABSTRACT

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1. INTRODUCTION

The design of equipment to withstand dynamic loading is a neglected feature of structural design. Equipment such as pumps, compressors, power generators, and piping systems is essential to lifeline systems and must be functional in the aftermath of a catastrophy such as a major earthquake, tornado, aircraft crash, or explosion, yet is rarely designed with the same care as the building within which it is housed. In addition, the cost of the equipment in many cases may be many times that of the building, see for example [1].

A rational approach to one aspect of equipment design, that of lightly damped relatively light equipment, is presented in this report. The model considered is an N-degree of freedom structure to which is attached a single-degree-of-freedom component. The frequency of the latter can be higher than the fundamental frequency of the structure. In previous work [2], we have described the response of such a system to steady-state ground shaking. Significant interaction effects were shown to occur in the case of tuning, the situation in which the equipment frequency is close or equal to one of the natural frequencies of the structure. If the equipment frequency is not tuned to a structural frequency, the response is roughly the superposition of the structural response and the equipment response with little interaction, in which case the conventional floor spectrum method should be valid for transient problems. If, on the other hand, the equipment frequency is tuned to a structural frequency, it was found that for the combined system there are two closely spaced frequencies on either side of the tuning frequency around which a band of high amplification appears, offering a substantial target for sympathetic oscillation. The significant equipment-structure interaction in this case means that the conventional floor spectrum method, which ignores that interaction, will not be valid for the transient analysis problem.

A typical result for the steady-state response of an equipment-structure system is shown in Fig. 1, which is taken from [2]. The equipment is tuned to the third natural frequency of the structure, and the curve shown is the ratio of equipment acceleration to input ground acceleration considered as a function of the frequency of input, normalized with respect to the natural frequency of the equipment. The mass of the structure in this example is a thousand times that of the equipment.

In this report the previous research is extended to transient analysis of the equipment-structure interaction problem. The peak response of the equipment is estimated by utilizing a design spectrum for a specified input to the structure, and fixed-base dynamic properties of the structure alone and of the equipment alone. By taking advantage of the mathematical structure of the equations and of asymptotic methods made possible by the smallness of the equipment mass in comparison with the structure mass, we obtain simple results that are valid for tuned, nearly tuned, and untuned systems.

The rationale for using design spectrum methods is that they are inexpensive and to a certain extent incorporate the probabilistic nature of the problem, *i.e.,* the uncertainty involved in specifying the structural parameters and the earthquake or other input. These uncertainties are accounted for in constructing a design spectrum and also in the way that the maximum values in each mode are combined to predict the maximum for the entire system.

For light equipment and small damping, the results obtained can be implemented easily and efficiently by a designer. A surprising feature of the analysis is its extreme simplicity; namely, if the response spectrum for the ground motion is available, the response spectrum for the equipment can be calculated merely by multiplying the former by an amplification factor.

This approach is in contrast to several earlier analyses of equipment response. A common approach to the design of equipment is based on the floor spectrum method, in which the equipment is treated as a single-degree-of-freedom system subject to a base motion that is taken to be that which the structure would experience at the attachment point in the absence of equipment. Not only does this method neglect interaction, it has the further disadvantage of requiring that an expensive time history analysis of the structure be conducted in order to determine a base motion. Approximate techniques that bypass associated computational problems have been proposed whereby floor response spectra are developed from ground spectra, but these are *ad hoc* methods whose accuracy cannot be evaluated.

An alternative approach is to consider an $N+1$-degree-of-freedom model for the equipment-structure system and subject it to time history analyses for a variety of specified ground motion inputs. This method has disadvantages also; the equipment may have a natural frequency somewhat higher than the fundamental frequency of the structure. Conventional methods of dynamic analysis are designed to compute lower-mode response, a response that is pertinent for structural design only. Thus, standard computer codes use implicit, unconditionally stable, time-integration algorithms with some form of numerical dissipation to damp out spurious participation from higher modes. Where equipment-structure interaction is important, the use of such codes can mask significant response.

Because time history analysis can be extremely expensive, response spectra methods for $N+1$-degree-of-freedom systems are frequently used, despite some uncertainty in the combination of peak modal values. The standard approach, in which the square root of the sum of the squares is used, is known to be inaccurate for closely spaced modal frequencies that occur in light equipment with a natural frequency close or equal to one of the natural frequencies of the structure. Penzien and Chopra [3] have studied this problem and have proposed an *ad hoc* method in which the response spectra of two-degree-of-freedom systems are numerically computed and used. In that approach, the N modes of a structure are determined and to each mode, considered as a single-degree-of-freedom system, the equipment is attached to yield N two-degree-of-freedom systems. The maximum response of the equipment for each of these N two-degree-of-freedom systems is obtained from the two-degree-of-freedom spectra previously constructed and the maximum response of the entire system evaluated using the square root of the sum of the squares of the maximum value for each. The analysis performed herein shows that the method is unnecessarily complicated and in principle incorrect.

2. MODAL ANALYSIS OF EQUIPMENT-STRUCTURE SYSTEMS

In this section we formulate the equations of motion governing the response of a general N-degree-of-freedom structure to which is attached equipment modeled as a single-degree-of-freedom oscillator (Fig. 2).

The equations of motion of the N-degree-of-freedom structural system take the form

$$\sum_{j=i}^{N} (M_{ij}\ddot{U}_j + C_{ij}\dot{U}_j + K_{ij}U_j) = \sum_{j=1}^{N} (C_{ij}R_j\dot{u}_g + K_{ij}R_ju_g) + Fe_i \quad , \quad i=1,2,...,N \tag{2.1}$$

where M_{ij}, C_{ij}, and K_{ij} are the mass, damping, and stiffness matrices, respectively. The vector R_i is a vector of influence coefficients introduced to couple the actual ground motion $u_g(t)$ to the structure, and e_i a vector whose components are zero at every degree of freedom except the one to which the equipment is attached, denoted by the index r, where it takes unit value. The term F is the interaction force between the equipment and the structure.

The natural frequency Ω_n and mode shape Φ_i^n of the n^{th} mode $(n=1,2,...,N)$ are obtained from the equations

$$\Omega_n^2 \sum_{j=1}^{N} M_{ij}\Phi_j^n = \sum_{j=1}^{N} K_{ij}\Phi_j^n \quad , \quad i=1,2,...,N \tag{2.2}$$

If the damping is assumed to be small enough not to introduce coupling between the modes, Eq. (2.1) becomes in modal coordinates

$$\ddot{q}_k + 2B_k\Omega_k\dot{q}_k + \Omega_k^2 q_k = \sum_{i=1}^{N} \Phi_i^k F_i / M_k \quad , \quad k=1,2,...,N \tag{2.3}$$

where

$$M_k = \sum_{i=1}^{N}\sum_{j=1}^{N} \Phi_i^k\Phi_j^k M_{ij} \quad , \quad 2B_k\Omega_k = \sum_{i=1}^{N}\sum_{j=1}^{N} \Phi_i^k\Phi_j^k C_{ij}/M_k$$

and

$$F_i = \sum_{i=1}^{N} (C_{ij}R_j\dot{u}_g + K_{ij}R_ju_g) + Fe_i$$

with M_k the generalized mass and B_k the fraction of critical damping for the k^{th} mode. The Laplace transform of the structural response $U_j(t)$ is given by

$$\overline{U}_j = \sum_{k=1}^{N} \frac{\sum_{i=1}^{N} \Phi_i^k \Phi_j^k \overline{F}_i}{M_k\left(p^2+2B_k\Omega_k p+\Omega_k^2\right)} \tag{2.4}$$

with

$$\overline{F}_i = \sum_{l=1}^{N} (C_{il}R_l p + K_{il}R_l)\,\overline{u}_g + \overline{Fe}_i$$

where p is the Laplace transform parameter and a bar above a function denotes its Laplace transform. The corresponding equation of motion for the equipment displacement u is

$$-m\ddot{u} = F = c(\dot{u} - \dot{U}_r) + k(u - U_r) \tag{2.5}$$

or, in Laplace transforms,

$$-p^2\overline{u} = \frac{\overline{F}}{m} = (2\beta\omega p + \omega^2)(\overline{u} - \overline{U}_r) \tag{2.6}$$

where m, c, and k are the mass, damping, and stiffness of the equipment, respectively, with β the fraction of critical damping. A relationship between u and U_r is obtained from Eq. (2.6) in the form

$$(p^2 + 2\beta\omega p + \omega^2)\,\overline{u} = (2\beta\omega p + \omega^2)\,\overline{U}_r \tag{2.7}$$

which, from Eq. (2.4), can be written as

$$\overline{u}\,(p^2+2\beta\omega p+\omega^2) = (2\beta\omega p+\omega^2) \sum_{k=1}^{N} \frac{\sum_{i=1}^{N} \Phi_i^k \Phi_r^k \left[\overline{Fe}_i + \sum_{l=1}^{N} (C_{il}R_l p+K_{il}R_l)\,\overline{u}_g\right]}{M_k\left(p^2+2B_k\Omega_k p+\Omega_k^2\right)}$$

Since $\overline{F} = -mp^2\overline{u}$, $\overline{F}$ can be eliminated. The final transformed equation for the equipment response is then

$$\bar{u}\left[(p^2+2\beta\omega p+\omega^2)+\sum_{k=1}^{N}\frac{mp^2(2\beta\omega p+\omega^2)\Phi_r^k\Phi_r^k}{M_k\left(p^2+2\beta_k\Omega_k p+\Omega_k^2\right)}\right]$$

$$=(2\beta\omega p+\omega^2)\sum_{k=1}^{N}\frac{\Phi_r^k\sum_{i=1}^{N}\Phi_i^k\sum_{l=1}^{N}(C_{il}R_l p+K_{il}R_l)}{M_k\left(p^2+2B_k\Omega_k p+\Omega_k^2\right)}\,\bar{u}_g \tag{2.8}$$

The expression $\sum_{i=1}^{N}K_{il}\Phi_i^k$ can be written as $\Omega_k^2\sum_{i=1}^{N}M_{il}\Phi_i^k$ and, given the assumption of small damping, $\sum_{i=1}^{N}C_{il}\Phi_i^k$ can be represented as $2B_k\Omega_k\sum_{i=1}^{N}M_{il}\Phi_i^k$. Thus, the solution for $\bar{u}$ for the multi-degree-of-freedom system takes the form

$$\bar{u}\left[(p^2+2\beta\omega p+\omega^2)+p^2\sum_{k=1}^{N}\frac{m\Phi_r^{k^2}(2\beta\omega p+\omega^2)}{M_k\left(p^2+2B_k\Omega_k p+\Omega_k^2\right)}\right]$$

$$=\sum_{k=1}^{N}\frac{\Phi_r^k\sum_{i=1}^{N}\Phi_i^k\sum_{l=1}^{N}M_{il}R_l\left(2B_k\Omega_k p+\Omega_k^2\right)(2\beta\omega p+\omega^2)}{M_k\left(p^2+2B_k\Omega_k p+\Omega_k^2\right)}\,\bar{u}_g \tag{2.9}$$

The zeroes of the term in brackets on the left-hand side of the equation must be determined to invert the Laplace transform by residue theory. These zeroes are the poles of the transfer function for the equipment response. The case considered here is that illustrated in Fig. 3, where the equipment frequency is close to a structural frequency, say Ω_n. The two expressions in the brackets on the left-hand side of Eq. (2.9) have been plotted separately; p was replaced by $i\Omega$ and the graph of the second function and the negative of the first function in the bracketed expression were then drawn. For simplicity, the undamped case ($\beta=B_1=\cdots=B_n=0$) has been plotted. The plot for the first function is a simple quadratic in Ω, zero when $\Omega=\omega$, the natural frequency of the equipment. The plot for the summation is a complicated curve that reaches $\pm\infty$ when $\Omega=\Omega_k$ and $k=1,2,\ldots,N$, the natural frequencies of the structure. Two such curves have been plotted, one for equipment of small mass, and another for equipment of larger mass.

The values of Ω at the intersections of these two curves locate the zeroes in the bracketed expression, where equipment-structure interaction is considered. When the equipment mass is small, these poles, all of which are simple, appear near the natural frequencies of the structure. Two closely spaced poles, referred to herein as tuning poles, are located near the equipment frequency and the frequency of the structure to which the equipment is nearly tuned, one below these frequencies and one above them as shown in the figure. These two poles coalesce into a double pole when $\omega = \Omega_n$ and $m \to 0$. Thus, the contribution to the sum of the residues at all poles is dominated by the residues associated with the two tuning poles. The contribution of the summation term to the residues at these two poles is dominated by the term where $k = n$ since the denominator of that term is nearly zero. Hence, in the region of $p = i\omega$, Eq. (2.9) can be approximated by

$$\bar{u}\left[(p^2 + 2\beta\omega p + \omega^2) + p^2 \frac{m}{M_n/(\Phi_r^n)^2} \cdot \frac{2\beta\omega p + \omega^2}{p^2 + 2B_n\Omega_n p + \Omega_n^2}\right]$$

$$= \frac{(2\beta\omega p + \omega^2)(2B_n\Omega_n p + \Omega_n^2)}{p^2 + 2B_n\Omega_n + \Omega_n^2}\left\{\sum_{i=1}^{N}\sum_{l=1}^{N}\Phi_r^n\Phi_i^n M_{il}R_l/M_n\right\}\bar{u}_g \tag{2.10}$$

This expression is identical to that for a two-degree-of-freedom system, as shown in Fig. 4. The equivalent expression for the system shown in that figure is

$$\bar{u}\left[(p^2 + 2\beta\omega p + \omega^2) + p^2\gamma \frac{(2\beta\omega p + \omega^2)}{p^2 + 2B\Omega p + \Omega^2}\right] = \left[\frac{(2\beta\omega p + \omega^2)(2B\Omega p + \Omega^2)}{p^2 + 2B\Omega p + \Omega^2}\right]\bar{u}_g \tag{2.11}$$

where $\gamma = m/M$ is the mass ratio. When we compare this expression to Eq. (2.10), we see that the effective mass ratio is

$$\gamma^{eff} = \frac{m}{M_n/(\Phi_r^n)^2} \tag{2.12}$$

and the effective ground motion

$$u_g^{eff} = C_r^n u_g \tag{2.13}$$

where

$$C_r^n = \Phi_r^n \sum_{i=1}^{N}\sum_{j=1}^{N}\Phi_i^n M_{ij}R_j/M_n \tag{2.14}$$

In the subsequent development, the contribution of the residues from the tuning poles (which are near $p = i\omega$) will be obtained from an analysis of the equivalent two-degree-of-freedom system defined by the above equations. The contributions at the other $(N-1)$ poles are straightforward and will be considered after the two-degree-of-freedom analysis has been completed. Although it is not essential that an equivalent two-degree-of-freedom system be considered, since it is only conceptual and introduces no further approximations beyond those made in passing from Eq. (2.9) to Eq. (2.10), the following development will be for such a system in order to simplify notation. We will use the notation of expression (2.11). Thus, B and Ω will refer to the structural quantities B_n and Ω_n, and γ and u_g to γ^{eff} and u_g^{eff}, as defined in Eqs. (2.12) and (2.13).

3. ANALYSIS OF TRANSFER FUNCTION FOR NEARLY TUNED TWO-DEGREE-OF-FREEDOM SYSTEM

The transformed equipment acceleration $\bar{\ddot{u}}(p)$ for the equivalent two-degree-of-freedom system takes, from Eq. (2.11), the form

$$\bar{\ddot{u}} = \left[N(p)/D(p)\right] \bar{\ddot{u}}_g \tag{3.1}$$

where

$$N(p) = (2\beta\omega p + \omega^2)\left[2\beta(1+\xi)\omega p + (1+\xi)^2\omega^2\right] \tag{3.2}$$

and

$$\begin{aligned} D(p) = p^4 &+ \omega p^3\left[2\beta(1+\gamma) + 2B(1+\xi)\right] \\ &+ \omega^2 p^2\left[2 + \gamma + 2\xi + \xi^2 + 4\beta B(1+\xi)\right] \\ &+ \omega^3 p\left[2\beta(1+\xi)^2 + 2B(1+\xi)\right] + \omega^4(1+\xi)^2 \end{aligned} \tag{3.3}$$

In the above $\xi = (\Omega-\omega)/\omega$, the detuning parameter. The following discussion concentrates on equipment acceleration; results for equipment displacement can be easily developed.

The nature of the solution strongly depends on the location of the zeroes of the denominator $D(p)$. The roots of $D(p)$ will be close to those of the system where γ, β, B, and ξ are taken to be zero because these parameters are small; that is,

$$p = \pm\, i\omega$$

To locate the poles of $D(p)$, we replace p in Eq. (3.3) by

$$p = i\omega(1+\delta) \tag{3.4}$$

where δ is a small quantity. We retain only the plus sign since the roots will appear as complex conjugates. Substitution of Eq. (3.4) into Eq. (3.3) yields the expression

$$\delta^4 + \left\{4 - i\left[2\beta(1+\gamma) + 2B(1+\xi)\right]\right\}\delta^3$$

$$+\left\{4-\gamma-2\xi-\xi^2-4\beta B(1+\xi) - i\left[6\beta(1+\gamma)+6B(1+\xi)\right]\right\}\delta^2$$

$$+\left\{-2\gamma-4\xi-2\xi^2-8\beta B(1+\xi) - i\left[2\beta(2+3\gamma-2\xi-\xi^2)+4B(1+\xi)\right]\right\}\delta$$

$$+\left\{-\gamma-4\beta B(1+\xi) - i\left[2\beta(\gamma-2\xi-\xi^2)\right]\right\} = 0 \qquad (3.5)$$

When $\beta = 0$, $B = 0$, and $\gamma \neq 0$, $\xi \neq 0$, this equation can be easily solved, *viz.*,

$$\delta = \left[1+\xi+\frac{\gamma}{2}+\frac{\xi^2}{2} \pm \left(\gamma+\xi^2+\xi^3+\gamma\xi+\frac{\gamma\xi^2}{2}+\frac{\gamma^2}{4}+\frac{\xi^4}{4}\right)^{1/2}\right]^{1/2} - 1 \approx \frac{1}{2}\left[\xi \pm (\gamma+\xi^2)^{1/2}\right]$$

Also, if $\gamma = 0$, $\xi = 0$, and $\beta \neq 0$, $B \neq 0$, then

$$\delta = (1-\beta^2)^{1/2} - 1 + i\beta \;\; , \;\; (1-B^2)^{1/2} - 1 + iB \approx i\beta \, , \; iB$$

Throughout the analysis it will thus be assumed that β, B, ξ, and $\gamma^{1/2}$ are all of the same order, say ϵ, and the various approximations for δ will be based on δ of order $\epsilon \ll 1$. When the parameters are not of the same order, the modifications required are obvious.

The solution of Eq. (3.5), where terms of order ϵ^2 are retained, is

$$\delta = \frac{\xi}{2} \pm \frac{\lambda}{2} + i\left(\frac{\beta+B}{2} \pm \frac{\mu}{2}\right) \qquad (3.6)$$

where here and throughout the remainder of the analysis the upper signs are taken together to give one root and the lower the other. The quantities λ and μ are given by

$$\lambda = \frac{1}{\sqrt{2}}\left(\left\{\left[\gamma+\xi^2-(\beta-B)^2\right]^2 + 4\xi^2(\beta-B)^2\right\}^{1/2} + \left[\gamma+\xi^2-(\beta-B)^2\right]\right)^{1/2} \qquad (3.7)$$

$$\mu = \frac{1}{\sqrt{2}}\left(\left\{\left[\gamma+\xi^2-(\beta-B)^2\right]^2 + 4\xi^2(\beta-B)^2\right\}^{1/2} - \left[\gamma+\xi^2-(\beta-B)^2\right]\right)^{1/2} \qquad (3.8)$$

For $\beta \neq 0$ and/or $B \neq 0$, the imaginary part is always positive and oscillations are therefore damped. Because this solution involves a large number of parameters, many special cases may exist; those of particular interest are examined more fully in sections that follow.

3.1 Undamped Tuned System

When $\beta = 0$, $B = 0$, and $\xi = 0$, the solution of Eq. (3.5), where terms of order ϵ^3 are retained, is

$$\delta = \pm \frac{\gamma^{1/2}}{2} \left(1 \pm \frac{\gamma^{1/2}}{4} \right) \tag{3.9}$$

The higher-order terms in δ are retained since they will be used to derive results that we will compare with results from the floor spectrum method. The expression for the roots p is then

$$p = i\omega \pm i\omega \frac{\gamma^{1/2}}{2} \left(1 \pm \frac{\gamma^{1/2}}{4} \right) \tag{3.10}$$

These roots are indicated in the root locus diagram Fig. 5, where the corresponding complex conjugate roots are also shown. The roots remain on the imaginary axis with the small spread between them equal to $\gamma^{1/2}\omega$ and lead to an undamped oscillatory solution.

3.2 Undamped Slightly Detuned System

The roots in this case are

$$\delta = \frac{\xi}{2} \pm \frac{1}{2} (\gamma + \xi^2)^{1/2} \tag{3.11}$$

which, in terms of p, are

$$p = i\omega \left(1 + \frac{\xi}{2} \right) \pm i\frac{\omega}{2} (\gamma + \xi^2)^{1/2} \tag{3.12}$$

(N.B. When $\gamma \to 0$, these become $p = \pm\, i\omega\,,\ \pm\, i\Omega$.)

These roots and the corresponding complex conjugate roots are indicated in Fig. 6. Again, an undamped oscillatory solution results and the spread between the closely spaced roots is now given by $(\gamma+\xi^2)^{1/2}\omega$.

3.3 Damped Tuned System

A double root of the equation $D(p)=0$ can appear for $\xi=0$ if certain relationships exist between the coefficients of the various powers of p in the expression for $D(p)$ given by Eq. (3.3) when $\xi=0$. These conditions are

$$\gamma\beta = 0 \tag{3.13}$$

and

$$\gamma+2\beta B = B^2+(1+\gamma)^2\beta^2 \tag{3.14}$$

For nonzero γ, β must be zero (from Eq. (3.13)), and $\gamma=B^2$ (from Eq. (3.14)).

The solution of Eq. (3.5) when $\xi=0$ and terms of order ϵ^2 are retained is

$$\delta = i\,\frac{\beta+B}{2} \pm \frac{1}{2}\left[\gamma-(\beta-B)^2\right]^{1/2} \tag{3.15}$$

With this as a first approximation and using iteration, the solution of Eq. (3.5) where terms of order ϵ^3 are retained is

$$\delta = i\,\frac{\beta+B}{2} \pm \frac{1}{2}\left(\gamma-(\beta-B)^2\mp\left[\gamma-(\beta-B)^2\right]^{1/2}(\beta^2+B^2-\gamma/2)\right.$$
$$\left.+\,i\left\{2\beta\gamma+(\beta+B)\left[\gamma-(\beta-B)^2\right]\right\}\right)^{1/2} \tag{3.16}$$

The term $\gamma-(\beta-B)^2$ dominates in the radical when of order ϵ^2, and use of Eq. (3.15) suffices to determine the roots δ. When $\gamma\rightarrow 0$, the two roots are

$$\delta = i\beta \quad\text{and}\quad \delta = iB$$

corresponding to the floor spectrum solution for the damped system. Since a double root can occur only if $\beta=0$, when $\gamma=(\beta-B)^2$ is of the order ϵ^3 or higher, the complete expression must be used. For $\gamma=(\beta-B)^2$, the roots are

$$\delta = i\,\frac{\beta+B \pm \beta^{1/2}\gamma^{1/2}}{2} \pm \frac{\beta^{1/2}\gamma^{1/2}}{2} \tag{3.17}$$

When $\beta = 0$, the roots are

$$\delta = i\,\frac{B}{2} \pm \frac{1}{2}\left[(\gamma-B^2) \mp (\gamma-B^2)^{1/2}(B^2-\frac{\gamma}{2}) + iB(\gamma-B^2)\right]^{1/2} \tag{3.18}$$

For fixed B, the roots range from

$$\delta = i\,\frac{B}{2} \pm \frac{1}{2}\gamma^{1/2} \quad \text{for} \quad \gamma >> B^2$$

to

$$\delta = i\,B - \frac{B^2}{2}\ ,\ 0 \quad \text{for} \quad \gamma \to 0$$

When $\gamma = B^2$, a double root occurs in Eq. (3.18) at

$$\delta = i\,\frac{B}{2}$$

However, if the complete expression (3.5) is considered and $\beta = 0$, $\gamma = B^2$ are substituted, we can show that the double root is given by

$$\delta = i\,\frac{B}{2} - \frac{B^2}{8} \tag{3.19}$$

The pattern of the roots in the p plane for $\beta = 0$ is shown in Fig. 7.

When $\gamma \neq (\beta-B)^2$, the solution depends on whether $\gamma > (\beta-B)^2$ or $\gamma < (\beta-B)^2$. For the former, the roots are given by Eq. (3.15). For the latter, they are, to lowest order,

$$\delta = \frac{i}{2}\left\{\beta + B \pm \left[(\beta-B)^2 - \gamma\right]^{1/2}\right\} \tag{3.20}$$

The roots in the p plane thus have the same imaginary value $i\omega$, but are equally spaced from $-\omega(\beta+B)/2$ on a line parallel to the real axis. Because both roots lie in the left-hand plane for all nonzero values of β, B, and γ, the transient response of the damped system always decays.

3.4 Damped Slightly Detuned System

Due to the large number of parameters in this case, it is convenient to illustrate the form of the solution by considering the special case $\beta = B$. Here

$$\lambda = (\gamma + \xi^2)^{1/2}$$

and $\mu = 0$, yielding

$$\delta = \frac{\xi}{2} \pm \frac{1}{2}(\gamma+\xi^2)^{1/2} + i\,\frac{\beta+B}{2} \tag{3.21}$$

and

$$p = i\omega\left[1 + \frac{\xi}{2} \pm \frac{1}{2}(\gamma+\xi^2)^{1/2}\right] - \omega\,\frac{\beta+B}{2} \tag{3.22}$$

These roots are similar to those shown in Fig. 6, except that they are shifted to the left by $\omega(\beta+B)/2$.

4. INVERSION OF TRANSFORM SOLUTION FOR TWO-DEGREE-OF-FREEDOM SYSTEM

The formal inversion of the transform expression (3.1) is

$$\ddot{u}(t) = \frac{1}{2\pi i} \int_{\Gamma} \frac{N(p)}{D(p)} \bar{\ddot{u}}_g(p) e^{pt} dp \tag{4.1}$$

where Γ is a suitable Bromwich path. If $\bar{\ddot{u}}_g(p)$ is taken to be 1, then the inversion directly yields Green's function $\ddot{u}_G(t)$ for the solution, the essential component of the subsequent analysis. The complete solution for the equipment acceleration for given ground motion $\ddot{u}_g(t)$ will take the form

$$\ddot{u}(t) = \int_0^t \ddot{u}_G(t-\tau) \ddot{u}_g(\tau) d\tau \tag{4.2}$$

Green's function will be obtained by residue theory, since there are no branch cuts in the p plane. The inversion of the transformed Green's function for the general case will be obtained, Eq. (3.6), for different ranges of the parameters γ, β, B, and ξ, corresponding to the special cases discussed in the previous chapter.

To obtain the inversion, the denominator $D(p)$ is written in the form

$$D(p) = (p-p_1)(p-\bar{p}_1)(p-p_2)(p-\bar{p}_2)$$

where

$$p_1 = i\omega\left(1 + \frac{\xi}{2} + \frac{\lambda}{2}\right) - \omega\left(\frac{\beta+B}{2} + \frac{\mu}{2}\right)$$

$$p_2 = i\omega\left(1 + \frac{\xi}{2} - \frac{\lambda}{2}\right) - \omega\left(\frac{\beta+B}{2} - \frac{\mu}{2}\right)$$

and $\bar{p}_1$ and $\bar{p}_2$ are the complex conjugates of p_1 and p_2. By evaluating the residues at each pole and collecting complex conjugate terms in pairs, we obtain the result, correct to dominant order,

$$\ddot{u}_G(t) = \frac{\omega}{\lambda^2+\mu^2} e^{-(\beta+B)\omega t/2} \left[\lambda \sinh\frac{\mu}{2}\omega t \cos\frac{\lambda}{2}\omega t \sin\left(1+\frac{\xi}{2}\right)\omega t \right.$$

$$- \lambda \cosh\frac{\mu}{2}\omega t \sin\frac{\lambda}{2}\omega t \cos\left(1+\frac{\xi}{2}\right)\omega t$$

$$- \mu \sinh\frac{\mu}{2}\omega t \cos\frac{\lambda}{2}\omega t \cos\left(1+\frac{\xi}{2}\right)\omega t$$

$$\left. - \mu \cosh\frac{\mu}{2}\omega t \sin\frac{\lambda}{2}\omega t \sin\left(1+\frac{\xi}{2}\right)\omega t \right] \qquad (4.3)$$

Several special cases are considered in the following sections.

4.1 Undamped Tuned System

When $\beta = 0$, $B = 0$, and $\xi = 0$, the solution where terms to the order ϵ^3 are retained is

$$\ddot{u}_G(t) = \frac{3\omega}{2} \sin\omega t \cos\eta t - \frac{\omega}{\gamma^{1/2}} \cos\omega t \sin\eta t \qquad (4.4)$$

where

$$\eta = \gamma^{1/2}\omega/2$$

This function represents an undamped beat solution with the beat frequency $\eta = \gamma^{1/2}\omega/2$ much smaller than the tuning frequency ω.

4.2 Undamped Slightly Detuned System

Green's function, where terms to order ϵ^2 are retained, takes the form

$$\ddot{u}_G(t) = -\frac{\omega}{(\gamma+\xi^2)^{1/2}} \cos\left(1+\frac{\xi}{2}\right)\omega t \sin\eta t \qquad (4.5)$$

where now

$$\eta = (\gamma+\xi^2)^{1/2}\omega/2$$

As for Eq. (4.4), this solution represents undamped beats.

4.3 Damped Tuned System

When $\xi = 0$, three ranges of the parameters γ, β, and B must be investigated.

i) For $\gamma > (\beta - B)^2$, $D(p)$ can be written in the form

$$D(p) = (p - i\omega + \epsilon_1\omega)(p + i\omega + \epsilon_1\omega)(p - i\omega + \epsilon_2\omega)(p + i\omega + \epsilon_2\omega)$$

where

$$\epsilon_1 = \frac{\beta+B}{2} + \frac{i}{2}\left[\gamma - (\beta-B)^2\right]^{1/2}$$

$$\epsilon_2 = \frac{\beta+B}{2} - \frac{i}{2}\left[\gamma - (\beta-B)^2\right]^{1/2}$$

By evaluating the residues at each pole and collecting complex conjugate terms in pairs, we obtain the result, correct to dominant order,

$$\ddot{u}_G(t) = -\frac{\omega e^{-(\beta+B)\omega t/2}\cos\omega t \sin\left[\gamma - (\beta-B)^2\right]^{1/2}\omega t/2}{\left[\gamma - (\beta-B)^2\right]^{1/2}} \tag{4.6}$$

This function represents a damped beat solution, where the beat frequency, $[\gamma - (\beta-B)^2]^{1/2}\omega/2$, is much smaller than the tuning frequency ω.

ii) For $\gamma < (\beta - B)^2$, $D(p)$ is written as before, the residues at each pole evaluated, and pairs of conjugate terms collected, leading to a Green's function of the form

$$\ddot{u}_G(t) = -\frac{\omega e^{-(\beta+B)\omega t/2}\cos\omega t \sinh\left[(\beta-B)^2 - \gamma\right]^{1/2}\omega t/2}{\left[(\beta-B)^2 - \gamma\right]^{1/2}} \tag{4.7}$$

Since $(\beta+B)^2 > (\beta-B)^2 - \gamma$ for nonzero values of β, B, and γ, the term $\exp[-(\beta+B)\omega t/2]$ dominates the term $\sinh[(\beta-B)^2 - \gamma]^{1/2}\omega t/2$. The solution can be interpreted as overdamped beats by analogy to the concept of overdamped vibrations. For large ωt, the solution is an oscillation of frequency ω damped by an exponential with factor

$$-\frac{1}{2}\left\{\beta + B - \left[(\beta-B)^2 - \gamma\right]^{1/2}\right\}\omega t$$

iii) For $\gamma = (\beta - B)^2$, the result in Eq. (3.15) suggests a double pole, which, as already shown, will not appear if $\beta \neq 0$. In fact, the more accurate location of the root yields

$$p = i\omega - \frac{\beta + B}{2}\,\omega \pm \frac{i\omega}{2}\,\beta^{1/2}\gamma^{1/2} \tag{4.8}$$

We obtain the following result by evaluating the residues

$$\ddot{u}_G(t) = -\frac{\omega e^{-(\beta+B)\omega t/2}\cos\omega t \,\sin\beta^{1/2}\gamma^{1/2}\omega t/2}{\beta^{1/2}\gamma^{1/2}} \tag{4.9}$$

As for Eq. (4.6), this solution represents damped beats. The beat period, of order $\epsilon^{-3/2}$, is very long.

iv) When $\beta = 0$ and $\gamma = B^2$, a genuine double root appears. Green's function for the solution takes the form

$$\ddot{u}_G(t) = -\frac{1}{2}\,\omega^2\, t\, e^{-B\omega t/2}\cos\omega t \tag{4.10}$$

This case can be interpreted as critically damped beats.

4.4 Damped Slightly Detuned System

For a detuned damped system where $\beta = B$, the Green's function for the solution is

$$\ddot{u}_G(t) = -\frac{\omega}{(\gamma + \xi^2)^{1/2}}\, e^{-(\beta+B)\omega t/2}\sin\eta t\,\cos\left(1 + \frac{\xi}{2}\right)\omega t \tag{4.11}$$

where

$$\eta = (\gamma + \xi^2)^{1/2}\omega/2$$

5. APPLICATION TO EQUIPMENT MOUNTING DESIGN: UNDAMPED TUNED SYSTEMS

The results obtained in the previous chapter for the response of various damped or undamped, tuned or untuned systems are applicable to the design of equipment or equipment mounting. The least complicated solution is that for the undamped tuned system; methods developed for damped and tuned or detuned systems are extensions of the method developed for this case. Equipment response can be more readily understood if the solution for the former case is examined in detail before more general cases are developed.

The results given in Eqs. (4.2) and (4.4) could in principle be used by a designer of equipment mounting if a specified ground acceleration history were available to estimate the forces that would develop in the equipment or its mounting. However, such information is not readily available and the numerical evaluation of these integrals may be inconvenient during the design process. Commonly, a designer begins with a design spectrum that may be specified by a code or determined by averaging several possible inputs as, for example, in seismic design [4], [5]. We are thus interested in determining the extent to which Eqs. (4.2) and (4.4) can be used to obtain estimates of maximum acceleration when the information available is the response spectrum of the ground motion $\ddot{u}_g$. In the following sections a number of alternative approaches are explored.

5.1 Floor Spectrum Analysis

When Green's function obtained in Eq. (4.4) for the undamped tuned system is substituted into Eq. (4.2), the response is

$$\ddot{u}(t) = \frac{\omega}{\gamma^{1/2}} \int_0^t \ddot{u}_g(\tau) \left[\frac{3}{2} \gamma^{1/2} \cos\eta(t-\tau) \sin\omega(t-\tau) - \sin\eta(t-\tau) \cos\omega(t-\tau) \right] d\tau \tag{5.1}$$

where $\eta = \gamma^{1/2}\omega/2$.

For small ηt, this reduces to

$$\ddot{u}(t) = \omega \int_0^t \ddot{u}_g(\tau) \left[\frac{3}{2} \sin\omega(t-\tau) - \omega(t-\tau) \cos\omega(t-\tau) \right] d\tau \tag{5.2}$$

This result is independent of γ and could be obtained directly by means of the floor spectrum analysis method whereby input to the structure is used to compute base motion at the equipment, assuming the latter to be absent, and equipment response is calculated with this motion as input. Thus, the floor spectrum analysis, in which equipment-structure interaction is neglected, is valid only for $\eta t << 1$.

Since the floor spectrum result can be obtained from the basic equations by setting $\gamma = 0$, a double pole will appear in the tuned case. (For an untuned system only simple poles occur.) This double pole leads to a term in $t\cos\omega t$ on inversion. Thus, the floor spectrum method cannot be used to determine maximum displacement or acceleration for undamped tuned systems since the response it yields grows without limit.

5.2 Modified Ground Motion Spectra

When ηt is not much smaller than unity, the term in Eq. (5.1) multiplied by $\gamma^{1/2}$ is negligible when compared with the other and it becomes

$$\ddot{u}(t) = -\frac{\omega}{\gamma^{1/2}} \int_0^t \ddot{u}_g(\tau)\sin\eta(t-\tau)\cos\omega(t-\tau)\,d\tau \tag{5.3}$$

Equation (5.3) can be written as follows when the term $\cos\omega(t-\tau)$ is expanded.

$$\ddot{u}(t) = -\frac{\omega}{\gamma^{1/2}}\cos(\omega t-\phi)\left\{\left\{\int_0^t\left[\ddot{u}_g(\tau)\cos\omega\tau\right]\sin\eta(t-\tau)\,d\tau\right\}^2+\left\{\int_0^t\left[\ddot{u}_g(\tau)\sin\omega\tau\right]\sin\eta(t-\tau)\,d\tau\right\}^2\right\}^{1/2} \tag{5.4}$$

where

$$\phi = \tan^{-1}\left\{\frac{\int_0^t\left[\ddot{u}_g(\tau)\sin\omega\tau\right]\sin\eta(t-\tau)\,d\tau}{\int_0^t\left[\ddot{u}_g(\tau)\cos\omega\tau\right]\sin\eta(t-\tau)\,d\tau}\right\} \tag{5.5}$$

The terms

$$\eta\int_0^t\left[\ddot{u}_g(\tau)\cos\omega\tau\right]\sin\eta(t-\tau)\,d\tau \tag{5.6}$$

and

$$\eta \int_0^t \left[\ddot{u}_g(\tau)\sin\omega\tau\right]\sin\eta(t-\tau)\,d\tau \tag{5.7}$$

can be interpreted as the acceleration response of an undamped single-degree-of-freedom system with frequency η to the modified ground input acceleration $\ddot{u}_g(t)\cos\omega t$ and $\ddot{u}_g(t)\sin\omega t$. Since $\eta << \omega$, the term $\cos(\omega t-\phi)$ is a rapidly oscillating function which achieves its maximum a great many times more than do the integrals which are slowly oscillating functions. The integrals are thus a slowly varying envelope of the more rapidly oscillating term. The maximum of the product is accordingly very close to the maximum of the envelope.

Thus, one way of estimating equipment motion is to construct spectra for the modified ground accelerations. Maximum acceleration would then be

$$\ddot{u}_{\max} = \frac{2}{\gamma}\left[S_A^c(\eta)^2 + S_A^s(\eta)^2\right]^{1/2} \tag{5.8}$$

A similar expression can be obtained for displacement. In the above, the terms $S_A^c(\eta)$ and $S_A^s(\eta)$ are undamped acceleration response spectra for the modified ground motions $\ddot{u}_g(t)\cos\omega t$ and $\ddot{u}_g(t)\sin\omega t$, respectively. In principle, then, if the time history $\ddot{u}_g(t)$ is available, a design technique can be developed by constructing low-frequency response spectra for the modified ground motions. Usually, only the response spectrum of $\ddot{u}_g$ and not the ground motion itself will be available. At present, the spectra for the modified ground motion cannot be computed if the only information available is the spectrum of the actual ground motion. Thus, we will develop an alternative approach in the following section.

5.3 Amplified Ground Motion Spectrum

The term $\sin\eta(t-\tau)$ is expanded, leading to

$$\ddot{u}(t) = -\frac{\omega}{\gamma^{1/2}}\sin(\eta t-\theta)\left\{\left[\int_0^t \ddot{u}_g(\tau)\cos\eta\tau\cos\omega(t-\tau)\,d\tau\right]^2 + \left[\int_0^t \ddot{u}_g(\tau)\sin\eta\tau\cos\omega(t-\tau)\,d\tau\right]^2\right\}^{1/2} \tag{5.9}$$

where

$$\theta = \tan^{-1}\left[\frac{\int_0^t \ddot{u}_g(\tau)\sin\eta\tau\,\cos\omega(t-\tau)\,d\tau}{\int_0^t \ddot{u}_g(\tau)\cos\eta\tau\,\cos\omega(t-\tau)\,d\tau}\right] \tag{5.10}$$

We are interested in situations where the ground motion has prescribed finite duration and for those frequencies ω where the maximum response of a single-degree-of-freedom oscillator, *i.e.* the response spectrum, is achieved late in or after the termination of the ground motion. These frequencies correspond to peaks in the response spectrum of a seismic ground motion. Design spectra, reflecting the probabilistic motion of the input, correspond closely to the peaks of actual spectra and thus presuppose late-occurring maxima. When ground motion is caused by a blast, which is of short duration, it is likely that the maxima of equipment response will occur long after the ground motion has ended.

Thus, for $\eta t_1 = 2\pi t_1/T << 1$, where t_1 is the duration of the ground motion and T is the beat period of the system, the first integral in Eq. (5.9) can be approximated by

$$\int_0^t \ddot{u}_g(\tau)\cos\omega(t-\tau)\,d\tau \tag{5.11}$$

and the second neglected since $\sin\eta t$ will be bounded by $\eta t_1 << 1$, and $\ddot{u}_g = 0$ for $t > t_1$. For $\eta t_1 << 1$, then, we have

$$\ddot{u}(t) = -\frac{\omega}{\gamma^{1/2}}\sin\eta t \int_0^t \ddot{u}_g(\tau)\cos\omega(t-\tau)\,d\tau \tag{5.12}$$

The term in the integral is a function that oscillates with frequency ω, which is high compared to η, and a maximum of that will nearly coincide with the maximum of $\sin\eta t$. Thus, an estimate of the maximum value of $\ddot{u}(t)$ is

$$|\ddot{u}|_{\max} = \frac{\omega}{\gamma^{1/2}}\max\left|\int_0^t \ddot{u}_g(t)\cos\omega(t-\tau)\,d\tau\right| \tag{5.13}$$

If the displacement, pseudo-velocity, and pseudo-acceleration response spectra as functions of frequency ω and damping parameter β are denoted by $S_D(\omega,\beta)$, $S_V(\omega,\beta)$, and $S_A(\omega,\beta)$, respectively, then

$$\max\left|\int_0^t \ddot{u}_g(\tau)\cos\omega(t-\tau)\,d\tau\right|$$

is the undamped relative velocity response spectrum, which is very nearly the pseudo-velocity response spectrum $S_V(\omega,0)$, for a single-degree-of-freedom system with frequency ω. Accordingly, we obtain the following estimate of maximum acceleration

$$|\ddot{u}|_{\max} = \frac{\omega}{\gamma^{1/2}}\, S_V(\omega,0) \tag{5.14}$$

Since $S_D = S_V/\omega$ and $S_A = \omega S_V$

$$|u|_{\max} = \frac{S_D(\omega,0)}{\gamma^{1/2}} \tag{5.15}$$

and

$$|\ddot{u}|_{\max} = \frac{S_A(\omega,0)}{\gamma^{1/2}} \tag{5.16}$$

If a designer is given only the response spectrum of the ground motion, the maximum displacement and force in the equipment can be estimated by using these spectra amplified by the factor $\gamma^{-1/2}$. These remarks refer to the equivalent two-degree-of-freedom system. Results for the general system are obtained by utilizing the factors in Eqs. (2.12), (2.13), and (2.14).

The simplicity of the result can be explained on physical grounds. In weakly coupled systems with the same frequency, the response of the system involves a perfect energy exchange between each component at a beat frequency much lower than the natural frequency of each component. The same phenomenon -- a classical beat phenomenon -- occurs here. The coupling is weak because the ratio of equipment mass to structure mass is small.

When a structure is subjected to a ground motion, the velocity imparted to the structure is mass-independent and determined only by the ground motion. Thus, if the same ground motion were applied directly to tuned equipment, the same velocity would be transmitted to it. Kinetic energy, on the other hand, is proportional to the mass of the system excited; in equipment, that energy would be much smaller than in a structure. However, if the equipment were attached to a structure and the structure subjected to a ground motion, the kinetic energy imparted to the latter would subsequently be wholly transmitted to the equipment, if tuned, and the velocity imparted would be amplified by the reciprocal of the square root of the mass ratio.

Damping is clearly important in this process because the energy transfer requires many cycles and much of the kinetic energy in a damped system could be dissipated before being transmitted. The transient analysis of damped tuned and nearly tuned systems follows in the next chapter.

6. APPLICATION TO EQUIPMENT MOUNTING DESIGN: DAMPED TUNED AND SLIGHTLY DETUNED SYSTEMS

The specified time history of a ground motion applied to a structure is often unavailable except in the restricted form of a design spectrum. The procedures developed in the previous chapter for undamped tuned systems enable a designer to utilize the design spectrum for the structure to estimate directly maximum values of the equipment acceleration and displacement.

Three approaches to the design problem were developed. These were the floor spectrum method, the modified ground motion spectrum method, and the amplified ground motion spectrum method. The floor spectrum method was demonstrated to be invalid for undamped tuned systems, and the modified ground motion spectrum method, while valid, to be inconvenient. The amplified ground motion spectrum was the most convenient to use in estimating the response of light undamped equipment.

If for a damped tuned system, $\gamma << 1$, then γ is negligible if $4\beta B >> \gamma$ (Eq. (3.3)), and equipment-structure interaction can be neglected. The floor spectrum method can, in principle, be used to determine the response of the equipment for this case. However, this method requires that time histories be computed, and the numerical time-integration algorithms available in structural dynamics computer programs are not known to be sufficiently accurate to calculate the peak response of equipment, which occurs late in time, reliably. Furthermore, if only the design spectrum and not the history of the ground motion is available, then the conventional floor spectrum method cannot be used. Approximations to be used in the design of equipment mounting for damped and slightly detuned systems will therefore be developed from the amplified response spectrum method.

6.1 Undamped Slightly Detuned Systems

The acceleration response $\ddot{u}(t)$ to imposed ground acceleration $\ddot{u}_g(t)$ is obtained from Eqs. (4.2) and (4.5) in the form

$$\ddot{u}(t) = -\frac{\omega}{(\gamma+\xi^2)^{1/2}} \int_0^t \ddot{u}_g(\tau) \sin\eta(t-\tau) \cos\left(1+\frac{\xi}{2}\right)\omega(t-\tau)\, d\tau \qquad (6.1)$$

where

$$\eta = (\gamma + \xi^2)^{1/2}\omega/2$$

By expanding the term $\sin\eta(t-\tau)$ as was done in Eq. (5.9) and neglecting analogous terms, we obtain an estimate of the maximum acceleration in the form

$$|\ddot{u}|_{\max} = \frac{S_A\left[\frac{\omega+\Omega}{2}, 0\right]}{(\gamma+\xi^2)^{1/2}} \tag{6.2}$$

This solution remains valid for $\gamma << \xi^2$, provided that $\xi << 1$. For such cases, the floor spectrum method is applicable, but cannot be used if the only information on the ground motion is a design spectrum. The above solution is equally valid, and clearly more convenient. Provided that the numerical computations employing the ground motion history are performed with suitable accuracy, the beat phenomenon between the two closely spaced frequencies ω and $\Omega = \omega(1+\xi)$, which is the physical basis of the result, will appear in the floor spectrum solution in the slightly detuned case.

6.2 Damped Tuned Systems

Four sets of the parameters γ, β, and B must be investigated for damped tuned systems.

<u>Case</u> <u>1:</u> $\gamma > (\beta-B)^2$

The acceleration response $\ddot{u}(t)$ to imposed ground acceleration $\ddot{u}_g(t)$ is given by

$$\ddot{u}(t) = -\frac{\omega}{\left[\gamma-(\beta-B)^2\right]^{1/2}}\int_0^t \ddot{u}_g(\tau)\, e^{-(\beta+B)\omega(t-\tau)/2}\cos\omega(t-\tau)\sin\eta(t-\tau)\,d\tau \tag{6.3}$$

where

$$\eta = \left[\gamma - (\beta - B)^2\right]^{1/2}\omega/2$$

When the term $\sin\eta(t-\tau)$ is expanded

$$\ddot{u}(t) = -\frac{\omega}{\left[\gamma-(\beta-B)^2\right]^{1/2}} \cos(\eta t-\theta)\left\{\left[\int_0^t \ddot{u}_g(\tau)e^{-(\beta+B)\omega(t-\tau)/2}\cos\omega(t-\tau)\cos\eta\tau\, d\tau\right]^2\right.$$

$$\left.+\left[\int_0^t \ddot{u}_g(\tau)e^{-(\beta+B)\omega(t-\tau)/2}\cos\omega(t-\tau)\sin\eta\tau\, d\tau\right]^2\right\}^{1/2} \tag{6.4}$$

where

$$\theta = \tan^{-1}\left\{-\frac{\int_0^t \ddot{u}_g(\tau)\cos\eta\tau\, e^{-(\beta+B)\omega(t-\tau)/2}\cos\omega(t-\tau)\,d\tau}{\int_0^t \ddot{u}_g(\tau)\sin\eta\tau\, e^{-(\beta+B)\omega(t-\tau)/2}\cos\omega(t-\tau)\,d\tau}\right\} \tag{6.5}$$

As in Section 5.3, we consider $\eta t_1 = 2\pi t_1/T << 1$, where t_1 is the duration of ground motion and T is the beat period of the system. Then, in analogy to the undamped cases, the first integral in Eq. (6.4) can be approximated by

$$\int_0^t \ddot{u}_g(\tau)\, e^{-(\beta+B)\omega(t-\tau)/2}\cos\omega(t-\tau)\,d\tau$$

and the second neglected. Thus, we take

$$\ddot{u}(t) = -\frac{\omega\sin\eta t}{\left[\gamma-(\beta-B)^2\right]^{1/2}}\int_0^t \ddot{u}_g(\tau)\, e^{-(\beta+B)\omega(t-\tau)/2}\cos\omega(t-\tau)\,d\tau \tag{6.6}$$

When the parameters $\gamma^{1/2}$, β, and B are small, this result may be interpreted in the following way: for $t > t_1$, the above expression can be written in the form

$$\ddot{u}(t) = -\frac{\omega^2\sin\eta t}{2\eta}\, e^{-(\beta+B)\omega t/2} R\cos(\omega t-\psi)$$

where

$$R = \left[A_1^2 + A_2^2\right]^{1/2}$$

with

$$A_1 = \int_0^{t_1} \ddot{u}_g(t)\, e^{+(\beta+B)\omega t/2} \cos\omega t \, dt$$

$$A_2 = \int_0^{t_1} \ddot{u}_g(t)\, e^{+(\beta+B)\omega t/2} \sin\omega t \, dt$$

and

$$\psi = \tan^{-1}\left(A_2/A_1\right)$$

The response indicated by the above is illustrated in Fig. 8. In the above, the terms R and ψ are constants independent of t for $t > t_1$, and $R\cos(\omega t - \psi)$ is a rapidly varying function of time. The term

$$\frac{\omega^2}{2\eta}\, e^{-(\beta+B)\omega t/2} \sin\eta t$$

is a slowly varying envelope curve whose maximum value must be determined. The maximum value of this envelope curve is attained at time t^*, expressed by

$$\tan\eta t^* = \frac{2\eta}{(\beta+B)\omega} \tag{6.7}$$

The value of t^* is thus

$$t^* = \arctan\left[2\eta/\omega(\beta+B)\right]/\eta \tag{6.8}$$

For lightly damped systems and light equipment, in general $t^* >> t_1$. The values of $\sin\eta t$ and $\exp[-(\beta+B)\omega t/2]$ when the envelope achieves its maximum are

$$\sin\eta t^* = \frac{\eta}{\left[\eta^2 + (\beta+B)^2\omega^2/4\right]^{1/2}}$$

$$e^{-(\beta+B)\omega t^*/2} = e^{-\kappa} \tag{6.9}$$

where

$$\kappa = (arctan\zeta)/\zeta \tag{6.10}$$

$$\zeta = \left[\gamma - (\beta - B)^2\right]^{1/2} / (\beta + B) \tag{6.11}$$

It follows that

$$|\ddot{u}|_{\max} = |\ddot{u}(t^*)| = \left\{ \frac{\omega}{\left[\gamma - (\beta - B)^2\right]^{1/2}} |\sin\eta t^*| e^{-\kappa} \right\}$$

$$\cdot \left\{ e^{(\beta+B)\omega\eta t^*/2} \left| \int_0^{t^*} \ddot{u}_g(\tau) e^{-(\beta+B)\omega(t^*-\tau)/2} \cos\omega(t^*-\tau) d\tau \right| \right\} \tag{6.12}$$

In order that this estimate of the peak acceleration be useful for design purposes, it is necessary that the second factor in braces be interpreted in terms of a ground response spectrum. To this end, we recognize that the integral is, to the order of $\beta+B$, the relative velocity response history, evaluated at time t^*, of a lightly damped single-degree-of-freedom oscillator of frequency ω and damping factor $(\beta+B)/2$ subjected to the ground acceleration $\ddot{u}_g(t)$. At some time $\tilde{t}$ during the ground motion or shortly after it ceases (so that $\tilde{t} << t^*$), the absolute value of the relative velocity will attain its global maximum, denoted as $|v(\tilde{t})|$. The relative velocity response at t^*, denoted as $v(t^*)$, can be thought of as that which would occur in a single-degree-of-freedom system (subjected to the ground acceleration $\ddot{u}_g(t)$) as a consequence of free vibration beginning at time $\hat{t}(>t_1)$ where the absolute value of the relative velocity of the oscillator attains its first local maximum, $|v(\hat{t})|$, after the end of the earthquake. This instant of time $\hat{t}$ is equal to $\tilde{t}$ if $\tilde{t}$ occurs after the end of the ground motion; otherwise, $\hat{t} > \tilde{t}$. In any event, $\hat{t} << t^*$. Thus, we can write

$$|v(t^*)| = |v(\hat{t})| e^{-(\beta+B)\omega(t^*-\hat{t})/2} |\cos\omega(t^*-\hat{t})| \tag{6.13}$$

where $|v(\hat{t})| \leqslant |v(\tilde{t})|$. It then follows that

$$|v(t^*)| \equiv |\int_0^{t^*} \ddot{u}_g(\tau) e^{-(\beta+B)\omega(t^*-\tau)/2} \cos\omega(t^*-\tau)\, d\tau|$$

$$= |v(\hat{t})| e^{-(\beta+B)\omega(t^*-\hat{t})/2} |\cos\omega(t^*-\hat{t})|$$

$$\leqslant |v(\tilde{t})| e^{-(\beta+B)\omega t^*(1-\hat{t}/t^*)/2}$$

$$\approx |v(\tilde{t})| e^{-(\beta+B)\omega t^*/2} \tag{6.14}$$

since $\hat{t}/t^* << 1$. From this we obtain the approximate result

$$|v(\tilde{t})| \approx e^{(\beta+B)\omega t^*/2} |\int_0^{t^*} \ddot{u}_g(\tau) e^{-(\beta+B)\omega(t^*-\tau)/2} \cos\omega(t^*-\tau)\, d\tau| \tag{6.15}$$

But, we recognize that to the order of β and B, $|v(\tilde{t})|$ is very nearly the pseudo-velocity response spectrum $S_V(\omega, (\beta+B)/2)$ for a lightly damped single-degree-of-freedom oscillator of frequency ω and damping factor $(\beta+B)/2$ subjected to the ground acceleration $\ddot{u}_g(t)$. Thus, an estimate of the maximum equipment acceleration is

$$|\ddot{u}|_{\max} = \frac{\omega |\sin\eta t^*| e^{-\kappa}}{\left[\gamma-(\beta-B)^2\right]^{1/2}} S_V\left(\omega, \frac{\beta+B}{2}\right)$$

With the value of $\sin\eta t^*$ from Eq. (6.9), we obtain the final estimate as

$$|\ddot{u}|_{\max} = \frac{e^{-\kappa}\omega S_V\left(\omega, \frac{\beta+B}{2}\right)}{(\gamma+4\beta B)^{1/2}} \tag{6.16}$$

Recalling that

$$\omega S_V = S_A = \omega^2 S_D$$

this estimate can be written in the alternative form

$$|\ddot{u}(t)|_{\max} = \frac{e^{-\kappa}}{(\gamma+4\beta B)^{1/2}} S_A\left(\omega, \frac{\beta+B}{2}\right) \tag{6.17}$$

and, similarly, for the displacement

$$|u(t)|_{\max} = \frac{e^{-\kappa}}{(\gamma+4\beta B)^{1/2}} S_D\left(\omega, \frac{\beta+B}{2}\right) \tag{6.18}$$

Thus, given only ground spectra at various damping values, the designer can estimate the maximum displacement and force in the equipment by using the spectra for a damping factor equal to the average of those in the structure and equipment amplified by the factor $(\gamma+4\beta B)^{-1/2}e^{-\kappa}$. Note that if $\beta+B$ is fixed, the maximum value of $4\beta B$ and the minimum value of $(\beta-B)^2$ are achieved when $\beta=B$, yielding the smallest value of the amplification factor. Thus, if total damping is fixed, it should optimally be shared equally by equipment and structure for this case.

Case 2: $\gamma < (\beta-B)^2$

In terms of the specified ground motion $\ddot{u}_g(t)$, the solution for $\ddot{u}(t)$ takes the form

$$\ddot{u}(t) = -\frac{\omega}{\left[(\beta-B)^2-\gamma\right]^{1/2}} \int_0^t \ddot{u}_g(\tau)\, e^{-(\beta+B)\omega(t-\tau)/2} \cos\omega(t-\tau) \sinh\eta(t-\tau)\, d\tau \tag{6.19}$$

where

$$\eta = \left[(\beta-B)^2-\gamma\right]^{1/2} \omega/2$$

which can be rewritten in a form analogous to that in Eq. (6.4) with the envelope now in the form $\exp[-(\beta+B)\omega t/2]\sinh\eta t$. When the envelope is analyzed as before for its maximum value, time $t=t^*$ is such that

$$\sinh\eta t^* = \frac{\eta}{\left[\eta^2+(\beta+B)^2\omega^2/4\right]^{1/2}}$$

$$\kappa = (arctanh\,\zeta)/\zeta$$

$$\zeta = \left[(\beta-B)^2-\gamma\right]^{1/2}/(\beta+B)$$

Note that since *arctan* $i\,x = i$ *arctanh* x, we may rewrite the above two equations in a form identical to that of the preceding case, *i.e.*

$$\kappa = (arctan\zeta)/\zeta$$

$$\zeta = \left[\gamma - (\beta - B)^2\right]^{1/2} / (\beta + B)$$

As in Case 1, we find that the amplification factor is $(\gamma + 4\beta B)^{-1/2} e^{-\kappa}$.

Case 3: $\gamma = (\beta - B)^2$

Here the solution takes the form

$$\ddot{u}(t) = -\frac{\omega}{\beta^{1/2}\gamma^{1/2}} \int_0^t \ddot{u}_g(\tau) e^{-(\beta+B)\omega(t-\tau)/2} \cos\omega(t-\tau) \sin\eta(t-\tau)\,d\tau \tag{6.20}$$

where now

$$\eta = \beta^{1/2}\gamma^{1/2}\omega/2$$

The envelope takes the form

$$e^{-(\beta+B)\omega t/2} \sin\eta t$$

and time t^* is given by

$$\tan\eta t^* = \frac{2\eta}{(\beta+B)\omega}$$

This expression and the appropriate value for η yield the following term for the amplification factor

$$\frac{e^{-\kappa}}{\left[(\beta+B)^2 + \beta\gamma\right]^{1/2}}$$

where $\kappa = (arctan\zeta)/\zeta$, $\zeta = \left[\beta\gamma/(\beta+B)^2\right]^{1/2}$.

The term $\beta\gamma$ is of order ϵ^3 and may be neglected in comparison to $(\beta+B)^2$. Thus, the amplification factor is $(\beta+B)^{-1}e^{-\kappa}$. Note, however, that because $\gamma=(\beta-B)^2$, $(\beta+B)^{-1}=(\gamma+4\beta B)^{-\frac{1}{2}}$. Similarly, ζ is of order $\epsilon^{\frac{1}{2}} << 1$, so that $\kappa=\lim_{\zeta\to 0}(arctan\zeta)/\zeta=1$. Thus, the expression for the amplification factor has the same form as was obtained for the two previous cases.

Case 4: $\gamma=B^2$, $\beta=0$

A true double root appears and the solution is a damped floor spectrum result

$$\ddot{u}(t) = -\frac{\omega^2}{2}\int_0^t \ddot{u}_g(\tau)\,e^{-B\omega(t-\tau)/2}\,(t-\tau)\cdot\cos\omega(t-\tau)\,d\tau \tag{6.21}$$

The envelope $t\,e^{-B\omega t/2}$ attains maximum at t^* given by $\omega t^*=2/B$, leading to an amplification factor of e^{-1}/B. However, because $\gamma=B^2$, $\beta=0$, and $\kappa=1$, the amplification factor is again

$$\frac{e^{-\kappa}}{(\gamma+4\beta B)^{\frac{1}{2}}}$$

Thus, Eqs. (6.17) and (6.18) obtained for Case 1 are in fact correct for all combinations of γ, β, and B, despite the difference in form of Green's function for each case.

6.3 Damped Slightly Detuned Systems

For a sightly detuned system, the response is considerably modified and is given by

$$\ddot{u}(t) = -\frac{\omega}{(\lambda^2+\mu^2)} \int_0^t \ddot{u}_g(\tau)\, e^{-(\beta+B)\omega(t-\tau)/2}$$

$$\Big\{ -\lambda \sinh\frac{\mu}{2}\omega(t-\tau) \cos\frac{\lambda}{2}\omega(t-\tau) \sin(1+\frac{\xi}{2})\omega(t-\tau)$$

$$+\lambda \cosh\frac{\mu}{2}\omega(t-\tau) \sin\frac{\lambda}{2}\omega(t-\tau) \cos(1+\frac{\xi}{2})\omega(t-\tau)$$

$$+\mu \sinh\frac{\mu}{2}\omega(t-\tau) \cos\frac{\lambda}{2}\omega(t-\tau) \cos(1+\frac{\xi}{2})\omega(t-\tau)$$

$$+\mu \cosh\frac{\mu}{2}\omega(t-\tau) \sin\frac{\gamma}{2}\omega(t-\tau) \sin(1+\frac{\xi}{2})\omega(t-\tau) \Big\} d\tau \qquad (6.22)$$

where λ and μ are defined in terms of ξ, γ, β, and B in Eqs. (3.7) and (3.8). To reduce the algebraic manipulation of this considerably more complicated expression, an illustrative case is discussed below.

The example selected is based on the optimal use of damping for the conditions of Case 1, $\beta = B$. For this damping, Eq. (6.22) then takes the form

$$\ddot{u}(t) = -\frac{\omega}{\lambda} \int_0^t \ddot{u}_g(\tau) e^{-(\beta+B)\omega(t-\tau)/2} \sin\frac{\lambda}{2}\omega(t-\tau) \cos(1+\frac{\xi}{2})\omega(t-\tau)\, d\tau \qquad (6.23)$$

where

$$\lambda = (\gamma + \xi^2)^{1/2}$$

When the term $\sin(\lambda/2)\omega(t-\tau)$ is expanded and we recall that for $t >> t_1$, the duration of the ground motion, the integral containing $\sin(\lambda/2)\omega\tau$ can be neglected, we obtain

$$\ddot{u}(t) = -\frac{\omega}{\lambda} e^{-(\beta+B)\omega t/2} \sin\frac{\lambda}{2}\omega t\, R \cos(\omega t - \psi)$$

where

$$R = (A_1^2 + A_2^2)^{1/2} \quad \text{and} \quad \psi = \tan^{-1}(A_2/A_1)$$

with

$$A_1 = \int_0^{t_1} \ddot{u}_g(t)\, e^{+(\beta+B)\omega t/2} \cos(1+\frac{\xi}{2})\omega t \, dt$$

$$A_2 = \int_0^{t_1} \ddot{u}_g(t)\, e^{+(\beta+B)\omega t/2} \sin(1+\frac{\xi}{2})\omega t \, dt$$

The slowly varying envelope function $\exp[-(\beta+B)\omega t/2]\sin(\lambda/2)\omega t$ reaches maximum at a time t^* such that

$$\sin\frac{\lambda}{2}\omega t^* = \frac{\lambda}{\left[\lambda^2+(\beta+B)^2\right]^{1/2}}$$

and

$$e^{-(\beta+B)\omega t^*/2} = e^{-\kappa}$$

where

$$\kappa = (\arctan\zeta)/\zeta$$

$$\zeta = \lambda/(\beta+B)$$

From this result and the argument used for the damped tuned cases, the estimated maximum response value is

$$|\ddot{u}|_{\max} = \frac{e^{-\kappa}}{(\gamma+\xi^2+4\beta B)^{1/2}}\, S_A\left(\frac{\omega+\Omega}{2}, \frac{\beta+B}{2}\right) \tag{6.24}$$

and

$$|u|_{\max} = \frac{e^{-\kappa}}{(\gamma+\xi^2+4\beta B)^{1/2}}\, S_D\left(\frac{\omega+\Omega}{2}, \frac{\beta+B}{2}\right) \tag{6.25}$$

For all cases considered a universal result applies: the appropriate response spectrum evaluated at the average damping and average frequency of structure and equipment is multiplied by the amplification factor

$$\frac{e^{-\kappa}}{(\gamma+\xi^2+4\beta B)^{1/2}}$$

where

$$\kappa = (arctan\zeta)/\zeta$$

$$\zeta = \left[\gamma+\xi^2-(\beta-B)^2\right]^{1/2}/(\beta+B)$$

7. COMPLETE SOLUTION INCLUDING OTHER POLES

In previous chapters, we determined the contribution of the tuning poles to the response of equipment-structure systems. These poles dominate response in the case of light equipment, but contributions from the other poles can be easily computed. To do so, recall that the non-tuning poles of Eq. (2.9) are close to their location for the structure alone, as indicated in Fig. 3. The poles for the m^{th} nontuned mode are

$$p = -B_m \Omega_m \pm i \Omega_m \tag{7.1}$$

If we evaluate the residues and drop negligible terms, we obtain, to dominant order, the contribution from the m^{th} structure poles as

$$\frac{C_r^m}{1-(\Omega_m/\omega)^2} \Omega_m e^{-B_m \Omega_m t} \sin \Omega_m t \quad , \quad m \neq n \tag{7.2}$$

and nondominant contributions of the same order from the tuning poles as

$$\sum_{\substack{m=1 \\ m \neq n}}^{N} \frac{C_r^m}{1-(\omega/\Omega_m)^2} \omega e^{-\beta \omega t} \sin \omega t \tag{7.3}$$

where C_r^m is defined in Eq. (2.14). The complete solution for the response of the equipment then takes the form

$$\ddot{u}(t) = \int_0^t \ddot{u}_g(\tau) \left\{ \sum_{\substack{m=1 \\ m \neq n}}^{N} \frac{C_r^m}{1-(\Omega_m/\omega)^2} \Omega_m e^{-B_m \Omega_m (t-\tau)} \sin \Omega_m (t-\tau) \right.$$

$$\left. + \sum_{\substack{m=1 \\ m \neq n}}^{N} \frac{C_r^m}{1-(\omega/\Omega_m)^2} \omega e^{-\beta \omega (t-\tau)} \sin \omega (t-\tau) + \ddot{u}_G(t-\tau) \right\} d\tau \tag{7.4}$$

where $\ddot{u}_G(t)$ is the dominant contribution from the tuning poles given in its various forms in Chapter 4.

The character of the two parts of the solution in Eq. (7.4) differs. The contributions from the nontuning poles and the nondominant contributions from the tuning poles are conventional and would attain their peaks during the ground excitation or shortly thereafter. The dominant response from the tuning poles, on the other hand, is controlled by the energy transfer from the structure to the equipment through beating, which takes a relatively long time (Chapters 5 and 6). The latter contribution achieves maximum considerably later than the former. It is pointless to add these contributions conventionally, such as by the square root of the sum of squares, or by a similar rule. In fact, they should not be added at all, but treated as separate maxima. The maximum response from the nondominant contributions can be estimated by the conventional square root of the sum of squares method.

Accordingly, the estimate of the maximum acceleration has two parts, namely, an early peak given by

$$|\ddot{u}|_{\max} = \left\{ \sum_{\substack{m=1 \\ m\neq n}}^{N} \left[\frac{C_r^m}{1-(\Omega_m/\omega)^2} S_A(\Omega_m,B_m) \right]^2 + \left[\sum_{\substack{m=1 \\ m\neq n}}^{N} \frac{C_r^m}{1-(\omega/\Omega_m)^2} \right]^2 S_A^2(\omega,\beta) \right\}^{1/2} \tag{7.5}$$

and a later peak, from the dominant contribution of the tuning poles, given by

$$|\ddot{u}|_{\max} = |C_r^n| \frac{e^{-\kappa}}{(\gamma^{eff}+\xi^2+4\beta B_n)^{1/2}} S_A\left(\frac{\omega+\Omega_n}{2},\frac{\beta+B_n}{2}\right) \tag{7.6}$$

where $\kappa = (arctan\zeta)/\zeta$, and $\zeta = |\gamma^{eff}+\xi^2-(\beta-B_n)^2|^{1/2}/(\beta+B_n)$. The detuning parameter is $\xi = (\Omega_n-\omega)/\omega$, and γ^{eff} is given by Eq. (2.12). For light equipment and lightly damped closely tuned systems, the second peak is likely to be more important. However, the early peak may be the larger and both should therefore be evaluated.

The methods developed here can also be used to estimate the peak response of grossly detuned systems, *i.e.*, where the equipment frequency is far from all structural frequencies. For light equipment, the structure poles are slightly shifted from their location for the structure alone, namely

$$p = -B_m\Omega_m \pm i\Omega_m \tag{7.7}$$

Additional poles due to the equipment occur at

$$p = -\beta\omega \pm i\omega \tag{7.8}$$

as shown in Fig. 9 where the response of the completely undamped system is illustrated. The residues at the structure poles are as before with the contributions from each $m=1$ to N set of poles given by Eq. (7.2). The residues at the equipment poles, Eq. (7.8), contribute to Green's function in the form, similar to Eq. (7.3),

$$\left\{ \sum_{m=1}^{N} \frac{C_r^m}{1-(\omega/\Omega_m)^2} \right\} \omega e^{-\beta\omega t} \sin\omega t \tag{7.9}$$

The derivation is standard and similar to the terms from the structure poles. The complete response for the equipment in the grossly detuned case is thus given by

$$\ddot{u}(t) = \int_0^t \ddot{u}_g(\tau) \left\{ \sum_{m=1}^{N} \frac{C_r^m}{1-(\Omega_m/\omega)^2} \Omega_m e^{-B_m \Omega_m (t-\tau)} \sin\Omega_m (t-\tau) \right.$$

$$\left. + \sum_{m=1}^{N} \frac{C_r^m}{1-(\omega/\Omega_m)^2} \omega e^{-\beta\omega(t-\tau)} \sin\omega(t-\tau) \right\} d\tau \tag{7.10}$$

The square root of the sum of the squares procedure is used to obtain the estimate of the peak response in the form

$$|\ddot{u}|_{\max} = \left\{ \sum_{m=1}^{N} \left[\frac{C_r^m}{1-(\Omega_m/\omega)^2} S_A(\Omega_m, B_m) \right]^2 + \left[\sum_{m=1}^{N} \frac{C_r^m}{1-(\omega/\Omega_m)^2} \right]^2 S_A^2(\omega, \beta) \right\}^{1/2} \tag{7.11}$$

This result can be used as an alternative to time history or modal analysis of the composite $N+1$-degree-of-freedom system, or as an alternative to the standard floor spectrum analysis for which time histories must be computed. Equation (7.10) is independent of the modal mass ratios, representing, in fact, the general closed-form solution of the floor spectrum method, the interpretation of which directly provides the simple estimate in Eq. (7.11). Indeed, the preceding analysis, which led to Eq. (7.10), mathematically justifies the use of the floor spectrum method for the grossly detuned system. All information needed for Eq. (7.11) is available from the building design, the equipment frequency and damping, and the design spectrum; it should thus be very convenient for practical design applications.

The methods developed for tuned poles can be used to determine the response of systems with closely spaced modes even if equipment is not considered. Of course, approximations based on small mass ratio could not be used in this case, but the envelope of beating response would be treated identically. Maximum response is not expected to differ significantly from that of the other modes, except that it will occur at a much later time. It is the light equipment mass that produces large amplification and the dominance of the late peak (Eq. (7.6)) over the early peak (Eq. (7.5)).

The approach employed by Penzien and Chopra [3], whereby the single-degree-of-freedom element representing the equipment is attached to each mode of the N-degree-of-freedom structure, can lead to a correct approximation of Green's function for the equipment response in the case of light equipment. Were this then used with response spectra, a correct estimate of the peak equipment response could be constructed. However, the method actually employed was to calculate the maximum response for each two-degree-of-freedom system thus formed first, and these were then added by the square root of the sum of the squares. That this *ad hoc* method can lead to a reasonable estimate of the peak equipment response in the case of a tuned system is a consequence of the fact that the particular two-degree-of-freedom system which is tuned dominates, as described earlier. This method is unnecessarily complicated, requiring as it does the numerical computation of a vast catalogue of response spectra for two-degree-of-freedom systems, and is in principle incorrect.

8. APPLICATION TO SYSTEMS WITH CLOSELY SPACED MODES

The peak response of structural systems in which two or more modes are closely spaced in frequency is difficult to estimate. The conventional square root of the sum of squares method of adding modal contributions is incorrect for this case and several alternatives, more or less *ad hoc,* have been proposed. The methods developed here for light tuned or nearly tuned equipment can give insight into the problem although the simplifications due to small mass ratio cannot be made.

In a general system with two closely spaced modes, Green's function for the contribution, $y_G(t)$, to the displacement response of any degree of freedom by the closely spaced pair is given by

$$y_G = A_1 e^{-\beta_1\omega_1 t}\sin\omega_1 t + A_2 e^{-\beta_2\omega_2 t}\sin\omega_2 t$$

where $\omega_1 = (1-\epsilon/2)\omega$, $\omega_2 = (1+\epsilon/2)\omega$, $\epsilon << 1$, and $\omega_1\beta_1 \approx \omega_2\beta_2 \approx \omega\beta$, say. The transient acceleration response of that degree of freedom due to ground acceleration $\ddot{u}_g$ can be expressed as

$$\ddot{y}(t) = -\omega^2 \int_0^t \ddot{u}_g(\tau) e^{-\beta\omega(t-\tau)} \left[A_1 \sin(1-\frac{\epsilon}{2})\omega(t-\tau) + A_2 \sin(1+\frac{\epsilon}{2})\omega(t-\tau) \right] d\tau$$

By expanding the sine functions and collecting terms, we obtain

$$\ddot{y}(t) = -\omega^2 \int_0^t \ddot{u}_g(\tau) e^{-\beta\omega(t-\tau)} \left[(A_1+A_2)\sin\omega(t-\tau)\cos\eta(t-\tau) + (A_1-A_2)\cos\omega(t-\tau)\sin\eta(t-\tau) \right] d\tau$$

where $\eta = \omega\epsilon/2$ is of low frequency compared to ω. The terms (A_1+A_2) and (A_1-A_2) may, in this case, be of the same order of magnitude, as opposed to the light equipment case in which the first was zero and the second of order $1/\epsilon$. Following the approach developed in Chapter 6, the terms $\cos\eta(t-\tau)$ and $\sin\eta(t-\tau)$ are expanded and approximated for small ϵ, leading to

$$\ddot{y}(t) = -\omega^2(A_1+A_2)\cos\eta t\int_0^t \ddot{u}_g(\tau)e^{-\beta\omega(t-\tau)}\sin\omega(t-\tau)\,d\tau$$

$$-\omega^2(A_1-A_2)\sin\eta t\int_0^t \ddot{u}_g(\tau)e^{-\beta\omega(t-\tau)}\cos\omega(t-\tau)\,d\tau$$

For each of these terms we obtain the envelope of the response and its maximum and note that when the envelope of one term achieves its peak value, the other is at a minimum; the peak of the first is separated from that of the second. The results for each peak are as follows:

early peak:

$$|\ddot{y}|_{\max} = \omega|A_1+A_2|S_A(\omega,\beta)$$

later peak:

$$|\ddot{y}|_{\max} = \frac{\omega\epsilon e^{-\kappa}}{\left(\epsilon^2+4\beta^2\right)^{1/2}}|A_1-A_2|S_A(\omega,\beta)$$

where $\kappa = (2\beta/\epsilon)\,arctan\,(\epsilon/2\beta)$.

If y applies to equipment response, the early peak does not appear since $A_1 = -A_2$ and the late peak is the result obtained previously. If, on the other hand, y applies to a structural degree of freedom in the case of light tuned equipment, then $A_1 = A_2$, the second peak disappears, and the first peak predominates. Although other late peaks will arise in the undamped case, they are unimportant when damping is present.

When response spectra are used in design, the early peak as calculated above should be combined with contributions from the other modes (using the square root of the sum of squares method of combination), while the second peak should be taken entirely on its own.

9. SUMMARY OF RESULTS

The purpose of the research presented herein was to develop results in an important area for which few rational results have been available and for which not all basic physical phenomena have been well understood. Practical methods have been developed for use in the design of light equipment attached to structures subject to ground motion.

The results obtained incorporate information readily available to designers, namely the fixed-base dynamic properties of the structure, the fixed-base dynamic properties of the equipment, and the ground shock or seismic response spectra.

The estimates of the peak response of the equipment may be summarized as follows:

Tuned or Nearly Tuned Systems

early peak (occurs during earthquake)

$$|\ddot{u}|_{\max} = \left\{ \sum_{\substack{m=1 \\ m \neq n}}^{N} \left[\frac{C_r^m}{1-(\Omega_m/\omega)^2} S_A(\Omega_m, \beta_m) \right]^2 + \left[\sum_{\substack{m=1 \\ m \neq n}}^{N} \frac{C_r^m}{1-(\omega/\Omega_m)^2} \right]^2 S_A^2(\omega, \beta) \right\}^{1/2}$$

late peak (occurs after earthquake)

$$|\ddot{u}|_{\max} = \frac{|C_r^n| e^{-\kappa}}{(\gamma+\xi^2+4\beta B_n)^{1/2}} S_A \left[\frac{\omega+\Omega_n}{2}, \frac{\beta+B_n}{2} \right]$$

where

β, ω, B_m, and Ω_m are the equipment and structural damping and frequencies, respectively;

n is the structural mode to which the equipment is tuned or nearly tuned;

r is the structural degree of freedom to which the equipment is attached;

$\xi \equiv (\Omega_n - \omega)/\omega \equiv$ detuning parameter;

$$\gamma \equiv \frac{m}{M_n/(\Phi_r^n)^2} \equiv \frac{\textit{mass of equipment}}{\textit{effective mass of structure in } n^{th} \textit{ mode}} \equiv$$

mass ratio;

$C_r^m \equiv \Phi_r^m \sum_{i,j=1}^{N} (\Phi_i^m M_{ij} R_j)/M_m \equiv$ participation factors;

R_j is the vector of influence coefficients that couples ground motion to the structural degrees of freedom;

Φ_i^m is the i^{th} component of the m^{th} mode shape;

M_{ij} is the structural mass matrix;

$M_m = \sum_{i,j=1}^{N} M_{ij}\, \Phi_i^m \Phi_j^m$ is the generalized mass for the m^{th} mode;

$\kappa = (arctan\,\zeta)/\zeta$; and

$\zeta = [\gamma + \xi^2 - (\beta - B_n)^2]^{1/2}/(\beta + B_n)$.

Untuned Systems

peak occurs during earthquake

$$|\ddot{u}|_{max} = \left\{ \sum_{m=1}^{N} \left[\frac{C_r^m}{1-(\Omega_m/\omega)^2} S_A(\Omega_m, B_m) \right]^2 + \left[\sum_{m=1}^{N} \frac{C_r^m}{1-(\omega/\Omega_m)^2} \right]^2 S_A^2(\omega, \beta) \right\}^{1/2}$$

In the above, S_A is the acceleration design spectrum; other responses such as maximum velocity or displacement can be obtained by using the appropriate design spectra.

The advantages of this approach are its simplicity and adaptability for practical application. A great deal of computational effort is avoided since time history analyses need not be performed. The equipment-structure need not be analyzed as an N+1-degree-of-freedom system either by modal or matrix-time-marching methods, and errors in estimates of peak response due to the possible unreliability of numerical time-integration schemes, or to uncertainty as to the appropriate procedure for summing the contributions of the two closely spaced modes, are thereby avoided. For tuned and nearly tuned systems it takes into account the important effect, completely neglected in the floor spectrum method, of equipment-structure interaction.

The method advanced here does not require new information to be generated. Data available from the building design alone (M_{ij}, R_j, Ω_m, Φ_i^m, B_m), the equipment alone (m, ω, β), and the ground shock spectra (S_D or S_V or S_A) are used. The estimates of peak response have been obtained from a rational analysis. Furthermore, these are easily evaluated and conveniently used during the design process.

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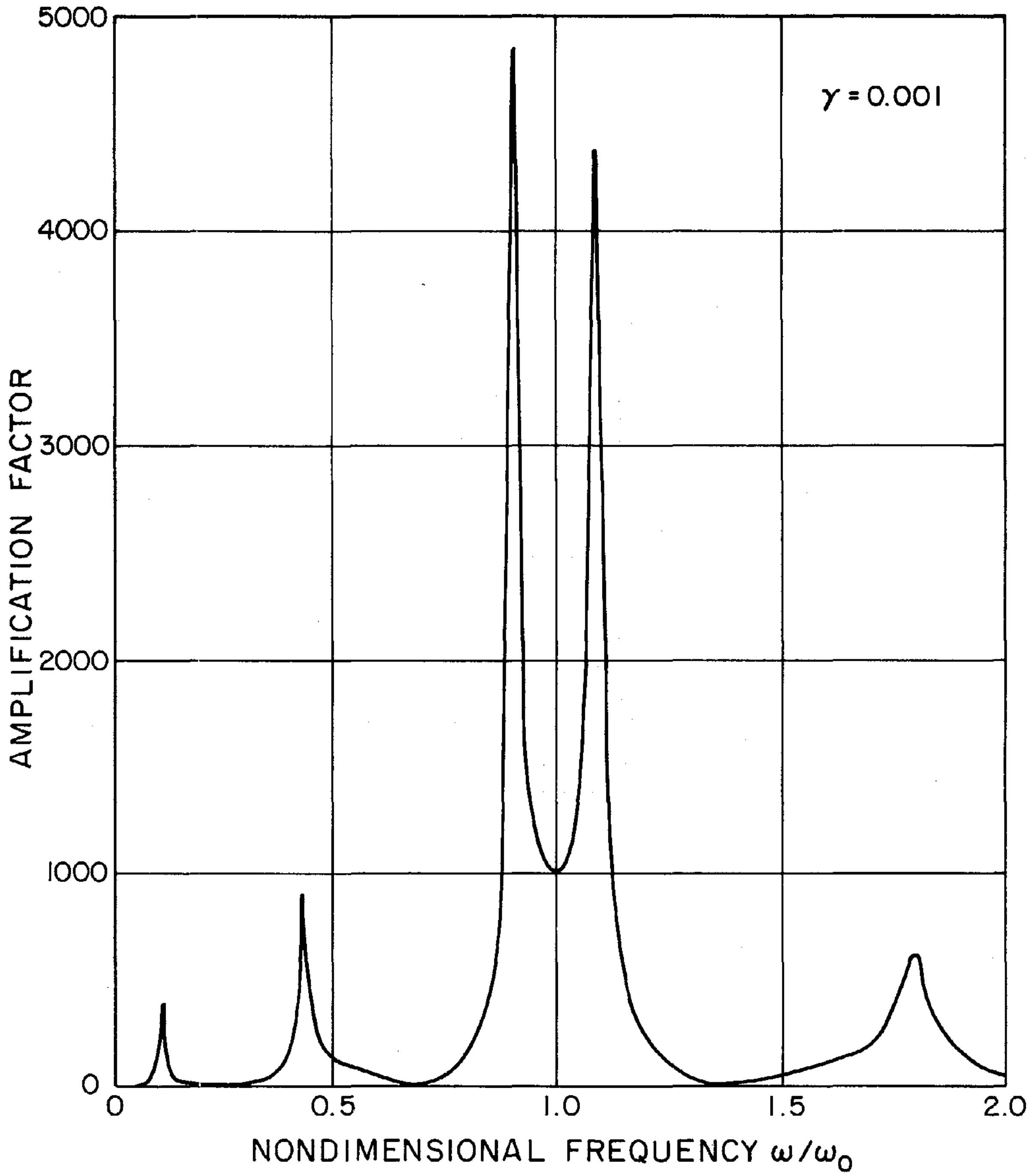

Fig. 1 Amplication factor for equipment acceleration as a function of frequency: equipment tuned to third structure frequency

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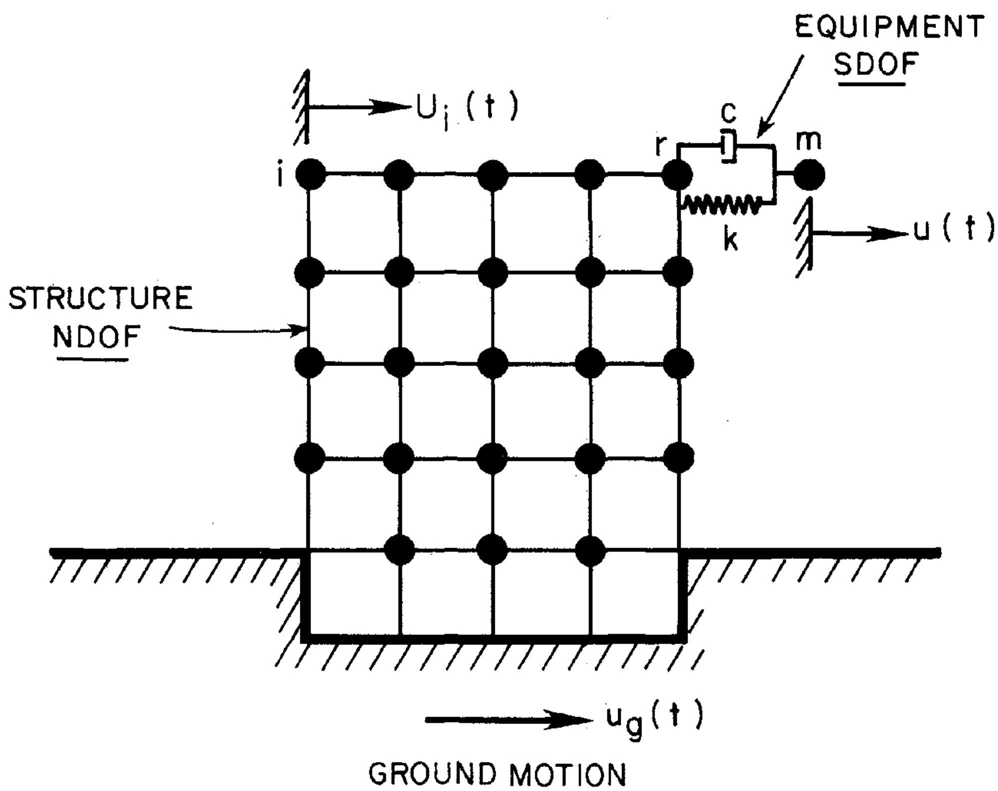


Fig. 2 Equipment-structure system

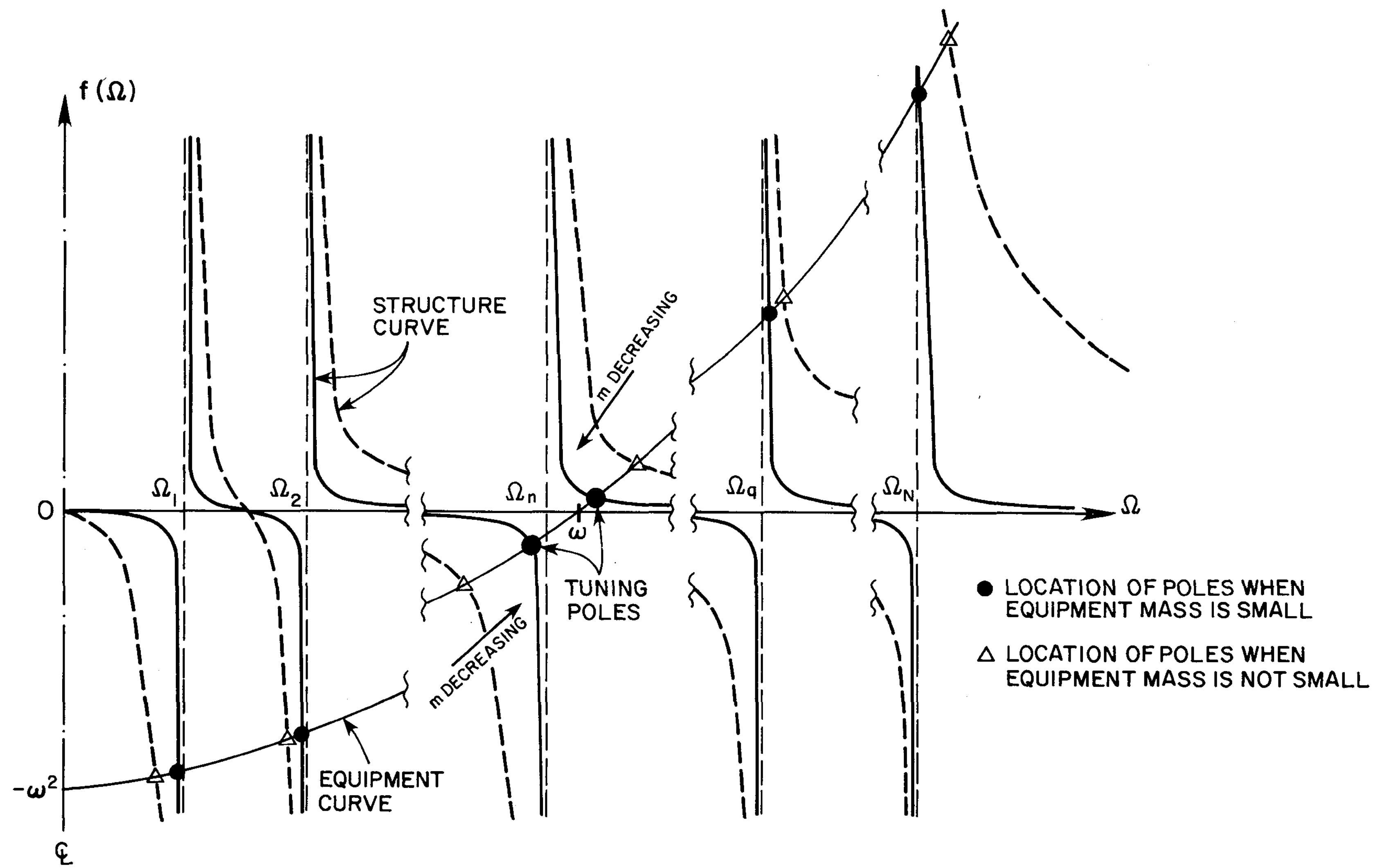
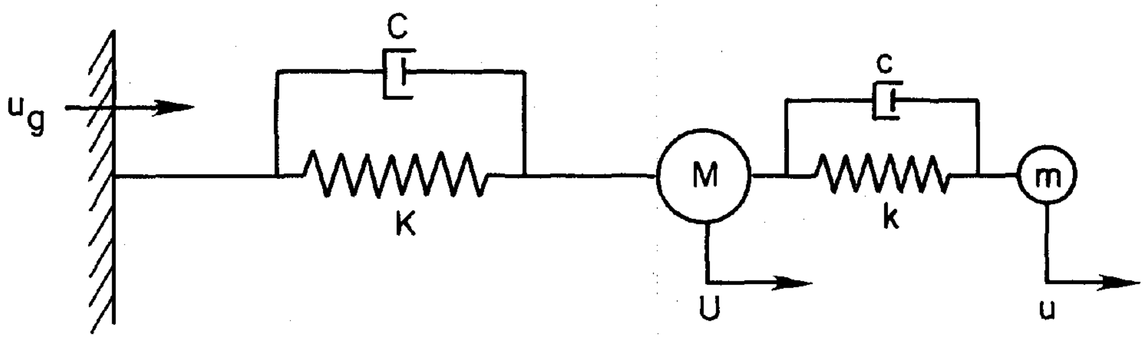


Fig. 3 Location of poles of equipment response transfer function with slightly detuned N-degree-of-freedom structure



$$\Omega^2 = \frac{K}{M} \quad \frac{k}{m} = \omega^2$$

$$2\beta\omega = \frac{c}{m} \quad 2B\Omega = \frac{C}{M}$$

Fig. 4 Two-degree-of-freedom system giving system parameters

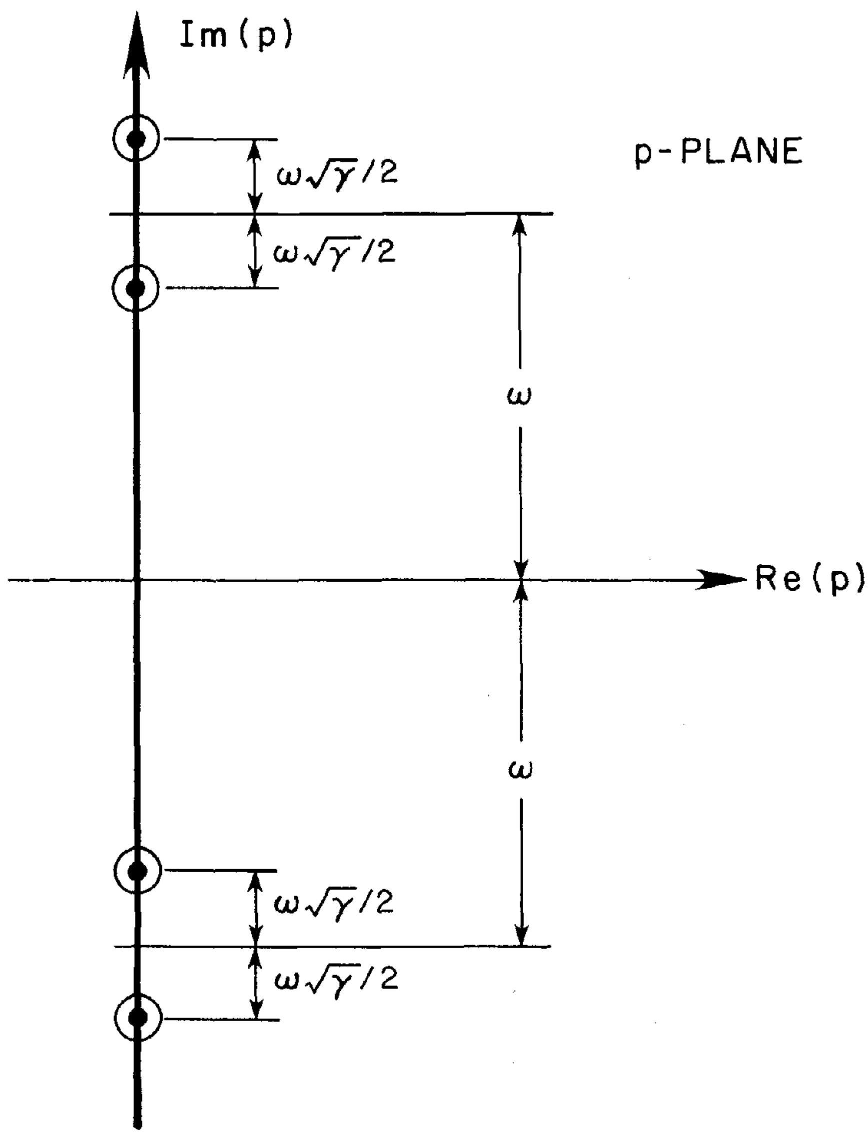


Fig. 5 Root locus diagram for undamped tuned two-degree-of-freedom system

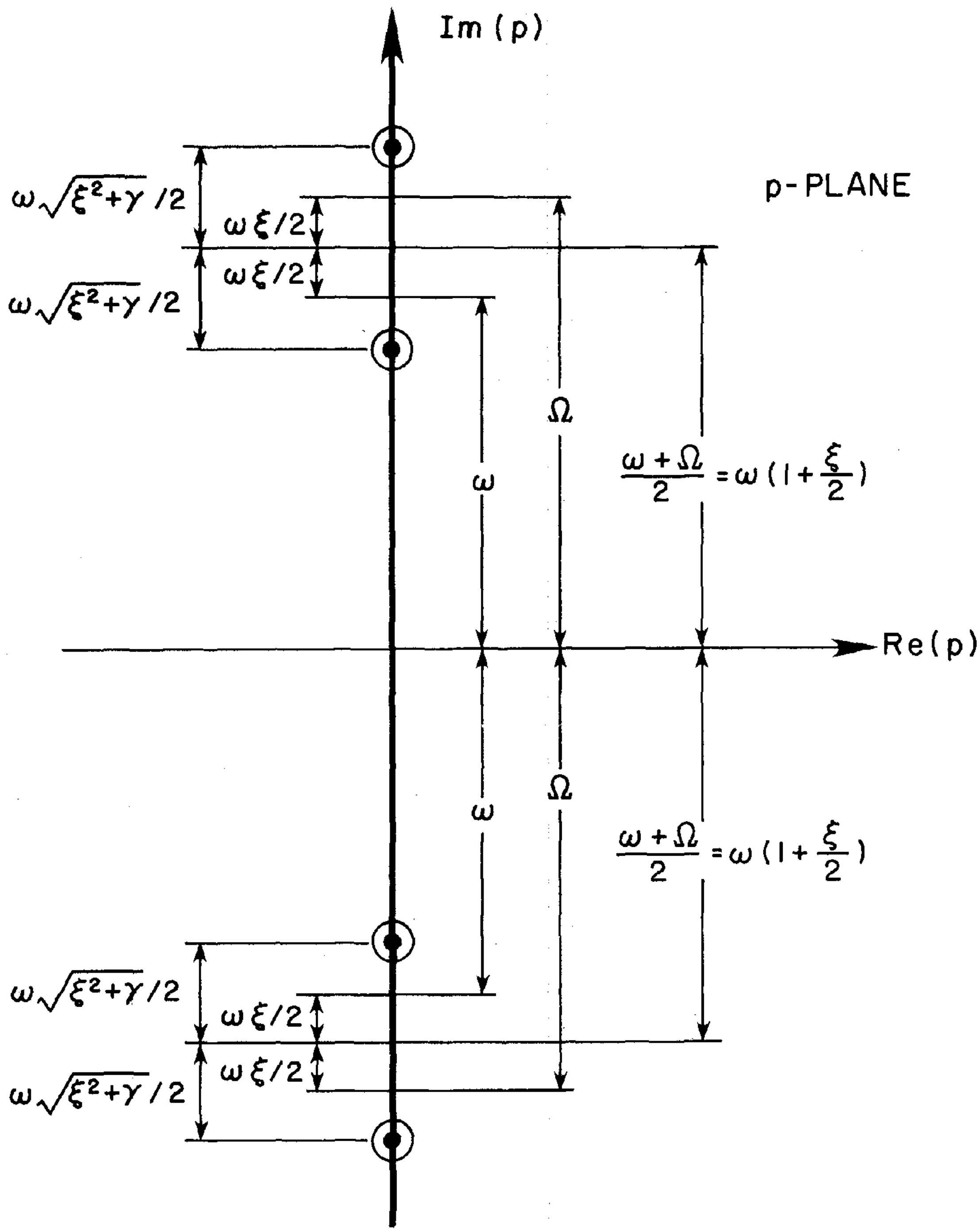


Fig. 6 Root locus diagram for undamped slightly detuned two-degree-of-freedom system

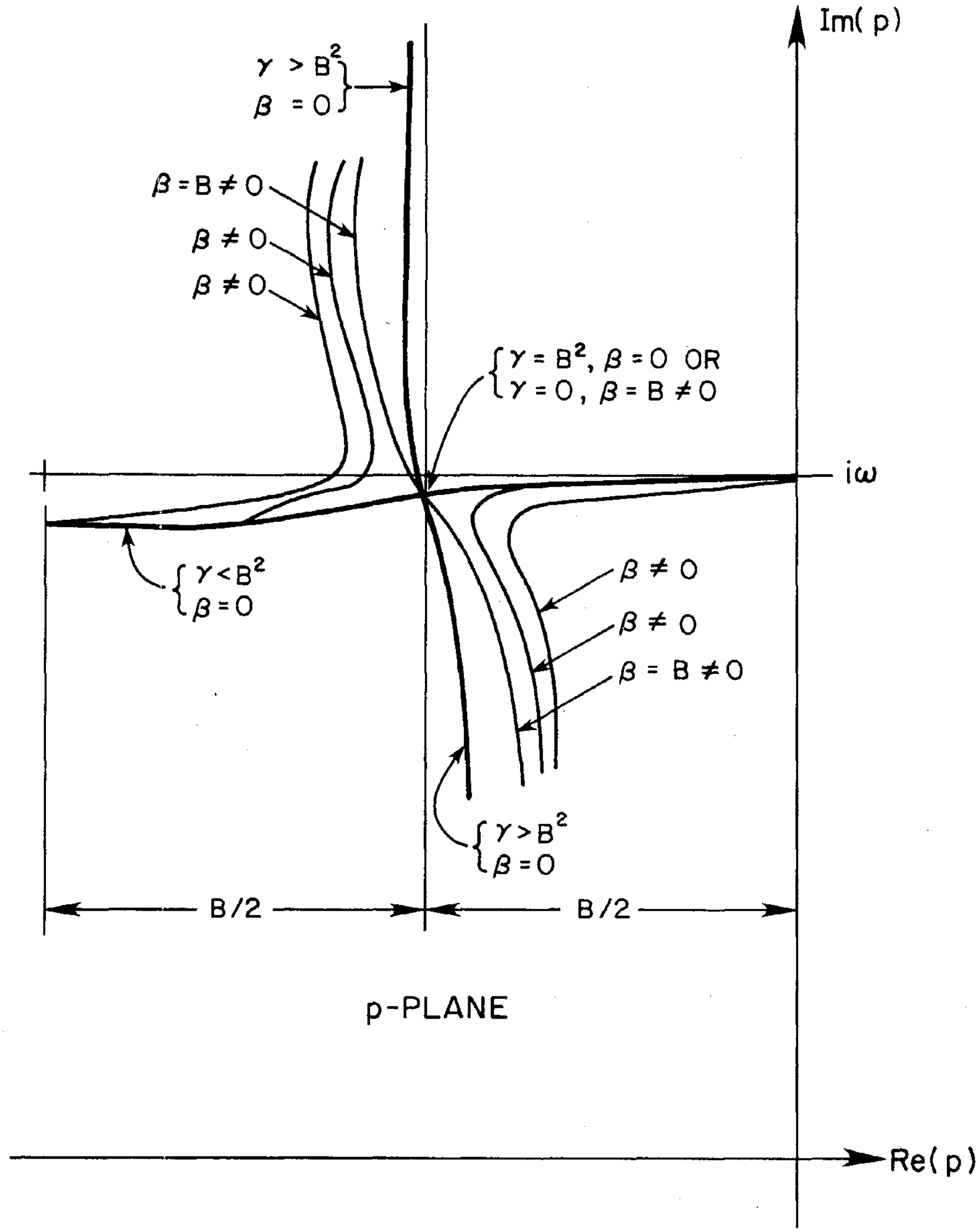


Fig. 7 Root locus for damped tuned two-degree-of-freedom system

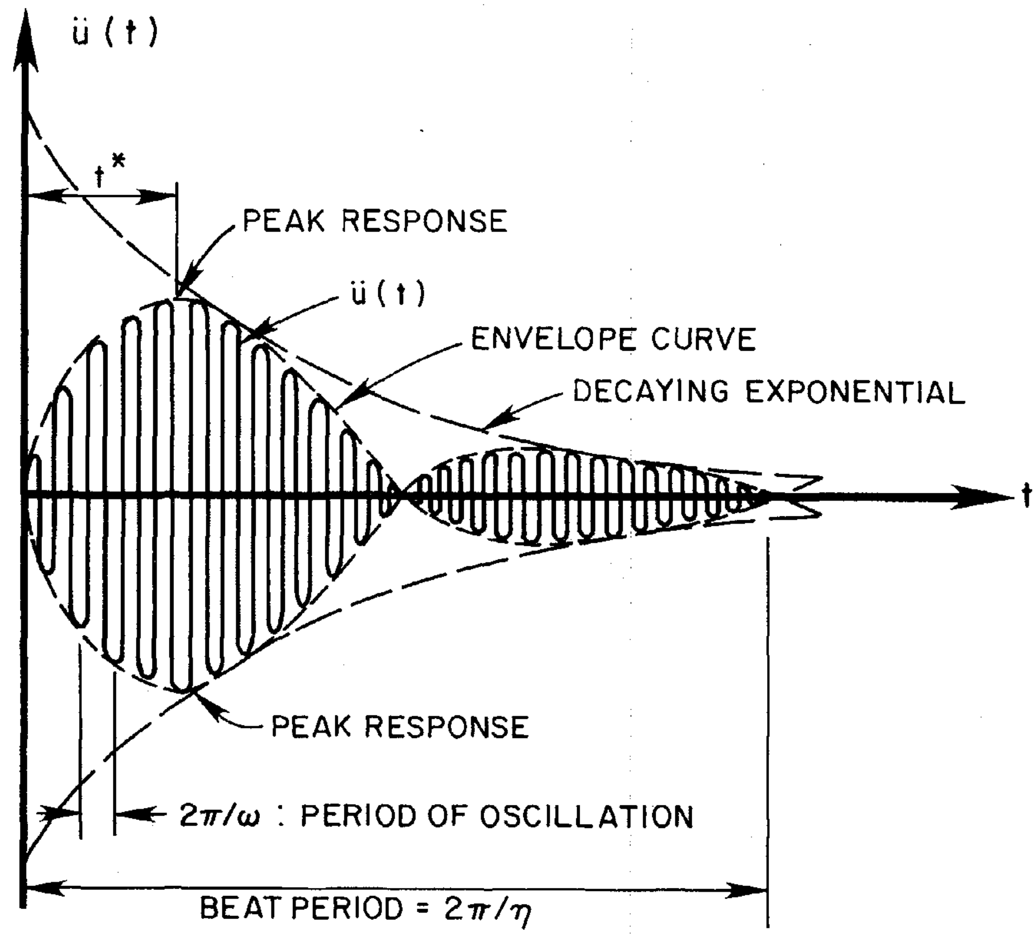


Fig. 8 Equipment response history in the case of damped beats

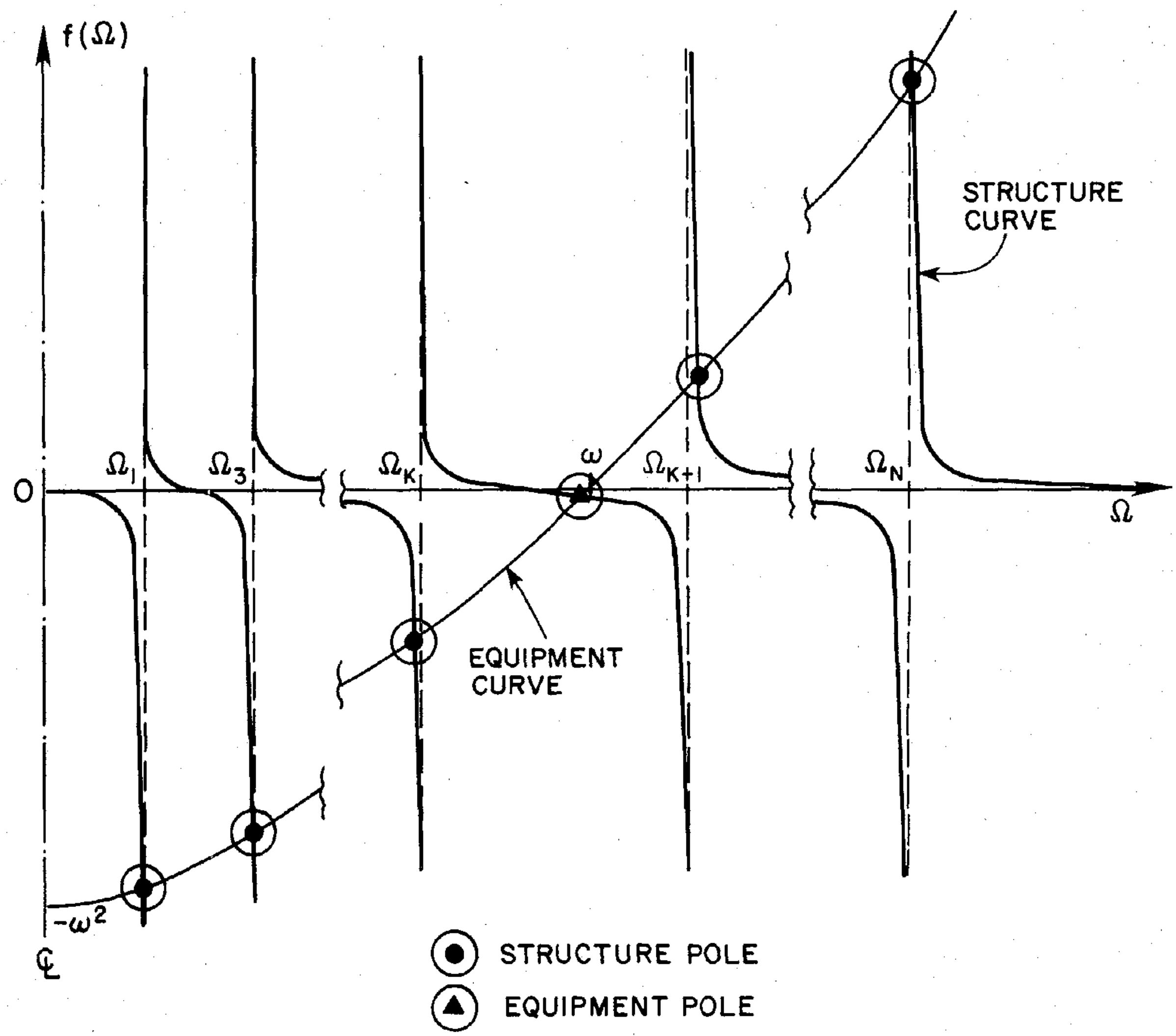


Fig. 9 Location of poles of equipment response transfer function for grossly detuned system

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