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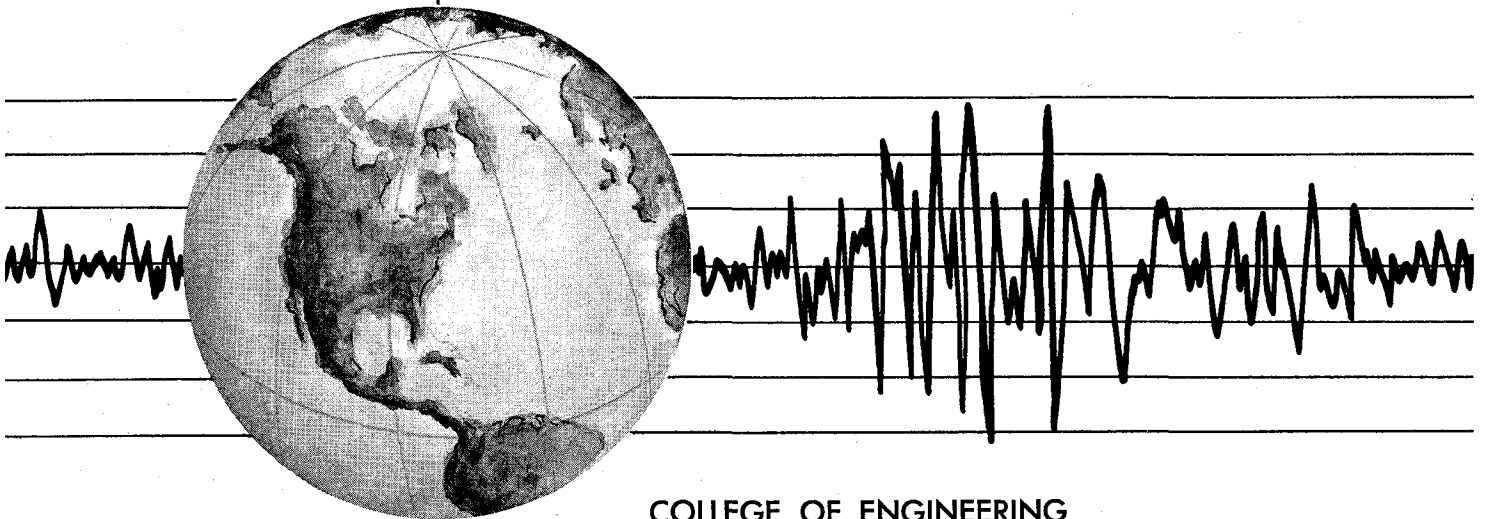
EARTHQUAKE ENGINEERING RESEARCH CENTER

# THREE DIMENSIONAL INELASTIC FRAME ELEMENTS FOR THE ANSR-I PROGRAM

by

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FOR THE ANSR-I PROGRAM

by

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Report to

National Science Foundation

Report No. UCB/EERC-78/06  
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## ABSTRACT

This report describes two elements developed to allow inelastic three-dimensional building frames to be analyzed using the ANSR-I computer program [1].

The first element is a beam-column which has two-dimensional stiffness and yield characteristics. This element may be located arbitrarily in a three-dimensional structure and is intended primarily for modelling beams. The second element is a beam-column which has three-dimensional stiffness and yield characteristics. This element can be used for modelling columns in which biaxial bending effects may be important and also for structures such as elevator shafts.

Both elements assume that inelastic behavior is concentrated in plastic hinges at the element ends. For the two-dimensional element the plastic hinges are affected by moment in the principal bending plane only, or by this moment interacting with axial force. For the three-dimensional element the plastic hinges are affected by bending moments about both axes, axial force and (if desired) torsional moment.

Allowance has been made for rigid floor diaphragms by means of a "slaving" feature. This slaving feature has been incorporated into both elements at the element level because ANSR-I cannot account for slaving at the nodal level. Both elements also allow for rigid end zones and for initial element actions.

The theoretical formulations are presented and the element characteristics are described. User's guides for both elements are included and an example analysis is described.



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## 1. INTRODUCTION

This report describes two elements developed to allow inelastic three-dimensional building frames to be analyzed using the ANSR-I computer program [1]. The elements are as follows:

- (1) A beam-column element which has two-dimensional stiffness and yield characteristics, but which may be located arbitrarily in a three-dimensional structure. This element is similar in concept to "element type 2" of the DRAIN-2D program [2], and is intended primarily for modelling beams.
- (2) A beam-column element which has three-dimensional stiffness and yield characteristics. This element is similar in concept to the element originally developed by Porter and Powell [3], but uses a continuous rather than a faceted yield surface. This element can be used for modelling columns in which biaxial bending effects may be important, and also for structures such as elevator shafts.

Both elements assume that inelastic behavior is concentrated in plastic hinges at the element ends. For the two-dimensional element the plastic hinges are affected by moment in the principal bending plane only, or by this moment interacting with axial force. For the three-dimensional element the plastic hinges are affected by bending moments about both axes, axial force and (if desired) torsional moment.

Allowance has been made for rigid floor diaphragms by means of a "slaving" feature. This slaving feature has been incorporated into both elements at the element level, because ANSR-I can not account for slaving at the nodal level. Both elements also allow for rigid end zones and for

initial element actions.

The element characteristics are described in Chapters 2 and 3, and an illustrative example is shown in Chapter 4. User's guides for the elements are contained in Appendices A and B.

## 2. TWO DIMENSIONAL BEAM-COLUMN ELEMENT

### 2.1 GENERAL CHARACTERISTICS

Beam column elements may be arbitrarily oriented in the global XYZ plane. If the slaving feature is to be used the Y axis must be vertical.

Each element must be assigned an axial stiffness plus a major axis flexural stiffness. Torsional and minor axis flexural stiffnesses may also be specified if necessary, as explained in Section 2.4. Elements of variable cross section can be considered by specifying appropriate flexural stiffness coefficients. Flexural shear deformations and the effects of eccentric end connections can be taken into account.

Yielding may take place only in concentrated plastic hinges at the element ends. Hinge formation is affected by the axial force and major axis bending moment only. That is, an element may be placed in a three-dimensional frame, but its yield mechanism is only two-dimensional, in the plane of major axis bending. The yield moments may be specified to be different at the two element ends, and for positive and negative bending. The interaction between axial force and moment in producing yield is taken into account approximately.

Strain hardening is approximated by assuming that the element consists of elastic and elasto-plastic components in parallel. With this type of strain hardening idealization, if the bending moment in the element is constant, and if the element is of uniform strength, then the moment-rotation relationship for the element will have the same shape as its moment-curvature relationship (Fig. 2.1a). This follows because curvature and rotation in this case are directly

proportional. If, however, the bending moment or strength vary, then the curvatures and rotations are no longer proportional, and the moment-rotation and moment-curvature variations may be quite different (Fig. 2.1b). With the parallel component procedure, a moment-rotation relationship is, in effect, being specified. Care must be taken in relating this to a moment-curvature relationship.

If static load analyses are carried out separately (i.e. outside the ANSR program), the results of these analyses may be included by specifying appropriate initial axial forces and bending moments in the elements. The P-delta effect can be considered by including a geometric stiffness.

## 2.2 ELEMENT DEFORMATIONS

The beam-column element has three primary modes of deformation, namely (a) axial extension and (b) flexural rotations in the major plane at ends i and j. The transformation relating increments of element deformation to increments of nodal displacement (Fig. 2.2) is

$$\underline{dv}_p = \underline{a}_p \underline{dr} \quad (2.1)$$

in which  $\underline{dv}_p^T = (dv_1, dv_2, dv_3)$  and  $\underline{dr}^T = (dr_1, dr_2, \dots, dr_{12})$

and the transformation  $\underline{a}_p$  is well known.

The element also has three secondary modes of deformation, which may have to be considered for reasons explained in Section 2.4. These consist of minor axis flexural deformations at ends i and j, and an angle of torsional twist. Again the transformation from displacements to deformations is well known (Fig. 2.2), and can be expressed as

$$\underline{dv}_s = \underline{a}_s \underline{dr} \quad (2.2)$$

in which  $\underline{dv}_s^T = (dv_4, dv_5, dv_6)$ .

It is assumed that these secondary deformations are elastic, and that they do not interact with the primary deformations in producing plastic hinges.

A plastic hinge forms when the moment in the elasto-plastic component of the element reaches its yield moment. A hinge is then introduced into this component, the elastic component remaining unchanged. The measure of flexural plastic deformation is the plastic hinge rotation.

For any increments of total flexural rotation,  $dv_2$  and  $dv_3$ , the corresponding increments of plastic hinge rotation,  $d\theta_{p2}$  and  $d\theta_{p3}$ , are given by

$$\begin{Bmatrix} d\theta_{p2} \\ d\theta_{p3} \end{Bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{Bmatrix} dv_2 \\ dv_3 \end{Bmatrix} \quad (2.3)$$

in which A, B, C and D are as given in Table 2.1. Unloading occurs at a hinge when the increment in hinge rotation is opposite in sign to the bending moment.

Inelastic axial deformations are assumed not to occur in beam-column elements of this type, to simplify the problem of interaction between axial and flexural deformations after yield. Only an approximate procedure for considering interaction effects is included, as explained in the following section. This procedure is not strictly consistent, but is believed to be reasonable for most practical applications.

### 2.3 INTERACTION SURFACES

Yield interaction surfaces of three types may be specified, as follows.

- (1) Beam type (shape code = 1, Fig. 2.3a). This type of surface should be specified where axial forces are small or are ignored. Yielding is affected by bending moment only.
- (2) Steel column type (shape code = 2, Fig. 2.3b). This type of surface is intended for use with steel columns.
- (3) Concrete column type (shape code = 3, Fig. 2.3c). This type of surface is intended for use with concrete columns.

For any combination of axial force and bending moment within a yield surface, the cross section is assumed to be elastic. If the force-moment combination lies on or outside the surface, a plastic hinge is introduced. Combinations outside the yield surface are permitted only temporarily, being compensated for by applying corrective loads in the succeeding load step or iteration.

This procedure is not strictly correct because the axial and flexural deformations interact after yield, and it is therefore wrong to assume that the flexural stiffness changes but the axial stiffness remains unchanged. However, this procedure is believed to be reasonable for practical analyses of buildings.

If a force-moment combination goes from the elastic range to beyond the yield surface in any load step or iteration, an equilibrium correction is made as shown in Fig. 2.4a. Also, because the axial stiffness is assumed to remain unchanged, the force-moment combination at a plastic hinge will subsequently move away from the yield surface

if yielding continues, as shown in Fig. 2.4b. An equilibrium correction, as shown, is therefore made in each succeeding step or iteration.

The axial force in an element with a column-type interaction surface can, in reality, never exceed the yield value for zero moment. However, because of the computational procedure being used, axial forces in excess of yield can be computed. For axial forces in excess of yield, the yield moments are assumed to be zero. The printed results from the program should be examined carefully and interpreted with caution. If axial forces approaching or exceeding yield are computed for a column, severe column damage is probably implied.

#### 2.4 ELEMENT STIFFNESS

The element is considered as the sum of an inelastic component and an elastic component in the major plane of bending, plus a further elastic component providing torsional and minor axis flexural stiffnesses. This third component is needed to avoid singular stiffness matrices in certain circumstances.

The element actions and deformations are shown in Fig. 2.2. The axial stiffness is constant, and is given by

$$dS_1 = \frac{EA}{L} dv_1 \quad (2.4)$$

in which  $E$  = elastic modulus, and  $A$  = effective cross sectional area.

The primary elastic flexural stiffness is given by

$$\begin{Bmatrix} dS_2 \\ dS_3 \end{Bmatrix} = \frac{EI}{L} \begin{bmatrix} k_{ii} & k_{ij} \\ k_{ij} & k_{jj} \end{bmatrix} \begin{Bmatrix} dv_2 \\ dv_3 \end{Bmatrix} \quad (2.5)$$

in which  $I$  = reference moment of inertia; and  $k_{ii}$ ,  $k_{ij}$ ,  $k_{jj}$  are coefficients which depend on the cross section variation. For a uniform element,  $I$  = actual moment of inertia,  $k_{ii} = k_{jj} = 4$ , and  $k_{ij} = 2$ . The coefficients must be specified by the program user, and may, if desired, account for such effects as shear deformations and nonrigid end connections as well as cross section variations.

After one or more hinges form, the coefficients for the elasto-plastic component change to  $k'_{ii}$ ,  $k'_{ij}$  and  $k'_{jj}$ , as follows

$$k'_{ii} = k_{ii}(1-A) - k_{ij} C \quad (2.6)$$

$$k'_{ij} = k_{ij}(1-D) - k_{ii} B \quad (2.7)$$

$$k'_{jj} = k_{jj}(1-D) - k_{ij} B \quad (2.8)$$

in which  $A$ ,  $B$ ,  $C$  and  $D$  are defined in Table 2.1.

If desired, effective flexural shear areas may be specified. The program then modifies the flexural stiffness to account for the additional shear deformations.

The minor axis flexural stiffness is obtained by multiplying the primary elastic stiffness by a user-specified factor,  $f$ . The torsional deformation is related to torque by

$$dS_6 = \frac{GJ}{L} dv_6 \quad (2.9)$$

in which it is assumed that  $G = 0.4E$  and

$$J = f(k_{ii} + k_{jj})I/8 \quad (2.10)$$

in which  $k_{ii}$  and  $k_{jj}$  are the primary flexural stiffness factors, after any modification for shear deformations.



The primary and secondary actions are related to their respective deformations by

$$\frac{dS_p}{dv_p} = k_p \quad (2.11)$$

and

$$\frac{dS_s}{dv_s} = k_s \quad (2.12)$$

## 2.5 GEOMETRIC STIFFNESS

The geometric stiffness which is used is exactly the same as for the ANSR truss element (see Reference [1], Section B1, Appendix B1). This is not the exact geometric stiffness for a beam column element, but is sufficiently accurate to account for the P-delta effect in building frames.

## 2.6 END ECCENTRICITY

Plastic hinges in frames and coupled frame-shear wall structures will form near the faces of the joints rather than at the theoretical joint centerlines. This effect can be approximated by postulating rigid, infinitely strong connecting links between the nodes and the element ends, as shown in Fig. 2.6. The displacement transformation relating the increments of node displacements,  $\underline{dr}_n$ , to increments of displacement at the element ends is easily established, and can be written as

$$\underline{dr} = \underline{a}_e \underline{dr}_n \quad (2.13)$$

## 2.7 GLOBAL STIFFNESS

The element stiffness,  $\underline{K}$ , relating global actions and displacements is obtained as

$$\underline{K} = \underline{a}_e^T \underline{a}_p^T \underline{K}_p \underline{a}_p \underline{a}_e + \underline{a}_s^T \underline{K}_s \underline{a}_s + \underline{K}_G \quad (2.14)$$

where  $\underline{K}_G$  is included only if geometric stiffness effects are to be considered. For simplicity, the secondary element is assumed to join the nodes  $i$  and  $j$  directly, without end eccentricities. The geometric stiffness is also formulated for a member connecting nodes  $i$  and  $j$  directly.

## 2.8 RIGID FLOOR DIAPHRAGMS

A frequently made assumption in the analysis of tall buildings is that each floor diaphragm is rigid in its own plane. To introduce this assumption, a "master" node at the center of mass of each floor may be specified, as shown in Fig. 2.7. Each master node has only three degrees of freedom as shown, which are the displacements of the diaphragm horizontally as a rigid body. If any beam-column member is connected to a diaphragm, its stiffness must be formulated partly in terms of these "master" displacements and partly in terms of displacements which are not affected by the rigid diaphragm assumption.

The displacement transformation relating the diaphragm displacements,  $\underline{dr}_d$ , to the displacements at a slaved node is as follows.

$$\begin{Bmatrix} dr_{n1} \\ dr_{n3} \\ dr_{n5} \end{Bmatrix} = \begin{bmatrix} 1 & 0 & d_z \\ 0 & 1 & -d_x \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} dr_x \\ dr_y \\ dr_\theta \end{Bmatrix} \quad (2.15)$$

or

$$\underline{dr}_{ns} = \underline{a}_d \underline{dr}_d \quad (2.16)$$

The slaved displacements at element nodes i and j can be expressed in terms of the displacements at the "master" node (or nodes). The corresponding coefficients of  $\underline{K}$  (eq. 2.14) are transformed to account for the slaving. The resulting element stiffness matrix is assembled in terms of the three master degrees of freedom plus the three local degrees of freedom  $dr_{n2}$ ,  $dr_{n4}$  and  $dr_{n6}$  at each node, which are not affected by slaving.

## 2.9 INITIAL FORCES

For structures in which static analyses are carried out separately, (i.e. outside the ANSR program), initial primary member forces may be specified. The sign convention for these forces is as shown in Fig. 2.5. These forces are not converted to loads on the nodes of the structure, but simply used to initialize the element end actions. For this reason, initial forces need not constitute a set of actions in equilibrium. The only effects they have on the behavior of the system are (a) to influence the onset of plasticity and (b) to affect the geometric stiffnesses.

Primary initial forces are defined as standard patterns. Each element can be identified with a standard pattern, and in addition a multiplication factor for scaling the standard pattern may be specified.

Fixed end forces, as permitted in the DRAIN-2D version of this element, are not currently permitted because ANSR-I is not able to consider such forces.

## 2.10 RESULTS OUTPUT

The following results are printed at the specified output intervals, during static and dynamic analyses, for those elements for which element force histories are requested.

- (1) Yield code at each end of element: zero indicates the element end is elastic, and 1 that a plastic hinge has formed.
- (2) Axial force, bending moment and shear force acting on each end, with the sign convention shown in Fig. 2.5.
- (3) Current plastic hinge rotations at each end. The sign convention is the same as for primary flexural actions and deformations (Fig. 2.2(a) and Fig. 2.5).
- (4) Accumulated positive and negative plastic hinge rotations up to the current time. These values are accumulated as shown in Fig. 2.8.

The maximum positive and negative values of axial force, bending moment, shear force, and plastic hinge rotation, with their times of occurrence, are printed at the time intervals requested for envelopes. The accumulated positive and negative plastic hinge rotations are also printed.

TABLE 2.1  
COEFFICIENTS FOR PLASTIC HINGE ROTATIONS

Yield Condition	A	B	C	D
Elastic ends	0	0	0	0
Plastic hinge at end i only	1	$k_{ij}/k_{ii}$	0	0
Plastic hinge at end j only	0	0	$k_{ij}/k_{jj}$	1
Plastic hinges at both ends i and j	1	0	0	1

Coefficients  $k_{ii}$ ,  $k_{ij}$ , and  $k_{jj}$  are defined by Eq. 2.5.

### 3. THREE DIMENSIONAL BEAM - COLUMN ELEMENT

#### 3.1 GENERAL CHARACTERISTICS

Three dimensional beam-column elements may be arbitrarily oriented in the global XYZ plane. If the slaving feature is to be used the Y axis must be vertical.

Each element must be assigned flexural stiffness and axial stiffness. Plastic hinges can form at the element ends. Interaction among the bending moments, torsional moment and axial forces at a hinge are taken into account in determining when hinges form. Displacements are assumed to be small, although the P-delta effect may be considered.

The orientation of the local element axes is as shown in Fig. 3.1. Node k, together with nodes i and j, defines the plane containing the local y axis.

Trilinear relationships can be specified for the moment-rotation relationship about the element y axis ( $M_y-\theta_y$ ), moment-rotation about the element z axis ( $M_z-\theta_z$ ), torque-twist ( $T-\phi$ ), and force-extension ( $F-\epsilon$ ) for the element, as indicated in Fig. 3.2. Different yield strengths can be specified at the two ends if desired. Different strengths can also be specified for axial tension and axial compression.

Each element is automatically divided into three parallel elements, two of which are elastic-perfectly plastic and the third elastic. The stiffnesses and strengths of these parallel elements are calculated by the program such that the action-deformation relationships for the combined element have the specified trilinear forms. Each elastic-perfectly plastic parallel element may develop a concentrated plastic hinge, with zero lengths, at one or both ends. The forces at each potential

hinge interact according to one of two available yield functions to produce yield. The two available functions define elliptical and parabolic yield surfaces, as shown in Fig. 3.3. The origin of the yield surface may be shifted along the force axis, as shown, so that the surface can be made to approximate that for a reinforced concrete column.

Elements with varying strengths can be modelled by specifying different action-deformation relationships at the two element ends, but the element stiffness is assumed to be constant along the element length. Shear deformations and interaction effects with shear are ignored.

Eccentric end connections, initial forces, and rigid diaphragm slaving, as for the 2-D beam-column element, may be specified.

### 3.2 TANGENT STIFFNESS FOR AN ELASTO-PLASTIC PARALLEL COMPONENT

A tangent stiffness matrix can be derived which relates increments in the element end actions to increments in the element deformations for a single parallel component. That is,

$$\underline{dS} = \underline{k}_t \underline{dv} \quad (3.1)$$

in which  $\underline{dS}$  = vector of element action increments,

$\underline{dv}$  = vector of element deformation increments, and

$\underline{k}_t$  = element tangent stiffness matrix.

The following basic assumptions are necessary for development of the theory.

- (a) Element deformation increments can be decomposed into

elastic and plastic components. That is,

$$\underline{dv} = \underline{dv}_e + \underline{dv}_p \quad (3.2)$$

in which  $\underline{dv}_e$  = vector of elastic deformation increments, and  
 $\underline{dv}_p$  = vector of plastic deformation increments.

- (b) Element action increments are related to the elastic deformation increments by the elastic action-deformation relationship. That is,

$$\underline{dS} = \underline{k}_e \underline{dv}_e \quad (3.3)$$

in which  $\underline{k}_e$  = initial elastic stiffness matrix.

- (c) The plastic increment of deformation is normal to the yield surface, directed outwards. For a hinge, at, say, element end  $i$ , this assumption can be expressed as

$$\underline{dv}_{pi} = \phi_{i,s} \cdot \lambda_i \quad (3.4)$$

in which  $\phi_{i,s}$  = gradient vector of yield function at end  $i$ , each term being a partial derivative of the yield function with respect to the corresponding element action; and  $\lambda_i$  = a (positive) scalar which determines the magnitude of the plastic deformations.

With these three assumptions, the tangent stiffness matrix for an element with hinges at either or both ends is formed as follows.



For the hinges at ends i and j, from Eq 3.4

$$\underline{dv}_{pi} = \phi_{i,s} \cdot \lambda_i \quad (3.5a)$$

and

$$\underline{dv}_{pj} = \phi_{j,s} \cdot \lambda_j \quad (3.5b)$$

Eqs. 3.5a and 3.5b can be combined to give

$$\begin{Bmatrix} \underline{dv}_{pi} \\ \underline{dv}_{pj} \end{Bmatrix} = \begin{bmatrix} \phi_{i,s} & 0 \\ 0 & \phi_{j,s} \end{bmatrix} \begin{Bmatrix} \lambda_i \\ \lambda_j \end{Bmatrix} \quad (3.6)$$

or

$$\underline{dv}_p = \phi_{,s} \lambda \quad (3.7)$$

At each hinge, the value of the yield function must remain constant

That is,

$$d\phi_i = 0 \quad (3.8a)$$

and

$$d\phi_j = 0 \quad (3.8b)$$

or

$$\phi_{i,s}^T \cdot d\underline{S}_i = 0 \quad (3.9a)$$

and

$$\phi_{j,s} \cdot \underline{dS}_j = 0 \quad (3.9b)$$

Eqs. 3.9a and 3.9b can be combined to give

$$\begin{bmatrix} \phi_{i,s} & \underline{0} \\ 0 & \phi_{j,s} \end{bmatrix}^T \begin{Bmatrix} \underline{dS}_i \\ \underline{dS}_j \end{Bmatrix} = 0 \quad (3.10)$$

or

$$\phi_{,s}^T \cdot \underline{dS} = \underline{0} \quad (3.11)$$

Substitution of Eq. 3.2 into Eq. 3.3 gives

$$\underline{dS} = k_e (\underline{dv} - \underline{dv}_p) \quad (3.12)$$

and substitution of Eq. 3.7 into Eq. 3.12 gives

$$\underline{dS} = k_e (\underline{dv} - \phi_{,s} \lambda) \quad (3.13)$$

Premultiplication of Eq. 3.13 by  $\phi_{,s}^T$  gives

$$\phi_{,s}^T \underline{dS} = \phi_{,s}^T k_e (\underline{dv} - \phi_{,s} \lambda) \quad (3.14)$$

Hence, from Eq. 3.11

$$\underline{\phi}_{,s}^T \underline{k}_e (\underline{dr} - \underline{\phi}_{,s} \lambda) = \underline{0} \quad (3.15)$$

or

$$(\underline{\phi}_{,s}^T \underline{k}_e \underline{\phi}_{,s}) \lambda = (\underline{\phi}_{,s}^T \underline{k}_e) \underline{dr} \quad (3.16)$$

Eq. 3.16 can be solved for  $\lambda$  to give

$$\lambda = (\underline{\phi}_{,s}^T \underline{k}_e \underline{\phi}_{,s})^{-1} (\underline{\phi}_{,s}^T \underline{k}_e) \underline{dr} \quad (3.17)$$

It can be shown that for certain cases, in particular when an element is yielding at both ends under axial force alone or under torque alone, the determinant of  $\underline{\phi}_{,s}^T \underline{k}_e \underline{\phi}_{,s}$  is zero. That is, the two simultaneous equations represented by Eq. 3.16 are linearly dependent. For such cases it can be assumed that the plastic deformation magnitudes at ends  $i$  and  $j$  (i.e.  $\lambda_i$  and  $\lambda_j$ ) are equal. This assumption avoids the singularity.

Substitution of Eq. 3.17 into Eq. 3.13 yields the elasto-plastic stiffness relationship

$$\underline{dS} = [\underline{k}_e - (\underline{k}_e \underline{\phi}_{,s})(\underline{\phi}_{,s}^T \underline{k}_e \underline{\phi}_{,s})^{-1} (\underline{\phi}_{,s}^T \underline{k}_e)] \underline{dr} \quad (3.18)$$

or

$$\underline{dS} = \underline{k}_t \underline{dr} \quad (3.19)$$

which is the required tangent stiffness relationship.

From equilibrium of axial forces and torques, it follows that

$$dF_i = dF_j \quad (3.20a)$$

and

$$dT_i = dT_j \quad (3.20b)$$

where  $F$  and  $T$  denote axial force and torque, respectively. Eqs. 3.20 must hold in all situations, whether an element yields at one or both

ends and whether there are equal or different yield strengths at the two ends. This requirement suggests that 6 element degrees of freedom can be used, as shown in Fig. 3.5, rather than the 8 degrees of freedom shown in Fig. 3.4. A theory similar to that outlined above can be derived, with the 8-by-2 matrix  $\underline{\phi}_{,s}$  compacted into a 6-by-2 matrix such that Eq. 3.11 still holds. Using this modified matrix, the increments of plastic axial and torsional deformation, as computed by Eq. 3.7, become the combined plastic deformations at both hinges.

Compaction from 8 degrees of freedom to 6 has the advantage of reducing the computational effort. The formulation with 6 degrees of freedom is used in the computer program.

The preceding derivation can be applied to the case with only one plastic hinge as well as the case with two hinges. For an element with one hinge, the column of the matrix  $\underline{\phi}_{,s}$  corresponding to the elastic end becomes zero and is deleted. The vector  $\underline{\lambda}$  then becomes a scalar.

### 3.3 TOLERANCE FOR STIFFNESS REFORMULATION

Each time a new hinge forms or an existing hinge unloads, the element stiffness changes. Moreover, because the yield surface is curved, the stiffness of a yielding element will generally change continuously. This change in stiffness results from differences in the directions of the tangents to the yield surface as the actions at the hinge change, as shown in Fig. 3.6 for successive states. If the angle  $\alpha$  is small, the change in stiffness is small and can be neglected, to

avoid recalculating the stiffness. In the computer program, an option is provided for the user to set a tolerance for the angle  $\alpha$ . If a non-zero tolerance is specified, the element stiffness is reformed only when the change in state is such that the angle between the current state and that at which the stiffness was last reformed exceeds the tolerance. For computational reasons, the program computes  $\alpha$  by adding the absolute values of changes in  $\alpha$  over succeeding steps. This is conservative, but provides a reasonable measure of the true angle.

Tangent stiffnesses for each of the two inelastic parallel elements are computed according to the preceding theory. These stiffnesses are then added together, and further added to the stiffness of the elastic parallel element. The change in stiffness since the preceding stiffness formulation is found, transformed to the global coordinate system, and returned for use by the ANSR base program.

### 3.4 P-DELTA EFFECT

Even for small displacements, changes in the shape of a structure can have a significant effect (the P-delta effect) on the equilibrium of the structure. This effect can be accounted for by adding a geometric stiffness to the element elastic or elasto-plastic stiffness. The geometric stiffness assumed for the element is that for a truss bar in three dimensions, which depends on the axial force only. The geometric stiffness is changed each time the elasto-plastic stiffness changes, using the current axial force, but is otherwise assumed to remain constant.

### 3.5 STATE DETERMINATION FOR AN ELASTO-PLASTIC PARALLEL COMPONENT

Increments of the element end actions are computed from Eq. 3.3. For an element with elastic ends,  $\underline{dv}_e$  is equal to  $\underline{dv}$ . For an element with one or two plastic hinges,  $\underline{dv}_e$  is obtained from Eq. 3.2. The plastic deformation,  $\underline{dv}_p$  is obtained from Eq. 3.4 after first computing  $\underline{\lambda}$  by Eq. 3.17. A negative value of  $\lambda$  for any end indicates that unloading has occurred at that end. If a plastic hinge unloads the matrix,  $\phi_s$  is recalculated without the hinge before applying the above equations.

During any load step, the element end actions may move outside the yield surface. This is not admissible and must be corrected. As an example, consider Fig. 3.7, showing paths which the actions at ends  $i$  and  $j$  might take during a single step. At the beginning of the step the actions are at points A, end  $i$  being elastic and end  $j$  plastic. Assuming linear behavior within the step, the final actions would be at points B. However, these actions are outside the yield surfaces at both ends, which is not correct.

It is assumed, first, that the actions reach points B along straight lines from A to B (that is, the deformations increase proportionately from A to B.) The action points C, along lines AB, are then obtained by computing the portion of the deformation increment which just brings end  $i$  to yield. The actions at end  $j$ , as shown by point C, will be slightly outside the yield surface because the true behavior at end  $j$  is not linear. These actions are scaled to give point D on the yield surface, along the line joining C to the centroid of the yield surface. At this stage, the actions at both ends are on the yield surface, at points D. For the remainder of the deformation

increment, hinges are present at both ends. The increments in element actions are computed, giving points E. Again, the actions are not exactly on the yield surface. A further correction is therefore made by scaling the actions to give points F.

The actions for each of the parallel elements are calculated in this way and are then added together to give the total element actions. Plastic deformations, consisting of plastic hinge rotations, twists, and extensions at each end, are also calculated. Because there are two elasto-plastic elements in parallel, two different sets of plastic deformations are present. The deformations printed by the program are those for the parallel element which yields first. The deformations for the other element are not printed.

### 3.6 END ECCENTRICITY

Rigid end zones may be specified as described in Section 2.6 for the 2-D beam-column element.

### 3.7 RIGID FLOOR SLAVING

The procedure for rigid floor slaving is as described in Section 2.8 for the 2-D beam-column element.

### 3.8 INITIAL FORCES

Initial forces may be specified as described in Section 2.9. The sign convention for positive forces is shown in Fig. 3.8.

### 3.9 RESULTS OUTPUT

The following items are printed for the elements for which results are requested:

- (1) Element number.
- (2) Node numbers at ends i and j.

- (3) Yield codes for ends i and j. This code is as follows for each end:
- 00: both elasto-plastic parallel elements are elastic.
  - 10: only one of the elasto-plastic parallel elements is plastic.
  - 11: both elasto-plastic parallel elements are plastic.
- (4) Moments about the element y and z axes, at each end, and torque and axial force. Positive directions of the actions are shown in Fig. 3.9.
- (5) Plastic rotations about the element y and z axes, plastic twist and plastic axial extension for each end of the element. Positive directions of the plastic deformations are shown in Fig. 3.9.

Envelope values of results (i.e., maximum positive and negative values of results and the corresponding times at which they occur) can also be printed if requested.



#### 4. EXAMPLE ANALYSIS

The single-story building frame shown in Fig. 4.1 was modelled and subjected to horizontal ground motions to demonstrate an application of the two beam-column elements described in this report.

The floor diaphragm is assumed to be rigid, with a center of mass located at the master node. The panel zones at the member intersections are assumed to be rigid and are modelled using the end eccentricity feature. The building was subjected simultaneously to the horizontal ground motions of Fig. 4.2.

All members were assumed to have identical initial elastic stiffnesses. All beams were proportioned with identical bilinear moment-curvature relationships, without moment-axial load yield interaction. The columns were proportioned as follows. Column 1 was assumed to have bilinear action-deformation relationships, and columns 2 and 3 were assumed to have identical trilinear relationships. A listing of the input data for the model is given in Table 4.1. All units are in kips and feet. The building's horizontal mass was lumped at the master node as follows:

$$\text{translational mass} = 1 \text{ kip-sec}^2/\text{ft}$$

$$\text{rotational mass} = 5 \text{ kip-sec}^2$$

Three time history analyses were performed for the duration of the ground acceleration (0-1 sec) as follows:

- (1) Linear elastic step-by-step (24 steps).
- (2) Nonlinear step-by-step with equilibrium correction (24 steps).
- (3) Nonlinear step-by-step with equilibrium correction (48 steps).

The x and z displacement histories of the diaphragm master node are plotted in Figs. 4.3 and 4.4, respectively.



Table 4.1 - Input Data (cont'd)

	3	1	2	2												
1	1574560.		71850.	0.			12.				1000.					
2	574560.		71850.	0.			12.				1000.					
3	574560.		71850.	0.			12.				1000.					
	1728000.		864000.	432000.			5.				7.5					
	2574560.		71850.	17955.			15.				20.					
	574560.		71850.	17955.			15.				20.					
	574560.		71850.	17955.			15.				20.					
	1728000.		864000.	432000.			5.				7.5					
1										-1.						
2										-1.						
1	1	2		5		7	1				1					1
2	5	3		1		7	1				1					1
3	6	4		8		7	2				2					1
	1	1		1		1					.1					.1

STOP

References

1. Mondkar, D. P. and Powell, G. H., "ANSR-I - General Purpose Computer Program for Analysis of Nonlinear Structural Response," Report No. EERC 75-37, Earthquake Engineering Research Center, University of California, Berkeley, 1975.
2. Powell, G. H., "DRAIN-2D User's Guide," Report No. EERC 73-22, Earthquake Engineering Research Center, University of California, Berkeley, 1973.
3. Porter, F. L. and Powell, G. H., "Static and Dynamic Analysis of Inelastic Frame Structures," Report No. EERC 71-3, Earthquake Engineering Research Center, University of California, Berkeley, 1971.

APPENDIX A

USER'S GUIDE

TWO-DIMENSIONAL BEAM-COLUMN ELEMENT

See Chapter 2 for description of element. Number of words of information per element = 199.

The global Y axis is assumed to be vertically upwards if the slaving feature is used.

A1. CONTROL INFORMATION (10I5, 6F5.0) - One card

- Cols. 5: Element group indicator. Punch 2 (to indicate that the group consists of two-dimensional beam-column elements)
- 6-10: Number of elements in this group
- 11-15: Element number of the first element in this group. If blank or zero, assumed to be 1.
- 16-20: Number of different element stiffness types (max 35)
- 21-25: Number of different and eccentricity types (max 15)
- 26-30: Number of different yield interaction surfaces for cross sections (max 40)
- 31-35: Number of different initial force patterns (max 30)
- 36-50: Blank
- 51-55: Initial stiffness damping factor,  $\beta_0$ . If blank or zero, assumed to be equal to the system  $\beta_0$  value input in card C6 of Reference [1].
- 56-60: Current tangent stiffness damping factor,  $\beta_T$ . If blank or zero, assumed to be equal to the system  $\beta_T$  value input in card C6 of Reference [1].

A2. STIFFNESS TYPES (I5,4F10.0,3F5.0,F10.0,2F5.0) - One card for each different stiffness type.

- Cols. 5: Stiffness type number, in sequence beginning with 1.
- 6-15: Young's modulus of elasticity
- 16-25: Strain hardening modulus, as a proportion of Young's modulus. Must be less than 1.
- 26-35: Average cross sectional area
- 36-45: Reference moment of inertia
- 46-50: Flexural stiffness factor  $k_{ij}$
- 51-55: Flexural stiffness factor  $k_{jj}$
- 56-60: Flexural stiffness factor  $k_{ij}$
- 61-70: Effective shear area. Leave blank or punch zero if shear deformations are to be ignored, or if shear deformations have already been taken into account in computing the flexural stiffness factors.
- 71-75: Poisson's ratio (used for computing shear modulus, and required only if shear deformations are to be considered).
- 76-80: Factor by which the major axis bending stiffness is multiplied to give the minor axis bending stiffness. The torsional stiffness is also obtained from this factor. See Section 2.4 for explanation. If zero or blank, zero torsional and minor axis bending stiffnesses are assigned.

A3. END ECCENTRICITIES (I5,6F10.0) - One card for each end eccentricity type.

Omit if there are no end eccentricities. See Fig. 2.6 for explanation. All eccentricities are measured from the node to the element end, in global coordinates.

- Cols. 1- 5: End eccentricity type number, in sequence beginning with 1.
- 6-15:  $X_i = X$  eccentricity at end i.
- 16-25:  $X_j = X$  eccentricity at end j.
- 26-35:  $Y_i = Y$  eccentricity at end i.

36-45:  $Y_j$  = Y eccentricity at end j.

46-55:  $Z_i$  = Z eccentricity at end i.

56-65:  $Z_j$  = Z eccentricity at end j.

A4. CROSS SECTION YIELD INTERACTION SURFACES (2I5,4F10.0,4F5.0) - One card for each yield surface.

See Fig. 2.3 for explanation.

Cols. 1- 5: Yield surface number, in sequence beginning with 1.

10: Yield surface shape code, as follows.

1: Beam type, without P-M interaction

2: Steel I-beam type

3: Reinforced concrete column type

11-20: Positive (sagging) yield moment,  $M_{y+}$ .

21-30: Negative (hogging) yield moment,  $M_{y-}$ .

31-40: Compression yield force,  $P_{yc}$ . Leave blank if shape code = 1.

41-50: Tension yield force,  $P_{yt}$ . Leave blank if shape code = 1.

51-55: M-coordinate of balance point A, as a proportion of  $M_{y+}$ . Leave blank if shape code = 1.

56-60: P-coordinate of balance point A, as a proportion of  $P_{yc}$ . Leave blank if shape code = 1.

61-65: M-coordinate of balance point B, as a proportion of  $M_{y-}$ . Leave blank if shape code = 1.

66-70: P-coordinate of balance point B, as a proportion of  $P_{yc}$ . Leave blank if shape code = 1.

A5. INITIAL ELEMENT FORCE PATTERNS (I5,6F10.0) - One card for each initial force pattern.

Omit if there are no initial forces.  
See Fig. 2.5a for force and moment direction.



- Cols. 1- 5: Pattern number, in sequence beginning with 1.  
6-15: Initial axial force,  $F_i$ .  
16-25: Initial shear force,  $V_i$ .  
26-35: Initial moment,  $M_i$ .  
36-45: Initial axial force,  $F_j$ .  
46-55: Initial shear force,  $V_j$ .  
56-65: Initial moment,  $M_j$ .

A6. ELEMENT GENERATION COMMANDS (11I4,2I3,2I5,2F5,0,I5,F5.0) - As many cards as needed to generate all elements in this group.

Cards must be in order of increasing element number. Cards for the first and last elements must be included. See Note A1 for explanation of generation procedure.

- Cols. 1- 4: Element number, or number of first element in a sequentially numbered series of elements to be generated by this command.  
5- 8: Node number at element end i, NODI  
9-12: Node number at element end j, NODJ  
13-16: Node number, NODK, not collinear with NODI and NODJ, which lies in the member's major bending (local xy) plane. If element generation is used, this node is the same for all elements in the series.  
17-20: Node number increment for element generation. If zero or blank, assumed to be 1.  
21-24: Number of node (diaphragm node) to which end I is slaved, NSI. If not slaved, leave blank.  
25-28: Number of node to which end J is slaved, NSJ. If not slaved, leave blank. If element generation is used, nodes NSI and NSJ are the same for all elements in the series. For a description of the slaving procedure see Section 2.8.  
29-32: Stiffness type number.  
33-36: End eccentricity type number. Leave blank or punch zero if there is no end eccentricity.

- 37-40: Yield surface number for element end i.
- 41-44: Yield surface number for element end j.
- 47: Code for including geometric stiffness. Punch 1 if geometric stiffness is to be included. Leave blank or punch zero if geometric stiffness is to be ignored.
- 50: Time history output code. If a time history of element results is not required for the element covered by this command, punch zero or leave blank. If a time history printout, at the intervals specified in Card D(a) of Reference [1], is required, punch 1.
- 51-55: Initial force pattern number. Leave blank or punch zero if there are no initial forces.
- 56-60: Scale factor to be applied to initial element forces.

NOTE A1. ELEMENT GENERATION

In the element generation commands, the elements must be specified in increasing numerical order. Cards may be provided for sequentially numbered elements, in which case each card specifies one element and the generation option is not used. Alternatively, the cards for a group of elements may be omitted, in which case the data for the missing group is generated as follows:

(1) All elements are assigned the same node k, strength type, interaction surface type, etc. as for the element preceding the missing group of elements.

(2) The node numbers for each missing element are obtained by adding the specified node number increment to the node numbers of each preceding element. The node number increment is that specified for the element preceding the missing set of elements.

In the printout of the element data, generated data is prefixed by an asterisk.

APPENDIX B  
USER'S GUIDE

THREE-DIMENSIONAL BEAM-COLUMN ELEMENT

See Chapter 3 for description of element. Number of words of information per element = 254.

The global y axis is assumed to be vertically upwards if the slaving feature is used.

B1. CONTROL INFORMATION (10I5,6F5.0) - One card.

- Cols. 5: Element group indicator. Punch 3 (to indicate that the group consists of inelastic beam-column elements).
- 6-10: Number of elements in this group.
- 11-15: Element number of the first element in this group. If blank or zero, assumed to be 1.
- 16-20: Number of different element strength types (max 20). If blank or zero, assumed to be 1.
- 21-25: Number of different end eccentricity types (max 15).
- 26-30: Number of different initial force patterns (max 30).
- 31-50: Blank
- 51-55: Initial stiffness damping factor,  $\beta_0$ . If blank or zero, assumed to be equal to the system  $\beta_0$  value input in card C6 of Reference [1].
- 56-60: Current tangent stiffness damping factor  $\beta_T$ . If blank or zero, assumed to be equal to the system  $\beta_T$  value input in card C6 of Reference [1].

B2. STRENGTH TYPES - One set of three cards, as follows, for each strength type.

B2(a). Bending Properties about Local y Axis (I5,5F10.0) - One card.

- Cols. 1-5: Strength type number, in sequence beginning with 1.
- 6-15: Flexural stiffness (effective elastic EI value) K1 about y axis. See Section 3.1 Figure 3.2 for explanation.
- 16-25: Flexural stiffness K2 about y axis.

26-35: Flexural stiffness K3 about y axis.

36-45: Yield moment YS1 about y axis.

46-50: Yield moment YS2 about y axis.

B2(b). Bending Properties about Local z Axis (5X,5F10.0) - One card.

Cols. 1-5: Blank

6-55: Bending stiffnesses and yield moments about z axis, in same sequence as in card B2(a).

B2(c). Torsional Properties (5X,5F10.0) - One card.

Cols. 1-5: Blank

6-55: Torsional stiffnesses (effective GJ) and yield moments, in same sequence as in card B2(a).

B2(d). Axial Properties (5X,6F10.0) - One card.

Cols. 1-5: Blank

6-55: Axial Stiffnesses (effective EA) and yield forces, in same sequence as in Section B2(a).

55-65: Yield strength YS3 (input as a positive value). See Fig. 2.3. If blank or zero, assumed to be equal to YS1. This allows for different tension and compression yield forces. Note that YS4 can not be specified, because  $YS4-YS3 = YS2-YS1$ .

B3. END ECCENTRICITIES (15,6F10.0) - One card for each end eccentricity type.

Omit if there are no end eccentricities. See Fig. 2.6 for explanation. All eccentricities are measured from the node to the element end, in global coordinates.

Cols. 1-5: End eccentricity type number, in sequence beginning with 1.

6-15:  $X_i = X$  eccentricity at end i.

16-25:  $X_j = X$  eccentricity at end j.

26-35:  $Y_i = Y$  eccentricity at end i.

36-45:  $Y_j = Y$  eccentricity at end j.

46-55:  $Z_i = Z$  eccentricity at end i.

56-65:  $Z_j = Z$  eccentricity at end j.

B4. INITIAL ELEMENT FORCE PATTERNS (15,6F10.0) - One card for each initial force pattern.

Omit if there are no initial forces. See Fig. 3.8 for force and moment direction.

Cols. 1-5: Pattern number, in sequence beginning with 1.

6-15: Initial moment,  $M_{yy}$  at end i.

16-25: Initial moment,  $M_{zz}$  at end i.

26-35: Initial moment,  $M_{yy}$  at end j.

36-45: Initial moment,  $M_{zz}$  at end j.

46-55: Initial Axial force, F.

56-65: Initial Torque,  $M_{xx}$ .

B5. ELEMENT GENERATION COMMANDS (10I5,F10.0) - As many cards as needed to generate all elements in this group.

Cards must be entered in order of increasing element number. Cards for the first and last element must be included. See Appendix A, Note A1 for explanation of generation procedure.

Cols. 1-5: Element number, or number of first element in a sequentially numbered series of elements to be generated by this card.

6-10: Node number at element end i.

11-15: Node number at element end j.

16-20: Node number increment for element generation. If zero or blank, assumed to be 1.

21-25: Number of a third node, k, lying in the xy plane, for definition of the local y axis orientation. Leave blank if y axis orientation is to be assigned automatically. See Note B1 for explanation of y axis orientation.

26-30: Number of node (diaphragm node) to which end I is slaved, NSI. If not slaved, leave blank.

- 31-35: Number of node to which end j is slaved, NSJ. If element generation is used, nodes NSI and NSJ are the same for all elements in the series. For a description of the slaving procedure, see Section 2.8.
- 36-40: Strength type number at element end i.
- 41-45: Strength type number at element end j.
- 46-50: End eccentricity type number. Leave blank or punch zero if there is no end eccentricity.
- 51-55: Initial Force pattern number. Leave blank or punch zero if there are no initial forces.
- 60: Interaction surface type. Punch 1 for elliptical yield surface, or 2 for parabolic yield surface.
- 65: Geometric stiffness code. Leave blank or punch zero if geometric stiffness is not to be included. Punch 1 if geometric stiffness is to be included.
- 70: Time history output code. Leave blank or punch zero for no time history. Punch 1 if time history output is required, at the intervals specified in card D(a) of Reference [1].
- 71-80: Stiffness reformulation angle tolerance,  $\alpha$  (in radians). See Section 3.3 for explanation.

NOTE B1. ELEMENT y AXIS ORIENTATION

The element y axis is oriented in a plane passing through nodes i and j and a third node, k, specified in the element generation card, as shown in Fig. 3.1. If a third node is not specified, the element y axis is assumed to be in a plane which is parallel to the global Y axis and passes through nodes i and j (typically the vertical plane containing the element). In effect node k is given a very large positive Y coordinate. If the element is parallel to the global Y axis, then this default procedure does not work, and the element y axis is assumed to be parallel to the global X axis.

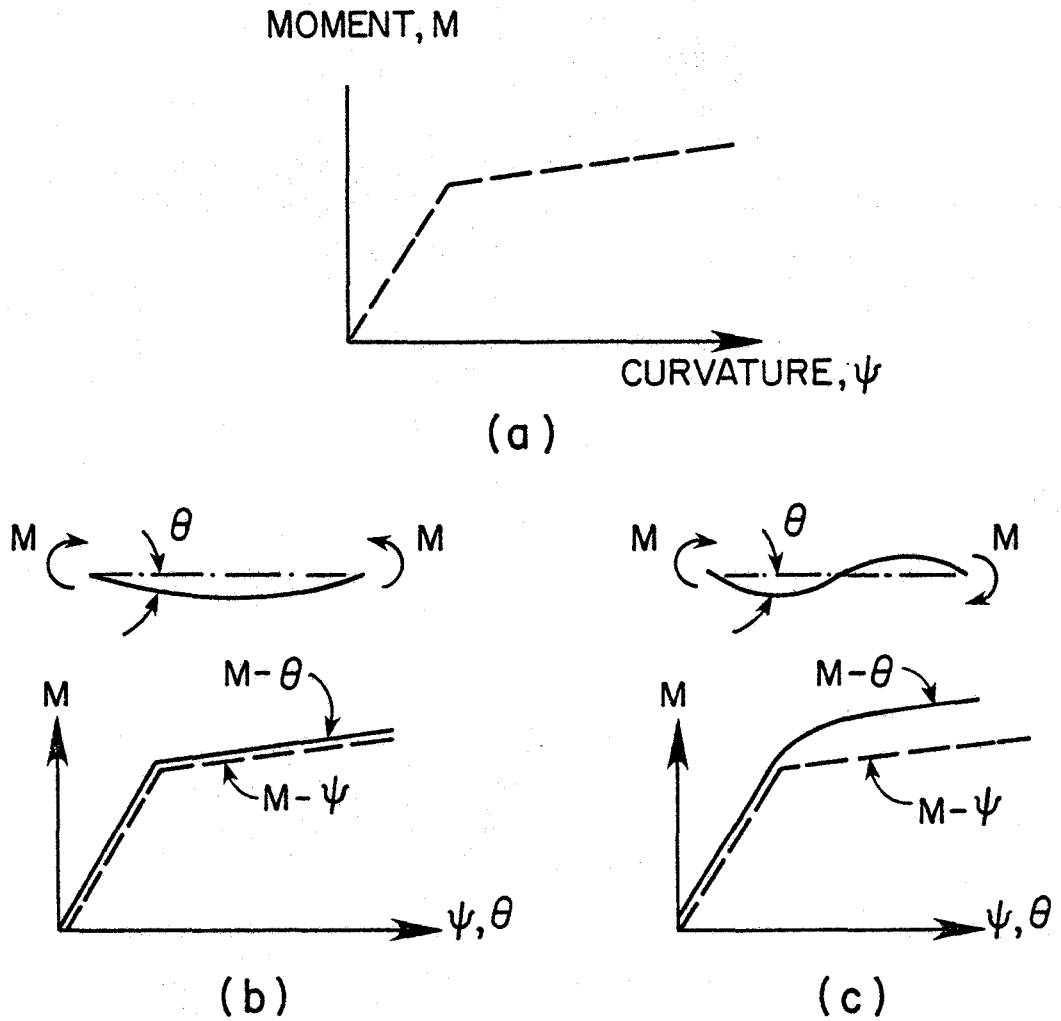


FIG. 2.1 MOMENT-CURVATURE AND MOMENT-ROTATION RELATIONSHIP



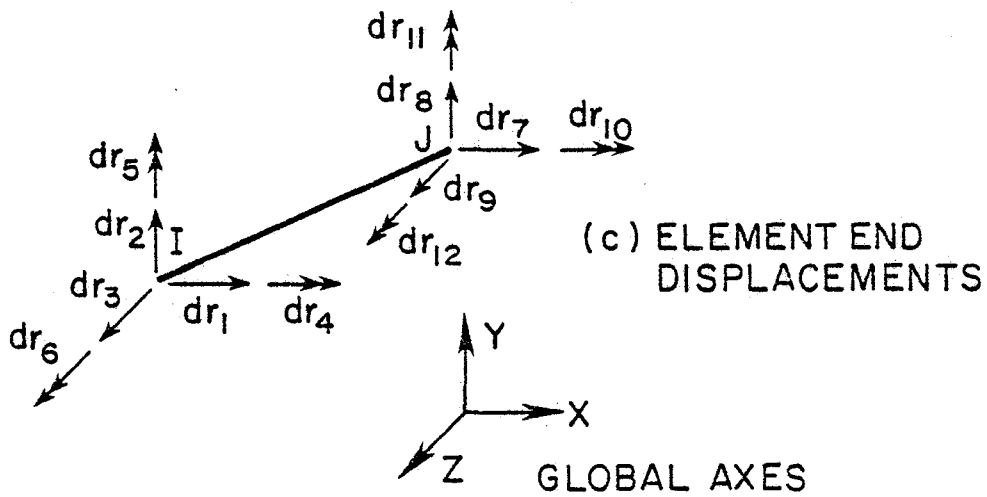
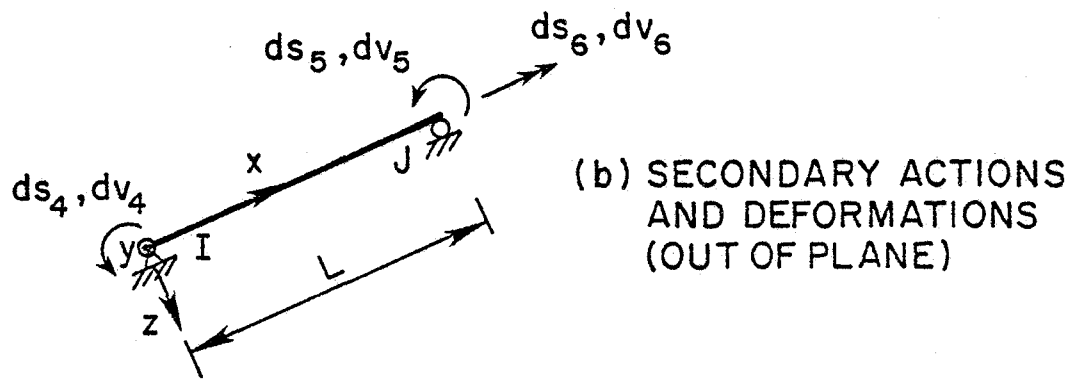
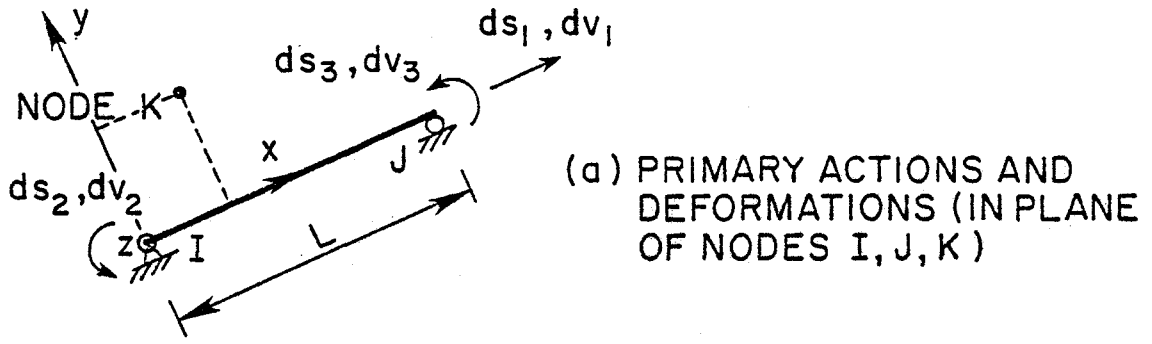
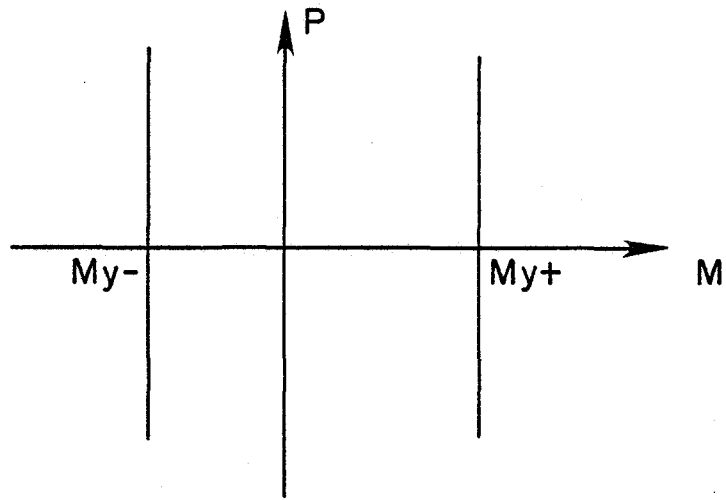
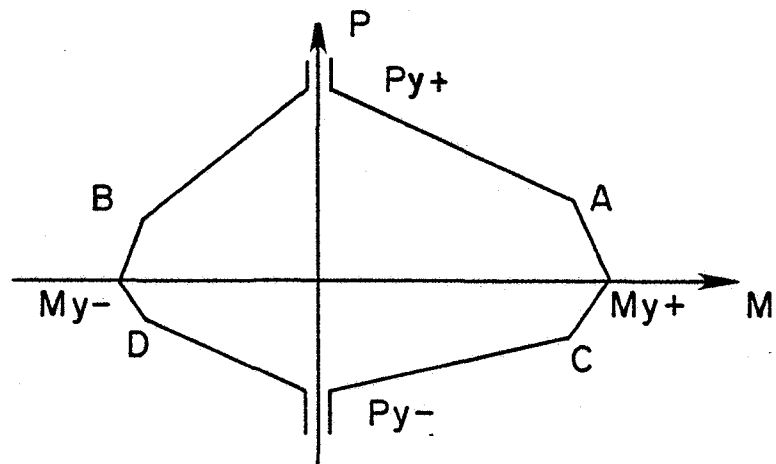


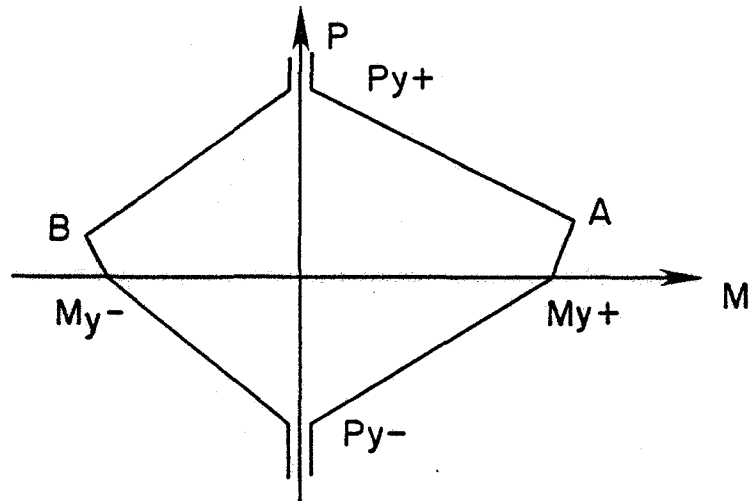
FIG.2.2 DEFORMATIONS AND DISPLACEMENTS



(a) SHAPE CODE = 1



(b) SHAPE CODE = 2



(c) SHAPE CODE = 3

FIG.2.3 YIELD INTERACTION SURFACES

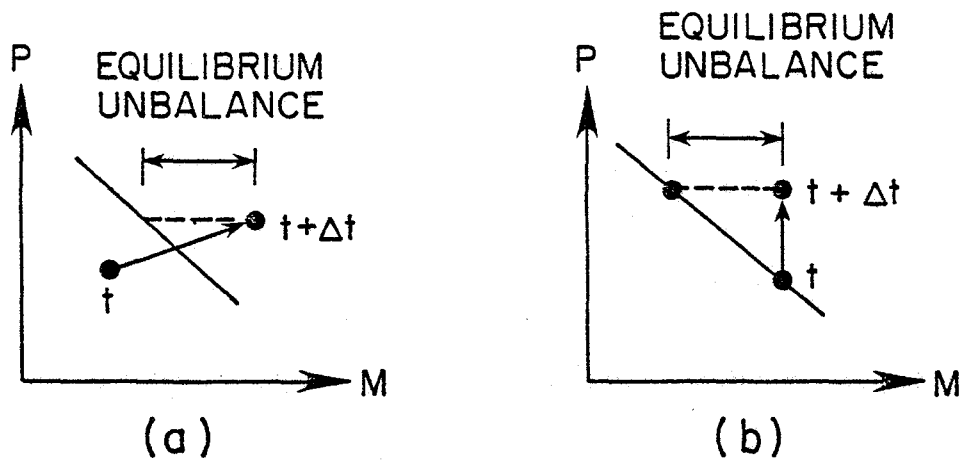


FIG. 2.4 EQUILIBRIUM CORRECTION FOR YIELD SURFACE OVERTSHOT

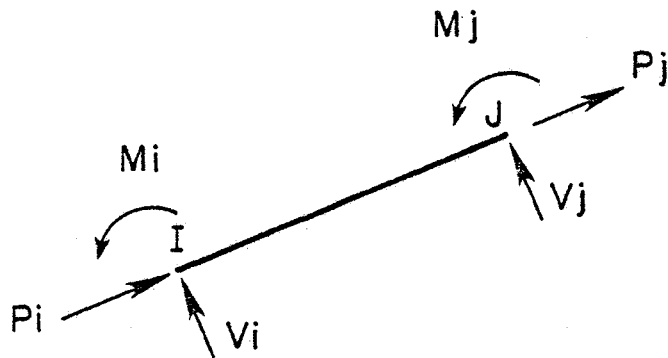


FIG. 2.5 POSITIVE DIRECTION FOR INITIAL FORCES AND OUTPUT RESULTS

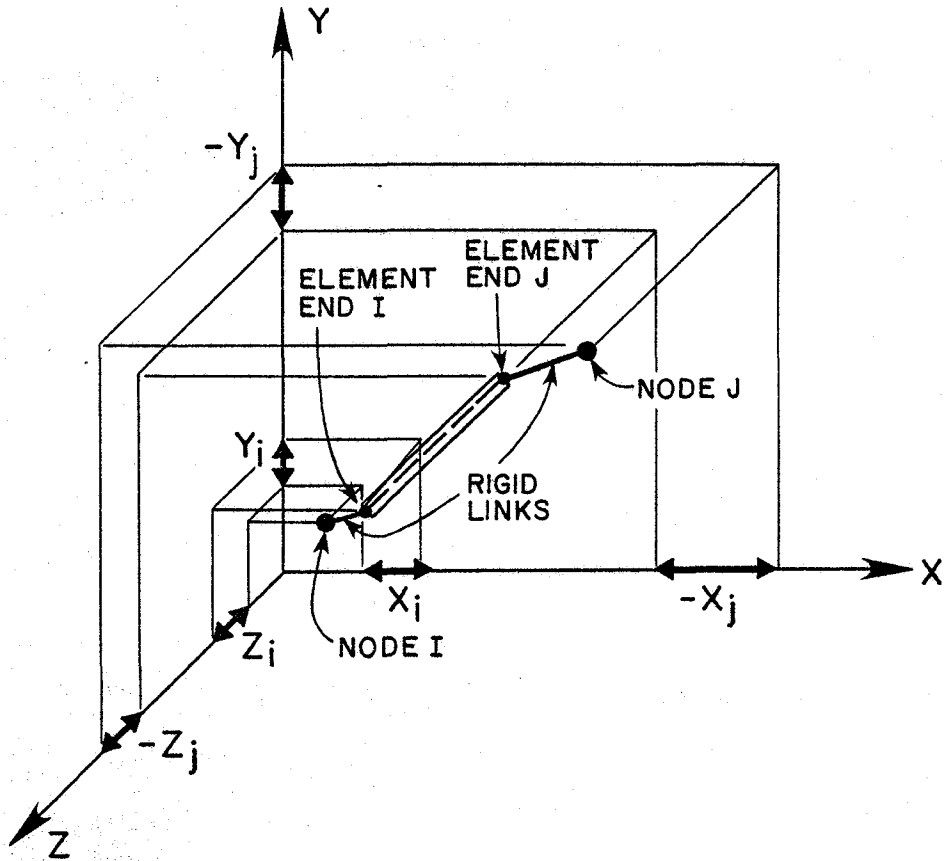


FIG. 2.6 END ECCENTRICITIES

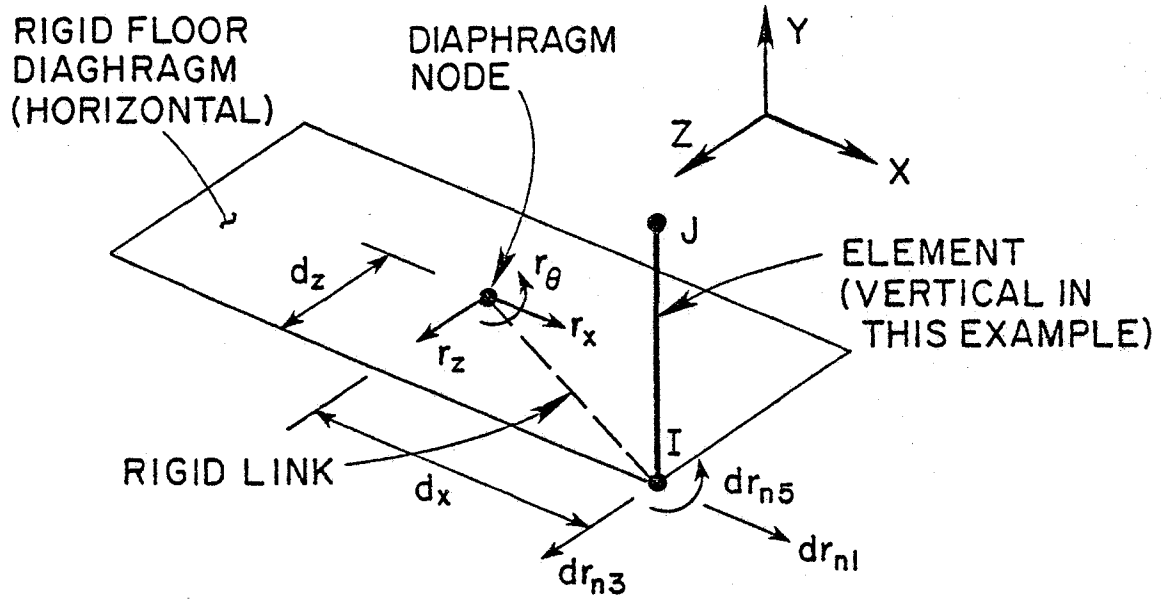
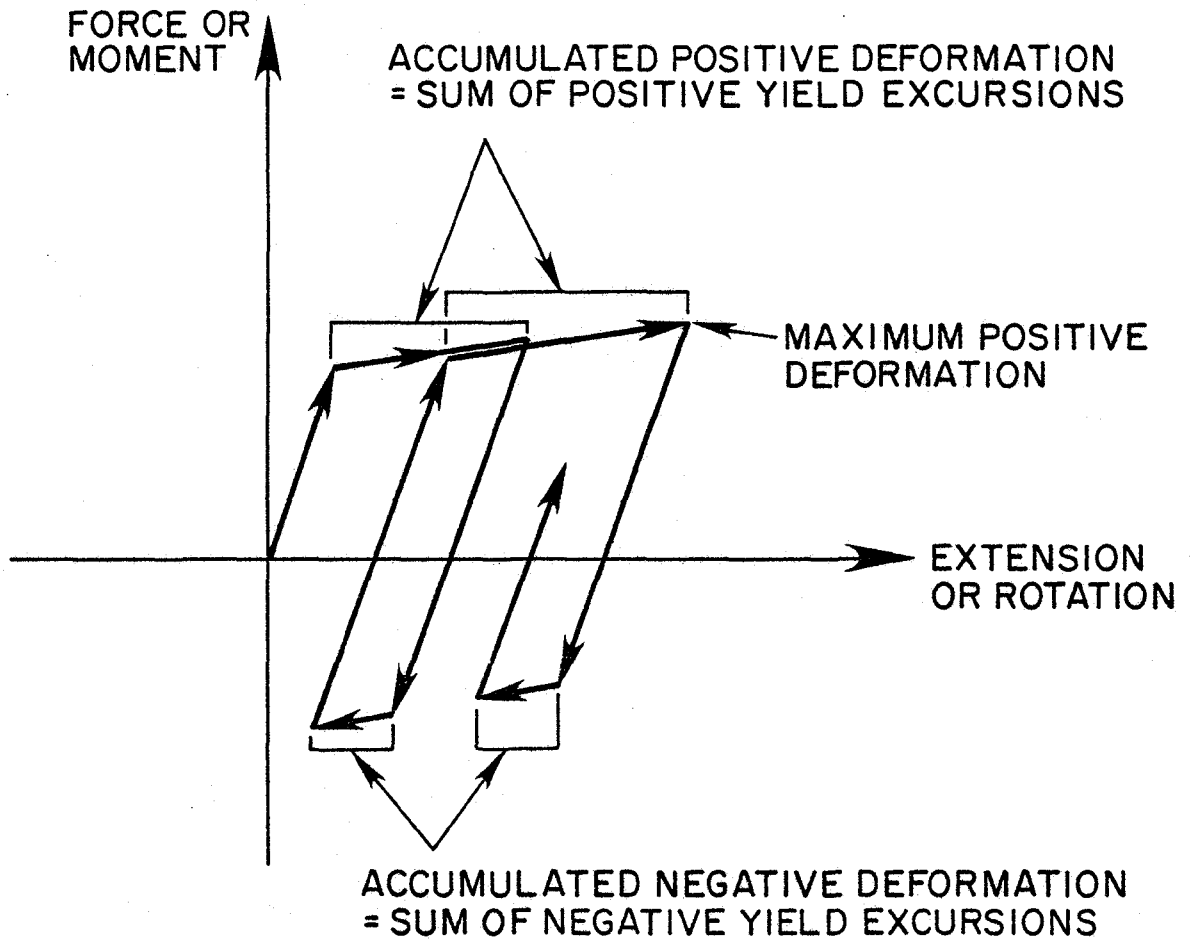


FIG. 2.7 RIGID FLOOR DIAPHRAGM MODELLING



NOTE THAT MAXIMUM NEGATIVE DEFORMATION IS ZERO,  
ALTHOUGH ACCUMULATED NEGATIVE DEFORMATION IS  
NOT ZERO

FIG. 2.8 PROCEDURE FOR COMPUTATION OF  
ACCUMULATED PLASTIC DEFORMATIONS

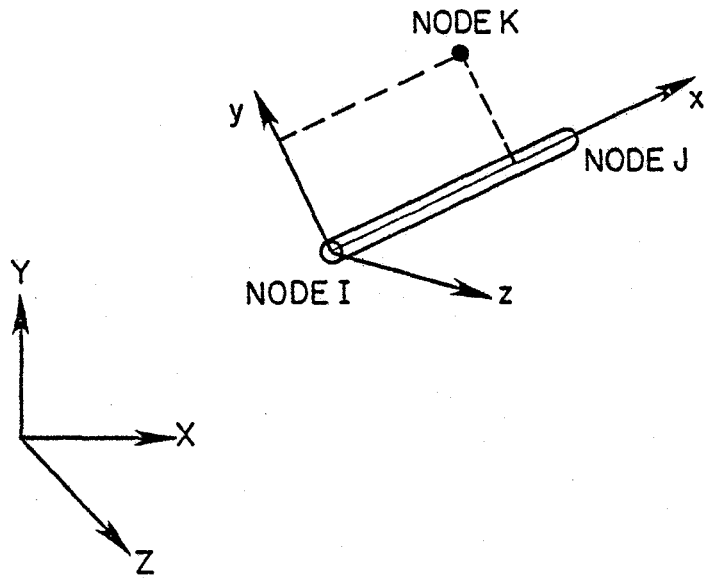


FIG. 3.1 ELEMENT COORDINATE AXIS

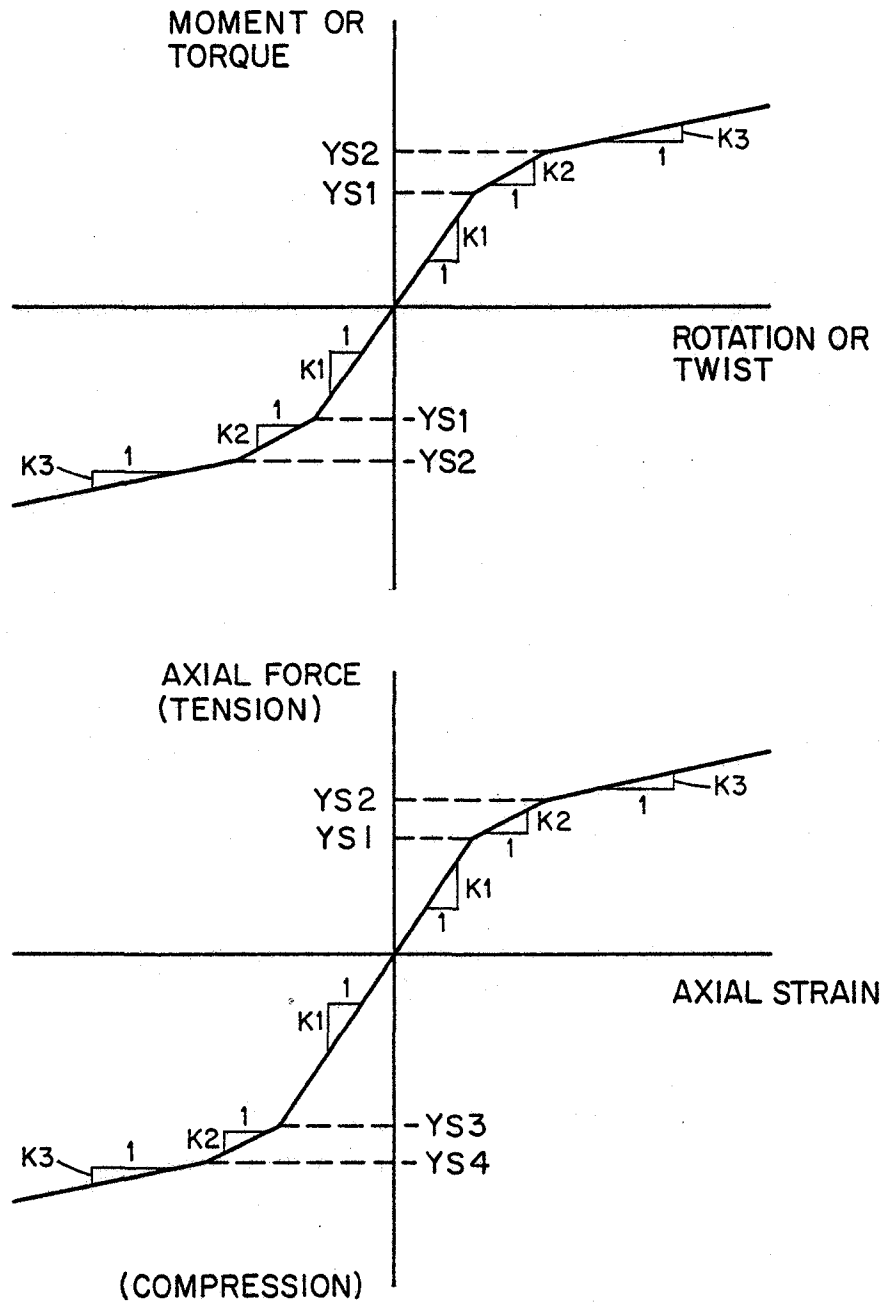
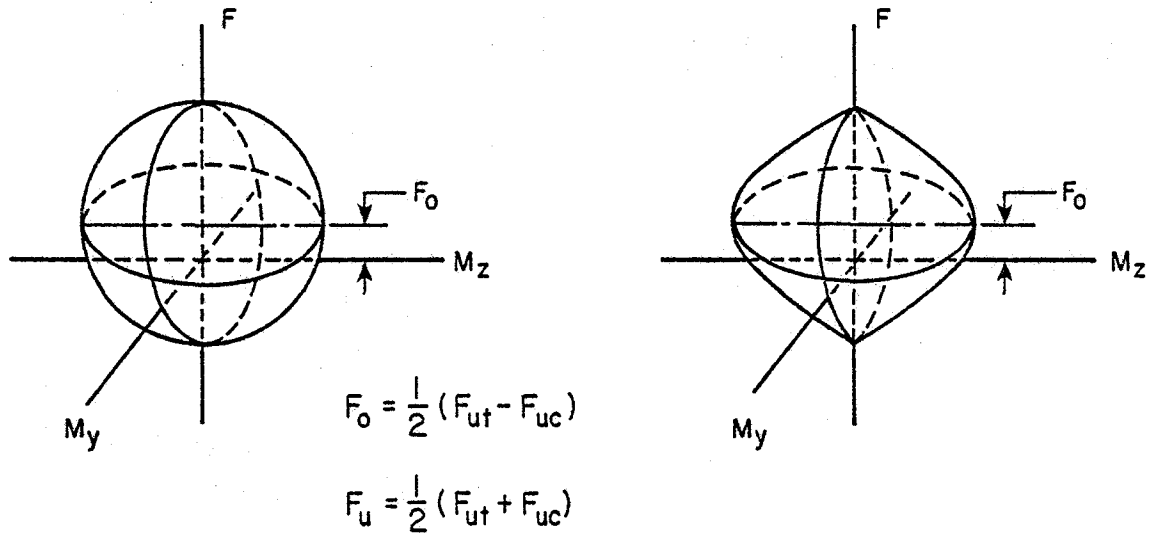


FIG. 3.2 ACTION-DEFORMATION RELATIONSHIPS





$$\phi = \left( \frac{M_y}{M_{yu}} \right)^2 + \left( \frac{M_z}{M_{zu}} \right)^2 + \left( \frac{T}{T_u} \right)^2 + \left( \frac{F - F_0}{F_u} \right)^2 = 1$$

INTERACTION SURFACE TYPE 1

$$\phi = \left[ \left( \frac{M_y}{M_{yu}} \right)^2 + \left( \frac{M_z}{M_{zu}} \right)^2 + \left( \frac{T}{T_u} \right)^2 \right]^{1/2} + \left( \frac{F - F_0}{F_u} \right)^2 = 1$$

INTERACTION SURFACE TYPE 2

$M_y$ ,  $M_z$ ,  $T$  and  $F$  denote bending moments about the element  $y$  and  $z$  axes, torque and axial force respectively. Subscript  $u$  denotes ultimate.  $F_{ut}$  and  $F_{uc}$  are axial ultimate strengths in tension and compression. Note that the interaction surfaces above are for a particular value of torque,  $T$ .

FIG. 3.3 INTERACTION SURFACES FOR ELASTO-PLASTIC COMPONENT

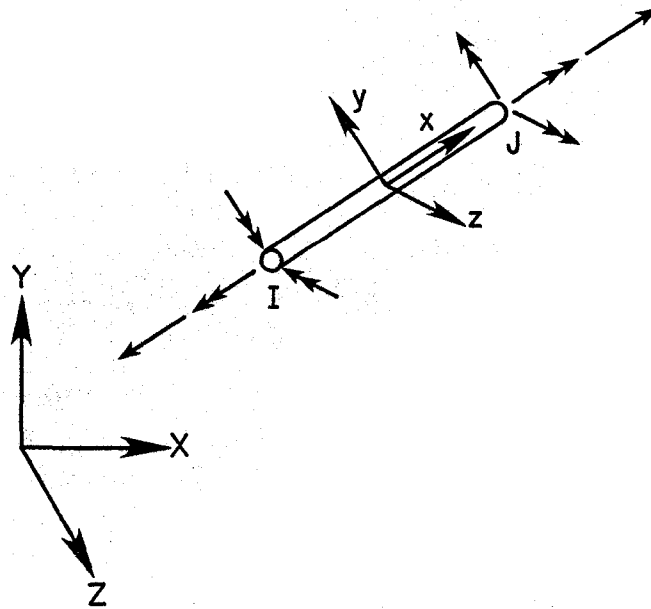


FIG. 3.4 ELEMENT NODAL FORCES AND DEFORMATIONS

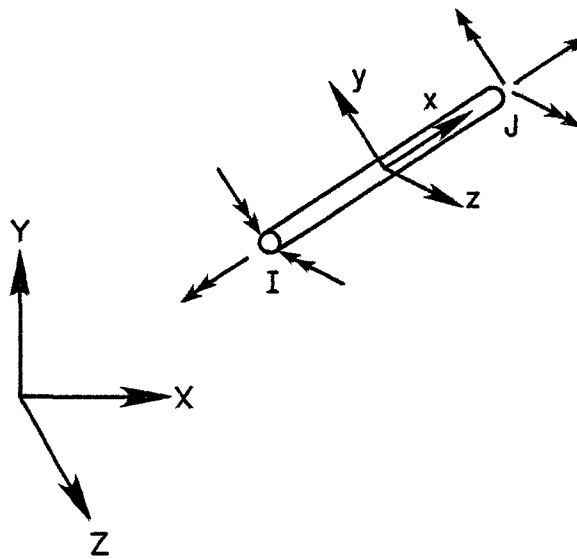


FIG. 3.5 COMPACTED ELEMENT NODAL FORCES AND DEFORMATIONS

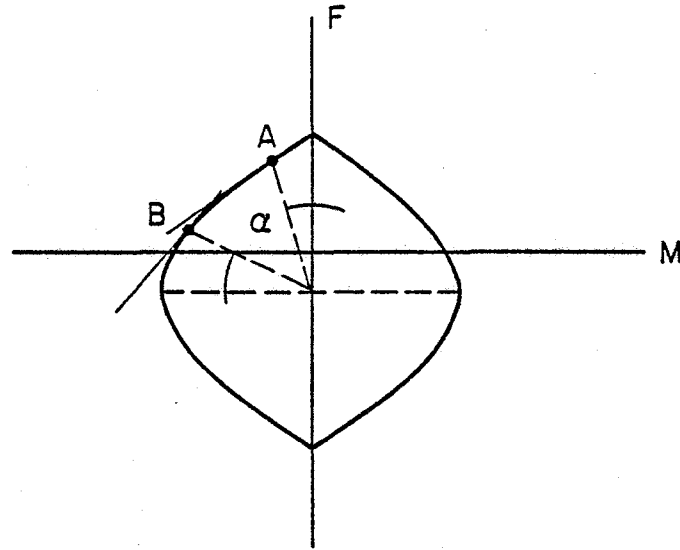
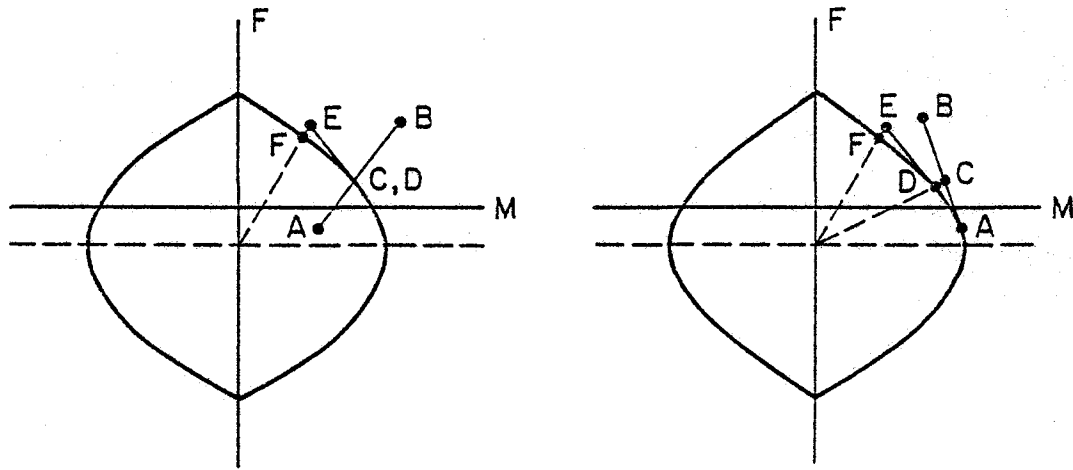


FIG. 3.6 STIFFNESS REFORMULATION ANGLE



YIELD SURFACE AT END I

YIELD SURFACE AT END J

FIG. 3.7 STATE DETERMINATION EXAMPLE

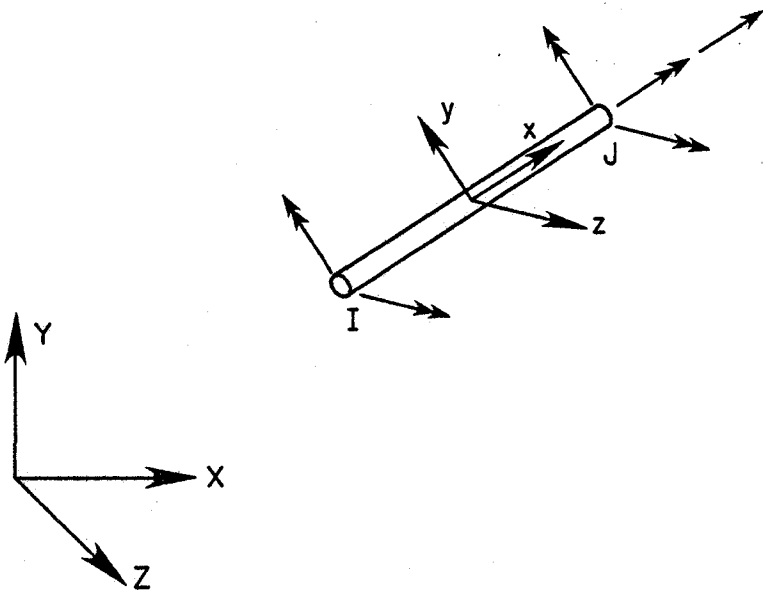


FIG. 3.8 POSITIVE DIRECTION OF INITIAL ELEMENT ACTIONS

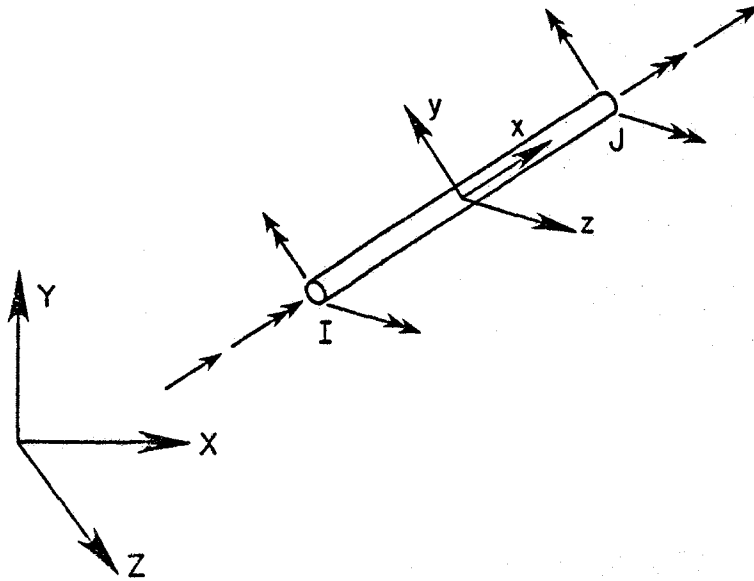


FIG. 3.9 POSITIVE DIRECTION OF NODAL FORCES AND DEFORMATION FOR OUTPUT RESULTS

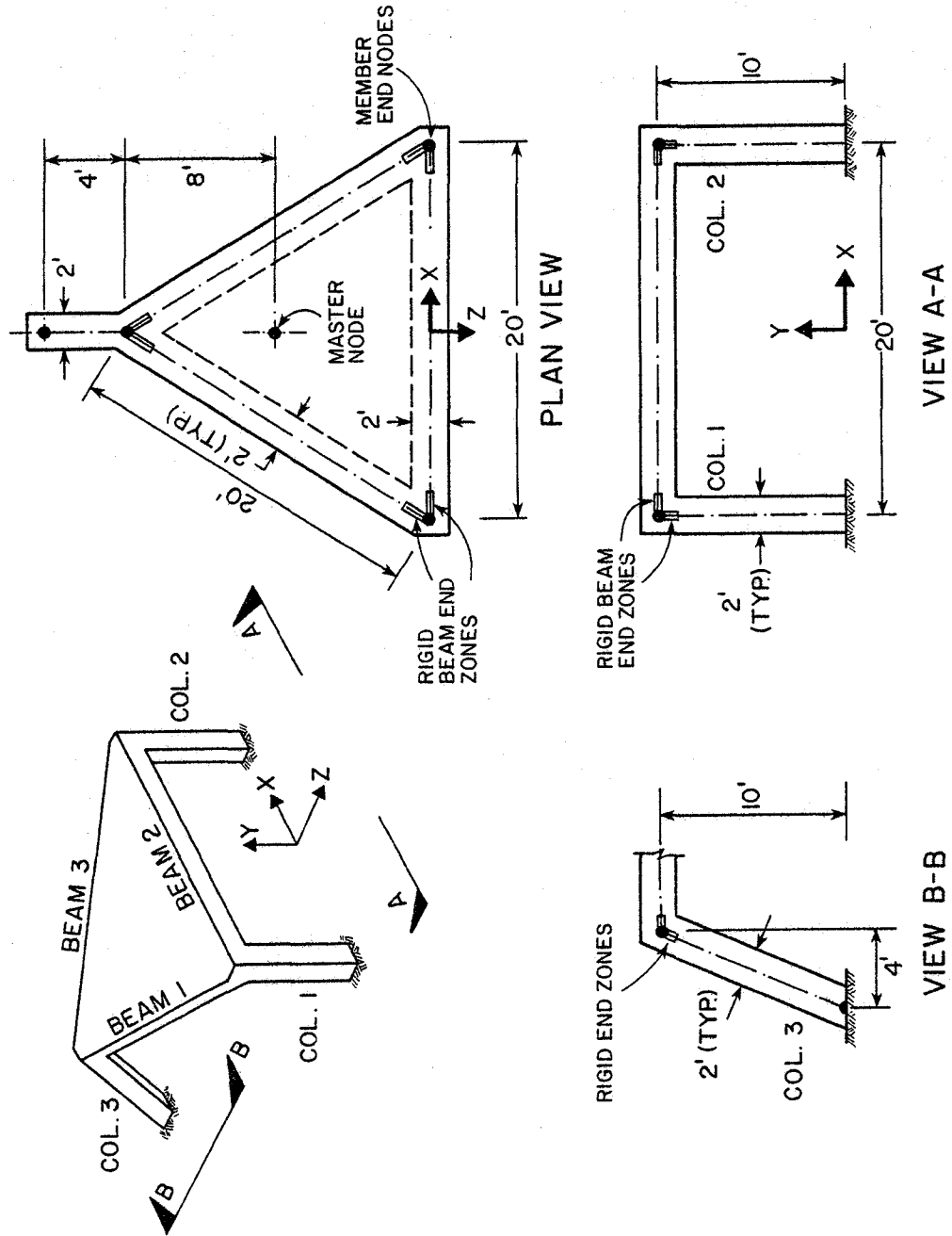


FIG. 4.1 EXAMPLE STRUCTURE

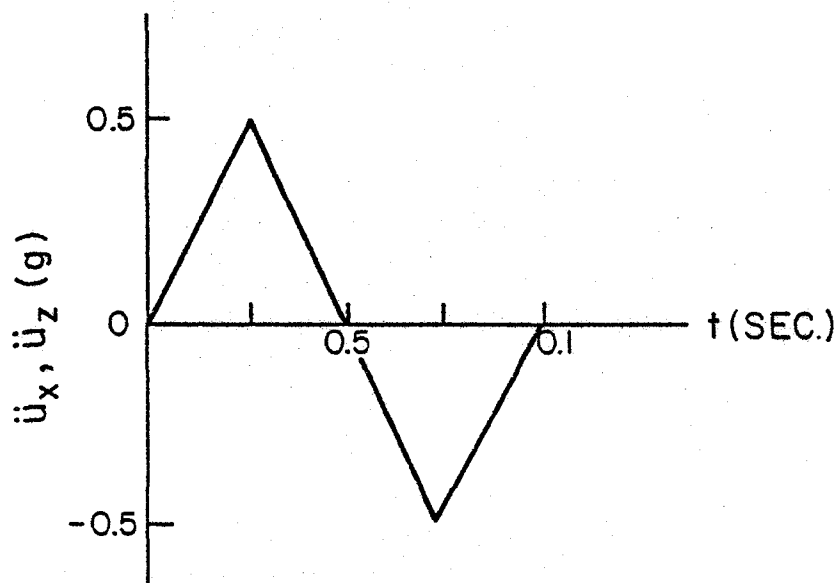


FIG. 4.2 X and Z GROUND ACCELERATION

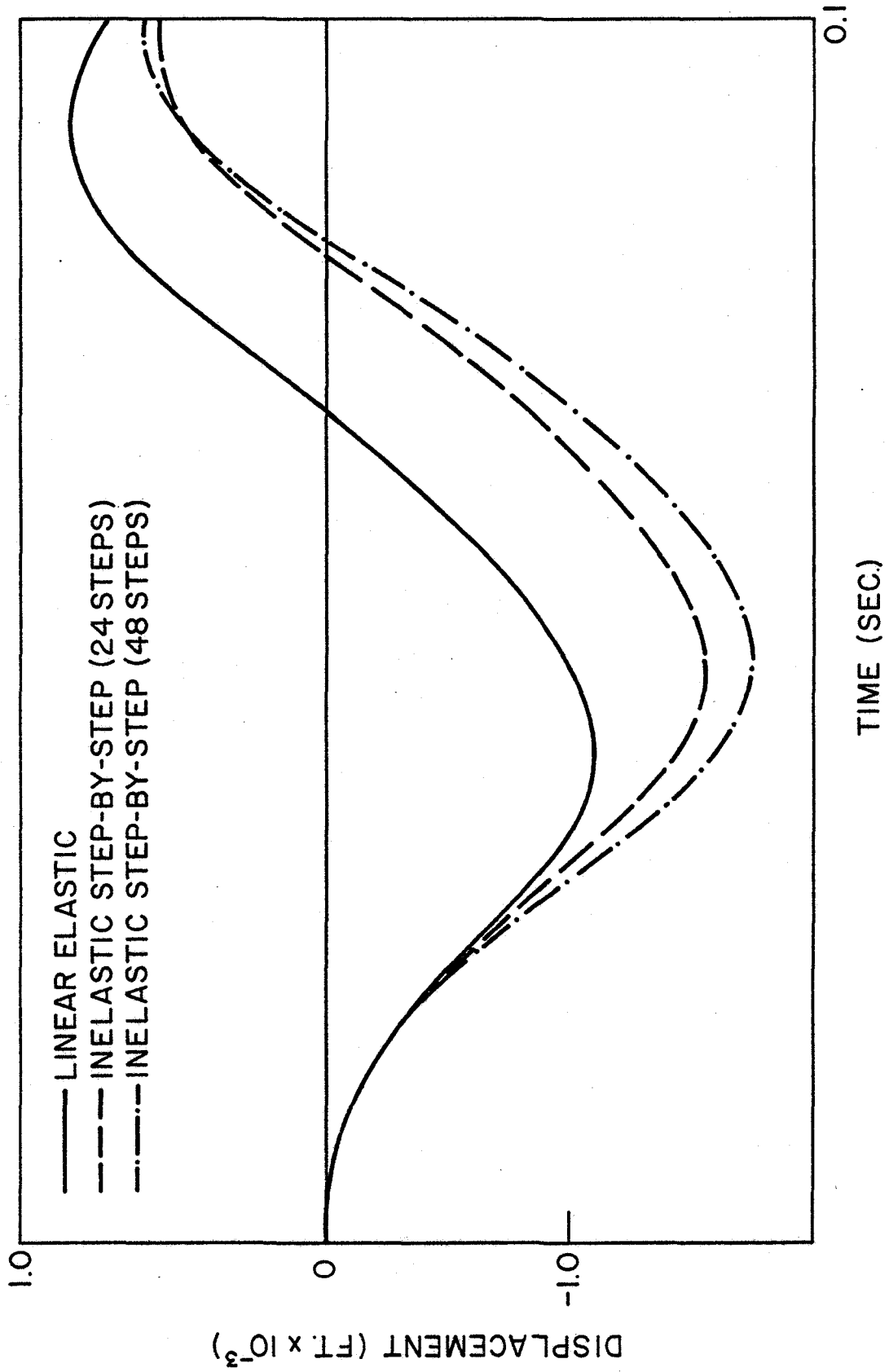


FIG. 4.3 X DISPLACEMENT OF MASTER NODE



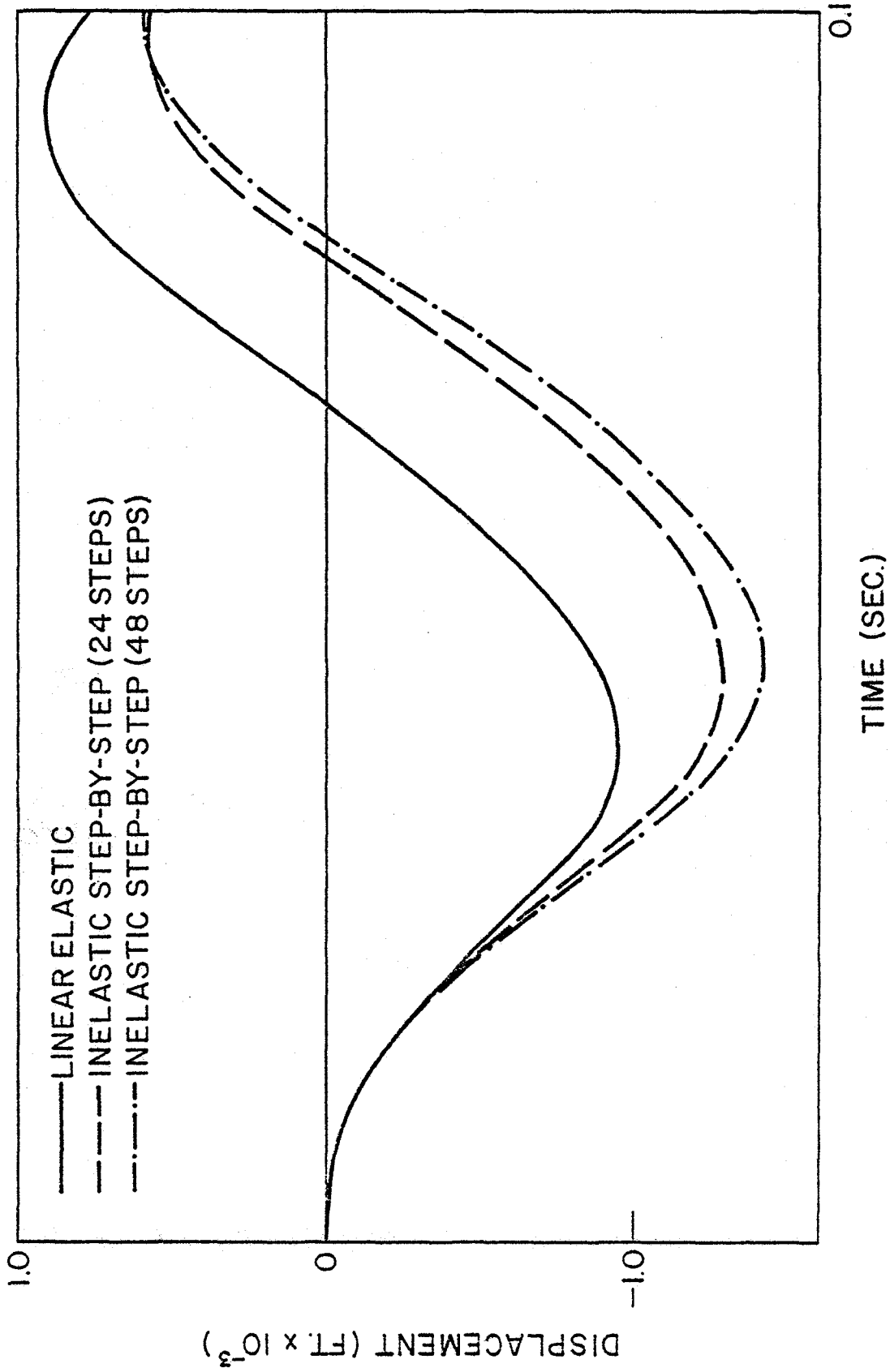


FIG. 4.4 Z DISPLACEMENT OF MASTER NODE



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