OPTIMUM SEISMIC PROTECTION FOR NEW BUILDING CONSTRUCTION IN EASTERN METROPOLITAN AREAS

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EVALUATION OF EXPECTED LOSSES AND TOTAL PRESENT COST: FURTHER PRELIMINARY SENSITIVITY ANALYSES

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Introduction

This report summarizes the results of some recent runs of the computer program which evaluates expected future losses due to earthquakes, and chooses an optimal seismic design strategy. The principal computations, the program listing, and the program input requirements are described in Internal Study Report No. 10. Sensitivity of the output is studied, primarily with respect to the following input data: (i) the earthquake occurrence probabilities, (ii) the damage probability matrices, (iii) the damage cost fractions, and (iv) the initial cost penalties.

Description of the Four Cases Studied

Four cases have been investigated, each for three different assumptions about the dependence of initial cost on design strategy, and for three different assumptions regarding physical and associated costs.

The earthquake occurrence probabilities used in these four cases are shown in Table 1, and are graphically depicted in Fig. 1. The occurrence probabilities for Cases 1 and 2 are identical; for Intensities below VII MMI, they correspond to the "high risk" probabilities referred to in I.S.R. No. 10. The Intensity VII and VII.5 risks have been increased substantially, and Intensity category VIII has been added. The occurrence probabilities for Case 3, which are 20 times those used in Cases 1 and 2, are reasonably representative of those that may be expected at some California site, for Intensities $\leq$ VII (they are too low at Intensities VII.5 and VIII, and higher Intensities are not included). The Case 4 occurrence probabilities represent Merz and Cornell's current best estimates of seismic risk in the Boston area.

The damage probability matrices used in Case 1 are shown in Table 2a. The estimates of damage probability are based on educated extrapolation from the San Fernando, Caracas and Alaska earthquake damage statistics reported in earlier Internal Study Reports. The matrices used in Cases 2, 3 and 4 (the same in all three cases) are shown in Table 2b. Only the high intensity, severe damage probabilities are different from the corresponding Case 1 probabilities.

Table 3 shows the average physical and non-physical costs for each damage category, expressed as fractions of the "Zone 0" (or strategy 0) construction cost. The annual expected physical cost (again normalized with respect to the Zone 0 construction cost) for the various design strategies j (j = 0, 1, 2, 3, 4), is denoted by c_j, and that for the corresponding non-physical cost by c'_j. The discounted total expected future cost L_j, can then be computed as a linear function of c_j and c'_j:

$$L_j = \frac{1}{\delta}(c_j + a\, c'_j)$$

where L_j = normalized discounted total expected future cost under strategy j, δ = discount factor, and a = weight given to the non-physical costs. In the present analysis, $\delta = 0.05$, and the choice of three different values of a (a = 0, 1 and 2) leads to three sets of values for L_j:

$$L_j^{(1)} = \frac{1}{0.05} c_j$$

$$L_j^{(2)} = \frac{1}{0.05}(c_j + c'_j)$$

$$L_j^{(3)} = \frac{1}{0.05}(c_j + 2c'_j)$$

To obtain the total present cost associated with a design strategy, j, the initial cost increment, A_j, must be added to the expected future cost, L_j. The initial cost increments are also expressed as fractions of the Zone 0 construction cost. Three arrays of values, denoted by $A_j^{(1)}$, $A_j^{(2)}$ and $A_j^{(3)}$, respectively, have been assumed; they are shown in Table 4 and Fig. 2

Summary of Results

The expected annual cost ratios, c_j and c_j', depend primarily on the earthquake occurrence probabilities and on the damage probability matrices, and to a much lesser degree on the initial cost increments. Tables 5a and 5b give the values of c_j and c_j' for the four cases under consideration. They are exact when the initial cost increments are given by $A_j^{(1)}$, and accurate to within a few percent when the initial cost increments $A_j^{(2)}$ and $A_j^{(3)}$ are used.

The total expected future losses, $L_j^{(1)}$, $L_j^{(2)}$ and $L_j^{(3)}$ (all normalized with respect to strategy 0 initial cost) are shown in Tables 6a, 6b and 6c, respectively.

The total expected cost ratio (TECR), for a given strategy j, equals the sum of the initial cost, A_j, and the expected future cost, L_j. The value of j which minimizes the total expected cost ratio is the optimal strategy. The values of TECR corresponding to the various assumptions about initial costs and damage costs, are listed in Tables 7a, 7b and 7c, and their minimum values are labeled.

Conclusions

There is little difference between Cases 1, 2 and 3, as far as optimal strategies is concerned. Small to moderate changes in the earthquake occurrence probabilities, or in the elements of the damage probability matrices, are not likely to affect the optimum decision much. Much higher occurrence probabilities (Case 3) lead to the choice to design for a maximum amount of protection (i.e., Zone 4 is optimal), if non-physical costs are included in the objective function. For the Boston setting, the input parameters which appear to affect the decision the most, are the initial cost increments. For example, Zone 2 will probably be optimal if the Zone 2 initial cost penalty is less than one percent.

TABLE 1

Earthquake Occurrence Probability

Intensity	Case 1	Case 2	Case 3	Case 4
IV	0.046	0.046	0.92	0.043
V	0.014	0.014	0.28	0.011
VI	0.003	0.003	0.06	0.0022
VI.5	0.0016	0.0016	0.032	0.0008
VII	0.0009	0.0009	0.018	0.0002
VII.5	0.0003	0.0003	0.006	1.8×10^{-5}
VIII	3×10^{-6}	3×10^{-6}	6×10^{-5}	1.0×10^{-9}

FIG. 1

Annual Earthquake Risk vs. Intensity Curve

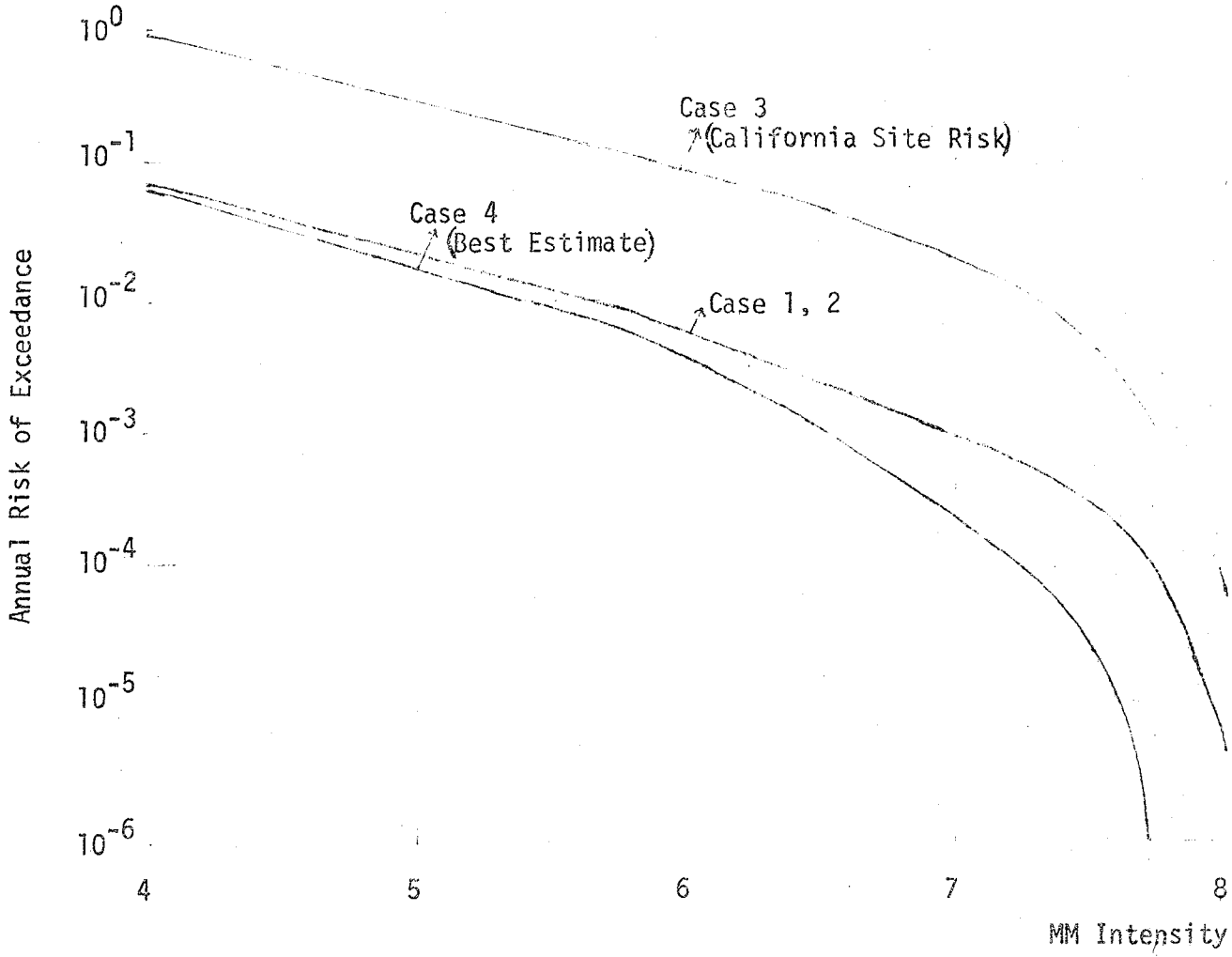

TABLE 2a

Damage Probability Matrices for Case 1

Strategy 0

IV	V	VI	VI.5	VII	VII.5	VIII
0.89000	0.75000	0.58000	0.24000	0.11000	0.03000	0.0
0.07000	0.14000	0.17000	0.34000	0.13000	0.12000	0.11000
0.04000	0.07000	0.14000	0.17000	0.34000	0.26000	0.13000
0.0	0.04000	0.07000	0.14000	0.17000	0.30000	0.34000
0.0	0.0	0.04000	0.07000	0.14000	0.15000	0.17000
0.0	0.0	0.0	0.04000	0.07000	0.09000	0.14000
0.0	0.0	0.0	0.0	0.04000	0.05000	0.07000
0.0	0.0	0.0	0.0	0.0	0.0	0.04000
0.0	0.0	0.0	0.0	0.0	0.0	0.0

Strategy 1

0.93	0.83	0.70	0.40	0.18	0.09	0.03
0.05	0.10	0.13	0.30	0.22	0.16	0.15
0.02	0.05	0.10	0.13	0.30	0.28	0.22
0.0	0.02	0.05	0.10	0.13	0.25	0.30
0.0	0.0	0.02	0.05	0.10	0.12	0.13
0.0	0.0	0.0	0.02	0.05	0.07	0.10
0.0	0.0	0.0	0.0	0.0	0.03	0.05
0.0	0.0	0.0	0.0	0.0	0.0	0.02
0.0	0.0	0.0	0.0	0.0	0.0	0.0

Strategy 2

0.97	0.90	0.81	0.59	0.24	0.15	0.07
0.03	0.07	0.09	0.22	0.35	0.25	0.17
0.0	0.03	0.07	0.09	0.22	0.29	0.35
0.0	0.0	0.03	0.07	0.09	0.18	0.22
0.0	0.0	0.0	0.03	0.07	0.08	0.09
0.0	0.0	0.0	0.0	0.03	0.05	0.07
0.0	0.0	0.0	0.0	0.0	0.0	0.03
0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0

Strategy 3

IV	V	VI	VI.5	VII	VII.5	VIII
0.98	0.93	0.87	0.70	0.29	0.20	0.10
0.02	0.05	0.06	0.17	0.41	0.30	0.19
0.0	0.02	0.05	0.06	0.17	0.34	0.41
0.0	0.0	0.02	0.05	0.06	0.08	0.17
0.0	0.0	0.0	0.02	0.05	0.05	0.06
0.0	0.0	0.0	0.0	0.02	0.03	0.05
0.0	0.0	0.0	0.0	0.0	0.0	0.02
0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0

Strategy 4

1.00	1.00	0.95	0.85	0.40	0.30	0.15
0.0	0.0	0.05	0.10	0.45	0.35	0.25
0.0	0.0	0.0	0.05	0.10	0.28	0.45
0.0	0.0	0.0	0.0	0.05	0.06	0.10
0.0	0.0	0.0	0.0	0.0	0.10	0.05
0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0

TABLE 2b

Damage Probability Matrices for Cases 2, 3 and 4

Strategy 0

IV	V	VI	VI.5	VII	VII.5	VIII
0.89	0.75	0.58	0.24	0.11	0.03	0.0
0.07	0.14	0.17	0.34	0.13	0.12	0.11
0.04	0.07	0.14	0.17	0.34	0.26	0.13
0.0	0.04	0.07	0.14	0.17	0.30	0.34
0.0	0.0	0.04	0.07	0.14	0.15	0.17
0.0	0.0	0.0	0.04	0.07	0.09	0.14
0.0	0.0	0.0	0.0	0.03	0.03	0.07
0.0	0.0	0.0	0.0	0.01	0.02	0.02
0.0	0.0	0.0	0.0	0.0	0.0	0.02

Strategy 1

0.93	0.83	0.70	0.40	0.18	0.09	0.03
0.05	0.10	0.13	0.30	0.22	0.16	0.15
0.02	0.05	0.10	0.13	0.30	0.28	0.22
0.0	0.02	0.05	0.10	0.13	0.25	0.30
0.0	0.0	0.02	0.05	0.10	0.12	0.13
0.0	0.0	0.0	0.02	0.05	0.07	0.10
0.0	0.0	0.0	0.0	0.01	0.02	0.05
0.0	0.0	0.0	0.0	0.01	0.01	0.01
0.0	0.0	0.0	0.0	0.0	0.0	0.01

Strategy 2

0.97	0.90	0.81	0.59	0.24	0.15	0.07
0.03	0.07	0.09	0.22	0.35	0.25	0.17
0.0	0.03	0.07	0.09	0.22	0.29	0.35
0.0	0.0	0.03	0.07	0.09	0.18	0.22
0.0	0.0	0.0	0.03	0.07	0.08	0.09
0.0	0.0	0.0	0.0	0.03	0.03	0.07
0.0	0.0	0.0	0.0	0.0	0.02	0.02
0.0	0.0	0.0	0.0	0.0	0.0	0.01
0.0	0.0	0.0	0.0	0.0	0.0	0.0

Strategy 3

IV	V	VI	VI.5	VII	VII.5	VIII
0.98	0.93	0.87	0.70	0.29	0.20	0.10
0.02	0.05	0.06	0.17	0.41	0.30	0.19
0.0	0.02	0.05	0.06	0.17	0.34	0.41
0.0	0.0	0.02	0.05	0.06	0.08	0.17
0.0	0.0	0.0	0.02	0.05	0.05	0.06
0.0	0.0	0.0	0.0	0.02	0.02	0.05
0.0	0.0	0.0	0.0	0.0	0.01	0.01
0.0	0.0	0.0	0.0	0.0	0.0	0.01
0.0	0.0	0.0	0.0	0.0	0.0	0.0

Strategy 4

1.00	1.00	0.95	0.85	0.40	0.30	0.15
0.0	0.0	0.05	0.10	0.45	0.35	0.25
0.0	0.0	0.0	0.05	0.10	0.28	0.45
0.0	0.0	0.0	0.0	0.05	0.06	0.10
0.0	0.0	0.0	0.0	0.0	0.01	0.03
0.0	0.0	0.0	0.0	0.0	0.0	0.02
0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0

TABLE 3

Damage States

	Description of Level of Damage	Repair Costs (Ratio to Const. Cost)	Non-Phys. Costs (Ratio to Const. Cost)
0	No damage	0	0.0
1	Minor non-structural damage - a few walls and partitions cracked, incidental mechanical and electrical damage	0.001	0.0
2	Localized non-structural damage - more extensive cracking (but still not widespread); possibly damage to elevators and/or other mechanical/electrical components	0.005	0.005
3	Widespread non-structural damage - possibly a few beams and columns cracked, although not noticeable	0.02	0.1
4	Minor structural damage - obvious cracking or yielding in a few structural members; substantial non-structural damage with widespread cracking	0.05	0.2
5	Substantial structural damage requiring repair or replacement of some structural members, associated extensive non-structural damage	0.10	0.5
6	Major structural damage requiring repair or replacement of many structural members, associated non-structural damage requiring repairs to major portion of interior; building vacated during repairs	0.30	1.3
7	Building condemned	1.0	2.5
8	Collapse	1.0	5

TABLE 4

Fractional Initial Cost Increases

Strategy	$A_j^{(1)}$	$A_j^{(2)}$	$A_j^{(3)}$
0	0.0	0.0	0.0
1	0.002	0.002	0.003
2	0.004	0.010	0.020
3	0.010	0.030	0.050
4	0.020	0.050	0.085

FIG. 2

Fractional Initial Cost Increases vs.
Design Strategy Curve

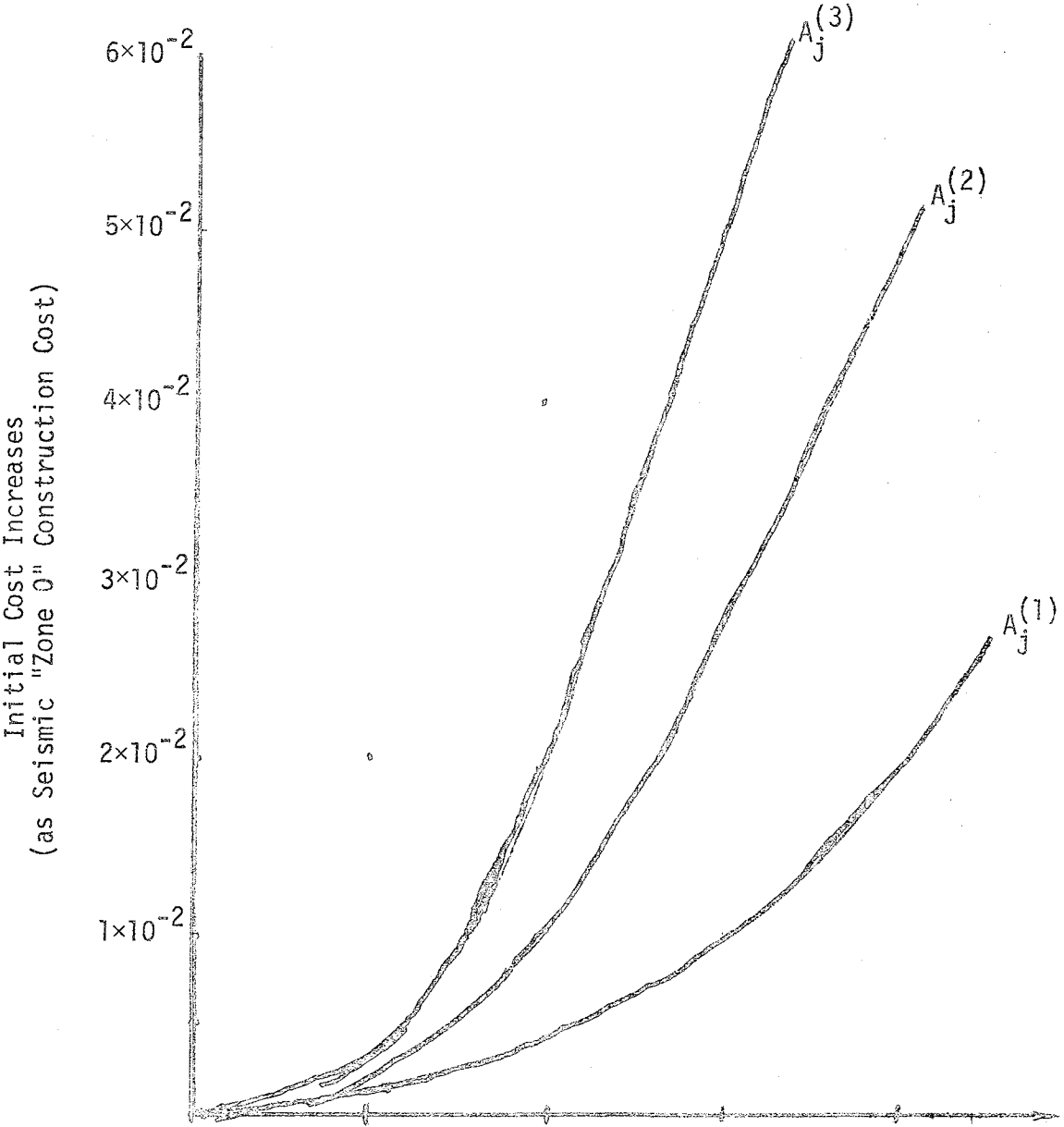

TABLE 5a

Strategy j	Case 1	Case 2	Case 3	Case 4
0	0.204×10^{-3}	0.225×10^{-3}	0.450×10^{-2}	0.106×10^{-3}
1	0.129×10^{-3}	0.145×10^{-3}	0.190×10^{-2}	0.652×10^{-4}
2	0.530×10^{-4}	0.554×10^{-4}	0.111×10^{-2}	0.223×10^{-4}
3	0.365×10^{-4}	0.377×10^{-4}	0.754×10^{-3}	0.154×10^{-4}
4	0.711×10^{-5}	0.712×10^{-5}	0.142×10^{-3}	0.170×10^{-5}

c_j = Ratio of Annual Expected Repair Costs to Zone 0 Construction Cost

TABLE 5b

Strategy j	Case 1	Case 2	Case 3	Case 4
0	0.736×10^{-3}	0.772×10^{-3}	0.154×10^{-1}	0.325×10^{-3}
1	0.457×10^{-3}	0.487×10^{-3}	0.974×10^{-2}	0.193×10^{-3}
2	0.175×10^{-3}	0.185×10^{-3}	0.370×10^{-2}	0.581×10^{-4}
3	0.116×10^{-3}	0.121×10^{-3}	0.242×10^{-2}	0.394×10^{-4}
4	0.166×10^{-4}	0.167×10^{-4}	0.334×10^{-3}	0.297×10^{-5}

c'_j = Ratio of Annual Expected Non-Physical Costs to Zone 0 Construction Costs

TABLE 6a

Strategy j	Case 1	Case 2	Case 3	Case 4
0	0.408×10^{-2}	0.449×10^{-2}	0.898×10^{-1}	0.212×10^{-2}
1	0.258×10^{-2}	0.291×10^{-2}	0.582×10^{-1}	0.130×10^{-2}
2	0.106×10^{-2}	0.111×10^{-2}	0.222×10^{-1}	0.446×10^{-3}
3	0.730×10^{-3}	0.755×10^{-3}	0.151×10^{-1}	0.307×10^{-3}
4	0.142×10^{-3}	0.142×10^{-3}	0.284×10^{-2}	0.340×10^{-4}

Ratio of Total Expected Future Loss to Zone 0 Construction Cost : $L_j^{(1)} = \frac{1}{0.05} c_j$

TABLE 6b

Strategy j	Case 1	Case 2	Case 3	Case 4
0	0.188×10^{-1}	0.199×10^{-1}	0.398	0.863×10^{-2}
1	0.117×10^{-1}	0.126×10^{-1}	0.252	0.516×10^{-2}
2	0.456×10^{-2}	0.480×10^{-2}	0.960×10^{-1}	0.161×10^{-2}
3	0.306×10^{-2}	0.318×10^{-2}	0.636×10^{-1}	0.110×10^{-2}
4	0.475×10^{-3}	0.476×10^{-3}	0.952×10^{-2}	0.934×10^{-4}

Ratio of Total Expected Future Loss to Zone 0 Construction Cost : $L_j^{(2)} = \frac{1}{0.05} c_j + \frac{1}{0.05} c'_j$

TABLE 6c

Strategy j	Case 1	Case 2	Case 3	Case 4
0	0.335×10^{-1}	0.354×10^{-1}	0.708	0.151×10^{-1}
1	0.209×10^{-1}	0.224×10^{-1}	0.448	0.903×10^{-3}
2	0.806×10^{-2}	0.850×10^{-2}	0.170	0.277×10^{-2}
3	0.538×10^{-2}	0.560×10^{-2}	0.112	0.189×10^{-2}
4	0.808×10^{-3}	0.809×10^{-3}	0.162×10^{-1}	0.153×10^{-3}

Ratio of Total Expected Future Loss to Zone 0 Construction Cost : $L_j^{(3)} = \frac{1}{0.05} c_j + \frac{1}{0.025} c'_j$

TABLE 7a

Sum of Initial and Expected Future Costs

(Expressed as a Fraction of Zone 0 Initial Cost)

Initial Costs $A_j^{(1)}$

$$TECR = A_j^{(1)} + L_j^{(1)}$$

Strategy j	Case 1	Case 2	Case 3	Case 4
0	0.0041	0.0045	0.089	0.002
1	0.0046	0.0049	0.060	0.003
2	0.0051	0.0052	0.026	0.004
3	0.011	0.011	0.025	0.010
4	0.020	0.020	0.023	0.020

$$TECR = A_j^{(1)} + L_j^{(2)}$$

Strategy j	Case 1	Case 2	Case 3	Case 4
0	0.019	0.020	0.398	0.009
1	0.014	0.015	0.254	0.007
2	0.009	0.009	0.100	0.006
3	0.013	0.013	0.074	0.011
4	0.020	0.020	0.029	0.020

$$TECR = A_j^{(1)} + L_j^{(3)}$$

Strategy j	Case 1	Case 2	Case 3	Case 4
0	0.034	0.035	0.708	0.015
1	0.023	0.024	0.450	0.011
2	0.012	0.012	0.174	0.007
3	0.015	0.016	0.122	0.012
4	0.021	0.021	0.036	0.020

TABLE 7b

Sum of Initial and Expected Future Costs

(Expressed as a Fraction of Zone 0 Initial Cost)

Initial Costs $A_j^{(2)}$

$$TECR = A_j^{(2)} + L_j^{(1)}$$

Strategy j	Case 1	Case 2	Case 3	Case 4
0	[0.0041]	[0.0045]	0.089	[0.002]
1	0.0046	0.0049	0.060	0.003
2	0.0111	0.0112	[0.032]	0.010
3	0.031	0.031	0.045	0.030
4	0.050	0.050	0.053	0.050

$$TECR = A_j^{(2)} + L_j^{(2)}$$

Strategy j	Case 1	Case 2	Case 3	Case 4
0	0.019	0.020	0.389	0.009
1	[0.014]	[0.015]	0.254	[0.007]
2	0.015	[0.015]	0.106	0.012
3	0.033	0.033	0.094	0.031
4	0.050	0.050	[0.059]	0.050

$$TECR = A_j^{(2)} + L_j^{(3)}$$

Strategy j	Case 1	Case 2	Case 3	Case 4
0	0.034	0.035	0.708	0.015
1	0.023	0.024	0.450	[0.011]
2	[0.018]	[0.018]	0.180	0.013
3	0.035	0.036	0.142	0.032
4	0.051	0.051	[0.066]	0.050

TABLE 7c

Sum of Initial and Expected Future Costs

(Expressed as a Fraction of Zone 0 Initial Cost)

Initial Costs $A_j^{(3)}$

$$TECR = A_j^{(3)} + L_j^{(1)}$$

Strategy j	Case 1	Case 2	Case 3	Case 4
0	[0.0041]	[0.0045]	0.089	[0.002]
1	0.0050	0.0059	0.061	0.004
2	0.021	0.021	[0.042]	0.020
3	0.051	0.051	0.065	0.050
4	0.085	0.085	0.088	0.085

$$TECR = A_j^{(3)} + L_j^{(2)}$$

Strategy j	Case 1	Case 2	Case 3	Case 4
0	0.019	0.020	0.398	0.009
1	[0.015]	[0.016]	0.255	[0.008]
2	0.025	0.025	0.116	0.022
3	0.053	0.053	0.114	0.051
4	0.085	0.085	[0.094]	0.085

$$TECR = A_j^{(3)} + L_j^{(3)}$$

Strategy j	Case 1	Case 2	Case 3	Case 4
0	0.034	0.035	0.708	0.015
1	[0.024]	[0.025]	0.451	[0.012]
2	0.028	0.028	0.190	0.023
3	0.055	0.056	0.162	0.052
4	0.086	0.088	[0.056]	0.085