OPTIMUM SEISMIC PROTECTION FOR NEW BUILDING CONSTRUCTION IN EASTERN METROPOLITAN AREAS

NSF GRANTS GK-27955 and GI-29936

Internal Study Report No. 40

MULTISTATE DAMAGE ANALYSIS OF NETWORKS

George Panoussis

May 1974

Department of Civil Engineering

Massachusetts Institute of Technology

Cambridge, Massachusetts

50272-101

REPORT DOCUMENTATION PAGE	1. REPORT NO. NSF-RA-E-74-532	2.	3. Recipient's Accession No. PB 80 116742
4. Title and Subtitle Multistate Damage Analysis of Networks (Optimum Seismic Protection for New Building Construction in Eastern Metropolitan Areas, Internal Study Report 40)			5. Report Date May 1974
			6.
7. Author(s) G. Panoussis			8. Performing Organization Rept. No. No. 40
9. Performing Organization Name and Address Massachusetts Institute of Technology, Department of Civil Engineering, Cambridge, Massachusetts 02139			10. Project/Task/Work Unit No.
			11. Contract(C) or Grant(G) No. (C) (G) GK27955 GI29936
12. Sponsoring Organization Name and Address Engineering and Applied Science (EAS), National Science Foundation, 1800 G Street, N.W., Washington, D.C. 20550			13. Type of Report & Period Covered
			14.

15. Supplementary Notes

16. Abstract (Limit: 200 words)

This report extends the two-state elements, survival and failure, to the case in which these elements have two or more possible partial damage states. Several schemes are explored for defining and evaluating the condition of an entire network as a single scalar value, given the conditions of the network's elements. The schemes are developed to facilitate seismic risk analysis. Damage states used in the study are: no damage; moderate damage; heavy damage; and total destruction/inoperative. A series system is presented in which three different methods for describing the network state as a function of the element's states are shown. Also described are three schemes analogous to the series system for which a system in parallel is developed. Finally, the report contains examples and results of complex systems and describes the concept of tie sets as a tool for analyzing networks.

17. Document Analysis a. Descriptors

Damage assessment	Prediction	Failure(Functional breakdown)
Analyzing	Risk	Buildings
Mathematical Models	Survival	Earthquake resistant structures

b. Identifiers/Open-Ended Terms

Seismic risk analysis
Tie sets

c. COSATI Field/Group

18. Availability Statement	19. Security Class (This Report)	21. No. of Pages
NTIS		20
	20. Security Class (This Page)	22. Price A02 - A01

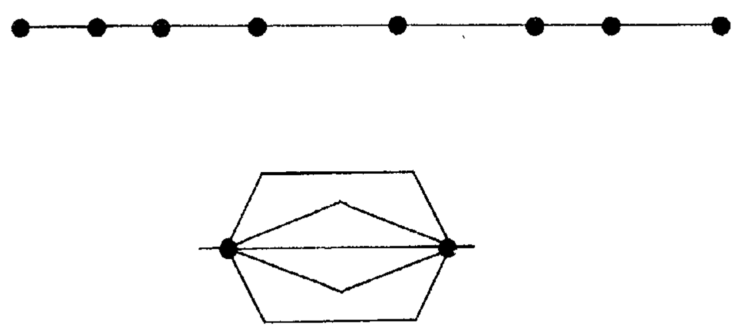
(See ANSI-Z39.18) See Instructions on Reverse OPTIONAL FORM 272 (4-77) (Formerly NTIS-35) Department of Commerce

PREFACE

This report is the second of a series of reports to be published for the Seismic Design Decision Analysis (SDDA) of lifelines project.

INTRODUCTION

Much is known about the reliability of systems of simple fail-survive (0-1) elements in series and in parallel (see Figure 1). The two fundamental concepts are



[a system of n elements in series will survive] =

$$[\text{each } i^{th} \text{ element survives}, i = 1, 2, \ldots, n] = [S_1 \cap S_2 \cap \ldots \cap S_n] \quad \ldots\ldots (1)$$

and

[a system of n elements in parallel will fail] =

$$[\text{all elements fail}] = [F_1 \cap F_2 \cap \ldots \cap F_n] \quad \ldots\ldots (2)$$

where S_i = event that element i survives

and F_i = event that element i fails.

Furthermore, Eqs. (1) and (2) can be translated into the following statements: "A series system will fail iff at least one element fails" and "a parallel system will survive iff at least one element survives."

So far, we have assumed two-state elements, the states being "survival" and "failure". The present study extends this to the case in which the elements have two or more possible partial damage states. Several schemes are explored for defining and evaluating the condition of an entire network as a single scalar value, given the conditions of the network's elements.

Such schemes will facilitate system seismic risk analysis. The problem is to find one or more such schemes that (1) make sense physically, (2) give correct results when reduced to the simple "survival-failure" case, and (3) give consistent results when alternative (equivalent) representations of the same network are analyzed. No unique answer can be expected; one scheme may seem more appropriate, for example, for transportation systems and another for water distribution networks.

DAMAGE STATES

It is recommended that the following damage states be used:

Designation	Damage State
0	no damage
1	light damage
2	moderate damage
3	heavy damage
4	total destruction/inoperative

This specific recommendation does not reduce the generality of the problem, nor are any implications made about the type of scale (regular cardinal, or other), except that the scale is ordered (e.g., damage state 3 is worse than damage state 2).

SERIES SYSTEM*

Assume a given system of n elements in series. Three different methods or describing the network state as a function of the elements' states are recommended for study:

1) $State_{TOT} = Max[State_i,\ i = 1, 2, 3, \ldots, n]$ .... (3)

where $State_{TOT}$ = state of network,

and $State_i$ = state of the i^{th} element.

This scheme is part of what will be called the "minimax" scheme.

2) $State_{TOT} = 4 - \sqrt[n]{\prod_{i=1}^{n} (4 - State_i)}$ .... (4)

This scheme will be referred to as the "p-norm" scheme.

3) $State_{TOT} = 4 - \dfrac{\sum_{i=1}^{n} L_i}{\sum_{i=1}^{n} \dfrac{L_i}{4 - State_i}}$ .... (5)

where L_i = an importance characteristic of the i^{th} element (a weighting function) perhaps equal to simply unity for all elements, perhaps not

This scheme will be referred to as the "resistivity" scheme.

---

* For the present assume that the network has only one input and one output.

All three schemes give identical results at the extremes. For instance, assume that all elements are at a 0 damage state. The total state of the network, as calculated from equations (3), (4) and (5), is also 0, as desired. Similarly, if any element is at a damage state 4, the total state of the network is 4, again as desired. These schemes are all applicable to the cases of 0-1 (survive-fail) states of elements.

For in-between values, however, the results vary from scheme to scheme. Also, the p-norm and resistivity schemes may produce non-integer values, which may cause difficulty in interpolation for certain definitions of the original damage state designations.

SYSTEM IN PARALLEL

Three schemes analogous to the series system are developed for a system with n elements in parallel.

1) $State_{TOT} = \mathrm{Min}[State_i,\ i = 1, 2, \ldots n]$ .... (6)

This is the second half of the "minimax" scheme.

2) The "p-norm" scheme:

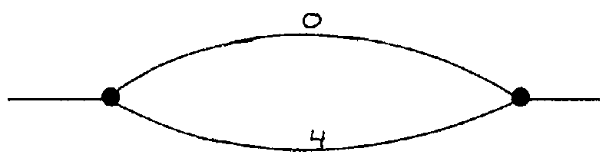
$$State_{TOT} = \sqrt[n]{\prod_{i=1}^{n} (State_i)} \qquad (7)$$

3) The "resistivity" scheme:

$$State_{TOT} = \frac{\sum_{i=1}^{n} (State_i) \cdot L_i}{\sum_{i=1}^{n} L_i} \qquad (8)$$

At the extremes, these schemes give the same results as a network of 0-1 type elements. That is, the system is in state 4 iff <u>all</u> elements are in state 4.

There is one exception: assume a two-element network (Figure 2)



with $State_1 = 0$ and $State_2 = 4$. From the first approach:

$$State_{TOT} = Min[0,4] = 0 \quad \ldots\ldots\ldots\ldots \quad (9)$$

From the second approach:

$$State_{TOT} = \sqrt[2]{0.4} = 0 \quad \ldots\ldots\ldots\ldots \quad (10)$$

From the third approach:

$$State_{TOT} = \frac{0 \cdot L_1 + 4 \cdot L_2}{L_1 + L_2}$$

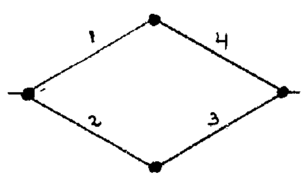
$$= 4 \cdot \frac{L_2}{L_1 + L_2} \neq 0 \quad \ldots\ldots\ldots\ldots \quad (11)$$

Although this last answer does not coincide with equations (9) and (10), the result may, in fact, for some cases be a preferred system description. For instance, if Figure 2 is a representation of two pipelines in parallel distributing oil, and one of the pipelines is totally destroyed, equation (11) suggests that the flow will not be the same as if no damage had occurred, as is implied in 0-1 state reliability models.

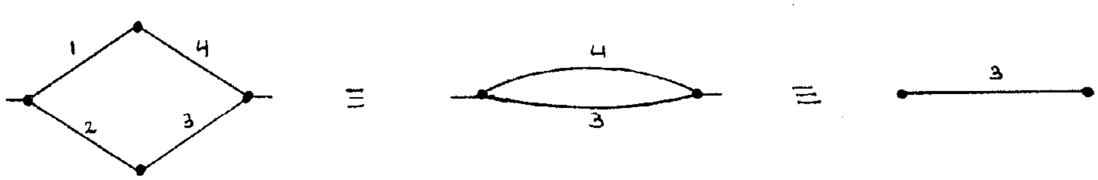
EXAMPLES OF COMPLEX SYSTEMS

It is assumed that the reader is familiar with reducing complex systems into subsystems of elements in series and/or in parallel.*

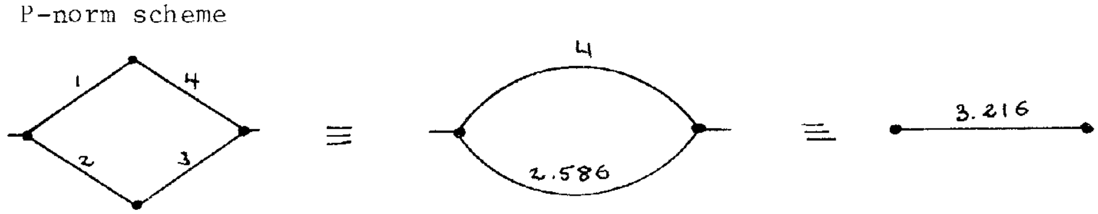
A. Find the damage state of the network in Figure 3. The damage state of each element is written adjacent to each element.



1) Minimax scheme



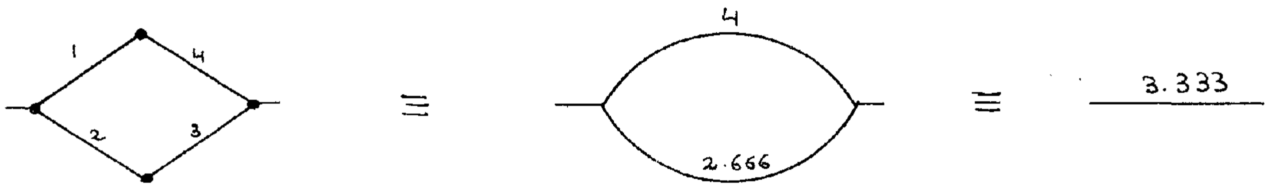
2) P-norm scheme



---

* See Probabilistic Reliability by M. L. Shooman, McGraw-Hill, New York, 1968, pp. 119-158, for review.

3) Resistivity scheme (equal L_i's)



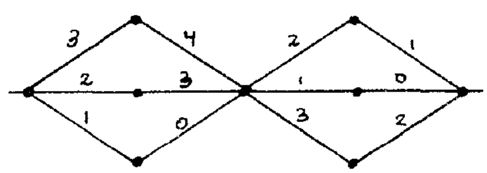
The damage states computed are, according to scheme

1) 3

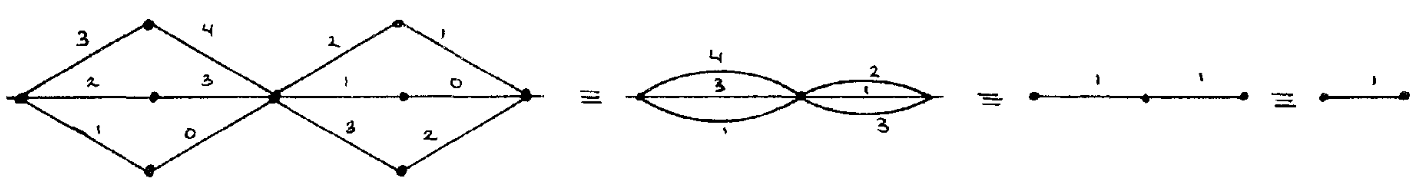
2) 3.216

3) 3.333

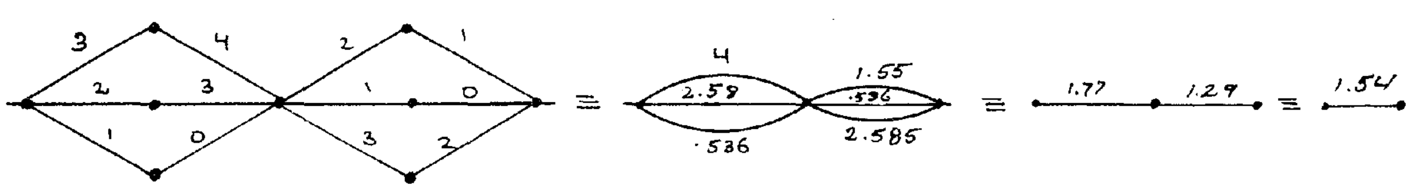
B. Find the damage state of the network in Figure 4.



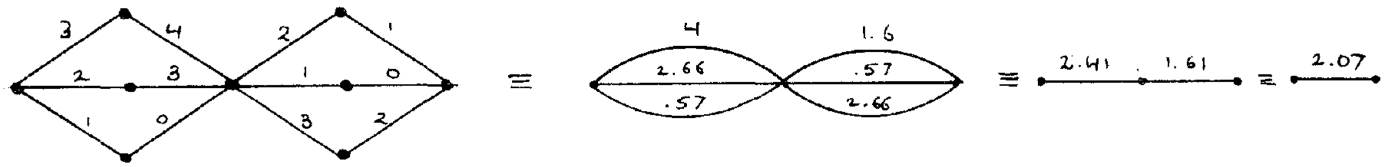
1) Minimax scheme



2) P-norm scheme



3) Resistivity scheme (equal L_i's)



The damage states computed are, according to scheme

1) 1

2) 1.54

3) 2.07

TIE SETS

The concept of a tie set is being introduced as a tool for analyzing networks. A <u>tie set</u>* is a set of elements and nodes one traverses from input to output without crossing the same element or node more than once. For example, the tie sets of example B are (as designated by damage state of elements):

3 - 4 - 2 - 1

3 - 4 - 1 - 0

3 - 4 - 3 - 2

2 - 3 - 2 - 1

2 - 3 - 1 - 0

2 - 3 - 3 - 2

1 - 0 - 2 - 1

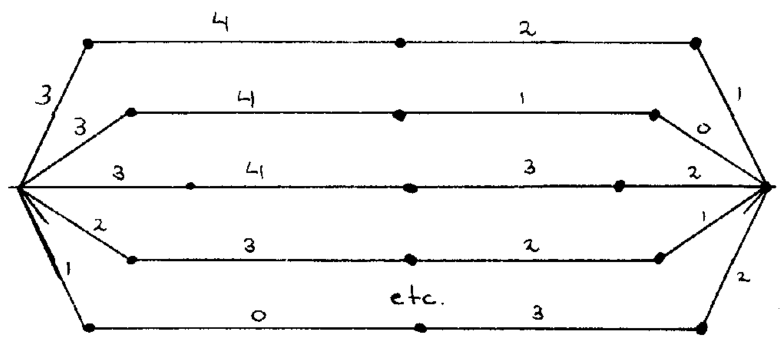
1 - 0 - 1 - 0

1 - 0 - 3 - 2

---

* See Internal Study Report No. 38.

The same network of example B is equivalent to a network consisting of all tie sets in parallel (Figure 5).



Using any one of the three schemes developed earlier in this report now becomes much easier due to the reduced complexity of the network, i.e., series systems in parallel.

NON-UNIQUENESS OF ANSWERS

Unfortunately two of the described schemes (namely the P-norm and resistivity schemes) yield different answers when processed by the two different methods, as tabulated in Table 1 for example B.

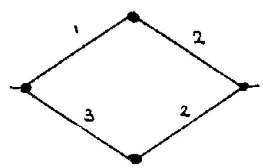
Scheme	Combinatorial Method	Tie Set Method
Minimax	1	1
P-norm	1.54	2.06
Resistivity	2.07	2.54

TABLE 1

RESULTS OF EXAMPLE B

The reason for the apparent discrepancy is the fact that each method places a different weight on each element depending on damage state and location in the network. This may be best illustrated by the following examples C and D.

C. Find the damage state of the network in Figure 6.



The results are tabulated in Table 2.

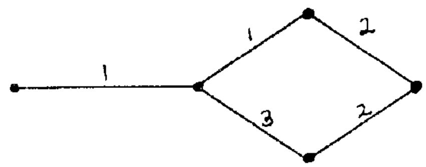
Scheme	Combinatorial Method	Tie Set Method
Minimax	2	2
P-norm	2.00	2.00
Resistivity	2.13	2.13

TABLE 2

RESULTS OF EXAMPLE C

In this case the results are, of course, the same, as the two methods use the same system representation.

D. Find the damage state of the network in Figure 7.



The results are tabulated in Table 3.

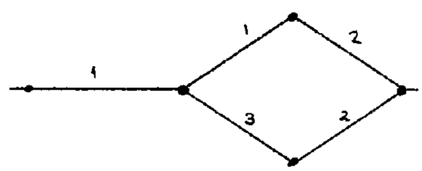
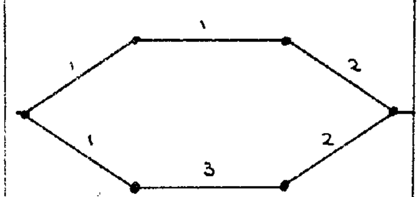
	Combinatorial Method	Tie Set Method
Equivalent Network		
Minimax	2	2
P-norm	1.551	1.736
Resistivity	1.509	1.896

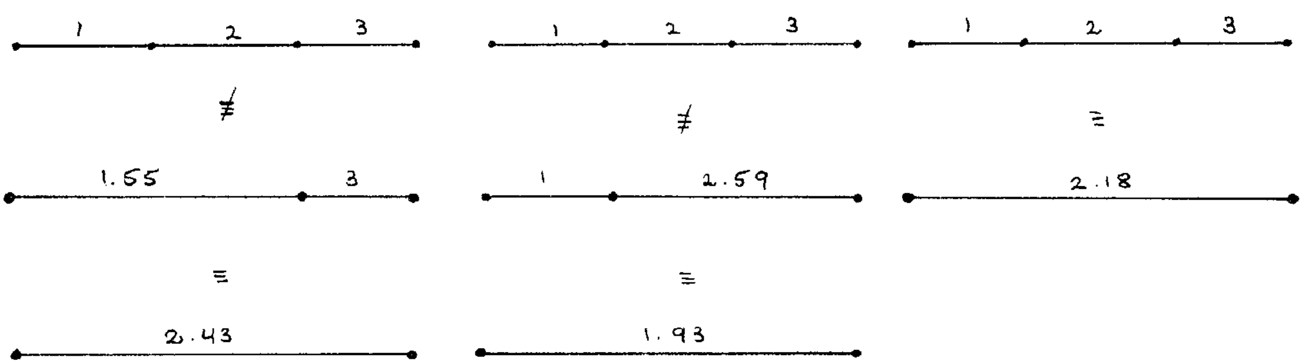
TABLE 3

RESULTS OF EXAMPLE D

The combinatorial method gives a higher weight to element (a) than the tie set method, hence the lower answers.

This apparent discrepancy should not be alarming for the following reason: a primary objective of this total study is the determination of the probability of (total) survival or (total) failure (i.e., states 0 or 4) of the given network and for both sets of methods these will be the same. For intermediate damage states, the concept of "system damage" is not well-defined. Therefore, in order to make these schemes unambiguous, we must <u>define</u> the P-norm and resistivity system analysis schemes as applying <u>only</u> to the reduced network of link systems in parallel (i.e., only to be used in conjunction with the tie set method). With this more inclusive definition, the system state for any particular scheme will be unique, given the link (or element) damage state.

Consideration must be given to reduce a series system of n links directly into a one-link system. One cannot, in general, replace two of the n links by an equivalent single link, thus reducing the system to (n-1) links, etc., up to the point where there is only one link in the system. For example, using the P-norm scheme for three links:



$$4 - \sqrt{(4 - (4 - \sqrt{(4-1)(4-2)}))(4-3)} = 2.43$$

$$\neq 4 - \sqrt{(4 - (4 - \sqrt{(4-3)(4-2)}))(4-3)} = 1.93$$

$$\neq 4 - \sqrt[3]{(4-1)(4-2)(4-3)} = 2.18$$

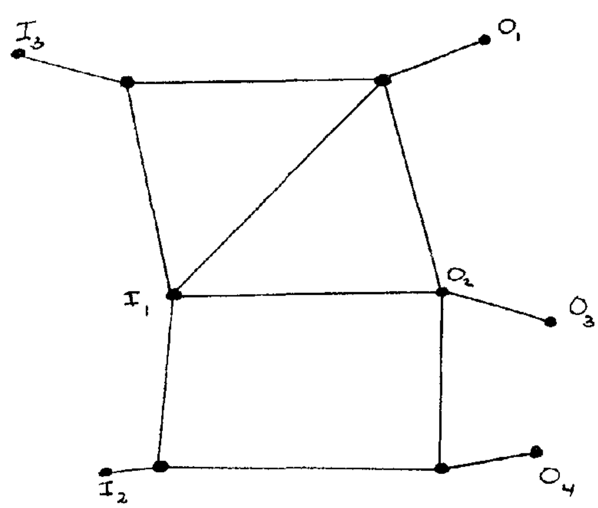
The procedure on the right is, by definition here, the "correct" one.

An analogous statement applies for parallel schemes. The extreme cases, however, (failure of series system if any link fails, survival of parallel system if any link survives) always work.

The minimax scheme gives a unique answer, regardless of the method (conbinatorial or tie set) by which the analysis is performed. This is a very desirable property; therefore, for the remainder of this study, the minimax scheme will be used. Perhaps new schemes can be developed in the future that satisfy all the intuitively desirable properties forementioned, as the minimax method does.

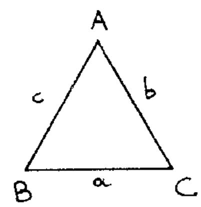
MULTI-INPUT/OUTPUT NETWORK

Given a network with many inputs (I_i) and many outputs (O_j) (Figure 8), one may wish to find its composite damage state. This is accomplished by finding all tie sets from each input to each output and then using all tie sets as chains in the new equivalent network. Study reports to be submitted in the very near future will focus on this subject.



Example

Given the network of Figure 9,



points A, B and C are defined to be both inputs and outputs and a, b and c are the elements of the network (two-dimensional).

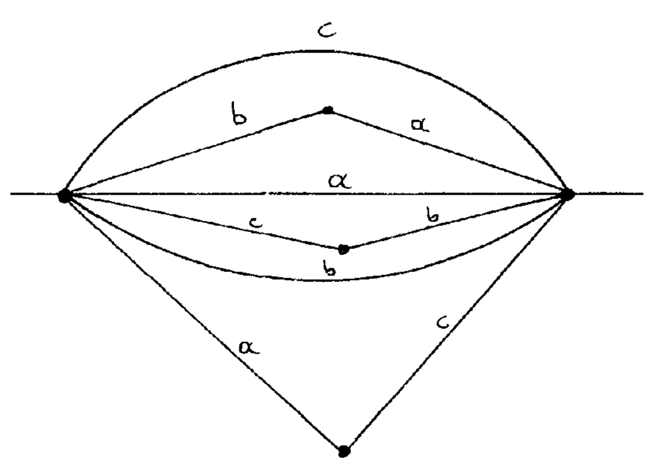
All tie sets are

$$\left.\begin{matrix} c \\ b - a \end{matrix}\right\}\ \text{joining A and B}$$

$$\left.\begin{matrix} a \\ c - b \end{matrix}\right\}\ \text{joining B and C}$$

$$\left.\begin{matrix} b \\ a - c \end{matrix}\right\}\ \text{joining C and A}$$

The equivalent network is



Given the damage states of a, b and c, we may calculate the network's total damage state.

Since paths from A to B and B to A are of equal importance but are obviously identical for the two-directional elements, they may or may not be included in the equivalent network. This arbitrariness is due to the dependence of the two sets of paths (A to B survives (fails) iff B to A survives (fails)).