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RELATIVE MOTION OF TWO SURFACE POINTS DURING AN EARTHQUAKE

by

John T. Christian

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Geotechnical Division
Department of Civil Engineering
Massachusetts Institute of Technology
Cambridge, Massachusetts 02139

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Abstract

The maximum relative displacement between two points on the surface during an earthquake is often an important consideration in the design of structures in seismic areas. Several proposals for obtaining engineering estimates of this quantity are examined. It is concluded that use of the maximum ground motion is very conservative but use of the ground motion spectrum directly is unconservative. A simple proposal of Newmark's gives excellent results: the maximum relative displacement divided by the applicable wave velocity is equal to or less than the maximum ground velocity. At large separations, the maximum relative displacement is limited by the maximum ground displacement.

Introduction

When a facility is to be built in an area of seismic risk, it often becomes necessary to estimate the **maximum** relative motion that two points on the surface of the soil or rock may undergo. This is important in the design of components that are to be connected by piping or conduits, as, for example, in a nuclear power point. The designer does not need a detailed time history or spectrum of the motion but an estimate of the maximum displacement or strain that his system must undergo.

This paper describes results for the situation shown in Fig. 1. The two points are separated by a distance b. If the seismic wave front were propagating vertically, the two points would move in phase with no relative displacement. The extreme case occurs when the wave front is moving horizontally with a velocity c, in which case the time lag between the motions of the points must be b/c. The velocity c would be either the shear wave or the Rayleigh wave velocity. Fig. 1(b) illustrates the typical time histories for the case when the wave front is propagating horizontally.

A somewhat different case is shown in Fig. 2. The energy there is transmitted through the firm lower layer at a velocity c_2 and then propagated up through the soil. The time lag between points a and b is then b/c_2. A similar treatment of the relative motions is possible in both cases, provided the appropriate value of the wave velocity is used in each case.

In the field the configurations of deposits can be quite involved, and the interaction in even simple inhomogeneous cases can give unexpected complications (Trifunac, 1971). Further, for closely spaced points the motions may be quite poorly correlated. The characteristics of the earthquake motions themselves could also be known only in a statistical way, so for most design problems a general description is the best available. A ground motion spectrum or design spectra are typical descriptions of a design earthquake. The present analyses concern a deliberately simple geometry for design purposes, and it is recognized that finite element methods or other analytical and probabilistic complexities may be necessary for more involved geometries. The extremely simple calculations used here are aimed at providing useful information for the designer. Horizontal motions are used, and the results apply equally to motions parallel to and transverse to the direction of propagation.

Previous Techniques

Several methods are now in use to estimate the relative motion between two points:

a. The relative displacement as twice the maximum ground displacement of the earthquake (d_{max}). This is obviously conservative.

b. Newmark (1967) proposed that the relative displacement must be less than or equal to the maximum positive displacement at a or b minus the maximum negative displacement at the other point. Where the two points have motions of the same shape, this is simply the sum of the magnitudes of the maximum positive and negative displacements. After base line correction most earthquake records have almost all the displacement in one sense, so this method is very close to using the maximum displacement (d_{max}) as a criterion.

c. Newmark also indicated that the maximum relative motion, R, could be approximated by

$$R \simeq -v_m \frac{b}{c} + a_m \frac{b^2}{2c^2} \qquad (1)$$

where v_m is the maximum velocity of the ground motion and a_m is the maximum acceleration. This formula is the result of simulating the motion at point b by a Taylor series expansion about point a and discarding all terms higher than second order.

d. If the motion consisted of a single sine wave, the maximum relative displacement would occur when b/c is exactly one half the period of the sine wave (Fig. 3a). The relative displacement would be twice the amplitude of the sine wave.

This suggests that, given the ground motion spectrum, the engineer could estimate the relative motion as twice the spectral displacement for a period T equal to twice b/c. The technique, illustrated in Fig. 3b, has been proposed by engineers active in nuclear power plant design.

All these methods involve some degree of approximation. The following section describes how they compare to direct calculation.

Results of Direct Calculation

To test the accuracy of the approximate methods direct calculations were made from these records of accelerations: the El Centro North-South component of motion recorded during the Imperial Valley Earthqueke of 1940, the Taft South-69°-East component recorded during the Kern County Earthquake of 1952, and the Golden Gate Park record from the San Francisco Earthquake of 1957. These were all base line corrected, but some errors are inevitable. Since all comparisons of motions, spectra, relative motions, etc. for a given input are computed from the same accelerogram, the conclusions about the comparisons are valid even if the actual motions are in error.

The calculations consisted of:

a. Direct integration of the accelerogram to obtain a ground motion history.

b. Direct calculations, from the ground motion time histories, of the maximum relative motion for various values of b/c. This is done by simply displacing the record by b/c seconds in time and directly subtracting the result from the undisplaced record.

c. Calculation of the ground motion spectrum by Fast Fourier Transform of the ground motion time history of displacement, which was found in a above.

d. Application of each of the methods described in the previous section to obtain estimates of the relative displacement.

The desired range of b/c is based on the realization that b will probably be less than 1500 ft. and c for soils will be between 500 and 2000 ft./sec. For rock c will be much larger. This gives a range of b/c between 0 and 3.0 sec. In fact, 20 values of b/c were distributed approximately logarithmically between 0.05 sec. and 3.0 sec.

Figs. 4, 5, and 6 show how the directly computed relative motions compared to those predicted by manipulating the ground motion spectra as suggested by method d of the previous section. The line identified as "2 x Amplitude of Fourier Transform ..." is what would be obtained from the ground motion spectra. In practice the ground motion spectra are used as smooth curves, but here the jagged Fourier Transform results are used directly. Through almost all of the range of interest the proposed method is wrong and unconservatively wrong. This is because the relative motion between two points is affected by sinusoidal motion with periods other than 2b/c. For example, if only one sinusoidal wave were present with a period T, points saparated by b/c other than 0.5T would feel significant relative motion, but the proposed technique would not account for it. Therefore, this method must be abandoned.

Figs. 4, 5, and 6 also show that method b is conservative and method a very conservative even for large values of b/c. Over the major portion of the significant range of b/c both methods are excessively conservative.

A further comparison was made by first normalizing all results to a maximum ground acceleration of 0.1g. The computed relative displacements were then plotted to logarithmic scales in Fig. 7. The maximum relative displacements estimated from the first term of Newmark's formula ($v_m b/c$) are also plotted along with the maximum displacements d_m. The figure shows that the two lines envelope the computed maximum relative displacements.

The effect of the second term in Newmark's formula ($a_m b^2/2c^2$) is shown in Table I. This term destroys the good fit that previously existed between the directly computed R and that estimated from $v_m b/c$. The reason is that v_m and a_m do not occur at the same time. Newmark suggested that the a_m term could be ignored, and these data indicate that it very much ought to be.

Proposed Approximate Method

On the basis of these calculations a simple method can be devised to predict the maximum relative displacements for design at a site for which there is given an a_m, v_m, and d_m for a design earthquake. The velocity of incident waves must also be known or estimated.

The maximum relative displacement, R, will be

$$|R| \leq |v_m \frac{b}{c}| \tag{2}$$

For large values of b/c, $|R|$ will also satisfy

$$|R| \leq |d_m| \tag{3}$$

for most earthquakes. This is outside the most commonly expected range of b/c.

The envelope specified by expressions (2) and (3) can be easily constructed on log-log paper as in Fig. 7 by drawing a line inclined 45° through the point ($b/c = 1.0$, $R = v_m$) and another horizontal line $R = \text{constant} = d_m$.

References

Newmark, N. M. (1967). Problems in Wave Propagation in Soil and Rock. Proceedings, International Symposium on Wave Propagation and Dynamic Properties of Earth Materials, Albuquerque, N. M., 7-26.

Trifunac, M. D. (1971). Surface Motion of a Semi-Cylindrical Alluvial Valley for Incident Plane SH Waves, Bull. Seis. Soc. Am. 61, 1755-1770.

TABLE I

Effect of Acceleration Term in Newmark Formula

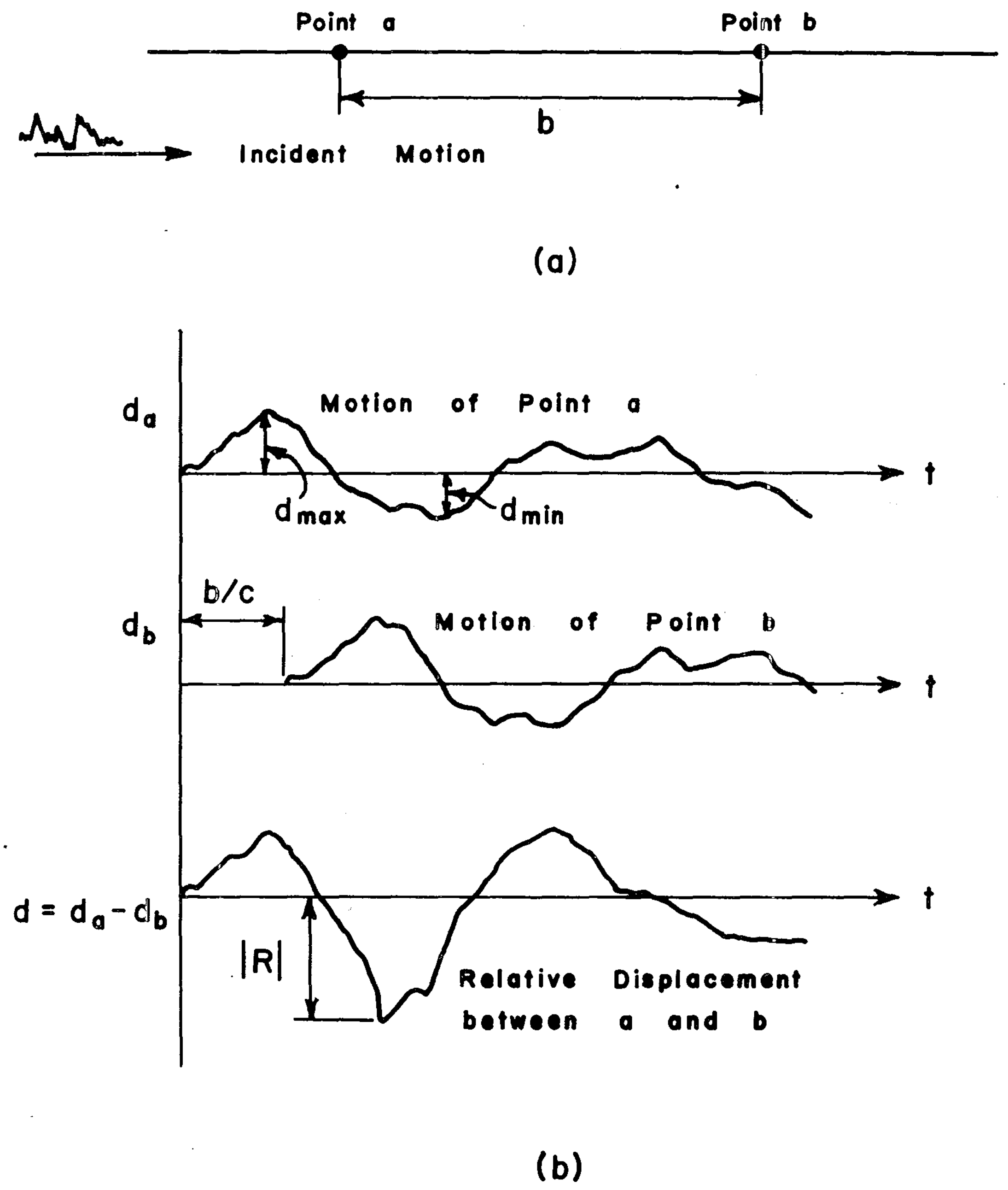
b/c	R Calculated			$R = v_m \frac{b}{c}$			$R = v_m\frac{b}{c} - a_m \frac{b^2}{2c^2}$			$R = v_m\frac{b}{c} + a_m \frac{b^2}{2c^2}$		
	EC	T	GG	EC	T	GGG	EC	T	GG	EC	T	GG
sec.	in.	in.	in.	in.	in.	in.	in.	in.	in.	in.	in.	in.
1.00	3.04	3.91	0.58	5.32	5.61	1.44	-14.00	-13.70	-17.90	24.60	24.90	20.70
0.50	1.74	2.29	0.38	2.66	2.80	0.72	-2.10	-2.00	-4.10	7.50	7.60	5.50
0.20	0.94	1.01	0.21	1.06	1.12	0.29	0.29	0.35	-0.48	1.83	1.89	1.06
0.10	0.53	0.55	0.13	0.53	0.56	0.14	0.34	0.37	-0.05	0.72	0.75	0.33
0.05	0.26	0.28	0.07	0.27	0.28	0.07	0.27	0.28	0.07	0.27	0.28	0.07

Notes: All a_m = 38.6 in./sec./sec. = 0.1g

EC = El Centro N-S record, normalized v_m = 5.32 in./sec.

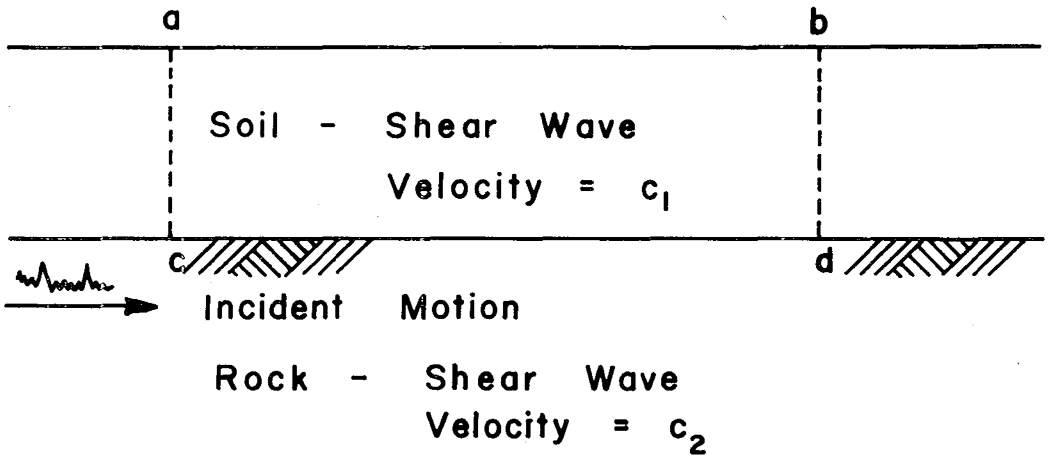
T = Taft S69E record, normalized v_m = 5.61 in./sec.

GG = Golden Gate record, normalized v_m = 1.44 in./sec.



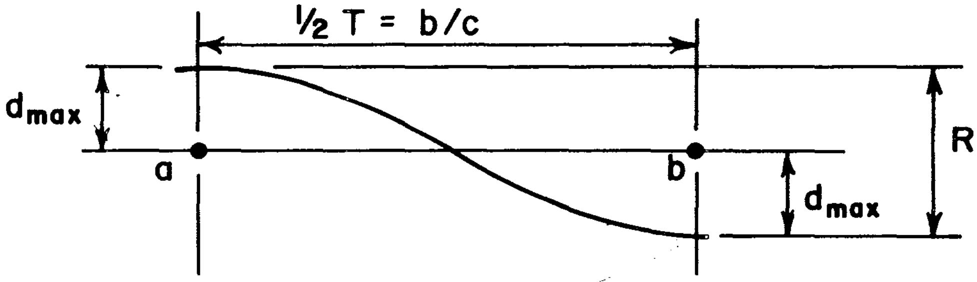
RELATIVE MOTION OF TWO POINTS

FIG. 1



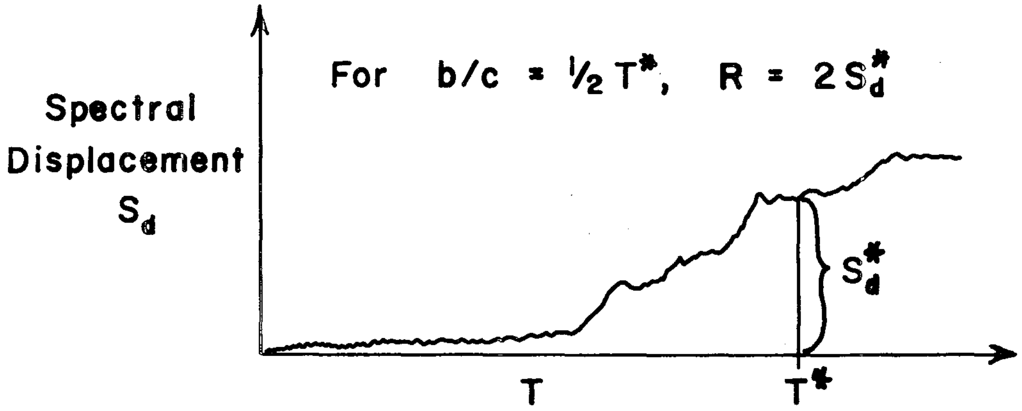
CASE OF SOIL LAYER OVER ROCK

FIG. 2



SINGLE SINUSOID

(a)

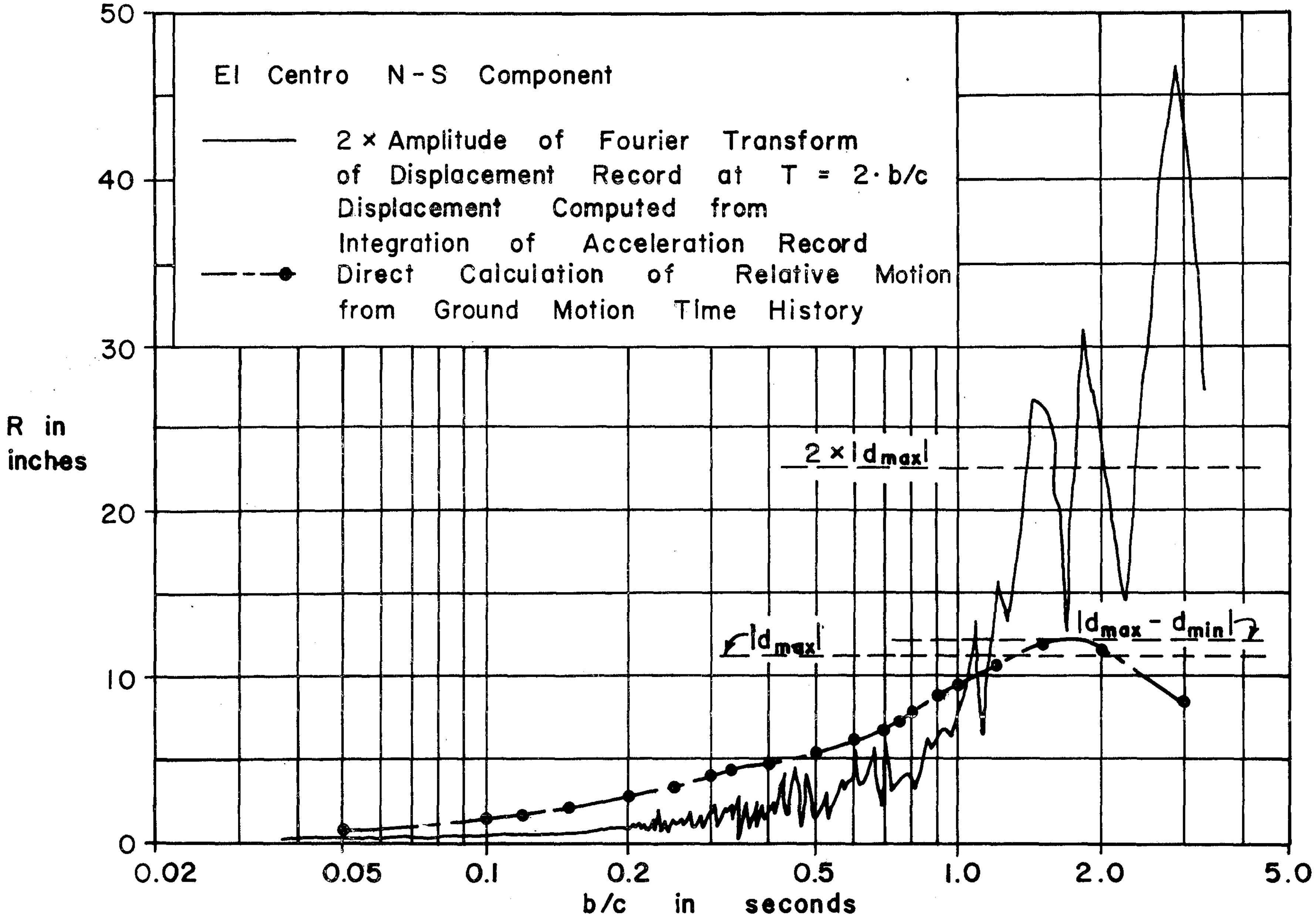


PROPOSED APPROXIMATE METHOD FOR GROUND MOTION SPECTRUM

(b)

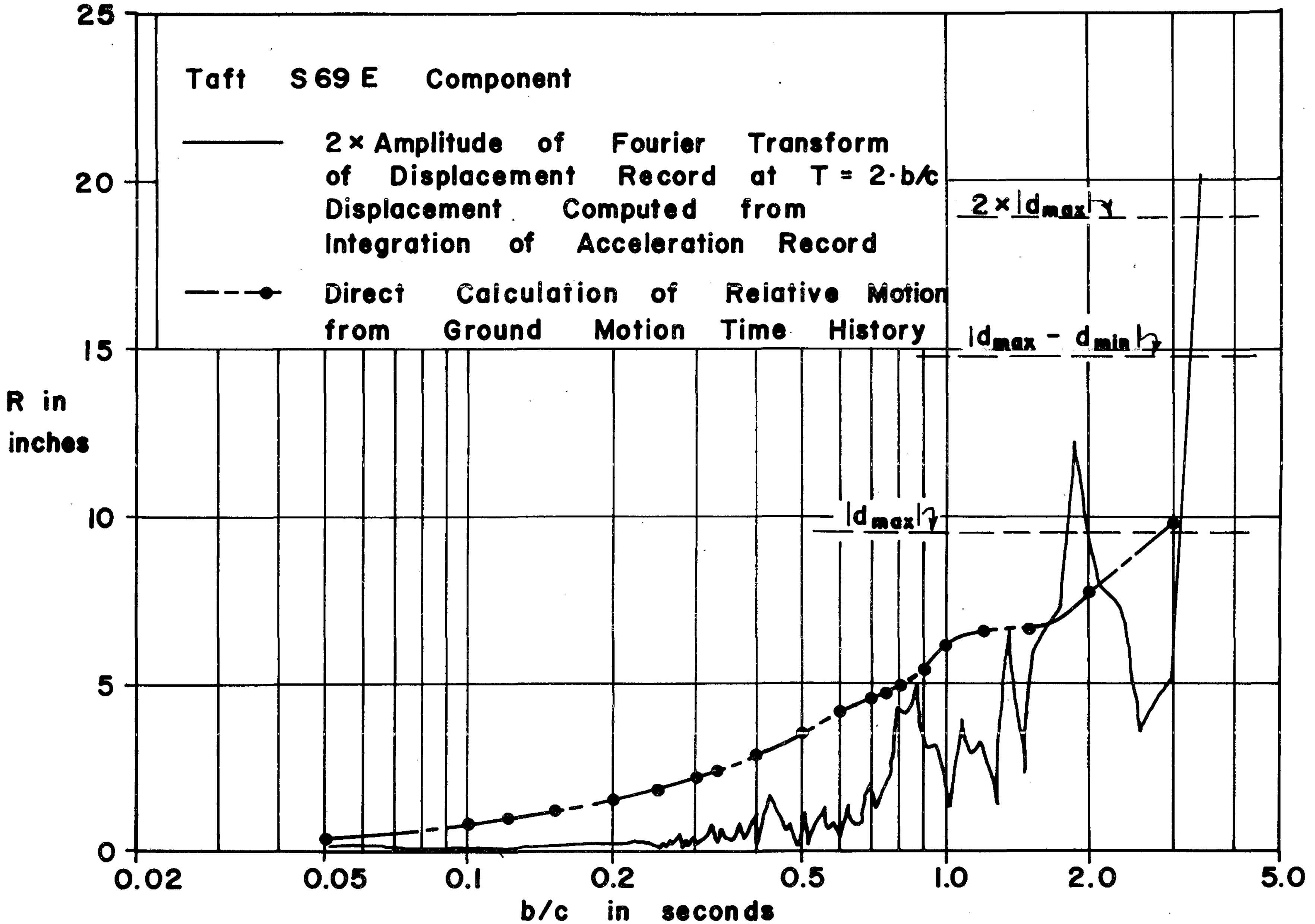
USE OF GROUND MOTION SPECTRUM TO ESTIMATE RELATIVE MOTION

FIG. 3



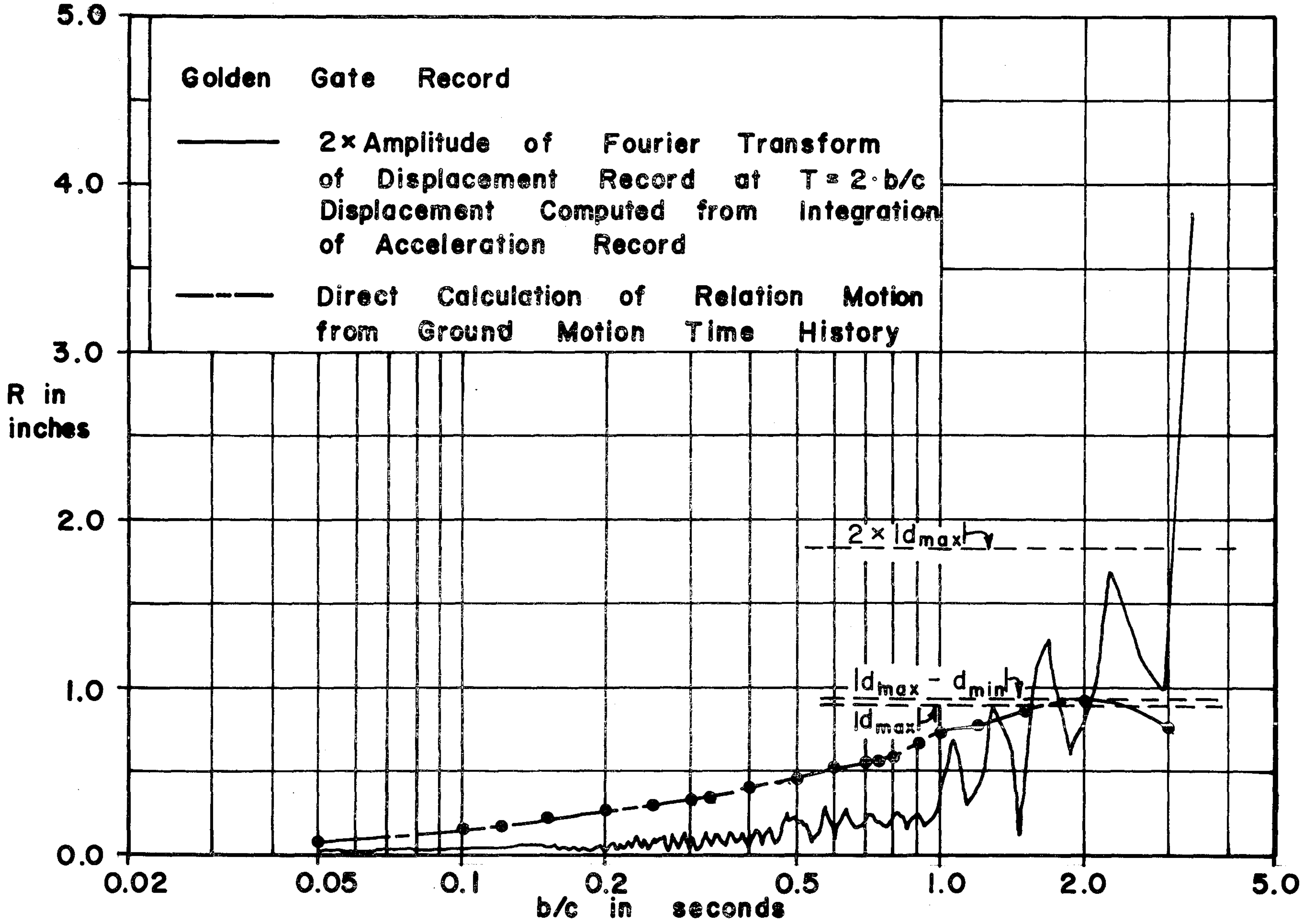
COMPARISON OF METHODS FOR EL CENTRO RECORD

FIG. 4



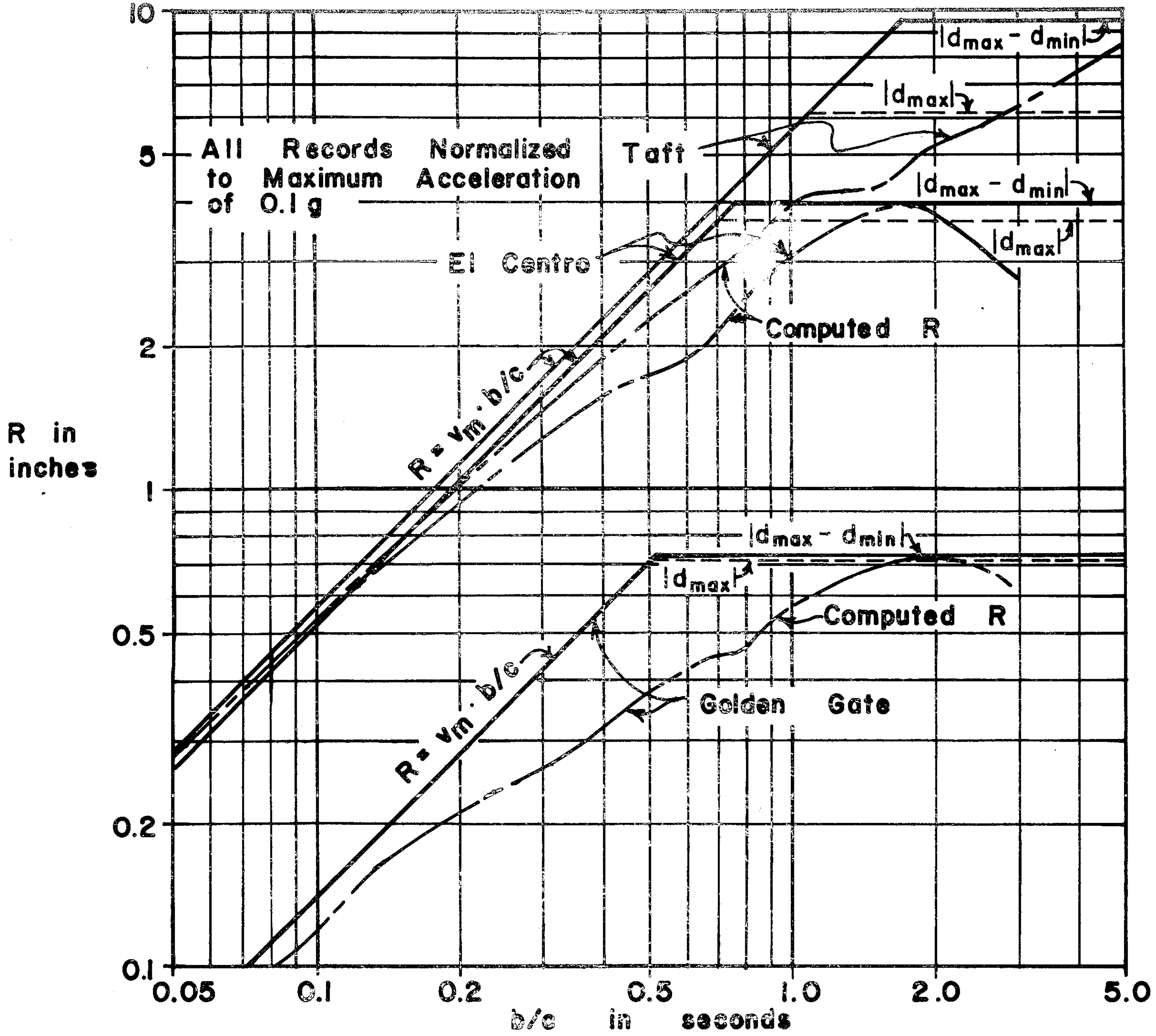
COMPARISON OF METHODS FOR TAFT RECORD

FIG. 5



COMPARISON OF METHODS FOR GOLDEN GATE RECORD

FIG. 6



COMPARISON OF RELATIVE MOTION AND $v_m \cdot b/c$

FIG. 7