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NETWORK RELIABILITY

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effort is made in this report to create an algorithm that will compute the relia-ity of a network of links in series. A given network with one input and one output assumed. Formulation of reliability expressions is considered. For a given network h many inputs and many outputs, the question arises of what "survival" means. In non terms survival is considered to mean that at least one input will reach one put. However, in most networks this may not constitute "survival." Instead, it suggested that "survival" may be defined as each input being able to reach each out-. "Survival" may also be defined as each input being connected with at least one put. Other definitions of "survival" are also reviewed. Multistate damage condi-ns are covered.

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INTRODUCTION

In Internal Study Report No. 41, the reliability of a system of links in series with regard to an earthquake threat was defined. An effort is being made here to create an algorythm that will compute the reliability of a network of links in series. For the time being, assume that each element (link) is a [0 - 1] ([survives, fails]) element.

FORMULATION OF RELIABILITY EXPRESSIONS

Assume a given network with one input and one output. In Internal Study Report No.40, we showed how it is possible to transform a given network into another network of parallel systems consisting of links in series (Figure 1).

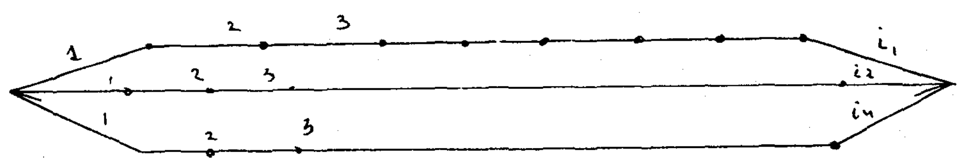

Figure 1

For the new network, and assuming independence of links, we say

$$P[\text{system survives}] = P[(T_1 \text{ survives}) \cup (T_2 \text{ survives}) \cup \ldots (T_n \text{ survives})] \quad \ldots\ldots\ldots (1)$$

where T_j = tie set j consisting of i_j elements.

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Example

Given a system of three elements, a, b and c (Figure 2).

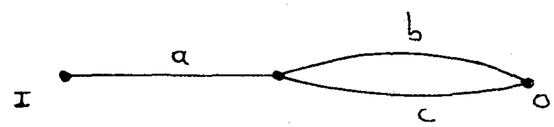


Figure 2

Given P[a survives], P[b survives] and P[c survives], conditioned to the same threat, find P[system survives] = P[at least one path exists between I and O].

Solution

The equivalent network is shown in Figure 3.

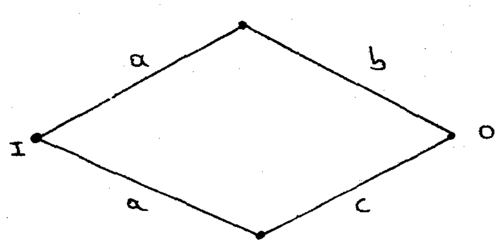


Figure 3

P[system survives] = P[ab survives) U (ac survives)]

= P[ab survives] + P[ac survives] - P[abc survives]

= P[a survives] • P[b survives] + P[a survives] • P[c survives]

- P[a survives] • P[b survives] • P[c survives]

(4)

By expanding equation (1),

P[system survives] =

$$\sum_{i=1}^{n} P[T_i \text{ survives}] - \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} P[T_i \text{ survives}) \cap (T_j \text{ survives})] +$$

$$\sum_{i=1}^{n-2} \sum_{j=i+1}^{n-1} \sum_{k=j+1}^{n} P[(T_i \text{ survives}) \cap (T_j \text{ survives}) \cap (T_k \text{ survives})] - \ldots$$

$$+ (-1)^{n-1} P[\cap (T_i \text{ survives, } i = 1,n)] \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (2)$$

and furthermore,

P[system survives] =

$$\sum_{k=1}^{n} \left[\prod_{L=1}^{i_N} P[\text{link L of } T_k \text{ survives}] \right] -$$

$$\sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \left[\prod_{i \neq j}^{\text{links of } T_i \text{ \& } T_j} P[\text{links of } T_i \text{ \& } T_j \text{ survive}] \right] + \ldots$$

$$+ (-1)^{n-1} \prod P[\text{each link in every } T_i \text{ survives}] \qquad (3)$$

Equation (3) is just an expansion of equation (2) into the sum of products of individual tie sets, T_k, or any combination of two, three, four, ..., N tie sets (e.g., $T_i \cap T_j \cap T_k$, $i \neq j \neq k$). If T_i and T_k have common elements, these are accounted for only once. An example is most useful and explanatory.

If P[a survives] = P[b survives] = P[c survives] = .9, then

P[system survives] = .81 + .81 - .729 = .891.

MULTI INPUT/OUTPUT SYSTEMS. DEFINITIONS FOR "SURVIVAL".

Given a network with many inputs (I_i) and many outputs (O_j) (Figure 4), the question of "what does survival mean?" arises. In Internal Study Report No. 40, a definition is given which in common terms means the following: "at least one input will reach one output". An example of a system whose survival coincides with the above statement is the following: Mayor A has 23 telephone lines coming out of City Hall. The National Guard has 250 lines coming into HQ's, 100 miles away from City Hall. If Mayor A's town is struck by an earthquake, and Mayor A needs help from the National Guard, the network of telephone lines will have survived, if Mayor A can use at least one of his telephone lines to reach one of the National Guard's telephones.

For this case, an equivalent network is defined by combining all tie sets leading from each input to each output into one system of tie sets in parallel.

This definition guarantees that at least one input will reach one output. In most networks, however, this may <u>not</u> constitute "survival". Most likely, "survival" may be defined as "each input is able to reach each output". A real life example of such a network survival is the following: City A has 5 fire stations with one truck each, scattered around its 20 neighborhoods. Given an earthquake, the city's road system survives if all 5 trucks can reach all 20 neighborhoods for a possible major fire.

In probabilistic terms, given a network (Figure 4):

$$P[\text{every input will reach every output}] = P[\cap(\cup T_{ij})] =$$

$$= P[(\cup T_{11}) \cap (\cup T_{12}) \cap (\cup T_{13}) \cap (\cup T_{21}) \cap (\cup T_{22}) \cap (\cup T_{23})$$

$$\cap (\cup T_{31}) \cap (\cup T_{32}) \cap (\cup T_{33}) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (5)$$

where $(\cup T_{ij})$ = union of all tie sets connecting input i to output j

and $P[T_{ij}]$ = probability that T_{ij} survives.

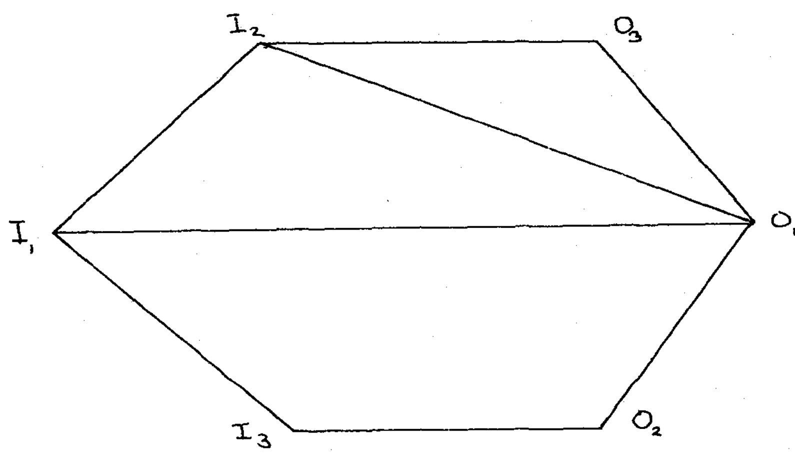


Figure 4

"Survival" may also be defined as "each output is connected with at least one input". In such a case, the probability of survival is

$$P[\text{system survives}] = P[((\cup T_{11}) \cap (\cup T_{21}) \cap (\cup T_{31}))$$

$$\cap ((\cup T_{12}) \cap (\cup T_{22}) \cap (\cup T_{32})) \cap ((\cup T_{13}) \cap (\cup T_{23}) \cap (\cup T_{33}))] \quad \ldots\ldots \quad (6)$$

where $(\cup T_{ij})$ = union of all tie sets connecting input i to output j.

Similarly, if survival is defined as "each input is connected with at least one output":

$$P[\text{system survives}] = P[((\cup T_{11}) \cap (\cup T_{12}) \cap (\cup T_{13}))$$

$$\cap ((\cup T_{21}) \cap (\cup T_{22}) \cap (\cup T_{23})) \cap ((\cup T_{31}) \cap (\cup T_{32}) \cap (\cup T_{33}))] \quad \ldots\ldots \quad (7)$$

The final definition of survival will be "each or some inputs are connected to some outputs (e.g., I_1 to O_1 and O_2, I_2 to O_1 and O_3, I_3 to O_1).

Table 1 gives a summary of all definitions considered. It also gives an example of the equivalent network from a network reliability point of view for the network in Figure 5, a real-life example and the expression for the probability of survival. Note that this equivalent network has one input and one output, an already familiar case.

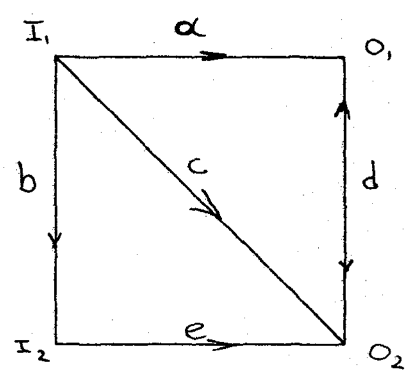


Figure 5

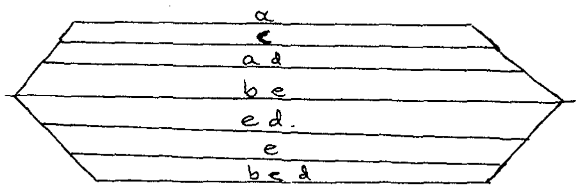
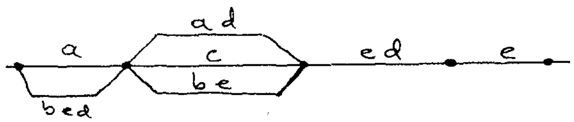
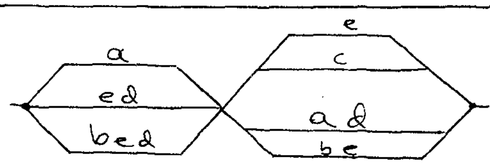
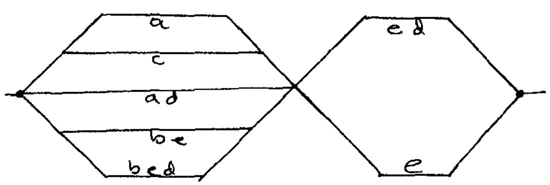
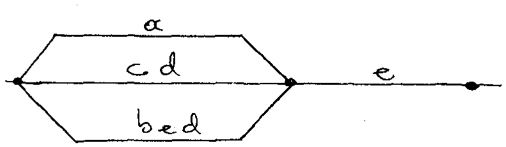
The next step in the analysis of the network is to transform the new network to one of series systems in parallel (Figure 1) and then proceed to find the probability of survival of the final equivalent network.

Recapitulation

For clarity, all required steps for the analysis of a given network with (0 - 1) elements are summarized:

TABLE 1

SOME DEFINITIONS OF "SURVIVAL"

SURVIVAL DEFINITION	EQUIVALENT NETWORK	REAL LIFE SITUATION	PROBABILITY OF SURVIVAL
At least one input will reach at least one output		One installation with many telephone lines is trying to reach another installation with many phone lines	$P[(I_1O_1) \cup (I_1O_2) \cup (I_1O_1O_2) \cup (I_1I_2O_1) \cup (I_2O_2O_1) \cup (I_2O_2) \cup (I_1I_2O_2O_1)]$
Every input reaches every output		Each of n fire trucks is able to reach each of m neighborhoods	$P[((I_1O_1) \cup (I_1I_2O_2O_1)) \cap ((I_1O_1O_2) \cup (I_1O_2) \cup (I_1I_2O_2)) \cap (I_2O_2O_1) \cap (I_1I_2O_2))]$
Each output is reached by at least one input		n power stations supply power to m neighborhoods	$P[((I_1O_1) \cup (I_2O_2O_1) \cup (I_1I_2O_2O_1) \cap ((I_2O_2) \cup (I_1O_2) \cup (I_1O_1O_2) \cup (I_1I_2O_2))]$
Each input reaches at least one output		Every household must be able to connect to some communications device	$P[((I_1O_1) \cup (I_1O_2) \cup (I_1O_1O_2) \cup (I_1I_2O_2) \cup (I_1I_2O_2O_1)) \cap ((I_2O_2O_1) \cup (I_2O_2))]$
Some inputs connected to some outputs (I_1 to O_1, and I_2 to O_2)		Each doctor must be able to reach his assigned hospital	$P[((I_1O_1) \cup (I_1O_2O_1) \cup (I_1I_2O_2O_1)) \cap (I_2O_2)]$

1 - Define survival (each input, each output, etc.),

2 - Draw equivalent network with one input and one output, according to definition,

3 - Draw final equivalent network of series systems in parallel and

4 - Find probability of survival.

Example

For the network in Figure 6, assume a, b and c are independent links.

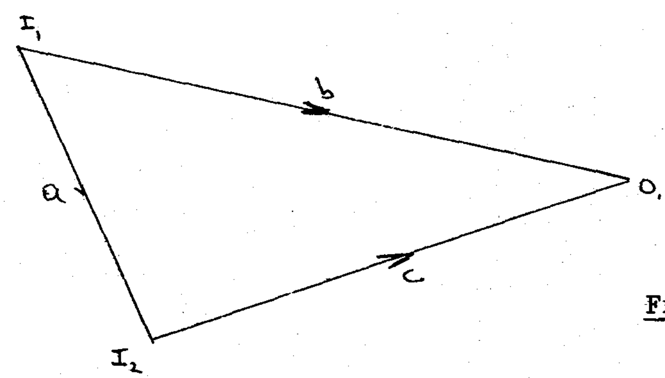


Figure 6

For a given earthquake threat, given P[a survives] = P[b survives] = P[c survives] = P[a] = P[b] = P[c] = .9, find the probability of

1) At least one input reaching 0,

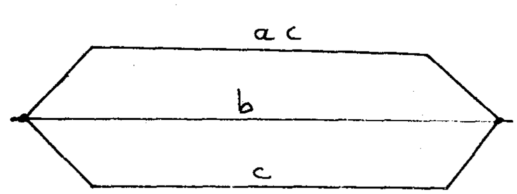
2) Every input reaching 0,

3) I_1 reaching 0,

4) I_2 reaching 0.

For notation purposes, $P(xy) = P(x \cap y) = P(x) \cdot P(y)$.

1) Equivalent network is



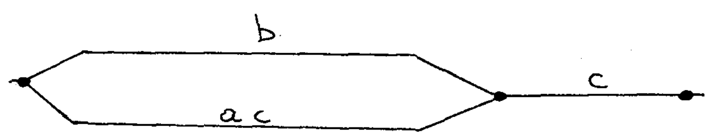
This is a network of series systems in parallel.

P[survival] = P[(ac) U (b) U (c)] =

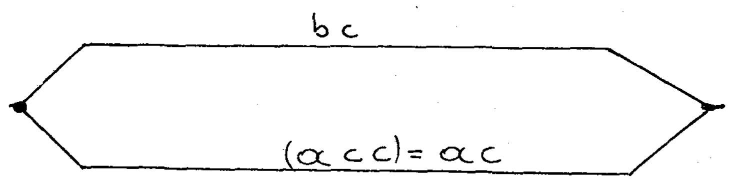
= P(ac) + P(b) + P(c) - P(abc) - P(ac) - P(bc) + P(abc) =

= .81 + .9 + .9 - .729 - .81 - .81 + .729 = .99

2) Equivalent network is



Transform to network of series system in parallel.

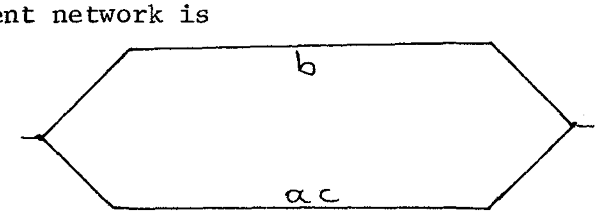


P[survival] = P[(bc) U (ac)] =

= P[bc] + P[ac] - P(abc) =

= .81 + .81 - .729 = .891

3) Equivalent network is



P[survival] = P[(b) U (ac)] = P(b) + P(ac) - P(abc) =

= .9 + .81 - .729 = .981

4) $P[I_2 \text{ reaches } O_1] = P(c) = .9$

It is expected that

$$P[\text{all inputs reach all outputs}] \leqslant P[\text{some inputs reach some outputs}]$$
$$\leqslant P[\text{an input reaches an output}]$$

MULTISTATE DAMAGE CONDITIONS*

The same analysis as for (0-1) elements is applied to multistate elements. For the reduced network of series systems in parallel, Fig. 1,

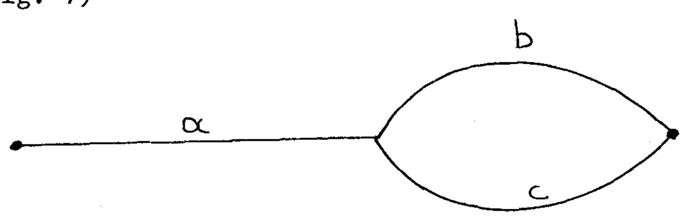
$$P[DS_T \leqslant n] = P[(DS_{T_1} \leqslant n) \cup (DS_{T_2} \leqslant n) \cup \ldots (DS_{T_n} \leqslant n)] \quad \ldots\ldots (8)$$

where DS_T = damage state of total system

and DS_{T_i} = damage state of i^{th} tie set

Further algebraic manipulation is similar to Eq. 2.

<u>Example</u> (See Fig. 7)



<u>Figure 7</u>

Given $P[DS_a \leqslant n] = P[DS_b \leqslant n] = P[DS_c \leqslant n] = .6 + \frac{n}{10}$ $(n = 0,1,2,3,4)$ for a given earthquake threat, find $P[DS_T \leqslant n]$.

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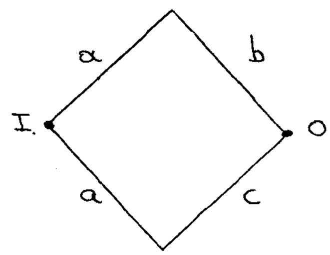
* See ISR #40 for definition of elemental damage states and definition of network damage state.

Solution

Equivalent network is



$$P[DS_T \leqslant n] = P[(DS_{ab} \leqslant n) \cup (DS_{ac} \leqslant n)] =$$

$$= P[DS_{ab} \leqslant n] + P[DS_{ac} \leqslant n] - P[DS_{abc} \leqslant n] =$$

$$= P[DS_a \leqslant n] \cdot P[DS_b \leqslant n] + P[DS_a \leqslant n] \cdot P[DS_c \leqslant n] -$$

$$- P[DS_a \leqslant n] \cdot P[DS_b \leqslant n] \cdot P[DS_c \leqslant n] =$$

$$= 2(.6 + \frac{n}{10})^2 - (.6 + \frac{n}{10})^3$$

$$\text{For} \quad n = \begin{Bmatrix} 0 \\ 1 \\ 2 \\ 3 \\ 4 \end{Bmatrix} \quad , \quad P[DS_T \leqslant n] = \begin{Bmatrix} .504 \\ .637 \\ .768 \\ .891 \\ 1.000 \end{Bmatrix}$$

$$\therefore \quad P[DS_T = \begin{Bmatrix} 0 \\ 1 \\ 2 \\ 3 \\ 4 \end{Bmatrix}] = \begin{Bmatrix} .504 \\ .133 \\ .131 \\ .123 \\ .109 \end{Bmatrix}$$

Conventional combinatorial reliability gives the same results.