OPTIMUM SEISMIC PROTECTION FOR NEW BUILDING CONSTRUCTION IN EASTERN METROPOLITAN AREAS

NSF GRANT GK-27955 and GI-29936

Internal Study Report No. 41

SEISMIC RISK ANALYSIS OF SIMPLE SERIES SYSTEMS

George Panoussis

May 1974

Department of Civil Engineering

Massachusetts Institute of Technology

Cambridge, Massachusetts

50272-101

REPORT DOCUMENTATION PAGE	1. REPORT NO. NSF-RA-E-74-533	2.	3. Recipient's Accession No. PB80-104755

4. Title and Subtitle Seismic Risk Analysis of Simple Series Systems (Optimum Seismic Protection for New Building Construction in Eastern Metropolitan Areas, Internal Study Report 41)	5. Report Date May 1974
	6.
7. Author(s) G. Panoussis	8. Performing Organization Rept. No. No. 41
9. Performing Organization Name and Address Massachusetts Institute of Technology Department of Civil Engineering Cambridge, Massachusetts 02139	10. Project/Task/Work Unit No.
	11. Contract(C) or Grant(G) No. (C) GK27955 (G) GI29936
12. Sponsoring Organization Name and Address Engineering and Applied Science (EAS) National Science Foundation 1800 G Street, N.W. Washington, D.C. 20550	13. Type of Report & Period Covered
	14.

15. Supplementary Notes

16. Abstract (Limit: 200 words)

A method is presented for the evaluation of the seismic risk at a site of an engineering project. A particular consideration is the geographical spread of the site, rather than the concentration of the system at one point. A site is assumed to be distributed geographically in a linear manner with a known random distribution of strength. It can then be assumed to be a series of "short" links. Each link has a resistance which is a known random variable and is independent of every other link. Four plots are detailed comparing various results: probability of failure of a one-site system, two-site system, and four-site system; probability of failure of a one-site system versus the location of the site; probability of failure of a one-site system versus increasing resistance; and probability of failure of a two-site system. The computer program employed is included in this report.

17. Document Analysis a. Descriptors

Risk
Earth movements
Earthquakes
Probability theory
Computer programs
Sites

b. Identifiers/Open-Ended Terms

c. COSATI Field/Group

18. Availability Statement	19. Security Class (This Report)	21. No. of Pages
NTIS		21
	20. Security Class (This Page)	22. Price PC A02 - A01

(See ANSI-Z39.18) *See Instructions on Reverse* **OPTIONAL FORM 272** (4-77) (Formerly NTIS-35) Department of Commerce

INTRODUCTION

An extension is made of a paper (3) introducing a method for the evaluation of the seismic risk at a site of an engineering project. The added feature is the geographical spread of the site, rather than the concentration of the system at one point.

DETERMINISTIC ANALYSIS

It is assumed that the particular form

$$Y = b_1 \, e^{b_2 M} \, d^{-b_3} \qquad \text{(1)}$$

recommended by Esteva and Rosenblueth (4) for peak ground acceleration ($Y = A$), peak ground velocity ($Y = v$), and peak ground displacement ($Y = D$) is exact and deterministic, given d, the straight-line distance between the point of interest and a point that just generated an earthquake of magnitude M. $\{b_1, b_2, b_3\}$ are suggested constants typically equal to {2000, .8, 1.7}, {16, 1.0, 1.7}, and {7, 1.2, 1.6} for A, V, and D respectively in southern California, with A, V, and D in units of centimeters and seconds, and d in kilometers.

Assume that a point site has a deterministic resistance R_0, and that a point earthquake source generates an earthquake of magnitude M_0. The coordinates of the site and the source are x_s and y_s, and x_e and y_e, respectively (Fig. 1). Then,

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$$d = \sqrt{(x_s - x_e)^2 + (y_s - y_e)^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (2)$$

and the peak-ground effect S_0 is

$$S_0 = b_1 \, e^{b_2 M} \, d^{-b_3} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3)$$

If S_0 is greater than or equal to R_0, the site will fail. If S_0 is less than R_0, the site will survive*.

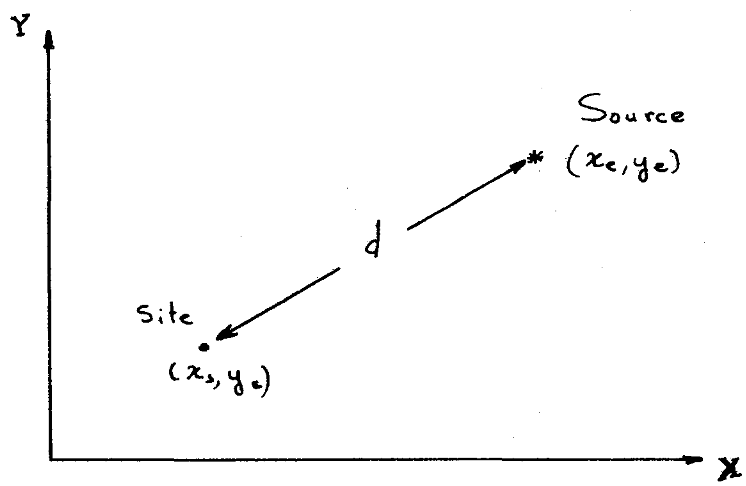


Figure 1

ASSUMPTION RELAXATIONS; INTRODUCE RANDOMNESS

A. Assume M is random, i.e., M = M(m).

Then

$$S_0 = b_1 \, e^{b_2 M(m)} \, d^{-b_3} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (4)$$

* The [survive,fail] or [0,1] type of site is assumed throughout this report.

and

$$\frac{S_0}{R_0} = \frac{b_1 \; eb_2^{M(m)} \; d^{-b_3}}{R_0} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (5)$$

$$P[\text{site survives}] = P[\frac{S_0}{R_0} < 1] = P[\frac{b_1 \; e^{b_2 M(m)} \; d^{-b_3}}{R_0} < 1] =$$

$$= P[M(m) < \frac{1}{b_2} \ln \frac{R_0}{b_1 \; d^{-b_3}}] = F_m(\frac{1}{b_2} \ln \frac{R_0}{b_1 \; d^{-b_3}}) \quad \ldots\ldots \quad (6)$$

where $F_m(m)$ is the cumulative distribution of the magnitude M.

B. M and R are random.

$$P[\text{site survives}] = P[\frac{R(r)}{S} > 1] = P[R(r) > b_1 \; e^{b_2 M(m)} \; d^{-b_3} \mid m] \cdot P[M=m] =$$

$$= \int_{M_{min}}^{M_{max}} [1 - F_R(b_1 \; e^{b_2 m} \; d^{-b_3})] f_m(m) dm \quad \ldots\ldots\ldots\ldots \quad (7)$$

where $F_R(r)$ is the cumulative distribution of the site resistance R.

C. Expand the earthquake source to a set of sources with the distribution of active source location given.

$$d = d(t) = \sqrt{(x_s + x_e(t))^2 + (y_s - y_e(t))^2} \quad \ldots\ldots \quad (8)$$

where $x_e(t)$ and $y_e(t)$ are the parametric equations of the earthquake sources. (See Fig. 2)

$$P[\text{site survives}] = P[\frac{S}{R} < 1] =$$

$$= P[\frac{b_1 \; e^{b_2 M} \; d(t)^{-b_3}}{R(r)} < 1 \,|\, m,t] \cdot P[T=t] \cdot P[M=m] =$$

$$= \int_t \int_m [1 - F_R(b_1 \; e^{b_2 m} \; d^{-b_3})] f_m(m) \cdot f_t(t) dmdt \;\; \ldots\ldots\ldots \;\; (9)$$

where $f_t(t)$ is the PDF of the active source location.

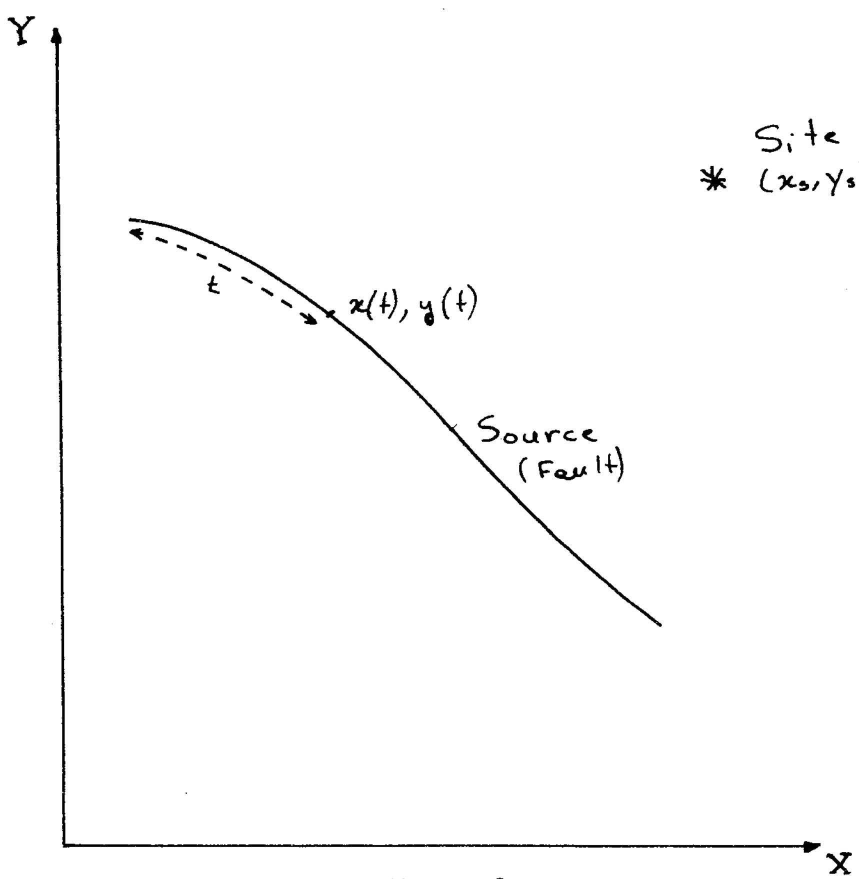


Figure 2

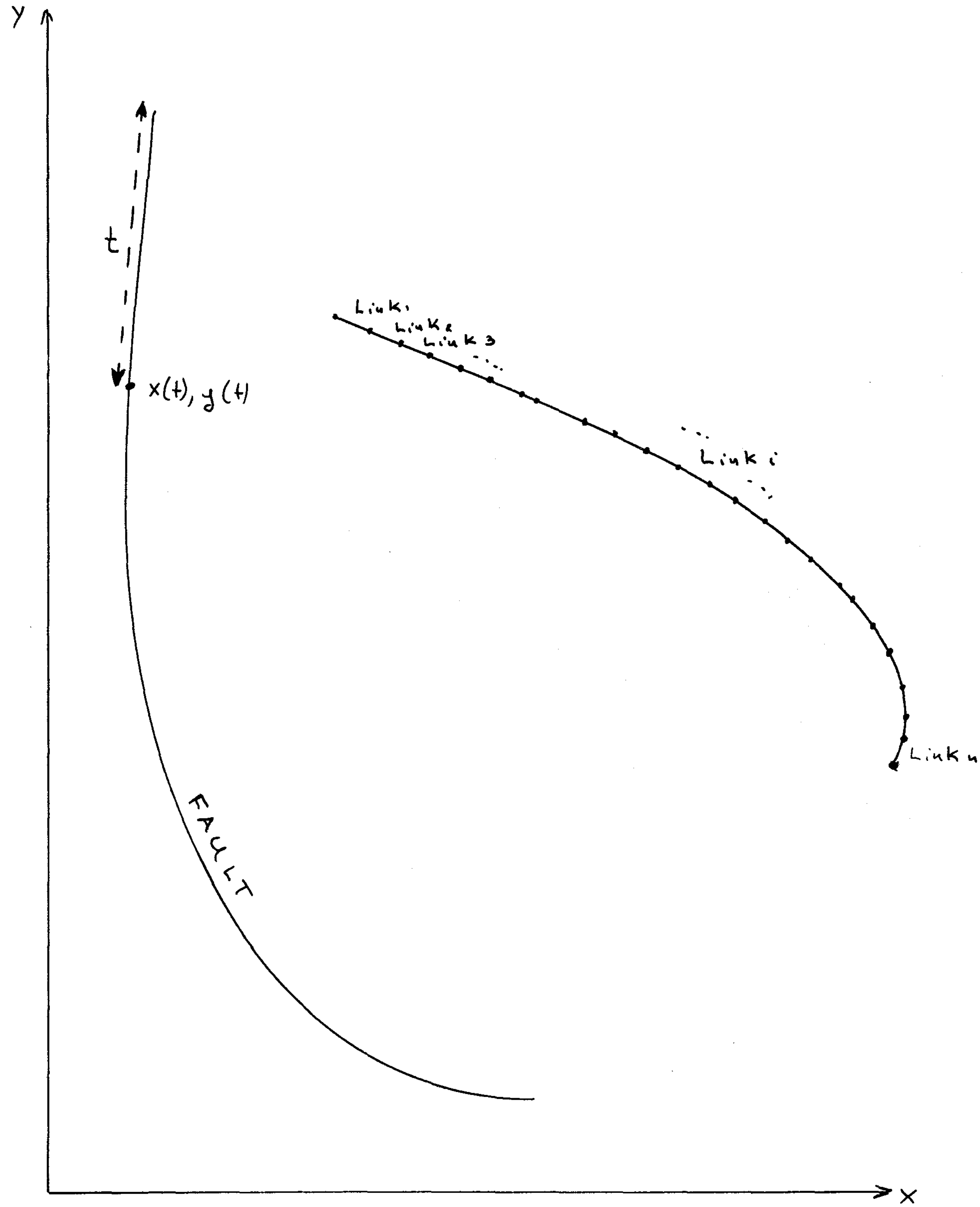


Figure 3

SYSTEM SITE RELIABILITY

Assume a fault with coordinates x(t), y(t), t $\in$ [0,L] where L is the length of the fault.

Also assume a site distributed geographically in a linear manner with a known random distribution of strength. The site can then be assumed to be a series of "short" links. (See Fig. 3) Each link has a resistance which is a known random variable and is independent of each other link. Then,

P[site system survives] =

$$= P[\text{site system survives} \mid m,t] \cdot P[M=m] \cdot P[T=t] =$$

$$= P[(\text{site link 1 survives}) \cap (\text{site link 2 survives}) \cap \ldots \mid m,t] \cdot$$
$$\cdot\ P[M=m] \cdot P[T=t] \ =$$

$$= \prod_{i=1}^{n} \{P[\text{site link } i \text{ survives}, i=1,2,\ldots,n \mid m,t]\} \cdot$$
$$\cdot\ P[M=m] \cdot P[T=t]$$

By substituting Eq. 1

$$= \int_m \int_t \prod_{i=1}^{n} [P(\frac{S_i}{R_i} < 1 \mid m,t)]\ f_M(m) \cdot f_T(t)dmdt \ =$$

$$= \int_m \int_t \prod_{i=1}^{n} \{ P[\frac{b_1\ e^{b_2 m}\ d_i(t)^{-b_3}}{R_i} < 1 \mid m,t] \}\ f_M(m) \cdot f_T(t)dmdt$$

$$= \int_m \int_t \prod_{i=1}^{n} \{ P[R_i > b_1\ e^{b_2 m}\ d_i(t)^{-b_3} \mid m,t] \}\ f_M(m) \cdot f_T(t)dmdt$$

$$= \int_m \int_t \prod_{i=1}^{n} \{ 1 - F_{R_i|m,t}\ (b_1\ e^{b_2 m}\ d_i(t)^{-b_3}) \} \cdot f_M(m) \cdot f_T(t) \cdot dm \cdot dt \qquad (10)$$

Equation 10 is the expression necessary to calculate the reliability of a continuous system, In many cases, however, it may be preferable to calculate the probability of failure directly, i.e.,

P[failure of system] = 1 - P[system survives] =

$$= 1 - \int_m \int_t \prod_{i=1}^{n} \{1 - F_{R_i|m,f} (b_1 e^{b_2 m} d_i(t)^{-b_3})\} \cdot f_M(m) \cdot f_T(t) \cdot dm \cdot dt =$$

$$= \int_m \int_t \{1 - \prod_{i=1}^{n} [1 - F_{R_i|m,t} (b_1 e^{b_2 m} d_i(t)^{-b_3})] \cdot f_M(m) \cdot f_T(t) \cdot dm \cdot dt \qquad (11)$$

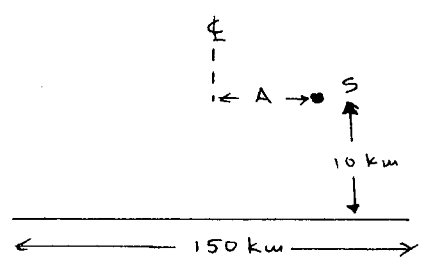
The purpose of this transformation is the ability to obtain more accurate significant figures of the computed reliability expressions.

EXAMPLES

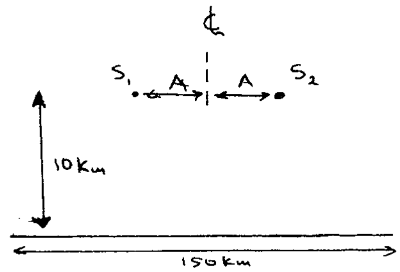
Four plots are presented comparing various results.

1. Probability of failure of a one-site system, two-site system, and four-site system. The geometric configuration of the system and source are:

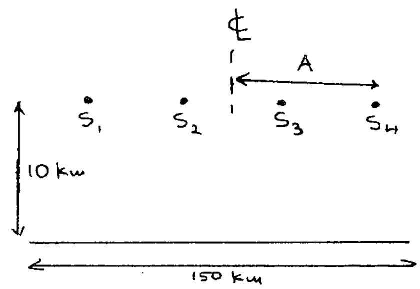
   a) 1-site system



b) 2-site system



c) 4-site system



Notice that the probability of failure for one site remains constant as long as A is less than the fault length minus some short length, say L_{eff}. In algebraic form

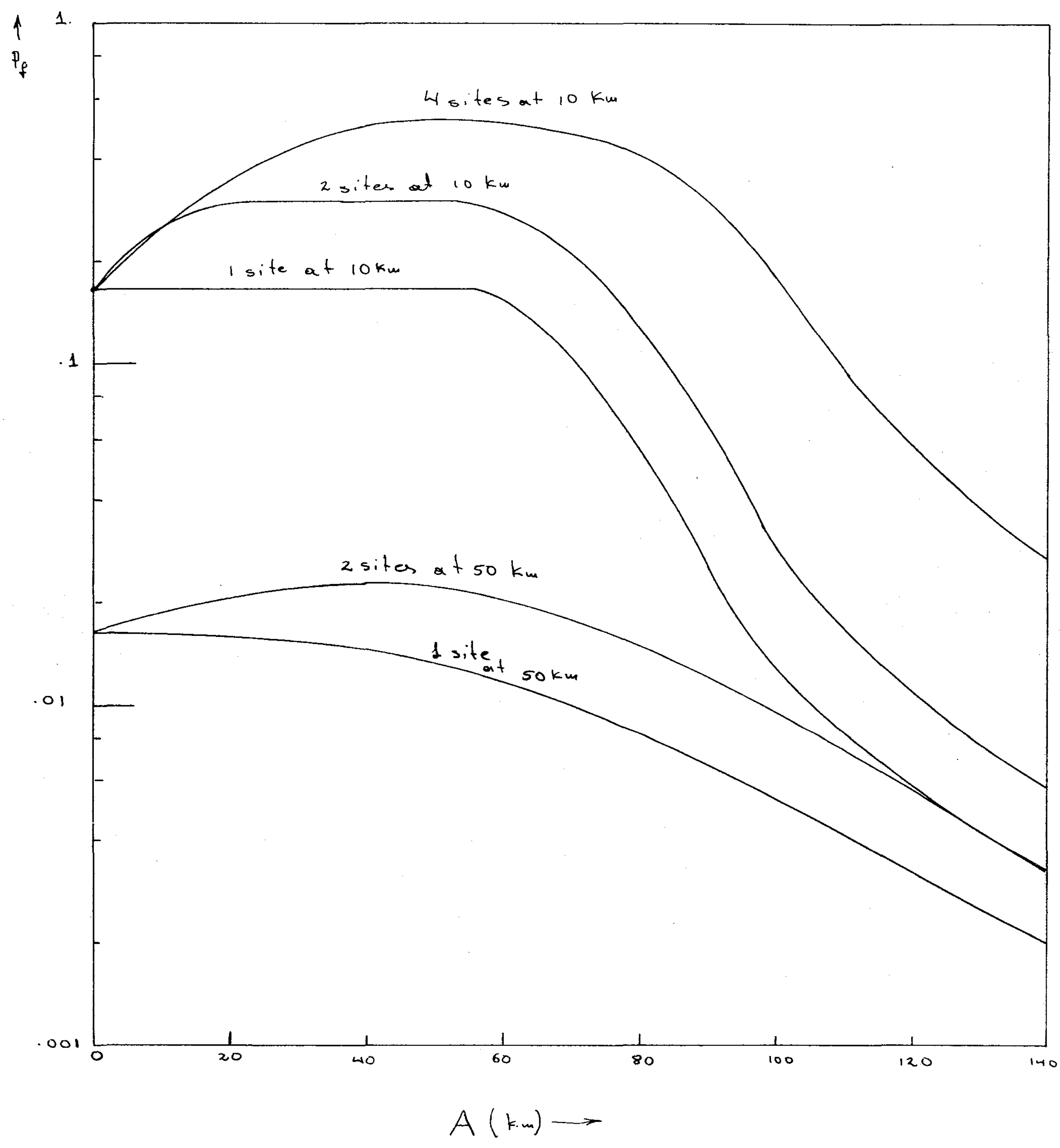
$$P[\text{failure} \mid A < \mid FLONG - L_{eff} \mid] = \text{constant} \quad .... \quad (12)$$

where FLONG = fault length.

Once A becomes much larger than the fault length, the probability of failure will decrease as a power function with a negative exponent.

For the two-site case, notice that the failure of one of the two sites is dependent on the failure of the second site for some small A, say

$$A < R_{eff} \quad .... \quad (13)$$



1.
p_f
.1
.01
.001
4 sites at 10 Km
2 sites at 10 Km
1 site at 10Km
2 sites at 50 Km
1 site at 50Km
0
20
40
60
80
100
120
140
A (km)

Figure 4

It seems that R_{eff} and L_{eff} are equal to 2.5 - 3.0 times the distance from the fault for the given set of parameters.

The probability of failure of a one-site and two-site system are also plotted for a different distance from the fault line (d = 50 km). Again, R_{eff} and L_{eff} are probably equal, but the short length of the fault, compared to d, hides this fact. (Fig. 4)

2. Probability of failure of a one-site system versus the location of the site. Note again, that, for small distances from the fault, the P[failure] is constant, but for large distances, the P[failure] drops. (Fig. 5)

3. Probability of failure of a one-site system versus increasing resistance. For all three examples, the resistance is a random variable, normally distributed with an extremely low standard deviation, thus making the site almost deterministic. Cornell's results are superimposed using the same parameters. (Fig. 6)

4. The probability of failure of a two-site system is plotted for $\sigma = 1$ (system almost deterministic) and $\sigma = 200$ (system with random normally distributed resistances). The only apparent change is at separation distances $d \simeq 0.$ to 10. This is expected to be, since the two sites are not totally dependent on each other (i.e., their resistances are not equal). (Fig. 7)

Table 1 shows the change of probability of failure for a one-site system with increasing standard deviation, σ, and mean equal to 1000 cm/sec^2.

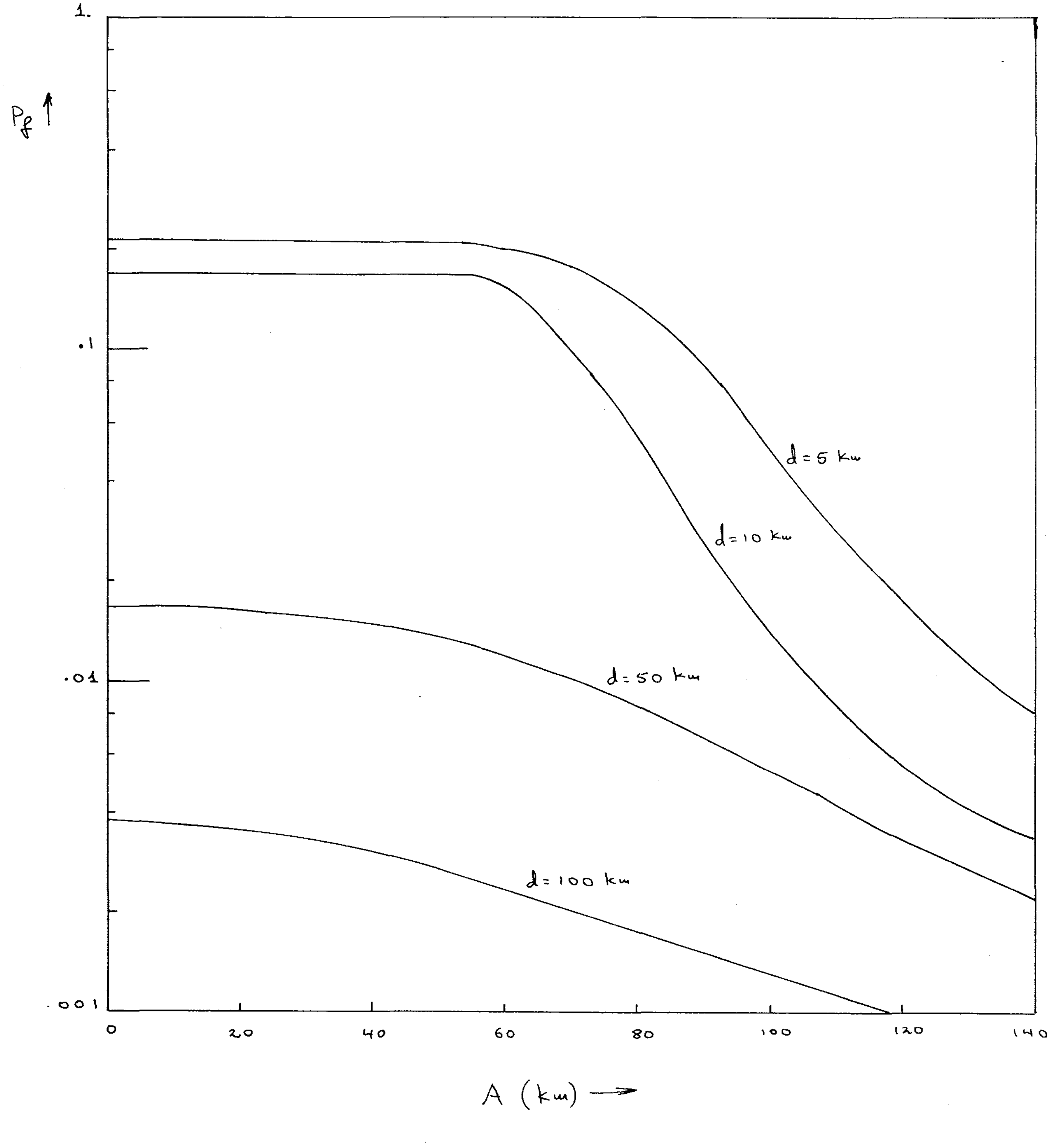

1.
P_f ↑
.1
.01
.001
0
20
40
60
80
100
120
140
A (km) →
d = 5 km
d = 10 km
d = 50 km
d = 100 km

Figure 5

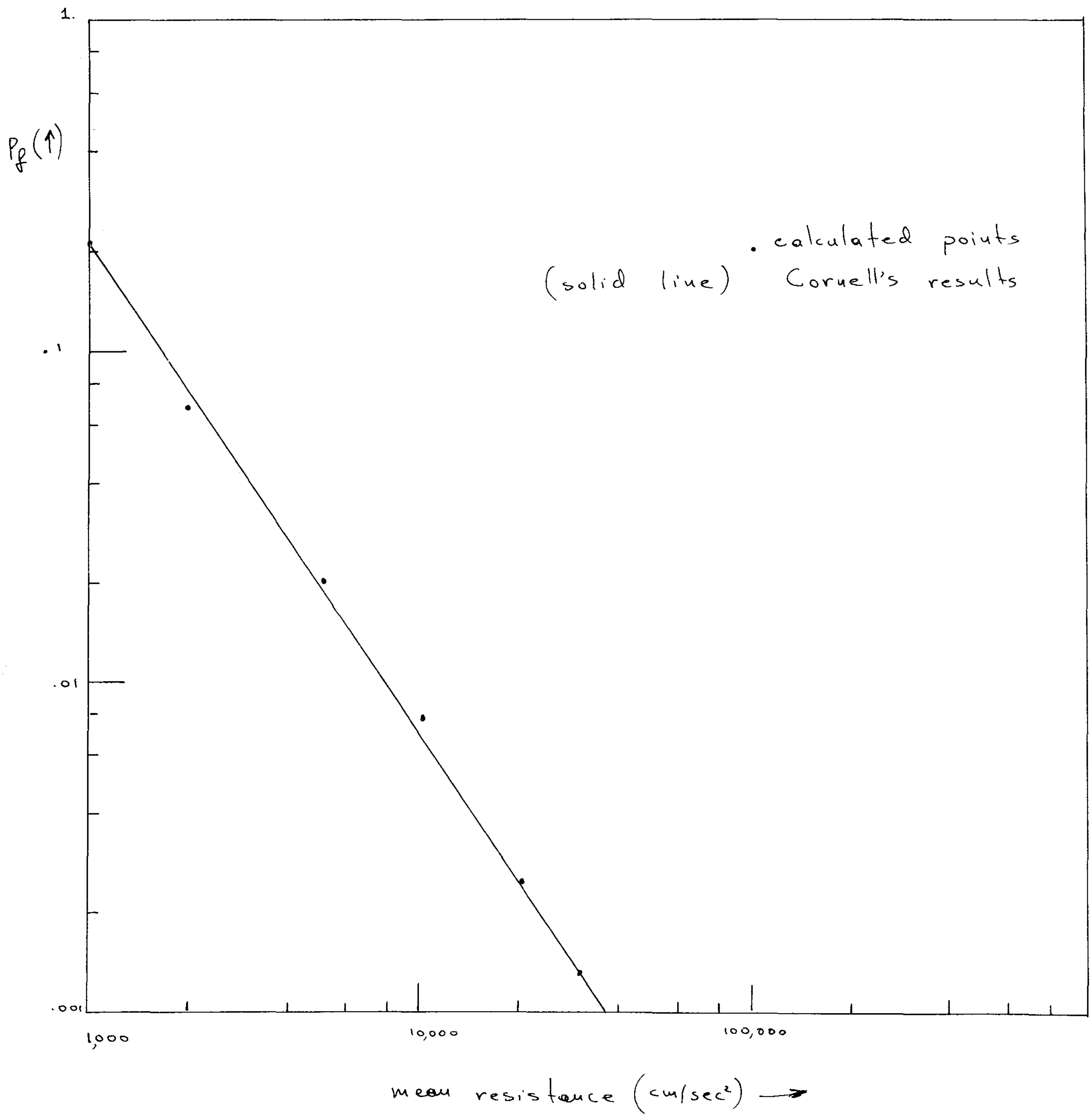

1.
$P_f(\uparrow)$
.1
.01
.001
1,000
10,000
100,000
. calculated points
(solid line) Cornell's results
mean resistance (cm/sec²) →

Figure 6

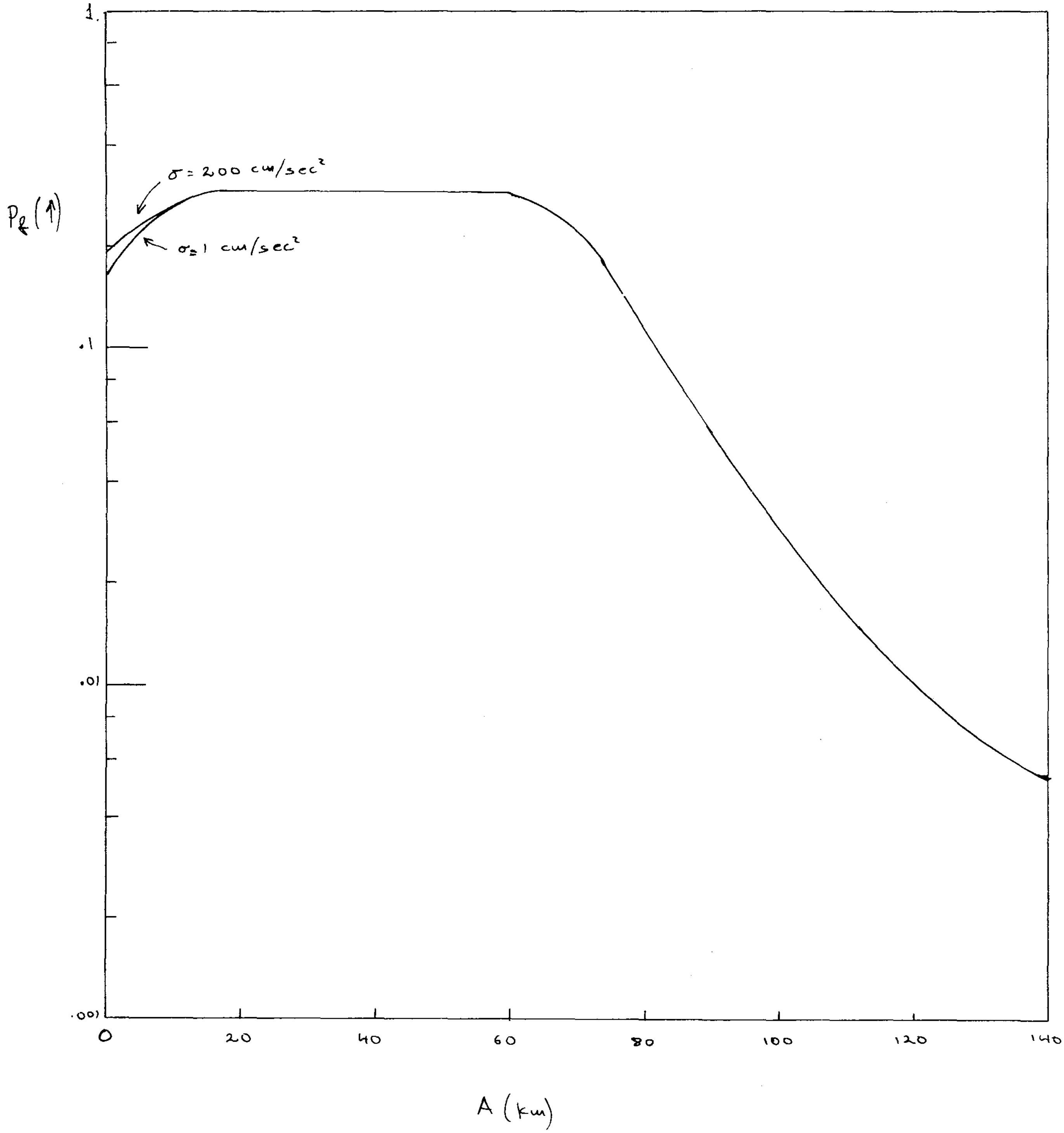

1.
.1
.01
.001
$P_f(\uparrow)$
σ = 200 cm/sec²
σ = 1 cm/sec²
0
20
40
60
80
100
120
140
A (km)

Figure 7

σ (cm/sec^2)	P(f)
1	.1681
2	.1676
5	.1666
20	.1665
50	.1669
70	.1672
150	.1691

Table 1

By no means are these results exhaustive, but they are an indication of the potential in the described analysis.

COMPUTER PROGRAM

```
C      DOUBLE INTEGRATION SCHEME FOR EVALUATING PROBABILITY OF SUCCESS
C      OF SITES IN SERIES SUBJECT TO AN EARTHQUAKE FROM A GIVEN FAULT.
       DIMENSION MEAN(100),SIGMA(100),X(100),Y(100)
       REAL NU,MMIN,MMAX,MEAN
       COMMON X,Y,NU,MEAN,SIGMA,FLONG,NSITES,MMIN,START
    18 READ(5,100) FLONG,START,END,MMIN,MMAX,NU
       IF(FLONG.EQ.0.)  CALL EXIT
   100 FORMAT(6F10.2)
       READ(5,101) NSITES
   101 FORMAT(I3)
       READ(5,200)(X(I),Y(I),MEAN(I),SIGMA(I),I=1,NSITES)
   200 FORMAT(4F10.4)
       WRITE(6,600) FLONG,NSITES
       WRITE(6,601)(X(I),Y(I),MEAN(I),SIGMA(I),I=1,NSITES)
   600 FORMAT(9(/),' THE FAULTS LENGTH IS',F20.5/' THE NUMBER OF SITES CON
      1SIDERED IS ',I3,' AT THE FOLLOWING COORDINATES',/'   X           Y
      2      MEAN AND STANDARD DEVIATION OF RESISTANCE',/)
   601 FORMAT(4F10.4)
       VOL=0.
       DO 1000 L1=1,49
       A=(MMAX-MMIN)*L1/50.+MMIN
       DO 1000 L2=1,49
       B=START+(END-START)*L2/50.
       CALL FUN(A,B,GFF)
  1000 VOL=VOL+GFF
       B=START
  1002 DO 1001 L1=1,49
       A=(MMAX-MMIN)*L1/50.+MMIN
       CALL FUN(A,B,GFF)
  1001 VOL=VOL+GFF/2.
       IF(ABS(B-END).LT..001) GO TO 1025
       B=END
       GO TO 1002
  1025 A=MMIN
  1004 DO 1003 L2=1,49
       B=START+(END-START)*L2/50.
       CALL FUN(A,B,GFF)
  1003 VOL=VOL+GFF/2.
       IF(ABS(A-MMAX).GT..001) GO TO 1026
       A=MMAX
       GO TO 1004
  1026 DO 1005 L1=1,2
       DO 1005 L2=1,2
       B=START+(END-START)*(L1-1)
       A=MMIN+(MMAX-MMIN)*(L2-1)
       CALL FUN(A,B,GFF)
  1005 VOL=VOL+GFF/4.
       VOL=VOL*(MMAX-MMIN)*(END-START)/2601.
       WRITE(6,301)VOL
   301 FORMAT('   PROBABILITY OF FAILURE = ',E10.4)
       GO TO 18
    19 CALL EXIT
       END
```

```
      SUBROUTINE FUN(EM,T,GFF)
      REAL NU,MMIN,MEAN
      DIMENSION MEAN(100),SIGMA(100),X(100),Y(100)
      COMMON X,Y,NU,MEAN,SIGMA,FLONG,NSITES,MMIN,START
      B1=2000.
      XT=T
      YT=0.
      FT=1./FLONG
      FM=NU*EXP(-NU*(EM-MMIN))
      B2=.8
      B3=1.7
      FS=1.
      DO 10 I=1,NSITES
      DEE=SQRT((X(I)-XT)**2+(Y(I)-YT)**2)
      ES=B1*EXP(B2*EM)/DEE**B3
   30 CALL DNDTR(ES,MEAN(I),SIGMA(I),P)
      P=1.-P
   10 FS=FS*P
      GFF=(1.-FS)*FT*FM
      RETURN
      END
```

```
      SUBROUTINE DNDTR(X,EX,SX,P)
C     THIS SUBROUTINE EVALUATES THE CUMULATIVE DISTRIBUTION  FOR A NORMALLY
C     DISTRIBUTED RANDOM VARIABLE, GIVEN ITS MEAN,AND STANDARD DEVIATION.
      AX=ABS((X-EX)/SX)
      T=1./(1.+.2316419*AX)
      D=0.
      IF(AX.GT.10.) GO TO 5
      D=.3989423*EXP(-AX*AX/2.)
    5 P=1.-D*T*(((((1.330274*T-1.821256)*T+1.781478)*T-0.3565638)*T+.31938
     1815    )
      IF(X.LT.EX) P=1.-P
      RETURN
      END
```

```
THE FAULTS LENGTH IS              150.00000
THE NUMBER OF SITES COSIDERED IS    1 AT THE FOLLOWING COORDINATES
 X          Y          MEAN AND STANDARD DEVIATION OF RESISTANCE

  50.0000    5.0000 1000.0000    1.0000
  PROBABILITY OF FAILURE = 0.2009E+00
```

```
THE FAULTS LENGTH IS              150.00000
THE NUMBER OF SITES COSIDERED IS    1 AT THE FOLLOWING COORDINATES
 X          Y          MEAN AND STANDARD DEVIATION OF RESISTANCE

 100.0000    5.0000 1000.0000    1.0000
  PROBABILITY OF FAILURE = 0.1541E-01
```

```
THE FAULTS LENGTH IS              150.00000
THE NUMBER OF SITES COSIDERED IS    1 AT THE FOLLOWING COORDINATES
 X          Y          MEAN AND STANDARD DEVIATION OF RESISTANCE

   0.0      50.0000 1000.0000    1.0000
  PROBABILITY OF FAILURE = 0.1780E-01
```

```
THE FAULTS LENGTH IS              150.00000
THE NUMBER OF SITES COSIDERED IS    1 AT THE FOLLOWING COORDINATES
 X          Y          MEAN AND STANDARD DEVIATION OF RESISTANCE

  50.0000   50.0000 1000.0000    1.0000
  PROBABILITY OF FAILURE = 0.1463E-01
```

Some typical results

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