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RESPONSE OF SIMPLE STRUCTURAL SYSTEMS TO TRAVELING SEISMIC WAVES

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16. Abstract (Limit: 200 words)

An investigation is undertaken of the possibility of using a traveling seismic wave procedure to study the translational and rotational responses of a building. This project was conducted with a view to assessing current building code procedures and to establish bounds for the interrelationship between torsional and translational response. The theory and procedure for both superposition and coupled reaction models used in the study are described together with results obtained. Three assumptions were made throughout the investigation: (1) only systematic motions over the base were taken into account, (2) only horizontally propagated plane waves with vertical wave fronts of motions were considered, and (3) only rigid foundation systems were considered. Results indicate that, in general, the coupling of translational and rotational response leads to increased scatter in the computed responses obtained and less certainty in the types of trends noted for the superposition model. This study indicates that the five percent building code value for accidental eccentricity appears reasonable. Results clearly indicate the need for instrumentation to obtain a more detailed picture of the translation and rotation experienced by a building during earthquake excitation.

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Earthquakes	Scattering	Shear stress

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Any opinions, findings, and conclusions or recommendations expressed in this report are those of the authors and do not necessarily reflect the views of the National Science Foundation or the U.S. Nuclear Regulatory Commission.

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1. INTRODUCTION

1.1 Motivation Behind Study

Over the years observations of earthquake damage suggest that structures on large foundations respond to ground motion with less intensity than do smaller structures. One of the first instrumented observations of such response reduction was for the Hollywood Storage building and adjacent P.E. lot. Response spectra for the Hollywood Storage building and P.E. lot for the 1971 San Fernando earthquake and the 1952 Kern County earthquake are shown in Figs. 1.1 and 1.2. While reduction for the earlier earthquake is not as apparent, it should be noted that this was a smaller and more distant earthquake than the San Fernando earthquake. One must appreciate that the building and P.E. lot instruments are over 100 feet apart and as discussed later even this clearly can lead to differences in response. Peak recorded ground motions can be expected to vary over a finite ground surface area. The reasons for this variation include the complicated interrelationships of the geology, source mechanism, transmission paths, and phasing, as well as random variation from point to point. As such this comparison may be only indicative of the true difference between building and free-field response; even so it is the best available from current data.

Although buildings also undergo rotational (horizontal torsion and rocking) as well as translational motion, no field measurements yet exist to permit cross-checking theory and observations. For purposes of simplicity in this study only horizontal torsion was considered. One of the simplest approaches that has been put forth for examining building response in the context of soil-structure interaction involving both translation and rotation is analysing a traveling wave as it passes a building. The reduction in point by point acceleration effects arising from free-field motion are obviously affected by the building. It is this effect on building response that is the subject of this study.

1.2 Early Work

The first paper that attempted to provide a rational explanation for observed reductions in response of buildings as compared to nearby free-field apparently was that by Yamahara in 1970 (Ref. 1). Similarly, Scanlan (Ref. 2) provides a general relationship between average acceleration over the width of the foundation as a function of the wave length of the acceleration pulse and the width of the foundation. Luco (Ref. 3) accomplished essentially the same thing except that whereas Scanlan had used a rigid block resting on a continuously distributed set of soil springs, Luco utilized elastic half space theory to provide resistance.

A measure of the effect of an earthquake on a large building can be obtained through calculations of a time-averaged acceleration over a transit time. As a basis for visualizing the effects that occur and the general techniques that are employed in making τ-averaging calculations, reference is made to Fig. 1.3 in which is shown the concept of the wave transit time (τ), the average translational acceleration ($\ddot{\phi}$), and the average linear acceleration at the edge of the foundation arising from rotation ($\ddot{\alpha}\tau/2$) as a wave propagates along the base of a building. The τ-averaging approach (Ref. 4) does not require an assessment of the frequencies included in the earthquake motion. Several individuals have investigated the traveling wave problem (Refs. 5 through 9); while it has limitations, the technique constitutes one systematic way for studying the response of structural systems in the light of observed behavior.

1.3 Outline of Investigation

This investigation has been an attempt to establish the feasibility of using the τ-average traveling seismic wave procedure to study the problems of combined motion (translation and rotation). It is not intended to be an exhaustive study; however an attempt has been made to determine reasonable upper bounds on the response for both translation and torsion.

In 1969 Newmark (Ref. 10) offered observations concerning torsion in symmetrical buildings that could arise from earthquake ground motions and pointed out the importance of accidental torsion. Concern with the effects of torsion in buildings is not new; as early as 1938 Ayre (Refs. 11 and 12) worked on the interconnection of translational and rotational motion and cited several reasons for desiring symmetry (i.e., independence of horizontal translation and rotation for translatory ground motion, and simpler analysis in all cases). On the last page of his 1938 paper (Ref. 11) Ayre states, "... in designing to resist such phenomena as earthquakes, for which there are relatively few engineering data, simplicity seems worth striving for." Today, more than forty years later, simplicity still seems worth striving for.

A sketch of the general techniques employed in making τ-averaging calculations is shown in Fig. 1.3. For calculational purposes, at any point in time some portion of the acceleration time history, treated as a shear wave propagating along the foundation, is positioned along one axis of the building. As used herein, the transit time (τ), is the time required for a point on the wave to travel the length of the base of the structure (i.e., the length of the base divided by the wave velocity). In effect, for this simple representation of motion, at each increment in time the acceleration time history is moved slightly ahead from its position at the previous time increment, and the corresponding average translation and rotation imparted to the building will be slightly different from those at the previous time increment. As the wave traverses the building the foundation attempts to conform to the shape of the wave. However, since the foundation is assumed rigid it conforms to the ground motion in a straight line. By using the principle of least square fitting of an acceleration record over a transit time one may obtain a simple mathematical model which is consistent with the assumed rigid conformity, and obtain time histories of average translation and induced rotation.

The use of least square fitting is not restricted to a straight line (i.e., the assumption of a rigid foundation) and indeed the averaging procedure can be applied whether or not the foundation is assumed to be rigid. Response reduction would occur for a rigid foundation when compared to that experienced by a point; nevertheless, in the case of a more realistic foundation with some degree of flexibility some reduction still would be expected to occur. Response curves for a real building would fall somewhere between those for a rigid foundation of the same size and a point in the free-field.

An analogy that may help in the visualization of the modification of motions that occur is that of a boat on the ocean. A small boat undergoes motions of much greater amplitude and higher frequency than does an ocean liner. The latter undergoes a highly averaged, and therefore reduced, response arising from the same wave motion. Even a rubber life raft of some size but relatively little stiffness experiences lower amplitude motion than does a particle on the water surface.

In the following chapters are detailed summaries of the theories used to generate the ground motion to be input into the system, to combine responses arising from translational and rotational motion in symmetric structures by taking advantage of superposition, and to generate and uncouple the equations of motion necessary for an eccentric structure (so that the effects of coupled translation and rotation on the overall response of a system may be studied).

1.4 Scope of Report

In an attempt to guide the reader through the discussion Figs. 1.4 through 1.6 have been presented as outlines of the procedures used herein. A presentation of the chronological sequence of events during the course of this investigation and the prime motivation which led the writer from one phase of the study to the next is given in Fig. 1.4. Shown in Fig. 1.5 is an outline of the responses computed for the superposition model and this can be used to aid understanding of Chapters 2 and 3. Outlined in Fig. 1.6 is the formulation and execution of the coupled motion model; it is hoped that this will depict the basis of the calculations in Chapters 4 and 5.

The procedure employed to determine the ground motion quantities utilized herein is briefly described in Chapter 2. Also in Chapter 2 there is presented a description of the superposition model along with a list of the various responses computed and the corresponding assumptions. Included in Chapter 3 is a discussion of the results obtained using the superposition model; in this discussion are explained the effects of translational averaging and induced rotation on a simple system, the differences in the results when total response is computed using the different combinational techniques (whether a rigorous summation in time, the square root of the sum of the squares of maximum individual responses, or the absolute sum of maximum individual responses), and the importance of which ratio of torsional to translational frequency is employed in the calculations.

A detailed derivation of equations of motion for a building having only one axis of symmetry and a description of the responses to be computed for this model is contained in Chapter 4. After a discussion of the results obtained for the coupled model comparisons of the results of the superposition model and the coupled model are given in Chapter 5. Perhaps the most important part of this investigation is the ability to ascertain, from comparisons of the superposition model with the coupled model, the importance of small eccentricities in the overall response of a structure and to determine the effect that phasing of the ground motion has on a structure with coupled degrees-of-freedom (translational and rotational). The results of this study are also compared to current seismic building code provisions. Finally, a summary of the important observations arising from this study and attempts to formulate guidelines for use by those wishing to consider the problems of combining torsion and translation and/or determining a reasonable method for computing induced torsion is given in Chapter 6.

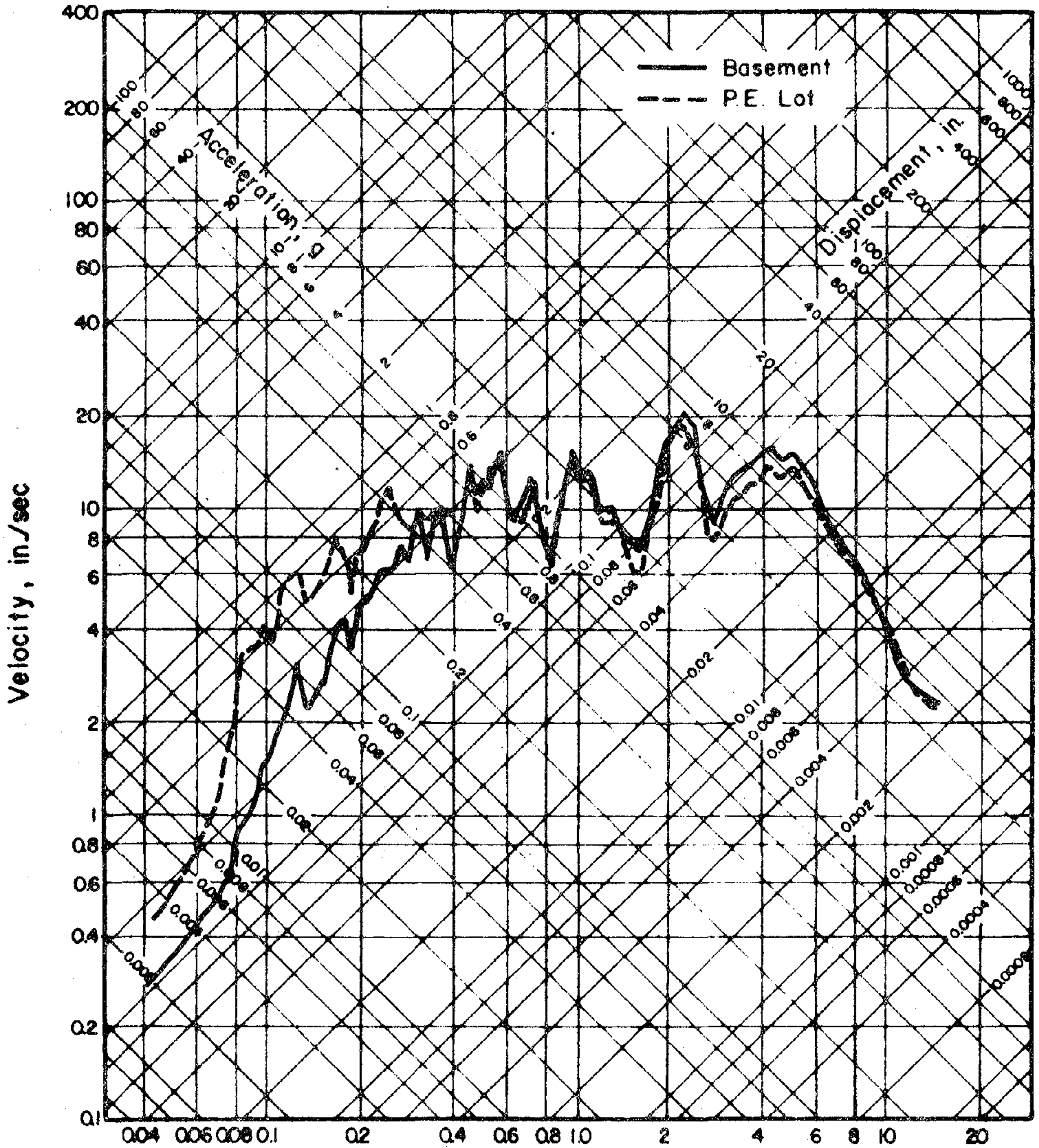


FIGURE 1.1 HOLLYWOOD STORAGE BUILDING BASEMENT AND P.E. LOT, SAN FERNANDO EARTHQUAKE, 9 FEBRUARY 1971 - 0600 PST, COMPONENT SOUTH, DAMPING 5% OF CRITICAL

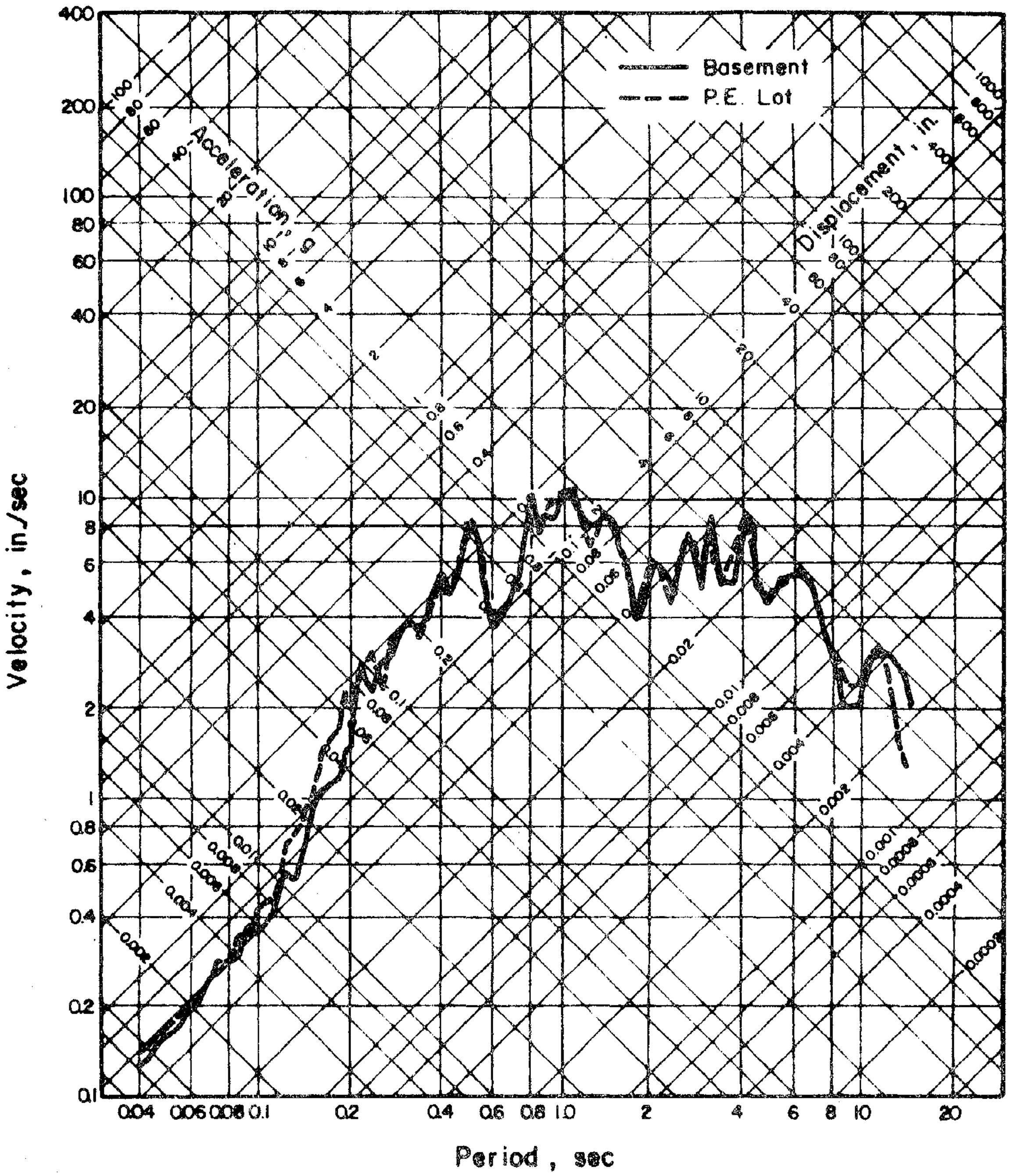


FIGURE 1.2 HOLLYWOOD STORAGE BUILDING BASEMENT AND P.E. LOT, KERN COUNTY EARTHQUAKE, 21 JULY 1952 - 0453 PDT, COMPONENT SOUTH, DAMPING 5% OF CRITICAL

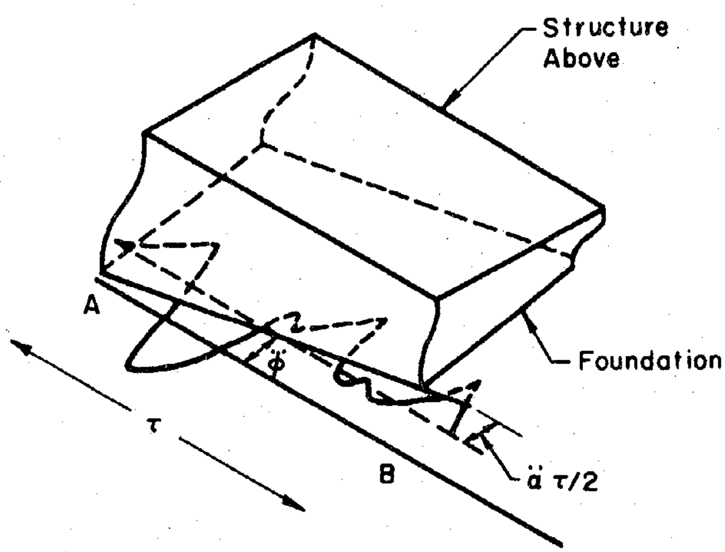


FIGURE 1.3 BASE TRANSLATION AND ROTATION ARISING FROM TRAVELING SEISMIC WAVES

STEPS IN PROCEDURE

CHAPTER 2

Generate Average Translational Acceleration Time History

Generate Induced Rotational Acceleration Time History

CHAPTER 3

Compute and Combine Responses Using Superposition Model

CHAPTERS 4 and 5

Compute and Combine Responses Using Coupled Motion Model

STEPS IN LOGIC

Need to Include Torsional Motion to Maintain Consistency with Averaging Concept.

Need to Address Problem of Combined Response

Need to Investigate Effects of Asymmetry (Eliminate Assumed Independence of Translation and Rotation)

FIGURE 1.4 OUTLINE OF PROCEDURE

PROCEDURE

1. COMPUTE RESPONSE ARISING FROM SEPARATE GROUND MOTION TIME HISTORIES

    a. Average translation as a function of time and compute response at f_x

    b. Translation at edge of foundation arising from induced rotation as a function of time and compute response at $f_\theta = f_x * (f_\theta/f_x)$; where $f_\theta/f_x = 1$, 1.189, or 1.414

2. ASSUME STATISTICAL INDEPENDENCE OF TRANSLATION AND TRANSLATION ARISING FROM ROTATION AND COMBINE RESPONSES AT EDGE OF BUILDING

    a. Sum of absolute individual maximum responses

    b. SRSS of individual maximum responses

    c. Maximum time sum of the individual response time histories

FIGURE 1.5 OUTLINE FOR SUPERPOSITION MODEL

PROCEDURE

1. GENERATE EQUATIONS OF MOTION FOR SIMPLE SYSTEM (FIG. 4.1)

2. EMPLOY CHUGHEY TECHNIQUE (REFS. 13 AND 14) TO GENERATE UNCOUPLED EQUATIONS OF MOTION WITH EQUAL DAMPING IN EACH MODE

3. COMPUTE RESPONSE OF UNCOUPLED EQUATIONS FOR

   a. Wave propagating in X direction

      i. free-field motion
      ii. averaged translation
      iii. induced rotation
      iv. averaged translation and induced rotation

   b. Wave propagating in Y direction (limited study)

      i. free-field motion
      ii. averaged translation and induced rotation

4. DETERMINE COMBINED RESPONSE

   a. Methods of combination

      i. absolute sum
      ii. SRSS
      iii. sum in time

   b. Locations of computed response (FIG. 5.1)

      i. center of mass
      ii. center of stiffness
      iii. + X edge
      iv. - X edge

FIG. 1.6 OUTLINE FOR COUPLED MOTION MODEL

2. THEORY AND PROCEDURE: SUPERPOSITION MODEL

2.1 Ground Input

As illustrated in the preceding chapter (Fig. 1.3) the model to be studied is that of a building whose foundation system is subjected to an earthquake ground motion which is treated as a horizontally traveling plane wave. At the same time the building system above is responding dynamically to this ground input.

The earthquake acceleration time histories used in this report are the California Institute of Technology corrected time histories (Ref. 15). However, since for the averaging procedure employed it is desirable to have an initial zero acceleration, a prefixed acceleration pulse developed by Pecknold and Riddell (Ref. 16) was incorporated into the procedure. An additional increment of zero acceleration also is added to the modified acceleration time history such that the record starts and ends with τ seconds of zero acceleration (the significance of τ will be explained subsequently). This transit time, τ, is taken as the time it takes for a wave to travel the length of a building (i.e., the length of the base divided by the wave velocity). The reason for these modifications is that the averaging technique employed in the calculations reported herein assumes that the ground motion corresponding to recorded acceleration time histories propagates as a plane wave which then passes a structure initially at rest. As such this procedure is essentially a time averaging of an acceleration time history over a transit time, τ. At a given instant in time some portion of the acceleration time history is positioned along an axis of the foundation as shown in Fig. 2.1. As time progresses the acceleration time history simply slides along the foundation such that at any time the accelerations imparted to the foundation are only slightly changed from those imparted at a time Δt earlier. For a given positioning of the τ interval on the acceleration time history the average translational acceleration can be computed from the velocities corresponding to the beginning and end of the interval. The average translational acceleration time history (assigned to the midpoint of the foundation) can be generated according to Eq. 2.1 given later.

Similarly, using the principle of least square fitting of an acceleration time history over a time interval, τ, between two points A and B, where $B = A + \tau$, an expression for the slope of the fitted straight line, with respect to the line representing the average acceleration in the interval, can be derived. The following theoretical derivation for computing translation and rotation is also given in Refs. 9 and 17 and is an outgrowth of earlier studies by Newmark on torsion in symmetrical buildings (Ref. 10).

Given an acceleration time history modified at the beginning as just discussed, $\ddot{\rho}$, and applying the principle of least squares to obtain the average translational acceleration, $\ddot{\phi}$, and the average slope of the fitted line, $\ddot{\alpha}$ (which is related to the average rotational acceleration), one can obtain the following relationships. At any point on the foundation the τ-averaging procedure models the input acceleration as $\ddot{\phi} + \ddot{\alpha}t$ where $\ddot{\phi}$ and $\ddot{\alpha}$ are as defined above and t is the distance, in the time domain (i.e., the distance divided by the velocity of wave propagation), between the midpoint of the foundation and the point in question. The principle of least squares requires that the square of the difference between this acceleration and the corresponding acceleration, $\ddot{\rho}$, obtained from the free field acceleration record summed over the width of the foundation be minimized. This can be represented by the following continuous relationship:

$$\int_A^B (\ddot{\phi} + \ddot{\alpha}t - \ddot{\rho})^2 \, dt = \text{minimum}$$

Minimizing a function of two variables is achieved by setting the partial derivatives with respect to each variable equal to zero (i.e., $\partial/\partial\ddot{\phi} = 0$ and $\partial/\partial\ddot{\alpha} = 0$), which yields for $\partial/\partial\ddot{\phi} = 0$

$$2 \; [\int \ddot{\phi}dt \; + \int \ddot{\alpha}tdt - \int \ddot{\rho}dt] = 0$$

and for $\partial/\partial\ddot{\alpha} = 0$

$$2 \; [\int \ddot{\phi}tdt + \int \ddot{\alpha}t^2dt - \int \ddot{\rho}tdt] = 0$$

with the upper and lower limits on the integrals being B and A in all previous cases and those that follow in this section. By placing the origin at the midpoint of the τ interval one obtains a simplification of these two equations in that $\int \ddot{\alpha}tdt = \int \ddot{\phi}tdt = 0$ since $A = -\tau/2$ and $B = \tau/2$. One may also note that the following relationships hold true:

$$\int \ddot{\phi}dt = \ddot{\phi} \; \{t(B) - t(A)\} = \ddot{\phi}\tau$$

$$\int \ddot{\alpha}t^2dt = \ddot{\alpha} \; \{\tfrac{1}{3} \; [t(B)^3 - t(A)^3]\} = \ddot{\alpha}\tau^3/12$$

$$\int \ddot{\rho}dt = \dot{\rho}(B) - \dot{\rho}(A)$$

$$\int \ddot{\rho}tdt \text{ integrating by parts} = {}^{\tau}/2 \; [\dot{\rho}(B) + \dot{\rho}(A)] - [\rho(B) - \rho(A)]$$

Substituting each of these into the above, one obtains

$$\ddot{\phi} = \frac{1}{\tau} \{\dot{\rho}(B) - \dot{\rho}(A)\} \tag{2.1}$$

$$\ddot{\alpha} = \frac{6}{\tau^2} \{\dot{\rho}(B) + \dot{\rho}(A)\} - \frac{12}{\tau^3} \{\rho(B) - \rho(A)\} \tag{2.2}$$

Note that in Eq. 2.2 as $\tau \to 0$, using Taylor Series expansions for $\dot{\rho}$ and ρ, $\ddot{\alpha} \to \dddot{\rho}\,(0)$ as would be expected. It should be noted that $\ddot{\alpha}$, the slope of the fitted acceleration curve, is the derivative of the acceleration with respect to time and therefore has units of length/time^3.

2.2 Response Calculation

Elastic response spectra were computed from ground acceleration time histories using the Z-transform method as presented by Stagner and Hart in 1970 (Ref. 18 and Appendix A). This procedure applies only to elastic response but is fast and its accuracy is comparable to that of other numerical integration techniques.

2.3 Response Combinations

With the generated time histories of averaged translational and rotational inputs the response of a single-degree-of-freedom (SDF) system may be computed and presented in the form of a response spectrum. The reason for using response spectra for comparison purposes is that the range of effects of the averaging procedure on the response of a SDF system is readily portrayed over a wide range of frequencies in the light of existing knowledge about the response of simple systems. This approach is in contrast to the limited information that would be readily available from a rigorous, time consuming, and tedius study of the time histories to obtain peak values, timing of peaks, frequency content, etc. Also, the more valuable and familiar information to a designer is that information regarding the response of a structure rather than that concerning the input to the structure.

The two averaged time histories generated from the τ-averaging procedure (Eqs. 2.1 and 2.2) are input into the Z-transform method described previously to compute the following responses (discussed in Chapter 3) in an effort to determine the relative importance of translational and rotational input on the overall response of a structure.

1. Response of SDF system to unaltered free-field record.
2. Response of SDF system to $\ddot{\phi}$ time history (i.e., arising from translational motion alone -- Figs. 3.3 through 3.10).
3. Response of SDF system to $\ddot{\alpha}\tau/2$ time history (i.e., arising at edge of structure from rotational motion alone -- Figs. 3.11 through 3.18).
4. Response of SDF system to $\ddot{\phi} + \ddot{\alpha}\tau/2$ time history (i.e., arising from combined translational and rotational motions, and taking advantage of phasing between ground inputs -- Figs. 3.20 through 3.27).
5. Combined SRSS response (i.e., square root of the sum of the squares of responses in 2 and 3 above).
6. Combined absolute sum response (i.e., absolute sum of responses computed in 2 and 3 above).

It will be noted that the SRSS combination of responses is commonly used in practice when the quantities are independent and that the absolute sum of responses is the most conservative possible combination.

It should be noted further that, in all of the aforementioned techniques of combination, it has been assumed that a simple superposition of the effects of translation and rotation can be used (i.e., that translational ground motion causes only translation and rotational motion causes only rotational response). Furthermore, it has been assumed that the ratio of torsional to translational frequencies is unity. It can be shown that the theoretical ratio of frequencies is unity if one assumes a rectangular floor plan with equal uniformly distributed resistance in the two principal directions (Ref. 10). In addition, if a more realistic resistance distribution (e.g., corner columns or end walls and columns) pattern is assumed, the theoretical ratio of torsional to translational frequencies is greater than unity. In order to illustrate the effects of an increased ratio of torsional to translational frequency four additional response quantities were computed.

7. Response calculated from the $\ddot{\alpha}\tau/2$ time history for a frequency ratio of 1.414 (i.e., torsional frequency = 1.414 x translational frequency in 1 above -- Figs. 3.28 and 3.29).

8. Response calculated from $\ddot{\phi}$ for $\omega_{translation}$ and $\ddot{\alpha}\tau/2$ for $\omega_{rotation} = 1.414\omega_{translation}$ (simultaneously computed and combined with respect to time).

9. Combined SRSS response (2 and 7 above).

10. Combined absolute sum response (2 and 7 above).

By comparing these four responses with the corresponding responses computed in the first set (i.e., 3 through 6) one can obtain a bound on the importance of a building's resistance distribution pattern on its response to ground excitation.

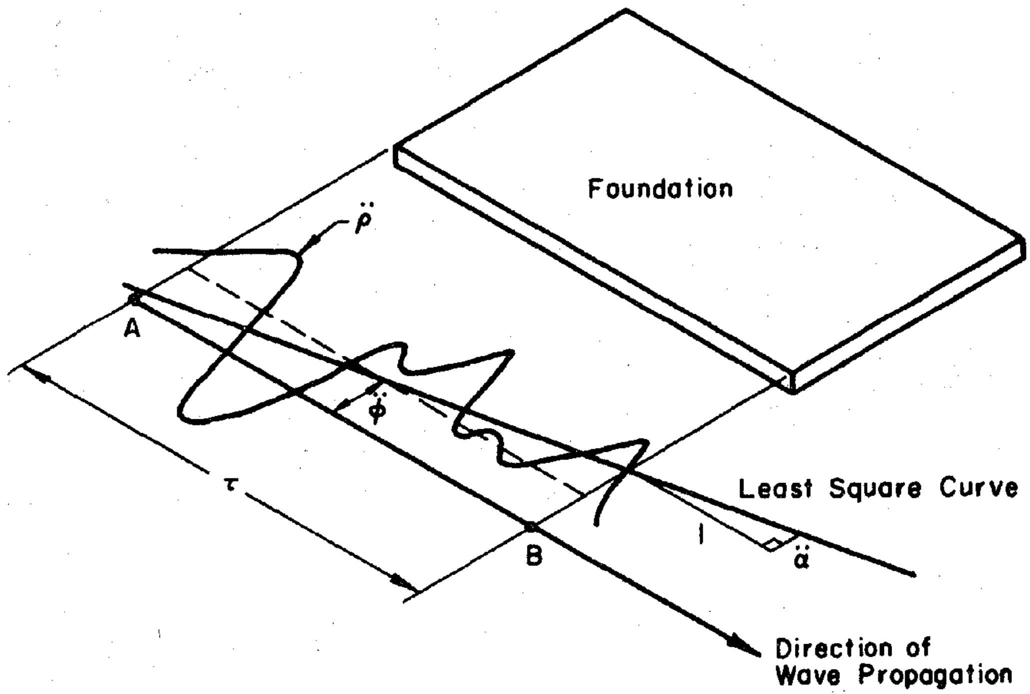


FIGURE 2.1 BASE TRANSLATION AND ROTATION
ARISING FROM TRAVELING SEISMIC WAVES

3. DISCUSSION OF RESULTS: SUPERPOSITION MODEL

3.1 Introduction

Studies of acceleration time histories and corresponding computed responses for buildings suggest that the accelerations imparted to large structures approach an average of the free-field motion over some transit time related to building size. The Hollywood Storage building and adjacent P. E. lot constitute one of the few sites where motions have been measured in the basement of a building and nearby in the free-field. The building itself is 51 feet in the N-S direction and 217.5 feet in the E-W direction (Fig. 3.1). The building is 150 feet high and is supported on piles. The basement accelerograph is located in the S-W corner of the building. The free-field instrument is located 112 feet due West of the S-W corner of the building. In Ref. 19 the shear wave velocity in the upper strata under the building is shown as being approximately 1500 to 2000 fps and this can be considered as possibly the wave propagation velocity in the near surface zone (Fig. 3.2).

Calculations were made for the Hollywood Storage P.E. lot records for both the San Fernando and Kern County earthquakes for various transit times for both horizontal components of motion (Figs. 3.3 and 3.4) using the averaging technique described in Chapter 2. The spectrum for a transit time of zero seconds is the unmodified free-field response spectrum for the P.E. lot. The other solid curves shown represent the averaged translational response for various transit times. The response spectrum computed from the Hollywood Storage building basement record for each earthquake is shown by the dashed line in each figure. As noted previously the instruments at Hollywood Storage are over 100 feet apart; nonetheless, the calculations herein lead to the trend observed. It appears from initial studies that either the longest dimension of the building or the mean or geometric mean of the dimension controls the effective transit time insofar as the reduction in response is concerned.

As a part of this study both horizontal components of earthquake ground motions from five different sites (both soil and rock), recorded during four different events were used as input for the τ-averaging procedure. For purposes of illustration one component from each of four sites (3 earthquakes) is presented, these four components were selected as being typical of the group studied. The other three records presented are the Pacoima Dam record, also from the San Fernando earthquake, one of the Cholame, Shandon array records from the 1966 Parkfield earthquake; and the Taft Lincoln School record from the 1952 Kern County earthquake. These records represent the maximum recorded acceleration, the record closest to associated fault motion, and a typical broad-banded spectrum earthquake, respectively.

Although there are no building records with which to compare, calculations were made with the acceleration time histories in addition to Hollywood Storage in an effort to demonstrate the τ effect for different time histories from different geologic conditions and at various distances from the source of the earthquake ground motion. It is hoped that this will minimize the bias in the observations and conclusions of this study. The discussion which follows is precisely as outlined in Chapter 2.

3.2 Response from Translation Only

As shown in previous work (Refs. 4, 9, 17) there is a significant reduction in the upper frequency range for averaged translational response as compared to free-field response. This reduction occurs at frequencies greater than 1 Hz and is illustrated in Figs. 3.5 and 3.6 for the Hollywood Storage P.E. lot, San Fernando earthquake record. The averaging that occurs is directly proportional to building size as would be expected (i.e., as the building gets larger the magnitude of response reduction also increases). These trends hold true for each of the records presented (see also Figs. 3.7 through 3.12).

3.3 Response at Edge from Rotation Alone

Torsional components of ground motion can be as important as the translational components in determining the base input to be used for design. The significance of torsional response increases with the size of the building (greater $\tau = b/c$ values) and with increasing frequency, as might be expected. In Figs. 3.13 and 3.14 the response at the building edge arising from rotation alone is plotted and compared to free-field response for the Hollywood Storage P.E. lot record. For both values of τ the rotational component can be seen to be important and in fact actually exceeds the free-field translational response in some regions (5-9 Hz for $\tau = 0.08$, 2.5-6 Hz for $\tau = 0.16$). Once again the same trend, i.e., the translational response arising from rotation is greater than the free-field translational response, holds true for the other records studied.

In all of the above comparisons of rotationally induced translational response with free-field translational response it has been tacitly assumed that the torsional response of the system is at the same fundamental frequency as the free-field or translational response. It has been found in this model study that this assumption overemphasizes the importance of torsion in systems whose torsional frequency is greater than its translational frequency (see also Ref. 9). A comparison of the rotational responses computed for a ratio of torsional to translational frequency of unity and for a ratio of 1.414 is shown in Fig. 3.21; as in all plots the frequency scale refers to the translational frequency. One will note that, when considering response in the acceleration controlled region of the spectrum (above 2 Hz), the response computed assuming a frequency ratio of 1.414 is not as large as that for a ratio of 1.0.

3.4 Combined Response

Of special importance in making design recommendations is the effect of combined motion, or in other words the total or maximum response arising from both translational and rotational effects. It has been shown that, in the high frequency region, consideration of translational averaging would lead to response reduction and consideration of torsional effects would lead to some response amplifications. When considering combined motion one would expect the effects of translational averaging and rotation to partially offset each other, as indeed they do at high frequencies. However, it has been found that in the mid-frequency (2-8 Hz) range the combined response is somewhat greater than the normally computed free-field response (see Figs. 3.22 through 3.29). There is, of course, some variation in the degree and region of amplified response (above 1.5 Hz for $\tau = 0.08$, and 1-6 Hz for $\tau = 0.16$) among the various records but the trend is unmistakable (Ref. 9). For each of the records studied there is no significant deviation from the trends observed for the Hollywood Storage P.E. lot (Figs. 3.20 and 3.21). Once again these comparisons assume a ratio of unity between torsional and translational frequencies. In Figs. 3.30 and 3.31 are shown the combined response for a torsional to translational frequency ratio of 1.414. It will be seen by comparing Fig. 3.31 with Fig. 3.25 that any amplifications, as compared to free-field response, are greatly reduced or eliminated and that any reductions are increased.

There are several methods which may be used to combine translational and torsional responses. The most conservative of these methods would be to combine the absolute values of maximum response arising from each type of motion. Alternately, one may wish to assume statistical independence of the two effects and use the square root of the sum of the squares of the maxima. Lastly, one may wish to take advantage of any phasing between ground motions and compute a rigorous algebraic summation in the time domain of the responses arising from averaged translation and induced rotation.

Little difference is observed between the total response somputed by the Square Root of the Sum of the Squares technique (SRSS) and that computed using the algebraic summation in time. In Fig. 3.32 there is shown a comparison of these two computations for $\tau = 0.08$ for one of the Pacoima Dam records from the 1971 San Fernando earthquake. It can be seen that only slight differences exist, as is true in all other cases studied. A comparison of the SRSS spectrum with the absolute summation of maximums for the same record is contained in Fig. 3.33. In this case one notes a significant difference in the two spectra, and as expected the summation of maximums leads to the larger computed response.

Assuming a torsional to translational frequency ratio of unity, an attempt was made (Ref. 9) to formulate a general procedure for constructing a design spectrum which includes both translational averaging and torsion.

As a result it was considered that a modification of a normal design spectrum in the acceleration region would be sufficient to include the combined effects. In a first attempt to indicate the notion of amplifying current design spectra, the regions and amplifications found as a part of this study are shown in Figs. 3.34 and 3.35. Transition zones were not shown since they could not be accurately defined at the time. The amplification values employed for the response spectra used for comparison purposes in Figs. 3.34 and 3.35 are taken from earlier studies by Hall, Mohraz and Newmark (Ref. 20).

3.5 Special Considerations

Certain trends were observed during this study which hold for records from different earthquakes and for sites on soil and rock. However, these trends can only be considered qualitative at this time. When considering the suggested trends shown in Figs. 3.34 and 3.35 (Ref. 9) one must also remember that these are from a study including only one torsional to translational frequency ratio, namely unity; unfortunately not enough computations were made to do this for a ratio of 1.414. As demonstrated in this chapter the frequency ratio is a very important parameter and for some frequency ratios the amplifications indicated in Figs. 3.34 and 3.35 may well be reductions. In fact it is suspected that the amplifications obtained for a frequency ratio of unity are the maximum that will occur since in practice one will encounter structures whose frequency ratio is greater than 1.

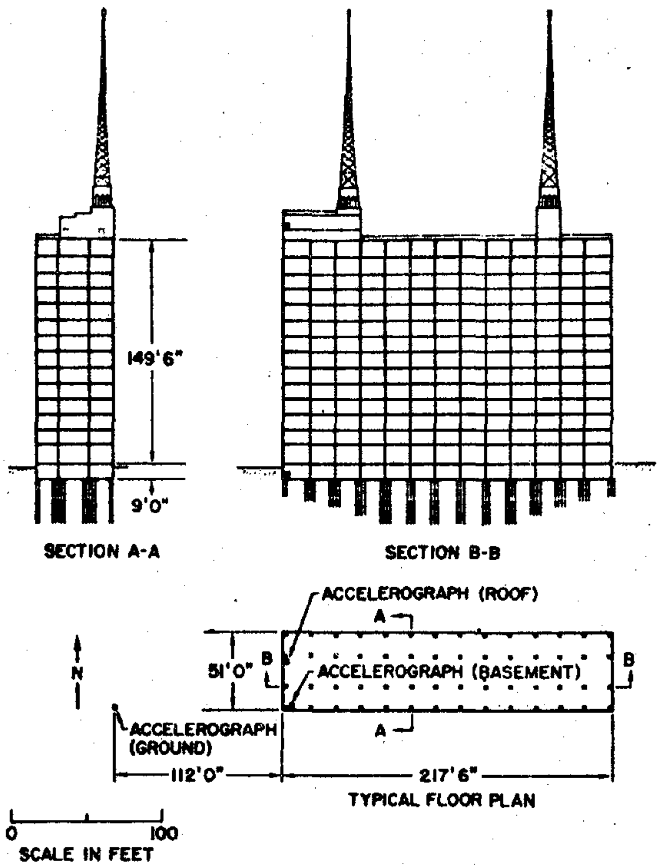
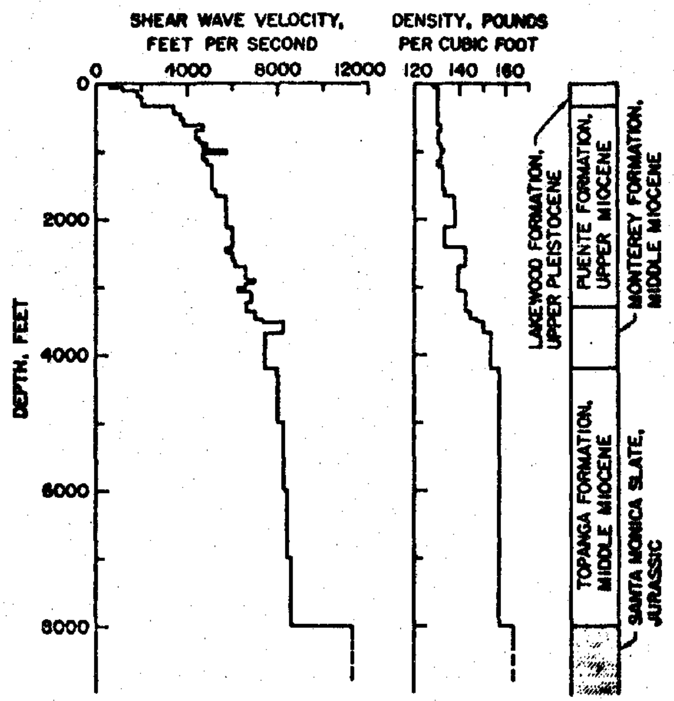


FIGURE 3.1 PLAN AND ELEVATION OF HOLLYWOCD STORAGE BUILDING

SOURCE (REF. 19)



Subsurface model.

Natural Frequencies of Building from Vibration Test*

Mode of Vibration	Frequency (cps) North-south	Frequency (cps) East-west
Fundamental translation	0.83	2.0
Second translational	2.7	
Third translational	4.5	
Fundamental torsional	1.57–1.67	
Second torsional	5.9	
Third torsional	9.1	
Others	1.0, 5.0	

* Source: Carder, 1964.

FIGURE 3.2 SHEAR WAVE VELOCITY PROFILE AND BUILDING FREQUENCIES, HOLLYWOOD STORAGE BUILDING

SOURCE (REF. 19)

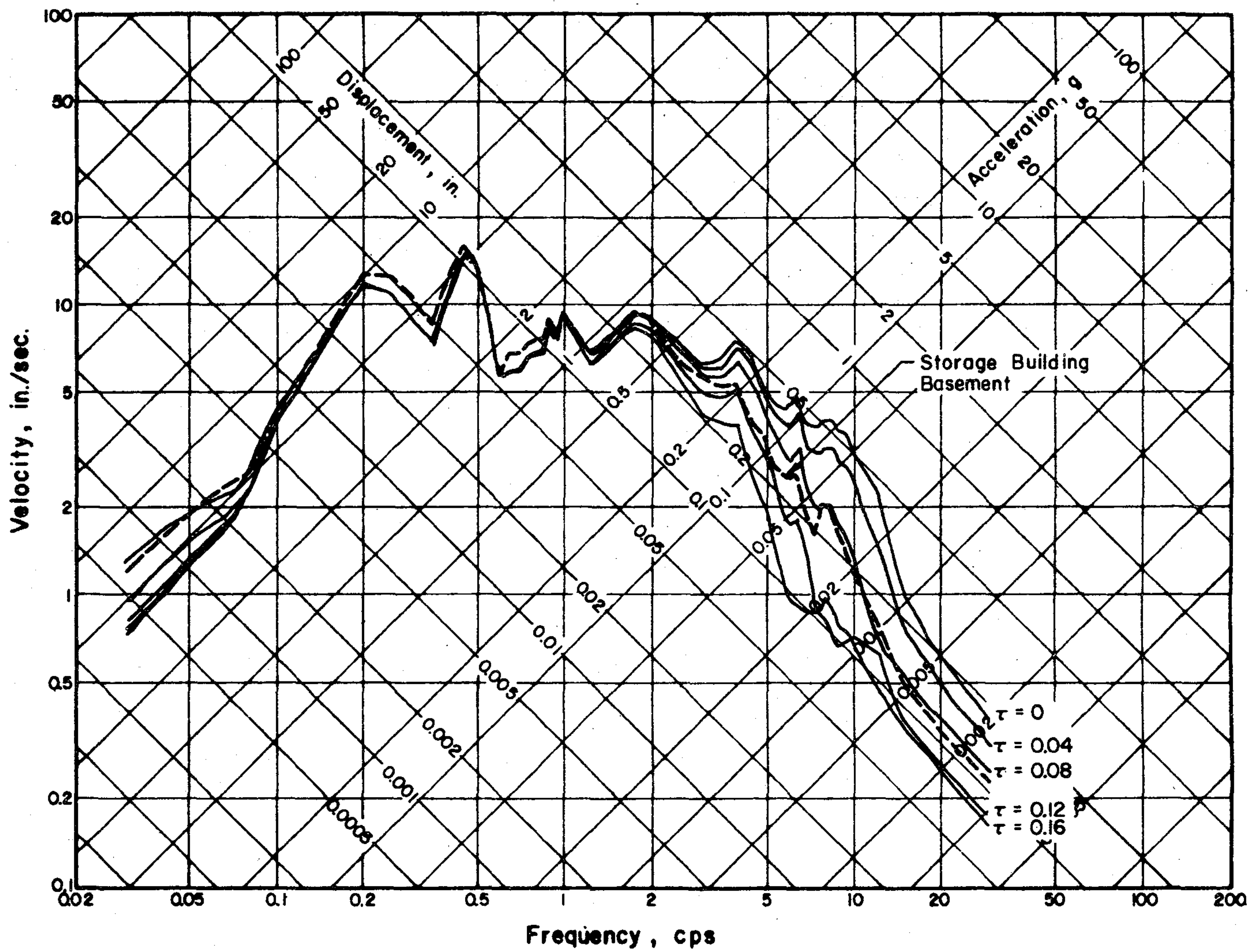


FIGURE 3.3 HOLLYWOOD STORAGE P.E. LOT, SAN FERNANDO EARTHQUAKE, 9 FEBRUARY 1971, COMPONENT SOUTH, DAMPING 2% OF CRITICAL, τ = 0, 0.04, 0.08, 0.12, AND 0.16 SECOND

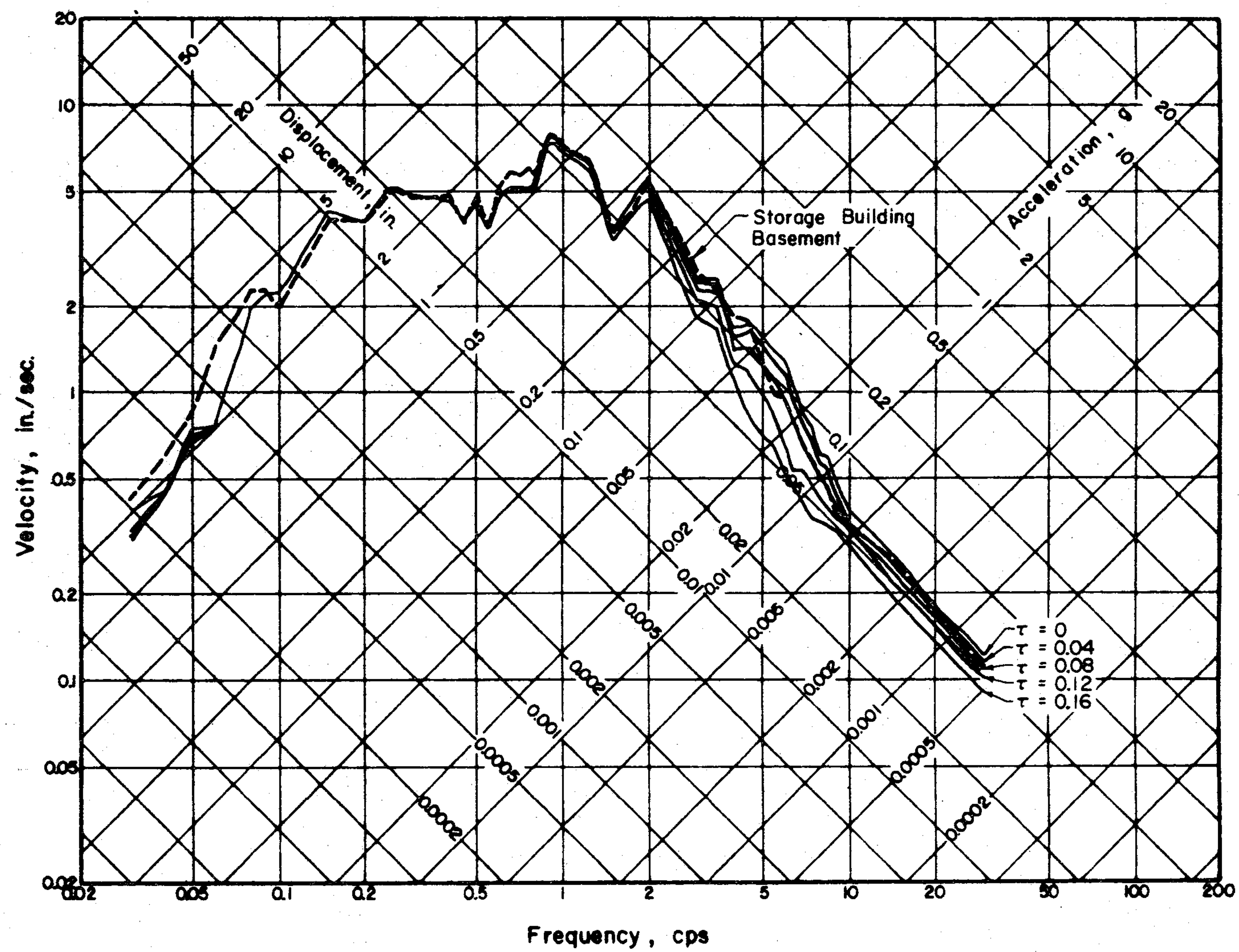


FIGURE 3.4 HOLLYWOOD STORAGE P.E. LOT, KERN COUNTY EARTHQUAKE, 21 JULY 1952, COMPONENT SOUTH, DAMPING 2% OF CRITICAL, τ = 0, 0.04, 0.08, 0.12, AND 0.16 SECOND

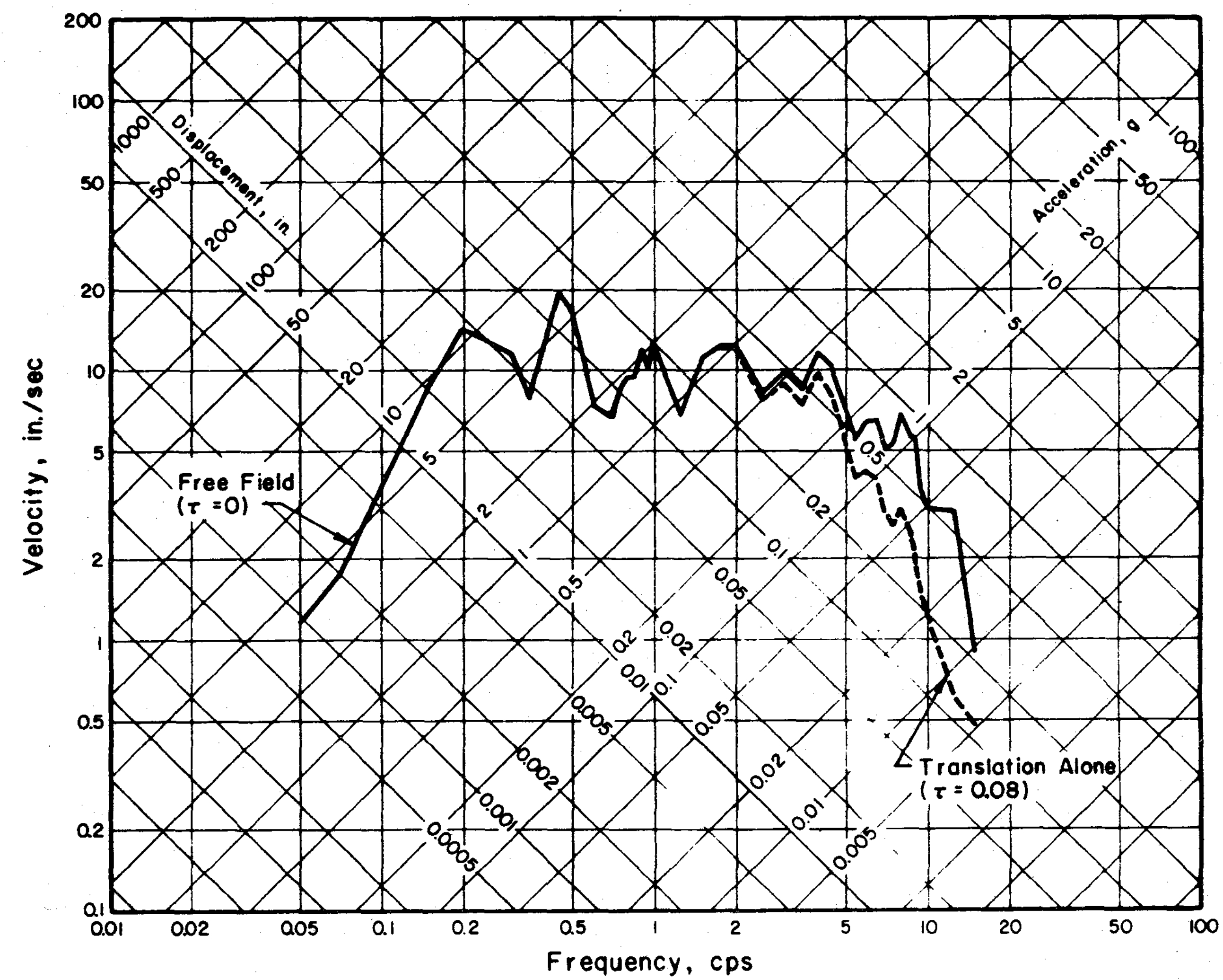


FIGURE 3.5 HOLLYWOOD STORAGE P.E. LOT, SOOW, SAN FERNANDO EARTHQUAKE, 9 FEBRUARY 1971, SPECTRUM COMPUTED USING 2.0 PERCENT CRITICAL DAMPING, $\tau = 0.08$

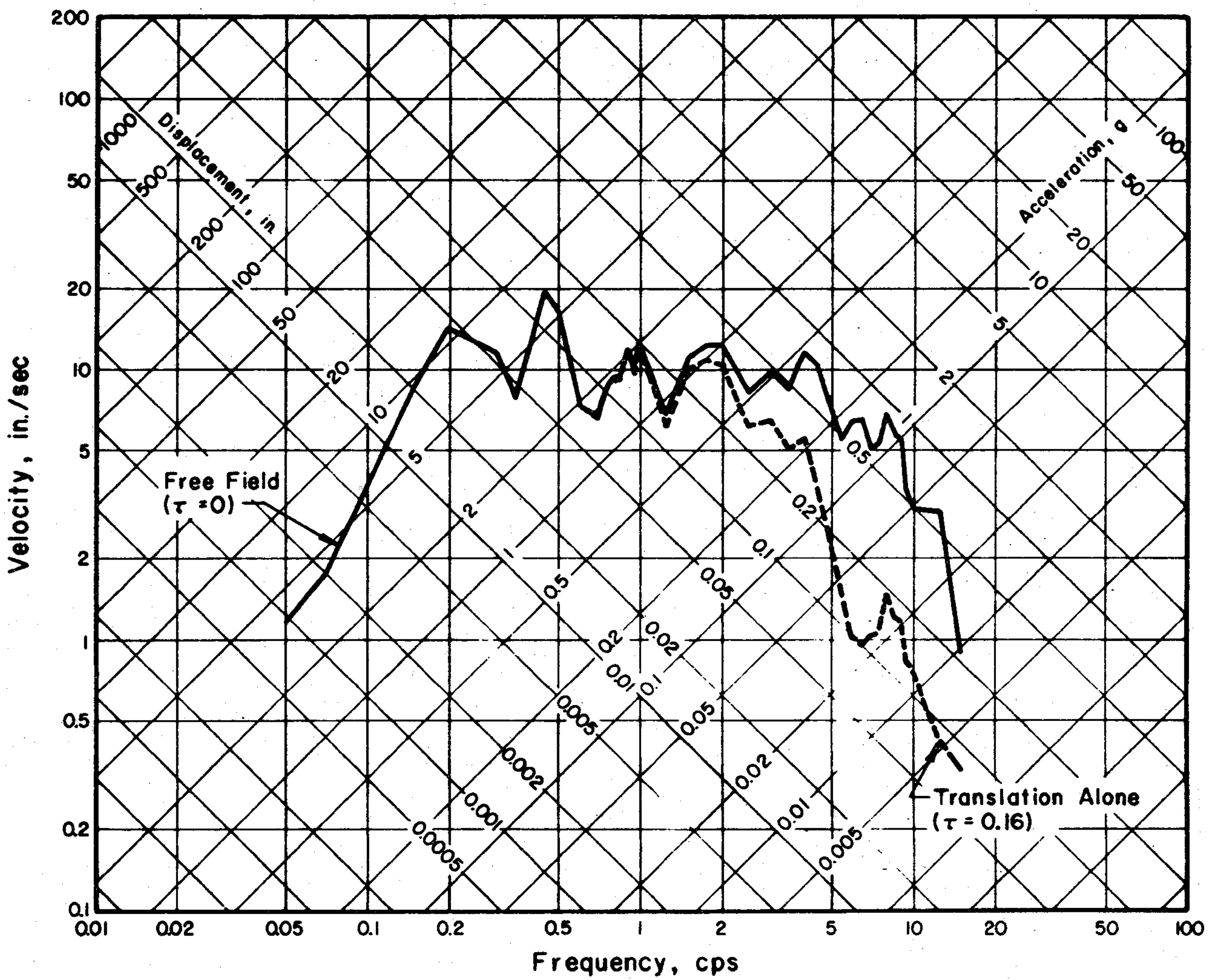


FIGURE 3.6 HOLLYWOOD STORAGE P.E. LOT, SOOW, SAN FERNANDO EARTHQUAKE, 9 FEBRUARY 1971, SPECTRUM COMPUTED USING 2.0 PERCENT CRITICAL DAMPING, $\tau = 0.16$

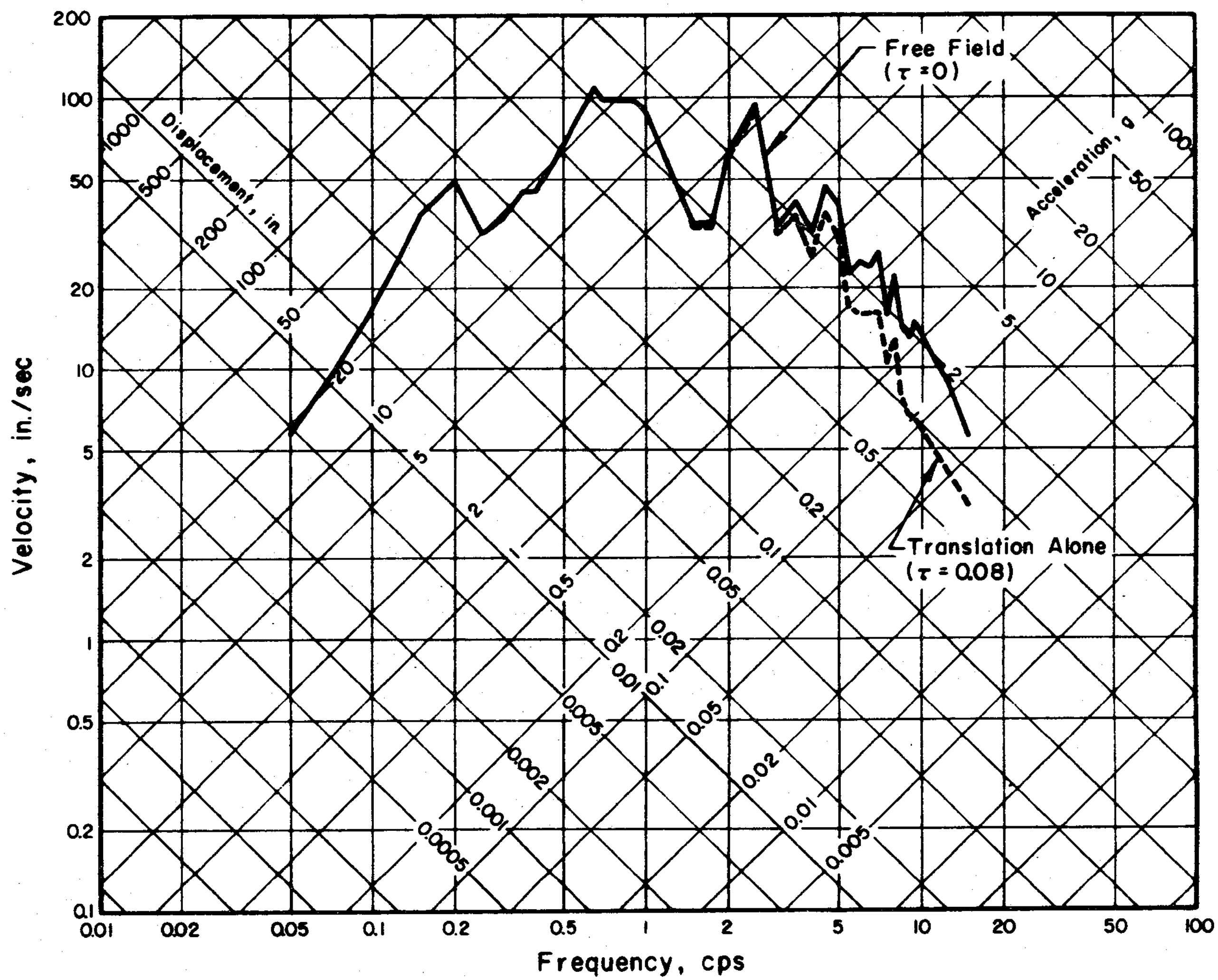


FIGURE 3.7 PACOIMA DAM, S16E, SAN FERNANDO EARTHQUAKE, 9 FEBRUARY 1971, SPECTRUM COMPUTED USING 2.0 PERCENT CRITICAL DAMPING, $\tau = 0.08$

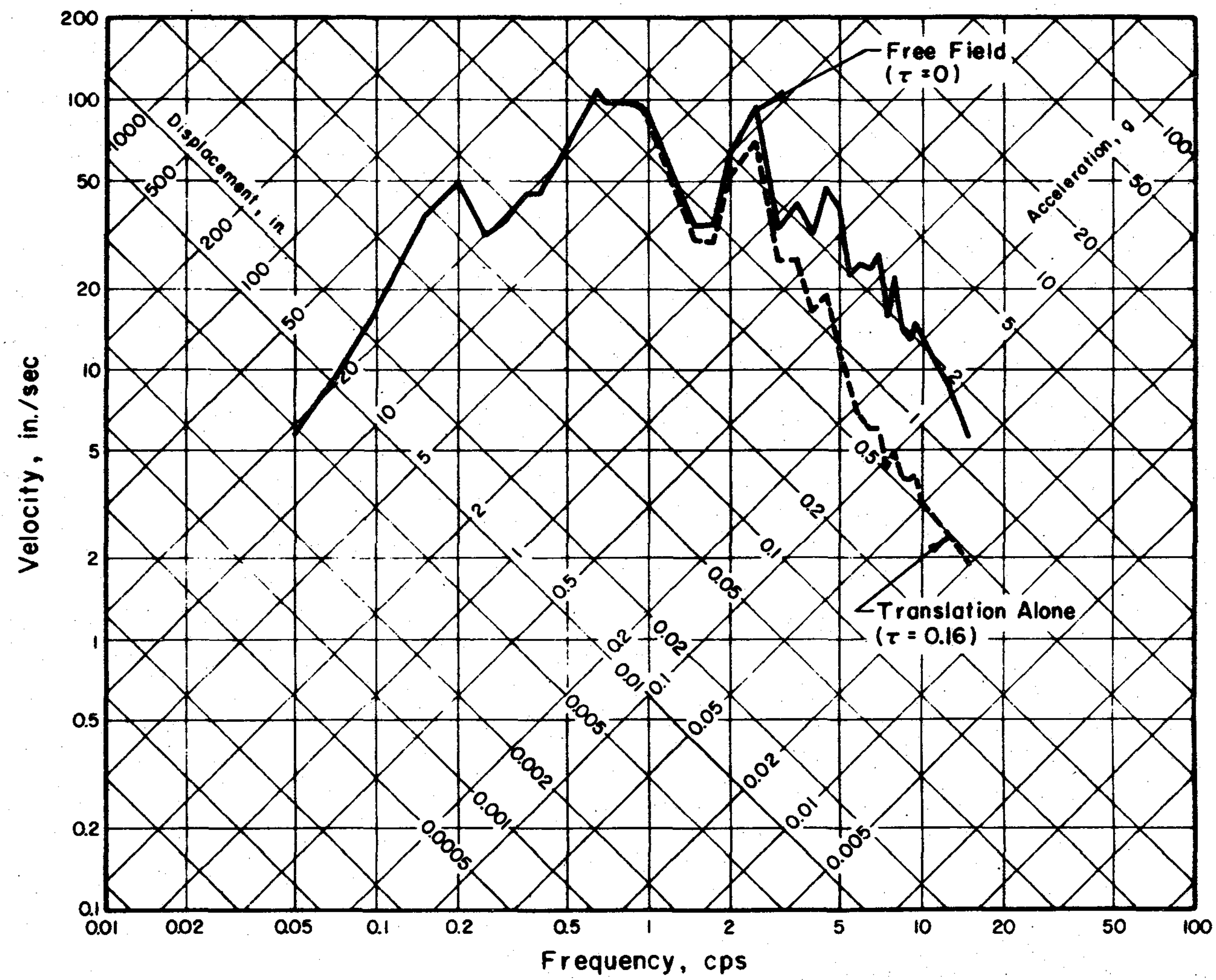


FIGURE 3.8 PACOIMA DAM, S16E, SAN FERNANDO EARTHQUAKE, 9 FEBRUARY 1971, SPECTRUM COMPUTED USING 2.0 PERCENT CRITICAL DAMPING, $\tau = 0.16$

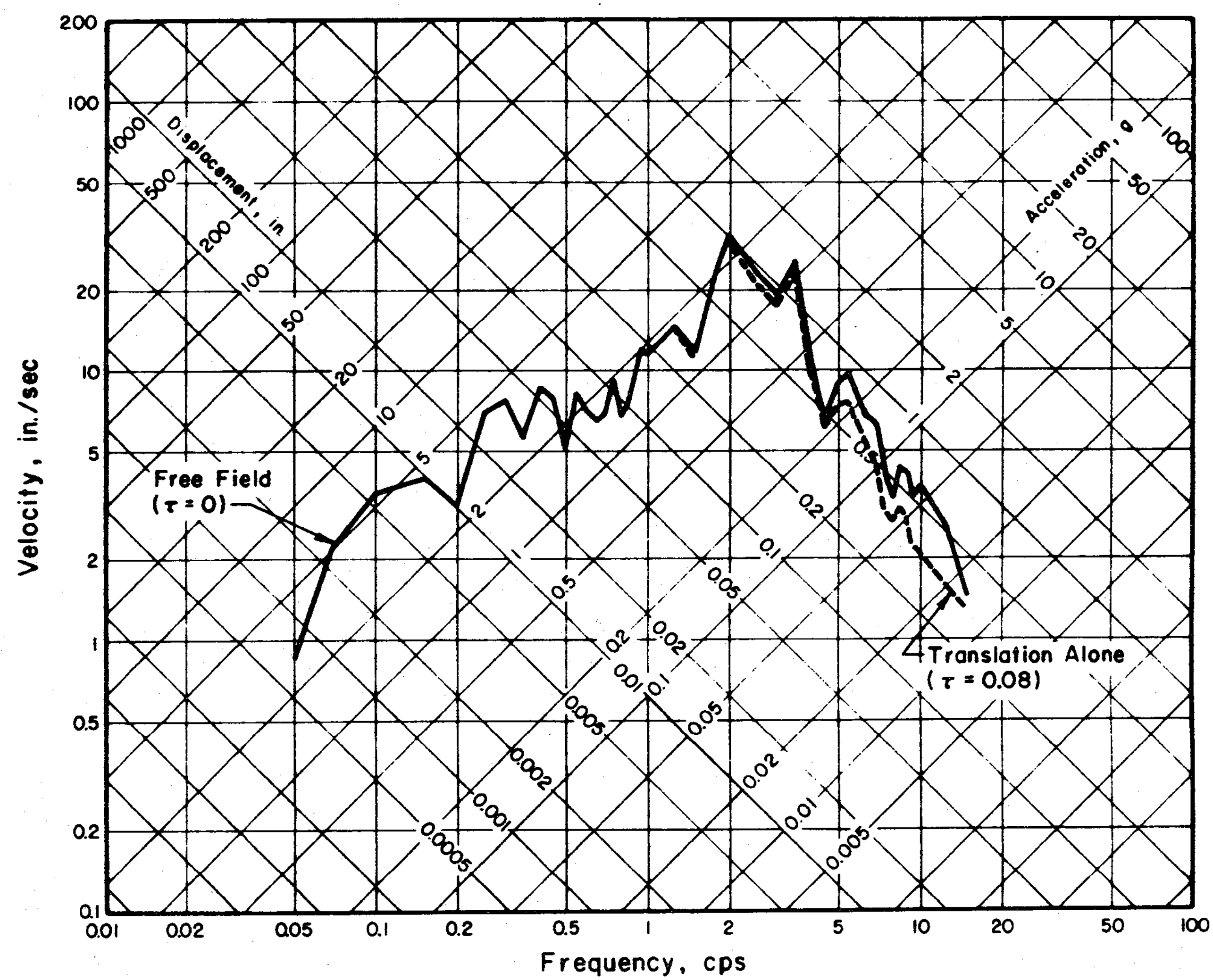


FIGURE 3.9 CHOLAME, SHANDON, ARRAY NO. 5, N05W, PARKFIELD EARTHQUAKE, 27 JUNE 1966, SPECTRUM COMPUTED USING 2.0 PERCENT CRITICAL DAMPING, $\tau = 0.08$

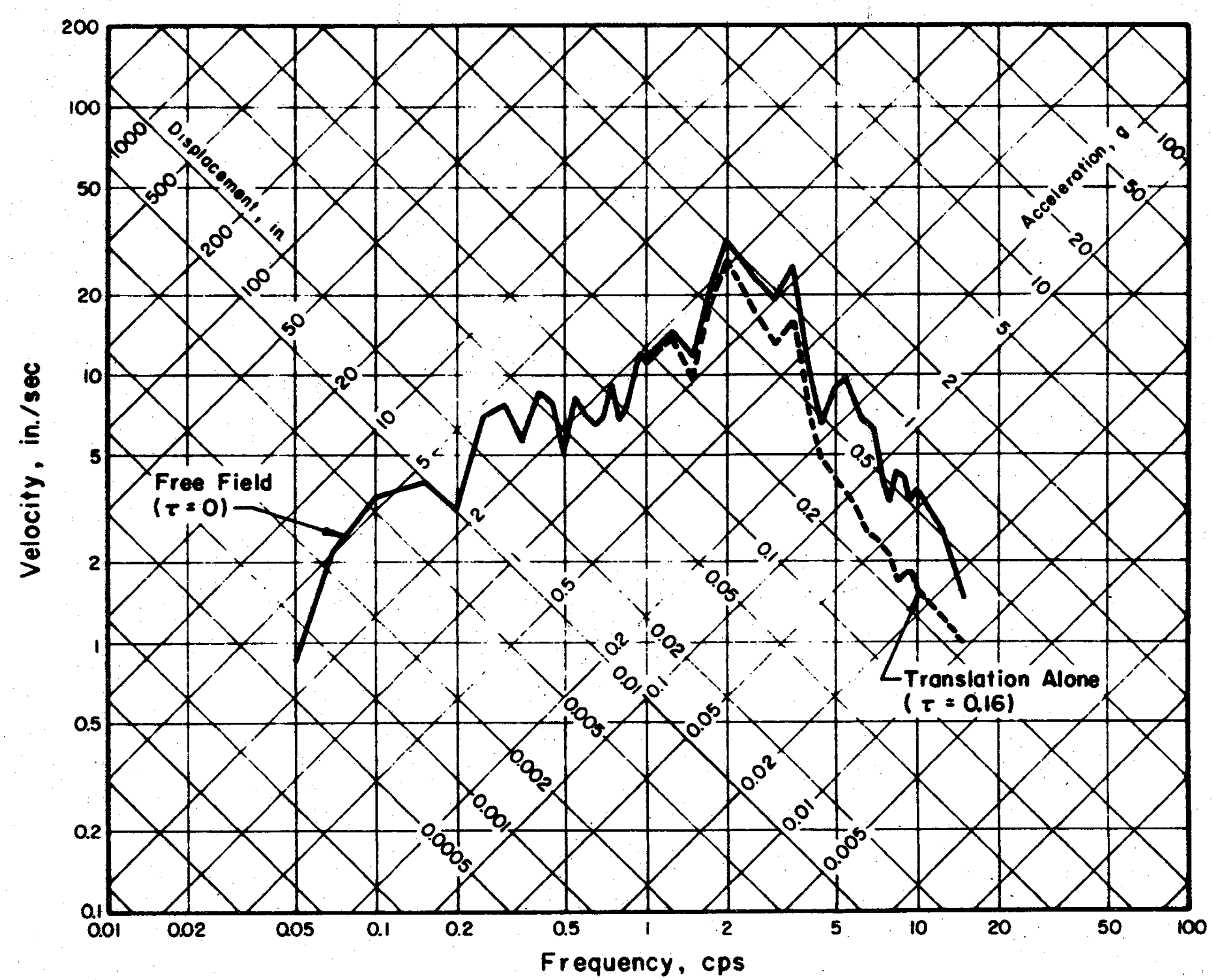


FIGURE 3.10 CHOLAME, SHANDON, ARRAY NO. 5, N05W, PARKFIELD EARTHQUAKE, 27 JUNE 1966, SPECTRUM COMPUTED USING 2.0 PERCENT CRITICAL DAMPING, $\tau = 0.16$

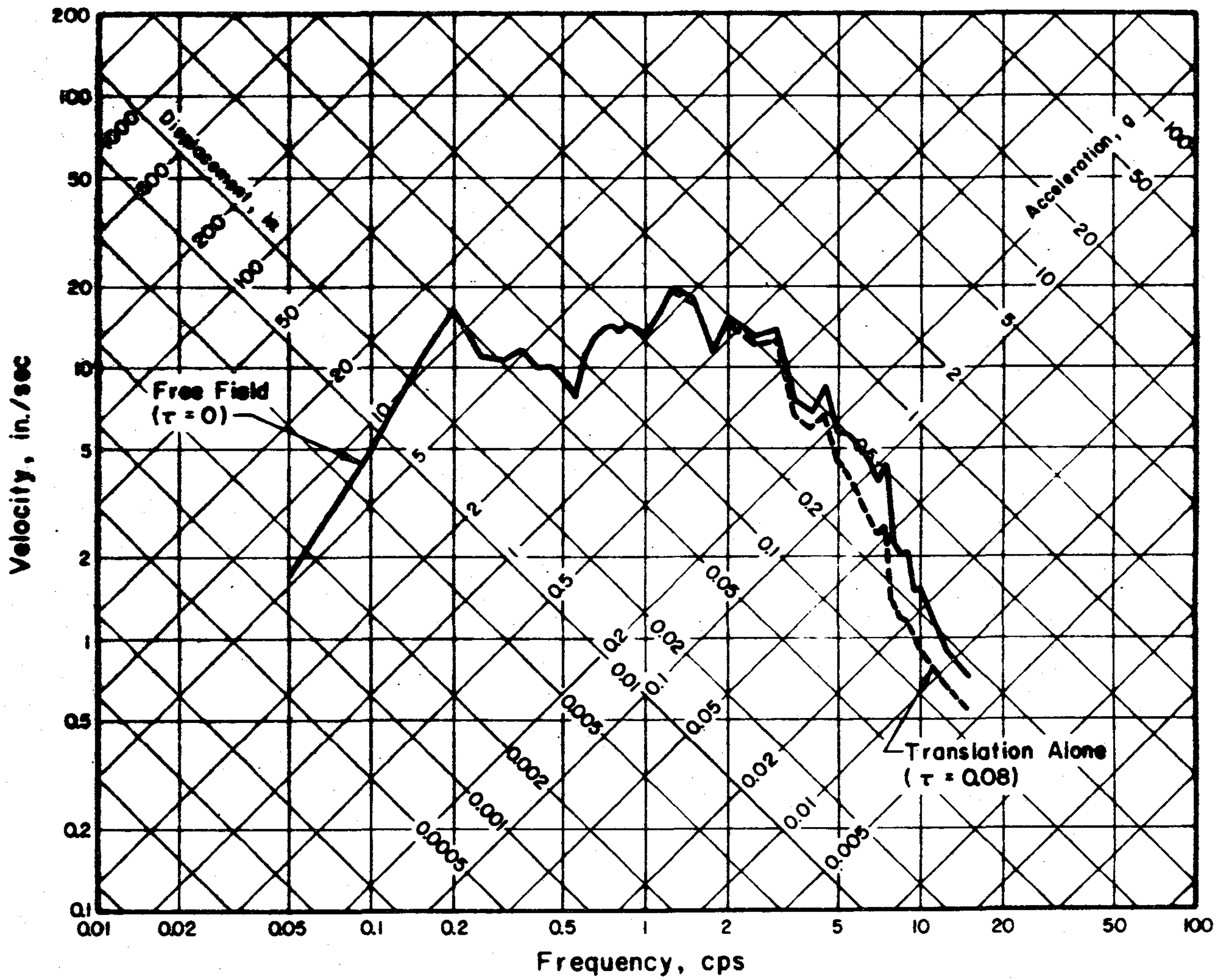


FIGURE 3.11 TAFT LINCOLN SCHOOL, N21E, KERN COUNTY EARTHQUAKE, 21 JULY 1952, SPECTRUM COMPUTED USING 2.0 PERCENT CRITICAL DAMPING, $\tau = 0.08$

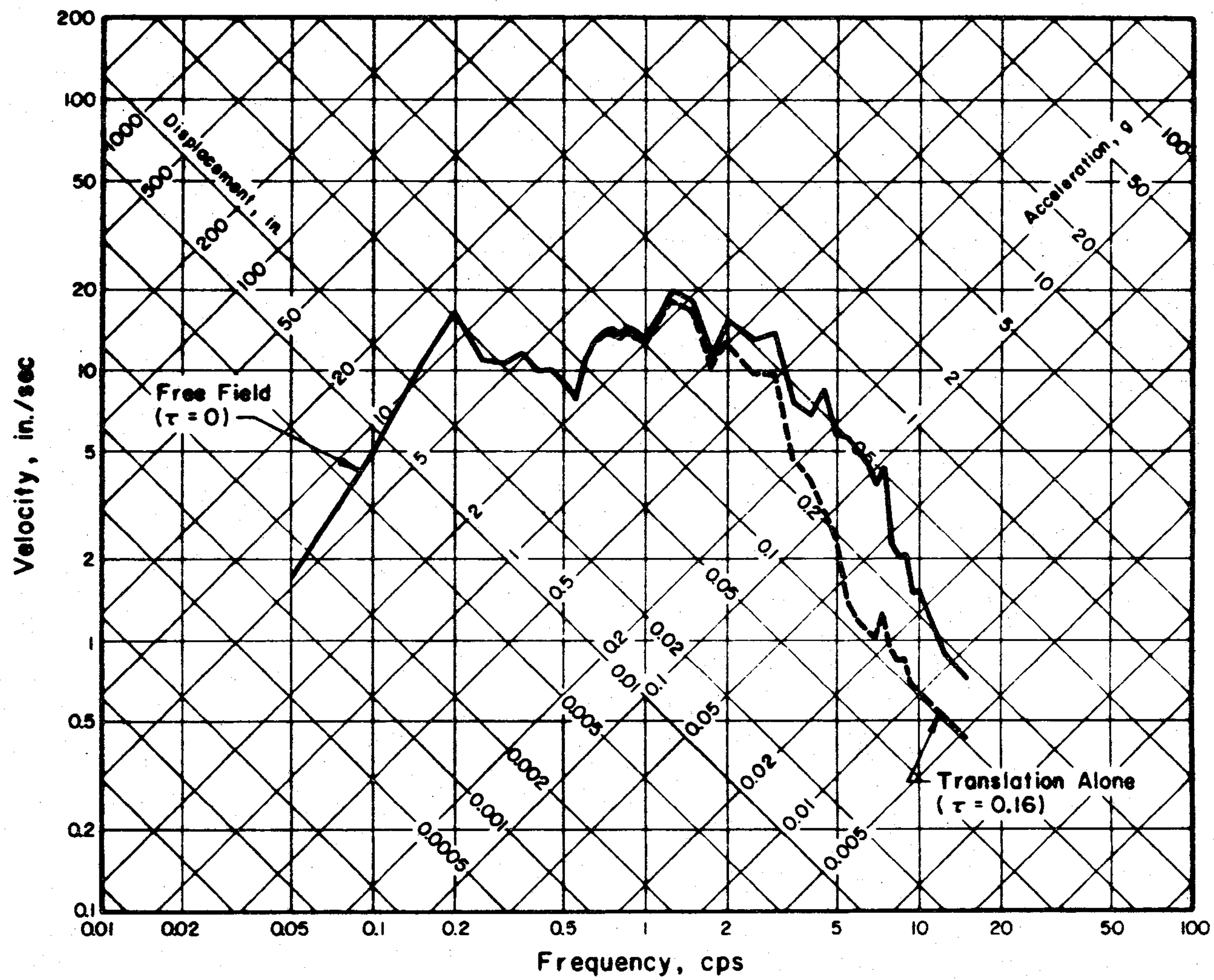


FIGURE 3.12 TAFT LINCOLN SCHOOL, N21E, KERN COUNTY EARTHQUAKE, 21 JULY 1952, SPECTRUM COMPUTED USING 2.0 PERCENT CRITICAL DAMPING, $\tau = 0.16$

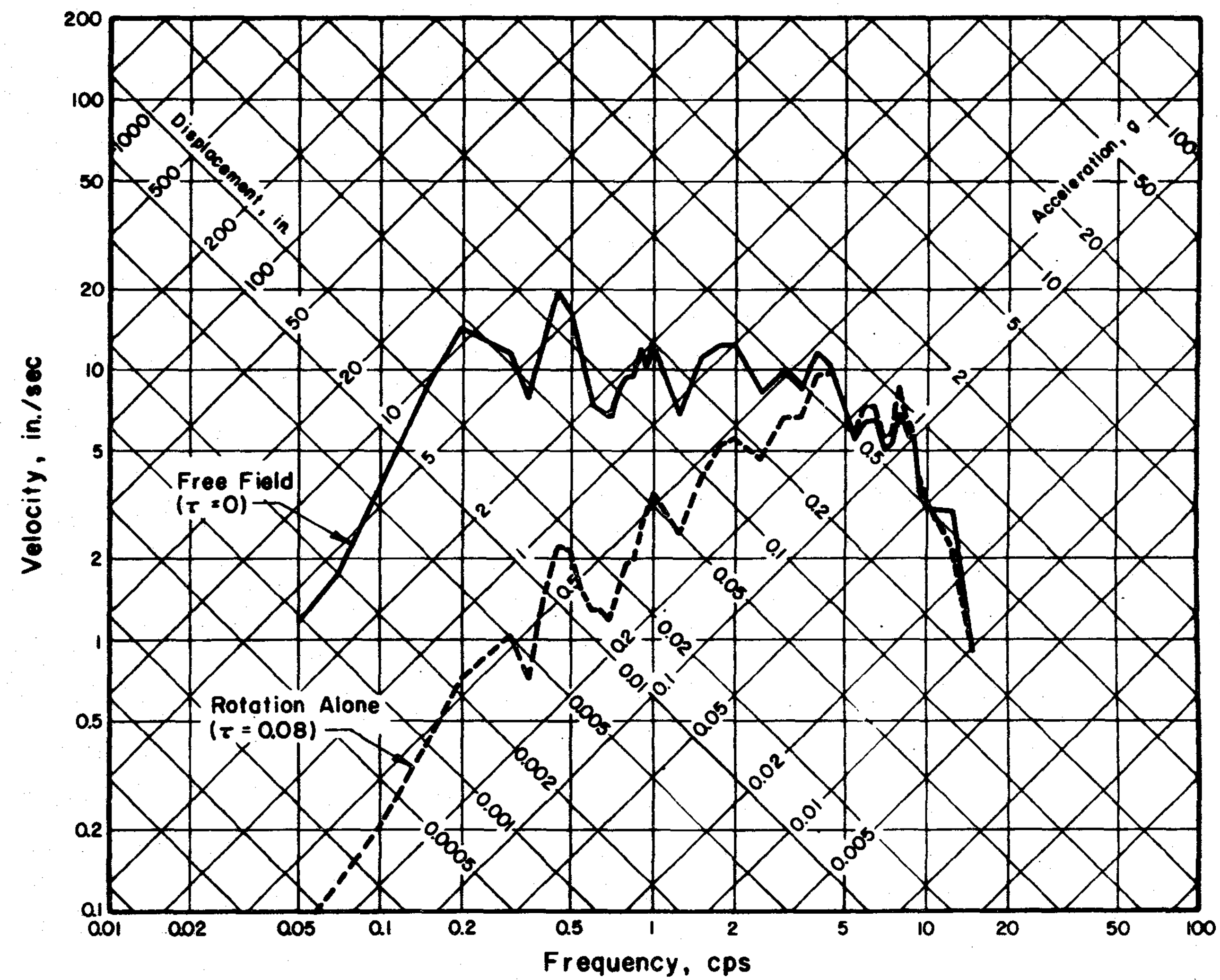


FIGURE 3.13 HOLLYWOOD STORAGE P.E. LOT, SOOW, SAN FERNANDO EARTHQUAKE, 9 FEBRUARY 1971, SPECTRUM COMPUTED USING 2.0 PERCENT CRITICAL DAMPING, $\tau = 0.08$, $f_\theta/f_x = 1$

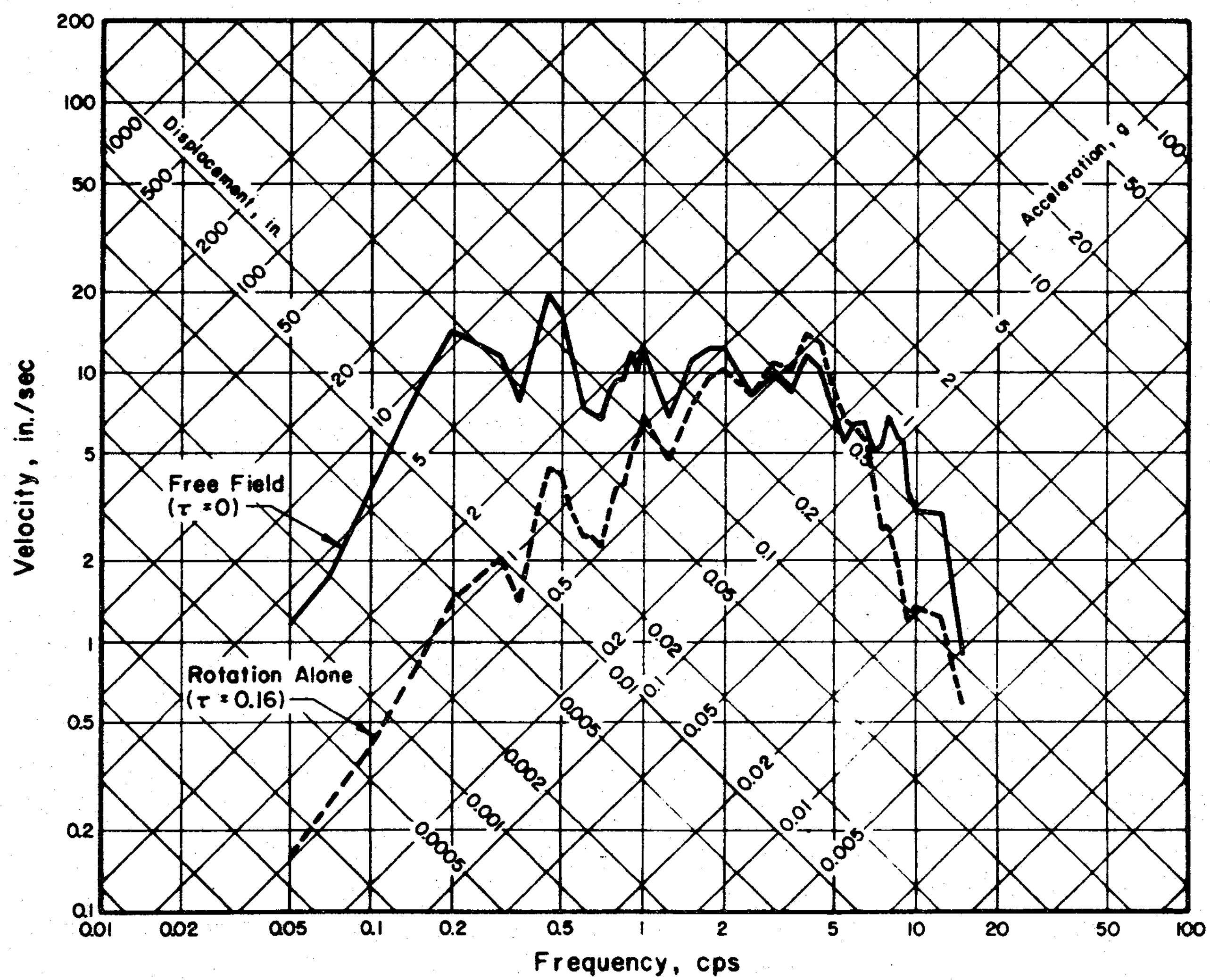


FIGURE 3.14 HOLLYWOOD STORAGE P.E. LOT, SOOW, SAN FERNANDO EARTHQUAKE, 9 FEBRUARY 1971, SPECTRUM COMPUTED USING 2.0 PERCENT CRITICAL DAMPING, $\tau = 0.16$, $f_\theta/f_x = 1$

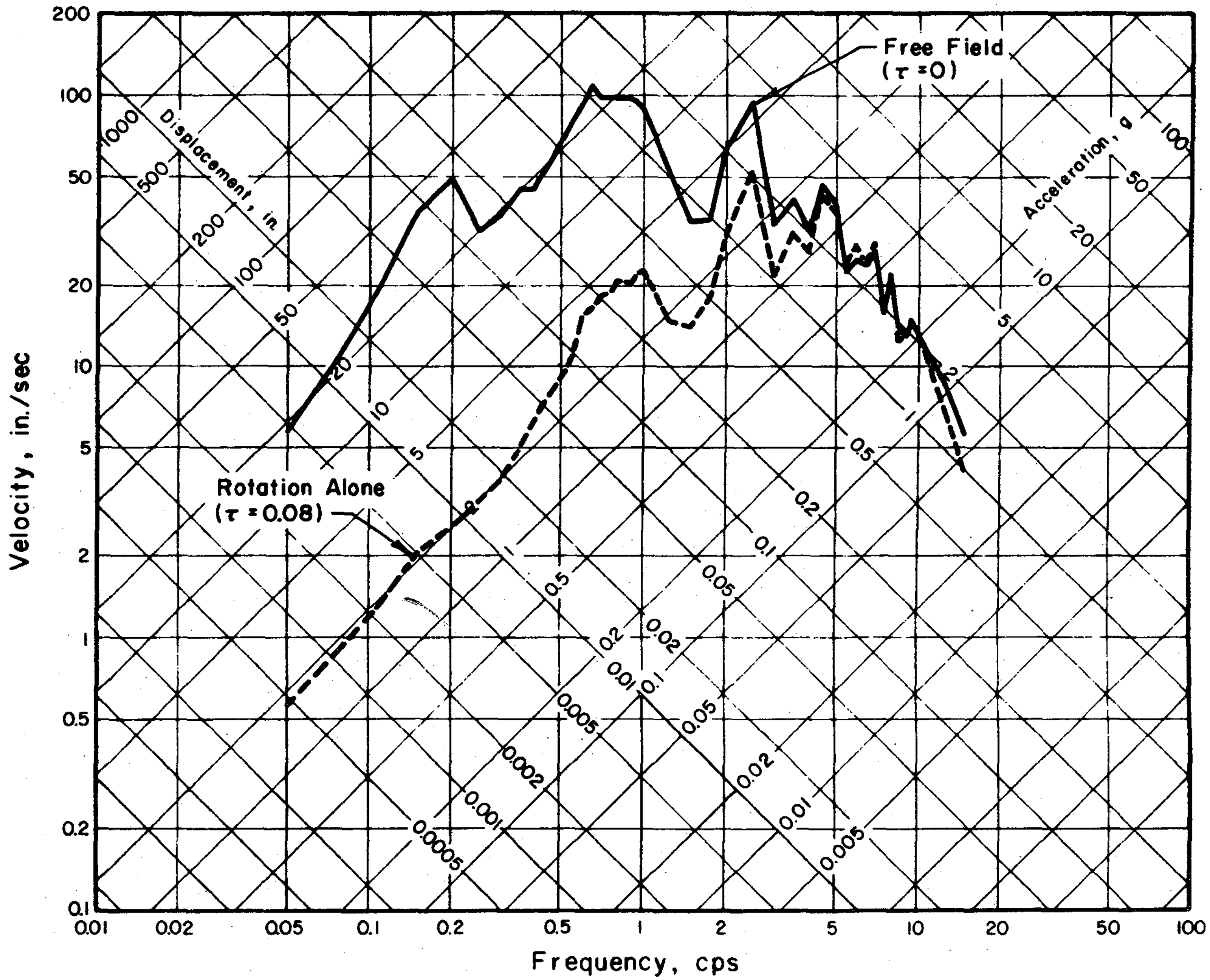


FIGURE 3.15 PACOIMA DAM, S16E, SAN FERNANDO EARTHQUAKE, 9 FEBRUARY 1971, SPECTRUM COMPUTED USING 2.0 PERCENT CRITICAL DAMPING, $\tau = 0.08$, $f_\theta/f_x = 1$

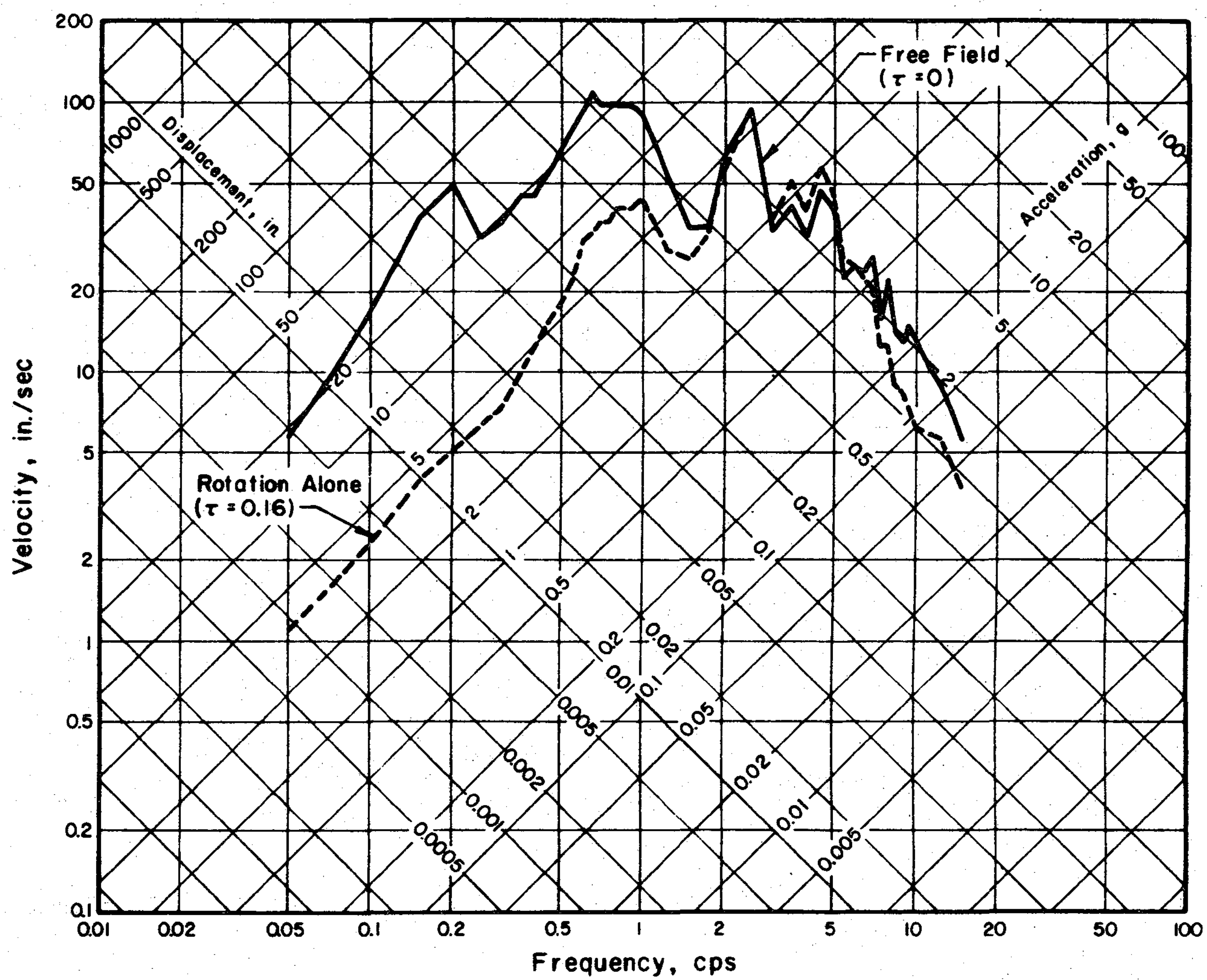


FIGURE 3.16 PACOIMA DAM, S16E, SAN FERNANDO EARTHQUAKE, 9 FEBRUARY 1971, SPECTRUM COMPUTED USING 2.0 PERCENT CRITICAL DAMPING, $\tau = 0.16$, $f_\theta/f_x = 1$

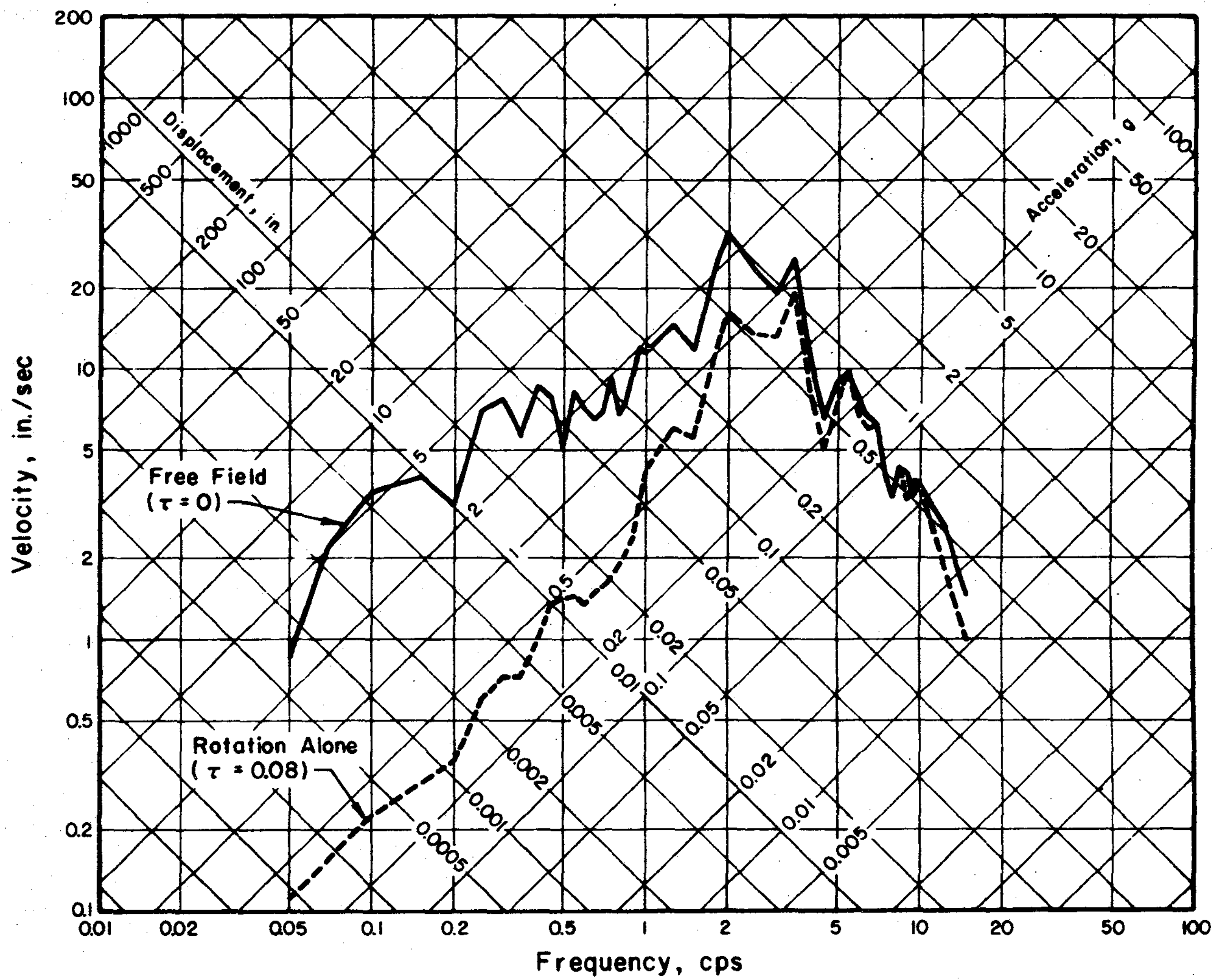


FIGURE 3.17 CHOLAME, SHANDON, ARRAY NO. 5, N05W, PARKFIELD EARTHQUAKE, 27 JUNE 1966, SPECTRUM COMPUTED USING 2.0 PERCENT CRITICAL DAMPING, $\tau = 0.08$, $f_\theta/f_x = 1$

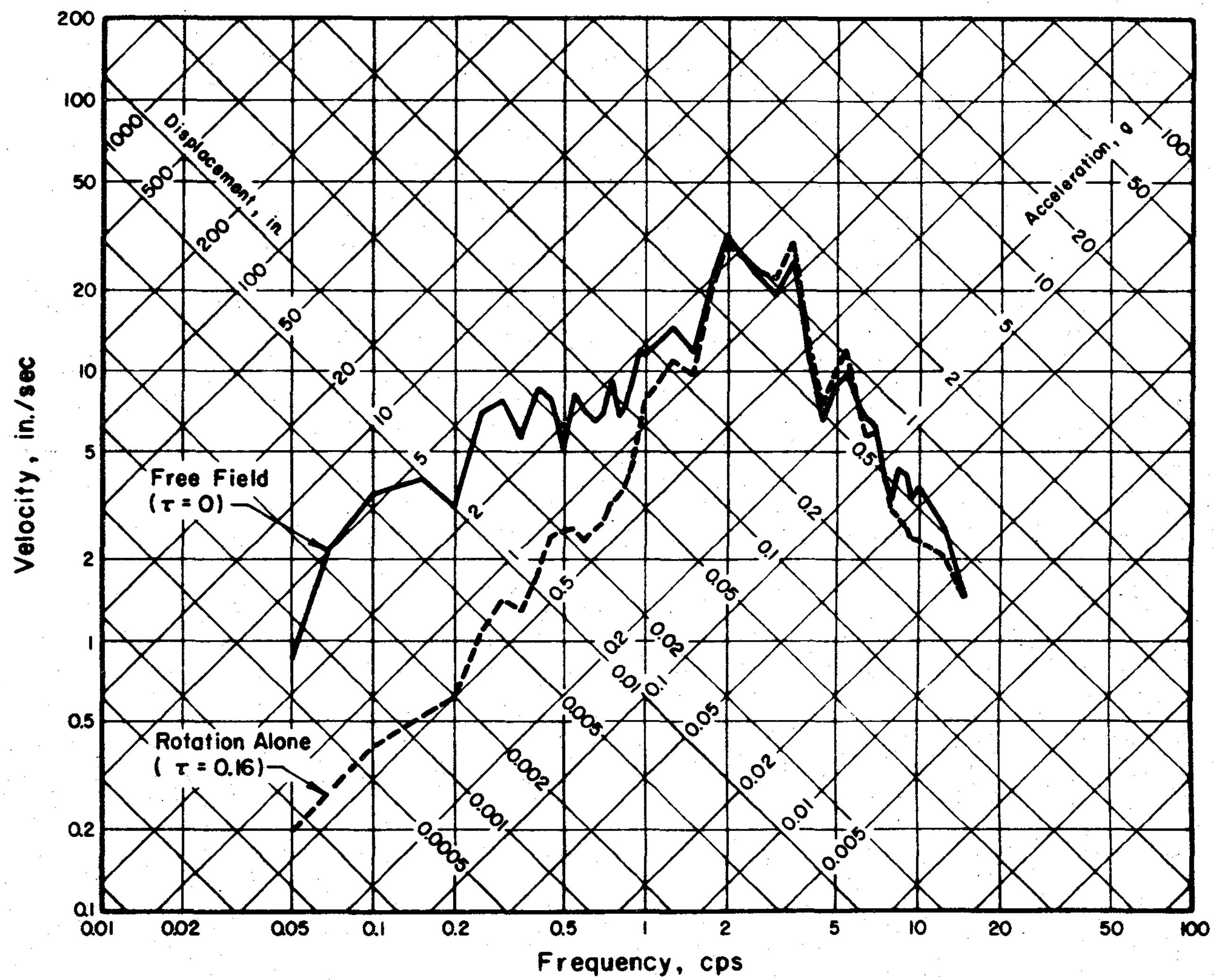


FIGURE 3.18 CHOLAME, SHANDON, ARRAY NO. 5, N05W, PARKFIELD EARTHQUAKE, 27 JUNE 1966, SPECTRUM COMPUTED USING 2.0 PERCENT CRITICAL DAMPING, $\tau = 0.16$, $f_\theta / f_x = 1$

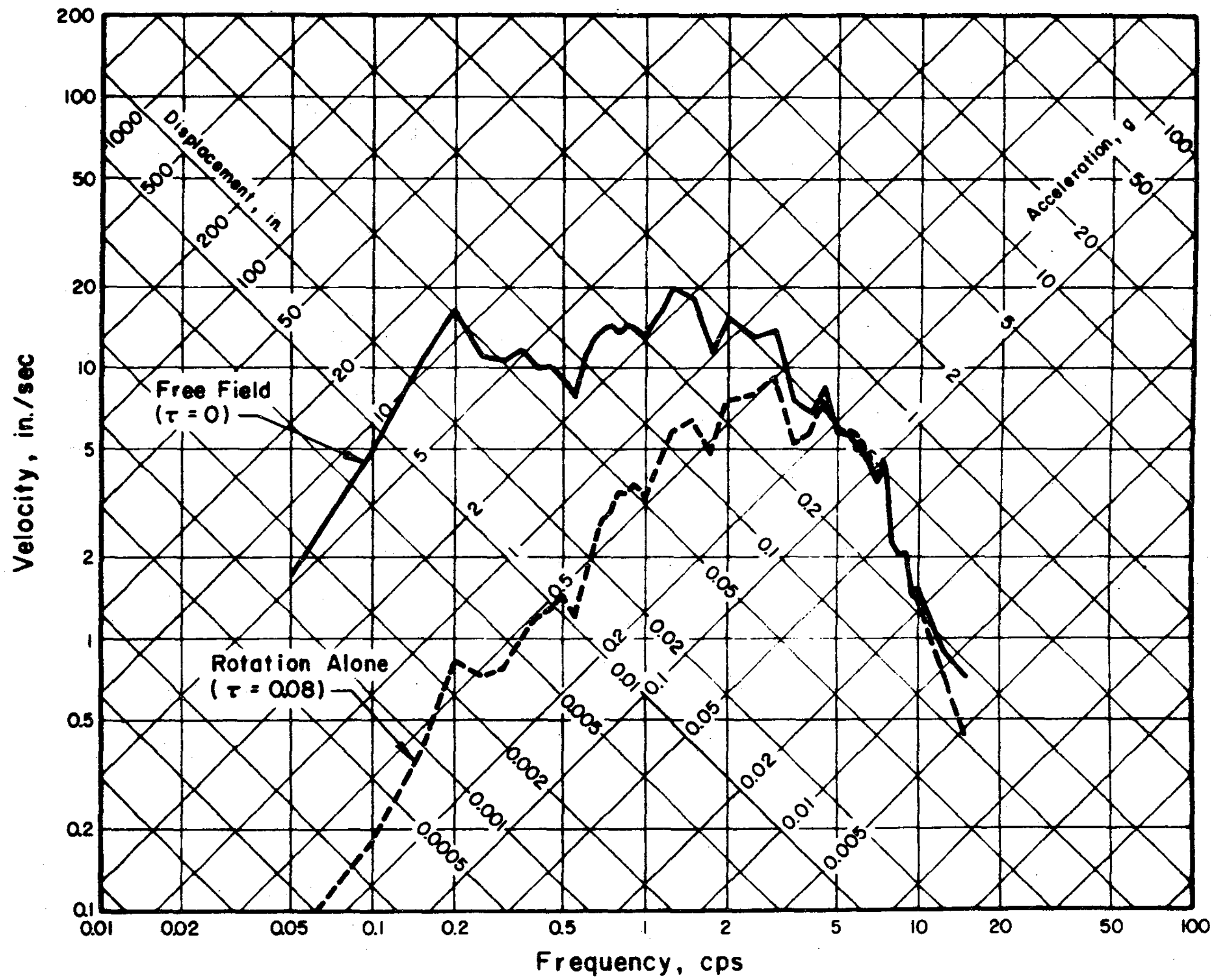


FIGURE 3.19 TAFT LINCOLN SCHOOL, N21E, KERN COUNTY EARTHQUAKE, 21 JULY 1952, SPECTRUM COMPUTED USING 2.0 PERCENT CRITICAL DAMPING, $\tau = 0.08$, $f_\theta/f_x = 1$

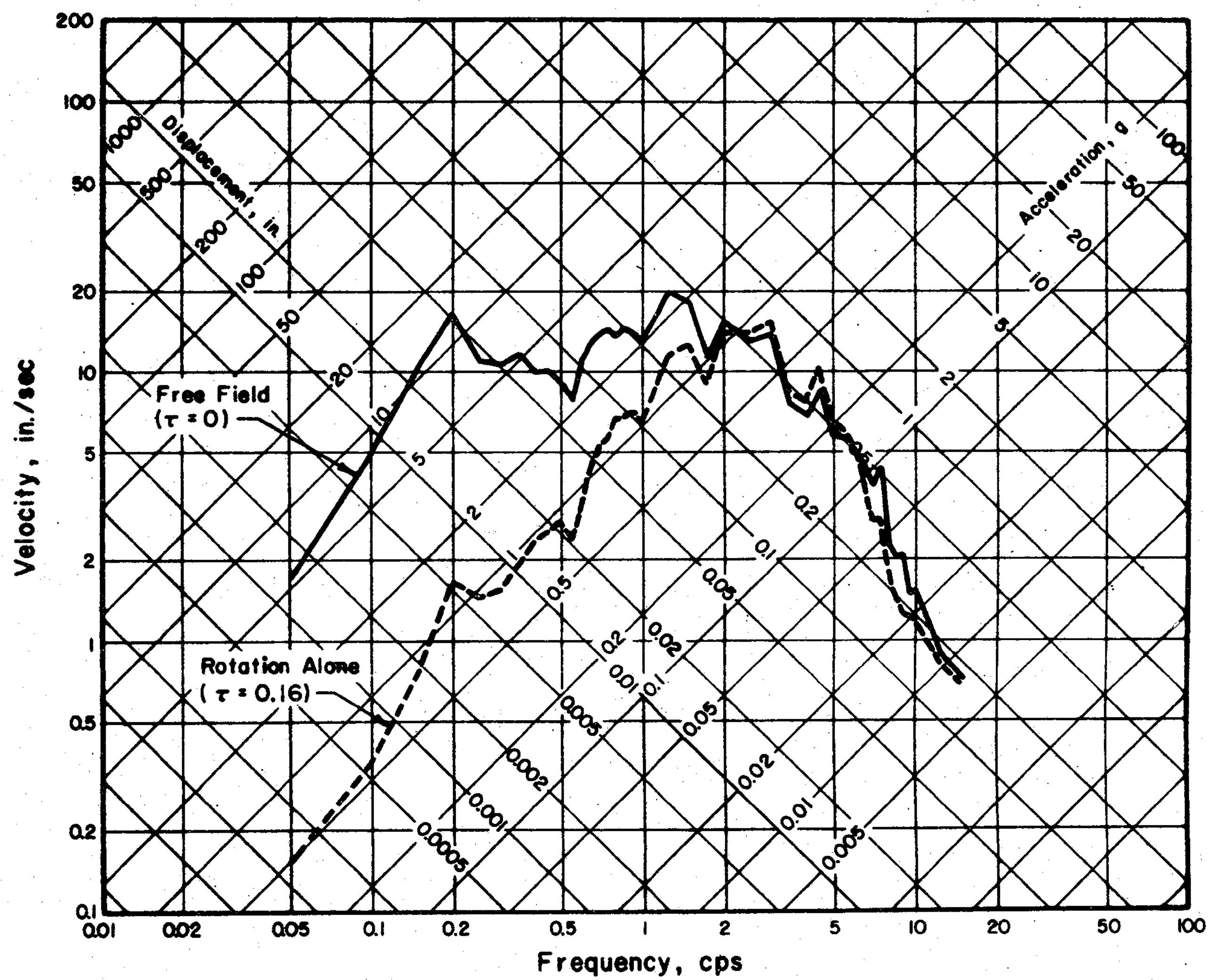


FIGURE 3.20 TAFT LINCOLN SCHOOL, N21E, KERN COUNTY EARTHQUAKE, 21 JULY 1952, SPECTRUM COMPUTED USING 2.0 PERCENT CRITICAL DAMPING, $\tau = 0.16$, $f_\theta/f_x = 1$

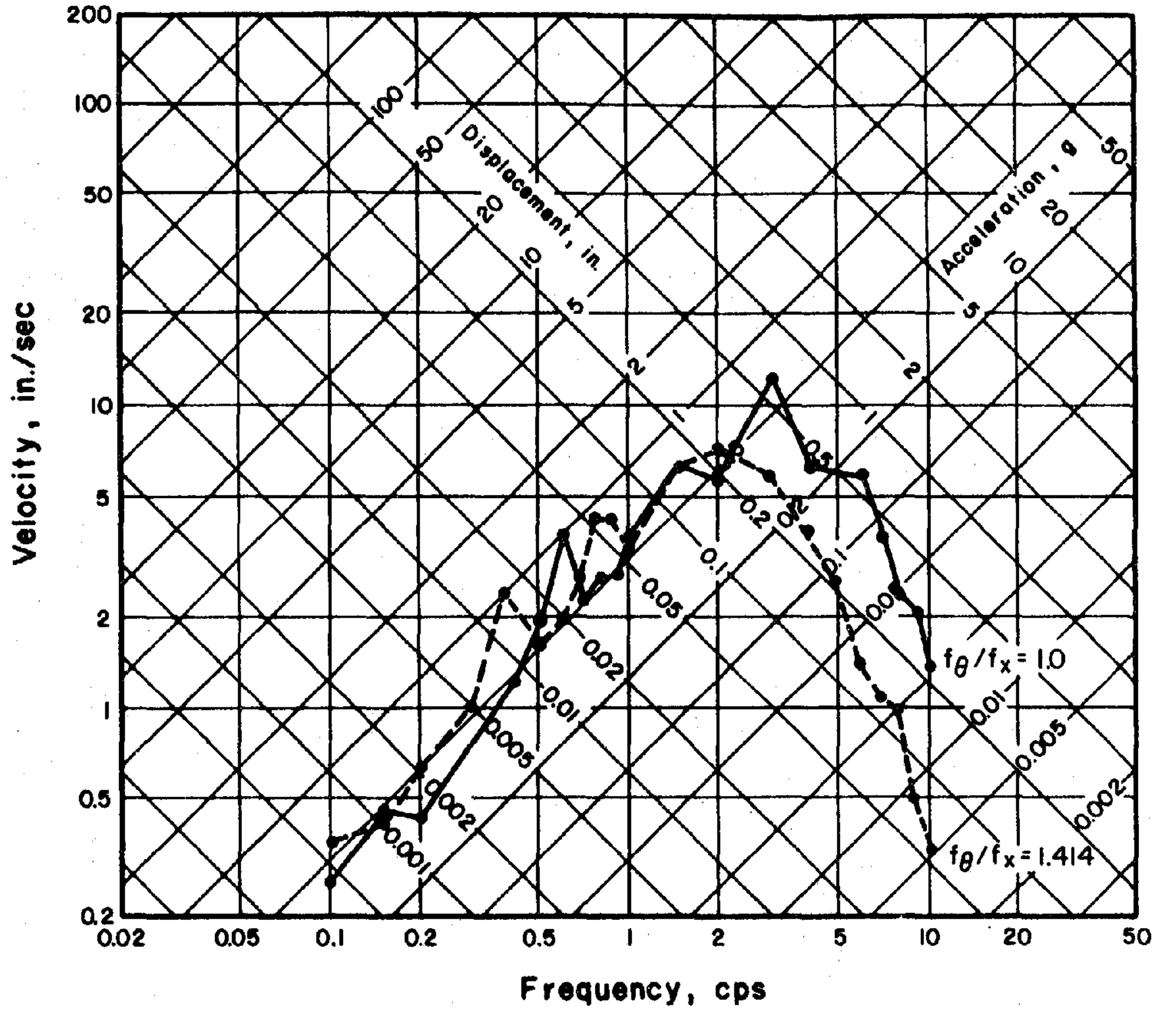


FIGURE 3.21 RESPONSE ARISING FROM ROTATION ALONE FOR $f_\theta/f_x = 1.0$ AND $f_\theta/f_x = 1.414$, TAFT LINCOLN SCHOOL, S69E, KERN COUNTY EARTHQUAKE, 21 JULY 1952, SPECTRUM COMPUTED USING 2.0 PERCENT CRITICAL DAMPING

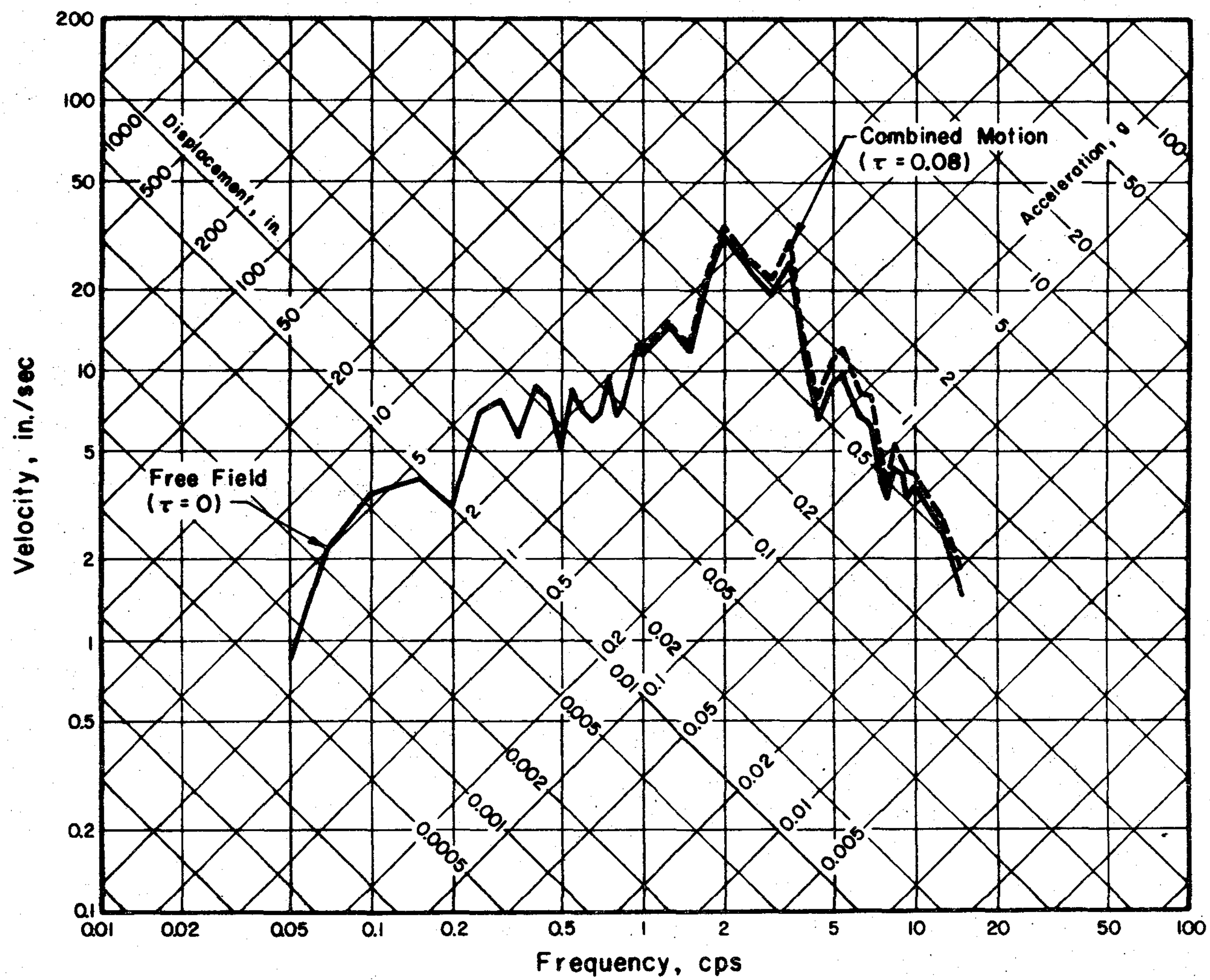


FIGURE 3.22 CHOLAME, SHANDON, ARRAY NO. 5, N05W, PARKFIELD EARTHQUAKE, 27 JUNE 1966, SPECTRUM COMPUTED USING 2.0 PERCENT CRITICAL DAMPING, $\tau = 0.08$, $f_\theta/f_x = 1$

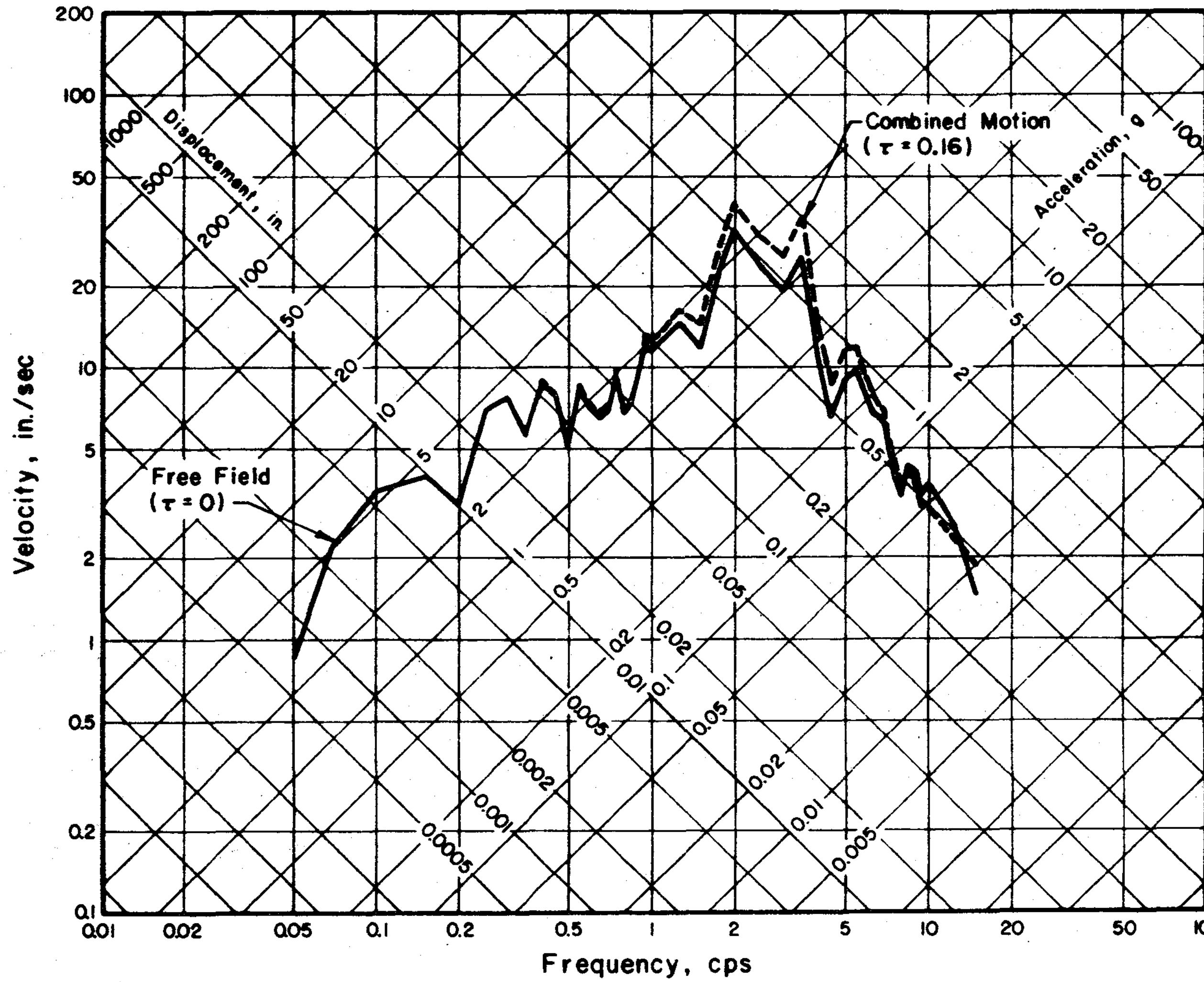


FIGURE 3.23 CHOLAME, SHANDON, ARRAY NO. 5, N05W, PARKFIELD EARTHQUAKE, 27 JUNE 1966, SPECTRUM COMPUTED USING 2.0 PERCENT CRITICAL DAMPING, $\tau = 0.16$, $f_\theta/f_x = 1$

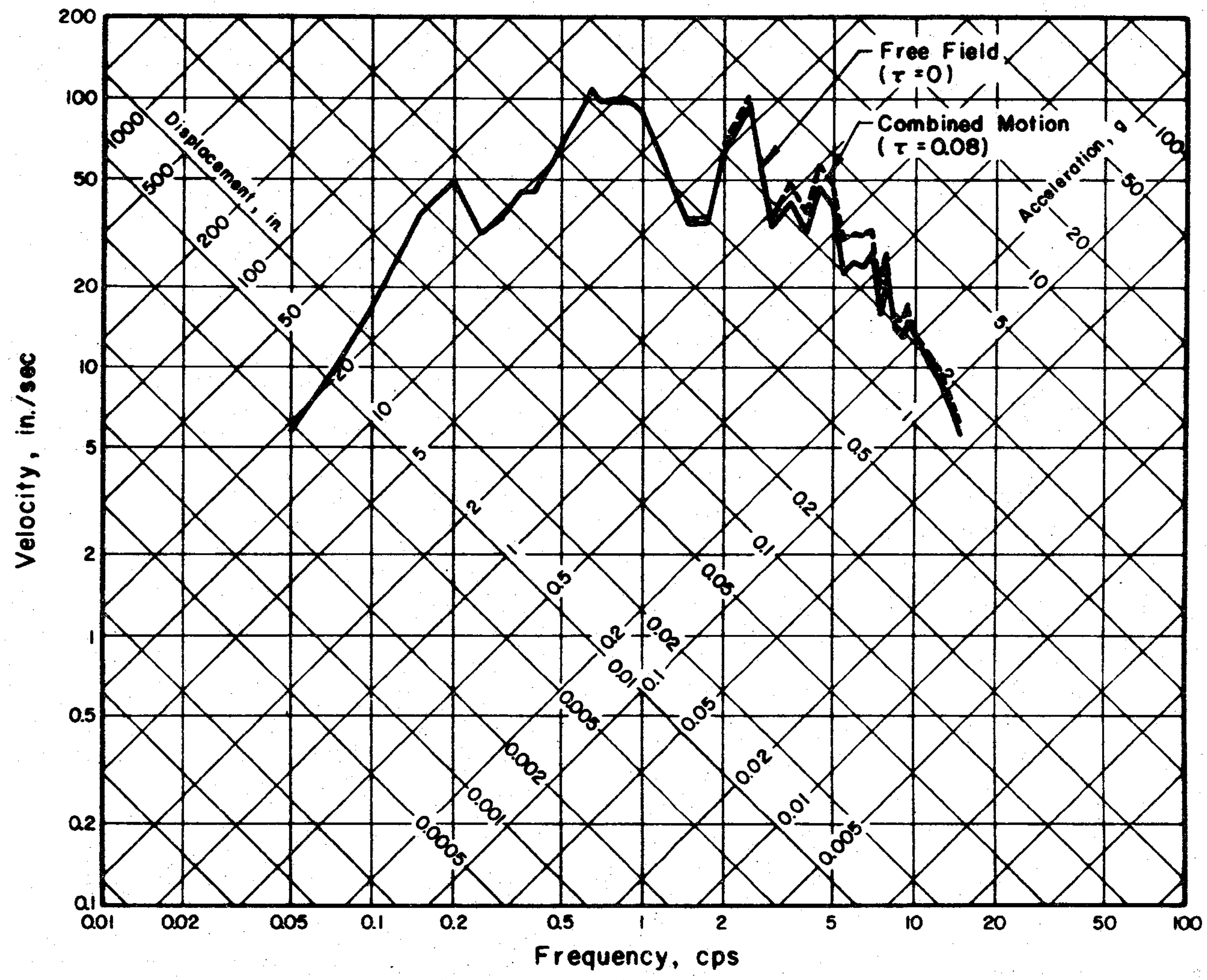


FIGURE 3.24 PACOIMA DAM, S16E, SAN FERNANDO EARTHQUAKE, 9 FEBRUARY 1971, SPECTRUM COMPUTED USING 2.0 PERCENT CRITICAL DAMPING, $\tau = 0.08$, $f_\theta/f_x = 1$

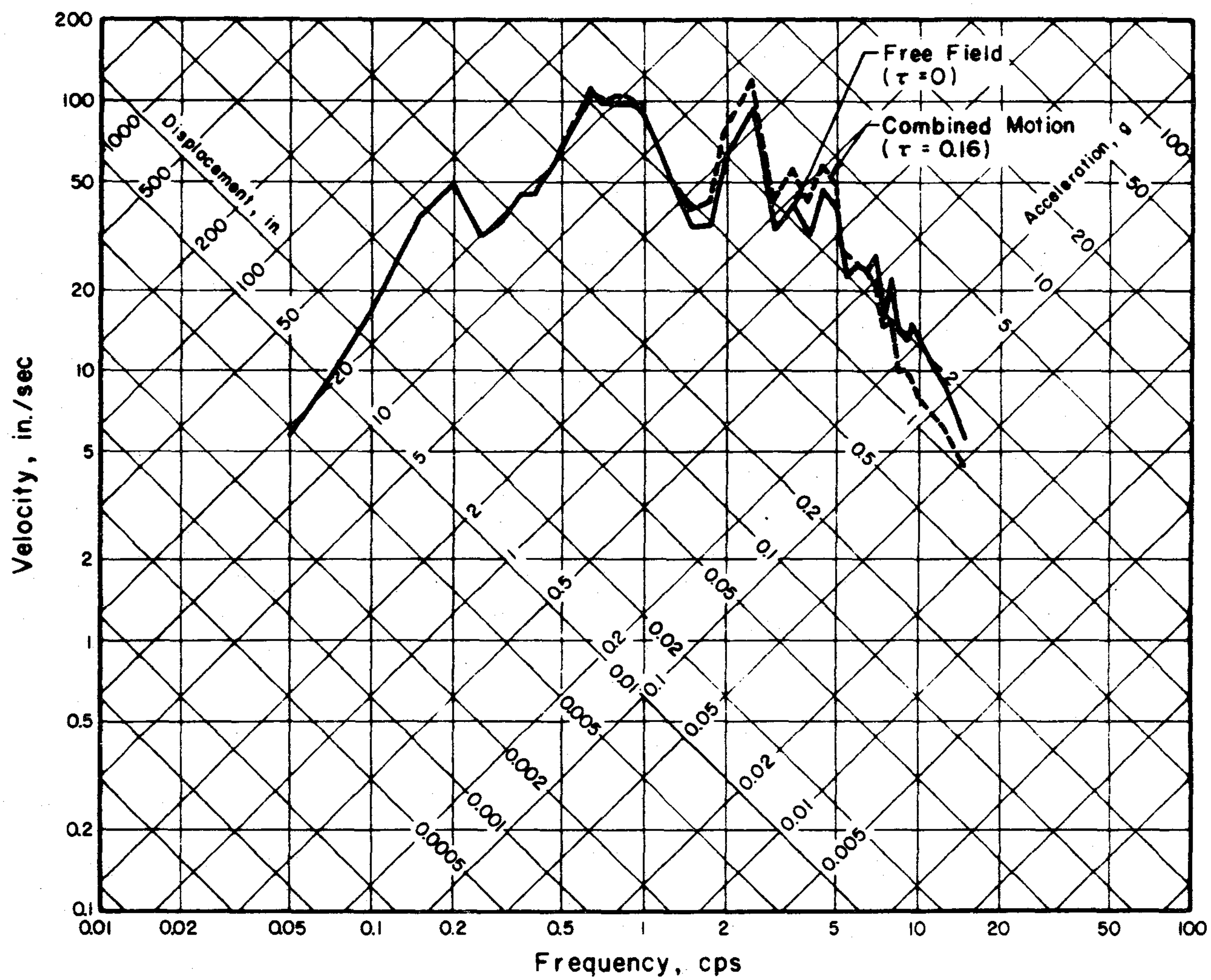


FIGURE 3.25 PACOIMA DAM, S16E, SAN FERNANDO EARTHQUAKE, 9 FEBRUARY 1971, SPECTRUM COMPUTED USING 2.0 PERCENT CRITICAL DAMPING, $\tau = 0.16$, $f_\theta/f_x = 1$

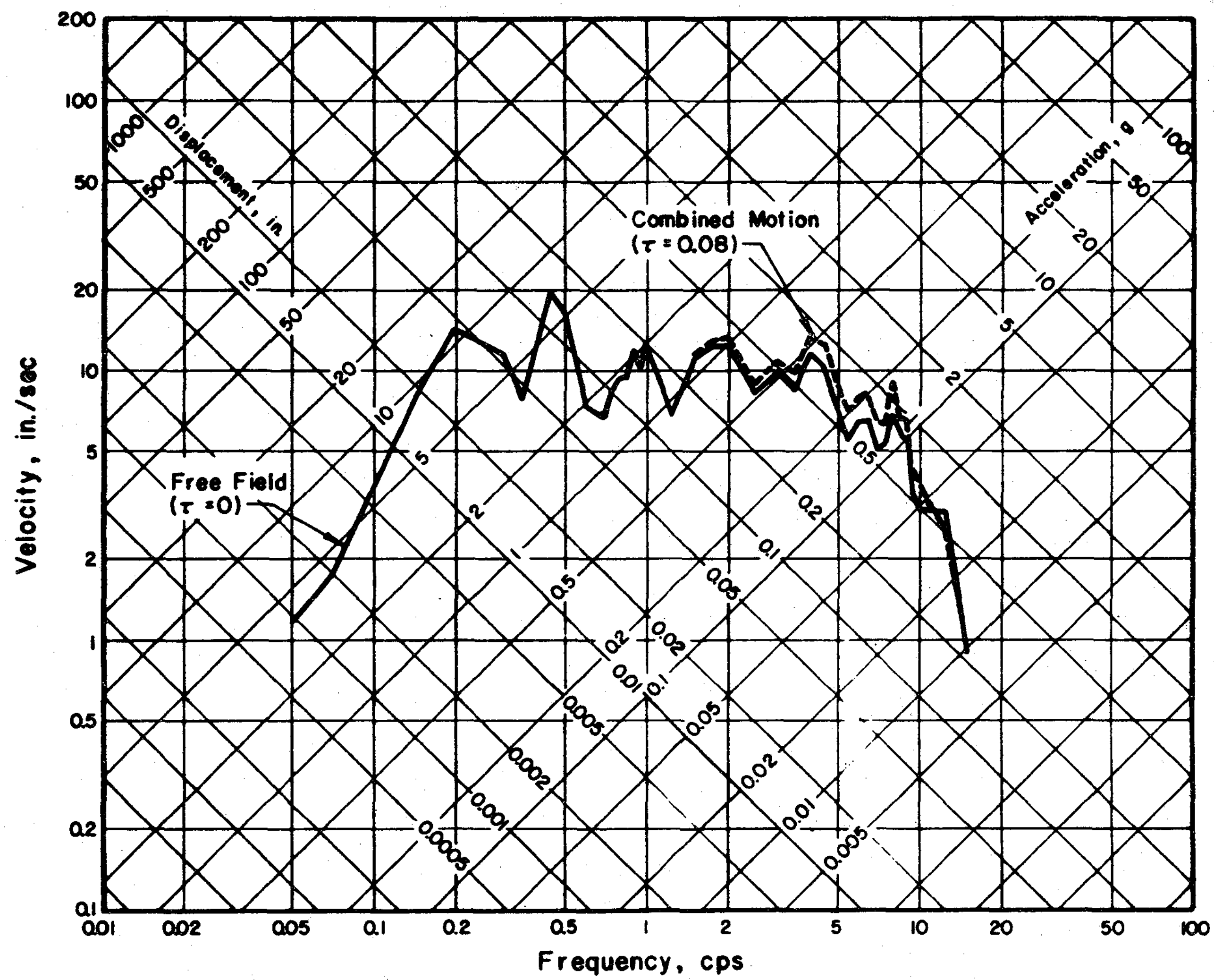


FIGURE 3.26 HOLLYWOOD STORAGE P.E. LOT, SOOW, SAN FERNANDO EARTHQUAKE, 9 FEBRUARY 1971, SPECTRUM COMPUTED USING 2.0 PERCENT CRITICAL DAMPING, $\tau = 0.08$, $f_\theta/f_x = 1$

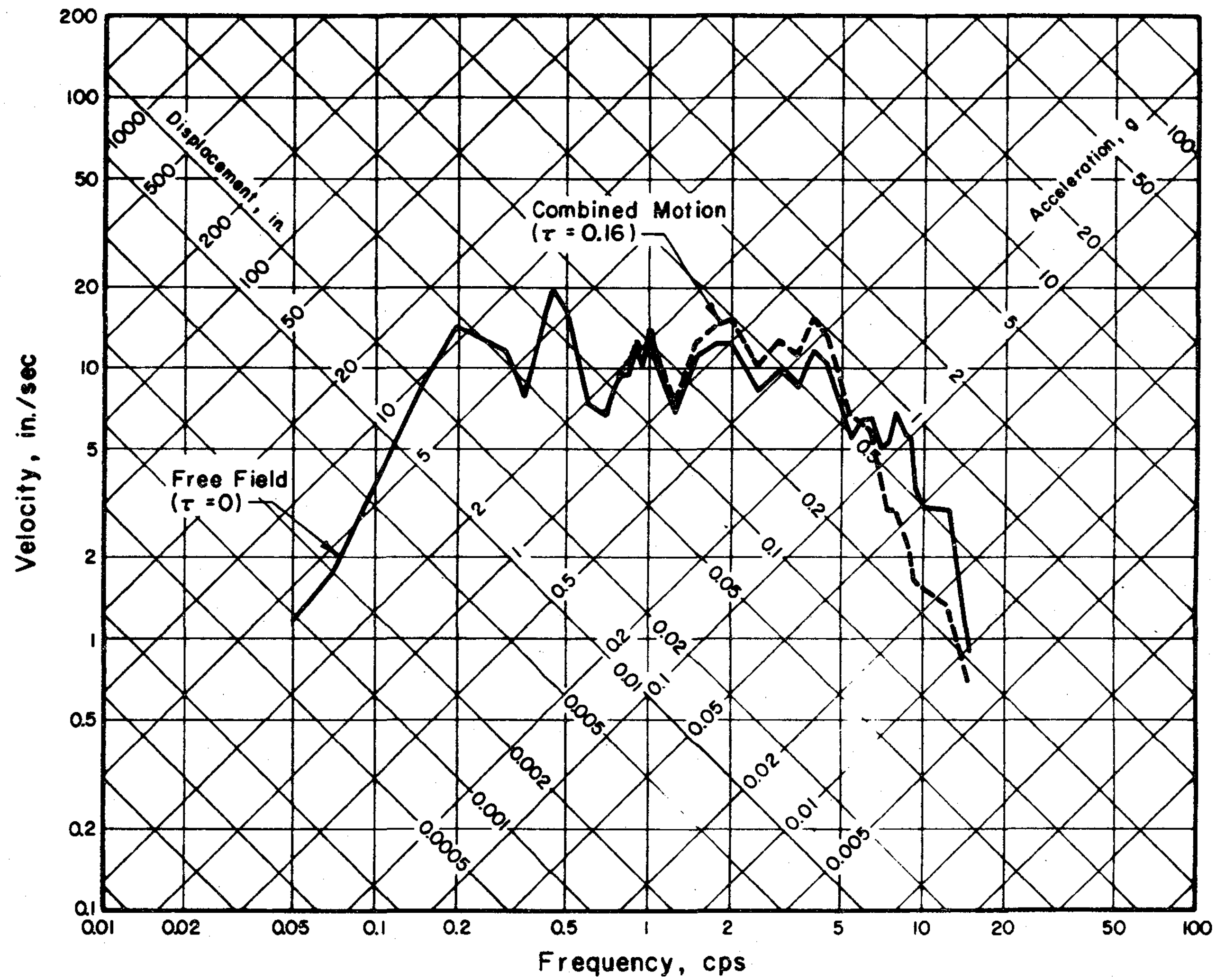


FIGURE 3.27 HOLLYWOOD STORAGE P.E. LOT, SOOW, SAN FERNANDO EARTHQUAKE, 9 FEBRUARY 1971, SPECTRUM COMPUTED USING 2.0 PERCENT CRITICAL DAMPING, $\tau = 0.16$, $f_\theta f_x = 1$

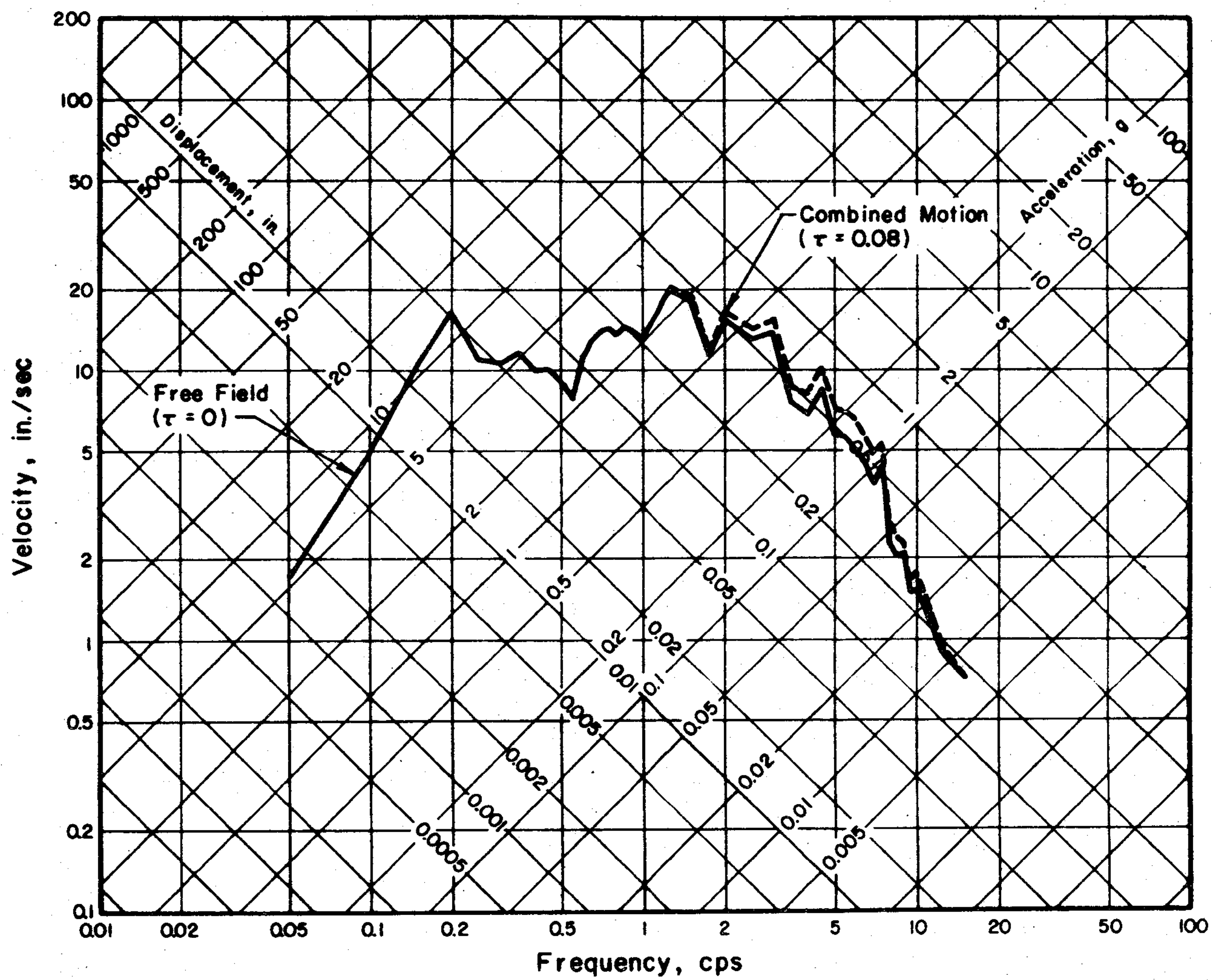


FIGURE 3.28 TAFT LINCOLN SCHOOL, N21E, KERN COUNTY EARTHQUAKE, 21 JULY 1952, SPECTRUM COMPUTED USING 2.0 PERCENT CRITICAL DAMPING, $\tau = 0.08$, $f_\theta/f_x = 1$

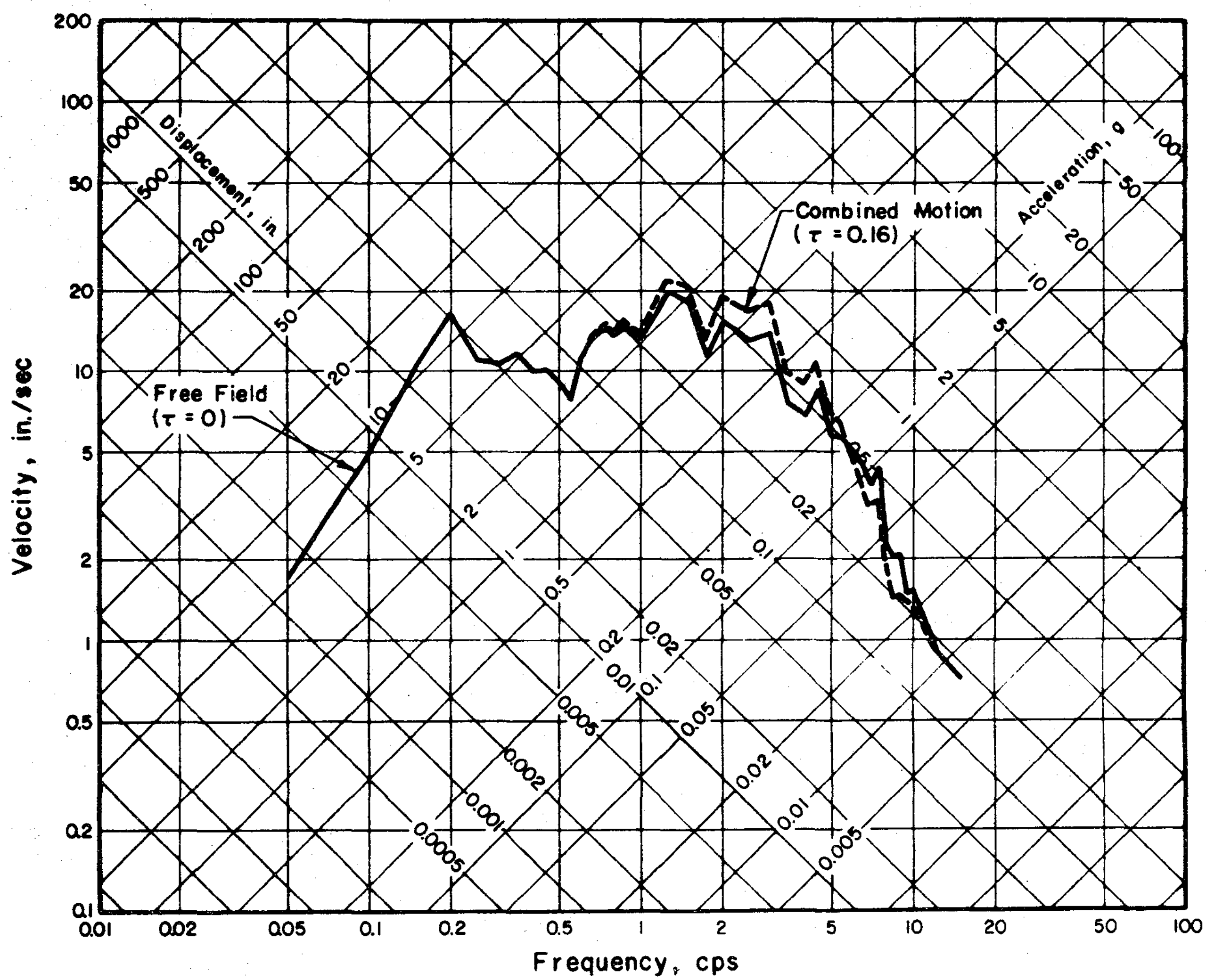


FIGURE 3.29 TAFT LINCOLN SCHOOL, N21E, KERN COUNTY EARTHQUAKE, 21 JULY 1952, SPECTRUM COMPUTED USING 2.0 PERCENT CRITICAL DAMPING, $\tau = 0.16$, $f_\theta/f_x = 1$

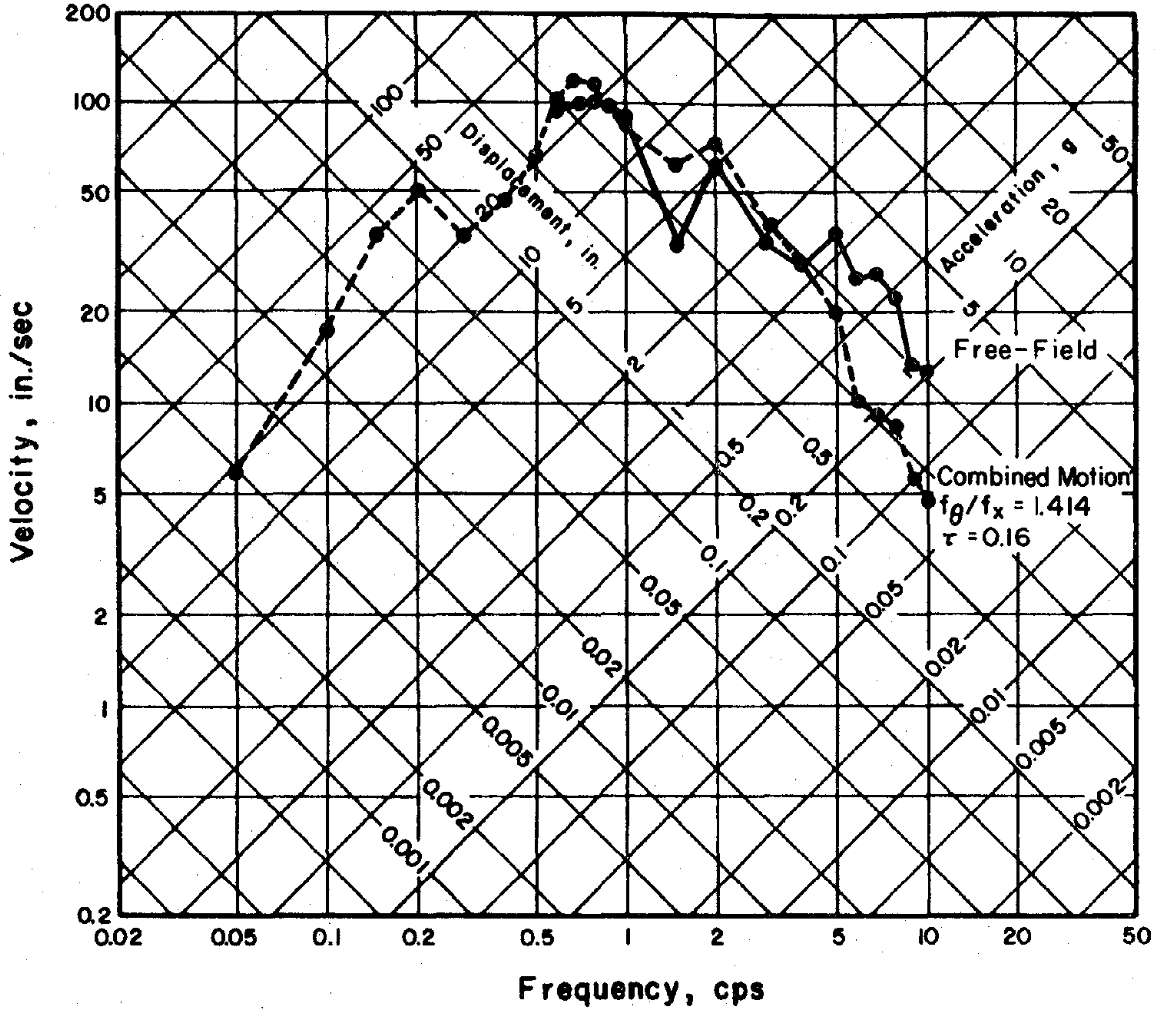


FIGURE 3.30 COMBINED MOTION SPECTRUM FOR $f_\theta/f_x = 1.414$, PACOIMA DAM, S16E, SAN FERNANDO EARTHQUAKE, 9 FEBRUARY 1971, SPECTRUM COMPUTED USING 2.0 PERCENT CRITICAL DAMPING, $\tau = 0.16$

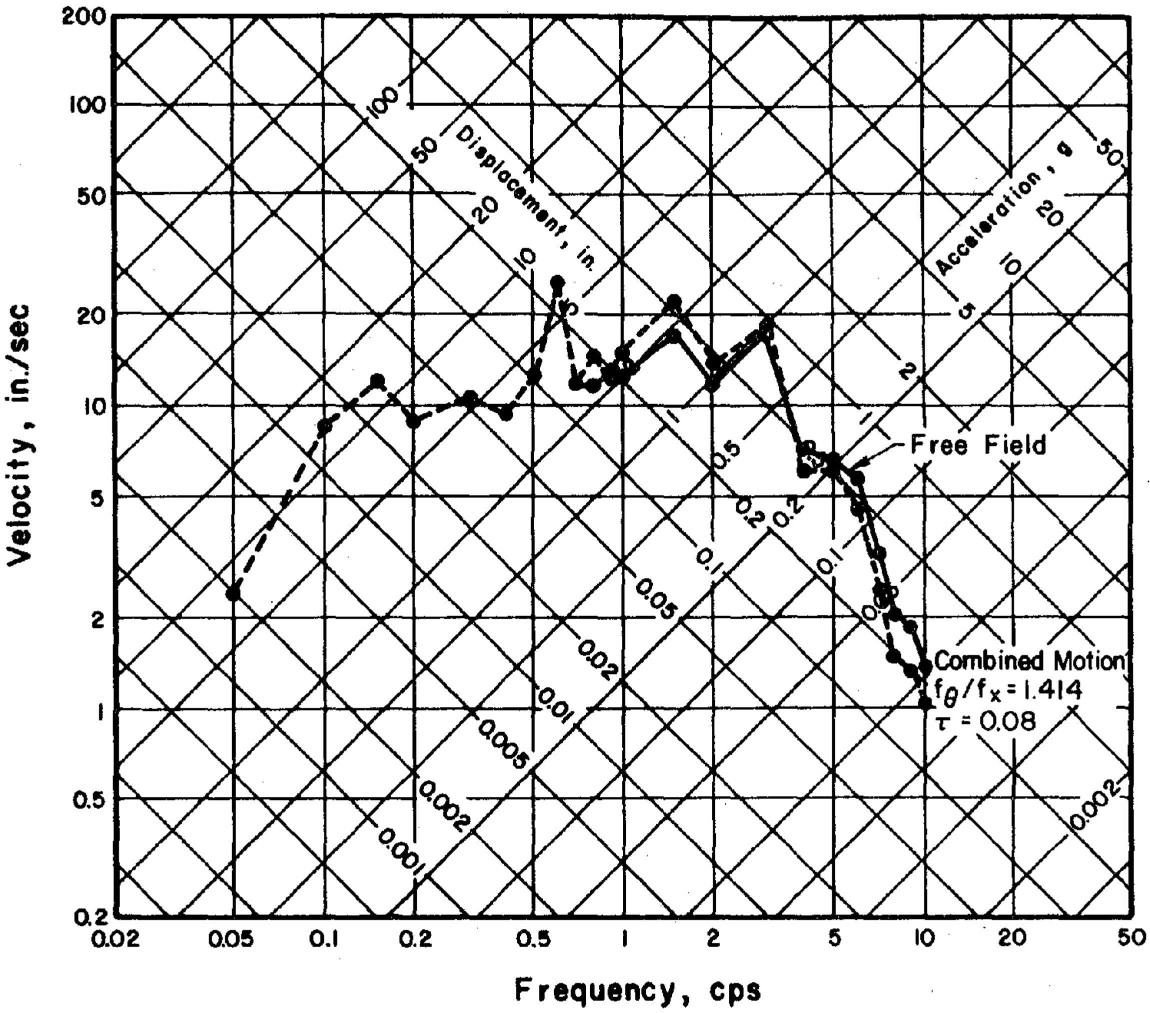


FIGURE 3.31 COMBINED MOTION SPECTRUM FOR $f_\theta/f_x = 1.414$, TAFT LINCOLN SCHOOL, S69E, KERN COUNTY EARTHQUAKE, 21 JULY 1952, SPECTRUM COMPUTED USING 2.0 PERCENT CRITICAL DAMPING, $\tau = 0.08$

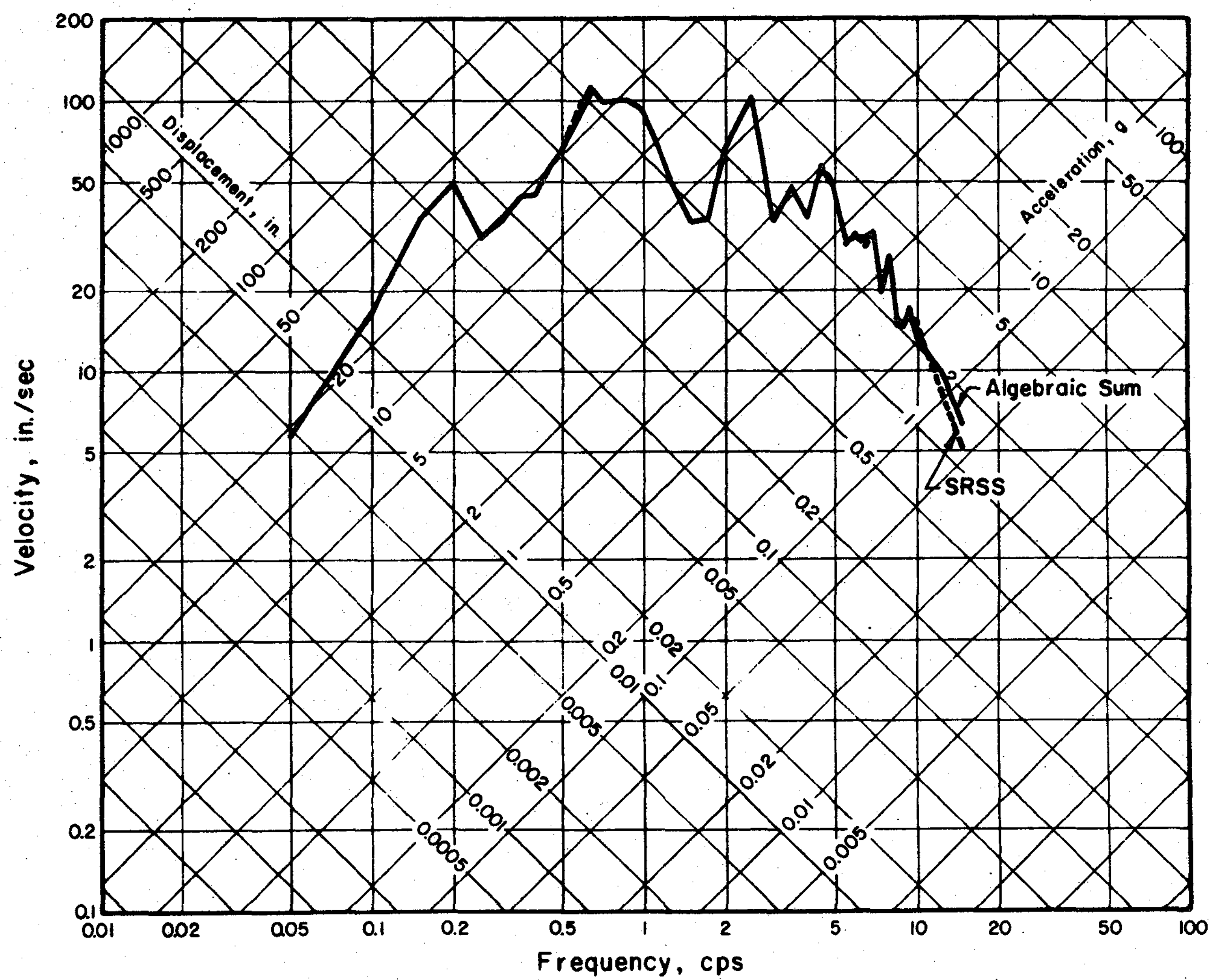


FIGURE 3.32 PACOIMA DAM, S16E, SAN FERNANDO EARTHQUAKE, 9 FEBRUARY 1971, SPECTRUM COMPUTED USING 2.0 PERCENT CRITICAL DAMPING, $\tau = 0.08$, $f_\theta/f_x = 1$

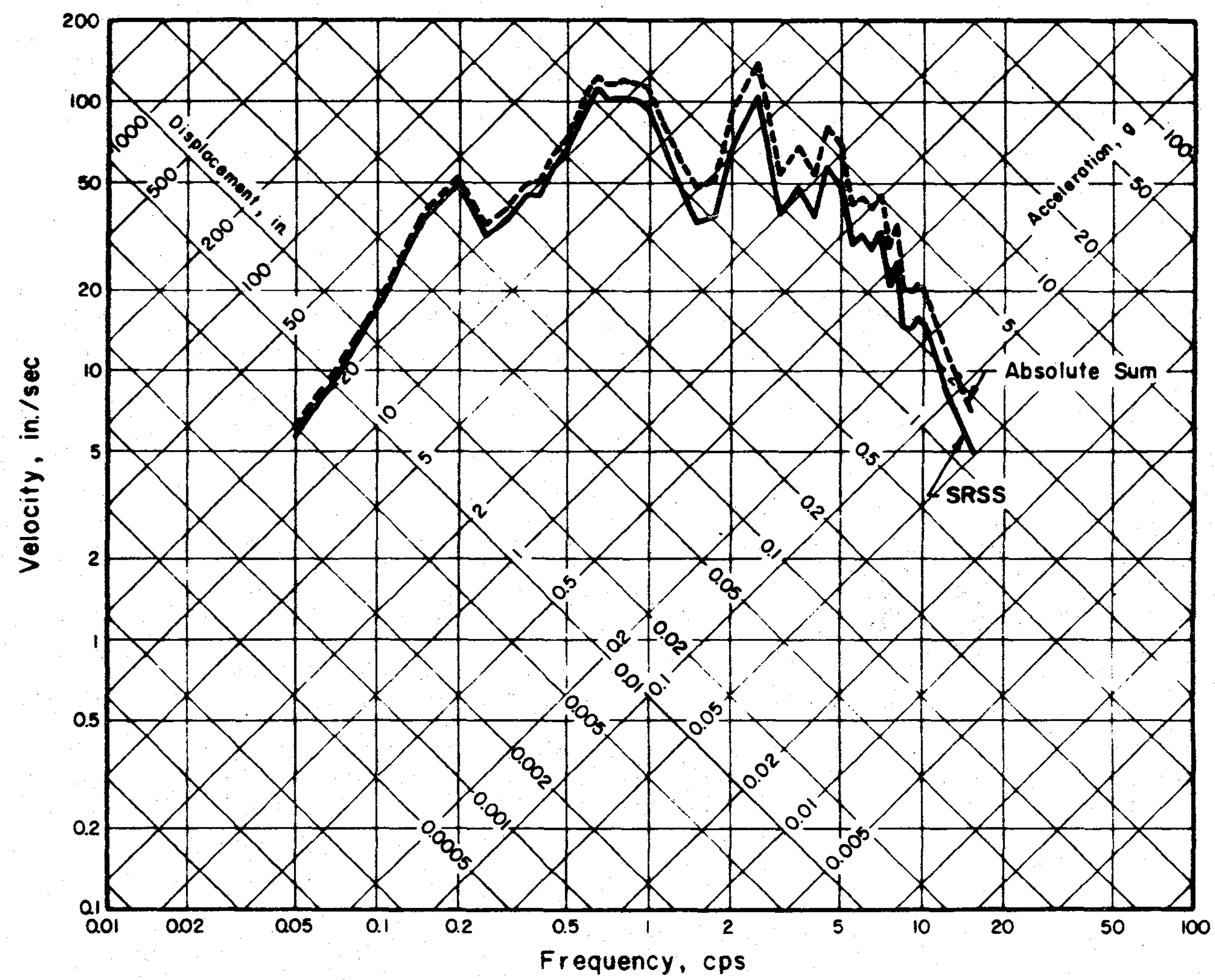


FIGURE 3.33 PACOIMA DAM, S16E, SAN FERNANDO EARTHQUAKE, 9 FEBRUARY 1971, SPECTRUM COMPUTED USING 2.0 PERCENT CRITICAL DAMPING, $\tau = 0.08$, $f_\theta/f_x = 1$

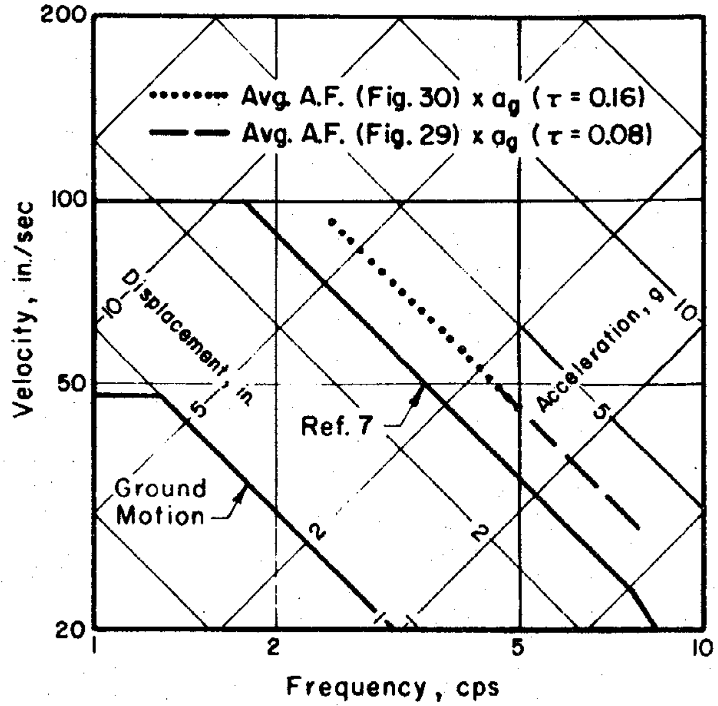


FIGURE 3-34 ILLUSTRATION OF RESPONSE AMPLIFICATION ARISING FROM COMBINED TRANSLATION AND TORSION (50 PERCENTILE), $f_\theta/f_x = 1$

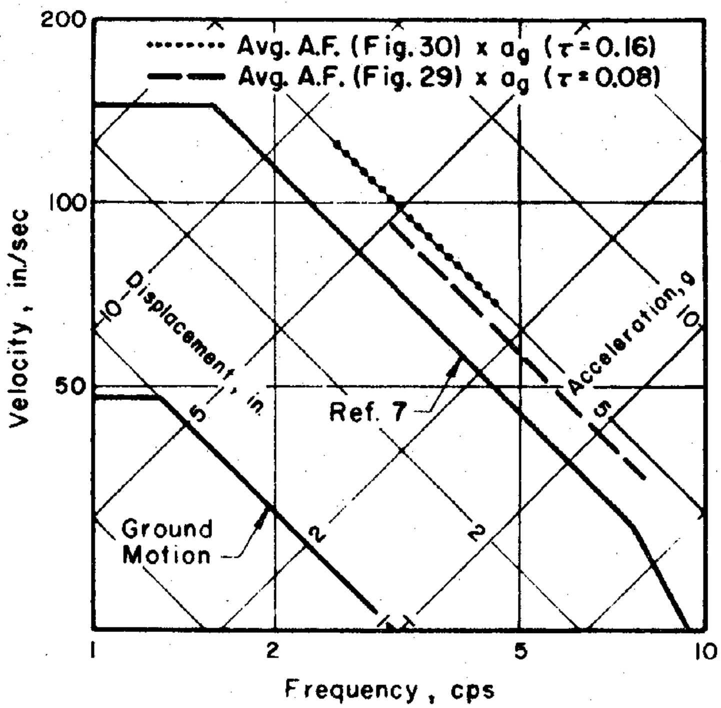


FIGURE 3.35 ILLUSTRATION OF RESPONSE AMPLIFICATION ARISING FROM COMBINED TRANSLATION AND TORSION (84.1 PERCENTILE), $f_\theta/f_x = 1$

4. THEORY AND PROCEDURE: COUPLED MOTION MODEL

4.1 Introduction

One of the major limiting assumptions inherent in the superposition model (Chapters 2 and 3) is that of independence of translational and rotational motions of the system. The effects of eccentricity between centers of mass and stiffness cannot be included in the superposition model. In order to study the coupling aspects of the problem a new model was developed and employed in the calculations. A single story three degree-of-freedom model with one axis of symmetry and some finite eccentricity between centers of mass and stiffness is a logical progression from the symmetrical building studied in previous chapters. For the sake of simplicity the mass is treated as a rigid thin plate with coordinate origin at the center of stiffness as shown in Fig. 4.1.

4.2 Undamped Equation of Motion

The Lagrangian equations of motion can be obtained from the kinetic and potential energies of the system. For a thin rigid plate the kinetic energy can be described by $T = \frac{1}{2}\, m v_{cm}^{2} + \frac{1}{2}\, m \rho_Z^{2}\, \dot{\theta}^2$ and the potential energy can be written as $V = \frac{1}{2}\, k_y\, y^2 + \frac{1}{2}\, k_x x^2 + \frac{1}{2}\, k_r\, \theta^2$

where

$$v_{cm} = \{\dot{x}^2 + (\dot{y} + e\dot{\theta})^2\}^{1/2} = \text{velocity of center of mass}$$

and

$$\rho_Z = \{ \frac{I_Z}{m} \}^{1/2} = \{ \frac{h^2 + b^2}{12} \}^{1/2} = \text{radius of gyration}$$

The Lagrangian equations of motion can then be derived as shown below. This procedure is described in detail in many places (see Ref. 21) but basically requires defining the Lagrangian function, L (where L = T - V) and then the action, $A = \int_{t_0}^{t_1} L dt$ where we require the variation of A to be zero, $\delta A = 0$. It then follows from Hamilton's principle that the Euler equations for the integral A are the differential equations of motion

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}} - \frac{\partial L}{\partial q} = 0 \quad \text{where } q \text{ is a coordinate}$$

therefore

$$m\ddot{x} + k_x x = 0$$

and

$$m\ddot{y} + me\ddot{\theta} + k_y y = 0$$

and

$$me\ddot{y} + me^2\ddot{\theta} + m\left(\frac{h^2 + b^2}{12}\right)\ddot{\theta} + k_r\theta = 0$$

These equations can be written more conveniently in matrix notation as shown in Eq. 4.1.

$$\begin{bmatrix} m & 0 & 0 \\ 0 & m & me \\ 0 & me & J_{cs} \end{bmatrix} \begin{Bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{\theta} \end{Bmatrix} + \begin{bmatrix} k_x & 0 & 0 \\ 0 & ky & 0 \\ 0 & 0 & k_r \end{bmatrix} \begin{Bmatrix} x \\ y \\ \theta \end{Bmatrix} = \bar{0} \qquad (4.1)$$

where for a thin plate

$$J_{cm} = \frac{m}{12}(b^2 + h^2)$$

$$J_{cs} = J_{cm} + me^2$$

$$J_{cs} = \frac{m}{12}(b^2 + h^2 + 12e^2)$$

4.3 Damping

It was decided to use equal small damping in each degree-of-freedom so that none of the possible effects of coupled motion would be lost or minimized. Following the example of Caughey (Ref. 13) we can, by transforming to appropriate coordinates (i.e., allowing the mass matrix to become uncoupled) and by forcing the damping matrix to be proportional to the frequencies, not only achieve equal damping in each mode but also obtain a damped dynamic system with classical normal modes, thereby uncoupling the equations of motion. In order to achieve the desired result we must obtain equations of motion in the following form

$$I\ddot{\xi} + 2\beta \; [\; \omega \;] \; \dot{\xi} + [\; \omega^2 \;] \; \xi = \bar{0} \qquad (4.2)$$

where β = critical damping ratio.

To achieve this from the current equation of motion,

$$M\ddot{q} + 2\beta C\dot{q} + kq = 0$$

first transform coordinates such that

$$q = \Phi\xi \qquad (4.3)$$

where Φ is the matrix of eigenvalues normalized such that $\Phi^T M\Phi = I$.

then $$M\Phi\ddot{\xi} + 2\beta C\Phi\dot{\xi} + k\Phi\xi = \bar{0}$$

Now <u>choose</u>

$$C = M\Phi \; [\; \omega \;] \; \Phi^T M$$

then $$M\Phi\ddot{\xi} + 2\beta M\Phi \; [\; \omega \;] \; \Phi^T M\Phi\ddot{\xi} + k\Phi\xi = \bar{0}$$

and premultiply by Φ^T thereby obtaining Eq. 4.4 since $\Phi^T M\Phi = I$ and $\Phi^T k\Phi = [\; \omega^2 \;]$ from orthogonality of normal modes.

$$I\ddot{\xi} + 2\beta \; [\; \omega \;] \; \dot{\xi} + [\; \omega^2 \;] \; \xi = \bar{0} \qquad (4.4)$$

4.4 Ground Input

The ground motion is input at the coordinate origin from the τ-averaged time histories obtained in Chapter 2. Since $\ddot{\alpha}$ is in units of length/time^3 and the coordinate is in radians we must first convert to angular acceleration (radian/time^2); this can be done easily by dividing $\ddot{\alpha}$ by the wave velocity C. Also, because the average translational time history is applied to the center of mass rather than the coordinate center, the ground motion input for the y direction must be modified such that $\ddot{y}_{cs} = \ddot{\Phi} - e\ddot{\alpha}/C$ as shown in Fig. 4.2. It must also be remembered that the equation of motion has been premultiplied by Φ^T; therefore the right hand side of the equation of motion becomes

$$\Phi^T \begin{Bmatrix} -m\ddot{x}_{cs} \\ -m\ddot{y}_{cs} \\ -J_{cs}\ddot{\theta} \end{Bmatrix}$$

For any one case studied at least one of the three ground inputs is equal to zero (i.e., one assumes a wave propagating as a plane wave in one of the two principle directions; if it travels in the x direction $\ddot{x}_{cs} = 0$, and if it travels in the y direction $\ddot{x}_{cs} = \ddot{\phi}$ and $\ddot{y}_{cs} = 0$).

4.5 Response Computation

Since the equations of motion uncouple the response is computed using the Z-transform procedure described in Appendix A. These responses are then transformed back into the original coordinates using the normalized matrix of eigenvalues (Eq. 4.3). Response quantities similar to those computed for the superposition model (Chapter 2) are computed for the coupled model. However, since the system is no longer symmetric, the computations must be made at four locations (positive and negative x edges and centers of mass and stiffness) instead of only two (center and edge) as before. The maximum responses plotted may arise from the maximum response from translational ground motion or the maximum response from rotational ground motion, or it may arise from a combination of translation and rotation at a point in time which may or may not be related to either of the individual maximums. It should be noted that the maximum response arises from the vector sum of the individual maximums since cases studied include responses in the X direction arising from ground motion in the X direction, responses n the Y direction arising from ground motion in the Y direction, and responses in the Y direction arising from ground motion in the θ direction.

It should be noted that the responses computed and tabulated in Chapter 5 were computed for a specific shear wave velocity, namely 500 fps. The edge responses arising from rotational motion is dependent on the length of the foundation and therefore on the wave velocity. However, the responses computed for other shear wave velocities (1000 fps, 1500 fps, and 2000 fps) were nearly identical to those responses presented in Chapter 5. This similarity indicates that rotationally induced response at least at the time of maximum overall response is not important relative to the component of response arising from translation.

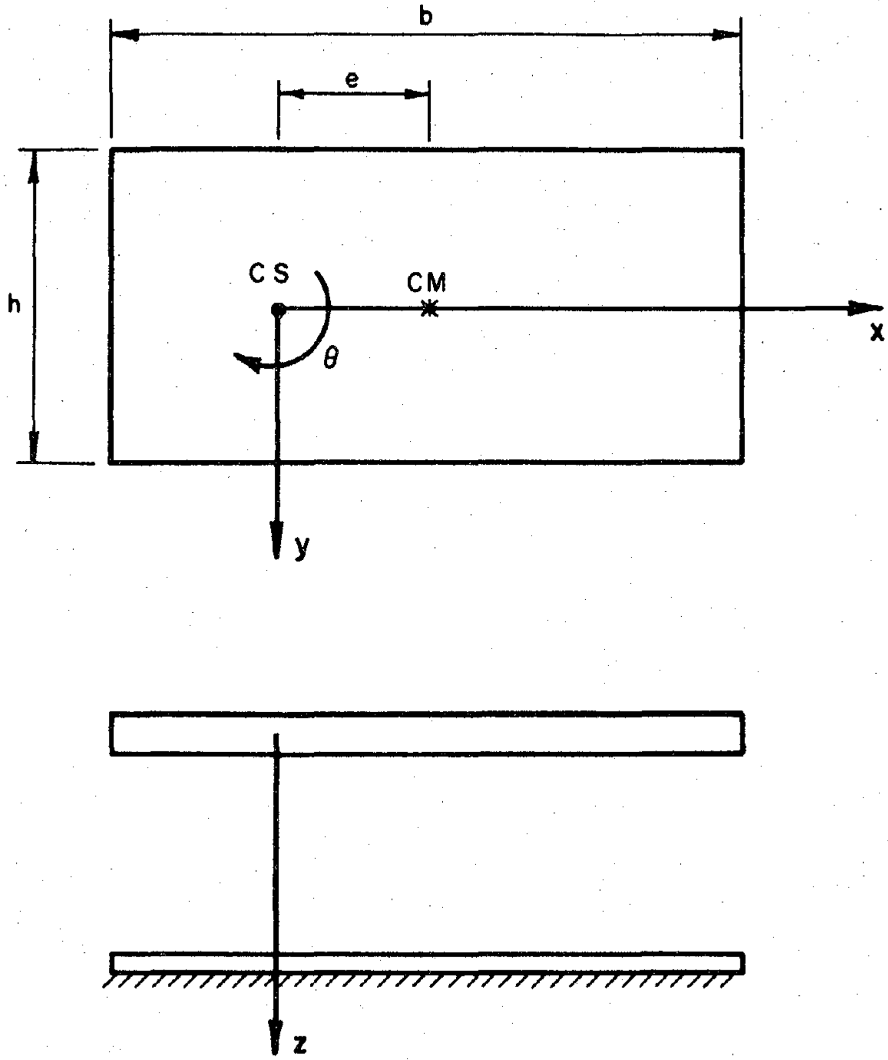


FIGURE 4.1 COUPLED MOTION MODEL

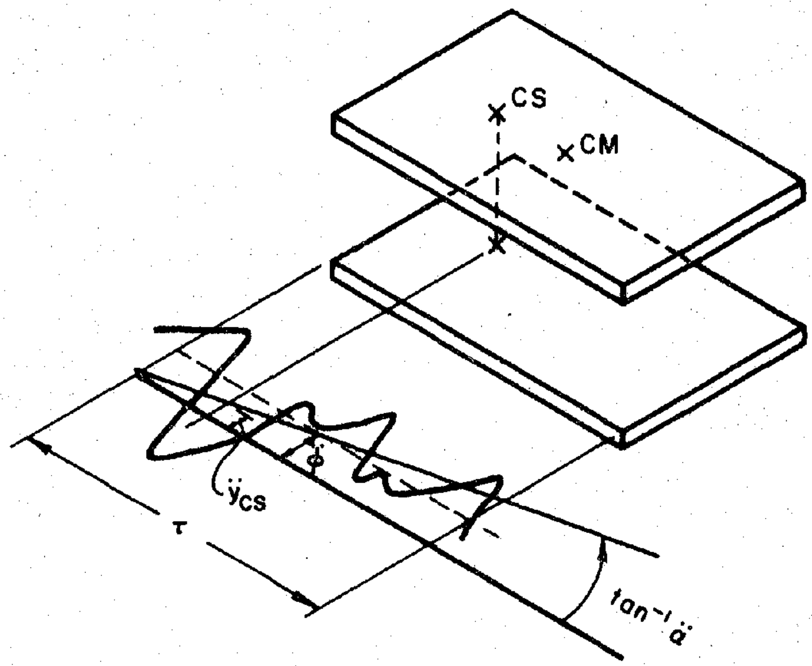


FIGURE 4.2 GROUND INPUT FOR COUPLED MOTION MODEL

5. DISCUSSION OF RESULTS: COUPLED MODEL

5.1 Introduction

The results presented in this chapter were obtained using a single story three degree-of-freedom (2 translation, 1 rotation) model with one axis of symmetry and allowance for some finite amount of eccentricity between centers of mass and stiffness as developed in Chapter 4. Because of limitations in the studies that may be undertaken with the superposition model it was decided to go to the more complex model described in order to investigate a number of parameters. This model collapses onto the superposition model if one forces the eccentricity to zero; therefore, to compare the results of the coupled model with those of the superposition model one need only examine the coupled model results for zero eccentricity.

Thus the intent of this chapter is to study to some limited extent a number of parameters believed to be important in assessing the translational and torsional responses arising from wave passage by a structure. These parameters were the eccentricity between centers of mass and stiffness as a percent of the building dimension along the axis of symmetry, the ratio of torsional to translational frequency for the structure, the aspect ratio of the building (i.e., the ratio of the horizontal dimensions of the building), and the ratio of stiffnesses in the two translational coordinate directions. Also, discussion is included on the effect that coupling in the model has on the results obtained. Finally, comparisons are made between the results obtained in this study and those obtained using current seismic building code procedures.

Most of the data presented in this chapter is in the form of tables. Because of the complexity of the problem being studied one can appreciate the difficulties in presenting the data in a rational and logical manner. There are a multitude of possibly enlightening plots that may be made but for the purposes of this discussion an attempt was made to present as few plots as possible. It is hoped that these plots along with the data available from the tables will permit the reader to obtain as much information as he desires.

5.2 Model Parameters Studied

In order to keep calculations and costs to a manageable level, only three basic frequencies ($f_x = \frac{1}{2\pi} \quad k_x/m = 1$, 3 and 5 Hz) were used in the response calculations. These frequencies were selected because they fell in the region of great interest from a structural point of view and as indicated by the results of the superposition model. It was hoped that, by condensing the area of the spectrum studied, sufficient interpretation could be accomplished to permit identification of important trends.

An attempt was made, with relatively few computations, to cover a wide range of possible structures. Rectangular structures with aspect ratios ranging from 1/2 to 2 (b/h = aspect ratio, see Fig. 5.1), and eccentricities ranging from 0% to 15% of the X dimension of the building were studied. In all cases, calculations were made assuming equal X and Y stiffnesses and then assuming that stiffnesses were proportional to building dimension (i.e., for $h/b = 2$ $k_y/k_x = 2$). Calculations also were made for two ratios of torsional to translational frequency; namely, unity and 1.414, with the latter corresponding to a more realistic structure than the former (see Ref. 10).

In Table 5.1 are presented the model parameters for all of the models studied and the corresponding simple frequencies for the various coordinates ($f_x = k_x/m$, $f_y = k_y/m$ and $f_\theta = k_\theta/J$). Note that the frequencies given in the table are the frequencies for the case with zero eccentricity. The shifts in frequency (i.e., shifts in eigenvalues) that arise from coupling of the Y and θ coordinates are dependent on the magnitude of the eccentricity in the model. An example of the shifts in eigenvalues arising from coupling of Y and θ in the characteristic equation is given in Table 5.2 for one of the cases studied. Similar shifts occur for all cases with non-zero eccentricity; however the degree of shift varies from case to case.

5.3 Eccentricity in Model

It generally would be expected that an eccentric building would have higher response, at least on the periphery, than would a symmetric building. A summary of the maximum responses computed at the edge of the building in this study is presented in Tables 5.3 through 5.6. A review of the data presented in these tables will reveal a surprising result. Intuition normally would imply that response listed for an eccentricity of 15% would be larger than that for an eccentricity of 10%, etc. The fact, over two-thirds of the cases studied do not follow this logical sequence. However, as can be seen in Tables 5.3 through 5.6, the differences in response magnitudes are generally small. Approximately one-third of the calculations at 1 and 5 Hz correspond to increasing response with increasing eccentricity, and for calculations at 3 Hz one-third of the cases lead to definite trends (split equally between the two extreme cases: 15 > 10 > 5 > 0 and 0 > 5 > 10 > 15).

This situation, i.e., absence of a logical trend, should not have been unexpected, in light of the limited studies made by Ayre (Ref. 12) wherein he found that with only translational input the induced motion arising from coupling of translation and rotation is more complex than for the symmetric (uncoupled) case. The current model is much more complex than that of Ayre since rotational input has been added which leads to additional translation and rotation which may be in or out of phase with that arising from the translational input. Illustration is made in Figs. 5.2 and 5.3 of the nature of the results obtained at various eccentricities for the range of frequencies studied. Irrespective of these problems there are real and important trends that emerge from this type of study.

Although the results presented in Tables 5.3 through 5.6 are the maximum responses at the positive X edge of the structure computed in the time domain, i.e., the algebraic sum with respect to time, similar results can be obtained from the maximum responses at the positive X edge computed from the maximum individual Y and θ components. The trends obtained and observations made for the two methods of computing maximum response will not be identical nor should they be expected to be. The responses computed and combined as the sum of the maximums (as opposed to the maximum sum in the time domain) are given in Tables 5.7 through 5.10. It should be noted that while there was no explicit time phasing present in this data the responses computed still arise from the coupled motions. The maximum Y response is both the Y arising from Y ground input and the Y arising from rotational ground input combined in the time domain. Similarly, the maximum θ response is both the θ arising from Y ground input and the θ arising from θ ground input. Thus it should be obvious that, in a coupled model it becomes very difficult to truly separate the components of response.

5.4 Frequency Ratio

The importance of the assumed ratio of torsional to translational frequency on the computed combined response at the edge of the building was quite apparent in the results of the superposition model. It was expected, based on the superposition model, that a frequency ratio of unity would lead to greater computed responses than would a larger frequency ratio. Furthermore, it was expected that the larger the frequency ratio assumed the smaller the computed response at the edge of the building. A review of the data in Tables 5.3 through 5.10 reveals that in well over half of the cases studied an assumed frequency ratio of unity leads to the larger computed response. It will be noted that the difference in magnitudes of the computed responses in those unexpected cases (where a frequency ratio of unity yields less than maximum response) is not large; therefore, the trend observed for the superposition model can be considered generally valid.

5.5 Building Aspect Ratio

As was noted in the derivation of the τ-averaging procedure a building with larger plan area can be expected to undergo lower overall response than would a smaller building subjected to the same free-field input. An examination of the data in Tables 5.3 through 5.10 will verify that in more than two-thirds of the cases studied larger building plan areas lead to lower response at the positive X edge of the building. This finding suggests that torsion may not be as important as originally presumed.

5.6 Stiffness Ratio

One would anticipate that a building with lower overall stiffness would undergo larger total response and indeed, in over ninety percent of the cases studied this is the case. Note that for all cases studied the stiffness in the X direction changes only with frequency, that is, all 1 Hz models have the same X stiffness. The stiffness ratios k_y/k_x in Tables 5.3 through 5.10 refer to the ratio of stiffness in the Y direction to that in the X direction; therefore, a stiffness ratio of 2 indicates greater overall stiffness generally than a stiffness ratio of 1 and a stiffness ratio of 1/2 indicates lower overall stiffness generally than a stiffness ratio of 1. As was expected, largest response arises for a stiffness ratio of 1/2 and smallest response arises for a stiffness ratio of 2.

5.7 Coupling of Response

It should be apparent by this point that the coupling among the various response components plays an important role in the determination of computed total responses. Phasing of the ground inputs, coupling of the Y and θ coordinates, and stiffness variations in the model all contribute to some degree of unpredictability in the results obtained.

As noted earlier, the response maximums are computed in two ways. First, a summation of all components of motion is made in the time domain and then the maximum edge response obtained. Alternately, the individual response components (X, Y and θ) are computed, maximum response values found and these maximums combined absolutely without respect to time. The latter approach leads to larger computed responses (approximately 30% on the average). This difference in magnitude arises from differences in time phasing of the response components, i.e., maximum translation and maximum rotation do not normally occur simultaneously; however, in those cases when they do, the two approaches lead to the same results (Tables 5.11 through 5.14).

For example, if one considers the responses arising from the Pacoima Dam record for ten percent eccentricity, the algebraic summation of Y and θ with respect to time leads to a maximum response of 1.379 feet (Table 5.3), the sum of absolute values of Y and θ responses without respect to time leads to a maximum response of 2.322 feet (Table 5.7), and the sum of responses arising from Y and θ individual ground motion, as discussed later, leads to a maximum response of either 1.904 or 3.148 feet depending on whether the responses are computed as the algebraic sum in the time domain or the sum of absolute values of the Y and θ components of motion.

This relatively wide range of possible maximum computed responses arises in total from time phasing of both the ground motions and the Y and θ responses. Lowest maximum response is computed for the case when both the phasing in ground motions and the phasing in the resulting response components (i.e., 1.379 algebraic summation in time) are included in the calculations. If one allows phasing of ground motions to be included in calculations but ignores phasing of response components (i.e., 2.322 sum of absolute values) a larger maximum response will be computed. Still larger computed maximum responses are obtained if neither phasing of ground motions nor phasing of response components is included (i.e., 3.148). If one includes phasing of response components but not phasing of ground motion a response will be computed which is between the two extremes. The relationship between the two intermediate responses is not uniform for all cases studied. It is hoped that these comparisons clearly indicate the importance of time phasing of both ground motions and response components on the responses computed.

With recognition of the fact that coupling between Y and θ motions takes place, a brief study of the individual responses arising from each of the two normally input ground motions (i.e., $\ddot{\phi}$ and $\ddot{\alpha}\tau/2$ as derived in Chapter 2) was made. In other words, for any given time history used two additional case studies were completed: in one case τ generated Y ground motion was input but θ input was forced to zero, and in the other case τ generated θ ground motion was input but Y input was forced to zero. These responses were compared to those obtained for simultaneous Y and θ ground inputs. As can be seen in Fig. 5.4 through 5.7 the response arising from θ input alone is of the same order of magnitude as the response arising from Y input alone. One also can see that the maximum response computed at the positive X edge for either Y or θ ground input in some cases is as great for only one of the ground inputs as for both components.

5.8 <u>Direction of Wave Propagation</u>

Another consideration which influences any conclusions drawn about the effect of coordinate coupling on the overall response of a system is that of direction of wave propagation. The majority of the cases studied consider a wave propagating in the X direction and therefore exciting the Y and θ components of motion. However, if one considers a wave propagating in the Y direction all three components of motion are excited, X and θ directly and Y because of Y-θ coupling in the model. One would expect the latter case (X, Y, and θ response) to lead to greater response; however, in most of the cases studied the opposite is true. As will be seen in Tables 5.3 through 5.10, the two responses from waves propagating in the two directions are quite close for all four earthquake excitations. The reasons for this tendency are not apparent.

5.9 Comparisons of Results for the Hollywood Storage Building and P. E. Lot

Presented in Figs. 5.8 through 5.11 are the zero percent eccentricity responses computed for the south component of the 1971 San Fernando earthquake for the Hollywood Storage Building and P. E. Lot records. In each figure the dashed line represents the response computed for the building basement record which, since it is located in the corner of the building, includes both translational and rotational components of motion. One will note that the responses computed using the τ-average ground motions are the same order of magnitude as those responses computed using the building basement record.

5.10 Comparisons of Results with Building Code Procedures

One of the most interesting comparisons in a study of this type is that with current building code procedures. A common approach used by building codes (Refs. 22 through 24) to include so-called accidental torsion arising from numerous effects, including ground rotation, is to compute response from the free-field ground motion, to position the resulting inertial force five percent of the building dimension away from the mass center and compute additional response arising from the equivalent static moment. As shown in Tables 5.15 through 5.18 there is a great deal of scatter in the ratio of current results from this study (from a summation in the time domain) to those from the above code procedure (using responses with maximum computed without respect to time).

It will be noted that two of the cases listed in the tables appear clearly out of line with other results; it was found that the ratio of torsional to appropriate translational frequency (f_y) is less than unity which is unrealistic for actual buildings. More specifically one will observe that whereas the ratios of f_θ/f_x = 1.0 and 1.189 respectively, since the ratio of f_y/f_x = 1.414, one finds the ratio of f_θ/f_y to be either .707 or .841 respectively; in all other cases this ratio is greater than or equal to 1. Ignoring these two cases, one finds that the results of this study are generally lower than code procedures (with a range of 22 to 131 per cent of the code values with less than 10% falling above code values) and average less than 70%. With realization that accidental torsion is intended to include several factors: irregularities in building plan; unforeseen differences in computed and actual values of stiffness, yield strength, dead load masses, etc.; unfavorable distributions of live load masses; and non-uniform ground motion, many of which have not been studied in this report; on the basis of the limited study herein it appears that a five percent accidental eccentricity code value is reasonable.

TABLE 5.1 MODEL PARAMETERS FOR CASES STUDIED

CASE NUMBER	H/B	k_y/k_x	f_x/f_θ	f_y^*, Hz			f_θ/f_y	f_θ^*, Hz		
1	1	1	1	1	3	5	1	1	3	5
2	1	1	1.189	1	3	5	1.189	1.189	3.568	5.946
3	1	1	1.414	1	3	5	1.414	1.414	4.242	7.070
4	2	1	1	1	3	5	1	1	3	5
5	2	1	1.189	1	3	5	1.189	1.189	3.568	5.946
6	2	1	1.414	1	3	5	1.414	1.414	4.242	7.070
7	2	2	1	1.414	4.242	7.070	.707	1	3	5
8	2	2	1.189	1.414	4.242	7.070	.841	1.189	3.568	5.946
9	2	2	1.414	1.414	4.242	7.070	1	1.414	4.242	7.070
10	1/2	1	1	1	3	5	1	1	3	5
11	1/2	1	1.189	1	3	5	1.189	1.189	3.568	5.946
12	1/2	1	1.414	1	3	5	1.414	1.414	4.242	7.070
13	1/2	1/2	1	.707	2.121	3.535	1.414	1	3	5
14	1/2	1/2	1.189	.707	2.121	3.535	1.681	1.189	3.568	5.946
15	1/2	1/2	1.414	.707	2.121	3.535	2	1.414	4.242	7.070
16	1/4	1	1	1	3	5	1	1	3	5
17	1/4	1	1.414	1	3	5	1.414	1.414	4.242	7.070
18	1/4	1/4	1	.5	1.5	2.5	2	1	3	5
19	1/4	1/4	1.414	.5	1.5	2.5	2.828	1.414	4.242	7.070

*f_y and f_θ are frequencies computed as if Y and θ were uncoupled

TABLE 5.2 EIGENVALUES FOR COUPLED MOTION MODEL FOR ONE CASE STUDIED

$H/B = 1/2$, $k_y/k_x = 1$, $f_\theta/f_x = 1.414$

PERCENT ECCENTRICITY	EIGENVALUES FOR COUPLED SYSTEM (frequencies in Hz)		
0	1	1	1.414
5	.989	1	1.447
10	.964	1	1.536
15	.937	1	1.664
0	3	3	4.243
5	2.967	3	4.341
10	2.892	3	4.607
15	2.811	3	4.993
0	5	5	7.071
5	4.945	5	7.235
10	4.820	5	7.679
15	4.685	5	8.322

TABLE 5.3 RESPONSE COMBINED ALGEBRAICALLY IN TIME FOR PACOIMA DAM, S16E, SAN FERNANDO EARTHQUAKE, 9 FEBRUARY 1971, $\beta = 0.02$, $\tau = 0.16$

RESPONSE IN FEET

f_x	1 Hz				3 Hz				5 Hz			
Eccentricity Case Number	0%	5%	10%	15%	0%	5%	10%	15%	0%	5%	10%	15%
1	1.266	1.241	1.379	1.480	.189	.270	.380	.459	.130	.147	.113	.083
3	1.123	1.273	1.360	1.419	.169	.219	.275	.290	.051	.045	.042	.048
4	1.266	1.281	1.272	1.236	.189	.287	.302	.390	.130	.185	.166	.142
6	1.123	1.184	1.227	1.256	.169	.177	.236	.244	.051	.047	.042	.040
7	.800	.841	.836	.866	.172	.203	.277	.341	.121	.148	.172	.197
9	.466	.438	.384	.421	.144	.132	.134	.142	.038	.044	.044	.045
10	1.266	1.419	1.700	1.633	.189	.250	.499	.479	.130	.131	.099	.069
12	1.123	1.353	1.475	1.551	.169	.256	.292	.253	.051	.045	.049	.051
13	1.256	2.654	2.788	2.589	.400	.481	.617	.433	.143	.131	.174	.177
15	1.811	2.014	2.171	2.280	.346	.376	.515	.550	.108	.106	.110	.121

NOTE: Responses are shown to three decimal places for comparative purposes; this does not imply accuracy to .001 feet.

TABLE 5.4 RESPONSE COMBINED ALGEBRAICALLY IN TIME FOR TAFT LINCOLN SCHOOL, S69E, KERN COUNTY EARTHQUAKE, 21 JULY 1952, $\beta = 0.02$, $\tau = 0.08$

RESPONSES IN FEET

f_x	1 Hz				3 Hz				5 Hz			
Eccentricity	0%	5%	10%	15%	0%	5%	10%	15%	0%	5%	10%	15%
1	.175	.149	.174	.223	.088	.068	.061	.056	.021	.024	.026	.024
3	.182	.205	.217	.219	.079	.079	.069	.077	.015	.017	.018	.020
4	.175	.163	.129	.146	.088	.066	.070	.060	.021	.026	.027	.027
6	.182	.191	.192	.195	.079	.077	.072	.063	.015	.015	.017	.017
7	.146	.135	.128	.119	.068	.072	.069	.070	.018	.021	.027	.028
9	.127	.115	.125	.154	.033	.027	.026	.028	.008	.009	.009	.012
10	.175	.175	.259	.274	.088	.078	.059	.061	.021	.027	.025	.024
12	.182	.216	.224	.232	.079	.080	.071	.079	.015	.018	.020	.021
13	.212	.262	.339	.502	.111	.115	.118	.101	.039	.038	.039	.047
15	.232	.246	.298	.311	.097	.101	.106	.093	.031	.034	.036	.038

TABLE 5.5 RESPONSE COMBINED ALGEBRAICALLY IN TIME FOR TAFT LINCOLN SCHOOL, S69E, KERN COUNTY EARTHQUAKE, 21 JULY 1952, $\beta = 0.02$, $\tau = 0.16$

RESPONSE IN FEET

f_x	1 Hz				3 Hz				5 Hz			
Eccentricity / Case Number	0%	5%	10%	15%	0%	5%	10%	15%	0%	5%	10%	15%
1	.185	.144	.178	.236	.103	.067	.060	.055	.022	.022	.023	.022
2	.173	.232	.203	.192	.050	.060	.059	.059	.014	.012	.012	.012
3	.204	.228	.217	.234	.067	.064	.058	.065	.008	.009	.008	.008
4	.185	.177	.138	.152	.103	.080	.087	.071	.022	.027	.029	.030
5	.173	.194	.190	.176	.050	.048	.055	.054	.014	.013	.012	.012
6	.204	.212	.215	.204	.067	.065	.059	.055	.008	.008	.009	.008
7	.168	.169	.166	.163	.088	.100	.105	.101	.021	.025	.032	.034
8	.224	.248	.247	.245	.047	.059	.059	.056	.014	.016	.017	.016
9	.140	.127	.140	.168	.037	.030	.030	.032	.005	.006	.007	.010
10	.185	.176	.255	.268	.103	.070	.053	.055	.022	.023	.022	.017
11	.173	.219	.227	.247	.050	.068	.061	.066	.014	.012	.010	.012
12	.204	.241	.246	.260	.067	.065	.064	.061	.008	.009	.009	.009
13	.218	.254	.334	.522	.120	.120	.119	.092	.032	.028	.030	.036
14	.303	.307	.352	.339	.094	.113	.098	.092	.025	.028	.027	.029
15	.243	.268	.321	.320	.088	.090	.099	.086	.021	.023	.024	.028

TABLE 5.6 RESPONSE COMBINED ALGEBRAICALLY IN TIME FOR HOLLYWOOD STORAGE P.E. LOT, SOOW, SAN FERNANDO EARTHQUAKE, 9 FEBRUARY 1971, $\beta = 0.02$, $\tau = 0.12$

RESPONSE IN FEET

f_x	1 Hz				3 Hz				5 Hz			
Eccentricity / Case Number	0%	5%	10%	15%	0%	5%	10%	15%	0%	5%	10%	15%
16	.177	.223	.182	.154	.054	.048	.043	.067	.025	.025	.032	.032
17	.147	.185	.196	.220	.056	.053	.059	.056	.014	.015	.016	.019
18	.451	.495	.530	.531	.126	.144	.131	.135	.046	.047	.056	.058
19	.427	.446	.494	.512	.112	.106	.122	.114	.037	.040	.038	.048

TABLE 5.7 RESPONSE COMBINED AS THE SUM OF ABSOLUTE MAXIMA FOR PACOIMA DAM, S16E, SAN FERNANDO EARTHQUAKE, 9 FEBRUARY 1971, $\beta = 0.02$, $\tau = 0.16$

RESPONSE IN FEET

f_x	1 Hz				3 Hz				5 Hz			
Eccentricity / Case Number	0%	5%	10%	15%	0%	5%	10%	15%	0%	5%	10%	15%
1	1.706	2.374	2.322	2.140	.274	.494	.566	.831	.153	.173	.149	.130
3	1.401	1.446	1.514	1.718	.247	.282	.390	.530	.065	.064	.065	.075
4	1.706	1.851	2.273	2.220	.274	.430	.627	.658	.153	.214	.228	.210
6	1.401	1.436	1.430	1.446	.247	.276	.353	.445	.065	.069	.065	.069
7	.937	.990	1.040	1.074	.212	.257	.330	.460	.132	.152	.178	.205
9	.632	.627	.731	.715	.185	.283	.382	.466	.044	.056	.060	.061
10	1.706	2.637	2.460	2.024	.274	.507	.613	.662	.153	.168	.143	.135
12	1.401	1.507	1.734	1.732	.247	.312	.467	.440	.065	.062	.069	.065
13	2.361	2.957	3.527	3.543	.463	.562	.821	.650	.216	.197	.224	.275
15	2.056	2.195	2.385	2.542	.436	.472	.610	.784	.128	.131	.130	.161

TABLE 5.8 RESPONSE COMBINED AS THE SUM OF ABSOLUTE MAXIMA FOR TAFT LINCOLN SCHOOL, S69E, KERN COUNTY EARTHQUAKE, 21 JULY 1952, $\beta = 0.02$, $\tau = 0.08$

RESPONSE IN FEET

f_x	1 Hz				3 Hz				5 Hz			
Eccentricity	0%	5%	10%	15%	0%	5%	10%	15%	0%	5%	10%	15%
Case Number												
1	.216	.292	.454	.576	.123	.101	.093	.101	.030	.036	.038	.036
3	.214	.247	.287	.324	.095	.098	.089	.097	.020	.023	.026	.022
4	.216	.228	.286	.391	.123	.114	.102	.099	.030	.035	.039	.042
6	.214	.223	.249	.272	.095	.096	.099	.094	.020	.023	.024	.027
7	.166	.179	.197	.238	.076	.084	.083	.080	.020	.023	.028	.031
9	.164	.194	.251	.261	.048	.047	.053	.060	.010	.013	.013	.015
10	.216	.370	.619	.447	.123	.108	.085	.102	.030	.037	.038	.031
12	.214	.272	.330	.368	.095	.098	.092	.101	.020	.024	.024	.025
13	.269	.324	.420	.722	.149	.148	.137	.133	.049	.052	.059	.075
15	.266	.293	.382	.390	.121	.123	.124	.115	.039	.040	.044	.040

TABLE 5.9 RESPONSE COMBINED AS THE SUM OF ABSOLUTE MAXIMA FOR TAFT LINCOLN SCHOOL, S69E, KERN COUNTY EARTHQUAKE, 21 JULY 1952, $\beta = 0.02$, $\tau = 0.16$

RESPONSE IN FEET

f_x	1 Hz				3 Hz				5 Hz			
Eccentricity Case Number	0%	5%	10%	15%	0%	5%	10%	15%	0%	5%	10%	15%
1	.256	.320	.477	.635	.141	.120	.105	.118	.028	.033	.032	.031
2	.312	.330	.330	.338	.094	.099	.091	.093	.020	.022	.022	.020
3	.248	.273	.327	.371	.087	.092	.085	.089	.012	.012	.013	.012
4	.256	.256	.337	.467	.141	.131	.138	.128	.028	.038	.042	.040
5	.312	.342	.327	.335	.094	.098	.110	.103	.020	.023	.024	.027
6	.247	.258	.290	.334	.087	.092	.095	.095	.012	.012	.013	.013
7	.204	.222	.246	.281	.101	.117	.124	.115	.023	.027	.033	.036
8	.260	.280	.352	.392	.054	.069	.086	.101	.016	.018	.018	.018
9	.196	.233	.301	.333	.047	.054	.061	.065	.007	.009	.009	.012
10	.256	.378	.627	.486	.141	.109	.088	.099	.028	.030	.033	.026
11	.312	.354	.326	.459	.094	.101	.090	.102	.020	.020	.020	.016
12	.248	.296	.351	.426	.087	.089	.085	.085	.012	.012	.012	.013
13	.308	.357	.472	.804	.170	.172	.151	.155	.044	.046	.047	.059
14	.363	.411	.445	.452	.123	.124	.121	.120	.036	.036	.039	.032
15	.300	.327	.438	.441	.116	.119	.121	.116	.028	.028	.029	.030

TABLE 5.10 RESPONSE COMBINED AS THE SUM OF ABSOLUTE MAXIMA FOR HOLLYWOOD STORAGE P.E. LOT, SOOW, SAN FERNANDO EARTHQUAKE, 9 FEBRUARY 1971, $\beta = 0.02$, $\tau = 0.12$

RESPONSE IN FEET

f_x	1 Hz				3 Hz				5 Hz			
Eccentricity / Case Number	0%	5%	10%	15%	0%	5%	10%	15%	0%	5%	10%	15%
16	.231	.375	.258	.288	.075	.068	.087	.150	.033	.035	.052	.044
17	.220	.269	.246	.312	.069	.081	.080	.083	.022	.022	.023	.031
18	.511	.535	.647	.597	.134	.147	.158	.182	.059	.060	.071	.079
19	.501	.525	.532	.528	.128	.140	.153	.153	.048	.048	.049	.065

TABLE 5.11 RESPONSE COMBINED ALGEBRAICALLY IN TIME AS A PERCENTAGE OF RESPONSE COMBINED AS THE SUM OF ABSOLUTE MAXIMA FOR PACOIMA DAM, S16E, SAN FERNANDO EARTHQUAKE, 9 FEBRUARY 1971, $\beta = 0.02$, $\tau = 0.16$

f_x	1 Hz				3 Hz				5 Hz			
Eccentricity Case Number	0%	5%	10%	15%	0%	5%	10%	15%	0%	5%	10%	15%
1	74	52	59	69	69	55	67	55	85	85	76	64
3	80	88	90	83	68	78	71	55	79	70	64	64
4	74	69	56	56	69	67	48	59	85	86	73	68
6	80	82	86	87	68	64	67	55	79	68	64	57
7	85	85	80	81	81	79	84	74	91	98	97	96
9	74	70	53	59	78	47	35	30	86	70	73	73
10	74	54	69	81	69	49	81	72	85	78	69	51
12	80	90	85	90	68	82	63	57	79	73	72	78
13	96	90	79	73	87	86	75	67	66	66	78	65
15	88	92	91	90	79	80	84	70	84	80	84	75

TABLE 5.12 RESPONSE COMBINED ALGEBRAICALLY IN TIME AS A PERCENTAGE OF RESPONSE COMBINED AS THE SUM OF ABSOLUTE MAXIMA FOR TAFT LINCOLN SCHOOL, S69E, KERN COUNTY EARTHQUAKE, 21 JULY 1952, $\beta = 0.02$, $\tau = 0.08$

f_x	1 Hz				3 Hz				5 Hz			
Eccentricity / Case Number	0%	5%	10%	15%	0%	5%	10%	15%	0%	5%	10%	15%
1	81	51	38	39	71	67	66	55	71	66	68	67
3	85	83	76	68	83	81	77	80	76	71	68	90
4	81	71	45	37	71	58	69	61	71	74	69	66
6	85	86	77	72	83	80	73	68	76	67	69	65
7	88	76	65	50	89	86	83	87	89	94	95	89
9	78	59	50	59	69	57	48	46	79	66	71	78
10	81	47	42	61	71	71	69	60	71	74	65	78
12	85	80	68	63	83	82	78	79	76	75	83	86
13	79	81	81	70	74	78	86	75	80	72	66	62
15	87	84	78	80	81	82	85	81	80	84	82	95

TABLE 5.13 RESPONSE COMBINED ALGEBRAICALLY IN TIME AS A PERCENTAGE OF RESPONSE COMBINED AS THE SUM OF ABSOLUTE MAXIMA FOR TAFT LINCOLN SCHOOL, S69E, KERN COUNTY EARTHQUAKE, 21 JULY 1952, $\beta = 0.02$, $\tau = 0.16$

f_x	1 Hz				3 Hz				5 Hz			
Eccentricity / Case Number	0%	5%	10%	15%	0%	5%	10%	15%	0%	5%	10%	15%
1	72	45	37	37	73	56	57	47	79	67	72	71
2	56	70	61	57	53	60	64	64	70	55	55	62
3	82	83	67	63	77	70	68	73	69	73	61	66
4	72	69	41	33	73	62	63	56	79	71	69	75
5	56	57	58	53	53	49	50	53	70	55	48	44
6	82	82	74	61	77	71	62	57	69	69	68	57
7	82	76	68	58	87	86	85	88	91	92	97	96
8	86	88	70	62	87	85	69	55	86	93	96	88
9	72	54	47	51	79	55	49	49	68	66	79	80
10	72	46	41	55	73	64	59	56	79	78	67	68
11	56	62	70	54	53	68	68	65	70	61	52	77
12	82	81	70	61	77	73	75	71	69	76	72	70
13	71	71	71	65	71	70	79	59	73	62	64	60
14	83	75	79	75	76	91	81	77	69	77	70	92
15	81	82	73	72	76	75	82	74	77	82	83	95

TABLE 5.14 RESPONSE COMBINED ALGEBRAICALLY IN TIME AS A PERCENTAGE OF RESPONSE COMBINED AS THE SUM OF ABSOLUTE MAXIMA FOR HOLLYWOOD STORAGE P.E. LOT, SOOW, SAN FERNANDO EARTHQUAKE, 9 FEBRUARY 1971, $\beta = 0.02$, $\tau = 0.12$

f_x	1 Hz				3 Hz				5 Hz			
Eccentricity / Case Number	0%	5%	10%	15%	0%	5%	10%	15%	0%	5%	10%	15%
16	77	59	70	53	72	72	50	45	76	71	61	72
17	67	69	80	71	66	65	74	67	65	69	68	60
18	88	94	82	89	95	98	83	74	78	78	79	71
19	85	85	93	97	88	76	80	74	77	83	77	87

TABLE 5.15 RESPONSE COMBINED ALGEBRAICALLY IN TIME AS A PERCENTAGE OF RESPONSE COMPUTED WITH FREE-FIELD GROUND MOTION AND STATIC MOMENT ARISING FROM A 5% ACCIDENTAL ECCENTRICITY FOR PACOIMA DAM, S16E, SAN FERNANDO EARTHQUAKE, 9 FEBRUARY 1971, $\beta = 0.02$, $\tau = 0.16$

f_x	1 Hz				3 Hz				5 Hz			
Eccentricity / Case Number	0%	5%	10%	15%	0%	5%	10%	15%	0%	5%	10%	15%
1	94	46	50	56	112	54	64	48	106	70	51	39
3	89	82	77	71	107	96	78	58	44	32	23	22
4	102	67	60	57	121	92	69	85	115	124	97	77
6	93	89	86	83	112	98	110	85	46	38	31	26
7	189	154	126	119	150	135	142	128	239	250	294	287
9	110	85	61	63	125	72	55	46	75	68	89	69
10	87	44	53	56	104	39	59	48	99	51	59	32
12	85	77	70	66	103	94	56	47	42	28	30	22
13	100	72	56	48	87	66	57	47	74	47	59	34
15	89	80	74	69	83	71	74	61	62	50	53	39

TABLE 5.16 RESPONSE COMBINED ALGEBRAICALLY IN TIME AS A PERCENTAGE OF RESPONSE COMPUTED WITH FREE-FIELD GROUND MOTION AND STATIC MOMENT ARISING FROM A 5% ACCIDENTAL ECCENTRICITY FOR TAFT LINCOLN SCHOOL, S69E, KERN COUNTY EARTHQUAKE, 21 JULY 1952, $\beta = 0.02$, $\tau = 0.08$

f_x	1 Hz				3 Hz				5 Hz			
Eccentricity / Case Number	0%	5%	10%	15%	0%	5%	10%	15%	0%	5%	10%	15%
1	89	45	30	31	98	63	57	44	105	68	62	53
3	99	79	70	61	95	79	75	78	81	65	58	62
4	96	68	41	33	106	77	76	71	114	96	96	89
6	103	92	79	74	99	89	81	79	84	73	68	64
7	112	83	63	44	227	186	158	131	292	274	286	222
9	98	60	48	59	111	75	66	63	131	102	100	107
10	82	38	30	39	91	56	51	42	97	65	44	54
12	95	70	57	56	91	73	70	61	78	61	60	55
13	76	62	54	46	88	69	64	53	86	57	47	46
15	92	73	67	63	86	77	76	82	76	69	64	62

TABLE 5.17 RESPONSE COMBINED ALGEBRAICALLY IN TIME AS A PERCENTAGE OF RESPONSE COMPUTED WITH FREE-FIELD GROUND MOTION AND STATIC MOMENT ARISING FROM A 5% ACCIDENTAL ECCENTRICITY FOR TAFT LINCOLN SCHOOL, S69E, KERN COUNTY EARTHQUAKE, 21 JULY 1952, $\beta = 0.02$, $\tau = 0.16$

f_x	1 Hz				3 Hz				5 Hz			
Eccentricity / Case Number	0%	5%	10%	15%	0%	5%	10%	15%	0%	5%	10%	15%
1	94	44	31	32	115	63	56	43	109	62	56	49
2	91	75	62	52	58	60	57	49	73	37	34	31
3	111	88	70	65	80	65	63	66	43	35	26	25
4	102	74	43	35	125	94	95	85	118	99	102	96
5	97	84	71	68	61	55	70	67	77	51	43	41
6	116	102	88	78	84	74	66	68	45	40	35	28
7	130	104	82	61	294	259	243	190	353	321	339	275
8	173	132	95	75	157	158	134	95	226	193	166	146
9	108	67	54	65	124	84	76	73	83	70	75	89
10	87	38	98	39	107	50	46	38	101	55	40	39
11	86	58	57	36	55	64	46	47	69	34	25	25
12	106	78	63	62	77	59	63	46	41	31	26	23
13	78	60	54	48	95	72	65	49	70	43	36	35
14	115	78	69	56	79	77	64	57	58	50	39	41
15	97	80	72	65	77	68	71	61	52	46	43	46

TABLE 5.18 RESPONSE COMBINED ALGEBRAICALLY IN TIME AS A PERCENTAGE OF RESPONSE COMPUTED WITH FREE-FIELD GROUND MOTION AND STATIC MOMENT ARISING FROM A 5% ACCIDENTAL ECCENTRICITY FOR HOLLYWOOD STORAGE P.E. LOT, SOOW, SAN FERNANDO EARTHQUAKE, 9 FEBRUARY 1971, $\beta = 0.02$, $\tau = 0.12$

f_x	1 Hz				3 Hz				5 Hz			
Eccentricity Case Number	0%	5%	10%	15%	0%	5%	10%	15%	0%	5%	10%	15%
16	83	48	50	36	96	52	38	33	104	48	42	41
17	77	66	67	59	113	72	62	56	65	51	41	35
18	89	64	51	48	100	77	53	46	83	62	55	44
19	84	72	69	66	99	75	70	57	75	67	56	55

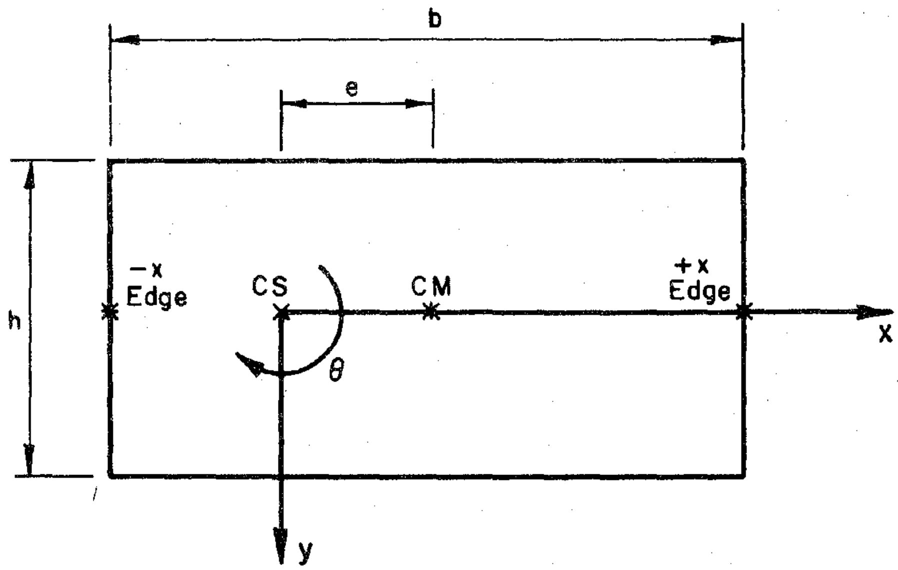


FIGURE 5.1 LOCATION OF RESPONSES COMPUTED
FOR THE COUPLED MOTION MODEL

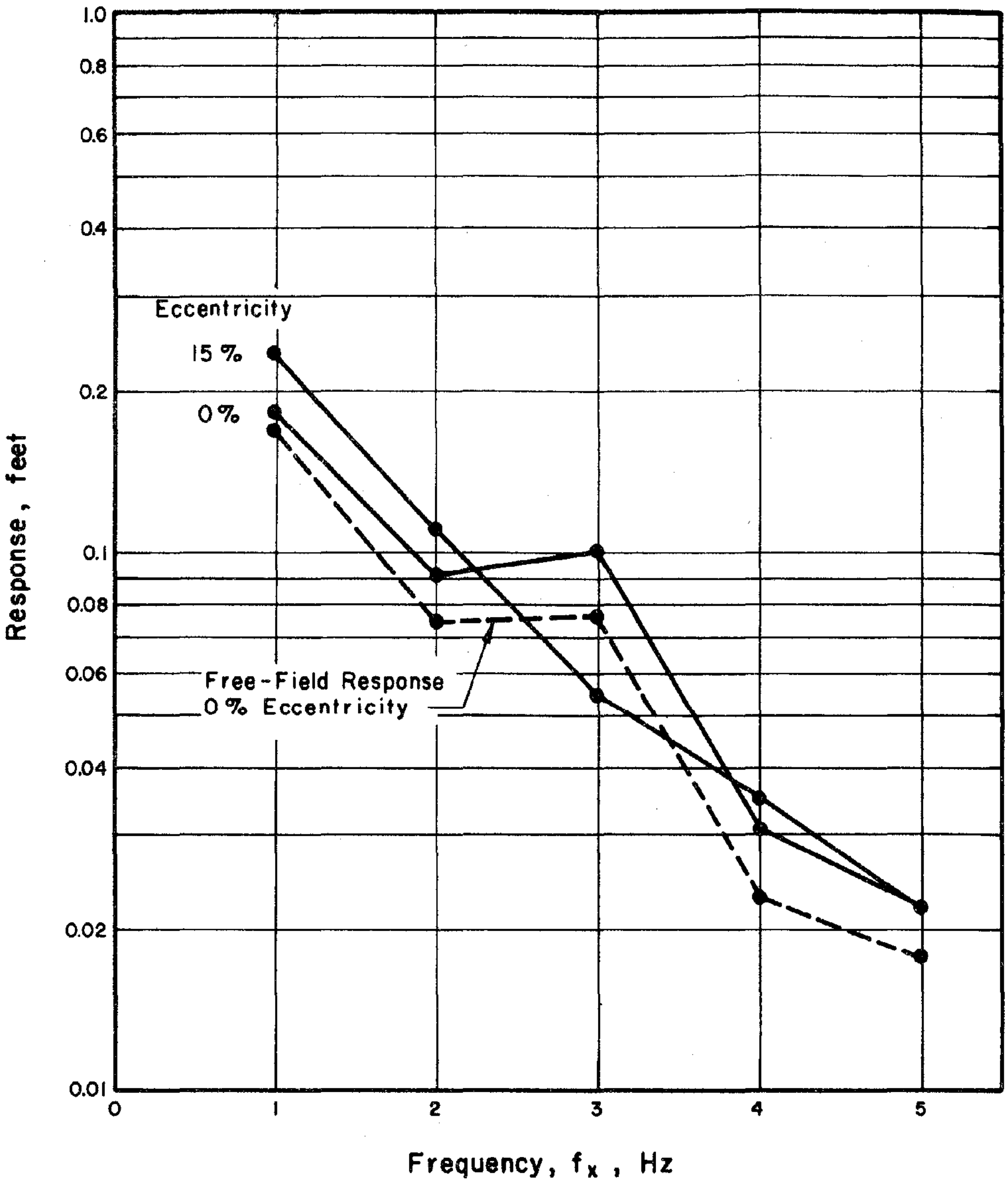


FIGURE 5.2 RESPONSE OF COUPLED MOTION MODEL, TAFT LINCOLN SCHOOL, S69E, KERN COUNTY EARTHQUAKE, 21 JULY 1952, SPECTRUM COMPUTED USING 2.0 PERCENT CRITICAL DAMPING, $\tau = 0.16$, $H/B = 1$, $k_y/k_x = 1$, $f_\theta/f_x = 1$

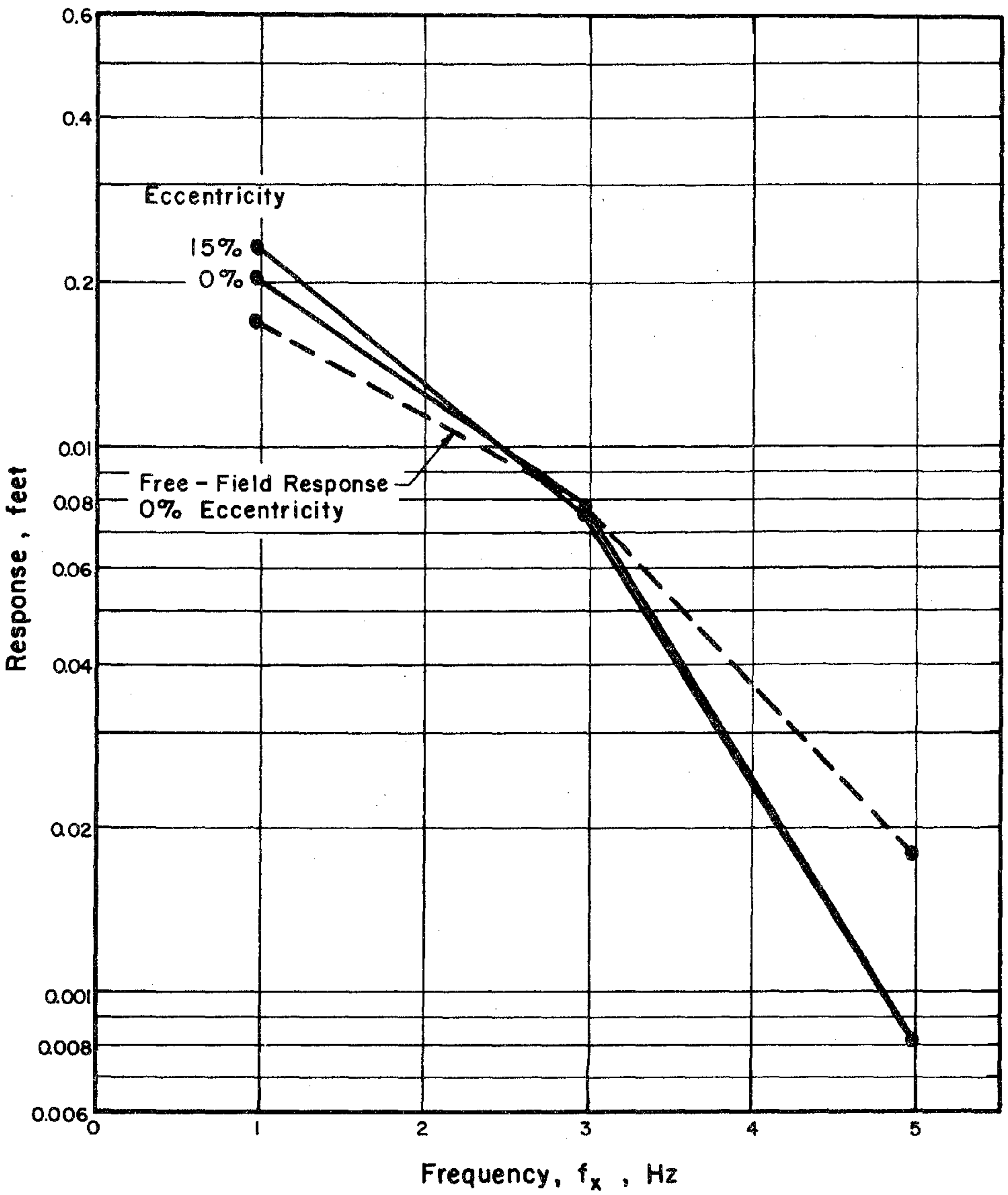


FIGURE 5.3 RESPONSE OF COMBINED MOTION MODEL, TAFT LINCOLN SCHOOL, S69E, KERN COUNTY EARTHQUAKE, 21 JULY 1952, SPECTRUM COMPUTED USING 2.0 PERCENT CRITICAL DAMPING, $\tau = 0.16$, $H/B = 1$, $k_y/k_x = 1$, $f_\theta/f_x = 1.414$

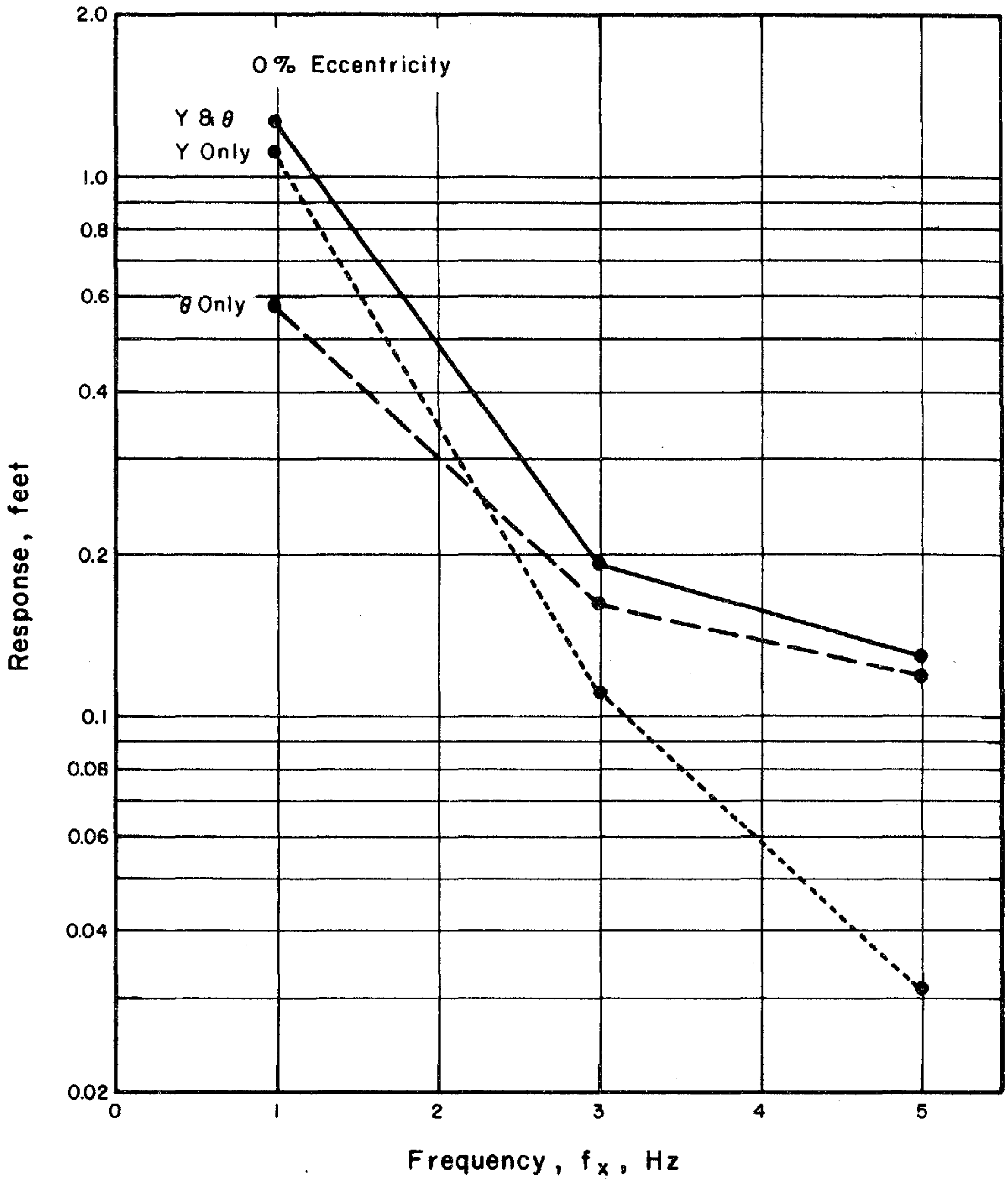


FIGURE 5.4 RESPONSE OF COMBINED MOTION MODEL TO INDIVIDUAL GROUND COMPONENT, PACOIMA DAM, S16E, SAN FERNANDO EARTHQUAKE, 9 FEBRUARY 1971, SPECTRUM COMPUTED USING 2.0 PERCENT CRITICAL DAMPING, $\tau = 0.16$, $H/B = 1$, $k_y/k_x = 1$, $f_\theta/f_x = 1$

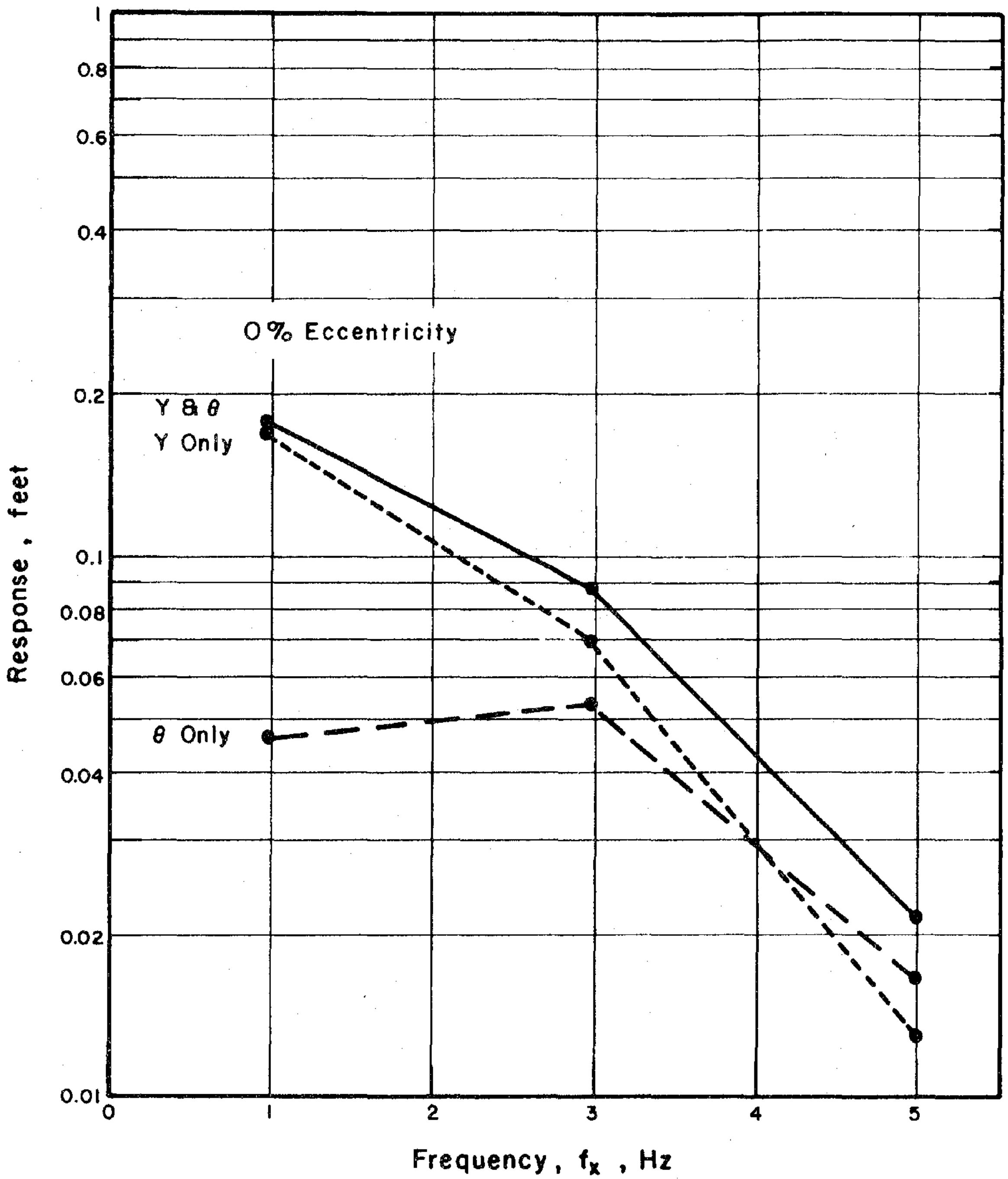


FIGURE 5.5 RESPONSE OF COMBINED MOTION MODEL TO INDIVIDUAL GROUND COMPONENTS, TAFT LINCOLN SCHOOL, S69E, KERN COUNTY EARTHQUAKE, 21 JULY 1952, SPECTRUM COMPUTED USING 2.0 PERCENT CRITICAL DAMPING, $\tau = 0.08$, $H/_B = 1$, $k_y/k_x = 1$, $f_\theta/f_x = 1$

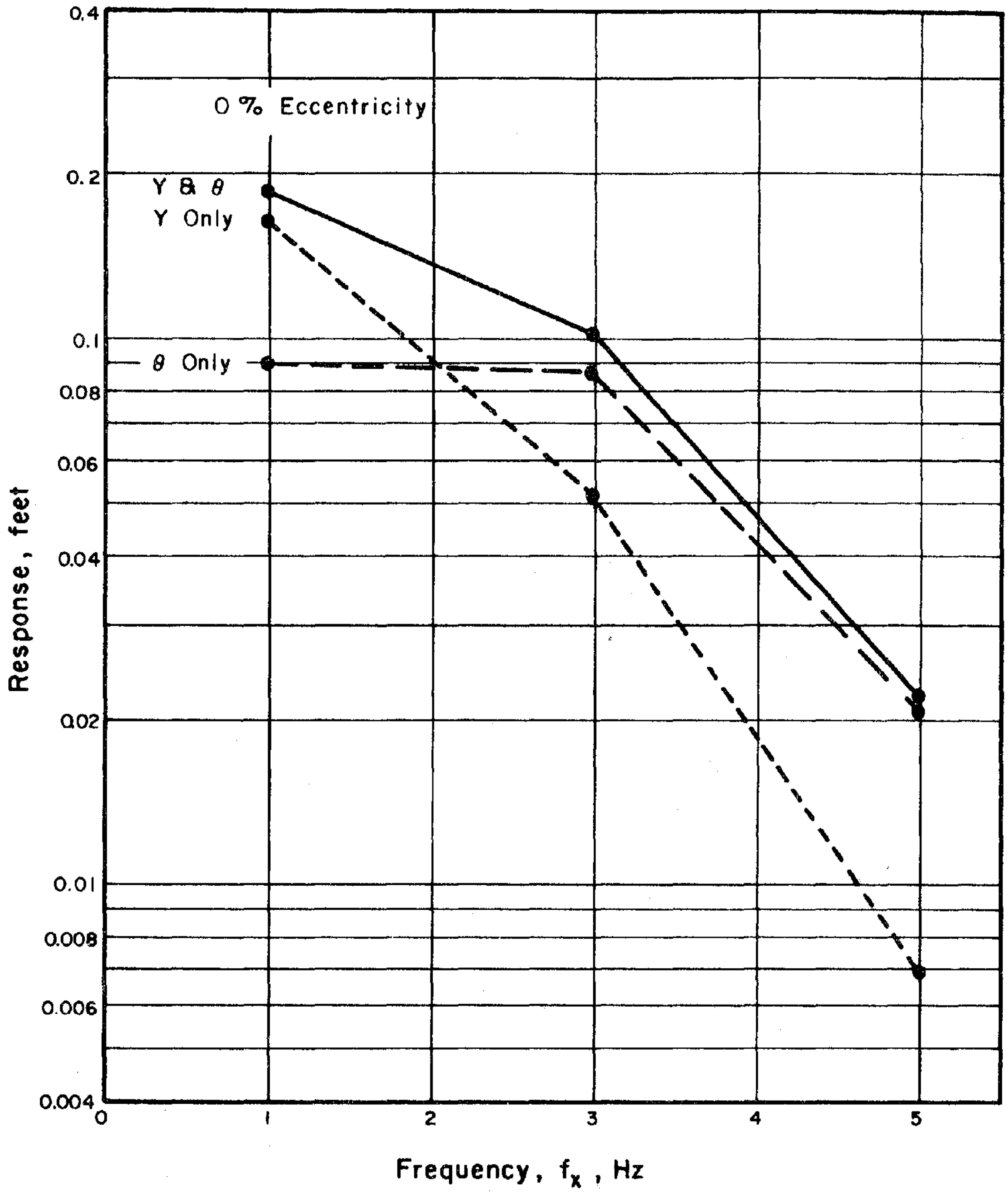


FIGURE 5.6 RESPONSE OF COMBINED MOTION MODEL TO INDIVIDUAL GROUND COMPONENTS, TAFT LINCOLN SCHOOL, S69E, KERN COUNTY EARTHQUAKE, 21 JULY 1952, SPECTRUM COMPUTED USING 2.0 PERCENT CRITICAL DAMPING, $\tau = 0.16$, $H/_B = 1$, $k_y/k_x = 1$, $f_\theta/f_x = 1$

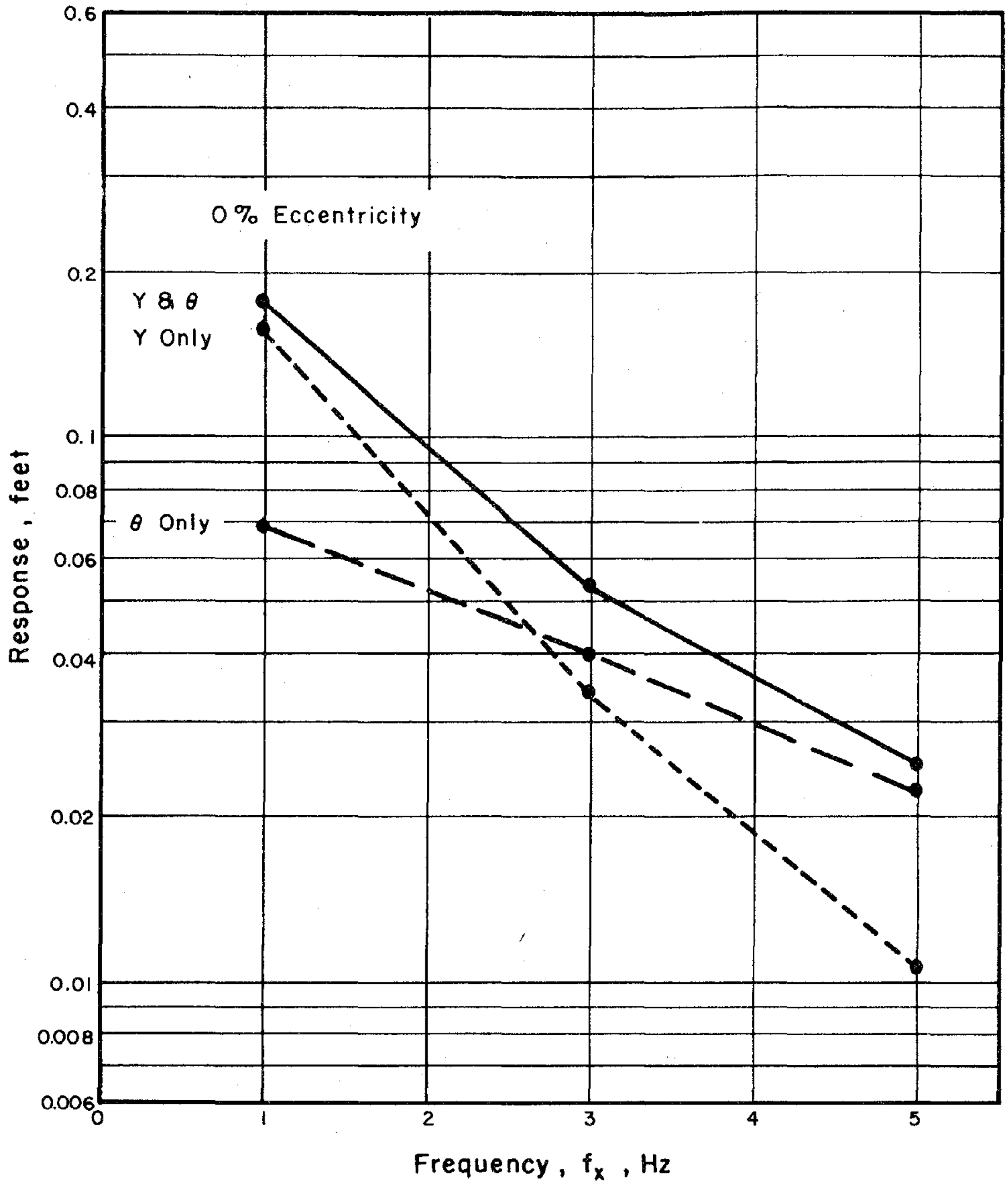


FIGURE 5.7 RESPONSE OF COMBINED MOTION MODEL TO INDIVIDUAL GROUND COMPONENTS, HOLLYWOOD STORAGE P.E. LOT, S00W, SAN FERNANDO EARTHQUAKE, 9 FEBRUARY 1971, SPECTRUM COMPUTED USING 2.0 PERCENT CRITICAL DAMPING, $\tau = 0.12$, $H/_B = 1/4$, $k_y/k_x = 1$, $f_\theta/f_x = 1$

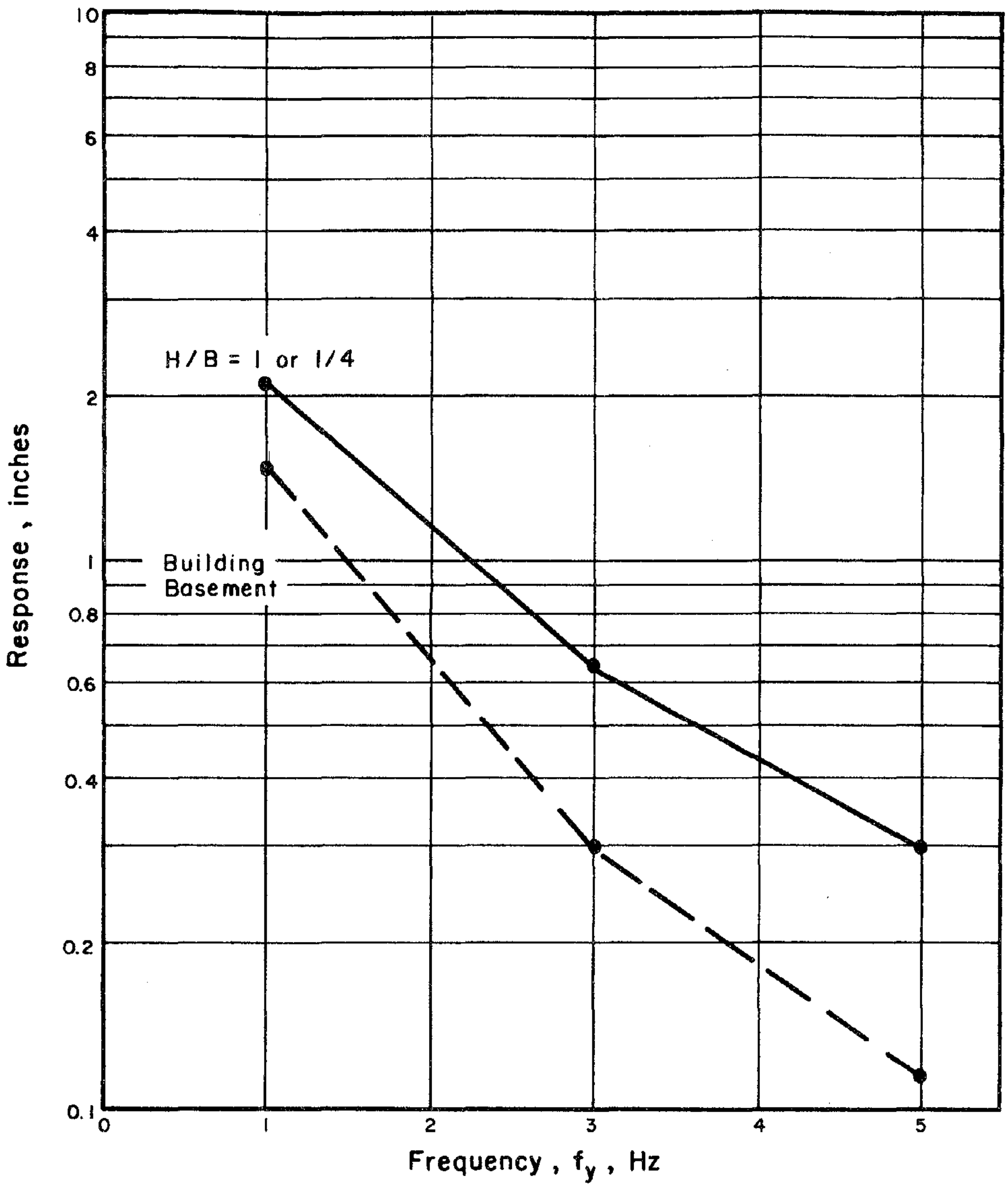


FIGURE 5.8 RESPONSE OF COMBINED MOTION MODEL, HOLLYWOOD STORAGE BUILDING BASEMENT AND P.E. LOT, S00W, SAN FERNANDO EARTHQUAKE, 9 FEBRUARY 1971, SPECTRUM COMPUTED USING 2.0 PERCENT CRITICAL DAMPING, $\tau = 0.12$, $k_y/k_x = 1$, $f_\theta/f_x = 1$

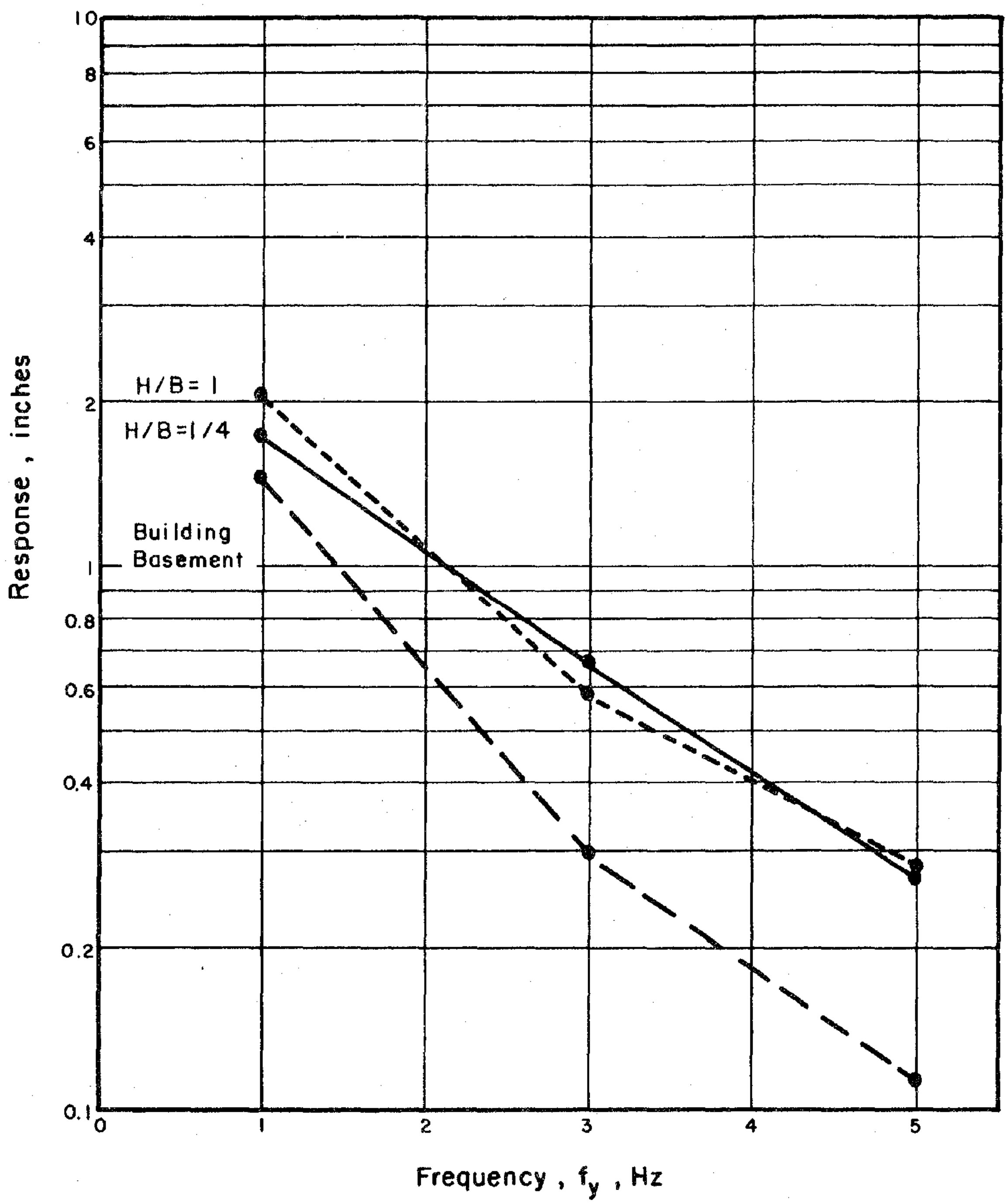


FIGURE 5.9 RESPONSE OF COMBINED MOTION MODEL, HOLLYWOOD STORAGE BUILDING BASEMENT AND P.E. LOT, SOOW, SAN FERNANDO EARTHQUAKE, 9 FEBRUARY 1971, SPECTRUM COMPUTED USING 2.0 PERCENT CRITICAL DAMPING, τ = 0.12, k_y/k_x = 1, f_θ/f_x = 1.414

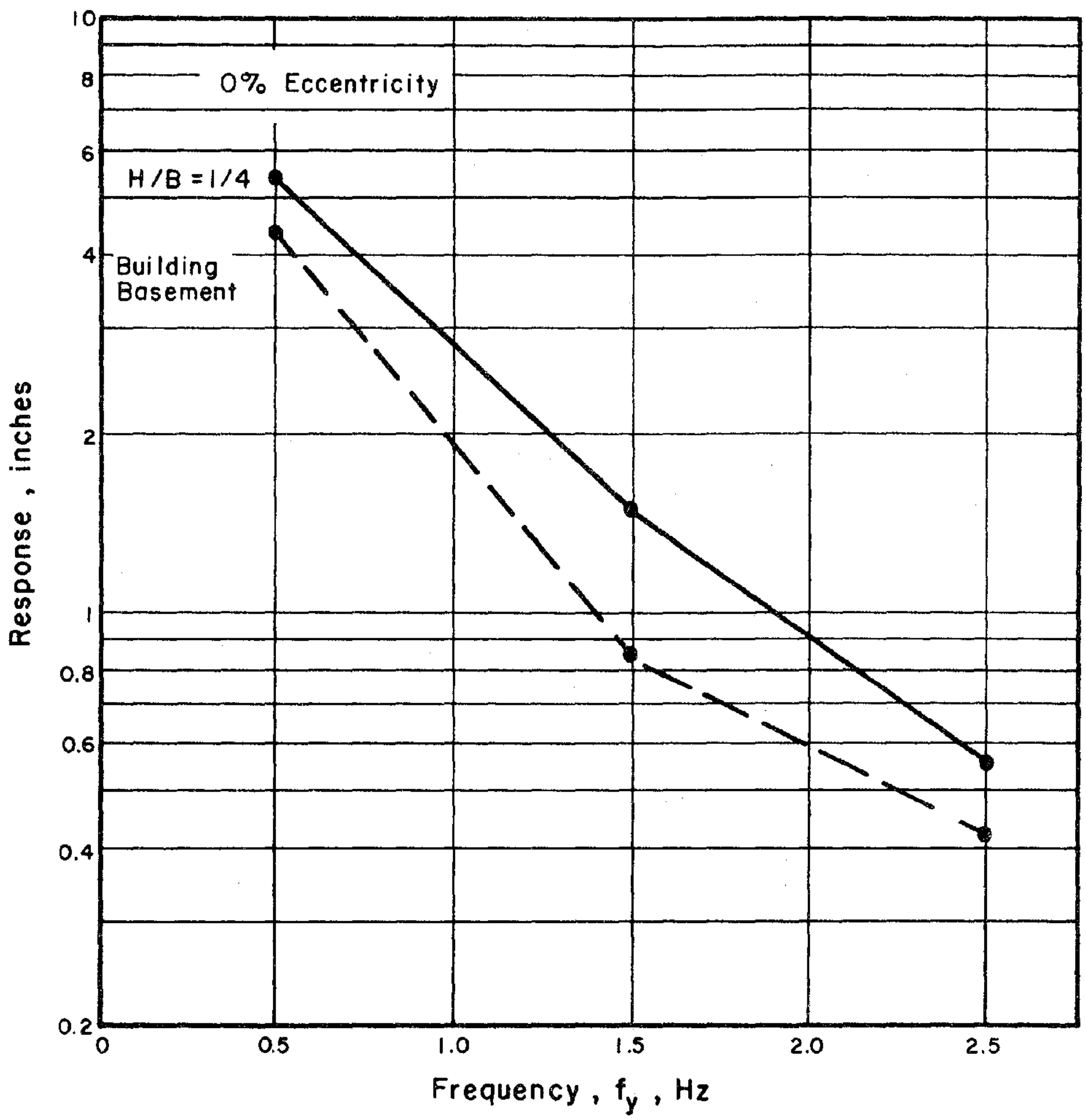


FIGURE 5.10 RESPONSE OF COMBINED MOTION MODEL, HOLLYWOOD STORAGE BUILDING BASEMENT AND P.E. LOT, S00W, SAN FERNANDO EARTHQUAKE, 9 FEBRUARY 1971, SPECTRUM COMPUTED USING 2.0 PERCENT CRITICAL DAMPING, $\tau = 0.12$, $k_y/k_x = 1/4$, $f_\theta/f_x = 1$

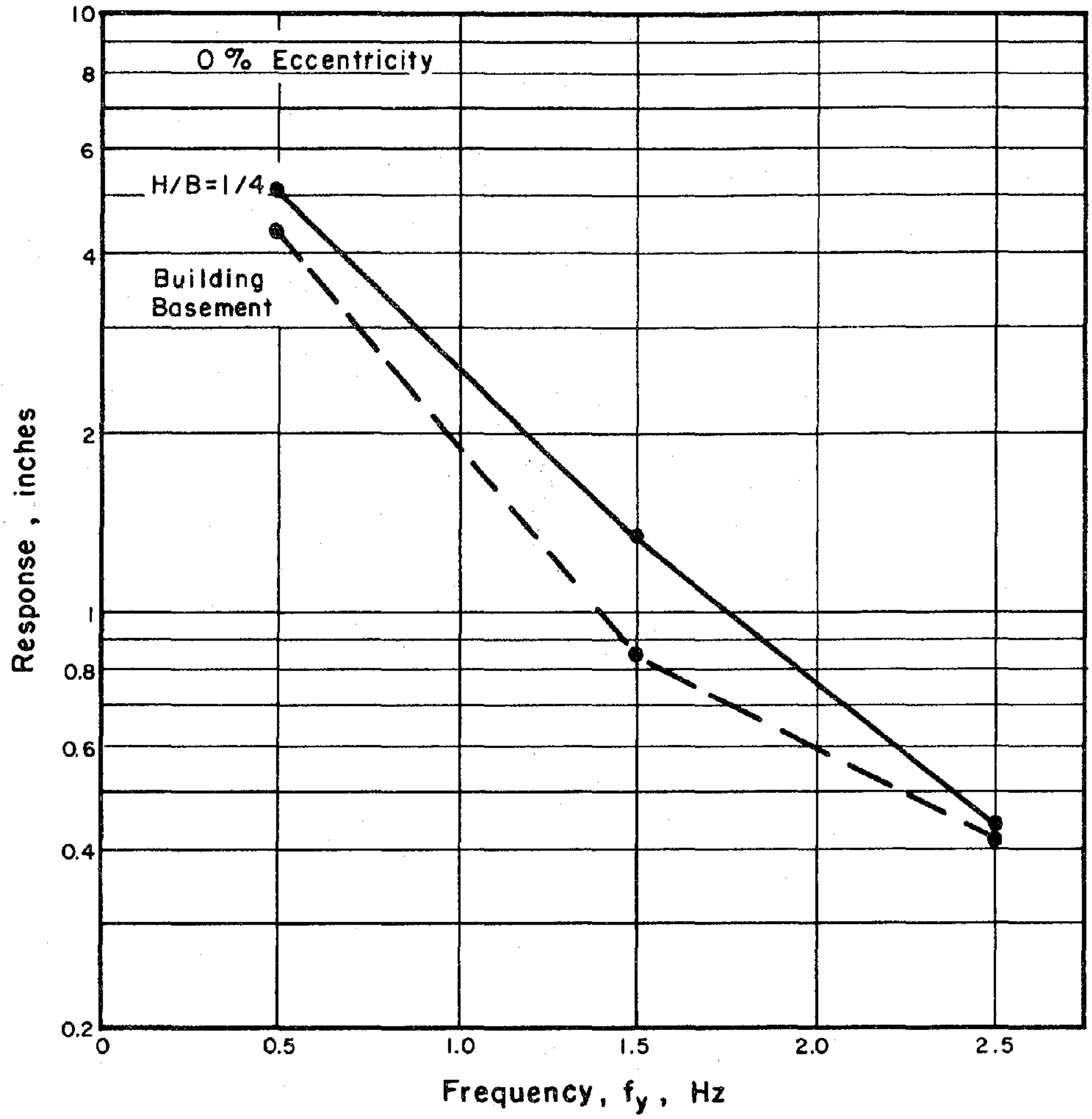


FIGURE 5.11 RESPONSE OF COMBINED MOTION MODEL, HOLLYWOOD STORAGE BUILDING BASEMENT AND P.E. LOT, SOOW, SAN FERNANDO EARTHQUAKE, 9 FEBRUARY 1971, SPECTRUM COMPUTED USING 2.0 PERCENT CRITICAL DAMPING, $\tau = 0.12$, $k_y/k_x = 1/4$, $f_\theta/f_x = 1.414$

6. FINDINGS AND RECOMMENDATIONS

6.1 Introductory Remarks

The following is a short summary of the principle findings based on the results of the foregoing studies. Several assumptions have been made throughout the study which warrant additional consideration

- Only systematic motions over the base are taken into account. The true motions are in large part random, and therefore cause much lower torsional responses, but about the same translational reduction.
- Only horizontally propagated plane waves with vertical wave fronts of motion are considered. Since only part of the motions in an earthquake is of this type, the torsional responses may well be exaggerated, but the translational reductions are not greatly affected.
- Only rigid foundation systems are considered. This assumption tends to exaggerate both the induced torsional responses and the translational reductions computed.

6.2 Summary of Findings: Superposition Model

Briefly, the conclusions to be drawn from the results of the superposition model are as follows:

- Translational averaging leads to significant response reductions depending on τ (i.e., width/wave velocity) when compared to response computed from free-field records, at frequencies above 1 Hz with little change in response for lower frequencies.

- The response arising from rotation induced from a traveling wave passing by a foundation is most important at frequencies above 1 Hz.
- For a torsional to translational frequency ratio of unity the reductions arising from translation are largely offset by induced rotational response and in some cases the computed combined response actually exceeds the computed free-field response.
- The use of frequency ratios greater than unity more closely simulates real buildings and leads to response in the regions of greatest interest which is less than that observed for a frequency ratio of unity, i.e., the amplifications noted above diminish.

6.3 Summary of Findings: Coupled Model

In general, the coupling of translational and rotational response leads to increased scatter in the computed responses obtained and less certainty in the types of trends noted for the superposition model. Briefly, the following observations can be made.

- The coupling of Y and θ coordinates no longer allows the separation of translational and rotational effects noted previously.
- Throughout the frequency range under study (1-5 Hz) and for the various other parameters investigated (building size, frequency ratio, and stiffness ratio) there was no clear dominant trend of translational or rotational effects.
- Although the trend is not as clear cut as for the superposition model, it was observed that an increase in torsional to translational frequency ratio (f_θ/f_x or f_θ/f_y) generally leads to lower computed edge responses.

- There is a trend toward the behavior that would be expected with respect to eccentricities; however, coupling of translational and torsional responses and phasing between Y and θ ground motions do not allow exact predictions of response.
- The results of the study clearly suggest that symmetry is desirable if possible. In such cases it is easier to predict the motions and they are in general more uniform throughout the structure.
- As a result of the various approaches for computing effects involving phasing of ground motion and/or phasing of response components (Tables 5.11 through 5.14), one observes that the techniques involving summation of maxima without respect to time in all cases studied gave motions that were larger than if they were summed with respect to time. This observation suggests that the code approaches (i.e., no allowance for time phasing) are of a conservative type.

6.4 Comparisons with Building Code Procedures

For the most part the results obtained in this study fall (averaging thirty percent) below those computed using a five percent accidental eccentricity approach; however, there is a great deal of scatter in the data ranging from an 80 percent reduction to a 31 percent increase of response computed over the building code approach.

It should be noted that the code approach intent is to include accidental torsion from all sources, of which ground rotation is only one item. As stated previously, based on the results of this study the 5 percent code value for accidental eccentricity appears reasonable.

6.5 Closing Remarks

This study has been an attempt to present a reasonable and systematic method for determining explicitly the ground motion effects including both translational averaging and induced rotation on a stiff structure of some size. It should be noted that these studies were restricted to motions in the horizontal plane. If one were to consider vertical motions additional factors which should be considered include: (a) bearing pressures; (b) lift-off; and, (c) need to define approximate shape function for vertical deformations. If the behavior becomes nonlinear the computation effort would become considerably greater than that presented herein.

In closing, the results of this study clearly indicate the need for instrumentation aimed at obtaining a more detailed picture of the translation and rotation experienced by a building undergoing earthquake excitation.

APPENDIX A. Z-TRANSFORM METHOD

This procedure is a recursive relationship (Eq. A.2) in the time domain for the elastic response of a single-degree-of-freedom oscillator to arbitrary base motion. Stagner and Hart (Ref. 18) solved the following differential equation of motion for a single-degree-of-freedom oscillator subjected to ground motion,

$$\ddot{x}(t) + 2\beta\omega_n\dot{x}(t) + \omega_n^{\,2}x(t) = -a(t) \qquad (A.1)$$

where β is the critical damping ratio, ω_n is the undamped natural frequency of the oscillator and a(t) is the ground acceleration. From this they derive the standard continuous Laplace transform S-plane representation of the ground motion -- oscillator displacement response relationship, and express this Laplace S-plane representation in terms of an equivalent sampled function Z-plane frequency representation. Then recognizing the parallelism between the Z-plane bilinear transform and finite difference operators Stagner and Hart derived the following recursive relation in the time domain for computing oscillator response,

$$\begin{aligned} x(j) = k_1^2/B_0\ \{a(j) + 2a(j-1) + a(j-2) + \\ + \frac{B_1}{B_0}\,x(j-1) + \frac{B_2}{B_0}\,x(j-2)\} \end{aligned} \qquad (A.2)$$

where

$$K_1 = \tan\,(\omega_n\Delta t/2)/\omega_n$$

$$K_2 = \tan\,(\omega_n\Delta t/2)$$

$$B_0 = 1 + 2\beta k_2 + k_2^2$$

$$B_1 = 2\ (k_2^2 - 1)$$

$$B_2 = 1 - 2\beta k_2 + k_2^2$$

Δt = sampling rate (i.e., spacing of acceleration data points in the time domain)

Calculations using this procedure give results that are very close to those obtained using other numerical integration techniques; however, use of the Z-transform method normally provides a substantial computational savings.

REFERENCES

1. Yamahara, H., "Ground Motions during Earthquakes and the Input Loss of Earthquake Power to an Excitation of Buildings," Soils and Foundations, V. 10, N 2, pp. 145-161, Tokyo, 1970.

2. Scanlan, R. H., "Seismic Wave Effects on Soil-Structure Interaction," Earthquake Engineering and Structural Dynamics, V. 4, pp. 379-388, 1976.

3. Luco, J. E., "Torsional Response of Structures to Obliquely Incident Seismic SH Waves," Earthquake Engineering and Structural Dynamics, V. 4, N 3, 1976.

4. Newmark, N. M., W. J. Hall and J. R. Morgan, "Comparison of Building Response and Free-Field Motion in Earthquakes," Proceedings 6th World Conference on Earthquake Engineering, New Delhi, India, V. 11, Topic 3 pp. 972-978, 1977.

5. Veletsos, A. S., M. O. Erdik and P. T. Kuo, "Response of Structures to Propagating Ground Motions," Proceedings 5th European Conference on Earthquake Engineering, Istanbul, Turkey 1975.

6. Whitley, J. R., J. R. Morgan, W. J. Hall and N. M. Newmark, "Base Response Arising from Free-Field Motion," Transactions 4th International Conference on Structural Mechanics in Reactor Technology, San Francisco, California, Paper K 2/15, in V. K(a), 10 pp., August 1977.

7. Ray, D. and D. P. Jhaveri, "Effective Seismic Input through Rigid Foundation Filtering," Transactions 4th International Conference on Structural Mechanics in Reactor Technology, San Francisco, California, Paper K 2/13, in V. K(a), 1 p., August 1977.

8. Werner S. D., L. C. Lee, H. L. Wong and M. D. Trifunac, "An Evaluation of the Effects of Traveling Seismic Waves on the Three-Dimensional Response of Structures," Report by Agbabian Associates, October 1977.

9. Hall, W. J., J. R. Morgan and N. M. Newmark, "Traveling Seismic Waves and Structural Response," Proceedings 2nd International Conference on Microzonation, San Francisco, California, V. III, pp. 1235-1246, 1978.

10. Newmark, N. M., "Torsion in Symmetrical Buildings," Proceedings 4th World Conference on Earthquake Engineering, Santiago, Chile, Vo. II, pp. A3-19 to A3-32, 1969.

11. Ayre, R. S., "Interconnection of Translational and Torsional Vibrations in Buildings," Bulletin of the Seismological Society of America, V. 28, pp. 89-130, 1938.

12. Ayre, R. S., "Experimental Response of an Asymmetric, One-Story Building Model to an Idealized, Transient Ground Motion," Bulletin of the Seismological Society of America, V. 33, N 2, April 1943.

13. Caughey, T. K., "Classical Normal Modes in Damped Linear Dynamic Systems," Journal of Applied Mechanics, pp. 269-271, June 1960.

14. Caughey, T. K. and M. E. J. O'Kelly, "Classical Normal Modes in Damped Linear Dynamic Systems," Journal of Applied Mechanics, pp. 583-588, September 1965.

15. "Analyses of Strong Motion Earthquake Accelerograms, Vol. II, Corrected Accelerograms and Integrated Ground Velocity Displacement," California Institute of Technology, Earthquake Engineering Laboratory, Pasadena, California, 1971 through 1975.

16. Pecknold, D. A. W., and R. Riddell, "Effect of Initial Base Motion on Response Spectra," Journal of the Engineering Mechanics Division, ASCE, V. 104, No. EM 2, April 1978, pp. 485-491, Technical Note.

17. Newmark, N. M., J. R. Morgan and W. J. Hall, "Response Spectra of Combined Translation and Torsion for a Traveling Seismic Wave," Unpublished Report to the Office of Nuclear Regulatory Research, U.S. Nuclear Regulatory Commission, September 1978.

18. Stagner, J. R. and G. C. Hart, "Application of the Bilinear Z-Transform Method to Ground Motion Studies," Bulletin of the Seismological Society of America, V. 60, N 3, pp. 809-817, June 1970.

19. Duke, C. M., J. E. Luco, A. R. Carriveau, P. J. Hradilek, R. Lastrico, and D. Ostrom, "Strong Earthquake Motion and Site Conditions: Hollywood," Bulletin of the Seismological Society of America, V. 60, No. 4, pp. 1271-1289, 1970.

20. Hall, W. J., B. Mohraz, and N. M. Newmark, "Statistical Analyses of Earthquake Response Spectra," Transactions 3rd International Conference on Structural Mechanics in Reactor Technology, London, England, September 1975.

21. Langhaar, H. L., Energy Methods in Applied Mechanics, John Wiley & Sons, 1962.

22. "Recommended Lateral Force Requirements and Commentary," Seismology Committee, Structural Engineers Association of California, 1975.

23. "Tentative Provisions for the Development of Seismic Regulations for Buildings," Applied Technology Council, Report No. ATC 3-06, June 1978.

24. "Uniform Building Code," International Conference of Building Officials, 1976.