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Internal Study Report No. 56

DETERMINATION OF THE FAILURE PROBABILITY OF ONE OR MORE STRUCTURES LOCATED AT A SITE

by

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May 1975

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16. Abstract (Limit: 200 words)

Using the resistance curves developed for the failure of a single structure, this report also investigates the probability of 1 out of 100 structures failing. An attempt is also made to determine how well damage can be predicted when instead of a known resistance function, just the mean and variance of resistance are given. That is, a specific distribution is not known for the resistance of a structure. These various formulations are discussed. This report deals with only a two damage state situation, either failure or non-failure. Therefore, the results are only useful in predicting whether one of these two damage states will occur, not the probabilities of different levels of damage occurring. The main purpose of this report was to investigate the effects of a probabilistic resistance function, as opposed to a deterministic resistance, in predicting damage. The conclusion is that the probabilistic analysis lowers the overall risk in a manner that is shown in this report. When the resistance function is known, the results are quite useful and by inspection of the graphs and tables in this report the magnitude of the effect of the probabilistic approach can be seen. The conclusion drawn from the calculations when only the mean and variance of the resistance are known is that without any assumption about the resistance distribution, one can do no better than to establish a broad range for the probability of failure.

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INTRODUCTION

The basic problem can be stated as follows: What is the probability of "failure" of a structure, given the probability that the "design" acceleration is exceeded? In order to answer this question, one needs two relationships, one of which relates probability of earthquake occurrence to acceleration, while the other relates probability of failure to the occurrence of a given acceleration. Combining these two relationships, we can predict damage probability over a given length of time.

Annual Damage Probability (ADP) will be defined as follows*:

$$ADP = \int_a P_a \times SR_a \, da \qquad (1)$$

where:

P_a = Probability that the structure will have damage at an acceleration a.

SR_a = Annual probability of occurrence of an earthquake with acceleration between a and a+da at a site.

For the first relationship (probability of occurrence vs. acceleration) the assumed form will be as follows:

$$p = ca^{-k}$$

where:

p = Probability that site acceleration a is exceeded.

a = acceleration

c = constant

k = slope of line on log-log plot of p vs. a

(k and c vary with different geographical regions)

If the mean rate of (significant) earthquakes is ν per year, then the annual mean rate of events with site acceleration greater than a is

$$\lambda_a = \nu p = \alpha a^{-k}$$

where:

$$\alpha = \nu c$$

*Reference 1, page 57.

The plot of λ_a vs. a is a straight line on a log-log graph (Fig. 1) with slope K.

For the other relationship mentioned, probability of failure (dependent on building resistance) vs. acceleration, we will assume curves of the general form as shown in Figure 2. The exact nature of these curves and the assumptions involved in constructing these curves will be discussed later. At this point, it is necessary, however, to define an acceleration a_1, below which there is zero probability of failure, and an acceleration na_1, above which there is certainty of failure. "Failure" in this context does not necessarily mean actual collapse of a structure. Failure can be other values of damage, say 10% of the replacement value of a structure. Failure could relate the structure's capacity to remain operable, that is, failure could be the damage at which the building could not remain open to the public. It can be said that a structure can exist in one of two states, that is, either fail or non-fail. This means if "failure" is established as 10% damage to the structure, a structure with 9% damage would not be considered as a failure.

In addition to this consideration of failure of one structure, this report also investigates the probability of n out of 100 structures failing. The only new aspect here is the construction of new resistance curves for n failures using the resistance curves developed for the failure of a single structure. Also, there will be an attempt to determine how well one can predict damage when instead of a known resistance function, just the mean and variance of resistance are given. That is, a specific distribution is not known for the resistance of a structure. These various formulations will all be discussed further on in this report.

2. RESISTANCE FUNCTIONS

The general form of the assumed resistance function is shown in Figure 2. This form was derived after looking at some historical data from past earthquakes. In the appendix, there is a list of M.I.T. Damage Probability Matrices*. These were derived from historical earthquake data. Plotting the damage probabilities against acceleration would give an approximate resistance function for different design requirements, represented here by the different zones of the Uniform Building Code (UBC). A problem exists in the fact that these matrices are in terms of damage vs. intensity, whereas we are concerned with damage vs. acceleration. This tends to "lump" damage into a few discrete intensity values instead of spreading them out along the acceleration axis as we would prefer.

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*Reference 1, Table 5.4.

The conversion law,

$$\log a = 1/3\ I - 1/2^{*}$$

was used in this case. With all these approximations, the accuracy of the curve will be limited but we are only trying to obtain a generalization of the shape of these functions and with this in mind, our approximations seem to be within reason.

Figure 3 shows a plot of some of these curves obtained in this manner. There are three different main shapes to these curves, and these shapes of the function seem to depend on the level of damage involved. The curves for light damage are of type A(i.e., of the general form of curves 1 and 2), whereas the curves for moderate damage are of type B, and for damage functions of heavy or worse damage, the general shape is that of type C. It is believed that the difference in shapes is due to assumptions about the end points. If instead of assuming that there are definite end points to the functions, it is assumed that there is never zero probability or, likewise, never certainty of failure. The functions would all tend to be of the same S-shape (i.e., type B).

Figure 4 replots the curves of Figure 3 with these new end point assumptions. It is seen that all the curves are of the same S-shape variety. However, for this report, finite end points for the resistance functions will be assumed, that is, there will be definite cut-off points of zero probability and certainty. The mathematical and physical consequences of this assumption will be discussed later.

In this report, the concern will be with the resistance functions of the heavy damage states (type C curves, Fig. 3). The structural engineer is not as much concerned with the lighter damages, since they may be due entirely to nonstructural damage. To represent these type C curves, two different curves will be assumed: (1) a parabolic function and (2) a cubic function for use in this report. The parabolic function will be the primary basis for comparison between probabilistic and deterministic resistance functions. The cubic function will give an idea of the sensitivity of the final result to the details of these curves; that is, how the damage probabilities will be affected due to a relatively slight shift in the resistance curve.

It is necessary here to define a new acceleration parameter, x, where

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* Reference 2.

$x = (a/a_1)$. The use of this term will help to generalize the problem for different situations. There are also definite mathematical advantages in using (x) as the acceleration parameter when expressing resistance functions or calculating damage probabilities as will be seen later.

Now, definite mathematical expressions for the resistance functions will be assumed. In particular, for the parabolic case the resistance will be assumed to be of the following form:

$$F_x(x) = 0 \qquad x < 1$$

$$F_x(x) = \frac{(x-1)^2}{(n-1)^2} \qquad 1 \le x \le n$$

$$F_x(x) = 1 \qquad x > n$$

These three curves establish the resistance function for the parabolic law assumption. The cubic law could be similarly expressed.

3. CALCULATION OF ADP

3.1 One Structure: $F_A(a)$ Known

With the resistance function known, the calculation of ADP is possible through direct integration. The probability of occurrence vs. acceleration curve will be generalized by using as the acceleration parameter $\log(a/a_1)$ instead of $\log(a)$, similar to the change in acceleration parameters for the resistance functions. Therefore we now have the following expression for probability of occurrence:

$$\lambda = \alpha\ (a/a_1)^{-k}$$

The value of $\lambda=1$ will be set a $a=a_1$, which makes the value of $\alpha=1$. Obviously, the probability of occurrence for any given value of acceleration, say ca_1, would be equal to $\lambda_c=(ca_1/a_1)^{-k} = c^{-k}$, on the normalized scale. If the true value of λ is known when $a=a_1$ (call this λ_1), then the actual probability of occurrence at any acceleration is given by

$$P_c = \lambda_c \lambda_1$$

where:

P_c = Probability of occurrence of ground acceleration ca_1.

λ_c = normalized probability of ground acceleration ca_1.

λ_1 = probability of occurrence of ground acceleration a_1.

It is obvious that using a normalized function will not alter our results in any way; as the multiplication by a known value of λ_1 will produce the actual probability of occurrence.

The actual calculation of the ADP by integration is shown in the appendix but an attempt will be made here to show the formulation of the problem. Figure 5 shows the two graphs involved in the integrand. The top graph (log λ vs. log a) is a complementary cumulative distribution function. The desired probability density function (probability of occurrence at acceleration a) is equal to the negative derivation of the complementary cumulative distribution function. Using the definition of ADP previously given

$$\text{ADP} = \int_a F_A(a) \ \frac{-d\lambda}{da} \ da$$

where $F_A(a)$ is our resistance function (bottom curve in Figure 5). The evaluation of this integral is shown in the appendix for a particular case. Note that in this calculation the acceleration parameter is shifted from (a) to (a/a_1). Also note the different contributions to the final figure of 0.38 for ADP/λ_1; of the 0.38, 0.25 (i.e. 66%) comes from the part of the curve for $(a/a_1) > n$. As the slope of K increases and the value of n increases this contribution decreases. For example when K=10 and n=8, the value of $\text{ADP}/\lambda_1 = 0.00057$ (see Table 1 for values of ADP/ $_1$ for various values of K and n). However, the contribution to ADP/λ_1 for $(a/a_1) > n$ is 9.3×10^{-10} (i.e. 0.0001 % of the total ADP). The reason for this is mathematically obvious. The values of λ rapidly decrease for high values of K as well as for values of acceleration greater than a_1. Therefore, the contribution to ADP/λ_1, at accelerations much greater than a_1 will be very small.

The knowledge of where the contributions to ADP/λ_1 are coming from is useful in deciding on a resistance curve. For low values of n and K, the actual curve is not that important. (Compare values for parabolic and cubic functions in Table 1.) However, for high values of K and n, the curve selection becomes critical, and the order of magnitude of your result can be changed by only a slight change in the resistance function. The tail of the resistance curve can now become extremely important and a little change in the tail of the resistance function will be magnified in the probability of failure. The choosing of endpoints, especially a_1, could also become quite critical.

This sensitivity also points out the disadvantages of a deterministic resistance function (see Fig. 6). The deterministic model has too sudden a shift in resistance. As an illustration, take a deterministic model with the shift from 0 to 1 of the resistance function at the mean, which is about the best approximation possible. The value of $\frac{ADP}{\lambda_1}$

$$ADP/\lambda_1 = K \int_m^\infty x^{-k-1}\, dx = m^{-k}$$

where m = mean. For the values of K=2 and n=2, the value of ADP/λ_1 is 0.359 which is a fair estimation of the previously calculated value of 0.38. However, for the case of n=8 and k=10, the resulting $ADP/\lambda_1 = 2.9 \times 10^{-8}$ which is in very poor agreement with the calculated value of 5.7×10^{-4} (Table 1).

Before proceeding any further, it may be of value to illustrate the physical meaning of the values in Table 1, as these values of ADP/λ_1 are used throughout the report. For example, if k=2 and n=2, what does $ADP/\lambda_1 = 0.38$ imply? There are several ways to interpret this result. First, if the value of λ_1 (probability of occurrence of an earthquake with acceleration a_1 or greater) is known, then the annual probability of failure of a structure in this case is $0.38\lambda_1$. Also, if a deterministic model with the damage threshold at $a = a_1$ had been assumed, the probability of failure would be λ_1. So another interpretation of the probabilistic damage threshold is that the probability of failure is 38% of the probability of failure for a deterministic model with a damage threshold of a_1. This reasoning can be extended to all values of k and n and a "feel" for the variation of annual damage probability with changes of k and n may be developed.

It was mentioned previously that definite end points have been assumed for the resistance functions. Now, it would be beneficial to illustrate the mathematical and physical implications of these assumptions. Figure 7 shows an identical resistance function to the ones previously used, except that now no point of zero probability is assumed. A constant value of c for a/a_1 values between 0 and 1 is assumed instead. This will establish a discontinuity at x = 1, but this will not affect the integration. The c value is assumed to be very small. As an example the particular case of n = 2 and k = 2 is used. The total value for ADP/λ_1 is now equal to 0.38 plus any additional contribution for values between $a/a_1 = 0$ and $a/a_1 = 1$. The calculation of ADP/λ_1 would be as follows:

$$ADP/\lambda_1 = 0.38 + k\int_0^1 c(x)^{-k-1} \, dx = cx^{-k}\Big]_1^0 = 0.38 + \infty - c = \infty$$

Obviously there is not an infinite probability and the reason is illustrated in Figure 8. The probability vs. acceleration graph is not always a line of constant slope. As the acceleration approaches zero, the slope of the line decreases. There is not an infinite probability of occurrence for low accelerations. However, it is still possible that small probabilities of failure at low accelerations will make a notable contribution to the ADP/λ_1 value, especially for high values of k and n. The following question arises: should failure probabilities for accelerations below a_1 be included?

To aid in the decision, take the failure probabilities at accelerations below a_1, and then consider the physical meaning of the contribution of these failure probabilities to ADP/λ_1. The value a_1 was originally meant to be a design acceleration. It may be reasoned that if we design all buildings so as to prevent failure for all accelerations less than a_1, there is zero probability of failure. However, due to poor design, poor materials, etc., it is possible that some building or buildings will fail at an acceleration less than than for which they were designed. In fact, it may even be possible that there are buildings that will eventually collapse, without any ground acceleration. But if there was an earthquake, the building's failure would probably be attributed to ground shaking. What to do in this situation is not obvious. Since the concern is with the straight line portion of the occurrence vs. acceleration plot, and since there is some question about accounting for "weak" buildings, the assumption of a definite endpoint at acceleration a_1 will be used in this report. Of course, a_1 can be as low a value of acceleration as necessary, but it must be remembered to keep the tail of the resistance curve small at low values of acceleration.

One final point with regard to the problem of one failure with $F_A(a)$ known is to notice a relationship between ADP/λ_1 and the parameter K ln n. This is illustrated in Figure 9. Although the relationship is not an exact one, by use of this graph a reasonably approximate value of ADP/λ_1 for any combination of k and n can be obtained. The graph is only good for one resistance function and in Figure 9 the assumed resistance function is $F_X(x) = \frac{(x-1)^2}{(n-1)^2}$. The graph is also a convenient way of seeing how rapidly the values of ADP/λ_1 fall off with increasing acceleration.

3.2 100 Structures: Resistance Function of Single Structures Known

Let us now assume the following problem. What is the probability of m or more of 100 structures failing, given that the resistance function for the individual structures is known? (Assume that all structures are statistically independent, but have the same resistance function.) The calculation of ADP/λ_1 is identical to the previous case, except that a new resistance function for n out of 100 buildings failing must be found, as that for single failures used in the previous section is not applicable here. It will not be assumed that the probability of m out of 100 structures failing follows a binomial distribution, that is,

$$P \text{ [that n of m will fail] } = \frac{m!}{n!(m-n)!} \rho^n (1-\rho)^{m-n}$$

where ρ = probability of one failure. Assuming different values of ρ one can get the probability of m or more of 100 buildings failing. The probability of n or more out of 100 buildings failing vs. probability of single failures may then be plotted (Fig. 10).

The resistance curve for one building must now be combined with the probability of n out of 100 structures failing. One can do this as follows. The parabolic resistance function,

$$\rho = \frac{(a/a_1 - 1)^2}{(n - 1)^2}$$

where ρ = probability of a single structure failing. For n=2 we get

$$\rho = (a/a_1 - 1)^2$$

and

$$a/a_1 = \sqrt{\rho} + 1 .$$

Therefore, starting with a value for ρ the probability that n out of 100 structures will fail can be calculated as well as the acceleration at which this probability occurs. In this way, one can construct a resistance curve for m out of 100 structures vs. acceleration. This was done for n_f=1 and n_f=5 in Figure 11. (To avoid confusion with the n or the previous section, n_f will be used, meaning

$n_{failures}$.) The integration procedure is identical to that in the previous section except that now there is not an exact mathematical formula for the resistance function and numerical integration must be used to calculate ADP. The results for these calculations for various values of n and k are shown in Table 2. These results can be compared with their counterparts in Table 1 to relate the risk of multiple failures out of 100 structures to the risk of single structures failing.

In Figure 12, ADP/λ_1 for the parabolic resistance vs. k ln n' is shown, where n' is not the n-value of the resistance function for single structures failing, but rather the new n value for the particular case of multiple failures under consideration, that is, the value of (a/a_1) at which the resistance reaches a probability of one. (See Figures 11 and 12). Again this graph (Fig. 12) is useful in obtaining an approximate value of ADP/λ_1 for multiple failures at various values of k and n and it shows the drop-off in ADP/λ_1 with increasing acceleration.

3.3 One Structure: Mean Resistance and Variance Known

Now let us assume that the mean resistance and variance of a structure are known and that the annual damage probability is desired. But with only the mean and variance of the resistance known a distribution of the resistance function can not be constructed. Therefore, only maximum and minimum limit curves of resistance can be drawn. This can be done using the Tchebycheff inequalities which will bound the resistance function.

The Tchebycheff inequality used for finding the maximum resistance is as follows:

$$P(|\beta - m| \geq k\sigma) \leq \frac{1}{k^2}$$

where β = random variable

m = mean

σ = standard deviation

k = constant

P = maximum probability.

A value of $k < 1$ leads to a probability greater than one, but, it will be assumed that for these cases, P=1. If we further assume that one half of the resistance is on either side of the mean, then,

$$P_{max} = \frac{1}{2k^2} \qquad \text{for } k > 1$$

$$P_{max} = \frac{1}{2} \qquad \text{for } k < 1$$

Using these equations we can construct a maximum resistance curve (Figure 13). Starting at the mean, k=0, and progressing to the left, that is, towards smaller accelerations, one can see that between k=0 and k=1 the maximum probability is one half. For values of k>1, the resistance follows the equation $P=1/2k^2$. On the right side of the mean (large accelerations) for all values of k, the maximum probability of failure is equal to one-half plus the value of P at k = 0, which is also equal to one-half. Therefore, the maximum probability of failure for acceleration greater than the mean is unity.

Similarly, the minimum resistance curve (Fig. 13) represents the minimum probability of failure that will insure the variance to remain at its known fixed value at any distance from the mean (i.e., at any acceleration). This curve could be constructed in a similar manner as the maximum resistance curve, but it may be easier to visualize that it would simply be the inverted mirror image (taken at the mean) of the maximum probability curve. Then, to the right of the mean (large accelerations),

$$P_{min} = 1 - \left(\frac{1}{2k^2}\right) \qquad k > 1$$

$$P_{min} = 1 - 1/2 \qquad k < 1$$

For values of accelerations to the left of the mean the P_{min} would equal 1-1/2-1/2=0.

Note, however, that these curves are continuous functions with no assumed end points. For our particular case the probability of failure below acceleration a_1 has been assumed equal to zero. Therefore, we have to cut our curves at a_1. The actual construction of these new curves (Figure 14) is not that important. The point to remember is that there is a slight change in the curves due to the assumption of a fixed end point, instead of extending the functions to infinity.

It must be remembered, however, that these resistance curves are limits and not actual resistance functions. The nature of these curves again makes it very difficult to integrate directly and therefore they are integrated numerically. The results of these integrations are given in Table 3. It is apparent from this table that the Tchebycheff limits are quite broad. The means and variances used in Table 3 are the means and variances of the parabolic and cubic functions in Section 3.1. Thus, it can be seen how the calculated Tchebycheff limits surround the actual resistance function of one of the previously assumed shapes. This gives an indication of where the previously assumed function fits into the range of distributions.

Figure 15 graphs the values of $\frac{\text{Probability of Failure, Max}}{\text{Probability of Failure, Min}}$ vs. the parameter k ln n, using the values in Table 3. The curve is quite steep and it can be seen that the quantities p_f max and p_f min diverge quite rapidly and that they are only of the same order of magnitude for values of k ln n $\lesssim$ 5. It is clear that without assuming a specific equation for the resistance distribution function, the only alternative is a broad range for the probability of failure. In the case of large values of k and n, this broad range becomes so wide, that its value is questionable.

3.4 100 Structures, Mean and Variance Known For Single Structures

As a final problem, consider the probability of failure of n out of 100 structures, with the mean and variance of the resistance of single structures known (again, it is assumed that the structures are independent but with the same mean and variance of resistance). The procedure for the calculation of a new minimum and maximum resistance function is identical to that in the previous sections. Using the maximum and minimum curves from Section 3.3, one can proceed exactly as in Section 3.2 to derive the minimum and maximum probability of failure curves for n_f out of 100 buildings failing. That is, the binomial distribution may be used to relate the maximum and minimum probability of n of 100 buildings failing to the maximum and minimum probability of single failures. Next, Figure 11 is used to find at what acceleration the particular probability of failure for a single structure occurs. Then the maximum and minimum probability of failure for a given acceleration is found. Finally, the construction of a graph of minimum and maximum probabilities of failure for n_f out of 100 structures failing vs. acceleration (Fig. 16) is possible. Calculations for ADP/λ_1 using these curves are in Table 4.

Figure 17 then graphs $\frac{P_{f\ max}}{P_{f\ min}}$ vs. acceleration for these values (Table 4) similar to what was done in the previous section. Again, the range of the probability of failure is great and except for low values of k and n, not much can be said about the probability of failure without assuming a distribution.

4. CONCLUSIONS

When drawing conclusions, all assumptions made must be remembered. This report deals with only a two damage state situation, that is, either failure or non-failure. Therefore, the results are only useful in predicting whether one of these two damage states will occur, not the probabilities of different levels of damage occurring. The main purpose of this report was to investigate the effects of a probabilistic resistance function, as opposed to a deterministic resistance, in predicting damage. The conclusion is that the probabilistic analysis lowers the overall risk in a manner that has been shown previously in this report. Recall for comparison that for a deterministic model at $a=a_1$, the ADP/λ_1 is always equal to one. When the resistance function is known, the results are quite useful and by inspection of the graphs and tables in this report the magnitude of the effect of the probabilistic approach can be seen.

The conclusion drawn from the calculations when only the mean and variance of the resistance are known is that without any assumption about the resistance distribution, one can do no better than to establish a broad range for the probability of failure. This range would be of use only in a very approximate calculation. It would be better to assume a distribution similar to the previously assumed distribution shapes for resistance, rather than to establish the mathematically broadest possible ranges for resistance. One should be able to narrow down the limits of the resistance function by having a general idea of the shape of actual resistance functions. The Tchebycheff limits approach covers the entire spectrum of distributions and one should be able to narrow the range of distributions covered. One approach would be to simply assume a distribution with a known mean and variance (as in Sections 3.1 and 3.2) and then one could calculate maximum and minimum probability of failure curves using more practical limits than Tchebycheff. The calculation of these minimum and maximum curves would have to be from experience. These would then be practical rather than mathematical limits. This would establish a range for ADP that would presumably cover all practical distributions.

In summary, it appears that the two-state damage approach works quite well and produces worthwhile results for the cases when the resistance function is known. However, when only the mean and variance of the resistance is known and no distribution is assumed, the results are not satisfactory and it seems it would be much better to assume a distribution based on previous knowledge about the shape of the resistance function distribution.

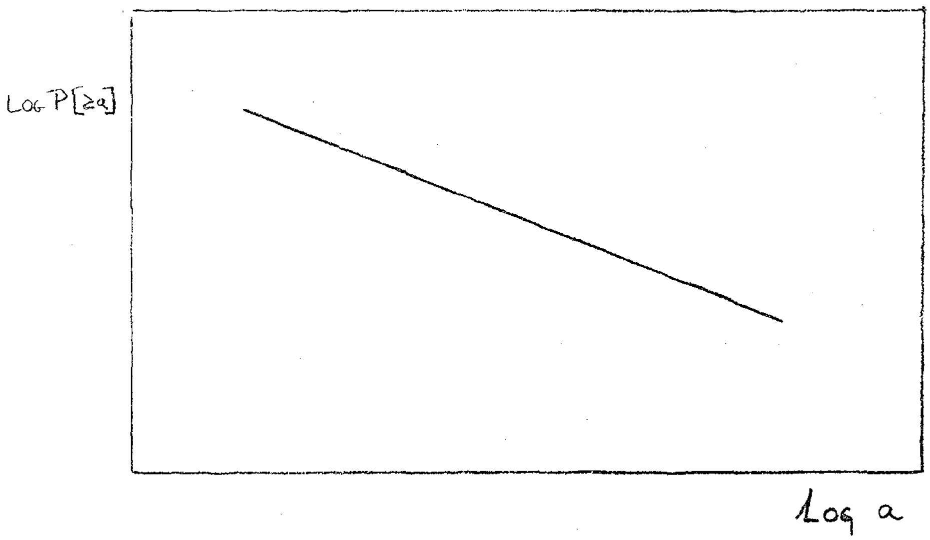

FIGURE 1 - ASSUMED FORM OF LOG λ_a VS. LOG a GRAPH

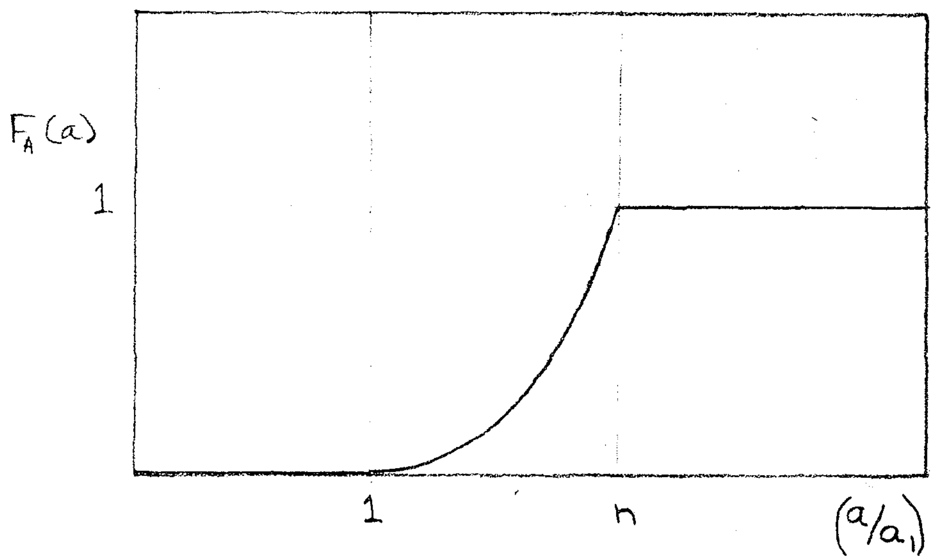

FIGURE 2 - TYPICAL RESISTANCE FUNCTION FOR A STRUCTURE

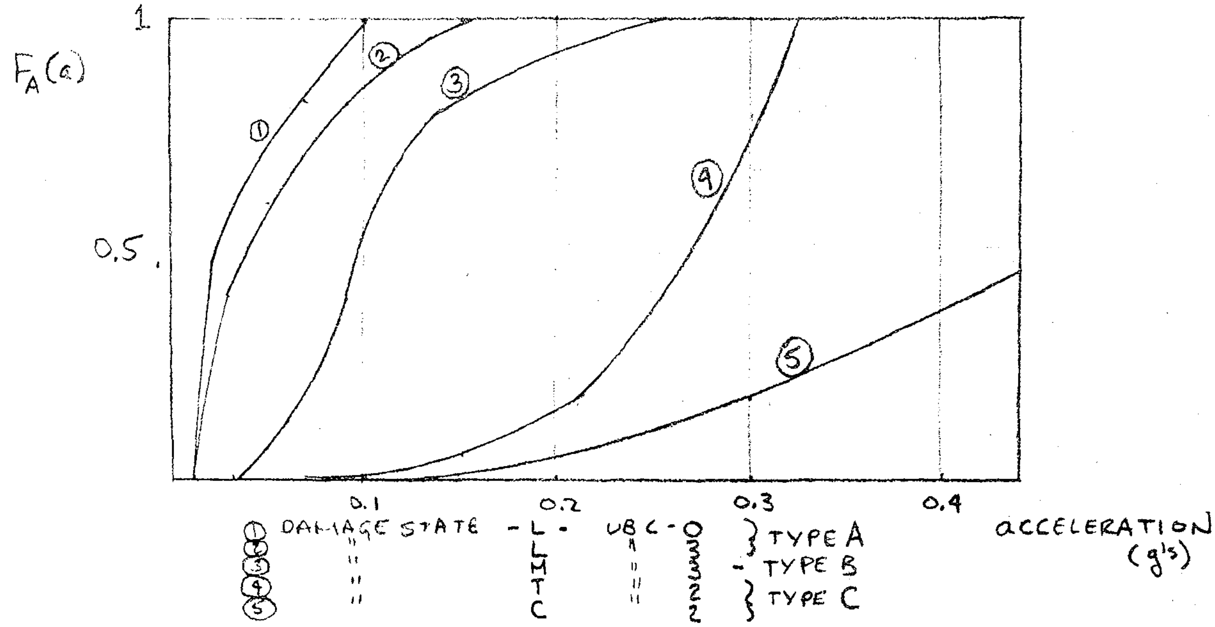

FIGURE 3 - PLOTS OF RESISTANCE VS. ACCELERATION FROM HISTORICAL DATA

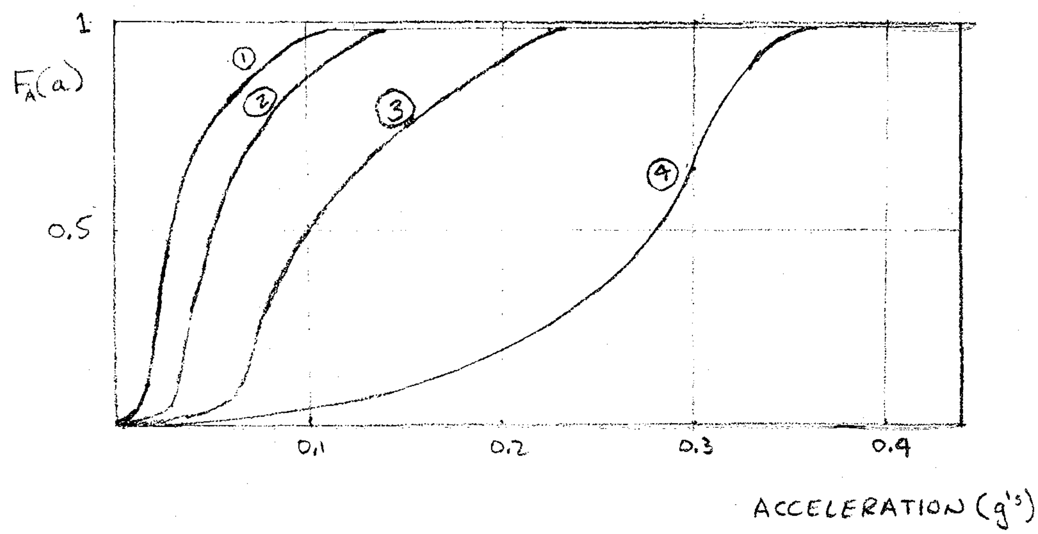

FIGURE 4 - PLOTS OF RESISTANCE VS. ACCELERATION

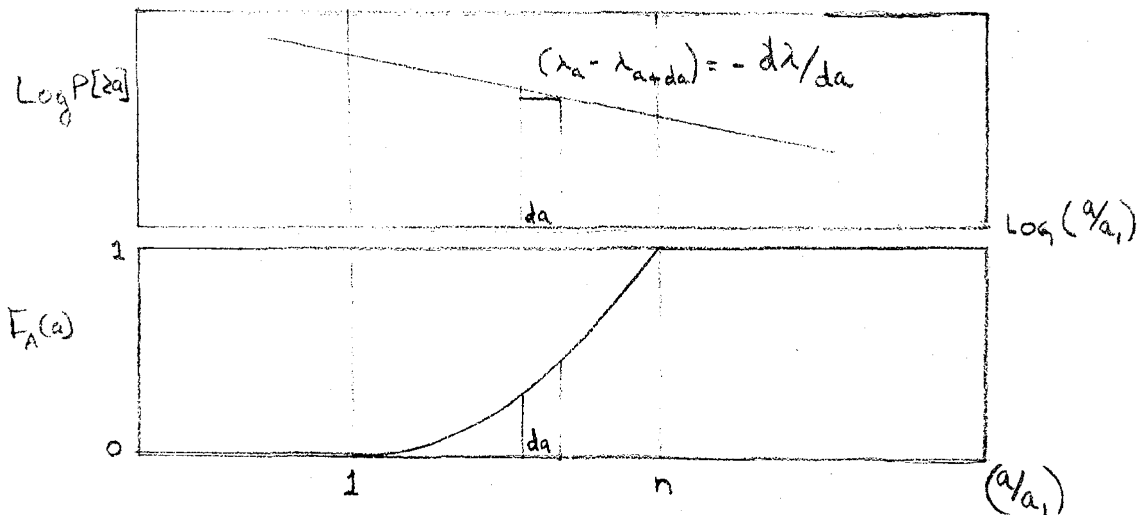

FIGURE 5 - RELATION BETWEEN GRAPHS OF PROBABILITY OF OCCURRENCE AND BUILDING RESISTANCE

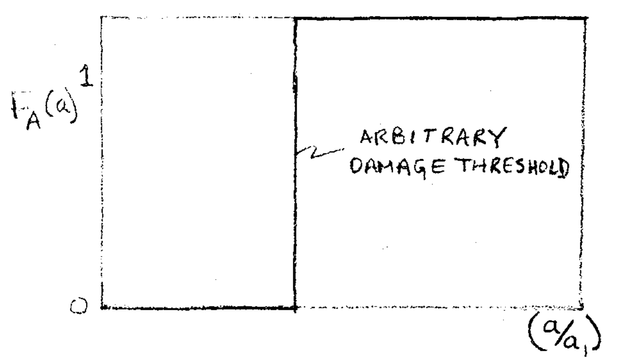

FIGURE 6 - DETERMINISTIC RESISTANCE FUNCTION

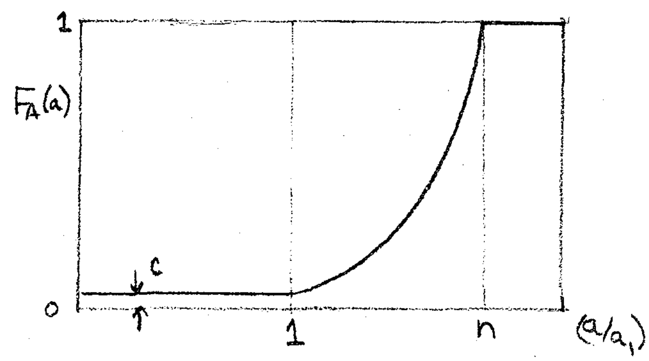

FIGURE 7 - RESISTANCE FUNCTION WITHOUT DEFINITE ENDPOINT AT ACCELERATION a_1

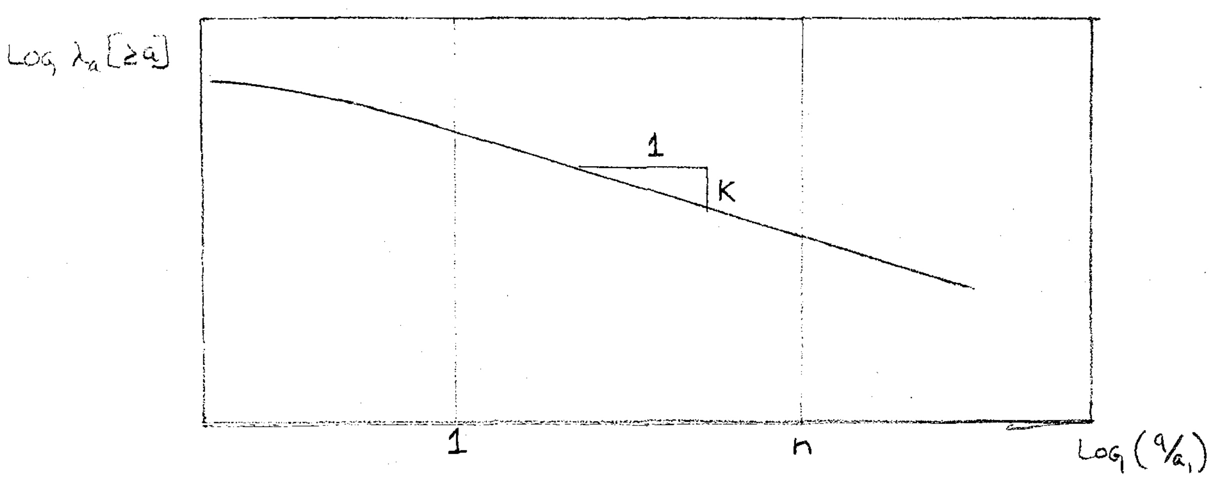

FIGURE 8 - GRAPH OF LOG λ_a VS LOG (a/a_1)

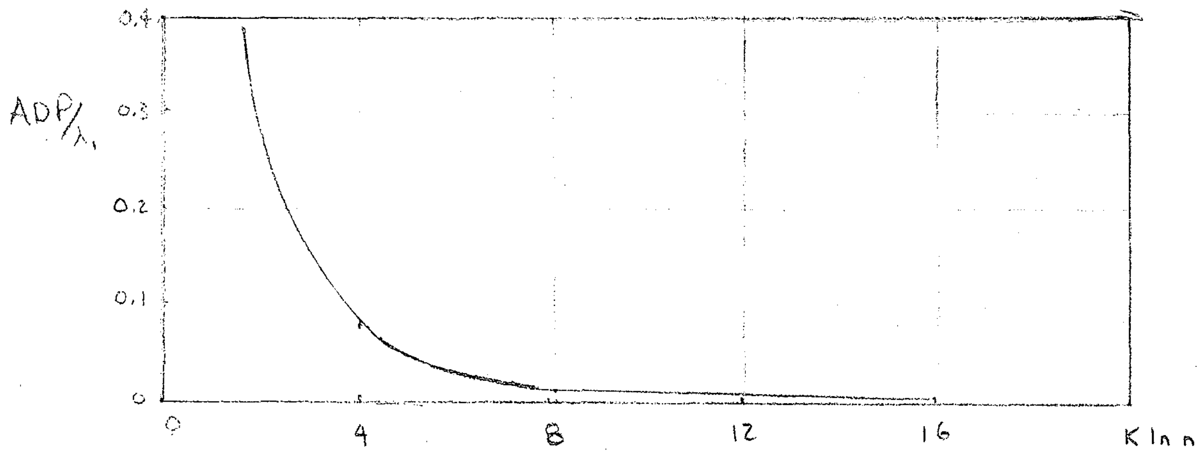

FIGURE 9 - GRAPH OF ADP/λ_1 VS. $k \ln n$ WITH $F_A(a) = (x-1)^2/(n-1)^2$ FOR THE PROBABILITY OF ONE OUT OF ONE STRUCTURES FAILING

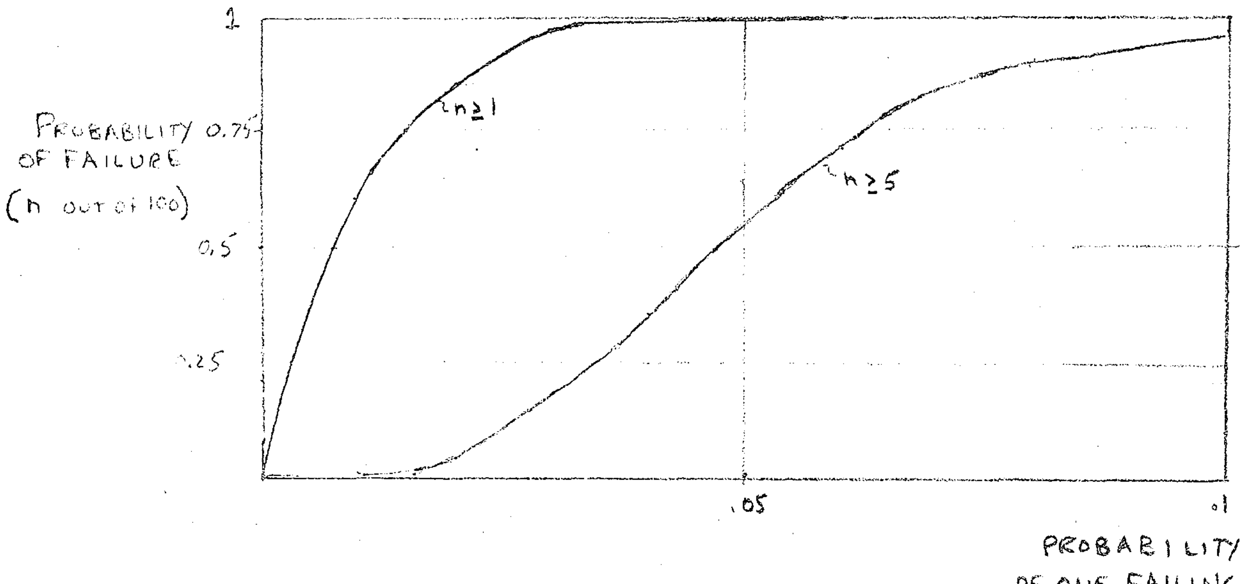

FIGURE 10 - PROBABILITY OF n OUT OF 100 FAILURES VS PROBABILITY OF SINGLE FAILURE

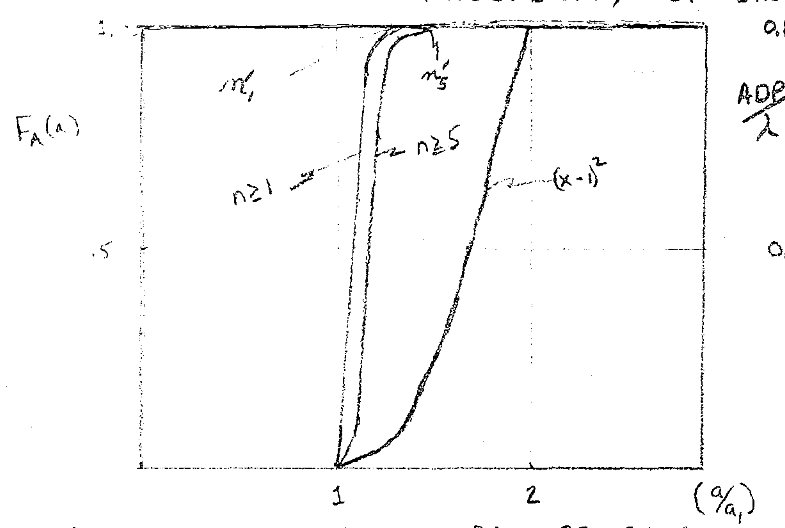

FIGURE 11 - CUMULATIVE FAILURE PROBABILITY FUNCTION FOR n OUT OF 100 BUILDINGS

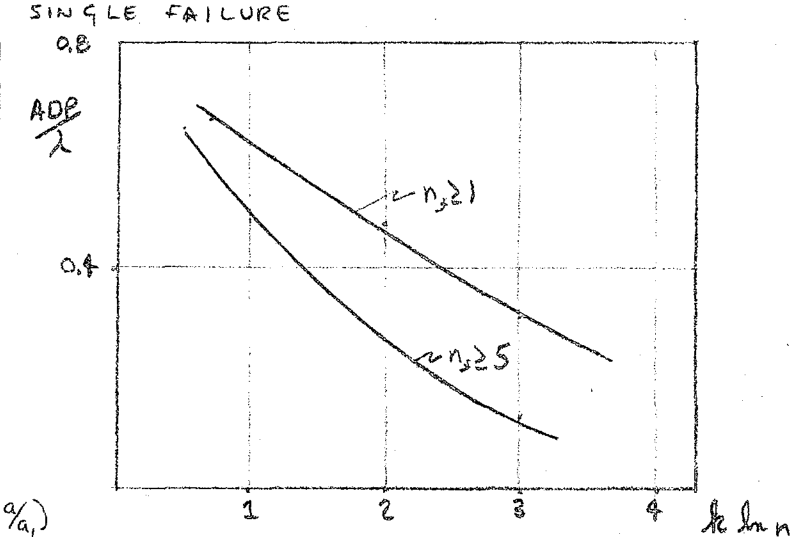

FIGURE 12 - PROBABILITY OF FAILURE FOR ($n_s \geq 1,5$) VS. $K \ln n$

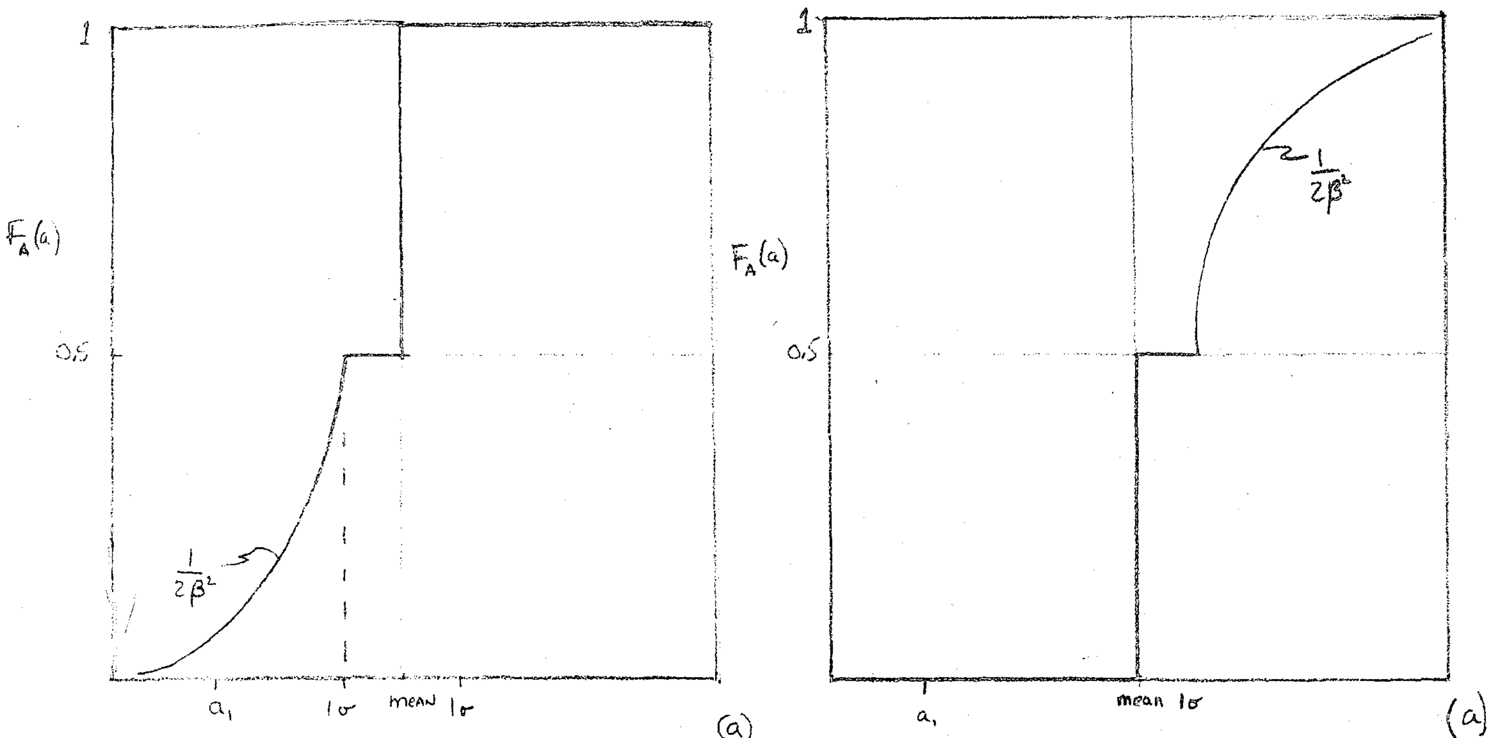

FIGURE 13 - MAXIMUM AND MINIMUM RESISTANCE FUNCTIONS FOR ONE STRUCTURE ASSUMING NO END POINTS

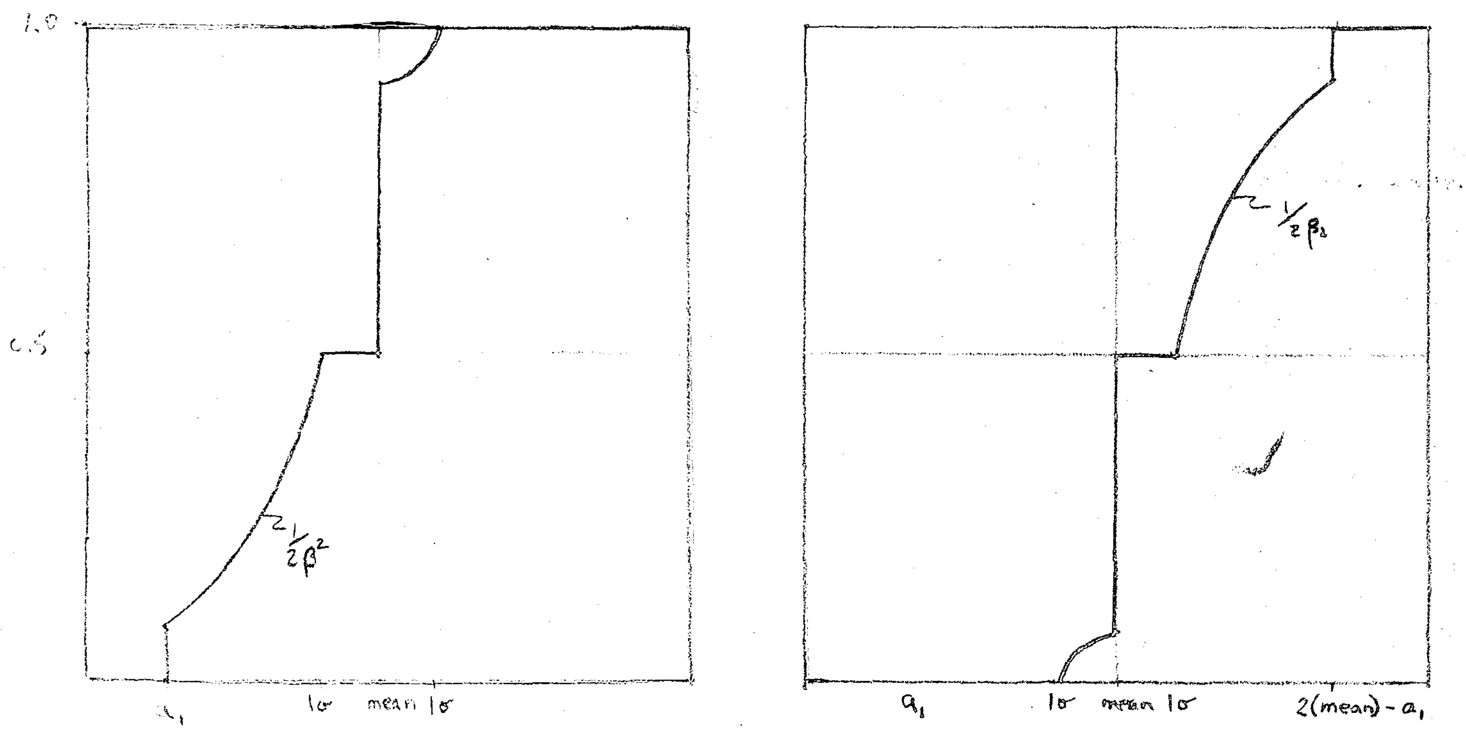

FIGURE 19 - MAXIMUM AND MINIMUM RESISTANCE FUNCTIONS FOR ONE STRUCTURE ASSUMING MINIMUM DAMAGING ACCELERATION, a_1

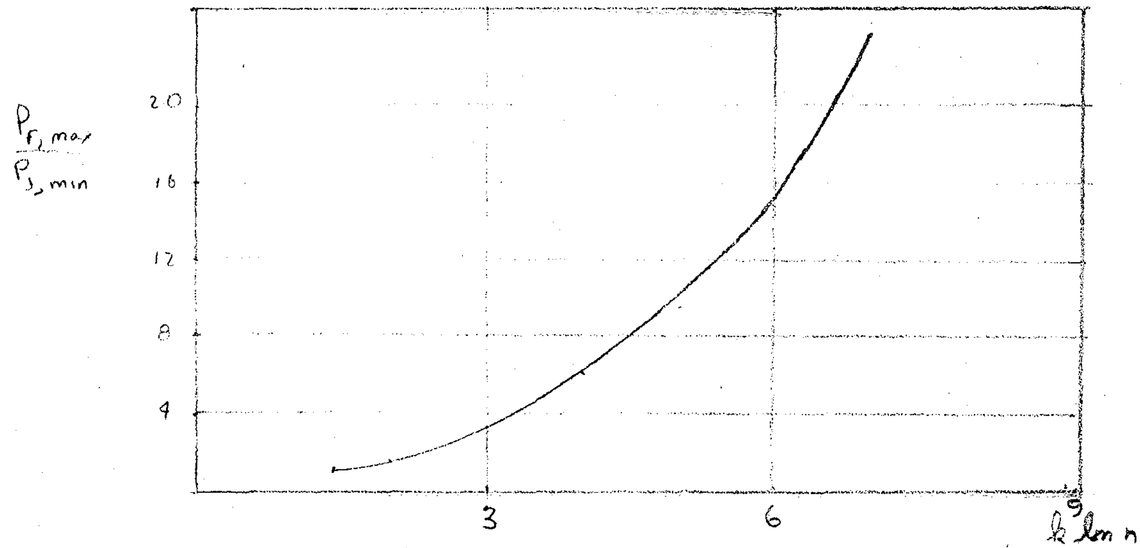

FIGURE 15 - GRAPH OF $\left(\frac{P_{f,max}}{P_{f,min}}\right)$ VS. K ln n FOR A SINGLE STRUCTURE.

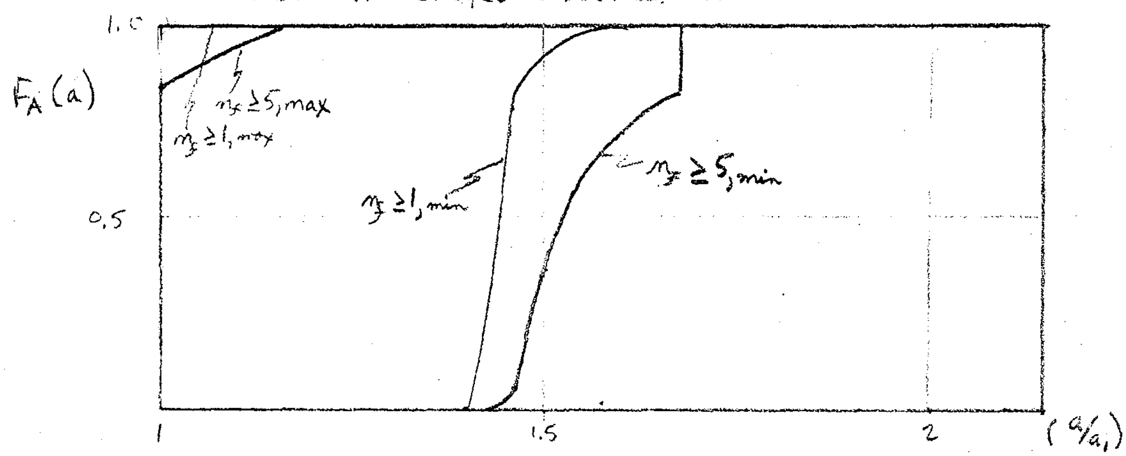

FIGURE 16 - GRAPH OF MINIMUM AND MAXIMUM RESISTANCES VS. ACCELERATION FOR ONE AND 5 FAILURES OUT OF 100 STRUCTURES ($F_A(a)$ FOR SINGLE STRUCTURE = $(x-1)^2$

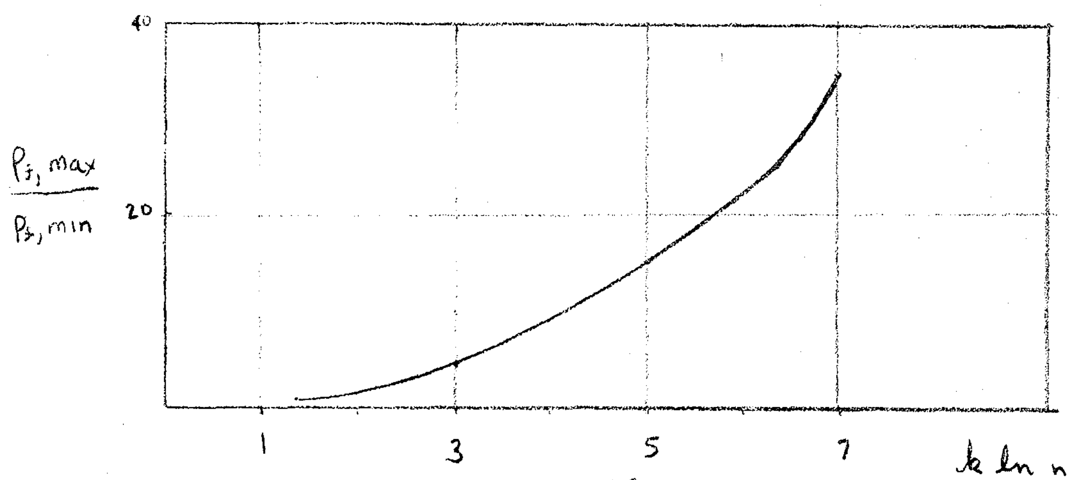

FIGURE 17 - GRAPH OF $\left(\frac{P_{max}}{P_{min}}\right)$ VS. K ln n FOR ONE OR MORE FAILURES OUT OF 100 STRUCTURES ($F_A(a) = (x-1)^2$ FOR ONE STRUCTURE)

TABLE 1. Values of ADP/λ_1 for Different n, k, and $F_A(a)$ Values

n \ k	2	3	4	5	7	10
2	0.386	0.250	0.167	0.115	0.059	0.027
3	0.216	0.111	0.062	0.037	0.016	0.0070
4	0.141	0.063	0.031	0.018	0.0074	0.0031
6	0.077	0.028	0.012	0.0066	0.0027	0.0011
8	0.049	0.015	0.0065	0.0034	0.0014	0.00057

$$F_A(a) = \begin{cases} 0 & x < 1 \\ (x-1)^3/(n-1)^3 & 1 < x < n \\ 1 & x > n \end{cases}$$

n \ k	2	3	4	5	7	10
2	0.341	0.204	0.125	0.078	0.033	0.0099
3	0.176	0.078	0.037	0.019	0.0056	0.0015
4	0.108	0.039	0.016	0.0068	0.0016	0.00044
6	0.054	0.0147	0.0046	0.0017	0.00040	0.000095
8	0.0325	0.0072	0.0020	0.00067	0.00015	0.000035

TABLE 2. Values of ADP/λ_1 for n_f or More out of 100 Buildings for Different n, k, and $F_A(a)$ Values

			K		
n	$F_A(a)$	$n_f \geq$	2	5	10
2	$(x-1)^2$	1	0.8439	0.6696	0.4725
		5	0.6811	0.3867	0.1551
	$(x-1)^3$	1	0.7262	0.4747	0.2169
		5	0.5446	0.2209	0.0502
3	$\frac{(x-1)^2}{4}$	1	0.7569	0.4990	0.2258
		5	0.4910	0.1761	0.0350

TABLE 3. Values of $P_{f\,max}$, $P_{f\,min}$ for n=2 and Various Values of k, $F_A(a)$, Mean and Variance

	K			
$F_A(a)$, mean, variance	2	5	7	10
$\sigma=0.25$ $F_A(a)=(x-1)^2$, $\bar{x}=1.67$	$P_{f\,min}=0.3176$ $P_{f\,max}=0.5463$	0.0480 0.2119	0.0150 0.1510	0.0063 0.1420
$(x-1)^3=F_A(a)$ $\bar{x}=1.75$ $\sigma=0.19$	$P_{f\,min}=0.2868$ $P_{f\,max}=0.4375$			0.0019 0.0599

TABLE 4. Values of p_{max} , p_{min} for n_f or more for 100

Structures Failing

$$F_A(a) = (x-1)^2 \qquad n=2$$

$$\bar{x} = 1.67 \qquad \sigma=0.25$$

k / $n_f \geq$	2	5	10
1	p_{min}=0.4818	0.1533	0.0267
	p_{max}=1.0	1.0	1.0
5	p_{min}=0.4117	0.0204	
	p_{max}=0.9795	0.9235	

APPENDIX

$$ADP = \int_a F_A(a) \quad \frac{-d\lambda}{da} \quad da$$

where $\lambda = \alpha a^{-k}$

$$ADP = k\alpha \int_a F_A(a) \; a^{-k-1} \; da$$

$$\lambda_1 = \alpha \; a_1^{-k} \quad \therefore \quad \alpha = \lambda_1 a_1^{k}$$

$$ADP = \lambda_1 k \int_{a_1} F_A(a) \; a_1^{k} \; a^{-k-1} \; da = k\lambda_1 \int_{a_1}^{\infty} F_A(a)(a/a_1)^{-k-1} \; d(a/a_1)$$

Let $x = a/a_1$

$$ADP = k\lambda_1 \int_1^{\infty} F_x(x) \; x^{-k-1} \quad dx$$

Example:

$$F_X(x) = \frac{1}{(n-1)^2} (x-1)^2 \qquad \begin{cases} x=1 & F_X(x)=0 \\ x=0 & F_X(x)=1 \end{cases}$$

then

$$\frac{ADP}{k\lambda_1} = \frac{1}{(n-1)^2} \quad \int_1^n \left\{ \left[x^2-2x+1\right] \; x^{-k-1} \; dx \right\} \; + \; \int_n^{\infty} x^{-k-1} \; dx$$

$$\frac{ADP}{\lambda_1} = \frac{K}{(n-1)^2} \int_1^n \left\{ (x^{-k-1} - 2x^{-k} + x^{-k-1} \;) \; dx - x^{-k} \right\}_n^{\infty}$$

$$= \frac{1}{(n-1)^2} \left\{ k \int_1^n \; x^{-k+1} \; dx + \left[\frac{-2k}{-k+1} \; x^{-k+1} \quad -x^{-k} \right]_1^n \right\} + n^{-k}$$

$$= \frac{1}{(n-1)^2} \left\{ k \int_1^n \; x^{-k+1} \quad \frac{2k}{k-1} \; (1-n^{-k+1}) \; + \; (1-n^{-k}) \right\} \; + \; n^{-k}$$

if k=2,

$$\frac{ADP}{\lambda_1} = \frac{1}{(n-1)^2}\left\{2 \ln n - 4(1-1/n) + (1 - \frac{1}{n^2})\right\} + \frac{1}{n^2}$$

if n=2,

$$\frac{ADP}{\lambda} = 2 \ln 2 - 1 \quad \underline{\sim} \quad \underline{0.38}$$

APPENDIX

DAMAGE PROBABILITIES (%) FOR PILOT APPLICATION
OF SEISMIC DESIGN DECISION ANALYSIS

DESIGN STRATEGY	DAMAGE STATE	MODIFIED MERCALLI INTENSITY V	VI	VII	VII.5	VIII	IX	X
	O	100	27	15	0	0	0	0
	L	0	73	48	21	0	0	0
UBC 0.1	M	0	0	33	45	20	0	0
	H	0	0	4	29	41	0	0
	T	0	0	0	5	34	75	25
	C	0	0	0	0	5	25	75
	O	100	47	20	0	0	0	0
	L	0	53	50	36	10	0	0
UBC 2	M	0	0	29	52	53	0	0
	H	0	0	1	11	31	0	0
	T	0	0	0	1	5	80	60
	C	0	0	0	0	1	20	40
	O	100	57	25	5	0	0	0
	L	0	43	50	48	25	0	0
UBC 3	M	0	0	25	41	53	20	0
	H	0	0	0	6	21	52	0
	T	0	0	0	0	1	23	80
	C	0	0	0	0	0	5	20
	O	100	67	30	10	0	0	0
	L	0	33	49	58	40	10	0
S	M	0	0	21	29	52	30	0
	H	0	0	0	3	8	58	0
	T	0	0	0	0	0	2	90
	C	0	0	0	0	0	0	10

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