

Massachusetts Institute of Technology  
Department of Civil Engineering  
Constructed Facilities Division  
Cambridge, Massachusetts 02139

Seismic Behavior and Design of Buildings

Report No. 1

ASEISMIC DESIGN PROCEDURES  
FOR REINFORCED CONCRETE FRAMES

by

John M. Biggs  
Wai K. Lau  
Drew Persinko

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ABSTRACT

Twelve reinforced concrete frames are designed for earthquake and gravity loads using three different procedures for determining the seismic design loads. The three procedures are: (1) The UBC static load approach, (2) modal analysis using inelastic response spectra, and (3) the Substitute Structure Method. The frames are of four, eight and ten stories.

The validity of each design procedure is evaluated by time-history analysis of each frame to determine maximum local ductility demands due to both real and artificial ground motions. Results are presented in the form of maximum ductility demands in each story and floor of the frames.

None of the three methods is found to be completely satisfactory because the ductility demands, or amount of computed yielding, are not the same as intended in the design and are not evenly distributed over the frame. Furthermore, the eight- and ten-story frames designed by all three methods yield excessively in the upper stories due to the "whip-lash" effect. However, the computed ductility demands are in no case large enough to indicate structural collapse and in this sense all three methods produce satisfactory designs.

Although the inelastic response spectrum approach produces slightly better results, the two more sophisticated methods do not produce significantly better designs than the simpler UBC approach.





ASEISMIC DESIGN PROCEDURES FOR REINFORCED CONCRETE FRAMES

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## CHAPTER I - INTRODUCTION

The purpose of the research reported herein was to evaluate the effectiveness of alternative aseismic design procedures for reinforced concrete frames. The evaluation of the procedures is based upon time-history analyses of the designed frames which provide computed maximum ductility demands in the individual members resulting from input ground motions. A satisfactory design is judged to be one which limits the maximum demands to the value intended by the design procedure, and produces a reasonably uniform distribution of demands throughout the frame.

This report is a summary of the results produced by two M.I.T. Master's Theses, one by Persinko (1) and the other by Lau (2). The investigations of a total of twelve frames are described herein. All frames are of regular geometry and considered independent of the rest of the building. They differ in the number of stories and the design procedure used. Further details of the designs and the time-history results may be found in the two theses referenced.

Three methods of design were applied to the frames: (i) The UBC equivalent static load approach (3); (ii) Design based upon modal analysis using inelastic response spectra constructed as proposed by Newmark and Hall (4); (iii) The Substitute Structure Method proposed by Shibata and Sozen (5). Although the Code designs were based on UBC-1973, it is believed that the same general conclusions would have been reached if later versions of the code had been used. The members of all frames were proportioned according to ACI-318 (6), although in some cases the provisions of Appendix A of that specification were eliminated.

Previous work in this general area includes studies of reinforced concrete frame behavior by Clough, Benuska and Wilson (7), Clough and Benuska (8), and Clough and Gidwani (10). The Applied Technology Council has studied the feasibility of inelastic design procedures in projects ATC-2 (3) and ATC-3 (11). Previous MIT investigations of design procedures for steel frames, using approaches similar to that described herein, are reported in References (12), (13), (14), and (15).

### 1.1 Design Approaches

Aseismic design procedures have two general objectives: (1) to limit damage from a moderate earthquake to that which can be repaired, and (2) to prevent collapse and loss of life due to a major earthquake. This research is directed primarily at the second objective. This obviously requires consideration of inelastic behavior even though the design procedure, as in the case of the Code approach, deals only with elastic behavior under reduced loads. Maximum ductility demand cannot be directly related to damage, but nevertheless serves a useful purpose in evaluating the effectiveness of the design procedure.

Because of its relative simplicity, the equivalent static load approach as specified by the UBC and other codes is used for most structures. It accounts for the dynamic properties of the structure and the expected ground motion only indirectly and crudely. However, experience in actual earthquakes seems to indicate that it provides a reasonable degree of protection. It has evolved over the years and represents the collective judgment of many engineers and researchers. One of the purposes of this research was to determine whether more sophisticated methods, made possible

by the development of advanced analytical techniques, offer a major improvement in aseismic design for conventional structures.

At the other end of the spectrum, design can be based on time-history analysis for the expected ground motion. This is an iterative process of design and analysis which is theoretically attractive but cumbersome in application. Aside from the cost in time and money, it involves some difficulties. The proper ground motion to be used as input cannot be determined, and the results of the analyses are not easily interpreted. This approach to design is not investigated herein.

A compromise between these two extremes is design based upon response spectra and modal analysis. Although this approach has been proven to be quite effective for elastic design, its reliability when applied to inelastic behavior is open to question. Ideally, it is attractive because it gives the designer control over the amount of yielding in the structure to be expected in the design earthquake. However, the construction of inelastic response spectra from a given elastic spectrum, and the application of elastic modal analysis to predict inelastic behavior, are of uncertain validity. Another purpose of this research is to further investigate the reliability of this procedure.

The Substitute Structure Method also utilizes response spectra. However, rather than modify the spectrum for inelastic behavior, the properties of the structure are changed, presumably to achieve the same result. The proposed method has the advantage that different design limits on ductility demands can be assigned to different members (e.g., columns and girders). This recently proposed method has not been widely tested and one of the purposes here is to provide further evaluation.

## 1.2 Scope of Report

The twelve frames discussed in this report are of four, eight and ten stories. Five were designed by Code, three by the inelastic response spectrum method, and four by the Substitute Structure Method (Table 3.1). The designs are described in terms of member sizes and member capacities as determined by the design analysis and member proportioning procedures used for each frame.

The designs were evaluated by computing local ductility demands due to strong ground motions using the computer program FRIEDA. Results are presented in the form of plots of these demands over frame height for columns and girders separately.

It should be noted that no attempt was made to optimize the frame designs. Better designs may be possible within the framework of the procedure used. However, the intent was to evaluate these procedures as they might be typically applied in practice without the benefit of special experience or expertise on the part of the designer. Attention was not paid specifically to joint details, it being assumed that the joint regions were detailed so that they could adequately respond to the ductility demands made of them. In some of the frame designs it might be desirable to increase member sizes so as to accommodate the intersecting steel of the beams and columns or for other practical reasons. Such considerations were not included in the study because they might have obscured the results in regard to the main purpose, i.e., the evaluation of the design procedures applied without modification.

## CHAPTER II - METHOD OF EVALUATING DESIGNS

The performances of the various designs were evaluated by subjecting the frames to inelastic time-history analysis. Artificial ground motions, generated to match a smoothed response spectrum, were used, except for the frames designed by the Substitute Structure Method which were analyzed for real earthquake motions. All analyses were made using the computer program FRIEDA, which computes ductility ratios at each end of each member of the frame.

The criteria for satisfactory performance has been taken to be the maximum ductility ratio, although it is recognized that this is not a complete indication of damage. However, the important consideration here is the relation between the computed response and that intended in the design, rather than any absolute measure of damage. It is assumed that the structures are well detailed, so that appreciable yielding can take place without complete failure.

### 2.1 Earthquake Ground Motions

Several artificial motions were employed and all were generated using the program SIMQKE (16). The target response spectrum was a Newmark-Blume-Kapur spectrum for 5% damping, normalized to a peak ground acceleration of 0.33g (17).

The SIMQKE program represents the ground acceleration by the sum of a series of sinusoids expressed by

$$Z(t) = I(t) \sum_{i=1}^n A_i \sin(\omega_i t + \phi_i)$$

where  $I(t)$  is a function which establishes the duration and time variation of the intensity of motion,  $A_i$  is the amplitude of the sinusoid with frequency  $\omega_i$ , and  $\phi_i$  is a random phase angle. The program first generates the power spectral density function from the given response spectrum and then selects values of  $A_i$  so as to match that function. In these studies durations of 10 and 20 seconds were used, the longer duration being used with the taller structures having longer natural periods.  $I(t)$  was taken to be constant over the middle 80% of the duration with linear rise and decay at the beginning and end of the motion. The phase angles,  $\phi_i$ , are selected by a random number generator and each set of values produces a different motion. Although all motions are generated to match the same response spectrum, they are basically different and cause somewhat different responses in a structure. The response spectrum for a typical artificial motion is shown in Fig. 3.1 along with the target spectrum. The match is quite good except at long periods.

The Substitute Structure Method as employed herein utilized the smooth design spectrum derived by Shibata and Sozen (5) as an approximate average of six real motions, all normalized to 0.5g peak ground acceleration. Therefore, three of these motions were used to evaluate the performance of these structures. The motions are: El Centro 1940 NS, Taft 1952 N21E, and Taft 1952 S68W.

## 2.2 Method of Analysis

The inelastic time-history analysis used to determine the response of all frames was performed by the computer program FRIEDA (Frame Inelastic Earthquake Dynamic Analysis). The program was written by Aziz (18) and later modified by Luyties, Anagnostopoulos and Biggs (19).

The dynamic analysis is executed by numerical integration of the equations of motion after the system has been condensed to one degree of freedom per floor. At each time step, joint rotations are obtained from the pseudo static condition and member forces are computed using the current member stiffnesses. These are checked against the member capacities and, if yielding has occurred, the stiffnesses are modified according to the nonlinear model selected for the elements. Once all elements have been checked, a new tangent stiffness matrix is assembled to represent the state of yield of the entire structure at that time step. The process is then repeated through the entire integration.

FRIEDA has several optional capabilities which take into account finite joint sizes,  $P-\Delta$  effects, nonlinear materials, non-prismatic members, foundation flexibility, etc. Discussed below are only those features used in the analyses reported herein.

The elements were represented by the dual component point hinge model first introduced by Clough et al. (20). This assumes that the member consists of two components in parallel, one elasto-plastic and one completely elastic. The sum of the two results in a bi-linear moment curvature relationship for the member (see Fig. 2.1). The stiffness of the second component,  $p \times EI$ , is a fraction  $p$  of the total elastic member stiffness and corresponds to the second slope of the bi-linear moment-curvature diagram. In these studies, the value of  $p$  is taken as .03. The hysteresis loop assumed considers neither strength nor stiffness degradation. The positive and negative yield moments are not necessarily equal.

For most of the frames, a constant 5% damping was assumed in each elastic mode. For those frames designed by the Substitute Structure Method,

values of 8% and 10% were used consistent with the assumptions of that design method.

Gravity loads were taken into account as initial axial forces and end moments on the individual elements (determined by a static frame analysis). The effect of axial load on the moment capacity of columns was taken into account by use of the standard interaction diagram. Since FRIEDA permits only one normalized interaction diagram for all members, an average was used. This is shown in Fig. 2.3, where the deviation due to differences in steel ratio (for a typical frame) is depicted. Studies indicated that the errors induced were not significant. During the time-history analysis the column moment capacities were recomputed at each time step, taking into account the current axial load due to both gravity and earthquake.

Throughout these studies the measure of inelastic response and the evaluation of structural performance were based upon the ductility demand at the ends of all elements in the frame. There is considerable uncertainty as to how this quantity should be determined, and three different definitions of ductility demand were used.

Rotational ductility is the ratio of maximum hinge rotation at the end of the member to the rotation at yield, or

$$\mu_{\theta} = \frac{\theta_{\max}}{\theta_{\text{yield}}} = 1 + \frac{\theta_{\text{plastic}}}{\theta_{\text{yield}}}$$

where  $\theta_{\text{plastic}}$  is the excursion on the second slope of the moment-curvature diagram (see Fig. 2.2) and

$$\theta_{\text{yield}} = \frac{M_y L}{6EI}$$

The expression for the yield rotation assumes that the member is bent in anti-symmetrical double curvature. This may be justified for the columns



of the frame, but probably not for girders, due to the effect of the gravity of the load. However, the use of rotational ductility for columns is complicated by the variation of  $M_y$  with axial load.  $M_y$  could be recomputed at each time step, but a study by Lai (15) indicated that more reasonable results were obtained if  $M_y$  were held constant at a value corresponding to the gravity axial load. In these studies the last procedure is used to determine the ductility demand in columns.

Moment ductility is the ratio of the moment that would occur if the member remained elastic to the yield moment, both at the same rotation. As shown in Fig. 2.4, this may be represented by

$$\mu_M = \frac{M_{el}}{M_y} = \frac{\theta_{max}}{\theta_y} = 1 + \frac{(\theta_m - \theta_y)}{\theta_y} = 1 + \frac{M - M_y}{pM_y} ,$$

where  $M$  is the maximum moment and  $p$  is the second slope of the moment-curvature relation. This definition has the advantage that it does not depend upon an assumed distortion of the member. It is used herein to compute the ductility demand for girders.

Damage ratio, which is part of the Substitute Structure Method, is used instead of ductility demand to evaluate structures designed by that method. As illustrated in Fig. 2.5, it is based upon the assumption that the slope of the line between the origin and the point of maximum moment on the moment-curvature curve is equal to the original stiffness divided by the damage ratio,  $\mu_D$ . Thus,

$$\mu_M = \frac{6EI}{\mu_D L} \theta_m$$

or

$$\mu_D = \frac{6EI}{ML} \theta_M .$$

This is closely related to rotational ductility since

$$\mu_{\theta} = \frac{\theta_m}{M_y L / 6EI} = \frac{6EI}{M_y L} \theta_m$$

$$\therefore \mu_D = \frac{M_y}{M} \mu_{\theta}$$

and the ratio of  $M_y$  to  $M$  is generally not far from unity.

None of these definitions is considered to be a satisfactory indication of structural damage. Current work under this project is considering other possible response parameters as indicators of damage. However, for the purpose of these studies, i.e., the evaluation of design procedures, these measures of yielding are useful.

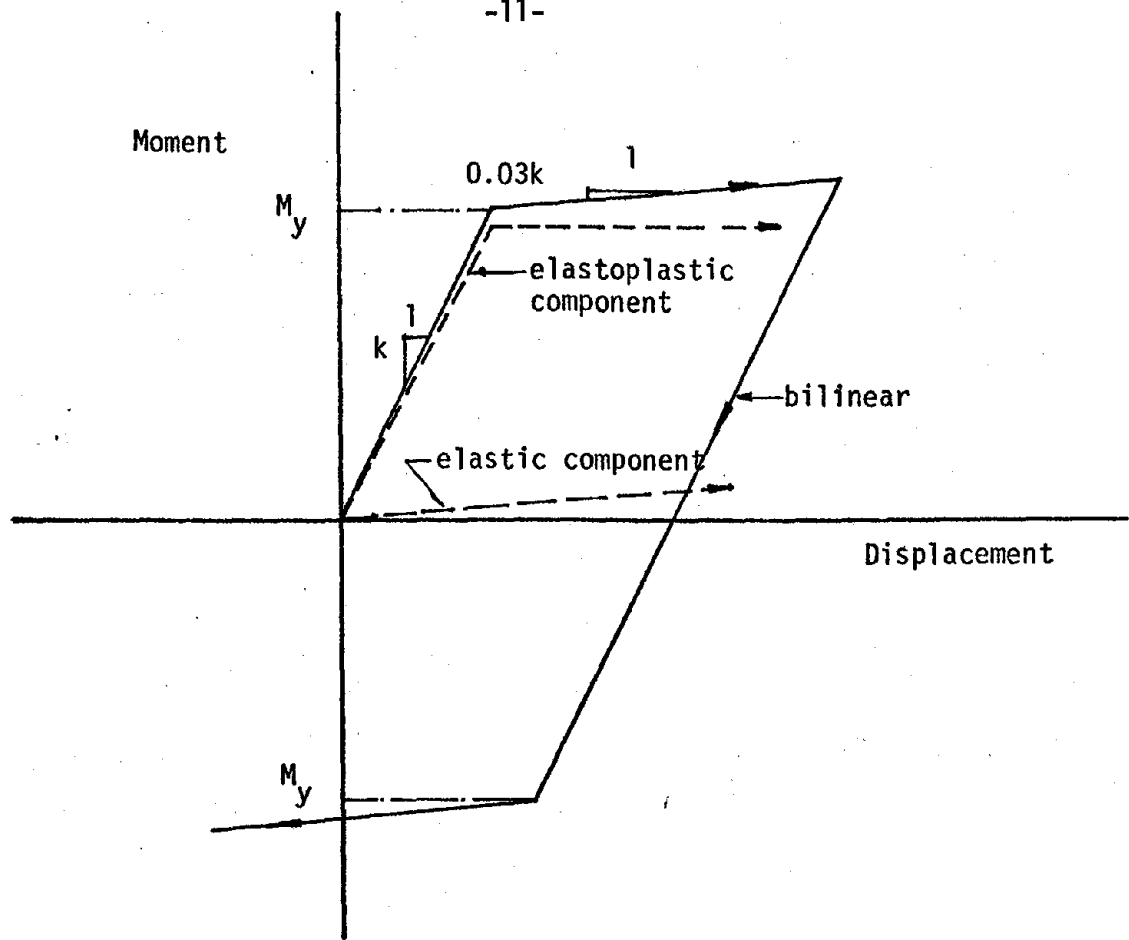


FIGURE 2.1 - DUAL COMPONENT MODEL

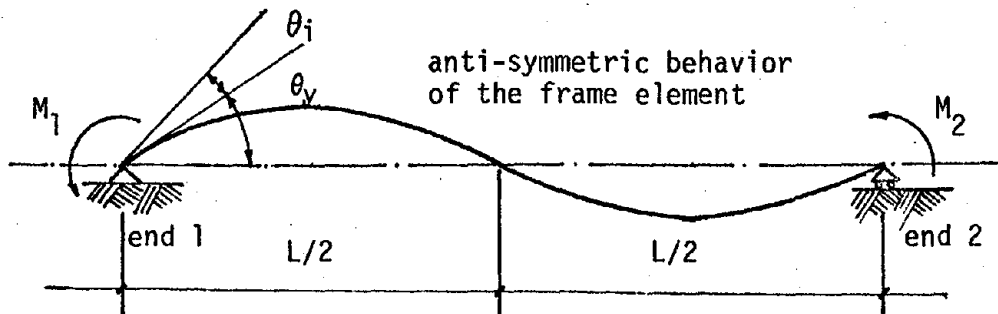
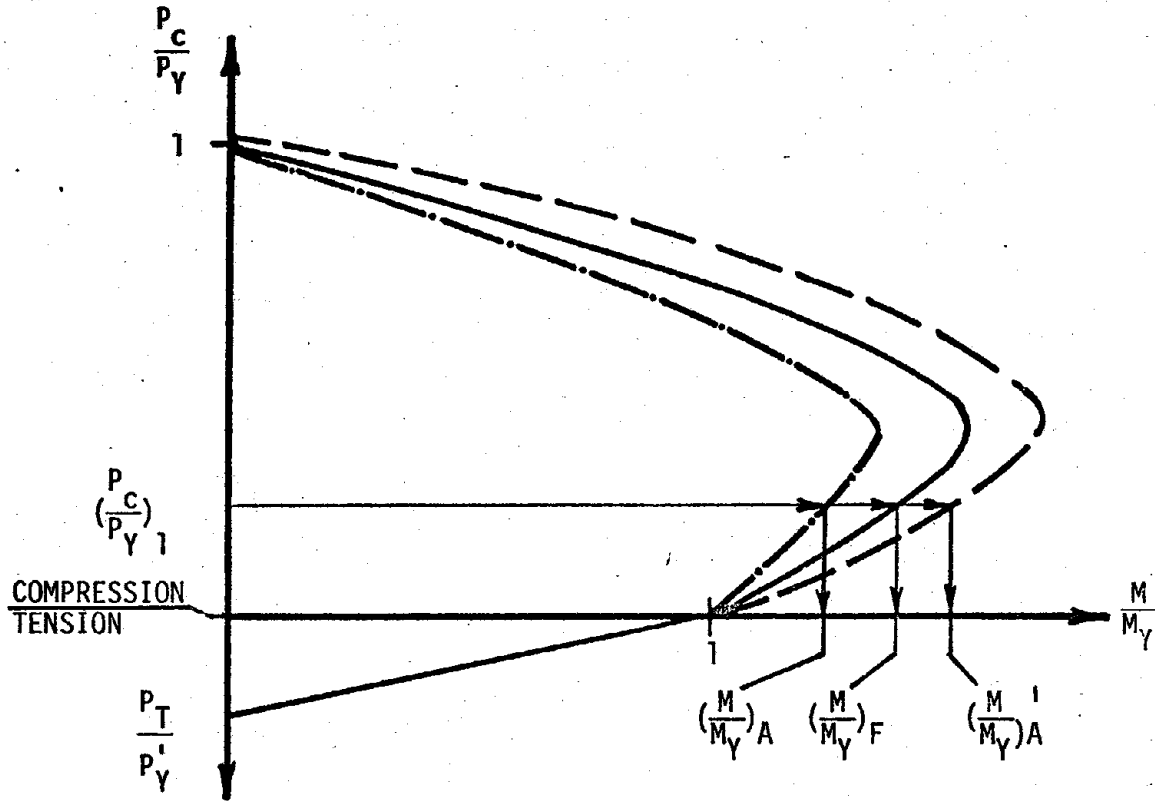


FIGURE 2.2 - DEFINITION OF ROTATIONAL DUCTILITY



- = AVERAGE DIAGRAM USED IN FRIEDA
- = ACTUAL SHAPE OF DIAGRAMS HAVING STEEL RATIO < AVERAGE STEEL RATIO
- = ACTUAL SHAPE OF DIAGRAMS HAVING STEEL RATIO > AVERAGE STEEL RATIO

$(M/M_Y)_F = M/M_Y$  USED BY FRIEDA

$(M/M_Y)_A, (M/M_Y)_A'$  = ACTUAL  $M/M_Y$  RATIOS

$(P_C/P_Y)_1 = P_C/P_Y$  RATIO FOR GIVEN AXIAL LOAD AND COLUMN.

FIGURE 2.3 - COLUMN INTERACTION DIAGRAMS (NORMALIZED) FOR REINFORCED CONCRETE

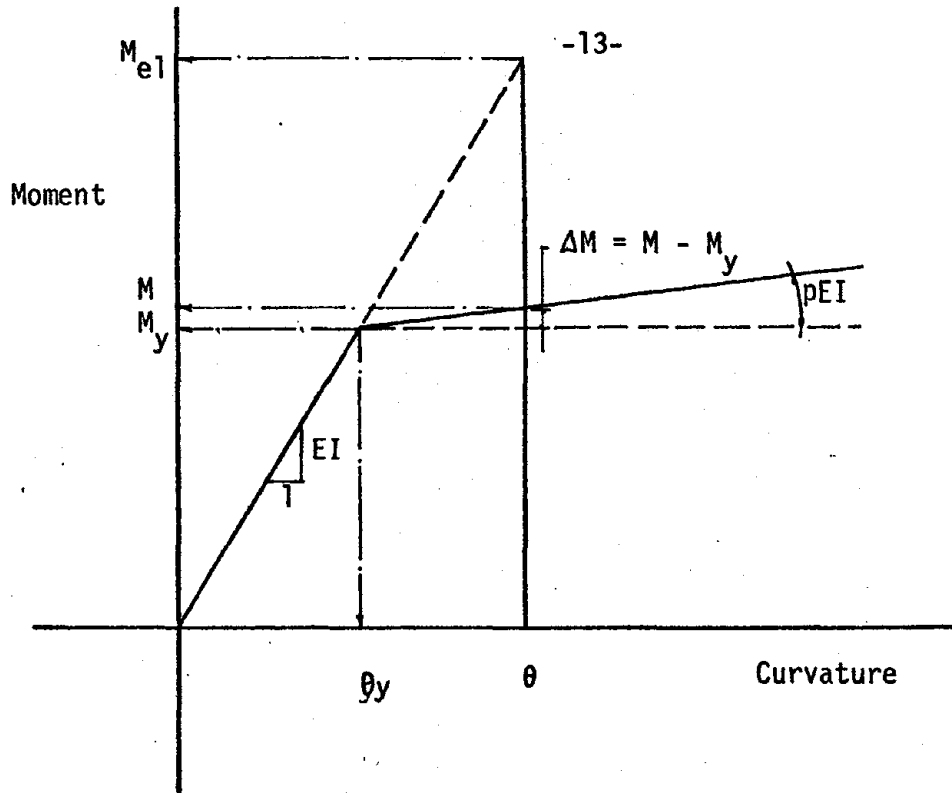


FIGURE 2.4 - DEFINITION OF MOMENT DUCTILITY

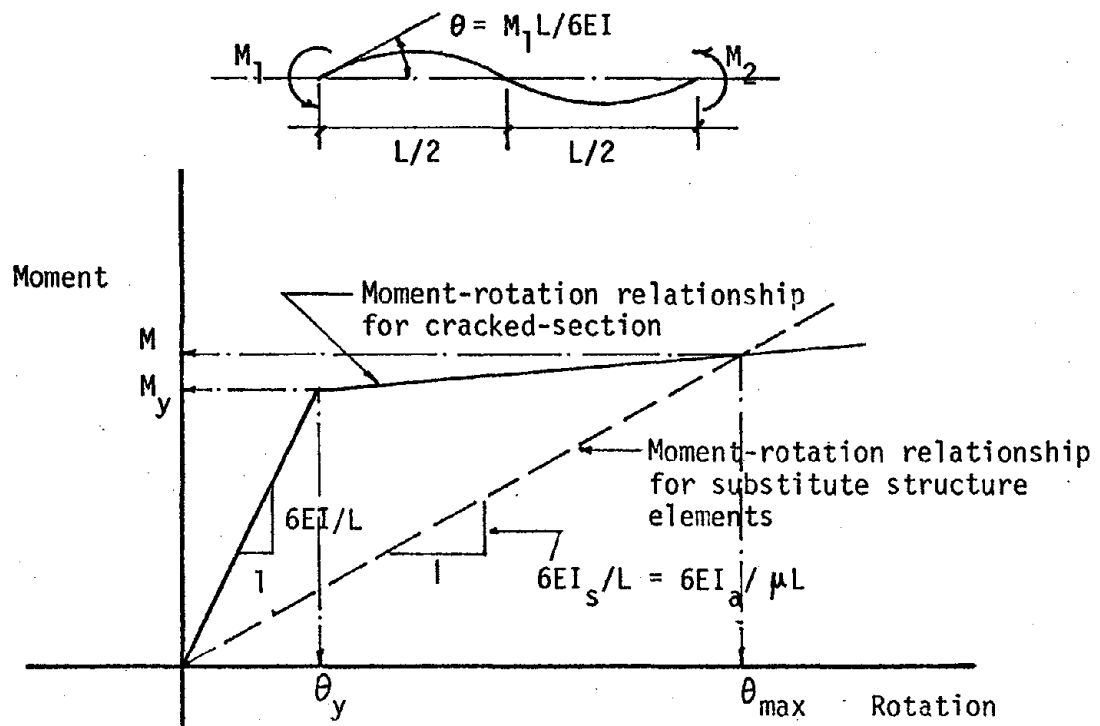


FIGURE 2.5 - DEFINITION OF DAMAGE RATIO



## CHAPTER III - RESULTS - PERFORMANCE OF FRAMES

### 3.1 General

This chapter contains both descriptions of the design procedures used for the twelve frames and the results of the inelastic analyses indicating behavior during earthquake ground motion. The objective of the design is to limit the amount of inelastic distortion during an earthquake of the expected intensity. The purpose of these studies is to determine how well each of the design procedures meets that objective.

The twelve frames being studied are listed in Table 3.1. They are grouped according to the three basic procedures used to determine the required member capacities. In all cases, members were proportioned by ACI-318-71 (6), although in some cases this was modified by ignoring some of the provisions of ACI Appendix A.

The results of the inelastic analyses are presented in the form of plots of girder and column ductility demands over the height of the frame. These are then compared with the presumed intent of the design procedure. The values plotted (e.g., see Fig. 3.3) are the maximum ductility demands at either end of all interior or exterior columns or girders in a story. They therefore represent the extreme values.

The two sets of frame designs by Lau and Persinko were done completely independently. The differences in member sizes, steel ratios, etc., reflect the philosophies and judgements of two individual designers. This results in some difference in frame performance, as would be expected in practice.

### 3.2 Frames Designed by Code

Five of the frames were designed in accordance with the 1973 Uniform Building Code (3), in which the total base shear is given by  $V = ZKCW$ . In these studies the following values were used:

$$Z = 1 \quad (\text{Zone 3})$$

$$K = 0.67 \quad (\text{Ductile frame})$$

$$C = 0.05 / \sqrt{T}$$

$$T = 0.1 N$$

$$N = \text{Number of stories}$$

$$W = \text{Total dead weight plus 25\% of live load.}$$

Member forces due to the seismic loads were determined using the computer program STRUDL.

The members of Frames A, C, and D were proportioned in accordance with ACI-318 and the alternative loading conditions

$$1.4 D + 1.7 L$$

or

$$\frac{3}{4} (1.4 D + 1.7 L + 1.87 E)$$

Frames B and E were designed for the UBC combination  $1.4 (D + L + E)$ .

The frames were analyzed for artificial ground motions matching the Newmark-Blume-Kapur response spectrum (see Fig. 3.1) with a peak ground acceleration of 0.33g. Although the basis for the UBC is not stated, this moderate to strong motion is believed to be a reasonable basis for evaluating performance. If the frames undergo considerable yielding, but do not collapse during this motion, it is considered that the intent of the Code has been achieved.



### 3.2.1 Frame A

This is a four-story frame with the dimensions shown in Fig. 3.2. It is an interior frame of a building with 20-ft. frame spacings. It was designed with a strict interpretation of the UBC-73 and ACI-71 Codes. Also shown in Fig. 3.2 are the overall member sizes and the ultimate capacities. Note that in each column stack the size (but not the reinforcement) is constant. The girders are of uniform size (but not strength) in each floor, and the positive moment capacities are one-half the negative. The member capacities, which were obtained by designing the member for the critical load combinations, and which were used in the dynamic analysis, do not include the ACI  $\phi$ -factors. In the case of columns, the P and M values given are the end points of the interaction diagram, i.e.,  $P_u$  for  $M = 0$  and  $M_u$  for  $P = 0$ . In this design the column strengths are largely determined by the ACI requirement that the column moment capacities shall exceed the girder capacities at a joint.

The design is based on a dead load of 100 psf of floor area and live loads of 20 psf on the roof and 40 psf on all floors.

The natural periods of the frame, as determined by FRIEDA, are given in Table 3.2.

The results of the FRIEDA analyses are shown in Figs. 3.3 and 3.4 for three artificial motions (0.33g). The largest ductility demand occurs in the first floor interior girder with a value of 9.8. The demands in the upper floor girders are relatively small since the design of these members was controlled by gravity loads. The largest demands for the columns (7.9) occur in the first story, but this is at the column base where the assumption of complete fixity may not be realistic. The non-uniform distribution

of column demands, i.e., larger in the third story than in the adjacent stories, is caused by the relative weakness in the columns of that story which is a natural result of this design procedure, which requires that column strengths not be less than girder strengths. As expected, the three motions give somewhat different results, although the difference is not extreme at most points.

The average ductility demands for the three motions and all four stories are given in Table 3.3.

In view of these rather modest average demands, and the fact that the largest demand (9.8) at a single point in a girder does not suggest collapse of the frame, it is concluded that the UBC design largely achieved its purpose. However, a more uniform distribution of demand over the frame would be desirable.

### 3.2.2 Frame B

This design has the same UBC/ACI basis as that of Frame A. The difference in member sizes and capacities is primarily due to the difference between the attitudes of two designers.

The dimensions of Frame B (see Fig. 3.5) are the same as Frame A, but the dead weight was taken to be 120 psf rather than 100 psf. Material properties were taken to be  $f'_c = 4000$  psi and  $f_y = 60,000$  psi as opposed to 3000 and 60,000 for Frame A. All girders are 12" x 20" and all columns are 12" x 18". The sum of the column design moments at a joint was not allowed to be less than 1.2 times the sum of the girder design moments. The critical design values of moment and axial force are given in Table 3.4. In spite of these differences, the two frames are quite similar.

The results of the FRIEDA analysis of this frame are shown in Fig. 3.6. The maximum ductility demands obtained were 8.5 in the girders and 9.7 in the columns, and the averages were 6.0 and 7.0 respectively. Compared to Frame A, the girder demands are smaller and the column demands greater (see Table 3.3). However, the differences are not great considering the reliability of any such prediction. Again, there is little yielding in the upper stories because gravity loading controls the member designs in this region. It was observed in this analysis that more yielding occurred in the bottom girder steel than in the top, even though the positive moment capacity was increased above the design value to one-half the negative capacity. Based on the computed response to a 0.33g ground motion, these results also suggest that the UBC/ACI design procedure produces a reasonable design.

### 3.2.3 Frame C

In the design of this frame the UBC provisions were applied as in the case of Frames A and B, but ACI provisions were modified by eliminating some of the provisions of Appendix A. Specifically, the limitation on steel ratio in the girders and the requirement that column strengths at a joint be at least as great as that of the girders, were ignored. This had the effect of reducing the size of all members and, most important, reducing the capacities of the columns. This can be observed by comparing the sizes and capacities shown in Fig. 3.7 (for Frame C) with those in Fig. 3.2 (for Frame A). The strengths of the columns, particularly in the upper stories, has been greatly reduced. The natural periods are of course lengthened (see Table 3.2).

The results of the FRIEDA analyses are shown in Figs. 3.8 and 3.9. As expected, the ductility demands are considerably increased as compared to Frame A because of the smaller member strengths. This is particularly true in the interior columns where one value at a column base reached 12. The largest ductility demand for girders was 9.5.

In this case Motion 1 produced much larger demands. This occurs by accident, and for obscure reasons, but confirms the belief that in time-history analysis one should not rely on the results for a single motion.

The average ductility demands (see Table 3.3), while larger than for frame A, are not excessive. The maximum values indicate considerable damage and are probably more than intended by the Code.

On the basis of this one comparison, it may be speculated that the provisions of ACI-318, Appendix A, which primarily increases column strength, results in improved performance, but not substantially. It should be noted that this marginal improvement requires a considerable increase in member sizes and strengths. However, the results for the taller Frames D and E seem to indicate that stronger columns are beneficial.

#### 3.2.4 Frame D

This eight-story frame was designed by the same procedures as was Frame B. The dimensions are shown in Figs. 3.10 and 3.11, and the design capacities of the members are given in Table 3.5. This is a rather unusual design in that the columns are much stronger than usual and considerably stronger than the girders. This resulted from the selection of relatively flexible girders so that, to some extent, the individual column stacks act like vertical cantilevers.

The results of the analyses for ground motions are shown in Figs. 3.12 and 3.13. The ductility demands are quite low in both columns and girders, with maximum values of 3.1 and 3.4 respectively. The exterior columns remained elastic over most of the height. One reason for the low ductility demands may be the long period of the frame (2.0 secs; see Table 3.2) compared to the value of 0.1 N or 0.8 secs used in the Code design. Although this design is not considered to be typical of Code designs, and may not be economical, it displayed great resistance to the seismic input.

#### 3.2.5 Frame E

This ten-story frame was designed on the same basis as Frame C (UBC/Modified ACI). The dimensions and member capacities are shown in Fig. 3.14. Note that, compared to the eight-story Frame D, the girders are slightly deeper and have about the same capacities, but the columns are shallower and have much less capacity. The natural periods of the two frames are about the same.

The analytical results (Figs. 3.15 and 3.16) show large ductility demands at the eighth floor with maxima of 15.5 in the columns and 12.0 in the girders. The demands in the lower stories are generally moderate. This behavior must be considered unacceptable.

This "whip-lash" effect in the upper stories has been observed by several other researchers. It is thought to be due to the effect of higher modes, which produce larger forces and shears in the upper stories. This may be particularly true in this case, because the frame is quite flexible, with a fundamental period of 1.99 secs. The higher modes (see Table 3.2) are therefore in a range of greater acceleration response (see Fig. 3.1).

The UBC attempts to correct for this condition by a concentrated force at the top of the frame. However, the 1973 UBC did not require that force in this case. The 1976 UBC would require such a force, but the magnitude is probably not enough to substantially improve the performance of this frame.

It is concluded that in this particular application the UBC approach did not produce a satisfactory result. However, it should be recognized that the maximum ductility demand may not be a reliable indication of the severity of the "whip-lash" effect.

### 3.2.6 Summary - Code Designs

The relatively simple Code approach to aseismic design produced frames which were reasonably well-behaved when subjected to a 0.33g ground motion. However, a more even distribution of yielding between the elements making up the frame would be desirable.

The requirement that the columns be at least as strong as the connected girders is marginally beneficial, but not essential to acceptable performance. This assumes that the columns are detailed so as to be sufficiently ductile.

The one exception to the above was the ten-story frame which displayed excessive ductility ratios in the upper stories. This frame could probably have been improved if the design had included a concentrated force at the top. However, that force should be even larger than that specified by UBC-76.

### 3.3 Frames Designed Using Inelastic Response Spectra

The three frames described below were designed using an inelastic response spectrum constructed according to rules proposed by Newmark and Hall (4).

In this procedure, the elastic design spectrum is modified by factors which depend upon the desired maximum ductility ratio. The required member capacities are then determined by a modal response spectrum analysis exactly the same as would be used in an elastic design. This approach presumably gives the designer control over the amount of yielding that would occur during the design earthquake.

The inelastic acceleration spectrum is constructed as shown in Fig. 3.17. Beginning with the Newmark-Blume-Kapur elastic spectrum, the inelastic acceleration is the same in the short period range ( $A_0$ ), is less by a factor of  $\sqrt{2\mu-1}$  in the intermediate range ( $A'$ ), and is less by a factor of  $\mu$  in the constant velocity and constant displacement range ( $V'$  and  $D'$ ). The inelastic displacement spectrum is at all points equal to  $\mu$  times the inelastic acceleration spectrum. The inelastic spectrum shown in Fig. 3.17, which was used for the designs discussed below, is based upon a peak ground acceleration of 0.33g, 5 percent damping, and a maximum ductility ratio of 4. The latter value, implying moderate structural damage, is thought to be a reasonable design basis.

The design procedure is based upon standard elastic modal analysis. For each mode of the elastic structure, the spectral acceleration is read, the inertia forces and the resulting member forces are computed, and then the modes are combined at the member level by the square root of the sum of the squares (SRSS) method. The members are then designed for these forces combined with the gravity load effects. The process is iterative since modifications to the assumed member sizes change the natural periods and hence the spectral accelerations.

This pseudo-elastic procedure is, of course, approximate. The rules for constructing the inelastic spectra are based upon analyses of one-degree

systems. Multi-degree analysis based upon elastic modes and modal superposition is, at best, an approximate indication of inelastic response. However, several studies have indicated that results so obtained are reasonable (12, 15).

The response spectrum approach is only slightly more expensive than the Code approach, given adequate computer facilities. The purpose here is to investigate the degree of control which this somewhat more sophisticated procedure gives the designer over the ductility demands on the members of a frame.

### 3.3.1 Frame F

This four-story frame has the same dimensions, gravity loads, material properties, and member sizes as Frame C. However, the member capacities are considerably larger as a result of the response spectrum analysis. (See Fig. 3.18). The larger capacities were achieved by increasing the steel ratios. The members were designed using the ACI load factors and load combinations, but not the special provisions of Appendix A. In other words, the design procedures used for Frames C and F are identical except that the seismic member forces were determined by Code in the former and by response spectrum analysis in the latter.

The results of the inelastic time-history analyses are shown in Figs. 3.19 and 3.20. The ductility demands for the columns are smaller than the target values (4) and at many points the columns remain elastic. The girders are subject to demands in the 4-5 range at the bottom stories, but to lesser demands in the upper stories, due to effect of gravity loads on the design. The average ductility demand (see Table 3.3) is considerably smaller than



the design value of 4. The maximum girder demand (5.7) is higher than desired but not excessive.

Compared to the comparable Code design (Frame C) the ductility demands are much smaller and more evenly distributed. The response spectrum approach, in this case, appears to result in an improved design. It produced yielding of the girders but little yielding in the columns, which is often considered desirable. On the average, it is conservative, but it did not prevent higher than desired demands in the lower girders.

### 3.3.2 Frame G

Perhaps a better test of the seismic design procedure would result if all safety factors were removed when proportioning members. The concept of a factor of safety may not be appropriate with the response spectrum approach, since presumably the maximum expected earthquake is the basis for design. To explore this possibility, Frame F was redesigned for the straight addition of dead, live and seismic effects without the ACI load factors. Furthermore, the  $\phi$ -factors were not included when determining member capacities. The result is shown in Fig. 3.21. Member sizes are the same but capacities are smaller, particularly in the columns, than those in Frame F.

The results of the test analyses are shown in Figs. 3.22 and 3.23. The column ductility demands are generally near the target value and fairly uniform over height, except for Motion 1, which produced larger demands. The same is true for the exterior girders, but the interior girders are subject to smaller demands except in the bottom story.

The average demands for the three motions are shown in Figs. 3.24 and 3.25. for both Frames F and G. On the whole, the average demands for

Frame G are not significantly larger, even though member strengths are less. The main difference is a shift of the major yielding from the girders to the columns.

It may be concluded from these results that, on the average, the response spectrum approach produces a reasonably good design, excluding the rather large demand required by Motion 1. It is an improvement, although perhaps not a major one, over the comparable Code design (Frame C).

### 3.3.3 Frame H

The ten-story Frame E was redesigned using the same procedure as for Frame G, i.e., the response spectrum approach and member proportioning without load factors or  $\phi$ -factors. The result is shown in Fig. 3.26. Compared to the Code-designed Frame E (Fig. 3.14), the column strengths are generally smaller, but the girder strengths are about the same in the upper floors but larger in the lower.

The results of the time-history analyses are shown in Figs. 3.27 and 3.28. The ductility demands are excessive in the 7th, 8th and 9th stories. Compared to the results for the Code design (Figs. 3.15 and 3.16) there is improvement. In fact, the ductility demands in the upper stories are generally greater. This is surprising, since the response spectrum method should take into account the effect of higher modes. Apparently, the higher-mode argument is not sufficient to explain the "whip-lash" effect in flexible frames.

It must be concluded that, at least in the case of this particular ten-story frame, the response spectrum approach, like the Code approach, is inadequate. Neither approach provides sufficient strength in the upper stories.

#### 3.3.4 Summary - Response Spectrum Designs

The inelastic response spectrum method of design is attractive because it enables one to design for the maximum expected earthquake and to control the amount of yielding due to that earthquake. In the case of the four-story frames it worked reasonably well. However, based upon maximum ductility demands, it failed to produce a good design for the ten-story frame. It is not known whether this frame is unique or the conclusion is generally valid for all tall, flexible frames.

Nevertheless, the response spectrum approach, with some modification, has the potential of becoming a relatively simple yet reliable method of design.

#### 3.4 Frames Designed by the Substitute Structure Method

The Substitute Structure Method (SSM), proposed by Shibata and Sozen (5), is an empirical, yet rational, procedure based upon response spectrum modal analysis. It differs from the inelastic response spectrum approach in that the properties of the structure are modified, rather than the response spectrum, to account for inelastic behavior. In this procedure the effective periods and the effective damping are both increased, and thus the spectral acceleration is generally decreased. This "substitute structure" is analyzed using the normal elastic modal analysis technique. Unlike the inelastic spectrum analysis method, it is possible to assign different levels of yielding to different members of the frame. To define these levels, the method uses "damage ratio" rather than ductility ratio (see Sec. 2.2).

The method will be described only in general terms here. The details of the procedure may be found in References (2) and (5).

The frames discussed herein were designed using the smoothed response spectrum developed by Shibata and Sozen. This is based upon six real motions normalized to a peak ground acceleration of 0.5g. These, together with the smoothed spectra, are shown in Fig. 3.29. For simplicity, the smoothed spectra are defined by mathematical expressions including the effect of damping. The frame designs discussed herein were evaluated by subjecting them to three of the real motions.

In the substitute structure the effective stiffness ( $EI$ ) of each member is assumed to be the elastic stiffness divided by the design damage ratio. Natural frequencies and mode shapes are obtained by linear analysis of the substitute structure. Each member of the frame is assigned a damping value which increases with the damage ratio which is assigned to that member. Modal damping values are then computed as an average of the member values weighted according to the strain energies in that mode.

Using the modal periods and damping values of the "substitute structure," modal member forces are determined in the usual way. These are finally combined by a modified SRSS procedure. The SSM also stipulates that the column moment capacities at a joint be at least 1.2 times the girder capacities.

The SSM requires somewhat more computational effort than the inelastic response spectrum approach. However, if the procedures are automated this may not be significant.

The frames described below are similar to the UBC-designed Frames B and D. They have the same dimensions, dead and live loads, and material properties. However, they were designed and evaluated for a peak ground acceleration of 0.5g. The natural periods of these frames are listed in

Table 3.2. These are for the elastic structure, not the substitute structure. They are based upon transformed cracked sections which were assumed to have one-third the moment of inertia of the gross section for girders and one-half the gross section for columns.

In general, members were proportioned for the combination

$$1.4(D+L) + E .$$

No factor was placed on the earthquake loading because the large input (0.5g) presumably includes a sufficient safety factor. However, where gravity moments are not required for stability (see below), they were not added to the earthquake moments.

#### 3.4.1 Frame I

This four-story frame was designed for damage ratios of one for all columns and six for all girders, the intent being to confine inelastic behavior to the latter elements. The girder design end moments were taken to be the larger of the earthquake and gravity values—these were not added. However, the member size was also required to be capable of resisting the simple-span gravity moment. Thus, the gravity end moments are not required to prevent collapse of the girders. The columns, since they were to remain elastic, were designed for the combined earthquake and gravity moments together with the gravity axial load plus or minus the earthquake load. The members were proportioned according to ACI-318 including the  $\phi$ -factors normally used.

The resulting member design forces are shown in Table 3.6. All columns are 28 ins. x 28 ins. and all girders are 16 ins. x 24 ins. Compared with the same frame designed by Code (Frame B, Table 3.4), the members are

considerably larger and the column moment capacities are several times larger. In fact, the required column capacities produced by the procedure seem unreasonable. This results from the attempt to keep the columns elastic during a strong earthquake.

The maximum damage ratios, which are related to ductility demand, resulting from the time-history analyses are shown in Fig. 3.30. The distribution over height is quite uniform but all values are below those intended in the design. For all three motions, the columns remain completely elastic, as intended, with a maximum damage ratio of 0.95 and an average (for all three motions) of 0.71. For the girders, the maximum was 3.5 and the average 3.0, compared with the target value of 6.0.

This design produced the desirable uniformity of yielding over the frame height and kept the columns elastic. However, it is conservative in that the damage ratios were smaller than intended.

#### 3.4.2 Frame J

This four-story frame was designed for a damage ratio of four in all members. Otherwise, the procedure was the same as for Frame I. However, the columns were designed for either the earthquake or the gravity moment, not the combination of the two. The resulting member design forces are shown in Table 3.7. All girders are 12 ins. x 20 ins. and all columns are 12 ins. x 18 ins. Compared to the preceding design, the exterior girder design forces are generally larger, those for the interior girders are smaller, and the columns are much weaker. The relative stiffnesses of the columns and girders in the real as well as in the substitute structure are quite different in the two cases. This had a great effect on the column moments.

The performance results for this frame are shown in Fig. 3.31. In this case the damage ratios are less uniform over the height, particularly in the columns. The maximum column ratio was 5.6 and the average was 2.3. The maximum girder ratio was 4.0 and the average 2.4.

Although the column demand ratios exceeded the target value for one motion, the SSM appears to be generally conservative in this case also.

### 3.4.3 Frame K

This is an eight-story frame designed for elastic column behavior and a damage ratio of six for the girders. Thus the procedure is in all respects the same as used for Frame I. The member design forces are given in Table 3.8. The girders are 16 ins. x 26 ins. in the upper five floors and 16 ins. x 30 ins. in the lower three. The columns are 25 ins. x 25 ins. in the upper five stories and 28 ins. x 28 ins. in the lower three.

The maximum damage ratios computed in the time-history analyses are plotted in Figs. 3.32 and 3.33. Except for the interior columns, the ratios are fairly uniform and less than the target values. The interior columns do not remain elastic as intended and reach a maximum damage ratio of 2.7. The average for all columns and all motions is 1.0. The maximum ratio in the girders was 4.0 and the average 2.9. This frame responded with the same "whip-lash" as the ten-story Frames E and H, although to a lesser extent.

The SSM procedure in this case did not produce the result desired, i.e., the columns did not remain elastic and the girders did not yield to the extent intended.

#### 3.4.4 Frame L

This eight-story frame was designed for a damage ratio of four in all members. The procedure was exactly the same as for Frame J. The member design forces are shown in Table 3.9. Compared to the preceding design, the girder forces are not appreciably different but the column forces, and hence actual capacities, are much smaller. The girders are all 12 ins. x 22 ins. The columns are 12 ins. x 22 ins. in the upper five stories and 12 ins. x 25 ins. in the lower three. All members are considerably smaller than in the previous case with a consequent increase in natural periods (see Table 3.2).

The response results are plotted in Figs. 3.34 and 3.35. The computed damage ratios indicate that the design was generally conservative, except for the upper stories of the columns where the "whip-lash" effect caused ratios near the target value. The maximum column ratio is 4.8 and the average for all columns and motions is 1.9. The maximum for girders was 2.7 and the average 1.8.

In this case the SSM procedure resulted in a generally conservative design but did not predict the "whip-lash" effect.

#### 3.4.5 Summary - Substitute Structure Method

The Substitute Structure Method, as interpreted and applied here, appears to be generally conservative and may result in uneconomical designs. In all four cases studied the girders failed to develop the amount of yielding which was intended in the design. On the average, the procedure also produced conservatively designed columns. However, like the inelastic response spectrum approach, it failed to allow for the "whip-lash" effect,



and larger than the desired ductility demands occurred in the upper stories of the columns in the ten-story frames. If the procedure were, on the average, less conservative, it is probable that the ductility demands caused by this effect would be as excessive as those found in the inelastic response spectrum study (Frame H).

Compared to the inelastic response spectrum approach, its main advantage is that it permits design for different ductility ratios in the members of the frame. However, this is not entirely achieved and requires some increase in the complexity of the procedure.

Frames Designed by Code

<u>FRAME I.D.</u>	<u>METHOD OF DESIGN</u>
Frame A (Persinko) (4 Stories)	UBC/ACI
Frame B (Lau) (4 Stories)	UBC/ACI
Frame C (Persinko) (4 Stories)	UBC/Modified ACI*
Frame D (Lau) (8 Stories)	UBC/ACI
Frame E (Persinko) (10 Stories)	UBC/Modified ACI*

Frames Designed by Inelastic Response Spectrum

Frame F (Persinko) (4 Stories)	RS**/Modified ACI* $\mu = 4$
Frame G (Persinko) (4 Stories)	RS**/Modified ACI* Without load factors or $\phi$ $\mu = 4$
Frame H (Persinko) (10 Stories)	RS**/Modified ACI* Without load factors or $\phi$ $\mu = 4$

Frames Designed by Substitute Structure Method

Frame I (Lau) (4 Stories)	SSM/ACI $\mu_D = 1$ Cols., $\mu_D = 6$ Girders
Frame J (Lau) (4 Stories)	SSM/ACI $\mu_D = 4$ Cols & Girders
Frame K (Lau) (8 Stories)	SSM/ACI $\mu_D = 1$ Cols., $\mu_D = 6$ Girders
Frame L (Lau) (8 Stories)	SSM/ACI $\mu_D = 4$ Cols. & Girders

TABLE 3.1 - DESCRIPTION OF FRAME DESIGNS

\*The limitation on steel ratio (Sect. A.5.1 of ACI-318-71) and the requirement that column strengths must exceed girder strengths at a joint (Sect. A.6.2) were not applied.

\*\*The Newmark-Hall inelastic response spectrum was used (4).

<u>Frame I.D.</u>	<u>Mode</u>			
	<u>1</u>	<u>2</u>	<u>3</u>	<u>4</u>
A	.641	.207	.118	.085
B	.872	.268	.143	.098
C	.882	.301	.179	.131
D	2.000	.645	.358	.233
E	1.986	.709	.411	.298
F	.882	.301	.179	.131
G	.882	.301	.179	.131
H	2.164	.780	.478	.336
I	.608	.169	.079	.049
J	1.096	.339	.184	.128
K	1.554	.528	.294	.189
L	2.489	.807	.452	.297

TABLE 3.2 - NATURAL PERIODS OF FRAMES  
(Secs)

<u>Frame I.D.</u>	<u>Ext. Cols.</u>		<u>Int. Cols.</u>		<u>Ext. Girds.</u>		<u>Int. Girds.</u>	
	<u>Max.</u>	<u>Av.*</u>	<u>Max.</u>	<u>Av.*</u>	<u>Max.</u>	<u>Av.*</u>	<u>Max.</u>	<u>Av.*</u>
<u>Code Design</u>								
A	7.9	3.1	7.9	3.1	5.9	3.2	9.8	5.3
B	9.7	3.4	7.6	2.9	4.8	2.3	8.5	3.6
C	8.9	4.0	12.0	6.1	9.5	5.0	9.0	5.2
D	2.0	1.2	3.1	1.5	3.4	2.3	3.1	2.3
E	9.0	2.4	15.5	3.7	12.0	5.4	6.6	3.2
<u>Inelastic Response Spectrum Design</u>								
F	2.9	1.8	2.0	1.2	4.9	3.1	5.7	3.2
G	6.2	3.4	7.3	3.9	5.1	2.8	2.9	1.7
H	12.0	5.8	16.3	5.5	12.1	5.9	3.5	1.9
<u>Substitute Structure Method</u>								
I	0.8	0.6	0.9	0.8	3.2	2.9	3.5	3.0
J	5.6	2.6	4.5	2.0	2.8	2.0	4.0	2.8
K	1.2	0.8	2.7	1.3	4.0	3.0	4.0	2.8
L	4.8	1.7	4.5	2.0	2.7	1.8	2.5	1.7

TABLE 3.3 - MAXIMUM AND AVERAGE DUCTILITY DEMANDS

\* Average of the maxima in the stories for the three motions.

GIRDER DESIGN MOMENTS

Story	Exterior Girders		Interior Girders	
	-ve Moment (kip-")	+ve Moment (kip-")	-ve Moment (kip-")	+ve Moment (kip-")
4	1730.4	865.2	1214.4	607.2
3	2130.0	1065.6	1521.6	760.8
2	2294.4	1147.2	1720.8	860.4
1	2318.4	1159.2	1734.0	867.6

COLUMN DESIGN FORCES

Story	Exterior Columns		Interior Columns	
	Axial Load (kips)	Moment (kip-")	Axial Load (kips)	Moment (kip-")
4	39.8	2076.0	70.3	2806.8
3	87.8	1381.2	150.7	1874.4
2	136.9	1832.4	232.1	1911.6
1	169.1	1562.4	313.7	1911.6

TABLE 3.4 - MEMBER DESIGN FORCES OF THE 4-STORY FRAME B (UBC-ACI)

GIRDER DESIGN MOMENTS

Story	Exterior Girders		Interior Girders	
Level	-ve Moment (kip-")	+ ve Moment (kip-")	-ve Moment (kip-")	+ve Moment (kip-")
8	1984.8	992.4	1984.8	992.4
7	2865.6	1432.8	2599.2	1299.6
6	3104.4	1552.8	2865.6	1432.8
5	3310.8	1656.0	3104.4	1552.8
4	3536.4	1768.8	3310.8	1656.0
3	3666.0	1833.6	3462.0	1731.6
2	3666.0	1833.6	3536.4	1768.8
1	3462.0	1731.6	3462.0	1731.6

COLUMN DESIGN FORCES

Story	Exterior Columns		Interior Columns	
Level	Axial Load (kips)	Moment (kip-")	Axial Load (kips)	Moment (kip-")
8	45.9	2382.0	85.8	3572.4
7	104.2	1887.6	186.5	2599.2
6	164.4	2445.6	287.6	2845.2
5	226.4	2973.6	388.6	2991.6
4	274.0*	3813.6	459.6*	3238.8
3	331.2*	3942.0	547.4*	3238.8
2	387.4*	5223.6	632.7*	3920.4
1	440.2*	6170.4	716.5*	5820.0

TABLE 3.5 - MEMBER DESIGN FORCES OF THE 8-STORY FRAME D (UBC/ACI)

\*Axial loads with live load reduction

GIRDER DESIGN MOMENTS

Story	Exterior Girder (kip-")		Interior Girder (kip-")	
Level	-ve Moment	+ve Moment	-ve Moment	+ve Moment
4	1765.2	1765.2	2289.6	2289.6
3	1869.6*	1839.6	2431.2	2431.2
2	1869.6*	1720.8	2266.8	2266.8
1	1880.4*	1231.2	1622.4	1622.4

\* Design governs by gravity loads.

COLUMN DESIGN FORCES

Story	Exterior Column		Interior Column	
Level	Axial Load (kips)	Moment (kip-")	Axial Load (kips)	Moment (kip-")
4	51.6	4255.2	80.9	4866.0
3	111.4	6163.2	173.3	6382.8
2	169.8	8570.4	265.2	9016.8
1	224.2	15332.4	354.6	15363.6

TABLE 3.6 - MEMBER DESIGN FORCES OF THE 4-STORY FRAME I-  
SSM/ACI. ( $\mu=1$  for columns and  $\mu=6$  for girders)

GIRDER DESIGN MOMENTS

Story Level	Exterior Girder (kip-")		Interior Girder (kip-")	
	-ve Moment	+ve Moment	-ve Moment	+ve Moment
4	1582.8*	792.0	1029.6*	614.4
3	1839.6*	1033.2	1159.2	1159.2
2	1854.0*	1342.8	1491.6	1491.6
1	1874.4*	1407.6	1518.0	1518.0

COLUMN DESIGN FORCES

Story Level	Exterior Column		Interior Column	
	Axial Load (kips)	Moment (kip-")	Axial Load (kips)	Moment (kip-")
4	33.9	1899.6*	67.1	2636.4**
3	70.6	1048.8*	140.9	2007.6**
2	105.5	1395.6*	237.9	2007.6**
1	140.1	1821.6	321.7	2107.2

\* Design governs by gravity loads.

\*\* The criterion that  $\Sigma M_C \geq 1.2 \Sigma M_G$  governs design.  
 ( $M_C$  = column moment strength at design axial load).  
 ( $M_G$  = girder moment strength).

TABLE 3.7 - MEMBER DESIGN FORCES OF THE 4-STORY FRAME J  
 ( $\mu=4$  for all members.)



GIRDER DESIGN MOMENTS

Story Level	Exterior Girder (kip-")		Interior Girder (kip-")	
	-ve Moment	+ve Moment	-ve Moment	+ve Moment
8	1702.8	1702.8	1759.2*	1648.8
7	2294.4*	1930.8	2166.0*	1923.6
6	2241.6*	2108.4	2170.8*	2085.6
5	2276.4	2276.4	2251.2	2251.2
4	2322.0	2322.0	2295.6	2295.6
3	3186.0	3186.0	3129.0	3129.0
2	2913.6	2913.6	2816.4	2816.4
1	2334.0	2334.0	2289.6	2289.6

\* Design governs by gravity loads.

COLUMN DESIGN FORCES

Story Level	Exterior Columns		Interior Columns	
	Axial Load (kips)	Moment (kip-")	Axial Load (kips)	Moment (kip-")
8	67.9	3493.2	97.5	4154.4
7	147.5	4236.0	210.1	3675.6
6	227.2	4674.0	323.0	4382.4
5	307.2	4531.2	436.3	4291.2
4	370.6	5020.8	519.3	5107.2
3	451.1	5378.4	620.8	5595.6
2	527.9	6231.6	720.1	6385.2
1	601.5	11976.0	820.6	12003.6

TABLE 3. 8 - MEMBER DESIGN FORCES OF THE 8-STORY FRAME K

( $\mu=1$  for columns and  $\mu=6$  for girders)

GIRDER DESIGN MOMENTS

Story Level	Exterior Girder (kip-")		Interior Girder (kip-")	
	-ve Moment	+ve Moment	-ve Moment	+ve Moment
8	1738.8*	870.0*	1742.4*	871.2*
7	2430.0*	1215.6*	2157.6*	1078.8*
6	2359.2*	1237.2	2162.4*	1108.8
5	2329.2*	1488.0	2162.4*	1334.4
4	2270.4*	1705.2	2167.2*	1518.0
3	2258.4*	1876.8	2169.6*	1706.4
2	2238.0*	1972.8	2170.8*	1834.8
1	2161.2*	1940.4	2178.0*	1736.4

COLUMN DESIGN FORCES

Story Level	Exterior Column		Interior Column	
	Axial Load (kips)	Moment (kip-")	Axial Load (kips)	Moment (kip-")
8	39.2	2086.8*	85.0	3134.4**
7	85.7	1414.8*	187.0	2042.4**
6	129.8	1629.6*	288.7	2119.2**
5	172.0	1836.0*	390.7	2277.6**
4	285.1	2080.8*	463.0	2368.8**
3	343.7	2239.2*	551.3	2486.4**
2	401.3	2674.8*	637.0	2486.4**
1	456.0	2926.8	721.1	3228.0

\* Design governs by gravity loads.

\*\* The criterion that  $\Sigma M_C \geq 1.2 \Sigma M_G$  governs design.

TABLE 3.9 - MEMBER DESIGN FORCES OF THE 8-STORY FRAME L  
( $\mu=4$  for all members.)

### RESPONSE SPECTRUM

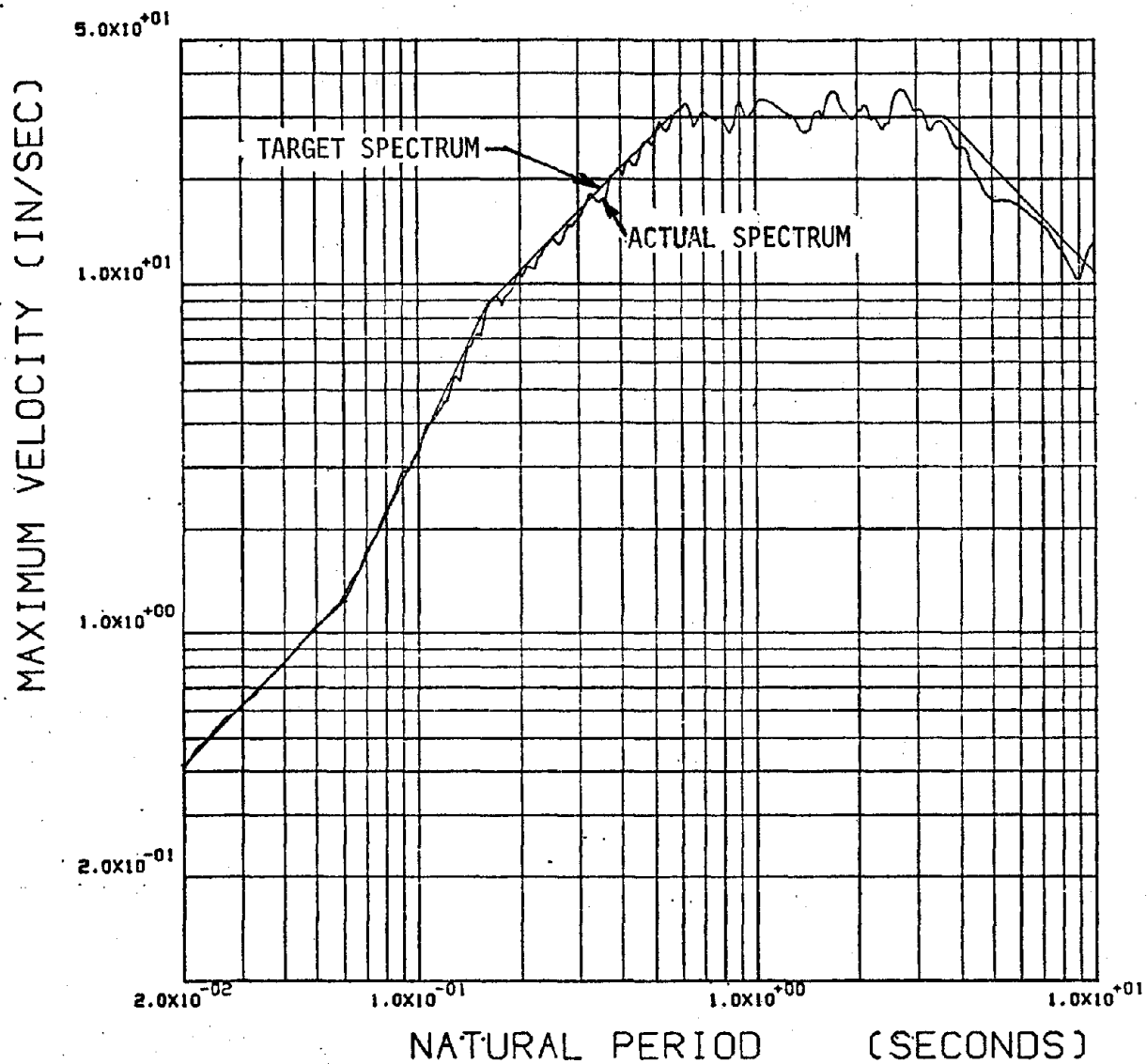


FIGURE 3.1 - RESPONSE SPECTRUM FOR MOTION 4

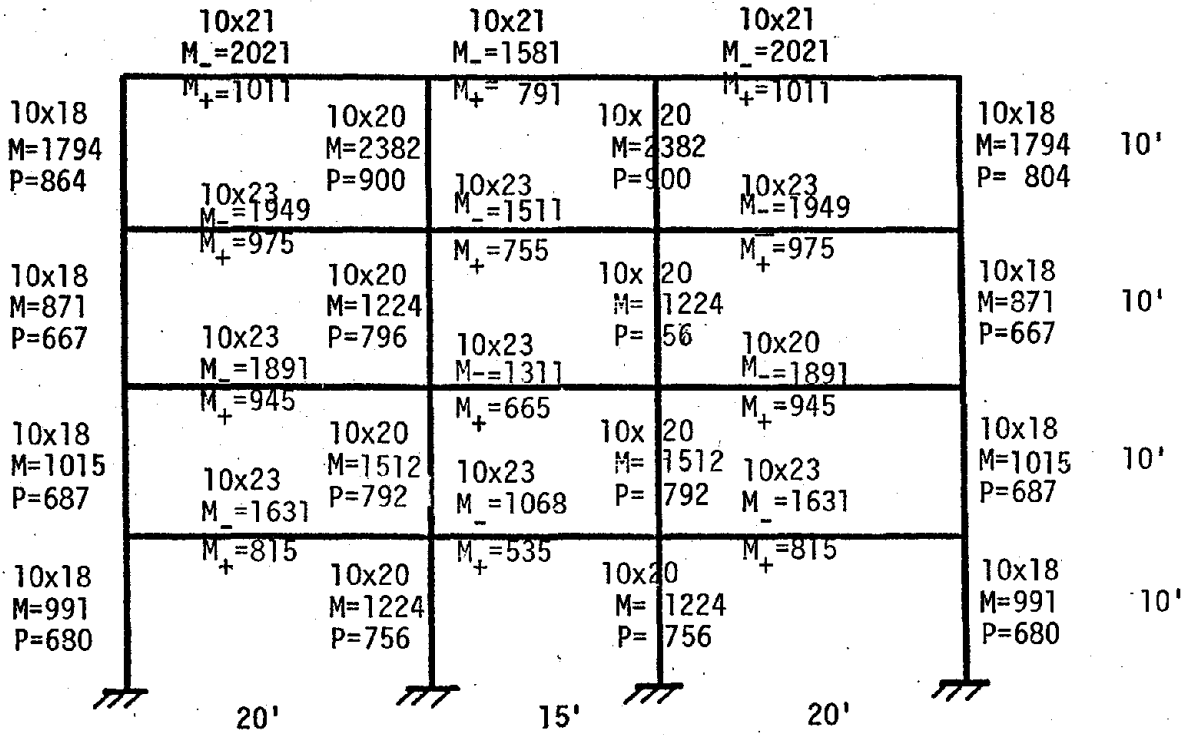


FIGURE 3.2 - FRAME A - UBC/ACI. MEMBER SIZES AND CAPACITIES (kips, ins.)

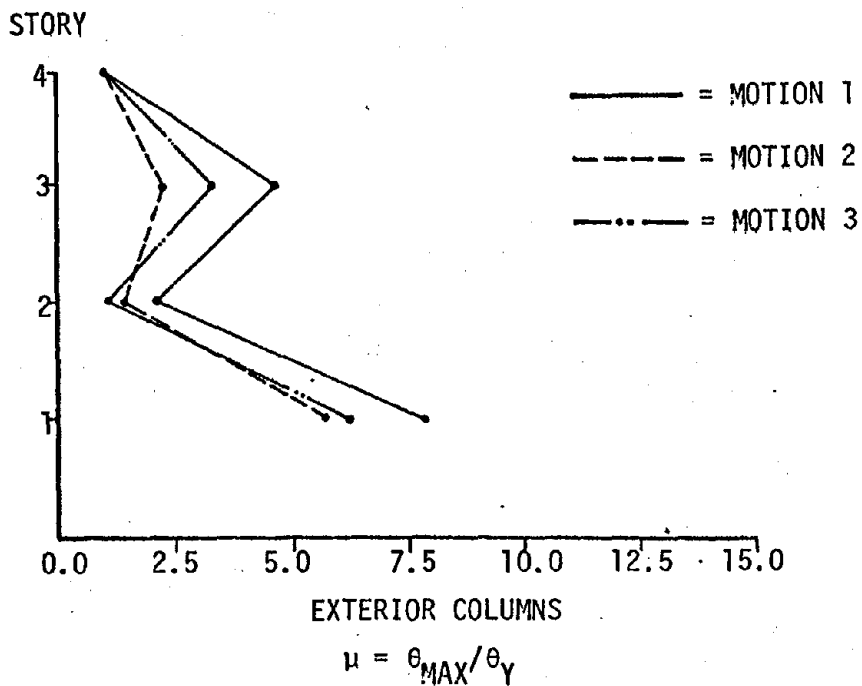
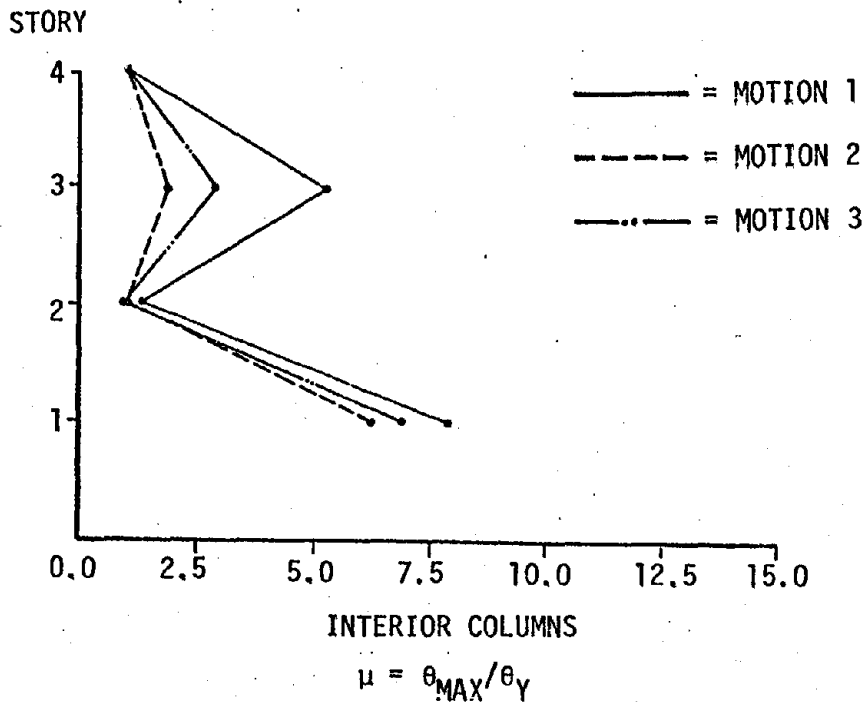


FIGURE 3.3 - FRAME A - UBC/ACI. DUCTILITY DEMANDS - COLUMNS

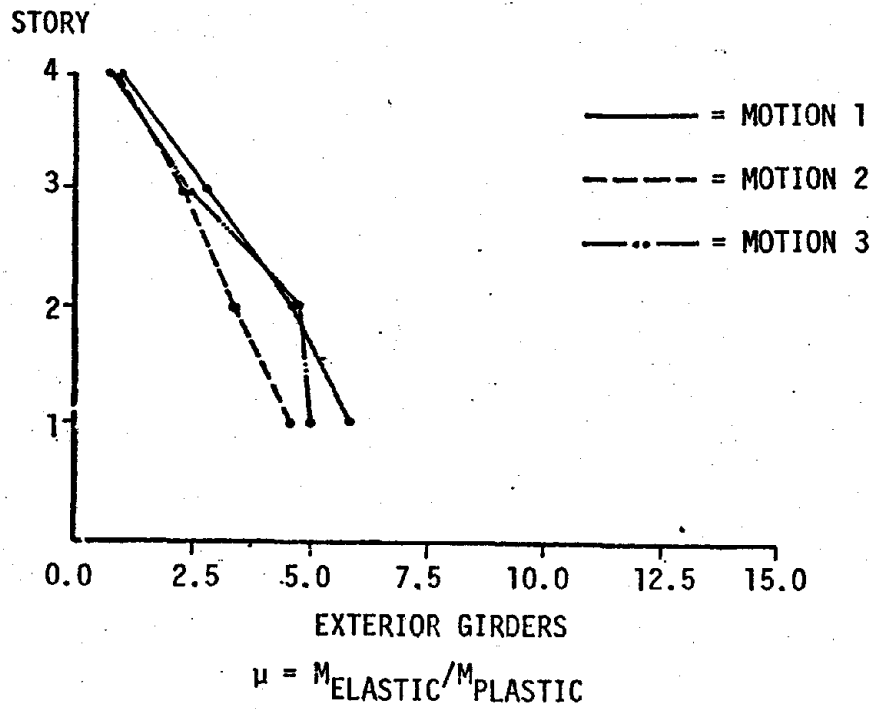
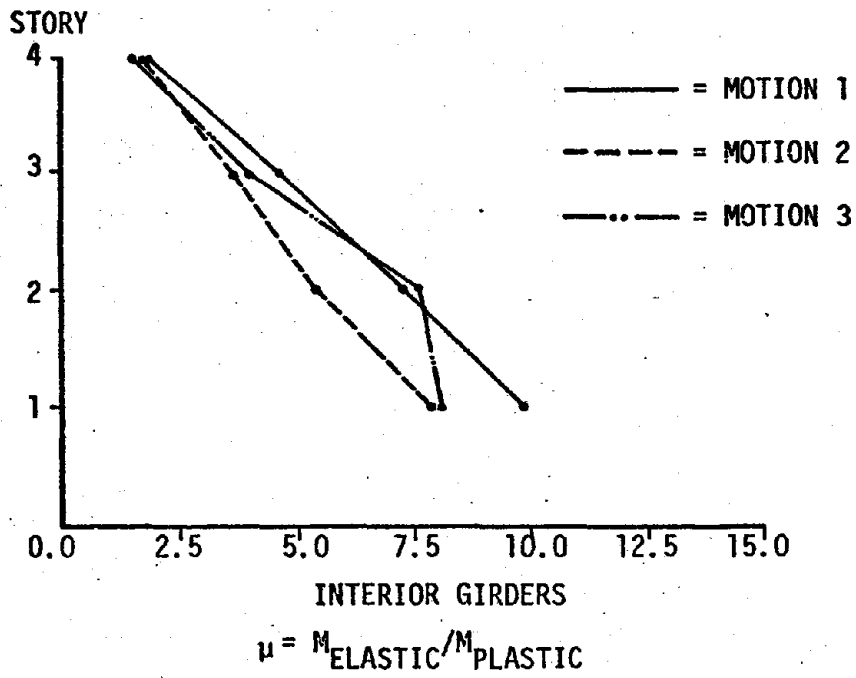


FIGURE 3.4 - FRAME A - UBC/ACI. DUCTILITY  
DAMANDS - GIRDERS

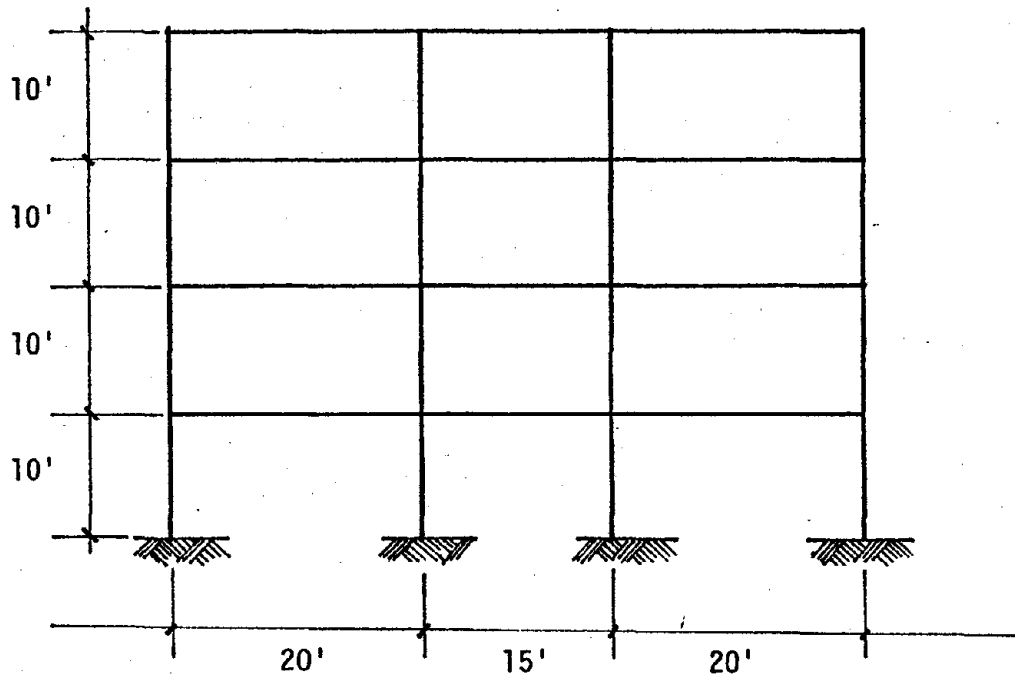


FIGURE 3.5a - AN ELEVATION VIEW OF THE 4-STORY FRAME B

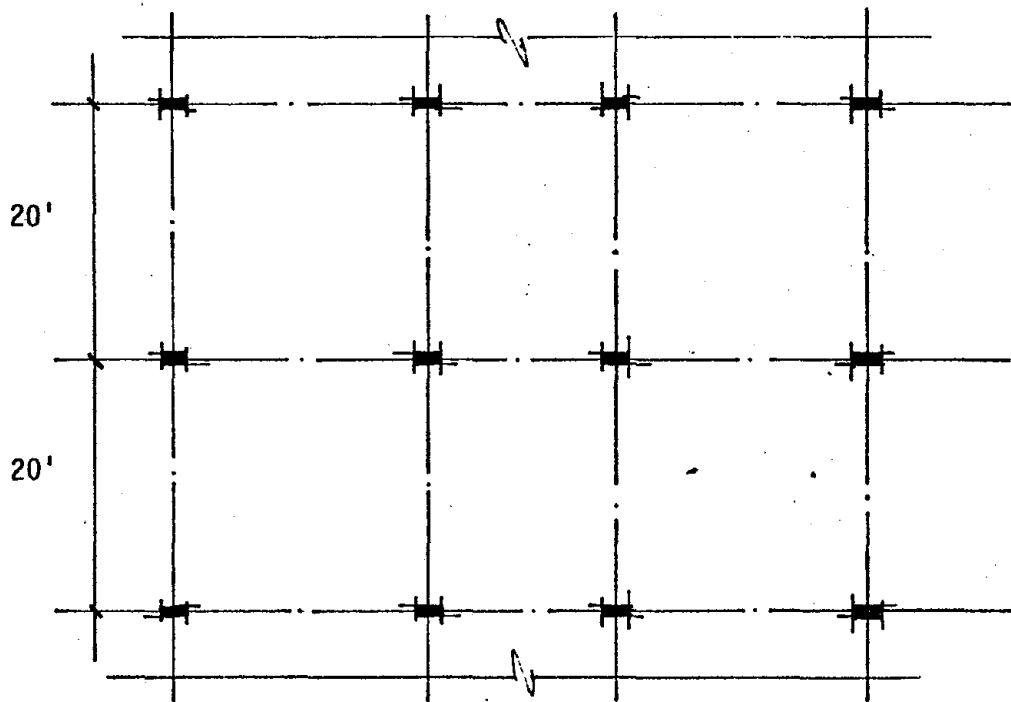
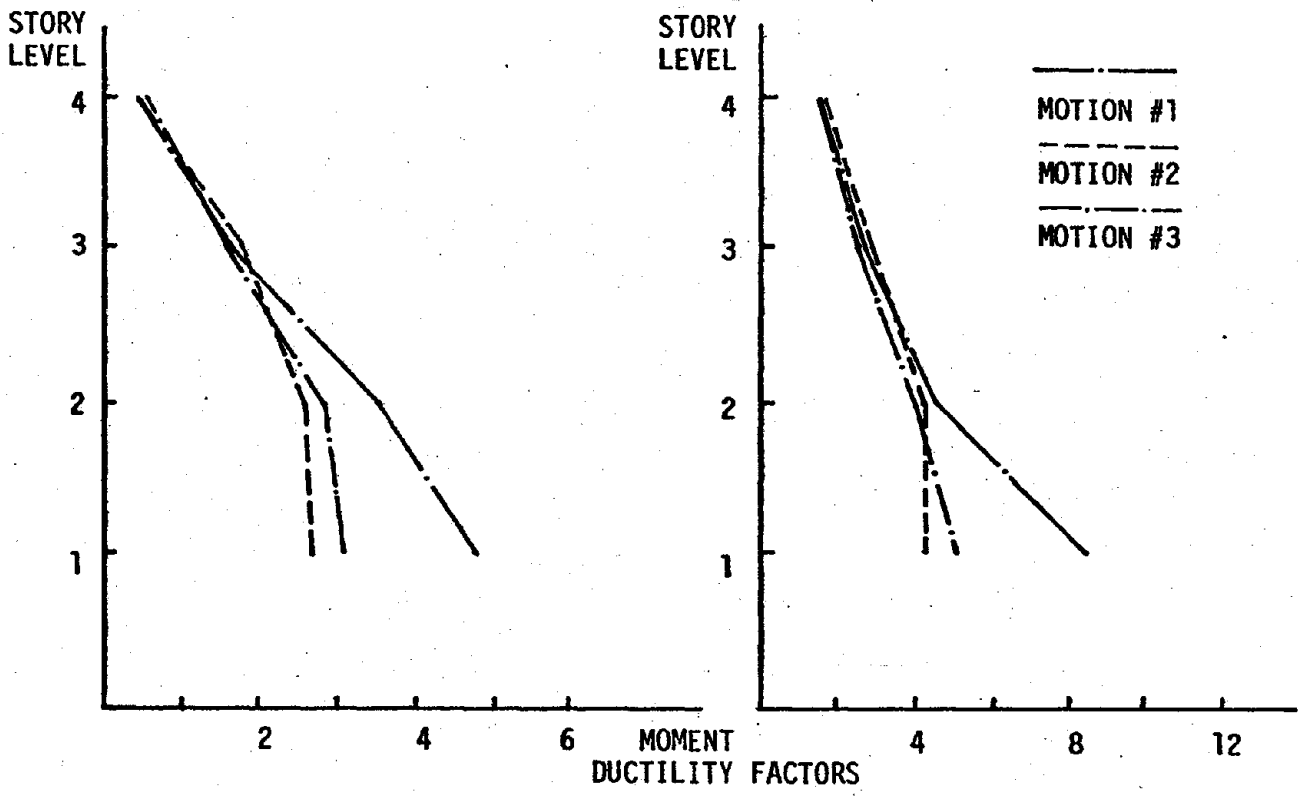
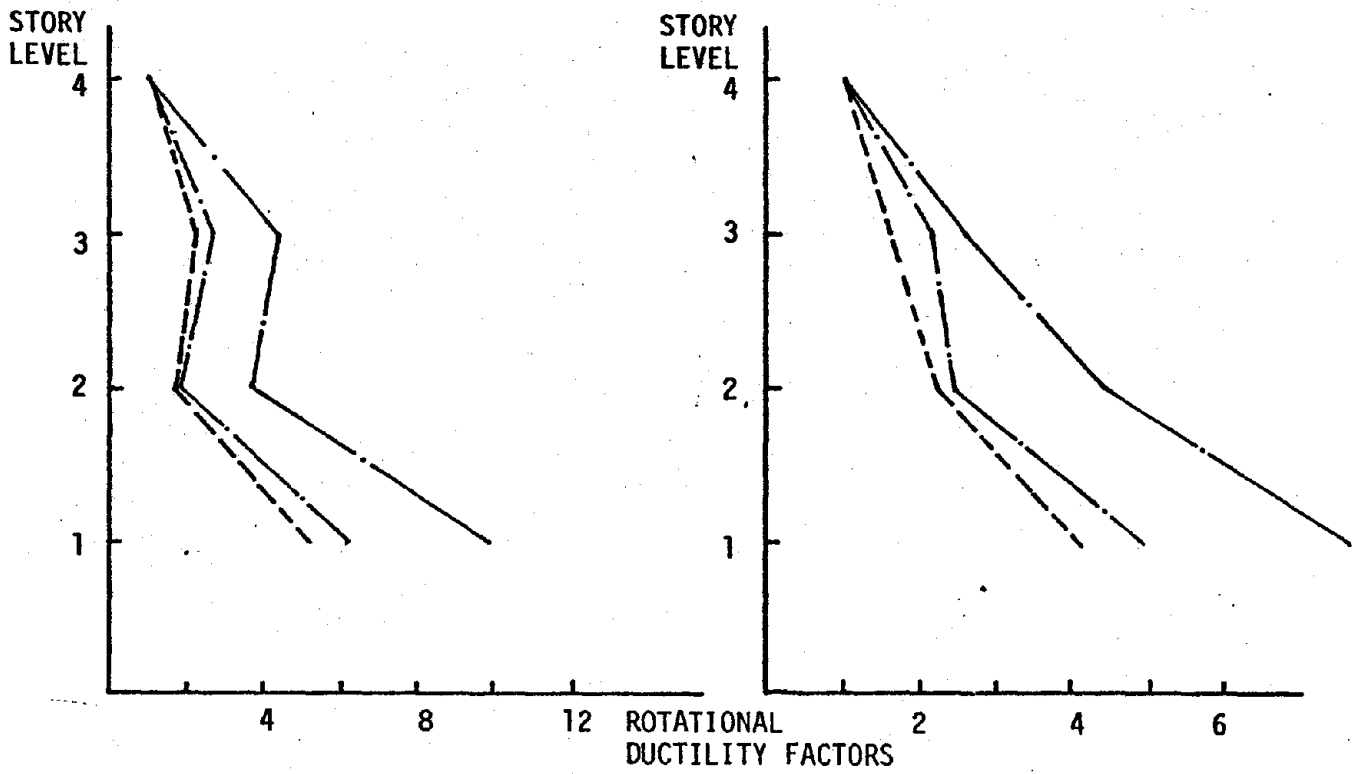


FIGURE 3.5b - A PLAN VIEW OF THE 4-STORY FRAME B



EXTERIOR GIRDER

INTERIOR GIRDER



EXTERIOR COLUMN

INTERIOR COLUMN

FIGURE 3.6 - FRAME B - UBC/ACI. DUCTILITY DEMANDS



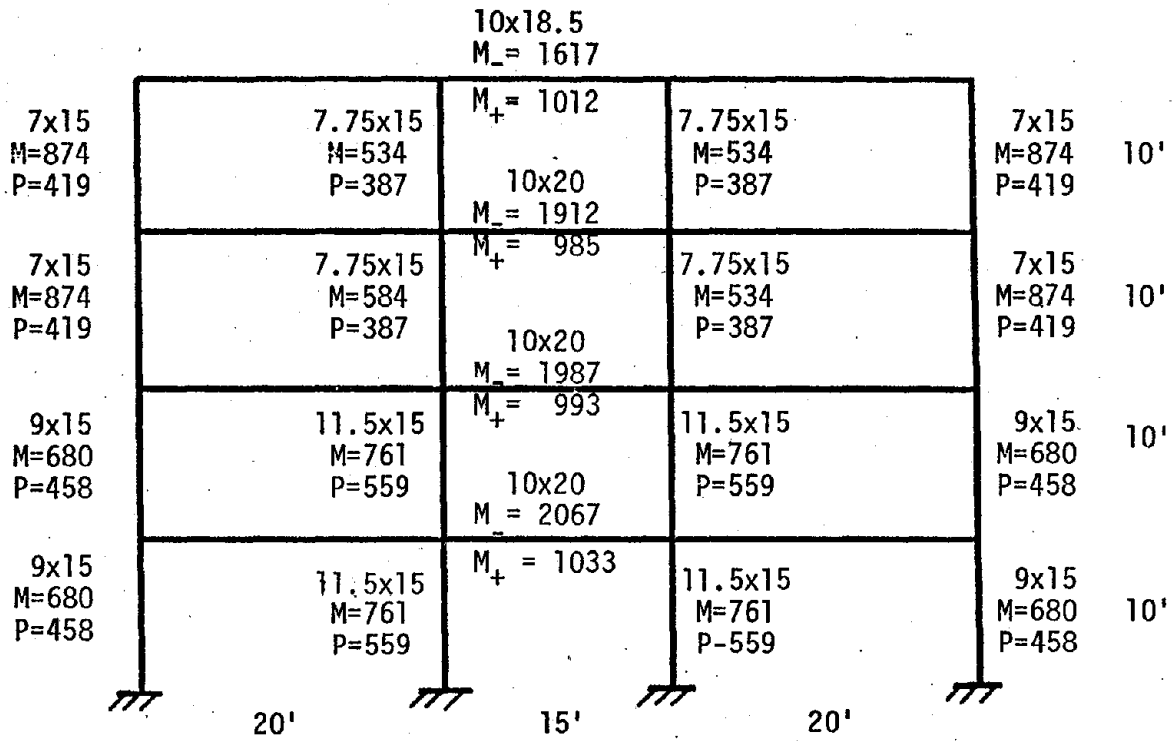


FIGURE 3.7 - FRAME C - UBC/Modified ACI. MEMBER SIZES AND CAPACITIES (kips, ins.)

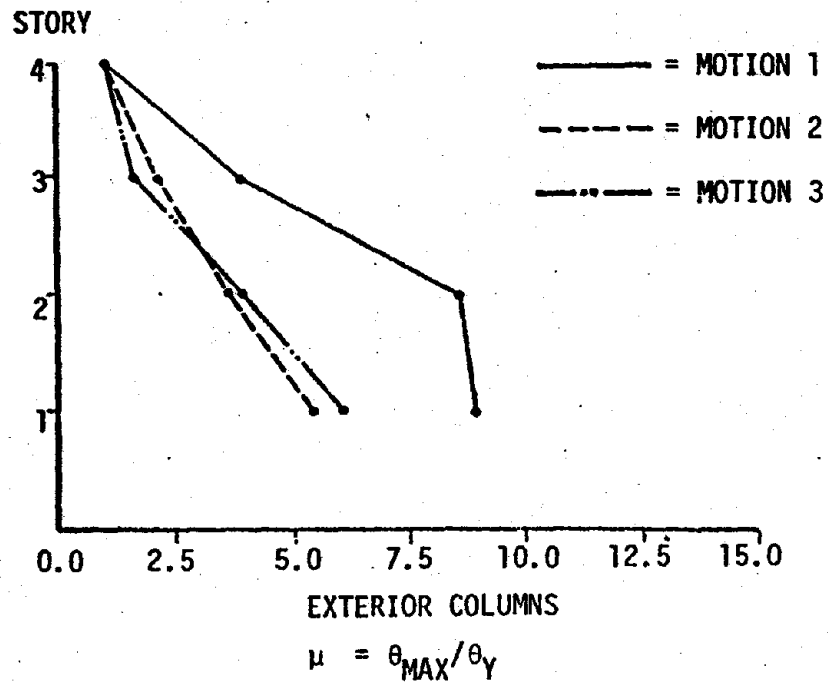
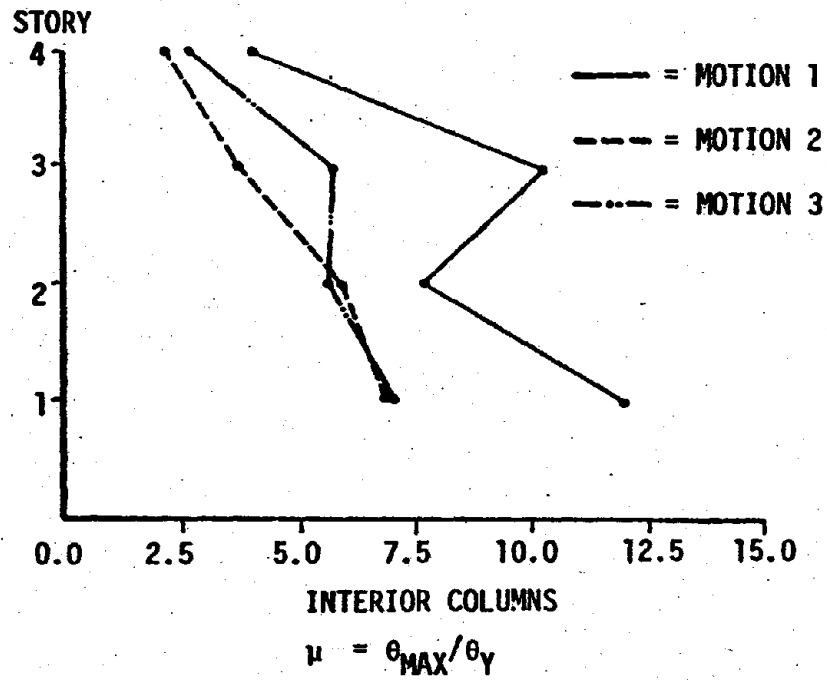


FIGURE 3.8 - FRAME C - UBC/Modified ACI  
DUCTILITY DEMANDS - COLUMNS

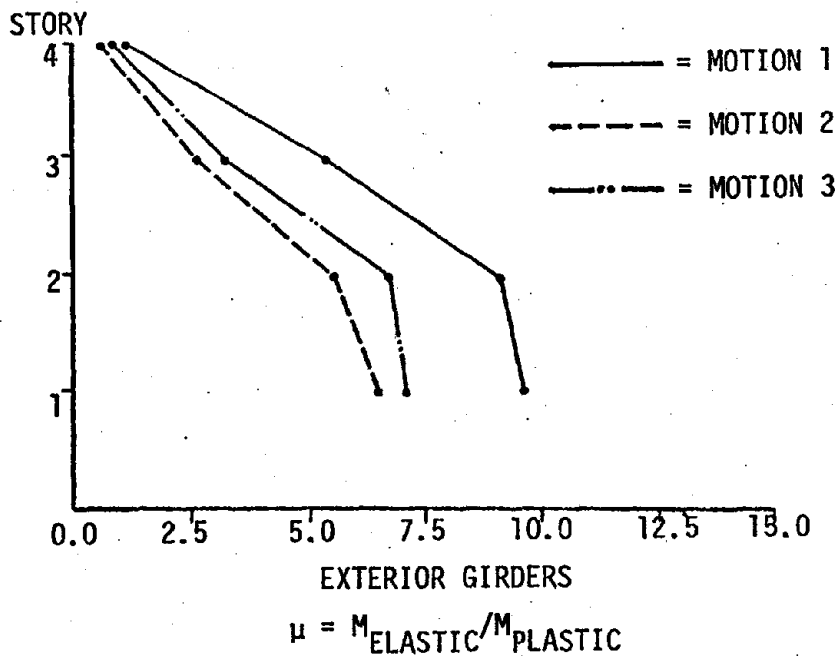
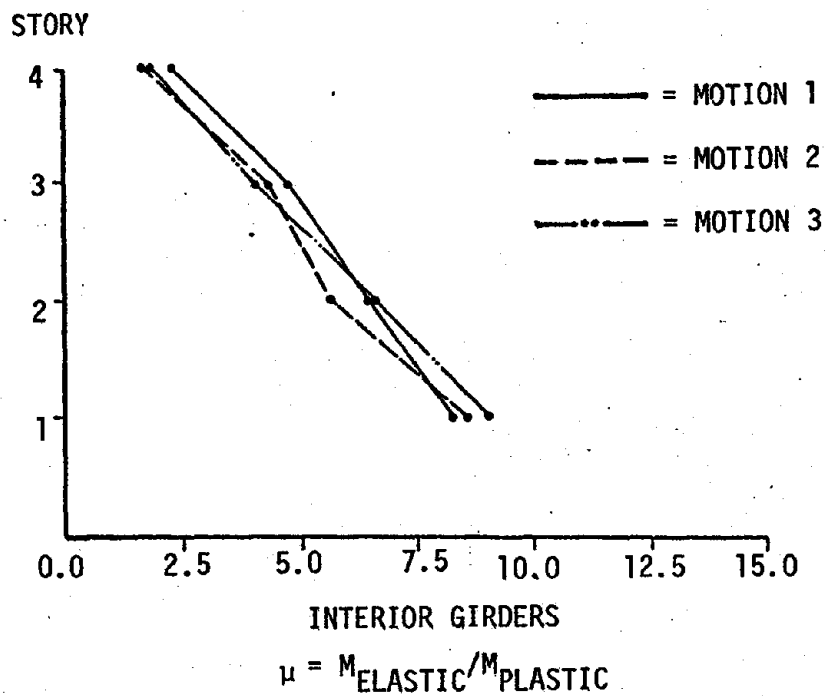


FIGURE 3.9 - FRAME C - UBC/Modified ACI  
DUCTILITY DEMANDS - GIRDERS

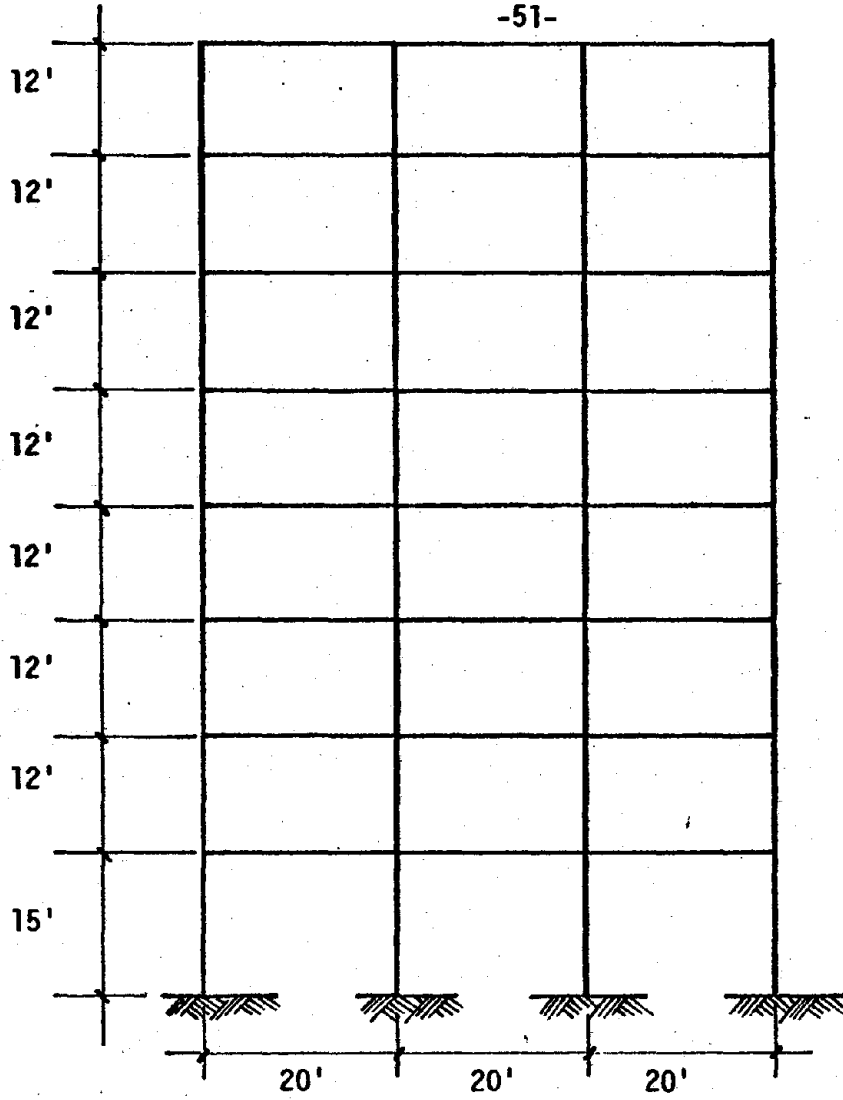


FIGURE 3.10a - AN ELEVATION VIEW OF THE 8-STORY FRAME D

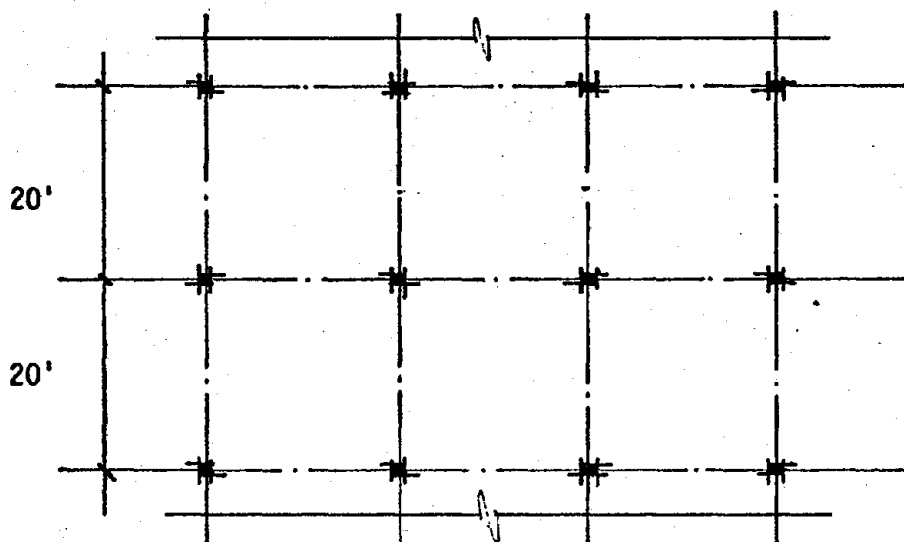


FIGURE 3.10b - A PLAN VIEW OF THE 8-STORY FRAME D

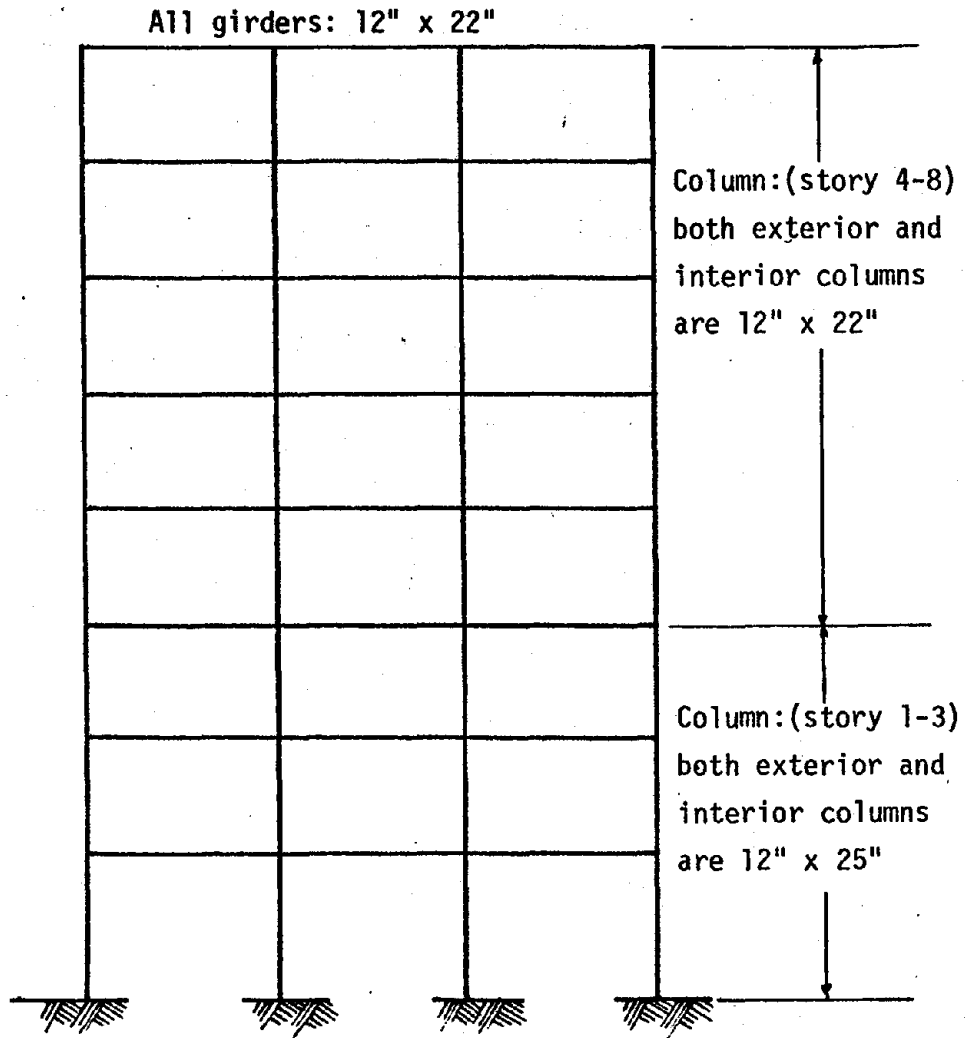


FIGURE 3.11 - MEMBER SIZES OF THE 8-STORY FRAME D

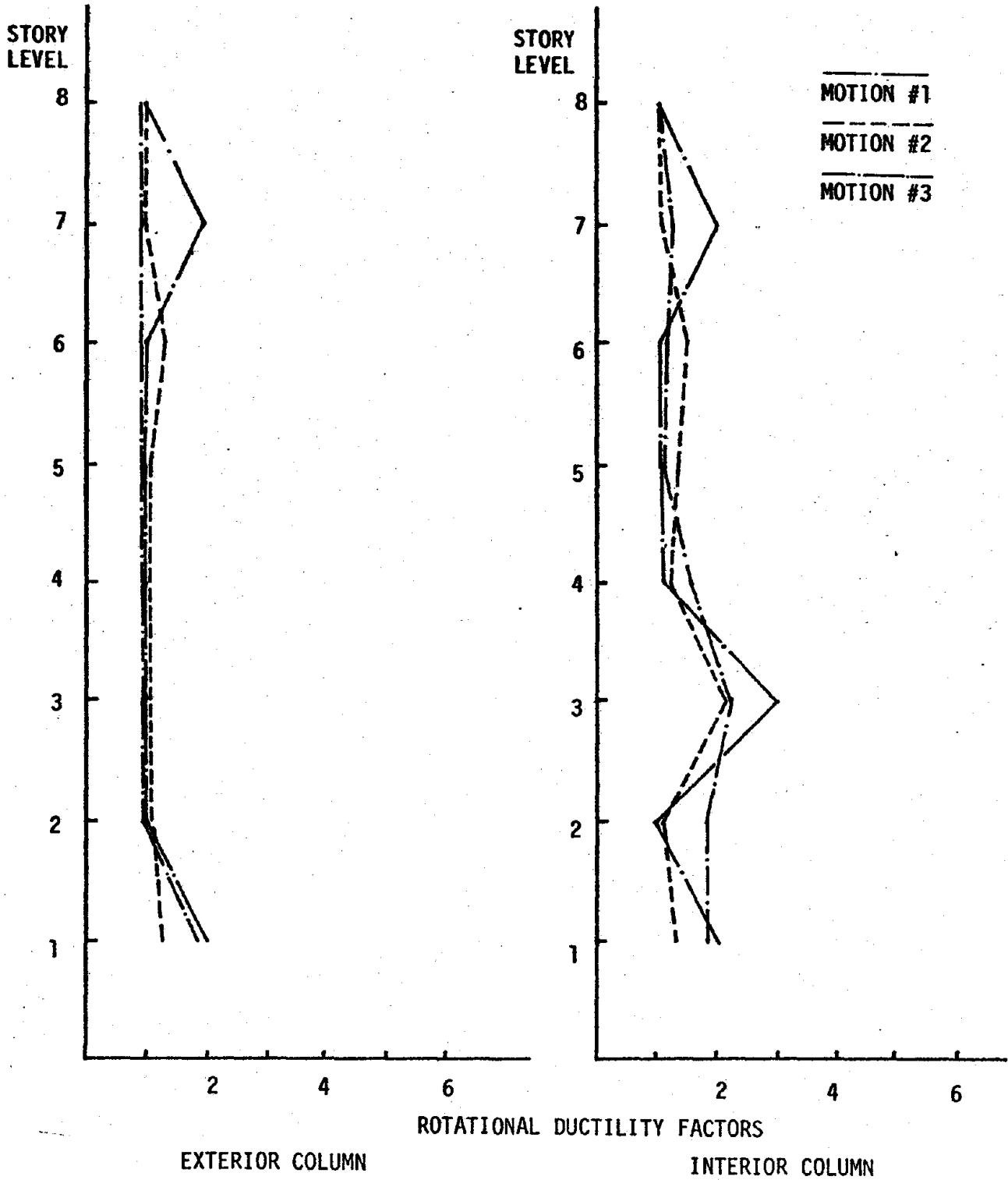


FIGURE 3.12 - FRAME D - UBC/ACI. DUCTILITY DEMANDS - COLUMNS

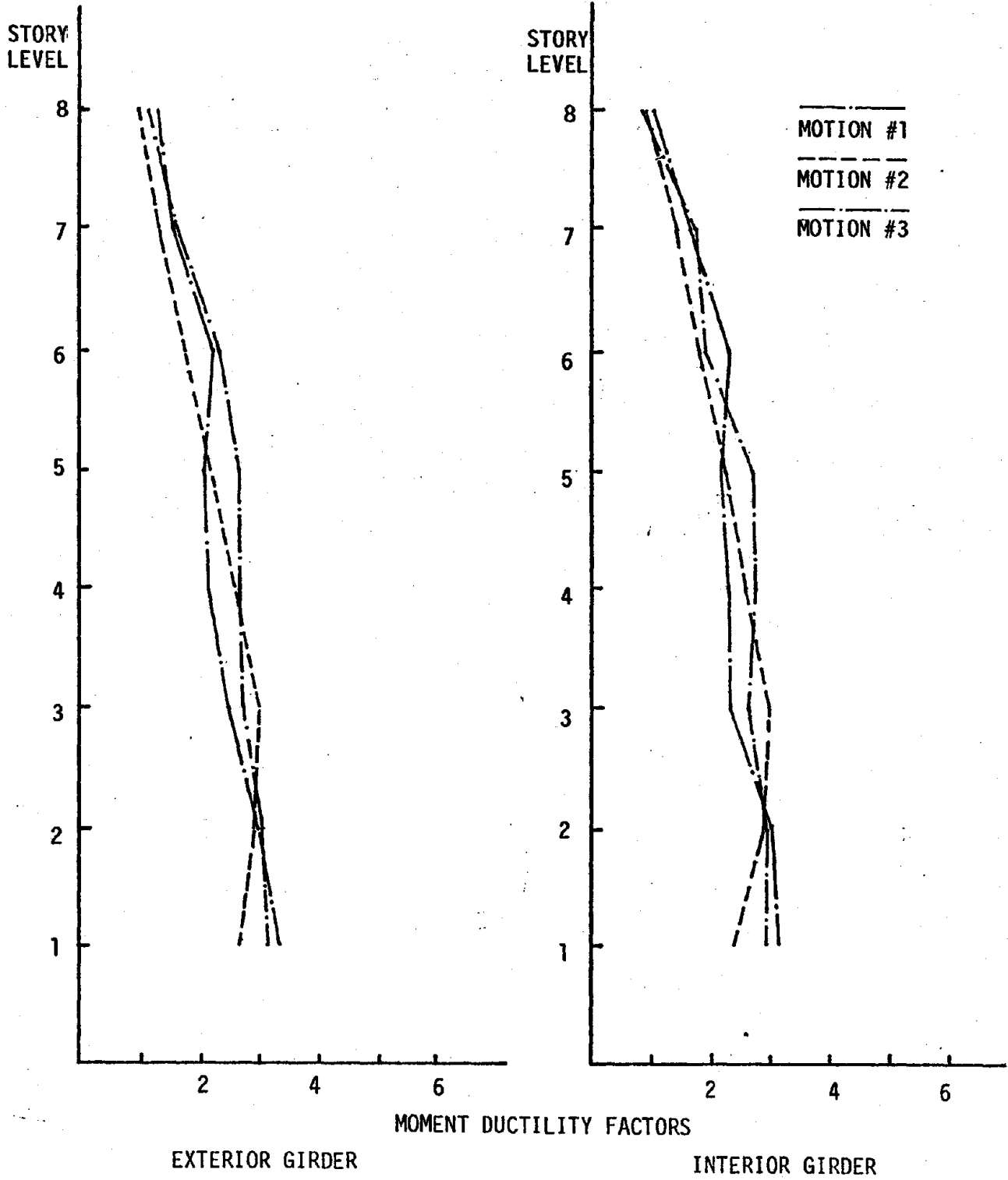


FIGURE 3.13 - FRAME D - UBC/ACI. DUCTILITY DEMANDS - GIRDERS

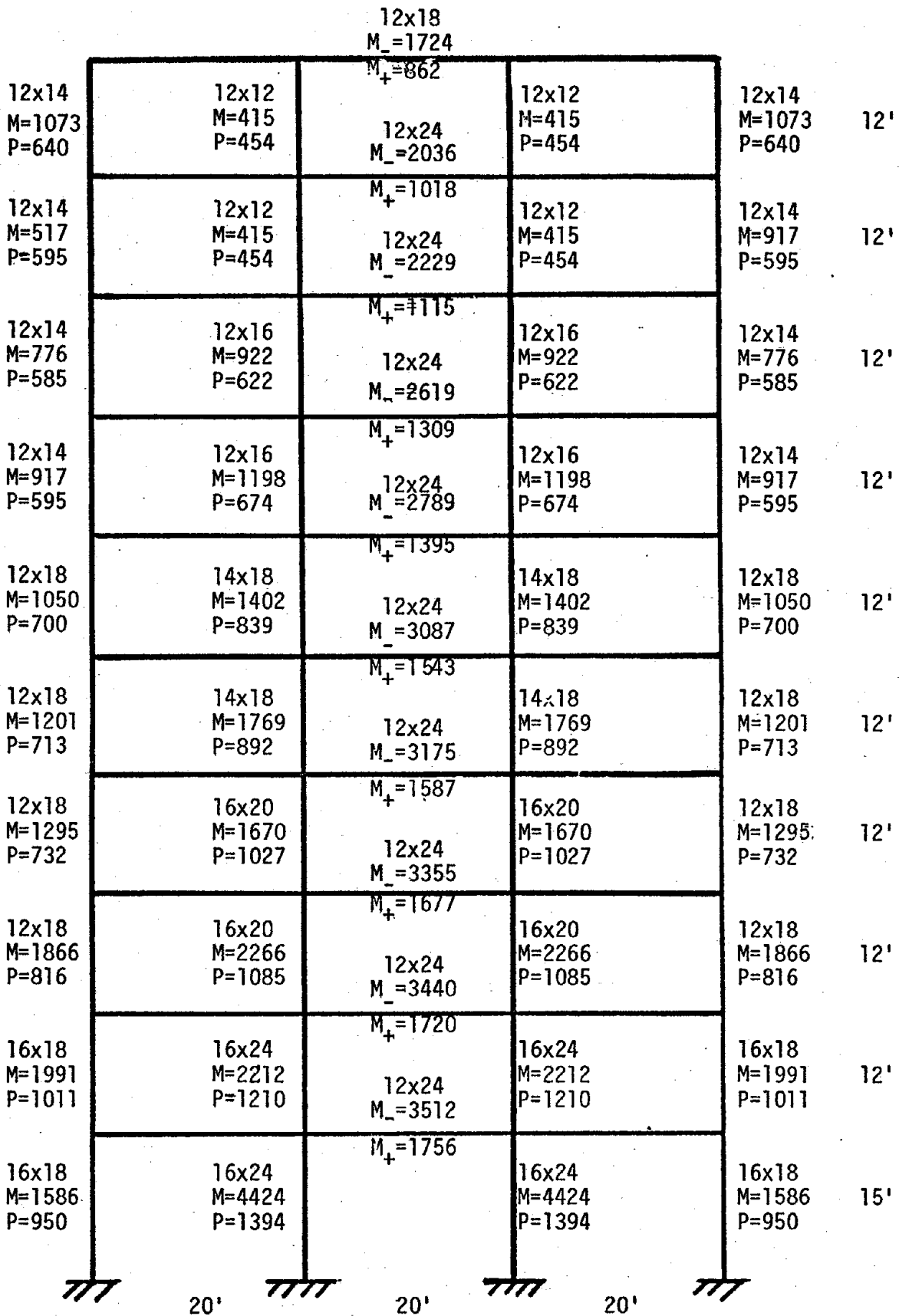


FIGURE 3.14 - FRAME E - UBC/Modified ACI. MEMBER SIZES AND CAPACITIES (kips, ins.)



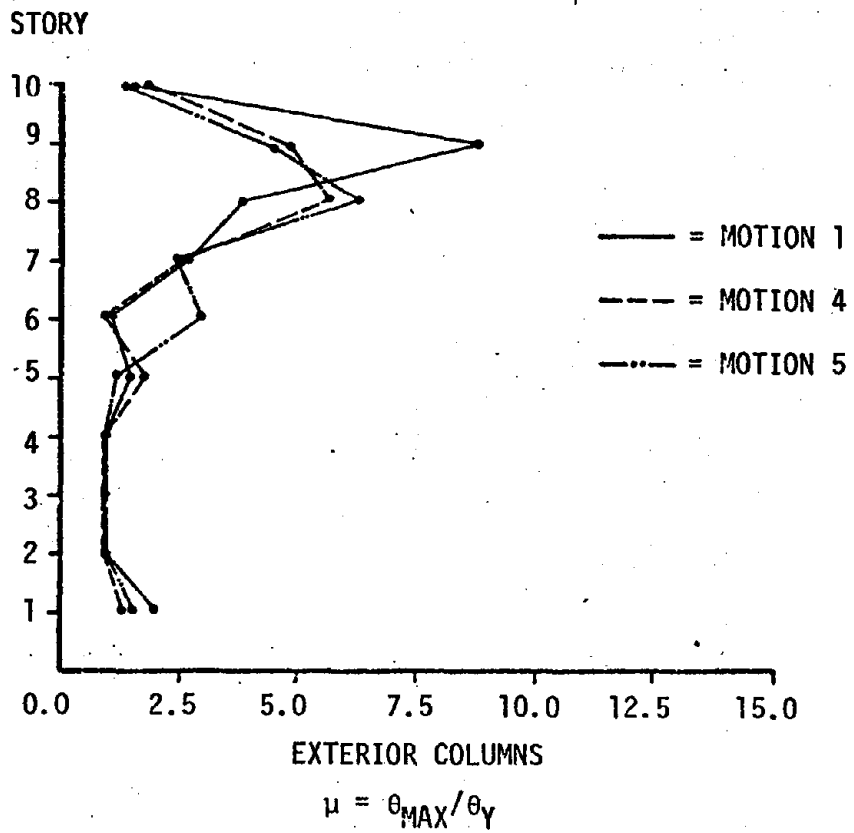
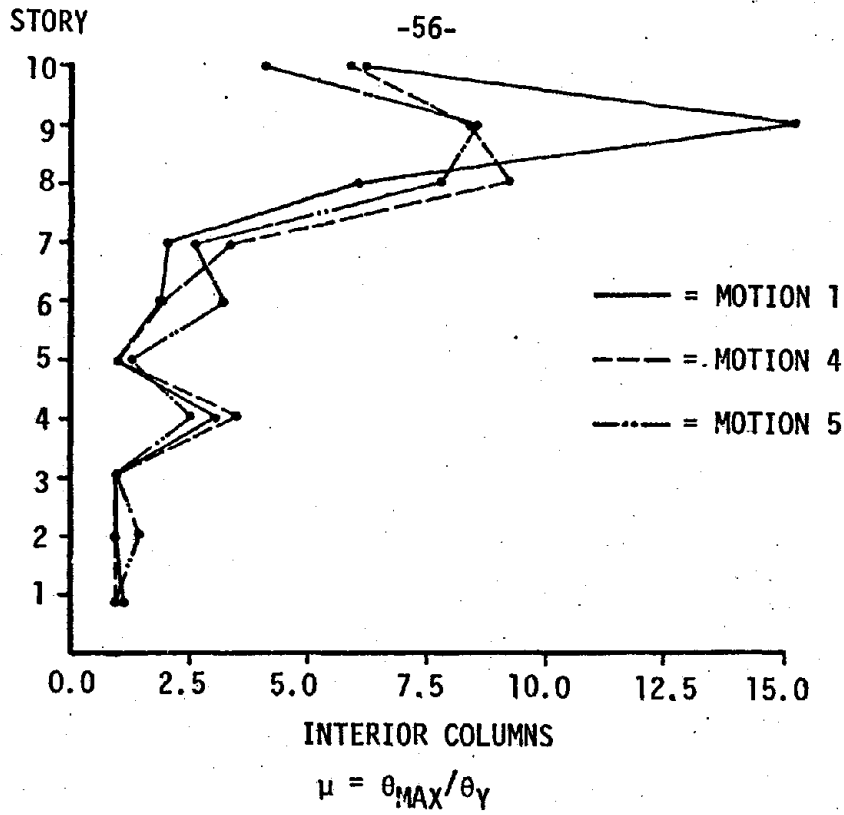
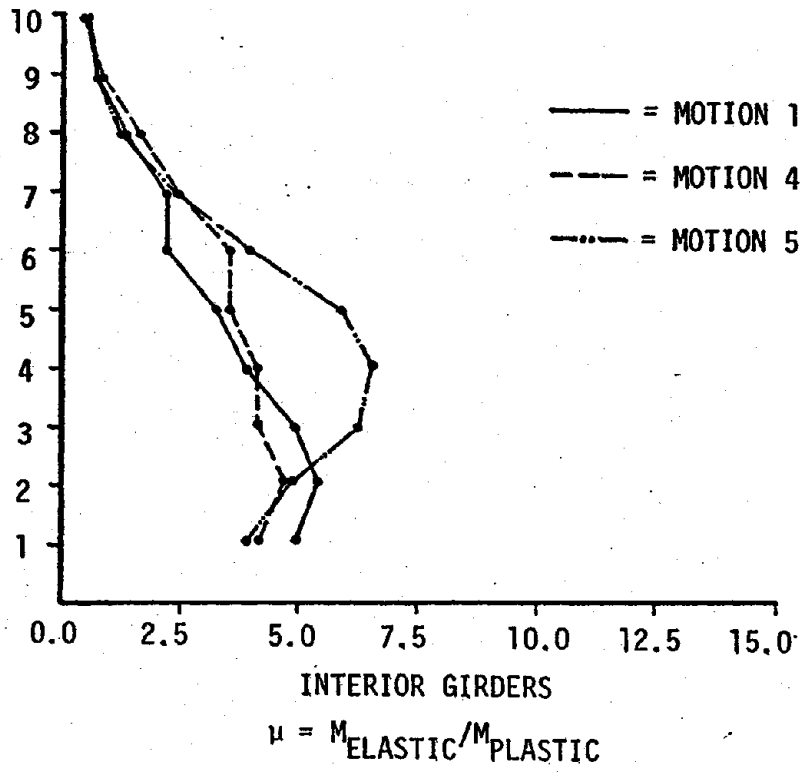


FIGURE 3.15 - FRAME E - UBC/Modified ACI  
DUCTILITY DEMANDS - COLUMNS

STORY

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STORY

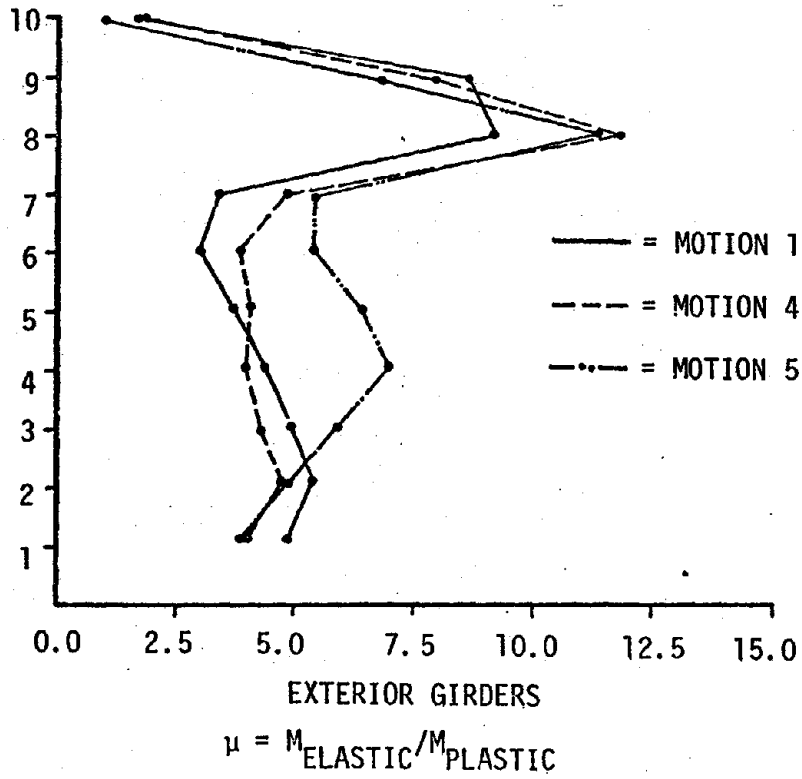


FIGURE 3.16 - FRAME E - UBC/Modified ACI  
DUCTILITY DEMANDS - GIRDERS

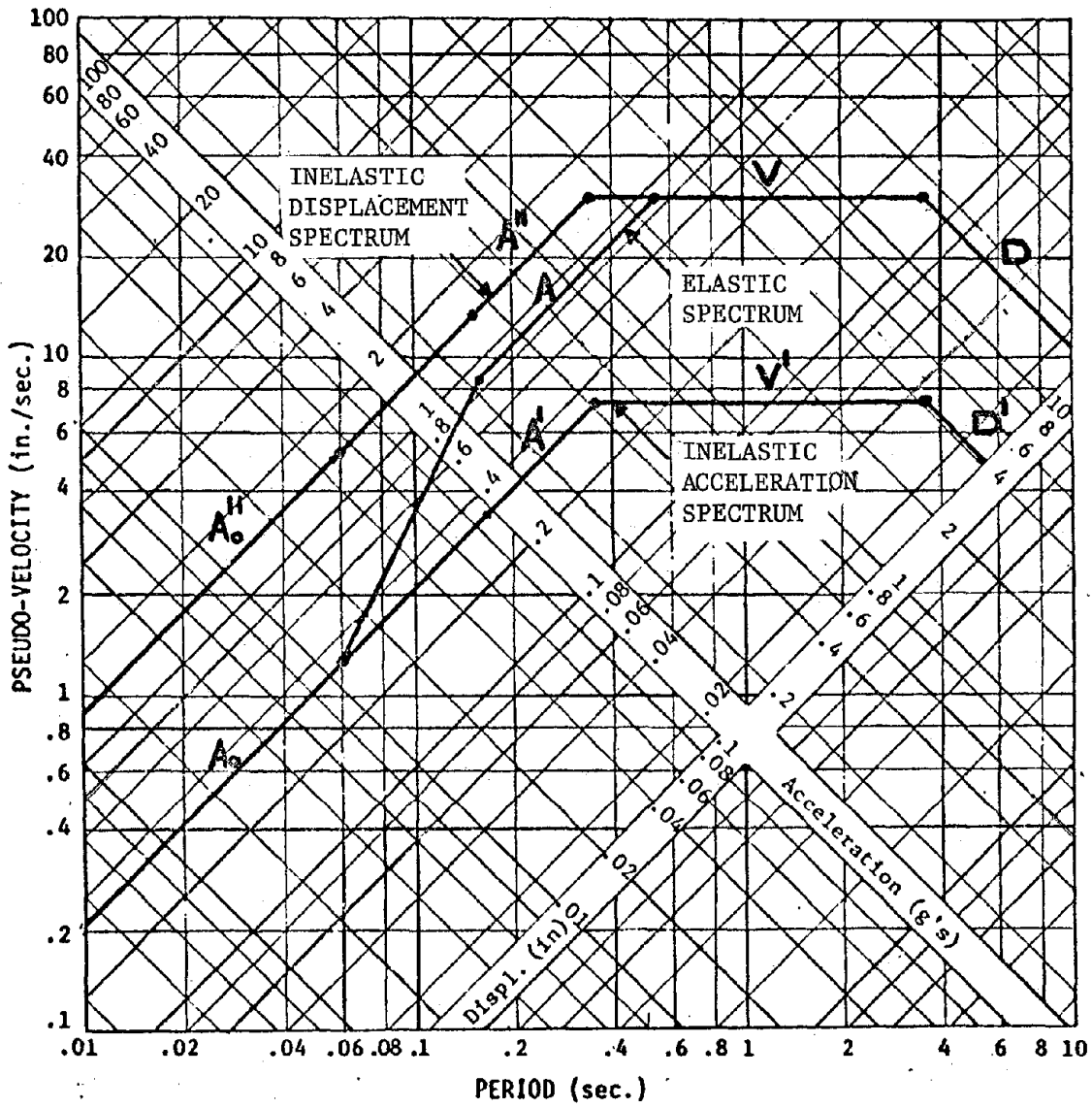


FIGURE 3.17 - ELASTIC AND INELASTIC DESIGN SPECTRA,  $\mu = 4$ ,  $\beta = 5\%$

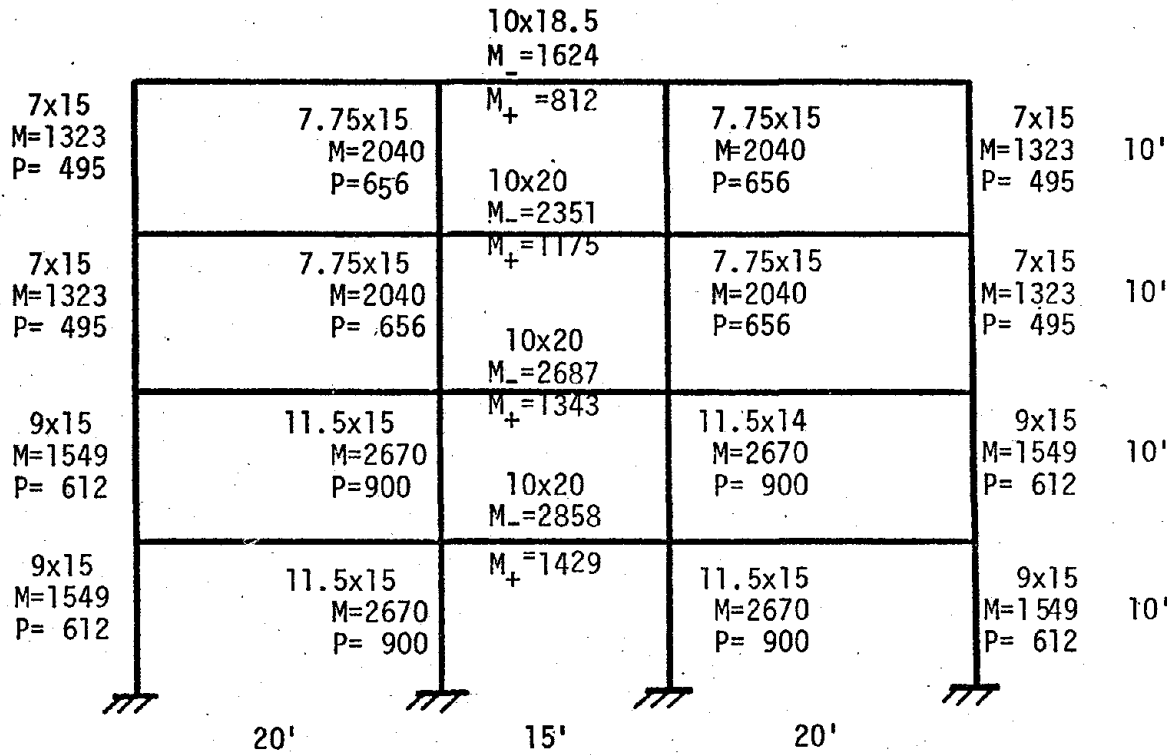


FIGURE 3.18 - FRAME F - RS/Modified ACI. MEMBER SIZES AND CAPACITIES (kips, ins.)

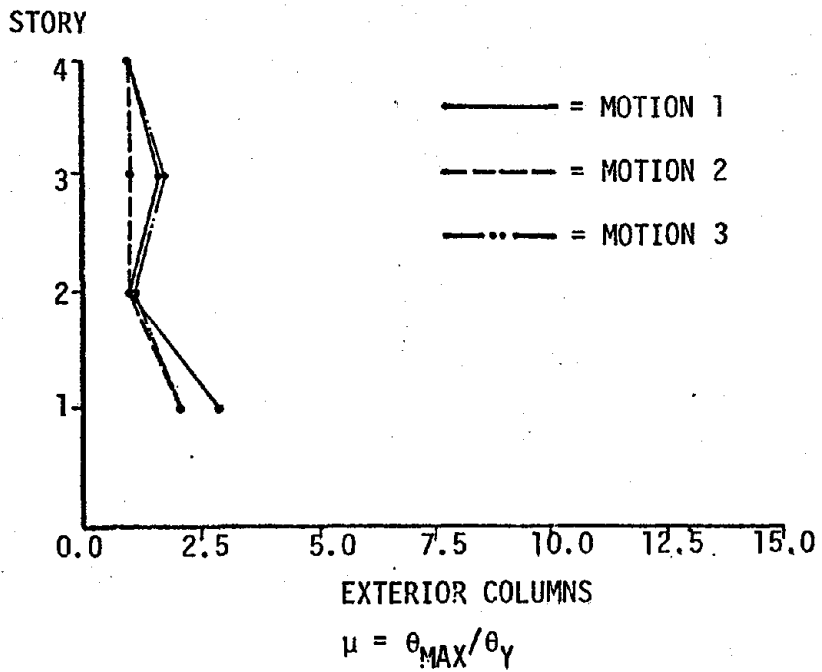
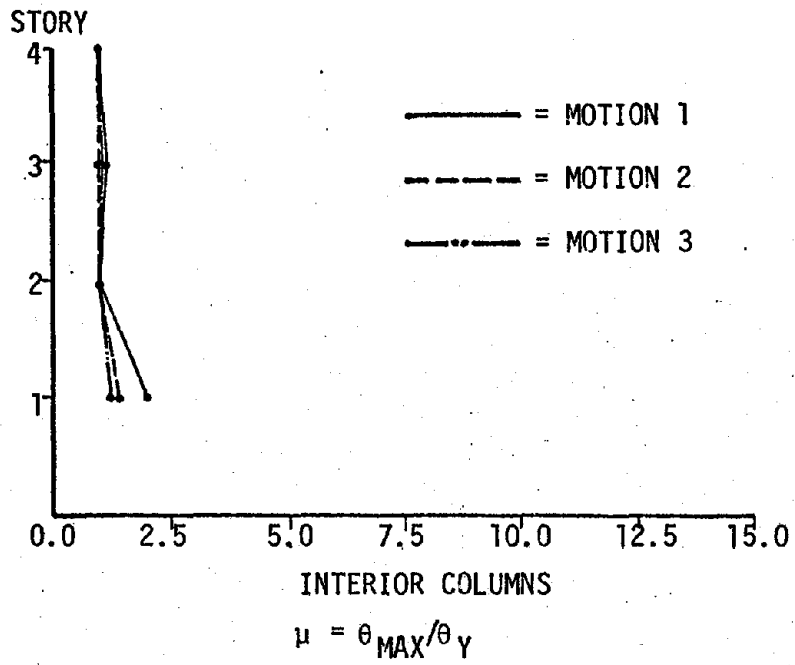


FIGURE 3.19 - FRAME F - RS/Modified ACI  
DUCTILITY DEMANDS - COLUMNS

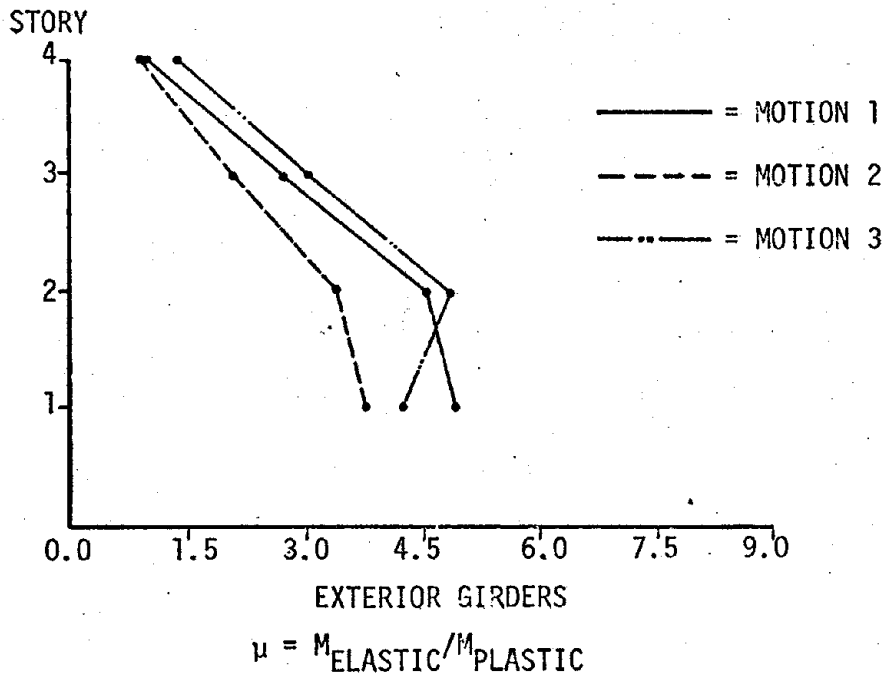
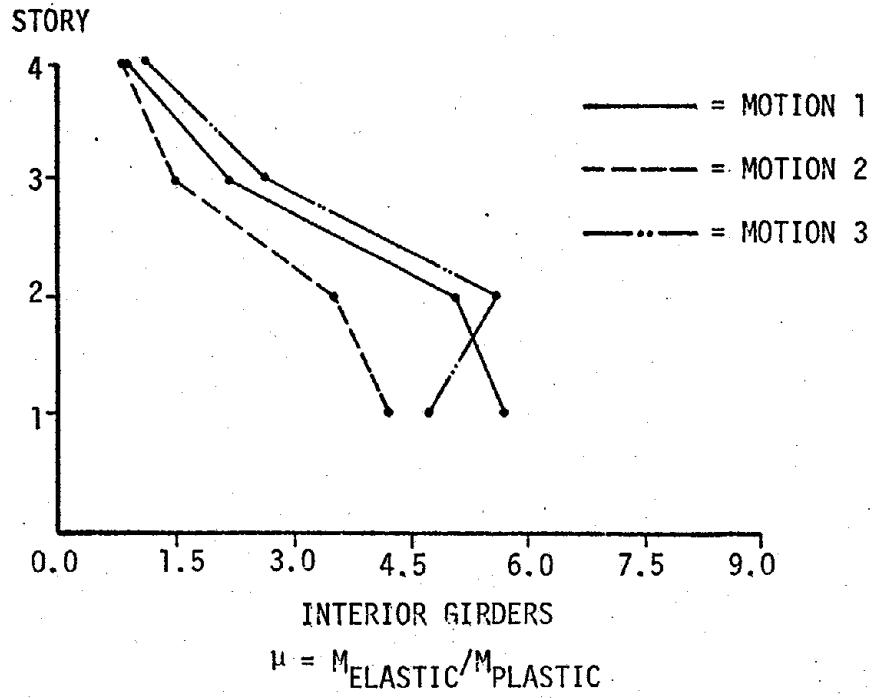


FIGURE 3.20 - FRAME F - RS/Modified ACI  
DUCTILITY DEMANDS - GIRDERS

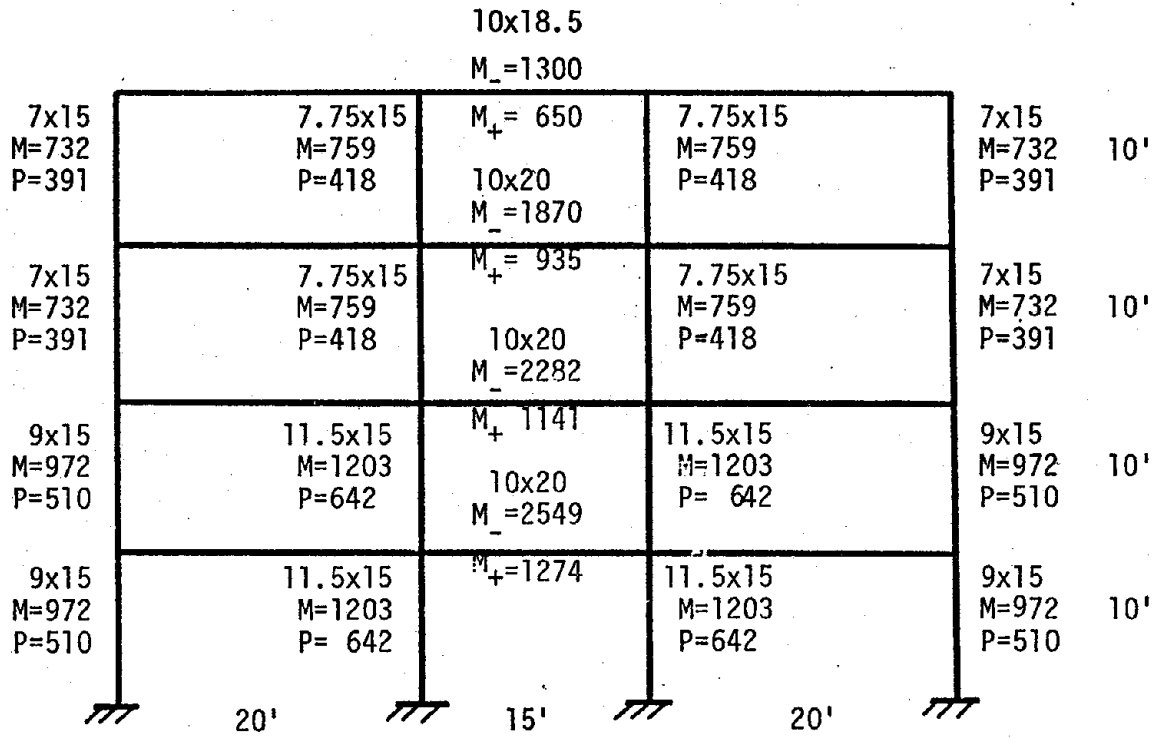


FIGURE 3.21 - FRAME G - RS/Modified ACI/No Factors  
MEMBER SIZES AND CAPACITIES (kips, ins.)

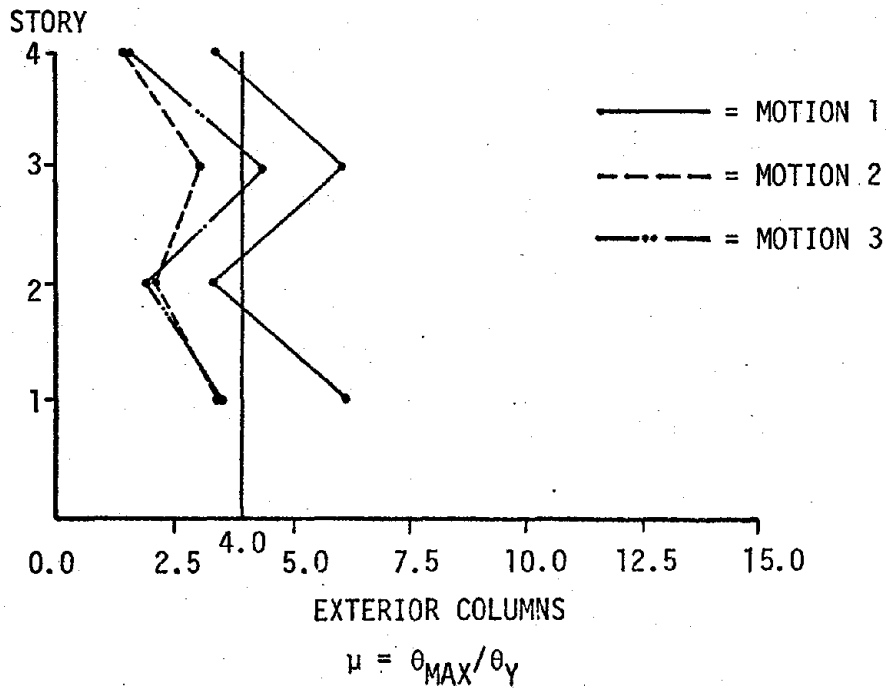
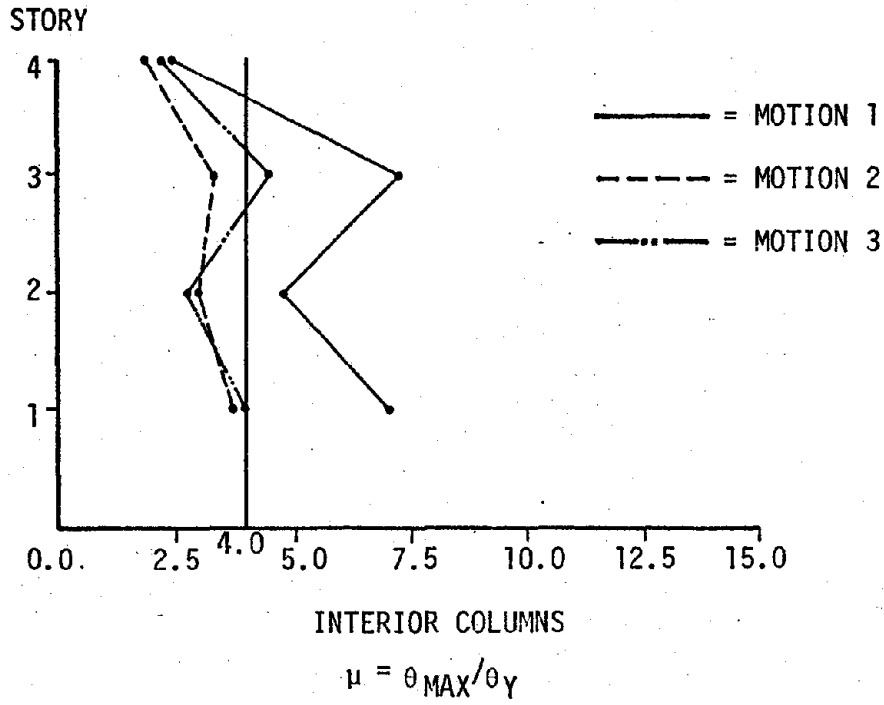


FIGURE 3.22 - FRAME G - RS/Modified ACI/No Factors  
DUCTILITY DEMANDS - COLUMNS



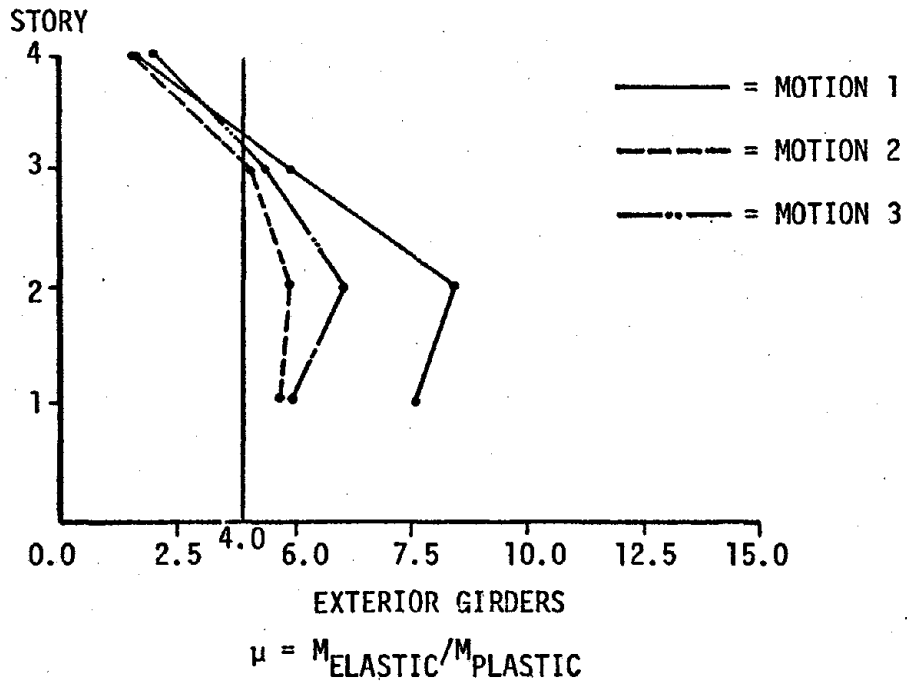
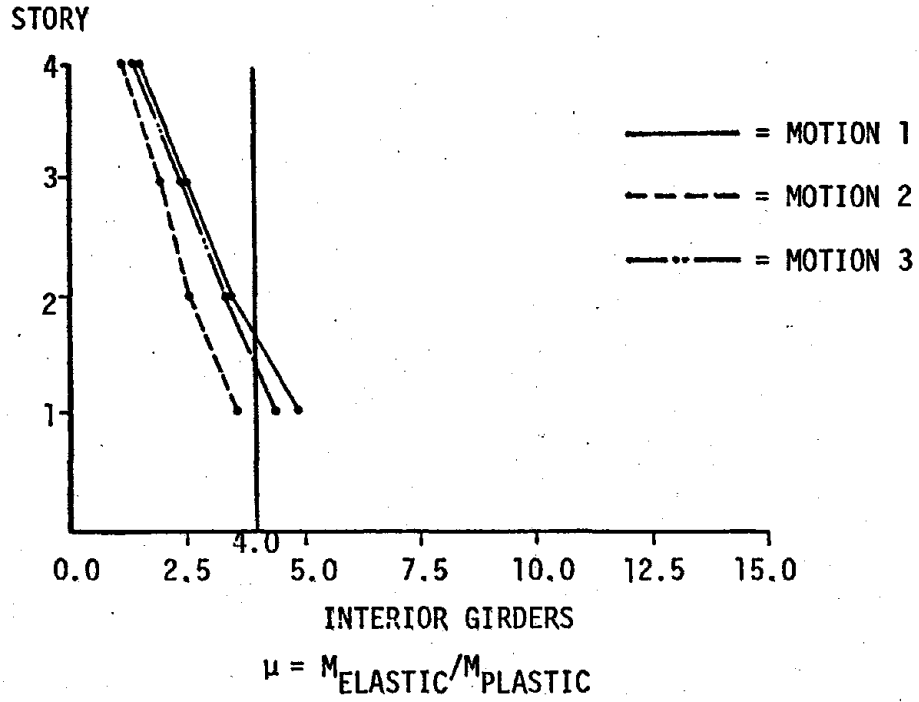


FIGURE 3.23 - FRAME G - RS/Modified ACI/No Factors  
DUCTILITY DEMANDS - GIRDERS

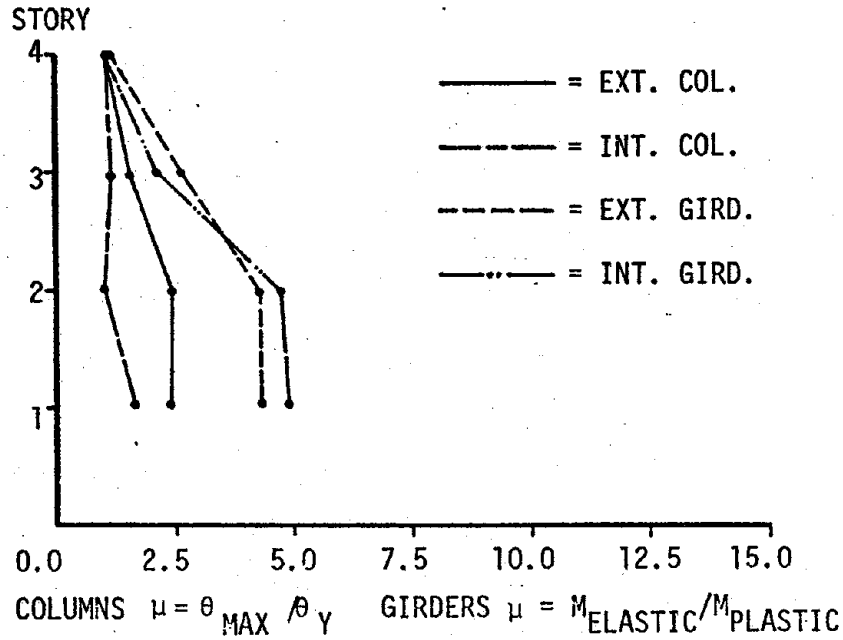


FIGURE 3.24 - FRAME F - RS/Modified ACI. AVERAGE DUCTILITY DEMANDS FOR THREE MOTIONS

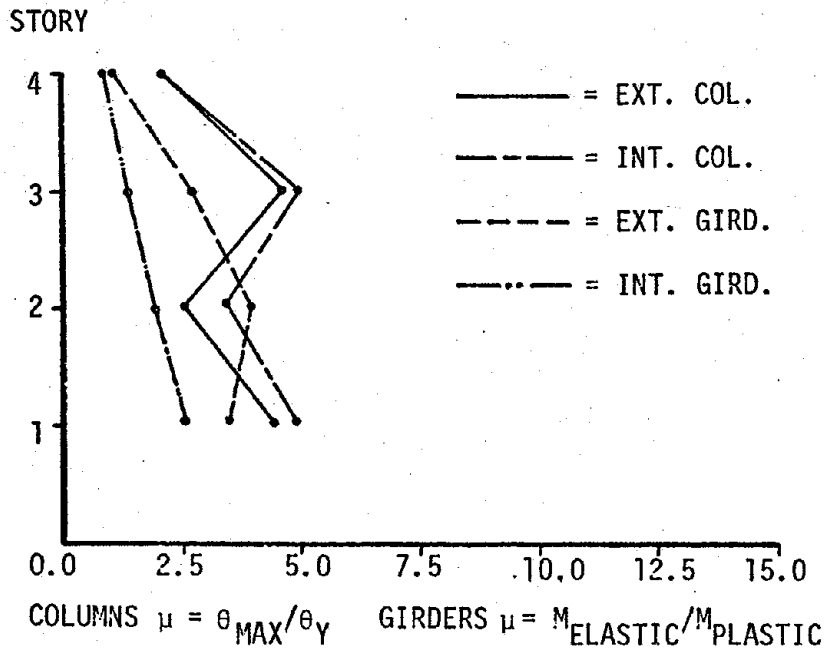


FIGURE 3.25 - FRAME G - RS/Modified ACI/No Factors AVERAGE DUCTILITY DEMANDS FOR THREE MOTIONS

12x18 M <sub>-</sub> =1314					
12x14 M=734 P=554	12x12 M=415 P=449	M <sub>+</sub> =657  12x24 M <sub>-</sub> =2034	12x12 M=415 P=449	12x14 M=734 P=554	12'
12x14 M=762 P=559	12x12 M=415 P=449	M <sub>+</sub> =1017  12x24 M <sub>-</sub> =2210	12x12 M=415 P=449	12x14 M=762 P=559	12'
12x14 M=621 P=534	12x16 M=737 P=524	M <sub>+</sub> =1105  12x24 M <sub>-</sub> =2649	12x16 M=737 P=524	12x14 M=621 P=534	12'
12x14 M=734 P=554	12x16 M=802 P=622	M <sub>+</sub> =1324  12x24 M <sub>-</sub> =2904	12x16 M=802 P=622	12x14 M=734 P=554	12'
12x16 M=876 P=622	12x16 M=1078 P=668	M <sub>+</sub> =1452  12x24 M <sub>-</sub> =3391	12x16 M=1078 P=668	12x16 M=876 P=622	12'
12x16 M=1014 P=645	12x16 M=1521 P=726	M <sub>+</sub> =1696  12x24 M <sub>-</sub> =3458	12x16 M=1521 P=726	12x16 M=1014 P=645	12'
12x16 M=986 P=639	14x18 M=1633 P=953	M <sub>+</sub> =1729  12x24 M <sub>-</sub> =3816	14x18 M=1633 P=953	12x16 M=986 P=639	12'
12x16 M=1429 P=720	14x18 M=2137 P=945	M <sub>+</sub> =1908  12x26 M <sub>-</sub> =4265	14x18 M=2137 P=945	12x16 M=1429 P=720	12'
14x16 M=1344 P=780	16x20 M=2112 P=1075	M <sub>+</sub> =2133  12x26 M <sub>-</sub> =4759	16x20 M=2112 P=1075	14x16 M=1344 P=780	12'
14x16 M=1806 P=853	16x20 M=3917 P=1325	M <sub>+</sub> =2380	16x20 M=3917 P=1325	14x16 M=1806 P=853	15'

FIGURE 3.26 - FRAME H - RS/Modified ACI/No Factors  
MEMBER SIZES AND CAPACITIES (kips, ins.)

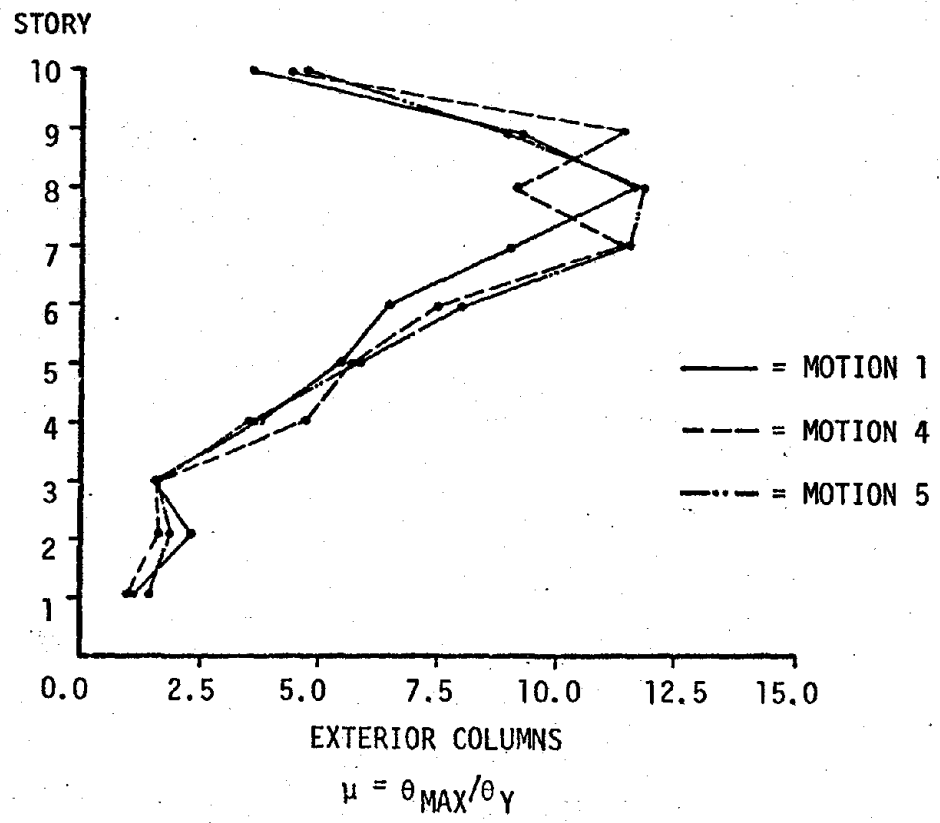
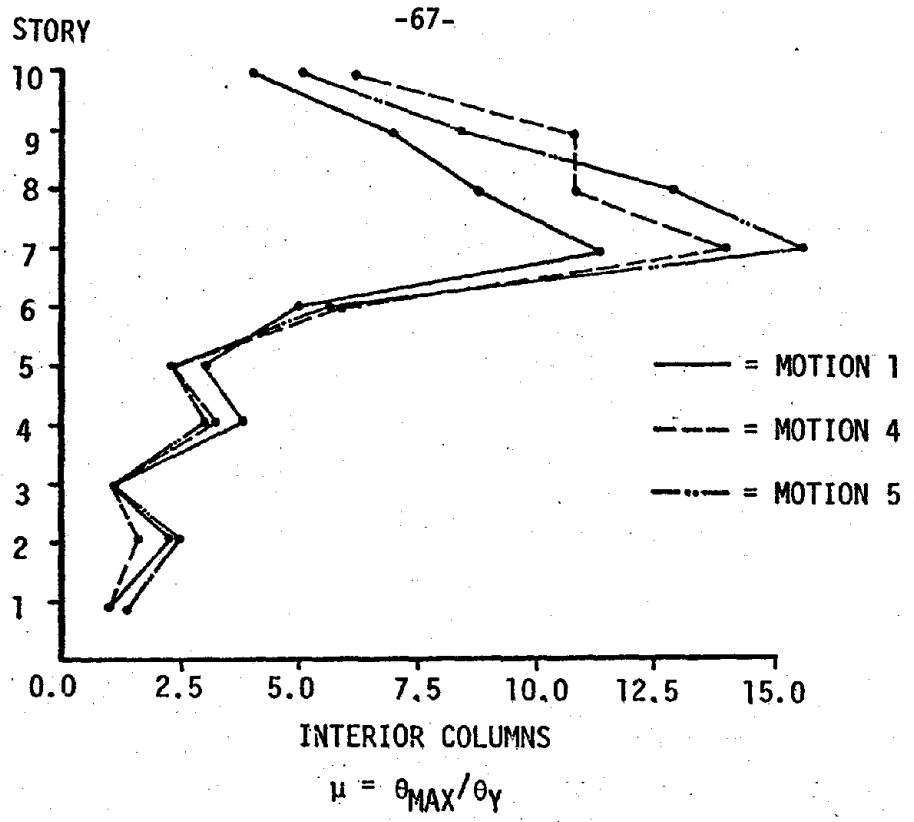


FIGURE 3.27 - FRAME H - RS/Modified ACI/No Factors  
DUCTILITY DEMANDS - COLUMNS

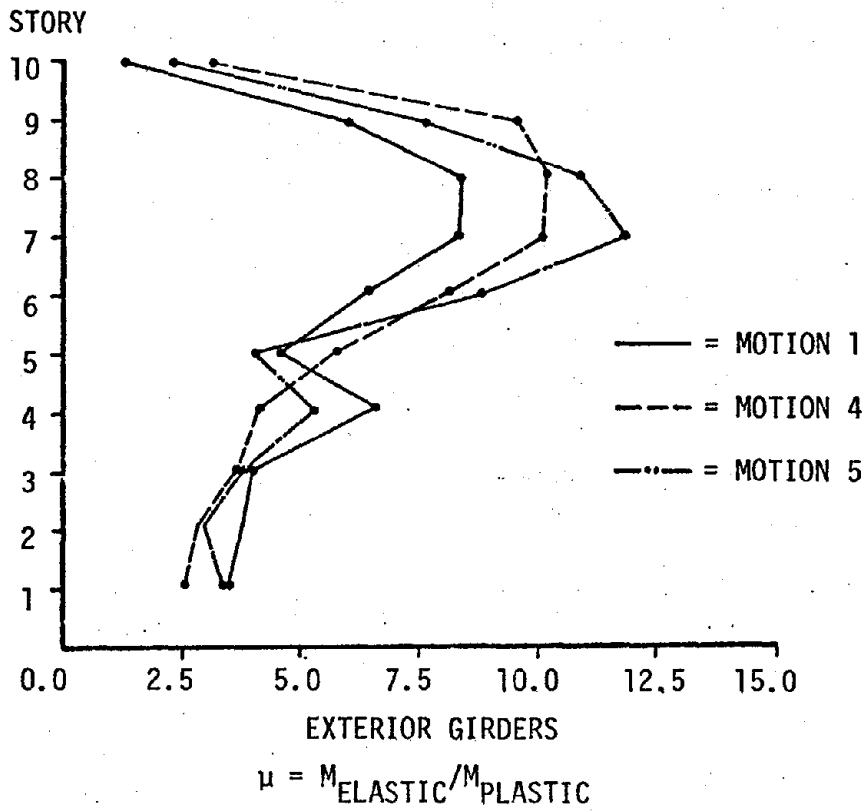
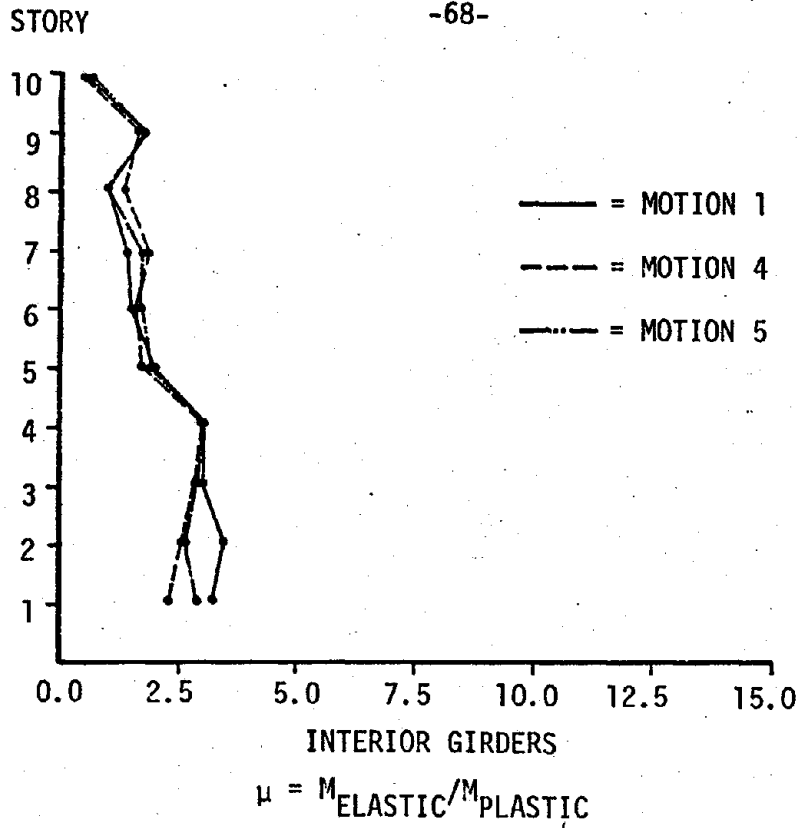


FIGURE 3.28 - FRAME H - RS/Modified ACI/No Factors  
DUCTILITY DEMANDS - GIRDERS

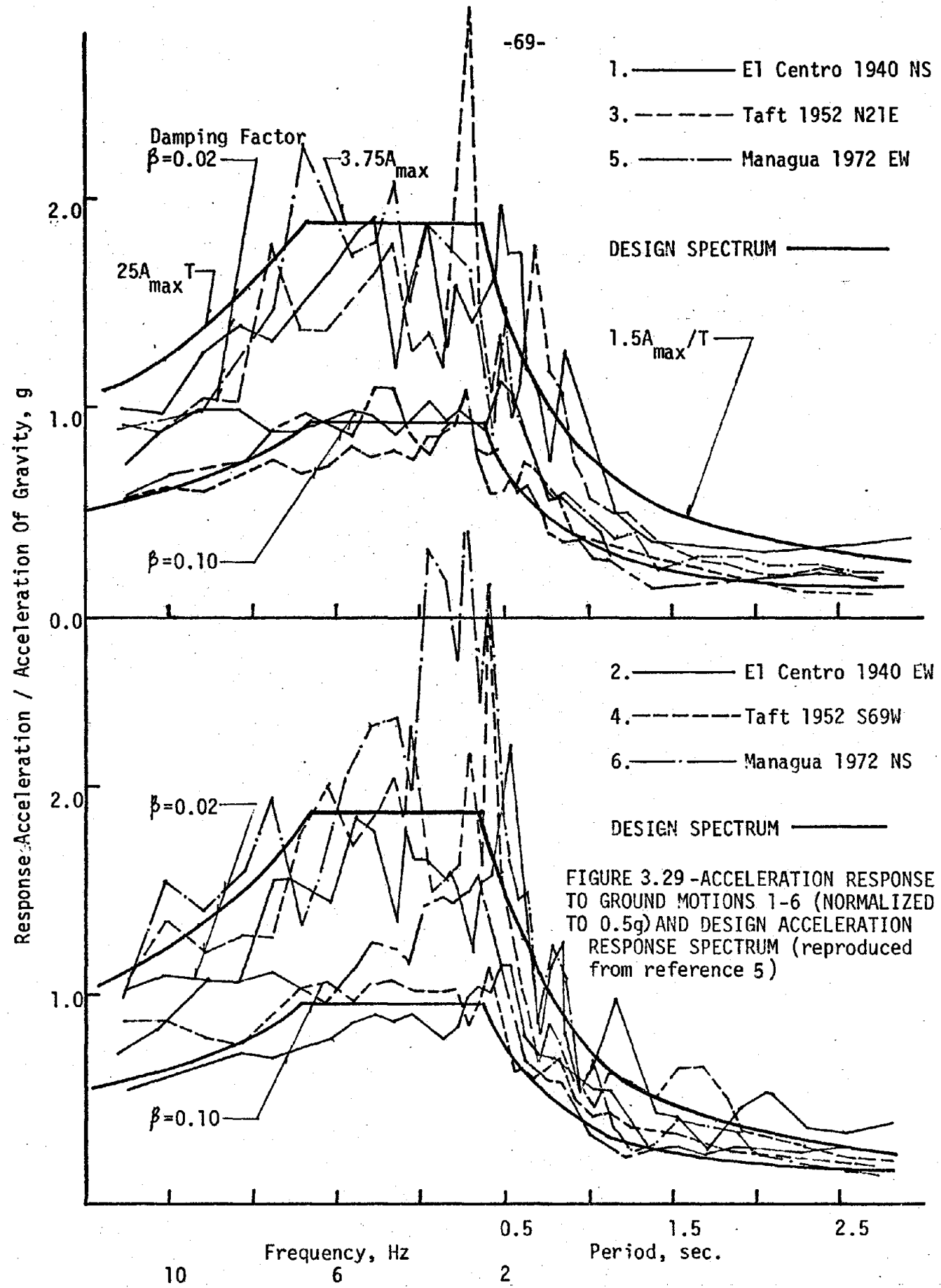


FIGURE 3.29 -ACCELERATION RESPONSE TO GROUND MOTIONS 1-6 (NORMALIZED TO 0.5g) AND DESIGN ACCELERATION RESPONSE SPECTRUM (reproduced from reference 5)

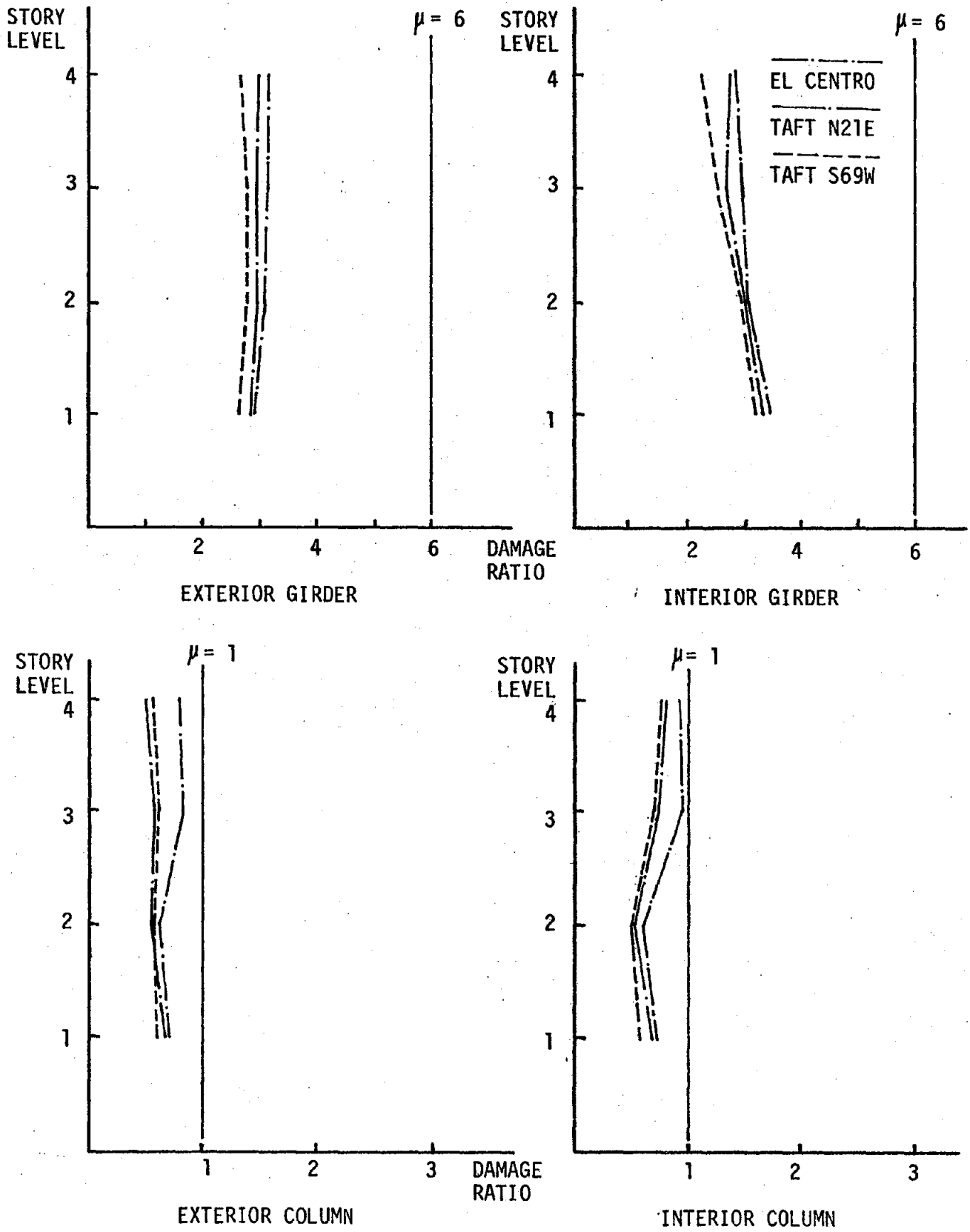


FIGURE 3.30 - FRAME I - SSM/ACI/ $\mu=1,6$   
MAXIMUM DAMAGE RATIOS

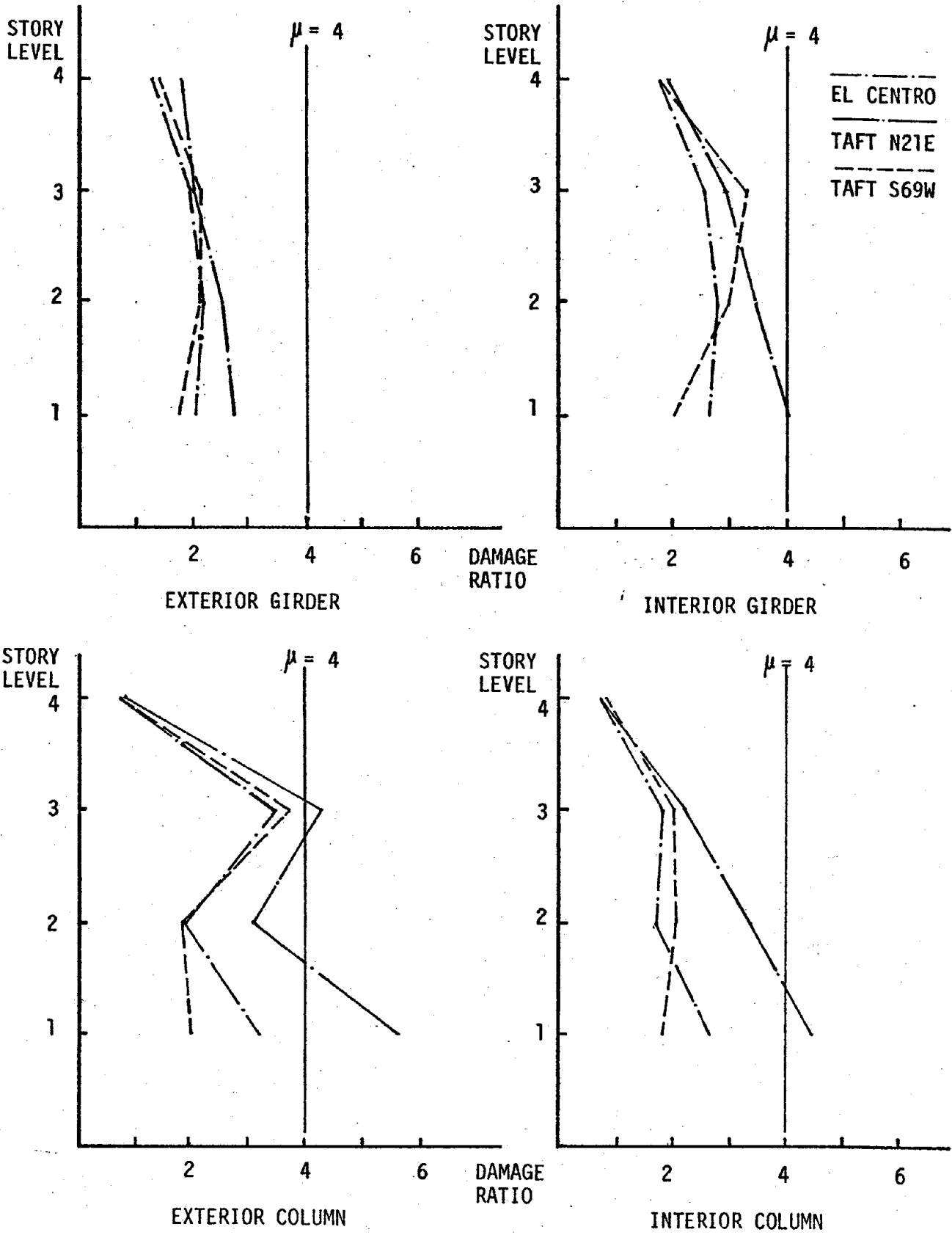


FIGURE 3.31 - FRAME J - SSM/ACI/ $\mu=4$   
MAXIMUM DAMAGE RATIOS



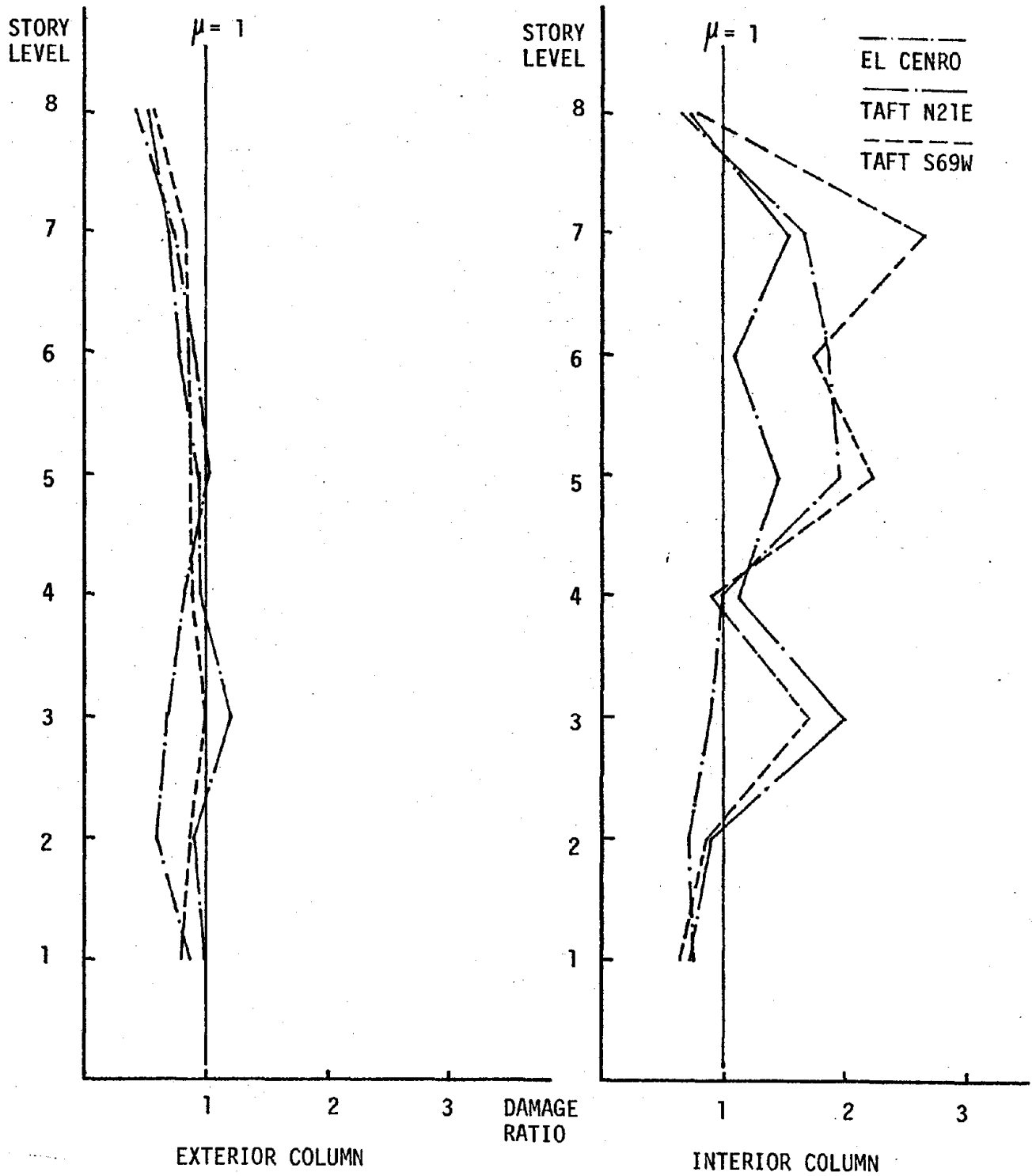


FIGURE 3.32 - FRAME K - SSM/ACI/ $\mu=1.6$   
MAXIMUM DAMAGE RATIOS - COLUMNS

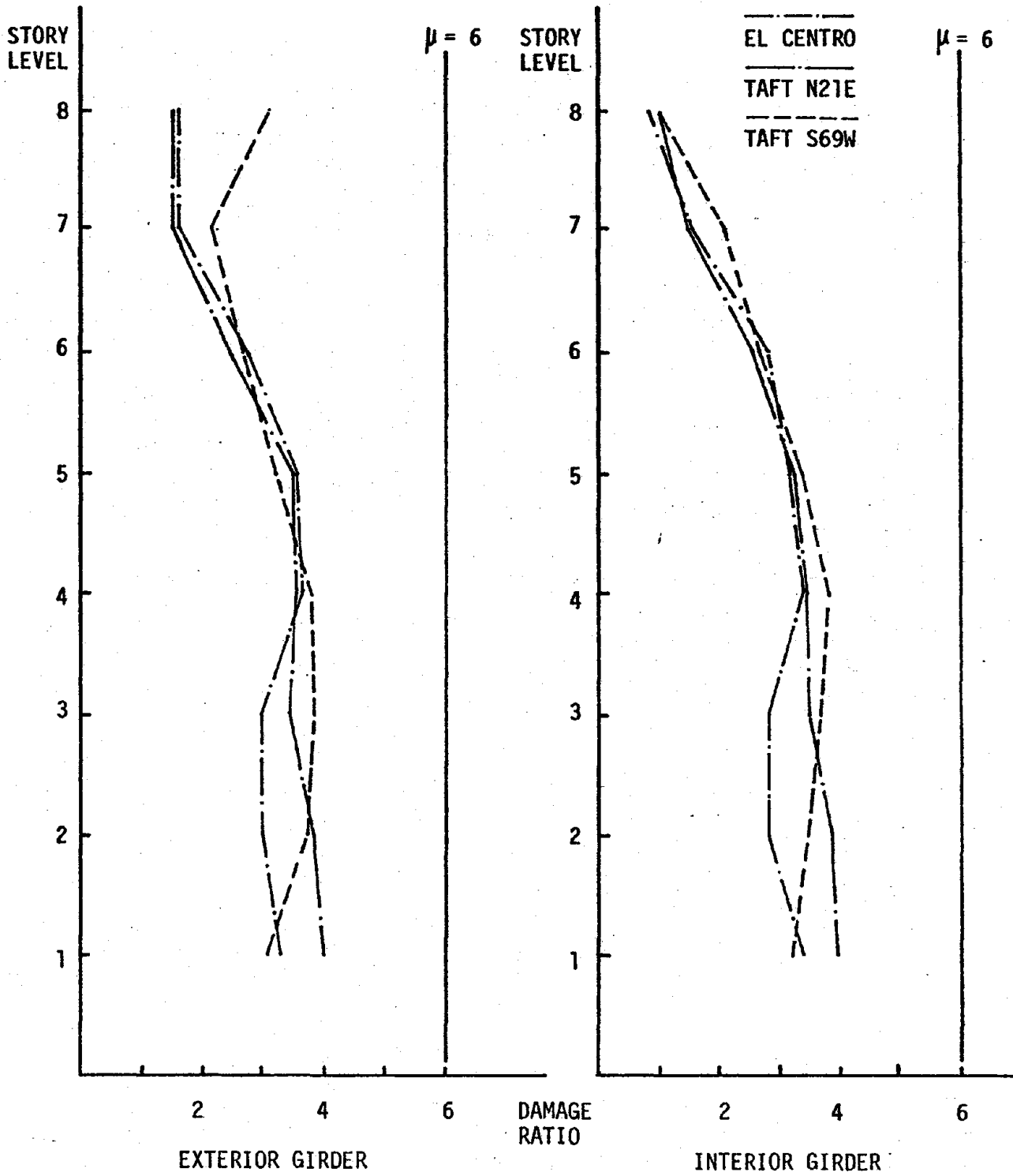


FIGURE 3.33 - FRAME K - SSM/ACI/ $\mu=1,6$   
MAXIMUM DAMAGE RATIOS - GIRDERS

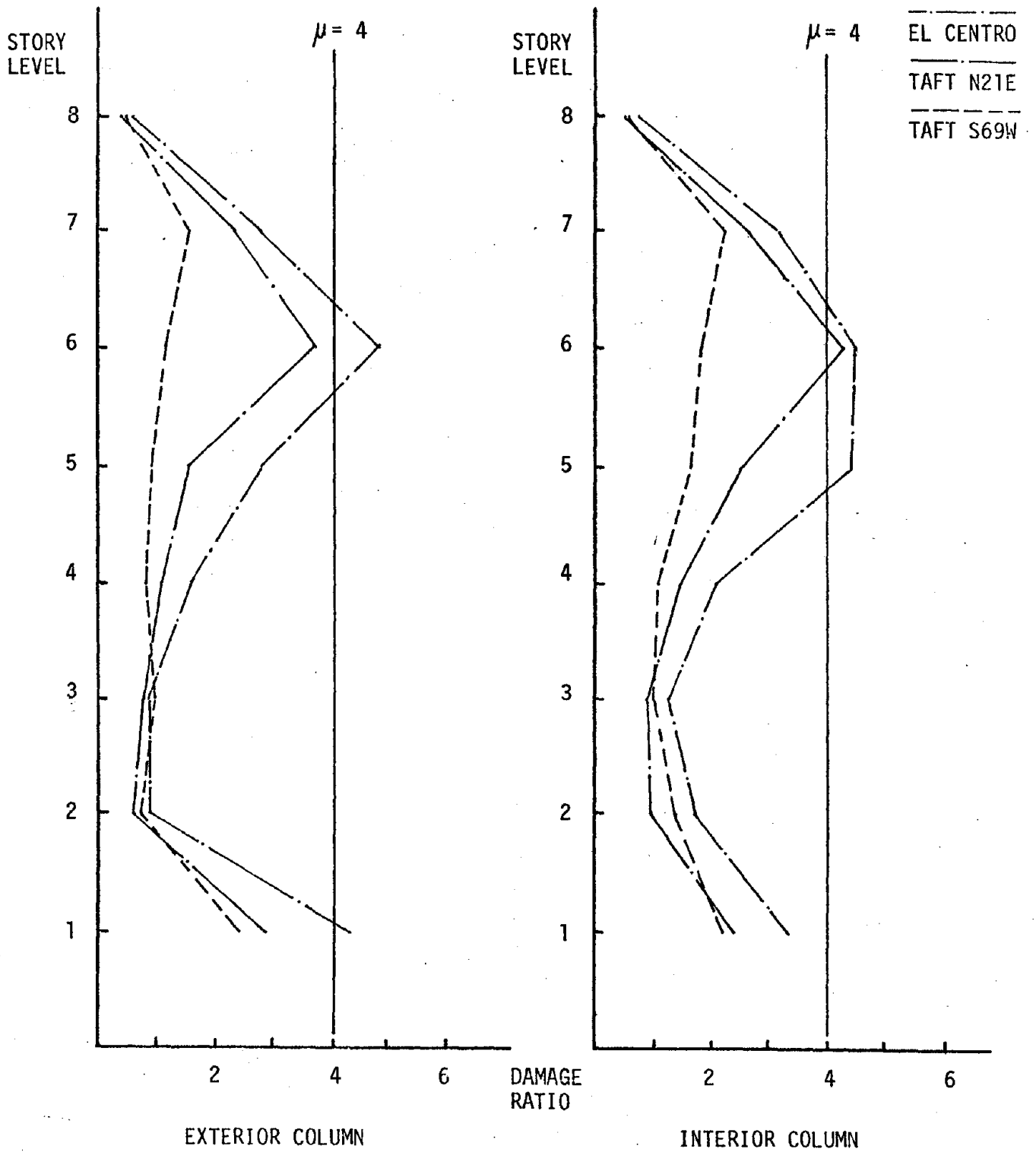


FIGURE 3.34 - FRAME L - SSM/ACI/ $\mu=4$   
MAXIMUM DAMAGE RATIOS - COLUMNS

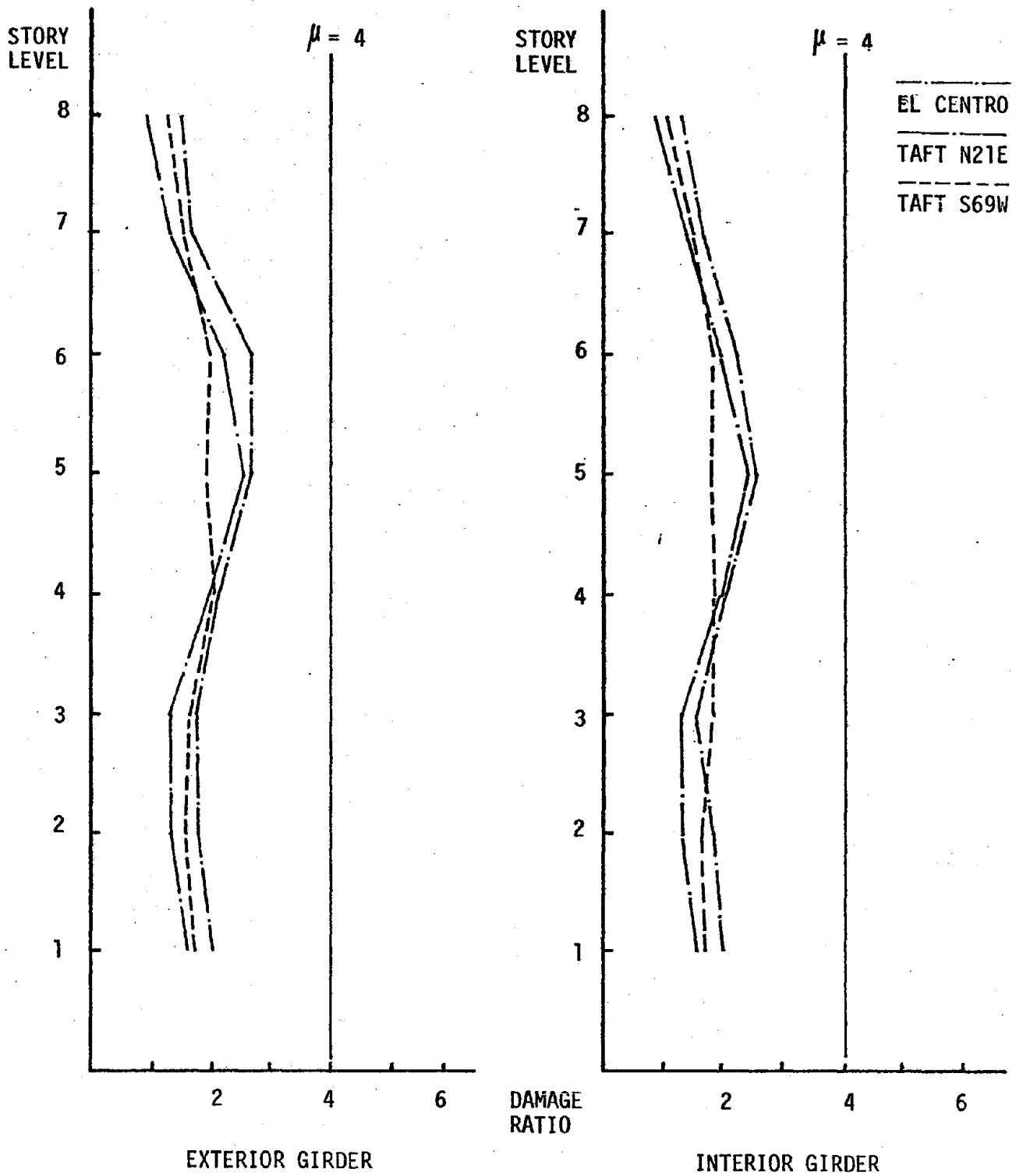


FIGURE 3.35 - FRAME L - SSM/ACI/ $\mu=4$   
MAXIMUM DAMAGE RATIOS - GIRDERS

## CHAPTER IV - CONCLUSIONS

The purpose of the studies summarized herein was to evaluate the effectiveness of three aseismic design methods for reinforced concrete frames. Effectiveness in this context is defined as the degree to which a method permits control over the amount and distribution of yielding in a frame when subjected to an earthquake ground motion.

The investigation was limited and any conclusions drawn must be qualified. Only one type of structure has been examined. The frame designs could probably be improved, e.g., by optimizing relative member stiffnesses. However, the designs were deliberately executed by a direct application of the method without modifications or special considerations which an experienced seismic designer might apply. On this limited basis, the following observations are made:

1. None of the three methods proved to be completely satisfactory. The desired amount of yielding (as measured by ductility demand or damage ratio) was not achieved and the distribution of yielding over the frame was, in most cases, quite uneven. This can best be seen in Table 3.3, where the average values are generally less than the target value but the maxima are considerably more. Perhaps it is impossible to achieve this objective, but none of the methods appears to be successful in this regard.

2. On the other hand, none of the computed responses indicates complete collapse of the frame. If prevention of collapse is the objective of aseismic design, all three methods are apparently successful.

3. Of all the frame designs, Frame G (4 stories, inelastic response spectrum approach with no load factors) was probably the most successful.

However, when the same procedure was applied to a 10-story frame, the maximum ductility demands were excessive.

4. The two more sophisticated methods, i.e., the inelastic response spectrum and Substitute Structure methods, do not produce significantly better designs than the UBC equivalent static load approach. At their present stage of development, these methods do not appear to be worth the extra effort required.

5. None of the methods sufficiently allowed for the "whip-lash" effect which produced excessive ductility demands in the upper stories of the eight- and ten-story frames. This problem requires further study. The UBC concentrated force at the top of frames should probably be made larger. The other two methods, even though they considered higher modes in the design analyses, did not provide protection against this effect.

6. In spite of the lack of theoretical rigor, the UBC procedure produced reasonably good results.

7. The use of inelastic response spectra is attractive because it gives the designer some control over the amount of yielding. However, further improvements are needed to make this method a reliable design procedure.

8. The Substitute Structure Method appears to be generally conservative, i.e., the ductility demands were less than intended. It is desirable to have the capability to design for different ductility demands in columns and girders. However, the method was not completely successful in this regard.

A design procedure which is based upon inelastic behavior and which reliably controls such behavior during a strong earthquake is highly desirable. However, it appears that this objective has not yet been achieved.

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