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MULTIPLE FAILURE RISK OF SPATIALLY DISTRIBUTED STRUCTURES

by

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INTRODUCTION

The problem being considered is the following: Given a set of N sites, each with an associated critical level of earthquake intensity (which may, for example, be a threshold intensity of damage,) what is the probability of simultaneously equalling or exceeding any q (=1,2,...,N) out of the N intensities within a specified period of time.

For instance, there are N nuclear power plants in a geographic region, and they each have an associated operating basis earthquake intensity. If the operating basis earthquake is exceeded at a plant, it will be necessary to shut the plant down for inspection. These power plants are subject to seismic risk from several seismically active areas, the locations of which, as well as the parameters describing their activity, are known. Of particular interest is the probability that any q out of the N power plants will have to be shut down for inspection at the same time during a given period of time, say during 5 years.

Given a point site S with a deterministic resistance r and a point earthquake source O_{EQ}, the acceleration experienced by point S due to any earthquake of epicentral magnitude m can be evaluated using the attenuation law recommended by Esteva and Rosenblueth[1]

$$A(m,d) = b_1 e^{b_2 m} d^{-b_3} \qquad (1)$$

in which b_1, b_2, and b_3 are constants and d is such that

$$d = \max \{d_o, D\}$$

where D is the distance between the site and the earthquake's epicenter, and d_o is a minimum distance (which depends upon soil conditions) which prevents the site acceleration from exceeding the epicentral acceleration. The acceleration A(m,d) at point S is a function of only the earthquake magnitude m and the distance d between the point site S and the earthquake source O_{EQ}.

[1] Esteva, L. and E. Rosenblueth (1964), spectra of earthquakes at moderate and large distances, Soc. Mex. de Ing. Sismica, Mexico, II, 1-18.

It is clear that the site will fail when the ground acceleration A(m,d) at the site is equal to or greater than the site resistance r, i.e.,

$$P_{fo} = P[\text{site S fails} | O_{EQ}] = P[A(m,d) \geq r | O_{EQ}]$$

$$= P[b_1 e^{b_2 m} d^{-b_3} \geq r | O_{EQ}]$$

$$= P[e^{b_2 m} \geq \frac{rd^{b_3}}{b_1} | O_{EQ}]$$

$$= P[m \geq \frac{1}{b_2} \ln\left(\frac{rd^{b_3}}{b_1}\right) | O_{EQ}]$$

The term $\overline{R} = rd^{b_3}$ can be called the pseudo-resistance of site S; that is, the resistance of an equivalent site at a distance of 1 Km from the source (see Figure 1).

If instead of a single site a set of N sites, $S = \left\{S_i | i=1,2,\ldots,N\right\}$, is considered next, where each element of the set, S_i, has an associated pair (r_i, d_i), the pseudo-resistances $\overline{R}_i$ can be evaluated such that

$$\overline{R}_i = r_i d_i^{b_3} \qquad i = 1, 2, \ldots, N$$

The pseudo-resistances can now be arranged in order of increasing magnitude, i.e.,

$$\overline{R}_{i_1} < \overline{R}_{i_2} < \ldots\ldots < \overline{R}_{i_N}$$

where the first subscript identifies the site, and the second subscript gives the position in the ordered sequence from weakest to strongest pseudo-resistance. (See Figure 2.)

For a given earthquake and a set of N sites there are only N possible failure modes,

1) $FM_1 \equiv$ only the site with the weakest pseudo-resistance $\bar{R}_{i_1}$ fails

2) $FM_2 \equiv$ only the sites with $\bar{R}_{i_1}$ and $\bar{R}_{i_2}$ fail

⋮

q) $FM_q \equiv$ only the sites with $\bar{R}_{i_1}, \bar{R}_{i_2}, \ldots, \bar{R}_{i_q}$ fail

⋮

N) $FM_N \equiv$ all sites fail.

It is clear that FM_q implies FM_i, $i = 1, 2, \ldots, (q-1)$, where $q=2,3,\ldots,N$, which can be written as

$$FM_1 \subset FM_2 \subset \ldots . \subset FM_N .$$

If the probability of a certain failure mode occurring, say that of q out of N sites failing, i.e. FM_q, is to be found, the first quantities that must be determined are the earthquake magnitudes which bound the range of magnitudes for which the acceleration one kilometer from the source causes FM_q, that is

$$\frac{1}{b_2} \ln\left(\frac{\bar{R}_{i_q}}{b_1}\right) \leq m \leq \frac{1}{b_2} \ln\left(\frac{\bar{R}_{i_{q+1}}}{b_1}\right)$$

These bounding magnitudes may be symbolized by $\bar{M}_q$ and $\bar{M}_{q+1}$, and used in the following equations

$$P[q \text{ or more of } N \text{ sites fail} | O_{EQ}] = P[m \geq \bar{M}_q] = 1. - F_M(\bar{M}_q) \quad (2)$$

$$P[(q+1) \text{ or more of } N \text{ sites fail} | O_{EQ}] = 1. - F_M(\bar{M}_{q+1}) \quad (3)$$

where $F_M(m)$ is the C.D.F. of the epicentral magnitude m.

Equations (2) and (3) can be combined to give

$$P[q \text{ of } N \text{ sites fail} | O_{EQ}]$$

$$= \left\{1. - F_M(\overline{M}_q)\right\} - \left\{1. - F_M(\overline{M}_{q+1})\right\}$$

$$= F_M(\overline{M}_{q+1}) - F_M(\overline{M}_q) \qquad (4)$$

A graphical representation of this would be as shown in Figures 3 and 4. Now taking $\overline{M}_q$ and $\overline{M}_{q+1}$, the probability of q out of N sites failing can be found graphically.

Assuming the range of possible epicentral magnitudes is bounded below by m_o and above by m_{max}, $F_M(m)$ can be found by integrating the P.D.F. of the magnitude in the magnitude-frequency law:

$$F_M(m) = \int_{m_o}^{m} \frac{\beta e^{-\beta\mu - m_o}}{1-\beta e^{-\beta(m_{max} - m_o)}} \, d\mu \qquad (5)$$

$$= \frac{1 - e^{-\beta(m - m_o)}}{1 - e^{-\beta(m_{max} - m_o)}}; \quad m_o \leq m \leq m_{max} \qquad (6)$$

Letting
$$X_{norm} = \frac{1}{1 - e^{-\beta(m_{max} - m_o)}}$$

$$F_M(m) = \left(1 - e^{-\beta(m-m_o)}\right) X_{norm}$$

$$= X_{norm} - X_{norm} \, e^{-\beta(m-m_o)}$$

$$= X_{norm} - e^{\gamma} e^{-\beta m} e^{\beta m_o}$$

where $\gamma = \ln(X_{norm})$. The C.D.F. can now be written in its final form,

$$F_M(m) = X_{norm} - e^{\alpha-\beta m} \qquad (7)$$

where $\alpha = \gamma+\beta m_o$.

Substituting (7) into (4) the following is obtained:

$$P[q \text{ of } N \text{ sites fail} | O_{EQ}]$$

$$= \{X_{norm} - e^{\alpha-\beta\overline{M}_{q+1}}\} - \{X_{norm} - e^{\alpha-\beta\overline{M}_q}\}$$

$$= e^{\alpha-\beta\overline{M}_q} - e^{\alpha-\beta\overline{M}_{q+1}}$$

$$= e^{\alpha}(e^{-\beta\overline{M}_q} - e^{-\beta\overline{M}_{q+1}})$$

where $\overline{M}_q = (1/b_2)\ln(\overline{R}_{iq}/b_1$, and $\overline{R}_{iq}$ is the q^{th} weakest (smallest) of N pseudo-resistances (the same applies to $\overline{M}_{q+1}$).

The above discussion dealt with only a single point source. If the scope is increased to cover a source area, which can be seen as an infinite number of point sources, the probability of failure will be

$$P[q \text{ or more of } N \text{ sites fail in any year source area} | A_K]$$

$$= \int_{A_K} P[q \text{ or more of } N \text{ sites fail} | x,y]\ \nu_K\ f_{A_K}(a)\ da\ (x,y) \qquad (8)$$

where (x,y) are the coordinates of each point source being integrated, ν_K is the rate of occurence for the source area, $f_{A_K}(a)$ is a P.D.F. of occurence over all the source points, and da(x,y) is the infinitesimal area of each point source.

To facilitate numerical integration, the source area can be approximated by a set of finite triangular elements, and all earthquakes occurring in an element can be considered to occur at the centroid of the element. For each of these point sources in a source area the pseudo-resistances of all sites can be evaluated. Then the probability of q or more of N sites failing simultaneously can be calculated, adjusting it by the occurence rate ν and the P.D.F. $f_{A_K}(a)$. The simplest kind of occurence distribution for a source area is that which gives to every unit area the same probability of an earthquake occurring, in which case $f_{A_K}(a)$ is the reciprocal of the total area of the source area. Equation (8) can be rewritten as

$$P'_{f_{q_K}} = P[\text{q or more of N sites fail in any year} | \text{source area } A_K]$$

$$= \sum_{i=1}^{NEL_K} P[\text{q or more of N sites fail} | (x,y)_i] \, \nu_K \frac{1}{\text{Area of } A_K} \Delta a(x,y)_i$$

where ELE_K is the number of triangular elements in source area A_K, $(x,y)_i$ are the coordinates of each point source representing a triangular element, $\frac{1}{\text{Area of } A_K}$ is $f_{A_K}(a)$, and $\Delta a(x,y)_i$ is the area of each of the triangular elements.

If the seismic activity in different source areas is taken as independent, then by extension the following is obtained:

$$P'_{f_q} = \sum_{k=1}^{NAREA} P'_{f_{q_K}} \qquad (9)$$

Finally,

$$P_{f_q} = P'_{f_q} - P'_{f(q+1)} \qquad (10)$$

where P_{fq} is now the probability that exactly q out of N sites will fail simultaneously in any given year.

Finally, if the probability of failure is to be found for a period of t years, the following equation may be used:

$$P_{f_q}(\text{in t years}) = 1 - e^{-P_{f_q} \cdot t}$$

The above development was based on the Esteva and Rosenblueth attenuation law, equation (1),

$$A(m,d) = b_1 \, e^{b_2 M} \, d^{-b_3}.$$

If an attenuation law based on modified Mercalli intensity is used, such as[2]

$$I_s = c_1 + c_2 I_o - c_3 \ln R \qquad (11)$$

then a parallel development can be set up to find P_{fq} after equation (11) is converted into exponential form, which gives an equation of the form of Equation (1).

$$e^{I_s} = e^{c_1} e^{c_2 I_o} R^{-c_3} \qquad (12)$$

The correspondence between Eq. (12) and Eq. (1) is as follows:

$$r \leftarrow e^{I_s} \; ; \quad b_1 \leftarrow e^{c_1} \; ; \quad b_2 \leftarrow c_2 \; ; \quad b_3 \leftarrow c_3 \; .$$

APPLICATION

The method developed is applied to a hypothetical problem which illustrates the possible application to a real problem. It is presumed that in New England there are nine nuclear power plants, as shown in Fig. 5. These nine are either already in operation or under construction or in initial stages of design. The operating basis earthquake intensity for these plants is considered to be M.M.I. VI, which corresponds by[3]

$$\log a = \frac{I}{3} - \frac{1}{2} \qquad (13)$$

to an acceleration (a) of 31.6 cm/sec^2. For the region considered there are three seismically active areas, and a seismically active background area (Fig. 5). This is the same problem treated in Internal Report No. 51, which uses a computer program developed in Russia.

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[2]Cornell, C.A.,"Probabilistic Analysis of Damage to Structures under Seismic Loads, Dynamic Waves in Civil Engineering, ed. by D.A. Howells, I.P. Haigh and C. Taylor, Wiley-Interscience, 1971, p. 474.

[3]Whitman, R.V.,"Damage Probability Matrices for Prototype Buildings," Department of Civil Engineering Research Report R73-57, M.I.T., Oct.'73,Fig.4.3.

[4]Schumacker, B., "Nuclear Power Plants and the Operating Basis Earthquake," Internal Study Report No. 51, M.I.T., January 1975.

The information for the seismic areas is given in Table 1. Conversion from MMI to magnitude is made by use of

$$M = (2/3)I + 1 \tag{14}$$

Two attenuation laws were used:

$$I_s = I_o + 3.219 - 1.3 \ln d \qquad \text{for firm ground}$$

$$I_s = I_o + 3.719 - 1.3 \ln d \qquad \text{for soft ground.}$$

For analysis in terms of magnitude and acceleration these attenuation laws were converted into the form of Eq. (1) using Equations (13) and (14). The slope of the ln N(I) vs. I_o relation was $\beta_{II} = 1.1$. For analysis in terms of magnitude this was converted to the equivalent value of β_{AM} using $\beta_{AM} = \frac{3}{2}\beta_{II} = 1.65$, where β_{AM} is the equivalent slope corresponding to the acceleration vs. magnitude relationship.

The results obtained when all this information is utilized in a computer program are summarized in Table 2. In the computer program that calculated the P_{fq}'s, the three seismic areas were divided into approximately 1600 triangular elements, and the background area into approximately 325 triangular elements. The median of the sides of a triangular element has a length between 14 and 18 kilometers.

The results obtained by the two approaches should be the same, since the equations and input data are entirely equivalent. The reasons for the slight discrepencies are not yet understood. Note that the discrepencies increase with q increasing.

Table 3 compares the results from the present study with those in Internal Study Report 51. The comparison involves the intensity approach and firm soil. In a general sense, the two sets of results are within the same range of order of magnitudes. The discrepencies between these results are partially attributed to the differences in the underlying assumptions in the mathematical models of the attenuation and frequency of occurence laws. For instance, the study in Report 51 used $I_s=I_o$ for $D \leq 16.1$ km while in the present study $I_s=I_o$ is used for $D \leq d_o = 11.9$ km. Thus, for epicentral distance between D =11.9 km. and 16.1 km., the former study assigned a higher intensity than the present study. This might explain the larger annual probability in the former study for q=1. The larger probabilities for q=2,3 and 4 in the present study may have resulted from using a larger number of cells to represent the source areas.

TABLE 1

SEISMIC AREA	I_{min} (M_{min})	I_{max} (M_{max})	ν EARTHQUAKES/YEAR
AREA 1	V (4.33)	VIII.3 (6.53)	0.06000
AREA 2	V (4.33)	VIII.3 (6.53)	0.02400
AREA 3	V (4.33)	VIII.7 (6.8)	0.13200
BACKGROUND	V (4.33)	VI.5 (5.3)	0.00967

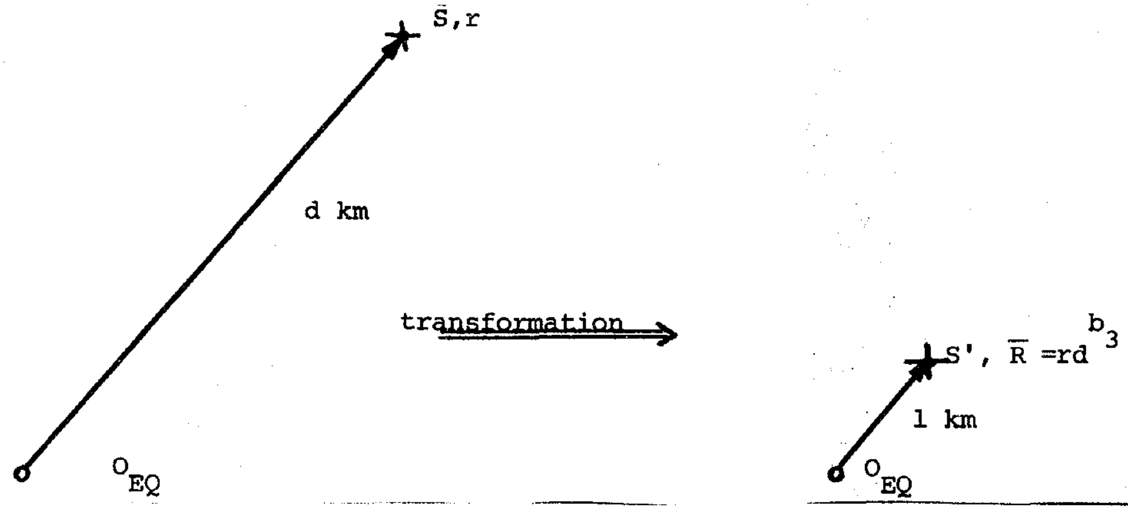
TABLE 2

	Magnitude approach equation (1)		M.M.I. approach equation (12)	
	Firm Soil	Soft Soil	Firm Soil	Soft Soil
q	P_{f_q}	P_{f_q}	P_{f_q}	P_{f_q}
1	5.08×10^{-3}	7.81×10^{-3}	5.24×10^{-3}	8.03×10^{-3}
2	6.88×10^{-4}	1.95×10^{-3}	7.28×10^{-4}	2.05×10^{-3}
3	1.60×10^{-4}	6.99×10^{-4}	1.75×10^{-4}	7.46×10^{-4}
4	6.02×10^{-6}	1.42×10^{-4}	7.52×10^{-6}	1.65×10^{-4}

P_{f_q} in any given year for 9 New England nuclear power plants

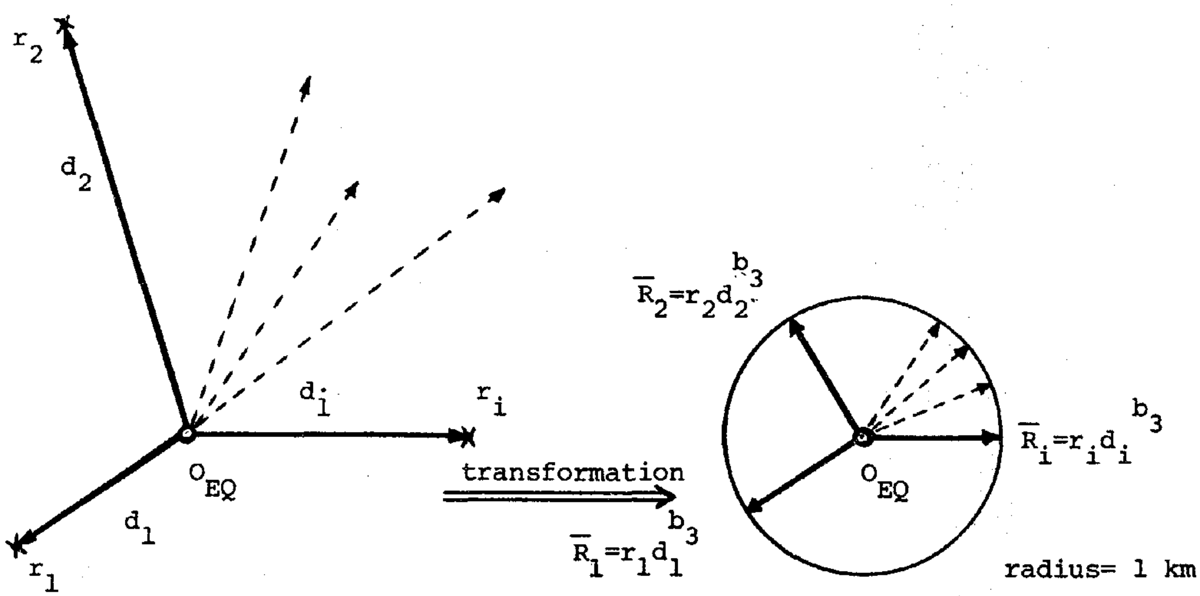
TABLE 3

	P_{f_q}	
q	Present Study	ISR 51
1	5.24×10^{-3}	9.18×10^{-3}
2	7.28×10^{-4}	6.07×10^{-4}
3	1.75×10^{-4}	8.04×10^{-5}
4	7.52×10^{-6}	0



$$P_{fo} = P[b_1 e^{b_2 m} d^{-b_3} \geq r] \qquad P_{fo} = P[b_1 e^{b_2 m} 1^{-b_3} \geq r d^{b_3}]$$

Figure 1



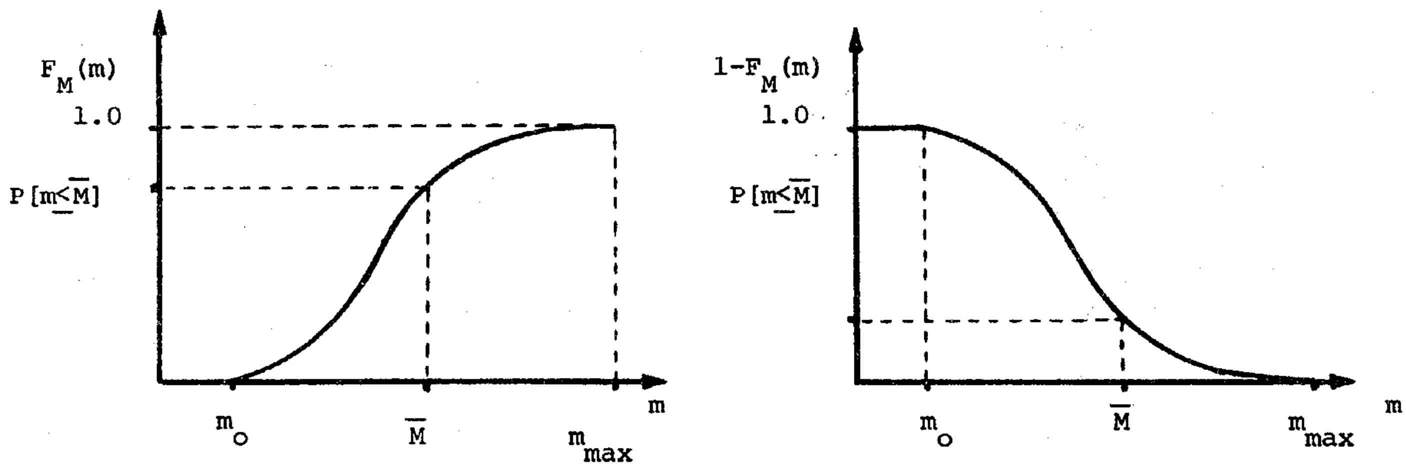


FIGURE 3

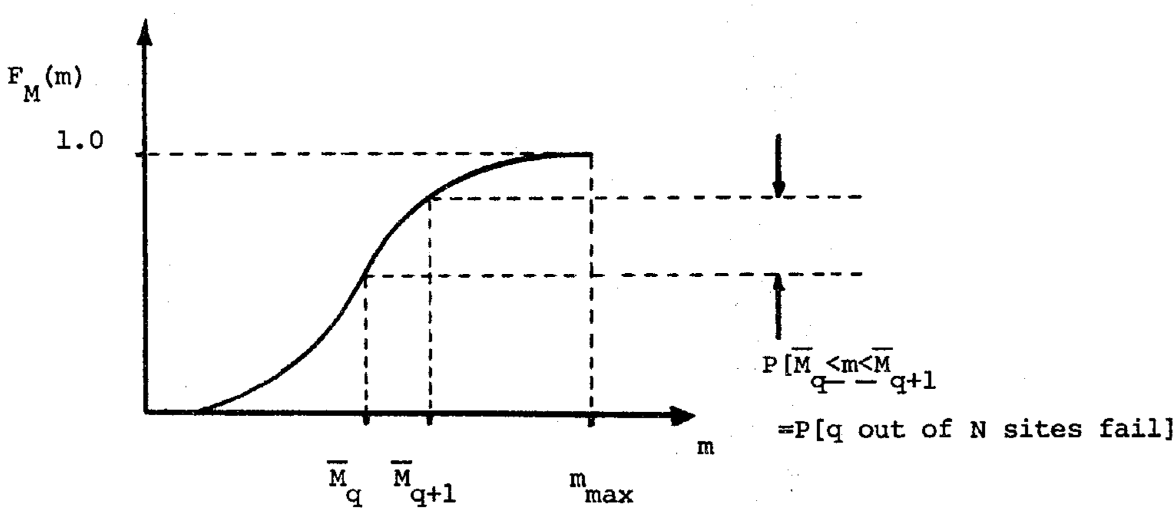


FIGURE 4

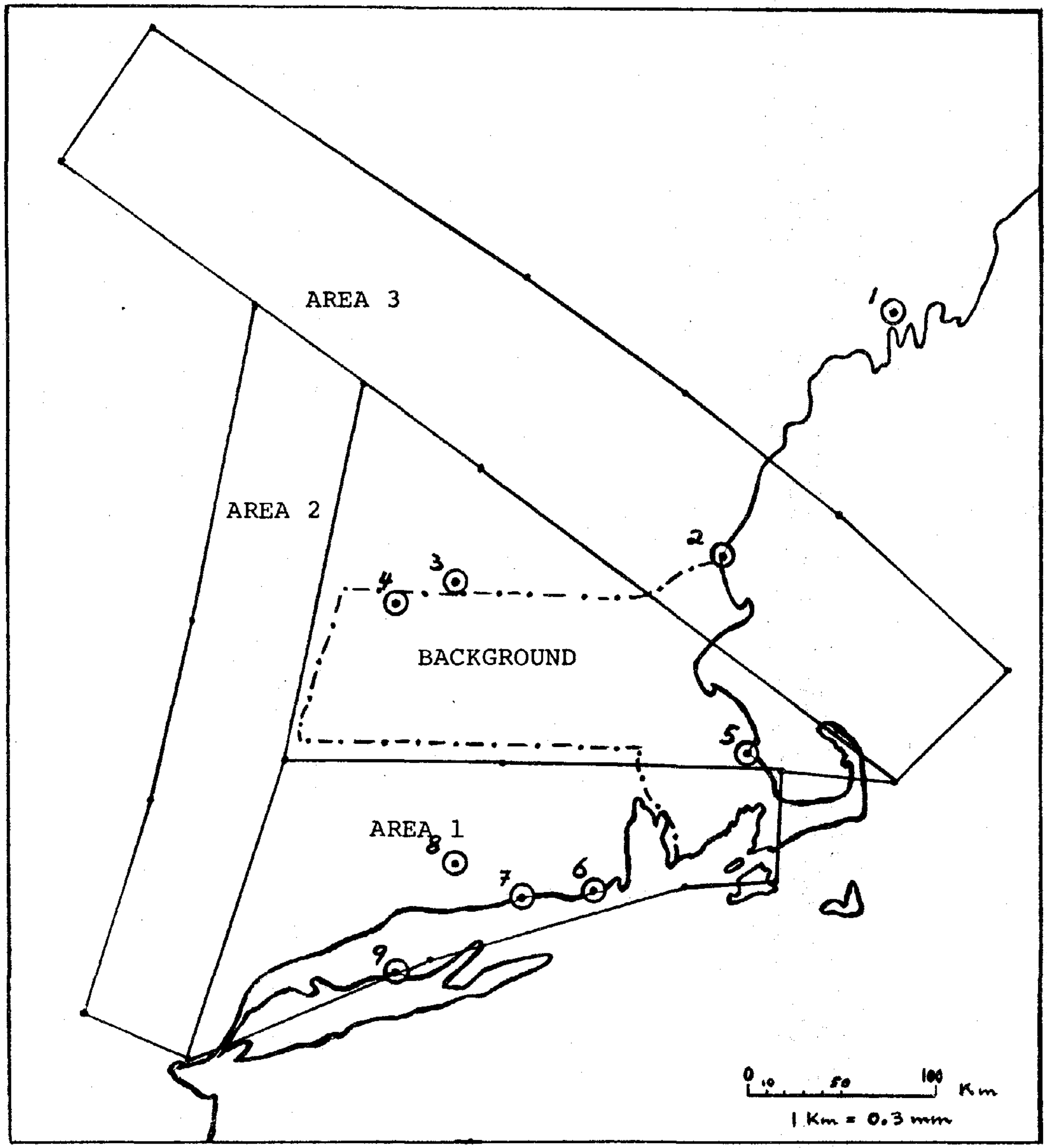


FIGURE 5.

Power stations	
1	Wiscasset, Maine
2	Seabrook, New Hampshire
3	Vernon, Vermont
4	Rowe, Massachusetts
5	Plymouth, Massachusetts
6	Charlestown, Rhode Island
7	Waterford, Connecticut
8	Haddam, Connecticut
9	Shoreham, L.I., N.Y.