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|  | CONSERVATISM IN SUMMATION RULES<br>For closely spaced modes               |
|  | by<br>JAMES M. KELLY<br>and<br>JEROME L. SACKMAN                          |
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## ABSTRACT

It is shown that the method recommended by the Nuclear Regulatory Commission to be used to combine spectral response in the case of closely spaced modes is unnecessarily conservative for certain systems. Closely spaced modes arise in structures from symmetry and where there is a light appendage with a frequency close to one of the natural frequencies of the structure. In the former case, the closely spaced modes do not interact and the Nuclear Regulatory Commission Guide is reasonable. The latter case, that is when there are closely spaced modes where interaction occurs as in the example of light appendages and in torsionally unbalanced buildings, must be treated by consideration of the interacting system. The approach proposed here is that the modes that are not closely spaced be treated by modal analysis and the closely spaced modes, in the case of two closely spaced modes, be treated as a coupled two-degree-of-freedom system. If this is done, the beat phenomenon, the most important characteristic of the interaction between the two closely spaced modes, is evident, as is the associated result that the peak response of the coupled system is developed much later than the peak responses obtained in the individual modes by standard analysis. It is shown that the square root of the sum of the squares procedure underestimates, as expected, the response for undamped and very lightly damped systems; but for damped systems, the square root of the sum of the squares method can be extremely conservative. It follows that the other methods specified by the Nuclear Regulatory Commission for closely spaced modes must be even more conservative.

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## INTRODUCTION

Response spectrum modal analysis continues to be the most widely accepted method for the seismic analysis of complex structural systems. The procedure whereby a representative maximum value of a particular response for design can be obtained from that response in the individual modes is outlined in the Nuclear Regulatory Commission Guide [1]. The method established for combining modal contributions is the well-known square root of the sum of the squares procedure [1], which is valid only so long as none of the modal frequencies are closely spaced. In the case of closely spaced modes, some studies (see reference 1) have suggested that when the square root of the sum of the squares procedure is used, the response can be significantly underestimated. Many methods have been proposed whereby the response when closely spaced modes exist can be combined. All such methods yield results that are greater than those of the square root of the sum of the squares method.

Closely spaced modes arise primarily from geometrical effects-such as symmetry in buildings--and when the natural frequency of an appendage is close or equal to one of the natural frequencies of the structure. Light equipment is an obvious example of the latter, but closely spaced modes also arise in the case of slightly eccentric buildings with a torsional frequency close to a lateral frequency. The important distinction between these two classes is that in the first, the geometrical case, there is no interaction between the closely spaced modes while in the second there is.

In treating geometrically closely spaced modes it is essential that the initial conditions for each mode and the resolution of the ground motion into the specific modes be known. In general this cannot be known for seismic loading. The best that a designer can do with available data is to treat each closely spaced mode as he would any other mode and to use the square root of the sum of the squares procedure. Since the modes do not interact, this will at worst underestimate the result by a factor of  $\sqrt{2}$ .

Closely spaced modes where interaction occurs, as with light appendages or torsionally unbalanced buildings, must be treated by consideration of the interacting system. Our approach is to treat the modes

-1-

that are not closely spaced by modal analysis and then to treat the closely spaced modes, in the case of two closely spaced modes, as a coupled two-degree-of-freedom system. If this is done, the beat phenomenon, which is the most important characteristic of the interaction between the two closely spaced modes, is evident. This is associated with the fact that the peak response of the coupled system is developed much later than the peak responses obtained in the individual modes by standard analysis. It will be shown that the peak values are very sensitive to the degree of damping and that the square root of the sum of the squares procedure underestimates, as expected, the response of undamped and very lightly damped systems; but for damped systems the square root of the sum of the squares procedure can yield extremely conservative results. It follows that the other methods specified by the Nuclear Regulatory Commission for closely spaced modes must be even more conservative. This is not to say that conservatism in design is bad, but only to point out that in matters of safety, rational assessment of the margin of safety is the essential point.

The physical explanation for this result is that the beat phenomenon involves an energy transfer between the two elements of the system. In the case of an undamped, perfectly tuned system, the energy transfer is complete, i.e., maximum response in one element is accompanied by zero response in the This energy transfer takes time; the beat period which controls other. the energy transfer is inversely proportional to the difference between the two closely spaced frequencies. The phenomenon can be interpreted geometrically in terms of the eigenvectors of the modes. The components of these modes can be thought of as vectors in a generalized state space. The components that represent the appendage are initially 180° out of phase and those of the structure are in phase. As the motion continues, the equipment components rotate and eventually align, while the structural components become out of phase. As the vectors rotate, the resultant for the equipment increases and if the system is undamped, attains a maximum when they are aligned. In the damped case, the peak values of the vectors will first increase and then diminish as they rotate and the resultant will achieve its maximum before they line up. Thus, it is important in the damped case to determine the time at which the maximum of the resultant occurs. In terms of this analogy, the square root of the sum of the squares

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procedure assumes that the peak values of both of the vectors are attained simultaneously when they are at 90°. The absolute sum method assumes that this occurs when they are lined up. The method described here evaluates the position at which the resultant actually attains its maximum.

## ANALYSIS OF EQUIVALENT TWO-DEGREE-OF-FREEDOM SYSTEM

We consider here a general system that contains a light subsystem or appendage whose natural frequency is close or equal to one of the frequencies of the main system. Closely spaced strongly interacting modes frequently occur in such systems. An equivalent two-degree-of-freedom model, in which the natural frequency  $\omega$  of each element is identical as is the modal damping factor  $\beta$  when each element is treated as a separate system, will be analyzed to determine the contribution of closely spaced modes to the response of the composite system. One element of the two systems will be taken to be much more massive than the other; the former will be termed the structure and the latter the appendage. The equations of motion of the combined system are:

$$\begin{bmatrix} m & o \\ o & M \end{bmatrix} \begin{pmatrix} \ddot{x} \\ \ddot{x} \end{pmatrix} + \begin{bmatrix} c & -c \\ -c & c+C \end{bmatrix} \begin{pmatrix} \dot{x} \\ \dot{x} \end{pmatrix} + \begin{bmatrix} k & -k \\ -k & k+K \end{bmatrix} = -\ddot{u}_{g} \begin{pmatrix} m \\ M \end{pmatrix}$$
(1)

where x, X are the relative motion of the appendage and the structure and  $\ddot{u}_g$  the ground acceleration. For perfect tuning we have  $K = \omega^2 M$ and  $k = \omega^2 m = \gamma K$  where  $\gamma$  is the ratio m/M. For equal damping in the appendage and the structure taken separately,  $c = \gamma C$ ; we take  $c = 2\beta\omega m$ .

The eigenvectors  $\phi_1^{},\phi_2^{}$  and the eigenfrequencies  $\omega_1^{},\omega_2^{}$  of the undamped system are:

$$\phi_{1} = \begin{cases} 1\\ \sqrt{\gamma} \end{cases}, \quad \phi_{2} = \begin{cases} -1\\ +\sqrt{\gamma} \end{cases}$$

$$\omega_{1} = (1 - \gamma^{1/2}/2)\omega, \quad \omega_{2} = (1 + \gamma^{1/2}/2)\omega$$
(2)

and

where the first component is associated with the equipment and the second with the structure. In these results terms of order  $\gamma$ ,  $\gamma^{3/2}$  have been neglected in comparison with terms of order  $\gamma^{1/2}$ . We use these modes to

represent x,X:

where

$$\ddot{q}_{1} + 2\beta\omega_{1}q_{1} + \omega_{1}^{2}q_{1} = -\frac{1}{2\gamma^{1/2}}\ddot{u}_{g}$$
 (4)

(3)

and

$$\ddot{q}_2 + 2\beta\omega_2 q_2 + \omega_2^2 q_2 = -\frac{1}{2\gamma^{1/2}}\ddot{u}_g$$
 (5)

The acceleration response of the appendage is thus:

 $\begin{cases} \mathbf{x} \\ \mathbf{x} \end{cases} = \mathbf{q}_1 \mathbf{\phi}_1 + \mathbf{q}_2 \mathbf{\phi}_2$ 

$$\ddot{x}(t) = \ddot{q}_{1}(t) - \ddot{q}_{2}(t)$$
 (6)

where

$$q_{1}(t) = -\frac{1}{2\gamma^{1/2}} \frac{1}{\omega} \int_{0}^{t} \ddot{u}_{g}(t) e^{-\beta\omega_{1}(t-\tau)} \sin\omega_{1}(t-\tau) d\tau$$
(7)

and

$$q_{2}(t) = -\frac{1}{2\gamma^{1/2}} \frac{1}{\omega} \int_{0}^{t} \ddot{u}_{g}(t) e^{-\beta \omega_{2}(t-\tau)} \sin \omega_{2}(t-\tau) d\tau$$
 (8)

The peak acceleration predicted by the square root of the sum of the squares procedure and the above is:

$$|\ddot{\mathbf{x}}|_{\max}^{srss} = \frac{1}{2\gamma^{1/2}} \{s_{A}^{2}(\omega_{1}, \beta) + s_{A}^{2}(\omega_{2}, \beta)\}^{1/2}$$
 (9)

If  $\omega_1$  is very close to  $\omega_2$ , then we may neglect the difference between the response spectra, especially if averaged or smoothed spectra are used:

$$\left|\ddot{\mathbf{x}}\right|_{\max}^{\text{srss}} = \frac{1}{\sqrt{2\gamma}} S_{\mathbf{A}}(\omega, \beta)$$
(10)

The group method of reference 1 becomes in this case the absolute sum method and yields

$$\left|\ddot{\mathbf{x}}\right|_{\max}^{\operatorname{asm}} = \frac{1}{\sqrt{\gamma}} \operatorname{s}_{A}(\omega, \beta) = \sqrt{2} \left|\ddot{\mathbf{x}}\right|_{\max}^{\operatorname{srss}}$$
(11)

On the other hand, if we combine the two expressions for  $q_1, q_2$ , retain terms of order  $\gamma^{1/2}$ , and assume that  $\beta$  is of the same order as  $\gamma^{1/2}$ , we obtain

$$\ddot{\mathbf{x}}(t) = \frac{\omega}{2\gamma^{1/2}} \int_{0}^{t} \ddot{\mathbf{u}}_{g}(\tau) e^{-\beta\omega(t-\tau)} \{\sin\omega_{1}(t-\tau) - \sin\omega_{2}(t-\tau)\} d\tau$$

$$= -\frac{2}{2\gamma^{1/2}} \omega \int_{0}^{t} \ddot{\mathbf{u}}_{g}(\tau) e^{-\beta\omega(t-\tau)} \{\sin(\frac{\omega_{2}-\omega_{1}}{2})(t-\tau)\cos(\frac{\omega_{2}+\omega_{1}}{2})(t-\tau)\} d\tau$$

$$= -\frac{1}{\gamma^{1/2}} \omega \int_{0}^{t} \ddot{\mathbf{u}}_{g}(\tau) e^{-\beta\omega(t-\tau)} \{\sin\eta(t-\tau)\cos(t-\tau)\} d\tau \qquad (12)$$
where  $n = (\gamma^{1/2}/2)\omega$ 

where  $\eta = (\gamma^{1/2}/2)\omega$ .

The kernel of the convolution, i.e. the Green's function of the response, is

$$-\frac{\omega}{\gamma^{1/2}}e^{-\beta\omega}sinntcos\omegat$$
(13)

which represents a damped beat motion with a beat frequency  $\eta$  that is very much less than the basic frequency  $\omega$ . Thus, there are many oscillations of the system within a single beat period,  $T = 2\pi/\eta$ .

When the term  $sin\eta(t-\tau)$  in eq. (12) is expanded, we obtain:

$$\ddot{u}(t) = -\frac{\omega}{\gamma^{1/2}} \cos(\eta t - \theta) \left\{ \left( \int_{0}^{t} \ddot{u}_{g}(\tau) e^{-\beta \omega (t - \tau)} \cos(\eta t - \tau) \cos(\eta \tau)^{2} + \left( \int_{0}^{t} \ddot{u}_{g}(\tau) e^{-\beta \omega (t - \tau)} \cos(\eta t - \tau) \sin(\eta \tau)^{2} \right\}^{1/2} \right\}$$
(14)

where

$$\theta = \tan^{-1} \begin{cases} \int_{0}^{\tau} \ddot{u}_{g}(\tau) \cos\eta\tau e^{-\beta\omega(t-\tau)} \cos\omega(t-\tau) d\tau \\ \int_{0}^{t} \ddot{u}_{g}(\tau) \sin\eta\tau e^{-\beta\omega(t-\tau)} \cos\omega(t-\tau) d\tau \end{cases}$$

It is a characteristic of the beat phenomenon that the peak response is achieved after many cycles of motion and thus the maximum acceleration of the appendage occurs after  $t = t_1$ , the termination of ground motion. Further, we are interested in situations where the ground motion has prescribed finite duration and in those frequencies  $\omega$  for which the maximum response of a single-degree-of-freedom oscillator, i.e. the response spectrum, is achieved late in or after the termination of ground motion. These frequencies correspond to peaks in the response spectrum of a seismic ground motion. Design spectra that reflect the probabilistic nature of the input correspond closely to the peaks of actual spectra and thus presuppose

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late-occurring maxima. When ground motion is caused by a blast of short duration, it is likely that the maxima of equipment response will be achieved after the ground motion has ended. Under these assumptions,  $nt_1 = 2\pi t_1/T$  and  $\beta\omega t$ , which is of the same order as nt, are both very much less than one, and  $\ddot{u}_g = 0$ ,  $t > t_1$ . Thus the first integral in eq. (14) can be approximated by:

$$\int_{0}^{t} \ddot{u}_{g}(\tau) e^{-\beta \omega (t-\tau)} \cos \omega (t-\tau) d\tau$$

and the second neglected since sinft will be bounded by  $\eta t_1$ . Thus, for  $\eta t_1 << 1$  and  $t > t_1$ , we have

$$\ddot{\mathbf{x}}(t) = -\frac{\omega \sin \eta t}{\gamma^{1/2}} \int_{0}^{t} \ddot{\mathbf{u}}_{g}(\tau) e^{-\beta \omega (t-\tau)} \cos \omega (t-\tau) d\tau$$
(15)

When the parameters  $\gamma^{1/2}$  and  $\beta$  are small, this result may be interpreted in the following way: for t > t<sub>1</sub>, the above expression can be written in the following form:

$$\ddot{\mathbf{x}}(t) = -\frac{\omega^2 \sin \eta t}{2\eta} e^{-\beta \omega t} A \cos(\omega t - \Psi)$$

where

$$A = (A_1^2 + A_2^2)^{1/2}$$

$$A_1 = \int_0^{t_1} \ddot{u}_g(t) e^{\beta \omega t} \cos \omega t \, dt$$

$$A_2 = \int_0^{t_1} \ddot{u}_g(t) e^{\beta \omega t} \sin \omega t \, dt$$

$$\Psi = \tan^{-1} (A_2/A_1)$$

and

The response indicated by the above is illustrated in Figure 1. The terms A and  $\Psi$  are constants independent of t for t > t<sub>1</sub>, and A cos( $\omega$ t -  $\Psi$ ) is a rapidly varying function of time. The term

$$\frac{\omega^2}{2\eta} e^{-\beta\omega t} \sin\eta t$$

is a slowly varying envelope curve whose maximum value must be determined. The maximum value of this envelope curve is attained at time t\*, expressed by

$$tan\eta t^* = \frac{\eta}{\beta \omega}$$

$$t^* = (\arctan \beta \omega) / \eta$$
 (16)

For lightly damped systems and light equipment, in general  $t^* >> t_1$ . The values of sinnt and exp(- $\beta\omega t$ ) when the envelope achieves its maximum are

singt\* = 
$$\frac{\eta}{(\eta^2 + \beta^2 \omega^2)^{1/2}}$$
  
e<sup>- $\beta\omega$ t\* = e<sup>- $\kappa$</sup></sup> 

where

$$\kappa = (\arctan \zeta) / \zeta$$
$$\zeta = \gamma^{1/2} / 2\beta$$

It follows that

$$\begin{aligned} \left| \ddot{\mathbf{x}} \right|_{\max} &= \left| \ddot{\mathbf{x}}(t^*) \right| \\ &= \frac{\omega}{\gamma^{1/2}} \left| \operatorname{singt*} \right| e^{-\kappa} \left\{ e^{\beta \omega t^*} \right| \int_{0}^{t^*} \ddot{\mathbf{u}}_{g}(\tau) e^{-\beta \omega (t^* - \tau)} \cos \omega (t^* - \tau) d\tau \right| \right\}$$
(17)

In order that this estimate of the peak acceleration be useful for design purposes, it is necessary that the second factor in braces be interpreted in terms of a ground response spectrum. To this end, we recognize that the integral is, to the order of  $\beta$ , the relative velocity response history, evaluated at time t\* , of a lightly damped single-degree-offreedom oscillator of frequency  $\omega$  and damping factor  $\beta$  subjected to the ground acceleration  $\ddot{u}_{\sigma}(t)$  . At some time  $\tilde{t}$  during the ground motion or shortly after it ceases (so that  $\tilde{t}$  << t\* ), the absolute value of the relative velocity will attain its global maximum denoted as  $|v(\tilde{t})|$  . The relative velocity response at  $t^*$ , denoted as  $v(t^*)$ , can be thought of as that which would occur in a single-degree-of-freedom system (subjected to the ground acceleration  $\ddot{u}_{\sigma}(t)$  ) as a consequence of free vibration beginning at time  $\hat{t}(>t_1)$  where the absolute value of the relative velocity of the oscillator attains its first local maximum  $|v(\hat{t})|$  after the end of the earthquake. This instant of time  $\hat{t}$  is equal to  $\tilde{t}$  if  $\tilde{t}$  occurs after the end of the ground motion; otherwise,  $\,\hat{t}\,>\,\tilde{t}$  . In any event,  $\hat{t} \ll t^*$  . Thus, we can write:

$$|\mathbf{v}(t^*)| = |\mathbf{v}(\hat{t})| e^{-\beta \omega (t^* - \hat{t})} |\cos \omega (t^* - \hat{t})|$$

where  $|v(\hat{t})| \leq |v(\tilde{t})|$  . It then follows that

$$|\mathbf{v}(t^*)| \equiv |\int_{0}^{t^*} \ddot{\mathbf{u}}_{g}(\tau) e^{-\beta\omega(t^*-\tau)} \cos\omega(t^*-\tau) d\tau|$$
$$= |\mathbf{v}(\hat{t})| e^{-\beta\omega(t^*-\hat{t})} |\cos\omega(t^*-\hat{t})|$$
$$\leq |\mathbf{v}(\hat{t})| e^{-\beta\omega t^* (1-\hat{t}/t^*)}$$
$$\simeq |\mathbf{v}(\hat{t})| e^{-\beta\omega t^*}$$

since  $\hat{t}/t^* \ll 1$  . From this we obtain the approximate result

$$|\mathbf{v}(\tilde{t})| \simeq e^{\beta \omega t^*} |\int_{0}^{t^*} \ddot{\mathbf{u}}_{g}(\tau) e^{-\beta \omega (t^*-\tau)} \cos \omega (t^*-\tau) d\tau|$$

We recognize that  $|v(\tilde{t})|$  is the relative velocity response spectrum  $S_{RV}(\omega, \beta)$  for a lightly damped single-degree-of-freedom oscillator of frequency  $\omega$  and damping factor  $\beta$  subjected to the ground acceleration  $\ddot{u}_{\alpha}(t)$ . Thus, an estimate of the maximum equipment acceleration is:

$$|\ddot{\mathbf{x}}|_{\max} = \frac{\omega |\operatorname{sinnt}^*| e^{-\kappa}}{\gamma^{1/2}} S_{\mathrm{RV}}(\omega, \beta)$$

With the value of sinnt\* from eq. (16), we obtain the final estimate:

$$\left|\ddot{\mathbf{x}}\right|_{\max} = \frac{e^{-\kappa} \omega S_{RV}(\omega, \beta)}{(\gamma + 4\beta^2)^{1/2}}$$
(18)

The derivation gives the result naturally in the form of the relative velocity spectrum, but design information is generally provided in the form of a pseudo-velocity spectrum. However, the pseudo-velocity response spectrum is nearly equal to the relative velocity spectrum for systems with moderate or high frequencies and differs only for very low-frequency systems [2]. Thus, for most cases,  $S_{RV}$  in eq. (18) can be replaced by  $S_V$ , the pseudo-velocity response spectrum.

We recall that

$$\omega s_{v} = s_{A} = \omega^{2} s_{D}$$

This estimate can be written in the alternative form:

$$\left|\ddot{\mathbf{x}}\right|_{\max} = \frac{e^{-\kappa}}{(\gamma + 4\beta^2)^{1/2}} \mathbf{s}_{\mathbf{A}}(\omega, \beta)$$
(19)



Similarly, for the relative displacement:

$$\left| x \right|_{\max} = \frac{e^{-\kappa}}{(\gamma + 4\beta^2)^{1/2}} S_D(\omega, \beta)$$
(20)

For ground motion of very short duration compared to the period of the tuned mode, such as that caused by blast loading, the interpretation in terms of a pseudo-velocity response spectrum is not strictly correct. From eq. (17) we see that if  $\ddot{u}_g(\tau)$  is nonzero for a time  $t_1$  that is short compared to  $2\pi/\omega$ , the terms in the integrand other than  $\ddot{u}_g$  do not change significantly over the duration of  $\ddot{u}_g$  and can be evaluated at  $\tau = 0$ . It follows that:

$$\left|\ddot{\mathbf{x}}\right|_{\max} = \frac{\mathrm{e}^{-\kappa}\omega}{(\gamma+4\beta^2)^{1/2}} \left|\int_{0}^{t_1} \ddot{\mathbf{u}}_{g}(\tau) d\tau\right|$$

The appropriate interpretation of the integral in the above is  $S_{V}(\omega, 0)$  so that for such cases the estimate of the peak acceleration is given by:

$$|\ddot{\mathbf{x}}|_{\max} = \frac{e^{-\kappa}\omega}{(\gamma+4\beta^2)^{1/2}} S_{\mathbf{v}}(\omega, 0)$$

These results have been derived for the precisely tuned system with equal damping in both elements. Analogous results have been presented for tuned undamped systems in [3]; and for slightly nontuned undamped systems in [4]; and for tuned and nontuned systems with different damping factors in each element in [5] and [6], and their application to nuclear plants in [7].

When different damping factors in the two elements of the system are considered, closely spaced modes still arise. The conventional modal approach, in which undamped modes are used, then presents problems. If the dampings in the two elements of the system differ, then in a strict sense conventional modes are coupled. If the argument is used that the damping will be sufficiently light not to couple the modes, then it is necessary to specify the modal damping factors  $\beta_1$  and  $\beta_2$  for the two modes of the combined system and there is no rational way to determine these. In references 5 and 6, Laplace transforms and residue theory are used; the peak response for the closely spaced modes is obtained by consideration of the contribution from the corresponding closely spaced poles in the transform space. This approach is equivalent to the use of complex modes. The result

for the nontuned system where the damping factors of the structure and appendage differ is:

$$\left|\ddot{\mathbf{x}}\right|_{\max} = \frac{e^{-\kappa}}{\left(\gamma + \xi^2 + 4\beta B\right)^{1/2}} S_{A}\left(\frac{\omega + \Omega}{2}, \frac{\beta + B}{2}\right)$$
(21)

where  $\omega$ ,  $\beta$  and  $\Omega$ , B are the structural frequencies and the damping factors for the appendage and main system when considered separately. The detuning parameter  $\xi$  is given by:

$$\xi = \frac{\Omega - \omega}{\omega}$$
(22)

Here

$$\kappa = (\arctan \zeta) / \zeta \tag{23}$$

where

$$\zeta = (\gamma + \xi^{2} - (\beta - B)^{2})^{1/2} / (\beta + B)$$
(24)

## COMPARISON OF ESTIMATES

The three estimates of peak acceleration developed in the previous section are:

$$\begin{aligned} \left| \ddot{\mathbf{x}} \right|_{\max}^{\text{srss}} &= \frac{1}{\sqrt{2\gamma}} \, \mathbf{s}_{A}(\omega, \beta) \\ \left| \ddot{\mathbf{x}} \right|_{\max}^{\text{asm}} &= \frac{1}{\sqrt{\gamma}} \, \mathbf{s}_{A}(\omega, \beta) \\ \left| \ddot{\mathbf{x}} \right|_{\max}^{\text{new}} &= \frac{e^{-\kappa}}{\sqrt{\gamma + 4\beta^{2}}} \, \mathbf{s}_{A}(\omega, \beta) \end{aligned}$$

We note immediately that if  $\beta \rightarrow 0$ , the square root of the sum of the squares estimate will be low by a factor of  $\sqrt{2}$ ; the absolute sum estimate and the new estimate are identical. However, the estimates differ for nonzero damping. We introduce an overestimation ratio

$$R = \frac{\left| \ddot{\mathbf{x}} \right|^{asm}}{\left| \ddot{\mathbf{x}} \right|_{max}^{new}} = (1 + 4\beta^2 / \gamma)^{1/2} e^{\kappa}$$
(25)

Since  $\kappa$  is given by:

$$\kappa = \arctan \frac{\gamma^{1/2}}{2\beta} / \frac{\gamma^{1/2}}{2\beta}$$

the overestimation ratio can be expressed in terms of the single parameter

 $\gamma^{1/2}/2\beta$  (Figure 2). The result has been evaluated for several values of  $\gamma$  as a function of  $\beta$ ; the resulting curves are shown in Figure 3. Clearly, for all values of  $\gamma$ , the overestimation parameter and thus the conservatism of the regulation steadily increase with  $\beta$ . If we fix  $\gamma$  and decrease  $\beta$  such that

$$\gamma^{1/2}/2\beta \rightarrow \infty$$

 $\kappa \rightarrow \frac{\pi}{2\beta}$ 

we find that

and

$$R \neq (1 + \frac{2\beta^2}{\gamma}) e^{\frac{\pi}{2}(2\beta/\gamma^{1/2})}$$

which indicates that R > 1 for all nonzero  $\beta$ .

On the other hand, if we fix  $\beta$  and decrease  $\gamma$  such that

$$\frac{\gamma^{1/2}}{2\beta} \rightarrow 0$$

we find that  $\kappa \to 1$  and  $R \to (2\beta/\gamma^{1/2})e$  which demonstrates that for light appendage cases, which correspond to very closely spaced modes, the response can be greatly overestimated.

We have performed a numerical experiment using a standard structural analysis program, TABS [8], to compute the response of a light appendage in a structure. The structure analyzed was a ten-story reinforced concrete frame building investigated in reference 9. The appendage is a singledegree-of-freedom oscillator attached to the top floor. The mass of the appendage was selected to provide a mass ratio of 0.001 with respect to the effective mass of the first mode of the building (see reference 5). The response of the appendage was calculated by using TABS to evaluate the eleven-degree-of-freedom system consisting of the structure and the appendage. The program computes the natural frequencies and mode shapes and then computes the time history in each mode. Two cases of damping were considered, undamped and 2% of critical damping in all modes. The response for several earthquakes, including the El Centro 1940, Pacoima Dam, and Taft ground motions, was computed in this way.

Results were obtained for appendage frequencies varying from 0 to 25 radians per second, which cover the first three frequencies of the

structure. Typical results are shown in Figures 4 and 5 for the Taft earthquake with 0% and 2% of critical damping, respectively. The program can also be used to compute peak response from response spectrum estimates for each mode which are combined by the square root of the sum of the squares method. The results in this case correspond to the square root of the sum of the squares estimate given in this paper. For appendage frequencies close or equal to structural frequencies, the TABS procedure would, under Nuclear Regulatory Commission Regulation 1.92, be replaced by the absolute sum. Since the response, when the appendage is tuned to the first mode, is dominated by the tuned system, the prediction of TABS should be multiplied by  $\sqrt{2}$  to yield the absolute sum result.

From Figure 4, for the undamped case, the TABS time history result when the appendage is tuned to the first mode is 7.5. The TABS spectrum result (SRSS) is 5.9, which implies an absolute sum method prediction of 8.34. The new estimate, derived using the method described here, is 8.3. Thus, we see that to the degree of accuracy possible here, the absolute sum method and the new estimate yield the same result and are more accurate estimates of the time history result than is the square root of the sum of the squares estimate.

For the 2% damped system, the TABS time history computation is 1.80. The TABS spectrum result (SRSS) is 4.67 from which the absolute sum method estimate would be 6.60. The overestimation ratio of the absolute sum method from this is thus 6.60/1.80 = 3.67. The new estimate predicts a response of 1.77 and an overestimation ratio of 6.60/1.77 = 3.73. It is interesting to note that for  $\gamma = 0.001$  and  $\beta = 0.02$ , the overestimation ratio R, from eq. (25), is 3.76.

A damping factor of 2% is low for nuclear plants (see Newmark [10] for typical damping factors in nuclear plants) and thus it is clear that the degree of conservatism through the use of Regulation 1.92 will be very much higher than that indicated in these numerical experiments.

### RELATIONSHIP TO FLOOR SPECTRUM METHOD

In many cases the floor spectrum method is used to predict the response of light appendages. The response at the appendage attachment point is computed from the ground motion and then the response of the

appendage as an isolated subsystem subject to this motion is computed. This is precisely equivalent to analyzing the combined system, assuming the mass ratio  $\gamma$  to be zero. Thus, we can obtain the floor spectrum estimate  $|\ddot{\mathbf{x}}|_{\max}^{\text{fs}}$  directly from the estimate given in eq. (19) by letting  $\gamma \neq 0$ . The estimate thus obtained is:

$$|\ddot{\mathbf{x}}|_{\max}^{\mathrm{fs}} = (e^{-1}/2\beta) S_{\mathrm{A}}(\omega, \beta)$$

which can be compared to the current estimate:

$$|\ddot{\mathbf{x}}|_{\max}^{\text{new}} = \frac{e^{-\kappa}}{(\gamma+4\beta^2)^{1/2}} \mathbf{S}_{\mathbf{A}}(\omega,\beta)$$

and which becomes, for  $\gamma^{1/2}/2\beta \to 0$  where  $\kappa \to 1 - \frac{1}{3} \frac{\gamma}{4\beta^2}$ ,  $|\ddot{\mathbf{x}}|_{\max}^{new} = \frac{e^{-1}e^{\frac{1}{3}(\gamma/4\beta^2)}}{(\gamma+4\beta^2)^{1/2}} S_{\mathbf{A}}(\omega,\beta)$ 

Thus, the floor spectrum method yields an accurate estimate if  $~\gamma~<<~4\beta^2$  .

If we use the general formula, eq. (21), for a nontuned system and retain the assumption that  $\gamma^{1/2}$ ,  $\xi$ ,  $\beta$ , B are all bounded by  $\varepsilon$ , where  $\varepsilon << 1$ , it can be established that a sufficient condition for the validity of the floor spectrum method is:

$$\gamma << \xi^2 + 4\beta B$$

This result is at first surprising in that intuition would suggest that the floor spectrum method would be valid if  $\gamma << 1$ , but this shows that in lightly damped, nearly tuned systems a much more restrictive constraint is operative. This indicates the dangers inherent in the naive application of the floor spectrum method.

It is also worth noting that the results indicated in Figures 3 and 4 are for the perfectly tuned case and for  $\beta = B$ . This suggests a further conservatism in the Regulation 1.92. Closely spaced modes are therein defined to be those whose frequencies differ by not more than 10% of the lower frequency. For mass ratios less than 0.01, a spread of 10% indicates a nonzero detuning parameter  $\xi$ . This parameter plays an important role in the new estimate in its general form as given in eq. (21), and provides a significant reduction in the peak response which will increase the over-

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estimation ratio R. Regulation 1.92 does not distinguish between tuned and nontuned systems provided that the frequencies differ by not more than 10%; the regulation thus incorporates a further conservatism. If  $\beta \neq B$ , the general form of the new estimate can yield even higher values of the overestimation ratio if

 $(\beta - B)^2 > \gamma + \xi^2$ 

The rational analysis of the interacting system has demonstrated that Regulation 1.92 is excessively conservative in its estimation of the maximum response of equipment, appendages, components, and piping systems for realistic values of mass ratio, damping, and detuning. Its conservatism arises from the neglect of the essential physics of the coupled system. It may further be noted that the validity of the floor spectrum method is questionable for lightly damped systems, but that, even when valid, it is unnecessarily complicated, requiring as it does that an expensive time history analysis of the main structure be carried out. When only a design spectrum is available, a series of spectrum-consistent ground motions must be generated; this itself is a matter of some controversy. The estimate here yields the floor spectrum result directly from the design spectrum with no computations needed other than those that define the structural properties and appendage properties separately.

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# FIGURE I EQUIPMENT RESPONSE HISTORY



FIGURE 2 OVERESTIMATION RATIO







FIGURE 4 APPENDAGE RESPONSE - TAFT EARTHQUAKE - 0% DAMPING

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2% DAMPING

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