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FINAL REPORT

PREDICTION OF EARTHQUAKE RESISTANCE OF STRUCTURES

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The prediction of structural resistance to earthquakes provides the focus of this report. Particular attention was paid to developing an upper bound or critical ground excitation for a structure of major importance, so that a high level of confidence in the prediction of structural resistance may be achieved. The term critical excitation is defined here as the one among a class of credible excitations at a given site that will produce the maximum response peak for a given structural design variable in question. Both linear and nonlinear structures were considered. To verify its practicality,the method was applied to several nuclear reactor structures. The results show that the method is conservative but not overly so. The responses obtained from the critical excitation are in the range of 1.1 to 2 times those produced by recorded earthquakes of the same intensity. To further enhance the practical applications, response spectra for different soil conditions were also produced.

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ABSTRACT

This report deals with the problem of prediction of earthquake resistance of structures. Particular attention was paid to developing an upper bound or critical ground excitation for a structure of major importance, so that a high level of confidence in the prediction of structural resistance may be achieved. The term critical excitation is defined here as the one among a class of credible excitations at a given site that will produce the maximum response peak for a given structural design variable in question. Both linear and nonlinear structures were considered. To verify its practicality the method was applied to several nuclear reactor structures. The results show that the method is conservative but not overly so. The responses obtained from the critical excitation is in the range of 1.1 to 2 times those produced by recorded earthquakes of the same intensity. To further enhance the practical applications, response spectra for different soil conditions were also produced.

1. INTRODUCTION

1.1 Background

The prediction of earthquake resistance of structures is an engineering procedure of decision-making based on uncertain or incomplete informations. Ideally, a person who is responsible for devising such a procedure is expected to arrive at a decision on a high confidence level that a structure will survive all credible ground motions which can occur at a particular location. For structures of major importance such as nuclear power plants, hospitals, school buildings, or strategic installations, even higher confidence level of safety must be achieved in the design against seismic excitations than those required for conventional structures. In the past, seismic assessment of structures was based on either statistically constructed design spectra or past ground motion records. Artificial time-histories generated from the design spectra were also used. However, it is not clear whether these approaches lead to designs that can be relied upon on the confidence levels that are presumably desired for structures of major importance mentioned above. In such cases it is not unrealistic to rely on the idea of the so called "worst-case analysis".

The present investigation in the prediction of earthquake resistance of structure is essentially a modified version of the "worst-case analysis" which greatly takes advantage of the past experience together with an intelligent prediction of the future events. The method relies on the concept of the critical excitation. The latter is defined as an excitation, among a certain class of excitations, which will produce the largest response peak for a structural design variable of interest. The major difficulty in this approach is the determination of the class of excitations that the critical one must be extracted from. It is reasonable to assume that a realistic class should be chosen in such a way that it includes all ground motions that are credible for the location under consideration, and includes as few others as possible. According to this assumption one should first of all consider, for a given site, all excitations that have already been recorded there or at some other locations with similar site conditions, focal distances, and macro or micro zoning. In addition one may also consider all linear combinations of these ground motions as credible ones.

The second difficulty in connection with this approach is the specification of a limiting intensity of the excitation. Many definitions have been proposed for the measurement of earthquake intensity [1] . A common definition used by engineers is the peak acceleration. Other definitions are the spectral intensity proposed by Housner [2], power spectral density [3] and square integral of the ground acceleration [4]. In the present investigation, partly for the convenience of calculation and partly from comparison studies, the square integral of ground acceleration was used as the intensity measurement. The comparison study was carried out by constructing response spectra using equal intensities defined first by the peak acceleration and then by the square integral of the acceleration history. There were less dispersions of the spectra when the second definition of intensity was used. With the class of excitations and the intensity measures defined, the critical excitation thus produced will excite the highest response peak of a prescribed design variable.

In case one wants to scale the design excitation to a lower level than the critical one, it can easily be done by arranging the class of excitations together with the critical one in a statistical distribution so that desired probabilistic level of a design excitation can be determined.

1.2 Review of Past Works

This investigation under NSF Grant No. PFR 76-14893 is a continuation of the effort initiated in a previous Grant No. AEN 72-00219 for developing a method in the prediction of earthquake resistance of structures based on the idea of "minimax procedure". The method is relatively conservative and thus has the potential of improving the design of socially or economically important structures such as nuclear power plants, hospital buildings or military installations. It is also based on a well founded mathematical formulation and thus will enhance the confidence on the part of the designers.

The original ideas of "critical" excitation or "minimax" procedure was developed on the basis that ground motions encompass all possible excitations with the intensity limited to a prescribed value E. The critical excitation is defined as the one among these excitations that will produce the maximum response peak for a design variable in question. The outcome from this assumption leads to a critical excitation proportional to the time reversed unit impulse response function of the design variable [4, 5]. Previous investigations show that this approach, although simple in concept, generally leads to assessments that are too conservative to be applicable to practical design.

Subsequent modifications were then carried out to improve the applicability of the method. The first attempt was done by way of least squares-fitting of the linear combinations of a selected set of past ground records (basis excitations) with the critical one previously defined. The new excitation is then called the sub-critical excitation which will have the least-squares difference from the critical one. The intensity constraint E is still maintained. By this modification not only the shape of the time history of the excitation appeared to be more realistic but also the over conservativeness of the response was reduced. Previous reports, [6, 7, ,8] show that the results are quite reasonable although inconsistency in the conservativeness may occur occasionally.

The present modification is aimed at improving the credibility of the critical excitation and at the same time to eliminate the occasional irregularties in conservativeness. The basis excitations are selected with special attention paid to the site conditions, epicentral distances, and other pertinent characteristic of a particular site in question. Then the critical excitation is obtained by the linear combination of these basis excitations in such a way so that the design variable in question exhibits the maximum response peak. The intensity limitation is posted as the constraint on the maximization procedure. The excitation thus produced are conservative but credible in all respects in comparison with recorded ground excitation both in the time domain and in the frequency domain.

1.3 Scope of the Investigation

As mentioned in the review of the past work, since the reversed-unit-impulse-response critical excitation and its least-squares fitted sub-critical excitation have drawbacks when applied to the real design applications, the first task in the present investigation is to search for an improved procedure to produce the critical excitation. This was indeed successfully accomplished at the begining of the investigation. Detailed presentations are given in several papers in the appendices, and a summary of the procedure is given in chapter 2. The criterion of selecting basis excitations was then studied in detail. A discussion of this is given also in chapter 2. Using the improved method, critical design response spectra were then constructed. This is presented in detail in Appendix A. Next, the critical excitation method was applied to the evaluation of nuclear reactor structures and the detailed presentation is given in Appendix B. Since inelastic behavior was inevitable in structural responses to strong earthquakes, a detailed discussion of critical excitation for inelastic structures is given in Appendix C and a summary is given in chapter 3. Theoretically, critical excitations can be different for different design variables. However, it is impractical to design a structure based on a great number of individual critical excitations for many design variables in question. In Appendix C methods of producing critical excitation based on the first few free vibration modes are discussed. In addition, Appendix C also presents a method of creating spectral conforming time-history as a single design critical excitation for a particular site. Appendix D is a paper dealing with the effective duration of the critical excitation. Appendix E includes other published papers and reports in connection with this investigation. Finally, computer programs to generate critical excitations are attached to form Appendix F.

2. METHOD OF GENERATING CRITICAL EXCITATION

2.1 Selection of the Basis Excitations

As mentioned in the introduction, critical excitation is created from a specific class of excitations suitable for the design of a particular structure at a particular site. These excitations then form the "basis". The question of how to select the basis excitations plays an important role in the final outcome. It is not precisely known just what characteristics an excitation must, or must not have in order to be considered a realistic (or credible) candidate for the design of a particular structure at a particular site. It is customary to assume that it is possible to set an upper bound E on the ground motion intensity at a given location which is so chosen that its exceedance is too unlikely an event to be taken into account. Several other characteristics are also widely accepted as distinguishing realistic candidates of ground motions from their opposites. Vanmarcke [9], for instance, lists the following:

(a) Duration of strong ground motion

(b) Variation of motion intensity with time

(c) Relative frequency content

(d) Effect of macro-zone, micro-zone, and site-soil

(e) Effect of focal distance

Some of these characteristics are clearly of a rather qualitative nature and it is not obvious how they are best converted into quantitative form. Discussions of this problem and a suggested method of selecting basis excitations were presented in reference [6] . This report is based on a modified and theoretically better founded version of the last. This stems from the following line of reasoning.

First of all consider any excitation to be realistic, that has been recorded at a site or at some other locations with similar soil conditions, focal distances, and marco or micro zoning. In addition , one should presumably admit any other excitation which has the same or similar characteristics (in the sense of the above list) as the basis excitations. Despite the vagueness of the list, it may be safe to say that these characteristics are shared by all linear combinations of the basis excitations, with the proviso that their intensities do not exceed the upper bound E appropriate for that location.

It is not known whether the class of excitations defined in this way comprises all that can be considered "realistic" or "credible". However, at the present state of knowledge regarding seismological disturbances, any further expansion of it seems difficult to justify. In this report, at any rate, the class of "realistic" basis excitations will be defined as just described.

2.2 Effective Duration of Excitation

The intensity measure $||\ddot{X}||$ adopted in this report is based on the square integral of the ground acceleration.

$$||\ddot{X}|| = \left[\int_0^T \ddot{x}_g^{\,2}(t)\, dt \right]^{\frac{1}{2}} \tag{2-1}$$

The limits of the integration defines the effective duration of the excitation. Real ground motions last a few seconds to a few minutes. However, the portion of the shake that influences the response of a particular structure is limited to a strong region of the shake, the duration of which depends on the rigidity and damping of the structure. A detailed discussion of this problem is given in Appendix D. A simple rule of selecting the effective duration based on the fundamental frequency of vibration ω (r.p.s.) and damping ratio β is

$$T = (-\ell n\, C)\,(\beta\,\omega)^{-1} \tag{2.2}$$

where C is the acceptable decay ratio (say 1/5) which indicates the fraction of the maximum value of the unit impulse response peak that can be disregarded with no appreciable error. In practical application and based on computational experience, the following rule was also used

$$4(\text{sec}) \leq \quad T = 8 \times T_1 \leq 80 \;(\text{sec}) \tag{2.3}$$

where T_1 is the fundamental period of the structure and is equal to $\frac{2\pi}{\omega}$.

2.3 Determination of the Critical Excitation

Although the detailed derivation of generating the critical excitation is given in Appendices A, B and C, it is not out of place to summarize it at this point.

In mathematical terminology, the selected class from which the critical excitation is generated lies in a linear manifold spanned by the basis excitations. In symbols, if $\ddot{x}_1(t)$, $\ddot{x}_2(t)$, ... $\ddot{x}_n(t)$ are the ground accelerations that form the basis, the manifold which is spanned by them contains all excitations

$$\ddot{x}(t) = \sum_{i=1}^{n} a_i \ddot{x}_i(t) \tag{2.4}$$

The class of allowable ones among these then includes all whose intensity $||\ddot{x}||$ does not exceed the given maximum E.

If the intensity is defined by (2.1), then the intensity constraint takes the following form

$$||\ddot{x}|| = \left[\int_0^T \ddot{x}^2(t)\, dt \right]^{1/2} < E \tag{2.5}$$

with E being a prescribed value.

The response y(t) of a linear structural system to an excitation in the manifold is

$$y(t) = \int_0^t \ddot{x}(\tau)\, h(t-\tau)\, d\tau = \sum_{i=1}^{n} a_i \int_0^t \ddot{x}_i(\tau)\, h(t-\tau)\, d\tau$$

$$= \sum_{i=1}^{n} a_i\, y_i(t) \tag{2.6}$$

where h(t) is the impulse response function of the design variable under consideration and $y_i(t)$ is its response to a basis excitation $\ddot{x}_i(t)$. The critical excitation $\ddot{x}_c(t)$ in the manifold is now defined as one which drives the response y(t) to its maximum value and which at the same time obeys the intensity constraint (2.5).

The problem of determining a critical excitation is the following. It is required to find an excitation $\ddot{x}_c(t)$ of the form (2.4) which obeys the constraint (2.5) and which drives the response y(t) in (2.6) to its largest peak. If the time t^* at which y(t) reaches its maximum were known, the problem would be that of determining a set of coefficients a_i, $i = 1, 2, \ldots n$, which achieves

$$\max_{a_i} |y(t^*)| = \max_{a_i} \left| \sum_{i=1}^{n} a_i \, y_i(t^*) \right|. \tag{2.7}$$

subject to the constraint

$$E^2 > \int_0^T \ddot{x}^2 (t) \, dt = \sum_{i=1}^{n} \sum_{j=1}^{n} a_i a_j \int_0^T \ddot{x}_i(t) \, \ddot{x}_j(t) \, dt$$

$$= \sum_{i=1}^{n} \sum_{j=1}^{n} a_i a_j \ddot{x}_{ij} \tag{2.8}$$

where

$$x_{ij} = \int_0^T \ddot{x}_i(t) \, \ddot{x}_j(t) \, dt \tag{2.9}$$

However, the time instant t^* is not known. Thus the maximization $y(t)$ must be carried out with respect to time t also, i.e.,

$$\max_t \max_{a_j} |y(t)| = \max_t \max_{a_j} \left| \sum_{i=1}^{n} a_i \, y_i(t) \right| \tag{2.10}$$

must be determined. The maximization with respect to a_i can be carried out by simple linear algebra. However, the one with respect to t can be done only by a numerical evaluation. These two procedures will be described now.

One can, first of all, ignore the absolute value in Eq. (2.10). For, if some set of a_i achieves the positive maximum, the set of $(-a_i)$ yields the negative one, and vice versa. Thus, $y(t)$ is to be maximized for a fixed time t and subject to Eq. (2.8). This can be done by setting

$$0 = \frac{\partial}{\partial a_j} \sum_{i=1}^{n} a_i\, y_i(t) + \lambda \sum_{i=1}^{n} \sum_{j=1}^{n} a_i\, a_j\, \ddot{x}_{ij}$$

$$= y_j(t) + 2\,\lambda \sum_{i=1}^{n} \ddot{x}_{ji} a_i \qquad j = 1,\ 2,\ \ldots\ n \tag{2.11}$$

where λ is a Lagrangian multiplier. This equation can be rewritten as

$$\sum_{i=1}^{n} \ddot{x}_{ji}\, a_i = -\frac{1}{2\lambda}\, y_j(t)\,, \quad j = 1,\ 2,\ \ldots\ n, \tag{2.12}$$

or, more comprehensively, in matrix form, namely

$$\ddot{x}\, a = -\frac{1}{2\lambda} Y(t) \tag{2.13}$$

where a and Y(t) are n-dimensional column vectors with the components a_i and $y_i(t)$, respectively, and $\ddot{x}$ is an n x n-dimensional matrix with the components $\ddot{x}_{ij}$. The solution is thus

$$a = -\frac{1}{2\lambda}\, \ddot{x}^{-1} Y(t) \tag{2.14}$$

where invertibility of $\ddot{x}$ can be shown in the following way. One assumes to start with that the basis excitations are linearly independent. In practice this is almost a matter of course and in theory it merely implies that none of those excitations can be omitted from the basis. Thus,

$$0 < \int_0^T \left[\sum_{i=1}^{n} a_i\, \ddot{x}_i(t)\right] dt = \sum_{i=1}^{n} \sum_{j=1}^{n} a_i\, a_j \int_0^T \ddot{x}_i(t)\, \ddot{x}_j(t)\, dt = \sum_{i=1}^{n} \sum_{j=1}^{n} a_i\, a_j\, \ddot{x}_{ij}\,.$$

The expression on the right can be interpreted as a quadratic form in the variables a_i , and in fact a positive definite form as the inequality shows. It follows that the matrix $\ddot{x}$ is positive definite also, hence has only positive eigenvalues. Since none of the eigenvalues vanish $\ddot{x}$ is invertible.

The Lagrangian multiplier can now be determined from the constraint equation Eq. (2.8) which, written in matrix form, is

$$a^T \ddot{x} a \leq E^2 \tag{2.15}$$

where a^T is the transpose of the vector a. Substituting Eq. (2.13) into Eq. (2.15) gives

$$\frac{1}{4\lambda^2} Y^T(t) (\ddot{x}^{-1})^T \ddot{x} \ddot{x}^{-1} Y(t) = \frac{1}{4\lambda^2} Y^T(t) \ddot{x}^{-1} Y(t) \leq E^2 \tag{2.16}$$

in which use has been made of the fact that $(\ddot{x}^{-1})^T = \ddot{x}^{-1}$ because $\ddot{x}$ is symmetric. Eq. (2.14) implies that the values of a_i as well as y(t) grow larger in magnitude as λ decreases. Thus, y(t) is maximized, for fixed t, if equality prevails in (2.15), i.e., if

$$\lambda = \frac{1}{2E} \left[Y^T(t) \ddot{x}^{-1} Y(t) \right]^{1/2} \tag{2.17}$$

This value, substituted into Eq. (2.14), gives the maximizing coefficients a_i

It remains to carry out the maximization in Eq. (2.10) with respect to the time t. This can be done numerically by subdividing the time period T of interest into subintervals Δt. In computational practice, to achieve a desired accuracy in the results without excessive effort, consideration must be given in the choice of Δt to the natural periods and decay time of the impulse response h(t) of the structural variable as well as to the frequency contents of the basis excitations. In this study, the subinterval Δt was taken as T/200 (i.e., approximately $T_1/20$).

2.4 Practicality of the Critical Excitation

The critical excitation produced according to the method presented above is a "worst-case" or upper bound approach. Its practicality certainly needs verification and it is done by three comparison studies versus real ground excitations:

(a) Comparison of the characteristics of time-histories.

(b) Comparison of the frequency contents of the Fourier Spectra.

(c) Comparison of the response peaks of various design variables.

These studies were carried out in reference [6], Appendix A, B and C. When the time-history and Fourier Spectrum were plotted separately for the critical excitation and for the real ground accelerations, there is no observable difference. The comparison of the peak responses of various design variables was also carried out in the same reference and appendices. The ratios of the peak responses obtained from the critical excitation to that obtained from the real ground accelerations were in the range of 1.1 to 2.0. It can be concluded that the critical excitation is a realistic but somewhat conservative ground motion. When it is applied to structures of social or economical importance it will enhance the confidence reliabity of the structure.

3. CRITICAL EXCITATION OF INELASTIC STRUCTURES

3.1 General Discription

The basic advantages of the critical excitation method is that it produces highly reliable assessments of structural earthquake resistance. Effective computational procedures exist by which such assessments can be derived, provided however the structure under study responds elastically. Evidently, this is unrealistic. Under severe ground shaking structures will exceed their elastic limits, and many are in fact designed to do so. It is accordingly highly desirable to develop computational procedures that are effective also for inelastically vibrating structures but which do not compromise the high reliability of the assessments to which they head.

Two studies exist which seek to achieve this goal. One was reported previously [10], and another in Appendix C of this report. Both extend the idea of the critical excitation into the inelastic domain and provide workable procedures. The first study was aimed at producing a critical excitation with intensity constraint only. The difficulty in regard to this approach is discussed in section 3.2. The second study is based on the equivalent linearlization of the nonlinear system. A brief summary is given in section 3.3 while a detailed description is given in Appendix C.

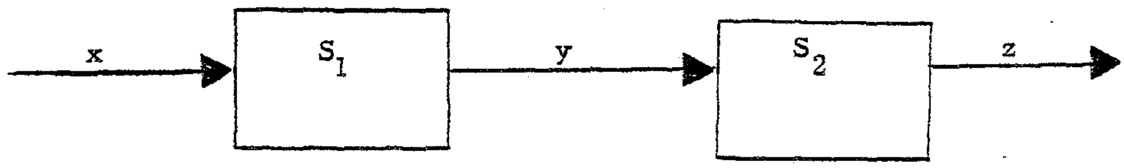
3.2 Discussion of Critical Excitation with Intensity Constraint only.

The first study follows the same path as that of the original idea for the linear structure, that is to determine the critical excitation with intensity constraint only. It did not achieve the desired assessment reliability on the same level as that for the elastic one. The reason for their short fall is fairly deep-seated. One can trace the trouble to the fact that the critical excitation method is based on a fundamental inequality (the Schwarz inequality) which can be exploited only when the structure is elastic. What would be needed therefore is an analogous inequality which applies to inelastic structure. No such inequality is known, to the writers' knowledge. It therefore seemed desirable to explore the possibility of deriving one.

Two avenues were explored towards this objective. As of this writting neither has completely produced results, but neither exploration seems exhaustive. Each can be stated as a conjectured theorem regarding certain very general system properties. They can be described as follows.

1. A Theorem Regarding Random Connections of Systems.

Suppose two systems S_1 and S_2 (not necessarily linear) are connected in series, as shown in the figure, and driven by an input X which is a unit impulse



applied at the time t = 0. The problem is as follows: for a given S_2, how should S_1 be chosen (subject to a certain normalization constraint), in order for the output Z to have the largest response peak at the time t = 0? The following conjecture arises: the system S_1 must obey the same (differential) equation as S_2 except the t in all is replaced with (- t).

This conjecture is valid for linear systems (i.e., elastic structures and the critical excitation method) but its extension to non-linear (inelastic) systems has not yet been proven.

2. A Representation Theorem for Random Processes

The second conjecture seeks to exploit a parallel between the critical excitation method and the response characteristics of systems to white noise inputs. This parallel was pointed out at one time for linear systems [4] but never exploited. The exploitation in the direction of nonlinear systems require the proof of a theorem conjectured to be roughly as follows:

Every random process whose distribution functions $F(x_t \mid X_{t-\epsilon})$, conditioned on the past $X_{t-\epsilon}$, are continuous in x_t for every $\epsilon > 0$, can be generated from white noise by a suitable nonlinear systems, unless the random process has a perfectly predictable component. In that case, only its imperfectly predictable component can be so generated.

A proof of this theorem may have been obtained recently for the simpler case of discrete time. It is fairly abstract however and needs to be carefully reviewed before publication is attempted.

3.3. Critical Excitation of Nonlinear System Replaced by Equivalent Linear System

The critical excitation of nonlinear system replaced by equivalent linear system is presented in Appendix C. However a brief summary is given below.

The typical nonlinear equation of motion of a single degree of freedom oscillator is

$$m\ddot{y} + c\dot{y} + ky + f = x \tag{3.1}$$

The term f is a nonlinear function associated with the nonlinearity of the oscillator and its presence does not allow a straight forward evaluation of its critical excitation for a given basis and reference intensity E. To overcome this, a linearization technique based on the concept of equivalent linearization [11] is employed. According to this approach the nonlinear oscillator is replaced by a linear one in such a way that the difference d between the two oscillators is minimized. The equivalent linear oscillator is:

$$m\ddot{y} + c^{\epsilon}\dot{y} + k^{\epsilon} y = x \tag{3.2}$$

where c^{ϵ} and k^{ϵ} are equivalent damping and stiffness:

$$\begin{cases} c^{\epsilon} = c + c_0 \\ k^{\epsilon} = k + k_0 \end{cases} \tag{3.3}$$

The new parameters c_0, k_0 are time independent but they do depend on the solution y_L of the equivalent linear oscillator as will be domonstrated below. The minimization of the difference d is expressed as:

$$\mathcal{A}\,(d^2) = \min \forall\,(c_0\,,\; k_0) \tag{3.4}$$

where $\forall$ is an averaging operator.

By carrying out the minimization operation, the following equation is obtained:

$$A \cdot \underline{p} = \underline{b} \tag{3.5}$$

where $\underline{p}$ and $\underline{b}$ are 2-vectors as:

$$\underline{p} = \begin{Bmatrix} k_0 \\ c_0 \end{Bmatrix} \qquad \text{and} \qquad \underline{b} = \begin{Bmatrix} \mathcal{A}\,(fy) \\ \mathcal{A}\,(f\dot{y}) \end{Bmatrix} \tag{3.6}$$

and A is a 2 x 2 matrix:

$$A = \begin{bmatrix} \mathcal{A}(y^2) & \mathcal{A}(\dot{y}y) \\ \mathcal{A}(\dot{y}y) & \mathcal{A}(\dot{y}^2) \end{bmatrix} \tag{3.7}$$

Eq. (3.5) assumes the equivalency of the two oscillators (Eqs. 3.1 and 3.2). The existence of the equivalency depends on the existence of the vector p , which in turn depends on the invertibility of A. †

It is clear, in general, the solutions from Eqs. (3.1) and (3.2) will differ. However, any trial value c_0 and k_0 result in trial solution y_L and $\dot{y}_L$ from Eq. (3.2). When they in turn are substituted into Eq. (3.5) a new set of values c_0 and k_0 can be found. Thus by successive computation an equivalent linear system will be obtained.

In order to apply the equivalent linear system for generating the critical excitations and then to construct the nonlinear response spectra, one proceeds in the same way as in the linear case; first select a set of basis excitations x_i's. For a given initial set of c_0 and k_0, equivalent linear critical excitation and $\underline{a}$ vector can be determined. With these prelininary values of vector $\underline{a}$, the A matrix in Eq. (3.7) is determined as follows:

$$A_{11} = \mathcal{A}(y^2) = \sum_{i=1}^{N} \sum_{j=1}^{N} a_i a_j \mathcal{A}(y_i y_j) = \underline{a}^T \cdot R \cdot \underline{a}$$

$$A_{12} = A_{21} = \underline{a}^T \cdot Q \cdot \underline{a}, \quad A_{22} = \underline{a}^T \cdot W \cdot \underline{a} \tag{3.8}$$

† The invertibility of A depends on the linear independence of y_i ; i = 1, ... , N, which depends on the selection of linearly independent basis excitations x_i; i = 1, ..., N.

where R, Q and W are N x N matrices:

$$R_{ij} = \mathcal{A}(y_i\, y_j)$$
$$Q_{ij} = \mathcal{A}(\dot{y}_i\, y_j) \qquad (3.9)$$
$$W_{ij} = \mathcal{A}(\dot{y}_i\, \dot{y}_j)$$

Similarly, the $\underline{b}$ vector becomes

$$b_1 = \sum_{i=1}^{N} a_i\, \mathcal{A}(f y_i) = \underline{a}^T \cdot \underline{e}$$
$$(3.10)$$
$$b_2 = \sum_{i=1}^{N} a_i\, \mathcal{A}(f \dot{y}_i) = \underline{a}^T \cdot \underline{g}$$

and thus new values of c_0 and k_0 can be computed from Eq. (3.5). Successive computations are carried out until two consecutive c_0 and k_0 values differ insignificantly.

The equivalent linearization of nonlinear system approach were employed to produce the nonlinear critical design spectrum. These were presented in detail in Appendix C.

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Pages 23-32 have been removed.

Because of copyright restrictions, the following article has been omitted: (Appendix A) "Site-Dependent Critical Design Spectra," by P. C. Wang and C. B. Yun, Earthquake Engineering and Structural Dynamics,* Vol. 7, 569-578 (1979).

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APPENDIX B

CRITICAL SEISMIC RESPONSE OF NUCLEAR REACTORS

by

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1. Introduction

This paper deals with the problem of how to assess the seismic resistance of important structures, particularly of nuclear reactor structures. Ideally, a person who is responsible for such an assessment is expected to certify on a high confidence level that a structure will survive all credible ground motions which can occur at its location, and perhaps also that it will do so on no more than a prescribed damage level.

Most of the design procedures presently in use or under investigation are based on design response spectra [1, 2] obtained by statistical evaluation of past ground excitations assuming certain probability distributions (e.g. normal or lognormal) of response peaks. Artificial time histories generated from these spectra are also used [4]. However, it is not clear whether these approaches lead to designs that can be relied upon on the confidence levels that are presumably desired for structures such as nuclear reactors whose integrity during an earthquake is of considerable importance. The trouble lies with the fact that the analysis of structural integrity seems highly sensitive to the assumptions regarding the nature of those probability distributions: small variations, especially in the tails of the distributions, can induce large changes in the desired results. This greatly weakens the reliance that can be placed in many assessments of earthquake resistance.

In this paper, a new method is developed which has the potential of avoiding those weaknesses. It is more specifically based on assumptions that seem well supported by seismological observations but side-steps others, especially those regarding the probability distribution of ground motions, which are more conjectural.

The avoidance of such assumptions is usually purchased at the expense of some conservatism in the results, and the new method is no exception. The assessments of earthquake resistance that are obtained through it are somewhat conservative. This makes its application most attractive to structures or devices whose integrity during a strong earthquake is of special importance, and nuclear reactors are felt to be good cases in point. The present paper therefore describes several such applications.

The method relies on the concept of the critical excitation. It is defined here as an excitation, among a certain class of excitations, that will produce the largest response peak for a structural variable of interest. The class in which critical one is determined should be chosen in such a way that it includes all ground motions that are credible for the location under consideration, and as few others as possible.

The definition of what does and does not constitute a credible excitation proved to be one of the major difficulties in the development of the method. This development is sketched in Sect. 2 below and the definition that was ultimately adopted is described there. It appears to represent a fairly realistic one in that it retains those features which are generally accepted as being characteristic of credible ground motion, while it excludes others which are of a more doubtful nature. Sect. 3 then outlines the computational procedure by which the critical excitations, and the response peaks they generate, are determined. Sect. 4 finally describes the application of the method to three reactor containment structures. The upshot or the results reported there is that there exist many credible ground motions that drive the structures to higher response peaks than any already recorded ground motions, and hence, that reliance on the latter may lead to assessments of earthquake resistance that are lower in some cases than that suggested by their importance.

2. Definition of Critical Excitations

A critical excitation has just been defined as one among a class of allowable excitations which will produce the most severe response peak in a structural variable. The nature of the critical excitation thus depends on the class of allowable excitations on which it is based and on the structural variable of interest. The choice of the latter is at the discretion of the structural engineer. That of the former however is not. In fact, the designer must in some way take into account all ground motions that can realistically be expected at the site of the structure.

Unfortunately, it is not quite clear just what characteristics an excitation must, or must not, have in order to be considered a realistic (or credible) candidate for a ground motion during an earthquake. It is customary to assume that it is possible to set an upper bound E on the ground motion intensity at a given location which is so chosen that its exceedance is too unlikely an event to be taken into account. Several other characteristics are also widely accepted as distinguishing realistic candidates of ground motions from their opposites. Vanmarcke [5], for instance, lists the following:

(a) Duration of strong gound motion

(b) Variation of motion intensity with time

(c) Relative frequency content

(d) Effect of macro-zone, micro-zone, and site-soil

(e) Effect of focal distance

Some of these characteristics are clearly of a rather qualitative nature and it is not obvious how they are best converted into quantitative form. Based on some early and inadequate ideas by one of the writers [6] [7] , Shinozuka [8] accomodated (c), and Iyengar (a) and (b). The present authors proposed another approach intended to allow for all five [10]. The one on which this paper is based is a modified and theoretically better founded version of the last. One is led to it by the following line of reasoning.

One should first of all consider any excitation to be realistic, for a given site, that has already been recorded there or at some other location with similar soil conditions, focal distances, and macro or micro zoning. These are called "basis" excitations below. In addition to these, one should presumably admit any other excitation which has the same or similar characteristics (in the sense of the above list) as the basis excitations. Despite the vagueness of the list, it may be safe to say that these characteristics are shared by all linear combinations of the basis excitations, with the proviso that their intensities do not exceed the upper bound E appropriate for that location.

It is not known whether the class of excitations defined in this way comprises all that can be considered "realistic" or "credible". However, at the present state of knowledge regarding seismological disturbances, any further expansion of it seems difficult to justify. In this paper, at any rate, the class of "realistic" excitations will be defined as just described.

In mathematical terminology, the class lies in a linear manifold spanned by the basis excitations, and it is a solid sphere within it namely the one with the maximum intensity E as radius. In symbols , if $\ddot{x}_1(t)$,

$\ddot{x}_2(t), \ldots \ddot{x}_n(t)$ are the ground accelerations that form the basis, the manifold which is spanned by them contains all excitations

$$\ddot{x}(t) = \sum_{i=1}^{n} a_1 \ddot{x}_1(t) \tag{1}$$

The class of allowable ones among these then includes all whose intensity $\|\ddot{x}\|$ does not exceed the given maximum E.

Several intensity measures have been employed for seismic excitations in the past. Among them, the peak ground acceleration

$$\|\ddot{x}\| = \max_t |\ddot{x}(t)| \tag{2}$$

is probably the one used most often. The square integral of the ground acceleration over the time period T of interest, i.e.

$$\|\ddot{x}\| = \left[\int_0^T \ddot{x}^2(t)dt\right]^{1/2} \tag{3}$$

is another. The second is more easily utilized towards obtaining peak responses of linear structural systems. Hence it is used in this study and a class of allowable excitations is defined as a set of ground accelerations of the form (1) whose intensities satisfy

$$\|\ddot{x}\| \triangleq \left[\int_0^T \ddot{x}^2(t)\, dt\right]^{1/2} \leq E \tag{4}$$

with E being a prescribed value.

The response y(t) of a linear structural system to an excitation in the manifold is

$$y(t) = \int_0^t \ddot{x}(\tau)\, h(t-\tau)\, d\tau = \sum_{i=1}^{n} a_i \int_0^t \ddot{x}_i(\tau)\, h(t-\tau)\, d\tau$$

$$= \sum_{i=1}^{n} a_i\, y_i(t) \tag{5}$$

where h(t) is the impulse response function of the design variable under consideration and $y_i(t)$ is its response to a basis excitation $\ddot{x}_i(t)$. The critical excitation $\ddot{x}_c(t)$ in the manifold is now defined as one which drives the response y(t) to its maximum value and which at the same time obeys the intensity constraint (4).

3. Determination of Critical Excitations

The problem of determining a critical excitation is the following. It is required to find an excitation $x_c(t)$ of the form (1) which obeys the constraint (4) and which drives the response y(t) in (5) to its largest peak. If the time t^* at which y(t) reaches its maximum were known, the problem would be that of determining a set of coefficients a_i, i = 1, 2, ... n, which achieves

$$\max_{a_i} |y(t*)| = \max_{a_i} \left| \sum_{i=1}^{n} a_i \, y_i(t*) \right| . \tag{6}$$

subject to the constraint

$$E \geq \int_0^T \ddot{x}^2 (t) \, dt = \sum_{i=1}^{n} \sum_{j=1}^{n} a_i \, a_j \int_0^T \ddot{x}_i(t) \, \ddot{x}_j(t) \, dt$$

$$= \sum_{i=1}^{n} \sum_{j=1}^{n} a_i \, a_j \, \ddot{x}_{ij} \tag{7}$$

where $$\ddot{x}_{ij} = \int_0^T \ddot{x}_i(t) \, \ddot{x}_j(t) \, dt \tag{8}$$

However, the time instant t^* is not known. Thus the maximization y(t) must be carried out with respect to time t also, i.e.,

$$\max_t \max_{a_i} |y(t)| = \max_t \max_{a_i} \left| \sum_{i=1}^{n} a_i y_i(t) \right| \tag{9}$$

must be determined. The maximization with respect to a_i can be carried out by simple linear algebra. However, the one with respect to t can be done only by a numerical evaluation. These two procedures will be described now.

One can, first of all, ignore the absolute value in Eq. (9). For, if some set of a_i achieves the positive maximum, the set of $(-a_i)$ yields the negative one, and vice versa. Thus, y(t) is to be maximized for a fixed time t and subject to Eq. (7). This can be done by setting

$$0 = \frac{\partial}{\partial a_j}\left(\sum_{i=1}^{n} a_i \, y_i(t) + \lambda \sum_{i=1}^{n} \sum_{j=1}^{n} a_i a_j \, \ddot{x}_{ij} \right)$$

$$= y_j(t) + 2\lambda \sum_{i=1}^{n} \ddot{x}_{ji} \, a_i \qquad j = 1, 2, \ldots n \tag{10}$$

where λ is a Lagrangian multiplier. This equation can be rewritten as

$$\sum_{i=1}^{n} \ddot{x}_{ji} \, a_i = - \frac{1}{2\lambda} \, y_j(t), \quad j = 1, 2, \ldots n, \tag{11}$$

or, more comprehensively, in matrix form, namely

$$\ddot{X} a = - \frac{1}{2\lambda} \, Y(t) \tag{12}$$

where a and Y(t) are n-dimensional column vectors with the components a_i and $y_i(t)$, respectively, and $\ddot{X}$ is an n x n-dimensional matrix with the components $\ddot{x}_{ij}$. The solution is thus

$$a = - \frac{1}{2\lambda} \, \ddot{X}^{-1} \, Y(t) \tag{13}$$

where invertibility of $\ddot{X}$ can be shown in the following way. One assumes to start with that the basis excitations are linearly independent. In practice this is almost a matter of course and in theory it merely implies that none of those excitations can be omitted from the basis. Thus,

$$0 < \int_0^T \left[\sum_{i=1}^{n} a_i \ddot{x}_i(t) \right]^2 dt = \sum_{i=1}^{n} \sum_{j=1}^{n} a_i a_j \int_0^T \ddot{x}_i(t) \ddot{x}_j(t) dt = \sum_{i=1}^{n} \sum_{j=1}^{n} a_i a_j \ddot{x}_{ij} .$$

The expression on the right can be interpreted as a quadratic form in the variables a_i , and in fact a positive definite form as the inequality shows. It follows that the matrix $\ddot{X}$ is positive definite also, hence has only positive eigenvalues. Since none of the eigenvalues vanish $\ddot{X}$ is invertible.

The Lagrangian multiplier can now be determined from the constraint equation Eq. (7) which, written in matrix form, is

$$a^T \ddot{X} a \leq E \tag{14}$$

where a^T is the transpose of the vector a. Substituting Eq. (13) into Eq. (14) gives

$$\frac{1}{4\lambda^2} Y^T(t) (\ddot{X}^{-1})^T \ddot{X} \ddot{X}^{-1} Y(t) = \frac{1}{4\lambda^2} Y^T(t) \ddot{X}^{-1} Y(t) \leq E^2 \tag{15}$$

in which use has been made of the fact that $(\ddot{X}^{-1})^T = \ddot{X}^{-1}$ because $\ddot{X}$ is symmetric. Eq. (13) implies that the values of a_i as well as Y (t) grow larger in magnitude as λ decreases. Thus, Y(t) is maximized, for fixed t, if equality prevails in (14), i.e., if

$$\lambda = \frac{1}{2E} \quad Y^T(t) \ddot{X}^{-1} Y(t) \quad ^{1/2} \tag{16}$$

This value, substituted into Eq. (12), gives the maximizing coefficients a_i .

It remains to carry out the maximization in Eq. (9) with respect to the time t. This can be done numerically by subdividing the time period T of interest into subintervals Δt . In computational practice, to achieve a desired accuracy in the results without excessive effort, consideration must be given in the choice of Δt to the natural periods and decay time of the impulse response h(t) of the structural variable as well as to the frequency contents of the basis excitations. In this study, based on computational experience, the time interval T in Eq. (4) is taken roughly as

$$4 \leq T = 8 \times T_1 \leq 40 \text{ (sec)} \tag{17}$$

where T_1 is the fundamental period of the structure. The subinterval Δt was taken as $T/200$ (i.e., approximately $T_1/20$). Computation of critical excitations for 6 structural variables consumed a computer time of roughly 2 minutes on an IBM 360/65 machine, an effort which is considered quite modest.

4. Analysis of Nuclear Reactor Structures

In order to illustrate the critical excitation approach to the practical design of structures, particularly of nuclear reactor structures, three analyses of containment structures are presented below. The first two example were drawn from the technical literature [6] [7] . The third was obtained from a civil engineering consulting firm which specializes in the design of such structures.

Effects of soil-structure interactions are represented in the analysis by introducing equivalent soil springs. In order to allow consideration of the geological properties of the construction sites, two sets of ground accelerations which were recorded respectively on stiff soil sites and on rock sites [11], are selected as basis excitations see Tables 1 and 2. Accordingly two classes of credible ground excitations are constructed from them in the way described in the previous section. The critical excitations are, then, computed for several design variables of each structure. Finally, comparisons are made between response peaks due to critical excitations and those produced by several actual ground motions.

The results obtained are as follows.

4.1 Reactor Structure I

The first example is a relatively simple model of a nuclear reactor building analyzed originally by Hamilton [12]. It consists of two substructures, namely the containment building and the auxiliary building. For the structural analysis it is idealized as a 2 dimensional two-sticks model with lumped masses, as shown in Fig. 1. The effects of soil-structure interaction are introduced by two equivalent soil springs which are attached to the foundation in horizontal and rotational directions.

For the dynamic analysis, the first six modes are used. The viscous damping is taken as 7% of the critical for each mode. The first three natural periods are .342, .132 and .067 sec.

In order to illustrate the assessment of the earthquake resistance of this structure, it is assumed that it is to be constructed on a stiff soil side in California. Accordingly the twelve ground accelerations recorded on stiff soil sites in that area are chosen as the basis excitations. They are listed in Table 1. The maximum intensity E of Eq. 4 is taken to be that of the NS-component of the Imperial Valley earthquake, as recorded at El Centro on May 18, 1940. This intensity to $E = 7.90$ ft/sec$^{3/2}$, with $T = 4$ sec as the relevant duration of strong ground motion. The maximizations described in the preceding section are carried out at intervals of $\Delta t = .02$ sec.

Critical excitations are computed for six design variables, namely top displacement, bottom moment and bottom shear for each substructure.

The critical excitation for the bottom moment of the containment building is shown in Fig. 2 along with the record of the San Francisco earthquake, as recorded at the Alexander Building on March 22, 1957. By inspection, at least, the former seems as realistic a sample of a possible ground motion as the latter. This impression is confirmed by a comparison of their Fourier spectra in Fig. 3.

Some of the results of the calculations are presented in Table 3. The first column lists the response peaks generated in several design variables by the critical excitations. Column 2 and 3 show the response peaks produced by the ground motions recorder at El Centro and at the Alexander building, normalized to the same intensity E_1 and column 4 the so-called "response envelope", i.e., the largest peaks due to the 12 basis excitations.

The comparison among these figures shows that there exist realistic excitations, namely the critical ones as well as others with similar characteristics, which produce response peaks that are higher by factors of 1.4 to 2 than those produced by recorded past earthquakes.

4.2 Reactor Structure II

The second example is a containment structure to be built on a stiff soil site in India, according to information presented by Arya, et al. [13].

This structure consists of three substructures, namely i) the outer containment, consisting of an outer cylindrical shell with a spherical dome at the top, ii) the inner containment consisting of an internal cylindrical shell and a cellular grid at the top, and iii) the internal structure which includes the reactor internal structural system and the raft. A vertical cross section of the structure is shown in Fig. 4 a. For the structural analysis, this structure is represented as a three-sticks model with lumped masses as shown in Fig. 4 b. The effect of soil stiffness on the imbedment portion is included by introducing one rotational and 8 translational soil springs.

Dynamic analyses are performed by using the first 6 modes. Damping ratios are taken as 5% for each mode. The first three natural periods are .699, .232 and .121 sec.. Because no earthquake records in India are available, to the authors the twelve ground accelerations in Table 2 recorded in the United States are used as the basis excitations. The maximum intensity E is taken as that of El Centro earthquake as for the preceeding structure but over a time interval T of 6 sec. Maximizations are carried out at intervals of $\Delta T = .03$ sec. The critical Excitations are computed for several design variables, namely top displacement, bottom moment and bottom shear face for each of two containment structures. The critical excitation for the bottom moment of outer containment is shown in Fig. 5, along with the El Centro ground motion. Fig. 6 displays their Fourier spectra. All are normalized to the same intensity E. Again, no characteristics are evident in either that are not present also in the other.

The response peaks due to the critical excitations are compared in Table 4 with those generated by two recorded ground motions, namely at El Centro on May 18, 1940 and at Castaic during the San Fernando earthquake on February 9, 1971. Also, the peaks from the response envelope are listed. As it happens, they are all produced by the same ground motion record, namely the one from El Centro.

As in the preceding example, the conclusion is that there are realistic ground motions which induce response peaks in these structures that are higher (in the present case by factors up to 1.77) than those considered in previous assessments of earthquake resistance.

4.3 Reactor Structure III

The last example to be discussed here is a relatively complex reactor structure for a nuclear power plant which is to be constructed on a rock site in the United States. It consists of three substructures, namely the containment building, the internal structure and the annulus building. A vertical cross section of this structure is shown in Fig. 7 a. For the analysis, it is idealized into a 3-dimensional stick model with lumped masses as shown in Fig. 7 b. Effects of soil-structure interaction are included by equivalent soil springs in three translational and three rotational directions.

The first fifteen modes are used and viscous damping is taken as 7% of the critical one for each mode. The first three natural periods are .275, .276 and .138 sec.

As the basis excitations, twelve ground accelerations recorded on rock sites are used. They are listed in Table 2. The intensity E assumed to be that of the ground motion recorded at the Pacoima Dam during the San Fernando earthquake over a period of T = 4 sec. i.e., $E = 17.89\ \mathrm{ft/sec}^{3/2}$. Maximizations are executed at intervals of $\Delta t = .02$ sec.

The critical excitations are computed for nine design variables, namely top displacement, bottom moment, and bottom shear face for each of the three substructures. The critical excitation for the bottom moment of the containment building is shown in Fig. 5, along with the Pacoima Dam ground motion.

The response peaks due to the critical excitations are compared in Table 5 with those recorded at other rock sites, namely at the Pacoima Dam during the San Fernando earthquake and at the Golden Gate Bridge during the San Francisco earthquake. All are of course normalized to the same intensity E. The peaks of the response envelope are also shown in the table.

The critical response peaks are seen to be higher than the other table entries by factors ranging from 1.10 to 1.87. The conclusion here is thus the same as in the preceding two examples: there are many credible excitations that exert higher stresses on this structure than those produced by recorded past earthquakes.

5. Conclusions

A new method has been described for the assessment of seismic resistance of structures, particularly of nuclear reactor structures. The method is based on the concepts of critical excitations and responses. It is derived under well-specified assumptions and by a well-defined procedure. The design ground excitations, namely the critical excitations, obtained are found to be quite realistic samples of possible ground motions.

The effects of these excitations were analyzed on three designs of reactor containment structures. It was found that the resulting response peaks are higher by factors ranging from 1.1 to 2.0, approximately, than those that would have been produced in these structures by already recorded ground motions. The conclusion therefore is that there exist many excitations which are realistic candidates for seismic ground motions at any location but which will drive a structure there to stronger responses than is evident from the history of such responses in the past.

Experience with similar analyses of other structures by this method indicates that competent structural engineering designs are typically adequate to accommodate these higher response peaks in ways provided for by the designers. It is considered very likely that the same is true of the structures discussed above.

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Table 1. Basis Excitations on Stiff Soil Sites

File number in CIT Report	Earthquake	Date	Record	Comp.
A001	Imperial Valley	5/18/40	El Centro	NS
A001	Imperial Valley	5/18/40	El Centro	S90W
A014	San Francisco	3/22/57	Alex. Bldg., S. F.	N09W
A016	San Francisco	3/22/57	State Bldg., S. F.	S09E
B024	Lower California	12/30/34	El Centro	NS
D056	San Fernando	2/09/71	Castaire	N69W
D058	San Fernando	2/09/71	Hollywood Storage L. A.	NS
D058	San Fernando	2/09/71	Hollywood Storage L. A.	N90E
E083	San Fernando	2/09/71	3407 6 St. L. A.	NS
E083	San Fernando	2/09/71	3407 6th St. L. A.	N90E
H115	San Fernando	2/09/71	15250 Ventura Blvd L. A.	N11E
Q233	San Fernando	2/09/71	14724 Ventura Blvd L. A.	N78W

Table 2. Basis Excitations on Rock Sites

File number in CIT Report	Earthquake	Date	Record	Comp.
A015	San Francisco	3/22/57	Golden Gate, S. F.	N10E
B025	Helena	10/31/35	Federal Bldg. S. F.	NS
B037	Parkfield	6/27/66	Temblor	S25W
C041	San Fernando	2/09/71	Pacoima Dam	S16E
C106	San Fernando	2/09/71	C. I. T., Seis. Lab.	EW
J141	San Fernando	2/09/71	Lake Hughes Sta. 1	N21E
J142	San Fernando	2/09/71	Lake Hughes Sta. 4	S69E
J144	San Fernando	2/09/71	Lake Hughes Sta. 12	N21E
L166	San Fernando	2/09/71	3838 Lankershim Blvd., L. A.	NS
0198	San Fernando	2/09/71	Griffith Park Observ.	SN
P221	San Fernando	2/09/71	Santa Anita Dam	N87W
W334	Lythe Creek	9/12/70	Wrightwood	S25W

Table 3. Maximum Responses of Reactor Structure I

Structural Variables	Critical	El Centro	Alex Bldg. S. F. 3/22/57	Envelope
Top Displ. (in)				
Containment Bldg.	1.185	.525	.820	.820*
Auxiliary Bldg.	.049	.027	.018	.037**
Bottom Moment (10^7 Kip-ft)				
Containment Bldg.	1.022	.439	.719	.719*
Auxiliary Bldg.	.141	.078	.052	.107
Bottom Shear (10^5 Kip)				
Containment Bldg.	.642	.300	.462	.462*
Auxiliary Bldg.	.380	.207	.138	.289**

Note:

*Response peaks due to ground motion at Alexander Building, S. F. 3/22/57

**Response peaks due to ground motion at Sixth Street, L. A. 2/9/71

Table 4. Maximum Responses of Reactor Structure II

Structural Variables	Critical	El Centro 5/18/40	Castair 2/9/71	Envelope
Top Displacement (m)				
at node 1	.227	.169	.132	.169*
at node 27	.139	.103	.082	.103*
Bottom Moment (10^6 Ton-m)				
member 25	.711	.546	.386	.546*
member 47	.471	.357	.264	.357*
Bottom Shear (10^4 Ton)				
member 25	1.09	.841	.587	.841*
member 47	1.39	1.075	.832	1.075*

Note:

*Response peaks due to ground motion at El Centro, 5/18/40

Table 5. Maximum Responses of Reactor Structure III

Structural Variables	Response Peak			
	Critical	Pacioma Dam 9/2/71	Golden Gate Bridge 3/22/57	Envelope
Top Displacements (inch)				
at node 6	.749	.415	.499	.636**
at node 16	4.183	2.351	3.955	3.955*
at node 21	3.201	1.769	3.059	3.059*
Moments (10^7 Kips-ft.)				
member 2, node 1	6.558	3.655	4.690	5.849**
member 6, node 2	3.848	2.140	3.605	3.605*
member 17, node 1	1.570	.794	1.004	1.349**
Shear Forces (10^5 Kips)				
member 2, node 1	5.736	3.181	4.110	5.122**
member 6, node 2	2.444	1.207	2.319	2.319*
member 17, node 1	1.993	1.014	1.280	1.712**

Note:

*Response peaks due to ground motion at Golden Gate Bridge, 3/22/57

**Response peaks due to ground motion at Santa Anita Dam, 2/9/71

Legends of Figures

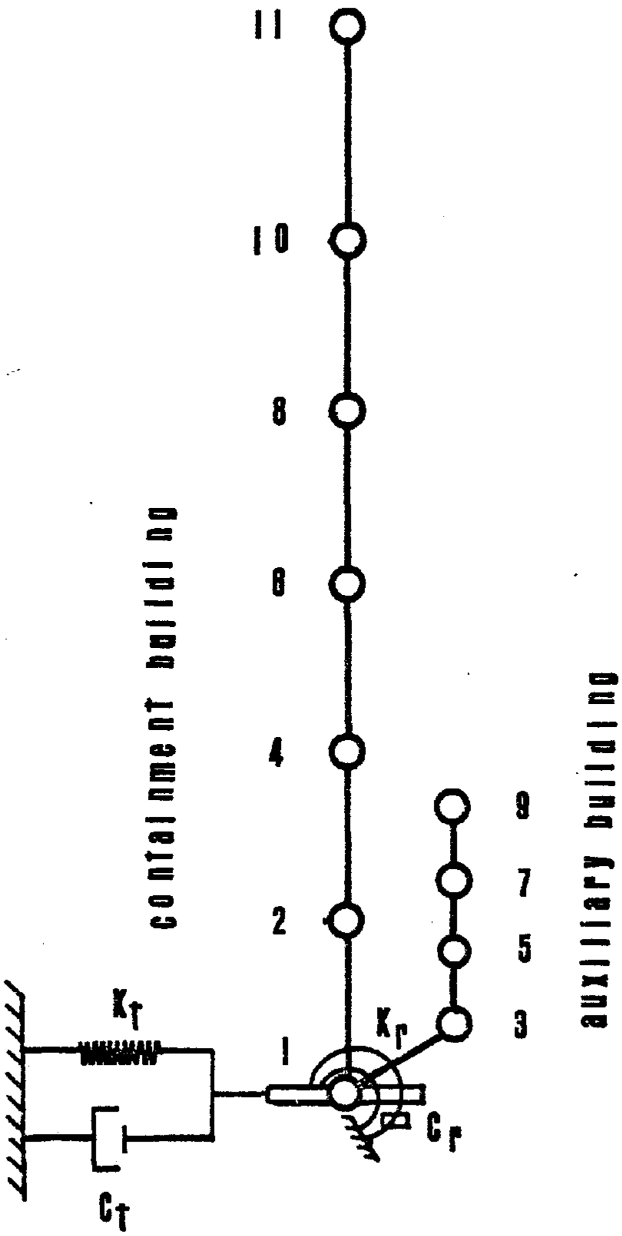

11
10
8
6
4
2
1
9
7
5
3
containment building
auxiliary building
K_t
C_t
K_r
C_r

Fig. 1

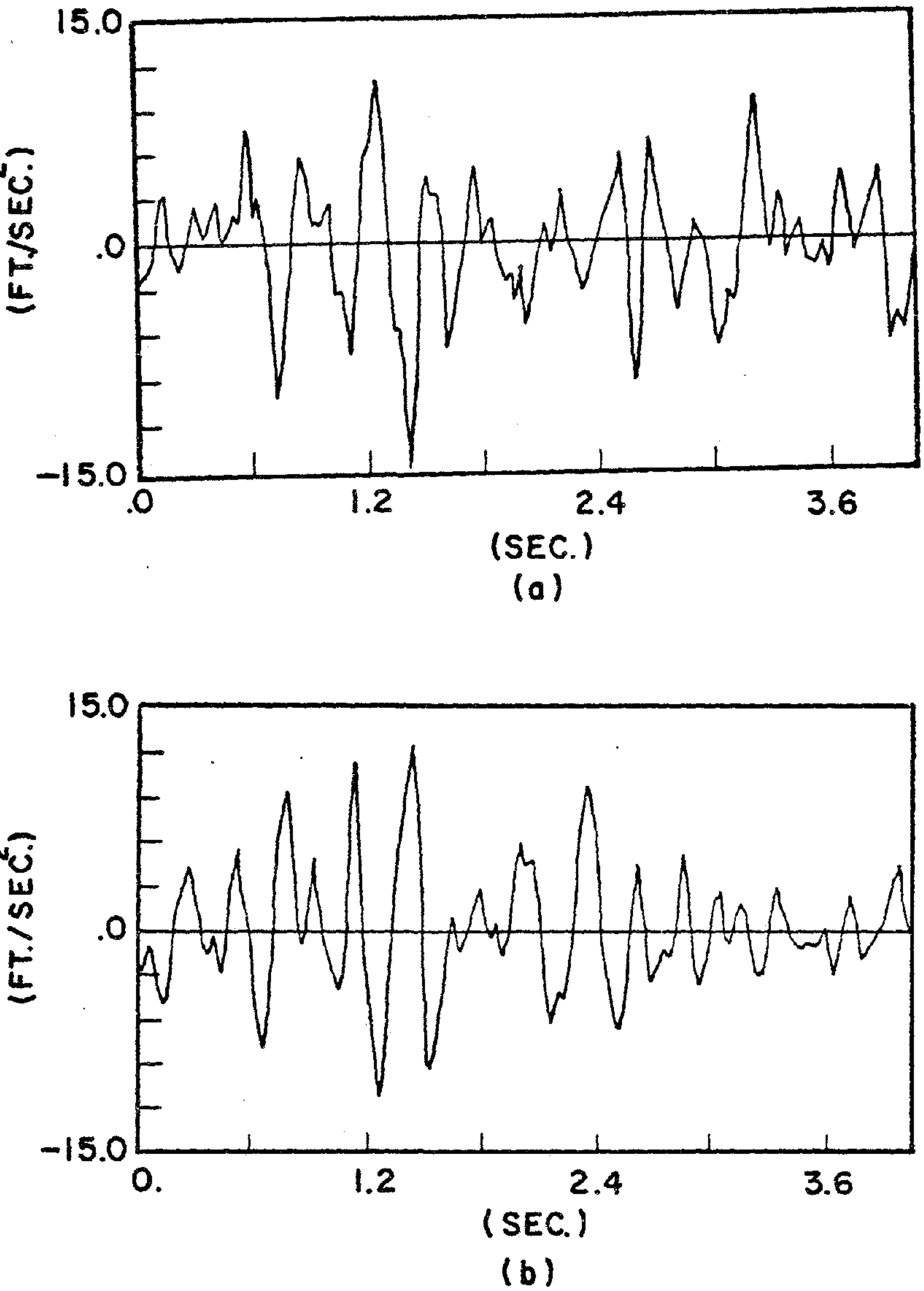


Fig. 2

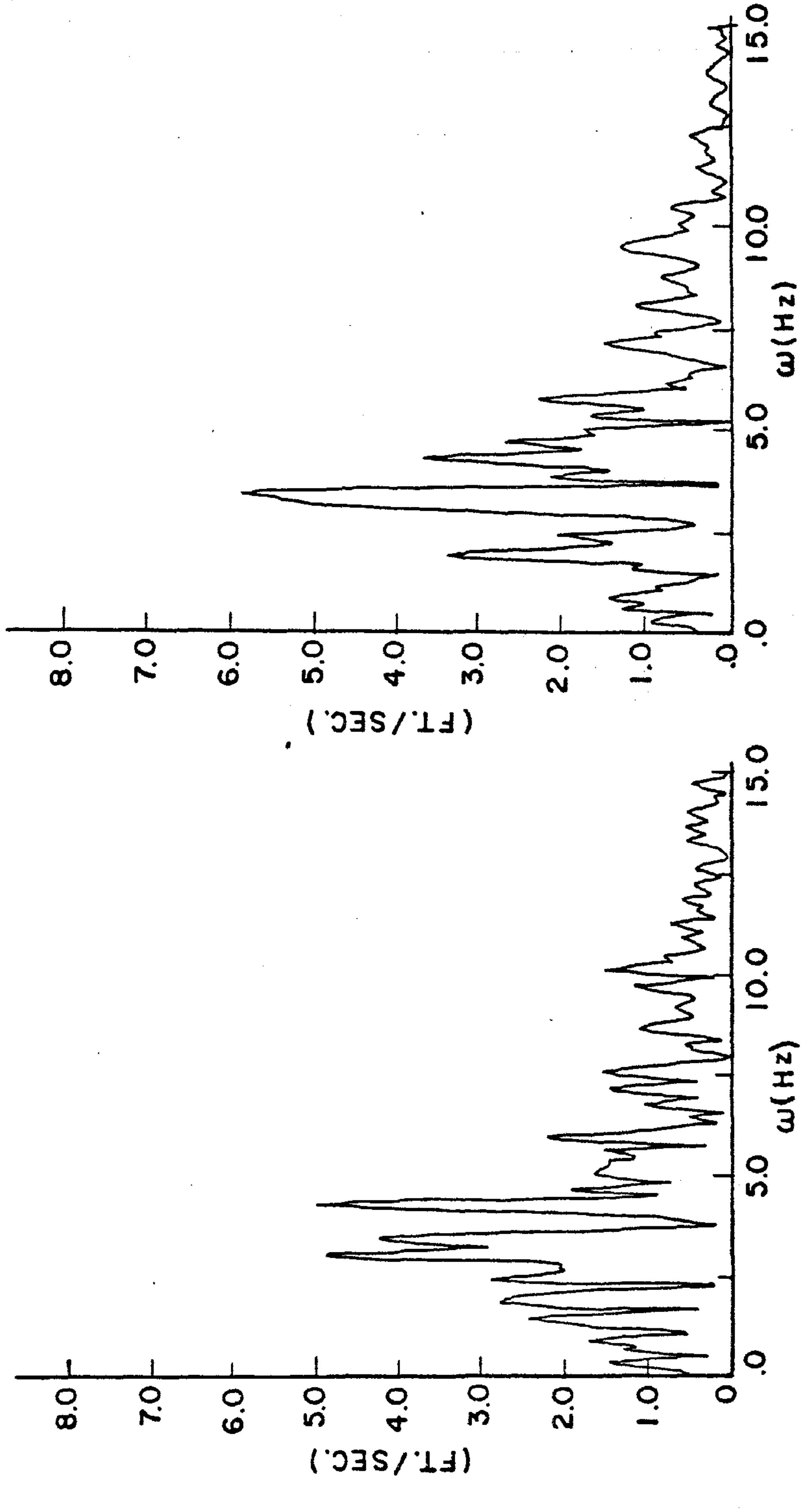
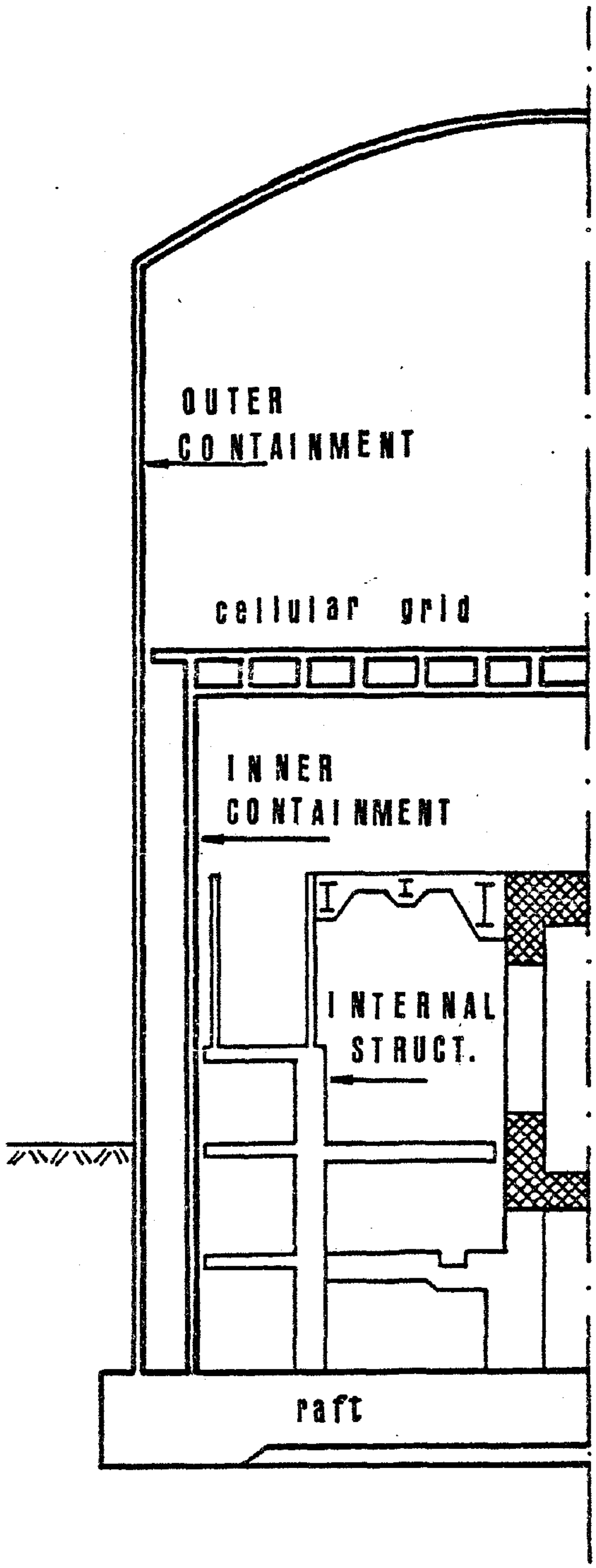
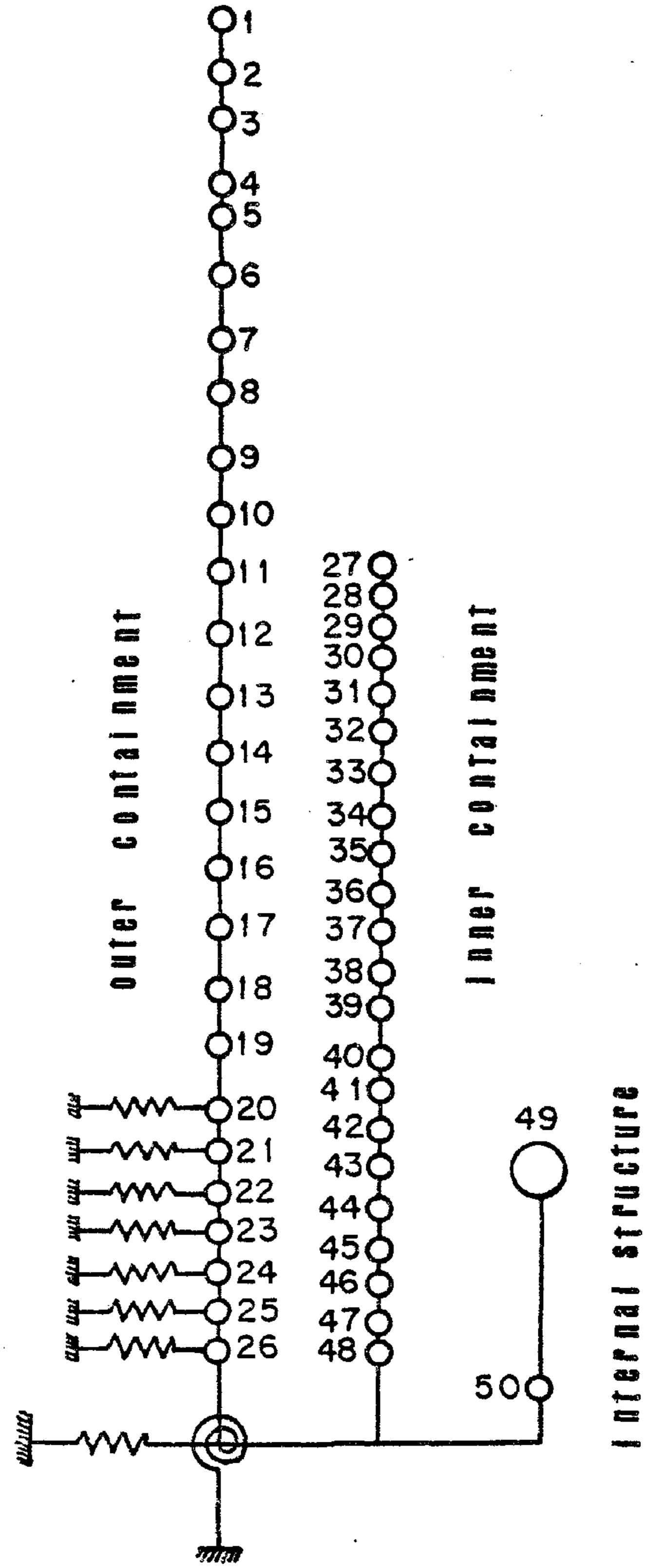


Fig. 3


OUTER
CONTAINMENT
cellular grid
INNER
CONTAINMENT
INTERNAL
STRUCT.
raft
[a]


outer containment
inner containment
internal structure
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21
22
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30
31
32
33
34
35
36
37
38
39
40
41
42
43
44
45
46
47
48
49
50
[b]

Fig. 4

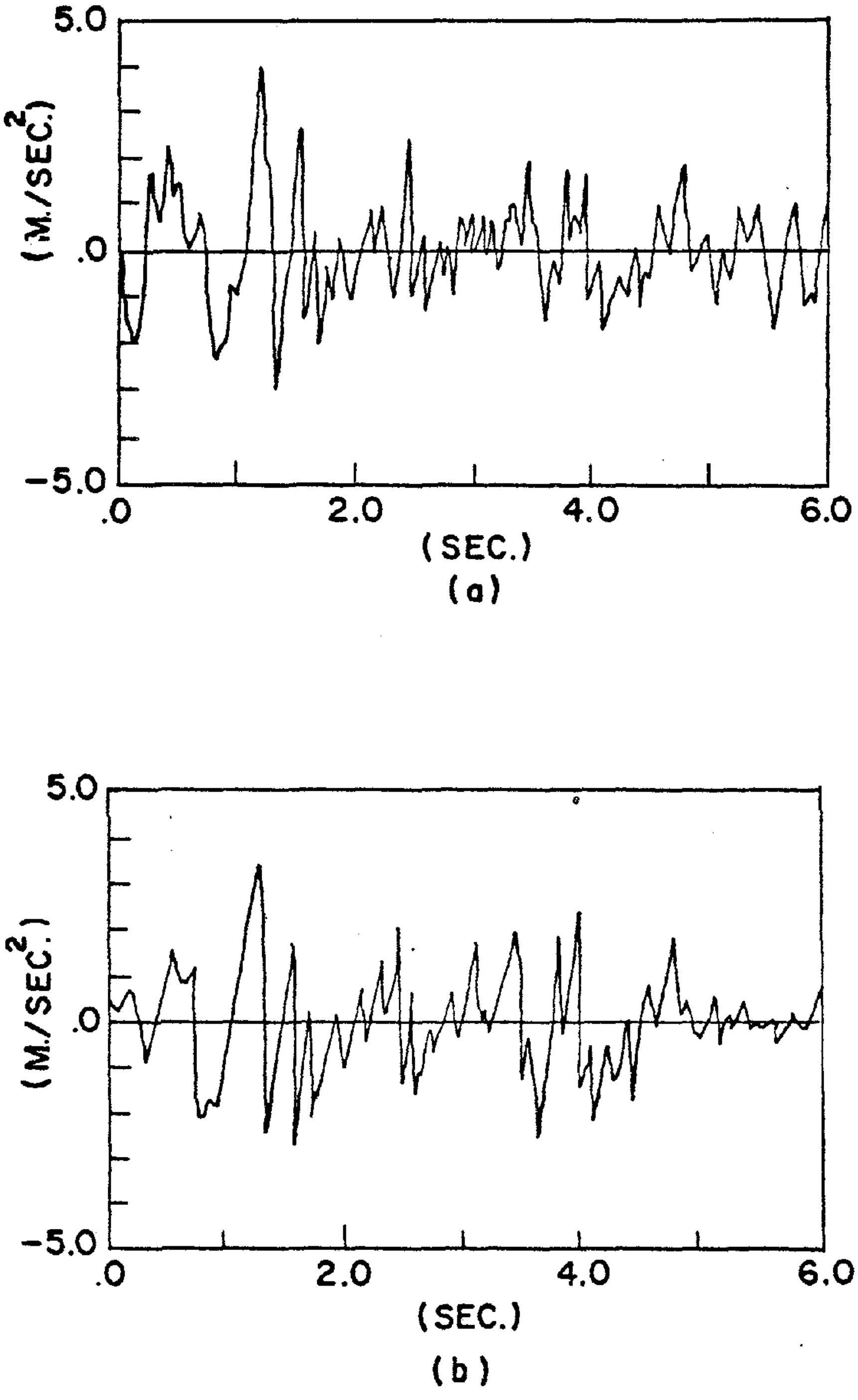


Fig. 5

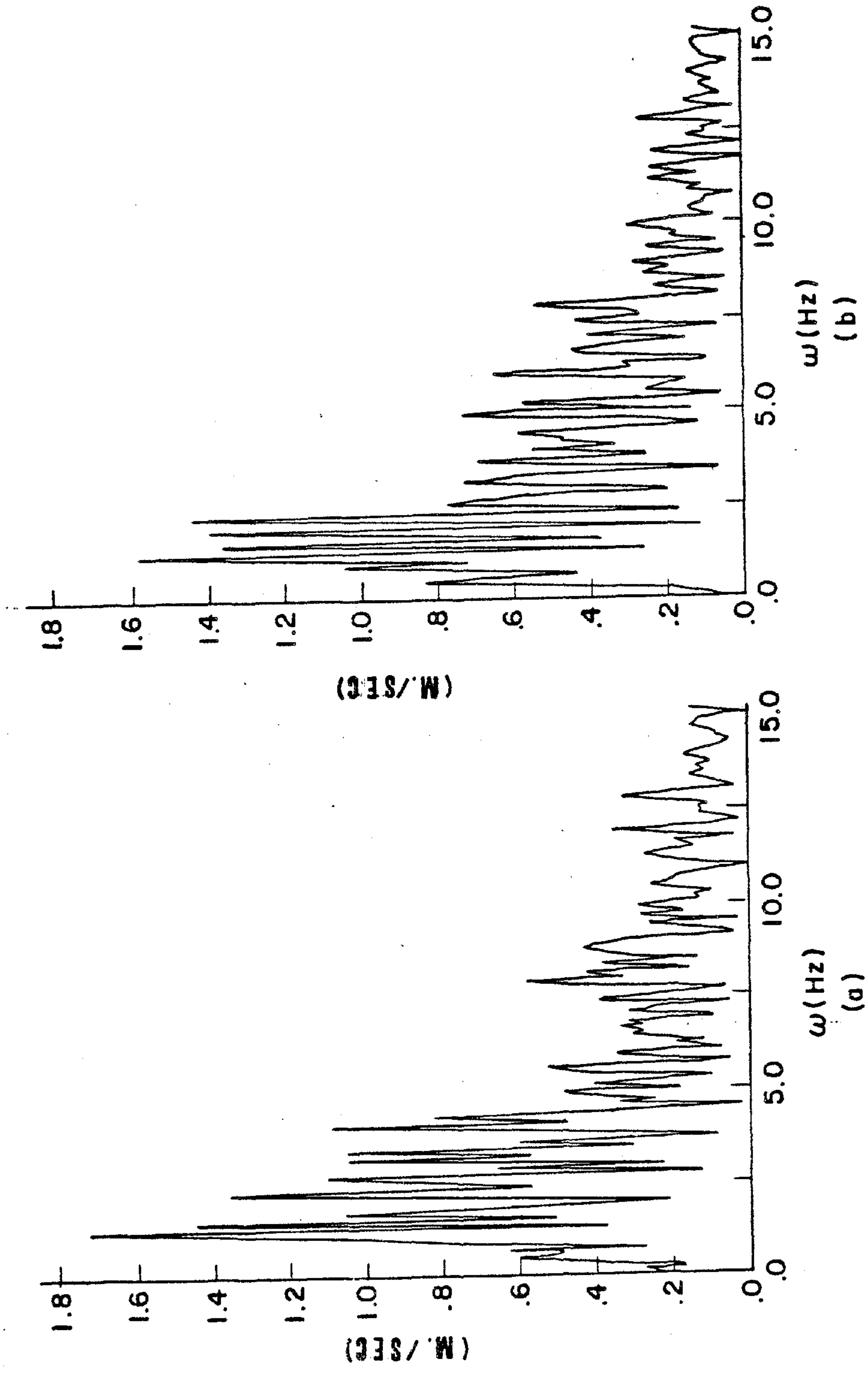


Fig. 6

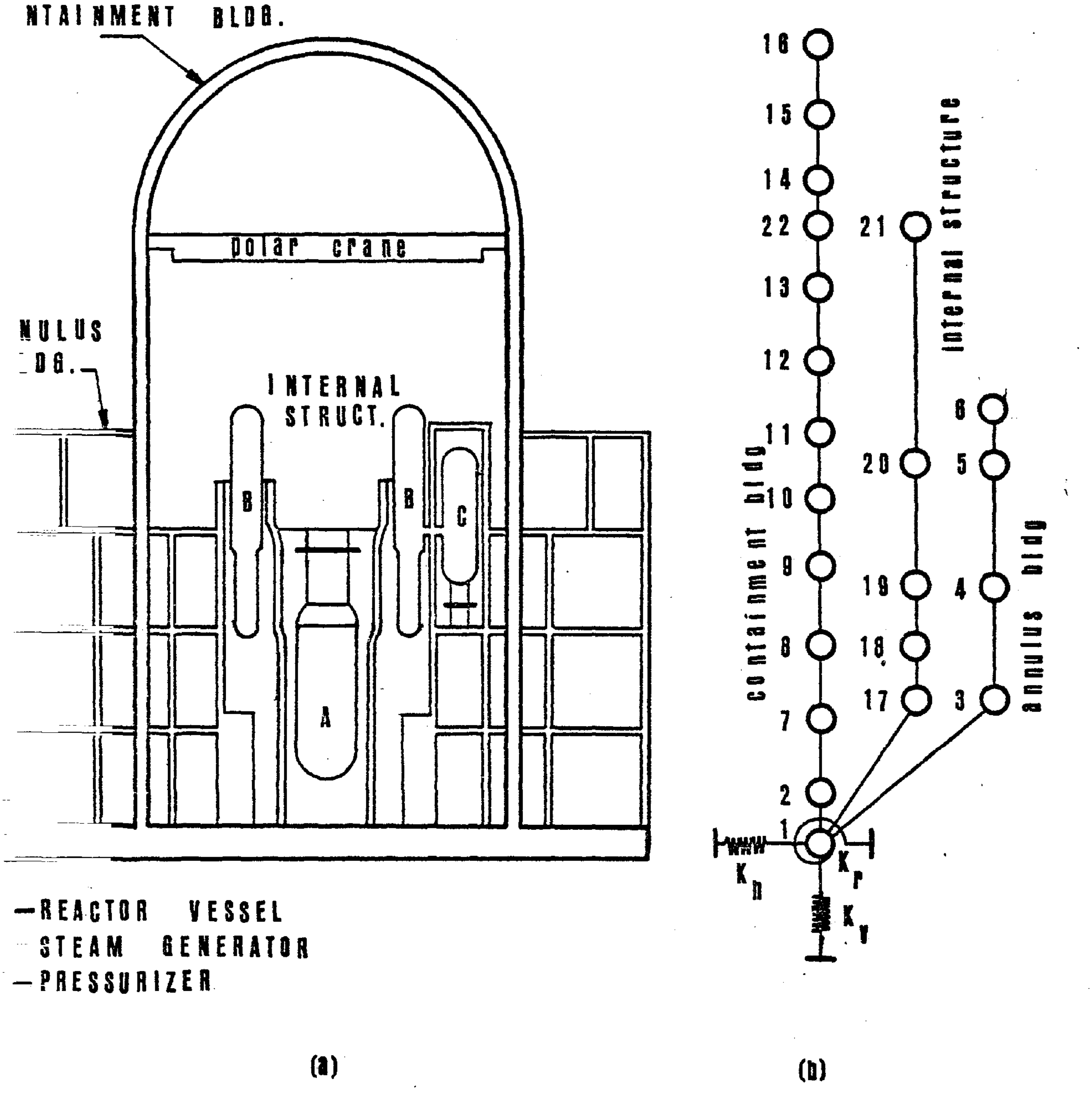

NTAINMENT BLDG.
polar crane
NULUS
DG.
INTERNAL
STRUCT.
B
B
C
A
—REACTOR VESSEL
STEAM GENERATOR
—PRESSURIZER
(a)
16
15
14
22
21
internal structure
13
12
6
11
20
5
containment bldg
10
9
19
4
annulus bldg
8
18
7
17
3
2
1
K_h
K_r
K_v
(b)

Fig. 7

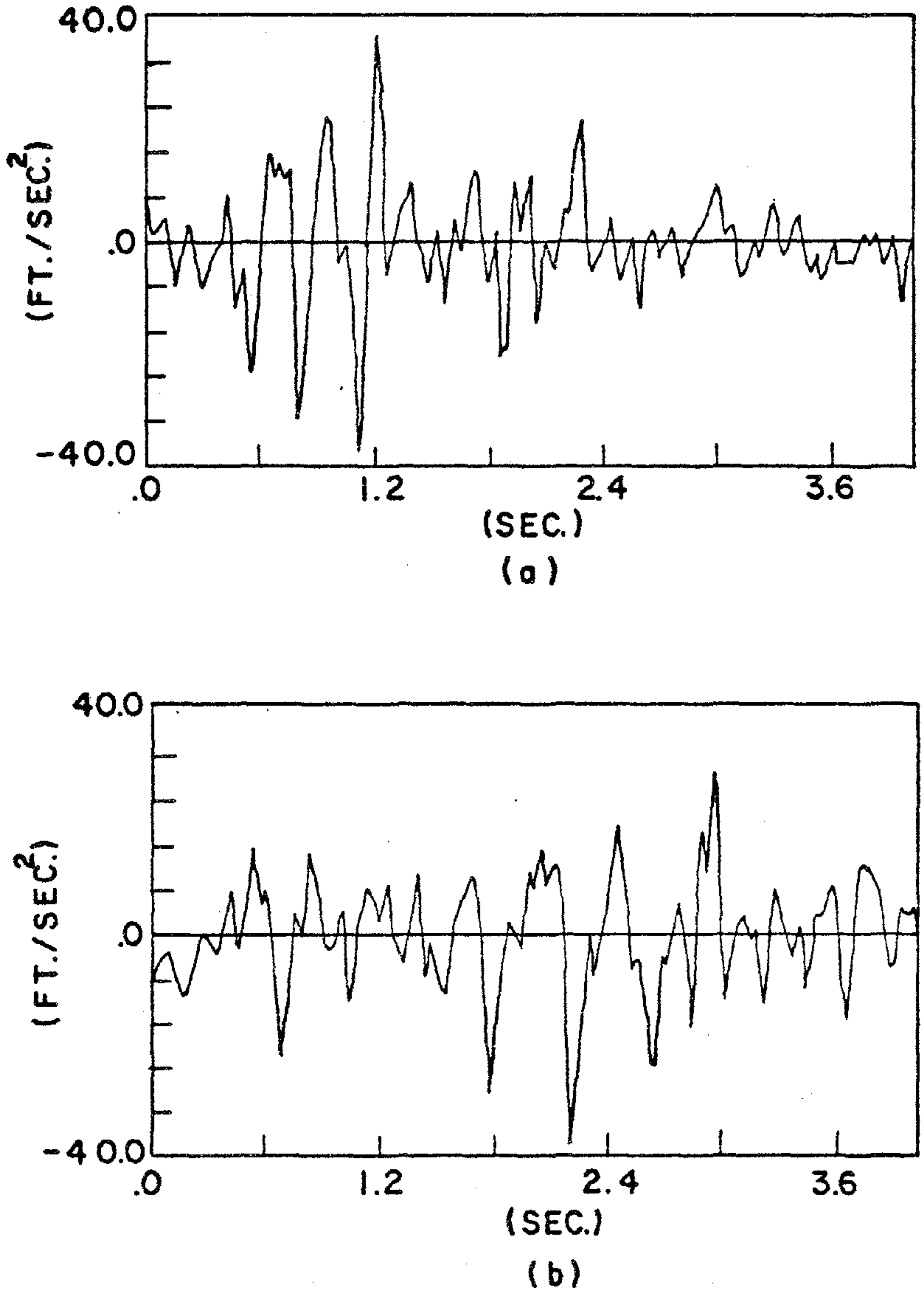
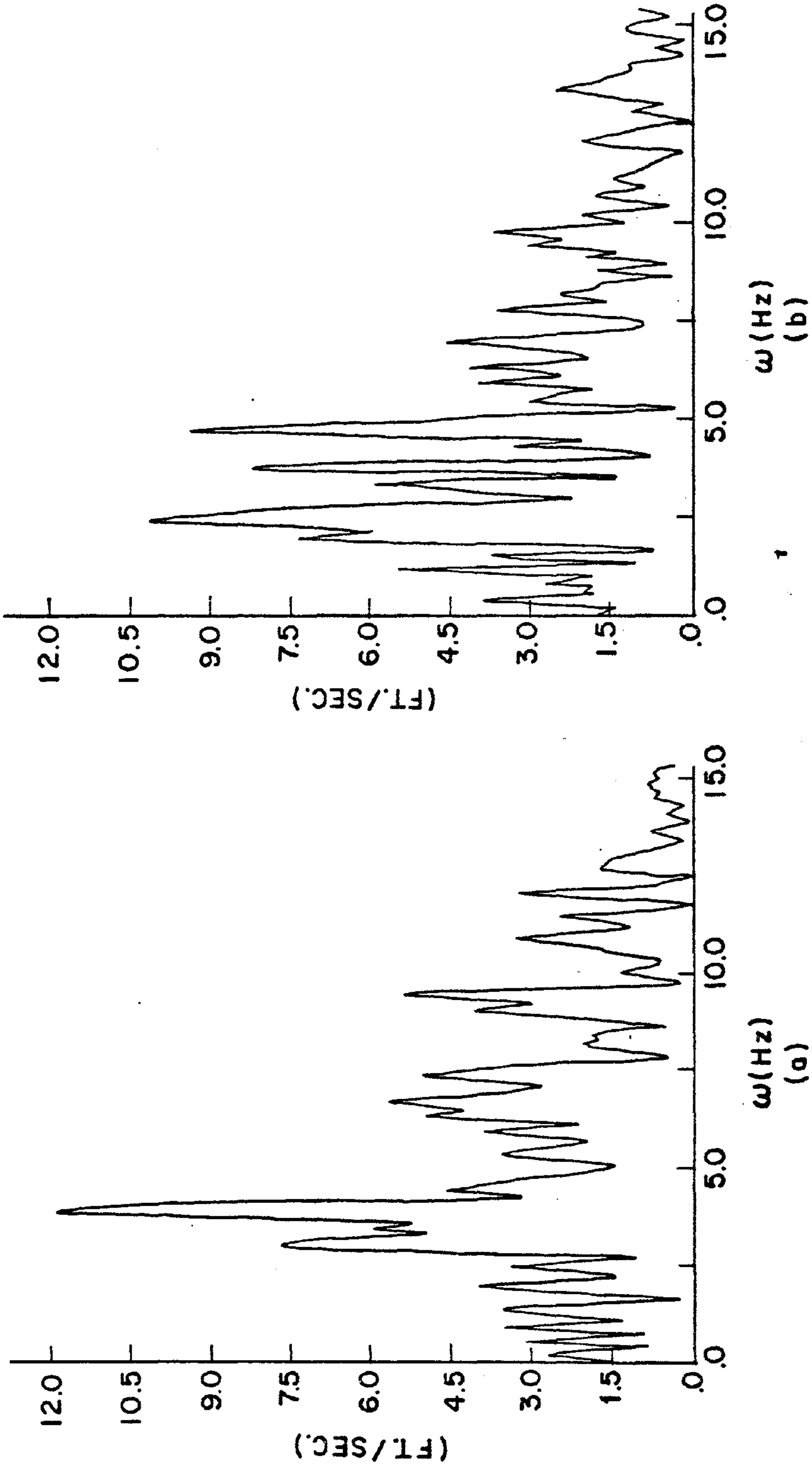


Fig. 8



(FT./SEC.)
12.0
10.5
9.0
7.5
6.0
4.5
3.0
1.5
.0
0.0
5.0
10.0
15.0
ω(Hz)
(b)
(FT./SEC.)
12.0
10.5
9.0
7.5
6.0
4.5
3.0
1.5
.0
0.0
5.0
10.0
15.0
ω(Hz)
(a)

Fig. 9

APP.·.·DIX C

CRITICAL EXCITATIONS FOR LINEAR AND NONLINEAR STRUCTURAL SYSTEMS

DISSERTATION

Submitted in Partial Fulfillment of the Requirements

for the Degree of

DOCTOR OF PHILOSOPHY (Civil Engineering)

at the

POLYTECHNIC INSTITUTE OF NEW YORK

by

Aristodimos J. Philippacopoulos

June 1980

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CHAPTER 1

THE CONCEPT OF EQUIVALENT LINEARIZATION

1.1 Introduction

The response of a nonlinear dynamical structural system is associated generally with the solution of a set of second order nonlinear differential equations. Techniques for exact closed form solutions are limited only for a few special cases of nonlinearity and types of inputs. A review of available techniques for nonlinear analysis oriented toward earthquake engineering area is given by Iwan [10].

The idea of replacing a nonlinear structural system by an "equivalent" linear one seems very attractive, for the reason that by this replacement the well-known results of linear analysis can be extended to nonlinear problems. There are various criteria according to which the equivalence of the two systems is assured. As such a criterion in this thesis, the minimization of the time average of the square of the difference between the two systems will be employed [8]. This point of view has the advantage of giving a very clear physical picture of the nature of the approximation. Furthermore it will be assumed that the equivalent linear system is time-independent and therefore the equivalent damping and stiffness matrices are constant in time.

Recently Spanos [11] and Mason [12] used this technique to develope approximate solutions for nonlinear systems subjected to deterministic and random inputs. In the present work, the equivalent linearization technique is employed in a different direction of application, namely to generate critical inputs for nonlinear structural systems. A brief summary of the linearization technique is presented in the first chapter. More detailed discriptions can be found in reference [8].

1.2 Formulation of the Problem

The response of a discrete n-degree-of-freedom nonlinear structural system subjected to dynamic inputs is described mathematically by the following set of differential equations:

$$M \cdot \ddot{\underline{y}} + C \cdot \dot{\underline{y}} + K \cdot \underline{y} + \underline{f} = \underline{x} \tag{1}$$

Where M is a n x n mass matrix, C is a n x n damping matrix and K is a n x n linear stiffness matrix. $\ddot{\underline{y}}$, $\dot{\underline{y}}$ and $\underline{y}$ are n-vectors of the system accelerations, velocities and displacements associated with its n-degrees-of-freedon. $\underline{f}$ is a n-vector of general function of $\dot{\underline{y}}$ and $\underline{y}$ expressing the nonlinearity of the system. $\underline{x}$ is a n-vector of the dynamic input.

The equivalent linear structural systems will be described mathematically by the following auxiliary system:

$$M \cdot \ddot{\underline{y}} + C^{\epsilon} \cdot \dot{\underline{y}} + K^{\epsilon} \cdot \underline{y} = \underline{x} \tag{2}$$

It is assumed that the equivalent linear system is time invariant and therefore the parameters C^{ϵ} and K^{ϵ}, which are called equivalent linear damping and stiffness, are time independent. They can be expressed as:

$$\left.\begin{aligned} C^{\epsilon} &= C + C^{o} \\ K^{\epsilon} &= K + K^{o} \end{aligned}\right\} \tag{3}$$

where C^{o} and K^{o} are n x n variable damping and stiffness matrices in connection with the nonlinearity of the system.

By proper selection of C^{o}, K^{o} the auxiliary structural system of Eq. (2) will become equivalent to the actual nonlinear one represented by Eq. (1). According to this formulation the determination of the equivalent linear structural system will be based on the evaluation of C^{o} and K^{o}. This will be done by minimizing the time average of the square of the difference between the two systems with respect to C^o and K^o. If $\underline{d}$ is a n-vector which represents this difference, i.e.

$$\underline{d} = \underline{f} - C^o \cdot \dot{\underline{y}} - K^o \cdot \underline{y} \tag{4}$$

By carrying out the minimization, matrices C^o and K^o will be determined in terms of some time averages between the n-vectors $\underline{y}^\epsilon$ and $\dot{\underline{y}}^\epsilon$, which are solutions of Eq. (2) and the n-vector f. This can be expressed as:

$$\left.\begin{aligned} c^o_{ij} &= \mathcal{A}\,(\underline{y}^\epsilon, \dot{\underline{y}}^\epsilon, \underline{f}) \\ k^o_{ij} &= \mathcal{A}\,(\underline{y}^\epsilon, \dot{\underline{y}}^\epsilon, \underline{f}) \\ &\text{where } i, j, = 1, 2, \ldots, n \end{aligned}\right] \tag{5}$$

The determination of C^o and K^o is by first assuming their trial values and then by solving Eq. (2) and Eq. (5) iteratively until the two succesive values of C^o and K^o differ within allowable limits. The derivation of Eq. (5) will follow.

1.3 Equivalent Linear Damping and Stiffness Matrices

The definition of the equivalent linear structural system is associated with the determination of the equivalent damping and stiffness matrices C^ϵ and K^ϵ or, according to Eqs. (3), the determination of C^o and K^o. This is based on a minimization technique similar to that given in reference [8].

The minimization criterion according to which the equivalency of Eqs. (1) and (2) is defined can be expressed mathematically as:

$$\min_{c^{o}_{ij},\; k^{o}_{ij}} \left| \mathcal{A}\,[\, \underline{d}^{T} \cdot \underline{d} \,] \right| \qquad \forall \quad \underline{y}(t) \in \mathcal{H} \tag{6}$$

where: i, j = 1, ..., n

where $\mathcal{H}$ is a class of solutions in which the solution of Eq. (2) is assumed to be a member. A particular member of the class $\mathcal{H}$ is identified by the parameters C^{o} and K^{o}. The symbol $\mathcal{A}$ denotes an averaging operator having the following properties:

I). $\frac{d}{dt}\, \mathcal{A}\,[\, z(t) \,] = 0$

II). $\mathcal{A}\,[\, z_1(t) + z_2(t) \,] = \mathcal{A}\,[\, z_2(t) \,] + \mathcal{A}\,[\, z_2(t) \,]$

III). $\mathcal{A}\,[\, z^2(t) \,] > 0 \; ; \quad \forall \; z(t) \neq 0$ and $\mathcal{A}\,[\, 0 \,] = 0$

IV). If :

$$Z = \begin{pmatrix} z_{11} \cdots & z_{1m} \\ \cdots\cdots & \cdots \\ z_{m1} \cdots & z_{mm} \end{pmatrix}$$

then:

$$\mathcal{A}[Z] = \begin{pmatrix} \mathcal{A}[\, z_{11} \,] \; \cdots & \mathcal{A}[\, z_{1m} \,] \\ \cdots\cdots & \cdots \\ \mathcal{A}[\, z_{m1} \,] \; \cdots & \mathcal{A}[\, z_{mm} \,] \end{pmatrix}$$

By using the property II and Eq. (4) :

$$\mathcal{A}\left[d^{T} \cdot d \right] = \mathcal{A}\left[\sum_{\ell=1}^{n} d_{\ell}^{2} \right] = \sum_{\ell=1}^{n} D_{\ell} \tag{7}$$

where:

$$D_\ell = \mathcal{A}\,[\,d_\ell^2\,] = \mathcal{A}\left[\,(f_\ell - \sum_{p=1}^{n} c^o_{\ell p}\cdot y_p - \sum_{q=1}^{n} k^o_{\ell p}\cdot y_q)^2\right] \quad (8)$$

From Eq. (8) and according to the property III it is concluded that:

$$D_\ell \geq 0 \quad ; \quad \forall\ \ell = 1, 2, \ldots, n \quad (9)$$

combining Eqs. (7) and (8) the minimization procedure can be expressed as follows:

$$\left.\begin{array}{l} \text{for fixed value of } i, \\ \min_{c^o_{ij},\, k^o_{ij}} (D_i) \quad \forall \quad y(t) \in \mathcal{H} \\ \text{where:} \quad i, j = 1, \ldots, n. \end{array}\right] \quad (10)$$

Eq. (8) shows that D_i ; $i = 1, \ldots, n$ is an explicit function of c^o_{ij} and k^o_{ij}. According to differential calculus the mimimization procedure results to the following 2n simultaneous linear equations

$$\forall\ i = 1, 2, \ldots, n: \begin{cases} \dfrac{\partial D_i}{\partial c^o_{ij}} = 0 \\ \dfrac{\partial D_i}{\partial k^o_{ij}} = 0 \end{cases} \quad \forall \quad j = 1, 2, \ldots, n \quad (11)$$

which can be simplified to

$$B \cdot \underline{p}_i = \underline{q}_i ; \quad i = 1, 2, \ldots, n \quad (12)$$

where,

$$\underline{p}_i = \begin{Bmatrix} \underline{k}^o{}_i \\ --- \\ \underline{c}^o{}_i \end{Bmatrix} \quad \underline{q}_i = \begin{Bmatrix} \underline{r}_i \\ --- \\ \underline{s}_i \end{Bmatrix}$$

and (13)

$$B = \left(\begin{array}{c|c} W & L \\ \hline L & Z \end{array}\right)$$

W, L and Z are n x n submatrices with typical elements $\mathcal{A}[\ y_i \cdot y_j\ ]$, $\mathcal{A}[\ y_i \cdot \dot{y}_j\ ]$ and $\mathcal{A}[\ \dot{y}_i \cdot \dot{y}_j\ ]$ respectively. The n-vectors $\underline{k}^o{}_i$ and $\underline{c}^o{}_i$ are the i-th rows of the K^o and C^o matrices and the n-vectors $\underline{r}_i$, $\underline{s}_i$ have typical j-th elements $\mathcal{A}[\ f_i \cdot y_j\ ]$ and $\mathcal{A}[\ f_i \cdot \dot{y}_j\ ]$ respectively. From the solution of Eqs. (12) the matrices C^o and K^o are obtained for a given nonlinearity vector $\underline{f}$ and a set of responses $\underline{y}(t)$ and $\dot{\underline{y}}(t)$.

The invertibility of matrix B is very important in the above formulation. In reference [8] it has been prooved that for linearly independent functions y_i (+), $\dot{y}_i$ (+) ; i = 1, ... , n the matrix B is positive definite.

CHAPTER 2

CRITICAL EXCITATIONS FOR STRUCTURAL SYSTEMS

2.1 Introduction

The seismic assessment of socially or economically important structures requires high confidence level of reliability and consequently demands carefully selected ground excitation input for computing responses. These excitations must be on one hand credible ones and on the other hand produce upper bound responses for the structure in question. As more records of strong ground motions become available, the information concerning the characteristics of credible earthquakes becomes more accessible. Based on these statistical informations many investigators have proposed artifical earthquake models which are stationary or nonstationary stochastic processes. However these works in general do not consider specific site conditions or particular structures in question. Consequently there is no assurance that the ground motion thus produced can produce upper bound responses. In this thesis a new type of generated ground motion which is termed as "critical excitation" is introduced. It is defined as an excitation among a class of credible excitations recorded at a specific site condition that will produce the highest response of a specific structural design variable. Furthermore the excitation is subjected to an intensity constraint. The concept of a critical excitation is described in section 2.2 and the intensity constraint is analyzed in section 2.3. The generation of critical excitations for linear structural systems is given in section 2.4. Finally the results obtained in section 2.4 are extented in section 2.5 for nonlinear structural systems.

2.2 The Concept of A Critical Excitation

A stated before a critical excitation is defined as an excitation among a class of "credible" excitations of the same characteristics that will produce the largest response peak for a given structural system.

For a given site condition, a number of earthquake records can be selected from the available past recorded ground motions occurring at similiar geological sites, focal distances and macro or micro zonations. These selected records are called "basis" excitations:

$$x_i(t) \quad ; \quad i = 1, 2, \ldots, N \tag{14}$$

An excitation $x_m(t)$ constructed by any linear combination of these may be considered as a member of the class of "credible" excitations $\mathcal{E}$ if its intensity does not exceed an upper bound E appropriate for the given site condition. This is expressed as:

$$x_m(t) \in \mathcal{E} : \begin{cases} x_m(t) = \sum_{i=1}^{N} a_i \cdot x_i(t) = \underline{a}^T \cdot \underline{x} \\ \text{subjected to the constraint} \\ \| x_m(t) \| \leq E \end{cases} \tag{15}$$

where the symbol $\| \quad \|$ denotes intensity and will be defined in the next section. In mathematical terminology the class $\mathcal{E}$ lies in a linear manifold spanned by the basis excitations $x_i(t)$; $i = 1, 2, \ldots, N$ and it is a solid sphere which has as radius the intensity E.

A critical excitation symbolized as $\chi(t)$ is a member of the class $\mathcal{E}$ which is identified by the property that when applied to a given structure, it will produce its highest response. From this definition it is clear that a critical excitation is site as well as structural system dependent. The site depending is expressed by the vector $\underline{x}$ and the system dependency by the vector $\underline{a}$. The latter is obtained by a maximization procedure as will be discussed in section 2.4.

2.3 Intensity Constraint

According to Eq. (15) the intensity constraint is an important consideration for the generation of the critical excitations. Several intensity measures have been employed for seismic excitations. Housner [13] has suggested using the spectrum intensity (SI). In their work for generation of artificial earthquakes, Housner and Jennings [35] have alterately used the root mean square (RMS) of the excitations over a duration of 30 sec. Peak ground acceleration has also been used as another definition of intensity. In the present work the square integral or root-square (RS) of an excitation over a duration t_e is considered as the intensity measure. Thus, so if x(t) is an excitation then its intensity is defined as follows:

$$\| x(t) \| = \sqrt{\int_0^{t_e} x^2(t) \cdot dt}$$

The time t_e is the effective duration of the excitation [38].

According to the above definition and Eq, (15) the intensity constraint is expressed as:

$$E^2 \geq \int_0^{t_e} \left[\sum_{i=1}^{N} a_i \cdot x_i(t) \right]^2 \cdot dt =$$

$$\sum_{i=1}^{N} \sum_{j=1}^{N} a_i \cdot a_j \cdot \int_0^{t_e} x_i(t) \cdot x_j(t) \cdot dt$$

or:

$$E^2 \geq \underline{a}^T \cdot G \cdot \underline{a} \tag{16}$$

where G is a N x N matrix with typical element

$$g_{ij} = \int_0^{t_e} x_i(t) \cdot x_j(t) \cdot dt \quad ; \qquad i, j=1, \ldots, N \tag{17}$$

and $\underline{a}$ is the N-vector of the weighing coefficients applied to the basis excitations.

2.4 Critical Excitations and Responses of Linear Structural Systems

In this section the generation of critical excitations associated with n-degree-of-freedom discrete linear structural systems is presented. The results which are obtained here will be extented to nonlinear case in section 2.5.

The differential equation which describes the system is:

$$M \cdot \ddot{\underline{y}} + C \cdot \dot{\underline{y}} + K \cdot \underline{y} = \underline{x} \tag{18}$$

where M, C and K are constant n x n matrices representing the its mass, damping and stiffness. The n-vectors $\underline{x}$ and $\underline{y}$ are the input and the response respectively. The individual responses of this system due to a given basis of excitations $x_j(t)$; $j=1, \ldots, N$ are described by defining two matrices Y and $\dot{Y}$. Both are n x N matrices with typical elements $y_{ij}(t)$ and $\dot{y}_{ij}(t)$ defined as the response and the velocity associated with the i-th degree-of-freedom of the linear structural system due to the j-th basis excitation. A critical excitation of this linear structural system will be defined with respect to its n-degrees-of-freedom $i=1, \ldots, n$. Let $\chi_i(t)$ to be the critical excitation associated with the i-th degree-of-freedom of the system. If a_{ij} represents the weighing coefficient applied to j-th basis excitation to obtain the $\chi_i(t)$ then according to Eq. (15)

$$\left.\begin{array}{l} \chi_i(t) = \sum_{j=1}^{N} a_{ij} \cdot x_j(t) = \underline{a}_i^T \cdot \underline{x} \\ \text{subjected to the constraint} \\ \| \chi_i(t) \| \leq E \\ i = 1, \ldots, n \end{array}\right\} \qquad (19)$$

where

Let $\psi_i(t)$ be the response of the system due to $\chi_i(t)$. This response is obtained by convolution operation as follows:

$$\psi_i(t) = \left\{ \sum_{j=1}^{N} a_{ij} \cdot x_j(t) \right\} * h_i(t)$$

$$= \sum_{j=1}^{N} a_{ij} \cdot y_{ij}(t) = \underline{a}_i^T \cdot \underline{y}_i(t) \qquad (20)$$

where $\underline{y}_i$ is the i-th row of the matrix Y. The symbol $*$ stands for convolution and $h_i(t)$ is the unit impulse response of the i-th degree-of-freedom of the system. The response $\psi_i(t)$ will be called the i-th critical response.

By extending the above, then for a given basis of ground excitations $x_j(t)$; $j=1, \ldots, N$, there are n critical excitations $\chi_i(t)$; $i=1, \ldots, n$ associated with the n degrees-of-freedom of Eq. (18) which are expressed as:

$$\underline{\chi}(t) = A \cdot \underline{x}(t) \qquad (21)$$

where $\underline{\chi}(t)$ is a n-vector of the n critical excitations and x(t) is a N-vector of the N basis excitations. Matrix A is a n x N matrix with typical element a_{ij}, where index i indicates the degree-of-freedom and j the basis excitation.

In order to evaluate a critical excitation associated with the i-th degree-of-freedom of the structural system of Eq. (18) the response $\psi_i(t)$ must be maximized under the constraint developed in section 2.3. When i is fixed and for a particular time t the mathematical formulation becomes:

$$\left.\begin{array}{l} \left| \sum_{j=1}^{N} a_{ij} \cdot y_{ij}(t) \right| = \max \quad \forall \quad a_{ij} \quad ; \quad j=1, \ldots, N \\ \text{subjected to constraint:} \\ \underline{a}_i^T \cdot G \cdot \underline{a}_i \leq E^2 \end{array}\right] \qquad (22)$$

According to the above expression, the determination of the critical excitation $\chi_i(t)$ requires a double maximization under a constraint. The maximization with respect to $\underline{a}_i$ for fixed time t is carried out by using a standard technique and that with respect to t by using a numerical evaluation.

The first maximization operation is done by using a Lagrangian multiplier λ_i as follows:

$$\left.\begin{array}{l} \text{For } i \text{ fixed and } k = 1, \ldots, n \\ \frac{\partial}{\partial a_{k\ell}} \left[\sum_{j=1}^{N} a_{ij} \cdot y_{ij}(t) + \lambda_i \cdot \sum_{p=1}^{N} \sum_{q=1}^{N} a_{ip} \cdot a_{iq} \cdot g_{pq} \right] = 0 \\ \forall \quad \ell = 1, \ldots, N \end{array}\right] \qquad (23)$$

The solution of which in matrix form can be written as:

$$\underline{a}_i = \underline{a}_i(t) = -\frac{1}{2\lambda_i} \cdot G^{-1} \cdot \underline{y}_i(t) \qquad (24)$$

The value of the Lagrangian multiplier λ_i is determined by substituting the value $\underline{a}_i$ into (16) and observing the symmetry of matrix G:

$$\frac{1}{4\lambda_i^2} \cdot \underline{y}_i^T(t) \cdot G^{-1} \cdot \underline{y}_i(t) \leq E^2$$

For a fixed value of time the response will be maximized when the equality prevails in the above relationship:

$$\lambda_i = \lambda_i(t) = \frac{1}{2E} \cdot \sqrt{\underline{y}_i^T(t) \cdot G^{-1} \cdot \underline{y}_i(t)} \qquad (25)$$

This value of λ_i is then substituted into Eq. (24) and the vector $\underline{a}_i$ is evaluated for the time t.

The second maximization, that is with respect to t, is done numerically by dividing the total duration into sufficiently representative steps and by comparing and selecting the largest value of the response at these steps.

According to the above the critical excitation $\chi_i(t)$ is obtained by the determination of the weighing coefficients or the vector $\underline{a}_i$ applied to the basis excitations $\underline{x}(t)$ so that the response $\Psi_i(t)$ is a maximum. By repeating this procedure for i=1, ..., n, the matrix A of Eq. (21) is obtained.

2.5 Critical Excitations of Nonlinear Structural Systems.

For a given basis excitations the maximization of the response of a nonlinear system described by Eq. (1) subjected to an excitation which belongs to the class $\mathcal{E}$ is not a straightforward problem. This is because of the presence of the nonlinear vector $\underline{f}$. Therefore only approximated critical excitations can be generated. In this section an extension of the results of section 2.4 is employed to obtain the critical excitation of an equivalent linearized system.

A class of auxiliary linear structural systems in the form of Eq. (2) is associated with a given nonlinear one of Eq. (1) based on the parameters C^ϵ and K^ϵ. The equivalent linear one is selected from this class according to the criterion that the values of C^{ϵ} and K^{ϵ} will minimize the time average of the square difference between the two systems. This is expressed by Eq. (12) of section 1.3. To find the critical excitations of the linear auxiliary systems of a nonlinear one, the basis excitations $x_j(t)$ and the coefficients a_{ij} where $i=1, \ldots, n$ and $j=1, \ldots, N$ have to be incorporated in the minimization criterion through Eqs. (12).

For a given basis of excitations $x_j(t)$; $j=1, \ldots, N$ and a reference intensity E the critical excitations $\chi_i(t)$ and the critical responses and velocities $\psi_i(t)$, $\dot{\psi}_i(t)$; $i=1, \ldots, n$ of the auxiliary linear system of Eq. (2) corresponding to the values C^{ϵ} and K^{ϵ} are obtained according to the method developed in section 2.4, i.e.

$$\chi_i(t) = \sum_{j=1}^{N} a_{ij} \cdot x_j(t) = \underline{a}_i^T \cdot \underline{x}(t) \tag{26}$$

$$\psi_i(t) = \sum_{j=1}^{N} a_{ij} \cdot y_{ij}(t) = \underline{a}_i^T \cdot \underline{y}_i(t) \tag{27}$$

$$\dot{\psi}_i(t) = \sum_{j=1}^{N} a_{ij} \cdot \dot{y}_{ij}(t) = \underline{a}_i^T \cdot \dot{\underline{y}}_i(t) \tag{28}$$

In the above Eqs. (26) and (27) the N-vectors $\underline{y}_i(t)$ and $\dot{\underline{y}}(t)$ are the i-th rows of the matrices Y and $\dot{Y}$ defined in section 2.4.

The criterion of equivalency between the actual nonlinear structural system of Eq. (1) and the auxiliary linear one of Eq. (2) as expressed by Eq. (12) can now be modified by including the weighing coefficients a_{ij}'s as follows:

The n x n submatrices W, L and Z of the matrix B of Eq. (12) will have typical elements as:

$$w_{ij} = \mathcal{A}\,[\psi_i(t) \cdot \psi_j(t)]$$

$$= \mathcal{A}\left[\left\{ \sum_{k=1}^{N} a_{ik} \cdot y_{ik}(t) \right\} \cdot \left\{ \sum_{m=1}^{N} a_{jm} \cdot y_{jm}(t) \right\} \right]$$

$$= \sum_{k=1}^{N} \sum_{m=1}^{N} a_{ik} \cdot a_{jm} \cdot \left\{ \mathcal{A}\left[y_{ik}(t) \cdot y_{jm}(t) \right] \right\}$$

or in matrix form:

$$w_{ij} = \underline{a}^T_i \cdot W^o_{ij} \cdot \underline{a}_j \tag{29}$$

where W^o_{ij} is a N x N matrix with typical $w^o_{k,m}$ element,

$$w^o_{km} = \mathcal{A}\left[y_{ik}(t) \cdot y_{jm}(t) \right] \tag{30}$$

Similarly,

$$l_{ij} = \underline{a}^T_i \cdot L^o_{ij} \cdot \underline{a}_j \text{ with typical } l^o_{km} = \mathcal{A}\left[y_{ik}(t) \cdot \dot{y}_{jm}(t) \right] \tag{31}$$

$$z_{ij} = \underline{a}^T_i \cdot Z^o_{ij} \cdot \underline{a}_j \text{ with typical } z^o_{km} = \mathcal{A}\left[\dot{y}_{ik}(t) \cdot \dot{y}_{jm}(t) \right] \tag{32}$$

The sub-vectors $\underline{r}_i$ and $\underline{s}_i$ of vector $\underline{q}_i$ can be derived as follows:
The j-th element of the $\underline{r}_i$; $i=1, \ldots, n$ vectors is:

$$\mathcal{A}\left[f_i \cdot \psi_j(t) \right]$$

$$= \mathcal{A}\left[f_i \cdot \left\{ \sum_{k=1}^{N} a_{jk} \cdot y_{jk}(t) \right\} \right]$$

or in matrix form:

$$\underline{a}_j^T \cdot \underline{r}_j^{(i)} \tag{33}$$

where $\underline{r}_j^{(i)}$ is a N-vector with typical k-th element:

$$\mathcal{A}\left[f_i \cdot y_{jk}(t)\right] \tag{34}$$

Similarly the j-th element of the $\underline{s}_i$ vector is:

$$\underline{a}_j^T \cdot \underline{s}_j^{(i)} \tag{35}$$

where $\underline{s}_j^{(i)}$ is a N-vector with k-th element as:

$$\mathcal{A}\left[f_i \cdot \dot{y}_{jk}(t)\right] \tag{36}$$

Further modification of submatrices W, L and Z leads to the following:

$$W = A_o^T \cdot Q \cdot A_o \tag{37}$$

$$L = A_o^T \cdot R \cdot A_o \tag{38}$$

$$Z = A_o^T \cdot S \cdot A_o \tag{39}$$

where A_o is a nN by n matrix

$$A_o = \begin{bmatrix} \underline{a}_1 & 0 & . & . & . & 0 \\ 0 & \underline{a}_2 & . & . & . & 0 \\ . & . & . & & & . \\ . & . & & . & & . \\ . & . & & & . & . \\ 0 & 0 & . & . & . & \underline{a}_n \end{bmatrix} \tag{40}$$

and Q, R and S are nN by nN square matrices with typical elements the N x N submatrices W^o_{ij}, L^o_{ij}, and Z^o_{ij}, given by Eqs, (30), (31) and (32) respectively.

By introduction of W, L and Z from Eqs. (37), (38) and (39) into matrix B it is obtained:

$$B = H^T \cdot B_o \cdot H \tag{41}$$

where:

$$B_o = \left[\begin{array}{c|c} Q & R \\ \hline R & S \end{array}\right] \tag{42}$$

and

$$H = \left[\begin{array}{c|c} A_o & 0 \\ \hline 0 & A_o \end{array}\right] \tag{43}$$

The vectors $\underline{r}_i$ and $\underline{s}_i$; $i = 1, \ldots, n$ are further modified according to Eqs. (33) and (35) as:

$$\underline{r}_i = A_o^T \cdot \underline{e}_i \tag{44}$$

and

$$\underline{s}_i = A_o^T \cdot \underline{u}_i \tag{45}$$

where A_o has been defined by Eq. (40) and $\underline{e}_i$ and $\underline{u}_i$ are nN-vectors with j-th elements the sub-vectors $\underline{r}_j^{(i)}$ and $\underline{s}_j^{(i)}$, given by Eqs. (34) and (36). By combining Eqs. (44) and (45) the vector $\underline{q}_i$ takes the form:

$$\underline{q}_i = H^T \cdot \underline{b}_i \tag{46}$$

where

$$\underline{b}_i = \begin{Bmatrix} \underline{e}_i \\ ---- \\ \underline{u}_i \end{Bmatrix} \tag{47}$$

In summary the criterion of the equivalency between the nonlinear system and equivalent linear one, leads to the following equations:

$$\left.\begin{array}{l} B \cdot \underline{p}_i = \underline{q}_i \quad ; \quad i=1, \ldots, n \\ \text{where} \\ B = H^T \cdot B_o \cdot H \quad ; \quad \underline{q}_i = H^T \cdot \underline{b}_i \\ \text{and} \\ \underline{p}_i = \begin{Bmatrix} \underline{k}^o_i \\ ----- \\ \underline{c}^o_i \end{Bmatrix} \end{array}\right] \tag{48}$$

It is noted that the matrix B_o depends only on the individual responses of the auxiliary system of Eq. (2) due to the basis excitations. On the other hand the vectors $\underline{b}_i$ depend, in addition, on the nonlinearity of the given nonlinear structural system (i.e. f_i). According to Eqs. (48) it can be seen that the matrix B and vector $\underline{q}_i$ are in a transformation form through the matrix H which depends on the coefficients a_{ij} of the critical excitations. From a computational point of view it is noted that for a specific auxiliary structural system the matrix B is computed only once for all the values of i in the derivation of the equations $B \cdot \underline{p}_i = \underline{q}_i$.

For a given basis of excitations and a reference intensity E, the critical excitations associated with a given nonlinear structural system are obtained by the following successive approximation procedure:
Any trial set of values C^{ϵ} and K^{ϵ} will define an auxiliary linear system. By employing the method described in section 2.4 the associated critical excitations as well as the individual responses due to the basis excitations can be obtained. By introducing the weighing coefficients a_{ij} and the individual responses into Eqs. (48) a new set of values for C^{ϵ} and K^{ϵ} is obtained and consequently a new auxiliary structrual system is defined. This procedure will be carried out continously until the successive values of C^{ϵ} and K^{ϵ} differ insignificantly. The last set of C^{ϵ} and K^{ϵ} will define the equivalent linear system from which the critical excitations of the nonlinear system are obtained.
The magnitude of the nonlinearity involved in the above formulation is very important. By comparison, it has been concluded that the equivalent linearization technique, when is employed for the determination of critical excitations for elastoplastic systems, gives good results for values of ductility factor smaller or equal to 3.0.

2.6 Applications

2.6.1. Critical Excitations for a Single Degree-Of-Freedom System

In order to illustrate the concept of critical excitation for a single degree-of-freedom system, the model of a one story frame structure shown in Fig. 1(a) is considered. The mass, damping and linear stiffness are 5.0 $k\text{-sec}^2/ft$, 2% and 8773 k /ft respectively. The linear elastic and the elastoplastic behavior of this structure are analyzed.

The first ten ground accelerations listed in Table 1, are chosen as the basis excitations occured at stiff soil sites. The reference intensity E is chosen to correspond the N-S component of the Imperial Valley Earthquake, recorded at El Centro on May 18, 1940. The value of this intensity is equal to 7.9 $ft/sec.^{3/2}$ A total duration of 4.0 sec with a time step equal to 0.00571 sec has been considered for the analysis. The results are summarized in Table 2.

The linear elastic response is based on the natural period of the frame equal to 0.15 sec. The elastic critical excitation is shown in Fig. 3(a). The elastoplastic response is characterized by the restoring force shown in Fig. 1(b) and the nonlinear function f shown in Fig. 1(c). For a fixed ductility factor $\mu = 3.0$ the equivalent linear system has been found after three iterations with natural period equal to 0.18 second and damping ratio equal to 2.5%.

The envelope response is defined as the maximum response among those produced by the basis excitations acting individually on the system. The envelope response for the linear elastic case is due to the N-S component of the Lower California Earthquake, as recorded at El Centro on December 30, 1934 and that for the elastoplastic case is due to the San Francisco Earthquake recorded at the Alexander Building S. F. on March 22, 1957.

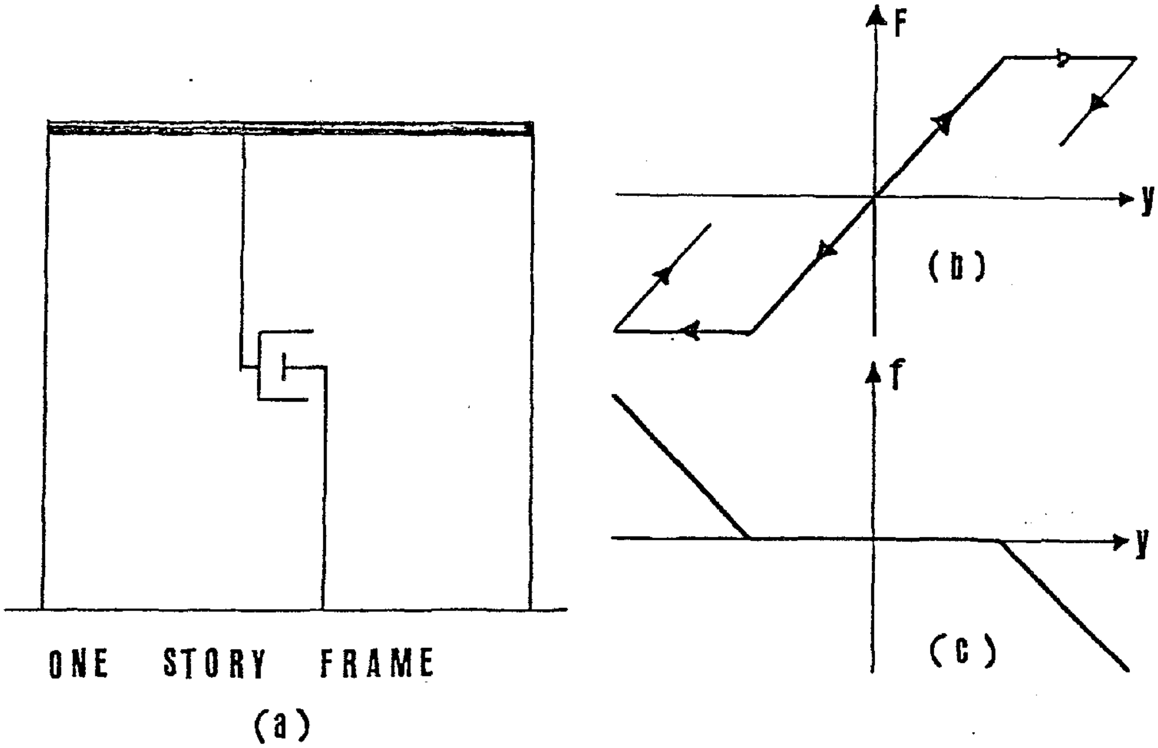


FIGURE 1

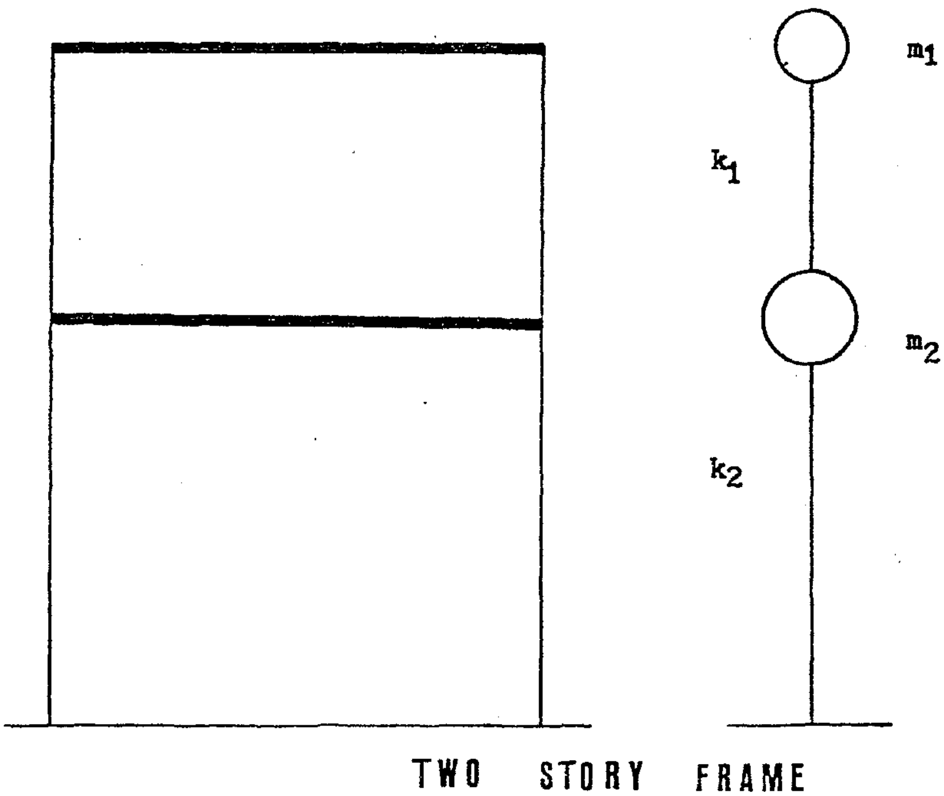


FIGURE 2

Table I. Basis Excitations for Stiff Soil Sites

File Number	Earthquake	Date	Record	Comp.
A001	Imperial Valley	5/18/40	El Centro	S00E
A001	Imperial Valley	5/18/40	El Centro	S90W
A014	San Francisco	3/22/57	Alexander Bldg, S. F.	N09W
A014	San Francisco	3/22/57	Alexander Bldg, S. F.	N81E
A016	San Francisco	3/22/57	State Bldg, S. F.	S09E
A016	San Francisco	3/22/57	State Bldg, S. F.	S81W
B024	Lower Calif.	12/30/34	El Centro	S00W
B024	Lower Calif.	12/30/34	El Centro	S90W
B034	Parkfield	6/27/66	Cholame Shandon 5	N05W
B034	Parkfield	6/27/66	Cholame Shandon 5	N85E
D056	San Fernando	2/09/71	Castai	N21E
D056	San Fernando	2/09/71	Castaic	N69W
D058	San Fernando	2/09/71	Hollywood Storage L. A.	S00W
D058	San Fernando	2/09/71	Hollywood Storage L. A.	N90E
E083	San Fernando	2/09/71	3407 6th St., L. A.	S00W
E083	San Fernando	2/09/71	3407 6th St., L. A.	N90E
H115	San Fernando	2/09/71	15250 Ventura Blvd. L. A.	N11E
H115	San Fernando	2/09/71	15250 Ventura Blvd. L. A.	N79W
Q233	San Fernando	2/09/71	14724 Ventura Blvd. L. A.	S12W
Q233	San Fernando	2/09/71	14724 Ventura Blvd. L. A.	N78W

TABLE 2: Peak Responses (inches) of One Story Frame Structure

	Linear	Non-Linear	
Iterations	1	2	3
El Centro	0.18	0.27	0.28
Envelope	0.27(1)	0.46(2)	0.47(3)
Critical	0.36	0.62	0.63

Note for envelope responses

(1) Response peak due to Lower California Earthquake at El Centro. (NS Component, December 30, 1934).

(2) Response peak due to San Fernando Earthquake at Castaire. (N69W component, February 9, 1971)

(3) Response peak due to San Francisco Earthquake at Alex. Bldg. (N09W component, March 22, 1957)

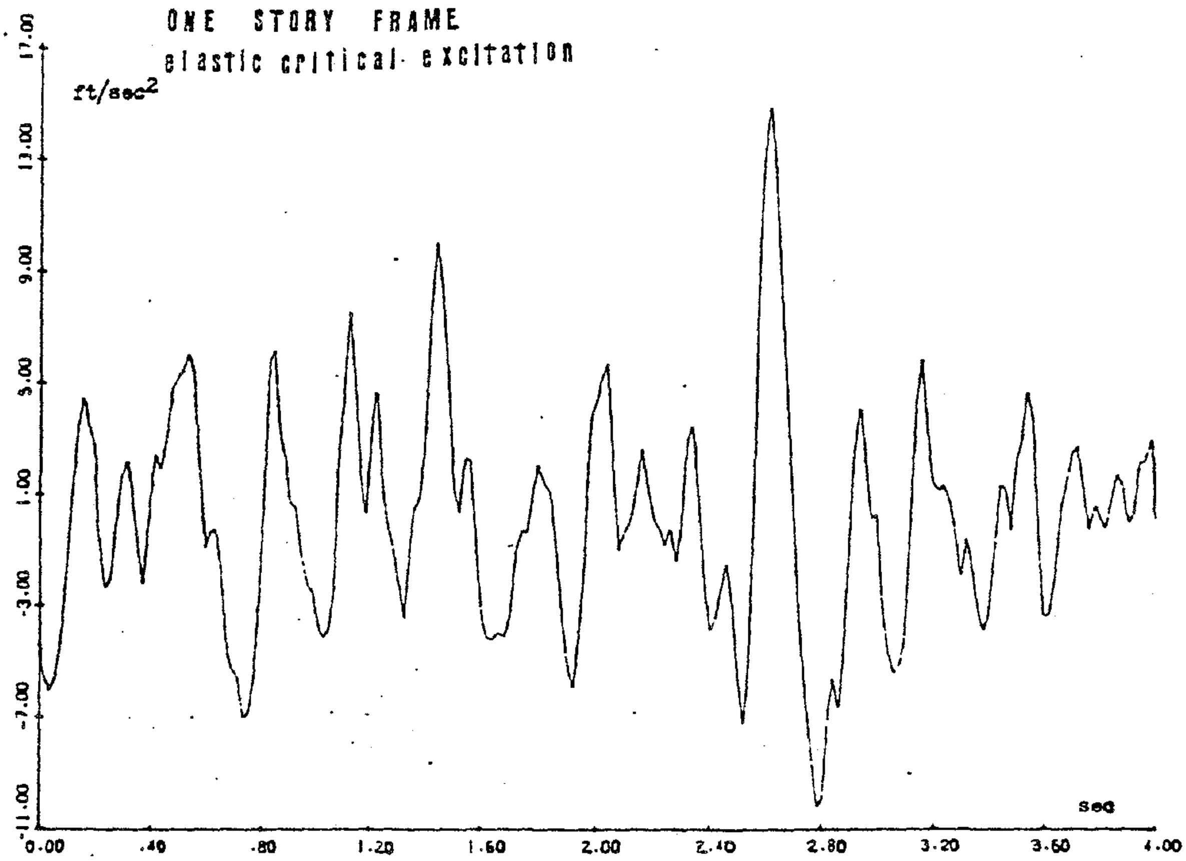
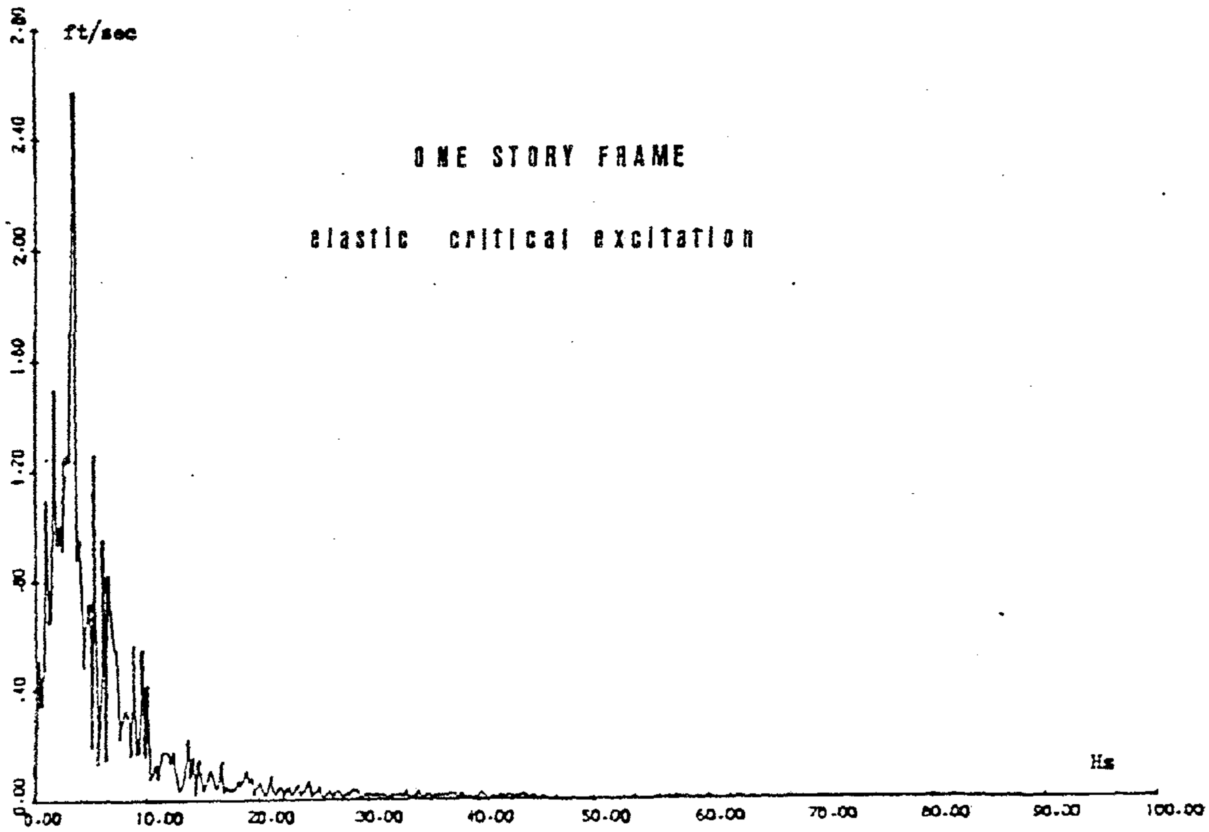

ONE STORY FRAME
elastic critical excitation
ft/sec²
17.00
13.00
9.00
5.00
1.00
-3.00
-7.00
-11.00
sec
0.00
.40
.80
1.20
1.60
2.00
2.40
2.80
3.20
3.60
4.00

FIG. 3(a)


ft/sec
ONE STORY FRAME
elastic critical excitation
2.80
2.40
2.00
1.60
1.20
.80
.40
0.00
Hz
0.00
10.00
20.00
30.00
40.00
50.00
60.00
70.00
80.00
90.00
100.00

FIC. 3(b)

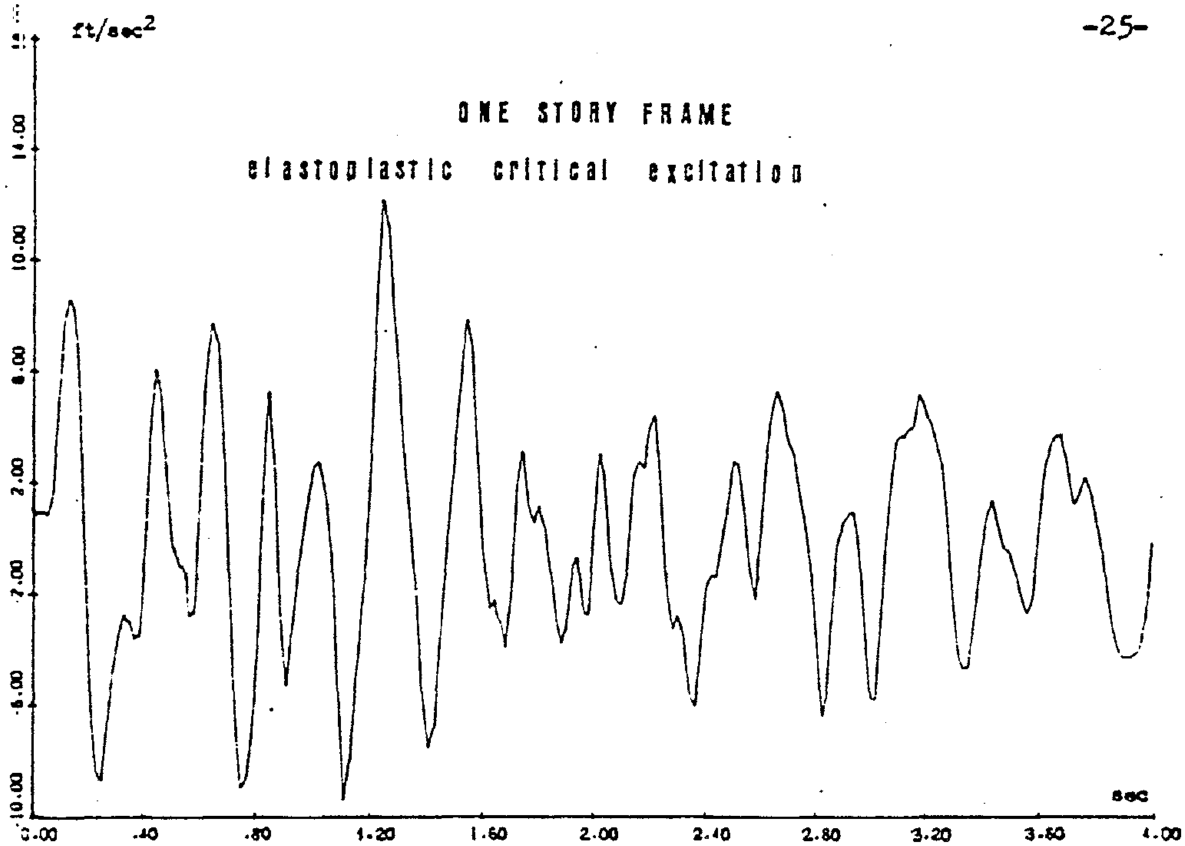

ft/sec²
ONE STORY FRAME
elastoplastic critical excitation
sec
0.00
.40
.80
1.20
1.60
2.00
2.40
2.80
3.20
3.60
4.00
14.00
10.00
6.00
2.00
-2.00
-6.00
-10.00

FIG. 4(a)

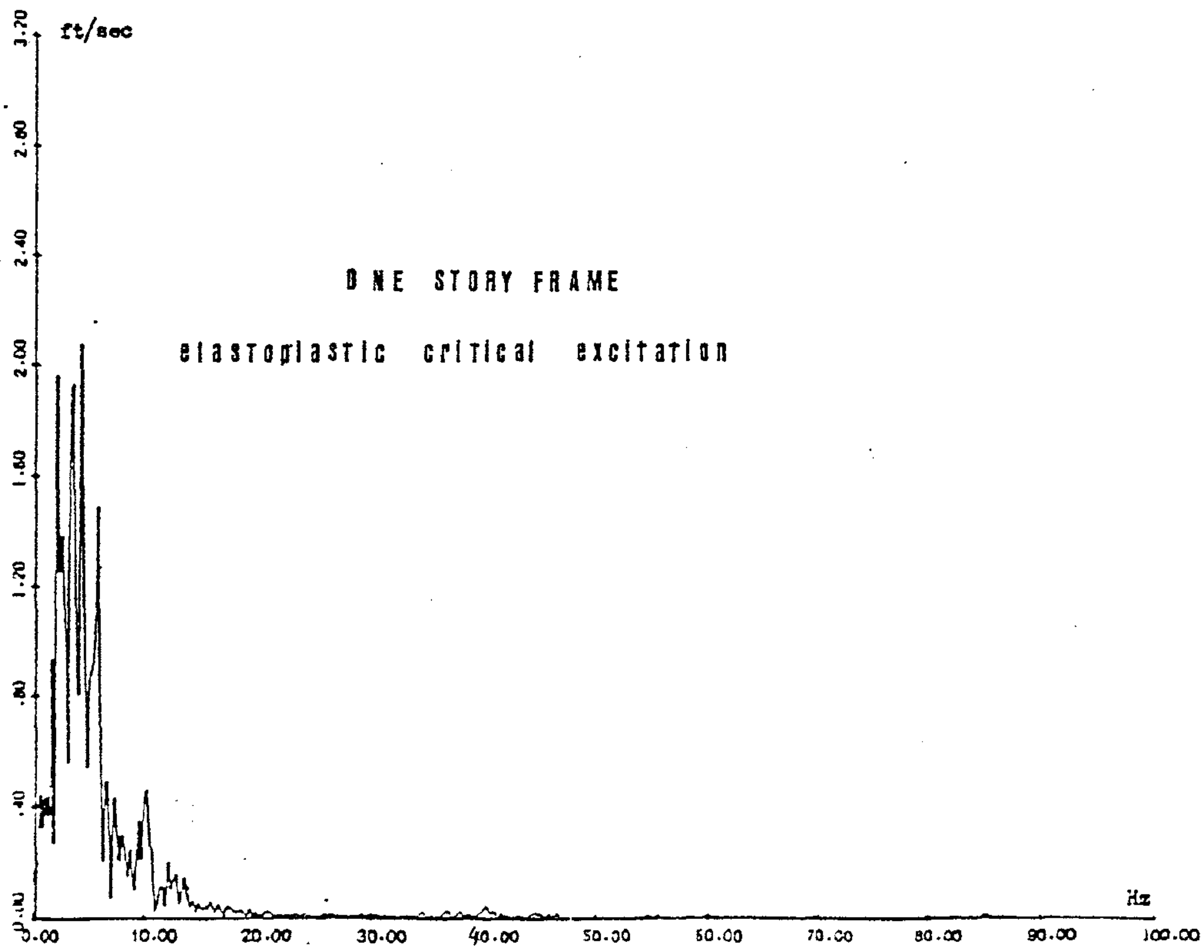

ft/sec
ONE STORY FRAME
elastoplastic critical excitation
Hz
0.00
10.00
20.00
30.00
40.00
50.00
60.00
70.00
80.00
90.00
100.00
3.20
2.80
2.40
2.00
1.60
1.20
.80
.40

FIG. 4(b)

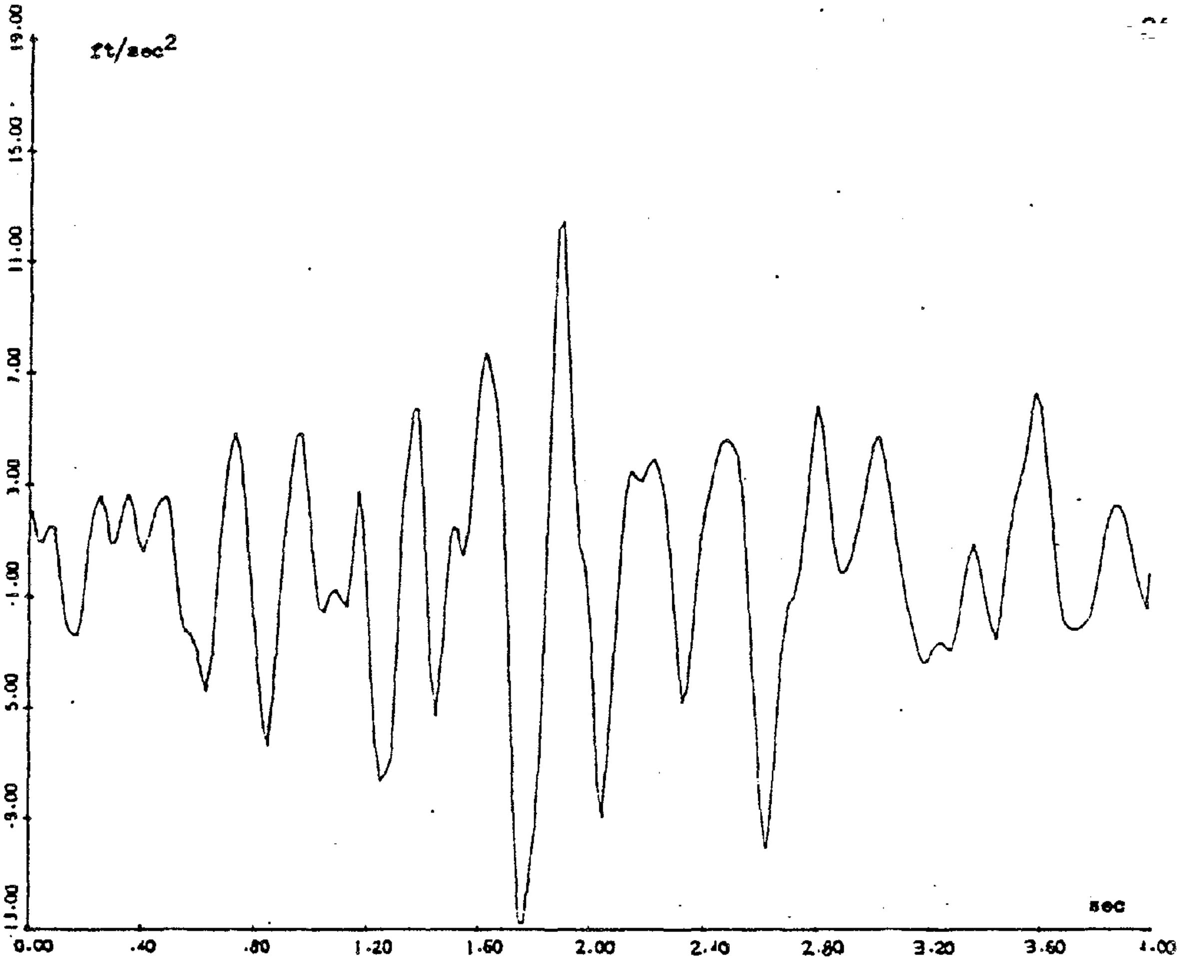

ft/sec^2
19.00
15.00
11.00
7.00
3.00
-1.00
5.00
9.00
13.00
sec
0.00
.40
.80
1.20
1.60
2.00
2.40
2.80
3.20
3.60
4.00

FIG. 5(a)

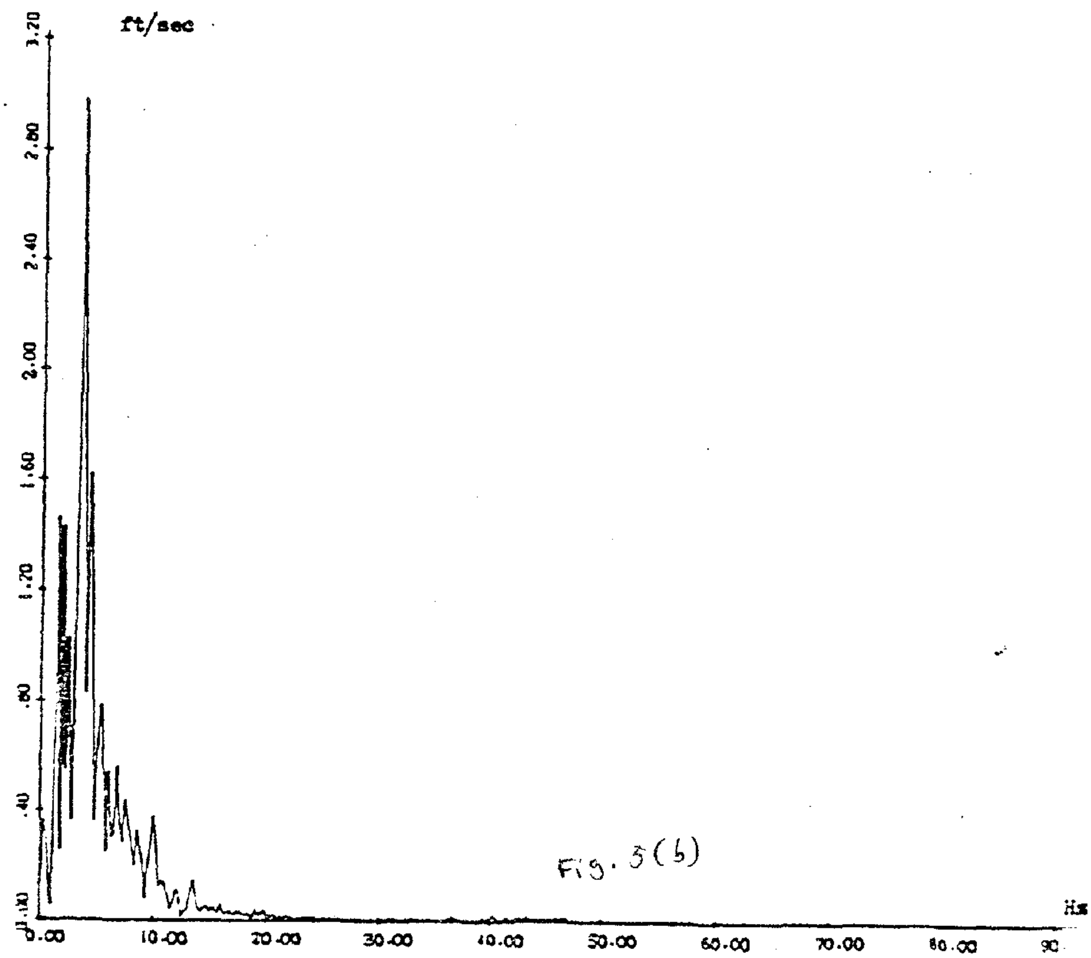

ft/sec
3.20
2.80
2.40
2.00
1.60
1.20
.80
.40
0.00
Hz
0.00
10.00
20.00
30.00
40.00
50.00
60.00
70.00
80.00
90.

Fig. 5(b)

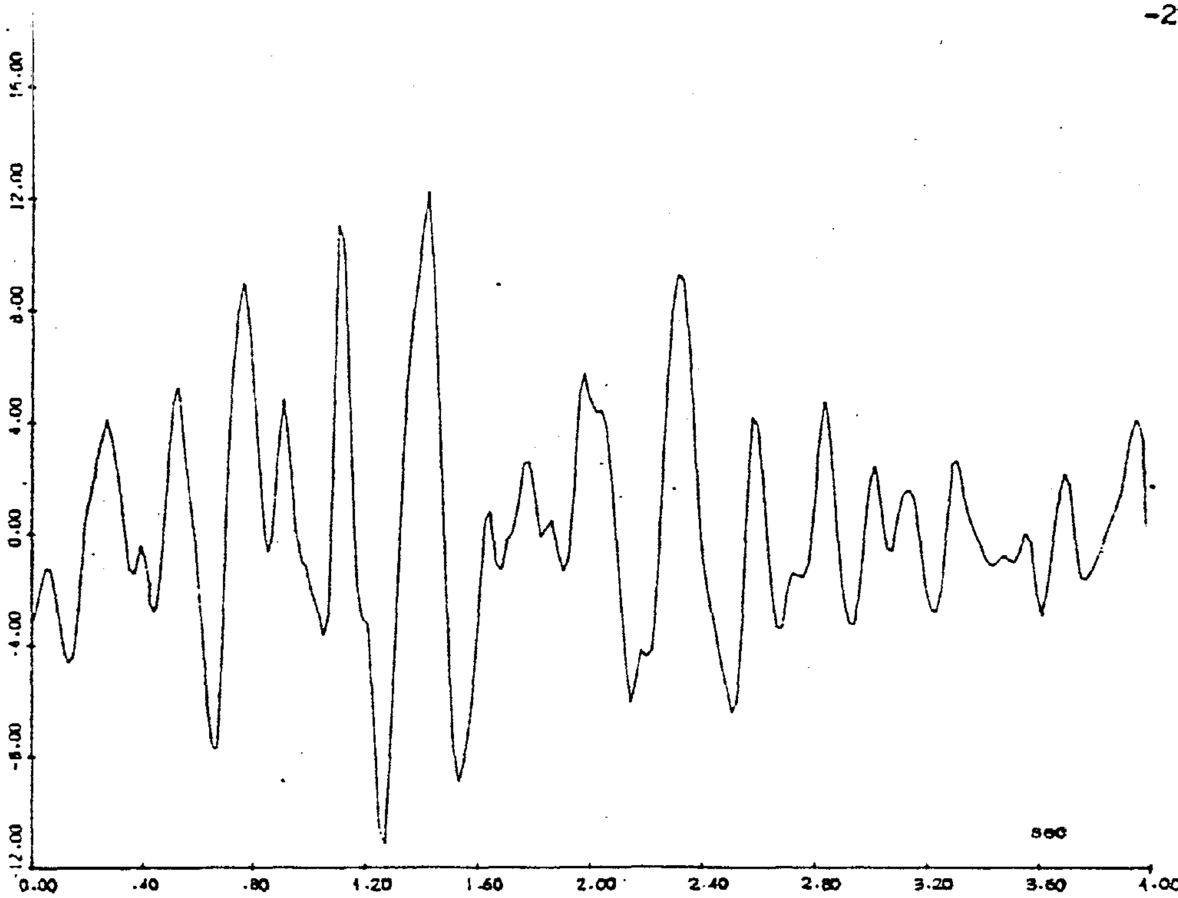

16.00
12.00
8.00
4.00
0.00
-4.00
-8.00
-12.00
sec
0.00
.40
.80
1.20
1.60
2.00
2.40
2.80
3.20
3.60
4.00

FIG. 6(a)

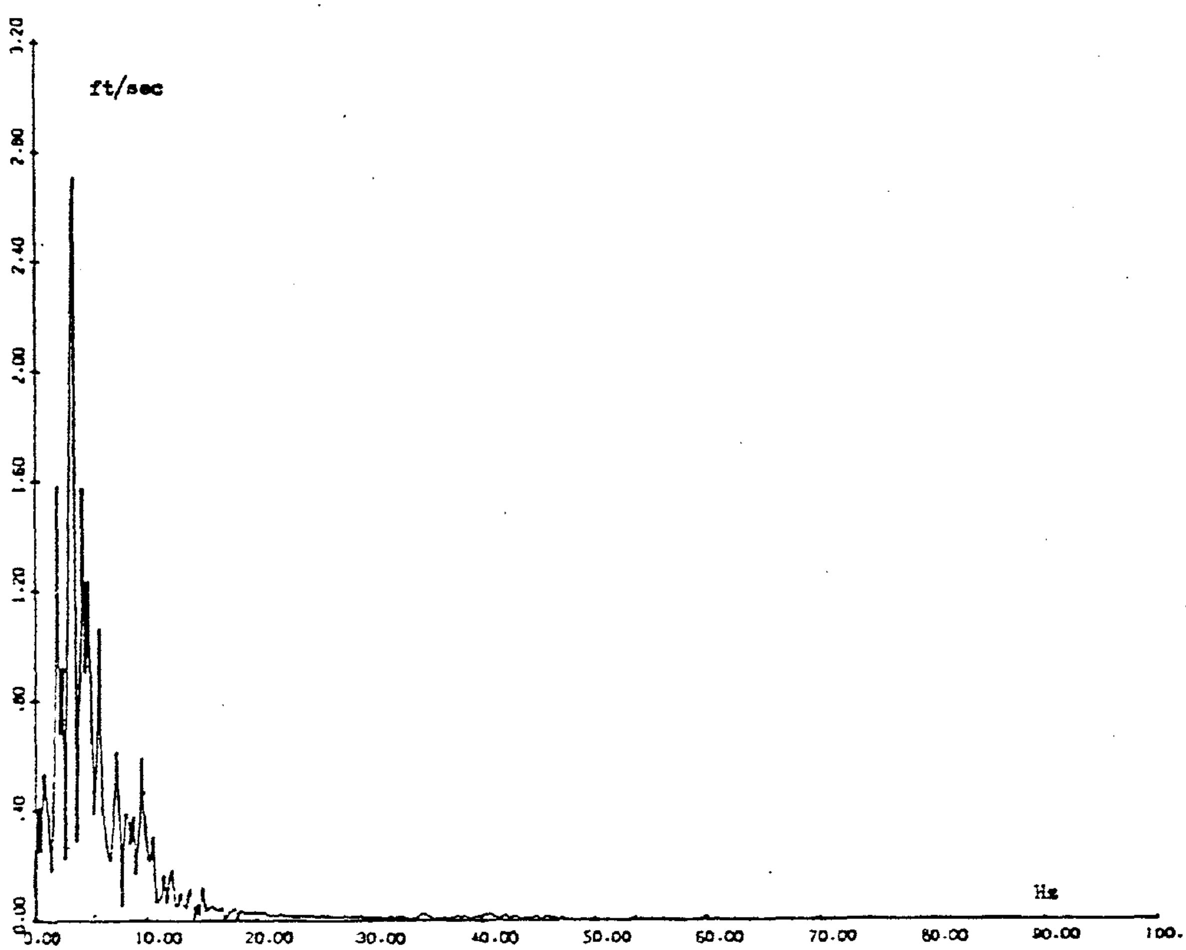

3.20
ft/sec
2.80
2.40
2.00
1.60
1.20
.80
.40
0.00
Hz
0.00
10.00
20.00
30.00
40.00
50.00
60.00
70.00
80.00
90.00
100.

FIG. 6(b)

From the peak responses listed in Table 2, it is concluded that the ratio of critical to the envelope response for the elastic and elastoplastic case is equal to 1.33 and 1.34 respectively. This ratio indicates a small degree of conservatism associated with the critical excitation and appears to be fully justified for important structures. The total displacement due to the elastoplastic critical excitation is greater than that due to the elastic critical excitation. This fact is consistent with the result obtained by other investigators [17] that for high frequency systems the elastoplastic system produces greater total displacements than that produced by elastic ones. By inspection of the time-histories shown in Figs. 3(a), 4(a), 5(a), 6(a) and the frequency contents given by Fourier amplitude spectra shown in Figs. 3(b), 4(b), 5(b), 6(b) it can be stated that the generated critical excitations appear to be realistic samples of possible ground motions. Thus, they should not be excluded from the consideration for the design of important structures.

2.6.2 Critical Excitations for a Two Degree-Of-Freedom System

Critical excitations for a two-story frame structure shown in Fig. 2 are generated here in order to demonstrate the application of the theory to a two degrees-of-freedom structural system. It is assumed that this structure is located in an area characterized by stiff soil conditions. Accordingly, the first ten earthquake ground motions listed in Table 1 are used to form the basis excitations. As the reference intensity the square integral of the ground acceleration of the N-S component of the El Centro 1940 Earthquake has been taken for the analysis and an effective duration equal to 6.0 seconds with a time step equal to 0.0086 seconds is used.

The structure is idealized by a stick model. It's masses are lumped at the first and second floors equal to 5.0 k-sec^2/ft and 2.0 k-sec^2/ft respectively. The lateral stiffness of the first floor is 1600.0 k/ft and that of the second floor is 900.0 k/ft. The system is considered undamped. The results of the analysis are summarized in Table 3, where peak values of the responses are given.

The first iteration respesents the linear elastic case based on the mass and linear stiffness matrices. The elastic translations of the two floors are governed by two modes with periods equal to 0.45 and 0.23 seconds. By comparison of the results, it is concluded that the ratios of the critical to the envelope peak responses are 1.50 for the first floor translation and 1.45 for the second floor translation. These values indicate a slightly higher and justifiable conservatism in connection with the design of important structures.

The critical elastoplastic response analysis is done for a fixed ductility factor $\mu = 3.0$, by employing an equivalent linear frame. The results are obtained at the fourth iteration. Two modes with periods 0.68 seconds and 0.29 seconds have been found from the free vibration analysis of the equivalent linear system. The difference between these natural periods with the ones obtained previously from the elastic analysis indicates the well known result that yielding modifies completely the elastic vibration mode characteristics.

Again by considering the ratios of the critical to the envelope peak responses from the results of the fourth iteration listed in Table 3, they are found to be equal to 1.60 and 1.67 for the ductile translations of the first and second floor respectively. Same justification for using critical excitations for designing important structures appear to be valid.

Summarizing the results of the elastic and elastoplastic critical response analysis of the two story frame, it is concluded that in both cases the critical response concept introduces an additional safety margin for its seismic assessment. This additional safety margin is expressed by the ratio of the response due to critical excitation to the envelope response due to the basis excitations. These ratios ranging from 1.45 to 1.67 have been found, which appear to be reasonable, especially when important structures are considered.

An additional insight into the elastic and elastoplastic critical excitations of the two-story frame structure the time histories and their frequency contents were plotted and comparison with real earthquake ground motions were made. Figures 7(a) and 8(b) shows the time histories of the elastic and elastoplastic critical excitations generated for the translation of the top floor of the two-story frame structure. Their frequency contents are shown in Figs 7(b) and 8(b) respectively. Time histories and frequency contents of the two basis excitations associated with the elastic and elastoplastic envelope response of the top floor are shown in Fig. 9 and 10. By comparison of the time histories and the frequency contents of the critical excitations with those of the basis excitations it is concluded that the critical excitations are realistic candidates of possible ground motions.

TABLE 3: Peak Responses of the Two Story-Frame Structure

Peak Responses		ITERATION			
		Linear	Non-Linear		
		1	2	3	4
El Centro	2nd Floor Translation	5.70	2.65	7.25	7.21
	1st Floor Translation	3.32	1.50	4.87	4.66
Envelope	2nd Floor Translation	6.96 (1)	5.06 (2)	7.62 (3)	7.55 (3)
	1st Floor Translation	3.97 (1)	2.85 (2)	5.14 (3)	5.04 (3)
Critical	2nd Floor Translation	10.11	7.86	12.53	12.63
	1st Floor Translation	5.98	4.43	8.33	8.09

Note:

(1) Response Peaks due to San Fernando Earthquake. (Castaire, February 9, 1971, component N69W).

(2) Response Peaks due to San Fernando Earthquake. (3407 6th St., L. A., February 9, 1971, Component N90E).

(3) Response Peaks Due to San Fernando Earthquake. (Hollywood Storage L. A. , February 9, 1971, Component NS).

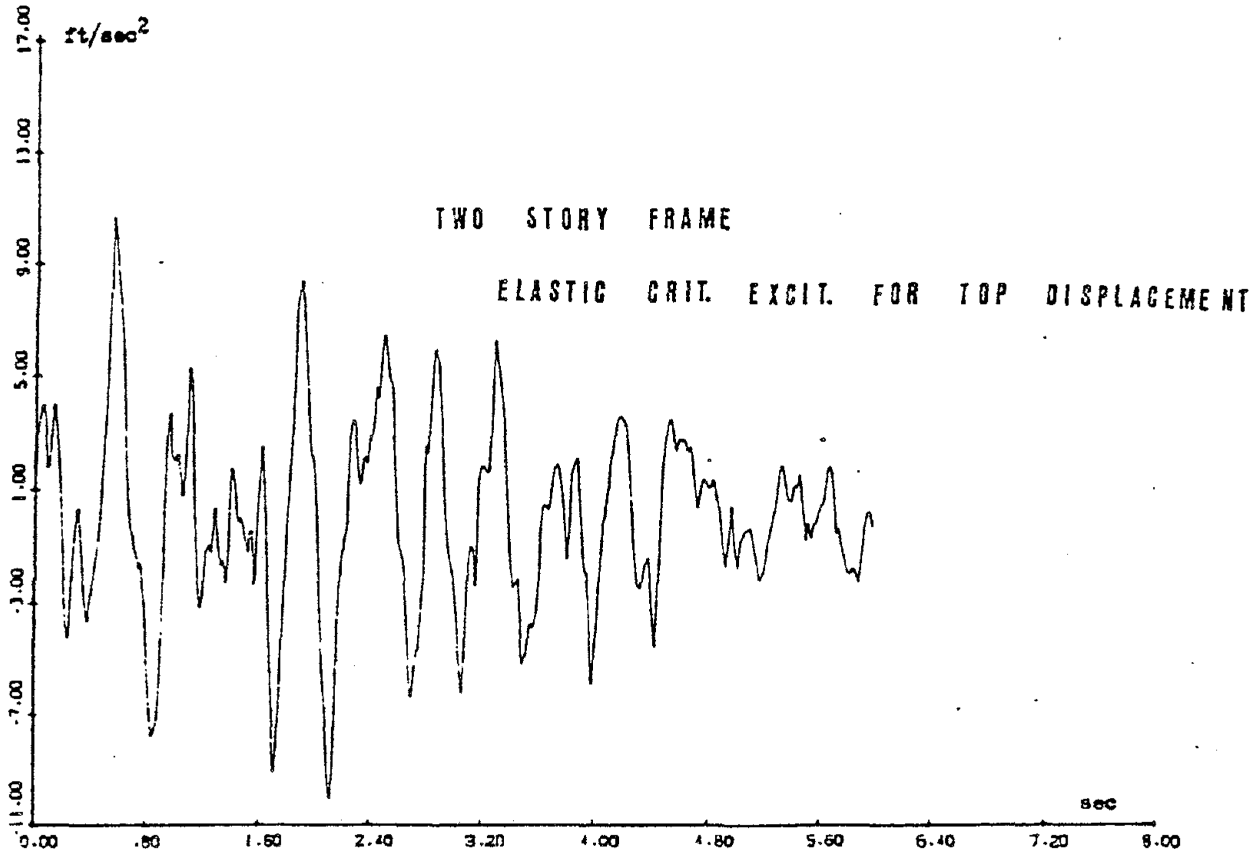

ft/sec^2
17.00
13.00
9.00
5.00
1.00
-3.00
-7.00
-11.00
TWO STORY FRAME
ELASTIC CRIT. EXCIT. FOR TOP DISPLACEMENT
sec
0.00
.80
1.60
2.40
3.20
4.00
4.80
5.60
6.40
7.20
8.00

FIG. 7(a)

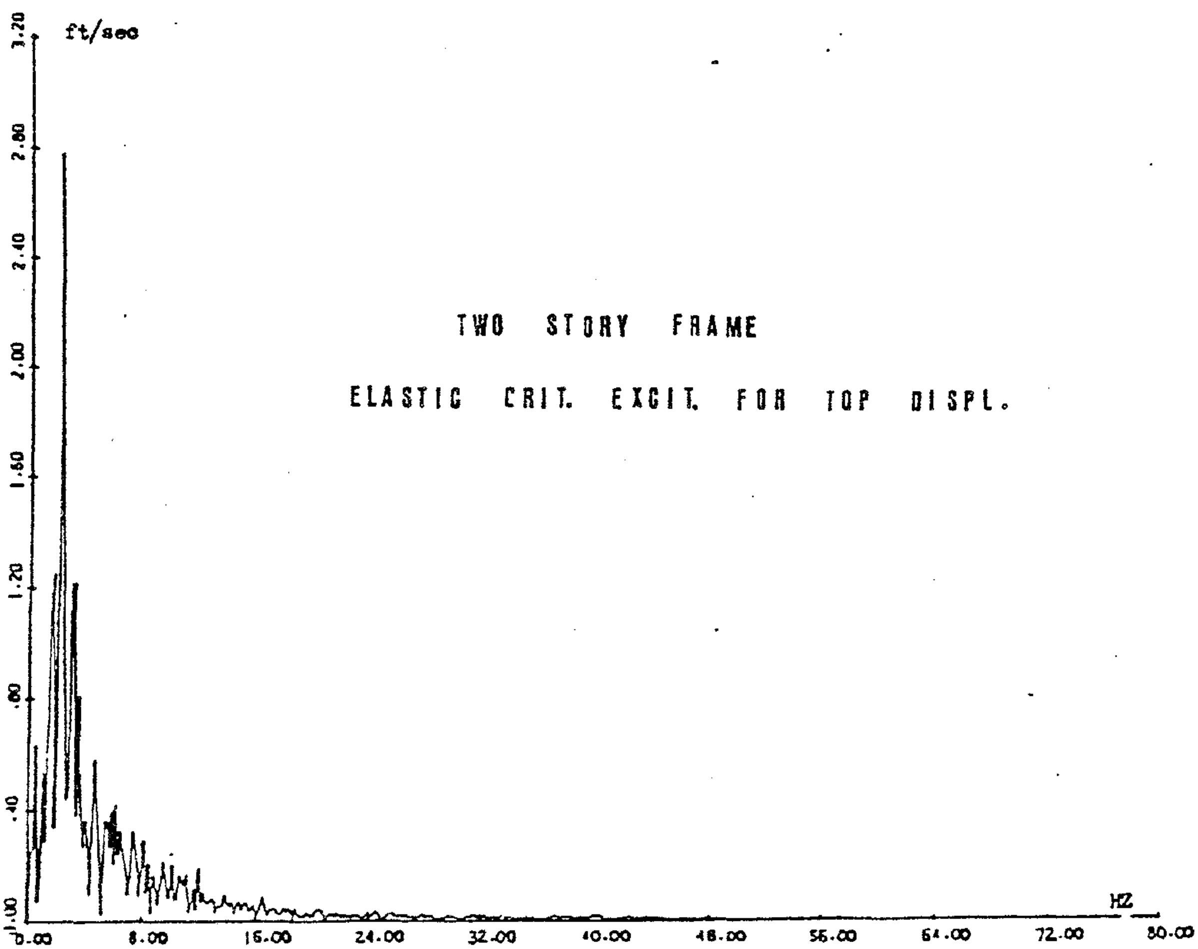

ft/sec
3.20
2.80
2.40
2.00
1.60
1.20
.80
.40
0.00
TWO STORY FRAME
ELASTIC CRIT. EXCIT. FOR TOP DISPL.
HZ
0.00
8.00
16.00
24.00
32.00
40.00
48.00
56.00
64.00
72.00
80.00

FIG. 7(b)

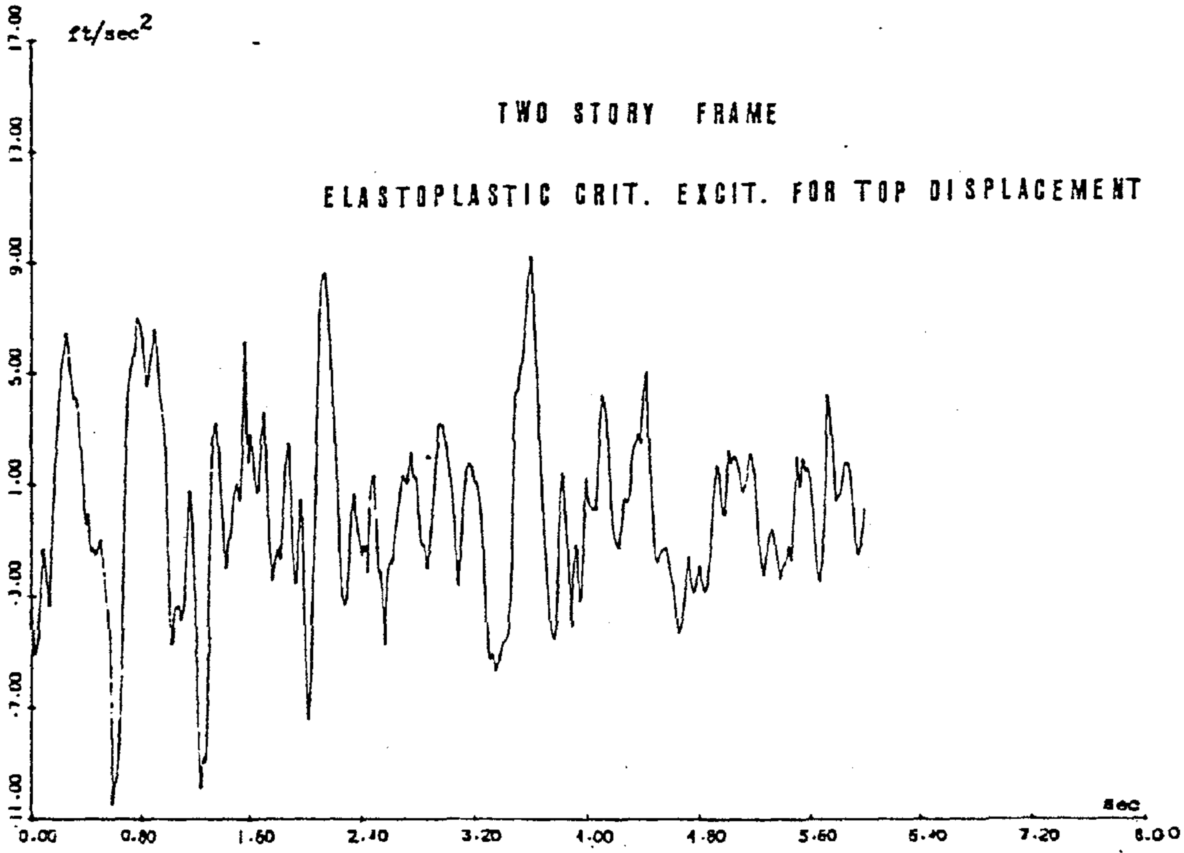

ft/sec²
TWO STORY FRAME
ELASTOPLASTIC CRIT. EXCIT. FOR TOP DISPLACEMENT
17.00
13.00
9.00
5.00
1.00
-3.00
-7.00
-11.00
0.00
0.80
1.60
2.40
3.20
4.00
4.80
5.60
6.40
7.20
8.00
sec

FIG. 8(a)

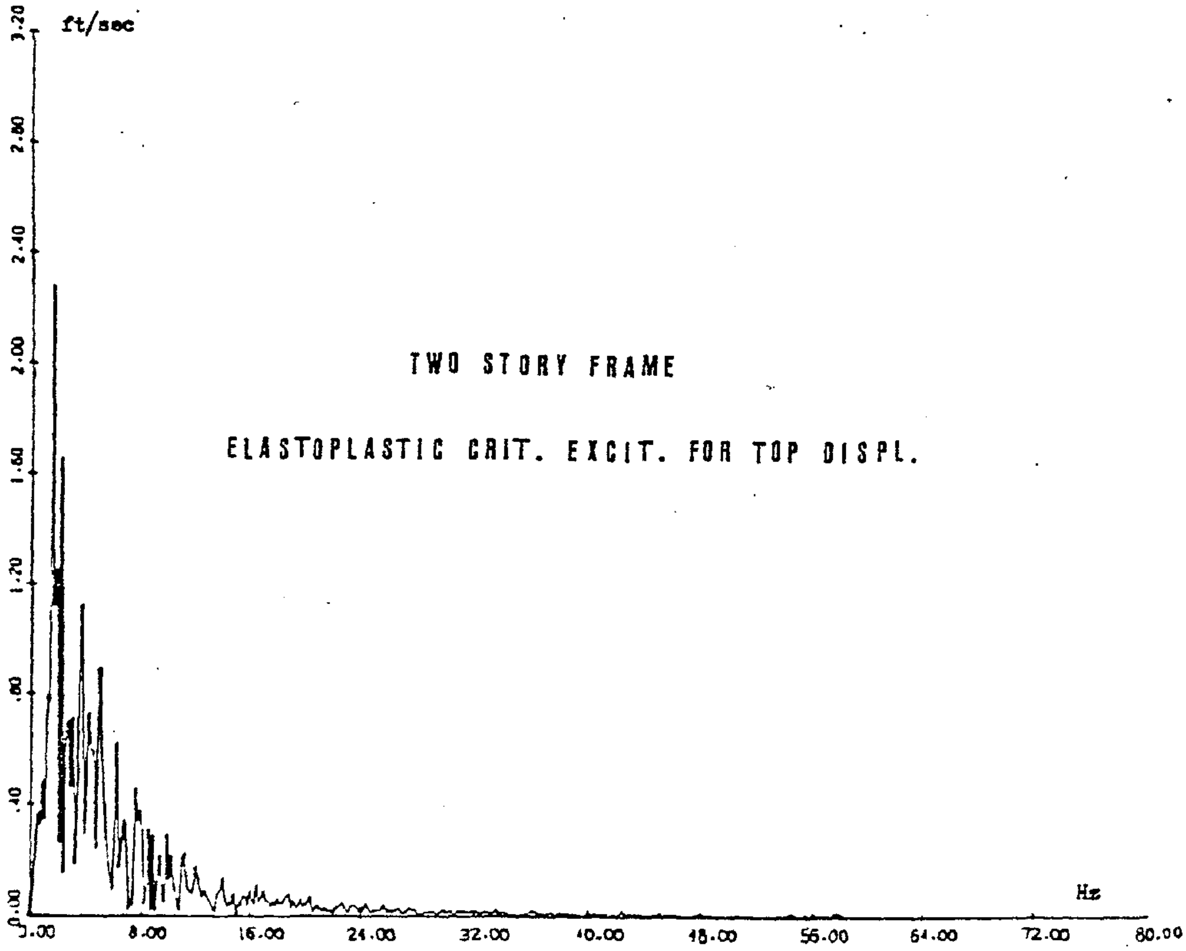

ft/sec
TWO STORY FRAME
ELASTOPLASTIC CRIT. EXCIT. FOR TOP DISPL.
3.20
2.80
2.40
2.00
1.60
1.20
.80
.40
0.00
0.00
8.00
16.00
24.00
32.00
40.00
48.00
56.00
64.00
72.00
80.00
Hz

FIG. 8(b)

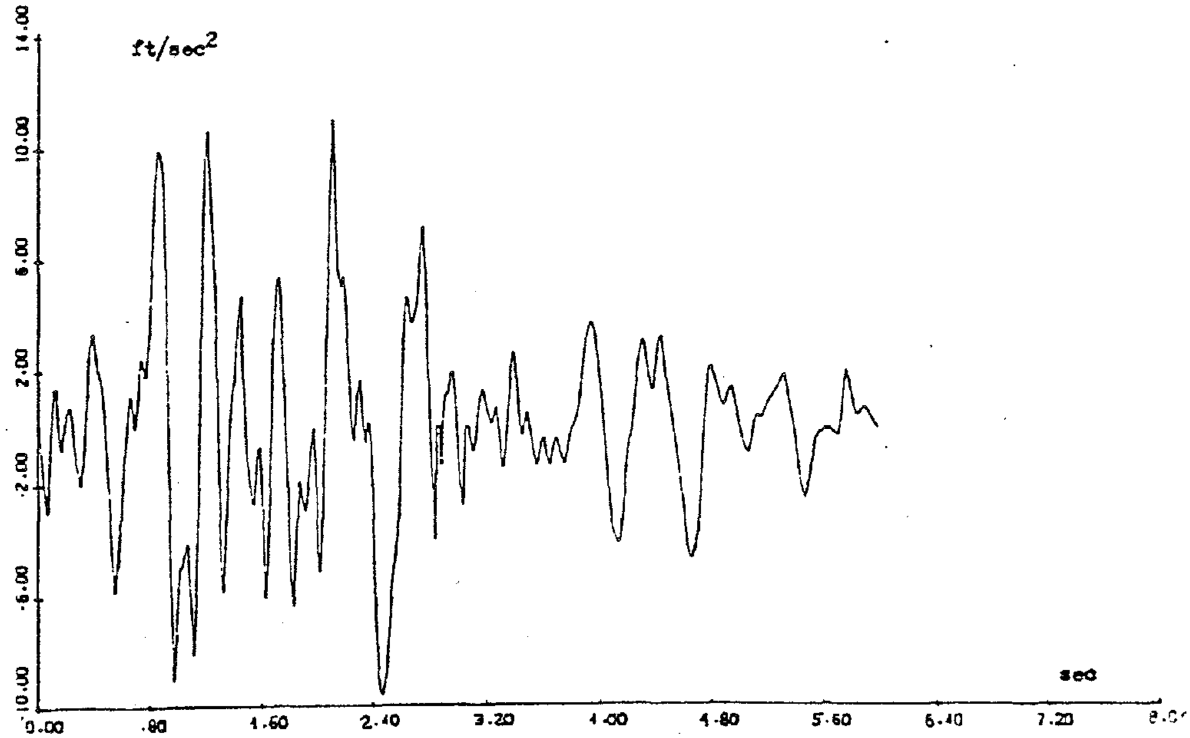
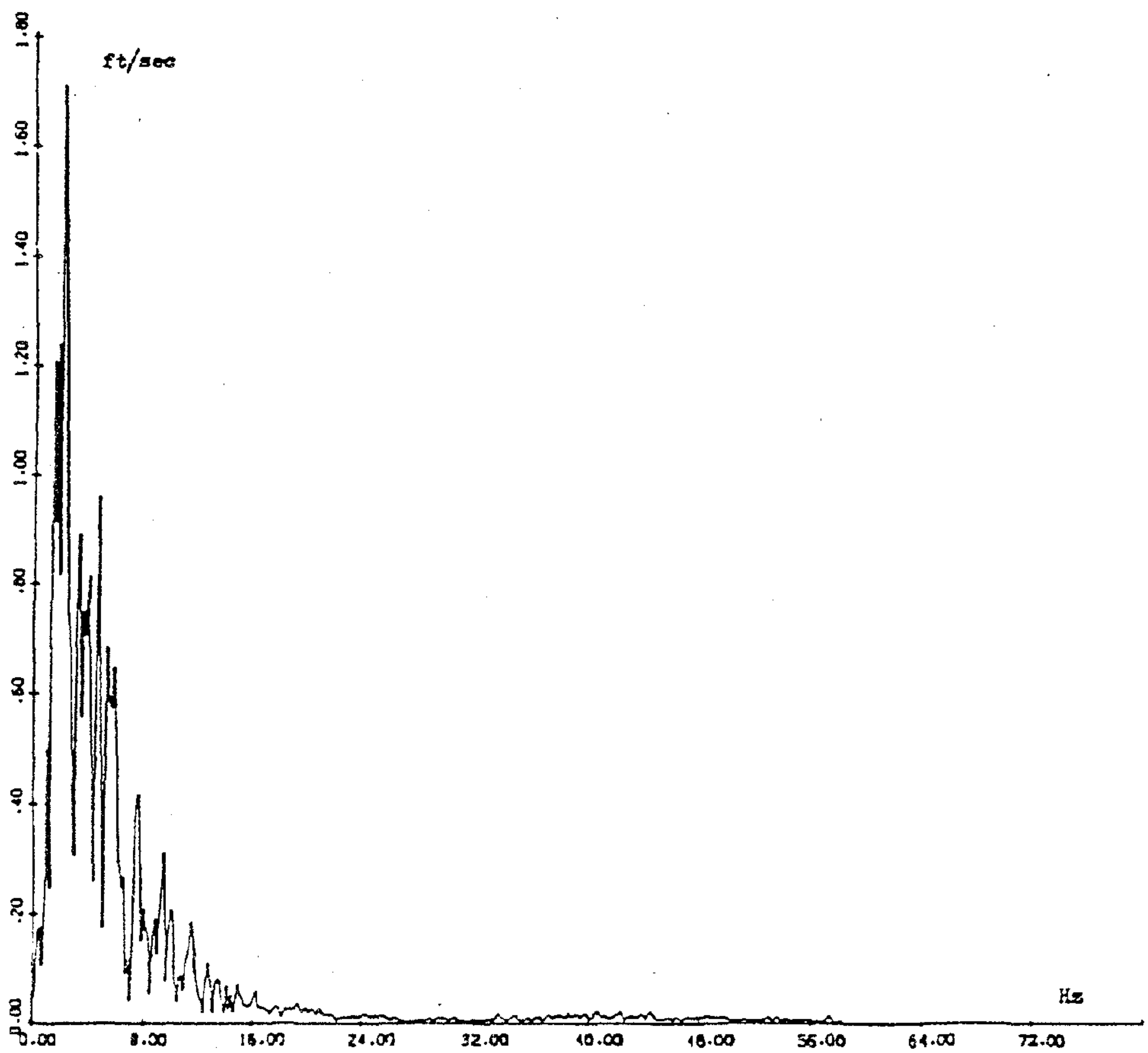

ft/sec^2
14.00
10.00
6.00
2.00
-2.00
-6.00
-10.00
0.00
.80
1.60
2.40
3.20
4.00
4.80
5.60
6.40
7.20
sec

FIG. 9(a)


ft/sec
1.80
1.60
1.40
1.20
1.00
.80
.60
.40
.20
0.00
8.00
16.00
24.00
32.00
40.00
48.00
56.00
64.00
72.00
Hz

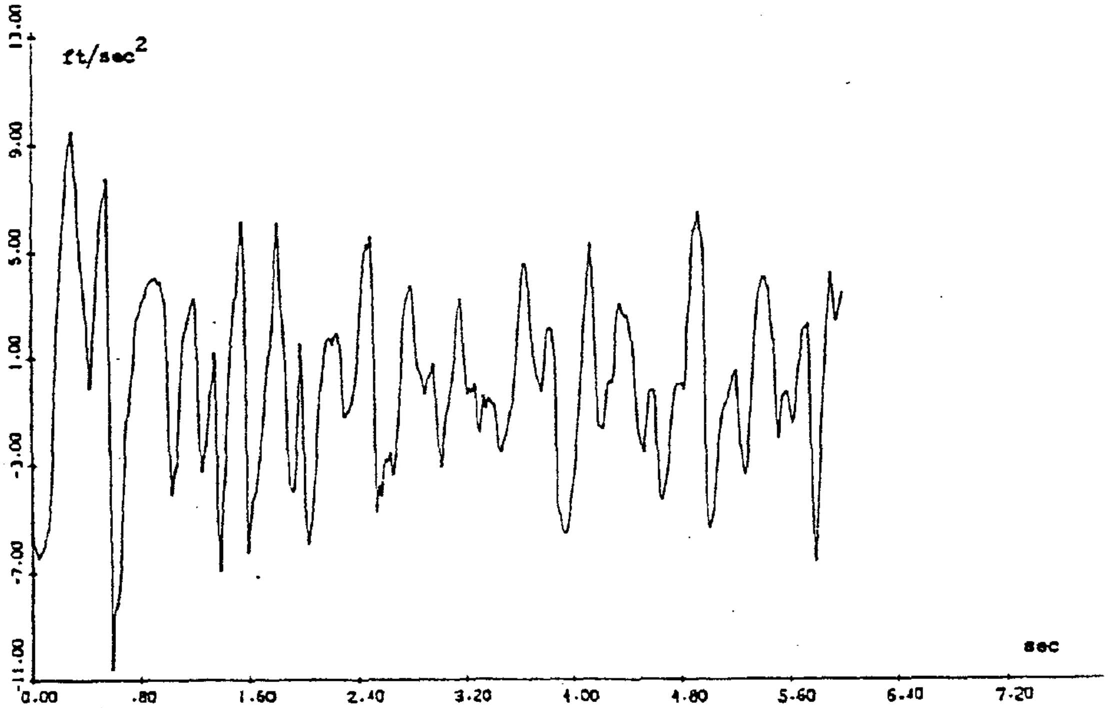


FIG. 10(a)

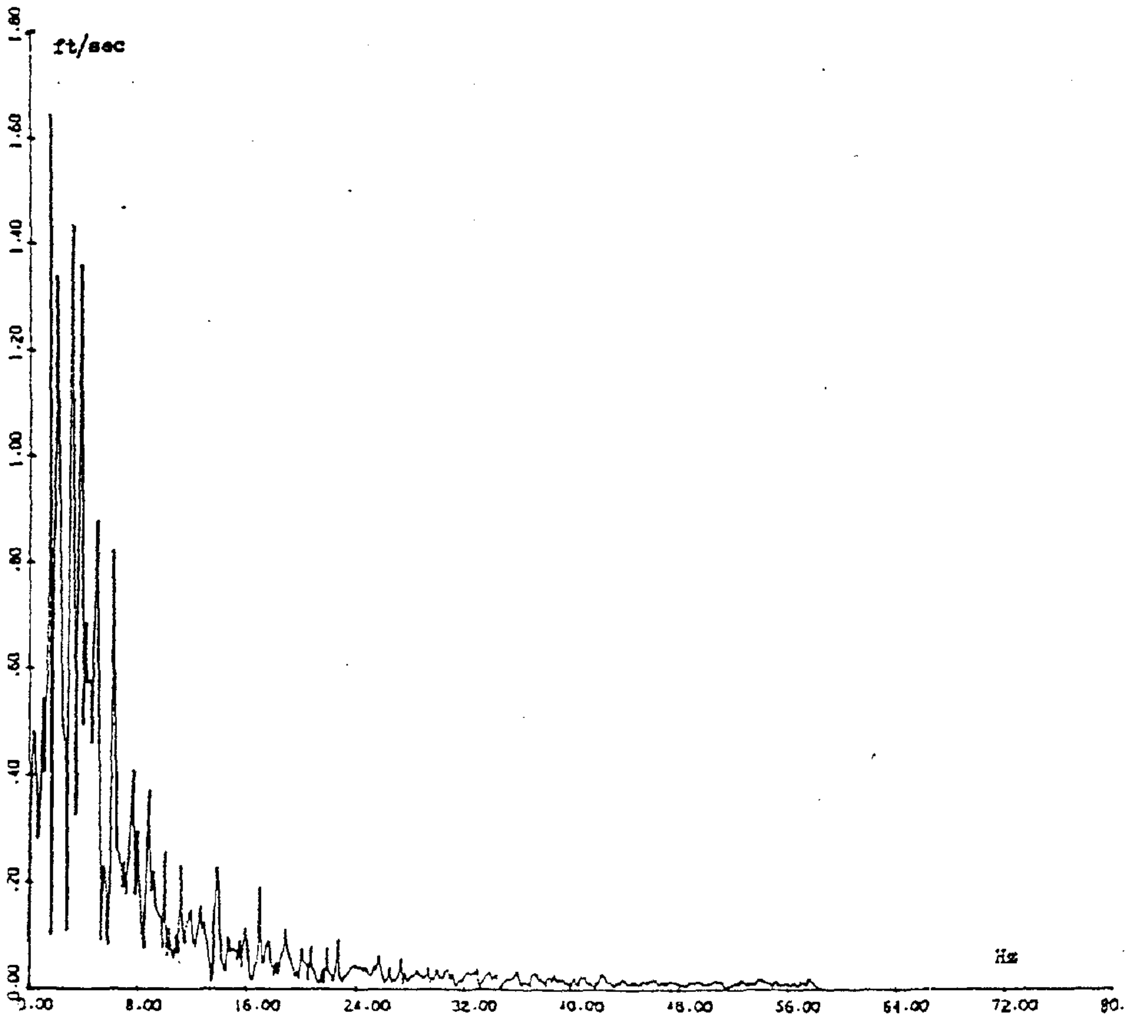


FIG. 10(b)

CHAPTER 3

CRITICAL RESPONSE SPECTRA FOR LINEAR AND NONLINEAR STRUCTURAL SYSTEMS

3.1 Introduction

The concept of the response spectrum for seismic excitation of structures was introduced by Housner and Biot in the early fourties. Since then, response spectra have been increasingly used for the analysis of the response of structural systems, due to seismic excitations.

One of the most significant studies in this area was done by Housner [13] who proposed in 1959 an average velocity spectrum. This spectrum was constructed by simply averaging the spectra of four strong ground motions using the two horizontal components of each of these motions. Later in 1969 Newmark and Hall [14] proposed another method of producing average response spectra and most recent studies by Newmark, Blume and Kapur [15] in 1973 recommended a single set of design spectrum. In these studies a large number of earthquakes were considered and normal or log-normal probability distributions were adopted for the analysis of spectral data.

In the area of inelastic response spectra the studies by Veletsos and Newmark [17] in 1960 and by Veletsos, Newmark and Chelepati [18] in 1965 are significant. The site-dependency of the response spectra was studied first by Seed [16] in 1974. Finally response spectra of artificial earthquakes were proposed by Housner and Jennings [35] in 1964. These earthquakes were generated by using a stationary Gaussian random process based on statistical data from known earthquake ground motions.

In this chapter a type of response spectra called "critical response spectra" is formulated and proposed for the seismic assessment of socially or economically important structures, which require a high confidence of reliability. Linear as well as nonlinear critical response spectra are considered.

Comparison of the critical response spectra with those previously derived by other investigators lead to the conclusion that they are conservative to a certain degree. When they are used for the seismic assessment of important structures, higher confidence levels will be attained.

3.2 The Concept of the Critical Spectrum

The critical response spectra are contructed by generating critical inputs according to the theory developed in chapter 2. In order to produce these spectra, the procedure starts with the search and collection of a set of representative ground accelerograms recorded at similiar geological sites. These records form the "basis" excitations:

$$x_i(t) \quad ; \quad i = 1, \ldots, N \tag{49}$$

A potential future ground excitation is postulated as the linear combination of these:

$$x(t) = \sum_{i=1}^{N} a_i \cdot x_i(t) = \underline{a}^T \cdot \underline{x}(t) \tag{50}$$

where $\underline{a}$ is the vector of the unknown weighing coefficients. The excitations given by Eq. 50 are subjected to an intensity constraint.

The response of a single degree-of-freedom oscillator due to this excitation is maximized with respect to the weighing coefficients $a_i \quad ; \quad i = 1, \ldots, N$ and with respect to the time. Thus it is clear that for a given set of basis excitations corresponding to a specific site condition the vector $\underline{a}$ must be determined for all the frequencies of the spectrum.

This determination implies that for each frequency of the spectrum there is a critical excitation associated with a particular set of the weighing coefficients $\underline{a}$. Damping and the type of nonlinearity of the single degree-of-freedom oscillator also play an important role in constructing the critical spectrum.

3.3 Linear Critical Response Spectra

Elastic critical response spectra are constructed by employing a linear oscillator with frequency ω and damping ratio ζ. The critical excitation of this single-degree-of-freedom oscillator is determined following the procedure described in section 2.4:

$$\chi(t) = \chi(\omega, \zeta, t) = \underline{a}^T(\omega, \zeta) \cdot \underline{x}(t) \tag{51}$$

where $\underline{a}(\omega, \zeta)$ is the vector of its weighing coefficients, determined from maximizing the response subject to the intensity constraint. If $S_v^c(\omega, \zeta)$ represents the critical pseudovelocity spectral value for frequency ω and damping ζ, then:

$$\left.\begin{aligned} &S_v^c(\omega, \zeta) = \max_t \left\{ \chi(t) * h_s(t) \right\} \\ &\text{where:} \\ &h_s(t) = h_s(\omega, \zeta, t) = \exp(-\zeta \omega t) \sin \omega t \end{aligned}\right] \tag{52}$$

The pseudovelocity curve for damping ζ in discrete form is obtained by extending the above for a range of frequencies ω_m ; $m = 1, \ldots, k$ as follows:

$$S_v^c(\omega_m, \zeta) = \max_t \left\{ \chi_m(t) * h_s(\omega_m, \zeta, t) \right\} \tag{53}$$

where $\chi_m(t)$ is the critical excitation associated with the m-th spectral frequency.

3.4 Nonlinear Critical Response Spectra

Nonlinear critical response spectra can be constructed by application of the procedure discussed in chapter 2. The structural system will be a single-degree-of-freedom nonlinear oscillator with damping coefficient c, stiffness k and nonlinear restoring force f. For an elastoplastic system, the above variables can be represented by the linear frequency ω, damping ratio ζ and ductility factor μ.

By employing the equivalent linearization approach an equivalent linear single-degree-of-freedom oscillator can be determined by computing the equivalent damping c^{ϵ} and stiffness k^{ϵ} for a given excitation. According to the procedure in chapter 1, this can be done by considering an auxiliary linear single-degree-of-freedom oscillator and by minimizing an average of the square of the difference between this oscillator and the nonlinear one. For the particular case of a critical excitation input this minimization results the following two simultaneous linear equations:

$$\left.\begin{aligned} &\mathcal{A}\,[\,\underline{Y}_{\epsilon}\cdot Y_{\epsilon}^{T}\,]\cdot\begin{pmatrix}k^{o}\\ c^{o}\end{pmatrix} = \mathcal{A}\,[\,f\cdot Y_{\epsilon}\,] \\ &\text{where:} \\ &\underline{Y}_{\epsilon} = \begin{pmatrix}\psi_{\epsilon}(t)\\ \dot{\psi}_{\epsilon}(t)\end{pmatrix}\end{aligned}\right\} \qquad (54)$$

The functions $\psi_{\epsilon}(t)$ and $\dot{\psi}_{\epsilon}(t)$ are the response and velocity of the equivalent linear oscillator due to the critical excitation $\chi(t)$. The Eq. (54) forms the basis of an iterative procedure for the determination of the equivalent linear single-degree-of-freedom oscillator associated with a given nonlinear one.

Based on the above formulation, for a given basis excitations and a reference intensity E, the equivalent linear oscillator for prescribed balues of spectral frequency, damping and nonlinearity can be computed as follows:

For a given initial set of c^o and k^o an auxiliary linear oscillator is defined. By application of the procedure presented in section 2.4 the critical excitation and the response and velocity due to it of this auxiliary linear oscillator can be computed. These results are then substituted into Eq. (54) from which a new set of values for c^o and k^o are obtained. Successive computations are carried out until two consecutive c^o and k^o values differ insignificantly. The last computed critical excitation defines the equivalent damping c^ϵ and stiffness k^ϵ. The spectral values are then computed by considering the above derived equivalent linear oscillator and its associated critical excitation and employing linear methods.

By repeating this procedure, the equivalent linear oscillator can be defined for all the spectral frequencies and consequently the corresponding critical excitations can be generated. The critical pseudovelocity spectral value $S_v^c(\omega, \zeta)$ for frequency ω and damping ζ can be expressed as follows:

$$S_v^c(\omega, \zeta) = \max_t \left\{ \chi^\epsilon_{(t)} * h_s^\epsilon(t) \right\}$$

where: (55)

$$h_s^\epsilon(t) = h_s^\epsilon(\omega^\epsilon, \zeta^\epsilon, t) = \exp(-\zeta^\epsilon \omega^\epsilon t) \sin \omega^\epsilon t$$

In Eq. (55) the superscript ϵ indicates that the corresponding values are the ones of the equivalent linear single degree-of-freedom oscillator. The symbol * stands for convolution operation. The pseudovelocity curve for fixed damping and type of nonlinearity is obtained for a range of frequencies

ω_m ; $m = 1, \ldots k$ as follows:

$$S_v^c(\omega_m, \zeta) = \max_t \left\{ \chi_m^{\epsilon}(t) * h_s^{\epsilon}(\omega_m^{\epsilon}, \zeta^{\epsilon}, t) \right\} \tag{56}$$

where $\chi_m^{\epsilon}(t)$ is the critical excitation associated with the equivalent linear oscillator corresponding to the m-th spectral frequency.

3.5 Applications

Critical response spectra have been constructed according to the procedure described in the previous sections. These spectra are developed by using the twenty ground motions listed in Table 1, to form the basis excitations representing stiff soil sites. The maximum intensity E appearing in the constraint of Eq. (19) is taken to be the intensity of the El Centro 1940 Earthquake, which is the first ground motion listed in Table 1. The time history and the frequency content of this Earthquake are shown in Figs. 16(a) and 16(b). For a duration of 4.0 sec. it has been found that $E = 7.90 \text{ ft/sec}^{3/2}$. For the computations, the participating time histories have been taken over a duration which depends on the value of the spectral period under consideration. For period T_n the duration of the records has been taken roughly equal to $7.T_n$, but not less than 4.0 sec. or more than 40.0 sec. This definition for the duration of the time histories of the ground motions has been found to give good results in regard to the response spectrum shape. Elastic and also elastoplastic critical response spectra have been constructed for the above defined basis excitations and reference intensity. Thus these spectra are proposed for the seismic assessment of structures to be constructed in stiff soil site conditions. These results can be easily extented to other site conditions by choosing proper basis excitations.

Elastic critical response spectra for damping 2, 5 and 10 % is shown in Fig. 11. A comparison between the critical, envelope and El Centro 1940 Earthquake response spectra for damping 2% is illustrated in Fig. 12. The envelope spectrum is constructed by considering the maximum spectral value among those produced by the basis excitations. It is concluded that the critical spectrum is a reasonable amplification of the envelope one. This amplification provides an additional safety for the structures designed according to the critical spectrum. In Fig. 13 the spectrum recommended by Newmark, Blume and Kapur [15] is compared with a smooth critical for damping 2%. Both these spectra have been normalized to 0. 2 g.

Critical deformation spectra for elastoplastic systems are shown in Fig. 14 for ductility factor $\mu = 1$, 1.5, 2 and 3. The value of damping has been taken equal to 2%. For the construction of these spectra an equivalent linear single-degree-of-freedom oscillator has been defined over the range of spectral frequencies and the associated critical excitations have been generated. Such a critical excitation for spectral frequency equal to 1 cps, damping $\zeta = 2\%$ and ductility $\mu = 1.5$ is shown in Figs 17(a) and (b). Finally in Figs. 15(a) and (b) a comparison between the critical, envelope and El Centro spectra is shown for ductility factor μ equal to 1.5 and 2 respectively and damping equal to 2%. The spectral curves of El Centro in both of the above cases have been computed by Newmark [20].

From the above it is concluded that both linear and elastoplastic critical response spectra are somewhat conservative compared with others that have been used in practice. However, this conservatism can be justified for the design of important structures.

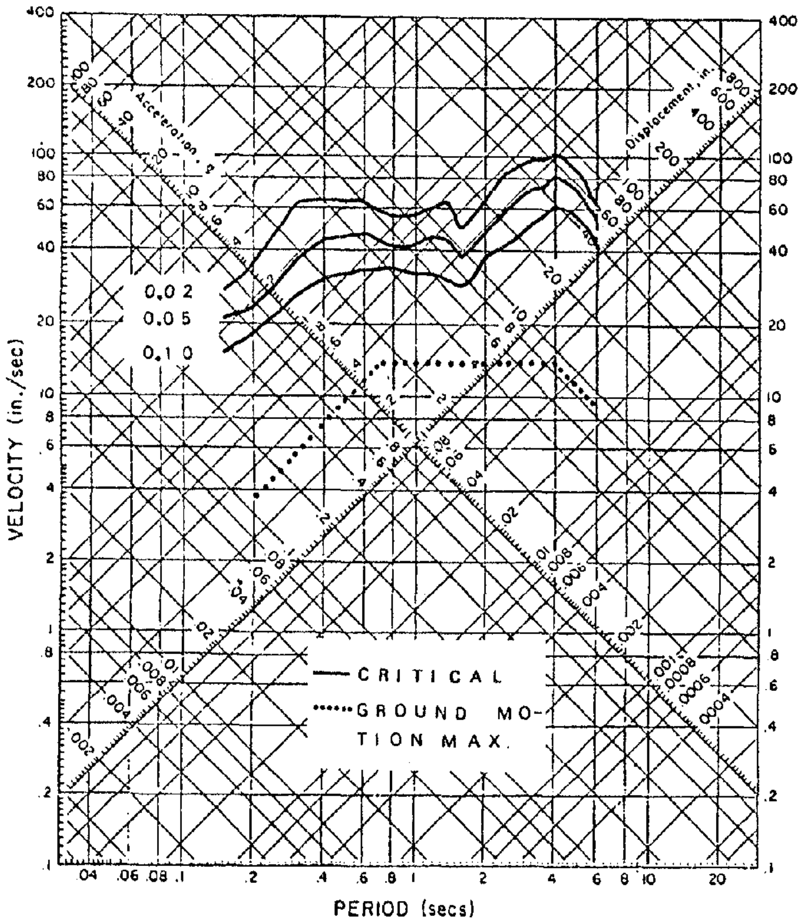


Fig. 11 Elastic Response Spectra

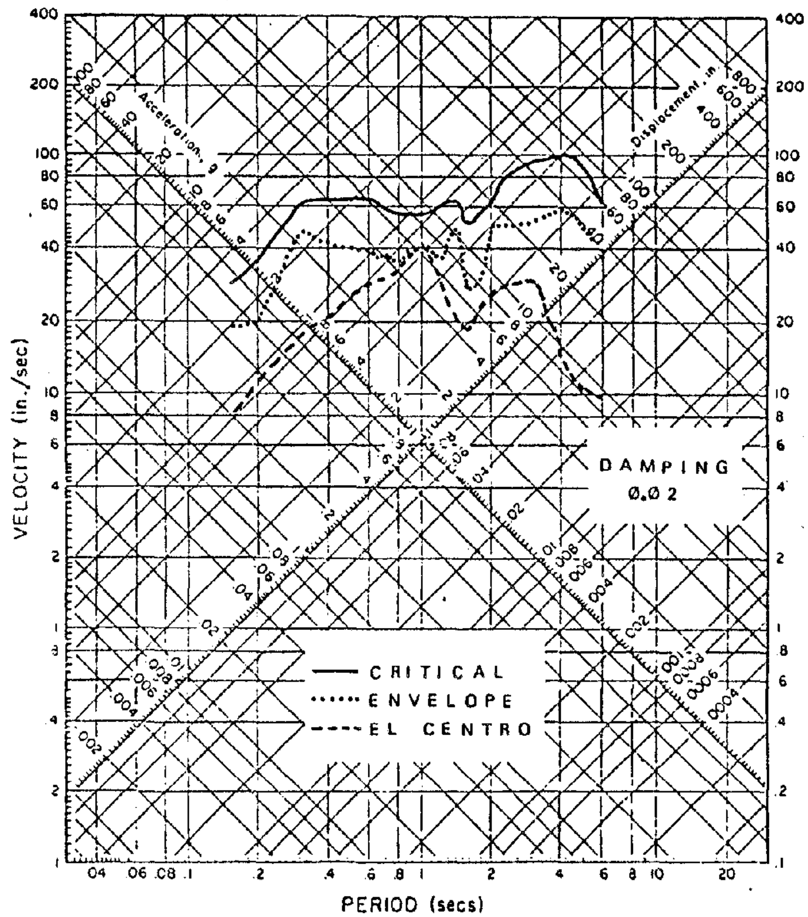


Fig. 12 Elastic Spectrum Comparison

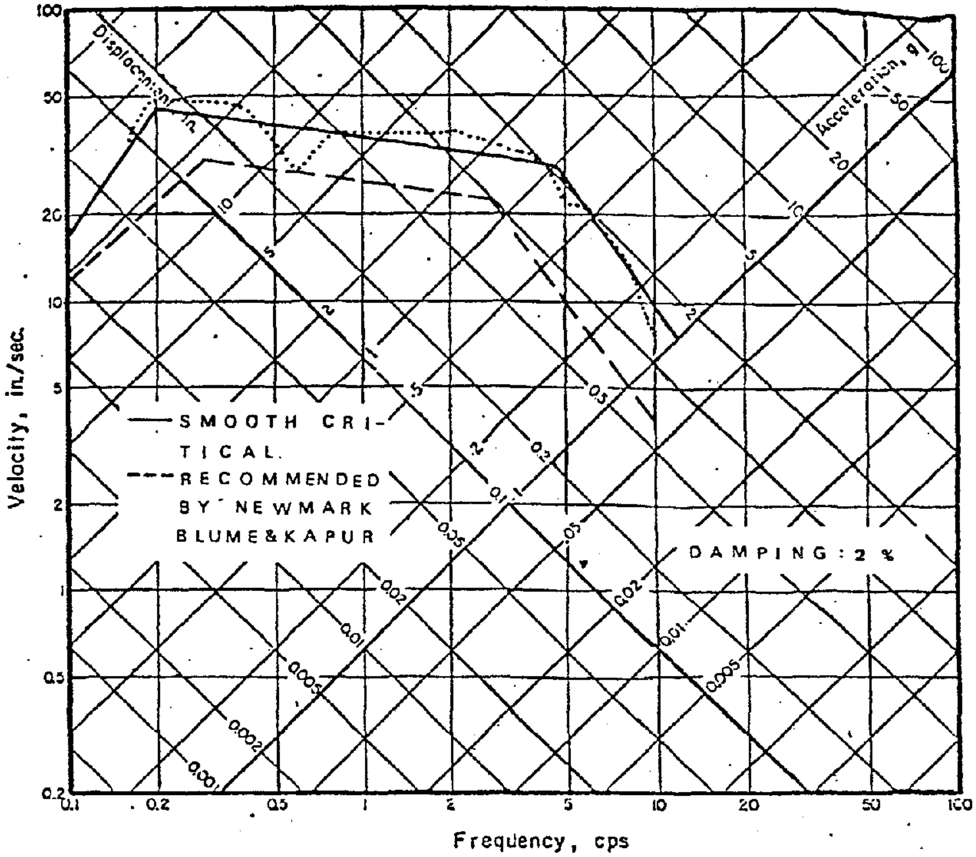


FIGURE 13

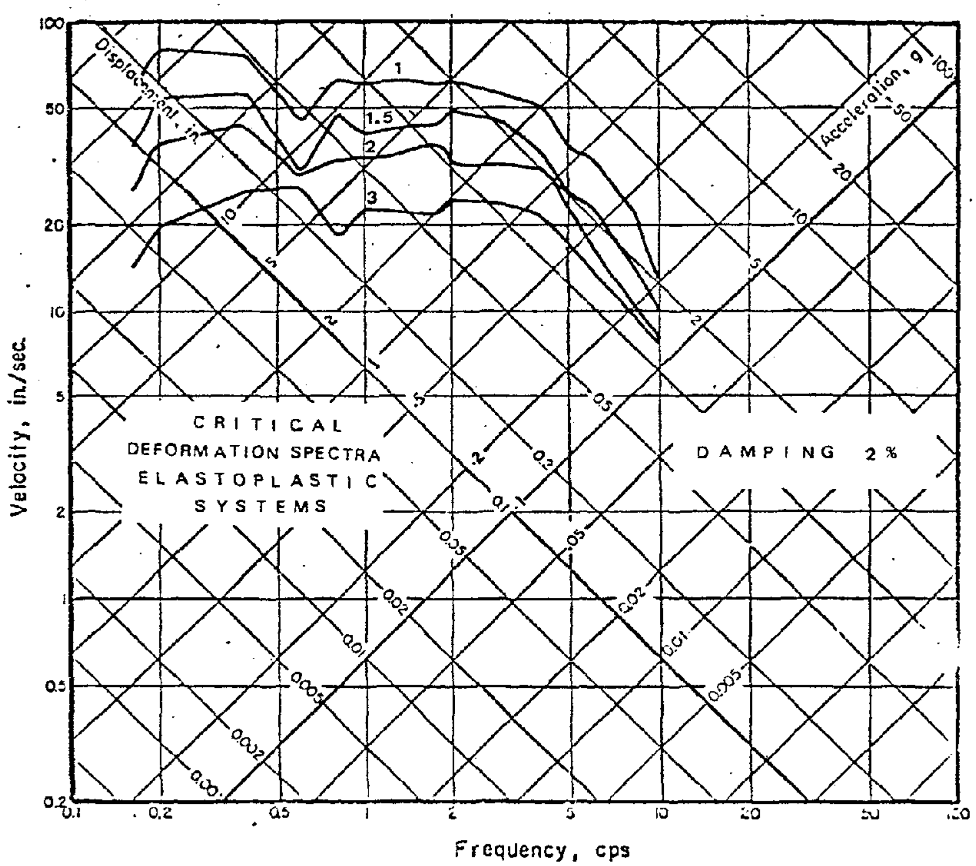


FIGURE 14

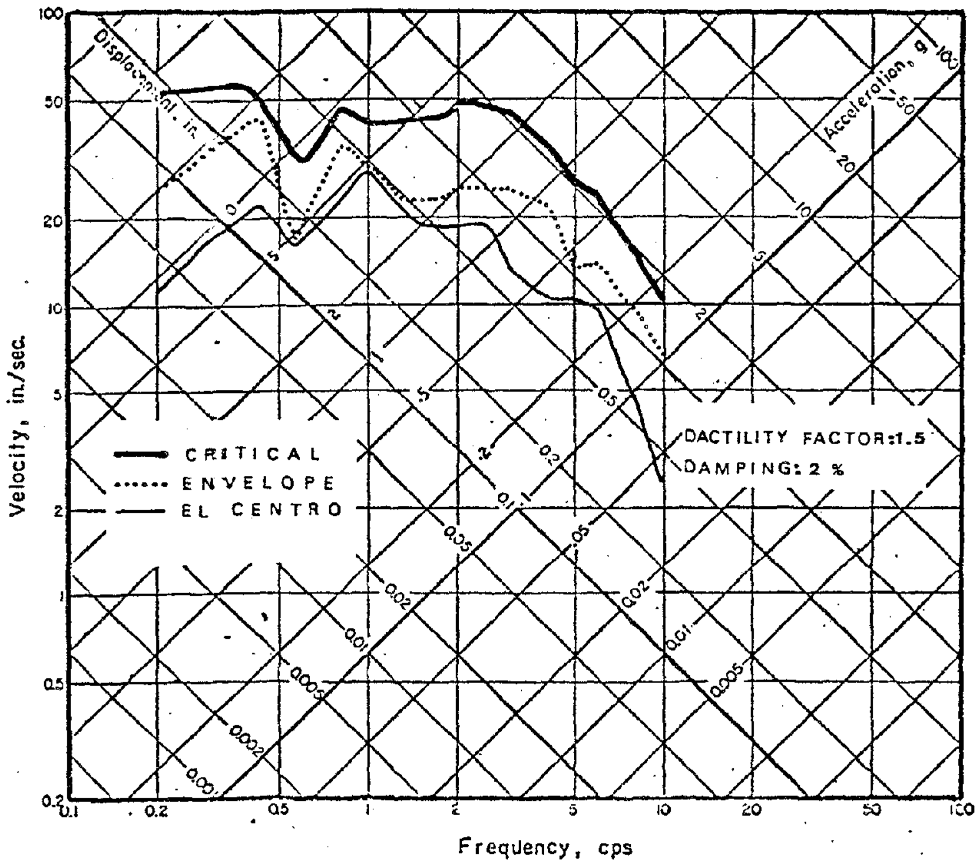


FIG. 15 (a)

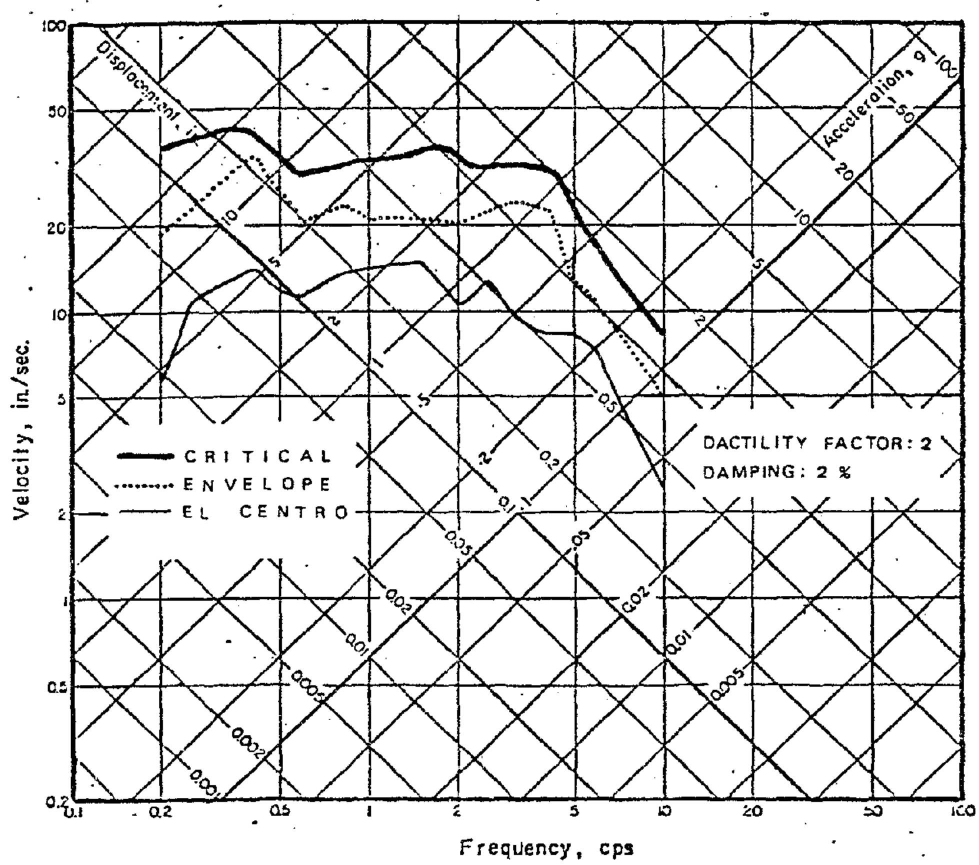


FIG. 15 (b)

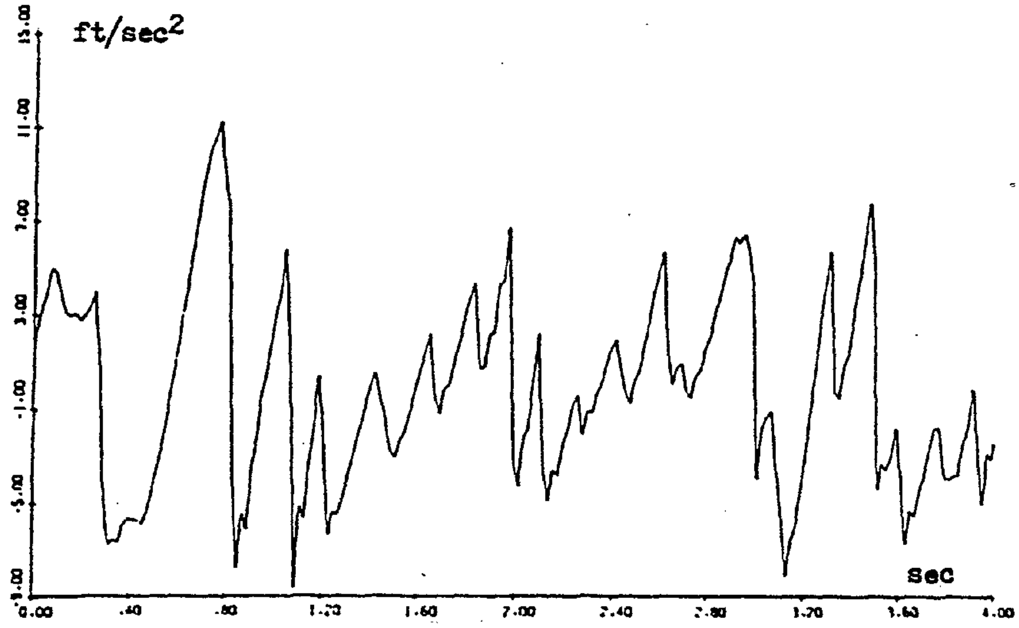


FIG. 16(a)

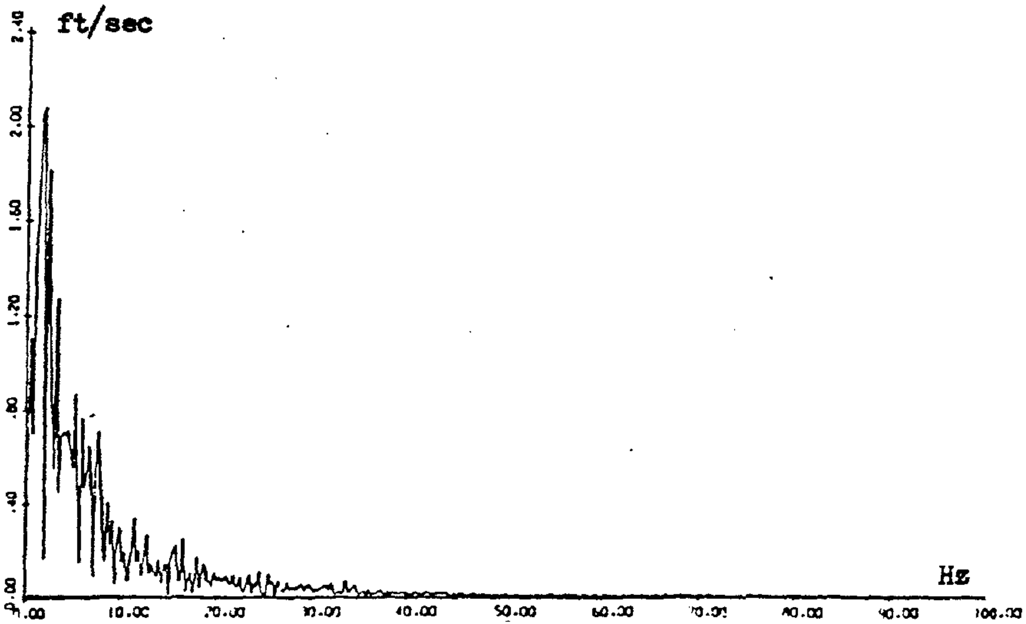


FIG. 16(b)

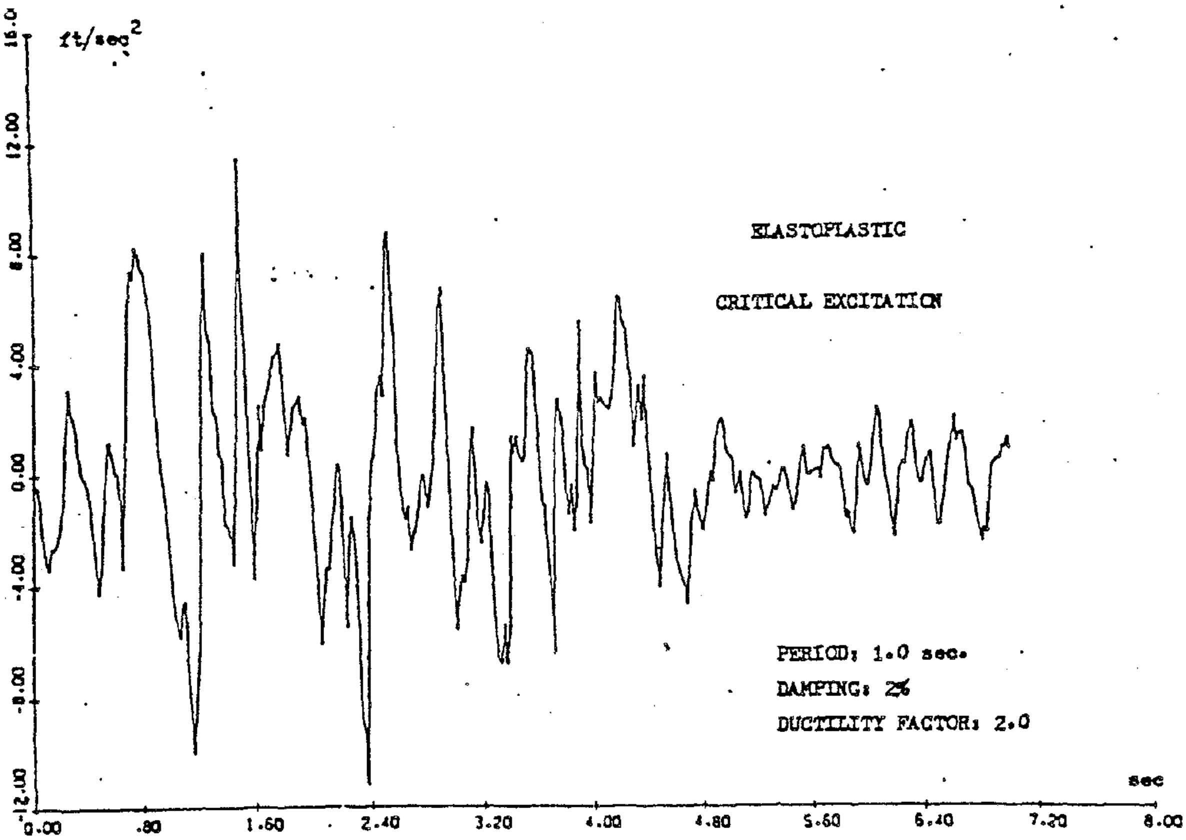


FIG. 17(a)

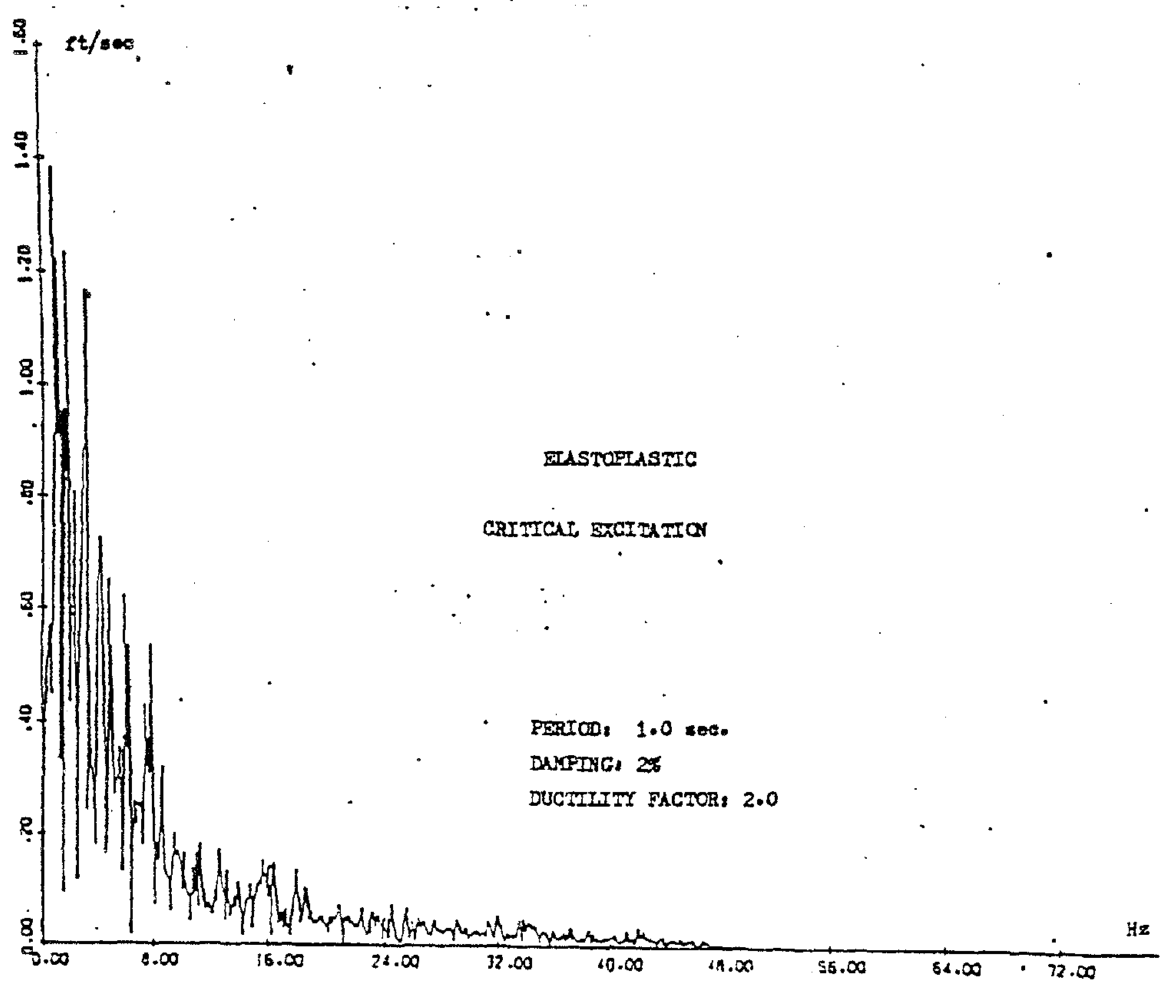


FIG. 17(b)

APPENDIX

SIMULATED CRITICAL EXCITATIONS

A.1 The Simulation Problem

The seismic assessment of structures employed commonly in industry is based on the given response spectra. However, there are special cases which require a time history approach for the determination of the response. Such cases are the calculation of "floor response spectra" and the analysis of the response of "soil-structure interaction" models, in which there is a need to generate time histories consistent with the given ground motion spectra. The generation of spectra consistent ground motions is done usually by applying simulation techniques. A review of the current methods is given by Vanmarcke [33] .

The critical excitation discussed in previous sections is associated with a specific structural variable (displacement, shear, moment etc.) of a structural system. It is generated by superposition of a selected number of ground motions. The weighing coefficients of this superposition are the coefficients expressed by the vector $\underline{a}$, which depends on the specific structural variable under consideration. Thus many critical excitations are required for many design variables of a single structure. To simplify the method, a statistical evaluation of the vector $\underline{a}$ defined over a large range of frequencies and different values of damping appears to be necessary.

However, an approximate solution of this problem can be achieved by synthesizing a time history consistent with the critical response spectra. This time history will represent a " simulated critical excitation", which may process the essential characteristics of such an excitation, defined for the range of spectral frequencies.

For the generation of such excitations the inverse problem of constructing response spectra for a given earthquake, must be solved. The solution of this problem is not unique. It means that for a given spectrum several time histories can be found which are consistent with it. Some of the criteria for selecting a "suitable" time history to represent the ground motion associated with a given spectrum, were discussed in reference [34].

In order to generate a "simulated critical excitation" compatible with the critical response spectra, the well-known simulation model of superposition of sinusoidal waves with random phase angles is employed here. According to this model the "simulated critical excitation" has the following form:

$$\chi_s(t) = \gamma(t) \cdot \underline{r}^T \cdot \underline{w}$$

where $\gamma(t)$ is an envelope function used to give a transient character in the motion. The vector $\underline{w}$ represents N sinusoidal waves with random phase angles and the vector $\underline{r}$ their amplitudes. An iterative procedure proposed by Scanlan and Sachs [36] was used for the simulation.

Next some applications were demonstrated.

A.2 Applications

In order to illustrate the concept of a "simulated critical excitation" for stiff soil sites was generated from the critical response spectra constructed in chapter 3. The dynamic responses of nuclear reactor structures were studied. First, a number of critical excitations associated with some design variables of these structures were synthesized by using the theory developed in chapter 2 and response peaks due to them were obtained. Second, response peaks of the same variables were determined from a simulated critical excitation. These results were summarized in Tables 4 and 5. It can be seen that the simulated critical excitation gives very similar results compared to those produced by the critical excitations determined by the rigorous analysis. Accordingly this excitation is suggested for the seismic assessment of structures to be constructed on stiff soil sites. These results can be easily extended for other soil conditions.

The first structure is a reactor for a nuclear power plant. This structure was originally studied by Hamilton and Hadjian [32]. The dynamic model of the structure is shown in Fig. 15. According to this model, the structure is idealized by two sticks with seven and four lumped masses respectively. The first stick represents the containment and the second the auxiliary building. The effect of soil-structure interaction is idealized by one translational and one rotational soil spring.

The second structure is also a reactor for a nuclear power plant which is shown in Fig. 18. The idealization of this structure consists of a three-stick model, which represents the three substructures, namely, the containment building, the internal structure and the annulus building. The effects of soil-structure interaction are included in this idealization by equivalent soil springs attached to the model in three translational and three rotational directions. This structure is more complex than the previous one.

To investigate the validity of the "simulated critical excitation", the following procedures were carried out:
First a simulated critical excitation shown in Fig. 21 was constructed by using critical response spectral values for frequencies between 0.1 and 10.0 cps and for values of damping 0, 2 and 5 percent. The time step was taken equal to 0.1 sec and the frequency interval equal to 0.1 cps. These

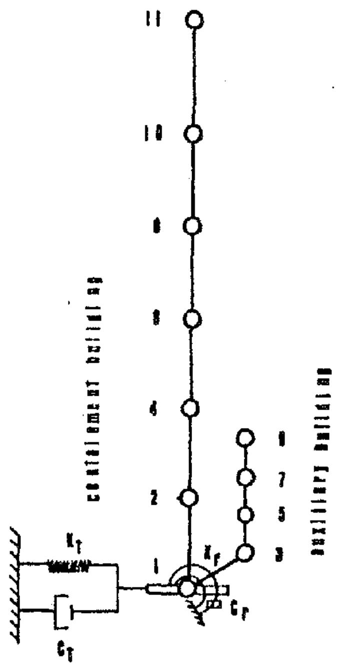

containment building
auxiliary building
11
10
2
1
8
7
5
3
K_T
C_T
K_F
C_F

Fig. 18

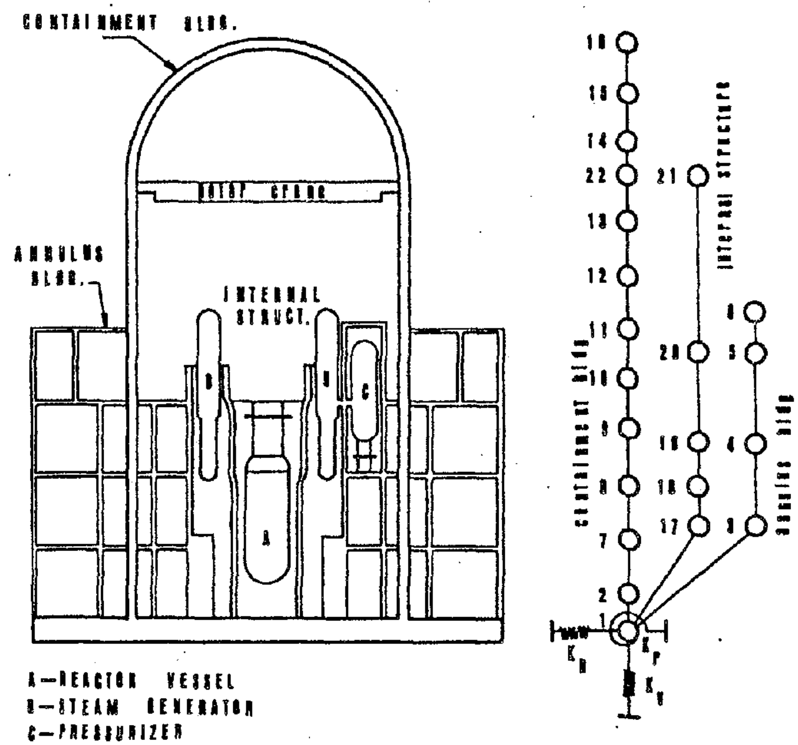

CONTAINMENT BLDG.
ANNULUS BLDG.
INTERNAL STRUCT.
A—REACTOR VESSEL
B—STEAM GENERATOR
C—PRESSURIZER
containment bldg
internal structure
annulus bldg
14
22
12
11
7
2
1
21
17
5
4
K_P

Fig. 19

spectral curves were constructed from the twenty basis ground motions listed in Table 1, according to the theory described in chapter 3. A total number of four iterations was used for each value of damping in the simulation procedure. These results are shown in Figs. 20(a), (b) and (c). The continuous curves represent the original critical response spectra and the dotted ones, those produced by the simulated excitation. It is noted that the fourth iteration gives a very good approximation for each value of the damping.

Next the individual critical excitations associated with the top displacement, bottom shear and moment of both containment and auxiliary buildings have been generated for the first reactor structure according to the theory of the second chapter. In this computation a viscous damping equal to 7% has been used for each mode and a total number of the first six modes was used. The responses due to these individual critical excitations were recorded in column 4 of Table 4. The responses of the above structural variables due to the simulated critical excitation shown in Fig. 21 were also computed and recorded in column 5 of Table 4. Together with the responses due to the El Centro 1940 earthquake and the envelope response, Table 4 presents the comprehensive comparison.

Similar computations for the second structural system shown in Fig. 19 were summarized in Table 5.

By comparison of the peak responses listed in Table 4, it can be seen that the critical response is higher than the envelope one by an average factor of 1.41. Furthermore, the responses due to the simulated critical excitations are very close to those of the critical excitations. In Table 5, the average ratio of critical response to the envelope response is 1.57.

From the application results the following conclusions can be drawn:

For the critical seismic assessment of linear structural systems a rigorous analysis based on individual critical excitation for each design variable can be performed without excessive computational effort. However in the case of critical assessment of nonlinear structural systems the computational effort based on rigorous individual critical excitations pertaining to each design variable becomes many folds higher than that employed to produce critical response spectra and then the simulated critical excitation for all the design variables. Thus the later appears to be the only feasible approach.

Table 4: Peak Responses of Nuclear Power Plant I

Structural Variables		El Centro Earthquake	Envelope	Critical Excitation	Simulated Critical Excitation
Containment Bldg.	Top Displ. (inch)	0.534	1.075 (11)	1.608	1.824
	Bottom Shear (10^5 kips)	0.304	0.596 (9)	0.840	0.942
	Bottom Moment (10^7 kips-ft)	0.442	0.929 (11)	1.380	1.560
Auxiliary Bldg.	Top Displ. (inch)	0.027	0.039 (16)	0.057	0.059
	Bottom Shear (10^5 kips)	0.208	0.305 (16)	0.444	0.455
	Bottom Moment (10^7 kips-ft)	0.078	0.144 (16)	0.165	0.169

Table 5: Peak Responses of Nuclear Power Plant II

Structural Variables	El Centro Earthquake	Envelope	Critical Excitation	Simulated Critical Excitation
Top Displ. (inch)	0.887	1.548 (3)	2.172	2.136
Bottom Shear (10^5 kips)	1.020	1.650 (16)	2.740	2.760
Bottom Moment (10^7 kips-ft)	1.180	1.890 (16)	3.140	3.160

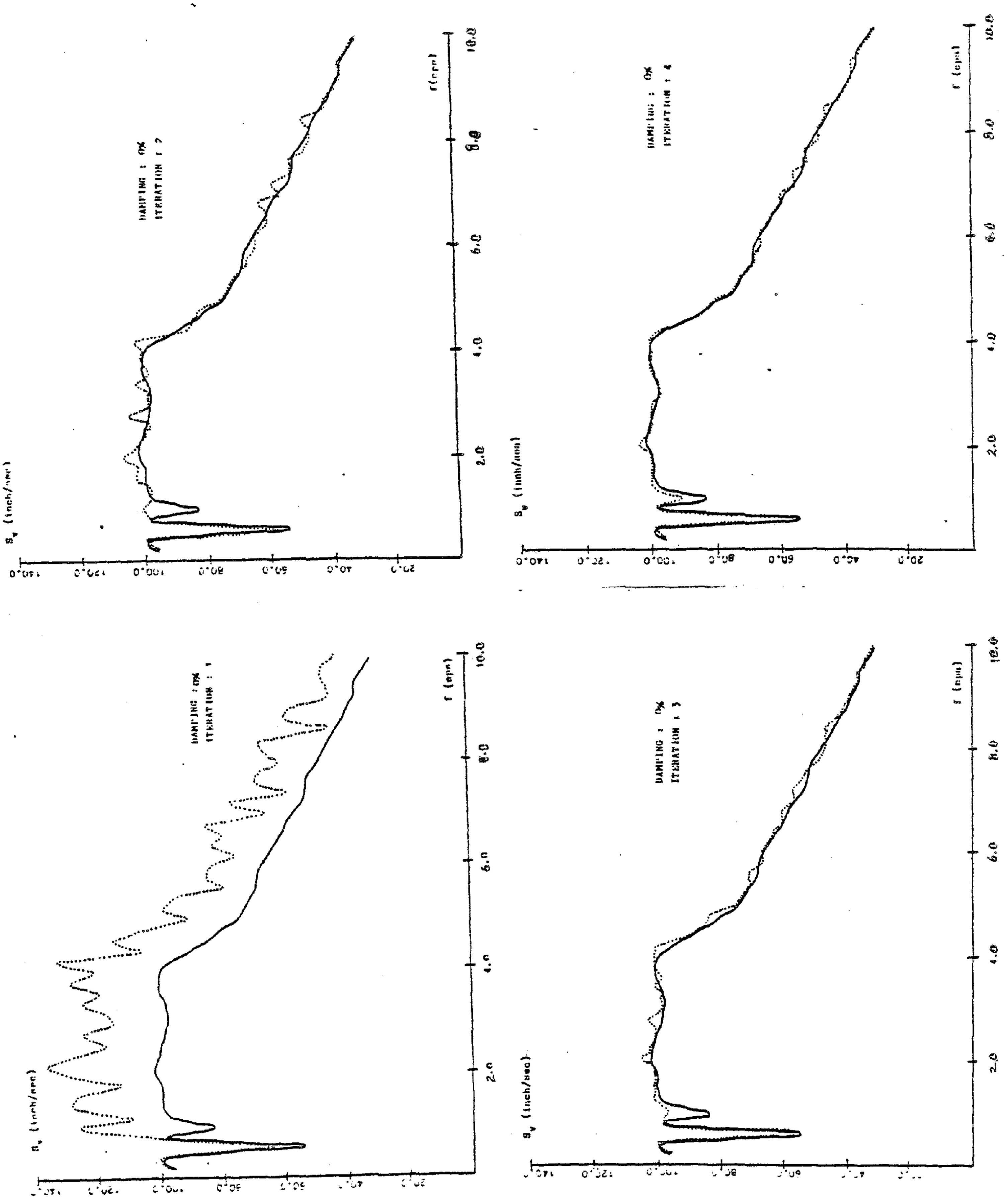

DAMPING : 0%
ITERATION : 2
DAMPING : 0%
ITERATION : 4
DAMPING : 0%
ITERATION : 1
DAMPING : 0%
ITERATION : 3

FIGURE 20 (a)

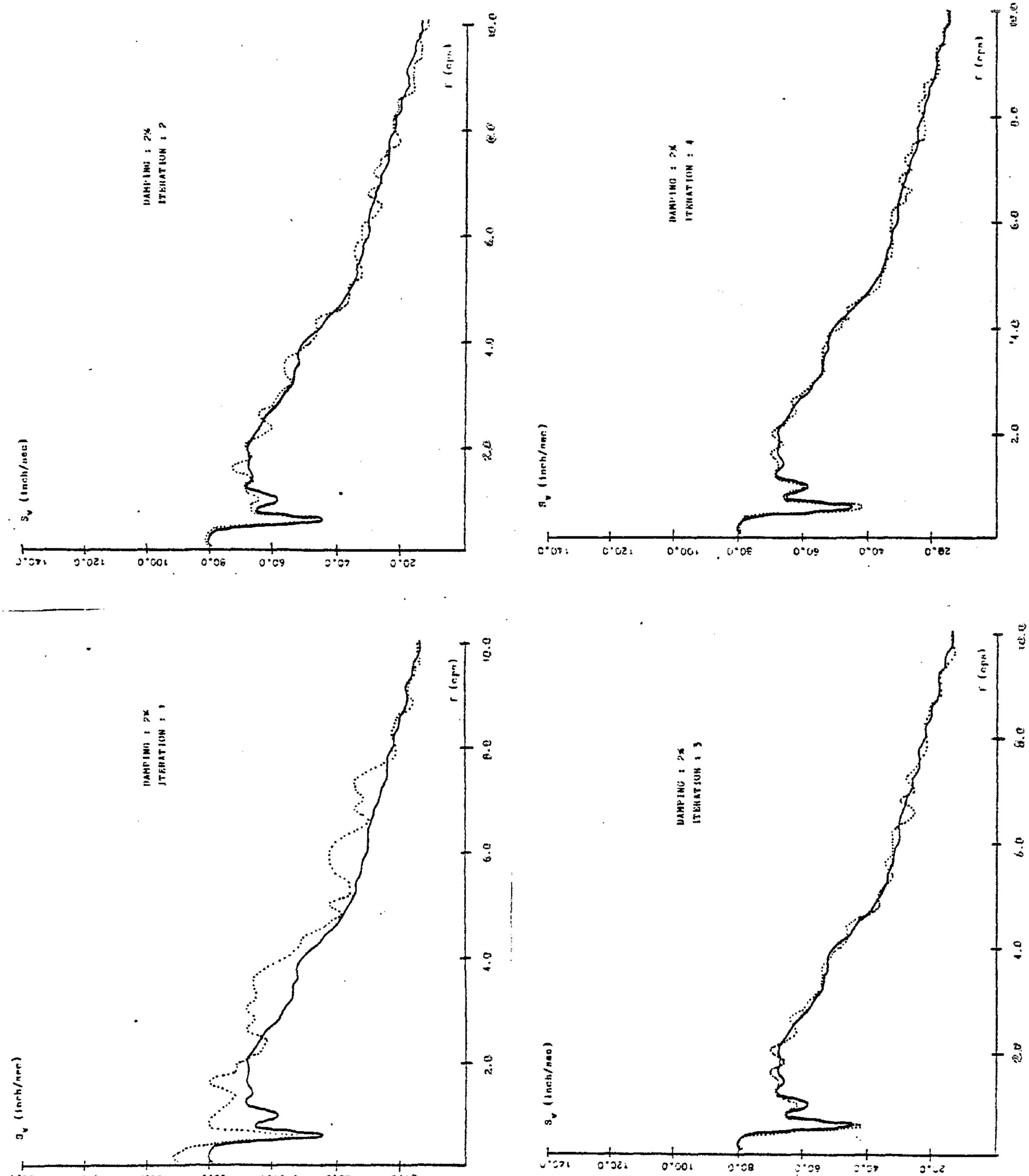


FIGURE 20 (b)

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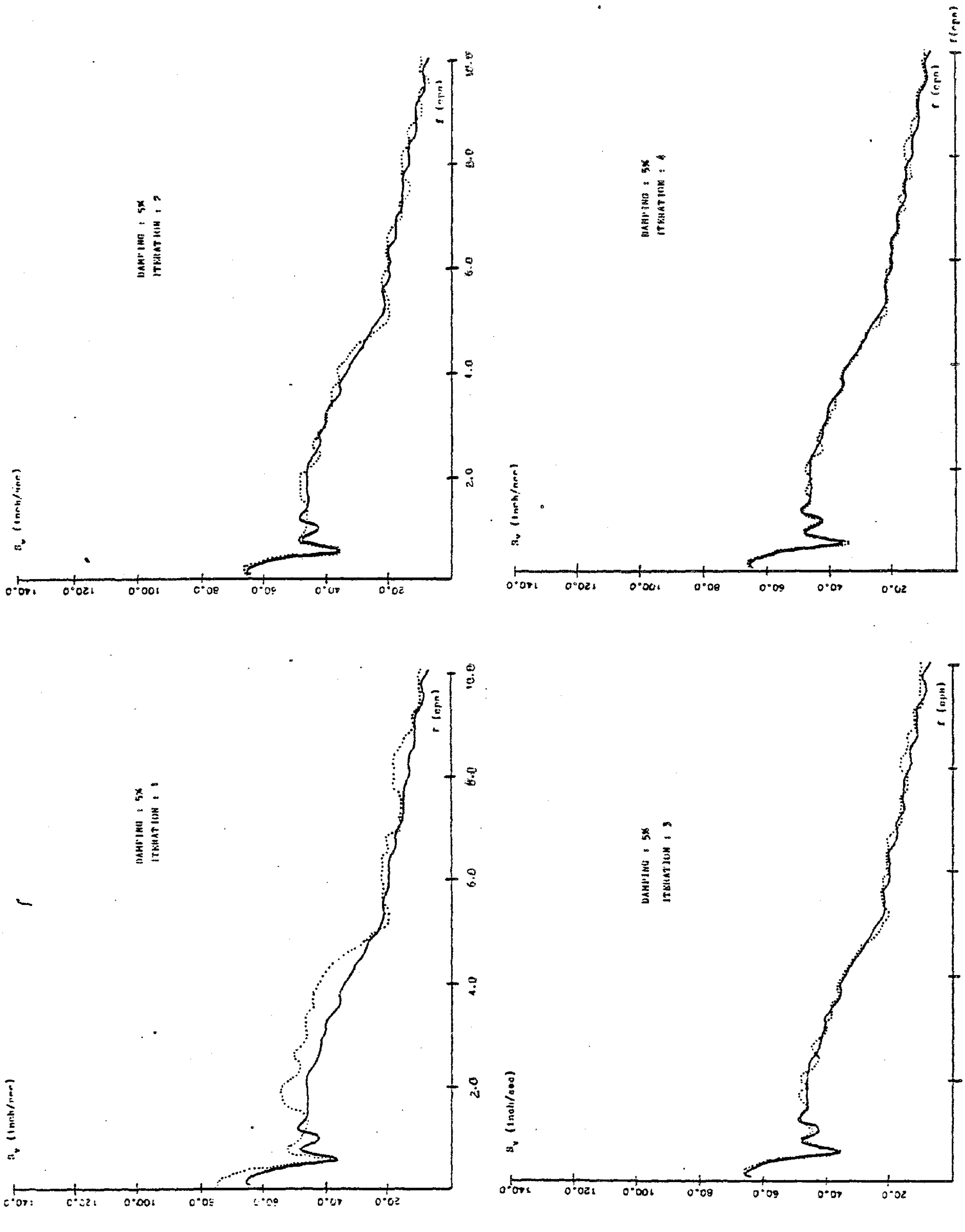

FIGURE 20 (c) 127

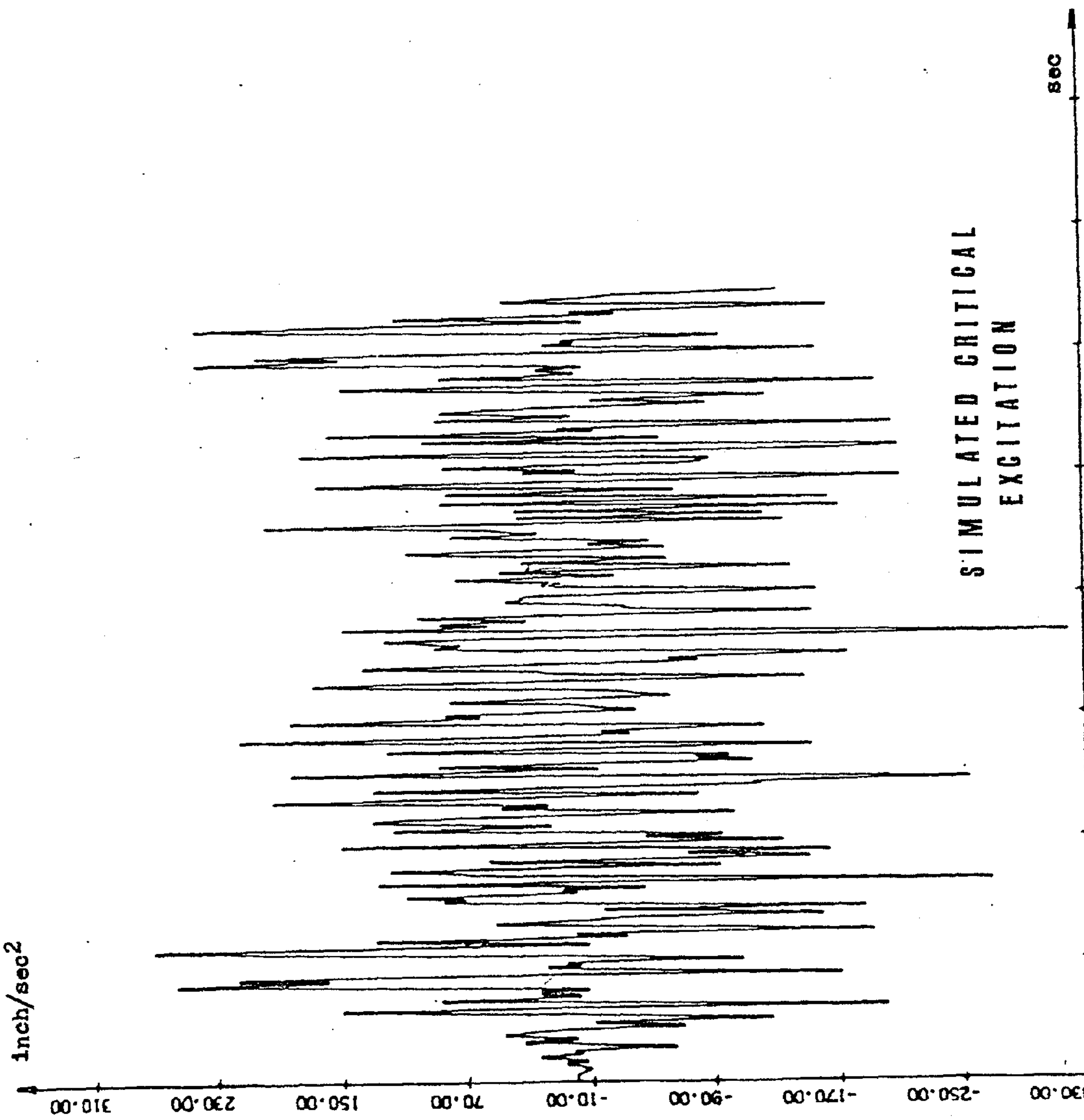
inch/sec^2
310.00
230.00
150.00
70.00
-10.00
-90.00
-170.00
-250.00
-330.00
0.00
2.00
4.00
6.00
8.00
10.00
12.00
14.00
16.00
sec
SIMULATED CRITICAL
EXCITATION

FIG. 21

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SYSTEM RELIABILITY ASSESSMENTS USING CRITICAL EXCITATIONS

R.F. Drenick and P.C. Wang*

Abstract - Critical and certain related excitations are applied to mechanical and structural reliability problems involving the assessment of the resistance of systems to dynamic loads whose characteristics are partly or largely unknown. The experience gained thus far in practical situations and possible extensions of the use of the technique are described. Dependable, but somewhat conservative, reliability assessments have been achieved that might be applicable to various systems.

A recurrent problem in many fields of engineering is that of assessing whether or not a system that has been designed to survive, perhaps with some tolerable level of damage, any of a large class of possible excitations can indeed survive. This problem arises in civil engineering with regard to the effects of earthquakes, wind forces, and wave motion; in aeronautical engineering with regard to the effects of wind gusts and air or jet turbulence; and in mechanical engineering in the study of engine vibrations and vibration effects on delicate instruments. The common factors in all cases are 1) the uncertain nature of the characteristics of the excitations to which the system might be subjected, and 2) the probabilities with which such excitations are likely to occur. These factors are of greatest significance in systems of great economic, social, or military value. In such cases, any statement regarding system integrity should be made with a high level of confidence and ought to be compared only with information known to be at a comparable level of confidence. Unfortunately, such information is often unreliable, particularly statistical data pertinent to a reliability assessment, as has been previously noted [1].

Critical and certain related excitations were first applied to the problem of assessing system reliability almost a decade ago [2]. Since then, the variations that have been developed and the practical applications that have been explored [3-7] indicate that the concept has considerable theoretical and practical potential. It is therefore of interest to report on the work thus far in this area and on some possible extensions.

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The technique is based on the assumption that it is possible to characterize, at a desired level of confidence, a certain class of excitations that a system should be able to withstand. The critical excitations within that class are used to drive the dynamical variables of the system to their highest response peaks. If those peaks are compatible with the damage level that can be tolerated in the system, the design is judged satisfactory.

The intuitive appeal of the technique lies in the fact that only reliable data regarding excitations of concern are used. In practical applications, however, problems are often encountered. It is frequently difficult to define the class of excitations that the system should be able to withstand. Design engineers usually have fairly definite notions of the excitations they consider realistic or credible and what their designs should be prepared to accommodate. It is another matter, however, to convert design concepts into mathematically manageable definitions. The compromise has been to define so-called subcritical excitations of a system.

Subcritical excitations have for the most part applied to earthquake engineering. This review describes both critical and subcritical excitations and some of the results that have been obtained in earthquake engineering. Partially solved and potential problems are surveyed.

The general conclusion is that the use of critical and subcritical excitations results in realistic, if somewhat conservative, reliability assessments, but that they can be used with greater assurance than those derived from others now in use or under consideration. The technique might eventually be used, either in its present or in some modified form, with systems whose survival and integrity is of considerable importance.

CRITICAL EXCITATIONS

In order to derive the critical excitations of a system, information available regarding the system under consideration must be collected, including the excitations the system should be capable of withstanding; the reliability of the information must also be established. The various structures studied thus far in earthquake engineering have included some already built, some in the process of design, and one after it collapsed. The analyses were based on the assumption that the equations of motion, established from engineering drawings and restricted to the elastic domain, did in fact adequately describe the structure. In other words, no allowances were made for uncertainties regarding system dynamics.

With regard to the excitations, it was initially assumed that only an upper bound on the intensities of the ground motions was known at the desired level of confidence. The idea was that a designer of a structure in, say, San Diego would be able to establish that earthquakes with intensities beyond a certain level could not be disregarded in his design. It was further assumed that he could establish this level with confidence because pertinent statistics are sufficiently reliable, and it was also assumed that no other ground motion statistics are reliable enough to be utilized. A class of admissible excitations was thus defined.

It was necessary to determine the critical excitations of the structure in that class. The critical excitations have intensities not exceeding an assumed maximum, and they drive selected structural variables to their highest response peaks. Such excitations are not very difficult to determine. The precise form of an excitation depends on the definition of its intensity. Table 1 shows three examples [3]. The symbol δ denotes the unit impulse and h the impulse response function of the variable under consideration. The first example shows a critical excitation that is, except for a constant factor, the time-reversed impulse response. The second example is a squared-off version of the first. (In undamped systems, this version is a combination of sine waves, as is sometimes expected.)

One disadvantage of examples such as those shown is that they can lead to preposterously large response peaks, especially for structures with relatively large fundamental periods. That is, the response induced in the structure by one of its critical excitations would be larger than could occur as a result of any realistic ground motion. Information regarding ground motions other than their intensity also lead to disqualification. Unfortunately, critical excitations derived without the benefit of information regarding ground motions are often disqualified.

Table 1. Examples of Critical Excitations

Intensity Definition (I)	Critical Excitation	Response Peak	Notation
$\left[\int_0^\infty x^2(t)dt\right]^{1/2}$	$(I/N)h(-t)$	IN	$N^2=\int_0^\infty h^2(t)dt<\infty$
$\max_t\lvert x(t)\rvert$	$\lvert h(-t)/\lvert h(-t)\rvert$	IN_1	$N_1=\int_0^\infty \lvert h(t)\rvert dt<\infty$
$\int_0^\infty \lvert x(t)\rvert\, dt$	$I\delta(t+t_m)$	IN_2	$N_2=\max_t\lvert h(t)\rvert = \lvert h(t_m)\rvert<\infty$

It has not yet been possible to establish unequivocally the additional information required and how to utilize it to determine critical excitations in earthquake engineering. A variation of the basic idea that has been somewhat successful is described in the next section.

SUBCRITICAL EXCITATIONS

Subcritical excitations are derived from critical ones. Although the characteristics of realistic ground motions have not been established, motions that have already been recorded are real -- although some might be more typical than others for a particular geographical site or geological environment. It might be surmised that any linear combination of recorded ground motions could be considered realistic, provided the intensity does not exceed the maximum assumed for a given location. These linear combinations thus define a manifold of all possible excitations. Consider those excitations that lie within this manifold -- and hence are realistic -- but differ least from the critical ones described above to be the subcritical excitations of the structure. (The least difference is taken as the least squares.)

RELIABILITY ASSESSMENTS

The earthquake resistance of various structures has been assessed by using many of their subcritical excitations. Twelve ground motion records, obtained in California during the past 40 years, were used as basis excitations to establish linear manifolds. All were recorded within 30 km from epicenters.

Some of these assessments are shown in Table 2. They are typical of others [6-8]. All have been normalized to the intensity of the ground motion of the NS component of the Imperial Valley earthquake, as recorded at El Centro on May 18, 1940. The subcritical excitations were derived from the critical ones shown in Table 1. The structural analyses were executed with modified versions of the STRUDL [9] and XTABS programs [10].

The response peaks listed in Table are from 2.5 to 3.5 times greater than those calculated for the El Centro ground motion. This implies that some realistic excitations – namely, subcritical ones – have the same intensity as the El Centro ground motion but induce response peaks in the structures that are higher by the factors cited. One such excitation, shown in Figure 1, drives the top floor of Office Building 1 (Table 2) to its highest peak. (Other peaks for the same building are similar.) On inspection, the excitation can pass for a realistic ground motion in the sense that no conspicuous traits distinguish it from recorded motions. (Nor does a Fourier amplitude spectrum reveal such traits.)

Table 2. Reliability Assessments

	Response-Peaks		Ductility Ratios	
	Due to El Centro	Due to Sub-critical Excitations	Due to El Centro	Due to Sub-critical Excitations
Office Building 1				
Top floor displ. (ft)	1.36	3.41		
Col. moment (ft-k)	972	2785	1.09	2.50
Col. axial force (k)	952	2500		
Laboratory Building				
Top floor displ. (ft)	0.53	1.87		
Col. moment (ft-k)	1021	3334	1.83	4.98
Col. axial force (k)	369	1144		
Office Building 2				
Top floor displ. (ft)	0.46	1.20		
Col. moment (ft-k)	721	1123	0.84	1.34
Col. axial force (k)	1096	2073		
Hospital				
2nd floor displ. (ft)	0.218	0.307		
Ext. Col.				
Moment (ft-k)	1922	2680	≅12	≅18
Shear (k)	307	428		

It is of interest whether or not the structures were designed with a ductility margin sufficient to absorb the motion described by the large peaks (see Table 2). The two office buildings are considered satisfactory. (Both were in fact designed by a consulting firm with broad experience in earthquake engineering.) The Laboratory Building and the Hospital are judged to fall short of what might be desired. In the case of the Laboratory Building, the same conclusion was independently reached by its owners, and a reinforcement program is underway. The collapse of the Hospital during the San Fernando earthquake of February 9, 1971, confirms the conclusion for this building.

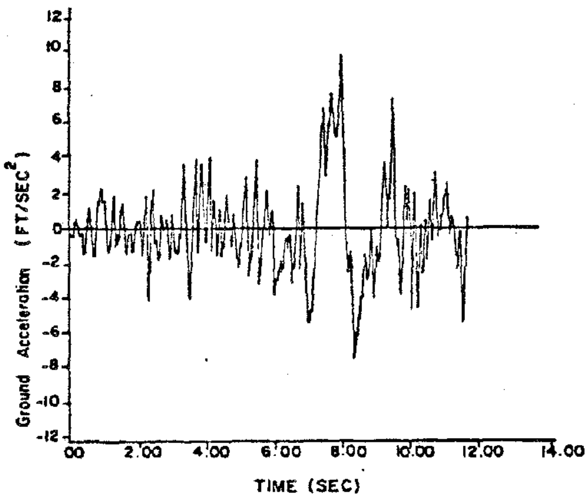

Figure 1. Example of a Subcritical Excitation with El Centro Intensity

DISCUSSION AND CRITIQUE

The results reported above, and others not reported in this review, support the conclusion that reliable, though somewhat conservative, assessments of structural earthquake resistance are possible by the method described. There is every reason to believe that similar assessments can be expected in other fields. These remarks should not be interpreted, however, to mean that modifications in the present method or variations on the original idea are not worthwhile. On the contrary, improvements and extensions are desirable in several directions.

First, the transition from a critical to a subcritical excitation contributes to the realism of the method, but at a price: the neat extremal properties of the critical excitation are lost. There is no guarantee that a subcritical excitation generates the highest response peak among all of those in the manifold of realistic ones. Computations have shown excitations that lie in the manifold but produce somewhat higher response peaks than the subcritical ones. This is not desirable. It would be better to determine the critical excitation in the manifold, but, although it can be done, no computational experience yet exists.

It would be even better to have a clear definition of what constitutes a realistic excitation. In earthquake engineering, several studies have been published [4, 5], but none has been practically applied. Success in this direction might eliminate a further disadvantage of assessments based on subcritical excitation: the sensitivity to the choice of the basis excitations. The elimination and/or addition of one such excitation can apparently bring about a non-negligible change in the response peaks that can be generated by the subcritical excitation. This is not desirable.

The nature of the geological overburden is an important factor in the assessment of earthquake resistance. Perhaps its importance would decrease if assessments were made using the critical excitations of a structure.

The computations in all case studies thus far have been comparable, perhaps slightly less than, those required for the reliability assessment of dynamical systems by other methods. Possible computational shortcuts are now being explored in an effort to economize, and additional study is desirable.

Evidently any mechanical system becomes nonlinear as it approaches failure. It is therefore desirable to extend the method to nonlinear systems. One theoretical extension has thus far been made [11], but no appreciable computation has been done. It is thus not clear that this particular extension will be suited to practical applications.

ACKNOWLEDGMENT

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Because of copyright restrictions, the following article has been omitted: (Appendix E.3) "Reliability of Seismic Resistance Predictions," by R. F. Drenick and C.-B. Yun, Journal of the Structural Division*, Proceedings of the American Society of Civil Engineers, Vol. 105, No. ST10, October, 1979.

*American Society of Civil Engineers, 345 E. 47th St. New York, N.Y. 10017.

APPENDIX F

COMPUTER PROGRAMS

FORWARD

****

These programs have been developed for the solution of problems related to the seismic response analysis of structural systems. They are written in FORTRAN IV language and used on the IBM 360/65 computer at Polytechnic.Institute of New York.

A data set with DSNAME=USER.PCWANG.BASIS including 60 records of earthquake time histories has been created and stored on the disk USER01.The first 20 of these records are selected to represent rock soil sites condition,the next 20 to represent stiff and the last 20 cohesionless soil site conditions.The subroutines used by the programs have been compiled and stored on the disk USER01 with DSNAME=USER.PCWANG.PHILIP.

In this text the description of input data and the listings of the subroutines and main programs are given.For each program the output of the results of the solution of some sample problems can be found.

Program : SP(1)

This program computes linear and nonlinear regular response spectra. records are limited to 701 points and a maximum number of 20 periods can be computed.

Card 1 :IANAL,NA,NF,INL,IFILE (8I10)

IANAL=0 : Linear spectrum computation.
IANAL=1 : Nonlinear spectrum computation.
NA : NO. of points of the records.Must be less than 701.
NF : NO. of periods to be computed.Must be less than 20.
INL : Type of nonlinearity.
=1 : Softening-Hardening spring case.
=2 : Elastoplastic case.
=3 : Bilinear case.
IFILE : NO. of the file of earthquake considered.Must be equal to 1 up to 60.

Card 2 :GETA (F10.5)

GETA : Damping ratio for the spectrum computation.

Card 3 :DUCT,BSK,SHC,GC,GK,ACRC,ACRK (8F10.5)

DUCT : Ductility factor for INL=2.
BSK : Ratio K_2/K_1 for INL=3.
SHC : Coefficient a for INL=1.($f=y \pm ay^3$).
GC : Assumed value for C_o.(Usually equal to 0).
GK : Assumed value for K_o.(Usually equal to 0).
ACRC : Accurancy for damping computation.
ACRK : Accurancy for stiffness computation.

Card(s) 4 :The values of the periods are given with format 8F10.5

Examples:Two cases of spectrum computation are listed here.First linear elastic response spectra are computed for the El Centro earthquake.Second elastoplastic spectra are also computed for the same earthquake with ductility factor equal to 3.0.

Program : SP(2)

This program computes linear critical response spectra.

Card 1 :DAMP,ISOIL,NPRS (F10.5,2I10)

DAMP :Damping ratio for the spectrum.

ISOIL :Type of soil condition.

=1 is for rock

=2 is for stiff

=3 is for cohes.

NPRS :No. of periods to be considered.Must be less than 20.

Card(s) 2 :Values of periods.(8F10.5)

Program : SP(3)

This program computes elastoplastic critical response spectra.

Card 1 :ISOIL,NITER (8I10)

ISOIL :Same as in program SP(2).

NITER :No. of iterations to be done.

Card 2 :DAMP,DUCT (8F10.5)

DAMP :Damping ratio to be considered.

DUCT :Ductility factor.

Card 3 :PERD (F10.5)

Value of the spectral period.

Program : SP(4)

This program computes critical excitations for linear structural systems.

Card 1 :DTHT,SFHT,ISOIL,IPUNCH (2F10.5,2I10)

DTHT :Time step for unit impulse response record:h(t).

SFHT :Scalar factor to be applied for h(t) record.

ISOIL :Soil condition type.Same as program SP(2).

IPUNCH :If =1,the time history of the critical excitation is punched out.

Card(s) 2 :The h(t) record with format (4(F13.8,7X)).

Program : SP(5)

This program computes critical excitations for elastoplastic S.D.O.F structural systems.

Card 1 : NE,ISOIL,NP,NITER,IPUNCH (8I10)

NE :No. of basis excitations.Max.=10.
ISOIL :Same as in program SP(2).
NP :No. of points of the records.Max.=701.
NITER :No. of iterations to be considered.
IPUNCH :Punch out case(=1).

Card 2 :TDUR,DUCT (8F10.5)

TDUR :Duration of the analysis.
DUCT :Ductility factor.

Card 3 :SM,DAMP,SK (8F10.5)

SM :Mass.
DAMP :Damping ratio.
SK :Stiffness.

Program : SP(6)

This program computes simulated earthquakes from given response spectra. (Velocity spectra S_v).

Card 1 :NITR,IPLT,IPUNCH (8I10)

NITR	:No. of iterations for each given spectrum.
IPLT	:Plotting case.
IPUNCH	:Punch out case. If =1 the simulated earthquake will be punched out.

Card 2 :DF,FBUILD,FDECAY,RAMDA,SCLF (8F10.5)

DF	:Frequency step in cps of the input spectrum.
FBUILD	:Percent of the period of the S.E used to build up its time history.
FDECAY	:Percent of the period of the S.E used in order to decay its time history.
RAMDA	:Coefficient of the exponetial decay.
SCLF	:Scalar factor to be applied for the input S_v spectra.

Card 3 :DMP(I),I=1,3 (8F10.5)

Three values of damping ratio for the given input S_v spectra.

Card(s) 4:Three sets of velocity spectra are to be given. (5(7X,F8.3))

Program : SP(7)

This program computes critical excitations of nonlinear M.D.O.F structural systems.

Card 1 :NDOF,NE,ISOIL,NP,IPUNCH (8I10)

NDOF	:No. of the degrees of freedom of the system.
NE	:No. of basis excitations.
ISOIL	:Same as program SP(2).
NP	:No. of points in each record for the analysis.
IPUNCH	:If =1 then linear and nonlinear critical excitations are punched out.

Card 2 :TDUR,DUCT (8F10.5)

TDUR	:Duration to be considered for the analysis.
DUCT	:Ductility factor.

Card(s) 3:Mass matrix.(8F10.5)

Card(s) 4:Stiffness matrix.(8F10.5)

DATA SET UTILITY - GENERATE

PROCESSING ENDED AT EOD

```
C                                                                    C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                                                                    C
C         PROGRAM    " SP(1) "                                       C
C                                                                    C
C         THIS IS TO COMPUTE LINEAR AND NONLINEAR                    C
C         REGULAR RESPONSE SPECTRA FOR A S.D.O.F. SYSTEM.            C
C                                                                    C
C         IANAL      =0........LINEAR SPECTRA                        C
C         IANAL      #0........NONLINEAR SPECTRA                     C
C                                                                    C
C         BY : A.J.PHILIPPACOPOULOS.-                                C
C                                                                    C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                                                                    C
      DIMENSION     X(701),Y(701),YY(701),F(701)
      DIMENSION     PERD(20),CNL(5)
C
      SCLF=32.174*12.00
C
      READ(5,10)    IANAL,NA,NF,INL,IFILE
      READ(5,20)    GETA
      READ(5,20)    DUCT,BSK,SHC,GC,GK,ACRC,ACRK
      READ(5,20)    (PERD(I),I=1,NF)
      WRITE(6,30)
      WRITE(6,40)   IANAL,NA,NF,IFILE,INL,DUCT,BSK,SHC,
     1              GC,GK,ACRC,ACRK,GETA
      WRITE(6,100)
      WRITE(6,20)   (PERD(I),I=1,NF)
C
C
      DO 1 IP=1,NF
      PRD=PERD(IP)
      WF=1.00/PRD
      WN=2.00*3.14159/PRD
      TDUR=7.00*PRD
      IF (TDUR.LT.4.)  TDUR=4.00
      IF (TDUR.GT.40.) TDUR=40.00
      NA1=NA-1
      DT=TDUR/NA1
C
C
      CALL     PINPUT     (X(1),TDUR,SCLF,NA,IFILE,DT,NSP,TSP,
     1                     SI,XMAX,TXMX,IXMX)
      SI=SI/SCLF
      XMAX=XMAX/SCLF
      WRITE(6,50)   IP,PRD,WF,WN,TDUR,DT,SI,XMAX
C
C
      IF  (IANAL.NE.0)  GO TO 2
      CALL     PRSPA      (X(1),Y(1),PRD,GETA,YM,NA,DT)
      SD=ABS(YM)
```

```
      GO TO 7
    2 CONTINUE
C
C
      AM=1.00
      AK=WN*WN*AM
      AC=2.00*AM*GETA*WN
      CNL(1)=AK
      CNL(2)=AK*BSK
      CNL(3)=SHC
      CNL(4)=DUCT
      CNL(5)=0.00
      SCI=GC
      SKI=GK
      INDEX=0
C
C
    4 CONTINUE
C
C
      CALL     ATHENS     (X(1),Y(1),YY(1),F(1),CNL(1),AM,AC,AK,
     1                    SCO,SKO,SCI,SKI,YM,INL,NA,DT)
      SD=ABS(YM)
      SD=SD/DUCT
      D1=SCO-SCI
      D2=SKO-SKI
      D1=ABS(D1)
      D2=ABS(D2)
      DD1=D1-ACRC
      DD2=D2-ACRK
      IF   (DD1)   5,5,6
    5 IF   (DD2)   3,3,6
    6 CONTINUE
      SCI=SCO
      SKI=SKO
      SKKK=AK+SKI
      SCCC=AC+SCI
      IF   (INDEX.EQ.30)        GO TO 1
      INDEX=INDEX+1
C
C
      GO TO 4
C
C
    3 CONTINUE
      IF   (INDEX.EQ.0)   GO TO 7
      WRITE(6,60)
      WRITE(6,70)   INDEX,AC,AK,SCCC,SKKK
    7 CONTINUE
      SV=WN*SD
      SA=WN*WN*SD/SCLF
      WRITE(6,30)   SD,SV,SA
C     WRITE(7,90)   PRO,WF,SD,SV,SA
    1 CONTINUE
C
C
   10 FORMAT   (8I10)
   20 FORMAT   (8F10.5)
   30 FORMAT   (1H1,//5X,12HINPUT DATA :,//)
   40 FORMAT   (/5X,38HTYPE OF ANALYSIS.....................:,I10,
```

```
     1          /5X,38HNO. OF POINTS IN THE RECORDS..........:,I10,
     2          /5X,38HNO. OF PERIODS........................:,I10,
     3          /5X,38HFILE NO. OF QUAKE RECORD..............:,I10,
     4          /5X,38HNONLINEARITY TYPE.....................:,I10,
     5          /5X,38HNONLINEARITY PARAMETERS...............:,
     6          3(/43X,F10.5),
     7          /5X,38HACCURANCY CONTROL DATA................:,
     8          4(/43X,F10.5),
     9          /5X,38HDAMPING RATIO.........................:,F10.5)
   50 FORMAT  (1H1,/5X,7HTABLE :,I5,/5X,12('='),//,
     1          /5X,38HPERIOD...........(SEC)...............:,F10.5,
     2          /5X,38HFREQUENCY........(CPS)...............:,F10.5,
     3          /5X,38HFREQUENCY........(RAD/SEC)...........:,F10.5,
     4          /5X,38HDURATION OF ANALYSIS........(SEC)....:,F10.5,
     5          /5X,38HTIME INCREMENT..............(SEC)....:,F10.5,
     6          /5X,38HINTENSITY OF QUAKE RECORD...(G)......:,F10.5,
     7          /5X,38HMAX. VALUE OF QUAKE RECORD..(G)......:,F10.5)
   60 FORMAT  (5(/),5X,10HCYCLE....:,12X,13HNONLINEAR SYSTEM :,
     1         11X,19HEQUIVALENT LINEAR :,
     2         /15X,2(6X,9HDAMPING :,4X,11HSTIFFNESS :)/)
   70 FORMAT  (5X,I10,4F15.4)
   80 FORMAT  (5(/),5X,17HSPECTRAL VALUES :,
     1         //5X,38HSD................(INCH)...............:,F10.5,
     2         //5X,38HSV................(INCH/SEC)...........:,F10.5,
     3         //5X,38HSA................(G)..................:,F10.5)
C  90 FORMAT(2F10.5,3F15.5)
  100 FORMAT  (/5X,38HTHE PERIODS ARE GIVEN BELOW...........:,//)
C
C
      STOP
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                                                                    C
C        PROGRAM   :   SP(2)                                         C
C                                                                    C
C        THIS IS TO COMPUTE :                                        C
C  L I N E A R   C R I T I C A L   R E S P O N S E   S P E C T R A   C
C                                                                    C
C        DAMP        :PERCENT OF THE CRITICAL DAMPING                C
C        ISOIL       :SOIL CONDITION                                 C
C        ISOIL       =1         ROCK       NRE=4      (PACOIMA)      C
C        ISOIL       =2         STIFF      NRE=1      (EL CENTRO)    C
C        ISOIL       =3         COHES.     NRE=4      (EUREKA)       C
C        NPRS        :NO. OF PERIODS FOR THE RESPONSE SPECTRUM       C
C        NA          :NO. OF POINTS FOR THE ANALYSIS
C        NE          :NO. OF EARTHQUAKE RECORDS                      C
C        NRE         :NO. FILE FOR THE REFERENCE EARTHQUAKE RECORD   C
C                                                                    C
C        SUBROUTINES USED HERE ARE :                                 C
C                                   PINPUT,PCP,PRSPA,PCRTL,MINV      C
C                                                                    C
C        BY :  A.J.PHILIPPACOPOULOS                                  C
C                                                                    C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                                                                    C
C
      DIMENSION     X(701,20),Y(701,20),B(20,20)
      DIMENSION     A(3000),ALPHA(20),ALFA(20),SSD(20)
      DIMENSION     BY(20),LB(20),MB(20),SI(20),PERD(20)
C
      DEFINE FILE   10(60,3000,U,KV)
```

```
C
      NTOT=701
      MTOT=20
C
C        INPUT DATA
C
      READ (5,1)    DAMP,ISOIL,NPRS
      READ (5,2)    (PERD(I),I=1,NPRS)
C
      NA=NTOT
      NE=MTOT
      SCLF=32.174
      IF  (ISOIL.EQ.1)  ISTRF=0
      IF  (ISOIL.EQ.2)  ISTRF=20
      IF  (ISOIL.EQ.3)  ISTRF=40
      IF  (ISOIL.EQ.1)  NRE=4
      IF  (ISOIL.EQ.2)  NRE=1
      IF  (ISOIL.EQ.3)  NRE=4
C
C        PRINT DATA
C
      WRITE (6,4)
      WRITE (6,5)  DAMP,ISOIL,NRE,NA,NE,NPRS
      WRITE (6,7)  (PERD(I),I=1,NPRS)
C
C
CCCCCCCCCC
      DO 8 IP=1,NPRS
CCCCCCCCCC
      WRITE (6,4)
      PRD=PERD(IP)
      WF=1.00/PRD
      WN=2.00*3.141593/PRD
      TDUR=7.00*PRD
      IF  (TDUR.LT.4.00)  TDUR=4.00
      IF  (TDUR.GT.40.0)  TDUR=40.00
      NA1=NA-1
      DT=TDUR/NA1
      WRITE (6,22) IP,PRD,WF,WN,TDUR,DT
      WRITE (6,4)
C
C        INPUT THE BASIS EXCITATIONS   X(I,J),I=1,NA,J=1,NE
C
      WRITE (6,23)
      DO 9 IF=1,NE
      IFILE=ISTRF+IF
      CALL     PINPUT  (X(1,IF),TDUR,SCLF,NA,IFILE,DT,NSP,TSP,
     1                  SSI,EXMAX,TEXMX,NPEMX)
      WRITE (6,10) IF,IFILE,NSP,TSP,SSI,EXMAX,TEXMX,NPEMX
      SI(IF)=SSI
    9 CONTINUE
C
C        NORMALIZE EXCITATIONS
C
      EM=SI(NRE)
      DO 12 J=1,NE
      IF  (J.EQ.NRE )  GO TO 12
      AMJ=EM/SI(J)
      DO 13 I=1,NA
      X(I,J)=X(I,J)*AMJ
```

```
   13 CONTINUE
   12 CONTINUE
C
C         COMPUTE MATRIX : B(I,J),I=1,NE,J=1,NE
C
      DO 14 I=1,NE
      DO 15 J=I,NE
      CALL     PCP        (X(1,I),X(1,J),B(I,J),NA,DT)
      IF  (I.NE.J)  B(J,I)=B(I,J)
   15 CONTINUE
   14 CONTINUE
      CALL     MINV       (B,NE,DB,LB,MB)
C
C         COMPUTE RESPONSES    : Y(I,J),I=1,NA,J=1,NE
C
      DO 16 J=1,NE
      CALL     PRSPA      (X(1,J),Y(1,J),PRD,DAMP,SD,NA,DT)
      SSD(J)=ABS(SD)
      SSD(J)=SSD(J)*12.00
   16 CONTINUE
C
C         COMPUTE THE COEFFICIENTS OF THE CRITICAL EXCITATION
C
      CALL     PCRTL      (Y,B,ALPHA,BY,ALFA,YCR,TYCR,IYCR,EM,DT,NA,NE)
      YCR=YCR*12.00
      SDCR=YCR
      SVCR=SDCR*WN
      SACR=SVCR*WN/(32.174*12.00)
      TYCRCL=TYCR
      IYCRCL=IYCR
C
C         PRINT RESULTS
C
      WRITE (6,24)
      DO 17 I=1,NE
      RD=SSD(I)
      RV=RD*WN
      RA=RD*WN*WN/(32.174*12.00)
      WRITE (6,25) I,ALPHA(I),RD,RV,RA
   17 CONTINUE
      WRITE (6,27)
      WRITE (6,28) SDCR,SVCR,SACR,TYCRCL,IYCRCL
CCCCCCCCCC
    8 CONTINUE
CCCCCCCCCC
C
C
    1 FORMAT (F10.5,2I10)
    2 FORMAT (3F10.5)
    4 FORMAT          (1H1)
    5 FORMAT          (10(/),5X,'COMPUTATION OF THE RESPONSE SPECTRUM :',
     1                //,5X,38('*'),10(/),
     2    /10X,'DAMPING.....................................:',F10.5,
     3    /10X,'SOIL CONDITION ROCK(1),STIFF(2),COH.(3).....:',I10,
     4    /10X,'REFERENCE EARTHQUAKE........................:',I10,
     5    /10X,'NO. OF POINTS FOR ANALYSIS..................:',I10,
     6    /10X,'NO. OF BASIS EXCITATIONS....................:',I10,
     7    /10X,'NO. OF PERIODS FOR THE RESPONSE SPECTRUM....:',I10)
    7 FORMAT          (//10X,'THE FOLLOWING PERIODS ARE USED:',
     1                 /,10F10.5,/,10F10.5,/,10F10.5)
```

```
   10 FORMAT        (3I10,4F10.3,I10)
   22 FORMAT        (20(/),10X,'TABLE NO. :',I5,/10X,16('='),5(/),
     1              /10X,'PERIOD..............................:',F10.5,
     2              /10X,'NATURAL FREQUENCY  (CPS)...........:',F10.5,
     3              /10X,'CIRCULAR FREQUENCY  (RAD/SEC).....:',F10.5,
     4              /10X,'DURATION OF THE RECORDS...........:',F10.5,
     5              /10X,'TIME INCREAMENT USED..............:',F10.5)
   23 FORMAT        (//5X,'QUAKE',6X,'FILE',5X,'START',5X,'START',
     1                 3X,'SQRT OF',7X,'MAX',8X,'AT',8X,'AT',/7X,'NO:',
     2                 7X,'NO:',4X,'POINT:',5X,'TIME:',2X,'SQ.INTEG',
     3                 4X,'ACCEL:',5X,'TIME:',4X,'POINT:',/)
   24 FORMAT        (//15X,'QUAKE',6X,'COEFF.  A',
     1    10X,'********* INDIVIDUAL RESPONSES *********',
     2    /44X,'* SD *',9X,'* SV *',9X,'* SA *',/)
   25 FORMAT        (10X,I10,4F15.5)
   27 FORMAT        (//10X,'MAX. OF THE CRITICAL RESPONSE',/)
   28 FORMAT (//5X,'SD=',F10.5,5X,'SV=',F10.5,5X,'SA=',F10.5,5X,
     1          'T=',F10.5,3X,'(',I5,')')
C
C
      STOP
      END
C                                                                      C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                                                                      C
C         PROGRAM :      SP(3)                                         C
C                                                                      C
C   C R I T I C A L      E L A S T O P L A S T I C                     C
C   R E S P O N S E      S P E C T R A.-                               C
C                                                                      C
C   NP..............:NO.OF POINTS IN EACH RECORD                       C
C   NE..............:NO. OF BASES EXCITATIONS                          C
C   IRE.............:FILE NO. OF THE REFERENCE EXCITATION              C
C   SCLF............:SCALAR FACTOR FOR THE EXCITATIONS                  C
C   DAMP............:DAMPING                                           C
C   PERD............:PERIOD                                            C
C   DUCT............:DUCTILITY FACTOR                                  C
C                                                                      C
C         BY  :  A.J.PHILIPPACOPOULOS.                                 C
C                                                                      C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                                                                      C
      DIMENSION     XG(701,20),RESP(701,20)
      DIMENSION     B(20,20),S(2,2)
      DIMENSION     YCRTL(701),VCRTL(701),FUNC(701)
      DIMENSION     SI(20),LB(20),MB(20),BB(20),ALPHA(20),ALFA(20),
     .              RMAX(20),LS(2),MS(2)
C
C
      IPTOT=701
      IETOT=20
      NOL=14
      SCLF=32.174
C
C         INPUT DATA
C
C
      READ (5,101) ISCIL,NITER
      READ (5,103) DAMP,DUCT
C
```

```
      IF (ISOIL.EQ.1)    ISTRF=0
      IF (ISOIL.EQ.2)    ISTRF=20
      IF (ISOIL.EQ.3)    ISTRF=40
      IF (ISOIL.EQ.1)    IRE=4
      IF (ISOIL.EQ.2)    IRE=1
      IF (ISOIL.EQ.3)    IRE=4
C
      NP=IPTOT
      NE=IETOT
C
C        PRINT DATA
C
      WRITE (6,105)
      WRITE (6,106)
      WRITE (6,107)      DAMP,ISOIL,IRE,NP,NE,DUCT
C
      DO 1000 IDL=1,NDL
C
      READ (5,102) PERD
C
C        COMPUTE THE DURATION FOR THE ANALYSIS
C
      PERD=1.00/PERD                                                   ******
      PRD=PERD
      WF=1.00/PRD
      WN=2.00*3.14159/PRD
      TDUR=7.00*PRD
      IF  (TDUR.LT.4.000)   TDUR=4.00
      IF  (TDUR.GT.40.00)   TDUR=40.0
      NP1=NP-1
      DT=TDUR/NP1
      WRITE (6,105)
      WRITE (6,110)         PRD,WF,WN,TDUR,DT
      WRITE (6,111)
      WRITE (6,112)
C
C        INPUT THE BASES EXCITATIONS : XG(I,J),I=1,NP,J=1,NE
C
      DO 2 IF=1,NE
      IFILE=ISTRF+IF
      CALL     PINPUT (XG(1,IF),TDUR,SCLF,NP,IFILE,DT,NSP,TSP,SSI,
     1                 XGMAX,TXGMX,IXGMX)
      WRITE (6,113)      IF,IFILE,NSP,TSP,SSI,XGMAX,TXGMX,IXGMX
      SI(IF)=SSI
    2 CONTINUE
C
C        NORMALIZE THE BASES EXCITATIONS TO THE REFERENCE INTENSITY
C
      SIRE=SI(IRE)
      DO 4 J=1,NE
      IF (J.EQ.IRE)   GO TO 4
      SIJ=SIRE/SI(J)
      DO 5 I=1,NP
      XG(I,J)=XG(I,J)*SIJ
    5 CONTINUE
    4 CONTINUE
C
C        COMPUTE MATRIX B(I,J),I=1,NE,J=1,NE
C
      DO 6 I=1,NE
```

```
      DO 7 J=I,NE
      CALL    PCP        (XG(1,I),XG(1,J),B(I,J),NP,DT)
      IF (I.NE.J)  B(J,I)=B(I,J)
    7 CONTINUE
    6 CONTINUE
C
      CALL    MINV       (B,NE,DB,LB,MB)
C
C        COMPUTE THE SPECTRUM VALUES
C
      SM=1.00
      SC=2.00*SM*DAMP*WN
      SK=WN*WN*SM
C
      GCO=0.0
      GKO=0.0
      SKO=GKO
      SCO=GCO
      INDEX=1
      WRITE (6,116)
C
C
   11 CONTINUE                                                    ++++++++
C
C
      SKI=SK+SKO
      SCI=SC+SCO
C
      OMEGA=SKI/SM
      OMEGA=SQRT(OMEGA)
      EQDMP=SCI/(2.00*SM*OMEGA)
      EQPRD=2.00*3.14159/OMEGA
C
      WRITE (6,105)
      WRITE (6,115)  INDEX,EQDMP,OMEGA
C
C        INDIVIDUAL RESPONSES
C
      DO 21 I=1,NE
      CALL    PRSPA      (XG(1,I),RESP(1,I),EQPRD,EQDMP,YMAX,NP,DT)
      YMAX=ABS(YMAX)
      YMAX=12.00*YMAX
      IF  (INDEX.EQ.1)   GO TO  41
      YMAX=YMAX/DUCT
   41 CONTINUE
      RMAX(I)=YMAX
   21 CONTINUE
C
C        CRITICAL RESPONSES
C
      CALL    PCRTL      (RESP,B,ALPHA,BB,ALFA,YCRT,TYCR,IYCR,
     .                    SIRE,DT,NP,NE)
      SD=YCRT*12.00
      IF  (INDEX.EQ.1)   GO TO  42
      SD=SD/DUCT
   42 CONTINUE
C
      DO 31 K=1,NP
      YC=0.0
      DO 32 I=1,NE
```

```
   32 YC=YC+ALPHA(I)*RESP(K,I)
      YCRTL(K)=YC
   31 CONTINUE
C
C        RESTORING FORCE
C
      DO 22 I=1,NP
      VCRTL(I)=SK*YCRTL(I)
   22 CONTINUE
C
C        NON-LINEAR FANCTION
C
      CALL     PAMAX      (VCRTL(1),RFMX,TMX,IMX,DT,NP)
      CALL     PFNL2      (VCRTL(1),RFMX,FUNC(1),DUCT,NP)
C
C        CRITICAL VELOCITIES
C
      NP1=NP-1
      DO 23 I=1,NP1
      VCRTL(I)=YCRTL(I+1)-YCRTL(I)
      VCRTL(I)=VCRTL(I)/DT
   23 CONTINUE
      VCRTL(NP)=VCRTL(NP1)
C
C        MATRIX S
C
      CALL     PCP        (YCRTL(1),YCRTL(1),S11,NP,DT)
      CALL     PCP        (YCRTL(1),VCRTL(1),S12,NP,DT)
      S21=S12
      CALL     PCP        (VCRTL(1),VCRTL(1),S22,NP,DT)
      S(1,1)=S11
      S(1,2)=S12
      S(2,1)=S21
      S(2,2)=S22
      CALL     MINV       (S,2,DS,LS,MS)
C
C        NEW VALUES FOR SCO,SKO
C
      CALL     PCP        (FUNC(1),YCRTL(1),P1,NP,DT)
      CALL     PCP        (FUNC(1),VCRTL(1),P2,NP,DT)
C
      SKO=S(1,1)*P1+S(1,2)*P2
      SCO=S(2,1)*P1+S(2,2)*P2
C
      SSD=SD
      SSV=WN*SSD
      SSA=WN*WN*SSD
      SSA=SSA/(32.174*12.00)
C
C        PRINT RESULTS
C
      WRITE (6,121)
      WRITE (6,117)
      DO 15 I=1,NE
      RD=RMAX(I)
      RV=RMAX(I)*WN
      RA=RMAX(I)*WN*WN
      RA=RA/(32.174*12.00)
      WRITE (6,118)      I,ALPHA(I),RD,RV,RA
   15 CONTINUE
```

```
      WRITE (6,119)
      WRITE (6,120)      SSD,SSV,SSA
C
      IF  (INDEX.GE.NITER)  GO TO 10
      INDEX=INDEX+1
C
C
      GO TO 11                                                          ++++++++
C
C
   10 CONTINUE
C
C
 1000 CONTINUE
C
  101 FORMAT  (8I10)
  102 FORMAT  (8F10.5)
  103 FORMAT  (4F10.5)
  105 FORMAT  (1H1)
  106 FORMAT  (///,5X,'RESPONSE SPECTRUM DATA:',/5X,23('='))
  107 FORMAT  (///5X,'DAMPING...............................:',F10.5,
     1    /5X,'SOIL CONDITION.....................:',I10,
     2    /5X,'REFERENCE EARTHQUAKE...............:',I10,
     3    /5X,'NO. OF POINTS FOR ALL RECORDS......:',I10,
     4    /5X,'NO. OF BASIS EXCITATIONS...........:',I10,
     5    /5X,'DUCTILITY FACTOR...................:',F10.5)
  110 FORMAT  (
     1////5X,'PERIOD  (SEC)  ....................:',F10.5,
     2    /5X,'FREQUENCY  (CPS)  .................:',F10.5,
     3    /5X,'FREQUENCY  (RAD/SEC)  .............;',F10.5,
     4    /5X,'DURATION  (SEC)  ..................:',F10.5,
     5    /5X,'TIME INCREMENT  (SEC)  ............:',F10.5)
  111 FORMAT  (///5X,'A). BASES EXCITATIONS DATA :')
  112 FORMAT         (//5X,'QUAKE',6X,'FILE',5X,'START',5X,'START',
     1                3X,'SQRT OF',7X,'MAX',8X,'AT',8X,'AT',/7X,'NO:',
     2                7X,'NO:',4X,'POINT:',5X,'TIME:',2X,'SQ.INTEG',
     3                4X,'ACCEL:',5X,'TIME:',4X,'POINT:',/)
  113 FORMAT  (3I10,4F10.3,I10)
  115 FORMAT(//5X,'ITER.:',I4,
     .                     //5X,'EQ. DAMP.=',F10.3,5X,'EQ. FREQ.=',F10.3,
     .    //5X,45('='))
  116 FORMAT  (///5X,'B). ITERATION :',//)
  117 FORMAT  (///10X,'QUAKE',4X,'COEF. ALPHA',5X,
     .     '******* INDIVIDUAL RESPONSES *******',//30X,
     .    6X,'* SD *',9X,'* SV *',9X,'* SA *',/)
  118 FORMAT  (5X,I10,4F15.4)
  119 FORMAT  (////5X,'II). CRITICAL SPECTRAL VALUES :',//)
  120 FORMAT  (5X,'SD=',F10.4,5X,'SV=',F10.4,5X,'SA=',F10.4)
  121 FORMAT  (////5X,' I). INDIVIDUAL SPECTRAL VALUES :',//)
C
C
      STOP
      END
C                                                                       C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                                                                       C
C         PROGRAM   :     SP(4)                                         C
C                                                                       C
C         THIS IS TO COMPUTE :                                          C
C         C R I T I C A L  E X C I T A T I O N S                        C
```

```
C      . F O R   L I N E A R   S Y S T E M S .                           C
C                                                                         C
C        ISOIL      :SOIL CONDITION                                       C
C        ISOIL      =1         ROCK       NRE=4      (PACOIMA)            C
C        ISOIL      =2         STIFF      NRE=1      (EL CENTRO)          C
C        ISOIL      =3         COHES.     NRE=4      (EUREKA)             C
C        NA         :NO. OF POINTS FOR THE ANALYSIS
C        NE         :NO. OF EARTHQUAKE RECORDS                            C
C        NRE        :NO. FILE FOR THE REFERENCE EARTHQUAKE RECORD         C
C                                                                         C
C        BY :  A.J.PHILIPPACOPOULOS                                       C
C                                                                         C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                                                                         C
C
      DIMENSION    X(701,20),Y(701,20),HT(701)
      DIMENSION    B(20,20),LB(20),MB(20)
      DIMENSION    ALPHA(20),ALFA(20),SI(20),BY(20)
C
C
      NTOT=701
      NTOT=301
      MTOT=20
C
C        INPUT DATA
C
      READ (5,1)    DTHT,SFHT,ISOIL,IPUNCH
      READ (5,2)    (HT(I),I=1,NTOT)
C
      NA=NTOT
      NE=MTOT
      SCLF=32.174
      SCLF=9.8067
C
      DT=DTHT
      NA1=NA-1
      TOUR=NA1*DT
C
      IF  (ISOIL.EQ.1)  ISTRF=0
      IF  (ISOIL.EQ.2)  ISTRF=20
      IF  (ISOIL.EQ.3)  ISTRF=40
      IF  (ISOIL.EQ.1)  NRE=4
      IF  (ISOIL.EQ.2)  NRE=1
      IF  (ISOIL.EQ.3)  NRE=4
C
C        PRINT DATA
C
      WRITE (6,4)
      WRITE (6,5)   DTHT,ISOIL,NRE,NA,NE,TOUR,SFHT
      WRITE (6,4)
      WRITE (6,37)
      WRITE (6,2)   (HT(I),I=1,NA)
      DO 32 I=1,NA
   32 HT(I)=HT(I)*SFHT
      WRITE (6,4)
C
C        INPUT THE BASIS EXCITATIONS    X(I,J),I=1,NA,J=1,NE
C
      WRITE (6,23)
      DO 9 IF=1,NE
```

```
      IFILE=ISTRF+IF
      CALL     PINPUT  (X(1,IF),TOUR,SCLF,NA,IFILE,DT,NSP,TSP,
     1                  SSI,EXMAX,TEXMX,NPEMX)
      WRITE (6,10) IF,IFILE,NSP,TSP,SSI,EXMAX,TEXMX,NPEMX
      SI(IF)=SSI
    9 CONTINUE
C
C        NORMALIZE EXCITATIONS
C
      EM=SI(NRE)
      DO 12 J=1,NE
      IF  (J.EQ.NRE )  GO TO 12
      AMJ=EM/SI(J)
      DO 13 I=1,NA
      X(I,J)=X(I,J)*AMJ
   13 CONTINUE
   12 CONTINUE
C
C        COMPUTE MATRIX : B(I,J),I=1,NE,J=1,NE
C
      DO 14 I=1,NE
      DO 15 J=I,NE
      CALL     PCP       (X(1,I),X(1,J),B(I,J),NA,DT)
      IF  (I.NE.J)  B(J,I)=B(I,J)
   15 CONTINUE
   14 CONTINUE
      CALL     MINV      (B,NE,DB,LB,MB)
C
C        COMPUTE RESPONSES    : Y(I,J),I=1,NA,J=1,NE
C
      WRITE (6,22)
      DO 16 J=1,NE
      CALL     PTDTR     (X(1,J),HT(1),Y(1,J),YMAX,TYMX,IYMX,DT,NA)
      WRITE (6,25) J,YMAX,TYMX,IYMX
   16 CONTINUE
C
C        COMPUTE THE COEFFICIENTS OF THE CRITICAL EXCITATION
C
      CALL     PCRTL     (Y,B,ALPHA,BY,ALFA,YCR,TYCR,IYCR,EM,DT,NA,NE)
      WRITE (6,27)
      WRITE (6,28) ((I,ALPHA(I)),I=1,NE)
      WRITE (6,30) YCR,TYCR,IYCR
      IF  (IPUNCH.NE.1)  GO TO 53
      DO 50 I=1,NA
   50 HT(I)=0.0
C
C        CRITICAL EXCITATION
C
      DO 54 K=1,NA
      XC=0.0
      DO 55 I=1,NE
   55 XC=XC+ALPHA(I)*X(K,I)
      HT(K)=XC
   54 CONTINUE
      WRITE (6,35)
      WRITE (6,7)  (HT(I),I=1,NA)
      WRITE (7,34) (HT(I),I=1,NA)
C
   53 CONTINUE
C
```

```
    1 FORMAT (2F10.5,2I10)
    2 FORMAT(8F10.1)
    4 FORMAT       (1H1)
    5 FORMAT       (10(/),5X,'INPUT DATA',//,5X,10('*'),/,
     2   /10X,'TIME INCREMENT...............................:',F10.5,
     3   /10X,'SOIL CONDITION ROCK(1),STIFF(2),COH.(3)......:',I10,
     4   /10X,'REFERENCE EARTHQUAKE.........................:',I10,
     5   /10X,'NO. OF POINTS FOR ANALYSIS...................:',I10,
     6   /10X,'NO. OF BASIS EXCITATIONS.....................:',I10,
     7   /10X,'DURATION FOR ANALYSIS........................:',F10.5,
     8   /10X,'SCALAR FACTOR FOR H(T) RECORD................:',F10.5)
    7 FORMAT(5E13.5)
   10 FORMAT       (3I10,4F10.3,I10)
   23 FORMAT       (//5X,'QUAKE',6X,'FILE',5X,'START',5X,'START',
     1                3X,'SQRT OF',7X,'MAX',8X,'AT',8X,'AT',/7X,'NO:',
     2                7X,'NO:',4X,'POINT:',5X,'TIME:',2X,'SQ.INTEG',
     3                4X,'ACCEL:',5X,'TIME:',4X,'POINT:',/)
   22 FORMAT  (///10X,'INDIVIDUAL RESPONSES',
     1   //5X,'QUAKE',12X,'RESPONSE',16X,'TIME',10X,'POINT',//)
   25 FORMAT  (5X,I5,10X,E10.4,10X,F10.3,10X,I5)
   27 FORMAT  (///10X,'COEF. OF THE CRITICAL EXCITATION',/)
   28 FORMAT  (10X,I10,F10.4)
   30 FORMAT  (//10X,'CRITICAL RESPONSE',/10X,'MAX. VALUE=',E10.4,
     1        5X,'TIME=',F6.3,5X,'POINT=',I5)
   34 FORMAT  (5E10.4)
   35 FORMAT  (///10X,'CRITICAL EXCITATION RECORD',//)
   37 FORMAT  (///10X,'UNIT IMPULSE RESPONSE RECORD',//)
C
C
      STOP
      END
C
C       *************************************************
C       *                                               *
3       *          PROGRAM  :  " SP (5) "               *
C       *                                               *
C       *          THIS IS TO COMPUTE                   *
C       *          CRITICAL EXCITATIONS OF              *
C       *          S.D.O.F ELASTOPLASTIC SYSTEMS        *
C       *                                               *
C       *          NE    :NO. OF BASIS EXCITATIONS      *
C       *          ISOIL:TYPE OF SOIL CONDITION         *
C       *          NP    :NO. OF POINTS FOR ANALYSIS    *
C       *          NITER:NO. OF ITERATIONS              *
C       *          TDUR :DURATION FOR ANALYSIS          *
C       *          RFMX :YIELDING FORCE                 *
C       *          SM    :MASS OF THE SYSTEM            *
C       *          DAMP :DAMPING RATIO (LINEAR)         *
C       *          SK    :STIFFNESS (LINEAR)            *
C       *          IPLOT=1 IS FOR PLOTTING CASE         *
C       *          IPUNCH=1 IS FOR PUNCHING OUT CASE    *
C       *                                               *
C       *          BY : A.J.PHILIPPACOPOULOS            *
C       *                                               *
C       *************************************************
C
      DIMENSION   XG(701,10),RESP(701,10)
      DIMENSION   YCRTL(701),VCRTL(701),ERF(701),FNL(701),TRF(701)
     .            ,HF(701),XCRTL(701),CRT(701)
      DIMENSION   S(10,10),AA(2,2)
```

```
      DIMENSION     ALPHA(10),AO(10),BC(10),LG(10),MG(10),LAA(2),MAA(2)
     .              ,SI(10)
C
C        INPUT DATA
C
      READ 100,NE,ISOIL,NP,NITER,IPUNCH,IPLOT
      READ 101,TDUR,RFMX,PLYM
      READ 101,SM,DAMP,SK
      SCLF=32.174
C
C        PRINT DATA
C
      PRINT 102,NE,ISOIL,NP,RFMX,SM,DAMP,SK
C
C        INPUT THE BASIS EXCITATIONS
C
      IF  (ISOIL.EQ.1)  ISTRF=0
      IF  (ISOIL.EQ.2)  ISTRF=20
      IF  (ISOIL.EQ.3)  ISTRF=40
      IF  (ISOIL.EQ.1)  IRE=4
      IF  (ISOIL.EQ.2)  IRE=1
      IF  (ISOIL.EQ.3)  IRE=4
C
      PRINT 103
      DO 1 IF=1,NE
      IFILE=ISTRF+IF
      CALL    PINPUT    (XG(1,IF),TDUR,SCLF,NP,IFILE,DT,
     1                   NSP,TSP,SIL,XMX,TMX,IMX)
      PRINT 104,IF,IFILE,TSP,NSP,SIL,XMX,TMX,IMX
      SI(IF)=SIL
    1 CONTINUE
C
C        NORMALIZE THE BASIS OF EARTHQUAKES
C
      SIRE=SI(IRE)
      DO 2 J=1,NE
      IF (J.EQ.IRE)  GO TO 2
      SIJ=SIRE/SI(J)
      DO 3 I=1,NP
      XG(I,J)=XG(I,J)*SIJ
    3 CONTINUE
    2 CONTINUE
C
C        COMPUTE MATRIX G
C
      DO 4 I=1,NE
      DO 5 J=1,NE
      CALL    PCP       (XG(1,I),XG(1,J),G(I,J),NP,DT)
      IF  (I.NE.J)  G(J,I)=G(I,J)
    5 CONTINUE
    4 CONTINUE
      CALL    MINV      (G,NE,DG,LG,MG)
      PRINT 105,DT
C
C        ITERATION PROCEDURE
C
      SCO=0.0
      SKO=0.0
      OMEGA=SK/SM
      OMEGA=SQRT(OMEGA)
```

```
      SC=2.00*DAMP*OMEGA*SM
      YIELD=RFMX/SK
      PRINT 124,YIELD
      PRINT 125,OMEGA
      ID=1
C
    7 CONTINUE
C
      PRINT 107,ID
      SCO=SCO+SC
      SKO=SKO+SK
      FRQ=SKO/SM
      FRQ=SQRT(FRQ)
      DMP=SCO/(2.00*SM*FRQ)
      PRINT 108,DMP,FRQ
C
C         UNIT IMPULSE RESPONSE
C
      CALL     PHT        (HT(1),SM,DMP,FRQ,NP,DT)
C
C         INDIVIDUAL RESPONSES
C
      PRINT 109
      DO 8 I=1,NE
      DO 85 J=1,NP
   85 XG(J,I)=-XG(J,I)*SM
      CALL     PTDTR      (XG(1,I),HT(1),RESP(1,I),YM,TY,IY,DT,NP)
      PRINT 110,YM,TY,IY
      DO 86 J=1,NP
   86 XG(J,I)=XG(J,I)/(-SM)
    8 CONTINUE
C
C         CRITICAL RESPONSE
C
      CALL     PCRTL      (RESP,G,ALPHA,BO,AO,YCR,TCR,ICR,SIRE,DT,NP,NE)
      PRINT 106
      PRINT 127
      PRINT 111,((I,ALPHA(I)),I=1,NE)
      DO 20 K=1,NP
      YC=0.0
      DO 21 I=1,NE
   21 YC=YC+ALPHA(I)*RESP(K,I)
      YCRTL(K)=YC
   20 CONTINUE
      CALL     PAMAX      (YCRTL(I),YCRMX,TCRMX,ICRMX,DT,NP)
      PRINT 112,YCRMX,TCRMX,ICRMX
      IF  (YCRMX.LT.YIELD)  GO TO 69
      DUCT=YCRMX/YIELD
      PRINT 123,DUCT
      IF  (IPLOT.NE.1)  GO TO 67
      PRINT 121
      CALL     PPLT       (YCRTL,NP,1,PLYM,1)
C
C         CRITICAL VELOCITY
C
   67 NP1=NP-1
      DO 70 I=1,NP1
      VCRTL(I)=YCRTL(I+1)-YCRTL(I)
   70 VCRTL(I)=VCRTL(I)/DT
      VCRTL(NP)=VCRTL(NP1)
```

```
C
C         ELASTIC RESTORING FORCES
C
      DO 30 I=1,NP
      ERF(I)=YCRTL(I)*SK
   30 CONTINUE
C
C         NONLINEAR FUNCTION
C
      DO 66 I=1,NP
      IF  (ABS(ERF(I)).LE.RFMX)   GO TO 61
      IF  (ERF(I))  62,63,64
   62 FNL(I)=-RFMX-YCRTL(I)*SK
      GO TO 65
   63 FNL(I)=0.0
      GO TO 65
   64 FNL(I)=+RFMX-YCRTL(I)*SK
      GO TO 65
   61 FNL(I)=0.0
   65 CONTINUE
   66 CONTINUE
C
C         TOTAL RESTORING FORCE
C
      DO 33 I=1,NP
   33 TRF(I)=ERF(I)+FNL(I)
      IF  (IPLOT.NE.1)  GO TO 68
      PRINT 122
      PLRF=1.5*RFMX
      CALL     PPLT       (TRF,NP,1,PLRF,1)
C
C         CRITICAL EXCITATION
C
      DO 40 K=1,NP
      XCR=0.0
      DO 41 I=1,NE
   41 XCR=XCR+ALPHA(I)*XG(K,I)
      XCRTL(K)=XCR
   40 CONTINUE
C
      IF  (ID.NE.1)  GO TO 80
      IF  (IPUNCH.NE.1)  GO TO 80
      PUNCH 120,((I,XCRTL(I)),I=1,NP)
      PUNCH 120,((I,TRF(I)),I=1,NP)
   80 CONTINUE
C
C         MATRIX AA
C
   68 IF  (ID.EQ.NITER)  GO TO 17
      CALL     PCP        (YCRTL(1),YCRTL(1),A11,NP,DT)
      CALL     PCP        (YCRTL(1),VCRTL(1),A12,NP,DT)
      A21=A12
      CALL     PCP        (VCRTL(1),VCRTL(1),A22,NP,DT)
      AA(1,1)=A11
      AA(1,2)=A12
      AA(2,1)=A21
      AA(2,2)=A22
      CALL     MINV       (AA,2,DAA,LAA,MAA)
C
C         NEW STIFFNESS & DAMPING
```

```
C
      CALL     PCP       (FNL(1),YCRTL(1),P1,NP,DT)
      CALL     PCP       (FNL(1),VCRTL(1),P2,NP,DT)
      SKO=AA(1,1)*P1+AA(1,2)*P2
      SCO=AA(2,1)*P1+AA(2,2)*P2
C
      IO=IO+1
      GO TO 7
C
C
   17 CONTINUE
C
C         OUTPUT RESULTS
C
      PRINT 113
      PRINT 114
      DO 18 I=1,NP
      T=(I-1)*DT
      PRINT 115,I,T,YCRTL(I),VCRTL(I),XCRTL(I),FNL(I),TRF(I)
   18 CONTINUE
      IF  (IPUNCH.NE.1)   GO TO 81
      PUNCH 120,((I,XCRTL(I)),I=1,NP)
      PUNCH 120,((I,TRF(I)),I=1,NP)
   81 CONTINUE
      GO TO 91
   69 PRINT 126
   91 CONTINUE
C
C
  100 FORMAT(3I10)
  101 FORMAT(3F10.3)
  102 FORMAT(1H1,//3X,12HINPUT DATA :,
     .    /3X,34HNO. OF EARTHQUAKES...............:,I10,
     .    /3X,34HSOIL TYPE........................:,I10,
     .    /3X,34HNO. OF POINTS....................:,I10,
     .    /3X,34HYIELDING FORCE...................:,F10.3,
     .    /3X,34HMASS.............................:,F10.3,
     .    /3X,34HDAMPING RATIO....................:,F10.3,
     .    /3X,34HSTIFFNESS........................:,F10.3///)
  103 FORMAT(/3X,19HBASIS EARTHQUAKES :,
     .  //7X,3HNO.6X,4HFILE,6X,9HSTARTS    ,1X,9HINTENSITY,6X,
     .    4HMAX.,5X,7HAT TIME/)
  104 FORMAT(5X,I5,I10,F10.3,'(',I3,')',3F10.3,'(',I3,')')
  105 FORMAT(///3X,17HTIME INCREAMENT :,F8.5,2X,3HSEC,//)
  106 FORMAT(///)
  107 FORMAT(1H1,
     .         //3X,25('*'),/3X,17H*  ITERATION CASE,I5,3H  *,
     .    /3X,25('*')///)
  108 FORMAT(//3X,
     .     12HEQ. DAMPING=,F10.3,5X,14HEQ. FREQUENCY=,F10.3,//)
  109 FORMAT(/3X,22HINDIVIDUAL RESPONSES :,/)
  110 FORMAT(3X,2F10.3,2X,1H(,I4,1H))
  111 FORMAT(3X,I10,F10.3)
  112 FORMAT(//3X,19HCRITICAL RESPONSE :,
     .  //3X,2F10.3,2X,1H(,I4,1H))
  113 FORMAT(///3X,15HFINAL RESULTS :,//)
  114 FORMAT(//5X,5HPOINT,6X,4HTIME,5X,5HRESP.,5X,5HVELT.,
     .    1X,9HCR. EXCT.,1X,9HNL FUNCT.,3X,7HTOT. RF,//)
  115 FORMAT(I10,6F10.3)
  118 FORMAT(I10,5F10.3)
```

```
  120 FORMAT(6(I4,F9.3))
  121 FORMAT(///3X,27HPLOT OF CRITICAL RESPONSE :)
  122 FORMAT(///3X,25HPLOT OF RESTORING FORCE :)
  123 FORMAT(//3X,17HDUCTILITY FACTOR=,F8.2,//)
  124 FORMAT(///3X,22HYIELDING DEFORMATION :,F10.3,//)
  125 FORMAT(///3X,18HLINEAR FREQUENCY :,F10.3,//)
  126 FORMAT(///3X,30HTHE OSCILLATOR REMAINS ELASTIC,//3X,30('$'))
  127 FORMAT(3X,34HCOEF. OF THE CRITICAL EXCITATION :/)
C
      STOP
      END
C
C        *************************************************
C        *                                               *
C        *          PROGRAM   : " SP(6) "                *
C        *                                               *
C        *          TO GENERATE SIMULATED EARTHQUAKES    *
C        *                                               *
C        *          BY : A.J.PHILIPPACOPOULOS            *
C        *                                               *
C        *************************************************
C
      DIMENSION    ACC(2001),RESP(2001),ACP(1024),AFOUR(1024)
      DIMENSION    AMPLT(101),SVINPT(101),SVOUTP(101),OMEGA(101)
     *             ,RANDF(101),FREQY(101),DMP(3)
      DIMENSION    GAMA(100)
C
C
      NTOT=2001
      MTOT=50
      NFT=1024
      PLTMX=200.0
C
C
      READ 3,NITR,IPLT,IPUNCH
      READ 1,DF,FBUILD,FDECAY,RAMDA,SCLF
      READ 1,(DMP(I),I=1,3)
C
C
      DW=2.00*3.14159*DF
C
      SEW=DW
      SEP=2.00*3.14159/SEW
      SEF=1.00/SEP
C
      DT=SEP/NFT
C
      SF=DT
      TBUILD=SEP*FBUILD
      TDECAY=TBUILD+SEP
      TT=TDECAY+FDECAY*SEP
C
C
      N=TT/DT+1.001
      NBUID=TBUILD/DT+1.001
      NDECAY=TDECAY/DT+1.001
      NPE=NDECAY-NBUID+1
      IF  (NPE.GT.NFT)  NPE=NFT
      M=MTOT
      PRINT 2,N,TT,NBUID,TBUILD,NDECAY,TDECAY,DT,SEP,SEF,SEW
```

```
      PRINT 5,M,DW,DF,IPUNCH,IPLT,SCLF
      IF  (N.GT.NTOT)   GO TO 4000
C
C
C
      DO 950 I=1,M
      SVINPT(I)=0.00
      SVOUTP(I)=0.00
      OMEGA(I)=0.00
      FREQY(I)=0.0
  950 AMPLT(I)=0.00
      OMEGA(1)=DW
      DO 2030 I=2,M
      OMEGA(I)=OMEGA(I-1)+DW
 2030 CONTINUE
      DO 350 I=1,M
      FREQY(I)=OMEGA(I)/(2.00*3.14159)
  350 CONTINUE
C
C     INPUT SPECTRUM
C
      IDMP=1
      DAMPG=DMP(IDMP)
      PRINT 41,DAMPG
      PRINT 42
C     READ 4,(SVINPT(I),I=1,M)
      READ 4,(GAMA(I),I=1,100)
      DO 5551 I=1,M
      L=2*I
 5551 SVINPT(I)=GAMA(L)
      DO 105 I=1,M
      SVINPT(I)=SVINPT(I)*SCLF
  105 CONTINUE
      DO 150 I=1,M
      AMPLT(I)=SVINPT(I)
  150 CONTINUE
      PRINT 36
      PRINT 38
      PRINT 37,((FREQY(I),SVINPT(I)),I=1,M)
      PRINT 39
      CALL    PPLT      (SVINPT(1),M,1,PLTMX,0)
C
C     GENERATE THE RANDOM ANGLE
C
      J=10000000L                                                    CHECK...
      DO 2053 I=1,M
      CALL    RAN360    (J,JO,R)
      RANDF(I)=2.00*3.14159*R
      J=JO
 2053 CONTINUE
C
C     EACH ITERATION STARTS HERE
C
      ITER=1
      COEF=2.00/SEP
 2000 CONTINUE
      PRINT 35,ITER
C
C     INITIALIZE S.E. RECORD
C
```

```
      DO 5000 I=1,N
      ACC(I)=0.00
 5000 CONTINUE
C
C     GENERATE THE RANDOM WAVE
C
      DO 2057 I=1,N
      T=(I-1)*DT
      C=0.
      DO 2059 II=1,M
      WW=OMEGA(II)*T
      ANGLE=WW+RANDF(II)
      SIN1=SIN(ANGLE)
      C=C+SIN1*AMPLT(II)
 2059 CONTINUE
      ACC(I)=C+ACC(I)
 2057 CONTINUE
C
C     GENERATE THE S.E. RECORD
C
      AO=1.0/TBUILD**2
      DO 550 I=1,N
      T=DT*(I-1)
      IF(I-NBUID) 60,60,70
   60 AMULT=AO*T**2
      GO TO 550
   70 IF(I-NDECAY) 80,80,90
   80 AMULT=1.0
      GO TO 550
   90 AMULT=EXP((TDECAY-T)*RAMDA)
  550 ACC(I)=ACC(I)*AMULT
      DO 5001 I=1,N
 5001 ACC(I)=ACC(I)*COEF
      CALL     PAMAX      (ACC(1),ACCMX,ATIME,IA,DT,N)
      CALL     PSQIL      (ACC(1),ASI,DT,N)
C
C     PRINCIPAL PART OF THE S.E RECORD
C
      K=1
      DO 2016 I=NBUID,N
      ACP(K)=ACC(I)
      IF  (K.EQ.NPE)  GO TO 2021
      K=K+1
 2016 CONTINUE
C
C     CALCULATE OUTPUT RESPONSE SPECTRUM
C
 2021 GETA=CAMPG
      PRINT 28,GETA
      DO 2001 I=1,M
      PRDD=2.00*3.14159/OMEGA(I)
      CALL     PRSPA      (ACC(1),RESP(1),PRDD,GETA,SD,N,DT)
      SD=ABS(SD)
      PSV=SD*OMEGA(I)
      SVOUTP(I)=PSV
 2001 CONTINUE
C
C     COMPARE INPUT & OUTPUT VELOCITY SPECTRA
C
      PRINT 32
```

```
      DO 2003 II=1,M
      PRINT 33,FREQY(II),SVINPT(II),SVOUTP(II),II
 2003 CONTINUE
      IF  (IPLT.NE.1)  GO TO 2005
      CALL     PPLT      (SVOUTP,M,1,PLTMX,0)
 2005 CONTINUE
      IF  (ITER.GE.NITR)  GO TO 302
C
C     FOURIER SPECTRUM
C
      CALL     PFTXT     (ACP(1),AFOUR(1),DT,DW,NPE)
C     PRINT 29
C     PRINT 31,((I,OMEGA(I),AFOUR(I)),I=1,M)
C
C     NEXT ITERATION
C
      FACT=1
      DO 2014 I=1,M
      SIO=SVOUTP(I)/SVINPT(I)
      SIO=1.00/SIO
      SIO=SIO*FACT
      AMPLT(I)=SIO*AFOUR(I+1)
 2014 CONTINUE
      ITER=ITER+1
      GO TO 2000
C
C        NEXT DAMPING RATIO
C
  302 CONTINUE
      IDMP=IDMP+1
      IF  (IDMP.GT.3)  GO TO 4001
      DAMPG=DMP(IDMP)
      PRINT 41,DAMPG
      PRINT 42
      ITER=1
C     READ 4,(SVINPT(I),I=1,M)
      READ 4,(GAMA(I),I=1,100)
      DO 5552 I=1,M
      L=2*I
 5552 SVINPT(I)=GAMA(L)
      DO 4002 I=1,M
 4002 SVINPT(I)=SVINPT(I)*SCLF
      PRINT 36
      PRINT 38
      PRINT 37,((FREQY(I),SVINPT(I)),I=1,M)
      PRINT 39
      CALL     PPLT      (SVINPT(1),M,1,PLTMX,0)
      GO TO 2021
 4001 CONTINUE
C
C     OUTPUT THE RESULTS
C
      IF  (IPUNCH.NE.1)  GO TO 360
      PUNCH 11,DT,TT,N
      PUNCH 12,(ACC(I),I=1,N)
  360 PRINT 15,ACCMX,ATIME,IA,ASI
      PRINT 24
      PRINT 23,((I,ACC(I)),I=1,N)
      CALL     PPLT      (ACC(1),N,1,ACCMX,1)
 4000 CONTINUE
```

```
C
C
    1 FORMAT(8F10.5)
    2 FORMAT(1H1,5X,14HCONTROL DATA :,/,
     .   /5X,41H NO. OF POINTS FOR S.E. RECORD..........:,I10
     .   /5X,41H DURATION OF THE S.E. RECORD............:,F10.5,
     .   /5X,41H POINT TO BUILD.........................:,I10,
     .   /5X,41H TIME TO BUILD..........................:,F10.5,
     .   /5X,41H POINT TO DECAY.........................:,I10,
     .   /5X,41H TIME TO DECAY..........................:,F10.5,
     .   /5X,41H DT FOR S.E. RECORD.....................:,F10.5,
     .   /5X,41H PERIOD OF THE S.E.....(SEC)............:,F10.5,
     .   /5X,41H FREQUENCY OF THE S.E. (CPS)............:,F10.5,
     .   /5X,41H FREQUENCY OF THE S.E. (RAD/SEC)........:,F10.5)
    3 FORMAT(3I10)
    4 FORMAT(5(7X,F8.3))
    5 FORMAT (
     .    5X,41H NO. OF FREQUENCIES TO BE COMPUTED......:,I10,
     .   /5X,41H FREQUENCY INCREAMENT..(RAD/SEC)........:,F10.5,
     .   /5X,41H FREQUENCY INCREAMENT..(CPS)............:,F10.5,
     .   /5X,41H PUNCH OUT CASE.........................:,I10,
     .   /5X,41H PLOTTING CASE..........................:,I10,
     .   /5X,41H SCALAR FACTOR .........................:,F10.5)
   11 FORMAT(25H ACCELERATION TIME INT. =F5.3,4X,11HTOTAL TIME=F5.1,13HT
     1OTAL NUMBER=I4)
   12 FORMAT(8F10.4)
   15 FORMAT  (///5X,22HSIMULATED QUAKE DATA :,/,
     .   /5X,41H MAX. VALUE.............................:,F10.5,
     .   /5X,41H TIME AT THE PEAK.......................:,F10.5,
     .   /5X,41H POINT AT THE PEAK......................:,I10,
     .   /5X,41H SQ. ROOT OF THE SQ. INTEGRAL...........:,F10.5)
   16 FORMAT(10F10.5)
   17 FORMAT(10F10.5)
   18 FORMAT(10F10.5)
   19 FORMAT(F7.3,4(F12.4,2F9.4))
   23 FORMAT  (5(I5,F11.4))
   24 FORMAT (////5X,30H SIMULATED EARTHQUAKE RECORD :,//,
     .    9X,6HPOINT:,19X,6HTIME:,7X,13HACCELERATION:,///)
   27 FORMAT(F6.2,3HHZ.,1X,10F10.5)
   28 FORMAT (////5X,37HOUTPUT VELOCITY SPECTRUM FOR DAMPING:,F5.3,/)
   29 FORMAT (////5X,18HFOURIER SPECTRUM:,//)
   31 FORMAT (4(I5,2X,'(',F6.2,')',2X,F10.3))
   32 FORMAT (////5X,17HTABLE TO COMPARE:,
     .   /28X,27HS  P  E  C  T  R  U  M    SV,
     .   /6X,9HFREQUENCY,            20X,5HINPUT,9X,6HOUTPUT//)
   33 FORMAT (5X,F10.5,10X,2(5X,F10.4),10X,I5)
   34 FORMAT(///,19HINTENSITY OF S.E. :,F10.5)
   35 FORMAT(1H1,//30X,11HITERATION :,I5,//5X,70('='),//)
   36 FORMAT (///5X,24HINPUT VELOCITY SPECTRUM:,//)
   37 FORMAT(4(2X,F8.4,3X,F12.4))
   38 FORMAT(4( 5X,5HFREQ.,4X,11HSV-SPECTRUM)//)
   39 FORMAT(///5X,25HPLOT OF INPUT SV-SPECTRUM,//)
   41 FORMAT(1H1,24(/),5X,15HDAMPING RATIO :,F5.3,//5X,20('*'))
   42 FORMAT(1H1)
C
C
      STOP
      END
C        *                                                    *
C        ******************************************************
```

```
C          *                                                     *
C          *     PROGRAM  :  " SP(7) "                           *
C          *                                                     *
C          *     THIS IS TO COMPUTE CRITICAL EXCITATIONS         *
C          *     OF AN UNDAMPED NONLINEAR MDOF SYSTEM            *
C          *     IN FORM OF A STICK MODEL.                       *
C          *                                                     *
C          *     NDOF        :NO. OF D.O.F. OF THE SYSTEM        *
C          *     NE          :NO. OF BASIS EXCITATIONS           *
C          *     ISOIL       :TYPE OF SOIL SITE                  *
C          *     NP          :NO. OF POINTS FOR ANALYSIS         *
C          *     TDUR        :DURATION FOR ANALYSIS              *
C          *     RFMX        :YIELD FORCE                        *
C          *     IPUNCH      :1 THEN LINEAR AND NONLINEAR CRIT.  *
C          *                  EXCITATIONS ARE PUNCHED OUT.       *
C          *                                                     *
C          *     BY : A.J.PHILIPPACOPOULOS.                      *
C          *                                                     *
C          *******************************************************
C          *                                                     *
      DIMENSION    XG(701,10),RESP(701,10),H(701,2),HS(701,2)
      DIMENSION    G(10,10),SI(10),AC(10),AD(10),BY(10),LG(10),MG(10)
      DIMENSION    SM(2,2),SK(2,2),SKO(2,2),SKE(2,2),SS(2,2),ALPHA(10,2)
     .             ,GR(2,2),GQ(2,2),EIGV(2),PF(2),SHP(2),SMM(2),
     .             LGR(2),MGR(2),PP(2)
C
C        INPUT DATA
C
      SCLF=32.174
      NITER=3
      READ 100,NDOF,NE,ISOIL,NP,IPUNCH
      READ 101,TDUR,RFMX
      READ 102,((SM(I,J),J=1,NDOF),I=1,NDOF)
      READ 102,((SK(I,J),J=1,NDOF),I=1,NDOF)
C
C        PRINT DATA.
C
      PRINT 93
      PRINT 94
      PRINT 95,NDOF,NE,ISOIL,NP,TDUR,RFMX
      PRINT 96
      DO 41 I=1,NDOF
      PRINT 99,(SM(I,J),J=1,NDOF)
   41 CONTINUE
      PRINT 98
      DO 43 I=1,NDOF
      PRINT 99,(SK(I,J),J=1,NDOF)
   43 CONTINUE
C
C        INPUT THE BASIS EXCITATIONS
C
      IF  (ISOIL.EQ.1)  ISTRF=0
      IF  (ISOIL.EQ.2)  ISTRF=20
      IF  (ISOIL.EQ.3)  ISTRF=40
      IF  (ISOIL.EQ.1)  IRE=4
      IF  (ISOIL.EQ.2)  IRE=1
      IF  (ISOIL.EQ.3)  IRE=4
C
C
      PRINT 103
```

```
      DO 1 IF=1,NE
      IFILE=ISTRF+IF
      CALL      PINPUT     (XG(1,IF),TDUR,SCLF,NP,IFILE,DT,
     1                     NSP,TSP,SIL,XMX,TMX,IMX)
      PRINT 104,IF,IFILE,TSP,NSP,SIL,XMX,TMX,IMX
      SI(IF)=SIL
    1 CONTINUE
C
C         NORMALIZE THE BASIS OF EARTHQUAKES
C
      SIRE=SI(IRE)
      DO 2 J=1,NE
      IF (J.EQ.IRE)  GO TO 2
      SIJ=SIRE/SI(J)
      DO 3 I=1,NP
      XG(I,J)=XG(I,J)*SIJ
    3 CONTINUE
    2 CONTINUE
C
C         COMPUTE MATRIX G
C
      DO 4 I=1,NE
      DO 5 J=1,NE
      CALL      PCP        (XG(1,I),XG(1,J),G(I,J),NP,DT)
      IF  (I.NE.J)  G(J,I)=G(I,J)
    5 CONTINUE
    4 CONTINUE
      CALL      MINV       (G,NE,DG,LG,MG)
      PRINT 105,DT
C
C         ASSUMED VALUES FOR SKO
C
      DO 6 I=1,NDOF
      DO 6 J=1,NDOF
      SKO(I,J)=0.0
    6 CONTINUE
C
C         ITERATION PROCEDURE
C
      ID=1
    7 CONTINUE
C
C
      DO 8 I=1,NDOF
      DO 8 J=1,NDOF
      SKE(I,J)=SK(I,J)+SKO(I,J)
    8 CONTINUE
      PRINT 106,ID
      PRINT 109
      DO 46 I=1,NDOF
      PRINT 108,(SKE(I,J),J=1,NDOF)
   46 CONTINUE
C
C         MODAL SHAPES  &  FREQUENCIES
C
      PRINT 110
      DO 70 I=1,NDOF
      DO 71 J=1,NDOF
      GR(I,J)=SM(I,J)
      GQ(I,J)=SKE(I,J)
```

```
   71 CONTINUE
   70 CONTINUE
      CALL     NROOT      (NDOF,GR,GO,EIGV,SS)
      DO 51 I=1,NDOF
      EV=EIGV(I)
      EV=1.00/EV
      EV=SQRT(EV)
      EIGV(I)=EV
   51 CONTINUE
      DO 47 I=1,NDOF
      PRINT 125,I,EIGV(I)
      PRINT 111,(SS(J,I),J=1,NDOF)
   47 CONTINUE
C
C         PARTICIPATION FACTORS
C
      DO 53 L=1,NDOF
      DO 54 I=1,NDOF
      SHP(I)=SS(I,L)
   54 CONTINUE
      RO=0.0
      DO 55 K=1,NDOF
      HO=0.0
      DO 56 J=1,NDOF
      HO=HO+SM(K,J)
   56 CONTINUE
      RO=RO+HO*SHP(K)
   55 CONTINUE
      PF(L)=-RO
C
C         MODAL MASSES
C
      QO=0.0
      DO 57 M=1,NDOF
      ZO=0.0
      DO 58 JM=1,NDOF
      ZO=ZO+SM(M,JM)*SHP(JM)
   58 CONTINUE
      QO=QO+ZO*SHP(M)
   57 CONTINUE
      SMM(L)=QO
C
C         MODAL UNIT IMPULSE RESPONSES
C
      AMA=SMM(L)
      AFR=EIGV(L)
      ADM=0.00
      DO 59 IL=1,NP
      H(IL,L)=PF(L)*H(IL,L)
   59 CONTINUE
   53 CONTINUE
      PRINT 122
      PRINT 123,(SMM(I),I=1,NDOF)
      PRINT 124
      PRINT 123,(PF(I),I=1,NDOF)
C
C         EQUIVALENT SYSTEM   :UNIT IMPULSE RESPONSES
C
      DO 65 IM=1,NDOF
      DO 66 I=1,NDOF
```

```
      SHP(I)=SS(IM,I)
   66 CONTINUE
      DO 67 K=1,NP
      Q=0.0
      DO 68 J=1,NDOF
      Q=Q+SHP(J)*H(K,J)
   68 CONTINUE
      HS(K,IM)=Q
   67 CONTINUE
   65 CONTINUE
      DO 33 J=1,NDOF
      DO 34 I=1,NP
   34 H(I,J)=0.0
   33 CONTINUE
C
C          EQUIVALENT SYSTEM   :INDIVIDUAL RESPONSES
C
      DO 17 II=1,NDOF
      PRINT 113,II
      PRINT 114
      DO 18 J=1,NE
      CALL     PTDTR      (XG(1,J),HS(1,II),RESP(1,J),YM,TM,IM,DT,NP)
      PRINT 115,J,YM,TM,IM
   18 CONTINUE
C
C          EQUIVALENT SYSTEM   :CRITICAL RESPONSES
C
      CALL     PCRIL      (RESP,G,AC,BY,AQ,YCR,TCR,ICR,SIRE,DT,NP,NE)
      DO 20 K=1,NP
      YC=0.0
      DO 21 I=1,NE
   21 YC=YC+AC(I)*RESP(K,I)
      H(K,II)=YC
   20 CONTINUE
      CALL     PAMAX          (H(1,II),AM,TM,IM,DT,NP)
      PRINT 116
      PRINT 117,((I,AC(I)),I=1,NE)
      PRINT 118,AM,TM,IM
      DO 701 KK=1,NE
  701 ALPHA(KK,II)=AC(KK)
C
   17 CONTINUE
C
      DO 72 J=1,NE
      DO 73 I=1,NP
      RESP(I,J)=0.0
   73 CONTINUE
   72 CONTINUE
C
C          EQUIVALENT SYSTEM   :CRITICAL EXCITATIONS
C
      DO 36 J=1,NDOF
      DO 37 I=1,NP
   37 HS(I,J)=0.0
   36 CONTINUE
      DO 704 KF=1,NDOF
      DO 702 K=1,NP
      XC=0.0
      DO 703 J=1,NE
  703 XC=XC+ALPHA(J,KF)*XG(K,J)
```

```
      HS(K,KF)=XC
  702 CONTINUE
  704 CONTINUE
C
C        PUNCH OUT THE LINEAR CASE
C
      IF  (ID.NE.1)  GO TO 706
      IF  (IPUNCH.NE.1)  GO TO 706
      DO 707 J=1,NDOF
      PUNCH 138,J
      PUNCH 139,(HS(I,J),I=1,NP)
  707 CONTINUE
  706 CONTINUE
C
      IF  (ID.GT.NITER)  GO TO 700
C
C        ELASTIC RESTORING FORCES
C
      DO 28 I=1,NDOF
      DO 29 L=1,NP
      Z=0.0
      DO 30 J=1,NDOF
   30 Z=Z+SK(I,J)*H(L,J)
      RESP(L,I)=Z
   29 CONTINUE
      CALL    PAMAX     (RESP(1,I),EO,TE,IE,DT,NP)
      PRINT 143,I,EO
   28 CONTINUE
C
C        NON LINEAR FUNCTIONS
C
      RFLM=RFMX
      DO 74 I=1,NDOF
      DO 75 J=1,NP
      R1=RESP(J,I)
      IF  (ABS(R1).GT.RFLM)  GO TO 310
      RESP(J,I)=0.0
      GO TO 75
  310 CONTINUE
      IF  (R1)  330,340,350
  330 RESP(J,I)=-R1-RFLM
      GO TO 75
  340 RESP(J,I)=0.0
      GO TO 75
  350 RESP(J,I)=-R1+RFLM
   75 CONTINUE
   74 CONTINUE
C
C        COMPUTATION OF MATRIX AA
C
      DO 210 I=1,NDOF
      DO 76 J=1,NDOF
   76 GR(I,J)=0.0
  210 CONTINUE
C
      DO 77 I=1,NDOF
      DO 78 J=1,NDOF
      CALL    PCP       (H(1,I),H(1,J),GR(I,J),NP,DT)
      IF  (I.NE.J)  GR(J,I)=GR(I,J)
   78 CONTINUE
```

```
   77 CONTINUE
      CALL     MINV      (GR,NOOF,DGR,LGR,MGR)
      DO 200 IK=1,NOOF
      DO 201  I=1,NOOF
      CALL     PCP       (RESP(1,IK),H(1,I),PP(I),NP,DT)
  201 CONTINUE
      DO 202 I=1,NOOF
      ZZ=0.0
      DO 203 J=1,NOOF
      ZZ=ZZ+GR(I,J)*PP(J)
  203 CONTINUE
      SKO(IK,I)=ZZ
  202 CONTINUE
  200 CONTINUE
C
C        PRINT DERIVED DAMPING & STIFFNESS MATRICES
C
      PRINT 135
      DO 211 I=1,NOOF
      PRINT 131,I
      PRINT 132,(SKO(I,J),J=1,NOOF)
  211 CONTINUE
C
      ID=ID+1
      GO TO 7
C
C        PRINT RESULTS OF THE LATEST ITERATION
C
  700 CONTINUE
      PRINT 93
      DO 709 J=1,NOOF
       PRINT 113,J
      PRINT 141
      DO 710 I=1,NP
      T=(I-1)*DT
      PRINT 142,I,T,HS(I,J),H(I,J)
  710 CONTINUE
  709 CONTINUE
      IF  (IPUNCH.NE.1)   GO TO 715
      DO 716 J=1,NOOF
      PUNCH 140,J
      PUNCH 139,(HS(I,J),I=1,NP)
  716 CONTINUE
  715 CONTINUE
   93 FORMAT (1H1,/)
   94 FORMAT(3X,11HINPUT DATA:,///)
   95 FORMAT(/3X,40HNO. OF D.O.F. OF THE SYSTEM............:,I10,
     .        /3X,40HNO. OF BASIS EARTHQUAKES...............:,I10,
     .        /3X,40HSOIL CONDITION CASE....................:,I10,
     .        /3X,40HNO. OF POINTS FOR ANALYSIS.............:,I10,
     .        /3X,40HDURATION OF ANALYSIS...................:,F10.5,
     .        /3X,40HYIELD FORCE............................:F10.5//)
   96 FORMAT(/3X,11HMASS MATRIX,/)
   97 FORMAT(/3X,14HDAMPING MATRIX,/)
   98 FORMAT(/3X,16HSTIFFNESS MATRIX,/)
   99 FORMAT(10X,7F10.3)
  100 FORMAT(8I10)
  101 FORMAT(8F10.5)
  102 FORMAT(8F10.3)
  103 FORMAT(/3X,17HBASIS EARTHQUAKES,
```

```
     .  //7X,3HNO.6X,4HFILE,6X,9HSTARTS    ,1X,9HINTENSITY,6X,
     .    4HMAX.,5X,7HAT TIME/)
 104 FORMAT(5X,I5,I10,F10.5,'(',I3,')',3F10.5,'(',I3,')')
 105 FORMAT(///3X,17HTIME INCREAMENT :,F8.5,2X,3HSEC,//)
 106 FORMAT(1H1,
     .        //3X,25('*'),/3X,17H*  ITERATION CASE,I5,3H  *,
     .    /3X,25('*')//)
 107 FORMAT(/3X,40HEQ. SYSTEM : DAMPING MATRIX                 ,/)
 108 FORMAT(10X,7F10.3)
 109 FORMAT(/3X,40HEQ. SYSTEM : STIFFNESS MATRIX               ,/)
 110 FORMAT(/3X,40HMODAL SHAPES & FREQ. BEFORE NORM            ,//)
 111 FORMAT (15X,5HSHAPE,6F10.4/)
 112 FORMAT(/3X,40HMODAL SHAPES & FREQ. AFTER NORM             ,//)
 113 FORMAT(///3X,40HRESULTS FOR OUTPUT D.O.F. CASE              ,I5,
     .    //3X,45('*'),//)
 114 FORMAT(/3X,20HINDIVIDUAL RESPONSES,
     . //10X,5HQUAKE,6X,4HMAX.1X,13HAT TIME-POINT,/)
 115 FORMAT(5X,I10,2F10.5,'(',I4,')')
 116 FORMAT(/3X,40HCOEFFICIENTS OF THE CRIT. EXCIT             ,/)
 117 FORMAT(5X,I10,F10.5)
 118 FORMAT(/3X,40HMAX. OF THE CRITICAL RESPONSE               ,
     .    /3X,4HMAX=,F10.5,3X,2HT=,F10.5,3X,6HPOINT=,I5)
 119 FORMAT(/3X,40HCRITICAL RESP. & VEL. RECORDS               /)
 120 FORMAT(/3X,14HFOR D.O.F. NO:,I5,/,
     .        /6X,4HTIME,3X,17HCRITICAL RESPONSE,3X,
     .    17HCRITICAL VELOCITY/)
 121 FORMAT(3X,F7.3,2F20.5)
 122 FORMAT(/3X,12HMODAL MASSES,/)
 123 FORMAT(8F10.5)
 124 FORMAT(/3X,40HPARTICIPATION FACTORS                        ,/)
 125 FORMAT(/3X,5HMODE:,I3,4X,5HFREQ:,F10.4,3X,7HRAD/SEC,/)
 126 FORMAT (///3X,14HNO. OF D.O.F :,I5,/3X,19('*'),//)
 127 FORMAT(/5X,5HPOINT,I6X,4HTIME,11X,9HCR. RESP.,
     .    3X,12HELASTIC R.F.,5X,15HNON LINEAR FUN.,4X,
     .    16HELASTOPLAS. R.F,//)
 128 FORMAT (3X,I7,5(5X,F15.4))
 129 FORMAT (//3X,30HMAX. ELASTIC RESTORING FORCE :,F10.4/)
 130 FORMAT(///3X,9HMATRIX A:,/)
 131 FORMAT(/3X,5HROW :,I5,/3X,10('=')/) .
 132 FORMAT(10F10.4)
 133 FORMAT(//3X,39HPLOT OF ELASTOPLASTIC RESTORING FORCE :,//)
 134 FORMAT(//3X,28HPLOT OF NONLINEAR FUNCTION :,//)
 135 FORMAT(//3X,26HDERIVED STIFFNESS MATRIX :,//)
 136 FORMAT(//3X,24HDERIVED DAMPING MATRIX :,//)
 137 FORMAT (//3X,30HMAX. ELASTOPL. RESTOR. FORCE :,F10.4/)
 138 FORMAT(5X,29HLINEAR CRIT. EXCIT. FOR DOF :,I5)
 139 FORMAT(10F8.3)
 140 FORMAT(5X,29HNONLIN CRIT. EXCIT. FOR DOF :,I5)
 141 FORMAT(5X,5HPOINT,6X,4HTIME,8X,12HCRIT. EXCIT.,8X,12HCRIT. RESP.,
     .        //)
 142 FORMAT(I10,F10.4,2F20.4)
 143 FORMAT(/3X,5HDOF :,I3,3X,21HMAX. RESTORING FORCE=,F10.3,/)
C
C
      STOP
      END
C
C
      SUBROUTINE  AIGION  (DATA,RESP,VELT,RFORCE,YCRTL,YCRVL,B,G,BY,
     1       ALFA,RMAX,ALPHA,CC,RE,YM,IY,NY,SSM,SSC,SSK,
```

```
     2              SCI,SKI,SCC,SKC,NLCASE,NA,NE,DT)
C                                                                          C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                                                                          C
C    THIS SUBROUTINE COMPUTES THE VALUES OF  SCO  &  SKO                   C
C    ACCORDING TO THE EQUIVALENT LINEARIZATION CONCEPT.-                   C
C    IT IS TO BE USED FOR CONSTRUCTION OF CRITICAL SPECTRA.-               C
C                                                                          C
C    SUBROUTINES USED ARE : PRSPA  PCRTL  PFNL  PCP  PETOE  MINV           C
C                                                                          C
C    NLCASE         :TYPE OF NONLINEARITY.                                 C
C    DATA(I,J)      :INPUT EXCITATIONS.                                    C
C    RESP(I,J)      :OUTPUT RESPONSES.                                     C
C    VELT(I,J)      :OUTPUT VELOCITIES.                                    C
C    RFORCE(I)      :NON-LINEAR FUNCTION ASSOCIATED WITH THE SYSTEM.       C
C    YCRTL(I)       :CRITICAL RESPONSE.                                    C
C    RMAX(I)        :MAX. RESPONSES DUE TO EXCITATIONS.                    C
C    CC(I)          :VECTOR ASSOCIATED WITH THE NONLINEARITY.              C
C    RE             :REFERENCE INTENSITY.                                  C
C    YM,TY,NY       :MAX. VALUES OF THE CRITICAL RESPONSE.                 C
C                                                                          C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                                                                          C
C
      DIMENSION     DATA(NA,NE),RESP(NA,NE),VELT(NA,NE)
      DIMENSION     RFORCE(NA),YCRTL(NA),YCRVL(NA)
      DIMENSION     B(NE,NE),G(NE,NE),ALPHA(NE),ALFA(NE),BY(NE),RMAX(NE)
      DIMENSION     S(2,2),CC(5),LS(2),MS(2),TNL(2),P(2)
C
      SM=SSM
      SC=SSC
      SK=SSK
C
      SCS=SC+SCI
      SKS=SK+SKI
C
      WW=SKS/SM
      WN=SQRT(WW)
      PERD=2.00*3.14159/WN
      WFR=1.00/PERD
      GETA=SCS/(2.00*WN*SM)
C
C        RESPONSES          RESP(I,J),I=1,NA,J=1,NE
      DO 2 J=1,NE
      CALL    PRSPA      (DATA(1,J),RESP(1,J),PERD,GETA,YMAX,NA,DT)
      YMAX=ABS(YMAX)
      RMAX(J)=YMAX
    2 CONTINUE
C
C        VELOCITIES VELT(I,J),I=1,NA,J=1,NE
C
      NA1=NA-1
      DO 5 J=1,NE
    5 VELT(NA,J)=0.00
      DO 4 J=1,NE
      DO 3 I=1,NA1
      VELT(I,J)=(RESP(I+1,J)-RESP(I,J))/DT
    3 CONTINUE
    4 CONTINUE
C
```

```
C         COEFFICIENTS OF THE CRITICAL EXCITATION ALPHA(I),I=1,NE
C
      CALL      PCRTL      (RESP,B,ALPHA,BY,ALFA,YCRL,TYCR,IYCR,
     1                      RE,DT,NA,NE)
      YM=YCRL
      TY=TYCR
      NY=IYCR
C
C         CRITICAL RESPONSE   YCRTL(I),I=1,NA
C
      DO 6 K=1,NA
      YC=0.00
      DO 7 I=1,NE
    7 YC=YC+ALPHA(I)*RESP(K,I)
      YCRTL(K)=YC
    6 CONTINUE
      CC(5)=YM
C
C         CRITICAL VELOCITY YCRVL(I),I=1,NA
C
      DO 19 I=1,NA1
      YCRVL(I)=(YCRTL(I+1)-YCRTL(I))/DT
   19 CONTINUE
      YCRVL(NA)=0.00
C
C         NON-LINEAR FUNCTION  RFORCE(I),I=1,NA
C
      CALL      PFNL       (RFORCE(1),YCRTL(1),YCRVL(1),CC(1),NLCASE,NA)
C
      DO 8 I=1,NE
      DO 9 J=1,NE
      CALL      PCP        (RESP(1,I),RESP(1,J),G(I,J),NA,DT)
    9 CONTINUE
    8 CONTINUE
C
      CALL      PETOE      (G,ALPHA,A11,NE)
C
      DO 10 I=1,NE
      DO 11 J=1,NE
      CALL      PCP        (VELT(1,I),RESP(1,J),G(I,J),NA,DT)
   11 CONTINUE
   10 CONTINUE
C
      CALL      PETOE      (G,ALPHA,A12,NE)
C
      DO 12 I=1,NE
      DO 13 I=1,NE
      CALL      PCP        (VELT(1,I),VELT(1,J),G(I,J),NA,DT)
   13 CONTINUE
   12 CONTINUE
C
      CALL      PETOE      (G,ALPHA,A22,NE)
C
      DO 14 I=1,NE
      CALL      PCP        (RFORCE(1),RESP(1,I),G(I,1),NA,DT)
      CALL      PCP        (RFORCE(1),VELT(1,I),G(I,2),NA,DT)
   14 CONTINUE
C
      P1=0.00
      DO 15 I=1,NE
```

```
      P1=P1+ALPHA(I)*G(I,1)
   15 CONTINUE
C
      P2=0.00
      DO 16 I=1,NE
      P2=P2+ALPHA(I)*G(I,2)
   16 CONTINUE
C
      S(1,1)=A11
      S(1,2)=A12
      S(2,1)=S(1,2)
      S(2,2)=A22
      CALL     MINV      (S,2,SD,LS,MS)
C
      P(1)=P1
      P(2)=P2
C
C        VALUES OF SCIO,SKIO
C
      DO 17 I=1,2
      TNL(I)=0.00
      DO 18 J=1,2
      TNL(I)=TNL(I)+S(I,J)*P(J)
   18 CONTINUE
   17 CONTINUE
C
C
      SKO=TNL(1)
      SCO=TNL(2)
C
C
      RETURN
      END
C
C
      SUBROUTINE    ATHENS     (X,Y,YY,F,CC,SSSM,SSSC,SSSK,
     1                         SCO,SKO,SCI,SKI,YM,ICASE,N4,DT)
C                                                                     C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                                                                     C
C        FOR A GIVEN SET :SCO,SKO THIS SUBROUTINE                     C
C        COMPUTES A NEW ONE :SCON,SKON ,ACCORDING                     C
C        TO THE EQUIVALENT LINEARIZATION CONCEPT.-                    C
C                                                                     C
C                                                                     C
C   SUBROUTINES CALLED :      PRSPA,,PFNL,,PCP,,MINV                  C
C                                                                     C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                                                                     C
      DIMENSION     X(1),H(1),Y(1),YY(1),F(1)
      DIMENSION     SS(2,2),LS(2),MS(2)
      DIMENSION     PP(2),TNL(2)
      DIMENSION     CC(5)
C
      SM=SSSM
      SC=SSSC
      SK=SSSK
C
      SCS=SC+SCI
      SKS=SK+SKI
```

```
C
      WW=SKS/SM
      WN1=SQRT(WW)
      PRD1=2.00*3.14159/WN1
      GETA1=SCS/(2.00*SM*WN1)
C
      CALL     PRSPA    (X(1),Y(1),PRD1,GETA1,YMAX,NA,DT)
      YM=ABS(YMAX)
C
      NA1=NA-1
      DO 1 J=1,NA1
      YY(J)=(Y(J+1)-Y(J))/DT
    1 CONTINUE
      YY(NA)=0.00
      CC(5)=YM
C
      CALL   PFNL  (F(1),Y(1),YY(1),CC(1),ICASE,NA)
C
      CALL     PCP  (YY(1),YY(1),A11,NA,DT)
      CALL     PCP  (Y(1),Y(1),A22,NA,DT)
      CALL     PCP  (YY(1),Y(1),A12,NA,DT)
C
      SS(1,1)=A11
      SS(1,2)=A12
      SS(2,1)=SS(1,2)
      SS(2,2)=A22
C
      CALL     PCP  (F(1),YY(1),P1,NA,DT)
      CALL     PCP  (F(1),Y(1),P2,NA,DT)
C
      PP(1)=P1
      PP(2)=P2
C
      CALL  MINV  (SS,2,DS,LS,MS)
      DO 2 I=1,2
      TNL(I)=0.0
      DO 2 J=1,2
      TNL(I)=TNL(I)+SS(I,J)*PP(J)
    2 CONTINUE
C
      SCO=TNL(1)
      SKO=TNL(2)
C
      RETURN
      END
C
C
      SUBROUTINE BITRV (DATA,NPREV,N,NREM)
C     SHUFFLE THE DATA BY *BIT REVERSAL*.
C     DIMENSION DATA(NPREV,N,NREM)
C     DATA(I1,I2REV,I3)=DATA(I1,I2,I3), ALL I1 FROM 1 TO NPREV,
C     ALL I2 FROM 1 TO N (WHICH MUST BE A POWER OF TWO),AND ALL
C     I3 FROM 1 TO NREM,WHERE I2REV-1 IS THE BIT REVERSE OF I2-1.
C     FOR EXAMPLE, N = 32, I2-1 = 10011 AND I2REV-1 = 11001.
      DIMENSION DATA(1)
      IP0=2
      IP1=IP0*NPREV
      IP4=IP1*N
      IP5=IP4*NREM
      I4REV=1
```

```
      DO 60 I4=1,IP4,IP1
      IF (I4-I4REV) 10,30,30
10    I1MAX=I4+IP1-IP0
      DO 20 I1=I4,I1MAX,IP0
      DO 20 I5=I1,IP5,IP4
      I5REV=I4REV+I5-I4
      TEMPR=DATA(I5)
      TEMPI=DATA(I5+1)
      DATA(I5)=DATA(I5REV)
      DATA(I5+1)=DATA(I5REV+1)
      DATA(I5REV)=TEMPR
20    DATA(I5REV+1)=TEMPI
30    IP2=IP4/2
40    IF (I4REV-IP2) 60,60,50
50    I4REV=I4REV-IP2
      IP2=IP2/2
      IF (IP2-IP1) 60,40,40
60    I4REV=I4REV+IP2
      RETURN
      END
C
C
      SUBROUTINE COOL2(DATA,NPREV,N,NREM,ISIGN)
C     FOURIER TRANSFORM OF LENGTH N BY THE COOLEY-TUKEY
C     ALGORITHM. BIT-REVERSED TO NORMAL ORDER.
C     DIMENSION DATA(NPREV,N,NREM)
C     COMPLEX DATA
C     DATA(I1,J2,I3)=SUM(DATA(I1,I2,I3)*EXP(ISIGN*2*PI*I*((I2-1)*
C     (J2-1)/N))),SUMMED OVER I2=1 TO N FOR ALL I1 FROM 1 TO
C     NPREV, J2 FROM 1 TO N AND I3 FROM 1 TO NREM.N MUST BE A
C     POWER OF TWO. FACTORING N BY 4S SAVES ABOUT 25 PERCENT
C     OVER FACTORING BY TWOS.
C     NOTE--IT IS UNNECESSARY TO REWRITE THIS ROUTINE INTO COMPLEX
C     FORM SO LONG AS THE FORTRAN COMPILER USED STORES REAL AND
C     IMAGINARY PARTS IN ADJACENT STORAGE LOCATIONS.  IT MUST ALSO
C     STORE ARRAYS WITH THE FIRST SUBSCRIPT INCREASING FASTEST.
      DIMENSION DATA(1)
      TWOPI=6.2831853072*FLOAT(ISIGN)
      IP0=2
      IP1=IP0*NPREV
      IP4=IP1*N
      IP5=IP4*NREM
      IP2=IP1
      IF(N-1)150,150,5
5     NPART=N
10    IF(NPART-2)50,30,20
20    NPART=NPART/4
      GO TO 10
C     DO A FOURIER TRANSFORM OF LENGTH TWO
30    IP3=IP2*2
      DO 40 I1=1,IP1,IP0
      DO 40 I5=I1,IP5,IP3
      J0=I5
      J1=J0+IP2
      TEMPR=DATA(J1)
      TEMPI=DATA(J1+1)
      DATA(J1)=DATA(J0)-TEMPR
      DATA(J1+1)=DATA(J0+1)-TEMPI
      DATA(J0)=DATA(J0)+TEMPR
40    DATA(J0+1)=DATA(J0+1)+TEMPI
```

```
      GO TO 140
C     FOURIER TRANSFORM OF LENGTH 4 (FROM BIT REVERSED ORDER)
 50   IP3=IP2*4
      THETA=TWOPI/FLOAT(IP3/IP1)
      SINTH=SIN(THETA/2.)
      WSTPR=-2.*SINTH*SINTH
C     COS(THETA)-1, FOR ACCURACY.
      WSTPI=SIN(THETA)
      WR=1.
      WI=0.
      DO 130 I2=1,IP2,IP1
      IF(I2-1)70,70,60
 60   W2R=WR*WR-WI*WI
      W2I=2.*WR*WI
      W3R=W2R*WR-W2I*WI
      W3I=W2R*WI+W2I*WR
 70   I1MAX=I2+IP1-IP0
      DO 120 I1=I2,I1MAX,IP0
      DO 120 I5=I1,IP5,IP3
      J0=I5
      J1=J0+IP2
      J2=J1+IP2
      J3=J2+IP2
      IF(I2-1)90,90,80
C     APPLY THE PHASE SHIFT FACTORS
 80   TEMPR=DATA(J1)
      DATA(J1)=W2R*TEMPR-W2I*DATA(J1+1)
      DATA(J1+1)=W2R*DATA(J1+1)+W2I*TEMPR
      TEMPR=DATA(J2)
      DATA(J2)=WR*TEMPR-WI*DATA(J2+1)
      DATA(J2+1)=WR*DATA(J2+1)+WI*TEMPR
      TEMPR=DATA(J3)
      DATA(J3)=W3R*TEMPR-W3I*DATA(J3+1)
      DATA(J3+1)=W3R*DATA(J3+1)+W3I*TEMPR
 90   T0R=DATA(J0)+DATA(J1)
      T0I=DATA(J0+1)+DATA(J1+1)
      T1R=DATA(J0)-DATA(J1)
      T1I=DATA(J0+1)-DATA(J1+1)
      T2R=DATA(J2)+DATA(J3)
      T2I=DATA(J2+1)+DATA(J3+1)
      T3R=DATA(J2)-DATA(J3)
      T3I=DATA(J2+1)-DATA(J3+1)
      DATA(J0)=T0R+T2R
      DATA(J0+1)=T0I+T2I
      DATA(J2)=T0R-T2R
      DATA(J2+1)=T0I-T2I
      IF(ISIGN)100,100,110
 100  T3R=-T3R
      T3I=-T3I
 110  DATA(J1)=T1R-T3I
      DATA(J1+1)=T1I+T3R
      DATA(J3)=T1R+T3I
 120  DATA(J3+1)=T1I-T3R
      TEMPR=WR
      WR=WSTPR*TEMPR-WSTPI*WI+TEMPR
 130  WI=WSTPR*WI+WSTPI*TEMPR+WI
 140  IP2=IP3
      IF(IP3-IP4)50,150,150
 150  RETURN
      END
```

```
C
C
      SUBROUTINE FOUR2 (DATA,N,NDIM,ISIGN)
C
C          THIS IS FOR MULTI-DIMENSIONAL FOURIER TRANSFORMATION
C
C          IF NDIM=1;
C                     N
C          DATA(J)=SUM DATA(K)*EXP(I*ISIGN*2*3.14*K*J/N)
C                     K=1
C
C     DATA   :N*DIM DIMENSIONAL INPUT MATRIX OF COMPLEX NUMBERS;
C                   LATER RESULT WILL BE STATED
C     N      :LENGTH OF COLUMN IN DATA;INDICATING TIME OR FREQUENCY POINTS
C     NDIM   :LENGTH OF ROW IN DATA;INDICATING DIMENSION OF F.T.
C     ISIGN :    -1 FORWARD TRANSFORM
C                +1 INVERSE TRANSFORM
C
      DIMENSION DATA(1), N(1)
      NTOT=1
      DO 10 IDIM=1,NDIM
 10   NTOT=NTOT*N(IDIM)
 20   NREM=NTOT
      DO 60 IDIM=1,NDIM
      NREM=NREM/N(IDIM)
      NPREV=NTOT/(N(IDIM)*NREM)
      NCURR=N(IDIM)
 40   CALL BITRV (DATA,NPREV,NCURR,NREM)
      CALL COOL2 (DATA,NPREV,NCURR,NREM,ISIGN)
 60   CONTINUE
      RETURN
      END
C
C
      SUBROUTINE  MINV(A,N,D,L,M)
C
C       THIS PROGRAM IS FOR MATRIX INVERSE
C          A:N*N DIMENSIONAL INPUT MATRIX, LATER REPLACED BY RESULT
C          N:LENGTH OF COLUMN OF A
C          D: DETERMINANT OF RESULT
C          L: N-DIMENSIONAL WORKING VECTOR
C          M: N-DIMENSIONAL WORKING VECTOR
C
      DIMENSION A(1),L(1),M(1)
C
C          SEARCH FOR LARGEST ELEMENT
C
      D=1.0
      NK=-N
      DO 80 K=1,N
      NK=NK+N
      L(K)=K
      M(K)=K
      KK=NK+K
      BIGA=A(KK)
      DO 20 J=K,N
      IZ=N*(J-1)
      DO 20 I=K,N
      IJ=IZ+I
   10 IF( ABS(BIGA)- ABS(A(IJ))) 15,20,20
```

```
   15 BIGA=A(IJ)
      L(K)=I
      M(K)=J
   20 CONTINUE
C
C        INTERCHANGE ROWS
C
      J=L(K)
      IF(J-K) 35,35,25
   25 KI=K-N
      DO 30 I=1,N
      KI=KI+N
      HOLD=-A(KI)
      JI=KI-K+J
      A(KI)=A(JI)
   30 A(JI) =HOLD
C
C        INTERCHANGE COLUMNS
C
   35 I=M(K)
      IF(I-K) 45,45,38
   38 JP=N*(I-1)
      DO 40 J=1,N
      JK=NK+J
      JI=JP+J
      HOLD=-A(JK)
      A(JK)=A(JI)
   40 A(JI) =HOLD
C
C        DIVIDE COLUMN BY MINUS PIVOT (VALUE OF PIVOT ELEMENT IS
C        CONTAINED IN BIGA)
C
   45 IF(BIGA) 48,46,48
   46 D=0.0
      RETURN
   48 DO 55 I=1,N
      IF(I-K) 50,55,50
   50 IK=NK+I
      A(IK)=A(IK)/(-BIGA)
   55 CONTINUE
C
C        REDUCE MATRIX
C
      DO 65 I=1,N
      IK=NK+I
      HOLD=A(IK)
      IJ=I-N
      DO 65 J=1,N
      IJ=IJ+N
      IF(I-K) 60,65,60
   60 IF(J-K) 62,65,62
   62 KJ=IJ-I+K
      A(IJ)=HOLD*A(KJ)+A(IJ)
   65 CONTINUE
C
C        DIVIDE ROW BY PIVOT
C
      KJ=K-N
      DO 75 J=1,N
      KJ=KJ+N
```

```
      IF(J-K) 70,75,70
   70 A(KJ)=A(KJ)/BIGA
   75 CONTINUE
C
C        PRODUCT OF PIVOTS
C
      D=D*BIGA
C
C        REPLACE PIVOT BY RECIPROCAL
C
      A(KK)=1.0/BIGA
   80 CONTINUE
C
C        FINAL ROW AND COLUMN INTERCHANGE
C
      K=N
  100 K=(K-1)
      IF(K) 150,150,105
  105 I=L(K)
      IF(I-K) 120,120,108
  108 JQ=N*(K-1)
      JR=N*(I-1)
      DO 110 J=1,N
      JK=JQ+J
      HOLD=A(JK)
      JI=JR+J
      A(JK)=-A(JI)
  110 A(JI) =HOLD
  120 J=M(K)
      IF(J-K) 100,100,125
  125 KI=K-N
      DO 130 I=1,N
      KI=KI+N
      HOLD=A(KI)
      JI=KI-K+J
      A(KI)=-A(JI)
  130 A(JI) =HOLD
      GO TO 100
  150 RETURN
      END
C
C
      SUBROUTINE   OUTP   (T,ANSR,FUN,N1,N2,CHECK)
C                                                                    C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                                                                    C
C        THIS IS TO PRINT THE ANSWERS AT EVERY VALUE OF T            C
C                                                                    C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                                                                    C
C
      DIMENSION    ANSR(1),FUN(1),CHECK(1)
C
      DD=CHECK(3)
      ID=T/DD+1
C
      WRITE(6,1) ID,T,ANSR(2),ANSR(1),N1
C     WRITE(7,2)  T,ANSR(2)
C
    1 FORMAT(5X,I5,3(5X,F15.7),5X,I5)
```

```
C    2 FORMAT(10X,2F15.6)
C
      RETURN
      END
C
C
      SUBROUTINE   PAMAX      (A,AB,TMX,JMX,DT,NU)
C                                                                C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                                                                C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                                                                C
      DIMENSION A(1)
C
      AB=ABS(A(1))
      DO 2 J=2,NU
      ABSM=ABS(A(J))
      IF(ABSM.LT.AB) GO TO 2
      AB=ABSM
      JMX=J
    2 CONTINUE
C
      TMX=(JMX-1)*DT
      RETURN
      END
C
C
      SUBROUTINE   PAUTO      (X,RX,NU,DT)
C                                                                C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                                                                C
C          THIS IS TO COMPUTE THE AUTOCORRELATION FUNCTION      C
C          OF A GIVEN SIGNAL.                                    C
C          RX(TT)=E(X(T)X(T+TT))                                 C
C          X(I),I=1,NU.........:GIVEN SIGNAL.                    C
C          RX(I),I=1,NP........:AUTOCORRELATION OF X(I).         C
C          RELATION BETWEEN RECORD LENGHTS : NP=NU/2             C
C                                                                C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                                                                C
      DIMENSION    X(1),RX(1)
C
      NP=NU/2
      NP1=NP-1
      DUR=(NP1-1)*DT
C
      DO 1 LR=1,NP
      IR=LR-1
      SUM=0.00
      DO 2 IX=1,NP1
      IX1=IX+1
      Q1=X(IX)
      Q2=X(IX1)
      R1=X(IX+IR)
      R2=X(IX1+IR)
      SUM=SUM+Q1*R2+Q2*R1+2.00*(Q1*R1+Q2*R2)
    2 CONTINUE
      SUM=SUM*DT/6.00
      SUM=SUM/DUR
      RX(LR)=SUM
```

```
    1 CONTINUE
C
      RETURN
      END
C
C
       SUBROUTINE   PCP (A,B,C,NT,DT)
C                                                 C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                                                 C
C         COMPUTATION CF:                         C
C         C= SUM(A(J)*B(J)*DT)   J=1,NT           C
C                                                 C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                                                 C
C
      DIMENSICN A(1), B(1)
      NT1=NT-1
      C=0.
      DO 100 I=1,NT1
      I1=I+1
      A1=A(I)
      A2=A(I1)
      B1=B(I)
      B2=B(I1)
      C=C+A1*B2+A2*B1+2.*(A1*B1+A2*B2)
  100 CONTINUE
      C=DT*C/6.
      RETURN
      END
C
C
      SUBROUTINE  PETDE     (D,E,ETDE,N)
C                                              C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                                              C
C                          T                   C
C                  ETDE= E  * D * E            C
C                                              C
C         AFTER THE MULTIPLICATION             C
C         MATRIX D IS DESTROYED    .           C
C                                              C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                                              C
C
      DIMENSION     D(N,N),E(N)
C
      DO 1 I=1,N
      S=0.00
      DO 2 J=1,N
      S=S+D(I,J)*E(J)
    2 CONTINUE
      D(I,1)=S
    1 CONTINUE
C
      C=0.0
      DO 3 I=1,N
      C=C+E(I)*D(I,1)
    3 CONTINUE
C
```

```
      ETDE=C                                                          PETDE300
C                                                                     PETDE310
      RETURN                                                          PETDE320
      END                                                             PETDE330
C
C
      SUBROUTINE   PFNL  (RF,RY,RYY,C,ICASE,NU)
C                                                C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                                                C
C         COMPUTATION OF THE NON LINEAR FORCE    C
C                                                C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                                                C
C         ICASE=1   :SOFTENING - HÅRDENING       C
C         ICASE=2   :ELASTOPLASTIC               C
C         ICASE=3   :BILINEAR ELASTIC            C
C                                                C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                                                C
      DIMENSION    RF(1),RY(1)
      DIMENSION    C(5)
C
      SK=C(1)
      SKK=C(2)
      RM=C(3)
      DF=C(4)
      YMAX=C(5)
      YLIM=YMAX/DF
      YLIM=ABS(YLIM)
C
      IF  (ICASE.EQ.1)  GO TO 1
      IF  (ICASE.EQ.2)  GO TO 2
      IF  (ICASE.EQ.3)  GO TO 3
      GO TO 50
C
C
    1 CM=RM
      DO 24 I=1,NU
      ISIGN=-1
      RF(I)=ISIGN*SK*CM*RY(I)*RY(I)*RY(I)
   24 CONTINUE
      GO TO 5
C
C
    2 RFY=YLIM*SK
      DO 6 I=1,NU
      IF  (ABS(RY(I)).LT.YLIM)  GO TO 7
      GO TO 8
    7 RF(I)=0.0
      GO TO 6
    8 IF  (RY(I))  9,10,11
    9 RF(I)=-SK*RY(I)-RFY
      GO TO 6
   10 RF(I)=0.00
      GO TO 6
   11 RF(I)=-SK*RY(I)+RFY
    6 CONTINUE
      GO TO 5
C
```

```
C
    3 RFY=YLIM*SK
      DO 16 I=1,NU
      IF  (ABS(RY(I)).LT.YLIM)  GO TO 17
      GO TO 18
   17 RF(I)=0.0
      GO TO 16
   18 IF(RY(I))  19,20,21
   19 RF(I)=-((SK-SKK)*RY(I)-SKK*YLIM+RFY)
      GO TO 16
   20 RF(I)=0.0
      GO TO 16
   21 RF(I)=-((SK-SKK)*RY(I)+SKK*YLIM-RFY)
   16 CONTINUE
    5 CONTINUE
      GO TO 52
   50 WRITE(6,51)
   51 FORMAT(//10X,'$$$ ERROR : CHECK VALUE OF ICASE $$$',//)
   52 CONTINUE
C
C
      RETURN
      END
      SUBROUTINE   PFTXT      (X,XF,DT,DW,N)
C
C         ***********************************************
C         *    XF(I),I=1,N  IS THE FOURIER AMPLITUTE        *
C         *    VECTOR OF X(I),I=1,N .-                      *
C         *    DT IS TIME INCR. OF X(T) RECORD              *
C         *    DW IS THE FREQ. INCR. OF XF(W) RECORD        *
C         *    DT*DW=2.0*3.14159/NF1                        *
C         *    LIMITATION : N MUST BE LESS THAN NF1         *
C         ***********************************************
C
      DIMENSION      XX(2048)
      DIMENSION      X(1),XF(1)
C
      NF1=1024
      NF2=2*NF1
      SCLF=DT
      DW=2.00*3.14159/(NF1*DT)
C
C         VECTOR     :XX(I),I=1,NF2
C
      DO 1 I=1,NF2
    1 XX(I)=0.00
      DO 2 I=1,N
      II=2*I-1
    2 XX(II)=X(I)
C
      CALL     FOUR2      (XX,NF1,1,-1)
C
C         FOURIER AMPLITUTE VECTOR
C
      DO 3 L=1,N
      LR=2*L-1
      LI=2*L
      XR=XX(LR)*SCLF
      XI=XX(LI)*SCLF
      D=XR*XR+XI*XI
```

```
      D=SQRT(D)
      XF(L)=D
    3 CONTINUE
C
C
      RETURN
      END
C
C
      SUBROUTINE   PFUNF  (Y,F,CF,T,DT,ICASE)
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                                                                  C
C          SYSTEM  :  DY/DX=F                                      C
C          THIS IS TO COMPUTE THE VECTOR F=F(X,Y(X))               C
C                                                                  C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
      COMMON        XG(701)
      DIMENSION     Y(1),F(1)
      DIMENSION     CF(10)
C
      VEL=Y(1)
      DISP=Y(2)
      TIME=T
C
      SM=CF(1)
      GETA=CF(2)
      SK=CF(3)
      SF=CF(4)
      DF=CF(5)
C
      WW=SK/SM
      WN=SQRT(WW)
C
      IX=TIME/DT+1
      EXCIT=XG(IX)
C
      IF  (ICASE)  1,2,1
    2 FF=WN*WN*DISP
      GO TO 3
    1 CALL  PRF  (FF,VEL,DISP,CF,ICASE)
C
    3 F(1)=-2.00*GETA*WN*VEL-FF-EXCIT
      F(2)=VEL
C
      Y(1)=VEL
      Y(2)=DISP
      T=TIME
C
       RETURN
      END
C
      SUBROUTINE   PFNL2      (RF,RFMX,F,DF,N)
C    *                                                   *
C    *****************************************************
C    *                                                   *
C    *    THIS COMPUTES THE NON-LINEAR VECTOR  F         *
C    *    FOR A M.D.O.F. NONLINEAR SYSTEM                *
C    *                                                   *
```

```
C     **************************************************
C     *                                                *
C
      DIMENSION     RF(1),F(1)
C
      RFLT=RFMX/CF
C
      DO 100 I=1,N
      R1=RF(I)
      IF (ABS(R1).GT.RFLT)  GO TO 10
      F(I)=0.0
      GO TO 20
   10 CONTINUE
      IF (R1) 30,40,50
   30 F(I)=-R1-RFLT
      GO TO 20
   40 F(I)=0.0
      GO TO 20
   50 F(I)=-R1+RFLT
   20 CONTINUE
  100 CONTINUE
C
      RETURN
      END
C
C
      SUBROUTINE    PATRA   (X,Y,F,H,YY,CC,SSSM,SSSC,SSSK,
     1                       SCO,SKO,SCON,SKON,YM,FY,NY,
     2                       ICASE,NA,DT)
C                                                                  C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                                                                  C
C         FOR A GIVEN SET :SCO,SKO THIS SUBROUTINE                 C
C         COMPUTES A NEW ONE :SCON,SKON ,ACCORDING                 C
C         TO THE EQUIVALENT LINEARIZATION CONCEPT.-                C
C                                                                  C
C                     |    A<YY*YY>       A<YY*Y>    |             C
C         SS    =     |                              |             C
C                     |    A<Y*YY>        A<Y*Y>     |             C
C                                                                  C
C                     |    A<F*YY>    |                            C
C         PP    =     |               |                            C
C                     |    A<F*Y>     |                            C
C                                                                  C
C         A : AVERAGING OPERATOR                                   C
C                                                                  C
C    SUBROUTINES CALLED :PHT,,PTDIR,,PFNL,,PCP,,MINV               C
C                                                                  C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                                                                  C
      DIMENSION     X(1),H(1),Y(1),YY(1),F(1)
      DIMENSION     SS(2,2),LS(2),MS(2)
C
      SM=SSSM
      SC=SSSC
      SK=SSSK
C
      SCS=SC+SCC
      SKS=SK+SKC
C
```

```
      WW=SKS/SM
      WN=SQRT(WW)
      GETA=SCS/(2.00*WN*SM)
C
C        H(T)        :UNIT IMPULSE RESPONSE RECORD
C
      CALL   PHT   (H(1),SM,GETA,WN,NA,DT)
C
C        Y(T)        :RESPONSE RECORD
C
      CALL   PTOTR   (X(1),H(1),Y(1),YMAX,TYMX,NPYMX,DT,NA)
C
      YM=YMAX
      TY=TYMX
      NY=NPYMX
C
C        RESTORING FORCE RECORD
C
      RMX=ABS(YM)*SK
      DO 2 I=1,NA
    2 YY(I)=Y(I)*SK
C
C        F(Y)        :NON-LINEAR FUNCTION RECORD
C
      CALL      PFNL2      (YY(1),RMX,F(1),CC,NA)
C
C        YY(T)       :VELOCITY RECORD
C
      NA1=NA-1
      DO 1 J=1,NA1
      YY(J)=(Y(J+1)-Y(J))/DT
    1 CONTINUE
      YY(NA)=YY(NA1)
C
C        MATRIX  SS(2,2)
C
      CALL      PCP   (YY(1),YY(1),A11,NA,DT)
      CALL      PCP   (Y(1),Y(1),A22,NA,DT)
      CALL      PCP   (YY(1),Y(1),A12,NA,DT)
C
      SS(1,1)=A11
      SS(1,2)=A12
      SS(2,1)=SS(1,2)
      SS(2,2)=A22
      CALL   MINV   (SS,2,DS,LS,MS)
C
C        COMPUTE SCON,SKON
C
      CALL      PCP   (F(1),YY(1),P1,NA,DT)
      CALL      PCP   (F(1),Y(1),P2,NA,DT)
C
C
      SCON=SS(1,1)*P1+SS(1,2)*P2
      SKON=SS(2,1)*P1+SS(2,2)*P2
C
C
      RETURN
      END
C
C
```

```
      SUBROUTINE    PHT  (H,SM,GTA,WN,NU,DT)
C                                                     C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C          COMPUTE UNIT IMPULSE RESPONSE : H(T)       C
C          FOR A GIVEN LINEAR SYSTEM                  C
C                                                     C
C          1.-MASS.......................SM           C
C          2.-DAMPING....................GTA          C
C          3.-NAT. FREQUENCY.............WN           C
C          4.-NO. OF POINTS IN H(T)......NU           C
C          5.-TIME INCREAMENT............DT           C
C                                                     C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                                                     C
      DIMENSION H(1)
C
      GT=GTA*WN
      WSQ=WN*SQRT(1.DO-GTA*GTA)
C
      DO 1 J=1,NU
      T=DT*(J-1)
      HJ=SIN(WSQ*T)
      H(J)=HJ*EXP(-GT*T)/(SM*WSQ)
    1 CONTINUE
C
      RETURN
      END
C
C
      SUBROUTINE  PINPUT  (EX,TDUR,SCLF,NU,IFILE,DT,NSP,TSP,
     1                     SI,EXMAX,TEXMX,NPEXMX)
C                                                                         C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                                                                         C
C          INPUT RECORD   :    A(I),I=1,3000       (DTO,TDURO)            C
C          OUTPUT RECORD  :    EX(I),I=1,NU        (DT,TDUR)              C
C                                                                         C
C                              WHERE:                                     C
C                                                                         C
C          THE PROGRAM READS THE RECORD A(I) FROM A GIVEN FILE            C
C          AND CREATES THE EX(I) RECORD ACCORDING TO A GIVEN              C
C          DURATION (TDUR) AND A GIVEN NO. OF POINTS (NU)                 C
C                                                                         C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                                                                         C
C          NSP         :STARTING POINT OF THE SELECTED PORTION            C
C          TSP         :TIME AT WHICH THE SELECTED PORTION STARTS         C
C          EXMAX       :MAX. VALUE OF EX(I) RECORD                        C
C          TEXMX       :TIME AT WHICH THE MAX. OCCURS                     C
C          NPEXMX      :POINT AT WHICH THE MAX. VALUE OCCURS              C
C          SI          :SQRT. OF THE SQUARE INTEGRAL OF EX(I) RECORD      C
C          DT          :TIME INTERVAL FOR EX(I) RECORD                    C
C          DTO         :TIME INTERVAL FOR A(I) RECORD                     C
C                                                                         C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                                                                         C
      DIMENSION    A(3000)
      DIMENSION    EX(1)
C
      DEFINE FILE  10(60,3000,U,KV)
```

```
C
      DTO=0.020
      NU1=NU-1
      DT=TDUR/NU1
      NTE=TDUR/DTO+1
      NTEN=2010-NTE
      DTRA=DT/DTO
      DO 44 K=1,NU
      EX(K)=0.00
   44 CONTINUE
C
      KV=IFILE
      READ (10'KV)   A
C
C
C
      IF(TDUR.GT.25.) GO TO 43
      SUM1=0.
      DO 34 K=1,NTE
   34 SUM1=SUM1+A(K)*A(K)
      SUM=SUM1
      IST=1
      DO 32 K=1,NTEN
      SUM2=SUM1+A(NTE+K)*A(NTE+K)-A(K)*A(K)
      IF(SUM2.LE.SUM) GO TO 32
      IST=K+1
      SUM=SUM2
   32 SUM1=SUM2
      SUM=SQRT(SUM)*SCLF
      TSS=(IST-1)*DTO
      GO TO 47
   43 IST=1
      TSS=0.
C
C
C
   47 CONTINUE
      IMXT=IST
      AIMX=0.
      DO 37 K=1,NTE
      KI=K+IST-1
      IF(ABS(A(KI)).LT.AIMX) GO TO 37
      IMXT=KI
      AIMX=ABS(A(KI))
   37 CONTINUE
C
      IML=(IMXT-IST)/DTRA-1
      IMR=NU-IML
C
      DO 36 K=1,IML
      RIS=IMXT-DTRA*K+.001
      JIS=RIS
      DJ=RIS-JIS
      EX(IML-K+1)=(A(JIS)+(A(JIS+1)-A(JIS))*DJ)*SCLF
   36 CONTINUE
C
      DO 38 K=1,IMR
      RIS=IMXT+DTRA*(K-1)+.001
      JIS=RIS
      DJ=RIS-JIS
```

```
      EX(IML+K)=(A(JIS)+(A(JIS+1)-A(JIS))*DJ)*SCLF
   38 CONTINUE
C
      NSP=IST
      TSP=TSS
C
C
C
      SS=0.
      DO 39 J=1,NU1
      S1=EX(J)*EX(J)
      S2=EX(J+1)*EX(J+1)
      S3=EX(J)*EX(J+1)
      SS=SS+DT*(S1+S2+S3)/3.00
   39 CONTINUE
      SI=SQRT(SS)
C
C
C
      AAA=ABS(EX(1))
      DO 40 J=2,NU
      BBB=ABS(EX(J))
      IF  (BBB.LT.AAA)  GO TO 40
      AAA=BBB
      JJJ=J
   40 CONTINUE
      EXMAX=AAA
      NPEXMX=JJJ
      TEXMX=(JJJ-1)*DT
C
      RETURN
      END
C
C
      SUBROUTINE   PICFT  (A,AA,SF,NP,ICASE)
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                                                                      C
C          THIS SUBROUTINE FORMS THE I/O VECTORS FOR #FOUR2#           C
C          DIMENSION FOR A & AA IS 2048                                C
C                                                                      C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
      DIMENSION     A(1),AA(1)
C
      N1=1024
      N2=2*N1
      IF  (ICASE)  1,2,3
C
C        ICASE=1    :FORM INPUT VECTOR AA(I),I=1,N2
C
    3 CONTINUE
      DO 4 I=1,N2
      AA(I)=0.00
    4 CONTINUE
C
      DO 5 J=1,NP
      JJ=2*J-1
      AA(JJ)=A(J)
    5 CONTINUE
      GO TO 6
```

```
C
C         ICASE=-1  :FORM OUTPUT VECTOR A(I)
C                   REAL PART             : I=1..........NP
C                   IMMAGINARY PART       : I=(NP+1)...2*NP
C
    1 K=1
      DO 7 L=1,NP
      LR=2*L-1
      A(K)=AA(LR)*SF
      K=K+1
    7 CONTINUE
C
      KK=NP+1
      DO 8 LL=1,NP
      LI=2*LL
      A(KK)=AA(LI)*SF
      KK=KK+1
    8 CONTINUE
C
    6 CONTINUE
    2 CONTINUE
C
      RETURN
      END
C
C
      SUBROUTINE   PPLT  (XX,IXX,JXX,XMAX,ISD)
C                                                C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                                                C
C         PPLT PLOTS THE ARRAY :                 C
C         XX(I,J),I=1,IXX,J=1,JXX                C
C                                                C
C         IXX  :COMMON LENGHT FOR ALL RECORDS    C
C         JXX  :NO. OF RECORDS TO BE PLOTTED     C
C         MAX.(JXX)=10                           C
C                                                C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                                                C
      REAL LINE
      DIMENSION    XX(IXX,JXX)
      DIMENSION    LINE(140),SYMBL(10),JQ(10)
      DATA BLANK,DOT,SYMBO/' ','.','-'/
      DATA SYMBL/'1','2','3','4','5','6','7','8','9','0'/
C
      JL=116
      JH=50
      JB=56
      IF  (ISD.EQ.0)  JH=100
      IF  (ISD.EQ.0)  JB=5
      JX=10
      RJX=JX
      JMAX=IXX
      NDIM=JXX
C
      DO 109 J=1,5
  109 LINE(J)=BLANK
      DO 110 J=6,JL
  110 LINE(J)=DOT
      WRITE(6,141) (LINE(I),I=1,JL)
```

```
      DO 112 J=1,JL
  112 LINE(J)=BLANK
C
C
      DO 120 J1=1,JMAX
      JT=J1/JX
      RI=ABS(JT*RJX-J1)
      IF(RI.GT..01) GO TO 83
      LINE(JB)=SYMBO
   83 CONTINUE
C
C        FORM ARRAY LINE(I),I=1,JL FOR A STEP : J1
C
      DO 80 JPT=1,NDIM
      XJ=XX(J1,JPT)/XMAX*JH
      J=XJ+JB
      JO(JPT)=J
   80 LINE(J)=SYMBL(JPT)
      WRITE(6,140) (LINE(I),I=1,JL)
C
C        CLEAR ARRAY LINE(I),I=1,JL FOR THE NEXT STEP : J1+1
C
      DO 81 JPT=1,NDIM
      J=JO(JPT)
   81 LINE(J)=BLANK
      LINE(JB)=DOT
  120 CONTINUE
C
  140 FORMAT(1H ,120A1)
  141 FORMAT(1H1,120A1)
C
      RETURN
      END
C
C
      SUBROUTINE   PRF  (F,V,D,CF,IC)
C                                                                     C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                                                                     C
C        THIS IS TO COMPUTE THE RESTORING FORCE                       C
C        OF A S.D.O.F. SYSTEM.-                                       C
C                                                                     C
C        IC=1      :SOFTENING SPRING RESTORING FORCE                  C
C        IC=2      :ELASTOPLASTIC RESTORING FORCE                     C
C        IC=3      :BILINEAR RESTORING FORCE                          C
C                                                                     C
C        F..........RESTORING FORCE                                   C
C        V..........VELOCITY                                          C
C        D..........DIDPLACEMENT                                      C
C                                                                     C
C        CF(I),I=1,10 IS THE VECTOR OF                                C
C        THE NONLINEARITY CHARACTERISTICS OF THE SYSTEM.-             C
C                                                                     C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                                                                     C
C
      DIMENSION    CF(10)
C
      MASS=CF(1)
      DAMP=CF(2)
```

```
      STIF=CF(3)
      SF1=CF(4)
      SF2=CF(5)
      COEF=CF(6)
      YO=CF(7)
      YLMT=CF(8)
C
      IF(IC.EQ.1)  GO TO 1
      IF(IC.EQ.2)  GO TO 2
      IF(IC.EQ.3)  GO TO 3
C
C          SOFTENING SPRING RESTORING FORCE:
C
    1 ISIGN=-1
      F=SF1*D+ISIGN*COEF*D*D*D
      GO TO 4
C
C          ELASTOPLASTIC RESTORING FORCE:
C
    2 FO=SF1*YO
      IF  (D)   6,5,6
    5 F=0.00
      GO TO 4
    6 ABD=ABS(D)
      IF  (ABD.GT.YO)  GO TO 7
      F=SF1*D
      GO TO 4
    7 IF  (D)   8,9,9
    9 F=FO
      GO TO 4
    8 F=-FO
      GO TO 4
C
C          BILINEAR RESTORING FORCE:
C
    3 FLMT=SF1*YLMT
      IF  (D)   11,10,11
   10 F=0.00
      GO TO 4
   11 ABD=ABS(D)
      IF  (ABD.GT.YLMT)  GO TO 12
      F=SF1*D
      GO TO 4
   12 F=SF2*D
C
    4 F=F/MASS
C
      RETURN
      END
C
C
      SUBROUTINE PRSPA   (A,UU,TT,BETA,SD,NU,DT)
C                                                                       C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                                                                       C
C       THIS IS FOR COMPUTING MAX. RESPONSE FOR SINGLE DEGREE   SYSTEM  C
C                                                                       C
C          INPUT EXCITATION      :A(I),I=1,NU                           C
C          OUTPUT RESPONSE       :UU(I),I=1,NU                          C
C          NATURAL RERLCO        :TT                                    C
```

```
C         DAMPING RATIO        :BETA                                          C
C         MAX. RESPONSE        :SD                                            C
C                                                                             C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                                                                             C
C
      DIMENSION A(1) ,UU(1)
C
      NU1=NU-1
      W=6.28318531/TT
      W2=W*W
      W3=W*W*W
      WD=W*SQRT(1.-BETA*BETA)
C
      Z=0.0
      DZ=0.0
      SD=0.0
C
C         COMPUTE RESPONSE UU AND VALUE OF SD
C
      DO 20 IP=1,NU1
      CA=A(IP)
      CB=(A(IP+1)-A(IP))/DT
      C=COS(WD*DT)
      S=SIN(WD*DT)
      Z1=(Z-CA/W2+2.*BETA*CB/W3)*C+(DZ+BETA*W*Z-BETA*CA/W+CB/W2
     1  *(2.*BETA*BETA-1.))*S/WD
      Z1=Z1*EXP(-BETA*W*DT)+(CA/W2-2.*BETA/W3*CB+CB/W2*DT)
      DZ=(DZ-CB/W2)*C+(CA-W2*Z-BETA*W*(DZ+CB/W2))*S/WD
      DZ=DZ*EXP(-BETA*W*DT)+CB/W2
C
      UU(IP)=Z1
C
      IF(ABS(Z1).GT.ABS(SD)) GO TO 30
      GO TO 40
   30 CONTINUE
      SD=Z1
   40 CONTINUE
      Z=Z1
   20 CONTINUE
C
C         REARRANGE VECTOR UU
C
      HOLD1=UU(1)
      DO 10 I=2,NU
      HOLD2=UU(I)
      UU(I)=HOLD1
      HOLD1=HOLD2
   10 CONTINUE
      UU(1)=0.00
C
      RETURN
      END
C
C
      SUBROUTINE  PSODE  (PRMT,Y,DERY,CF,NDIM,IHLF,AUX,ILN)
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                                                         C
C         SYSTEM OF ORDINARY DIFF. EQUATIONS IN FORM:     C
```

```
C          DY/DX=F(X,Y)                                                 C
C          WHERE:                                                       C
C                         |Y1|                                          C
C                       Y=|  |                                          C
C                         |Y2|                                          C
C                                                                       C
C                         |F1(X,Y1,Y2)|                                 C
C                  F(X,Y)=|           |                                 C
C                         |F2(X,Y1,Y2)|                                 C
C                                                                       C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
      COMMON          XG(701)
      DIMENSION       AUX(8,1),Y(1),DERY(1),PRMT(1)
      DIMENSION       AS(4),BS(4),CS(4),CF(10)
C
      DO 1  I=1,NDIM
    1 AUX(8,I)=0.06666667*DERY(I)
      X=PRMT(1)
      XEND=PRMT(2)
      H=PRMT(3)
      PRMT(5)=0.00
      DDX=PRMT(3)
C
      CALL    PFUNF  (Y,DERY,CF,X,DDX,ILN)
C
      IF  (H*(XEND-X))  38,37,2
    2 AS(1)=0.50
      AS(2)=0.2928932
      AS(3)=1.707107
      AS(4)=0.1666667
      BS(1)=2.00
      BS(2)=1.00
      BS(3)=1.00
      BS(4)=2.00
      CS(1)=0.50
      CS(2)=0.2928932
      CS(3)=1.707107
      CS(4)=0.50
C
      DO  3  I=1,NDIM
      AUX(1,I)=Y(I)
      AUX(2,I)=DERY(I)
      AUX(3,I)=0.00
    3 AUX(6,I)=0.00
C
      IREC=0
      H=H+H
      IHLF=-1
      ISTEP=0
      IEND=0
C
C        START A STEP
C
    4  IF  ((X+H-XEND)*H)  7,6,5
    5 H=XEND-X
    6 IEND=1
C
    7 CALL   OUTP (X,Y,DERY,IREC,NDIM,PRMT)
C
```

```
      IF  (PRMT(5))  40,8,40
    8 ITEST=0
C
    9 ISTEP=ISTEP+1
C
      J=1
   10 AJ=AS(J)
      BJ=BS(J)
      CJ=CS(J)
      DO  11  I=1,NDIM
      R1=H*DERY(I)
      R2=AJ*(R1-BJ*AUX(6,I))
      Y(I)=Y(I)+R2
      R2=R2+R2+R2
   11 AUX(6,I)=AUX(6,I)+R2-CJ*R1
      IF  (J-4)  12,15,15
   12 J=J+1
      IF  (J-3)  13,14,13
   13 X=X+0.50*H
C
   14 CALL    PFUNF   (Y,DERY,CF,X,DDX,ILN)
C
      GO TO 10
C
C        TEST
C
   15 IF  (ITEST) 16,16,20
   16 DO 17 I=1,NDIM
   17 AUX(4,I)=Y(I)
      ITEST=1
      ISTEP=ISTEP+ISTEP-2
   18 IHLF=IHLF+1
      X=X-H
      H=0.50*H
      DO 19 I=1,NDIM
      Y(I)=AUX(1,I)
      DERY(I)=AUX(2,I)
   19 AUX(6,I)=AUX(3,I)
      GO TO 9
   20 IMOD=ISTEP/2
      IF (ISTEP-IMOD-IMOD)   21,23,21
C
   21 CALL    PFUNF   (Y,DERY,CF,X,DDX,ILN)
C
      DO 22 I=1,NDIM
      AUX(5,I)=Y(I)
   22 AUX(7,I)=DERY(I)
      GO TO 9
C
C        ERROR CALCULATION
C
   23 DELT=0.00
      DO 24 I=1,NDIM
   24 DELT=DELT+AUX(8,I)*ABS(AUX(4,I)-Y(I))
      IF  (DELT-PRMT(4))  28,28,25
C
C        ERROR > GIVEN (=PRMT(4))
C
   25 IF (IHLF-10)   26,36,36
   26 DO 27 I=1,NDIM
```

```
   27 AUX(4,I)=AUX(5,I)
      ISTEP=ISTEP+ISTEP-4
      X=X-H
      IEND=0
      GO TO 18
C
C         ERROR = OR < GIVEN (=PRMT(4))
C
   28 CALL    PFUNF   (Y,DERY,CF,X,DDX,ILN)
C
      DO 29 I=1,NDIM
      AUX(1,I)=Y(I)
      AUX(2,I)=DERY(I)
      AUX(3,I)=AUX(6,I)
      Y(I)=AUX(5,I)
   29 DERY(I)=AUX(7,I)
C
      CALL OUTP   (X-H,Y,DERY,IHLF,NDIM,PRMT)
C
      IF   (PRMT(5))   40,30,40
   30 DO 31 I=1,NDIM
      Y(I)=AUX(1,I)
   31 DERY(I)=AUX(2,I)
      IREC=IHLF
      IF   (IEND)   32,32,39
   32 IHLF=IHLF-1
      ISTEP=ISTEP/2
      H=H+H
      IF (IHLF)   4,33,33
   33 IMOD=ISTEP/2
      IF   (ISTEP-IMOD-IMOD) 4,34,4
   34 IF   (DELT-0.02*PRMT(4))   35,35,4
   35 IHLF=IHLF-1
      ISTEP=ISTEP/2
      H=H+H
C
      GO TO 4
C
   36 IHLF=11
C
      CALL    PFUNF   (Y,DERY,CF,X,DDX,ILN)
C
      GO TO 39
   37 IHLF=12
      GO TO 39
   38 IHLF=13
C
   39 CALL OUTP (X,Y,DERY,IHLF,NDIM,PRMT)
C
   40 RETURN
      END
C
C
      SUBROUTINE    PSQIL       (A,SUM,DT,NU)                               PSQIL010
C                                                                  C        PSQIL020
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC        PSQIL030
C                                                                  C        PSQIL040
C        SUM= SQRT(SUM(A(J)**2*DT))    J=1,NU                      C        PSQIL050
C                                                                  C        PSQIL060
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC        PSQIL070
```

```
C                                                              C
C
      DIMENSION A(1)
C
      N=NU-1
      SUM=0.
C
      DO 60 J=1,N
      A1=A(J)*A(J)
      A2=A(J+1)*A(J+1)
      A3=A(J)*A(J+1)
      SUM=SUM+DT*(A1+A2+A3)/3.
   60 CONTINUE
C
      SUM=SQRT(SUM)
C
      RETURN
      END
C
C
      SUBROUTINE  PTDTR  (X,H,Y,YMAX,TYMAX,NPYMX,DT,NU)
C                                                              C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                                                              C
C         THIS SUBROUTINE COMPUTES THE RESPONSE OF A           C
C         S.D.O.F. SYSTEM IN TIME DOMAIN.                      C
C                                                              C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                                                              C
C         X(J),J=1,NU          :INPUT RECORD                   C
C         H(J),J=1,NU          :UNIT IMPULSE RESPONSE RECORD   C
C         Y(J),J=1,NU          :RESPONSE RECORD                C
C         YMAX        :MAX. VALUE OF THE RESPONSE              C
C         NPYMX       :POINT AT WHICH THE PEAK RESP. OCCURS    C
C         TYMAX       :TIME AT WHICH THE PEAK RESP. OCCURS     C
C         NU          :NO. OF POINTS FOR X(I),H(I),Y(I)        C
C         DT          :TIME INTERVAL FOR ALL RECORDS  (X,H,Y)  C
C                                                              C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C                                                              C
C
      DIMENSION X(1),H(1),Y(1)
C
      NPT=NU-1
      YMAX=0.00
C
C
      DO 200 J=1,NPT
      YY=0.
      JN=J
C
      DO 100 K=1,JN
      K1=K+1
      JK=JN-K+2
      JK1=JK-1
      YY=YY+(X(K)*H(JK1)+X(K1)*H(JK)+2.*(X(K)*H(JK)+X(K1)*H(JK1)))
  100 CONTINUE
C
      Y(J)=      YY*DT/6.
      IF(ABS(Y(J)).LE.YMAX)  GO TO 200
```

```
      YMAX=ABS(Y(J))
      NPYMX=J
  200 CONTINUE
C
C
      HOLD1=Y(1)
C
      DO 300 I=2,NU
      HOLD2=Y(I)
      Y(I)=HOLD1
      HOLD1=HOLD2
  300 CONTINUE
C
      Y(1)=0.00
C
      TYMAX=NPYMX*DT
      NPYMX=NPYMX+1
C
      RETURN
      END
C
      SUBROUTINE   RAN360     (IX,IY,YFL)
C
C        **************************************************************
C
C        IX   :FOR THE FIRST ENTRY MUST CONTAIN ANY ODD INTEGER
C              NUMBER WITH NINE OR LESS DIGITS.
C              AFTER THE FIRST ENTRY IX SOULD BE THE PREVIOUS VALUE
C              OF IY COMPUTED BY THE SUBROUTINE./
C        YFL  :IS THE RESULTANT UNIFORMLY DISTRIBUTED FLOATING
C              POINT RANDOM NUMBER IN THE RANGE  0 TO 1.0  ./
C
C        **************************************************************
C
      IY=IX*65539
      IF   (IY)    1,2,2
    1 IY=IY+2147483647+1
    2 YFL=IY
      YFL=YFL*0.4656613E-9
C
      RETURN
      END
```