AN APPROACH TO DAMAGE ASSESSMENT
OF EXISTING STRUCTURES

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Abstract

Important structures are frequently tested and inspected by structural engineers following the occurrence of a hazardous event such as a strong-motion earthquake. Usually, voluminous data are obtained from such an inspection. On the other hand, the expected conclusion of the study of these data can be a simple statement such as "this structure has been severely damaged". While it is relatively simple to understand various experimental and analytical procedures in the investigation of any particular structure, the complex decision-making process summarizing the many results of such an investigation into a simple concluding statement remains as privileged and specialized knowledge for highly qualified structural engineers.

In this report, an attempt is made to explore the application of fuzzy sets as an alternative and/or supplementary approach to assessing the damage state of existing structures. This proposed methodology can be used to incorporate the experience, intuition, and judgement of various experts, who may be willing to verbalize their valuable knowledge in advancing the state of the art of our profession.

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James T. P. Yao[1]

1. INTRODUCTION

1.1 General Remarks

Following the occurrence of a strong-motion earthquake, it is desirable to make safety evaluations by inspecting and testing important structures in the region. Resulting information and data from such inspections and tests are then analyzed and used by structural engineers as the bases for their recommendations concerning any necessary repairs.

In current practice, a given structure can be investigated both analytically and experimentally (1,2)*. Analytical investigations consist of the examination of original design calculations and drawings, the review of project specifications, and the analysis of the structure using additional field observations and test data. Experimental investigations include the determination of locations of damaged members, the application of nondestructive testing techniques to the structure, the search for defective workmanship and construction details, the proof-loading and other types of load testing of the structure, and the examination and testing of samples of the structural materials which are collected from the field.

Voluminous data usually result from these analytical and experimental studies. On the other hand, a typical conclusion that is expected from these studies can be a relatively simple statement, such as "this particular structure has been severely damaged by the recent earthquake". For most structural engineers, the various analytical and experimental procedures in a

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*Numerals in parentheses refer to the list of references as given in Appendix A.

given investigation can be readily understood. However, the complex decision-making process summarizing the many results of such an investigation into a simple concluding statement remains as privileged and specialized knowledge for a relatively few highly qualified structural engineers. Moreover, the transmission of the precious knowledge of such a decision-making process to younger engineers depends primarily on many years of close working relationships between experienced engineers and their apprentices.

1.2 Objective and Scope

An attempt is made herein to explore the application of fuzzy sets as an alternative and/or supplementary approach to assessing the damage state of existing structures. The state-of-theart of damage identification of existing structures is summarized and discussed. The fundamental elements of fuzzy sets are then presented with structural engineering examples. Finally, an approach using fuzzy sets is formulated and discussed. It is hoped that such a pilot study will help to stimulate interest among structural engineers in this direction.

2. DAMAGE IDENTIFICATION OF EXISTING STRUCTURES

In structural engineering practice, both analysis and design usually involves iterative procedures. Prior to the construction phase, it is necessary to make use of mathematical representations which result from generalizations of available knowledge in the structural engineering profession. Following the completion of the construction process, each structure has its own characteristics, which can no longer be described with the same initial mathematical models used in the design phase. During this past decade, techniques of system identification have been applied to modify and improve such mathematical representations for subsequent dynamic analyses. Available literature in this subject area have been reviewed by several investigators to-date (3-9).

In the classical theory of system identification, the order, form, and parameters of the system differential equation are estimated from excitation and response measurements, which are often noise-polluted. To improve the mathematical representation of a real structure, response records with or without known excitation have been collected and analyzed with the use of system identification techniques. In reality, these tests are usually performed at small response amplitudes to avoid the exceedence of any serviceability or safety limit states (10). Consequently, the applicability of the resulting mathematical model is restricted to the linear or slightly nonlinear range of the structural behavior. Although such mathematical representations are more realistic for making further linear analysis of the structure under consideration, these mathematical models are not applicable for making safety analyses involving catastrophic loading conditions such as strong-motion earthquakes. In addition, it is well known that nonlinear structural behavior is load-history dependent. Therefore, it is difficult to obtain a simple mathematical relationship to simulate the nonlinear, load-dependent, and time-variant behavior of complex structures subject to natural hazards. One alternative is to assess the extent of structural damage following each catastrophic event, and then use the results of such an assessment to modify and up-date the corresponding mathematical representation (11,12).

Liu and Yao (13) presented a comprehensive literature review of damage functions and discussed the general problem of structural identification. For structural engineers, it is important to estimate the damage state or structural reliability at the time of the test and inspection in addition to obtaining a set of differential equations or generalized impulse response functions. Recently, Gorman (14) considered the undesirable consequence of structural damage as a measure of risk. Nevertheless, it is still difficult to clearly define the degree of damage of a prototype complex structural system which exists in the real world.

Most civil engineering structures are massive, stiff, and individuallly designed and constructed. Consequently, it is much more costly to conduct full-scale tests than other types of structures such as airplanes. Nevertheless, many such structures have been tested but usually under small-amplitude dynamic loading conditions (8,10). Recently, destructive and dynamic full-scale tests were performed on an 11-story reinforced concrete building (15) and a 3-span steel highway bridge (16). Experimental data from such full-scale destructive tests are considered to be very important in the development of a rational approach to damage assessment of existing structures (17). In the formulation as described in the following,procedures for the practical application of such full-scale test data will be introduced and discussed (see Chapter 5).

3. ELEMENTS OF FUZZY SETS

3.1 General Remarks

According to Zadeh (18,19), our ability for making both precise and significant statements concerning a given system diminishes with increasing system complexity. He concluded that, the closer one looks at a real-world problem which is usually complex, the fuzzier its solution becomes. Although the theory of fuzzy sets is relatively new, the calculus of fuzzy sets is well developed with various applications (20,21). The application of fuzzy sets to several civil engineering problems was reviewed recently (22). In the following, fundamental elements of the theory of fuzzy sets as given by Zadeh (19) and Kaufmann (20) are summarized along with several structural engineering examples from Yao (22) and a simplified version of an example on structural reliability from Brown (23).

A linguistic variable is defined as a variable, the values of which are words, phrases, or sentences in a given language. For example, structural damage can be considered as a linguistic variable if the values of this variable such as "severely damaged", "moderately damaged", etc., may not be clearly defined but are meaningful classifications. In many situations, a complex problem can be divided into simpler questions. Some of these questions can best be answered by experienced engineers with descriptive words such as "large" or "medium", which are values of a given linguistic variable. The theory of fuzzy sets can be used to interpret such adjuctives with membership functions, which can be manipulated in a logical manner to obtain an answer to the original and complex problem.

3.2 Basic Definitions

A fuzzy set A in a given sample space Ω is a set of ordered pairs $\{(x|\mu_A(x))\}$, for each $x \in \Omega$, where $\mu_A(x)$ is called the membership function which takes its values in a membership set M. If the membership set consists of only two elements, say 0 and 1, i.e., $M = \{0,1\}$, then A is said to be an ordinary (or nonfuzzy, or crisp) set. For fuzzy sets, the membership set usually consist of the continuous interval 0 to 1, i.e, $M = [0,1]$. As an example, let N be the set of natural numbers, i.e., $N = \{0,1,2,\ldots\}$. Consider the fuzzy set A of "small" natural numbers as follows:

$$A = \{(0|1),\ (1|0.8),\ (2|0.6),\ (3|0.3),\ (4|0),\ldots\} \qquad (1)$$

In words, we say that the number "0" has a nonfuzzy membership, "1" has a "strong" membership, "2" has a "fairly strong" membership, "3" has a "weak" membership, "4" and higher numbers have non-memberships of the fuzzy set A of "small" natural numbers.

As another example, let y denote the proportion of cracked width of an uniaxially-loaded plate, and let B denote the "severely damaged" state of this plate. Then, we may write

$$B = \{(y<0.1|0),\ (0.1\le y\le 0.6|2(y-0.1)),\ (y>0.6|1)\} \qquad (2)$$

or

$$\mu_B(y) = \begin{cases} 0, & y < 0.1 \\ 2(y-0.1), & 0.1 \le y \le 0.6 \\ 1 & y > 0.6 \end{cases} \tag{3}$$

Such a description as shown in Figure 1 can be the result of compiling and analyzing the subjective evaluation of a number of experts. As it is given in this contrived example, this plate specimen is clearly (nonfuzzy) severely damaged whenever $y > 0.6$, i.e., the crack length exceeds six-tenths of the width of the plate. On the other hand, the plate specimen is not considered to be in a severely damaged state when $y < 0.1$. In the range $0.1 \le y \le 0.6$, there exists a fuzziness about the definition of the "severely damaged" state, which are reflected by the linear membership function in this case.

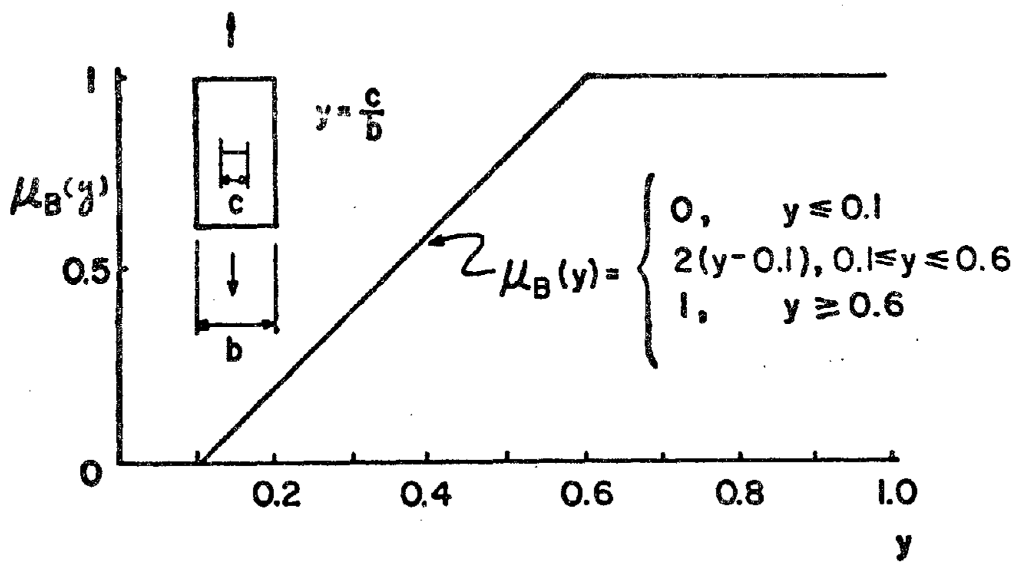

Fig. 1. Sample Membership Function

The complement of a fuzzy set A is denoted by $\bar{A}$, and is given by

$$\bar{A} = \{(x|\mu_{\bar{A}}\ (x))\} \tag{4}$$

where

$$\mu_{\bar{A}}(x) = 1 - \mu_A(x) \tag{5}$$

The membership function for the intersection of two fuzzy sets, say A and B, is given as follows:

$$\mu_{A \cap B}(x) = \min\ (\mu_A(x),\ \mu_B(x)) \tag{6}$$

On the other hand, the membership function for the union of two fuzzy sets, say A and B, is as follows:

$$\mu_{A \cup B}(x) = \max\ (\mu_A(x),\ \mu_B(x)) \tag{7}$$

The algebraic sum of two fuzzy sets, say A and B, is denoted by A + B and has the following membership function:

$$\mu_{A+B}(x) = \mu_A(x) + \mu_B(x) - \mu_A(x)\ \mu_B(x) \tag{8}$$

or, for given values of x,

$$\mu_{A+B} = \mu_A + \mu_B - \mu_A \cdot \mu_B \tag{9}$$

Note that, more generally,

$$\mu_{\sum_{i=1}^{n} A_i} = 1 - \prod_{i=1}^{n} [1 - \mu_{A_i}] \tag{10}$$

The membership of the algebraic product of two sets, say A and B, is given by

$$\mu_{AB}(x) = \mu_A(x)\ \mu_B(x) \tag{11}$$

Therefore, for $\alpha > 0$,

$$\mu_{A^\alpha}(x) = [\mu_A(x)]^\alpha \tag{12}$$

As an example, consider an axially-loaded plate in which n cracks (with length C_i, $i = 1,\ldots,n$,) have been detected. Let D_i denote the severe-damage state of this plate due to the ith crack, $y_i = \frac{c_i}{b}$, and

$$\mu_{D_i} = \begin{cases} 0, & y_i < 0.1 \\ 2(y_i - 0.1), & 0.1 \leq y_i \leq 0.6 \\ 1, & y_i > 0.6 \end{cases} \tag{13}$$

Let B denote the overall damage state of this plate due to all these n cracks. If these cracks are far apart, we may say that $B_1 = \bigcup_{i=1}^{n} D_i$, then

$$\mu_{B_1} = \max\,(\mu_{D_1},\ldots,\mu_{D_n}) \tag{14}$$

If these cracks are fairly close to each other, we may say that $B_2 = \sum_{i=1}^{n} D_i$, then

$$\mu_{B_2} = 1 - \prod_{i=1}^{n} [1 - \mu_{D_i}] \tag{15}$$

Or,

$$\mu_{B_1} \leq \mu_B \leq \mu_{B_2} \tag{16}$$

More specifically, say that 3 cracks are detected with the following data; $y_1 = 0.05$, $y_2 = 0.5$, $y_3 = 0.4$. Using Equation 13, we find that $\mu_{D_1} = 0$, $\mu_{D_2} = 0.8$, $\mu_{D_3} = 0.6$, then

$$\mu_{B_1} = \max\,(0,\ 0.8,\ 0.6) = 0.8 \tag{17}$$

and

$$\mu_{B_2} = 1 - [1-0]\ [1-0.8]\ [1-0.6] = 0.92 \tag{18}$$

or,

$$0.8 \leq \mu_B \leq 0.92 \tag{19}$$

In this case, this plate with these three detected cracks is said to have a "strong" membership in the "severely damaged" category.

3.3 Fuzzy Relation

Let P be a product set of n sets and M be its membership set. A fuzzy n-ary relation is a fuzzy set of P taking its values in M. As an example, let X = {Building A, Building B}, and Y = {Bridge C, Bridge D}. Then, a binary fuzzy relation of "similar damage state" between members of X and Y may be expressed as

$$\mathcal{R} = \begin{array}{cc} & \begin{array}{cc} C & D \end{array} \\ \begin{array}{c} A \\ B \end{array} & \begin{bmatrix} 0.8 & 0.1 \\ 0.3 & 0.9 \end{bmatrix} \end{array} \tag{20}$$

in which the $(i,j)^{th}$ element is the value of the binary membership function $\mu_{\mathcal{R}}(x,y)$ for the i^{th} value of x and the j^{th} value of y. For this numerical example, Building A and Bridge C are said to have a strong membership of 0.8 to be in a similar damage state. These number are arbitrarily selected for the purpose of illustration.

The union of two relations, say $\mathcal{R}$ and $\mathcal{S}$, is denoted by $\mathcal{R} \cup \mathcal{S}$ and has the following membership function,

$$\mu_{\mathcal{R}\cup\mathcal{S}}(x,y) = \mu_{\mathcal{R}}(x,y) \vee \mu_{\mathcal{S}}(x,y) = \max[\mu_{\mathcal{R}}(x,y), \mu_{\mathcal{S}}(x,y)] \tag{21}$$

where "v" denotes maximum. On the other hand, the intersection of two relations has the following membership function:

$$\mu_{\mathcal{R}\cap\mathcal{S}}(x,y) = \mu_{\mathcal{R}}(x,y) \wedge \mu_{\mathcal{S}}(x,y) = \min[\mu_{\mathcal{R}}(x,y), \mu_{\mathcal{S}}(x,y)] \tag{22}$$

where "$\wedge$" denotes minimum. More generally if R_i, i=1,...,n, are relations, then

$$\mu_{\underset{i}{\cup} R_i}(x,y) = \bigvee_i \left[\mu_{R_i}(x,y)\right] \tag{23}$$

$$\mu_{\underset{i}{\cap} R_i}(x,y) = \bigwedge_i \left[\mu_{R_i}(x,y)\right] \tag{24}$$

The complement, algebraic sum, algebraic product are represented respectively with the following membership functions:

$$\mu_{\bar{R}}(x,y) = 1 - \mu_R(x,y) \tag{25}$$

$$\mu_{R+S}(x,y) = \mu_R(x,y) + \mu_S(x,y) - \mu_R(x,y)\mu_S(x,y) \tag{26}$$

$$\mu_{RS}(x,y) = \mu_R(x,y)\mu_S(x,y) \tag{27}$$

If R is a fuzzy relation from X to Y, and S is a fuzzy relation from Y to Z, then the composition of R and S is a fuzzy relation which is described with the following membership function:

$$\mu_{R\bullet S}(x,y) = \bigvee_y \left[\mu_R(x,y) \wedge \mu_S(y,z)\right] = \max_y \left[\min\left(\mu_R(x,y),\ \mu_S(y,z)\right)\right] \tag{28}$$

Recall the example relating similar damage states of buildings and bridges as given in Equation 20. Let Z = {Dam E, Dam F, Dam G}, and

$$S = \begin{array}{c c} & \begin{array}{ccc} E & F & G \end{array} \\ \begin{array}{c} C \\ D \end{array} & \begin{bmatrix} 0.7 & 0.5 & 0.4 \\ 0.2 & 0.6 & 0.5 \end{bmatrix} \end{array} \tag{29}$$

Then,

$$R\bullet S = \begin{bmatrix} 0.8 & 0.1 \\ 0.3 & 0.9 \end{bmatrix} \bullet \begin{bmatrix} 0.7 & 0.5 & 0.4 \\ 0.2 & 0.6 & 0.5 \end{bmatrix}$$

$$= \begin{array}{c c} & \begin{array}{ccc} E & F & G \end{array} \\ \begin{array}{c} A \\ B \end{array} & \begin{bmatrix} 0.7 & 0.5 & 0.4 \\ 0.3 & 0.6 & 0.5 \end{bmatrix} \end{array} \tag{30}$$

To interpret this result, we say that Building A and Dam E have similar damage state with a degree of membership 0.7, whic is obtained from the operation $[(0.8 \wedge 0.7) \vee (0.1 \wedge 0.3)]$ or $\max[\min(0.8,0.7), \min(0.1,0.2)]$

3.4 A Simplified Example in Structural Reliability

Since Professor Freudenthal presented a rational approach to the structural safety problem more than thirty years ago (24), the theory of structural reliability has become a significant tool of the civil engineering profession. For most structures, the calculated probability of failure, p_f, using available statistics is generally smaller than 10^{-6}. Brown (23) indicated that his perceived failure rate is on the order of 10^{-3} for a certain type of structures. Following Blockley (25) in part, Brown (23) applied the theory of fuzzy sets in an attempt to bridge this gap between the calculated and observed probabilities of failure. The proposed procedure includes the following steps: (i) compute the objective failure probability $p_F^{(0)} = 10^{-n}$ using all available objective statistical data on load, strength, etc.; (ii) list the gravity g and consequence c, for each subjective factor which can affect the structural safety; (iii), assign linquistic safety statements into fuzzy sets and obtain the total effect $T(g,c)$; (v) obtain a relation $R(c,n)$ between the consequence of the combined parameters C and safety measure n; (vi) find the composite of T and R, i.e., $T \bullet R$, to provide the subjective safety measure for this structure $F_S = F_S(g,n)$; and (vii) extract a subset $K(n)$ from F_S for an element $g \varepsilon G$ which yields the fuzzy safety measure.

A simplified version of the numerical examples as given by Brown (23) is presented below for the purpose of illustration. Consider two subjective factors as follows: (1) the effect of mathematical modeling, numerical calculations, and design experience, and (2) the effect of human factors, and construction experience and process. For each factor, the gravity (or importance) of the adverse effect G and the consequence of this effect C are estimated by experts with linguistic statements as listed in Table 1.

Table 1. Estimates for Subjective Factors.

	Factor	Gravity, G_i or importance	Consequence, C_i
(1)	Mathematical Modeling, Numerical Calculations, Design Experience	small	large
(2)	Human Factors, Construction Experience and Process	medium	grave

To expressly translate linguistic terms such as "small" in terms of fuzzy sets, let

$$G_1 = \text{small} = \{(0|1),\ (0.1|0.9),\ (0.2|0.5)\}$$

$$\begin{aligned} G_2 = \text{medium} = \{&(0.2|0.1),\ (0.3|0.2),\ (0.4|0.8),\ (0.5|1), \\ &(0.6|0.8),\ (0.7|0.2),\ (0.8|0.1)\} \end{aligned} \tag{31}$$

$$C_1 = \text{large} = \{(0.8|0.5),\ (0.9|0.9),\ (1|1)\}$$

$$\begin{aligned} C_2 = \text{grave} &= (\text{large})^2 \\ &= \{(0.8|0.25),\ (0.9|0.81),\ (1|1)\} \end{aligned}$$

Thus,

$$G_1 \cap C_1 = \begin{array}{cc|ccc} & & \multicolumn{3}{c}{\text{Consequence = large}} \\ & & 0.8 & 0.9 & 1 \\ \hline \text{Gravity = small} & \begin{array}{c} 0 \\ 0.1 \\ 0.2 \end{array} & \multicolumn{3}{c}{\begin{bmatrix} 0.5 & 0.9 & 1 \\ 0.5 & 0.9 & 0.9 \\ 0.5 & 0.5 & 0.5 \end{bmatrix}} \end{array} \tag{32}$$

$$G_2 \cap C_2 = \begin{array}{cc|c} & & \text{Consequence = grave} \\ & & \begin{array}{ccc} 0.8 & 0.9 & 1.0 \end{array} \\ \hline \text{Gravity = medium} & \begin{array}{c} 0.2 \\ 0.3 \\ 0.4 \\ 0.5 \\ 0.6 \\ 0.7 \\ 0.8 \end{array} & \begin{bmatrix} 0.1 & 0.1 & 0.1 \\ 0.2 & 0.2 & 0.2 \\ 0.25 & 0.8 & 0.8 \\ 0.25 & 0.81 & 1 \\ 0.25 & 0.8 & 0.8 \\ 0.2 & 0.2 & 0.2 \\ 0.1 & 0.1 & 0.1 \end{bmatrix} \end{array} \tag{33}$$

The total effect of both factors can be obtained by taking the union of Equations 32 and 33, i.e.,

$$T = (G_1 \cap C_1) \cup (G_2 \cap C_2) = \begin{array}{cc|c} & & \text{Consequence} \\ & & \begin{array}{ccc} 0.8 & 0.9 & 1.0 \end{array} \\ \hline \text{Gravity} & \begin{array}{c} 0 \\ 0.1 \\ 0.2 \\ 0.3 \\ 0.4 \\ 0.5 \\ 0.6 \\ 0.7 \\ 0.8 \end{array} & \begin{bmatrix} 0.5 & 0.9 & 1 \\ 0.5 & 0.9 & 0.9 \\ 0.5 & 0.5 & 0.5 \\ 0.2 & 0.2 & 0.2 \\ 0.25 & 0.8 & 0.8 \\ 0.25 & 0.81 & 1 \\ 0.25 & 0.8 & 0.8 \\ 0.2 & 0.2 & 0.2 \\ 0.1 & 0.1 & 0.1 \end{bmatrix} \end{array} \tag{34}$$

To establish a fuzzy relation $R(c,n)$ between the fuzzy sets of consequences C and safety measures N, let

$$R:\quad N = \begin{cases} \text{very large} = \{(n|.04),\ (n-1|0.64),(n-2|1)\}, & \text{if C is large} \\ \text{large} \qquad = \{(n|0.2),\ (n-1|0.8),\ (n-2|1)\}, & \text{if C is medium} \\ \text{small} \qquad = \{(n|1),\ (n-1|0.8),\ (n-2|0.2)\}, & \text{if C is small} \end{cases} \tag{35}$$

Then,

$$
R_1 = C_1 \cap N_1 = \begin{array}{c|c} & N_1 = \text{very large} \\ & \begin{array}{ccc} n & n-1 & n-2 \end{array} \\ \hline \text{Consequence} = \text{large} \begin{array}{c} 0.8 \\ 0.9 \\ 1 \end{array} & \begin{bmatrix} 0.04 & 0.5 & 0.5 \\ 0.04 & 0.64 & 0.9 \\ 0.04 & 0.64 & 1 \end{bmatrix} \end{array} \tag{36}
$$

$$
R_2 = C_2 \cap N_2 = \begin{array}{c|c} & N_2 = \text{large} \\ & \begin{array}{ccc} n & n-1 & n-2 \end{array} \\ \hline \text{Consequence} = \text{medium} \begin{array}{c} 0.2 \\ 0.3 \\ 0.4 \\ 0.5 \\ 0.6 \\ 0.7 \\ 0.8 \end{array} & \begin{bmatrix} 0.1 & 0.1 & 0.1 \\ 0.2 & 0.2 & 0.2 \\ 0.2 & 0.8 & 0.8 \\ 0.2 & 0.8 & 1 \\ 0.2 & 0.8 & 0.8 \\ 0.2 & 0.2 & 0.2 \\ 0.1 & 0.1 & 0.1 \end{bmatrix} \end{array} \tag{37}
$$

$$
R_3 = C_3 \cap N_3 = \begin{array}{c|c} & N_3 = \text{small} \\ & \begin{array}{ccc} n & n-1 & n-2 \end{array} \\ \hline \text{Consequence} = \text{small} \begin{array}{c} 0 \\ 0.1 \\ 0.2 \end{array} & \begin{bmatrix} 1 & 0.8 & 0.2 \\ 0.9 & 0.8 & 0.2 \\ 0.5 & 0.5 & 0.2 \end{bmatrix} \end{array} \tag{38}
$$

Using Equations 36 through 38, we obtain

$$R = R_1 \cup R_2 \cup R_3 = \begin{array}{cc|c} & & \begin{array}{ccc} & N & \\ n & n-1 & n-2 \end{array} \\ \hline \text{Consequence} & \begin{array}{c} 0 \\ 0.1 \\ 0.2 \\ 0.3 \\ 0.4 \\ 0.5 \\ 0.6 \\ 0.7 \\ 0.8 \\ 0.9 \\ 1 \end{array} & \begin{bmatrix} 1 & 0.8 & 0.2 \\ 0.9 & 0.8 & 0.2 \\ 0.5 & 0.5 & 0.2 \\ 0.2 & 0.2 & 0.2 \\ 0.2 & 0.8 & 0.8 \\ 0.2 & 0.8 & 1 \\ 0.2 & 0.8 & 0.8 \\ 0.2 & 0.2 & 0.2 \\ 0.1 & 0.5 & 0.5 \\ 0.04 & 0.64 & 0.9 \\ 0.04 & 0.64 & 1 \end{bmatrix} \end{array} \qquad (39)$$

To provide the subjective safety measure, use Equations 34 and 39 to find the composite as follows:

$$F_S = T \bullet R = \begin{array}{cc|c} & & \begin{array}{ccc} & N & \\ n & n-1 & n-2 \end{array} \\ \hline \text{Gravity} & \begin{array}{c} 0 \\ 0.1 \\ 0.2 \\ 0.3 \\ 0.4 \\ 0.5 \\ 0.6 \\ 0.7 \\ 0.8 \end{array} & \begin{bmatrix} 0.1 & 0.64 & 1 \\ 0.1 & 0.64 & 0.9 \\ 0.1 & 0.5 & 0.5 \\ 0.1 & 0.2 & 0.2 \\ 0.1 & 0.64 & 0.8 \\ 0.1 & 0.64 & 1 \\ 0.1 & 0.64 & 0.8 \\ 0.1 & 0.2 & 0.2 \\ 0.1 & 0.1 & 0.1 \end{bmatrix} \end{array} \qquad (40)$$

From Equation 40, choose a subset K(n) of the composition F_S, where K(n) is called a fuzzifier. As an example, choose the largest element in each column and we have,

$$F = \{(n|0.1), (n-1|0.64), (n-2|1)\} \tag{41}$$

In this case, if the objective failure probability is 10^{-6}, the inclusion of subjective factors produces a failure probability on the order of 10^{-4} which is closer to Brown's perceived value.

In summary, two subjective factors are evaluated linguistically in terms of the gravity and consequency of each factor as shown in Table 1. Appropriate membership functions are then assigned to such linguistic descriptions in Equation 31. The total effect of both factors is given in Equation 34. A fuzzy relation between the consequence and a safety measure is established in Equation 39. Finally, the fuzzy relation between the gravity and the safety measure is obtained by taking composition of Equations 34 and 39. Results of this example are used to illustrate a rational evaluation of subjective factors in the structural reliability analysis.

4. FORMULATION

4.1 General Remarks

Recently, Fu and Yao (26) considered the problem of damage assessment in the context of pattern recognition (27,28). The theory of pattern recognition is the study of mathematical techniques to build machines to aid human experience (27). Essentially, the process of pattern recognition can be illustrated in a schematic diagram as shown in Figure 2. The physical world consisting of infinite dimensions are measured through the use of transducers to produce a measurement space with m dimensions. These measurements are then analyzed to obtain a feature space with n(<m) dimensions. Finally, a classifier is needed to yield the desired classification.

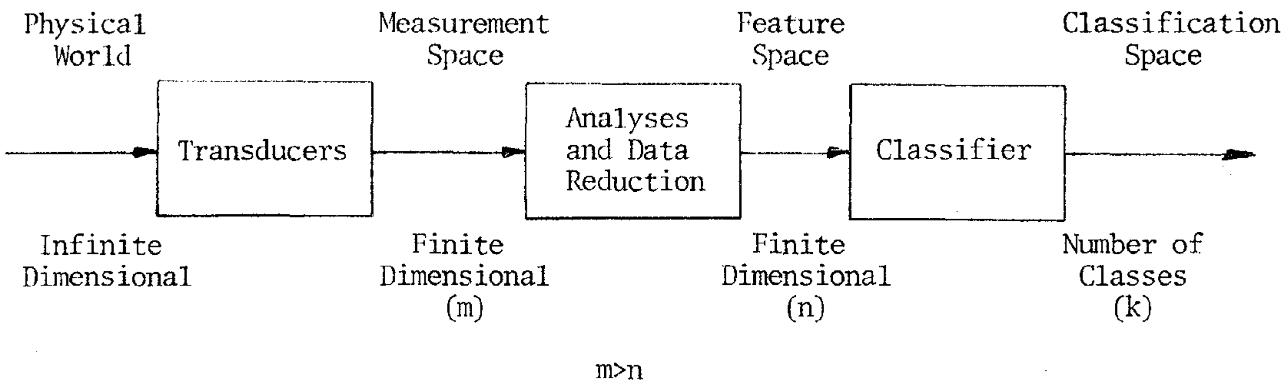

Figure 2. Schematic Diagram of Pattern Recognition

In general, data are collected from the inspection and testing of an existing building structure with the use of transducers. Such data may include (a) the size, number, and location of cracks, and (b) time-history of measured ground motions and structural response in the form of accelerograms. An example of crack patterns is given by Abrams and Sozen (29). Data such as accelerograms can be analyzed to extract a pattern or feature space. As examples, Beck (30) and Chen and Yao (31) have developed methods for the estimation of the changing natural frequency using records of ground motions and structural response during a given earthquake. In the following, an attempt is made to formulate a decision function or classifier for the determination of the damage state on the basis of the resulting pattern space.

4.2 Classifiers

In general, there are two types of data from the inspection and testing of the structure. One type of observations is made from local phenomena such as cracks in certain structural members. Such information can be incorporated in a logical manner to obtain an estimate of the damage state of the whole structure. The other type of data are taken from global behavior of the structure such as the structural response and ground-motion records.

Let B denote the event that the whole structure has been severely damaged, and B_i denote the severely-damaged state of the structure using ith group of data. For example, i=1 corresponds to the information on detected cracks in the structure, and i=2 corresponds to the features extracted from recorded accelerograms. Therefore, for m groups of data, we have,

$$B = \bigcup_{i=1}^{m} B_i \tag{42}$$

or,

$$\mu_B = \bigvee_i (\mu_{B_i}) \tag{43}$$

Furthermore, for ith group of data which are related to the jth component of the structure consisting of a total of n components, let D_{ij} denote the severely-damaged state of the jth component. Then B_i can be considered as the algebraic sum of the damage of each component, i.e.,

$$B_i = \sum_{j=1}^{n} D_{ij} \tag{44}$$

or

$$\mu_{B_i} = 1 - \prod_{j=1}^{n} [1 - \mu_{D_{ij}}] \tag{45}$$

For the purpose of illustration as noted above, let B_1 denote the severely-damaged state of the structure from carck detection and measurements, and B_2 denote the severely-damaged state of the structure from a reduction of the natural (fundamental) frequency of the structure. Say that there are 3 major components with detected cracks, and we have $\mu_{D_{11}} = 0$, $\mu_{D_{12}} = 0.8$, $\mu_{D_{13}} = 0.6$, then

$$\mu_{B_1} = 0.92 \tag{46}$$

Meanwhile, we find that the calculated reduction of measured natural frequency is 25%. Through the use of an hypothetically established membership function, we obtain

$$\mu_{B_2} = 0.78 \tag{47}$$

The determination of this membership can be based on full-scale destructive test data such as those of Galambos and Mayes (15) as shown in Figure 3 plus advice from various experts. Then, the membership of the structure in the severely-damaged state is given by

$$\mu_B = \max\ (\mu_{B_1}, \mu_{B_2}) = 0.92 \tag{48}$$

As another possible approach, let $X = \{x_1, x_2, \ldots, x_k\}$ be a set of k features. For example, x_1 = many cracks, x_2 = large cracks, and x_3 = excessive deformation. Also, let $Y = \{y_1, y_2, \ldots, y_\ell\}$ be a set of ℓ potential failure modes. For example, y_1 = fatigue and fracture failure, y_2 = creep, y_3 = instability, and y_ℓ = progressive collapse. Furthermore, let Z = the severely-damaged state. If we can find the fuzzy relations R (from X to Y) and S (from Y to Z), we can relate features X to the severely-damaged state of the structure Z by taking the composition $R \bullet S$. For the purpose of illustration, let R and S be given as follows:

$$R = \begin{array}{l|cccc} & \begin{array}{c}y_1:\\ \text{Fatigue}\\ \&\\ \text{Fracture}\end{array} & \begin{array}{c}y_2:\\ \text{Creep}\end{array} & \begin{array}{c}y_3:\\ \text{Instability}\end{array} & \begin{array}{c}y_4:\\ \text{Progressive}\\ \text{Collapse}\end{array} \\ \hline x_1: \text{ many cracks} & \left[\begin{array}{c}0.9\\ \\ 0.8\\ \\ 0.3\end{array}\right. & \begin{array}{c}0.2\\ \\ 0.3\\ \\ 0.8\end{array} & \begin{array}{c}0.4\\ \\ 0.7\\ \\ 0.9\end{array} & \left.\begin{array}{c}0.4\\ \\ 0.8\\ \\ 0.7\end{array}\right] \\ x_2: \text{ large cracks} & & & & \\ x_3: \text{ excessive deformation} & & & & \end{array} \tag{49}$$

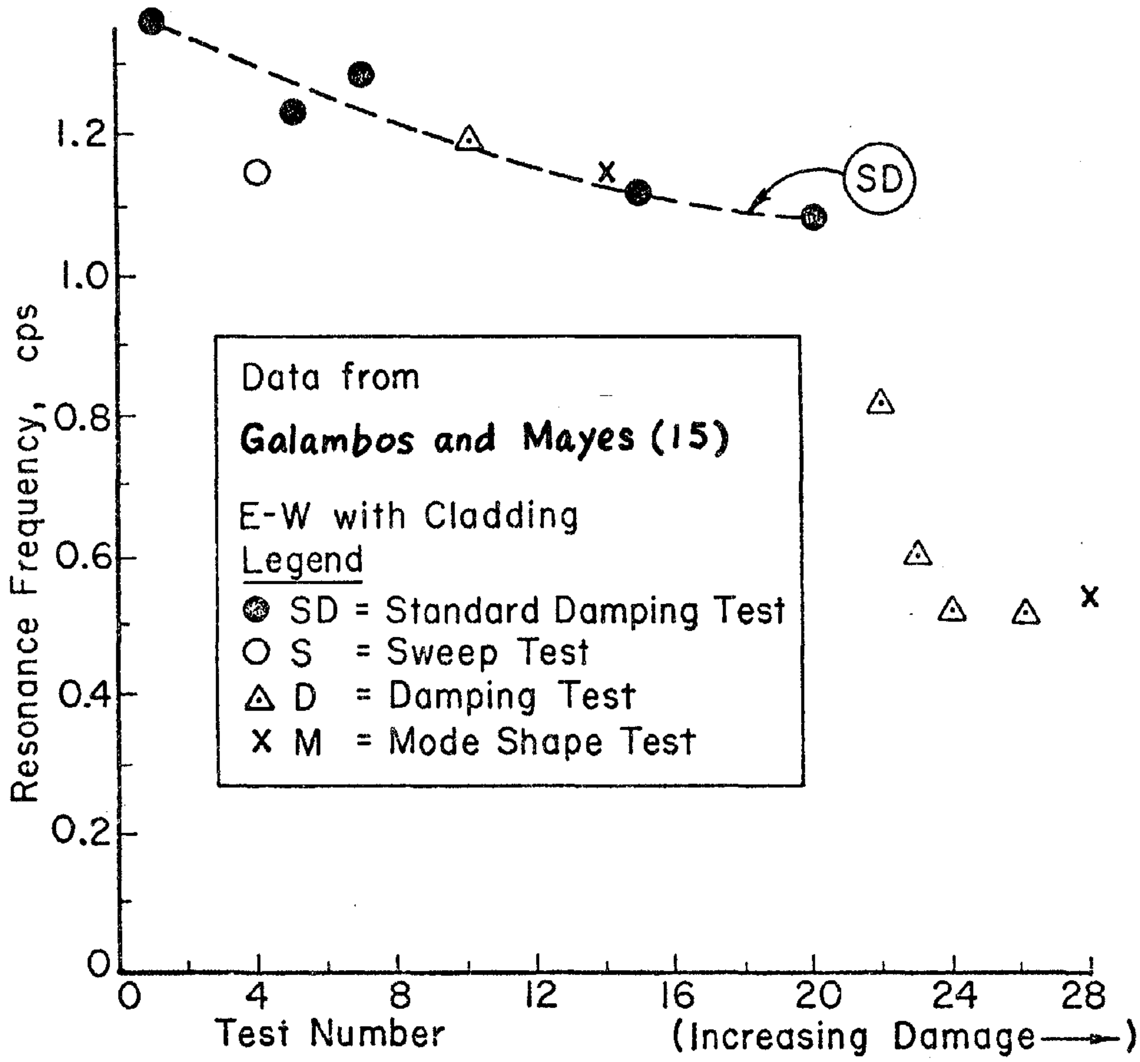

Figure 3. Variation of National Frequency with a Damage Measure

$$
S = \begin{array}{c|c} & \begin{array}{c} z \\ \text{severely damaged} \end{array} \\ \hline \begin{array}{c} y_1 \\ y_2 \\ y_3 \\ y_4 \end{array} & \begin{bmatrix} 0.4 \\ 0.3 \\ 0.8 \\ 1.0 \end{bmatrix} \end{array} \tag{50}
$$

Then

$$
R \bullet S = \begin{array}{c|c} & z \\ \hline \begin{array}{c} x_1 \\ x_2 \\ x_3 \end{array} & \begin{bmatrix} 0.4 \\ 0.8 \\ 0.8 \end{bmatrix} \end{array} \tag{51}
$$

Results as given in Equation 51 indicate that the presence of features x_2 (large cracks) and x_3 (excessive deformation) would constitute a strong membership of the structure being in the severely damaged state. In other words, if large cracks and excessive deformations are present, the structure can be classified as being "severely damaged".

5. DISCUSSION

An important step in the implementation of this approach is to establish various membership functions. Provided that the overall and complex problem can be divided into a series of detailed questions, qualified persons can answer these questions with simple descriptive words. With the assistance of experts, these descriptive words can be interpreted with suitable memberhsip functions. The calculus of fuzzy sets can then be applied to obtain the relevant classification in the end. It is believed that such an approach can become practical and useful with (a) the collarboration of various experts; and (b) the accumulation of information from evaluations of structural damage.

An attempt has been made to apply the theory of fuzzy sets to the complex problem of damage assessment of existing structures. In this pilot study, only elementary fuzzy relations have been used. It is hoped that this report will serve the purpose of stimulating interest among structural engineers who are concerned with the identification of structural damage. Furthermore, the report may be useful in introducing the problems of structural identification to experts of fuzzy sets and pattern recognition so that some of them will collaborate with structural engineers in fully developing such applications in the near future.

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