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ACCURACY OF MODAL SUPERPOSITION FOR ONE-DIMENSIONAL SOIL AMPLIFICATION ANALYSIS

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ABSTRACT

The nature and possible magnitude of the errors involved in the determination of soil amplification effects, using modal analysis of a discrete lumped mass system, are discussed. These errors are mainly due to the treatment of damping and arise from the nature of the damping (hysteretic rather than viscous), the possible lack of normal modes in the classical sense, and the radiation effect in the underlying rock. It is shown that in many cases these errors are either negligible or they can be accounted for by relatively simple procedures (use of weighted modal damping and an additional term corresponding to the radiation effect). There are cases, however, where these corrections do not yield satisfactory results (in particular when the stiffness and damping of two layers are very different). It is then advisable to use the continuous solution in the frequency domain instead of the lumped mass model.

INTRODUCTION

In studying the effect of local soil conditions upon earthquake ground motions, a soil profile often is represented as a linear shear beam. Several types of error are thereby introduced:

1. Replacing a 3-dimensional non-linear problem by a linear one-dimensional problem.
2. Poor choice of inputs: soil properties and input earthquake motion.
3. Use of approximate mathematical solutions.

This paper is concerned with the last of these errors; specifically, the errors resulting from use of modal superposition. In most cases, these "mathematical errors" are the least important of the three categories. However, when they are important, it is because of the way in which damping is treated in the modal superposition method. Thus, when such errors occur, they are very fundamental in nature, and must be understood by any engineer wishing to take advantage of the convenience of the modal superposition method.

To study these errors, a number of comparisons have been made between results obtained by that method and by an "exact" method. (Of course, the "exact" method is only as good as the assumption of a linear shear beam and the choice of soil properties and input earthquake.) The methods of solution have been compared using the same soil properties. Thus, no use has been made of the iterative procedure (Seed and Idriss, 1968) in which soil properties are adjusted to be consistent with the level of strain. Use of such an iterative procedure tends to reduce the errors discussed in this paper, since any error in calculated response is compensated for by a change in the assumed damping.

The discussion in this paper parallels a similar treatment concerning use of modal superposition for analysis of soil-structure interaction (Roesset et al, 1972).

THE EXACT SOLUTION

An "exact" solution for the response of a linear shear beam may be obtained using Fourier analysis and transfer functions (Roesset and Whitman, 1969; Roesset, 1970). The method which had been used earlier in Japan, Mexico and Chile is illustrated in Fig. 1. The diagrams on the figure show the time history of the input motion and corresponding Fourier spectra, and the computed transfer function. Multiplying the transfer function times the Fourier spectrum for the input motion gives the Fourier spectrum for motion at the top of the profile. This spectrum can then be converted into the time history for motion at the surface. Actually, both the amplitude and phase angle of the Fourier spectra and transfer function must be used for the computation, although for convenience only the amplitudes are shown in the figure.

A computer program embodying Fourier analysis and transfer functions is very simple and uses very little computer time, especially if the computer will accept complex arithmetic. Such a program can readily handle profiles with many soil strata having different properties, and any linear damping may be utilized. This method also makes it possible to account for the effect of the non-rigidity of the earth below the bottom of the soil profile.

Some error may be introduced into this method by the numerical integration schemes used to convert from time history of motion to Fourier spectra, and vice versa. In work at MIT, the Cooley-Tukey Fast Fourier Transform algorithm has been used. This algorithm has been tested by starting with an input time history, computing surface motion for a soil profile, and then reversing the process to compute base motion from the surface motion. The final computed motion was virtually indistinguishable from the input motion.

MODAL SUPERPOSITION

Modal superposition is one of several methods that may be used to solve the equations governing the linear response of a shear beam to input motion at its base. This method separates the two variables in the basic equations: time, and the depth within the profile. The mode shapes express variation of motion with depth, while the time responses of each mode to the input motion are calculated and then added.

The method of modal superposition has been used extensively to solve the shear beam problem. It has some important advantages:

1. It is familiar to many engineers involved in earthquake engineering.
2. It is economical in terms of computer running time.

No doubt extensive utilization of this method will continue into the future.

However, mathematically speaking modal superposition is not valid for many soil amplification problems--even when linear equations are used. That is, the governing differential equations usually do not satisfy the requirements which permit separation of the time and depth variables. There are several reasons for this difficulty:

1. Differences in damping among various parts of the soil profile.
2. The effect of the properties of the earth lying below the bottom of the profile selected for analysis. This effect is equivalent to introducing additional damping.
3. To determine modal response by time-step integration, it is necessary to assume that damping in soil is viscous, whereas actually the damping is much more nearly hysteretic in character.

These three difficulties mean that "errors" are introduced whenever modal superposition is used. All of these difficulties are, in one way or another, associated with the treatment of damping.

Lumping

In most practical solutions using modal superposition, it is necessary to replace the continuous soil profile by a system of lumped masses and springs. Many of the comparisons presented in this paper are between exact results for a continuous shear beam and results obtained by applying modal superposition to a lumped shear beam. Hence, it is necessary to be sure that purely numerical errors introduced by lumping do not obscure the fundamental errors described above. The errors described in the Introduction can occur even if modal superposition is applied to a continuous shear beam.

When a continuous shear beam is replaced by a discrete system of lumped masses and springs, the governing partial differential equation is converted into a system of ordinary differential equations which may readily be solved by numerical techniques. The lumped system provides an accurate representation of frequency components whose wave lengths are long compared to the spacing of the masses. High frequency components of the ground motion will be distorted. Hence, the first step in satisfactory lumping is to choose the highest frequency which is to be represented; then enough masses are taken to provide a satisfactory representation of this frequency.

Idriss and Seed (1968) have compared solutions obtained by lumped systems with those obtained for continuous systems for the two special cases of a uniform stratum and a stratum in which the wave velocity increases as the one-third power of depth. Damping was assumed to be constant with depth and to be viscous in both lumped and continuous systems. For these conditions, modal superposition is rigorously correct for the continuous shear beam. Based upon this study, rules for choosing the number of masses in practical problems were suggested. Alternate versions for these rules have been suggested by Hagmann and Whitman (1969). These several rules provide an accuracy which is adequate for engineering purposes, and the number of masses they require can reasonably be handled by computer programs.

HYSTERETIC VS. VISCOUS DAMPING

Damping in soil is caused primarily by relative slipping and sliding among soil particles. This non-linear behavior is shown by the hysteresis loop which develops during a cycle of loading (Fig. 2). Hysteretic damping is conveniently expressed in terms of specific damping capacity, ψ , defined as:

$$\psi = \Delta W/W \quad (1)$$

where ΔW = energy lost during cycle of loading
W = maximum strain energy stored during cycle.
For many soils tested in the laboratory, it has been found that ψ is substantially independent of frequency, for the range of frequencies important in earthquake ground motions (Hardin and Drnevich, 1970). That is to say, for earth materials the size of the hysteresis loop is independent of frequency.

Thus, it is desirable to utilize a linear stress-strain (σ vs. ε) relation which will simulate this observed damping behavior. A convenient form of relation from visco-elastic theory is:

$$\sigma = G\varepsilon + \eta \frac{d\varepsilon}{dt} \quad (2)$$

where G is a shear modulus and η is a viscosity coefficient. When τ varies sinusoidally with time, Eq. 2 predicts that the stress-strain relation during one cycle will be a loop very similar to the observed hysteresis loops. The value of ψ for such a material is:

$$\psi = 2\pi\eta\omega/G \quad (3)$$

where ω = circular frequency of applied load. To simulate the observed behavior, η must thus be made inversely proportional to frequency. It is convenient to introduce a new variable D, called hysteretic damping ratio, defined as:

$$D = \psi/4\pi = \eta\omega/2G \quad (4)$$

For strains typical during important earthquakes, D for soil typically is between 0.02 and 0.15, while D for rocks varies from 0.05 to 0.03.

This stress-strain model for soil is called the linear-hysteretic model. It is a visco-elastic model, but with viscosity chosen in such a way as to simulate non-linear hysteretic behavior. It is the simplest linear model which adequately simulates actual behavior, and indeed the simulation often is quite good (Dobry et al, 1971). In the remainder of this section, it will be presumed that the linear-hysteretic model is the "correct" form of damping, and errors introduced by substituting a mathematically more convenient form of damping (η = constant) will be examined.

Response of 1-DOF System

Consider two single-degree-of-freedom systems with the same mass M and spring constant k, one having hysteretic damping (D = constant) and the other having viscous damping (η = constant). Fig. 3 shows amplification and phase angle curves for sinusoidal excitation of these two systems. The values of D and η have been chosen such that the peak amplification is the same for both systems. This condition is met when:

$$D = \eta/2\sqrt{kM} = \beta \qquad (5)$$

where β is the critical damping ratio for the viscously damped system. That is, if β is set numerically equal to D, then a viscously damped 1-DOF system will have the same peak amplification as a hysteretically damped system. As may be seen from Fig. 3, the two systems will also respond very similarly over a wide range of excitation frequencies; significant differences appear only at frequencies much greater than the resonant frequency.

If a 1-DOF system with linear hysteretic damping is subjected to earthquake base motion, the response of the system may be computed by Fourier analysis. However, because η varies with frequency, the governing differential equation cannot be obtained by the usual method of numerical integration with respect to time. On the other hand, response of a viscously damped 1-DOF system may be evaluated by the usual time-step integration methods (as well as by Fourier analysis). Hence, the "correct" hysteretically damped system often is replaced by an "equivalent" viscously damped system. This substitution may be done without introducing much error, provided that the predominant frequency in the input motion is not very large compared to the resonant frequency of the system. Comparisons have been made of response time history inputs and the results for a viscous system are indeed virtually indistinguishable from the results for a hysteretic system.

Response of Soil Profile with Uniform Damping

If the damping is the same at all points of a soil profile, then the rules for the existence of classical modes are satisfied (Dobry et al, 1971). This is true if the damping is linear hysteretic as well as when the damping is viscous. If the hysteretic damping ratio is D at all points of the profile, then the damping ratio is D for each mode. However, if damping is hysteretic, it is necessary to use Fourier analysis to find the response of each mode. Hence, as a practical matter, an exact solution using modal superposition is not useful if damping is hysteretic.

The usual method is to approximate the exact behavior by calculating the response of each mode using time-step integration, as though each mode were viscously damped. Two questions then occur: how should the viscous damping for each mode be chosen, and what error arises from this procedure?

The answer to the first question is: use the same critical damping ratio β = D for each mode. This conclusion follows from the preceeding discussion for a 1-DOF system. In effect, the amplification curve for the viscously damped mode is made equal to that of the hysteretic mode at the resonant frequency for the mode. This conclusion is different than that which would be reached if damping in soil really were viscous; then the critical damping ratio would increase in proportion to the natural frequency of the mode. Use of the same damping in each mode is a "trick" whereby hysteretic damping may be approximated by mathematically convenient viscous damping.

The general effect of making this approximation may be inferred from the discussion for a 1-DOF system. The response of each mode is very nearly correct except at frequencies much greater than the natural frequency of the mode. Thus there should be very little error for frequencies near the fundamental frequency, but some error may appear at higher frequencies. This conclusion is borne out by the amplification curves in Fig. 4.

To indicate the actual magnitude of the possible error, calculations were first made for two uniform soil profiles (Hagmann and Whitman, 1969). The damping was 15% and the input earthquake was the N69W component of the 1952 record from Taft. Table 1 gives other properties of the strata and compares peak computed surface accelerations. Response spectra from the surface motions for one of these profiles are compared in Fig. 5.

An additional set of computations were made for the layered profiles in Fig. 6. The wave velocity for the lower layer was always 800 fps while the velocity of the upper layer was varied. The inputs were a family of four artificial motions whose spectra fitted a smoothed response spectra. Fig. 7 compares peak accelerations. For all cases, the agreement of response spectra was as good as in Fig. 5.

In all of these examples, the agreement between the exact and approximate methods is quite good--certainly good enough to permit practical use of the modal superposition method. Some of the error results from lumping, but most of it is caused by use of viscous damping to compute modal response. In all cases there were only minor differences between the time histories computed by the two methods. As the fundamental frequency decreases, so that the predominant frequencies in the input are larger than the fundamental frequency, there is a tendency for modal superposition to underestimate response.

<u>Summary</u>

The main conclusion from this section is that very little error is introduced as the result of assuming that each mode is viscously damped so as to permit use of time-step integration for computing modal response. Of course, it is necessary to assign the proper amount of viscous damping to each mode. When damping is constant throughout the soil profile, the viscous damping ratio should be the same for each mode, and equal to the hysteretic damping ratio.

If damping in soil really were viscous, then the errors discussed in this section would not exist. However, the critical damping ratio for each mode would then vary from mode to mode.

NON-UNIFORM DAMPING

If the damping is different in various parts of the soil profile, then two difficulties arise:

1. From the practical standpoint, how should the modal dampings be chosen.
2. From the theoretical standpoint, the criteria for the applicability of modal damping in general are not satisfied.

These difficulties apply whether the damping is thought to be hysteretic or viscous.

Choosing Modal Damping

The problem may be illustrated by reference to profile I in Fig. 6. Suppose that the wave velocity C_S is uniform throughout, but that D = 30% for the upper stratum and only 5% for the lower stratum. In many analyses, an equivalent uniform damping is obtained by averaging the damping ratios, weighting each ratio by the thickness of the corresponding stratum. Thus, for this case (30 x 1/4) + (5 x 3/4) = 11.2%. Now we can compare the results for an exact analysis using the actual dampings with a modal superposition solution using 11.2% damping in each mode. The ratio of peak accelerations in this example proves to be about 0.7. This is a rather unsatisfactory result.

A much better procedure is to use weighted modal damping (Biggs and Whitman, 1970; Roesset et al, 1972). In this procedure the damping ratio for the nth mode is computed as:

$$D_n = \frac{\Sigma D_i \; E_{in}}{\Sigma E_{in}} \tag{6}$$

where D_i = damping ratio of ith stratum and E_{in} = energy stored in the ith stratum for the nth mode. The summation is over all strata. Eq. 6 thus weights the damping ratios for the various strata by the fraction of total energy that is stored in the strata. The relative energy stored in each stratum depends upon the mode shape, and is different for each mode. Thus D_n varies from mode to mode. For the example used above, the computed modal dampings for the first three modes are: D_1 = 5.6%, D_2 = 9.2%; D_3 = 12.0%. Thus the large damping in the upper stratum has little effect upon the damping of the first mode, but does contribute to the damping of the higher modes. The ratio of peak accelerations, as computed by the exact method and by modal superposition with weighted modal damping, now proves to be 0.98. The response spectra for the two calculated motions are in excellent agreement.

A series of calculations has been made, using the profiles in Fig. 6, to test the validity of weighted modal damping. In each case, the shear wave velocity C_{SL} in the lower layer was 800 fps, and the damping ratio D_L in this layer was 5%. The shear wave velocity C_{SU} and damping D_U of the upper layer were varied. Calculations were made for the combination of variables shown by the dots in Fig. 8. Also shown on this figure are:

1. The ratio of peak surface accelerations computed by modal superposition with weighted modal damping and by the exact method.
2. A subjective rating of the agreement of the response spectra derived from the surface motions computed by the two methods. Examples of response spectra appear in Fig. 9; the rating for each spectra appears in the lower right hand corner of each diagram. Usually the agreement is quite good for periods greater than 0.8 sec.

Based upon these results, curves have been drawn on Fig. 8 separating zones of good,fair and poor agreement between the two methods. The agreement becomes poor when both the damping and stiffness of the two layers are very different. Agreement is sufficient for practical purposes in both the good and fair zones.

Theoretical Considerations

The errors appearing in Figs. 8 and 9 occur primarily because the criteria necessary to permit modal superposition are not met when there are arbitrarily different dampings in different parts of the soil profile. This point is discussed in more detail by Roesset et al (1972) in connection with the problem of soil-structure interaction. Put in simple mathematical terms, the governing differential equations cannot be decoupled. The coupling appears in the damping matrix, and use of modal superposition with weighted modal damping is equivalent to neglecting the off-diagonal terms in the damping matrix. In some cases, where both damping and wave velocity differ markedly within the profile, these off-diagonal terms are simply too important to neglect.

Summary

It is clear that use of modal superposition can lead to significant errors when damping varies considerably within the profile, especially when there is also considerable variation of wave velocity. These errors can be reduced, but not always eliminated, by use of weighted modal damping.

While it is tempting to seek some better rule for selecting modal damping, it would seem better to recognize that normal modes simply do not exist in some situations. In such situations, modal superposition should be abandoned and a more exact method used. One possible approach is numerical integration of the system of coupled equations (Idriss and Seed, 1970); however, this method works only for a mathematically convenient (but physically unrealistic) way of representing damping. The Fourier analysis approach should be used.

EFFECT OF EARTH BENEATH SOIL PROFILE

Use of Fourier analysis to solve the one-dimensional amplification problem has led to one important overall conclusion: the ground motion at any depth in a soil profile is affected by the nature of the soil above this point. Thus, it is not really correct to assume that the input motion is known at the base of a soil profile. For convenience, this earth material below the bottom of the soil profile will be referred to as rock. Starting with the motion that would exist at an outcropping of this underlying rock and using Fourier analysis and transfer functions, it is possible to calculate the motion that would exist at an interface between this rock and some soil profile, and then the corresponding motion at the top of the soil profile. Results from this procedure are illustrated in Fig. 10. Here the soil profile is 100 ft. thick, having C_S = 800 fps and D = 5%. The shear wave velocity in the underlying rock is assumed to be 2000 fps. The differences between the input motion and the motion at the interface are small and rather subtle. Yet there are large differences between the surface motions computed including the interface effect (part b of the figure) and the surface motions computed assuming that the input motion occurs at the interface (part a).

The example in Fig. 10 indicates that consideration of the rock below the soil profile introduces an additional damping effect. This additional damping arises because some of the energy of vibration within the soil "leaks" into the underlying rock. This situation is perhaps best understood by imagining waves bouncing back and forth between the boundary and the surface; each time a wave comes to the boundary, only part of the wave energy is reflected back into the soil while a part passes into the underlying rock. Based upon the theory for reflecting waves, the amount of additional damping should be related to the ratio of shear wave velocities above and below the boundary (actually, to the ratio of the product ρC_S above and below, where ρ is mass density). This is illustrated by the results in Table 2, which gives the peak computed surface accelerations when the wave velocity in the rock is varied.

There have been several studies of the potential importance of this so-called radiation or impedance damping (Roesset and Whitman, 1969; Lysmer, Seed and Schnabel, 1971). Certainly this effect is important in some practical problems, although it is unimportant in many others.

Approximate Solution Using Modal Superposition

The effect of radiation damping cannot be accounted for exactly in a modal superposition solution. However, it is possible to approximate its effect by introducing an additional damping into each mode (Roesset and Whitman, 1969). For a uniform soil profile:

$$D_n = \frac{2}{\pi} \frac{1}{2n-1} \frac{(\rho C_S)_{soil}}{(\rho C_S)_{rock}} \tag{7}$$

The damping is added to the internal damping ratio. Various comparisons have been made of the motions computed by the exact method and by modal superposition using Eq. 7 (Hagmann and Whitman, 1969), and for uniform strata the agreement is excellent.

When C_S varies within the soil profile, it is necessary to use some average value. In most studies, this average has been obtained by weighting the values of ρC_S for each layer according to the thickness of the layer. Using this rule, exact and approximate results have been compared for the profiles in Fig. 6. In this study, the internal damping was 5% in both layers, while C_{SU} = 250 fps and C_{SL} = 800 fps. Three shear wave velocities were used in the rock: 2000, 4000 and 6000 fps. Typical response spectra for surface motions appear in Fig. 11. For profiles I and III, the results by the exact and approximate methods were in fair to good agreement. However, with profile II the agreement was fair to poor. Usually, the approximate method underestimates the response of the first mode of the soil. In these cases, there would be better agreement if more weight were given to the shear wave velocity of the upper layer. However, in profile III the reverse situation is true: where the first mode is controlled by the properties of a very thick lower layer, the surface layer has little influence upon radiation damping. A better method of weighting wave velocities is needed. Perhaps an equation of the form of Eq. 6 might be used to calculate $(\rho C_S)_{ave}$ where D_i is replaced by $(\rho C)_i$. However, this idea has not been tested as yet.

Summary

The effect of impedance or radiation damping can be important in some problems, especially where the wave velocities of the soil profile and underlying rock are not too dissimilar. This additional damping effect can be incorporated into the modal superposition solution in an approximate way. The approximation is very good for a uniform soil profile, but may not be entirely satisfactory for a profile in which the wave velocity varies greatly. The exact method easily accounts for this effect.

CONCLUSIONS

The method of modal superposition is a powerful method for analyzing the one-dimensional soil amplification problem, and because of its advantages it likely will continue to receive extensive use in the future. However, the method may be inaccurate for several reasons having to do with the treatment of damping. (There may also be errors associated with the number of modes to be retained and with conversion of a continuous soil profile into an equivalent lumped system.) This paper has illustrated and analyzed these errors:

1. Error caused by inability to do time-step integration when a realistic representation of damping (linear hysteretic) is used. This error is insignificant in nearly all cases of interest.
2. Error caused by averaging damping when damping differs within the soil profile. This error can be quite significant. It may be reduced by use of weighted modal damping, but even so the errors still may be significant when both damping and wave velocity vary considerably within the profile.
3. Additional damping effect associated with non-rigidity of underlying rock. This error also can be quite significant in some cases. The error may be greatly reduced, and even effectively eliminated, by use of an additional damping term.

In all cases where there is concern about the errors involved in modal superposition, an easily usable exact method (Fourier analysis) is available.

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Table 1

COMPARISON OF RESULTS BY EXACT AND MODAL SUPERPOSITION METHODS - UNIFORM STRATUM WITH CONSTANT DAMPING

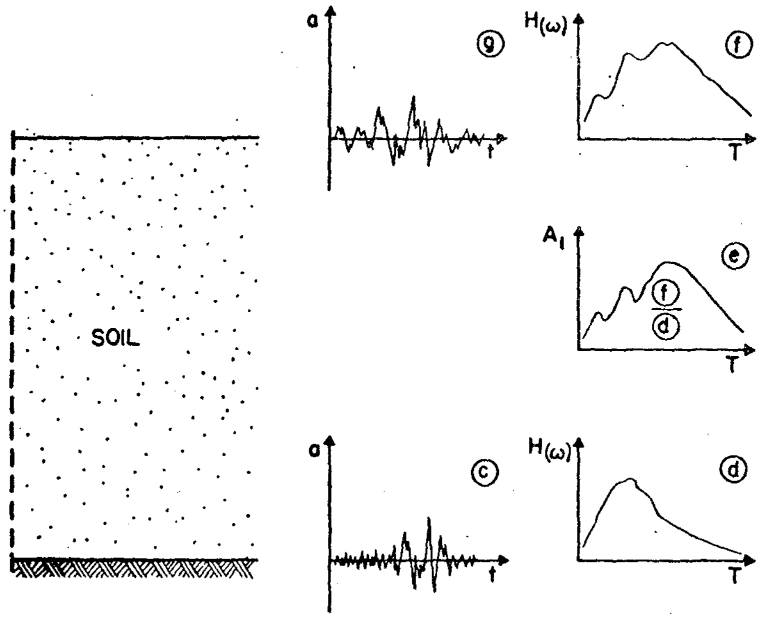
Thickness ft.	Shear wave vel. - fps	Fund. freq. cps	Ratio peak accel. by exact and modal methods
100	750	1.88	1.04
1000	1250	0.31	1.00

Table 2

PEAK SURFACE ACCELERATIONS FOR STRATUM 100 FEET THICK WITH C_S = 800 CPS AND D = 5%

C_S of rock - fps	For input 1	For input 2
Infinite	0.28g	0.37g
8000	0.23g	0.25g
5000	0.20g	0.21g
3000	0.17g	0.17g
2000	0.15g	0.15g

Peak acceleration of both inputs was 0.1g.



A = Amplification factor
a = Absolute acceleration
H(ω) = Fourier coefficient

T = Period
t = Time

FIGURE 1: FOURIER ANALYSIS METHOD FOR SOLUTION OF SHEAR BEAM

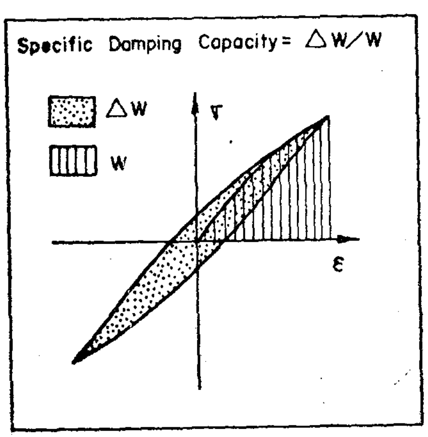


FIGURE 2: DEFINITION OF SPECIFIC DAMPING

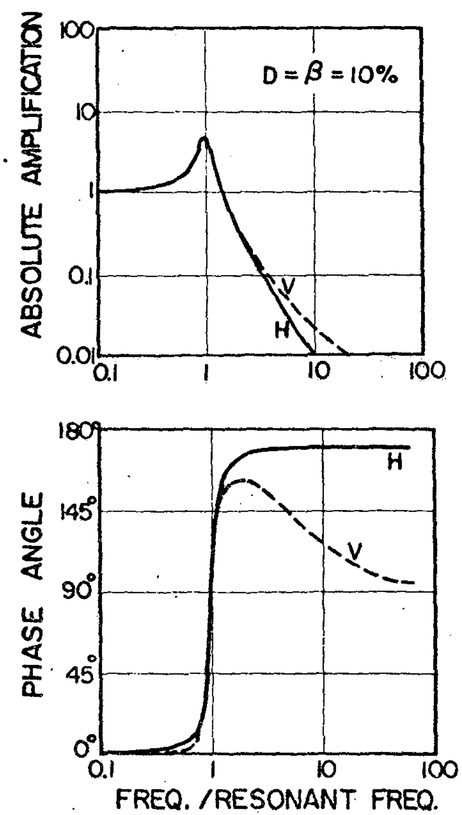


FIGURE 3: TRANSFER FUNCTIONS FOR 1-DOF VISCOUS AND HYSTERETIC SYSTEMS.

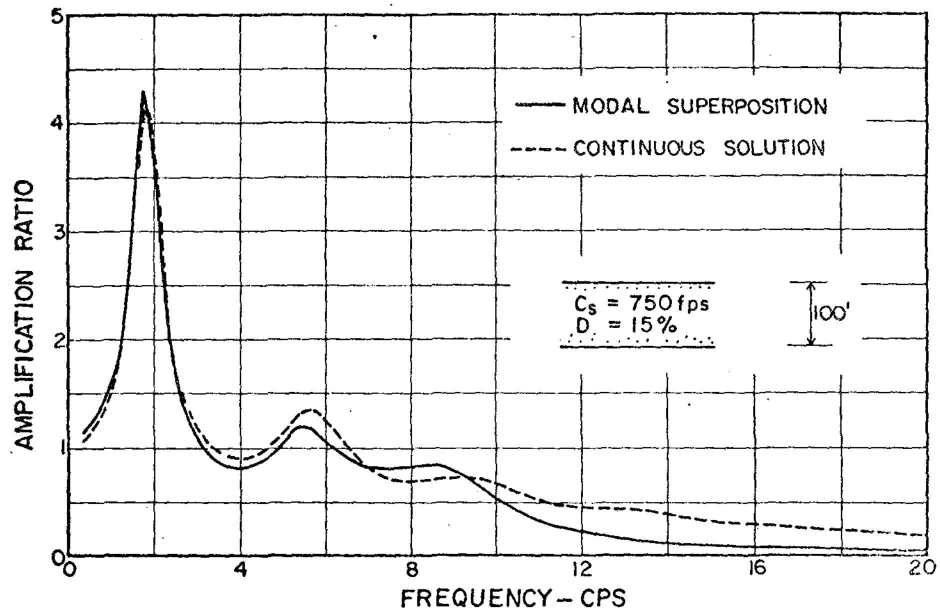


FIGURE 4 : AMPLIFICATION SPECTRA FOR UNIFORM PROFILE, BY EXACT AND MODAL SUPERPOSITION SOLUTIONS.

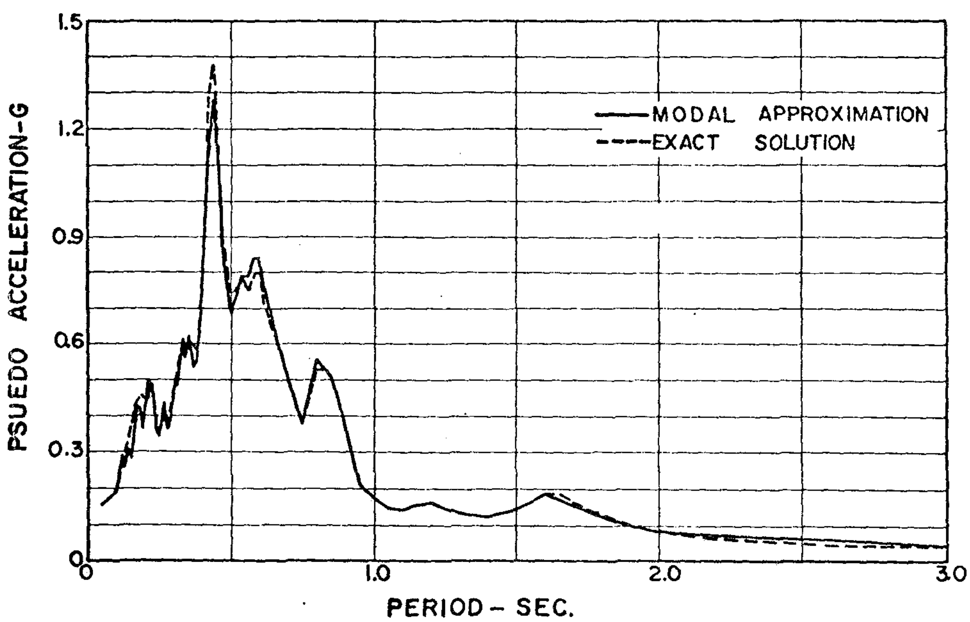


FIGURE 5: RESPONSE SPECTRA (2% structural damping) FOR SURFACE OF UNIFORM PROFILE OF FIG. 4.

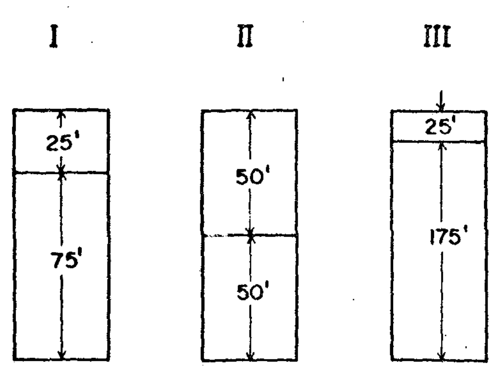


FIGURE 6: 2 - LAYER PROFILES.

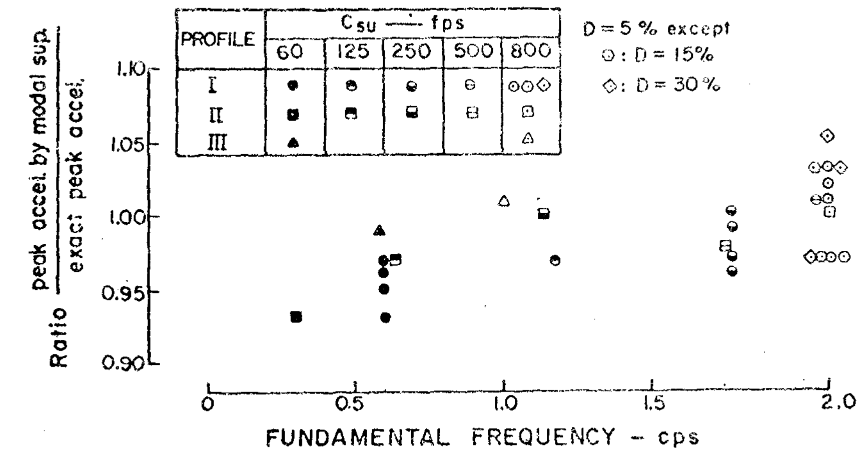


FIGURE 7: COMPARISON OF PEAK ACCELERATIONS FOR PROFILES WITH UNIFORM DAMPING.

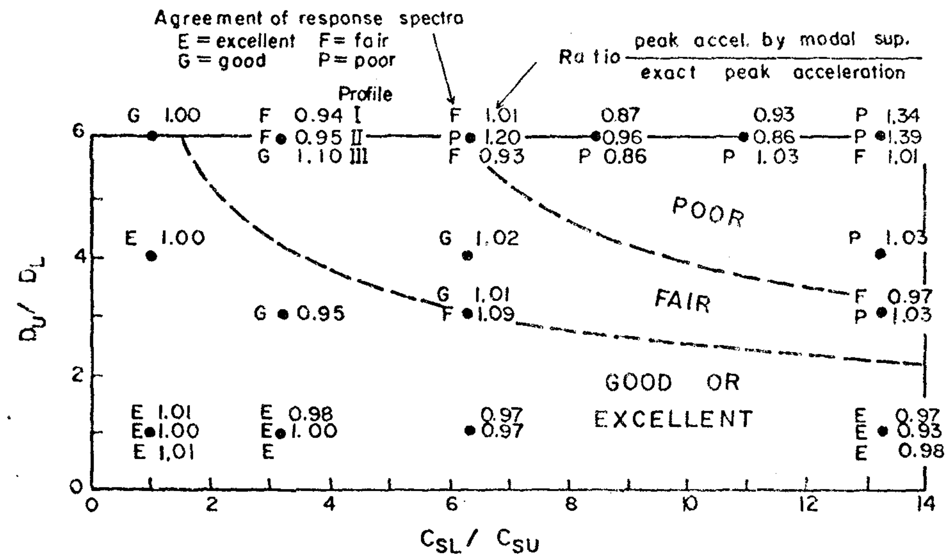


FIGURE 8: COMPARISON OF RESPONSE BY MODAL SUPERPOSITION FOR 2-LAYER PROFILES.

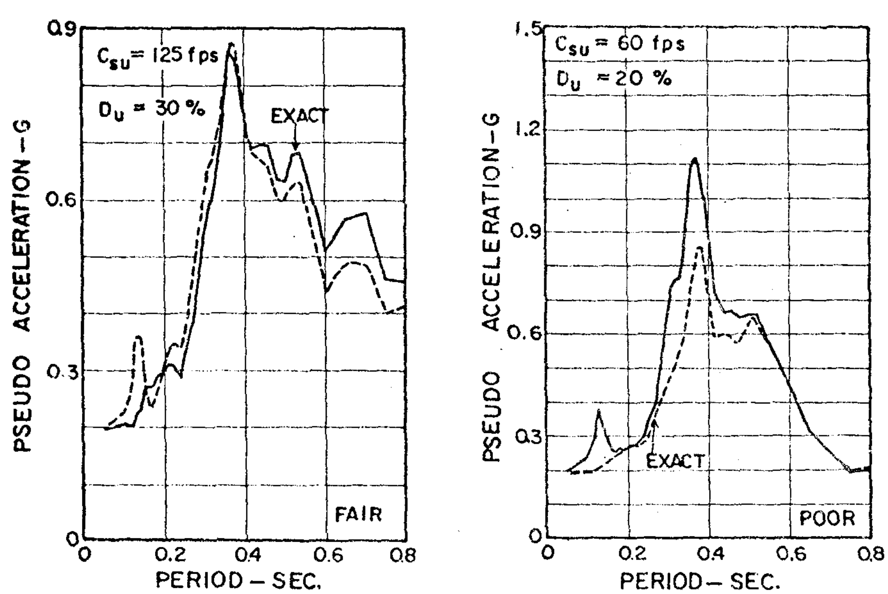


FIGURE 9: COMPARISON OF RESPONSE SPECTRA FOR PROFILE II

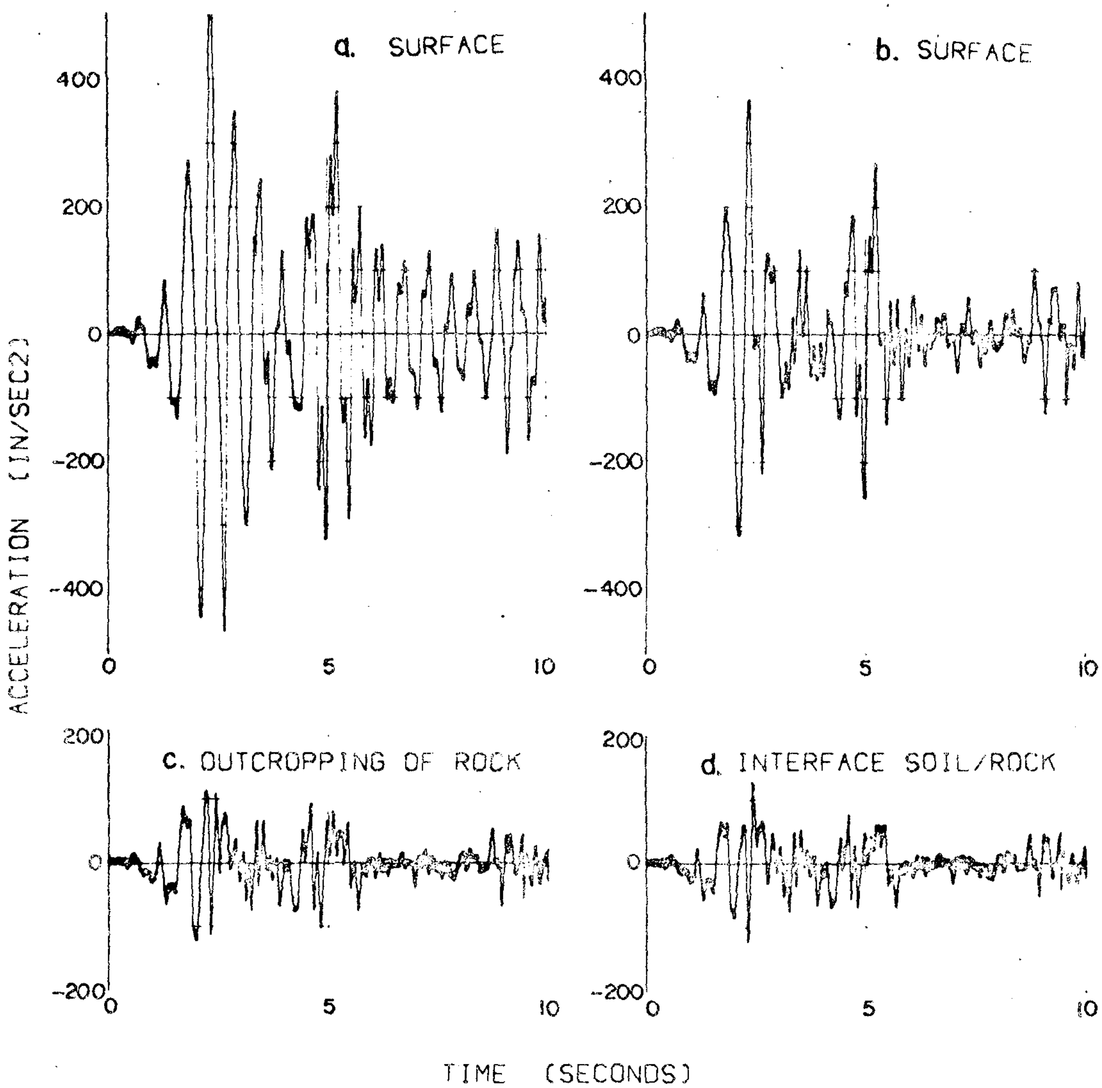


FIGURE 10: SURFACE ACCELERATIONS WITH AND WITHOUT INTERFACE EFFECT.

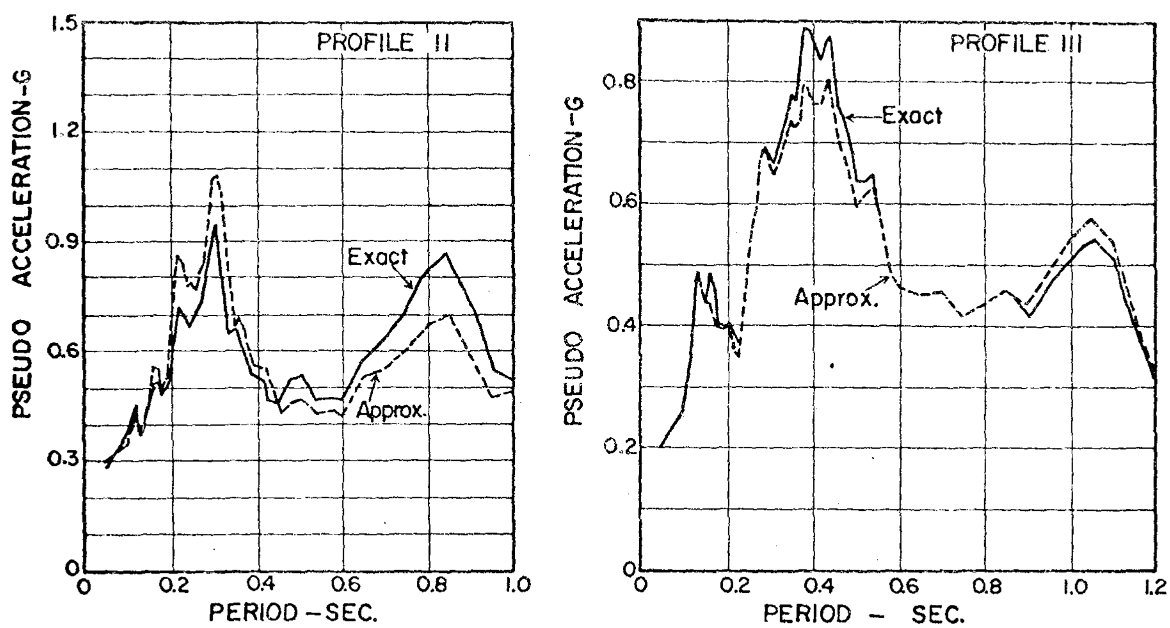


FIGURE 11: RESPONSE SPECTRA FOR PROFILES OVER ROCK WITH WAVE VELOCITY OF 4000 fps.