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DUCTILITY REQUIREMENTS OF MULTIDEGREE FREEDOM SYSTEMS

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This study addresses the issue of providing and mobilizing work capacity of a structure under strong ground motion so that no structure will collapse by excessive deformation of a certain story while the rest of the building suffers little or no damage. Structures used in the study are a hypothetical 6-story reinforced concrete frame and a hypothetical 6-story steel frame. Both structures are assumed to be of shear-type and are modeled by close-coupled mass-spring systems with masses concentrated at floor levels. Methods of analysis using digital computer simulation for estimating the inelastic behavior of a multidegree freedom system are reported. Input to the program contains the characteristics of the spring system and the earthquake time history. Output of the analysis consists of the natural periods, mode shapes, maximum interstory displacements, ductility factors required, maximum shear forces, and time of occurrence. Analyses of results and conclusions drawn from 31 computer runs are presented. Tables show characteristics of computer runs and a summary of results. Ductility and intensity ratio graphs are included.

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INTRODUCTION

To design a structure which will behave elastically during strong ground motion is economically unfeasible. The goals of earthquake design can only be limited to providing structures with a capability to resist minor earthquakes without damage and to resist major earthquakes without collapse. Damages are expected during moderate to severe earthquakes. However, the damage should not be so great that the structure is rendered unrepairable even in a major earthquake. To ensure the design goals, structures have to be designed under the philosophy of two levels of performance; elastic for service levels and the frequent earthquakes, and inelastic for ultimate strength and major earthquakes. Dynamic analysis of an elastic system is well developed and many computer programs have been made available, however, reliable methods for predicting inelastic behavior of a multidegree system are still lacking. A major issue in earthquake resistant design is thus to provide and to mobilize energy absorption capability of a structure under earthquake excitation. Evidently, the work capacity of a structure reaches its maximum when all components which comprise the structure reach their maximum allowable inelastic deformation simultaneously. The components must therefore be proportioned so that the ductility factors, the ratios of the ultimate deformation to the yielding deformation, reach their allowable values under the excitation of the design earthquake.

For regular steel or concrete buildings, maximum ductility factors which may be achieved are more or less uniform for all stories. A uniform ductility factor distribution over the stories may be most effective in developing the work capacity of a structure. However, the soft-story concept is of practical value in building design if the allowable ductility factor of a story can be made much larger than the others. The soft story is not necessarily restricted to the bottom story.

The study reported here addresses the question of how to provide and to mobilize work capacity of a structure under strong ground motion so that no structure will collapse by excessive deformation of a certain story while the rest of the building suffers little to no damage. Structures studied here are a hypothetical 6-story reinforced concrete frame and a hypothetical 6-story steel frame. Both structures are assumed to be of shear-type and are modelled by close-coupled mass-spring systems with masses concentrated at floor levels. The difference between the concrete frame and the steel frame is that the concrete frame is represented by a system with uniform spring constant and the steel frame by a system with a variable spring constant such that the first mode shape of the system is a straight line.

STRUCTURAL RESPONSE DURING EARTHQUAKE EXCITATION

The structural responses during earthquake excitation may be estimated by using either the response spectrum technique or time history analysis. The response spectrum method requires little effort, but the solution is approximate for multidegree systems. Time history analysis is an expensive procedure, but the solution is exact in that the structure is properly modelled and errors from numerical integration are negligible. Within the elastic range, solutions derived from the response spectrum method approximate those from time history analysis (1, 2). While the inelastic response of a single degree system may be approximated by an inelastic response spectrum (3, 4), the time history analysis remains the only method to assess the inelastic responses of a multi-degree system since normal modes do not exist in an inelastic system. The ductility factor cannot yet be considered quantitatively in earthquake design processes. An excellent summary of what has been done in the area of inelastic behavior of single and multidegree systems through 1969 is given by Newmark and Rosenblueth (5).

METHODS OF ANALYSIS

Since no close-form analytical solution is available, inelastic behavior of a multidegree freedom system is too complex to be treated analytically. Digital computer simulation of the system behavior appears to be the only method and is used for this study. In essence, properties of a mechanical system and the time history of a ground motion are input to a computer program and the system responses are calculated by numerical integration performed step by step at small time increments.

Two 6-degree-of-freedom systems, designated Systems C and S, are used as physical models simulating a 6-story reinforced concrete frame and 6-story steel frame. Since it is reasonable to assume that masses at each floor level are identical and column sizes are uniform on all floors, the concrete frame is modelled as a 6-degree spring system with uniform spring constants and masses. Steel buildings, on the contrary, usually have a straight line first mode shape and therefore the steel frame is represented by a 6-degree spring system with uniform masses and variable spring constants such that a straight line first mode is preserved.

To compare the results from both systems, the first fundamental periods are made identical for both (T_1 = 0.638 seconds) by lowering the masses in System S. The ratio of masses of System S to System C is 0.82 assuming the same stiffness at the first story level. The stiffness ratio and the first mode shape of the systems are given in Table 1.

Yielding displacements at each floor level are assumed to be either uniform or in a shape proportional to the elastic maximum interstory drifts calculated by the response spectrum method which equates the bottom story yielding displacement to the uniform yielding displacement. The former is termed uniform yielding displacement and the latter, the R. S. yielding displacement. Even though both uniform and R. S. yielding displacement may not be typical in a real structure, they are used to show that the pattern of allowable yielding displacements greatly influence the ductility requirements and that a structure's work capacity may be fully utilized by properly adjusting the pattern.

Bilinear spring with dashpot damping is the most popular model used to simulate inelastic behavior of a reinforced concrete frame or a steel frame (6) and is used here. For simplicity, a bilinear spring with a flat second segment is usually employed for structural analysis even though experimental data have shown that post-yielding stiffness exists for both frames (7, 8). This assumption may not distort the results significantly for a spring system in parallel, while it may influence the results to a certain extent for a spring system in series. The effect of the post-yielding stiffness is studied by varying it among 0, 3, and 10% of the initial stiffness.

Stiffness degrading has long been observed to occur in concrete frames loaded beyond yield point. To study the effect of stiffness degrading on maximum ductility requirement, an elasto-plastic (bilinear with flat second segment) stiffness degrading model is also used in this study. The models are shown in Figures 1 and 2.

The input excitation used is an artificial earthquake time history generated to simulate a target response spectrum as shown in Figure 3. This response spectrum was constructed to be typical of what might be appropriate for design in the Boston Area (9). Note that the response spectrum used to estimate the elastic response of the system is the target response spectrum but not the actual one computed from the time history. Since the response spectrum solution is approximate, this estimation is justifiable.

The intensity of the earthquake is regulated by multiplying the whole time history by a common factor. To determine the effect of earthquake intensity on the maximum and average required ductilities, three intensity levels with maximum ground accelerations at 0.0135, 0.27 and 0.54 g are studied. The intensity level at which yield in the systems begins is calculated and then the intensity ratios (intensity level/yielding intensity) are found for the levels of 0.27 and 0.54 g.

A real earthquake motion record from the San Fernando earthquake of 1971 is also used to study the effect of earthquake characteristics on ductility requirements. The record is chosen arbitrarily. The response spectrum of this record is shown in Figure 4.

In addition to the factors mentioned, damping is also thought to be influential in ductility requirements. A value of 4% of critical is used for most of the runs: 1 and 10% of critical are used to test the effect of damping on ductility requirements: 4 and 10% are chosen because they are believed to be typical for steel and concrete frames at high strain range. For obtaining another data point, 1% is used.

The computer program used was developed by Anagnostopoulos (7) for nonlinear dynamic analysis of buildings. The first step of the numerical integration is performed by the 4th order Runge-Kutta procedure while all subsequent steps are done by the constant velocity method. Input to the program contains the characteristics of the spring system and the earthquake time history. Output of the analysis consists of the natural periods, mode shapes, maximum interstory displacements, ductility factors required, maximum shear forces and the time of occurrence.

Thirty-one computer runs were made. An index of the characteristics of the systems is shown in Table 2a and the results are summarized in Table 2b. In Table 2a are the run number, system type, spring type, post-yielding stiffness, yielding displacement pattern, damping value, maximum ground acceleration, and type of earthquake. Table 2b presents results on floor ductilities, intensity ratio, average ductility, ratio of maximum ductility to average ductility, ratio of maximum ductility to minimum ductility, ratio of maximum story drift to maximum pseudo-elastic story drift, ratio of average story drift to average pseudo-elastic story drift and location of maximum story drift. The pseudo-elastic solution is obtained by multiplying the elastic solution at first yield by the intensity ratio. In other words, the pseudo-elastic solutions are calculated assuming that the yielding strengths of the springs are extended to infinity.

ANALYSIS OF THE RESULTS

Effect of Yielding Displacements, Damping, Stiffness Degrading and Earthquake Characteristics on Ductility

Figure 8 compares ductility requirements for System C with uniform yielding displacements and R. S. yielding displacements. Even though the system with R. S. yielding displacement is structurally weaker than the system with uniform yielding displacement, it undergoes less inelastic deformation and its work capacity is nearly fully utilized, mainly because the response spectrum solution closely approximates the elastic solution and the uniform ductility is nearly achieved in the elastic range. A comparison of ductility requirements for Systems C and S with uniform yielding displacements is shown in Figure 9. System S is superior to System C in reducing the maximum required ductility.

Figure 11 shows the effect of damping on the required ductility. The behaviors of the systems with 4 and 10% damping appear to be very consistent and the maximum ductilities are nearly identical. The pattern of the ductility requirement of the system with 1% damping is somewhat different and the maximum ductility factor is larger than the other two. The behavior of systems with low damping value is not of our concern, since we are interested in the range where high damping values prevail.

The effect of stiffness degrading on ductility requirements is shown in Figure 12. Stiffness degrading appears to have an adverse effect on the required maximum ductility at lower levels of excitation and a favorable effect at higher levels. Reasons for these effects are unclear and further work is needed for substantiation. However, the required maximum ductility clearly relies on the type of model used.

The required story ductilities during two different earthquakes are shown in Figure 13 at two levels of earthquake intensity. The curves, as expected, do not coincide. The shapes, however, are similar and the maximum ductilities are of the same order.

Effect of Post-Yielding Stiffness on Ductility

Effects of post-yielding stiffness of a bilinear spring on story ductilities are depicted in Figure 10. The effect is negligible in the region where inelastic deformation is less than the pseudo-elastic solution. However, significant differences exist at the bottom story where maximum ductilities are required. Further, the reduction of maximum ductility from 0 to 3% post-yielding stiffness is greater than that from 3 to 10%. The reduction is highly nonlinear with the increase of post-yielding stiffness as higher rates prevail in the range of low post-yielding stiffness: the rate of reduction decreases as the post-yielding stiffness increases. To prevent an excessive inelastic deformation, 3% of initial stiffness appears sufficient.

Effects of post-yielding stiffness on maximum ductilities, average ductilities and their ratios are summarized in Figures 14, 15 and 16 for the systems considered. In general, the maximum ductility and average ductility increase more rapidly than does the intensity ratio, especially when the ratio of maximum ductility to average ductility is high. A scrutiny of the figures reveals that the nonlinearity may be minimized by increasing the post-yielding stiffness or equalizing the ductility requirements. Linear relations exist between intensity ratio and ductility requirements for the systems with a 10% post-yielding stiffness. High post-yielding stiffness may be achieved by special design, e.g., by using high strength steel in some components.

For most cases, the ratio of maximum ductility to average ductility increases with, but much more slowly than, the intensity ratio. In other words, the increase in the maximum ductility factor is greater than the increase in the average ductility factor. The increase is significant at lower earthquake intensities and levels off at higher intensities. For the cases with post-yielding stiffness, the curves are practically flat at higher intensity levels.

Use of Elastic Analysis to Predict Inelastic Response

Figures 5 to 7 show the comparisons of inelastic maximum interstory displacements and the pseudo-elastic maximum interstory displacements. As soon as the force in one of the springs reaches its yielding strength, the spring becomes softer and deforms excessively, while the other unyielded springs deform less than the pseudo-elastic drifts as shown by the first pair of the curves in Figure 5. The deviation between inelastic and pseudo-elastic deformation grows as the earthquake intensity increases; the pattern and magnitude of the gap depend on the relative stiffness and yielding displacements of the system. Observations reveal that the pattern of deviation between inelastic and pseudo-elastic deformation depends primarily on the pattern of ductility factors in the elastic range. If the ductilities are basically uniform, as in the case of System C with R. S. yielding displacements and System S with uniform yielding displacement, many floors will surpass the psuedo-elastic deformation at high levels of excitation. The maximum ratio of inelastic to pseudo-elastic solution is somewhat limited to less than 1.5. On the other hand, the bottom story is the only floor to deform beyond the pseudo-elastic limit and the amount of excessive displacement is significantly higher for system C with uniform yielding displacements. In all cases, the maximum story drifts and the maximum ductility required occur at the bottom story, which suggests that the bottom story is more likely to collapse during a strong earthquake. Thus, perhaps, the yielding strength of the bottom story should be increased or the story should be made more ductile.

To predict the required maximum ductility factor without an inelastic time history analysis, a relation between the maximum inelastic story drift and the maximum pseudo-elastic story drift is required. To make a preliminary study of this relationship, the ratios of maximum and average story drifts to the corresponding pseudo-elastic solutions against intensity ratio are plotted in Figure 17. The maximum story drifts are larger than the corresponding pseudo-elastic solutions in all cases where $K_2 = 0$ or $0.03\ K_1$. The ratios are mostly less than 1.5. Note that the maximum story drifts are near or less than the pseudo-elastic values for the cases when $K_2 = 0.1\ K_1$. The effectiveness of post-yielding stiffness in reducing the maximum ductility factor is clearly demonstrated.

The average story drifts, generally, are less than the corresponding pseudo-elastic solutions. Therefore, it is safe to approximate the inelastic drift of the top story by the pseudo-elastic solution in designing for maximum building drift during earthquake excitation. The pseudo-elastic solution may be easily obtained by the response spectrum model.

SUMMARY AND CONCLUSIONS

From the results of the 31 computer runs reported here, the following conclusions may be drawn.

1. The pattern of yielding displacements significantly affects the required ductilities. A pattern which results in a uniform ductility in the elastic range appears to ensure a low value (less than 1.5) of the ratio of the maximum inelastic story drift to the maximum pseudo-elastic story drift.
2. Post-yielding stiffness has a great effect on the required maximum ductility. A value of 3% of initial stiffness seems

to effectively reduce the excessive inelastic deformation. The ratios of maximum story drift to the corresponding pseudo-elastic solution are less than 1.25 for the cases studied. A value of 10% of initial stiffness would be ideal since it results in a linear relationship between the required ductility and intensity ratio. Also, the interstory drifts are less than the corresponding pseudo-elastic solutions for cases considered.

3. Ductility requirements appear to be less sensitive to damping for values higher than 4% of critical.
4. The ratio of the maximum inelastic story drift to the corresponding pseudo-elastic solution is usually larger than 1. For cases where the yielding displacement is distributed so that the maximum difference between two-story ductility in the elastic range is less than 10%, the ratio appears to be less than 1.5.
5. The average inelastic story drift is usually less than the corresponding pseudo-elastic solution. Therefore, one may approximate the top-story displacement by the pseudo-elastic solution in designing for maximum drift requirement during earthquake excitation. The pseudo-elastic solution may be approximated with the response spectrum technique.

Note that the conclusions are based only on the results of computer simulation and are by no means extensive. Extrapolations of the results to other cases must be done carefully.

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TABLE 1

Stiffness Ratios and First Mode Shapes

of the Physical Systems

	SYSTEM C		SYSTEM S	
Floor	Stiffness Ratio	1st Mode Shape	Stiffness Ratio	1st Mode Shape
6	1	4.16	0.286	6
5	1	3.91	0.481	5
4	1	3.44	0.714	4
3	1	2.77	0.857	3
2	1	1.94	0.952	2
1	1	1	1	1

RUN NO	SYSTEM TYPE	SPRING TYPE	POST-YIELDING STIFFNESS	YIELDING DISPLACEMENT PATTERN	DAMPING (% OF CRITICAL)	MAXIMUM ACCELERATION (g)	TYPE OF EARTHQUAKE
1a	C	1, 2	0%	U.N.	4%	0.1185	A
1b	"	"	"	R.S.	"	0.108	"
2	"	1	"	U.N.	"	0.27	"
3	"	"	"	"	"	0.54	"
4	"	"	"	"	1%	"	"
5	"	"	"	"	10%	"	"
6	"	"	"	R.S.	4%	0.27	"
7	"	"	"	"	"	0.54	"
8	"	"	3%	U.N.	"	0.27	"
9	"	"	"	"	"	0.54	"
10	"	"	"	"	1%	"	"
11	"	"	"	"	10%	"	"
12	"	"	"	R.S.	4%	0.27	"
13	"	"	"	"	"	0.54	"
14	"	"	10%	U.N.	4%	0.27	"
15	"	"	"	"	"	0.54	"
16	"	"	"	R.S.	"	0.27	"
17	"	"	"	"	"	0.54	"
18	S	1, 2	0%	U.N.	4%	0.142	"
19	"	1	"	"	"	0.27	"
20	"	"	"	"	"	0.54	"
21	"	"	3%	"	"	0.27	"
22	"	"	"	"	"	0.54	"
23	"	"	10%	"	"	0.27	"
24	"	"	"	"	"	0.54	"
25	"	"	0%	"	"	1	S
26	"	"	"	"	"	2	"
27	"	"	"	"	"	4	"
28	C	2	"	"	"	0.27	A
29	"	"	"	"	"	0.45	"
30	"	"	"	"	"	0.54	"

TABLE 2a

CHARACTERISTICS OF COMPUTER RUNS

SUMMARY OF RESULTS

	RUN	1a	1b	2	3	4	5	6	7	8
FLOOR DUCTILITY	1	1	0.91	2.64	8.86	5.83	7.82	2.41	6.54	2.37
	2	0.914	0.89	1.43	4.15	5.33	3.61	1.63	3.96	1.46
	3	0.8	0.89	1.42	3.12	3.95	2.08	2.48	5.43	1.37
	4	0.699	0.95	1.07	2.14	4.35	1.11	2.28	5.92	1.05
	5	0.534	1	0.89	1.05	1.12	0.95	1.77	5.99	0.89
	6	0.289	1	0.59	0.76	0.77	0.61	2.19	6.06	0.59
INTENSITY RATIO		1	1	2.29	4.58	4.58	4.58	2.5	5.0	2.29
AVERAGE DUCTILITY		0.71	0.96	1.34	3.35	3.56	2.36	2.13	5.65	1.29
MAXIMUM DUCTILITY / AVERAGE DUCTILITY		1.42	1.04	1.97	2.64	1.64	3.31	1.16	1.16	1.84
MAXIMUM DUCTILITY / MINIMUM DUCTILITY		3.46	1.12	4.47	11.66	7.57	12.82	1.52	1.65	4.02
M.S.D. / M.P.E.S.D. *		1	1	1.15	1.93	1.27	1.71	1.06	1.44	1.03
A.S.D. / A.P.E.S.D. +		1	1	0.83	1.03	1.14	0.83	0.77	1.20	0.79
LOCATION OF M.S.D.		1	1	1	1	1	1	1	1	1

* M.S.D. = Maximum Story Drift

M.P.E.S.D. = Maximum Pseudo-Elastic Story Drift

\+ A.S.D. = Average Story Drift

A.P.E.S.D. = Average Pseudo-Elastic Story Drift

TABLE 2b

SUMMARY OF RESULTS

	RUN	9	10	11	12	13	14	15	16	17
FLOOR DUCTILITY	1	5.54	6.84	5.23	2.33	5.27	2.19	4.61	2.16	4.34
	2	4.03	3.86	3.75	1.79	4.29	1.61	4.0	1.67	4.13
	3	3.15	3.34	2.18	1.90	4.66	1.32	2.74	1.73	3.90
	4	2.08	3.45	1.17	2.01	5.19	1.05	1.99	1.93	4.06
	5	1.21	1.40	0.92	1.71	5.43	0.87	1.18	1.68	3.80
	6	0.73	0.79	0.59	1.84	5.32	0.57	0.65	1.74	4.18
INTENSITY RATIO		4.58	4.58	4.58	2.50	5.0	2.29	4.58	2.50	5.0
AVERAGE DUCTILITY		2.79	3.28	2.31	1.93	5.03	1.27	2.53	1.82	4.07
MAXIMUM DUCTILITY / AVERAGE DUCTILITY		1.99	2.08	2.26	1.21	1.08	1.72	1.82	1.19	1.07
MAXIMUM DUCTILITY / MINIMUM DUCTILITY		7.59	8.66	8.86	1.30	1.26	3.84	7.09	1.29	1.14
M.S.D. / M.P.E.S.D. *		1.21	1.49	1.14	1.02	1.16	0.96	1.01	0.95	0.95
A.S.D. / A.P.E.S.D. +		0.86	1.01	0.71	0.86	1.07	0.79	0.78	0.80	0.89
LOCATION OF M.S.D.		1	1	1	1	1	1	1	1	1

* M.S.D. = Maximum Story Drift

M.P.E.S.D. = Maximum Pseudo-Elastic Story Drift

\+ A.S.D. = Average Story Drift

A.P.E.S.D. = Average Pseudo-Elastic Story Drift

TABLE 2b (continued)

SUMMARY OF RESULTS

	RUN	18	19	20	21	22	23	24	25	26
FLOOR DUCTILITY	1	1	2.33	5.34	2.20	4.69	1.85	3.66	0.88	2.08
	2	0.95	1.57	2.15	1.57	2.44	1.55	2.81	0.83	1.25
	3	0.91	1.50	2.51	1.48	2.60	1.43	2.28	0.84	1.14
	4	0.87	1.26	3.15	1.22	2.90	1.35	2.56	0.89	1.21
	5	0.95	1.60	4.47	1.51	3.81	1.45	3.29	0.97	1.71
	6	0.93	1.99	3.98	1.75	3.98	1.55	3.12	1	2.52
INTENSITY RATIO		1	1.9	3.8	1.9	3.8	1.9	3.8	1	2
AVERAGE DUCTILITY		0.93	1.71	3.43	1.62	3.40	1.53	2.95	0.90	1.65
MAXIMUM DUCTILITY / AVERAGE DUCTILITY		1.07	1.36	1.56	1.36	1.38	1.21	1.24	1.11	1.53
MAXIMUM DUCTILITY / MINIMUM DUCTILITY		1.15	1.85	2.48	1.80	1.80	1.37	1.60	1.20	2.21
M.S.D. / M.P.E.S.D. *		1	1.22	1.40	1.16	1.23	0.97	0.96	1	1.26
A.S.D. / A.P.E.S.D. +		1	0.96	0.96	0.91	0.89	0.86	0.82	1	0.91
LOCATION OF M.S.D.		1	1	1	1	1	1	1	6	6

* M.S.D. = Maximum Story Drift

M.P.E.S.D. = Maximum Pseudo-Elastic Story Drift

+ A.S.D. = Average Story Drift

A.P.E.S.D. = Average Pseudo-Elastic Story Drift

TABLE 2b (continued)

SUMMARY OF RESULTS

	RUN	27	28	29	30					
FLOOR DUCTILITY	1	4.06	3.85	4.13	7.46					
	2	3.21	1.50	3.04	3.62					
	3	2.70	1.09	1.83	3.31					
	4	2.94	0.94	1.22	2.07					
	5	2.84	0.73	0.91	0.92					
	6	4.34	0.41	0.58	0.58					
INTENSITY RATIO		4	2.29	3.41	4.58					
AVERAGE DUCTILITY		3.35	1.42	1.95	2.99					
MAXIMUM DUCTILITY / AVERAGE DUCTILITY		1.30	2.71	2.12	2.49					
MAXIMUM DUCTILITY / MINIMUM DUCTILITY		1.50	9.39	7.12	12.86					
M.S.D. / M.P.E.S.D. *		1.09	1.68	1.21	1.63					
A.S.D. / A.P.E.S.D. +		0.93	0.88	0.81	0.93					
LOCATION OF M.S.D.		6	1	1	1					

* M.S.D. = Maximum Story Drift

M.P.E.S.D. = Maximum Pseudo-Elastic Story Drift

\+ A.S.D. = Average Story Drift

A.P.E.S.D. = Average Pseudo-Elastic Story Drift

TABLE 2b (continued)

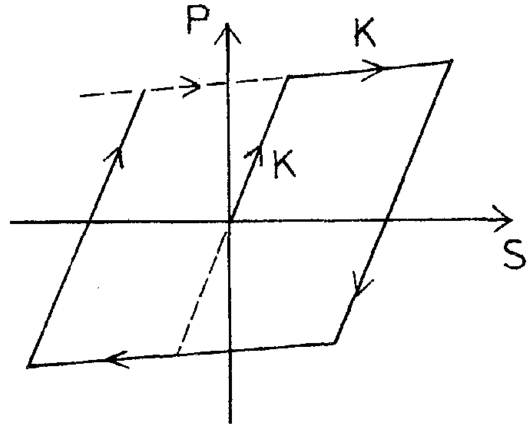

Figure 1. Bilinear Model.

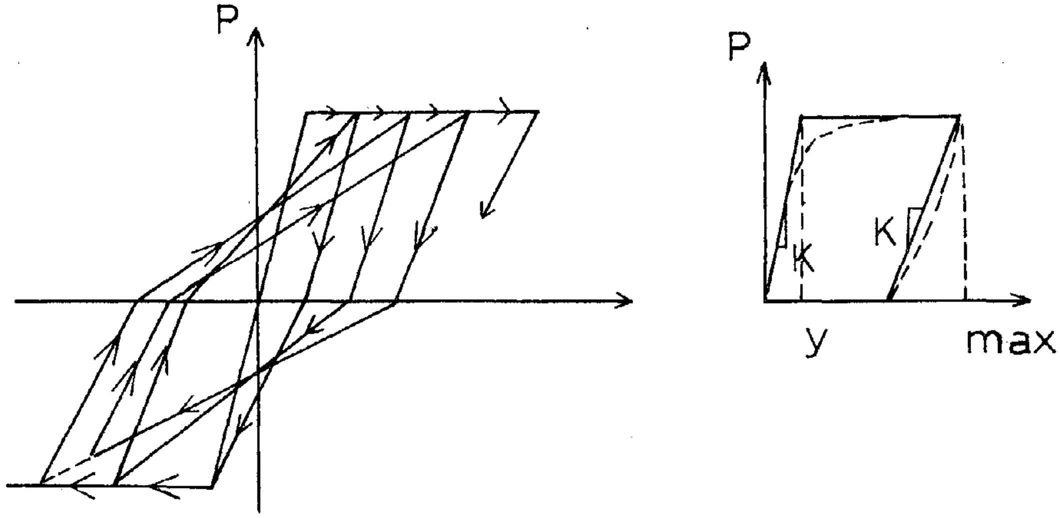

Figure 2. Stiffness Degrading Model.

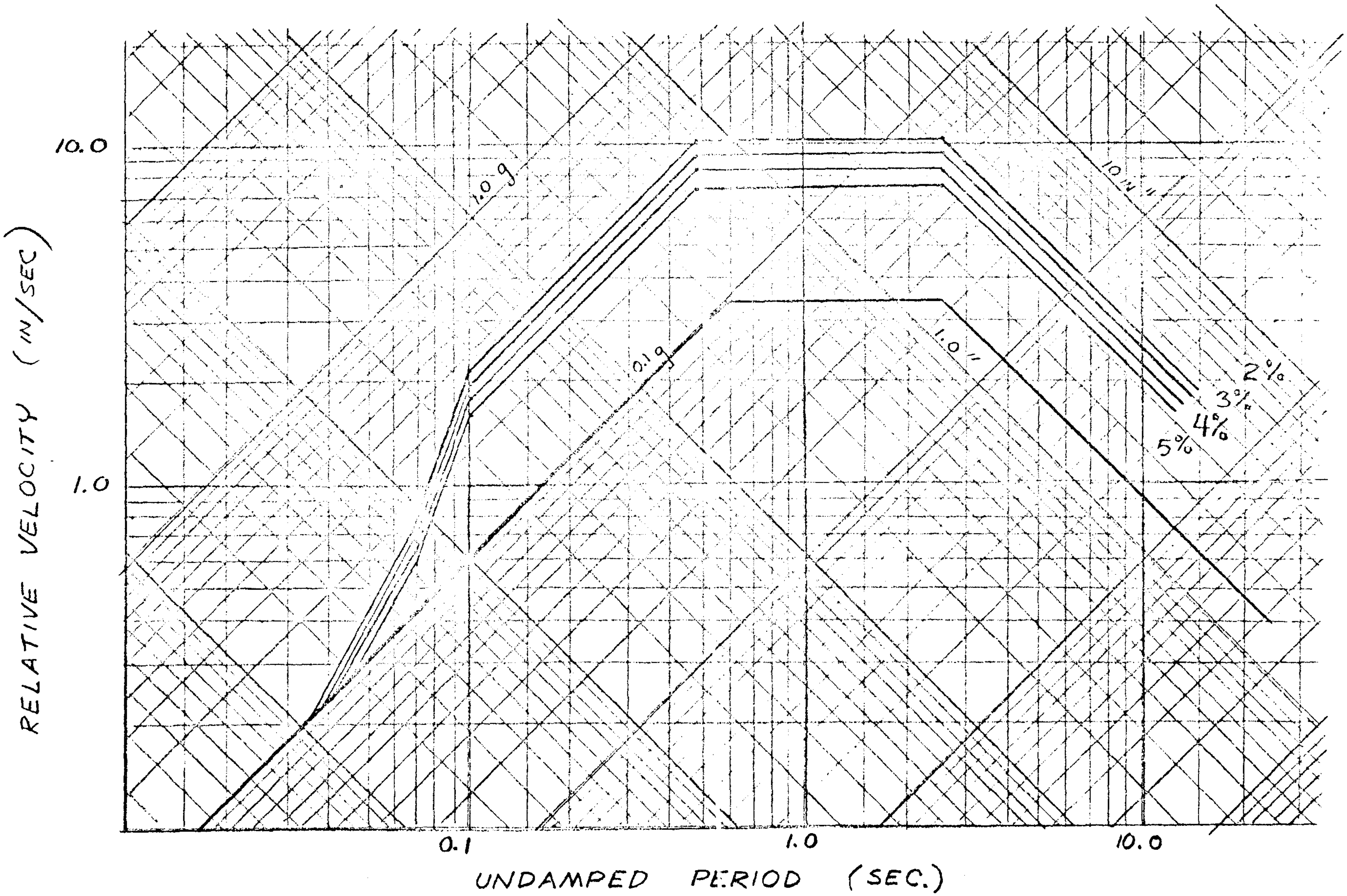

Figure 3. Ground Response Spectra of the Artificial Earthquake (0.1 g Peak Ground Acceleration).

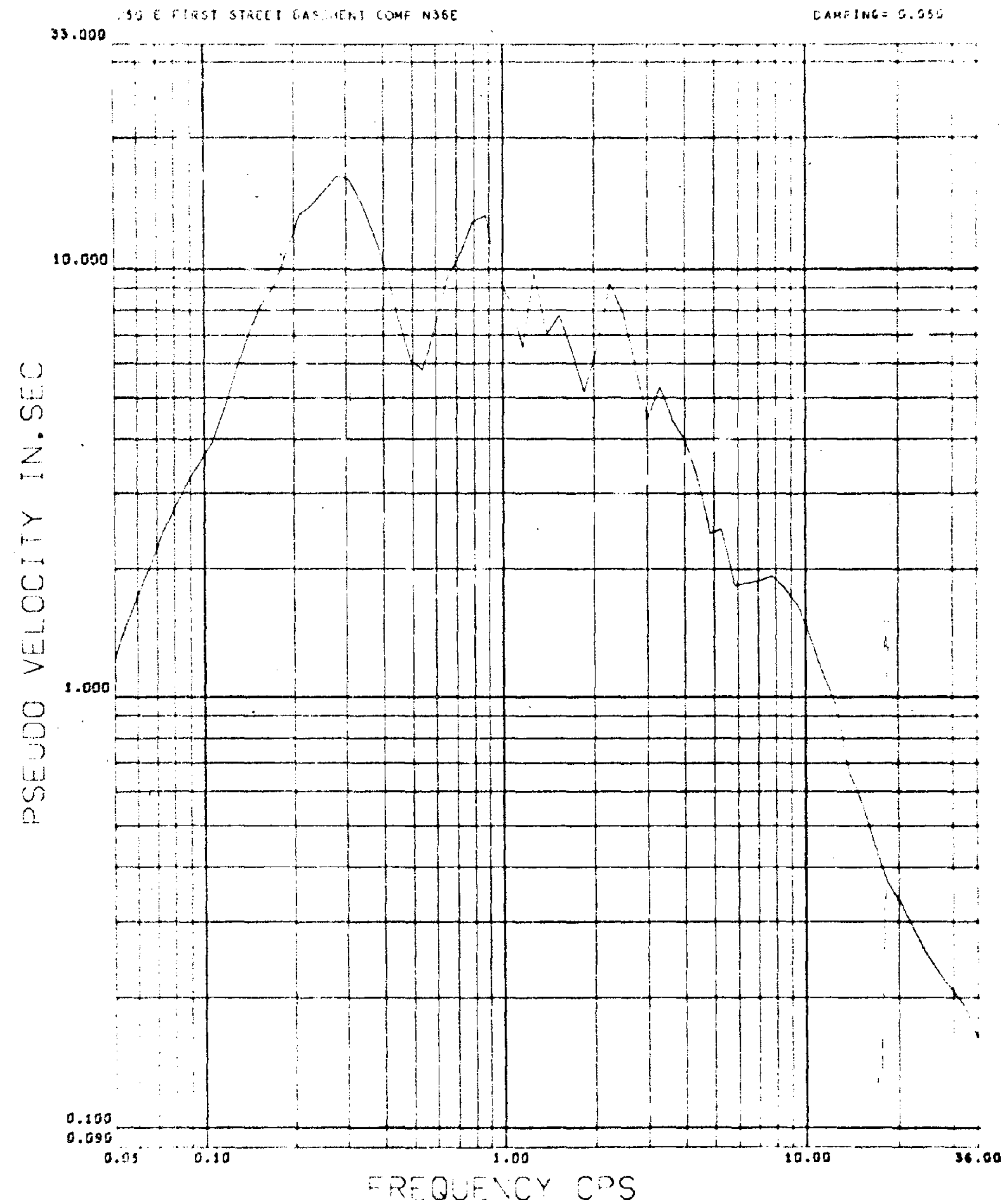

Figure 4. Ground Response Spectrum of the Real Earthquake (San Fernando Earthquake of 1971).

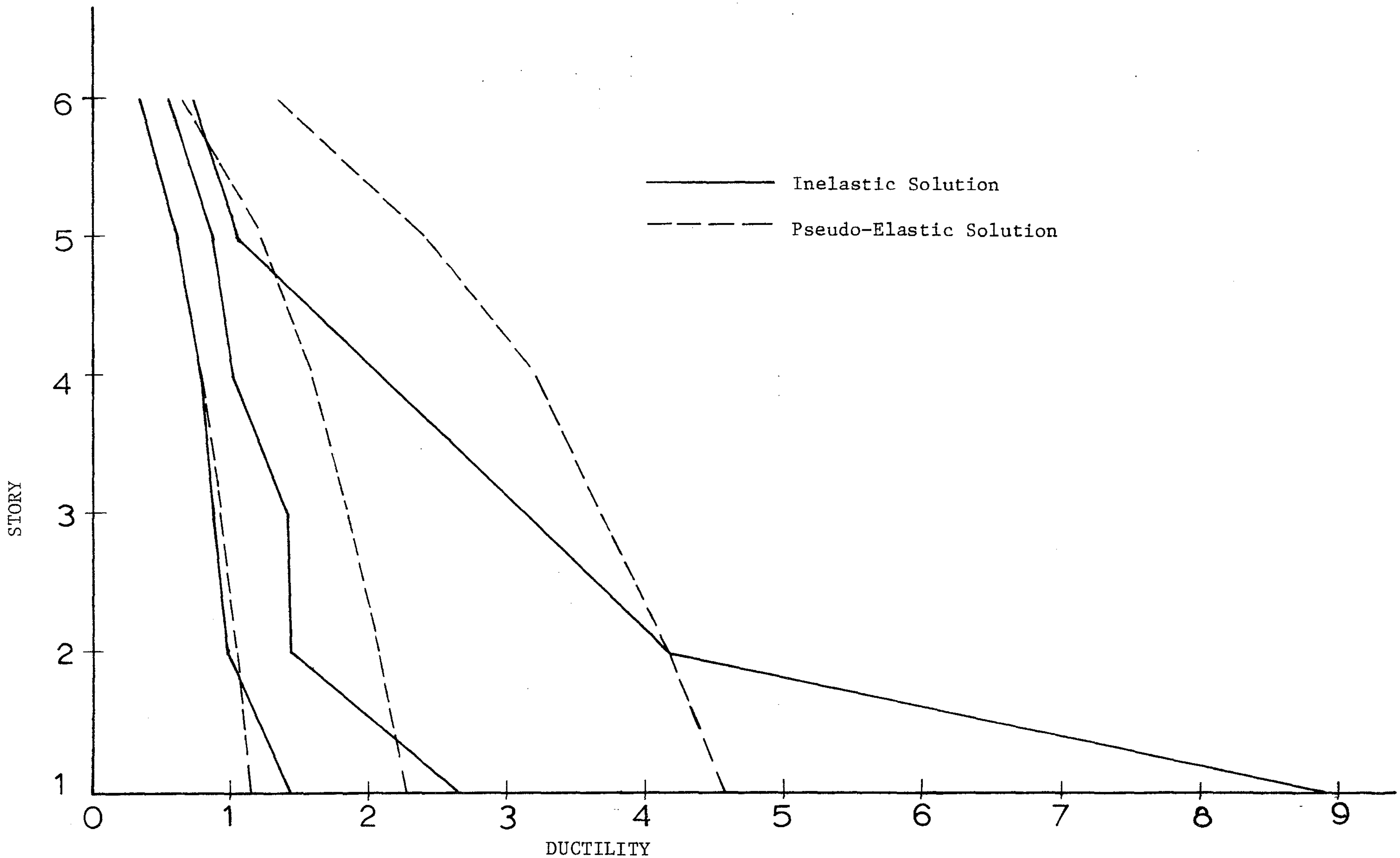

Figure 5. Comparison of Inelastic and Pseudo-Elastic Solution; System C with Uniform Yielding Displacements, Zero Post-Yielding Stiffness and 4% Damping.

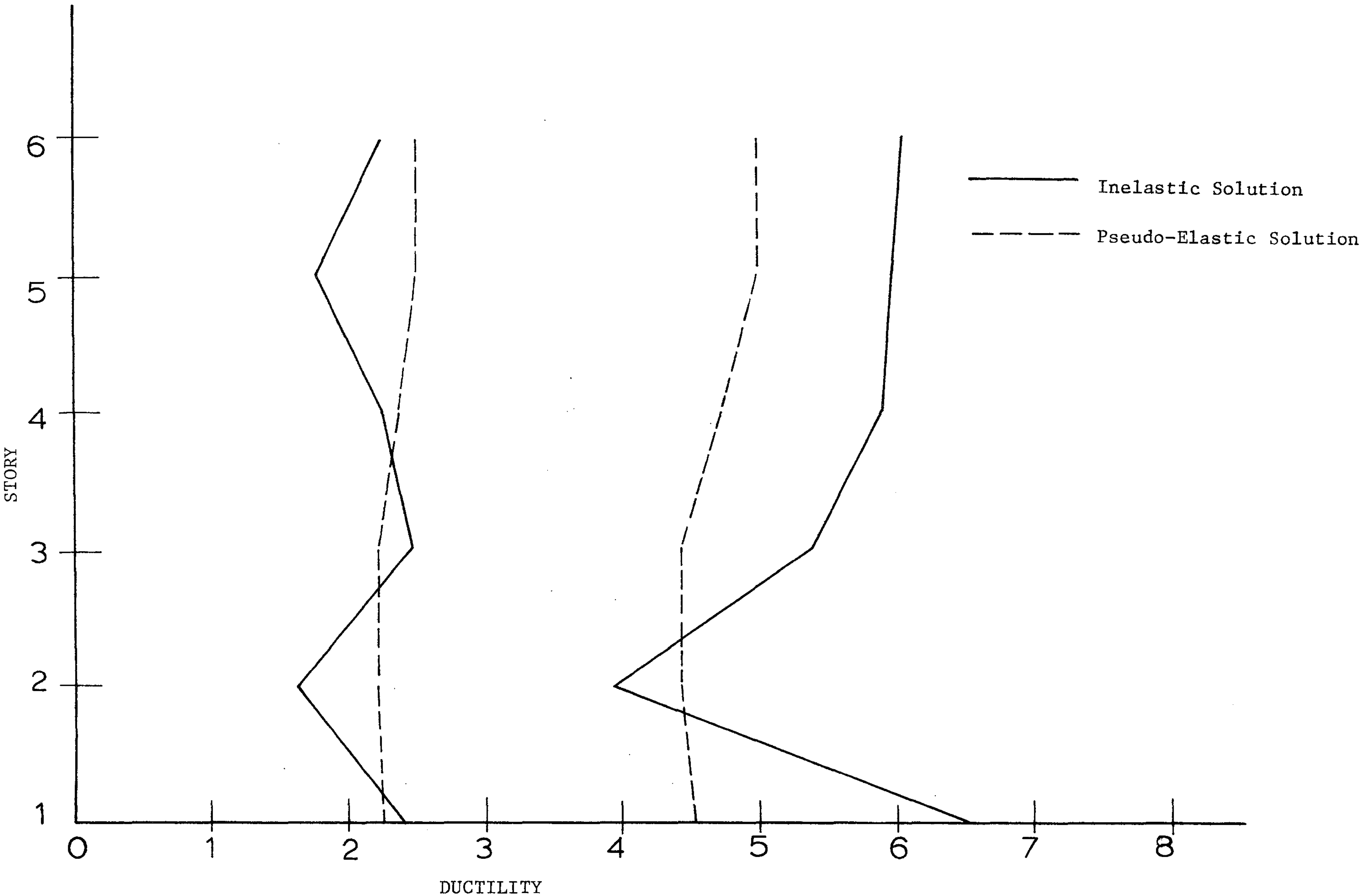

Figure 6. Comparison of Inelastic and Pseudo-Elastic Solution; System C with R.S. Yielding Displacements, Zero Post-Yielding Stiffness and 4% Damping.

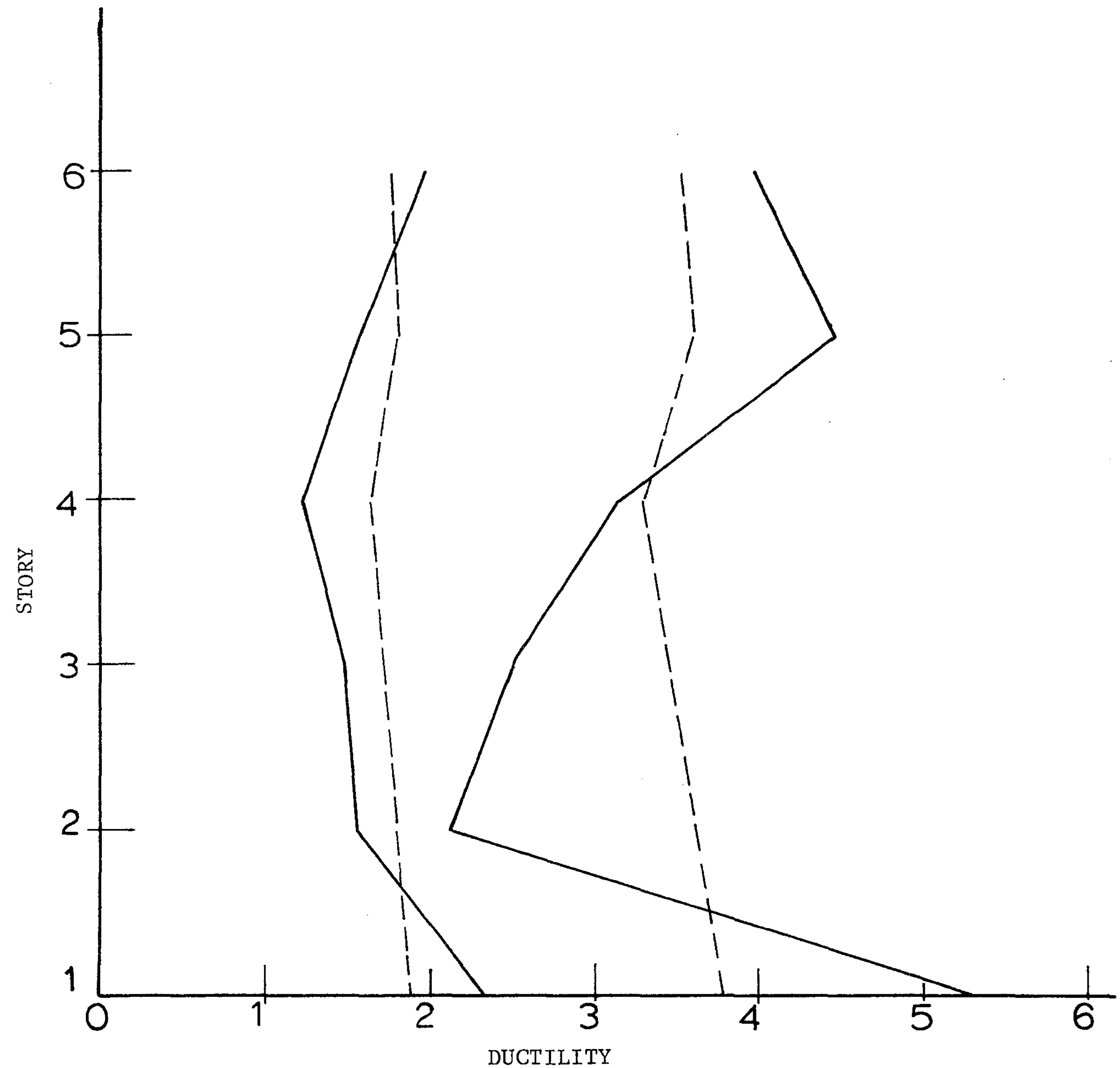

Figure 7. Comparison of Inelastic and Pseudo-Elastic Solution; System S with Uniform Yielding Displacements, Zero Post-Yielding Stiffness and 4% Damping.

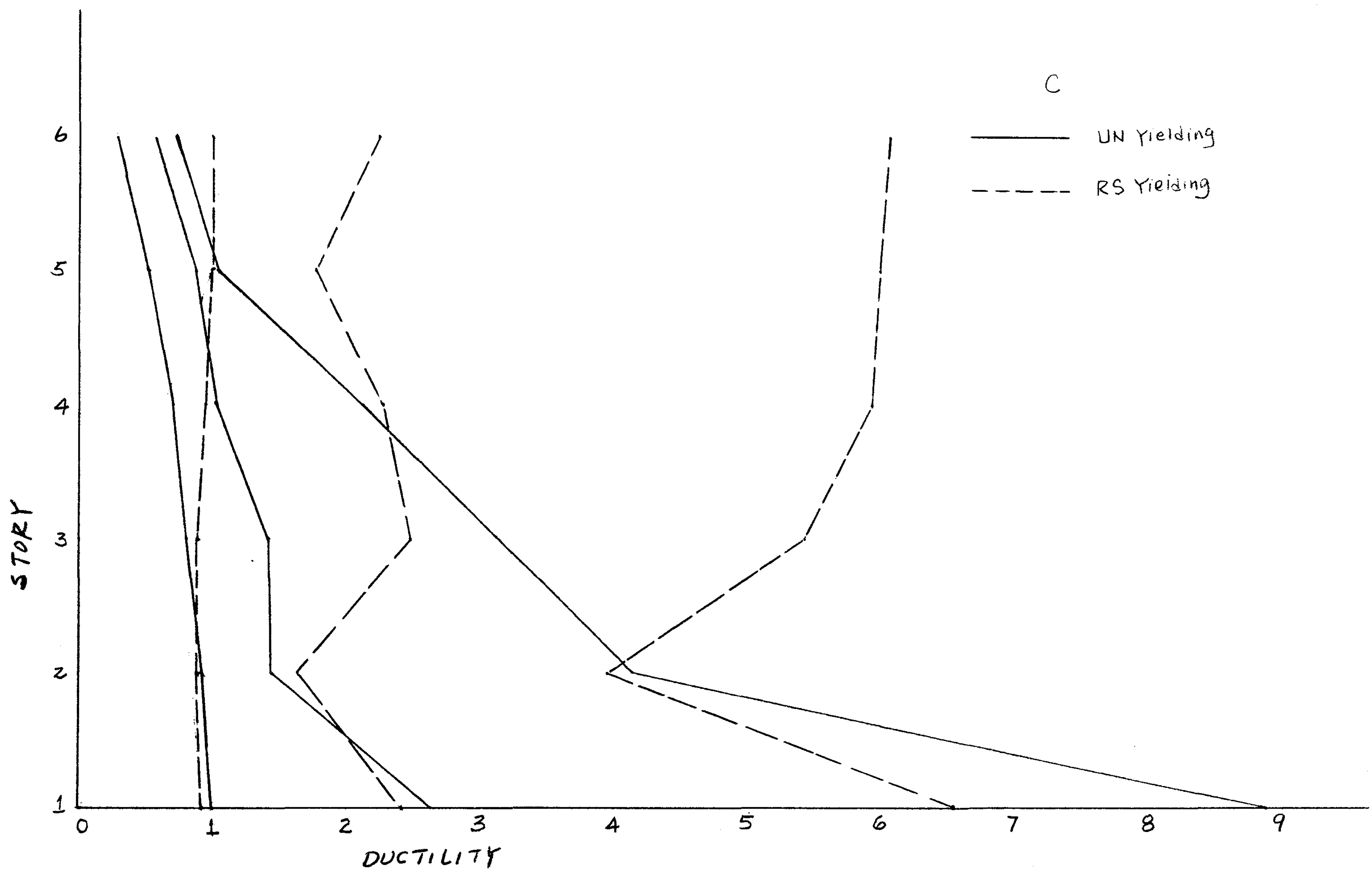

Fig. 8 Effect of Yielding Displacements on Ductility Requirement;
System C with zero post-yielding Stiffness and 4% Damping

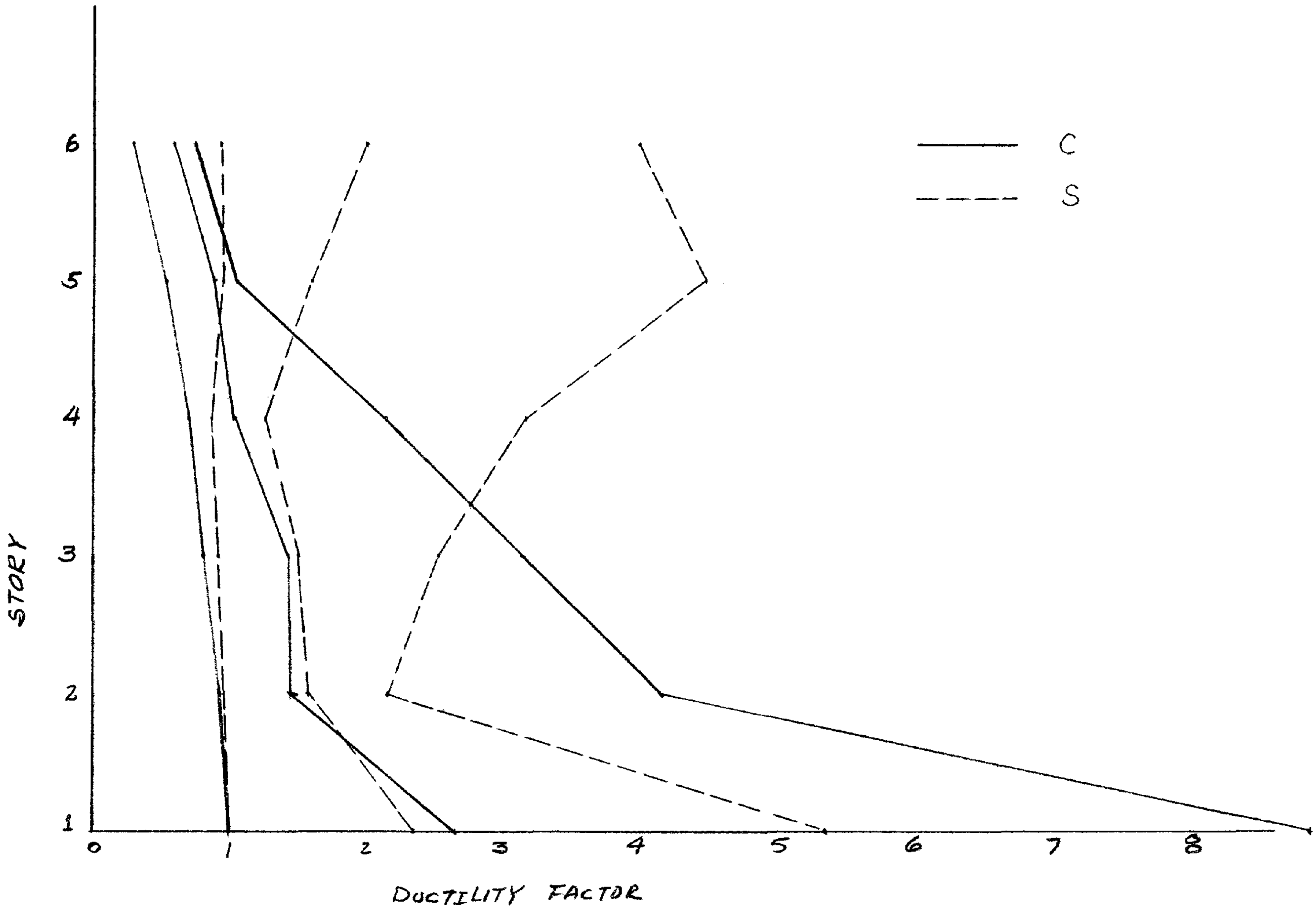

Fig 9 Comparison of Ductility Requirements for system C and S with uniform yielding displacement, Zero post-yielding stiffness and 4% Damping

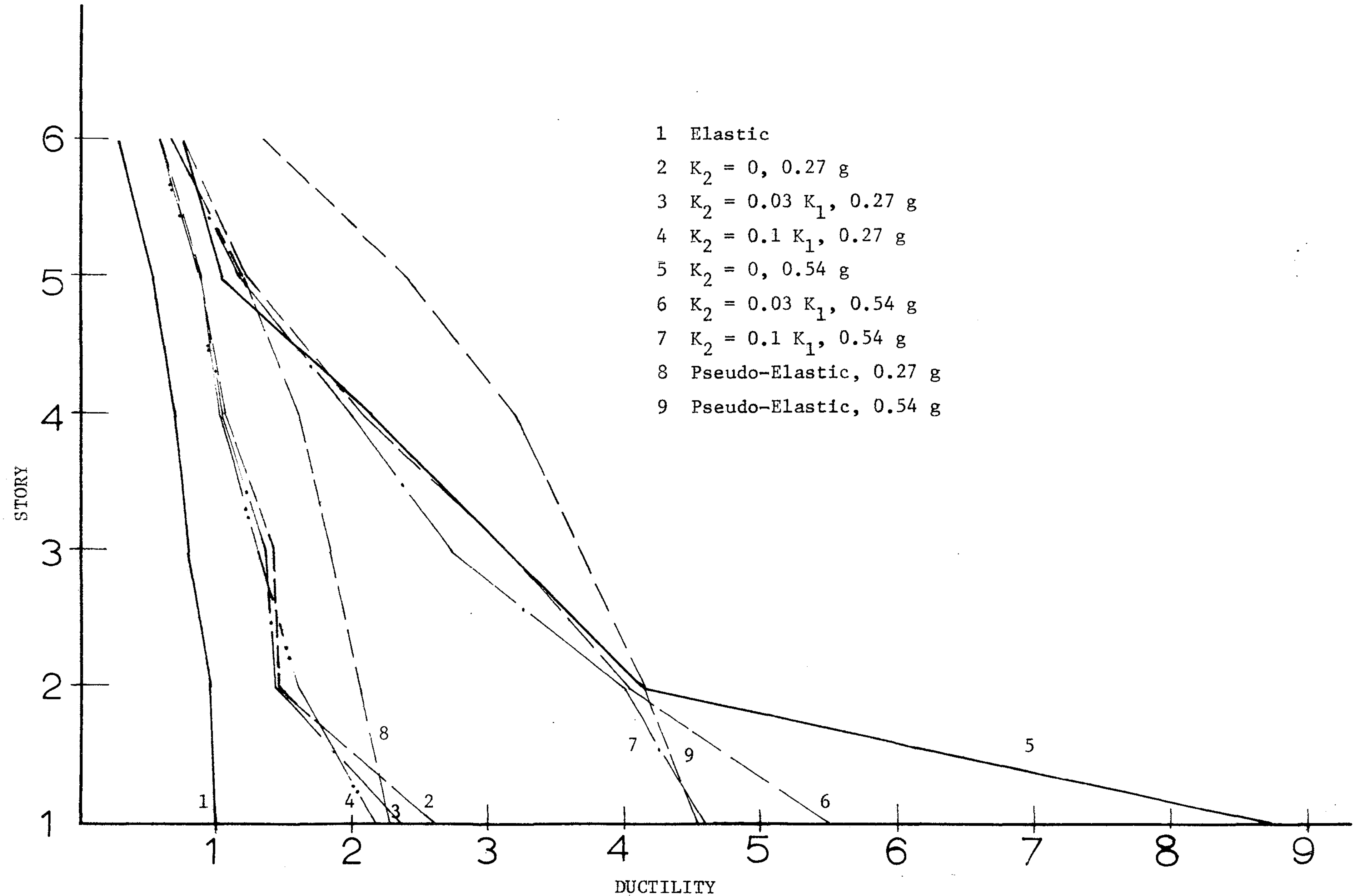

Figure 10. Effects of Post-Yield Stiffness of a Bilinear Spring on Ductility Requirement; System C with 4% Damping.

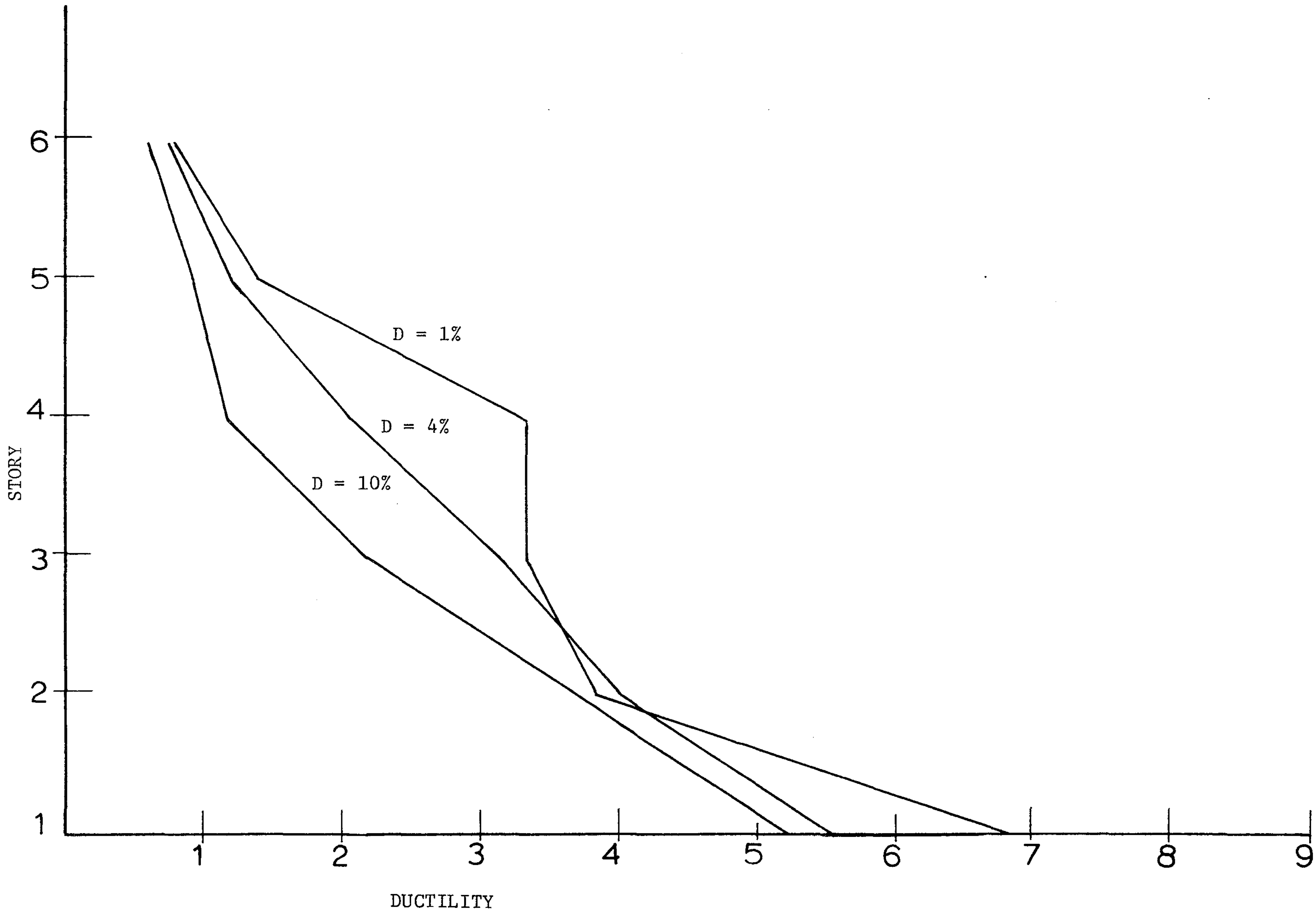

Figure 11. Effect of Damping on Ductility Requirement; System C with Uniform Yielding Displacements and 3% Post-Yielding Stiffness.

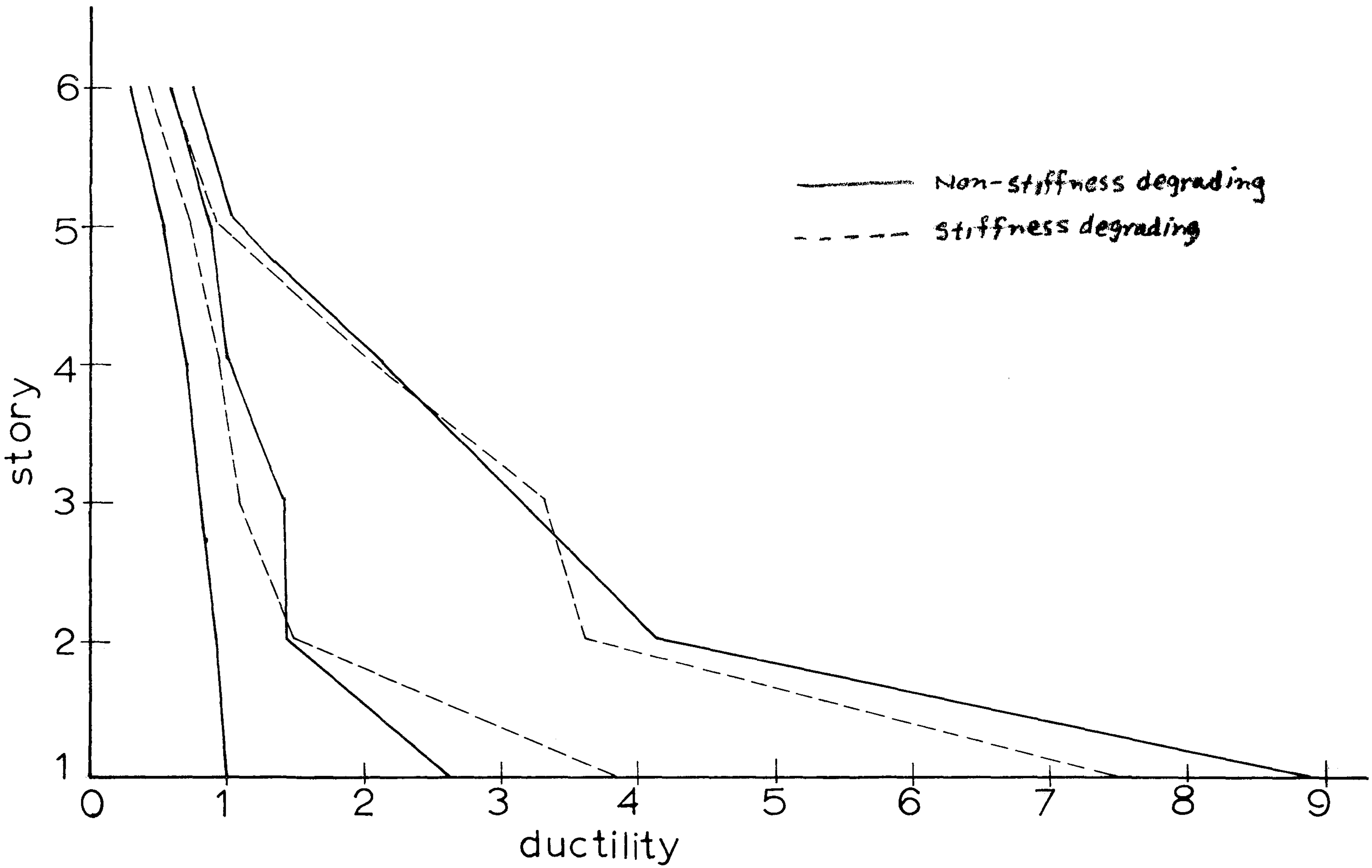

Figure 12. Effect of Stiffness Degrading on Ductility Requirements; System C with Uniform Yielding Displacements, Zero Post-Yielding Stiffness and 4% Damping.

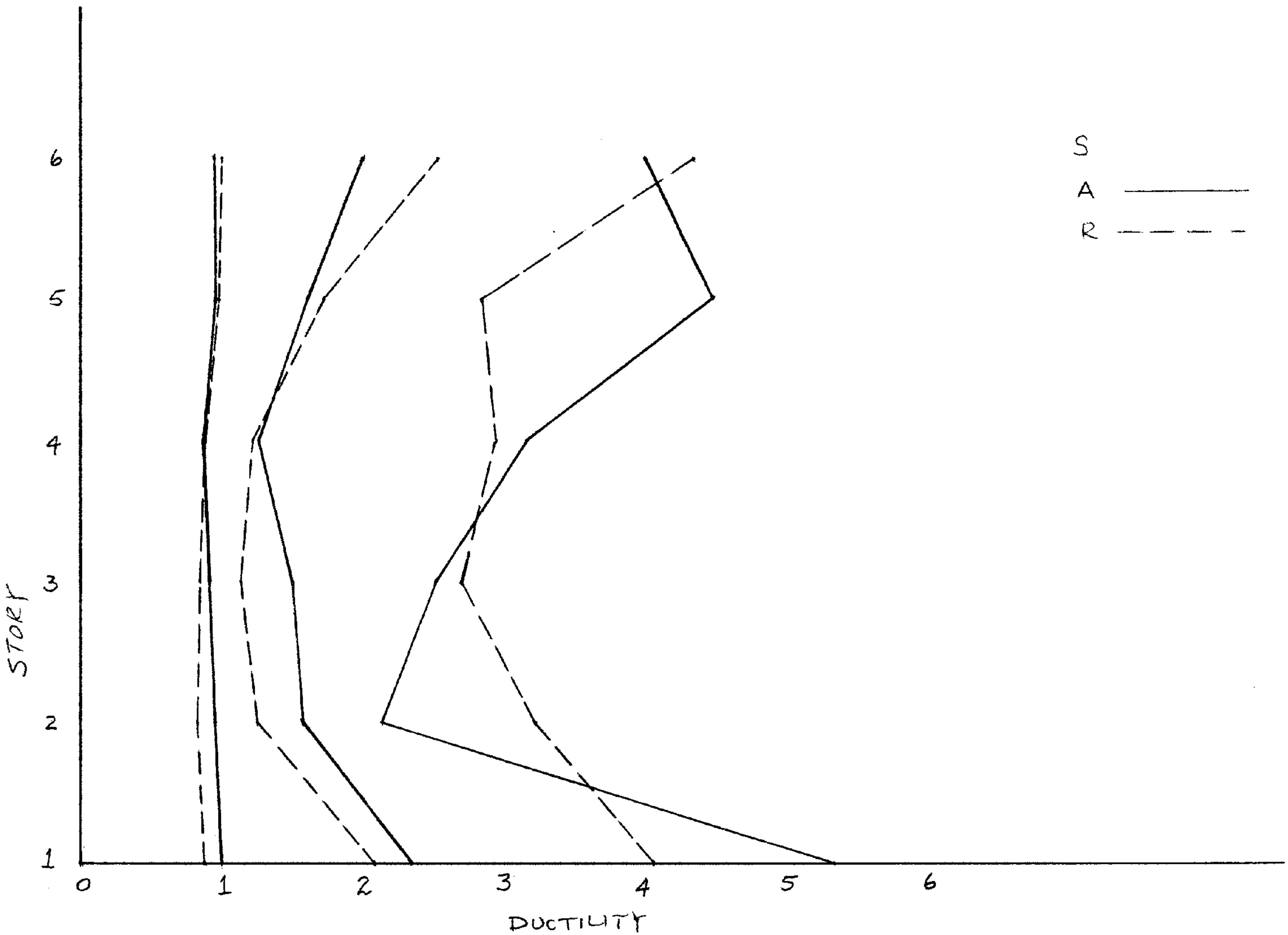

Fig. 13 Effect of Earthquake characteristics on Ductility Requirements
Systems with Uniform Yielding Displacements, Zero post-yielding stiffness and 4% Damping

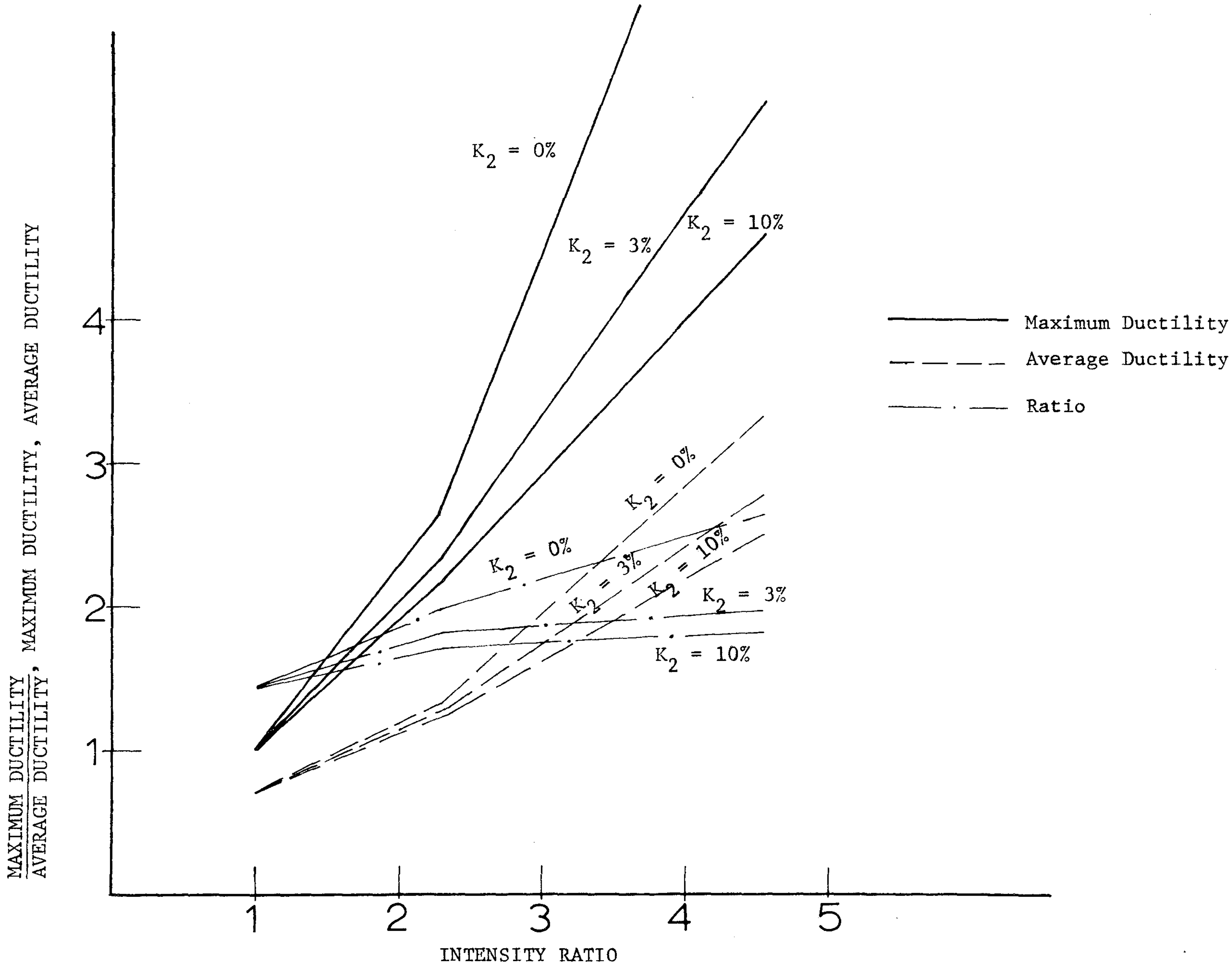

Figure 14. Effect of Intensity Ratio and Post-Yielding Stiffness on Maximum Ductilities and their Ratios; System C with Uniform Yielding Displacements.

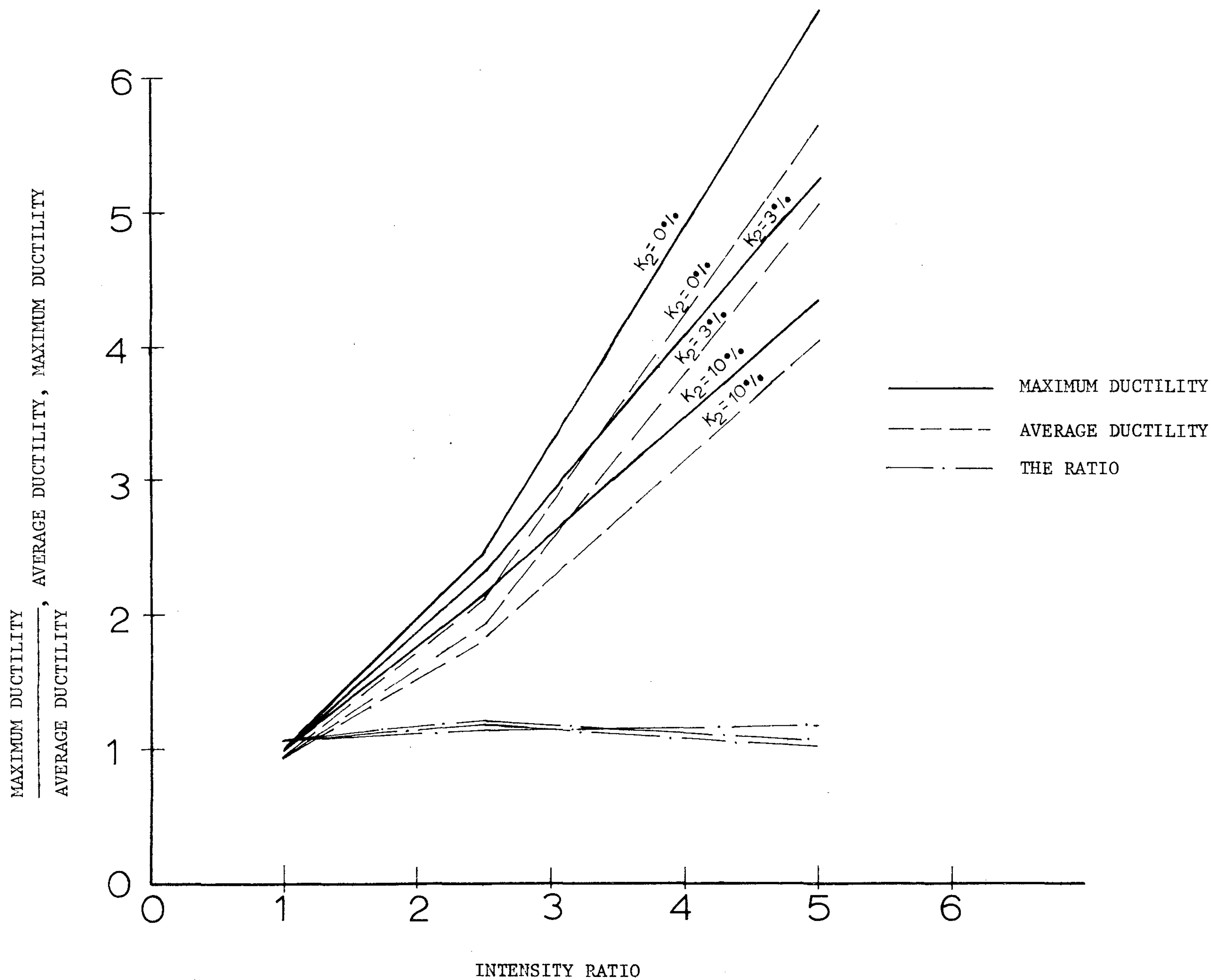

Figure 15. Effects of Intensity Ratio and Post-Yielding Stiffness on Maximum Ductilities, Average Ductilities and Their Ratios; System C with R.S. Yielding Displacements

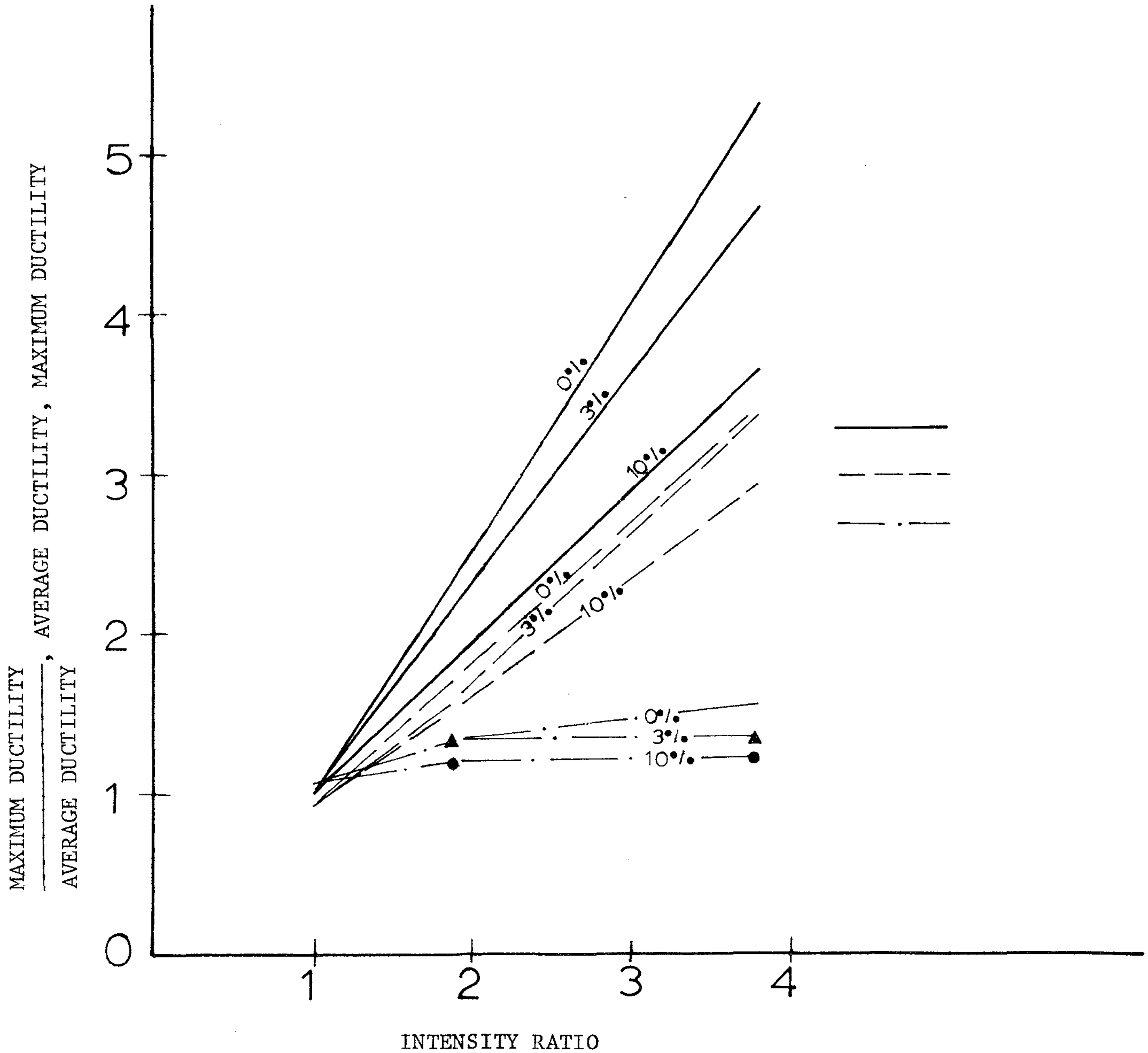

Figure 16. Effect of Post-Yielding Stiffness and Intensity Ratio on the Maximum Ductilities, Average Ductilities and Their Ratios; System S with Uniform Yielding Displacements and 4% Damping.

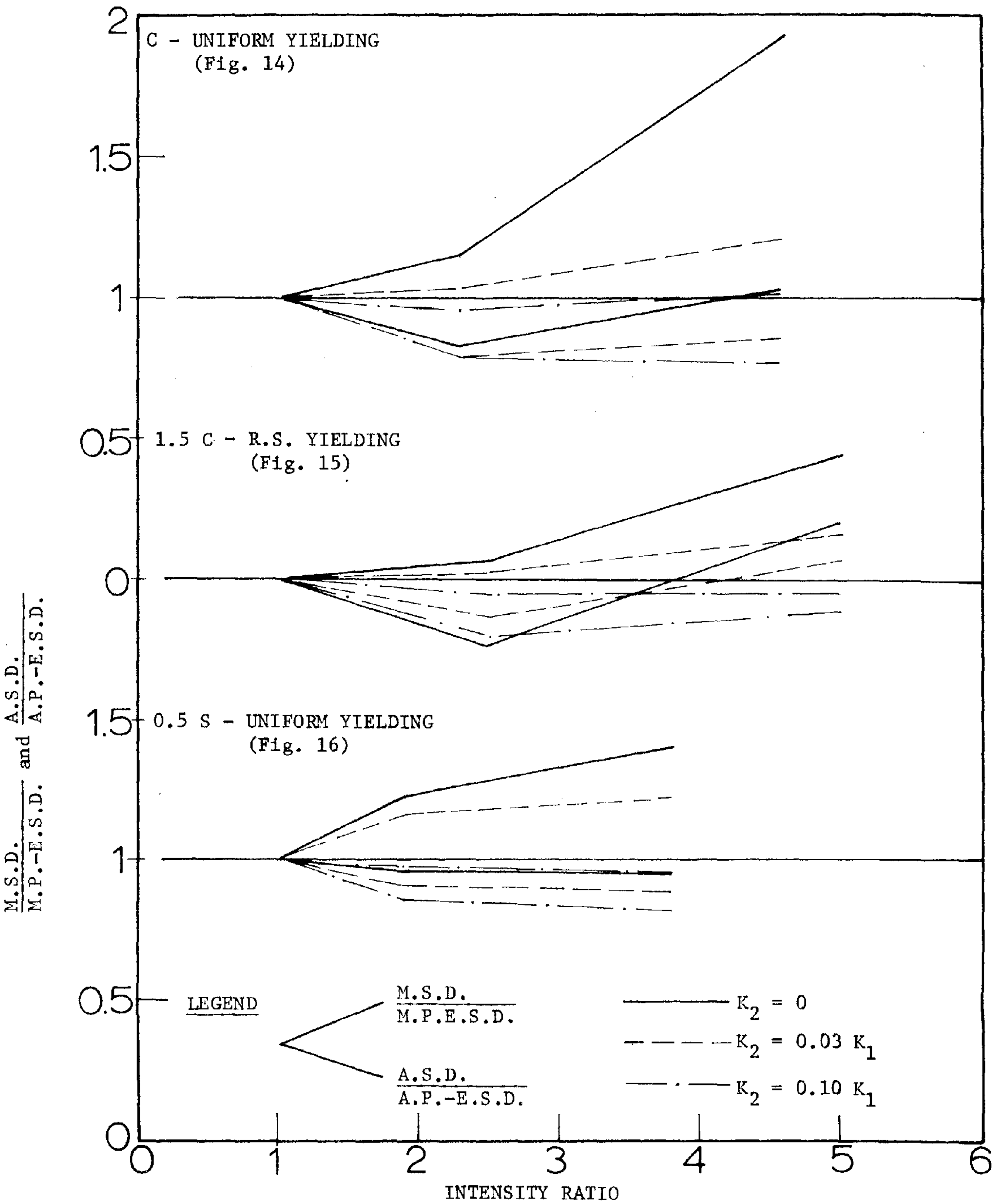

Figure 17. Effects of Intensity Ratio and Post-Yielding Stiffness on the Ratios of Maximum Story Drift (M.S.D.) to Maximum Pseudo-Elastic Story Drift (M.P.-E.S.D.) and the Ratio of Average Story Drift (A.S.D.) to Average Pseudo-Elastic Story Drift (A.P.-E.S.D.).