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Strudures Publication No. 358

Optimum Seismic Protection and Building Damage Statistics

Report No.4

# **SEISMIC RESPONSE OF BUILDINGS DESIGNED BY CODE FOR DIFFERENT EARTHQUAKE INTENSITIES**

by

John M. Biggs Peter H. Grace

January 1973

Sponsored by National Science Foundation Grants GK·27955 and GI·29936

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# OPTIMUM SEISMIC PROTECTION AND BUILDING DAMAGE STATISTICS

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Report No. 4

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### ABSTRACT

Seismic analyses have been made for five multi-story buildings of different types, each designed for five different levels of protection under the UBC Code. Both elastic and inelastic analyses have been made. The former is based directly upon a response spectrum, and the latter is based upon an artificial time-history input generated from the same response spectrum.

For the buildings investigated, it is concluded that, for both elastic and inelastic response, the average peak interstory displacement agrees very closely with that predicted by the firstmode elastic response. The elastic limit earthquake and the average maximum ductility ratio bear an approximate linear relationship with the ratio of first-mode elastic base shear to bottom story resistance.

It is further concluded that increasing the equivalent static lateral design force, as in conventional code procedures, does not appreciably reduce the interstory displacements. It does, however, increase the elastic limit earthquake and, to a lesser extent, decrease the maximum ductility ratio for a larger earthquake.

PREFACE

This is the 4th in a series of reports concerning optimum seismic protection and building damage statistics, prepared under Grants GK-27955 and GI-29936 from the National Science Foundation. The previous reports were:

- 1. Whitman, R.V., Cornell, C.A., Vanmarcke, E.H., and Reed, J.W.; "Methodology and Initial Damage Statistics," Department of Civil Engineering Research Report R72-17, M.I.T., March 1972.
- 2. Leslie, S.K., and Biggs, J.M., "Earthquake Code Evolution and the Effects of Seismic Design on the Cost of Buildings," Department of Civil Engineering Research Report R7 -20, M. 1. T., May 1972.
- 3. Anagnostopoulos, S.A., "Non-Linear Dynamic Response and Ductility Requirements of Building Structures Subjected to Earthquakes," Department of Civil Engineering Research Report R72-54, M.I.T., September 1972.

Co-principal investigators for this effort are Professors John M. Biggs, C. Allin Cornell and Robert V. Whitman, all of the Department of Civil Engineering. Mr. Grace is a Research Assistant in that Department.

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#### I. INTRODUCTION

Reported herein are the results of seismic analyses of several typical buildings designed according to the Uniform Building Code for different levels of protection (zones). This effort is part of a larger project (Optimum Seismic Protection and Building Damage Statistics) the objective of which is to develop procedures by which an optimum seismic design strategy might be determined for a community, i.e., one which would minimize total cost including initial construction costs and probable future losses due to earthquake.

The purpose of the effort reported here was to determine analytically the responses of buildings of different types, and resistance levels, to ground motions of varying intensity. These results were then used to estimate the amount of damage. The actual damage calculations are presented elsewhere (Ref. 1), but this report contains general observations regarding the nature of the responses and their qualitative relationship to damage.

In addition to this immediate objective, the study was also intended to gain further information on the behavior of actual structures under earthquake excitation. Perhaps most significantly, it serves to evaluate the effectiveness of current Code design procedures by determining the variation in response with seismic design level (zone).

The buildings studied are hypothetical but have been designed

by a practicing structural engineering firm using conventional procedures. They are therefore believed to be typical. Five basic buildings are studied. They all represent apartment house construction but include three types of framing systems and three building heights. Each building has been designed for five different levels of seismic protection. Therefore, twenty-five different buildings are investigated.

Each building has been analyzed both elastically, to represent behavior under small earthquakes, and inelastically to investigate behavior under more intense motions. The analyses are based upon a response spectrum which is believed to be appropriate for seismic design in the Boston area. The elastic analyses use the response spectrum approach and modal superposition. Inelastic response is determined for a severe earthquake using time-history analysis for an artificial ground motion generated from the postulated ground response spectrum. The results are presented in the form of interstory displacements, ductility ratios, and floor accelerations. It should be noted that these results are for a single set of ground motions, as represented by the response spectrum, and are not necessarily typical of all possible earthquakes.

#### II. DESCRIPTION OF TEST BUILDINGS

The buildings studied, and the methods used in this design, are described in Reference 2. The designs were executed by Le Messurier Associates, Consulting Engineers, Cambridge, Massachusetts. The intent in the designs was to produce buildings which are typical of competent, but conventional, engineering practice. Special refinements, which might be appropriate for research on seismic design, were deliberately avoided. On the other hand, care was taken to ensure that the structural systems developed were typical of good earthquake engineering practice. For this purpose the designs were reviewed by S. B. Barnes and Associates, Consulting Structural Engineers, Los Angeles.

The buildings were designed in sufficient detail to determine the parameters required in the dynamic analysis. All members were proportioned so that their stiffness and strength parameters could be determined. Typical connections and joint details were investigated to ensure that the systems adopted were feasible and reasonable in cost.

A single, apartment house floor plan was adopted for all buildings. As shown in Fig. 1, it has a double-loaded corridor and is 60 x 200 feet in plan dimensions. The architecture and structure of the buildings was intentionally made symmetrical and very simple. Although the plan is typical of many apartment buildings, most real buildings have some structural complications which would make the dynamic response peculiar to that particular structure. It is believed that the study of these hypothetical buildings provides



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more representative results than would a study of actual buildings.

In order to economize in the design effort, each building incorporates two structural systems. With one exception, lateral resistance in the short directions is provided by concrete shear walls or braced steel frames, and in the long direction by concrete or steel moment-resisting frames. Thus a seismic analysis of a single building in two directions provided data on two types of framing systems. Since the geometry and loading of the structural systems are inherently different, the analytical results for the various structural types are not directly comparable. However, it is not the purpose of this study to compare the efficiency of different types of structures.

The design of the buildings is described in Reference 2. It is based upon the ACI and AISC specifications supplemented by the seismic provisions of the Uniform Building Code. Wind load was taken to be 20 psf as specified by the Boston Building Code for buildings of these heights. With respect to seismic forces, five designs were made for each building. These correspond to Zone o (no seismic provisions), Zones 1, 2 and 3 as defined by the UBC, and a more severe condition, herein identified as Zone 4, in which the seismic coefficient is taken as twice that for Zone  $3<sub>z</sub>$ The last was added in order to study the effect of a more severe seismic loading than that now employed. The structures were checked for drift using drift ratio criteria of 1/600 for wind and 1/300 for earthquake.

Floor plans of the five buildings included in this study are shown in Figs. 2-6. Their description follows:

- **1.** l7-Story RC Shear Wall Building (Fig. 2, short direction). Lateral resistance is provided by shear walls, each extending over the full height, ranging in number from two exterior walls in the Zone 0 design to a total of eight walls in the Zone 4 design. The interior columns, which support a flat plate floor, are not considered to provide lateral resistance in the design but are considered in the dynamic analysis.
- 2. ll-Story Steel Moment Resisting Frame Building (Fig. 3, long direction). The lateral resistance is provided by two exterior rigid frames running the length of the building. The interior columns, which support a metal deck and concrete floor, are not rigidly framed. The building is braced in the short direction but that system is not considered here.
- 3. ll-Story RC Moment Resisting Frame Building (Fig. 4, short direction). Here the lateral resistance is provided by eleven beam and column rigid frames which also support the floor slabs. The frames are not identical since the orientation of the exterior columns varies.
- 4. ll-Story RC Shear Wall Building (Fig. 5, short direction). This is similar to the l7-Story shear wall structure except for the number of shear walls.







Figure 2-Typical Framing Plan for 17-Story Concrete Shear Wall<br>Building (CSW-17, Short Direction)





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Figure 4 - Typical Framing Plan for ll-Story Concrete Frame Building (CMRF-ll Short Direction)







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5. 6-Story RC Moment Resisting Frame Building (Fig. 6, long direction). In this case lateral resistance is provided by the two exterior rigid frames in the long direction.

In the proportioning of members for these buildings standard practice was followed even though theoretically that may not provide the most efficient lateral resisting system. Shear walls are maintained at a constant thickness and the reinforcement is varied with height to provide the required resistance at each level. In many cases beams and columns of the same cross section are used over several stories even though the requirements may vary somewhat. These common practices, which result in economy of construction, have a noticeable effect on dynamic response.

Other buildings have been designed but are not included in the present study of seismic response. These five buildings were selected to provide a range of structural systems and building heights. They are not intended to provide a comparison of structural systems. To provide data for the optimization study, cost estimates (Ref. 2) have been made but that information is not included here.

### III. METHODS OF ANALYSIS

Two seismic analyses have been made for each of the twentyfive building designs. The first is an elastic analysis for a relatively small earthquake (O.lg) and the second is an inelastic analysis for a large earthquake (0.27g). The analyses are based on the response spectra shown in Fig. 7 (Ref. 3). This was constructed to be typical of what might be appropriate for design in the Boston area. It is based upon a seismological study of the region (Ref. 4). Fig. 7 is constructed for a peak ground acceleration of O.lg and it is assumed that all ordinates are proportional to the peak ground acceleration. The elastic analyses make use of the response spectrum directly. However, for the inelastic analyses, an artificial time-history of ground motion is employed. This was generated to match the ground response spectra (Ref. 3).

Shown in Fig. 8 is a comparison of the ground response spectrum and the UBC seismic coefficient used in the design of the buildings. Although the magnitudes are not to be compared, it is obvious that the shapes of the two spectra are radically different. In addition, the natural periods for design purposes were computed using the approximate UBC formulas, whereas a rigorous computation was used in the dynamic analyses. Therefore, one cannot expect any direct correlation between the code design levels and the computed dynamic response. The purpose of this study is to determine if any correlation exists.

The analyses have been executed by a computer program which



Figure 7 - Assumed Ground Response Spectra (0.1g Peak Ground Acceleration)





was developed by S. Anagnostopoulos (Ref. 3). The program utilizes simplified models which make feasible the dynamic analysis of complete buildings. The studies reported in Reference 3 indicate that the simplified procedures are reliable and sufficiently accurate for practical purposes. The input consists of individual member properties and the program computes the parameters of the dynamic model.

The models have one degree of freedom per floor, the masses being lumped at these points. Since the buildings are symmetrical, the two horizontal directions are uncoupled and there is no torsion about the vertical axis. The rotational inertias about the horizontal axis are not significant and are ignored. The frame structures are assumed to be "shear" buildings, i.e., springs are placed only between adjacent masses. The total force in each spring at any time is the sum of the corresponding story shears in the individual frames providing lateral resistance. The story resistance function of a single frame is assumed to be elasto-plastic. The stiffness is computed by an appropriate procedure which takes into account the stiffness of the individual columns and the beams in the floors above and below the story. The ultimate resistance is computed by assuming a story-shear mechanism in which the hinge moments are taken to be the smaller of the column or beam moment capacity.

Since the behavior of the shear wall building is more like a cantilever beam than a shear beam, a far-coupled system is used in these cases, i.e., the full stiffness matrix is generated. The moment resistance function at any floor in a wall is assumed to be

elasto-plastic. However, the shear resistance is assumed to have no ductility, i.e., if the shear capacity is exceeded the wall is assumed to have failed and to provide no resistance thereafter. The total internal forces at any time are taken to be the sum of those provided by the individual walls. The forces developed in the interior frames, although small, are superimposed on those developed by the shear walls. The frame action becomes significant if the shear walls fail.

Although it is known that non-structural elements (block walls, partitions, etc.) contribute significantly to both strength and stiffness when the building distortion is small, this effect has been ignored in the analysis. The analyses reported here involve substantial distortions and it is probable that most of the nonstructural resistance would have been destroyed. In any case, it is impossible to predict this effect with any reliability.

After the parameters of the dynamic model have been completed, the program first determines the eigen values and vectors for all modes. In the case of inelastic analysis, the response to the timehistory input is then computed by numerical integration of the equation of motion. At each step, the internal forces in each resisting element are determined from its distortion and the assumed elasto-plastic resistance function. These are then superimposed to obtain the total force on each floor mass. Thus, a time-history of the displacement and acceleration of each floor is computed.

Even with the simplified model, the numerical analysis is

expensive, considering the number of analyses to be made. Therefore, the program by S. Anagnostopoulos was extended to include a response spectrum analysis to be used for the elastic cases. This is a conventional technique in which the maximum response in each mode is determined by the product of the response spectrum ordinate (relative displacement or absolute acceleration, Fig. 7), the modal participation factor, and the eigenvector for the point being considered. To estimate the total response, the modal components are combined by the square root of the sum of the squares method. Comparison of the two methods (time-history and response spectrum) for elastic solutions indicate reasonable agreement.

Constant modal damping was assumed for all analyses but a distinction was made between steel and concrete structures and between elastic and inelastic responses. The following values were used.

Steel Buildings



For the time-history analysis, the inelastic values were used to generate a proportional damping matrix, assuming that these elastic damping coefficients were also valid in the inelastic range.

The input to the program includes moment of inertia, shear

area, ultimate moment capacity, and ultimate shear capacity for each member. The methods used are described in Reference 2. In computing capacities, only a very small live load was assumed. This has an important effect on column moment capacity, particularly in reinforced concrete. ACI and AISC interaction formulas were used to determine moment capacities. The removal of most of the design live load, and the elimination of load factors on both dead and live, results in lateral resistances much greater than those corresponding to the design seismic forces. The moment of inertia of concrete beams was taken as 0.4 times that of the gross cross section to allow for cracking. For shear walls this figure was assumed to be 0.5. For concrete columns, the full cross section was assumed to be effective.

The story parameters for the twenty-five buildings, as computed by the program, are tabulated in Appendix A. In the case of shear wall buildings, these parameters are total moment resistance and total shear resistance. In the case of frame buildings, they are total story stiffness and shear resistance. It may be observed that in the shear wall buildings the shear resistance is constant over the building height since the total wall area and horizontal steel are constant. The moment resistance varies because the amount of reinforcement decreases with elevation. In the concrete frame buildings the story stiffnesses are identical (except for the bottom story) because the dimensions of the members are constant over the full height. This is not true of the steel frame

building. For both the steel and the concrete frames, it may be noted that the resistance is constant over three or four stories. This results from using identical members for economy through duplication. All of the data characteristics mentioned, which result from normal design practices, have an influence on the dynamic response of the buildings.

It may also be noted that the ll-story steel building has much smaller stiffness and resistance than the 11-story concrete frame building. This results from the fact that the steel frames are in the long direction of the building whereas the concrete frames are in the short direction. In the former case, the lateral resisting frames are only the exterior frames which carry a smaller total dead and live load. This results in smaller member sizes and hence less stiffness and ultimate capacity to resist lateral forces. The two buildings are therefore not comparable.

It is assumed in the analysis that the lumped masses at all floors and the roof are equal. The total weight at each floor was computed to be 2160 kips for the concrete buildings and 1290 kips for the steel buildings.

#### IV. ANALYTICAL RESULTS

The results of all analyses are summarized in Tables 1 and 2. They are given in terms of peak values of displacement at top of building  $(X_T)$ , interstory displacement  $(X)$ , and floor acceleration (X). The elastic values in Table 1 are for a peak ground acceleration of O.lg, even though this is beyond the elastic limit for some buildings. The inelastic values in Table 2 are for an 0.27g input which causes inelastic behavior in all cases.

The detailed results from which these summaries are derived are given in Appendix B. where the individual peak interstory displacement and floor accelerations are tabulated. Note that average peak interstory displacement  $(x_{av})$  is an average of the peak values in the stories and is always somewhat more than the top displacement  $(x_T)$  divided by the number of stories.

The Elastic Limit  $(X_e)$  given in Table 1 is the ground acceleration which would cause first yielding in the structures. For frame structures, this is the intensity which would cause the shear in some story to just reach the ultimate shear resistance (see Appenidx A). For shear wall structures, it is the intensity which causes the bending moment or the shear to reach the capacity of the walls in some story. In both cases, all elements are lumped together for this purpose, i.e., the total building stiffness and total shear capacity are used to define an elastic limit. Thus the limit is actually an average value for all elements of the structure.



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Table 1 - Summary of Elastic Analysis Results (0.1g)

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DESIGN ZONE	NTAL UNDAME ERIOD 11 正	$D$ ISPLACEMENT INTER-STORY $\frac{\lambda}{\lambda}$ ۱É م AVG.	$\mathcal{E}_{\mathcal{O}}$ CCELERATI AVG. PEAK FLOOR $\sigma$	INTER-STORY DISPLACEMENT WAXIMUM	x' $\frac{1}{2}$ FLOOR	FLOOR CCELERATION WAXIMUM ⋖	$\ddot{\times}$ $\frac{1}{\sqrt{2}}$ FLOOR	ACENIT MAXIMUM TOP STORY $SPL$ . $\sum_{i=1}^{n}$	DUCTLITY AVG.
Z	$\top_{1}$ SEC	$\times$ av FT.	$\frac{1}{x}$ $g$ .	$\chi_{\mathcal{M}}$ FT		$\ddot{\ddot{x}}_{av}$ $g$ .		$\chi_{\tau}$ FT	$\mu_{\mathsf{M}}$
$CSW-17$	SHORT DIRECTION $\beta$ = .05								
Ō	3.34	,0452	.1968	.0647	4	.3065	T	.535	2.66
$\sqrt{ }$	3.34	.0459	.1677	.0581	3	.2755	$\sqrt{ }$	.508	2.58
$\boldsymbol{z}$	2.68	.0640	, 3416	.1030	17	.4842	4	.993	3.13
$\overline{\mathfrak{Z}}$	2.35	.0556	.3504	.0822	17	.5317	17	.834	2.04
4	2.11	.0414	.3525	.0610	17	.5937	17	.663	1.02
$SNRF -11$	$\beta$ = DIRECTION LONG .03								
O	3.53	0860	.1742	.2436	9	.2484	T	.6710	3.78
$\mathbf{I}$	3.53	.0860	.1742	.2436	9	.2484	$\mathbf 2$	.6710	3.78
$\boldsymbol{z}$	3.12	,0820	.2009	.1592	9	.2496	$\overline{2}$	.6210	2.58
$\mathcal{Z}_{\mathcal{A}}$	2.40	.0905	.2540	.1934	9	.3148	$\overline{I}$	.6822	2.69
4	1.71	.0693	.3015	.1791	9	.3535	2	.5574	2.06
$CMRF -11$	SHORT DIRECTION F ß .05								
O	2.66	0872	.2221	,1399	$\mathbf{z}$	.2974	$\prime$	,8286	1.36
Ť	2.66	0872	.2221	.1399	$\overline{2}$	.2974	I	.8288	1.36
$\overline{2}$	2.66	.0897	.2321	.1585	$\prime$	.3034	$\mathcal{L}_{\mathcal{L}}$	.8864	1.24
3	2.05	.0710	.2752	.1030	$\prime$	, 3136	$\eta$	.6104	1.19
4	1.47	.0496	.3230	.0630	ł	.4273	$\overline{11}$	.5202	1.14
$CSW-H$	SHORT DIRECTION $\beta$ = .05								
O <sub>1</sub>				$1.84 + 0.0570 + 2683 + 0701 + 111.4907$			$\mathcal{L}$	.6041	8.60
T	1.84	.0570	.2655	.0690	I	.4658	$\overline{1}$	.5986	8.20
$\mathcal{Z}$	1.84	.0188	, 2143	.1622	$\left  \right $	.4059	3	.7538	6.50
3	1.37	.0361	, 3354	.0481	$\vert$ $\vert$ $\vert$	.6180	1	.3840	2.20
4	1.11	1.0340	, 3261	$.0430$ [11]		,4783	$\mu$	.3012	1.70
$CMRF-6$	<b>DIRECTION</b> LONG $\beta$ = .05								
0.		$2.81$ $1.1493$	$.1589$	$.2579$ $14$ $1.2154$			$\sqrt{ }$	.7741	2.10
T	2.81	.1493	.1589	.2579	$\frac{4}{ }$	.2154	$\overline{I}$	.7741	2.10
$\mathbf{z}$	2.81	.1517	$.1594$	.2519	4	.2148	$\sqrt{ }$	.7839	2.2
3	2.07	.1243	.2136	.2758	$\overline{3}$	.2954	$\boldsymbol{z}$	.6030	2.0
4	1.38	.0773	.2741	.1184	L	.3156	6	.4210	1.6

Table 2 - Summary of Inelastic Analysis Results (0.27g)

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The ductility ratio ( $\mu$ av) given in Table 2 is the ratio of maximum total displacement to elastic limit displacement. For frame buildings this is the average of the values for the individual stories which are tabulated in Appendix B. It may be observed that in most cases the maximum story ductility is considerable larger than the average. Also, no attempt has been made to determine local ductilities in individual members.

For shear wall buildings, the ductility ratio cannot be associated with inter-story displacements, and the values given are based upon the displacement at the top of the building. In other words, it is the ratio of the maximum top displacement to the value of that displacement corresponding to the elastic limit as previously defined.

Although the results are analyzed in more detail in later sections, it is appropriate to comment here on the range of fundamental periods. First, the periods are generally much larger than those estimated by the approximate UBC formulas. It is not known why this is true unless the UBC anticipates the contributions of non-structural elements to stiffness or uses this device to introduce conservatisms. The periods of course decrease with design zone, that for the Zone 4 design being approximately half, or a little more, of that for Zone O. Considering the ll-story buildings, the steel frame has a very long fundamental period because of its flexibility. As mentioned previously, this is partly due to the fact that these are exterior frames in the long direction of the building. As expected, the

concrete frame has a much longer period than the shear wall building of the same height. The 6-story concrete frame has a long period, also because these frames are in the long direction.

The average peak inter-story displacements are plotted against intensity of ground motion in Figs. 9 thru 13. The elastic portions of the curves are properly constructed as straight lines passing through the analytical result for O.lg. However, since there are only two data points, the curves between the elastic limit and 0.27g are merely indicative of the trend. It is important to note that there is a consistent increase in the elastic limit, and a decrease in slope of the elastic curve, as the design zone increases from  $0$  to  $4$ . In general, the points at 0.27g lie below the extension of the straight elastic curve. This is primarily due to the larger damping assumed in the inelastic analysis. As discussed in the next section,  $\mathsf{X}_{\mathsf{av}}$  does in fact vary almost linearly with intensity of ground motion through both the elastic and inelastic ranges.

For the 0.27g results there is not a consistent relation between  $x_{av}$  and design zone, e.g.,  $x_{av}$  for Zone 3 may be greater than for Zone O. This scattering of results is believed to be real and caused by the following factors:

> 1. The time-history input used for the inelastic analysis does not have a smooth response spectrum. Therefore, there is not a continuously decreasing displacement response with decreasing period.

- 2. Most of the buildings do not have a uniform variation of story resistance with height. This results in "soft" stories where the peak inter-story displacements are larger than would otherwise be expected. When the design is changed for a different zone, these "soft" spots are moved or eliminated and the nature of the overall response is affected.
- 3. In the shear wall buildings, the shear walls completely fail in the lower zone designs. The resistance is then taken over by the interior frames which are very flexible. This sudden transition from a very rigid to a very flexible building completely changes the response and destroys any relation to the design level.

Although the above factors make it difficult to interpret the results, they in no way invalidate the analyses. Real earthquakes do not have smooth response spectra, real buildings do not have uniform variation in properties over their heights, and shear walls do fail. Therefore, it is believed that the results presented are at least representative of the behavior of real buildings. Furthermore, as shown in the next section, there is considerable consistency in the results.



# Figure 9 - Variation of Inter-Story Displacement with Ground Motion Intensity 17-Story Shear Wall Building (CSW-17, Short Direction)











Figure 12 - Variation of Inter-Story Displacement with Ground Motion Intensity 11-Story Shear Wall Building (CSW-11, Short Direction)



Figure 13 - Variation of Inter-Story Displacement with Ground Motion Intensity 6-Story Concrete Frame Building (CMRF-6, Long Direction)

#### V. COMMENTS ON BUILDING BEHAVIOR

In this section an attempt is made to draw some general conclusions, based upon the limited number of buildings studied, regarding the behavior of building structures under seismic disturbance. This is done by relating the elastic and inelastic responses to the strength and stiffness of the buildings and to the assumed ground response spectra. It should be noted here that all of the buildings studied have relatively long periods  $(T_{1} > 1 \text{ sec.})$  and fall in the constant-velocity or constantdisplacement regions of the response spectrums. The observations and conclusions based on these data are not necessarily valid for buildings with shorter periods.

Elastic Response. In Table 3 the computed elastic disp1acements are related to the first mode responses. The spectral displacement  $(S_1)$  is read directly from Fig. 7 and the modal participation factor  $(\Gamma_1)$  has been normalized for a unit eigenvector at the top of the building. Thus, the product  $\Gamma_1 \text{S}_1$  is the first mode component of the roof displacement  $(X_T)$ . It may be noted that in all cases there is essentially no difference between  $\Gamma_1 S_1$  and  $X_T$ . Therefore, it is concluded that for elastic displacements, only the fundamental mode is significant and that a reliable prediction may be made by considering only the first mode.

As would be expected, the average of the peak interstory displacement  $(x_{av})$  is always slightly greater than the roof displacement divided by the number of stories. This is true

 $\Gamma_i$  = FIRST MODE PARTICIPATION FACTOR<br> $S_i$  = FIRST MODE RESPONSE SPECTRUM DISPLACEMENT

 $N = NUMBER OF STORIES$ 

 $\beta$  = DAMPING RATIO



Table 3 - Comparison of Computed Elastic Displacements with First-Mode, Response Spectrum Displacements (0.1g)

because the higher modes make a noticeable contribution to the individual interstory displacements. However, as shown in Table 3,  $X_{\alpha V}$  is only slightly more than  $X_{\tau}/N$  and the first mode top displacement could be used as a reasonable basis for computing the average peak interstory displacement.

The first mode cannot be used as a reliable estimate of the peak floor accelerations because the higher modes do make a significant contributions in this case. On the average, the total accelerations are about 50 percent higher than the first mode components.

The relationship between the variations in natural period with design zone and the response spectrum is significant. Referring to Fig. 7, it is seen that the spectral displacement is constant for periods larger than 2.7 sees. For all five buildings the periods for Zones 0, 1 and 2 are either identical or larger than 2.7 secs. Therefore, the interstory displacements are essentially the same for all three design zones.

Inelastic Response. It has been concluded by several investigators that, in the constant-displacement and constantvelocity regions of the response spectrum, the total displacement is essentially the same whether the structure responds elastically or inelastically. In other words, the elastic response spectrum can be used as a reliable indication of inelastic displacement response. This conclusion is largely based on studied of onedegree systems. However, the results presented here indicate that the same equivalence, at least approximately, exists between

the inelastic response of a multi-story building and the first mode elastic response.

Table 4 compares the first mode elastic response  $(\Gamma_1 S_1)$  with the computed inelastic top story and average interstory displacements. Comparing the displacements at the top of the building  $(\Gamma_1 S_1$  vs  $x_T$ ), the agreement is reasonable except for the tall shear building (CSW-17) and the steel frame building (SMRF-ll). This is partially explained by the following: For the lower design zones of CSW-17, the shear walls fail completely and this produces an erratic, non-uniform behavior; for the steel frame building, the "soft" stories yield (stories 7-9, see Table 8.28) and this tends to reduce the inertia forces in the other stories even though they remain elastic. In **all** cases, the elastic top story response is greater than the inelastic displacement, probably because of the relieving effect of an individual story yielding.

In spite of the above, the average peak interstory displacements by the two methods agree quite closely. This is concluded by comparing the inelastic value  $(x_{av})$  with the top deflection predicted for elastic behavior divided by the number of stories (T<sub>1</sub>S<sub>1</sub>/<sub>N</sub>). The agreement between these two values is demonstrated in Fig. 14. It appears to be quite good. The points which depart furthest from the 45-degree line are associated with the shear wall and steel frame buildings and the reasons for the departures are probably those cited above.



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Table 4 - Comparison of Computed Inelastic Displacements with First-Mode Response Spectrum Displacements (0.27g)

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 $\mathcal{A}^{\text{max}}_{\text{max}}$ 



Figure 14 - Correlation of Inelastic Inter-Story Displacement with Response Spectrum Displacement

It is concluded that, for the multi-story buildings considered, a reasonable estimate of the average interstory displacement for inelastic response can be obtained from the elastic spectral disolacement for the first mode.

Elastic Limit and Ductility Ratio. It might be exoected that these two response parameters would be related to the ratio of the total elastic inertia force to the ultimate resistance of the building. An attempt to establish such a correlation is shown in Figs. 15 and 16 where elastic limit and average maximum ductility ratio are plotted against a force-resistance ratio (FR), defined herein as one-half the total weight times maximum first mode  $\sim 10^7$ acceleration (from the elastic response spectrum) divided by bottom story resistance. This is not a precise ratio. The numerator includes only the first mode, although this probably dominates base shear and moment, and assumes a uniform (from zero at the bottom) variation of inertia force with height. It is therefore only an approximate base shear. The denominator, which is the bottom story resistance (total shear for frame buildings and total moment for shear wall buildings' overall resistance. Never-the-less, the force-resistance ratio is an indicator of the relation between applied forces and strength.

In Fig. 15 there appears to be a correlation between the resistance-force ratio (inverse of FR) and the elastic limit. This is to be expected, although the relation is clouded by the changing period. If the resistance of the building could be

changed without changing the stiffness and period, there would be a direct proportion between elastic limit and resistance. However, increasing resistance generally increases stiffness, which results in decreasing period and hence increasing inertia forces. Therefore, the straight line in Fig. 15 is somewhat flatter, i.e., the elastic limit does not increase as rapidly as the resistance. However, there does appear to be an almost linear relationship.

The elastic limits in Fig. 15 are all less (by about 45%) than the value which would be predicted on the basis of the first mode spectral acceleration and corresponding inertia forces. In the Figure this would be represented by a straight line,  $X_{\alpha} = \frac{0.27}{FR}$  (0.27 appears because S, has been computed for a ground acceleration of 0.27g). This is not surprising because of the way the elastic limit has been defined, i.e., first yield in some story, not general yielding in all stories. Further study might indicate that a different definition would produce a better correlation between the elastic limit and FR.

The same general comments apply to the average maximum ductility ratio plotted in Fig. 16, although there is more scatter in the results. This scatter may primarily result from the difficulty in defining ductility for the entire building as discussed previously. However, there appears to be an almost linear relationship between the ductility ratio and the force-resistance ratio, beginning with unit ductility for an FR of one.

The points in Fig. ]6 are not too far from the equality line, i.e., the line for which  $\mu_{av}$  = FR. This line represents



Figure 15 - Correlation of Resistance - Force Ratio and Elastic Limit



the proposal by others that one may design for an acceleration equal to the elastic response value divided by  $\mu$ . In other words, if the shear resistance were made equal to that indicated by the first mode elastic shear divided by the desired maximum ductility ratio, the response would indeed produce the desired  $\mu$ . Again, further study is required to determine the best definitions of ductility ratio for a multi-story building.

From the above it appears that for a given period and hence level of response, as the resistance is increased there is a proportional decrease in ductility ratio. Both of these have implications as to the performance of the building.

It is not possible to relate response displacements to the force-resistance ratio. As was discussed previously, displacements, both elastic and inelastic, are almost entirely a function of the fundamental period and not the strength of the building.

Some of the scatter in the results plotted in Figs. 14, 15 and 16 is due to the fact that the elastic response predictions are based upon the smooth response spectrum, whereas the inelastic computation is based upon a time-history which does not have a smooth response spectrum. It is difficult to correct for this effect because for inelastic response the effective value of natural period is unknown.

However, it does appear possible to establish simple relationships between the first mode elastic response and the inelastic response (displacements, elastic limit, and maximum ductility ratio) of a multi-story building.

#### VI. EFFECT OF CODE DESIGN

With respect to the response parameters discussed herein, the only effect of seismic code design (UBC) is to increase the lateral resistance of the building in accordance with the seismic coefficient specified for the zone. In addition, the drift ratio for the equivalent static forces is limited, although this may not control the design. It is assumed herein that for design purposes the fundamental period is determined by the approximate code formulas, not by actual analysis, and hence depends only on geometry and not the level of resistance provided. Thus the actual stiffness and period is changed with design zone, but not systematically.

As discussed in the preceding section, interstory displacements are a function of natural period and not resistance. Hence. one would not expect a direct correlation between design zone and displacements. If the drift ratio controlled all design, such a relationship might exist since natural period is related to drift. However, the actual drift ratio normally varies between zone designs.

In Fig. 17 the elastic limit (ground acceleration just causing yield) is plotted against design level for each building. There is a consistent and appreciable increase in elastic limit with seismic coefficient or zone. This presumably means a somewhat improved performance of the structure, *even* though the inters tory displacements are not appreciably reduced. However, as discussed in the preceding sections, the increase in resistance between zones

is proportionally more than the resulting increase in elastic limit. The slopes of the Fig. 17 curves for shear wall buildings are steeper because the resistance increases more rapidly relative to the applied forces, i.e., the force-resistance ratio decreases more rapidly (see Table 5 and Fig. 15).

The average peak interstory displacement for elastic behavior (computed for O.lg in all cases) is plotted against design zone in Fig. 18. Except in one case (CMRF-6), the decrease with design zone is not large. As demonstrated in the previous section, these decreases are entirely due to the decrease in natural period. Because of the shape of the ground response spectrum, there is essentially no difference for the zones 0, 1 and 2 designs. The slope of the CMRF-6 curve is steeper because the natural period changes more radically with design zone in this case. All of these effects may be observed in the response spectrum data of Table 3.

The same general trends are apparent in Fig. 19 where the inelastic (0.27g) average interstory displacement is plotted. Again there is not a significant decrease with design zone except for CMRF-6. However, the Zone 4 design consistently produces the smallest displacement. In no case do the Zones 1, 2 and 3 designs provide any substantial improvement over the Zone 0 design. The results for Zones 2 and 3 are somewhat erratic. In some cases this is due to the effects of shear wall failure, and in other cases to the interaction between stories, i.e., the yielding of a "soft" story may reduce the forces in other stories.



Figure 17 - Effect of Design Zone on Elastic Limit



Figure 18 - Effect of Design Zone on Elastic Inter-Story Displacement



Figure 19 - Effect of Design Zone on Inelastic Inter-Story Displacement



Figure 20 - Effect of Design Zone on Average Ductility Ratio

 $\ddot{S}_1$  = FIRST MODE RESPONSE SPECTRUM ACCELERATION (6.27g)  $R_B = BOTTOM$  STORY RESISTANCE \*

W = TOTAL WEIGHT OF BUILDING

FR= RATIO OF PSEUDO-INERTIA FORCE TO TWICE THE BOTTOM STORY RESISTANCE =  $WFS/2R_B$ 



\*For rigid frame systems,  $R_B$  = shear resistance in bottom story. For shear wall systems,  $R_B$  = moment resistance in bottom story divided by two thirds the building height.

Table 5 - Force-Resistance Ratios (FR)

It is concluded that conventional code design, in which only lateral resistance is considered, is not an effective way to control interstory displacements.

It was shown previously that the maximum ductility ratio generally decreases with force-resistance ratio (Fig. 16). Therefore, one would expect it to decrease with design zone, but to a lesser extent because the applied force as well as resistance increases with zone. This effect is shown in Fig. 20. The average maximum ductility ratio generally decreases with zone, but not appreciably except for CSW-ll. The reasons for the difference between buildings is apparent in the force-resistance ratios computed in Table 5. For example, CSW-ll has the most rapid decrease in FR with zone.

Implications Regarding Damage. It is concluded in Reference 1 that the best indicator of total damage (structural and nonstructural) to a building is the interstory displacement. It is apparent from the above that increasing the code seismic coefficient (or zone) is not an effective way to reduce these displacements. Therefore, one would not expect a significant decrease in damage with design zone.

To the extent that floor accelerations cause damage, increasing the design zone may be detrimental, because acceleration always increases with zone.

On the other hand, increasing the seismic coefficient has the effect of increasing the elastic limit earthquake and decreasing

the maximum ductility ratio for larger earthquakes (although not greatly in all cases). This must improve the performance of the structure: First, it will remain elastic under a larger earthquake, and second, for an even larger earthquake, the structural damage will presumably be less, and third, a larger earthquake is probably required to cause collapse. The last two improvements mentioned are not significant in all cases. It should also be noted that increasing the zone implies, at least indirectly, a greater attention to construction details, both structural and nonstructural. This may be very effective in reducing damage.

Controlling the drift ratio in design has an effect on damage only as it is reflected in the natural period. The dynamic interstory displacements do not decrease in proportion to drift. The period is approximately prooortional to the square root of the drift and the displacements are determined by the response spectrum. For example, if the drift were halved the period would be reduced by 0.7. Referring to Fig. 7, if the period remained above 2.7 secs. there would be no decrease in displacement. If the structure is in the constant velocity region of the spectrum  $(T>2.7)$ , the displacement would decrease by 0.7, still less than the 0.5 decrease in drift.

Improved Design Procedures. It is apparent from all of the above that more effective (i.e., better design control over the response parameters affecting damage and resistance to collaose) seismic design could be achieved if two refinements were made in the procedure: (1) Use of an actual response spectrum such as

Fig. 7, and (2) a more exact computation of natural period. These improvements, which are certainly not an original proposal, would not significantly increase the design effort required. The response spectrum could be inserted in the Code. The fundamental period can be estimated with sufficient accuracy from the drift calculations which are normally made.

Using the correlations established in Section IV, the suggested improvements would enable the designer to directly control the average interstory displacement. Making use of a parameter similar to the force-resistance ratio (Table  $5)$ , he could also control the elastic limit earthquake and the maximum ductility ratio. These parameters are probably sufficient to provide a satisfactory structural performance and a limitations to damage much more directly and effectively than current code procedures.

## **VII.** SUMMARY AND CONCLUSIONS

The following observations and conclusions are drawn from the results of elastic and inelastic analyses of the twenty-five building designs. Because of the limited nature of this study, further verification of these conclusions is required. Furthermore, all buildings studied had relatively long periods (T<sub>1</sub>>l sec.), and some of the conclusions are not valid for buildings with shorter periods.

Regarding General Behavior

- **1.** To determine displacements in the elastic range only the fundamental mode need be considered.
- 2. For both elastic and inelastic response, the average peak interstory displacement is closely aoproximated by the elastic spectral displacement  $(\Gamma_1 S_1)$  for the first mode.
- 3. Floor accelerations are not easily predicted because they are influenced by higher modes.
- 4. The elastic limit earthquake and the average maximum ductility ratio for a larger earthquake both disolay an almost linear relationship with the ratio of bottom story resistance to first mode elastic base shear (or the inverse).
- 5. The distribution of story reistances with height may have a significant influence on resoonse.

## Regarding the Effectiveness of Code Design

- **1.** The increase in resistance provided by increasing the seismic coefficient (zone) only indirectly affects the natural period, the parameter which has the greatest influence on response.
- 2. Increasing the design zone results in only small decreases in interstory displacements. Zones 1, 2 and 3 provide very little improvement in this regard.
- 3. Increasing the design zone increases the elastic limit earthquake and slightly decreases the maximum ductility ratio for a larger earthquake.
- 4. Since most damage is related to interstory displacements, seismic design by current code procedures does not greatly reduce damage.
- 5. However, by increasing the elastic limit earthquake, code design improves the structure's performance in small earthquakes. By decreasing the maximum ductility ratio, it probably increases the earthquake required to cause collapse, but not always by a significant amount.

## Regarding Improved Design Procedures

The effectiveness of seismic design to limit damage and prevent collapse could be substantially improved over current procedures if  $(1)$  an actual response spectrum were used for analysis, and  $(2)$  if the fundamental period were computed more accurately. This would give the designer more direct control over the response oarameter which are related to damage.

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## APPENDIX A

## BUILDING PARAMETERS

This appendix contains tabulations of story resistances and/or stiffnesses for all buildings. In the case of shear wall buildings, the total moment and shear resistances for all· walls are given. For frame buildings, the total story shear resistance and stiffness for all frames are tabulated.



Table A.1 - Story Properties 17-Story Shear Wall Building (CSW-17)

BENDING RESISTANCE, KIP-FT<br>ULTIMATE SHEAR RESISTANCE, KIPS TOTAL<br>TOTAL  $M_{p} =$ <br> $R_{S} =$ 



 $K = STORY$  STIFFNESS, KIPS  $/FT$ <br> $R = TOTAL UITIMATE$  STORY RESISTANCE, KIPS

Table A.2 Story Properties 11-Story Steel Frame Building (SMRF-11)
Table A.3 Story Properties ll-Story Concrete Frame Building (CMRF-11)

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 $K = STORY$  STIFFNESS,  $KIPS/FT$ 

R= TOTAL ULTIMATE STORY RESISTANCE, KIPS



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Table A.4 - Story Properties 11-Story Shear Wall Building (CSW-11)



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Table A.5 - Story Properties 6-Story Concrete Frame Building (CMRF-6)

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## APPENDIX B

## TABULATED RESULTS

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This appendix contains the detailed results of all analyses. This includes the peak interstory displacement and the ductility ratio for each story and the peak floor accelerations.

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 $X = INTER - STORY$  DISPLACEMENT, FT  $x = FLOOR$  ACCELERATION,  $FT/sec^2$ 

Table B.1A - Elastic Analysis Results by Floor 17-Story Shear Wall Building (CSW-17) 0.1g Peak Ground Acceleration

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 $X = INTER - STQRY$  DISPLACEMENT, FT X = FLOOR ACCELERATION  $FT/sec^k$ 

Table B.1B - Inelastic Analysis Results by Floor 17-Story Shear Wall Building (CSW-17) 0.27g Peak Ground Acceleration

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Table B.2A - Elastic Analysis Results by Floor

11-Story Steel Building (SMRF-11)

0.1g Peak Ground Acceleration





Table B.2B - Inelastic Analysis Results by Floor

11-Story Steel Building (SMRF-11)

0.27g Peak Ground Acceleration



Table B.3A - Elastic Analysis Results by Floor

11-Story Concrete Frame Building (CMRF-11)

0.1g Peak Ground Acceleration





Table B.3B-Inelastic Analysis Results by Floor

11-Story Concrete Frame Building (CMRF-11)

0.27g Peak Ground Acceleration



Table B.4A - Elastic Analysis Results by Floor

11-Story Shear Wall Building (CSW-11)

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0.1g Peak Ground Acceleration





Table B.4B - Inelastic Analysis Results by Floor 11-Story Shear Wall Building (CSW-11) 0.27g Peak Ground Accleration

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Figure B.5A - Elastic Analysis Results by Floor 6-Story Concrete Frame Building (CMRF-6) 0.10g Peak Ground Acceleration





Figure B.5B - Inelastic Analysis Results by Floor 6-Story Concrete Frame Building (CMRF-6) 0.27g Peak Ground Acceleration



Table B.6 - Average Floor Ductilities from Inelastic Analysis of<br>Frame Buildings (.27g)  $\ddot{\phantom{a}}$ 

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