

AXISYMMETRIC BUCKLING OF BURIED PIPELINES
BY SEISMIC EXCITATION

by

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ABSTRACT

A quasi-bifurcation theory of dynamic buckling and a simple flow theory of plasticity are employed to analyze the axisymmetric, elastic-plastic buckling behavior of buried pipelines subject to seismic excitations. Using the seismic records of the 1971 San Fernando earthquake, a series of numerical results have been obtained, which show that, at strain rates prevalent in earthquakes, the dynamic buckling axial stress or strain of a buried pipe is only slightly higher than that of static buckling.

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INTRODUCTION

It has been observed (1) that buried pipelines may buckle by earthquakes. A buried pipe is not likely to buckle in a beam-column bending mode because it is restrained by the surrounding soil, except where transverse ground movement or faulting occurs. The available records of earthquake damage to pipelines (1) show that a pipe may buckle locally in an essentially axisymmetric buckling mode of a thin shell. It is the purpose of this paper to analyze such dynamic buckling.

The real physical environment of a buried pipeline may be very complex, especially when undergoing seismic excitations. To simplify the problem, it is assumed that the pipe is surrounded by a homogeneous and isotropic, elastic soil medium which may develop only a radial pressure on the pipe. The pipe itself is made of an elastic-plastic material having constitutive relationships which may be predicted by the simple flow theory of plasticity. The actual ground motion due to propagating seismic waves may also be complex and vary with time in amplitude as well as in direction. The motions at different points of the ground at a given time, are obviously not identical. The difference between the ground motions at two points of a pipe segment may produce stresses and strains in the pipe. A long pipe is laterally flexible but stiff in the axial direction; hence it may develop a comparatively high axial stress produced by the axial components of an average amplitude of the ground motions. Therefore, only the effects of axial components of the ground motions on the buckling of a pipe are considered.

It is also to be noted that seismic waves propagate in a soil medium at a wave speed of an order of about 700 m/sec depending on the soil stiffness and density (2). The longitudinal wave speed of a steel or iron pipe is about 5,000 m/sec. The higher wave speed allows the pipe to match the motion of

the surrounding soil quickly. In other words, it is assumed that the axial strain and strain rate in the pipe and that of the surrounding soil are identical. The wavelength of ground motion at the Rayleigh - wave speed of 700 m/sec is about 140 meters (2). Away from an epicenter, the seismic waves are generally continuous and do not have strong discontinuities typical in shock waves. Therefore, it is further assumed that, in a relatively short segment of pipe, the strain and strain rate are constant along the length but may vary with time.

Buried pipes used in natural gas, water and oil piping network systems are usually made of steel or ductile iron and of dimensions having radius over thickness ratios of less than 60. Such a cylindrical shell under axial compression may buckle statically in the plastic range, and predominately in an axisymmetric mode (3,4). Under a relatively low rate of loading, the shell buckles dynamically also in an axisymmetric mode. The dynamic buckling process is determined by a recently developed quasi-bifurcation theory (5,6), which is briefly described in the paper. Numerical results for a number of ductile iron pipes subject to the 1971 San Fernando earthquake are presented and discussed.

KINEMATICS

Consider a cylindrical shell of uniform thickness \bar{h} and length \bar{L} . Let \bar{x} , \bar{z} , θ denote the axial, radial and circumferential coordinates of the undeformed middle surface of radius R . The uniform axial and radial motions of the middle surface before buckling may be expressed in terms of the time-dependent axial and circumferential strains $e_x(t)$ and $e_\theta(t)$ respectively. The perturbed axisymmetric motion may be expressed in terms of the additional axial and radial displacements of a point on the middle surface of the shell, $\bar{u}(\bar{x}, t)$ and $\bar{w}(\bar{x}, t)$, respectively. \bar{z} and \bar{w} are positive radially inward.

The following assumptions usually considered in the theories of thin shells are made:

(a) The displacements are small in comparison with the length or radius of the cylinder, but may be of magnitudes comparable to the thickness;

(b) The stresses in the radial direction are negligible and lines originally normal to the middle surface of the shell remain so after deformation; and

(c) The effects of rotational inertia of the shell element on the motion are negligible.

To simplify the subsequent analysis, the following dimensionless quantities are introduced:

$$x = \frac{\bar{x}}{R}, \quad z = \frac{\bar{z}}{R}, \quad u = \frac{\bar{u}}{R}, \quad w = \frac{\bar{w}}{R}, \quad h = \frac{\bar{h}}{R}, \quad L = \frac{\bar{L}}{R}$$

The additional axial and circumferential strains may be shown to be:

$$\begin{aligned} \epsilon_x &= u_{,x} - zw_{,xx} \\ \epsilon_\theta &= -w \end{aligned} \tag{1}$$

STRESS-STRAIN RELATIONSHIPS

The simple flow theory of plasticity (4) is employed to describe the stress-strain relationships. The strain rate tensor, $\dot{\epsilon}_{KL}$, may be considered as the sum of the elastic and plastic parts. For an isotropic and homogeneous material, the elastic strain rate tensor is given by

$$\dot{\epsilon}_{KL}^e = \frac{1}{E} \{ (1+\nu) \dot{S}_{KL} - \nu \dot{S}_{MM} \delta_{KL} \} \tag{2}$$

where E and ν are, respectively, Young's modulus and Poisson's ratio, S_{KL} is the stress tensor, δ_{KL} is the Kronecker delta and repeated indices denote summation over the range of values of the index. The plastic strain rate tensor is given by

$$\begin{aligned} \dot{\epsilon}_{KL}^P &= \frac{3}{4J_2} \left(\frac{1}{E_T} - \frac{1}{E} \right) S_{KL}' \dot{J}_2 \quad \text{when } \dot{J}_2 > 0 \\ \dot{\epsilon}_{KL}^P &= 0 \quad \text{when } \dot{J}_2 \leq 0 \end{aligned} \quad (3)$$

where

$$\begin{aligned} S_{KL}' &= S_{KL} - \frac{1}{3} \delta_{KL} S_{MM} \\ J_2 &= \frac{1}{2} S_{KL}' S_{KL}' \end{aligned}$$

and E_T , a function of J_2 , is the tangent modulus obtained from the uniaxial stress-strain curve.

It is assumed that the cylindrical shell has simply-supported edges and is surrounded by a homogeneous soil medium behaving as distributed radial springs with a spring constant k N/m/m². The outer surface of the shell is smooth and it is subjected to an internal pressure p . It is further assumed that the shell is subjected to axial seismic excitations which may be expressed in terms of the axial strain rate $\dot{\epsilon}_x(t)$

$$\dot{\epsilon}_x(t) = \frac{\dot{U}_2(t) - \dot{U}_1(t)}{d} \quad (4)$$

where \dot{U}_2 and \dot{U}_1 are the axial velocities of two points on the pipelines at a distance d apart. The axial displacement $U_1(t)$ may be that from a seismic record. If the travel time of an undistorted signal from point 1 to point 2 in the soil medium is denoted by t^* , we may have

$$U_2(t) = U_1(t-t^*) \quad (5)$$

Employing eqs. (2) and (3), the average axial and circumferential stress rates, $\dot{S}_x(t)$ and $\dot{S}_\theta(t)$, accompanying the prebuckling, uniform axial and radial motions of the shell caused by seismic excitations may be expressed as

$$\dot{S}_x = \frac{C_{12} \dot{p}}{\kappa + hC_{22}} + \left(C_{11} - \frac{C_{12}C_{21}h}{\kappa + hC_{22}} \right) \dot{\epsilon}_x^E \quad (6)$$

$$\dot{s}_\theta = \frac{\dot{p}}{h} - \frac{\kappa}{\kappa + hC_{22}} \left(\frac{\dot{p}}{h} - C_{12} \dot{\epsilon}_x E \right) \quad (7)$$

where

$$\begin{aligned} \kappa &= \frac{kR}{E} \\ C_{11} &= \{4(1-\xi + \xi^2) + (\lambda-1)(2\xi-1)^2\}/D \\ C_{12} &= \{4\nu(1-\xi + \xi^2) - (\lambda-1)(2-\xi)(2\xi-1)\}/D = C_{21} \\ C_{22} &= \{4(1-\xi + \xi^2) + (\lambda-1)(2-\xi)^2\}/D \\ D &= 4(1-\nu^2)(1-\xi + \xi^2) + (\lambda-1)\{5-8\xi+5\xi^2 - 2\nu(2-5\xi+2\xi^2)\} \\ \xi &= \frac{s_\theta}{s_x} \\ \lambda &= \begin{cases} \frac{E}{E_T} & \text{for } J_2 = J_2^* \text{ and } \dot{J}_2 > 0 \\ 1 & \text{for } J_2 < J_2^* \text{ or } \dot{J}_2 \leq 0 \end{cases} \end{aligned} \quad (8)$$

Here, $J_2^* > s_y^2/3$ is the maximum value of J_2 previously attained and s_y is the initial yield stress from the uniaxial stress-strain curve.

When the uniform motions of shell are perturbed, there may be additional, nonuniform axisymmetric motions, $u(x,t)$ and $w(x,t)$, which lead to additional axial and circumferential stresses $s_x(x,z,t)$ and $s_\theta(x,z,t)$. If the additional motions are relatively small, the total stress path of a shell element follows essentially the stress path of that of the uniform motion in either the loading ($\dot{J}_2 > 0$) or unloading ($\dot{J}_2 \leq 0$) condition. In that case, the additional stresses and strains in the shell have the following relationships:

$$\begin{aligned} s_x &= E(C_{11} \epsilon_x + C_{12} \epsilon_\theta) \\ s_\theta &= E(C_{21} \epsilon_x + C_{22} \epsilon_\theta) \end{aligned} \quad (9)$$

where C_{11} , C_{12} , C_{21} and C_{22} are given by eqs. (8).

STABILITY CONCEPT

A certain motion of a system of n degrees of freedom is considered stable, if after a sufficiently small disturbance, the system remains to follow the undisturbed motion. In other words, the undisturbed motion is stable, if the deviated motion, $\zeta_r(t)$, $r=1, \dots, n$, the difference between the disturbed and undisturbed motions in n generalized coordinates, remains small. When the generalized forces, P_r , of the system are configuration-dependent and when the deviated motion is relatively small, the deviated motion is governed by the following variational equations.

$$\ddot{\zeta}_r = a_{rs} \zeta_s = \Delta P_r, \quad r=1, \dots, n; \quad s=1, \dots, n \quad (10)$$

where the coefficients a_{rs} are known functions of time depending on the undisturbed motion only. The stability of such system may be determined by analyzing the coefficients $a_{rs}(t)$ (5,6). It may be shown that the deviated motion is not bounded when a quadratic term

$$Q = \Delta P_r \zeta_r = \ddot{\zeta}_r \zeta_r = a_{rs} \zeta_r \zeta_s > 0 \quad \text{for } t > t_{cr} \quad (11)$$

is positive, after a critical time t_{cr} , for any possible deviated motion.

Furthermore, the variation of Q yields the following relationship

$$\delta Q = 2a_{rs} \zeta_s \delta \zeta_r = 2\ddot{\zeta}_r \delta \zeta_r \quad (12)$$

The quadratic form Q at a given time can always be reduced to a linear combination of squares such as

$$Q = \alpha_m \eta_m^2, \quad m=1, \dots, n \quad (13)$$

where $\alpha_m(t)$ are the time-dependent eigenvalues. η_m are related to ζ_r by an orthogonal transformation

$$\zeta_r = \ell_{mr} \eta_m \quad (14)$$

where ℓ_{mr} are the directional numbers of n mutually orthogonal vectors in the ζ_r space known as eigenvectors. Such transformation is called a transformation to principal axes. If ζ_r and η_r are normalized, Q assumes the greatest value α_1 in the ζ_r space by a theorem of Weierstrass. If $\alpha_1 > 0$ for $t > t_{cr}$, then the part of the initial disturbance having the mode η_1 grows unbounded. Such phenomenon is called a quasi-bifurcation phenomenon. The above concept and approach are applied to the solution of the present problem.

Of an elastic-plastic solid of a volume V , let the undisturbed motion be described by the displacement $U_K(X_M, t)$ in the X_M spatial coordinate system. The deviated (or additional) displacement is denoted by $u_K(X_M, t)$. The corresponding Lagrangian strains are e_{KL} and ϵ_{KL} and the corresponding Piola-Kirchhoff stresses are S_{KL} and s_{KL} respectively. When u_K is relatively small, the additional strain ϵ_{KL} is given by

$$\epsilon_{KL} = \frac{1}{2}(u_{K;L} + u_{L;K} + U_{;K}^M u_{M;L} + U_{M;L} u_{;K}^M) \quad (15)$$

and the deviated motion is governed by

$$\Delta P_M = \{S_{M;L}^{KL} u_{M;L} + s_{M;L}^{KL} (\delta_{ML} + U_{M;L})\}_{;K} = \rho \ddot{u}_M \quad (16)$$

where ρ is the initial mass density. The deviated motion satisfies the following boundary conditions: $u_K(X_M, t) = 0$ on that part of the boundary with prescribed kinematic conditions and

$$\{S_{M;L}^{KL} u_{M;L} + s_{M;L}^{KL} (\delta_{ML} + U_{M;L})\} N_K = 0 \quad (17)$$

on that part of the boundary with prescribed surface forces, where N_K is the outward unit normal to the surface. The quadratic term for the elastic-plastic continuum is given by

$$Q(u_K) = \int_V \Delta P_M u_M dV \quad (18)$$

For a body which has no interior discontinuities of the variables, the quadratic term may be written as

$$Q(u_K) = - \int_V (S^{KL} u_{M;K} u_{M;L} + s^{KL} \epsilon_{KL}) dV \quad (19)$$

The motion and its stability of the solid may be determined by analyzing the functional $Q(u_K)$.

DYNAMIC BUCKLING OF SHELL

Employing eqs. (1), (9) and (19), the quadratic functional Q for the cylindrical shell in axisymmetric motion may be written as

$$Q = -2\pi R^3 E \int_0^L \left[h \left\{ \frac{S}{E} + C_{11} \right\} u_{,x}^2 - 2C_{12} u_{,x} w + \frac{S}{E} w_{,x}^2 + C_{22} w^2 \right] \\ + \left(\frac{S}{E} + C_{11} \right) \frac{h^3}{12} w_{,xx}^2 + \kappa w^2 \Big] dx \quad (20)$$

After integrating by parts and employing the boundary conditions, the variation of Q is given by

$$\delta Q = -4\pi R^3 E \int_0^L \left[h \left\{ - \left(\frac{S}{E} + C_{11} \right) u_{,xx} + C_{12} w_{,x} \right\} \delta u \right. \\ \left. + \left\{ \left(\frac{S}{E} + C_{11} \right) \frac{h^3}{12} w_{,xxxx} - \frac{S}{E} h w_{,xx} + C_{22} h w + \kappa w \right. \right. \\ \left. \left. - C_{12} h u_{,x} \right\} \delta w \right] dx \quad (21)$$

From eqs. (12) and (21), the axial deviated motion of the shell is found to be governed by

$$u_{,\tau\tau} = h \left\{ \left(\frac{S}{E} + C_{11} \right) u_{,x} - C_{12} w \right\}_{,x} \quad (22)$$

where τ is a dimensionless time given by

$$\tau = \sqrt{\frac{E}{\rho h R^2}} t \quad (23)$$

If the effects of non-uniform axial waves on the lateral motion of the shell are negligible, the axial deviated acceleration may be very small and taken as nil. Observing that S_x/E is small in comparing with C_{11} , the term at the right side of eq. (22) leads to the expression

$$s_{x,x} = E(C_{11}u_{,x} - C_{12}w)_{,x} = 0 \quad (24)$$

In order to match the undisturbed motion at the boundaries, $s_x = 0$. Thus, eq. (24) yield the following relationship.

$$u_{,x} = \frac{C_{12}}{C_{11}} w \quad (25)$$

Substituting eq. (25) into eq. (20) and omitting second ordered terms, the functional Q may be reduced to

$$Q = -2\pi R^3 E \int_0^L \left\{ \frac{S_x}{E} h w_{,x}^2 + \frac{C_{11} h^3}{12} w_{,xx}^2 + \frac{(C_{11}C_{22} - C_{12}^2) h w^2}{C_{11}} + \kappa w^2 \right\} dx \quad (26)$$

When the shell is in a state of uniform loading or unloading, i.e. C_{11} , C_{22} and C_{12} are constants along the length of the shell segment, the functional Q may be further simplified by the following transformation:

$$w = \sum_{m=1}^{\infty} \eta_m(t) \sin \frac{m\pi x}{L} \quad (27)$$

Substitution of eq. (27) into eq. (26) yields

$$Q = (\pi R^3 L E) \alpha_m \eta_m^2 \quad (28)$$

where

$$\alpha_m = - \left\{ \frac{S_x(t) h}{E} \left(\frac{m\pi}{L}\right)^2 + \frac{C_{11} h^3}{12} \left(\frac{m\pi}{L}\right)^4 + \frac{(C_{11}C_{22} - C_{12}^2) h}{C_{11}} + \kappa \right\} \quad (29)$$

The variation of the functional Q with respect to a particular η_m leads to

the following set of uncoupled equations of motion

$$\frac{d^2 \eta_m}{d\tau^2} = \alpha_m \eta_m, \quad m=1,2,3,\dots \quad (30)$$

where the underscores are placed under the indices to suspend the summation.

Equation (30) as well as the functional Q in (28) shows when any one of the α_m becomes positive, that particular η_m mode of motion may grow and the undisturbed motion may be unstable. In fact, eq. (30) shows that the radial displacement will grow when $\alpha_m = 0$, the axial stress corresponding to $\alpha_m = 0$ is a static eigenvalue given by

$$S_x = - \frac{\frac{C_{11} h^3}{12} \left(\frac{m\pi}{L}\right)^4 + \frac{C_{11} C_{22} - C_{12}^2}{C_{11}} h + \kappa}{\frac{h}{E} \left(\frac{m\pi}{L}\right)^2} \quad (31)$$

where the leading minus sign indicates a compressive stress.

Minimization of S_x in equation (31) with respect to $(m\pi/L)$ yields the static axial buckling stress

$$(S_x)_{cr} = - \frac{E}{\sqrt{3}} \sqrt{(C_{11} C_{22} - C_{12}^2) h^2 + \kappa C_{11} h} \quad (32)$$

The corresponding length of half-waves into which the shell buckles is

$$\frac{L}{m} = \pi \left[\frac{C_{11}^2 h^3}{12 [(C_{11} C_{22} - C_{12}^2) h + C_{11} \kappa]} \right]^{1/4} \quad (33)$$

It is to be noted that the circumferential buckling stress $(S_\theta)_{cr}$ is implicit in the expressions C_{11} , C_{12} and C_{22} in eqs. (32) and (33), which are pertinent only when S_θ is in tension or a comparatively small compressive stress under which the shell will not buckle by the lateral external pressure.

Under dynamic loadings, a particular mode of motion corresponding to the algebraically largest value of $\alpha_m(t)$ grows instantaneously at the highest rate. When the largest α_m changes from negative to positive, the corresponding

mode of motion changes from an oscillatory to a divergent motion. The time, t_{cr} , at which the largest $\alpha_m = 0$ indicates that the static buckling stress or strain has been reached and a quasi-bifurcation of motion begins, i.e. the critical mode of motion begins to be divergent. To determine the dynamic behavior of a shell subjected to a complex loading history, it is necessary to integrate eq. (30) by a numerical procedure. A computer program based on the fourth order Runge-Kutta method has been prepared for this purpose. A score of cases with various combinations of parameters have been numerically analyzed and are described as follows.

NUMERICAL RESULTS

Numerical results have been obtained by the foregoing procedure for a number of ductile iron pipes (7) having the following common properties: $\bar{L} = 12.19$ m, $R = 0.6096$ m, $E = 1.6547 \times 10^{11}$ Pa, $S_y = 3.3095 \times 10^8$ Pa. The uniaxial stress-strain curve of ductile iron may be idealized by a bi-linear representation having a constant tangent modulus $E_T = 3.3095 \times 10^9$ Pa. The mass density of the pipe together with an added mass density of the surrounding soil is taken as $\rho = 10.629$ g/cm³. All the pipes have a common thickness of $\bar{h} = 1.2954$ cm, except for Case 7062, $\bar{h} = 1.6510$ cm. All the pipes are buried in a common soil medium with an elastic constant $k = 190.10$ N/cm³, except for Case 3064, $k = 54.29$ N/cm³ and for Case 4070, $k = 271.45$ N/cm³. All the pipes are subjected to no internal pressure, except for Case 5065, $p = 6.8948 \times 10^5$ Pa, $\dot{p} = 0$. All the pipes are assumed to have the simply supported edge conditions and they are subjected to identical seismic excitations.

In order to make a realistic application, the horizontal displacement records shown in Fig. 1 of the San Fernando earthquake of February 9, 1971 in the east-west direction in the 6400th block of Sunset Boulevard, Los Angeles

were utilized. The distance between Station 6464 and Station 6430 is about 50 meters. Within the 20-second duration of the records the notable axial strains obtained by eq. (4) occur in the time interval between 9 and 13 seconds. It has been found that the maximum strain is less than 2×10^{-4} and the maximum strain rate less than 2×10^{-3} /sec which agree with that of other records in the literature (2). It has also been found that, under such excitation, the radial oscillatory motion of any one of the pipes considered remains small in the entire duration and of an amplitude of an order of the given initial disturbance: $\eta_m(0) = 1 \times 10^{-6}$ and $\dot{\eta}_m(0) = 0$ for any mode m . This agrees with the fact that no pipe was damaged in the Sunset Boulevard region, about 15 km from the epicenter, during the San Fernando earthquake. However, near the epicenter, there were many damages to pipes which would be subjected to higher strain and strain rates. The magnitude of the San Fernando earthquake was 6.6 in Richter scale. The largest known earthquake in the world had a magnitude of 8.8. It is conceivable that buried pipes may be subjected to seismic excitations 100 times stronger than that shown in Fig. 1. Therefore, the records shown in Fig. 1 were magnified 100 times in displacement amplitude and used as input excitation. The corresponding typical results are shown in Figs. 2, 3 and 4.

Figure 2 shows the time variation of the amplitude of the critical mode of radial motion of Case 1068 and that of Case 7062. For a case, such as Case 1068, the critical mode, $m=68$, is identified by the last two digits. t_{cr} indicates the time when the static buckling stress is reached. The results show when the axial stress is approaching to the static buckling stress, the frequency of oscillation is decreasing and when the axial stress exceeds the static buckling stress, the motion becomes divergent. The dynamic buckling stress is somewhat higher than the static buckling stress depending on the

strain rate and other factors. Figure 2 shows that the thicker shell, Case 7062 has a higher buckling stress and longer spatially buckling wave length.

Figure 3 shows that the pipe denoted by Case 5065 and subjected to an internal pressure of $p = 6.948 \times 10^5$ Pa has a higher static as well as dynamic buckling stress in comparing with that of Case 1068 having $p=0$. Figure 4 shows that a soil medium having a larger value of elastic constant k tends to stiffen a pipe and increase its buckling stress. Each of the pipes considered is subjected to an average axial strain rate of about .1/sec and buckled under an axial strain within a range of 0.002 to 0.02 in the plastic range. The initial uniaxial yield strain is 0.002. The dynamic axial compressive stress at $\eta_m = 8 \times 10^{-4}$ and the static buckling stress at t_{cr} of each case are given respectively in 1×10^8 Pa in the parentheses as follows: Case 1068 (3.568, 3.500), Case 3064 (3.346, 3.310), Case 4070 (3.725, 3.650), Case 5065 (3.718, 3.562) and Case 7062 (4.433, 4.215). The average ratio of the two stresses is 1.029. The results show that at a strain rate prevalent in earthquakes; a buried pipe buckles dynamically under an axial compressive stress or strain practically identical to the static buckling stress or strain.

CONCLUDING REMARKS

Figures 2, 3 and 4 show only the growth of a critical mode of radial motion of each pipe. In reality, there may be a number of modes of motion which may be excited by the initial disturbances, growing at rates less than that of the critical. Furthermore, a pipe may have local imperfections and surrounded locally by softer soil which allow the pipe to develop a local bulge. If the variations in the various parameters caused by the imperfections are small, the bulge would develop at an axial stress close to that predicted

by eq. (32) and with a half-wave length by eq. (33). In using the stress-strain relationships based on the simple flow theory of plasticity, a somewhat higher predicted buckling stress (4) in comparing with that of the deformation theory or experimental results is usually obtained. However, of a material having a large λ value, the difference can be shown not significant. Therefore, the axisymmetric, elastic-plastic buckling behavior of buried pipes subject to seismic excitations can be reasonably predicted by the analytical approach presented in this paper.

It is to be noted that a number of papers appeared recently on the behavior of a buried pipe undergoing seismic excitation (9). Most of the papers have modelled the system as a beam on a visco-elastic foundation. However, it is known that two of the observed failure modes in buried pipelines under seismic excitations are buckling and fracture (10). Therefore it is also the intent of this paper to extend the model of a buried pipe as a thin circular cylindrical shell in a resisting soil medium (9) to the buckling analysis.

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REFERENCES

- (1) BENFER, N. A. and COFFMAN, J. L.; Editors, 'San Fernando, California Earthquake of Feb. 9, 1971,' U. S. Department of Commerce, Washington, D. C., 1974.
- (2) O'ROURKE, M. I., SINGH, S. AND PIKUL, R., 'Seismic Behavior of Buried Pipelines,' Lifeline Earthquake Engineering - Buried Pipelines Seismic Risk and Instrumentation, Edited by Ariman, T., Liu, S. C., and Nickell, R. E., 1979, 49, PVP-34, ASME, New York.

- (3) LEE, L. H. N., 'Inelastic Buckling of Initially Imperfect Cylindrical Shells Subject to Axial Compression,' J. Aerospace Sci., 1962, 29, 87.
- (4) MURPHY, L. M. and LEE, L. H. N., 'Inelastic Buckling Process of Axially Compressed Cylindrical Shells Subject to Edge Constraints,' Int. J. Solid Structures, 1971, 7, 1153.
- (5) LEE, L. H. N., 'Quasi-bifurcation in Dynamics of Elastic-Plastic Continua,' J. Appl. Mech., 1977, 44, 413.
- (6) LEE, L. H. N., 'On Dynamic Stability and Quasi-Bifurcation,' Proceedings of 16th Annual Meeting of Society of Engineering Science, 1979, 208 (Northwestern University, Evanston, Illinois).
- (7) 'Handbook: Ductile Iron Pipe, Cast Iron Pipe,' Cast Iron Pipe Research Association, Fifth Edition, Oak Brook, Illinois, 1978.
- (8) 'Listing of 329 Earthquake Records, Volume II Part A Through Y,' Earthquake Engineering Research Center, University of California, Berkeley, California, 1978.
- (9) MULESKI, G. E., ARIMAN, T., and AUMEN, C. P., 'A Shell Model of a Buried Pipe in a Seismic Environment,' J. Press. Vess. Tech., 1979, 101, 44.
- (10) ARIMAN, T., and MULESKI, G. E., 'A Review of the Response of Buried Pipelines under Seismic Excitations,' Lifeline Earthquake Engineering - Buried Pipelines, Seismic Risk and Instrumentation, 1979, PVP-34, ASME, New York.

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- Fig. 4 Displacement-Time Relationships.

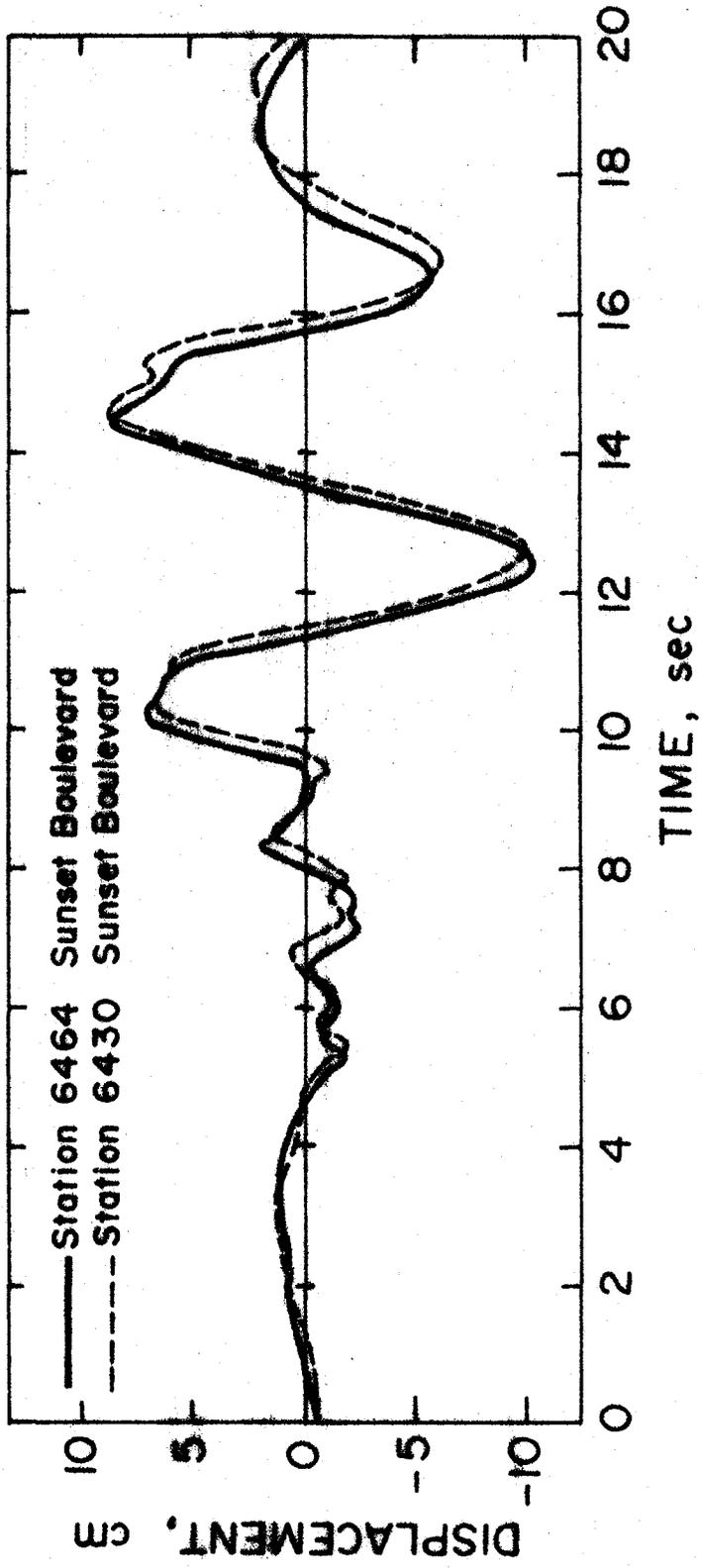


Fig. 1. Horizontal Displacement Records of 1971 San Fernando Earthquake.

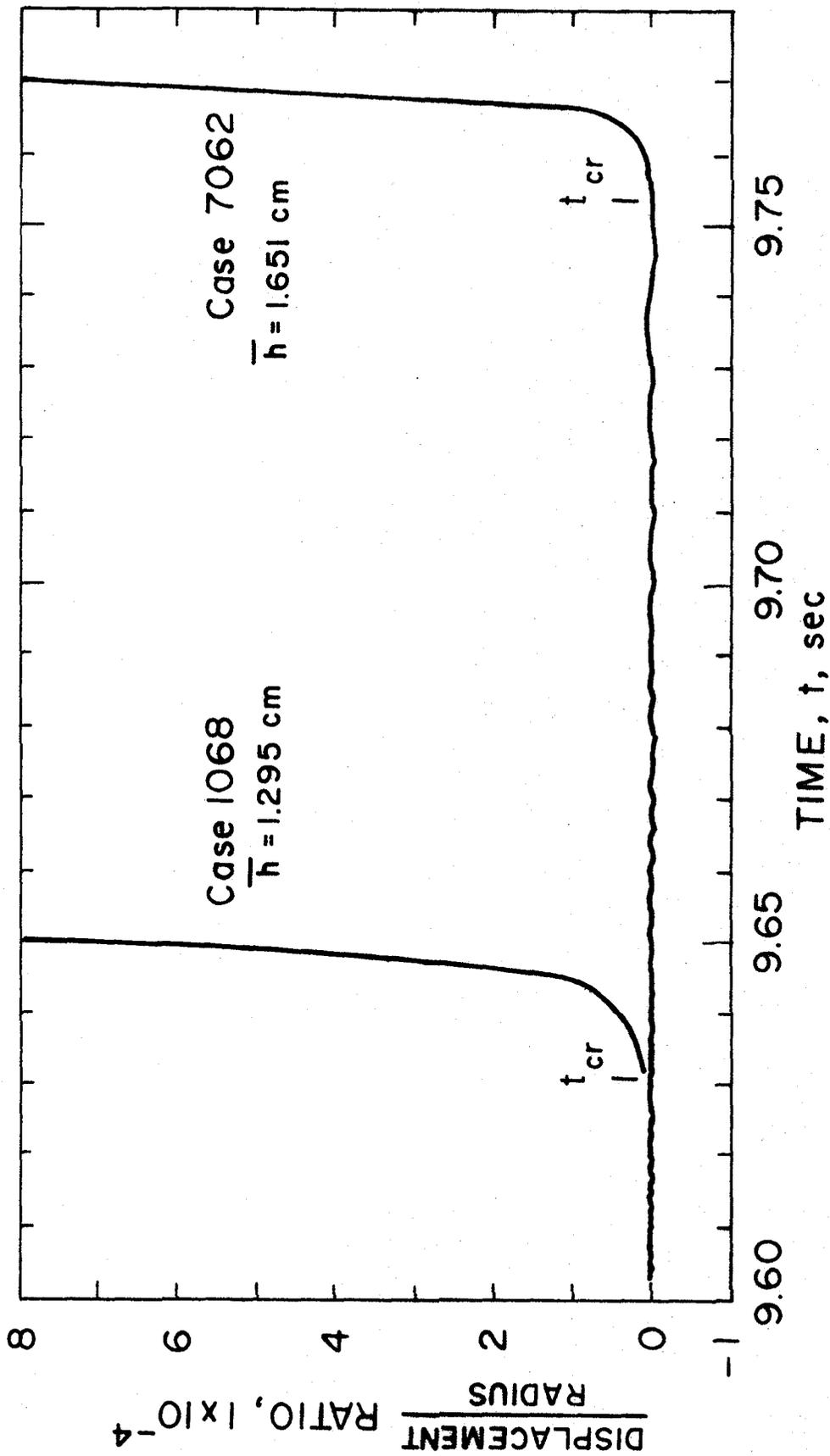


Fig. 2 Displacement-Time Relationships.

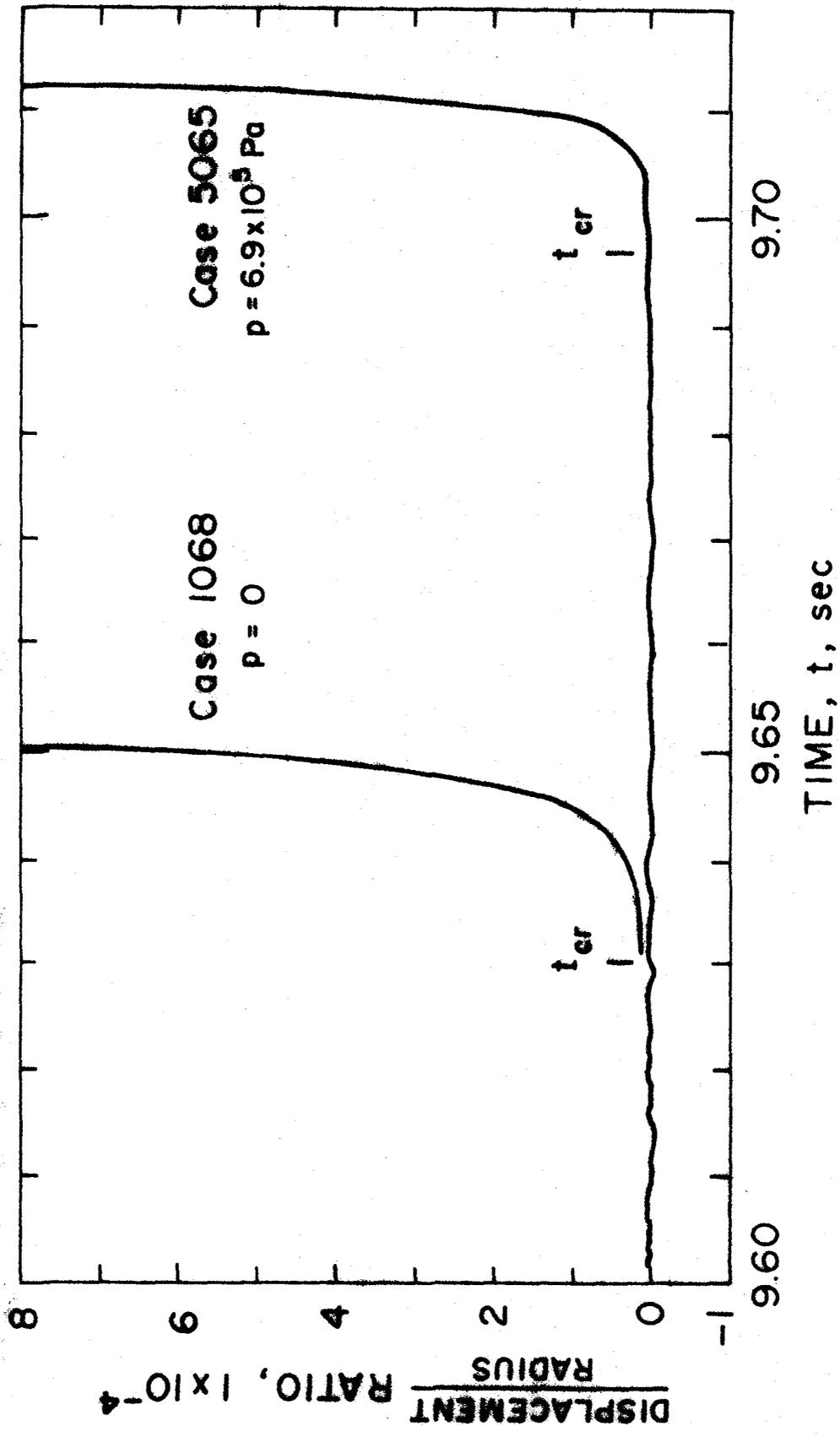


Fig. 3 Displacement-Time Relationships.

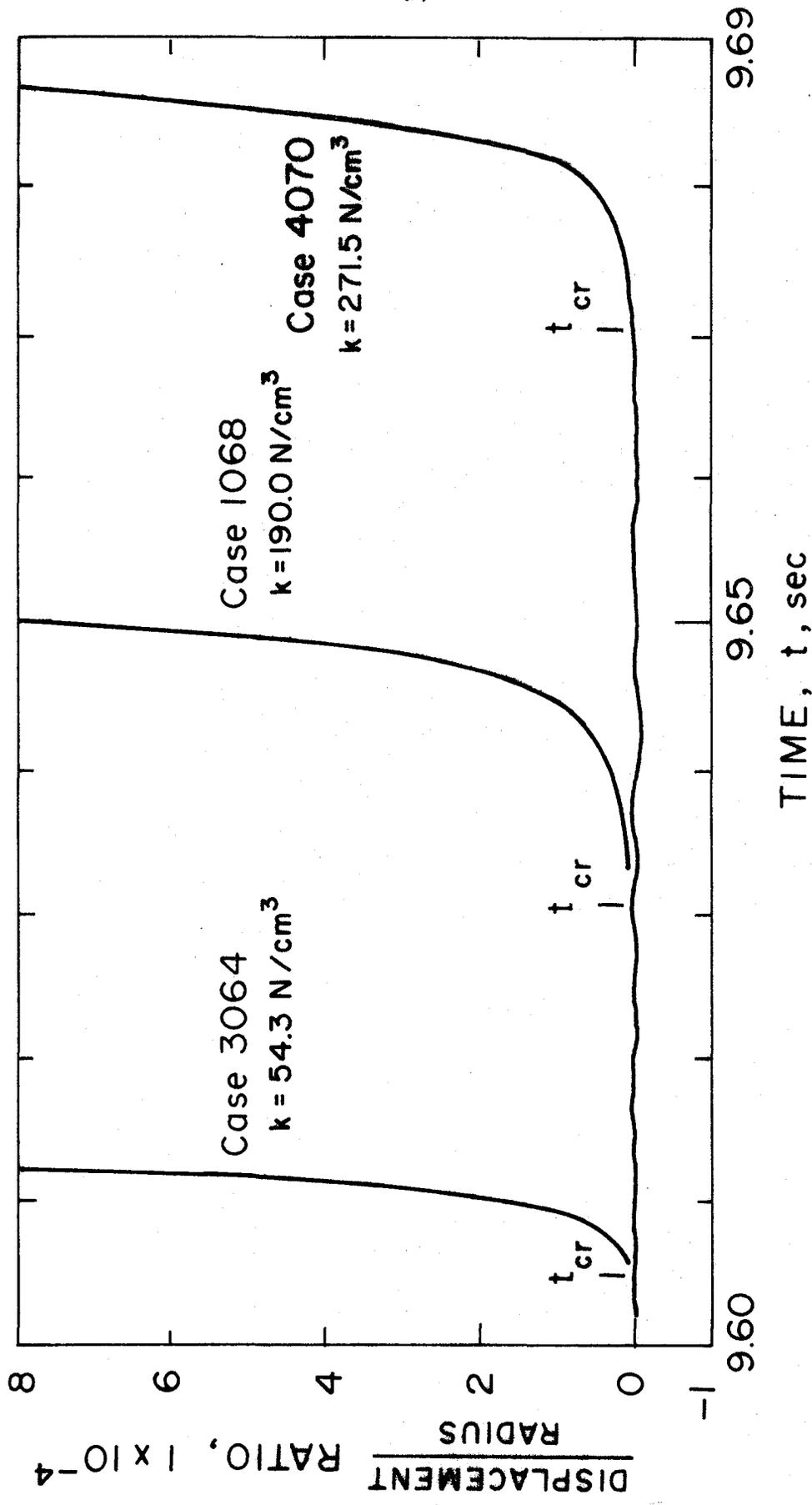


Fig. 4 Displacement-Time Relationships

