A SHELL MODEL OF A BURIED PIPE IN A SEISMIC ENVIRONMENT

by

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ABSTRACT

In a number of recent investigations, a buried pipe undergoing seismic excitation was modelled as a beam on a visco-elastic foundation. However, it is known that two of the observed failure modes in the buried pipelines under seismic loads are buckling and fracture. Therefore, in this paper a thin circular cylindrical shell model in a resisting soil medium is used for a buried pipe. The coupled equilibrium equations arising from this model are modified to yield three decoupled equations which somewhat simplify the analysis. Then, an application for a long buried pipe is presented and the results are discussed and compared with those of the beam model.

NOMENCLATURE

a	=	Radius of shell
A, A_1, B, C	=	Constants defined in the text
b_{ij}	=	Constant defined in the text
c_i	=	Roots of the polynomial
D	=	Bending rigidity
d_{ij}	=	Constant defined in the text
E	=	Modulus of elasticity
e_x, e_θ	=	Normal strain components of the shell
f_{ij}	=	Constant defined in the text
F_r, F_θ, F_x	=	Force per unit area
g_i	=	Roots of the polynomial
h	=	Thickness of the shell
k, k_r, k^*	=	Spring constants defined in the text
K	=	Extensional rigidity
L_d	=	Donnell-Mushtari operator
L_m	=	A modifying operator used in conjunction with L_d
m, n, p, q, r	=	Constants defined in the text
$M_x, M_\theta, M_{x\theta}, N_x, N_\theta, N_{x\theta}, Q_x, Q_\theta$	=	Stress resultants per unit length
R_u, R_w	=	Ratios of displacement
s	=	Constant
t	=	Time
u, v, w	=	Pipe displacement components
u_i	=	Relative displacement of pipe to soil
u_p	=	Particular form of the solution
U, V, W	=	Ground displacement components
$\bar{U}$	=	Amplitude of ground deformation

U', V', W' etc	=	Dimensionless quantities defined in the text
x	=	Shell coordinate
x^*	=	A dimensional coordinate defined in the text
x'	=	Dimensionless quantity defined in the text
$\alpha, \chi, \Omega, \beta$	=	Second order spatial operator
ε	=	A dimensionless parameter
ε_o	=	A constant
ρ	=	Mass Density of Shell Material
∇^2	=	Laplacian
$\bar{\nabla}^4$	=	Fourth-order spatial operator
Θ	=	Shell coordinate
$\kappa_x, \kappa_\theta, \tau$	=	Curvature and twist related teams
λ	=	Wavelength
λ_d, λ_{d_r}	=	Damping constants
η, η_r, μ, μ_r	=	First and second-order time operators
ν	=	Poisson's ratio
ξ^2	=	Shell parameter, $= \dfrac{h^2}{12a^2}$
ϕ_1	=	Excitation function
Φ, χ	=	Operators defined in the text
$\gamma_{x\theta}$	=	Shear strain
ω	=	$2\pi/\lambda$

Introduction

Due to the great potential for destruction, damage and disruption, the seismic problems of utility systems have recently attracted the attention of earthquake engineers and researchers [1,2]*. It became apparent that the seismic behavior of buried pipeline systems is quite different than that of above ground structures. For example bridges and dams, for which horizontal inertia force is the most important factor, are mostly designed by the seismic coefficient method. However, in the seismic resistant design of buried pipelines very small pipe stresses are obtained by this method. This is because the seismic load is mainly resisted by the surrounding soil. Therefore, it is not suitable in the design of underground piping systems. Seismic damage to underground piping systems is caused primarily by ground movement and faulting, traveling seismic waves, liquefaction of sandy soil, or difference in stiffness of two horizontally adjacent soil layers [3].

Furthermore, since utility systems are networks having sources, transmission lines, storage facilities and distribution systems within themselves, damage to single locations in a utility network often affects significant portions of the entire system.

Recently a number of investigators [4 , 5 through 9] have examined the behavior of a buried pipe undergoing seismic excitation. An extensive and up-to-date review of the subject is recently provided by Ariman and Muleski [10]. The vast majority of this work has modelled the system as a beam on a visco-elastic foundation with the exciting function ϕ_i in the form

$$\phi_i = (k + \lambda_d \frac{\partial}{\partial t}) u_i \tag{1}$$

* References are presented at the end of the report.

where $i = 1,2,3; k$ is a spring constant and λ_d is a coefficient of viscous damping. The relative displacement (in the i-th direction) of the pipe to the soil is denoted by u_i.

It is known that two of the observed failure modes in buried pipelines under seismic excitations are buckling and fracture (Figs. 1 and 2). Clearly the buckling phenomenon exhibited in Fig. 1 cannot be described with the use of the beam model. Furthermore the same beam model also fails in the formulation of fracture problem of a pipe. Therefore, in order to examine these failure modes, it is the intent of the present paper to model a buried pipe as a thin circular cylindrical shell in a resisting soil medium. The coupled equilibrium equations arising from this model are modified to yield three decoupled equations which somewhat simplify the analysis. Then the governing equations are utilized for the analysis of long buried pipe when the ground is deformed sinusoidally in the direction of the pipe axis. The results are discussed and compared with those of the beam model.

General Theory

It is assumed that the reader is acquainted with the equilibrium equations for a thin isotropic shell and is aware that these may be written in terms of the displacements u, v, and w in the x, θ, and r (radial) directions, respectively, as shown in Figure 3. The equilibrium equations may be presented as follows [11]

$$\begin{aligned} &\frac{\partial N_x}{\partial x} + \frac{1}{a}\frac{\partial N_{\theta x}}{\partial \theta} + q_x = 0 \\ &\frac{\partial N_{x\theta}}{\partial x} + \frac{1}{a}\frac{\partial N_\theta}{\partial \theta} + \frac{Q_\theta}{a} + q_\theta = 0 \\ &\frac{\partial Q_x}{\partial x} + \frac{1}{a}\frac{\partial Q_\theta}{\partial \theta} - \frac{N_\theta}{a} - q_r = 0 \end{aligned} \tag{2}$$

$$\frac{\partial M_x}{\partial x} + \frac{1}{a}\ \frac{\partial M_{\theta x}}{\partial \theta} - Q_x = 0$$

$$\frac{\partial M_{x\theta}}{\partial x} + \frac{1}{a}\ \frac{\partial M_\theta}{\partial \theta} - Q_\theta = 0 \tag{2}$$

Here q_i represents the external loading and force and moment stress resultants (Figs. 4 and 5) are given as:

$$N_x = K\ (e_x + \nu\ e_\theta),\quad N_\theta = K\,(e_\theta + \nu\ e_x)$$

$$N_{x\theta} = N_{\theta x} = \frac{E}{2(1+\nu)}\ h\ \gamma_{x\theta}\quad , \tag{2a}$$

$$M_x = D_s(\kappa_x + \nu\ \kappa_\theta)\quad ,\quad M_\theta = D_s(\kappa_\theta + \nu\ \kappa_x)\quad ,$$

in which $D_s = \dfrac{Eh^3}{12(1-\nu^2)}$, $M_{x\theta} = M_{\theta x} = \dfrac{E}{2(1+\nu)} \cdot \dfrac{h^3}{12}\ \tau$. In these expressions h, a, E and ν represent the thickness, radius, Young's modulus and Poisson's ratio for the cylinder respectively. Furthermore

$$e_x = \frac{\partial u}{\partial x}\ ,\ e_\theta = \frac{\partial v}{a\partial\theta} + \frac{w}{a}\ ,\quad \gamma_{x\theta} = \frac{\partial u}{a\partial\theta} + \frac{\partial v}{\partial x}$$

$$\kappa_x = -\ \frac{\partial^2 w}{\partial x^2}\ ,\ \kappa_\theta = \frac{1}{a}\frac{\partial}{\partial\theta}\ \left(\frac{v}{a} - \frac{1}{a}\frac{\partial w}{\partial\theta}\right) \tag{2b}$$

and

$$\tau = \frac{1}{a}\frac{\partial v}{\partial x} - 2\ \frac{1}{a}\ \frac{\partial^2 w}{\partial x\partial\theta}$$

The equilibrium equations in operator notation are presented as

$$[L_d + \xi^2 L_m] \begin{bmatrix} u \\ v \\ w \end{bmatrix} = -\ \frac{1}{K} \begin{bmatrix} F_x \\ F_\theta \\ F_r \end{bmatrix} \tag{2c}$$

where

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*Equations (2), (2a) and (2b) correspond to the case of L_m=0.

$$L_d = \begin{bmatrix} \Phi & \frac{1+\nu}{2a}\frac{\partial^2}{\partial x \partial \theta} & \nu \frac{\partial}{\partial x} \\ & \chi & \frac{1}{a}\frac{\partial}{\partial \theta} \\ \text{symmetric} & & 1 + \xi^2 \nabla^4 \end{bmatrix} \tag{3}$$

in which

$$\xi^2 = \frac{1}{12}\left(\frac{h}{a}\right)^2 , \qquad K = \frac{Eh}{1-\nu^2}$$

$$\Phi = \nabla^2 - \frac{1+\nu}{2a^2}\frac{\partial^2}{\partial \theta^2} , \qquad \chi = \nabla^2 - \frac{1+\nu}{2}\frac{\partial^2}{\partial x^2} \tag{4}$$

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{1}{a^2}\frac{\partial^2}{\partial \theta^2} , \qquad \nabla^4 = \nabla^2\nabla^2$$

The forces per unit area are denoted by F_x, F_θ, and F_r.

The operator L_d in (2c) and (3) is known as the Donnell-Mushtari operator, with time dependence through forcing terms, while L_m represents a "modifying operator," which allows extension of (2c) to other shell theories. Leissa [12] lists nine L_m's in this 3 x 3 notation, from a minimum of two non-zero entries involving four operations (Houghton-Jones Theory) to a maximum of nine non-zero elements encompassing a total of 28 operations (Epstein-Kennard Theory).

Donnell's Equations and Complete First-Order Equations

In the Donnell-Mushtari theory, $L_m = 0$ and thus presents the simplest set of equilibrium equations represented by (2c). Nevertheless, each equation involves all three displacements u, v and w. By suitable operations, these equations may be decoupled, resulting in three other equations, one involving only u and w, another only v and w, and, finally, a differential equation in w alone. This decoupling is important in the application of thin shell theory in that, given the loading situation, the third equation may be solved for w and, knowing w, u and v are found from the first two equations directly.

However, in the derivation of these equations (commonly referred to as Donnell's equations), a number of assumptions must be made [13]. These amount to the neglecting of

1. The Q_θ term in the second equation of (2)

2. The ν terms in the last two expressions of equations (2b).

In the present paper, the terms neglected by Donnell are retained and carried through a series of similar operations to obtain the following set of decoupled equations:

$$\frac{1-\nu}{1+\nu}(1+\xi^2)\nabla^4 u + \frac{2}{K^2(1+\nu)}\eta^2(u) - \frac{3-\nu}{K(1+\nu)}\nabla^2\eta(u) - \frac{2\xi^2}{K(1+\nu)}\chi\,\eta(u)$$

$$+\ \xi^2\frac{1+\nu}{2a^2}\frac{\partial^4 u}{\partial x^2\partial\theta^2} = \frac{1}{Ka}\mu\left(\frac{\partial^2 V}{\partial x\partial\theta}\right) + \frac{2}{K^2(1+\nu)}\eta\mu(U)$$

$$-\ \frac{2(1+\xi^2)}{K(1+\nu)}\chi\,u(U) + \frac{2\nu}{Ka(1+\nu)}\eta\left(\frac{\partial w}{\partial x}\right)$$

$$+\ \frac{d}{a^3}\frac{\partial^3 w}{\partial x\partial\theta^2} - \frac{2\nu(1+\xi^2)}{a(1+\nu)}\chi\left(\frac{\partial w}{\partial x}\right)$$

$$-\ \xi^2\frac{1}{a}\left(\frac{1}{a^2}\frac{\partial^5 w}{\partial\theta^4\partial x} + \frac{\partial^5 w}{\partial x^3\partial\theta^2}\right) \qquad (5a)$$

$$[(\frac{1-\nu}{1+\nu}(1+\xi^2)\ \nabla^4 v + \frac{2}{K^2(1+\nu)}\ \eta^2(v) - \frac{3-\nu}{1+\nu}\nabla^2\eta(v)$$

$$+ \xi^2\ \frac{1+\nu}{2a^2}\ \frac{\partial^4 v}{\partial x^2\partial\theta^2} - \frac{2\xi^2}{K(1+\nu)}\ \chi\eta(v)] = [\frac{1}{Ka}\ \mu(\frac{\partial^2 U}{\partial x\partial\theta}) + \frac{2}{K^2(1+\nu)}\ \eta\mu(V)$$

$$- \frac{2}{K(1+\nu)}\ \Phi\mu(V) + \frac{2}{Ka^2(1+\nu)}\ \eta(\frac{\partial w}{\partial\theta})$$

$$+ \frac{\nu}{a^2}\ \frac{\partial^3 w}{\partial x^2\partial\theta} - \frac{2}{a^2(1+\nu)}\ \Phi\ (\frac{\partial w}{\partial\theta})$$

$$+ \xi^2\ \frac{2}{1+\nu}\ \{\frac{-1}{Ka^2} + \Phi\ \}\ \nabla^2\ (\frac{\partial w}{\partial\theta})] \qquad (5b)$$

and finally,

$$\frac{h^2}{12}\nabla^8 w + \frac{1-\nu^2}{a^2}\frac{\partial^4 w}{\partial x^4} + \frac{\xi^2}{1-\nu}\{\frac{2(2+\nu)(1-\nu)}{a^2}\ \frac{\partial^6 w}{\partial x^4\partial\theta^2} - \frac{6(1+\nu)(2\nu-3)}{a^4}\ \frac{\partial^6 w}{\partial x^2\partial\theta^4}$$

$$+ \frac{2(1-\nu)}{a^6}\ \frac{\partial^6 w}{\partial\theta^6}\} = \frac{2}{K^2(1-\nu)}\ \{\ \frac{1}{K}\ \eta^2\eta_r - \frac{h^2}{12}\nabla^4\ \eta^2 - \frac{1}{a^2}\ \eta^2\ \}(w) + \frac{1}{Ka^2}\ \nabla^2\ \eta(w)$$

$$- \frac{3-\nu}{K(1-\nu)}\ \{\nabla^2\ \eta\ \frac{1}{K}\ \eta_r - \frac{h^2}{12}\ \nabla^6\eta\}(w) + \frac{2(1+\nu)}{Ka^2}\ \eta\ (\frac{\partial^2 w}{\partial x^2}) + \frac{1}{K}\ \nabla^4\ \eta_r(w)$$

$$+ \frac{4\xi^2}{Ka^2(1-\nu)}\ \eta\ (\nabla^2\ \frac{\partial^2 w}{\partial\theta^2}) \qquad - \frac{2}{K(1+\nu)}\{\chi\mu\ (\frac{\nu}{a}\ \frac{\partial U}{\partial x}) + \Phi\mu(\frac{1}{a^2} - \xi^2\nabla^2)\frac{\partial V}{\partial\theta}\}$$

$$+ \frac{1}{K}\ \mu\{\frac{1}{a}\ \frac{\partial^2}{\partial x\partial\theta} + \frac{2}{K(1+\nu)}\ \eta\}\ [\frac{\nu}{a}\ \ \frac{\partial U}{\partial x} + (\frac{1}{a^2} - \xi^2\nabla^2)\ \frac{\partial V}{\partial\theta}]$$

$$+ \frac{1+\nu}{1-\nu}\ \{\frac{2}{K^3(1+\nu)}\ \eta^2 - \frac{3-\nu}{K^2(1+\nu)}\ \nabla^2\eta + \frac{(1-\nu)(1+\xi^2)}{(1+\nu)\ K}\ \nabla^4 + \frac{2\xi^2}{K^2(1+\nu)}\ \chi\eta$$

$$- \frac{\xi^2(1+\nu)}{2a^2K}\ \frac{\partial^4}{\partial x^2\partial\theta^2}\ \}\ (\mu_r(W)) \qquad (5c)$$

Here the forces per unit area are introduced as

$$F_x = \mu(U) - \eta(u)$$

$$F_\theta = \mu(V) - \eta(v) \tag{6}$$

$$F_r = \mu_r(W) - \eta_r(w)$$

Here U, V, and W represent the ground displacements in the x, θ and r directions respectively and μ, μ_r and η, η_r are first- and second-order time operators as:

$$\mu = k + \lambda_d \frac{\partial}{\partial t} \quad , \quad \eta = k + \lambda_d \frac{\partial}{\partial t} + \rho h \frac{\partial^2}{\partial t^2}$$

(6a)

$$\mu_r = k_r + \lambda_{d_r} \frac{\partial}{\partial t} \quad , \quad \eta_r = k_r + \lambda_{d_r} \frac{\partial}{\partial t} - \rho h \frac{\partial^2}{\partial t^2}$$

The k and λ_d implied in μ and η by equations (1) and (6a) are assumed to be constant. This situation greatly facilitates the decoupling process in that the various operators then commute and allow the simplification of the expressions. However, k_r and λ_{d_r} in μ_r and η_r may be considered functions of time and space. Namely

$$k_r = k_r \ (x,\theta,t) \quad \text{and} \quad \lambda_{d_r} = \lambda_{d_r} \ (x,\theta,t) \tag{7}$$

Note that these operators include inertia terms as well as the visco-elastic forces implied in equation (1).

It will be noted that, by setting the shell parameter $\xi^2 = 0$, the dynamic Donnell equations result. While it is customary to neglect the shell parameter with respect to unity for thin shells, there are clearly terms in (5) in which ξ^2 stands alone.

Theory of Flügge-Lur'ye-Byrne.

It is well known [11, 13-15] that Donnell's equations can prove inadequate, especially for cases in which the circumferential distortion's wavelength increases. With this in mind, Morley [13] proposed a different spatial operator for the w-equation in the static case. A similar dynamic expression is now derived.

Morley proposed a spatial operator for the third equation in the form

$$\nabla^4(\nabla^2 + 1)^2 + \frac{12\, a^2(1-\nu^2)}{h^2} \frac{\partial^4}{\partial x^4} \tag{8}$$

Examination of (5c) reveals that such terms cannot be had by the simple retention of the terms neglected by Donnell. As such, one must appeal to another shell theory. The Flügge-Lur'ye-Byrne theory gives the desired results.

The modifying operator L_m in this case and in non-dimensional form may be written as

$$L_m = \begin{bmatrix} \frac{1-\nu}{2} \frac{\partial^2}{\partial\theta^2} & 0 & -\frac{\partial^3}{\partial x^3} + \frac{1-\nu}{2} \frac{\partial^3}{\partial x \partial\theta^2} \\ & \frac{3(1-\nu)}{2} \frac{\partial^2}{\partial x^2} & -\frac{3-\nu}{2} \frac{\partial^3}{\partial x^2 \partial\theta} \\ \text{symmetric} & & 1 + 2 \frac{\partial^2}{\partial\theta^2} \end{bmatrix} \tag{9}$$

These additional terms in the Flügge-Lur'ye-Byrne (FLB) [12] theory result from the relaxation of the thinness assumption. In writing strain-displacement relations, one encounters terms involving the radial coordinate divided by a radius of curvature. In the Donnell-Mushtari theory, this is neglected with respect to unity. However, the fact that this quantity is less than unity enables one to expand the quotient by a geometric series which may then be truncated at a suitable order. This is precisely where the FLB theory differs from the Donnell-Mushtari and why the FLB theory is termed a "higher-order approximation."

The equilibrium equations in terms of the displacements

$$[\, L_d + \xi^2 L_m \,] \begin{bmatrix} u \\ v \\ w \end{bmatrix}$$

appear in Flügge's 1934 text, Statik und Dynamik der Schalen [16] (interestingly, the year after Donnell's equations were published). The same results were found independently by Lur'ye [17], Byrne [18], and Biezeno and Grammel [19]. In 1955, Kempner [15] showed that, for circular cylindrical shells, Flügge's equations [16,20] could be expressed in a form analogous to Donnell's equations. It is his approach that is now pursued for the dynamic situation.

Decoupled FLB Equations

Consider first the non-dimensional quantities defined as

$$x' = \frac{x}{a} \qquad u' = \frac{u}{a} \qquad v' = \frac{v}{a} \qquad w' = \frac{w}{a}$$

$$U' = \frac{U}{a} \qquad V' = \frac{V}{a} \qquad W' = \frac{W}{a} \tag{10}$$

and then the operators

$$\nabla'^2 = \frac{\partial^2}{\partial x'^2} + \frac{\partial^2}{\partial \theta^2} \qquad \begin{matrix} \eta' = a\eta & \mu' = a\mu \\ \eta'_r = a\eta_r & \mu'_r = a\mu_r \end{matrix} \tag{11}$$

The primes are now dropped for ease of notation. The operators $\bar{\nabla}^4, \alpha, \beta$, and Ω are also defined as

$$\bar{\nabla}^4 = \nabla^4 + 3\xi^2 \frac{\partial^4}{\partial x^4} + \frac{1-\nu}{2}\xi^2 (4+3\xi^2)\frac{\partial^4}{\partial x^2 \partial\theta^2} + \xi^2 \frac{\partial^4}{\partial\theta^4}$$

$$\Omega = \nabla^2 - \frac{3-\nu}{2}\frac{\partial^2}{\partial\theta^2}$$

$$\alpha = \frac{1}{\xi^2} - \frac{3-\nu}{2}\frac{\partial^2}{\partial x^2} \qquad (12)$$

$$\beta = \frac{\nu}{\xi^2} - (\nabla^2 - \frac{3-\nu}{2}\frac{\partial^2}{\partial\theta^2})$$

or

$$\beta = \frac{\nu}{\xi^2} - \Omega$$

The first two equations in (2c) can be solved for one term each. From the first, an expression for $\frac{\partial^2 v}{\partial x \partial\theta}$ in terms of derivatives of u and w is obtained; from the second, $\frac{\partial^2 u}{\partial x\partial\theta}$ is expressed in terms of v and w. Taking $\frac{\partial^2}{\partial x\partial\theta}$ of each of the first two equations and using the previous results gives the first two decoupled equations:

$$\nabla^{-4}u + \frac{2}{1-\nu}\frac{a^6}{D_s^2}\xi^4\eta^2(u) - \frac{2}{1-\nu}\frac{a^3}{D_s}\xi^2\eta\{(\frac{3-\nu}{2} + \xi^2\frac{1-\nu}{2})\nabla^2 u + \xi^2(1-\nu)\frac{\partial^2 u}{\partial x^2}\}$$

$$= \frac{1+\nu}{1-\nu}(\overline{LU}) + \frac{-2}{1-\nu}\frac{a^3}{D_s}\xi^4\eta\beta(\frac{\partial w}{\partial x}) + \{(1+3\xi^2)\nu\frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial\theta^2}$$

$$+ \xi^2\{(\frac{\partial^4}{\partial\theta^4} - (1+3\xi^2)\frac{\partial^4}{\partial x^4} + \frac{3\xi^2(1-\nu)}{2}\frac{\partial^4}{\partial x^2\partial\theta^2})\}(\frac{\partial w}{\partial x}) \qquad (13a)$$

$$\bar{\nabla}^4 \bar{v} + \frac{2}{1-\nu}\ \frac{a^6}{D_s^2}\ \xi^4 \eta^2(v) \qquad - \frac{2}{1-\nu}\ \frac{a^3}{D_s}\ \xi^2 \eta\ \{ (\frac{3-\nu}{2} + \xi^2\ \frac{1-\nu}{2})\ \nabla^2 v + \xi^2\ (1-\nu)\ \frac{\partial^2 v}{\partial x^2}\ \} =$$

$$(\overline{LV}) \qquad + \frac{-2}{1-\nu}\ \frac{a^3}{Ds}\ \xi^4\ \eta\alpha(\frac{\partial w}{\partial \theta}) + \{ (2+\nu)\frac{\partial^2}{\partial x^2} + (1+\xi^2)\ \frac{\partial^2}{\partial \theta^2} - 2\xi^2 (\frac{\partial^4}{\partial x^4} + (1+ \frac{3-\nu}{4}\ \xi^2)\ \frac{\partial^4}{\partial x^2 \partial \theta^2}) \}(\frac{\partial w}{\partial \theta}) \tag{13b}$$

where the following quantities are presumed known

$$(\overline{LU}) = \frac{a^3}{D_s}\ \xi^2 \{ \mu(\frac{\partial^2 V}{\partial x \partial \theta}) - \frac{2}{1+\nu}(X + \xi^2\ \frac{3(1-\nu)}{2}\ \frac{\partial^2}{\partial x^2})\mu(U) + \frac{2}{1+\nu}\ \frac{a^3}{D_s}\ \xi \eta \mu(U) \}$$

$$(\overline{LV}) = \frac{a^3}{D_s}\ \xi^2 \{ \mu(\frac{\partial^2 U}{\partial x \partial \theta}) - \frac{2}{1+\nu}(\Phi + \xi^2\ \frac{1-\nu}{2}\ \frac{\partial^2}{\partial \theta^2})\mu(V) + \frac{2}{1+\nu}\ \frac{a^3}{D_s}\ \xi^2 \eta \mu(V) \} \tag{14}$$

in which

$$D_s = \frac{Eh^3}{12(1-\nu^2)} = \xi^2 a^2 K \tag{15}$$

and all operators are as given after non-dimensionalization.

The decoupling is completed by taking $\beta \frac{\partial}{\partial x}$ of the u-equation in (13) and $\alpha \frac{\partial}{\partial \theta}$ of the v-equation and adding the results. The third equation in (2c) implies that

$$\alpha(\frac{\partial v}{\partial \theta}) + \beta\ (\frac{\partial u}{\partial x}) = \nabla^4 w + (\frac{1}{\xi^2} + 1)\ w + 2\ \frac{\partial^2 w}{\partial \theta^2}\ \frac{a^3}{Ds}\ \{\mu_r(W) - \eta_r(w)\} \tag{16}$$

This term appears as the argument of several operators in the final result of the last decoupling operation. Substitution of equation (16) gives one equation in terms of w alone. The terms in this equation will be divided into two groups: (1) terms that involve only spatial derivatives of w, and (2) terms that are either purely temporal or mixed (time and space). Letting {S} denote the first set and {T} the second, we have

$$\{S\} = \{T\} \tag{17}$$

Consider first the {S} term. This expression is eighth-order and expansion reveals that

$$\{S\}=[\nabla^8 + 2\nu(1+3\xi^2)\frac{\partial^6}{\partial x^6} + [6+3\xi^2(2-\nu-\nu^2)]\frac{\partial^6}{\partial x^4 \partial\theta^2}$$

$$+ [2(4-\nu) + (7-5\nu)\xi^2 + 3(1-\nu)\xi^4] \frac{\partial^6}{\partial x^2 \partial\theta^4}$$

$$+ 2(1+\xi^2) \frac{\partial^6}{\partial\theta^6} + \frac{1}{\xi^2} (1+3\xi^2)(1+\xi^2-\nu^2) \frac{\partial^4}{\partial x^4}$$

$$+ [2+ \frac{1-\nu}{2} (1+\xi^2)(4+3\xi^2)] \frac{\partial^4}{\partial x^2 \partial\theta^2} + (1+\xi^2) \frac{\partial^4}{\partial\theta^4}$$

$$+ \xi^2 \{(2-3\xi^2) \frac{\partial^8}{\partial x^8} + \frac{1}{2} [(11-3\nu) + 9(1-\nu)\xi^2] \frac{\partial^8}{\partial x^6 \partial\theta^2}$$

$$+ [3(2-\nu) - \nu^2\xi^2] \frac{\partial^8}{\partial x^4 \partial\theta^4}$$

$$+ \frac{1}{2} [(7-3\nu) + 3(1-\nu)\xi^2] \frac{\partial^8}{\partial x^2 \partial\theta^6} + \frac{\partial^8}{\partial\theta^8})\}](w) \qquad (18)$$

If $\xi^2 << 1$, equation (18) may be written (noting that all terms in the last part of the operator have counterparts in the first),

$$\{S\} = \{\nabla^8 + 2\nu \frac{\partial^6}{\partial x^6} + 6 \frac{\partial^6}{\partial x^4 \partial\theta^2} + 2(4-\nu) \frac{\partial^6}{\partial x^2 \partial\theta^4}$$

$$+ 2 \frac{\partial^6}{\partial\theta^6} + \frac{1-\nu^2}{\xi^2} \frac{\partial^4}{\partial x^4} + [2+2(1-\nu)] \frac{\partial^4}{\partial x^2 \partial\theta^2}$$

$$+ \frac{\partial^4}{\partial\theta^4} \} (w) \qquad (19)$$

or,

$$\{S\} = \{\nabla^8 + 2\nabla^6 + \nabla^4 + \frac{1-\nu^2-\xi^2}{\xi^2}\ \frac{\partial^4}{\partial x^4}\}\ (w)$$

$$+\ 2(1-\nu)\{\frac{\partial^4 w}{\partial x^2 \partial\theta^2} + \frac{\partial^6 w}{\partial x^2 \partial\theta^4} - \frac{\partial^6 w}{\partial x^6}\} \tag{20}$$

To maintain consistency, we note that the shell parameter will be neglected in favor of unity. Furthermore, the last term is neglected in Morley's static equations; we follow suit. Thus,

$$\{S\} = \nabla^4\ (\nabla^2+1)^2\,w\ + \frac{1-\nu^2}{\xi^2}\ \frac{\partial^4 w}{\partial x^4} \tag{21}$$

which is recognized as Morley's operator as previously given by equation (8).

The term involving time derivatives, $\{T\}$, is now considered. Expansion reveals that, if

$$(\overline{LW}) = \frac{a^3}{D_s}\,\bar{\nabla}^4 \mu_r(W) - \frac{2}{1-\nu}\,\frac{a^6}{D^2}\,\xi^2\{(\frac{3-\nu}{2} + \xi^2\,\frac{1-\nu}{2})\nabla^2 + \xi^2\,(1-\nu)\frac{\partial^2}{\partial x^2}\}[\eta\mu_r(W)]$$

$$+\ \frac{2}{1-\nu}\,\frac{a^9}{D^3}\,\xi^4\eta^2\mu_r(W)\ + \frac{1+\nu}{1-\nu}\ \{\alpha\ \frac{\partial}{\partial\theta}\ (\overline{LV}) + \beta\ \frac{\partial}{\partial x}\ (\overline{LU})\} \tag{22}$$

then,

$$\{T\} = (\overline{LW}) - \frac{a^3}{D_s}\,\bar{\nabla}^4\ \eta_r(w)\ - \frac{2}{1-\nu}\,\frac{a^9}{D^3}\,\xi^4\ \eta^2\eta_r\ (w)$$

$$+\ \frac{2}{1-\nu}\ \frac{a^6}{D_s^2}\,\xi^2\ \eta\{(\frac{3-\nu}{2} + \xi^2\,\frac{1-\nu}{2})\nabla^2 + \xi^2(1-\nu)\ \frac{\partial^2}{\partial x^2}\}\ \eta_r(w)$$

$$-\ \frac{2}{1-\nu}\ \frac{a^6}{D_s^2}\,\xi^4\eta^2\ \{\nabla^4_w + (\frac{1}{\xi^2} + 1)w + 2\ \frac{\partial^2 w}{\partial\theta^2}\}$$

$$+\ \text{"}\eta\ \text{term"} \tag{23}$$

where

$$"\eta\ \text{term}" = \frac{2}{1-\nu}\ \frac{a^3}{D_s}\ \xi^2\eta[[(\frac{3-\nu}{2} + \xi^2\ \frac{1-\nu}{2})\nabla^2 + \xi^2 (1-\nu)\ \frac{\partial^2}{\partial x^2}\ \}(\nabla^4 w + (\frac{1}{\xi^2}+1)w$$

$$+ 2\ \frac{\partial^2 w}{\partial\theta^2}) - \xi^2\beta^2\ \frac{\partial^2 w}{\partial x^2} - \xi^2\ \alpha^2\ \frac{\partial^2 w}{\partial\theta^2}] \qquad (24)$$

Expansion of the argument of the "η term" gives

$$\arg\eta = \{[(1+\frac{1}{\xi^2})A_1 - \frac{\nu^2}{\xi^2}]\ \frac{\partial^2}{\partial x^2} + (1+\frac{1}{\xi^2})\ A_1\ \frac{\partial^2}{\partial\theta^2}$$

$$+ 2\nu\ \frac{\partial^4}{\partial x^4} + [\nu^2 - 3\nu + 6 + 3\xi^2\ (1-\nu)]\ \frac{\partial^4}{\partial x^2\partial\theta^2}$$

$$+ A_1\ \frac{\partial^4}{\partial\theta^4} + [A_1 - \xi^2\nu]\ \frac{\partial^6}{\partial x^6}$$

$$+ [3A_1 + \xi^2(2(1-\nu) - \frac{(1-\nu)^2}{4} - 1)]\ \frac{\partial^6}{\partial x^4\partial\theta^2}$$

$$+ [3A_1 + \xi^2(1-\nu - \frac{(1-\nu)^2}{4})]\ \frac{\partial^6}{\partial x^2\partial\theta^4} + A_1\ \frac{\partial^6}{\partial\theta^6}\ \}(w) \qquad (25)$$

in which

$$A_1 = \frac{3-\nu}{2} + \xi^2\ \frac{1-\nu}{2} \qquad (26)$$

Assuming that ξ^2 can be neglected with respect to one, equation (25) becomes

$$\arg\eta = [\frac{1-\nu}{2\xi^2}\ \nabla^2 + \frac{1-\nu^2}{\xi^2}\ \frac{\partial^2}{\partial x^2} + \frac{3-\nu}{2}\ \nabla^6 + (3-\nu)\ \nabla^4$$

$$+ (\nu-1)\{3\frac{\partial^4}{\partial x^4} + \nu\ \frac{\partial^4}{\partial x^2\partial\theta^2}\ \}](w) \qquad (27)$$

and equation (23) becomes

$$\{T\} = (LW) - \frac{a^3}{D_s} \nabla^4 \eta_r(w) - \frac{2}{1-\nu} \frac{a^9}{D^6} \xi^4 \eta^2 \eta_r(w)$$

$$+ \frac{3-\nu}{1-\nu} \frac{a^6}{D_s^2} \xi^2 \eta \nabla^2 \eta_r(w)$$

$$- \frac{2}{1-\nu} \frac{a^6}{D_s^2} \xi^4 \eta^2 \{\nabla^4 w + \frac{1}{\xi^2} w + 2 \frac{\partial^2 w}{\partial\theta^2}\} \tag{28}$$

$$+ \frac{2}{1-\nu} \frac{a^3}{D_s^2} \xi^2 \eta[\arg \eta]$$

where

$$LW = \frac{a^3}{D_s} \bar{\nabla}^4 \mu_r (W) - \frac{2}{1-\nu} \frac{a^6}{D^2} \xi^2 (\frac{3-\nu}{2} \nabla^2)[\eta \mu_r(W)]$$

$$+ \frac{2}{1-\nu} \frac{a^9}{D_s^3} \xi^4 \eta^2 \mu_r (W) + \frac{1+\nu}{1-\nu} [\alpha \frac{\partial}{\partial\theta} (LV) + \beta \frac{\partial}{\partial x} (LU)] \tag{28a}$$

with

$$LU = \frac{a^3}{D_s} \xi^2[\mu(\frac{\partial^2 V}{\partial x\partial\theta}) - \frac{2}{1+\nu} \chi \mu(U) + \frac{2}{1+\nu} \frac{a^3}{D_s} \xi^2 \eta \mu(U)]$$

and

$$LV = \frac{a^3}{D_s} \xi^2[\mu(\frac{\partial^2 U}{\partial x\partial\theta}) - \frac{2}{1+\nu} \Phi \mu(U) + \frac{2}{1+\nu} \frac{a^3}{D_s} \xi^2 \eta \mu(U)] \tag{28b}$$

Equation (27) is somewhat reminiscent of equation (20); three $\bar{\nabla}$ operators are involved, with an additional x derivative term corresponding to the lowest order $\bar{\nabla}$ operator. We choose to neglect the last term in equation (27) with the following justification: If the term is operated upon by the Laplacian (which would result in an expression even more reminiscent of (20)), then two derivatives in the final bracket are those neglected by Morley. The other term would be the second derivative with respect to x of the third term Morley neglected. This is dropped because, as Morley remarks, terms involving θ derivatives are more important than derivatives with respect to x. With this assumption, equation (27) becomes

$$\arg \eta = [\frac{3-\nu}{2} \nabla^4(\nabla^2 + 2) \quad + \frac{1-\nu}{2\xi^2} \nabla^2 + \frac{1-\nu^2}{\xi^2} \frac{\partial^2}{\partial x^2}](w) \tag{29}$$

Finally the equation for w is given by

$$\nabla^4(\nabla^2 + 1)^2 w + \frac{1-\nu^2}{\xi^2} \frac{\partial^4 w}{\partial x^4} = (LW) - \frac{a^3}{D_s}\{ \nabla^4 - \frac{3-\nu}{1-\nu} \frac{a^3}{D} \xi^2 \nabla^2 \eta$$

$$+ \frac{2}{1-\nu} \frac{a^6}{D_s^2} \xi^4 \eta^2 \} \eta_r(w) - \frac{2}{1-\nu} \frac{a^6}{D_s^2} \xi^4 \eta^2 [\nabla^4 + \frac{1}{\xi^2} + 2 \frac{\partial^2}{\partial \theta^2}](w) \tag{30}$$

$$+ \frac{2}{1-\nu} \frac{a^3}{D_s} \xi^2 \eta \{\frac{3-\nu}{2} \nabla^4(\nabla^2+2) + \frac{1-\nu}{2\xi^2} \nabla^2 + \frac{1-\nu^2}{\xi^2} \frac{\partial^2}{\partial x^2} \}(w)$$

Morley's static equations differ from Donnell's only in the w equation; the equations for u and v are identical to Donnell's. With this in mind, we complete the system of governing equations by listing the dynamic Donnell equations after non-dimensionalization (recall that these are had by setting $\xi^2 = 0$ in (5)):

$$\{ \nabla^4 - \frac{3-\nu}{1-\nu} \xi^2 \frac{a^3}{D_s} \nabla^2 \eta + \frac{2}{1-\nu} (\xi^2 \frac{a^3}{D_s})^2 \eta^2 \} (u)$$

$$= \frac{1+\nu}{1-\nu} (LU) + \{ \frac{2\nu}{1-\nu} \xi^2 \frac{a^3}{D_s} \eta + \frac{\partial^2}{\partial \theta^2} - \nu \frac{\partial^2}{\partial x^2} \} (\frac{\partial w}{\partial x}) \tag{31a}$$

$$\{ \nabla^4 - \frac{3-\nu}{1-\nu} \xi^2 \frac{a^3}{D_s} \nabla^2 \eta + \frac{2}{1-\nu} (\xi^2 \frac{a^3}{D_s})^2 \eta^2 \} (v)$$

$$= \frac{1+\nu}{1-\nu} (LV) + \{ \frac{2}{1-\nu} \xi^2 \frac{a^3}{D_s} - (2 + \nu) \frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial \theta} \} (\frac{\partial w}{\partial \theta}) \qquad (31b)$$

An Application to a Buried Pipe

As a simplified application of the equation (30) and (31a), consider an axisymmetric example in which

$$\begin{aligned} &\frac{\partial}{\partial t} = \frac{\partial}{\partial \theta} = 0 \qquad && \eta_r = \mu_r = 0 \\ &\eta = \mu = k && \omega = \frac{2\pi}{\lambda} \\ &V = W = 0 \\ &U = \bar{U} \cos\omega x \end{aligned} \qquad (32)$$

where $\bar{U}$, k, and λ are constants. Further,it is assumed that the pipe is infinitely long. Thus, this example is the steady-state solution when the ground is deformed sinusoidally in the direction of the pipe axis.

In light of the axial symmetry, we have

$$\nabla^2 = \frac{d^2}{dx^2}$$

$$\beta = \frac{\nu}{\xi^2} - \frac{d^2}{dx^2}$$

$$\alpha = \frac{1}{\xi^2} - \frac{3-\nu}{2}\frac{d^2}{dx^2} \tag{33}$$

$$\chi = \frac{1-\nu}{2}\ \frac{d^2}{dx^2}$$

The "loading term" (LW) is given by

$$(LW) = \{\omega^3 A - \omega B - \omega^5 C\}\sin\omega x \tag{34}$$

where,

$$A = \bar{U}\ \frac{1+\nu}{1-\nu}\ \frac{a}{K}\ \{\frac{-\nu(1-\nu)}{(1+\nu)\xi^2}\ k - \frac{2}{1+\nu}\ \frac{a}{K\xi^2}\ k^2\}$$

$$B = \bar{U}\ \frac{1+\nu}{1-\nu}\ \frac{a}{K}\ (\frac{-2\nu a k^2}{K(1+\nu)\xi^2}) \tag{35}$$

$$C = \frac{a}{K}\ k\bar{U}$$

Equation (30) becomes

$$(D^8 + pD^6 + qD^4 + rD^2 + s)(w) = (LW) \tag{36}$$

where

$$D = \frac{d}{dx}$$

and

$$p = 2 - \frac{3-\nu}{1-\nu}\frac{a}{K}k$$

$$q = \frac{1-\nu^2}{\xi^2} - 2ak \frac{3-\nu-\frac{ak}{K}}{K(1-\nu)}$$

$$r = -(2\nu+3)\frac{a}{K\xi^2}k$$

$$s = \frac{2a^2}{K^2\xi^2(1-\nu)} k^2$$

This polynomial in D in (36) clearly has at most eight distinct roots. Let n be the number of distinct roots, j_i the order of the i-th root, and let g_i be the i-th root itself. It will be noted that

$$\sum_{i=1}^{n} j_i = 8$$

A complementary solution for w is

$$w_c = \sum_{i=1}^{n} \sum_{j=1}^{j_i} b_{ij}\, x^{(j-1)}\, e^{g_i x}$$

where the b_{ij} are constants. It should be noted that, if any root has a non-zero real part or if any root has an order greater than one, then the constant b_{ij} corresponding to that root must vanish because $-\infty < x < \infty$.

A particular solution for w is found by setting[1]

$$w_p = P\sin\omega x + Q\cos\omega x$$

where P and Q are constants. Substitution into (36) yields

$$Q = 0$$

$$P = \frac{\omega^3 A - \omega B - \omega^5 C}{\omega^8 - p\beta^6 + q\omega^4 - r\omega^2 + s} \tag{37}$$

[1]Note that this implies that $g_i \neq \omega$ for all i=1,...,n

The complete solution is thus

$$w = w_c + P\sin\omega x \tag{38}$$

Thus a ground displacement in the axial direction gives rise to a radial pipe displacement.

Now, equation (31a) gives, for b_{ij} all zero,

$$\begin{aligned}(D^4 + mD^2 + n)u \\ &= \{(\frac{2\omega\nu}{1-\nu}\ \frac{ka}{K} + \nu\omega^3)P \\ &+ (\omega^2 \frac{ka}{K} + \frac{2}{1-\nu} (\frac{ka}{K})^2\ )\ \overline{U}\} \cos\omega x\end{aligned} \tag{39}$$

where

$$D = \frac{d}{dx}$$

and

$$\begin{aligned} m &= - \frac{3-\nu}{1-\nu}\frac{ka}{K} \\ n &= \frac{2}{1-\nu} (\frac{ka}{K})^2 \end{aligned} \tag{40}$$

The solution to (40) is found as before and is given by

$$u = \sum_{i=1}^{N} \sum_{j=1}^{j_i} d_{ij}(x)^{j-1} e^{c_i x} \tag{41}$$

where N is the number of distinct roots of the fourth-degree polynomial, c_i the roots, j_i the order of i-th root, and d_{ij} constants. Furthermore, u_p is the particular solution

$$u_p = F \cos\omega x + G \sin\omega x$$

where F and G are constants. Equation (40) gives

$$G = 0$$

$$F = \frac{[\omega^2 \frac{ka}{K} + \frac{2}{1-\nu} (\frac{ka}{K})^2]\overline{U} + [\nu\omega^3 + \frac{2\nu\omega}{1-\nu} \frac{ka}{K}] P}{\omega^4 - m\omega^2 + n} \tag{42}$$

Thus,

$$u = \sum_{j=1}^{N} \sum_{j=1}^{j_i} d_{ij} x^{(j-1)} e^{c_i x} + F \cos\omega x \tag{43}$$

and again, if any real part of the c_i is nonzero or if the order is greater than one, the d_{ij} term corresponding to that root must vanish.

Finally, v is seen to have the solution

$$v = \sum_{i=1}^{N} \sum_{j=1}^{j_i} f_{ij} x^{(j-1)} e^{c_i x} \tag{44}$$

where the c_i are the same as in (41) and the f_{ij} are constants. The same admonition following equation (43) also applies here.

The shell model thus gives displacements in three directions for only axial ground displacement. O'Rourke and Wang have investigated a similar problem using the beam model, which gives only axial displacements [9].

An Example

Let the dimensionless quantity ε be defined as

$$\varepsilon = \frac{ak}{K} \tag{45}$$

With this definition, A, B and C from equation (35) become

$$A = \overline{U} \left(\frac{-\nu}{\xi^2} \varepsilon - \varepsilon^2 \frac{1+\nu}{1-\nu}\right)$$

$$B = \frac{2\nu}{\xi^2(1-\nu)} \varepsilon^2 \overline{U}$$

$$C = \varepsilon\overline{U}$$

Thus

$$R_w = \frac{P}{\overline{U}} = \frac{\omega^3 A - \omega B - \omega^5 C}{(\omega^8 - p\omega^6 + q\omega^4 - r\omega^2 + s)\,\overline{U}} \tag{46}$$

where p, q, r and s are now given by

$$p = 2 - \frac{3-\nu}{1-\nu}\,\varepsilon$$

$$q = \frac{1-\nu^2}{\xi^2} - \frac{2(3-\nu)}{1-\nu}\,\varepsilon + \frac{2}{1-\nu}\,\varepsilon^2$$

$$r = -\,(2\nu + 3)\,\frac{\varepsilon}{\xi^2}$$

$$s = \frac{2}{1-\nu}\;\frac{\varepsilon^2}{\xi^2}$$

As an example, consider a 30" (76.2 cm) diameter steel pipe with

$$\begin{aligned} a &= 15 \text{ in } (38.1 \text{ cm}) \\ h &= 0.375 \text{ in } (.95 \text{ cm}) \\ \nu &= 0.3 \\ a\lambda &= 1000 \text{ ft } (304.8 \text{ cm}) \\ \omega &= a2\pi/12000 \text{ in } (a\ 2\pi/30480 \text{ cm}) \end{aligned} \tag{47}$$

and let all b_{ij}, d_{ij}, f_{ij} be zero. An order of magnitude analysis simplifies (46) to

$$R_w = \frac{-\dfrac{\nu}{\xi^2}\,\omega^3\varepsilon - \dfrac{2\nu}{\xi^2(1-\nu)}\,\omega\varepsilon^2}{\dfrac{1-\nu^2}{\xi^2}\,\omega^4 + (2\nu+3)\,\dfrac{\varepsilon}{\xi^2}\,\omega^2 + \dfrac{2}{1-\nu}\,\dfrac{\varepsilon^2}{\xi^2}} \tag{48}$$

It will be noted that

1) $\varepsilon = 0$ gives $R_w = 0$

2) For suitably large ε, $R_w \approx -\,\nu\omega$

Furthermore, a plot of R_w and ε is to be found in Figure 6.

Similarly, if the ratio of pipe axial displacement amplitude to ground displacement amplitude is defined as

$$R_u = \frac{F}{\bar{U}} \tag{49}$$

then

$$R_u = \frac{\nu\omega^3 R_w + [\frac{2\nu}{1-\nu}\,\omega\, R_w + \omega^2]\varepsilon + \frac{2}{1-\nu}\,\varepsilon^2}{\omega^4 + \frac{3-\nu}{1-\nu}\,\omega^2\varepsilon + \frac{2}{1-\nu}\,\varepsilon^2} \tag{50}$$

Here it will be noted that if k=0 (i.e., the soil and pipe are not interconnected), $R_w = R_u = 0$, which is to say, the ground's deformation does not give rise to either axial or radial displacements in the pipe. As seen in Figure 7, R_u is approximately one for $\varepsilon > 10^{-3}$. In this range of ε, then, the pipe's displacement follows that of the ground.

Conclusions

It will be noted that in Figures 6 and 7 three regions of ε are defined, I, II and III. In region I, k < 8.3 psi (57.2 KPa) and the ratio R_u is quite small. In III, k > 830 psi (5.72 M Pa) and $R_u \approx 1$, indicating that the pipe moves with soil. Region II represents a transition from I to III.

It is well known that, during seismic excitation, a buried pipe moves with the ground [3,10]. Thus, demanding that ε lie in Region III, we have

$$830 \text{ psi } (5.72 \text{ M Pa}) \leq k \tag{51}$$

For k satisfying (51) it would appear that the thin shell theory of buried pipelines give results consistent with observation [3,10]. However, use of the thin shell model does give rise to other displacements (and hence, stresses) in the pipe. In the present model, account is taken of the coupled displacements arising from the curvature of the pipe and therefore it appears that the shell model is not only necessary for the investigation of the buckling and failure modes of the buried pipelines but it also provides a more realistic stress field than the beam model in the seismic investigation of pipelines.

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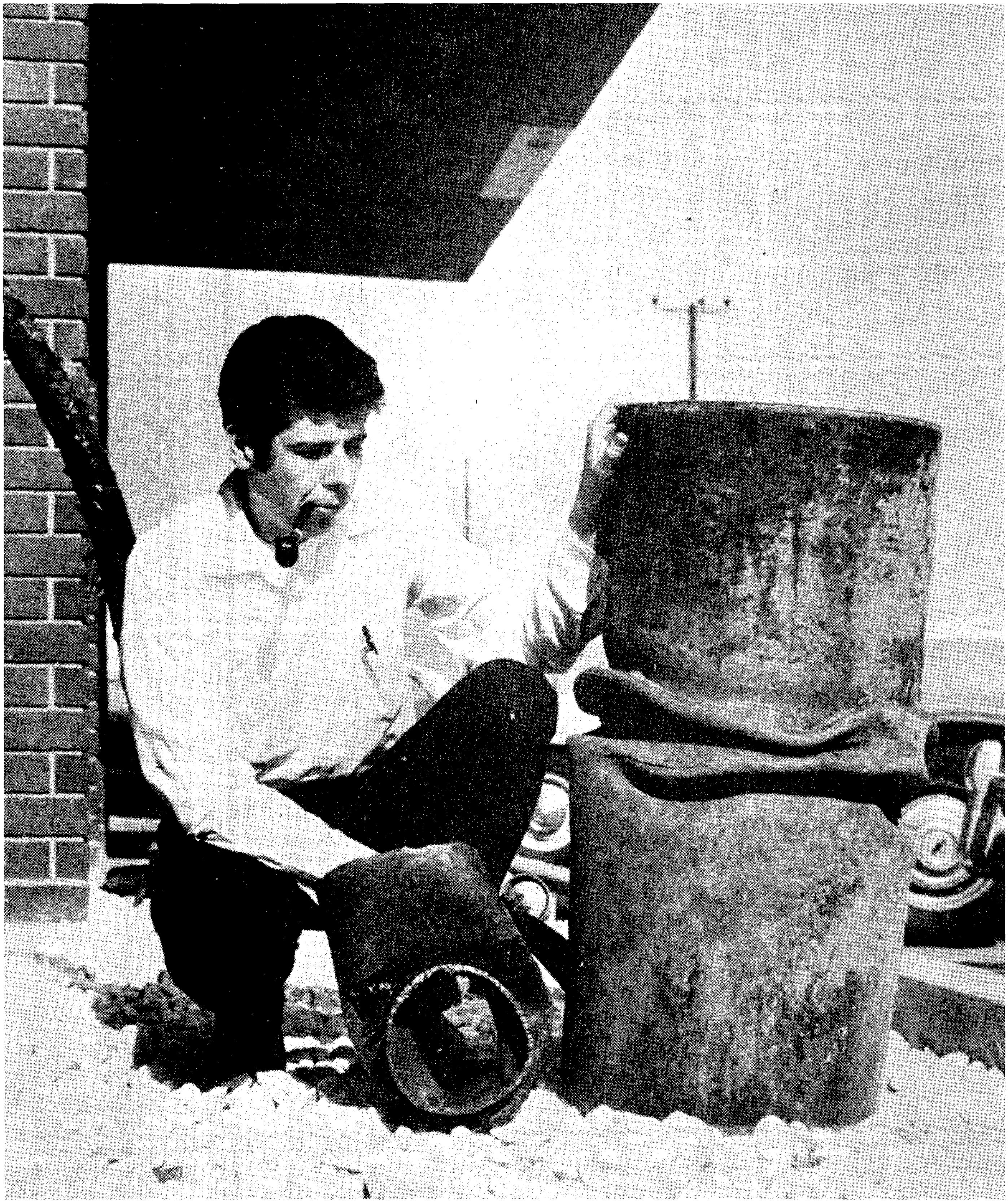

Figure 1 Compression Forces Buckled 16 inch Steel Pipe and 10 inch T.

Figure 2 Brittle Crack Propagation in a Concrete Pipe.

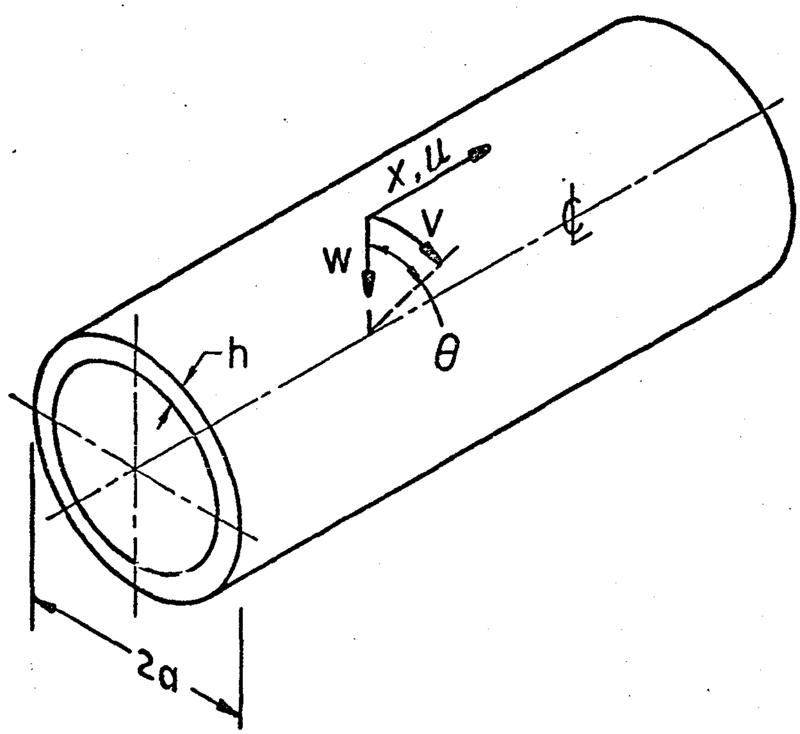

Figure 3. Geometry of the Cylindrical Shell.

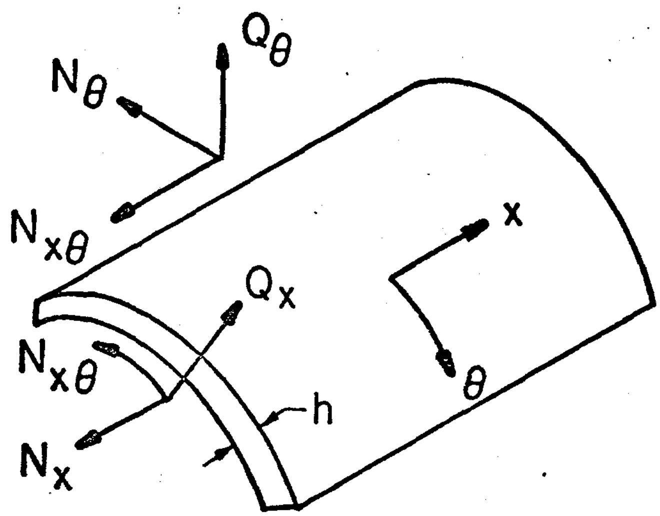

Figure 4. Force Stress Resultants on Shell Element.

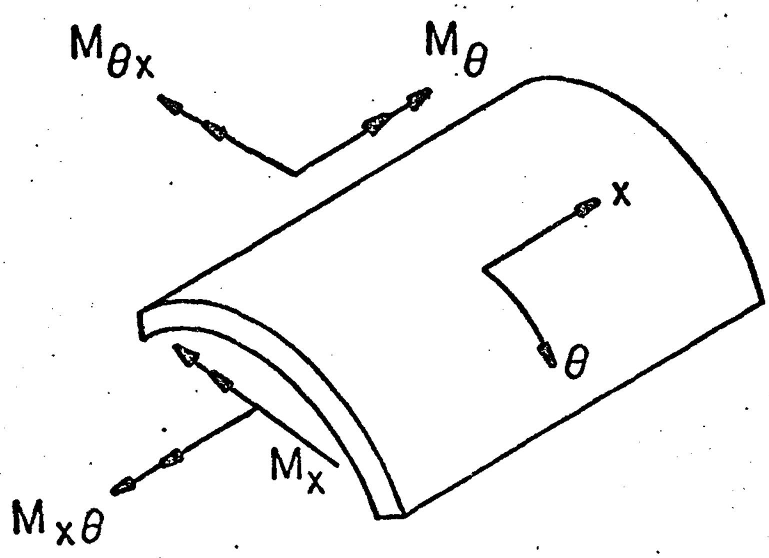

Figure 5. Moment Stress Resultants on Shell Element.

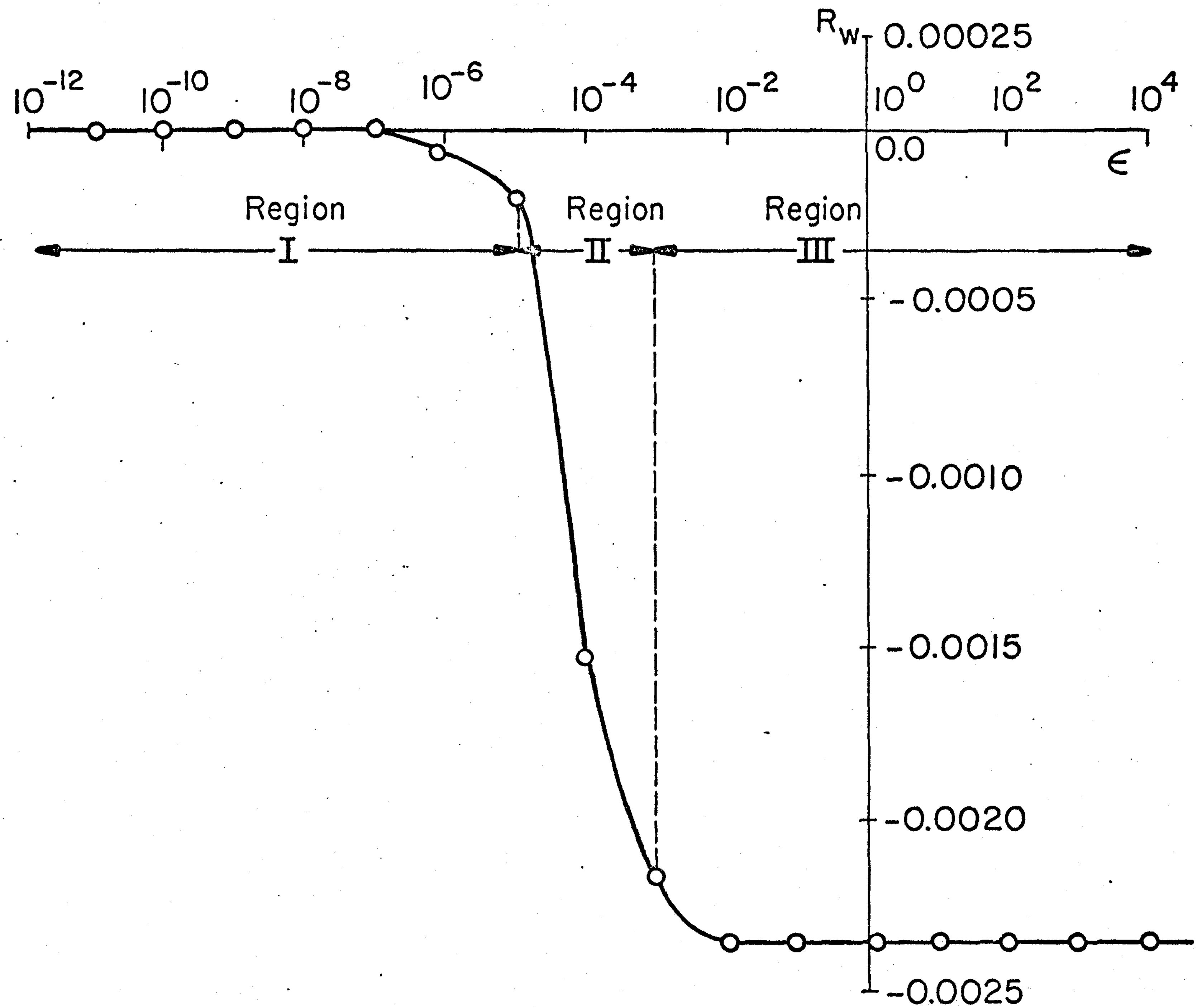

Figure 6. Ratio of Radial Displacement Amplitude to Ground Axial Displacement Amplitude in Axial Direction for Varying ε.

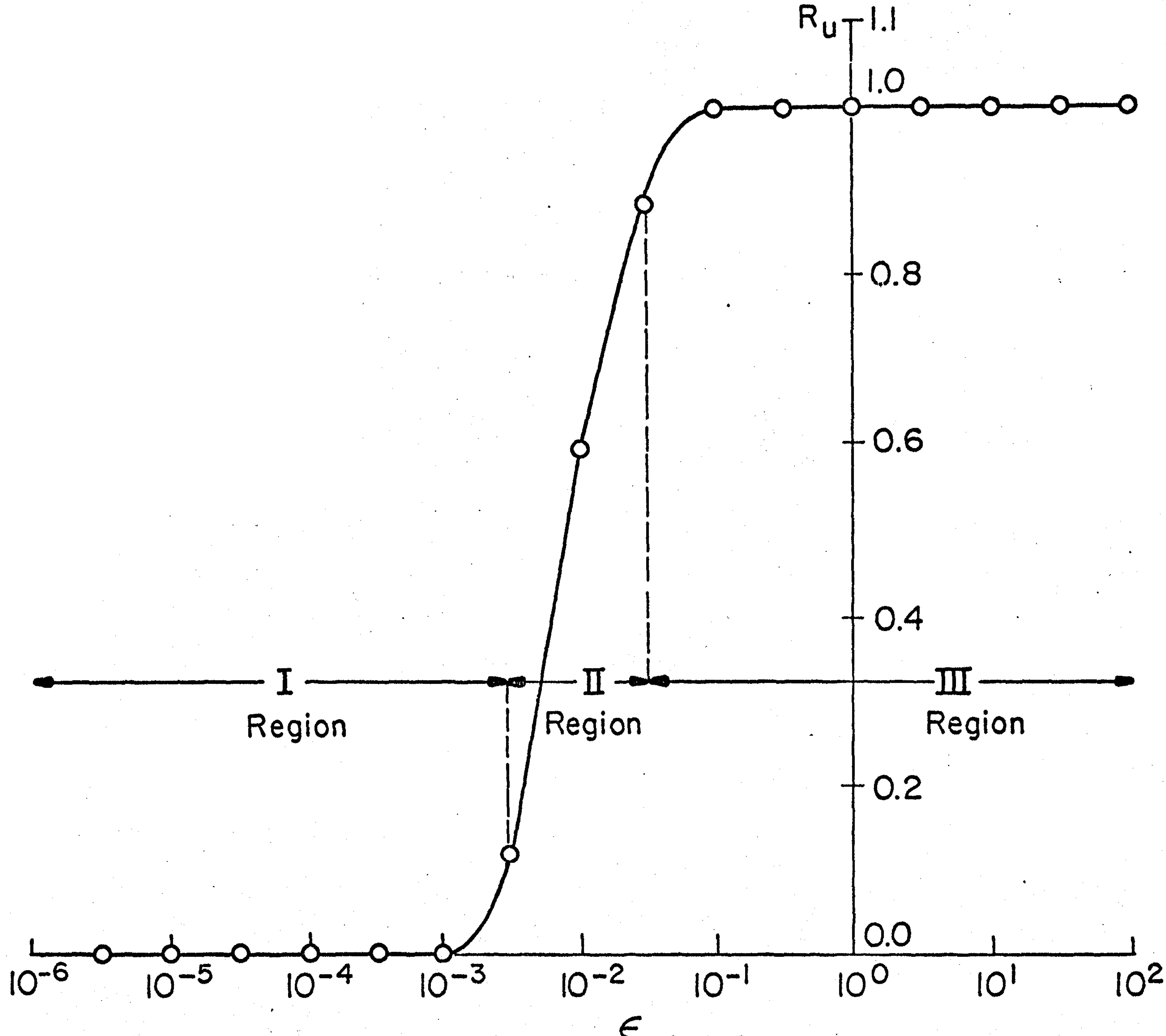

Figure 7. Ratio of Axial Displacement Amplitude to Ground Axial Displacement Amplitude for varying ε.

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15. Supplementary Notes

16. Abstract (Limit: 200 words)

In a number of recent investigations, a buried pipe undergoing seismic excitation was modelled as a beam on a visco-elastic foundation. However, it is known that two of the observed failure modes in the buried pipelines under seismic loads are buckling and fracture. Therefore, in this paper, a thin circular cylindrical shell model in a resisting soil medium is used for a buried pipe. The coupled equilibrium equations arising from this model are modified to yield three decoupled equations which somewhat simplify the analysis. Then, an application for a long buried pipe is presented and the results are discussed and compared with those of the beam model.

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