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STATIC ANALYSIS OF AN EMBEDDED PIPE SUBJECTED TO PERIODICALLY SPACED LONGITUDINAL FORCES

by

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16. Abstract (Limit: 200 words)

The dynamic response of buried pipelines to earthquakes is best expressed in terms of dynamic amplification factors, i.e. as the ratio of dynamic to static response. In the present report, the required static response of pipes of diameter D subjected to periodic longitudinal forces at intervals L, acting in alternate directions, is obtained. Such a load pattern corresponds to the incoherent motion occurring in pipes due to earthquakes. The static displacements and interacting stresses of a pipe-soil system are established, and are found to be dependent, for a given soil, on the ratio of stiffness of the soil and pipe as well as on the aspect ratio D/L. Numerical results are presented for a series of pipes governed by the above non-dimensional parameters.

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Pipelines
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Earthquake resistant structures
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Static structural analysis

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ABSTRACT

The dynamic response of buried pipelines to earthquakes is best expressed in terms of dynamic amplification factors, i.e. as the ratio of dynamic to static response. In the present report, the required static response of pipes of diameter D subjected to periodic longitudinal forces at intervals L, acting in alternate directions, is obtained. Such a load pattern corresponds to the incoherent motion occurring in pipes due to earthquakes.

The static displacements and interacting stresses of a pipe-soil system are established, and are found to be dependent, for a given soil, on the ratio of stiffness of the soil and pipe as well as on the aspect ratio D/L. Numerical results are presented for a series of pipes governed by the above non-dimensional parameters.

STATIC ANALYSIS OF AN EMBEDDED PIPE
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I. INTRODUCTION

A problem which has received considerable attention has been the effect of earthquakes on buried pipes in the earth [1-3]. In particular, attempts have been made to establish the degree of interaction within the pipe-soil system and to determine the damping characteristics of such a system.

In order to demonstrate effectively the dynamic effects of earthquakes, the response is best expressed in terms of dynamic amplification factors, i.e. as the ratio of dynamic to static response. Thus it is necessary to obtain the static response to equivalent forces acting upon the pipe. In the present report the static solution to an interacting pipe-soil system is established.

In the model considered below, it is assumed that longitudinal forces act at intervals L. Since the relevant effect is the incoherent motion occurring during an earthquake [4], these forces are taken in alternating directions.

The pipe is represented by a linear elastic bar and the soil by an elastic isotropic material. It is observed that, for a given soil, the static response obtained is expressed in terms of the ratio of the moduli of the two elastic materials as well as in terms of an aspect ratio defined as the ratio of radius to length L. Values of the relevant parameters may be taken to correspond to either continuous or segmented pipes.

where $\tau_{rz}(a,z)$ represents the interacting shear stress at the interface.

With the assumptions stated above, together with the requirements on continuity of displacements at the pipe-medium interface, the boundary conditions on the medium displacements become

$$U_r(a,z) = 0 \quad , \; U_z(a,z) = U_p(z) \tag{2a,b}$$

The behavior of the surrounding medium can be readily formulated in terms of a Love strain function $\psi(r,z)$ [6]. Expressing the radial and axial displacements of the medium for the axi-symmetric case respectively by

$$U_r = -\frac{1}{2\mu}\frac{\partial^2\psi}{\partial r\partial z} \tag{3a}$$

$$U_z = \frac{1-\nu}{\mu}\nabla^2\psi - \frac{1}{2\mu}\frac{\partial^2\psi}{\partial z^2}, \tag{3b}$$

the equations of equilibrium in the medium are then satisfied if

$$\nabla^4\psi(r,z) = 0 \tag{4}$$

Appropriate solutions of the bi-harmonic equation which decay as $r \to \infty$ are [7]

$$\psi(r,z) = \sum_{m=1}^{\infty} X_m(r)\ \text{Cos}\ \alpha_m z \tag{5}$$

where

$$X_m(r) = A_m K_0(\alpha_m r) + B_m r K_1(\alpha_m r). \tag{6}$$

In the above, $K_n(\alpha_m r)$ are modified Bessel functions of the second kind , and α_m is a parameter to be evaluated subsequently.

Substitution in eqs. (3) yields

$$U_r(r,z) = \sum_{m=1}^{\infty} U_{rm}(r,z) = -\frac{1}{2\mu}\sum_{m=1}^{\infty}\alpha_m^2[A_m K_1(\alpha_m r)+B_m r K_0(\alpha_m r)]\text{Sin}\ \alpha_m z \tag{7}$$

$$\delta_p(z) = \frac{4}{L}\sum_{m=1}^{\infty} \mathrm{Cos}\left[\frac{(2m-1)z}{L}\right] \tag{13}$$

Upon setting

$$\alpha_m = (2m-1)\pi/L , \tag{14}$$

eq. (12) may then be satisfied term by term.

It is noted here in passing that the interval $0 \leq z \leq L$ represents a half Fourier interval and hence the analysis of the infinite pipe problem is given by the solution in a periodic interval $0 \leq z \leq L$, with $\lambda = 2L$ being the total Fourier interval [See Fig. (2)].

Using the representation of eq. (13) in eq. (12), the constants B_m are readily determined; viz

$$B_m = -\frac{4F_0K_1(\alpha_m a)}{A\alpha_m^2 Q_m}\cdot\frac{a}{L} \tag{15}$$

where

$$Q_m = \frac{\bar{E}}{\mu}\alpha_m a\left\{4(1-\nu)K_0(\alpha_m a)K_1(\alpha_m a)+\alpha_m a\left[K_0^2(\alpha_m a)-K_1^2(\alpha_m a)\right]\right\}+8(1-\nu)K_1^2(\alpha_m a) \tag{16}$$

Upon evaluating the constants A_m and B_m, the axial displacements of the pipe, $U_p(z) = U_z(a,z)$ are known according to eq. (8). However, at this point, it is advantageous to express the solution in terms of non-dimensional quantities. To this end, let

$$\eta = a/\lambda \quad , \text{ where } \lambda = 2L \tag{17a}$$

$$R = \mu/\bar{E} \tag{17b}$$

$$v = \alpha_m a = 2\pi(2m-1)\eta = 2\pi\zeta\eta \quad \text{where } \zeta = 2m-1 \tag{17c}$$

$$\xi = z/L \tag{17d}$$

At the interface, $\rho=1$, the interacting stress becomes

$$\frac{\tau_{rz}(1,\xi)}{(F_0/2A)} = -32(1-\nu)\eta \sum_{m=1}^{\infty} \frac{K_1^2(v)\mathrm{Cos}\pi\zeta\xi}{Q_m} \tag{23}$$

A limiting case of particular interest is that for which the soil becomes infinitely weak; i.e. $\mu \to 0$ or $R \to 0$. This case then represents a simple bar free to displace with no restraint from the surrounding soil. For this case, eq. (18) becomes

$$\left(\frac{U_p(\xi)}{L}\right)\left(\frac{\bar{E}A}{F_0/2}\right) = \frac{4}{\pi^2}\sum_{m=1}^{\infty} \frac{\mathrm{Cos}\ \pi\zeta\xi}{(2m-1)^2} \tag{24a}$$

while eq. (23) yields

$$\tau_{rz}(a,\xi) = 0 \tag{24b}$$

At the point of load application, $\xi = 0$,

$$\left[\frac{U_p(\xi=0)}{L}\right]\left[\frac{\bar{E}A}{F_0/2}\right] = \frac{4}{\pi^2}\sum_{m=1}^{\infty} \frac{1}{(2m-1)^2} = 1/2 \tag{25a}$$

since the series [8, No. 339]

$$\sum_{m=1}^{\infty} (2m-1)^{-2} = \pi^2/8. \tag{25b}$$

Note that here

$$U_p^f(0) \equiv U_p(0)\Big|_{R=0} = \frac{F_0 L}{4A\bar{E}} \tag{26}$$

values of η, U^* approaches unity; i.e. the behavior approaches that of a free pipe. For larger values of R, stronger interaction occurs thus causing greater attenuation of the displacement.

The relative effect of R and λ is demonstrated in Fig. (4) where the displacement U^* is plotted as a function of R for a family of values of η. Here again, it is noted that the attenuation of the displacement is greater for small values of η. In a given pipe-soil system with R = 0.5, the attenuation with respect to a free pipe is seen to be over 60% for values $\eta \leq 0.1$.

The variation of $U^*(\xi)$ along the longitudinal axis $\xi = z/L$(*), obtained from eq. (18), is shown in Fig. (5) for the case $\eta = 0.1$ and a family of values of R, while in Fig. (6), similar curves are presented with R held constant, R = 1. In both figures, it is observed that the displacement U^* at points away from the applied force varies almost linearly and increases more rapidly as ξ approaches zero.

The interacting stress $\tau_{rz}(1,\xi)$ acting at the pipe-soil interface is evaluated from eq. (23). In Fig. (7), the variation of τ_{rz} along the longitudinal axis is shown for a value of $\eta = 0.10$ with stiffness ratios R = 0.05, 0.1 and 1.0. The shear stress is seen to be significant only for small values of $\xi = z/L$ (i.e. in the neighborhood of the application of F_0) and decays rapidly to zero at distances away from the applied force.

In Fig. (8), the variation of τ_{rz} along the axis is shown for R = 1.0 and for typical values of η, $\eta = 0.05$ and 0.1. It is observed that the interacting shear stress increases with η. However, this is not in contradiction

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(*) All variations along the longitudinal axis are given in the range $0 \leq \xi = z/L \leq 0.5$. Values outside this range are merely Fourier extensions of the range.

For a realistic set of parameters corresponding to actual concrete or cast iron pipes encountered in practice [9-10], the stiffness ratio R in the case of continuous pipes is found to lie in the range $0.01 < R < 0.1$, while for segmented pipes (with $L = 20$ ft.) the range of R is found to be $0.1 < R < 1.0$.

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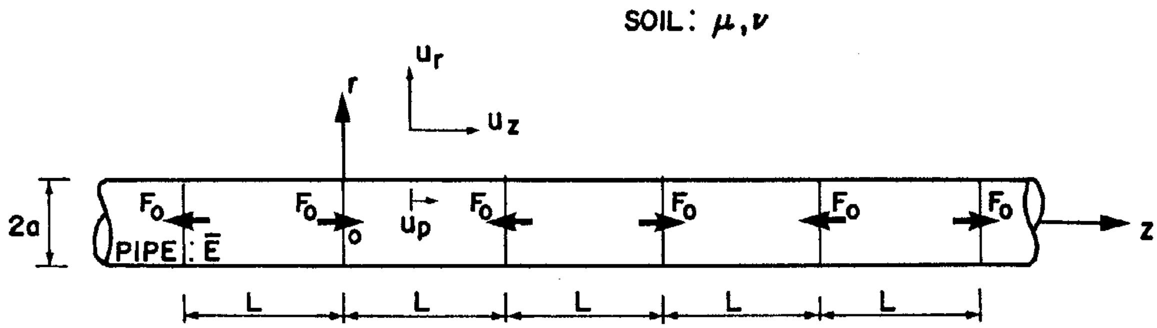

FIG. 1 GEOMETRY OF PROBLEM

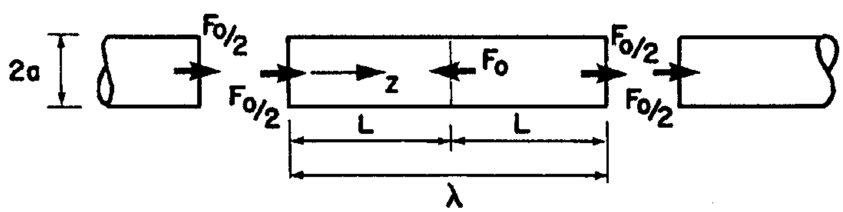

FIG. 2 PERIODIC INTERVAL

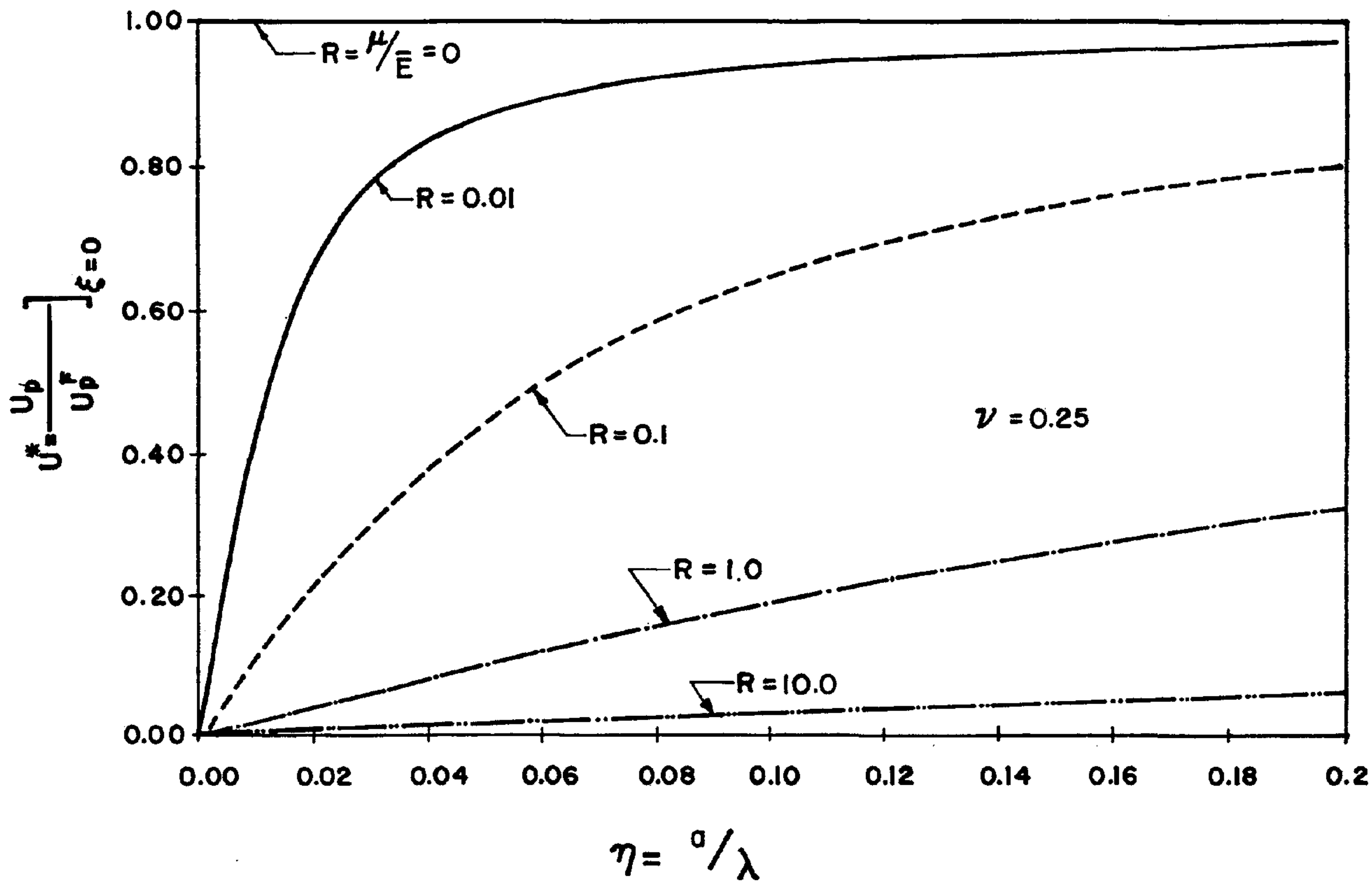

FIG. 3. NORMALIZED DISPLACEMENT VERSUS ASPECT RATIO.

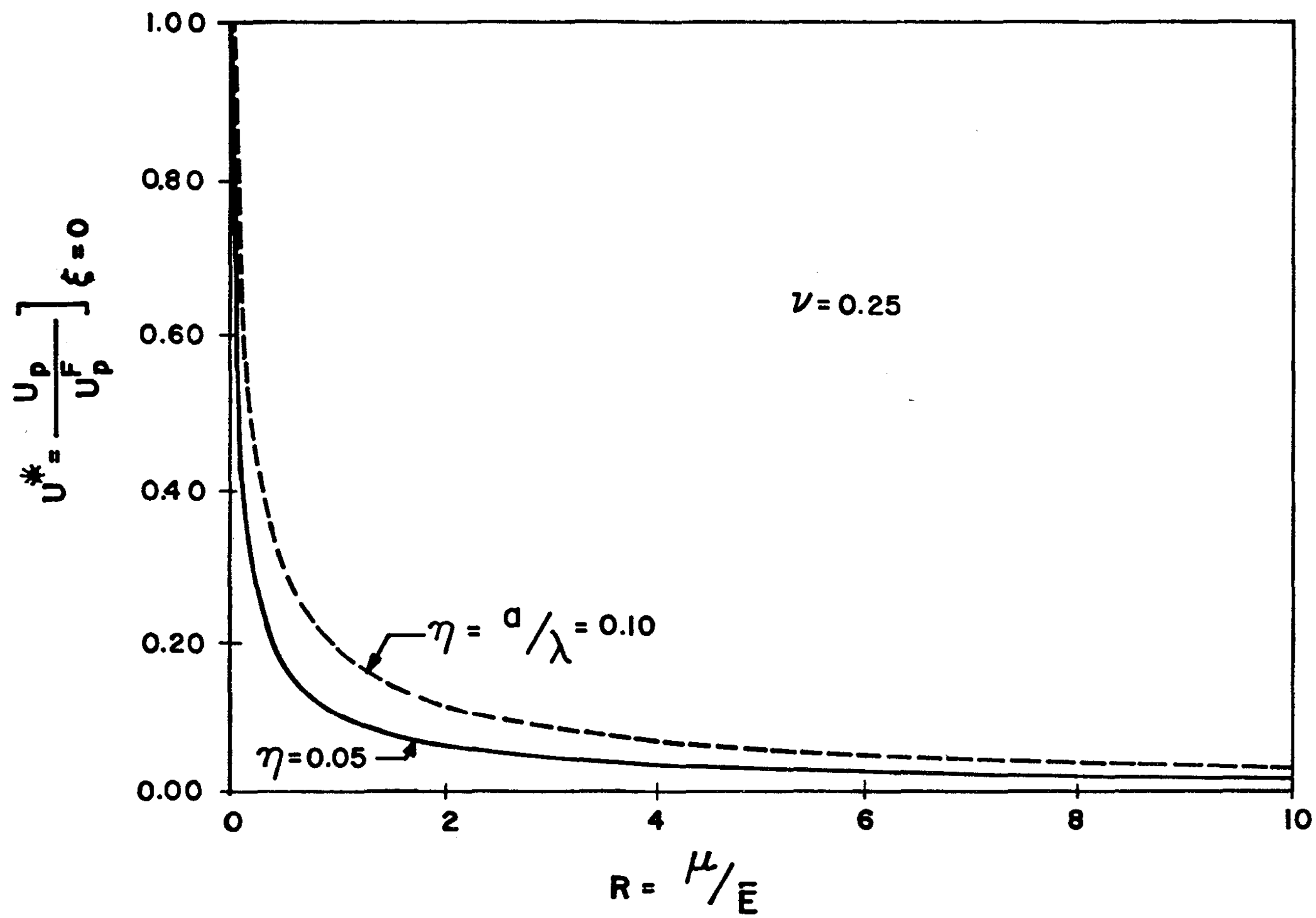

FIG. 4. NORMALIZED DISPLACEMENT VERSUS RELATIVE STIFFNESS RATIO.

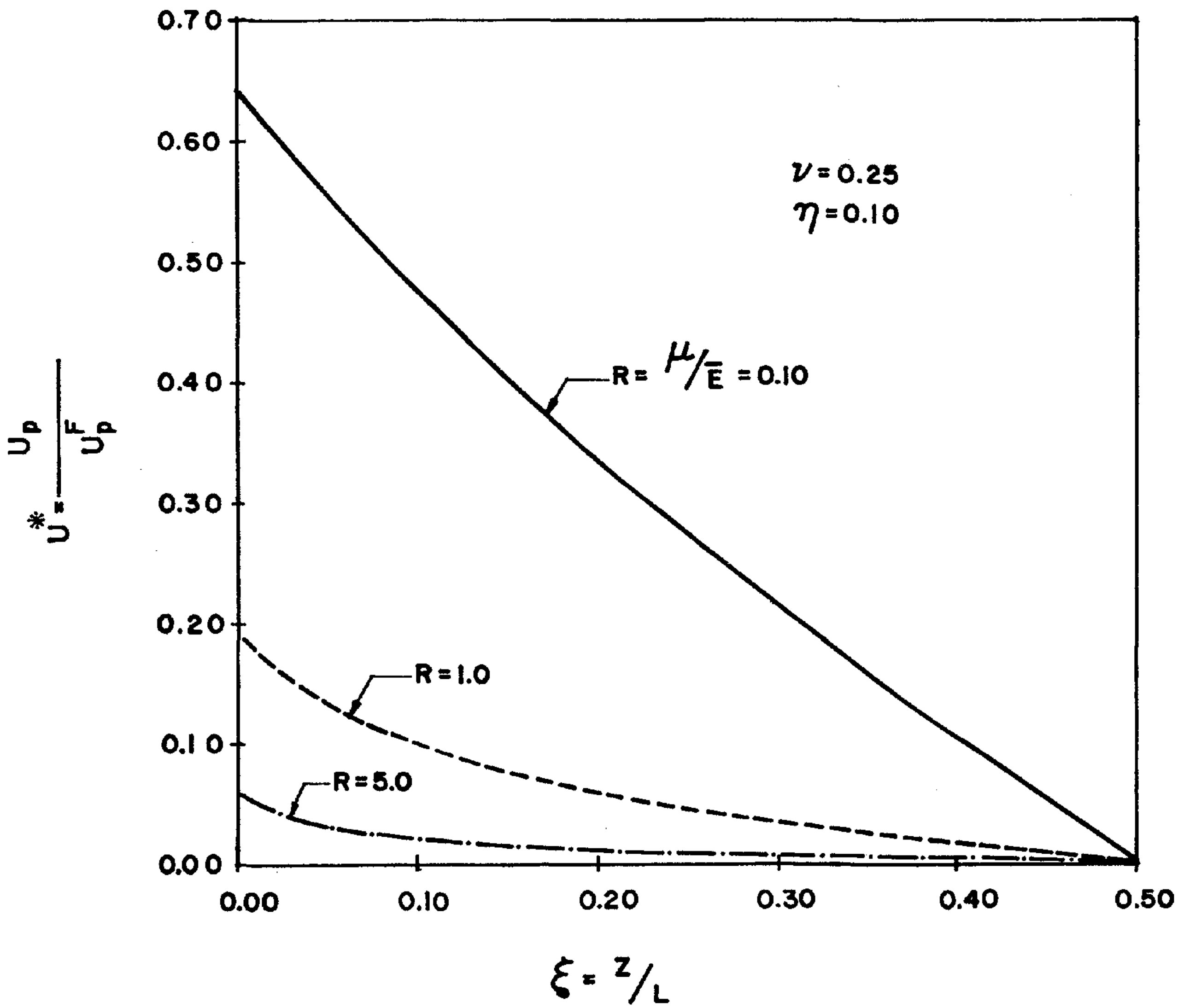

FIG. 5 NORMALIZED DISPLACEMENT VERSUS ξ; $\eta = 0.10$

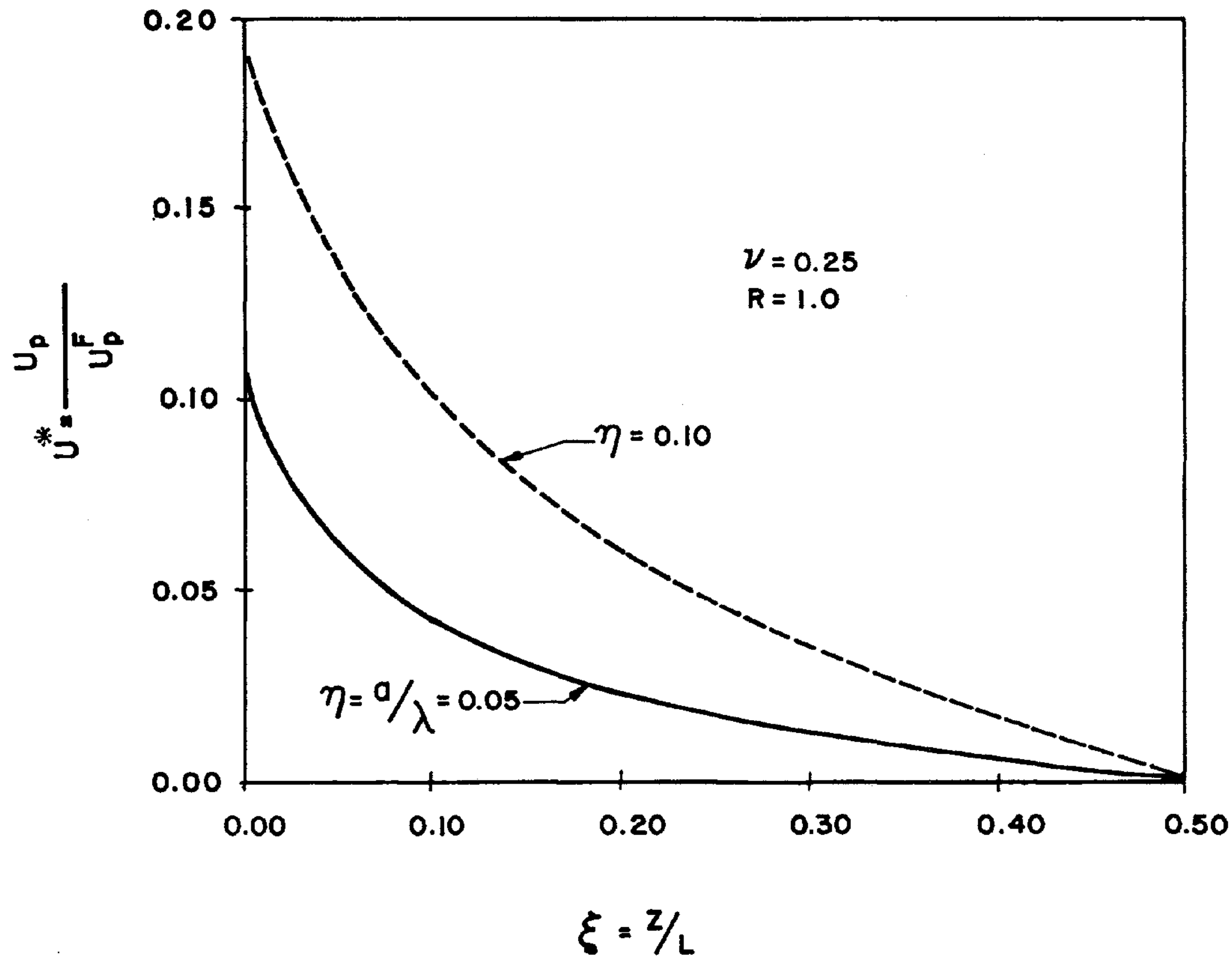

FIG. 6 NORMALIZED DISPLACEMENT VERSUS ξ, R = 1.0

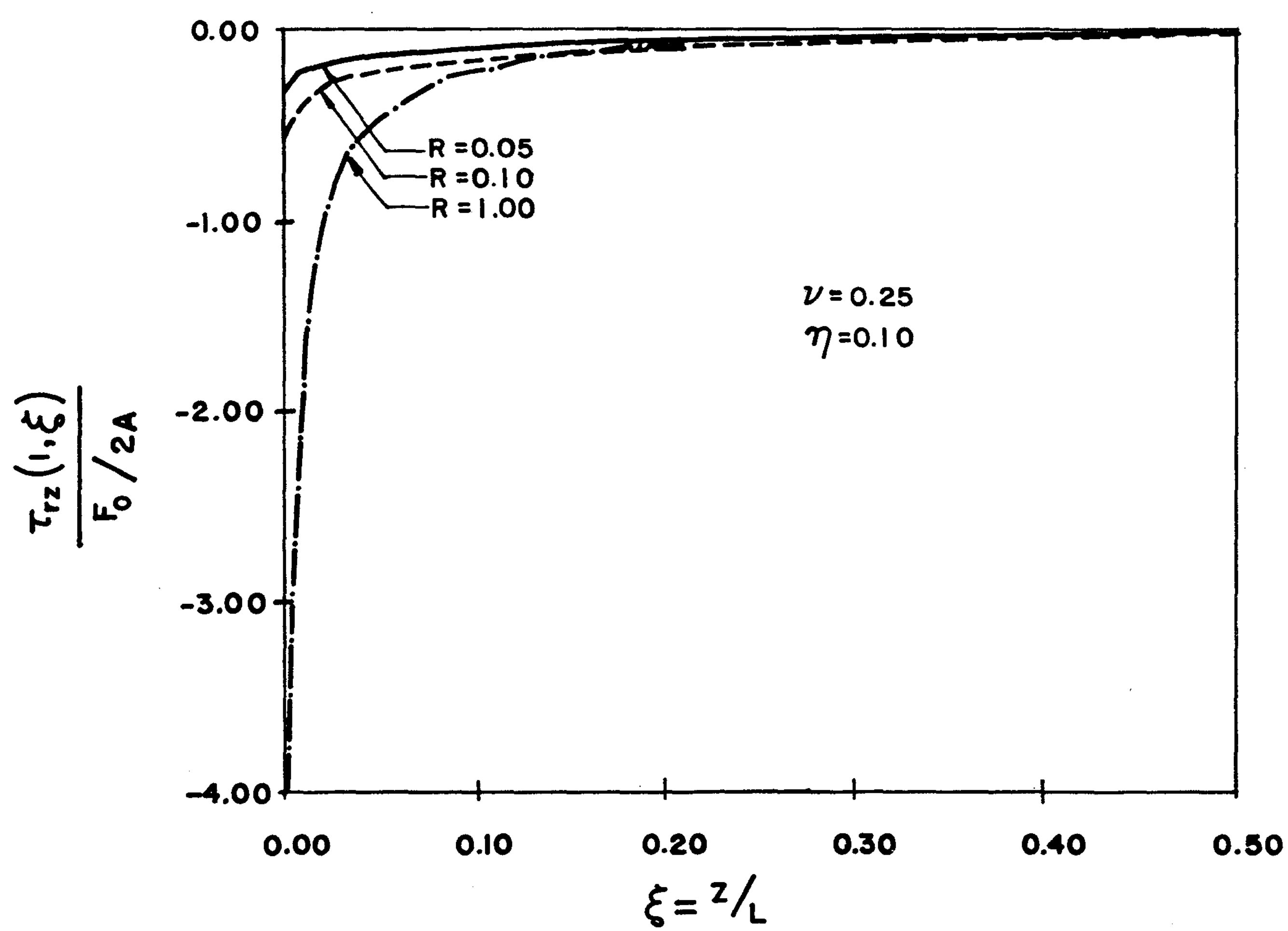

FIG. 7 INTERACTING SHEAR STRESS VERSUS ξ; η = 0.10.

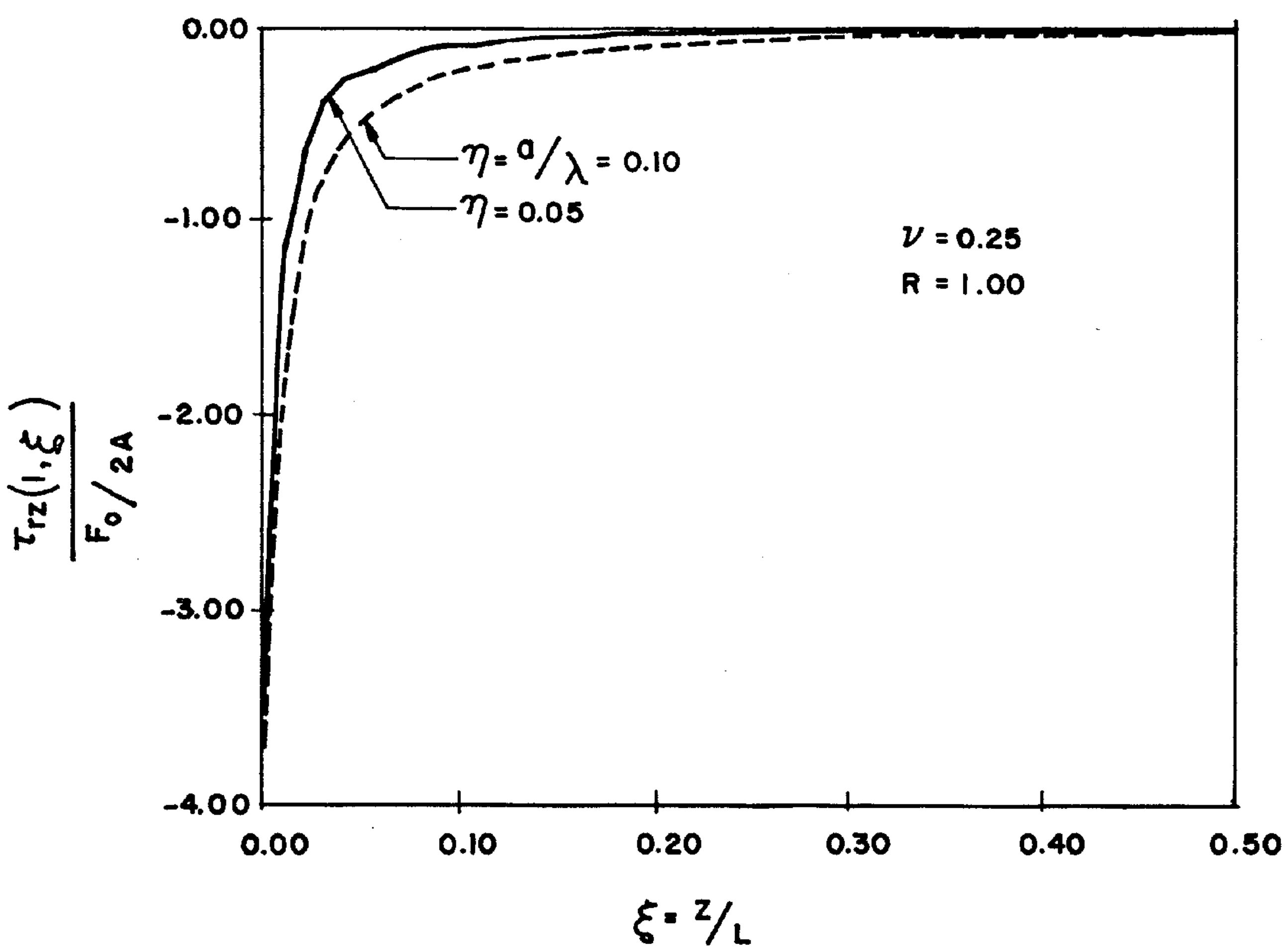

FIG. 8 INTERACTING SHEAR STRESS VERSUS ξ; R = 1.0