MEASURES OF DUCTILITY FOR STRUCTURAL WALLS IN EARTHQUAKE-RESISTANT MULTISTORY BUILDINGS

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This paper attempts to establish, for the particular case of slender structural walls subject predominantly to flexure, a relationship among displacement, rotational and curvature ductilities which refer to an entire structure, to critical regions within a structure, and to critical sections within those regions, respectively. The relationships are developed on the basis of a simplified model. Limited results correlating displacement and rotational ductilities, and based on actual seismic analyses of isolated structural walls, are presented. Design implications of the reported information are briefly discussed. The results of this study represent part of the data obtained during an analytical investigation carried out to determine the force and deformation requirements in structural walls and wall systems required to withstand strong earthquake excitation. The dynamic analyses were carried out using the computer program DRAIN-2D.

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PORTLAND CEMENT ASSOCIATION
5420 Old Orchard Road, Skokie, Illinois 60076 / Area Code 312 / 966-6200

July 8, 1977

Professor H. Y. Fang
Geotechnical Engineering Division
Lehigh University, Building 13
Bethlehem, Pa. 18015

Dear Professor Fang:

Enclosed please find the original manuscript and a xerox copy of the paper "Measures of Ductility for Structural Walls in Earthquake Resistant Multistory Buildings" by S. K. Ghosh, A. T. Derecho, M. Iqbal and M. Fintel, which has been accepted by the Technical Committee of the Central American Conference on Earthquake Engineering.

I will look forward to seeing you at the conference in El Salvador.

Yours sincerely,

SKGhosh

Senior Structural Engineer
Advanced Engineering Services

S. K. Ghosh/sr

cc: M. Fintel
A. T. Derecho
M. Iqbal
W. G. Corley

MEASURES OF DUCTILITY FOR STRUCTURAL WALLS IN EARTHQUAKE RESISTANT MULTISTORY BUILDINGS

S. K. Ghosh
Senior Structural Engineer
Advanced Engineering Services
Portland Cement Association,
Skokie, Illinois

A. T. Derecho
Manager
Structural Analytical Services
Portland Cement Association,
Skokie, Illinois

M. Iqbal
Structural Engineer
Structural Analytical Services
Portland Cement Association,
Skokie, Illinois

M. Fintel
Director
Advanced Engineering Services
Portland Cement Association,
Skokie, Illinois

ABSTRACT

This paper attempts to establish, for the particular case of slender structural walls subject predominantly to flexure, a relationship among displacement, rotational and curvature ductilities which refer to an entire structure, to critical regions within a structure and to critical sections within those regions, respectively. The relationships are developed on the basis of a simplified model, and are examined in the light of results of seismic analysis of isolated structural walls.

INTRODUCTION

In most reinforced concrete structures, it is uneconomical to resist the forces, generated during strong seismic ground excitations, within the limits of elastic response of the structures. It is accepted that during rare ground accelerations of large intensity, yielding and consequent plastic deformations may occur at some or all critical areas within the structure. Because prevention of collapse is a fundamental design requirement, it is necessary to ensure that post-elastic deformations in all parts of the structure can occur while the lateral and vertical load capacities of the structure are substantially maintained, [Paulay (1975)].

The ability of a structure to perform past the elastic limit has often been measured in terms of ductility. In general, ductility in reinforced concrete structures is defined as the ratio of a specified distortion at a particular stage of the loading to that at the onset of yielding. More specific definitions of ductility depend on whether the entire structure, or some critical region(s) within it, or some critical section(s) within those regions is being considered. This paper attempts to establish relationships among displacement, rotation and curvature ductilities which relate to the structure, the critical region and the section, respectively. The particular case of slender structural (shear) walls, which act predominantly in flexure, is considered. The relationships are developed on the basis of a simplified model, along lines suggested by Paulay and Uzumeri (1975). Limited results correlating displacement and rotation ductilities, and based on actual seismic analyses of isolated structural walls, are presented. Design implications of the reported information are briefly discussed.

DEFINITIONS OF DUCTILITY

To assess the overall behavior of a structural system, such as a highrise building, it is convenient to use the term displacement ductility. This is the ratio of the lateral displacement, Δ_u, corresponding to a suitably defined ultimate load stage, to the deflection at a suitably defined yield load stage, Δ_y:

$$\mu_\Delta = \Delta_u / \Delta_y \qquad (1)$$

Both deflections are measured or calculated at some convenient location, usually at the roof level, of the structure.

For the purposes of design and proportioning, it is almost necessary to express ductility in terms of sectional properties. Accordingly, the term curvature ductility, which is the ratio of the curvature at a suitably defined ultimate stage, φ_u, to the curvature at yield, φ_y, is commonly used:

$$\mu_\varphi = \varphi_u / \varphi_y \qquad (2)$$

While curvature ductility is easily derived from first principles, it is hardly possible to measure it in experiments. What can be measured is the rotational ductility of the critical region, which may be defined as the ratio of the rotation occurring within the region at the ultimate stage, θ_u, to the rotation of yield, θ_y:

$$\mu_\theta = \theta_u / \theta_y \qquad (3)$$

Since most inelastic action in a structure takes place within certain critical regions, the rotational ductility is particularly useful as an index of such inelastic action.

DISPLACEMENT AND ROTATIONAL DUCITILITIES

The slender cantilever wall of Fig. 1(a) is considered in an attempt to establish a relationship between displacement and rotational ductilities. The wall carries a single lateral point load at the roof level, and is of uniform section, having flexural rigidity EI. It is assumed that all sections of the wall respond according to the moment-curvature diagram of Fig. 1(b), and that shear distortions are negligible. Fig. 1(d) shows the deflected shape of the wall at the attainment of yield moment at the critical base section; the corresponding bending moment and curvature diagrams are shown in Fig. 1(c). The inelastic deformations subsequent to yielding may be looked upon as resulting from the rotation θ_p in the plastic hinge with length ℓ_p, as shown in Fig. 1(e) and (c).

From Figs. 1(c) and (d), the rotation over the length ℓ_p, corresponding to the attainment of My at base, is:

$$\theta_y = \frac{\ell_p}{2}\left(\frac{M_y}{EI} + \frac{M_y}{EI}\frac{H-\ell_p}{H}\right)$$

$$= \varphi_y \ell_p (1 - \ell_p/2H) \qquad (4)$$

where $\varphi_y = M_y/EI$ is the yield curvature. Also, from Fig. 1(e):

$$\theta_p = \frac{\Delta_p}{H - \ell_p/2} = \frac{\Delta_u - \Delta_y}{H - \ell_p/2} = \frac{\mu_\Delta - 1}{H - \ell_p/2}\Delta_y$$

$$= \frac{\mu_\Delta - 1}{H - \ell_p/2}\ \frac{M_y H^2}{3EI} = \frac{\mu_\Delta - 1}{3(1 - \ell_p/2H)}\ \varphi_y H \qquad (5)$$

Thus, rotational ductility is:

$$\mu_\theta = \frac{\theta_u}{\theta_y} = 1 + \frac{\theta_p}{\theta_y} = 1 + \frac{\mu_\Delta - 1}{3\frac{\ell_p}{H}\ (1 - \frac{\ell_p}{2H})^2} \qquad (6)$$

or,

$$\frac{\mu_\Delta - 1}{\mu_\theta - 1} = 3\frac{\ell_p}{H}\ (1 - \frac{\ell_p}{2H})^2 \qquad (6a)$$

As is obvious from Fig. 1, the above analysis neglects for simplicity the elastic deformations beyond the onset of yielding at the base. The relationship between μ_Δ and μ_θ is seen to be a function of the plastic hinge length, ℓp. It is very difficult to accurately determine this length in a practical situation. Typical values of plastic hinge length for the model shown in Fig. 1(a), based on the suggestion of Mattock (1967), are:

$$\ell_p = 0.4\ \ell_w + 0.05\ H \qquad (7)$$

where ℓ_w and H are defined in Fig. 1(a). ℓ_p values based on the suggestion of Sawyer (1964) are:

$$\ell p = 0.2\ell_w + 0.075\ H \qquad (8)$$

Eqs. (7) and (8) are likely to be conservative for walls with small height to width ratios, because in these the effect of shear becomes more dominant and the extent of yielding in the flexural tension steel over the height is larger due to diagonal cracking. For these cases, more representative values of ℓ_p may be:

$$\ell_p = \ell_w \qquad (9)$$

Table 1 lists the values of ℓ_p/H according to Eqs. (7), (8) and (9) for the practical range of values of the slenderness ratio, H/ℓ_w. The practical range of variation of $\ell p/H$ is seen to be from 0.06 to 0.5. For this range of values of ℓ_p/H, the values of μ_θ corresponding to $\mu_\Delta = 2$, 3, 4, 5, and 6 are listed in Table 2. The values of μ_θ in five rows of Table 2, corresponding to $\ell_p/H = 0.1, 0.2, 0.3, 0.4$, and 0.5, are also plotted in Fig. 2 against the corresponding values of μ_Δ. It can be seen that for any value of ℓ_p/H, $\mu_\Delta - 1$ and $\mu_\theta - 1$ are linearly related, the function of ℓ_p/H, relating the two quantities being as given in Eq. 6(a). Fig. 3 is a plot of this function over the practical range of variation of ℓ_p/H. The discrete points marked by symbols in Fig. 2 should be disregarded for the moment.

ROTATIONAL AND CURVATURE DUCTILITIES

If the distribution of curvatures over the plastic hinge length at the ultimate stage is assumed to be uniform, then a corresponding average curvature may be defined as follows:

$$\varphi_{uav} = \theta_u/\ell_p = \ell_p\ (\varphi_u + \varphi_y)/2\ell_p = (\varphi_u + \varphi_y)/2 \qquad (10)$$

from Fig. 1(c). An average yield curvature may likewise be defined as follows:

$$\varphi_{yav} = \theta_y/\ell_p = \varphi_y\ (1 - \ell_p/2H) \qquad (11)$$

from Eq. (4). Since only φ_{uav} and φ_{yav} as defined in Eqs. (10), (11) can be determined from test results, it appears sensible to define curvature ductility in terms of these curvatures, rather than as in Eq. (2). Thus:

$$\mu_\varphi = \frac{\varphi_{uav}}{\varphi_{yav}} \quad \frac{\theta_u}{\theta_y} = \mu_\theta \qquad (12)$$

The relationships between φ_{uav}, φ_{yav} and the φ_u, φ_y calculated from first principles are as given in Eqs. (10) and (11). These relationships must be confirmed through careful correlation between analytical and experimental results.

RESULTS OF SEISMIC ANALYSIS

The results reported in this section represent part of the data obtained during an analytical investigation carried out at the Portland Cement Association with the aim of determining the force and deformation requirements in structural walls and wall systems required to withstand strong earthquake excitation. Extensive dynamic inelastic analyses of structural walls were undertaken within the framework of a parametric study. The basic structure considered was a 20-story building consisting of a series of parallel walls. A reference structural wall with a fundamental period of 1.4 sec. was first designed in accordance with current code provisions. Subsequent cases had only one parameter varied at a time, with the other structural and ground motion parameters held constant, due regard being given to the practical range of variation of each parameter. Among the structural parameters considered were fundamental period, yield level, post-yield stiffness ratio, stiffness and strength taper, rate of stiffness degradation under cyclic loading, damping coefficient, degree of base fixity, and building height. Among the ground motion parameters considered were the intensity, duration and frequency characteristics. Variations in the fundamental period and the yield level only are considered in the results presented here.

The ground motion used in the dynamic analyses had the frequency characteristics of the E-W component of the 1940 El Centro record. The duration of the motion was set at 10 seconds, since preliminary studies indicated that maximum response under most input accelerograms occurred during this interval. The intensity was normalized to 1.5 times the spectrum intensity corresponding to the first 10 seconds of the N-S component of the 1940 El Centro record.

The dynamic analyses were carried out using the computer program DRAIN-2D, developed by Kanaan and Powell (1973) at the University of California, Berkeley, with modifications introduced at PCA. Inelasticity was allowed by means of flexural 'point hinges' which formed at member ends. The hysteretic moment-rotation relationship for these hinges was an extended version of a model proposed by Takeda (1970), which accounts for the observed decrease in reloading stiffness subsequent to yielding in reinforced concrete members subjected to reversed inelastic loading.

Table 3 lists the ductilities based on top displacements as well as the rotational ductilities of the 'hinging regions' of sixteen isolated structural walls with varying fundamental period, T_1 and yield level, M_y (yield moment of the critical section at the base). The displacement ductility is equal to:

$$\mu_\Delta = \Delta_{max}/\Delta_y \qquad (13)$$

where Δ_{max} is the maximum top displacement during the response period, and Δ_y is the top displacement corresponding to the attainment of M_y at the base. The rotational ductility is equal to:

$$\mu_\theta = \theta_{max}/\theta_y \qquad (14)$$

where θ_{max} is the maximum rotation in the hinging region during the response period and θ_y is the rotation in this region when the base critical section reaches M_y. If the primary moment-rotation characteristic of the hinging region is of the bilinear form as shown in Fig. 4, as is assumed in program DRAIN-2D, then:

$$\mu_\theta = \theta_{max}/\theta_y = 1 + (M_{max} - M_y)/r_y M_y \qquad (14a)$$

It should be noted that a hinging length is implicit in the moment-rotation relationship of Fig. 4.

The displacement and rotation ductilities from Table 3 are plotted as discrete points in Fig. 2, different symbols being used for walls with different fundamental periods. The plastic hinge length to height ratios corresponding to these points and the model of Fig. 1 are computed using Fig. 3, and listed in Table 3. Although the single static point load at the top, assumed in Fig. 1, is not a realistic representation of seismically induced forces, the $\mu_\Delta - \mu_\theta$ relationships derived on the basis of the simplified model appear to be reasonably realistic when viewed against the discrete points obtained from seismic analysis. Indeed it appears that if an appropriate plastic hinge length (to wall height ratio) can be chosen for use in conjunction with the simplified model, the corresponding μ_Δ - μ_θ relationship may give a fair indication of the relative magnitudes of these two quantities under actual seismic conditions. If one disregards the very low ductilities in the vicinity of $\mu_\Delta = 1$ as being of little interest from the point of view of design, then the appropriate plastic hinge length, for the walls analyzed appears to be $\ell_p/H = 0.2$ (Fig. 2).

DESIGN IMPLICATIONS

Minimum deformability requirements are often specified in terms of displacement ductilities in the various buildings codes. It is then necessary to know the order of rotational ductility that may have to be developed in the hinging region, so that this region may be reinforced and detailed accordingly. It is seen, as indeed has been known for some time, that the rotational ductility requirements can be considerably larger than the corresponding displacement ductility requirements, since most inelastic action is usually confined to the hinging region. The results presented in this paper give an idea as to how much larger the μ_θ requirements are likely to be in comparision with the μ_Δ requirements. If a plastic hinge length $\ell_p/H = 0.2$ is used in conjunction with the model of Fig. 1, then from Eq. (6a),

$$\mu_\theta - 1 \approx 2 (\mu_\Delta - 1) \qquad (15)$$

One word of caution, however, may be in order. Many more inelastic dynamic anaylsis results need to be examined before a numerical relationship such as above can be used in actual design.

ACKNOWLEDGMENT

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Table 1: Plastic hinge length versus wall width

H/ℓ_w	ℓ_p/H		
	Eq. (7)	Eq. (8)	Eq. (9)
2	0.250	0.175	0.500
4	0.150	0.125	0.250
6	0.117	0.108	0.167
8	0.100	0.100	0.125
10	0.090	0.095	0.100
12	0.083	0.092	0.083
14	0.079	0.089	0.071
16	0.075	0.088	0.063

Table 2: Values of rotational ductility

ℓ_p/H ↓ \ μ_Δ →	2	3	4	5	6
0.06	6.90	12.81	18.71	24.62	30.52
0.08	5.52	10.04	14.56	19.08	23.61
0.10	4.69	8.39	12.08	15.77	19.47
0.12	4.14	7.29	10.43	13.57	16.72
0.14	3.75	6.51	9.26	12.01	14.76
0.16	3.46	5.92	8.38	10.85	13.31
0.18	3.24	5.47	7.71	9.95	12.18
0.20	3.06	5.12	7.17	9.23	11.29
0.25	2.74	4.48	6.22	7.97	9.71
0.30	2.54	4.08	5.61	7.15	8.69
0.35	2.40	3.80	5.20	6.60	8.00
0.40	2.30	3.60	4.91	6.21	7.51
0.45	2.23	3.47	4.70	5.93	7.17
0.50	2.19	3.37	4.56	5.74	6.93

Table 3: Results of seismic analysis

T_1 (sec)	0.8			1.4			2.0			2.4		
M_{Y_n} (k-in)	μ_Δ	μ_θ	$\frac{\ell_p}{H}$	μ_Δ	μ_θ	$\frac{\ell_p}{H}$	μ_Δ	μ_θ	$\frac{\ell_p}{H}$	μ_Δ	μ_θ	$\frac{\ell_p}{H}$
500,000	7.6	13.6	0.22	4.1	8.1	0.18	3.4	6.2	0.19	3.6	5.7	0.24
750,000	2.9	6.3	0.14	2.9	4.9	0.20	2.6	2.9	0.50	2.6	3.9	0.24
1,000,000	1.8	4.1	0.10	1.2	2.9	0.04	1.6	2.5	0.16	1.01	2.1	0.002
1,500,000	1.1	2.6	0.02	1.06	1.1	0.27	1.05	1.4	0.04	1.0	1.0	-

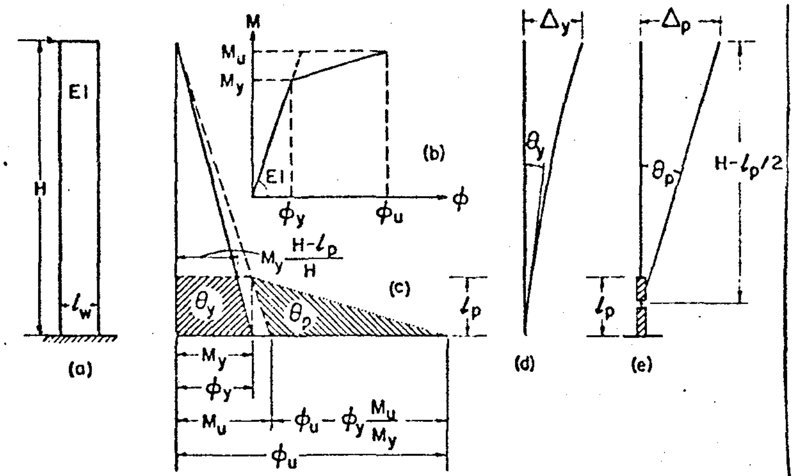

Fig. 1: Moments and deformations in a cantilever wall

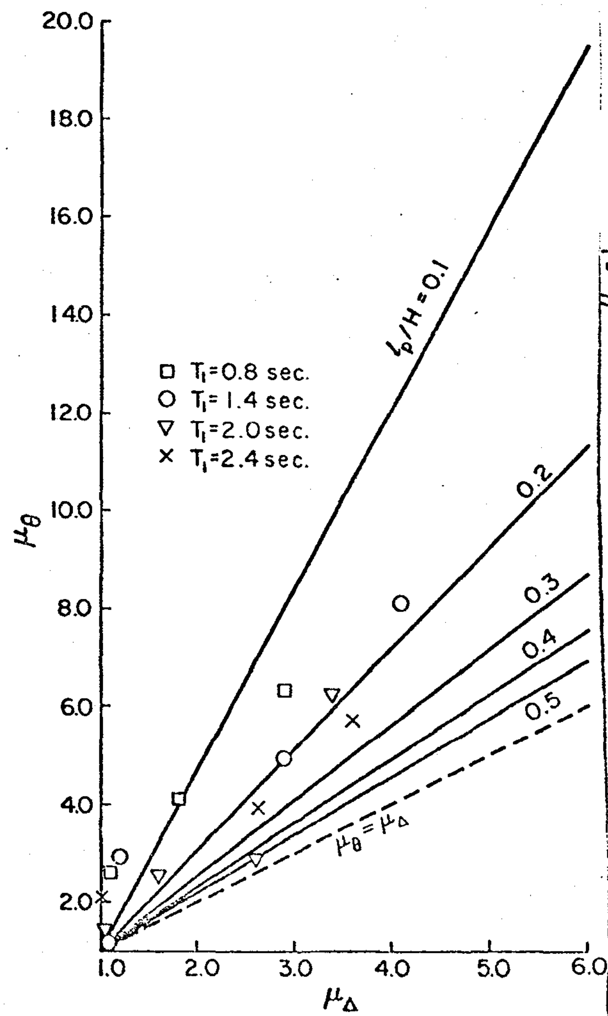

Fig. 2: Rotational ductility vs. displacement ductility

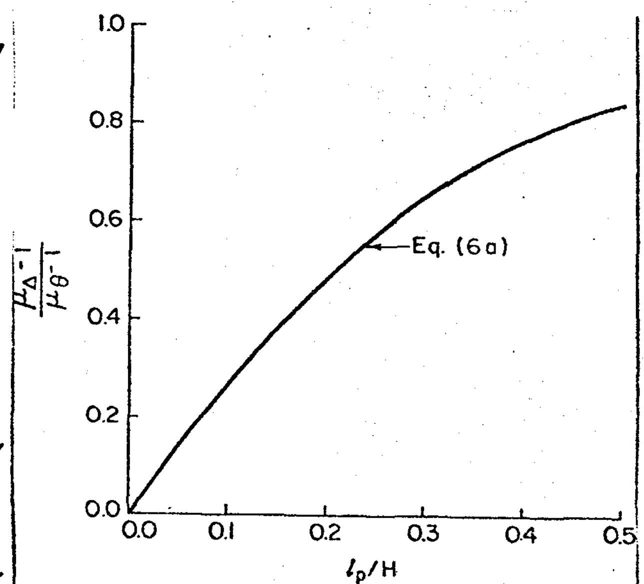

Fig. 3: Ductilities and plastic hinge length

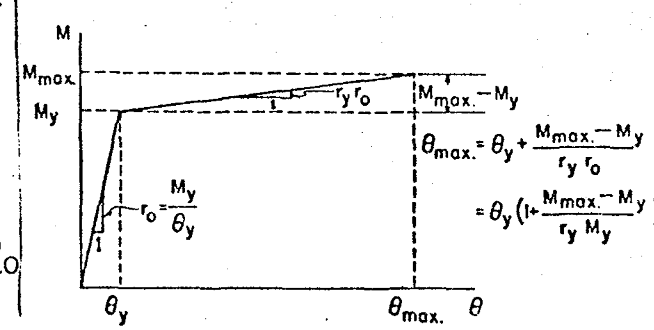

Fig. 4: Moment-rotation characteristics of plastic hinge