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Report No. 30

MODELS OF THRESHOLD EXCEEDANCE AND LOSS COMPUTATIONS OF NON-HOMOGENEOUS, SPATIALLY DISTRIBUTED FACILITIES

by

Betsy Schumacker

Robert V. Whitman

March 1977

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16. Abstract (Limit: 200 words)

Data and procedures are developed for balancing the increased cost of more resistant construction against the risk of losses during future earthquakes. Such information is relevant in considering measures for monitoring earthquake hazards which utilize knowledge of the probability that different thresholds of loss might be exceeded during any single earthquake. Different sets of probabilities are considered to establish the likelihood of proscribed threshold exceedance. Life safety, of course, is the paramount factor in deciding on measures for earthquake hazard mitigation, but building failures sometimes can be used as an indirect measure of potential for life loss. Life loss is conditional to building failure; and therefore, any model for threshold exceedance of life loss must also be a model for building failure. The report describes various loss thresholds which might be of interest together with a model developed for situations in which a two-state analysis is reasonable. Methods are described which are applicable for multi-state analysis. Applications of some of the methods are given for the areas of greater Boston and eastern Massachusetts.

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PREFACE

This is the thirtieth in a series of reports under the general title of Seismic Design Decision Analysis. The overall aim of the research is to develop data and procedures for balancing the increased cost of more resistant construction against the risk of losses during future earthquakes. The research has been sponsored in part by the Earthquake Engineering Program of National Science Foundation Research Applied to National Needs (RANN) under Grant GI-27955. A list of previous reports follows this preface.

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1. Whitman, Cornell, Vanmarcke, and Reed, "Methodology and Initial Damage Statistics", Department of Civil Engineering Research Report R72-17, Structures No. 339, March 1972.

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I. INTRODUCTION

When losses from earthquakes are averaged over the years, such losses appear modest - at least in the United States - when compared with those caused by other natural hazards. Earthquakes, however, exceed other hazards in their capacity to cause losses of catastrophic proportions in a few moments of intense shaking which occurs without warning. Hence, in considering measures to mitigate the earthquake hazard, it is desirable to estimate the probability that different thresholds of loss might be exceeded, in a city or a region, during any one single earthquake. Since any one eathquake can cause different intensities of shaking at various places within the city or region and since different types of buildings respond differently to a given intensity of shaking, it is necessary to combine together different sets of probabilities in determining the likelihood of any proscribed threshold exceedance.

Even though life safety is the primary factor in deciding on measures for earthquake hazard mitigation, building failure can sometimes be used as an indirect measure of the potential for life loss. In fact, it is found that life loss is conditional on building failure in the main and thus any model for threshold exceedance of life loss must also be a model for building failure.

This report describes the various loss thresholds which might be of interest, describes in detail a model which has been developed for situations in which a 2-state analysis is reasonable, describes an approach for scenario development, and describes a couple of methods which could be used for multistate analyses. Applications of some of these methods are given for the areas of greater Boston and eastern Massachusetts.

II. Description of Problems

The long term objective of the modeling of threshold exceedence due to earthquake is to model the threat to life safety posed to a city or region of the country by activity within seismic source zones. The threshold exceedence parameter here is number of lives lost. Since the general model (based on work done to date) is rather complex and costly in terms of solution time, building "failure" can quite often be used as an indirect measure of life loss. In fact, if damage states for buildings are developed as life safety damage states, building failure becomes a relatively good measure of life loss. This was done, as a first approximation, in developing the first set of examples described in Chapter 5, and was the rationale behind the 2-state developments of Chapters 3 and 4.

Since the general case is a combination of life loss given damage state and damage state given an event and intensity levels, various approximations were considered. One such is described in Chapter 6 as is the general model and potentially fruitful avenues for further work.

The several problems are briefly described below and are further developed in the remaining chapters of this report.

A. 2-State Problem

The 2-state problem arises from situations in which the concern is "success" or "failure", where failure is defined as being in a given state or greater (worse) state. Failure could thus be collapse of a structure, a given state or greater of any facility, etc. In these situations there are 2 and only 2 states in which the threshold parameter of each (assumed) independent facility can be as a result of an earthquake of given magnitude and given site intensity. Thus each such conditional result of each facility can be modeled as a binomial occurrence, and the likelihood of exceeding some number of failures of these facilties determined by combining all possible combinations of such over all possible events. The modeling and implementation of this problem with example applications is pursued in detail in the next three chapters of this report.

B. Multi-State Problem

The multi-state problem occurs in situations where, for the threshold parameter of interest, there are many (discrete) states in which each (assumed) independent facility can be as a result of a given magnitude and site intensity. Thresholds dealing with lives lost fall into this category since life loss is a function not only of occupancy and time of day but also of the damage state of the facility itself. If life loss within a single facility could be modeled discretely and directly as a function of site intensity, then this problem could be modeled using the multinomial distribution and transform theory used to develop reasonable methods for combining the distributions over all possible combinations for an event, over all events. Thus far, however, this has not been done. A gross approximation, using expected values, has been developed and is briefly discussed in the last section of Chapter six.

C. Single Event

It is sometimes of interest to study what the life safety threat would be in a given city or region to a single specific event. This approach is frequently used in developing scenarios. Part of Chapter six illustrates how parameters developed for the multi-state, life loss problem can be used in an easy fashion for estimating the life safety threat to a city by a specific level of shaking. This particular problem has been included as an interesting by-product of work performed rather than as a form of threshold exceedence model - which it is not.

III. Mathematical Modeling of 2-State and Assumptions

A. General Model

The probability that n facilities will fail during an earthquake is

$$p(n) = \int_{\text{Area}} \int_{\text{Mag}} p(n|\text{event, } M) \cdot p(M|\text{event}) \cdot p(\text{event})\, dM dA \qquad (3.1)$$

where the event is greater than some minimum magnitude. Since in most practical cases a closed form solution does not exist, the computation must be done numerically by means of discretizing both the area and the magnitude range. This leads to a solution of the following form:

$$p(n) = \sum_{\text{Location}} p(\text{event}|\text{loc}) \sum_{M} [p(n|\text{event, } M) \cdot p(M|\text{event})] \qquad (3.2)$$

If the area is discretized into equal sized cells, if each source zone is uniform in its rate of occurrence, if all earthquakes in a source cell are lumped at the center of the cell, then the above can be stated as

$$p(n) = \sum_{\substack{\text{Source}\\\text{zones}}} p^1(\text{event}|\text{loc.}) \sum_{\text{cells}} \sum_{M} [p(n|\text{event, } M) \cdot p^1(M|\text{event})] \qquad (3.3)$$

where

$$p^1(M|\text{event}) = \frac{e^{a-bM}(1 - e^{-b\Delta M})}{\text{no. events/yr. in that source zone}}$$

= fraction of earthquakes with mag. $M \rightarrow M + \Delta M$ in that source zone

$p^{1}(\text{event}|\text{loc})$ = no. events/yr./cell

ΔM = discretization interval for magnitude range

e^{a-bM} = no. events $\geq M$ per year per source zone

Simplifying further and normalizing the rate of occurrence, we get finally

$$p(n) = \sum_{\substack{\text{Source}\\ \text{zones}}} \frac{1}{(1 - e^{a-b\overline{M}})} \sum_{\text{Cells}} \sum_{M} p(n|\text{event, M}) \cdot \frac{e^{a-bM}(1 - e^{-b\Delta M})}{\text{no. cells in source zone}} \qquad (3.4)$$

where the first term is the normalization factor and $\overline{M}$ is the maximum possible magnitude in that source zone (1,2).

If the rate of occurrence in any zone varies in its logarithmic slope for different intervals of magnitude, the above can be stated as

$$p(n) = \sum_{\substack{\text{Source}\\ \text{zones}}} \frac{1}{(1 - e^{a_m - b_m \overline{M}})} \sum_{\text{Cells}} \sum_{M} p(n|\text{event, M}) \; \frac{e^{a_i - b_i M}(1 - e^{-b_i \Delta M})}{\text{no. cells in source zone}} \qquad (3.5)$$

where M is in the i^{th} slope interval of the source zone and there are m such intervals in that zone

The sum of p(n) over all n (n = 0, 1, ...,N) is equal to λ, the rate of occurrence of earthquakes in the total area (in all source zones).

The first term in the sum can be logically thought of as having two terms, namely

$$p(n|\text{event, M}) = p(n|\text{event, M}) \cdot f(\text{cell resistance}) \qquad (3.6)$$

where f(cell resistance) is a function of the level of ground shaking at the cell (given an event, an M and an attenuation relationship for intensity or acceleration) and the smallest resistance of the facilities in the cell. In other words

f(cell resistance) = 0 if level of shaking at cell is < the minimum resistance in the cell and if n = 0

= 1 otherwise

In the case where the level of shaking at the cell is < min (resistances in cell) and n = 0, then p (n | event, M) = p(o | event, M) = 1.

B. Computation of Probability Given an Event

This section discusses the method for computing p(n | event, M), or the probability of n facilities failing given an event, an epicentral magnitude, and an attenuation relationship. For notational ease, $p_n(n_o)$ will be used to refer to this conditional probability.

If we have only one target cell, and only one kind of facility in that cell (i.e. the probability of failure of a single facilitiy is the same for all facilities), then we have

$$p_n(n_o) = \binom{N}{n_o} p^{n_o}(1 - p)^{N-n_o} \tag{3.7}$$

which is the standard binomial distribution where

N = total number of facilities in the cell

p = probability of failure of a single facility given an event and a level of shaking at the facility

n_o = 0, 1, ..., N

However, when working with area (or distributed) targets, one does not have this kind of situation. Rather, one has more than one kind of facility in a cell as well as having different levels of ground shaking at different target cells. Hence one has a total population with subgroups, each of which has a different failure probability caused by being of different kinds, or receiving different excitation levels, or both.

Thus, for any event, the population of facilities will be a non-homogeneous set in that the probability of failure varies from facility to facility. The distribution remains binomial, however, in the following fashion.

Since the failure curve (probability of failure vs level of ground shaking) for each kind of facility is discrete (by breaking the ground shaking into intensity or acceleration ranges), the total population (given an event) can be grouped into k homogeneous sets each having its own probability of failure and each binomially distributed. Thus, since the facilities are assumed independent of one another, the probability of n out of N failing can be written:

$$p_n(n_o) = p_n(n_{o_1}, n_{o_2}, \ldots, n_{o_k}) \tag{3.8}$$

where the variable n_o is the sum of random variables

$$n_o = n_{o_1} + n_{o_2} + \ldots + n_{o_k}$$

and is itself a random variable. The i^{th} random variable of this set, n_{o_i}, would have a binomial mass function $p_{n_i}(n_{o_i})$ where $n_{o_i} = 0, 1, \ldots, N_i$. The sum of all N_i, $i = 1, 2, \ldots, k$, would be N, the total number of facilities in all target cells.

The combining of these k distributions can be performed by convolution or by discrete transform using the z-transform. The z-transform of binomial mass function $p_{n_i}(n_{o_i})$ is $p_{n_i}{}^T(z_i)$ and is given by (3)

$$p_{n_i}{}^T(z_i) \equiv E(z_i^{\,n_i}) = \sum_{n_i=0}^{N_i} z^{n_i}\, p_{n_i}(n_{o_i}) \tag{3.9}$$

where $E(z_i^{\,n_i})$ is the expected value of the transform variable z_i for the distribution on n_i. The coefficients of this polynomial, namely $p_{n_i}(n_{o_i})$, are the probabilities of exactly n_{o_i} out of N_i failing in the i^{th} group above i.e. they are values for the binomial probability mass function for group i.

Since our desired distribution is based on the sum of random variables, we apply transform theory again to compute our random variable sum as the product of the transforms of the individual variables.

In other words, if (as we have)

$$n_o = n_{o_1} + n_{o_2} + \ldots + n_{o_k}$$

is a sum of random variables, then its discrete transform is

$$p_{n_o}{}^T(z) = p_{n_{o_1}}{}^T(z) p_{n_{o_2}}{}^T(z) \ldots p_{n_{o_k}}{}^T(z). \tag{3.10}$$

Since $p_{n_o}{}^T(z) = \sum_{n_o=0}^{N} p_n(n_o) z^{n_o}$.

$$= \sum_{n_{o_1}=0}^{N_1} p_{n_1}(n_{o_1}) z^{n_{o_1}} \sum_{n_{o_2}=0}^{N2} p_{n_2}(n_{o_2}) z^{n_{o_2}} \ldots \sum_{n_{o_k}=0}^{N_k} p_{n_k}(n_{o_k}) z^{n_{o_k}} \tag{3.11}$$

then the coefficient of z^{n_o} in $p_{n_o}{}^T(z)$ is equal to our wanted probability, i.e. is equal to $p_n(n_o)$ or to exactly n_o failing out of the total N.

This coefficient of z^{n_o} can be computed as one of the terms in the products of the polynomials from the transforms of the probability mass functions of the k groups.

Thus the general procedure is to (given an event and an M)

a) Determine the number of homogeneous groups

b) Compute the probability mass function for each group

c) Compute the product of the transforms of the k mass functions as the polynomial products of the k mass functions

d) The coefficients just computed are the desired probability $p_n(n_o)$, n_o = o, 1, ..., N.

C. Summary of Assumptions

In the prior discussion, the following assumptions were made:

- Non-homogeneity of facilities and/or spatial distribution of targets. This resulted in the facilities being grouped by specific probability of failure given an event since different facilities fail differently for a single level of shaking and/or experience different levels of shaking during a single event.

- Independence of facilities. Each facility was assumed to be statistically independent of all other facilities. In other words, the failure of one building has no implication on the success or failure of any other building.
- Two-state environment. As the result of an event, each facility could be in one of two states - failure or non-failure. This in combination with statistical independence resulted in the model being a binomial distribution.

IV. Implementation of 2-State and Approximations Used

A. General Procedure

The implementation of the 2-state analysis will be described in terms of the inputs to the program, the results from the program, and the computational procedures.

The user needs to specify information of the following types (the meaning of which will be discussed in the paragraphs which follow):

1. The building damage probability matrices (DPMs),
2. The boundary of the total (sources and targets) area and the interval for discretization of the area,
3. The earthquake models for each seismic source zone,
4. The interval for discretization of magnitude,
5. The configuration of buildings in the target area,
6. The level of damage to be considered failure.

These will now be described more fully in the order just stated which is also the order in which the data is given to the program.

1. The number of building types, the number of MMI levels, the value of the first MMI level, and the number of damage states in the DPMs are given followed by a DPM for each building type. Each DPM is of size number of damage states by number of MMI levels and they should be given in a row-wise fashion. The number of building types is the product of the number of basic types and the number of soil types. This is to permit soil effects of other than a simple one-MMI shift for good to bad soil.

2. The boundary of the total area is specified as a rectangle by giving the min and max x and y coordinates of the area in miles. This area must include all source zones to be considered as well as all target areas. The area dis-

cretization interval is also given here and it is in miles as well. It serves to define the length of each side of the square cells into which the area is divided.

3. The earthquake models are defined as follows. First, the number of source zones is given. Then, for each source zone the following are specified:

a. the number of cells which comprise that zone

b. the indices of these cells. The lower left hand cell in the entire area specified in 2. above has an X index of 1 and a Y index of 1 and so forth. The indices are specified by giving for each Y which contains at least 1 cell in this source zone the range of X indices for the cells in this zone. This is done for as many Y's as necessary.

c. We next give the range of magnitude of interest for this zone in terms of the minimum and maximum we want the magnitude to be and/or the minimum and maximum values the magnitude can be.

d. The frequency of occurrence for this zone is now specified. It is given as the number of events per year for each of a set of magnitude intervals. This permits one to have several bends (or slopes) in the frequency of occurrence curve if desired. This information, then, is specified first by giving the number of intervals of magnitude for which occurrences are to be given, then the value of the magnitude at the beginning of each interval and finally for each such magnitude the number of events per year for that magnitude or greater. This in effect gives a_i and the b_i is computed from a_i and $a_i + 1$. The end points of the magnitude intervals can be fractional parts of magnitude units. However, it gains nothing to have them more precise than the interval for discretization of the magnitude.

e. The attenuation information for this source zone is specified. First the type of attentuation law to use is defined. This can be either a magnitude-intensity relationship or a magnitude-acceleration relationship. Note that even though the program is set up to handle both types, only the intensity relationship is fully implemented. After the type of law is given the constants are then specified. The intensity relationship has four constants, namely

$$\begin{cases} I = b_1 + b_2M - b_3\ln R & R \geq b_4 \\ I = \frac{3}{2}(M - 1) & R < b_4 \end{cases} \tag{4.1}$$

where R is in miles.

The information in (a) through (e) above is repeated for each source zone and serves to fully define the earthquake models for each zone.

4. The interval for discretization of magnitude is given as a fraction of a Richter unit.

5. The target areas are now defined and they are described on a cell by cell basis. For each cell which is a target cell, the indices of that cell are given and then the number of buildings of each type which are in that cell are specified. The number of buildings information must be given in the same order as the DPMs were given. Thus, the first count given corresponds to the building type of the first DPM, etc. Note that these counts are for all types, not just basic types.

6. The last thing given is the failure level to use, i.e. the definition of failure for this analysis. This is specified as a row index in the DPMs saying from this damage state on failure occurs. Thus the probability of failure for this analysis will be the column sum of the damage probabilities from the specified damage state to the last damage state for each MMI level.

Note that analyses for several different specifications under items 5 and 6 may be accomplished in a single computer run.

The output from the program consists of several things. First the input is printed including the computed frequency of occurrence constants in the relationship

$$N(\geq M) = e^{a-bM} \tag{4.2}$$

These constants are computed to force a log-linear relationship in each specified magnitude interval. The computed probability of M for each discretized magnitude interval is printed and the number of events per year within the magnitude ranges of interest are also printed.

The contents of each cell in the total area is listed indicating what buildings are in each cell and in what souce zone each cell is. The computed (summed) failure probability vectors are listed for each building type, using the failure level to decide where to sum.

The last sets of output are the computed probabilities, the first set being the probability mass function of the probabiity of exactly n failures for all n, the second set being the cumulative distribution function of the probability of n or more failures for all n. These 2 functions are printed and plotted.

The computational part is divided into two logical phases - the computations necessary to transform the input parameters into more usable forms, and the computations for the integration.

The input transformations consist of the computation of the FPVs (which is straight forward) and computations based on the earthquake models. The parameters specified for the models are transformed as follows. First, the number of events per year per unit area (cell) is computed for each source zone. Then the coefficients of the frequency relationship are computed for each zone. These are determined for each magnitude range for which occurrences were given. They are based on a log-linear relationship in each range in each zone and determined by the relationships:

$$N(\geq M) = e^{a-bM} \tag{4.3}$$

$$N(M \rightarrow M + \Delta M) = e^{a-bM}(1 - e^{-b\Delta M}) \tag{4.4}$$

$$\text{or } \ln N_{ij} = a_{ij} - b_{ij}M_{ij}$$

$$b_{ij} = \frac{\ln(N_{i,j+1}) - \ln(N_{i,j})}{M_{ij} - M_{i,j+1}} \tag{4.5}$$

$$a_{ij} = \ln(N_{ij}) + b_{ij}M_{ij} \tag{4.6}$$

where i refers to the i^{th} source zone and j to the j^{th} magnitude range for which occurrences were given.

The probability of M for each discretized magnitude interval in each zone is next determined. It is computed as

$$\{PM_{i\ell}\} = \frac{(1 - e^{-b_{ij}\Delta M})e^{a_{ij}-b_{ij}M_\ell}}{NCELL_i} \tag{4.7}$$

where ℓ refers to the ℓ^{th} discretized magnitude interval, M_ℓ is the magnitude value at the beginning of that interval, ΔM is the interval size, and $NCELL_i$ is the number of cells in source zone i. This is the probability of an earthquake of size M to M + ΔM occurring in a cell in source zone i.

The last input transformation is the determination of the maximum meaningful radius for each magnitude interval of each source zone. The maximum meaningful radius is that distance from the center of a source cell beyond which the intensity would be too low to cause any damage. This is determined from the attenuation relationship for the source zone and the FPVs. This ends the input transformations.

Now begins the integration phase. This consists logically of the following

1. For a source cell generate an earthquake of magnitude M_ℓ.

2. Determine the intensity of that earthquake at each target cell which falls within the maximum meaningful radius of M_ℓ , the total number of buildings

of each type in each intensity, and the probability of failure for each type and intensity. This produces the groups of buildings which must be binomially convolved.

3. Compute the probability function that n fail for all n given the magnitude and intensity.

4. Multiply the probabilities determined in (3) above by the probability of that magnitude interval occurring, i.e. by $PM_{i\ell}$.

The values obtained in (4) above are summed over all magnitude intervals which can produce damage at at least one cell and over all source cells. If an earthquake is generated for which no cells are susceptible, the probability of that earthquake is added to a special probability of zero building failing value. In other words, the probability of zero failing is divided into two categories (and printed out as two values) - the first meaning the probability of all earthquakes for which the ground shaking at all target cells meant zero probability of any building failing, the second meaning the probability of zero failing in all situations other than the first. Note that the probability of no earthquake occurring is never added in.

The computation in (2) and (3) above will now be described in a little more detail. This is the part in which the groups are determined and the group probabilities and transfer functions are computed. The intensity of shaking at each target cell is determined by the attenuation law

$$\begin{cases} I = b_1 + b_2(M_\ell + \frac{\Delta M}{2}) - b_3 \ln R & R \geq b_4 \\ I = \frac{3}{2}(M_\ell + \frac{\Delta M}{2} - 1) & R < b_4 \end{cases} \quad (4.8)$$

For each different level of shaking, the number of buildings of each type affected by that level of shaking is determined. The failure probability for each group is obtained from the appropriate FPV (for that type and that intensity). This step ignores all buildings which, for that level of shaking, have a failure probability of zero. At the end of this step we have k groups of buildings, the number of buildings in each group, and the failure probability for each group. We can thus at this point proceed to compute the needed binomial distributions and the convolved (product transform) distribution of exactly n failing given M_ℓ and I_k.

The binomial PMF is computed for each group, producing k mass functions of the form

$$p_{n_i}(n_{o_i}) = \binom{N_i}{n_{o_i}} p_i^{n_{o_i}} (1 - p_i)^{N_i - n_{o_i}} \tag{4.9}$$

where N_i = total number of buildings in group i

p_i = failure probability of a building in group i

Each PMF is computed recursively, namely by

$$p(i) = f[p(i - 1)] \tag{4.10}$$

$$\text{or} \quad p(i) = f[p(i + 1)]$$

and the recursive process is started by p(0), or by p(N) if $p(0) < 10^{-76}$, or by $p(m_x)$ if p(0) and p(N) are $< 10^{-76}$, where 10^{-76} is the smallest magnitude which can be used in the computer.

After the group PMFs are computed (with some approximations which are described in the next section), the final PMF for this earthquake is computed as the polynomial product of the coefficients of the transform, where there are k polynomials and the coefficients of each polynomial are the values of the PMF for that group.

B. Approximations

Three approximations were used in implementing the two-state model on the computer. The first two appear in the computation of the binomial mass functions, the third appears in the attenuation computation.

Since the binomial probability mass function of each grouping is a highly "spiked" function, i.e. there is a large difference in order-of-magnitude between the minimum and maximum values of the function, the function can be truncated at 6 standard deviations about the mean without any appreciable loss in significance of the final mass function for the probability of n facilities failing. This truncation at $\pm 6\sigma$ assures a minimum difference of about 6 orders of magnitude at either $+ 6\sigma$ or $- 6\sigma$ and the value at the mean. For the types of computations being performed, this is a sufficient range of magnitude to guarantee reasonable accuracy in the resultant function. This truncation serves to considerably reduce the amount of computations involved as it is performed not only on each of the k binomial mass functions but also on the polynomial product computation for the transform.

The second approximation is used for the computation of n! when $p(n_{o_i}=0)$ and $p(n_{o_i} = N_i)$ are less than 10^{-76} for some group i. This obviously relates to the portion of the program where the probability mass function is being computed for each of the k groups and $p(n_{o_i})$ is the mass function for group i, $i = 1,\ldots, k$, $n_{o_i} = 0, 1, \ldots, N_i$. The computation is recursive for a group, using $p(n_{o_i} = j)$ to compute $p(n_{o_i} = j + 1)$ or to compute $p(n_{o_i} = j - 1)$. It must get started some-somewhere, however, and the attempt is made to start it at either $n_{o_i} = 0$ or $n_{o_i} = N_i$. Due to the magnitude range limitation within the computer - namely 10^{-76} to 10^{+76} - this fails when both $p(n_{o_i} = 0)$ and $p(_{o_i} = N_i)$ are less than 10^{-76} (which for the types of problems bring run here is fairly often). Thus a different starting point is chosen in this case, namely when this occurs we begin at the mode point - $n_{o_i} = p_{f_i} N_i$ where p_{f_i} is the probability of failure for group i. To ensure computation success in this case with a minimum of effort, the approximation for the factorial

$$n! \simeq e^{-n} n^n \sqrt{2\pi n} \tag{4.11}$$

is used in computing $p(n_{o_i} = p_{f_i} N_i)$. This is a good approximation for $n > 50$.

The last approximation was the use of the midpoints of the intervals of discretization for the magnitude in the attenuation formula.

V. Examples of 2-State

A. Basic Data Used

The model described in Chapters III and IV was run for several configurations of data pertaining to portions of Eastern Massachusetts. The area chosen included six cities as target areas and four seismic source zones. The whole region (targets plus sources) was discretized into 5 square mile cells. At this level of discretization, 5 of the cities were defined as 1 target cell each, and 1 was defined as 4 target cells. Figure 5.1 shows the discretized area of interest with source zones and target cells labeled. For purposes of illustration the target areas chosen were heavily industrialized and populated cities in the eastern part of the state in which a fair amount of damage would be expected due to the numbers of brick buildings and, in some cases, the poor soil conditions. For these purposes the cell labels in Figure 5.1 correspond to real cities as follows.

Cell Label	City	Number of Cells
1	Lowell	1
2	Lawrence	1
3	Haverhill	1
4	Worcester	1
5	Springfield	1
6	Boston	4

The 4 cells of Boston comprise some metropolitan areas around Boston as well as the city itself. The area for these cells was defined to be that area within 5 miles of the State House dome.

One construction type - brick - was chosen and two soil types - good and bad - were used, giving a total of 2 building types for the model. In order to obtain the data necessary for the model - i.e. in order to get the number of buildings of each type in each cell - a building inventory had to be obtained. After contacting the following organizations and/or individuals

Boston Redevelopment Authority (BRA)

Building Inspector in each city or town

Sanborn maps

1970 Census

Civil Defense

Boston Housing Authority (BHA)

DCPA, Maryland

Fire Department of Boston

Real Estate Companies

Assessor's Office in Boston

it was determined that no single place contained the necessary data in an easily accessible form and, for the purposes for which this example was intended, it was unwise to spend the time and money which would have been necessary to get the information from several sources. Therefore the following was done for the Boston Target area.

1. Obtained estimates of the total number of structures in their city/town from building inspectors.

2. Obtained estimates of the percent of the total structures which were wood, masonry, steel/concrete from the same sources as in (1) but only for a few towns.

3. Applied the percentages in (2) to other towns which appeared to be similar.

4. Got the total number of residences in each town from the Census except for Boston where this figure was obtained from the BRA.

5. Got the total area of each city and town.

6. Determined the number of residences per square mile and the number of non-residences per square mile for each city and town and then determined the number of residences and non-residences in each one mile square cell in the area. The center of these cells for the Boston target area was the State House dome.

7. Used a soils map for the area to determine the percentage of bad soil in each 1 mile square cell and from this - assuming a uniform distribution of structures in each 1 mile square cell - determined the number of residential and non-residential structures on good soil and on bad soil in each 1 mile square cell.

8. Using (2) and (3) estimated the number of brick, wood , steel and concrete structures in the area as a percent of the total residential and a percent of the total non-residential.

9. Combined these 1 mile square numbers into 5 mile square numbers.

The numbers derived here, and the percentages, etc. are given in Chapters VI of this report. For the purposes of this example in this section, only brick was studied and a rough figure was used. The actual numbers used for each type and each target cell are given in Table 5.1. Numbers for each type for the other 5 cities of interest were obtained by applying Boston area numbers to the residential and non-residential guesses for those 5 cities. Type 1 corresponds to good soil, type 2 to bad soil, and the numbers are the rough figures divided by 15. This division by 15 was done to economize on computer time and does not affect the purpose of this example, which is to illustrate the use of the model and the interpretation of the results obtained from the model.

The earthquake models for the four source zones correspond to the models developed by Cornell and Merz (1) for their seismic study of Boston. The attenuation law used was the same for all 4 source zones and was

$$\begin{cases} I_s = 1.6 + 1.5M - 1.3\ln R & R > 10.8549 \\ I_s = \frac{3}{2}(M - 1) & R \leq 10.8549 \end{cases} \tag{5.1}$$

where R is in miles. The occurrence and magnitude range parameters for each source zone are given in Table 5.2. The background zone is included to allow for a certain amount of random small earthquakes around the target cells. The magnitude ranges correspond to an epicentral intensity range of MMI V through MMI VIII, and the minimum MMI of interest was specified as V. The magnitude discretization interval was 0.1.

Damage probability matrices were given for each type and are shown in Tables 5.3 and 5.4. These DPMs were based on work done by Peter McMahon (4) and indicate 3 damage states which are a measure of fatalities. State X corresponds to 1% of the occupants killed by an event, State Y to 10% killed by an event, and State Z to 50% killed. Each state is actually a range so that the probability, on the average, of 1% or more of the occupants killed by an event of a given intensity is the sum of the probabilities for X, Y, and Z. Since for this example we were interested in the results from two levels of failure, two classes of runs were made. Class 1 runs had failure defined as State X or greater, class 2 runs had failure as State Z. The failure probability vectors corresponding to these 2 classes are shown in Table 5.5 and Figure 5.2.

For each class, the following runs were made

1. All 6 cities together as a single run,
2. Each of the 6 cities run separately, and
3. Each of the 9 target cells run separately.

In all of them we were interested in the final distribution function, the effects of changes in parameters on the distribution function, and the relationship between the results when run separately and the results when run together, i.e. between simple summing and convolving. The three types of runs stated above are described in the next three sections, and the comparisons and summaries of these runs are given in the section following.

B. Six Cities Separately

Each of the six cities was run as a separate problem for class 1 ($\geq$ state X) and for class 2 (= state Z). The final distribution functions, both mass and cummulative functions, are shown in Figures 5.3 through 5.15 and a summary of the results appears in Table 5.6. The graphs in the figures are semi-log plots with the Y-axis giving the probabilities and the X-axis the values for n: the number of buildings failing. The labeling on the X-axis is one-tenth the number of buildings used due to the workings of the library program used for axis labelling. The zero values (the values for zero failing) are category 2 zero values only, i.e. the probability for zero failing does not include those earthquakes which have zero probability of causing any failures, which for these runs are those earthquakes which do not produce a site intensity of V or greater for class 1 or a site intensity of VII or greater for class 2.

Even though these computations (for each city individually except Boston) could have been done more simply, they are included here for several reasons, of which the most important at this point is to illustrate the cause-effect interpretation of the results. The appearance of the results is primarily due to the use of the binomial model.

The binomial mass function is a very spiked or steep function, varying greatly in its order of magnitude values over a relatively short interval of its total population N. The largest value occurs at its mean, m_x, where

$$m_x = pN \tag{5.2}$$

For 5 of the cities (Boston excluded) there is only one target cell; hence for each generated earthquake in the integration, there will be only one level of shaking and only 2 groups of buildings - one on good soil and one on bad soil. Thus, for each such earthquake, the mean of the mass function for that earthquake will be

$$m_{x_i} = p_{1_i} N_1 + p_{2_i} N_2 \tag{5.3}$$

where

i is the intensity at the target

p_{1_i} is the probability of failure for MMI i for type 1 buildings

p_{2_i} is the probability of failure for MMI i for type 2 buildings

N_1 is the number if type 1 buildings

N_2 is the number of type 2 buildings

Since there are at most 4 intensities (with non-zero likelihood of causing damage) which can occur at the target cell(for 1 of the 5 cities) for the $\geq$ X runs, there will be at most 4 peaks in the final mass function. The first peak will correspond to the effect of MMI V shaking, the second to MMI VI shaking, etc. For these simple cases it can easily be predicted where each peak will occur and how many such peaks there will be. For example, if we look at Lowell ($\geq$ X) we know there will be 4 peaks since it is close enough to source zone C to get intensities of VIII. Thus the peaks will occur at

for MMI V $m_x = .0045 \times 470 + .0092 \times 70 = 3$

VI $= .0545 \times 470 + .157 \times 70 = 37$

VII $= .2 \times 470 + .45 \times 70 = 126$

VIII $= .425 \times 470 + .8 \times 70 = 256$

Since we are truncating our computations for each generated earthquake at $\pm 6\sigma$ about the mean, we get separable peaks or mass functions in our final distribution. If N, the total number of buildings of all types, is large enough, each such peak will be completely separate and distinct and will have a binomial-like shape. If N is small with respect to the difference in the value of the failure probabilities for each of the MMI's then the peaks will not be distinct since one will overlap near the high point of the other. This will be shown when we discuss the Boston results.

For single cells and reasonably separate peaks, we can thus also predict the spread of each peak and the amount of overlap since we know the computation is performed to $\pm 6\sigma$ and, for the binomial, σ is

$$\sigma_x = [p(1 - p)N]^{1/2} \tag{5.4}$$

for one group, or for two groups and intensity i

$$\sigma_{x_i} = [p_{1_i}(1 - p_{1_i})N_1 + p_{2_i}(1 - p_{2_i})N_2]^{1/2} \tag{5.5}$$

Thus for Lowell we get the following ranges for the 4 peaks for $\geq$ X:

MMI	m_x	$6\sigma_x$	n_{MIN}	n_{MAX}
V	3	10	0	13
VI	37	35	2	72
VII	126	58	68	184
VIII	256	67	189	323

We can also predict, for single cells and separate peaks, the height of the level places or benches in the cummulative function. Since the benches correspond to the peaks of the mass function, which peaks correspond to the different intensities, the height of each bench [p ($\geq$n) failing] corresponds to the probability of that intensity or greater occurring at the site given that an earthquake of $M \geq M_{min}$ occurs.

The mass functions for the 5 cities for the =Z runs show only a single peak. This is predictable since the only MMIs of interest in this case are VII and VIII and the ranges for both overlap one another to such a degree that they are not distinct at all. It should be noted here that the values for Worcester are extremely small and this is due to the fact that it is located such that no VIII intensities can occur there and only relatively few VII intensities can occur. The highest risk sites in the =Z class should be and are the cells which are closest to or in source zone C, which is the highest risk zone for this class problem.

The results for Boston serve to introduce the effects due to multiple target cells. When there are multiple target cells, a generated earthquake can cause different levels of shaking at the target cells. If N is large enough, there will be $i^j - 1$ peaks or peak values in the final mass function, where

i = number of effective MMIs + 1

j = number of target cells.

Note that if $j = 1$, then i = number of effective MMIs. Also note that the +1 provides for intensities other than effective intensities occurring at one or more cells and the −1 removes the case where there is no cell with an effective intensity. It is thus possible, in some cases, to relate the peaks on the final mass function to level of shaking and particular target cell(s) by knowing that these peaks come from the expected value of each product binomial, i.e. from each generated earthquake and by knowing that only $\pm 6\sigma$ is computed for each earthquake. In fact if N is large enough, this can be done in a fairly straightforward, even though tedious, fashion.

In the Boston $\geq$ X case, there will in principal be a maximum of $5^4 - 1$ peaks. However, due to the geometric relationship of the 4 target cells, there can never be more than a 1 intensity variance among the intensities of the Boston target cells for a given earthquake giving a total possible of 60. Taking into account the spatial relationsip between the 4 target cells and the souce zones reduces this even further to 41. This latter relationship (in conjunction with the attenuation relationship) says that the largest intensity at any Boston target cell due to source zone A activity is MMI VI, the largest due to source zone B activity is MMI V, the largest due to background activity is VI, and the largest due to activity in C is VIII. Using this plus the relationsip among the 4 cells, the following are impossible events

CELL

IV	V	V	IV
V	IV	IV	V
V	VI	VI	V
VI	V	V	VI
VI	VII	VII	VI
VII	VI	VI	VII
VII	VI	VI	VI
VII	VI	VII	VI
VII	VII	VI	VI
VII	VII	VII	VI
VII	VIII	VII	VII
VII	VIII	VIII	VII

as well as all events with VIII at cell 1 and VII at at least 1 other cell. This produces the maximum total of 41. Moreover, since N is not that large (2600), we find in the plot that we can readily distinguish only 18 of these and also that some of these 18 overlap one another to a fair degree. This does not prevent us, however, from determining some cause-effect relationsips.

We can do the following basic interpretation of the Boston $\geq$ X results. The range from 800 on is the range where MMI VIIIs start to enter the picture. Entering means that from then on at least one of the 4 cells has an intensity VIII and the other have intensity VII. The last peak which occurs at 1424, is due to all 4 cells having MMI VIII. The range from 260 to 800 is where MMI VIIs enter the picture, the range from 25 to 260 is due to MMI VIs coming into play and the < 25 groups is caused by MMI V.

The =Z curves for the single cell cities all have single peaks around 2 due to MMI VII. In the first three cities, MMI VIII starts entering around 5. The =Z curve for Boston has two groupings, the first, going to about n = 20, due to VII and the second due to VIIIs occurring.

C. Six Cities Together

All six cities were run together for class 1 and for class 2. The results appear in Figures 5.16 through 5.19 and are summarized in Table 5.7. The dominating cells are the three northeast cities and Boston which is what one would expect due to their proximity to source zone C, the highest risk zone.

D. Nine Target Cells Separately

This set of runs was an expansion of the runs for class 1 in section B in order to compare the effects of convolution with merely summing the results from each individual target cell to get an approximate combined result. Five of the nine target cells were presented in Section B and the results for the other four (those for the city of Boston) are presented in Figures 5.20 through 5.27. Each has four peaks corresponding to the effect from MMI V through VIII. The results for all nine targets are summarized in Table 5.8.

E. Convolution versus Summation

In cases where the target cells are geographically distributed in such a way that no more than 1 cell can have failures from a single event, the cells can be treated independently by computing the probability distribution for each cell and then summing these to get the distribution for the entire area. In cases where the spatial distribution is not of this type, there can be sizeable differences between the distribution obtained by convolving all the target cells together and the one obtained by summing.

Table 5.9 compares the values obtained from the convolved or 6-city together run (Section C) with the summed values from the 6-city separate runs (Section B). Except for the large numbers of buildings failing (beyond 200), the two sets of values are similar. This is due to geographical separation and to the domination of Boston in the results. Note that in the separate and summed values, Boston was convolved.

Table 5.10 compares the values obtained from the 6-city together run with the summed values from the 9 target cells separately runs (Section D). Note that in this summation, Boston was not convolved but consisted of 4 separate entities. The 2 results here are not similar due to geometric proximity. It is also interesting to note that the summed value for P ($\geq$ 1) is larger than the earthquake occurence would allow. What the earthquake occurrence would allow can be computed by subtracting from the lambda (which for the models used is 0.23) the sum or the probabilities of the earthquakes which do not produce a minimum intensity at a target cell, i.e. subtracting 0.207, the probability of our category 1 zeros. This gives the maximum of what P ($\geq$ 0) can be; in this case that value is .023. The value for P ($\geq$ 1) must of course be less than or equal to that. The larger than allowed effect is true in both tables, but is noticeably so in Table 5.10 and is due to the fact that the summation is treating the effect of a single earthquake at a target cell as independent from the effect of that same earthquake at another target cell when in fact that treatment is erroneous. This treatment thus causes more earthquakes to be produced than actually modelled or, in actuality, causes the area under the risk curve to be multiplied by some factor rather than merely determining the area.

This multiplicative effect of assuming independence where great dependence exists (Boston as 4 rather than Boston as 1) is the reason why the summed results in Table 5.10 start out much larger than the convolved results. The summed results in Table 5.9 also start out larger, but the difference is small enough to be ignored. There is enough geometrical separation between the 4 cells of Boston (treated here as one) and the three northeastern Massachusetts cities so that assuming independence does not hurt until large numbers of buildings failing are involved.

After some crossover point, the summed values will always be less than the convolved values. This is due to the fact that if there are 2 cells with N_1 and N_2 buildings respectively, in the independent summed case there can never be more than the larger of N_1 and N_2 failing whereas in the dependent convolved case there can be up to N_1 plus N_2 failing. Again we see this crossing over and the sum being less than the convolution in both tables. It is immaterial in Table 5.9 since the dominant influence, namely Boston, has already been convolved. It becomes material only when we get to large numbers of buildings resulting from dependencies between Boston and the Northeast cities.

F. Improvement of Failure Curve

Several runs were made to investigate the effect of improving the failure curve (the failure probability vector) which improving would correspond to upgrading all existing buildings of a given type. To do this, the DPMs in the SDDA Report 10 for UBC 0/1, 2, 3 and S were chosen as the basis for the failure curves (5). All other parameters remained the same. The DPMs were converted to DPMs for damage states X, Y, and Z using the correspondences

State X = Damage States L + 1/2M

State Y = Damage States 1/2M + H

State Z = Damage States T + C

This results in the DPMs shown in Table 5.11. These DPMs are for good soil; bad soil DPMs result in shifting the good soil DPMs one MMI to the left. This will produce the FPVs for class 1 ($\geq$ State X) shown in Table 5.12.

Two sets of runs were made with these FPVs - one set for Lowell and one set for Boston. Figures 5.28 through 5.35 contain results for Lowell, Figures 5.36 through 5.43 contain the results for Boston, and both are summarized in Table 5.13.

The Lowell runs are very easy to interpret. The 3 peaks on the mass function correspond to MMI V, VI, and VII respectively and there is a point on the PMF at n=540 which correesponds to MMI VIII. This last peak is all buildings failing, i.e. $P(I=VIII|M\geq 4.3)$. The major effect of improving the failure curve is the visible effect of shifting the peaks to the left (with the exception of the single point at 540). The reason for this is that for the FPVs used here, the probability of failure decreases for each MMI from one UBC zone to the next (better) zone. This results in the mean value for each MMI (corresponding to each peak) becoming smaller and hence shifting to the left. Even though in the general case an improved failure curve would not contain all failure probabilities less than the non-improved curve values and hence the resulting PMF would not have all peaks shifted left (some could remain the same and some could be shifted right except the right most one since lessening the damage due to MMI VIII could increase the damage due to MMI VII, etc.), the center of mass of the resulting function from the improved case would always be less than the center of mass previously.

The Boston runs show, on a gross scale, an effect similar to the Lowell runs, namely a shifting left of the peaks or peak combinations. Since the Boston runs involved convolving 4 target cells, these runs also show peaks appearing and disappearing with changes in the failure probability and with changes in the shape of the individual binomial curves (from left-skewed to normal to right-skewed).

The general case could have a peak totally disappearing. This would occur if the probability of failure at the minimum effective MMI became zero. In our cases here, the failure probability of the minimum effective MMI, namely V, becomes smaller with each improved curve but never becomes zero. Thus there is always a peak corresponding to MMI V occurring and the probability of zero or more failing remains the same for all failure curves. In fact, the probability of n or more failing is the same for all the failure curves where n is less than $m_x - 3\sigma_x$ and where m_x and x_o are the mean and standard deviation of the smallest failure probability for the minimum effective MMI. This is true in the general case and thus, for a small n, improvement of the resistance of a class of structures may not reduce the probability of n or more failing.

G. Summary of Parameter Effects

Changes to various parameters in the model can have different effects on the final distribution, the underlying cause being the innate behavior of the binomial distribution.

Small variations in the rate of occurrence cause virtually no change in the final distribution. Large variations in the rate of occurrence cause shifts in the curve up or down but do not appreciably change the shape of the curve.

Variations in the values of the failure probability vectors can cause major changes in the final distribution. This is caused by the change in shape of the binomial when the value for the probability of failure p_f changes. As p_f goes from .1 to .5 to .9 the binomial goes from a right-tailed Poisson to a normal to a left-tailed Poisson. The mode point also changes greatly since the mean = $p_f N$. The most visible effect of changing the failure curve is the shift of the peaks due to the change in the value of the means.

Small variations ($\pm$ 10%) in the total population (N) has little effect on the final distribution.

An increase in the total population causes more "binomial peaks" to come into view if there is more than one target cell, the probability of the expected value of each binomial curve to be lowered, the spread of each binomial curve to be increased, and the mode point (expected value) of each binomial curve to be increased or shifted to the right. This right shift of each contributing binomial is a cumulative shift in the final distribution.

Our lowering of the population by a factor of 15 for the examples in this chapter may seem wrong in light of the effects just stated of population increases. If our purpose in making the runs had been to obtain reasonably precise results across the entire real range of N(from 0 to 15 x 2600), then it would indeed have been wrong. However, our purpose was to obtain a reasonable picture of what happens, and this can be obtained by using a smaller population. We can extrapolate to the real population as long as we extrapolate from a wide range of n rather than a small range so that the sum of the probability mass function in that range remains the same, be it for the factored down range n_1 to n_2 or the real range, $15n_1$ to $15n_2$.

VI. LIFE LOSS COMPUTATIONS

Even though it is necessary many times to determine the likelihood of n buildings failing, what is quite often of prime concern is the life safety threat posed by an earthquake or the likelihood of some range of people being killed. The general statement of this would be "what is the annual probability of n people being killed due to an earthquake?" Even though this is a similar statement to that posed for building failure, the same assumption of independence cannot be made. In other words, assuming that what happens to one building is independent of what happens to another building is more often than not a reasonable physical assumption; doing the same thing with each person is not reasonable physically. The threat to people is a function of the damage state and the structural type of the facility in which the people are occupants at the time of the earthquake. Thus, threat to occupants of different buildings can be assumed independent but threat to individual occupants of a building cannot. A model of this life safety threat could be developed and implemented. It was decided not to do so, however, due to the cost of running such a model. Rather, it was decided to use expected values in developing methods for studying the threat to life safety. This section develops that approach.

A. Expected Life Loss Ratios

As stated above, life safety is a function of the number of occupants in each building at the time of the earthquake, the level of damage suffered by the building, and the general risk posed by any structure of a given type (say as a function of height and building material). All except occupancy are independent of time of day. The first thing we wish to develop, then, is a set of expected life loss ratios (ELLR) in the form fraction of occupants expected to be killed by a certain level of intensity within a structure of a given type and risk class.

To do this, two things need to be developed before the ELLRs can be determined - first a set of DPMs based on type and second a set of probability density functions for the probability of x fraction of the occupants being killed given the type and risk and the level of damage.

In developing the DPMs, the general classification of the MSK scale (6) was used as a guide. Structures were divided into three classes, A, B, and C, based on their general resistance to shaking. In class A we have poorly constructed masonry and RC structures, in class B we have unreinforced but well constructed brick, average RC and poorly constructed wooden, and in class C are well built wooden and reinforced structures. Table 6.1 shows the DPMs developed for each of these 3 classes. Note that the damage levels are the same as used throughout most of the SDDA work.

The probability density functions were taken to be 3rd order parabolic curves (7). It was decided that fatalities would not occur until damage state M, that less than 100% of the occupants would be killed given damage states M, H, or T, and that there were spikes in the function at 0% being killed for all damage states and at 100% killed for damage state C. The PDFs used are:

for states M, H, and T:

$$\begin{cases} p(x) = 4\,\dfrac{1 - p_o}{x^4_{max}}\,(x_{max} - x)^3 & 0 < x \leq x_{max} \\ p(0) = p_o & \end{cases} \tag{6.1}$$

for state C

$$\begin{cases} p(x) = 4(1 - p_o - p_1)(1 - x)^3 & 0 < x < 1 \\ p(0) = p_o & \\ p(1) = p_1 & \end{cases} \tag{6.2}$$

where x is the fraction of occupants killed.

Incorporating the risk factor into the structural types generated 5 "life safety" types of buildings, namely wooden, RC less than or equal to 5 stories tall, RC over 5 stories, low rise brick residential, and high rise brick or brick warehouse type structures. If steel structures were also being considered, they would be divided into 2 categories, over 5 stories and 5 stories or less. Table 6.2 shows the parameters used for each PDF and the computed expected value (m_x) and standard deviation (σ_x) for each type.

Expected life loss ratio (ELLR) was defined in SDDA Report 10 as

$$ELLR_I = \sum_{D.S.} P_{DSI} \times CLLR_{DS} \tag{6.3}$$

where

$ELLR_I$ is the expected (or mean) life loss ratio for MMI I

P_{DSI} is probability of damage state given MMI I

$CLLR_{DS}$ is central life loss ratio for damage state and is the same as the expected values (m_x) of our PDFs shown in Table 6.2.

One additional thing should be added to this, however, that being the provision for different soil types. This provision affects the building performance, not the PDF. For the purposes of this study, 2 soil types were assumed - good and bad - and the DPMs for good soil were defined as those in Table 6.1 and the DPMs for bad soil were defined as those in Table 6.1 shifted 1 MMI to the left. In other words, damage probability for bad soil for MMI I equals damage probability for good soil for MMI (I + 1). Applying this 2 soil type separation to the formula of Eq. (6.3) and the data in Tables 6.1 and 6.2 gives the set of ELLRs shown in Table 6.3. Note that the ELLR for a building type is a linear function of the fraction of buildings of that type on good soil, and the ELLR values given in Table 6.3 are the 2 end point values for each of those straight lines. Also note that MMI V is not given. V on good soil is zero, V on bad soil = VI on good soil. Since the only fatalities which can occur given MMI V are from bad soil MMI VI class A damage state M and hence are very small, they have been ignored since they have virtually no effect in any practical situation.

B. Application to Boston as a Single Target

If we apply the ELLRs of Table 6.3 to the Boston area in which 34% of the area is bad soil, we get the set of ELLRs shown in Table 6.4. If we now take Boston as a single target with seismic risk values of

$$= \text{VI} \quad 2.4 \times 10^{-3}$$

$$= \text{VII} \quad 3.1 \times 10^{-4}$$

$$= \text{VIII} \quad 1.6 \times 10^{-5}$$

we can develop a set of expected annual life loss ratios (EALLR) defined by

$$\text{EALLR} = \sum_{I} \text{ELLR}_I \times \text{SR}_I \tag{6.4}$$

where SR_I is the seismic risk for MMI I. These EALLRs for Boston are shown in Table 6.5. It can be seen from this that the expected annual life loss from MMI VII is the largest contributor in class A structures and in class B except for RC structures where VIII is the largest contributor to the EALLR.

We can now determine for Boston the expected killed for each level of intensity and the annual expected killed. To do this, totals of buildings for each type and class were determined. Two sets were used, one with 0% of the brick buildings in class A (shown in Table 6.6a) and the other with 20% of the brick buildings in class A (Table 6.6b). The totals of each building type and the occupancy factor for each is summarized in Table 6.7.

Applying the building counts and occupancy factors to the ELLRs of Table 6.4, we get the expected killed for each MMI for 0% class A brick and 20% class A brick as shown in Tables 6.8a and 6.8b. Multiplying these by the seismic risk values as given above, we get expected annual killed for each of the 2 class A brick assumptions. These appear in Tables 6.9a and 6.9b.

Here we see that for Boston MMI VII is the greatest contributor to the expected annual life loss.

C. Boston as 4-Cell Target

We can refine the computation of the prior section by using the 4-cell model of Boston. For this model, we need building counts for each of the 4 cells (Tables 6.10a and 6.10b) and the seismic risk for each cell (Table 6.11) The occupancy levels remain the same as in the single target example of the prior section.

Applying Tables 6.10a or 6.10b and 6.7 to Table 6.4, we get expected killed for each cell and each MMI with 0% class A brick (Table 6.12a) and with 20% Class A brick (Table 6.12b). Multiplying each of these by the seismic risk values of Table 6.11 we get expected annual life loss by cell for each of the 2 class A brick cases, as shown in Tables 6.13a and 6.13b.

We notice some difference between the results obtained using the 4 cell model and the ones obtained using the single cell model. The 4 cell model illustrates well the sensitivity of Boston to the definition (geometrically) of the seismic source zones, in particular to the location of source zone C from where the MMI VIII intensitites which affect Boston originate. It can be seen that the cell farthest away from source zone C (namely cell 1) has much lower seismic risk for VIII than does the cell closest to zone C. If zone C were lowered (moved southwest) 1 or 2 cells (5 to 10 miles), this difference would change some and cause the contribution from VIII in the expected annual life loss to be much greater.

The prior applications to Boston were included to illustrate how the ELLRs could be used to develop scenarios about the effects of specific earthquakes. Time of day considerations could be included by modifying the occupancy factors appropriately.

D. Annual Risk of Range of Life Loss

We now wish to determine the annual probability that n_1 to n_2 people will be killed in an earthquake, where n_1 to n_2 could be 1-10, 11-50, etc. There are 2 approximate methods we could use. The first is based on expected values of the number of buildings of each type being in each damage state given an event, location, and cell intensity and on the expected values of the number of deaths given the number of buildings in each damage state. In this approach, an event is generated, the intensity at each target cell is determined, and the expected life loss for that event is determined by

$$\text{ELL} = \sum_{\substack{\text{Target} \\ \text{Cells}}} \sum_{\text{Types}} \text{no. builindgs type}_j \times \text{ELLR}_{I,j} \tag{6.5}$$

where $\text{ELLR}_{I,j}$ for each cell is computed from the % bad soil in each cell.

The appropriate range for ELL for that event is determined (e.g. ELL is between 1 and 10) and the probability for that event is associated with that range. This is performed for all events generated and the probability of all events producing life loss in each range is summed for each range. This results in an annual risk of range of life loss.

The approach was used with the 4-cell Boston model, resulting in risks shown in Table 6.14.

The second approach is very close to the actual model. The approximation used is the discretization of the fatalities PDFs into say 1% ranges. With this model, we have a multinomial distribution for the number of buildings of a group in each damage state given an event and site intensity. We can then get

p(T_ℓ people killed in group ℓ| event, building damage and % killed configurations)

$$= \prod_{\text{bldgs.}} p(\text{\% killed in each bldg.}) \times \prod_{\text{d.s.}} p(\text{n bldgs. in each damage state})$$

$$= \left[\prod_{i=1}^{4} \prod_{j=1}^{n_{\ell i}} q_{\ell i}(x_{ij})\right] \frac{N_\ell! P_{\ell 1}^{n_{\ell 1}} P_{\ell 2}^{n_{\ell 2}} P_{\ell 3}^{n_{\ell 3}} P_{\ell 4}^{n_{\ell 4}} (1-P_{\ell 1}-P_{\ell 2}-P_{\ell 3}-P_{\ell 4})^{N_\ell - n_{\ell 1}-n_{\ell 2}-n_{\ell 3}-n_{\ell 4}}}{n_{\ell 1}! n_{\ell 2}! n_{\ell 3}! n_{\ell 4}! (N_\ell - n_{\ell 1} - n_{\ell 2} - n_{\ell 3} - n_{\ell 4})!} \qquad (6.6)$$

where

k: number of groups

i: damage state M, H, T, or C

$n_{\ell i}$: number of buildings in damage state i in group ℓ, $\ell = 1,2,\ldots,k$

x_{ij}: fraction killed in building j in damage state i (in group ℓ)

$q_{\ell i}(x_{ij})$: probability of x_{ij} fraction of occupants killed in building j, damage state i, group ℓ, given building j in damage state i

and $$T_\ell = occ_\ell \sum_{i=1}^{4} \sum_{j=1}^{n_i} x_{ij} \qquad (6.7)$$

Multiplying the above by the probability of the event gives the probability of T_ℓ fatalities for a certain configuration of buildings in each damage state and a certain configuration of fraction of fatalities in each building in group ℓ.

The probability of T killed is

$$p(T) = p(T_1, T_2, \text{---}, T_k)$$

where $\sum T_\ell = T$ and the set of T_ℓ are random variables

The probability of ranges of fatalities is obtained by summing the p(T)'s in the appropriate range for T over all combinations of x_{ij} for all combinations of $n_{\ell i}$ over all events. This can be a time-consuming approach and thus other approximations are being pursued. One such could be the use of the multivariate normal for the multinomial. Even though the values for $p_{\ell i}$ are not in the range for which the multivariate normal would be a good mathematical approximation, it still may be good enough considering the fact that many of the parameters of the model are merely best-guess values. This certainly is an area in which more work should be done. Another approach to this same problem is to re-evaluate the damage probability matrices of buildings with the intent of deriving continuous distributions for damage, rather than the discrete distributions used to date. This approach might lead toward developing single distributions (of a more manageable form) for life loss given an event and a site intensity.

In other words, more work needs to be done to develop a model which, while reasonably depicting the physical circumstances, is numerically convenient in terms of time and cost for running the model. A model which transforms nicely (either discretely or continuously) or is itself nice would be ideal.

VII. REFERENCES

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CITY	TYPE 1	TYPE 2
1	470	70
2	500	100
3	500	100
4	470	70
5	500	120
6–1	500	200
6–2	150	100
6–3	600	370
6–4	500	180

Table 5.1 Building Population in Target Cells

SOURCE ZONE	NO. $\geq$ 4.3 PER YEAR PER MI^2	MINIMUM MAGNITUDE	MAXIMUM MAGNITUDE	NO. OF CELLS
A	.0578743	4.3	6.53	398
B	.0229156	4.3	6.53	346
C	.126257	4.3	6.8	758
Bkgd.	.00934	4.3	5.2	467

Table 5.2 Earthquake Model Parameters

STATE	V	VI	VII	VIII	IX
X	.0044	.05	.186	.365	.49
Y	.0001	.0045	.010	.048	.275
Z	0.0	0.0	.004	.012	.035

Table 5.3 DPM for Type 1 - Good Soil

STATE	V	VI	VII	VIII	IX
X	.0085	.152	.403	.458	.1
Y	.0007	.005	.042	.307	.62
Z	0.0	0.0	.005	.035	.28

Table 5.4 DPM for Type 2 - Bad Soil

CLASS	TYPE	V	VI	VII	VIII	IX
1	1	.0045	.0545	.2	.425	.8
	2	.0092	.157	.45	.8	1.0
2	1	0	0	.004	.012	.035
	2	0	0	.005	.035	.28

Table 5.5 Failure Probability Vectors

CITY	$\geq X$							
	$P(\geq 1)$	Max n	$P(\geq 50)$	$P(\geq 100)$	$P(\geq 150)$	$P(\geq 200)$	$P(\geq 250)$	$P(\geq 300)$
Lowell	.00775	323	1.49×10^{-4}	1.31×10^{-4}	3.7×10^{-6}	2.79×10^{-6}	1.98×10^{-6}	1.41×10^{-10}
Lawrence	.0101	363	5.81×10^{-4}	3.18×10^{-4}	1.13×10^{-4}	1.27×10^{-5}	1.27×10^{-5}	3.5×10^{-6}
Haverhill	.01219	363	8.26×10^{-4}	4.95×10^{-4}	1.75×10^{-4}	1.86×10^{-5}	1.86×10^{-5}	5.13×10^{-6}
Worcester	.00459	183	5.39×10^{-6}	8.41×10^{-7}	5.96×10^{-9}			
Springfield	.00545	217	1.91×10^{-4}	3.05×10^{-5}	2.03×10^{-5}	3.57×10^{-10}		
Boston	.00928	1566	1.4×10^{-3}	1.09×10^{-3}	9.8×10^{-4}	7.79×10^{-4}	1.8×10^{-4}	1.35×10^{-4}

CITY	$= Z$		
	$P(\geq 1)$	Max n	$P(\geq 10)$
Lowell	.000117	25	7.35×10^{-7}
Lawrence	.000292	27	5.65×10^{-6}
Haverhill	.000442	27	9.06×10^{-6}
Worcester	7.5×10^{-7}	11	7.1×10^{-11}
Springfield	.000028	13	8.86×10^{-9}
Boston	.000132	91	3.23×10^{-5}

Table 5.6 Summary Results from 6 City Separate; Class 1 and Class 2

$\geq X$

$P(\geq 1)$	Max n	$P(\geq 50)$	$P(\geq 100)$	$P(\geq 150)$	$P(\geq 200)$	$P(\geq 250)$	$P(\geq 300)$
.0227	2371	3.79×10^{-3}	2.16×10^{-3}	1.32×10^{-3}	1.11×10^{-3}	8.9×10^{-4}	7.14×10^{-4}

$= Z$

$P(\geq 1)$	Max n	$P(\geq 10)$
.000595	120	9.04×10^{-5}

Table 5.7 Summary of Results from 6 City Together Runs

$\geq$ X

CITY	P($\geq$1)	P($\geq$50)	P($\geq$100)	P($\geq$150)	P($\geq$200)	P($\geq$250)	P($\geq$300)
1	.00775	.000149	.000131	.0000037	.0000028	.000002	0
2	.0101	.000581	.000318	.000113	.0000127	.0000127	.0000035
3	.01219	.000826	.000495	.000175	.0000186	.0000186	.0000051
4	.00459	.0000054	.0000008	0	0	0	0
5	.00545	.000191	.0000305	.0000203	0	0	0
6-1	.00725	.000794	.000035	.000035	.0000071	.00000004	.00000004
6-2	.00644	.000067	.0000004	.00000007	0	0	0
6-3	.00787	.001072	.000240	.0000821	.0000821	.0000819	.0000148
6-4	.00831	.001065	.000131	.000131	.0000092	.0000028	.0000028

Table 5.8 Summary of Results from Nine Target Cells Separately

	P(≥ 1)	P(≥ 50)	P(≥ 100)	P(≥ 150)	P(≥ 200)	P(≥ 250)	P(≥ 300)
Summed	.0494	.00315	.00207	.00129	.00081	.00213	.000144
Convolved	.0227	.00379	.00216	.00132	.00111	.00089	.000714

Table 5.9 Summation of 6-City Separately vs 6-Cities Together

	P(≥ 1)	P(≥ 50)	P(≥ 100)	P(≥ 150)	P(≥ 200)	P(≥ 250)	P(≥ 300)
Summed	.06995	.00475	.00138	.00056	.00013	.00012	.000026
Convolved	.0227	.00379	.00216	.00132	.00111	.00089	.000714

Table 5.10 Summation of 9 Target Cells Separately vs 6-Cities Together

STATE	V	VI	VII	VIII	IX	X	
X	0	.73	.64	.10	0	0	
Y	0	0	.21	.51	0	0	UBC 0,1
Z	0	0	0	.39	1.0	1.0	
X	0	.53	.63	.36	0	0	
Y	0	0	.17	.58	0	0	UBC 2
Z	0	0	0	.06	1.0	1.0	
X	0	.43	.62	.51	.10	0	
Y	0	0	.13	.48	.62	0	UBC 3
Z	0	0	0	.01	.28	1.0	
X	0	.33	.59	.66	.25	0	
Y	0	0	.11	.34	.73	0	UBC S
Z	0	0	0	0	.02	1.0	

Table 5.11 DPMs for Failure Curve Improvement Comparisons

TYPE	MMI V	VI	VII	VIII	IX	
1	0	.73	.85	1.0	1.0	UBC 0,1
2	.73	.85	1.0	1.0	1.0	
1	0	.53	.80	1.0	1.0	UBC 2
2	.53	.80	1.0	1.0	1.0	
1	0	.43	.75	1.0	1.0	UBC 3
2	.43	.75	1.0	1.0	1.0	
1	0	.33	.70	1.0	1.0	UBC S
2	.33	.70	1.0	1.0	1.0	

Table 5.12 FPVs for Failure Curve Improvement Comparisons for State $\geq$ X.

	UBC	P(≥ 0)	P(≥ 1)	P(≥ 10)	P(≥ 50)	P(≥ 100)	P(≥ 150)	P(≥ 200)	P(≥ 250)	P(≥ 300)
Lowell	0/1	.00819	.00819	.00819	.00591	.00124	.00124	.00124	.00124	.00124
	2	.00819	.00819	.00819	.00125	.00124	.00124	.00124	.00124	.000896
	3	.00819	.00819	.00819	.00124	.00124	.00124	.00124	.000878	.000131
	S	.00819	.00819	.00819	.00124	.00124	.00124	.000866	.00131	.000131
Boston	0/1	.00933	.00933	.00933	.00933	.009199	.00859	.00844	.00797	.00777
	2	.00933	.00933	.00933	.009298	.00881	.00830	.00784	.00774	.00730
	3	.00933	.00933	.00933	.00921	.00854	.00794	.00775	.00723	.00677
	S	.00933	.00933	.00933	.00917	.00813	.00775	.00715	.00661	.00186

Table 5.13 Summary of Failure Curve Improvement Runs

	MMI				
	VI	VII	VIII	IX	X
O	.35				
L	.60	.30	.05		
M	.05	.50	.25	.05	
H		.15	.35	.15	.05
T		.05	.30	.30	.20
C			.05	.50	.75

CLASS A: Poor Masonry, Poor RC

	MMI				
	VI	VII	VIII	IX	X
O	.95	.15			
L	.05	.55	.30		
M		.25	.40	.15	.05
H		.05	.25	.30	.15
T			.05	.50	.30
C				.05	.50

CLASS B: Unreinforced Brick, Average RC
Poor Wood

	MMI				
	VI	VII	VIII	IX	X
O	1.0	.60	.10		
L		.40	.45	.20	.05
M			.40	.35	.15
H			.05	.40	.30
T				.05	.40
C					.10

CLASS C: Well Built Wooden, RC

Table 6.1 DPMs for 3 Classes

	M				H				T				C			
	p_o	x_{max}	m_x	σ_x	p_o	x_{max}	m_x	σ_x	p_o	x_{max}	m_x	σ_x	p_o	p_1	m_x	σ_x
Brick Residential	.97	.03	.00018	.0013	.90	.10	.002	.0079	.70	.30	.018	.0384	.20	.05	.2	.245
Brick Storage or Tall	.95	.05	.0005	.0028	.85	.15	.0045	.0143	.65	.35	.0245	.0475	.16	.29	.4	.408
Wooden	.99	.05	.0001	.0013	.95	.10	.001	.0057	.84	.25	.008	.0245	.85	.05	.07	.2275
RC $\leq$ 5 stories	.98	.03	.00012	.0011	.92	.10	.0016	.0071	.65	.30	.021	.0407	.16	.29	.4	.408
RC $>$ 5 stories	.98	.03	.00012	.0011	.92	.10	.0016	.0071	.60	.30	.024	.0427	.12	.53	.6	.44

Table 6.2 Expected Values and Standard Deviations of PDFs for Fatalities

		VI		VII		VIII	
		x = 0	x = 1	x = 0	x = 1	x = 0	x = 1
CLASS A	Brick Residential	.00129	.000009	.016145	.00129	.10571	.016145
	Brick Storage	.00215	.000025	.02905	.00215	.20805	.02905
	RC $\leq$ 5	.00135	.000006	.02689	.00135	.20655	.02689
	RC > 5	.00150	.000006	.03779	.00150	.30745	.03779
CLASS B	Brick Residential	.000145	0	.001472	.000145	.019627	.001472
	Brick Storage	.00035	0	.00255	.00035	.033675	.00255
	Wooden	.000075	0	.00069	.000075	.007815	.00069
	RC $\leq$ 5	.00011	0	.001498	.00011	.030998	.001498
	RC > 5	.00011	0	.001648	.00011	.042498	.001648
CLASS C	Wooden	0	0	.00009	0	.000835	.00009
	RC $\leq$ 5	0	0	.000128	0	.001732	.000128
	RC > 5	0	0	.000128	0	.001732	.000128

Table 6.3 Expected Life Loss Ratios (ELLR)

x = fraction of buildings on good soil

ELLR = a - bx

CLASS		VI x 10^{-5}	VII x 10^{-4}	VIII x 10^{-3}
A	Brick Residence	44.5	63.4	46.6
	Brick Storage	74.75	113.0	89.9
	RC $\leq$ 5	46.3	100.3	87.97
	RC > 5	51.4	138.4	129.5
B	Brick Residence	4.93	5.96	7.64
	Brick Storage	11.9	10.98	13.1
	Wooden	2.55	2.84	3.08
	RC $\leq$ 5	3.74	5.82	11.5
	RC > 5	3.74	6.33	15.5
C	Wooden	0	0.306	0.343
	RC $\leq$ 5	0	0.435	0.673
	RC > 5	0	0.435	0.724

Table 6.4 Boston $ELLR_I$ 34% Bad Soil

CLASS		VI $\times 10^{-8}$	VII $\times 10^{-8}$	VIII $\times 10^{-8}$	EALLR $\times 10^{-8}$
A	Brick Residence	106.8	196.54	74.56	377.9
	Brick Storage	179.4	350.3	143.84	673.54
	RC $\leq$ 5	111.12	310.93	140.75	562.8
	RC > 5	123.36	429.04	207.2	759.6
B	Brick Residence	11.83	18.48	12.22	42.53
	Brick Storage	28.56	34.04	20.96	83.56
	Wooden	6.12	8.8	4.93	19.85
	RC $\leq$ 5	8.98	18.04	18.4	45.42
	RC > 5	8.98	19.62	24.8	53.4
C	Wooden	0	.949	.549	1.5
	RC $\leq$ 5	0	1.35	1.08	2.43
	RC > 5	0	1.35	1.16	2.51

Table 6.5 Boston EALLR 34% Bad Soil, Single Target

CLASS	Brick Residence	Brick Storage	Wooden	RC $\leq$ 5	RC > 5	TOTALS
A				652	163	815
B	30563	10187	31785	3260	815	76610
C			74165	2608	652	77425
TOTALS	30563	10187	105950	6520	1630	154850

Table 6.6a Boston Building Count - 0% Brick Class A

CLASS	Brick Residence	Brick Storage	Wooden	RC $\leq$ 5	RC > 5	TOTALS
A	6113	2037		652	163	815
B	24450	8150	31785	3260	815	76610
C			74165	2608	652	77425
TOTALS	30563	10187	105950	6520	1630	154850

Table 6.6b Boston Building Count - 20% Brick Class A

	Occupancy Factor	Number Buildings	Total Exposed
Brick Residence	4	30,563	122,252
Brick Storage	20	10,187	203,740
Wooden	2	105,950	211,900
RC $\leq$ 5	20	6,520	130,400
RC > 5	100	1,630	163,000
TOTALS		154,850	831,292

Table 6.7 Summary - Boston Assumptions

		VI	VII	VIII
A	RC $\leq$ 5	6	131	1147
	RC >5	8	226	2110
B	Brick Residence	6	73	935
	Brick Storage	24	224	2676
	Wooden	2	18	196
	RC $\leq$ 5	2	38	752
	RC >5	3	52	1266
C	Wooden	0	5	51
	RC $\leq$ 5	0	2	35
	RC > 5	0	3	47
TOTALS		51	772	9215

Table 6.8a Boston Expected Killed - 0% Class A Brick

		VI	VII	VIII
A	Brick Residence	11	155	1139
	Brick Storage	30	460	3663
	RC $\leq$ 5	6	131	1147
	RC > 5	8	226	2110
B	Brick Residence	5	58	748
	Brick Storage	19	179	2141
	Wooden	2	18	196
	RC $\leq$ 5	2	38	752
	RC > 5	3	52	1266
C	Wooden	0	5	51
	RC $\leq$ 5	0	2	35
	RC > 5	0	3	47
TOTALS		86	1327	13295

Table 6.8b Boston Expected Killed - 20% Class A Brick

		VI $\times 10^{-3}$	VII $\times 10^{-3}$	VIII $\times 10^{-3}$	TOTALS
A	RC ≤ 5	14	41	18	.073
	RC > 5	19	70	34	.123
B	Brick Residence	14	23	15	.052
	Brick Storage	58	69	43	.170
	Wooden	5	6	3	.014
	RC ≤ 5	5	12	12	.029
	RC > 5	7	16	20	.043
C	Wooden	0	2	1	.003
	RC ≤ 5	0	1	1	.002
	RC > 5	0	1	1	.002
	TOTALS	122	241	148	.511

Table 6.9a Boston Expected Annual Killed - 0% Class A Brick

		VI $\times 10^{-3}$	VII $\times 10^{-3}$	VIII $\times 10^{-3}$	TOTALS
A	Brick Residence	26	48	18	.092
	Brick Storage	72	143	59	.274
	RC ≤ 5	14	41	18	.073
	RC > 5	19	70	34	.123
B	Brick Residence	12	18	12	.042
	Brick Storage	46	55	34	.135
	Wooden	5	6	3	.014
	RC ≤ 5	5	12	12	.029
	RC > 5	7	16	20	.043
C	Wooden	0	2	1	.003
	RC ≤ 5	0	1	1	.002
	RC > 5	0	1	1	.002
	TOTALS	206	413	213	.832

Table 6.9b Boston Expected Annual Killed - 20% Class A Brick

		1	2	3	4
A	RC ≤ 5	202	52	274	124
	RC > 5	51	13	68	31
B	Brick Residence	9475	2445	12836	5807
	Brick Storage	3158	815	4279	1935
	Wooden	9853	2543	13350	6039
	RC ≤ 5	1011	261	1369	619
	RC > 5	253	65	342	155
C	Wooden	22991	5933	31149	14092
	RC ≤ 5	808	209	1095	496
	RC > 5	202	52	274	124

Table 6.10a Boston Building Count Per Cell - 0% Class A Brick

		1	2	3	4
A	Brick Residence	1895	489	2567	1162
	Brick Storage	631	163	856	387
	RC ≤ 5	202	52	274	124
	RC > 5	51	13	68	31
B	Brick Residence	7580	1956	10269	4645
	Brick Storage	2527	652	3423	1548
	Wooden	9853	2543	13350	6039
	RC ≤ 5	1011	261	1369	619
	RC > 5	253	65	342	155
C	Wooden	22991	5933	31149	14092
	RC ≤ 5	808	209	1095	496
	RC > 5	202	52	274	124

Table 6.10b Boston Building Count Per Cell - 20% Class A Brick

CELL	= VI	= VII	= VIII
1	2.2812 x 10-3	2.2895×10^{-4}	1.6591×10^{-6}
2	2.3891×10^{-3}	2.9532×10^{-4}	9.9545×10^{-6}
3	2.3393×10^{-3}	3.1854×10^{-4}	1.6591×10^{-5}
4	2.4455×10^{-3}	3.8823×10^{-4}	3.65×10^{-5}

Table 6.11 Seismic Risk for Each of 4 Cells of Boston

		VI				VII				VIII			
		1	2	3	4	1	2	3	4	1	2	3	4
A	RC $\leq$ 5	1.87	.48	2.54	1.15	40.5	10.4	55.	24.9	335.4	91.5	482.1	218.2
	RC > 5	2.62	.67	3.49	1.59	70.6	18.	94.1	42.9	660.3	168.3	880.4	401.4
B	Brick Residence	1.87	.48	2.53	1.15	22.6	5.8	30.6	13.8	289.7	74.8	392.5	177.6
	Brick Storage	7.52	1.94	10.2	4.61	69.4	17.9	94.	42.5	829.4	214.1	1123.9	508.2
	Wooden	.50	.13	.68	.31	5.6	1.4	7.6	3.4	60.7	15.7	82.2	37.2
	RC $\leq$ 5	.76	.2	1.02	.46	11.8	3.	15.9	7.2	233.1	60.2	315.6	142.7
	RC > 5	.95	.24	1.28	.58	16.	4.1	21.7	9.8	393.1	101.	531.4	240.8
C	Wooden	0	0	0	0	1.4	.4	1.9	.9	15.8	4.1	21.4	9.7
	RC $\leq$ 5	0	0	0	0	.7	.2	1.	.4	10.9	2.8	14.7	6.7
	RC > 5	0	0	0	0	.9	.2	1.2	.5	14.6	3.8	19.9	9.
	TOTALS	16.1	4.1	21.7	9.9	239.5	61.4	323	146.3	2863	736.3	3864.1	1751.5
		52				770				9215			

Table 6.12a Boston Expected Life Loss Per Cell - 0% Class A Brick

		VI				VII				VIII			
		1	2	3	4	1	2	3	4	1	2	3	4
A	Brick Residence	3.4	.9	4.6	2.1	48.1	12.4	65.1	29.5	353.2	91.1	478.5	216.6
	Brick Storage	9.4	2.4	12.8	5.8	142.6	36.8	193.4	87.4	1134.7	293.1	1539.3	695.9
	RC $\leq$ 5	1.9	.5	2.5	1.2	40.5	10.4	55.	24.9	355.4	91.5	482.1	218.2
	RC > 5	2.6	.7	3.5	1.6	70.6	18.	94.1	42.9	660.3	168.3	880.4	401.4
B	Brick Residence	1.5	.4	2.	.9	18.1	4.7	24.5	11.1	231.8	59.8	314.	142.
	Brick Storage	6.	1.6	8.1	3.7	55.5	14.3	75.2	34.	663.7	171.2	899.1	406.6
	Wooden	.5	.1	.7	.3	5.6	1.4	7.6	3.4	60.7	15.7	82.2	37.2
	RC $\leq$ 5	.8	.2	1.	.5	11.8	3.	15.9	7.2	233.1	60.2	315.6	142.7
	RC > 5	1.	.2	1.3	.6	16.	4.1	21.7	9.8	393.1	101.	531.4	240.8
C	Wooden	0	0	0	0	1.4	.4	1.9	.9	15.8	4.1	21.4	9.7
	RC $\leq$ 5	0	0	0	0	.7	.2	1.	.4	10.9	2.8	14.7	6.7
	RC > 5	0	0	0	0	.9	.2	1.2	.5	14.6	3.8	19.9	9.
	TOTALS	27.1	7	36.5	16.7	411.8	105.9	556.6	252.	4127.3	1062.6	5578.6	2526.8
		87				1326				13295			

Table 6.12b Boston Expected Life Loss by Cell - 20% Class A Brick

		VI x 10^{-3}				VII x 10^{-3}				VIII x 10^{-3}				TOTAL x 10^{-3}
		1	2	3	4	1	2	3	4	1	2	3	4	
A	RC $\leq$ 5	4.3	1.1	5.9	2.8	9.3	3.1	17.5	9.7	.6	.9	8.0	8.0	71
	RC > 5	6.	1.6	8.2	3.9	16.2	5.3	30.	16.7	1.1	1.7	14.6	14.7	120
B	Brick Residence	4.3	1.1	5.9	2.8	5.2	1.7	9.7	5.4	.5	.7	6.5	6.5	50
	Brick Storage	17.2	4.6	23.9	11.3	15.9	5.3	29.9	16.5	1.4	2.1	18.6	18.5	165
	Wooden	1.1	.3	1.6	.8	1.3	.4	2.4	1.3	.1	.2	1.4	1.4	12
	RC $\leq$ 5	1.7	.5	2.4	1.1	2.7	.9	5.1	2.8	.4	.6	5.2	5.2	29
	RC > 5	2.2	.6	3.	1.4	3.7	1.2	6.9	3.8	.7	1.0	8.8	8.8	42
C	Wooden	0	0	0	0	.3	.1	.6	.3	.03	.04	.4	.4	2
	RC $\leq$ 5	0	0	0	0	.2	.1	.3	.2	.02	.03	.2	.2	1
	RC > 5	0	0	0	0	.2	.1	.4	.2	.02	.04	.3	.3	2
	TOTALS	36.7	9.8	50.8	24.2	54.8	18.1	102.9	56.8	4.8	7.3	64.1	63.9	
		122				233				140				494

Table 6.13a Boston Expected Annual Life Loss By Cell - 0% Class A Brick

		VI x 10^{-3}				VII x 10^{-3}				VIII x 10^{-3}				TOTAL
		1	2	3	4	1	2	3	4	1	2	3	4	x 10^{-3}
A	Brick Residence	7.8	2.2	10.8	5.1	11.	3.7	20.7	11.5	.6	.9	7.9	7.9	90
	Brick Storage	21.4	5.7	29.9	14.2	32.6	10.9	61.6	33.9	1.9	2.9	25.5	25.4	266
	RC $\leq$ 5	4.3	1.1	5.9	2.8	9.3	3.1	17.5	9.7	.6	.9	8.	8.	71
	RC > 5	6.	1.6	8.2	3.9	16.2	5.3	30.	16.7	1.1	1.7	14.6	14.7	120
B	Brick Residence	3.4	1.	4.7	2.2	4.1	1.4	7.8	4.3	.4	.6	5.2	5.2	40
	Brick Storage	13.7	3.8	18.9	9.	12.7	4.2	24.	13.2	1.1	1.7	14.9	14.8	132
	Wooden	1.1	.3	1.6	.8	1.3	.4	2.4	1.3	.1	.2	1.4	1.4	12
	RC $\leq$ 5	1.7	.5	2.4	1.1	2.7	.9	5.1	2.8	.4	.6	5.2	5.2	29
	RC > 5	2.2	.6	3.	1.4	3.7	1.2	6.9	3.8	.7	1.	8.8	8.8	42
C	Wooden	0	0	0	0	.3	.1	.6	.3	.03	.04	.4	.4	2
	RC $\leq$ 5	0	0	0	0	.2	.1	.3	.2	.02	.03	.2	.2	1
	RC > 5	0	0	0	0	.2	.1	.4	.2	.02	.04	.3	.3	2
	TOTALS	61.6	16.8	85.4	40.5	94.3	31.4	177.3	97.9	7.	10.6	92.4	92.3	
		204				401				202				807

Table 6.13b Boston Expected Annual Life Loss By Cell - 20% Class A Brick

	1 - 10	11 - 50	51 - 100	101 - 500	501 - 1,000	1,001 - 5,000	5,001 - 10,000	>10,000
0% Class A Brick	1.59×10^{-4}	6.67×10^{-4}	1.85×10^{-3}	1.76×10^{-4}	2.27×10^{-4}	2.32×10^{-5}	1.49×10^{-5}	0
20% Class A Brick	4.48×10^{-5}	4.2×10^{-4}	2.21×10^{-3}	1.05×10^{-4}	1.08×10^{-4}	2.12×10^{-4}	1.49×10^{-5}	1.66×10^{-6}

Table 6.14 Annual Risk of Range of Life Loss in Boston Based on Expected Life Loss in Each Cell

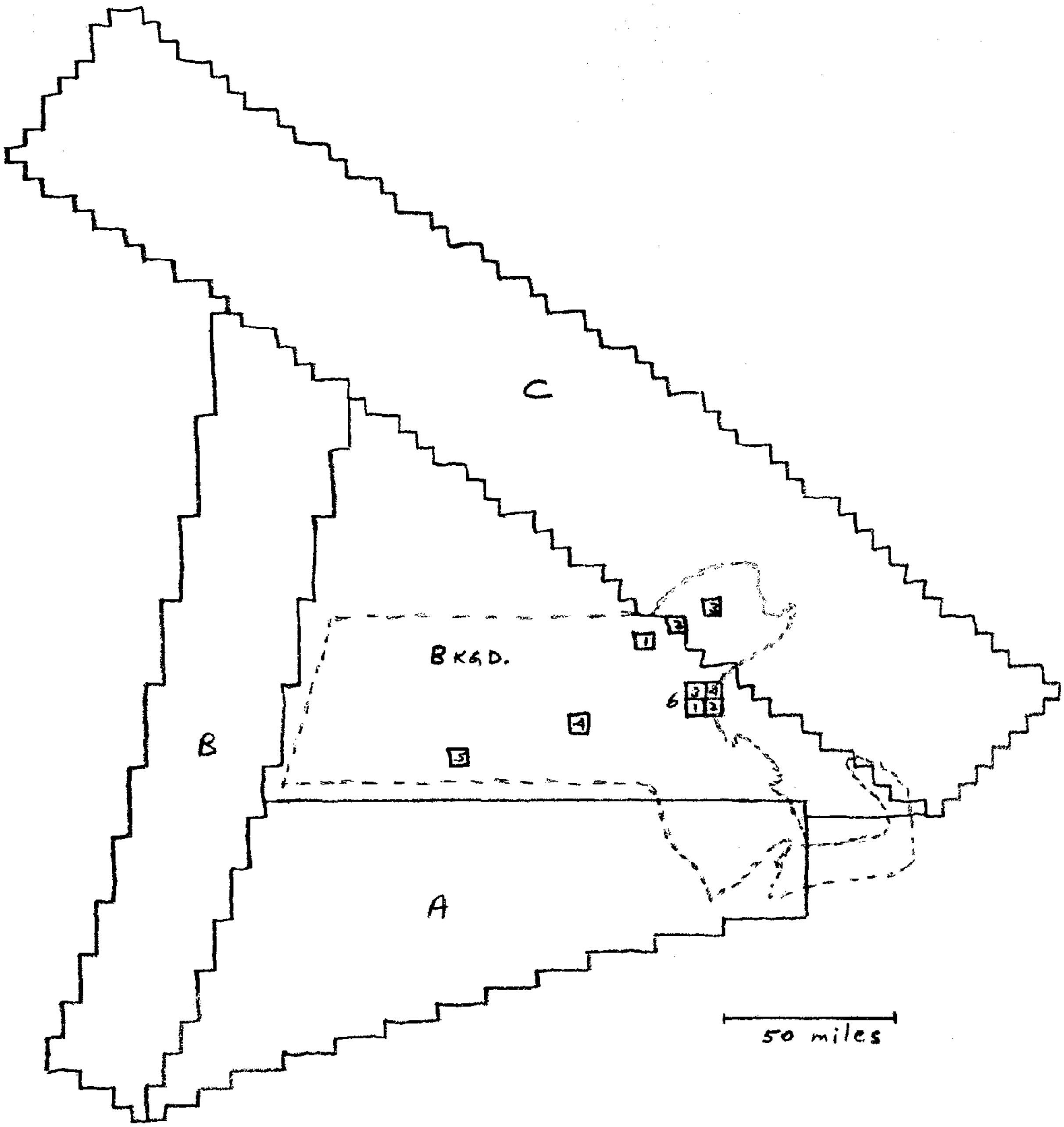
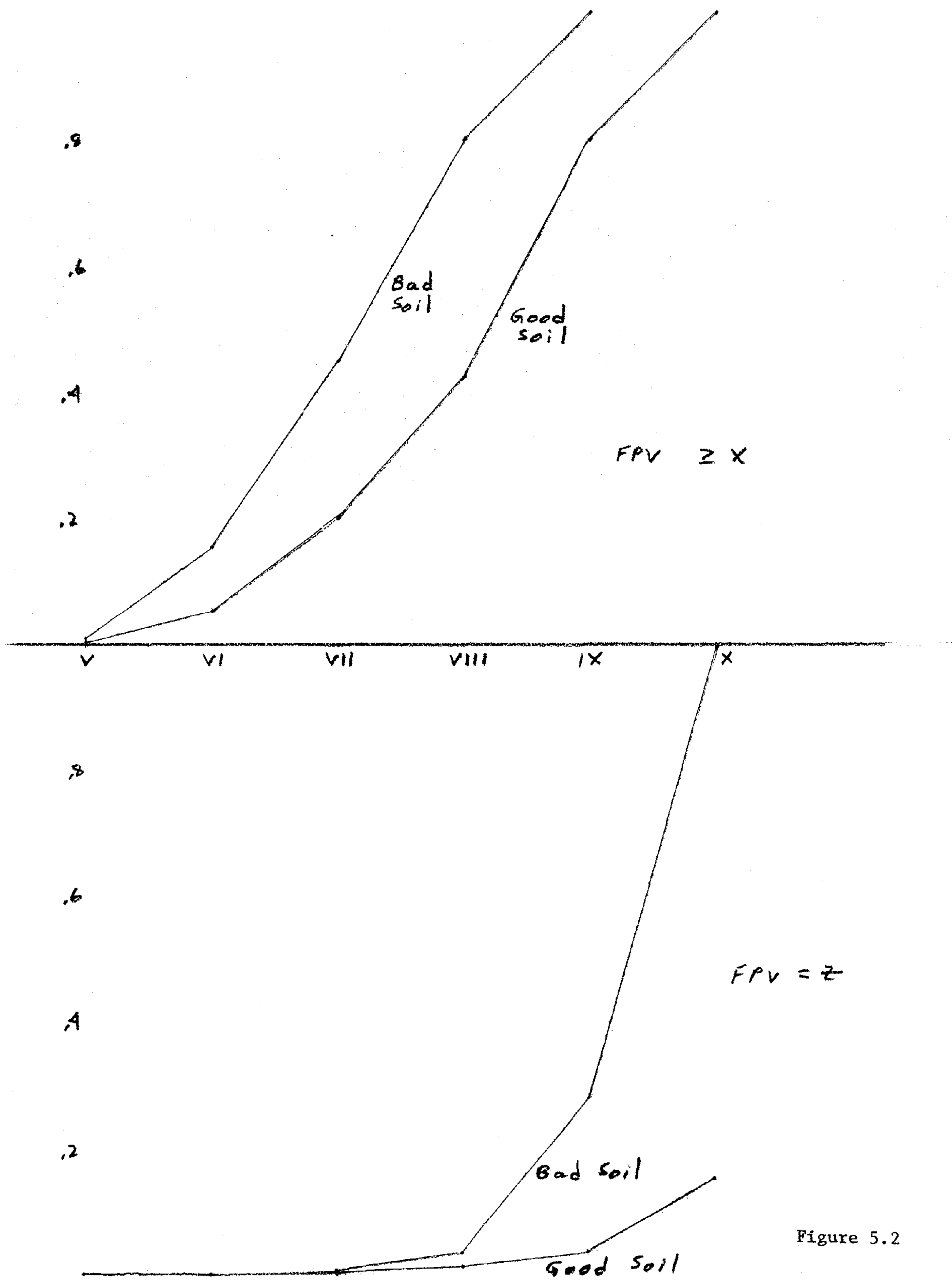


Figure 5.1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Good soil
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V
VI
VII
VIII
IX
X
.8
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.4
.2
FPV = Z
Bad Soil
Good Soil

Figure 5.2

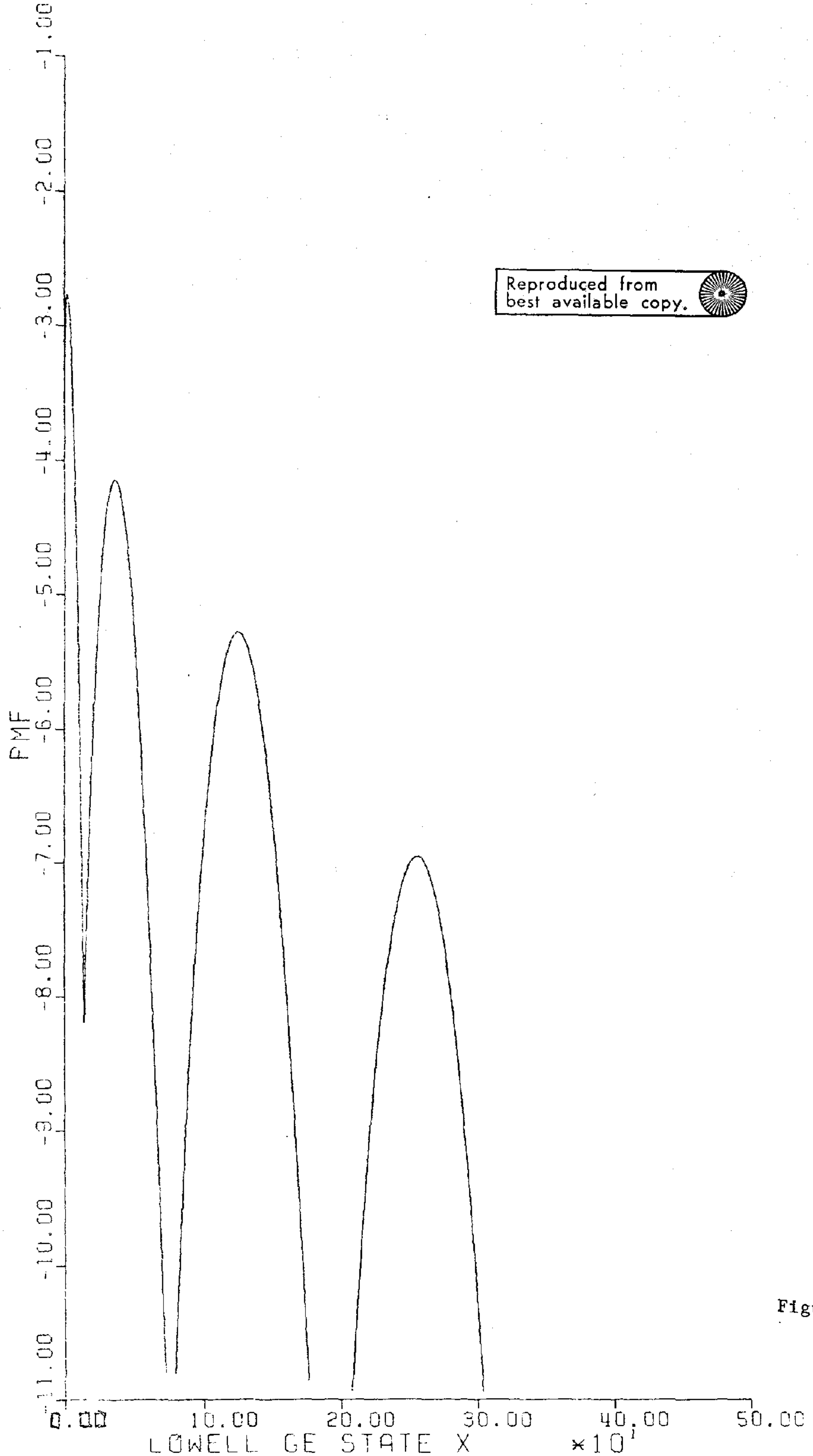
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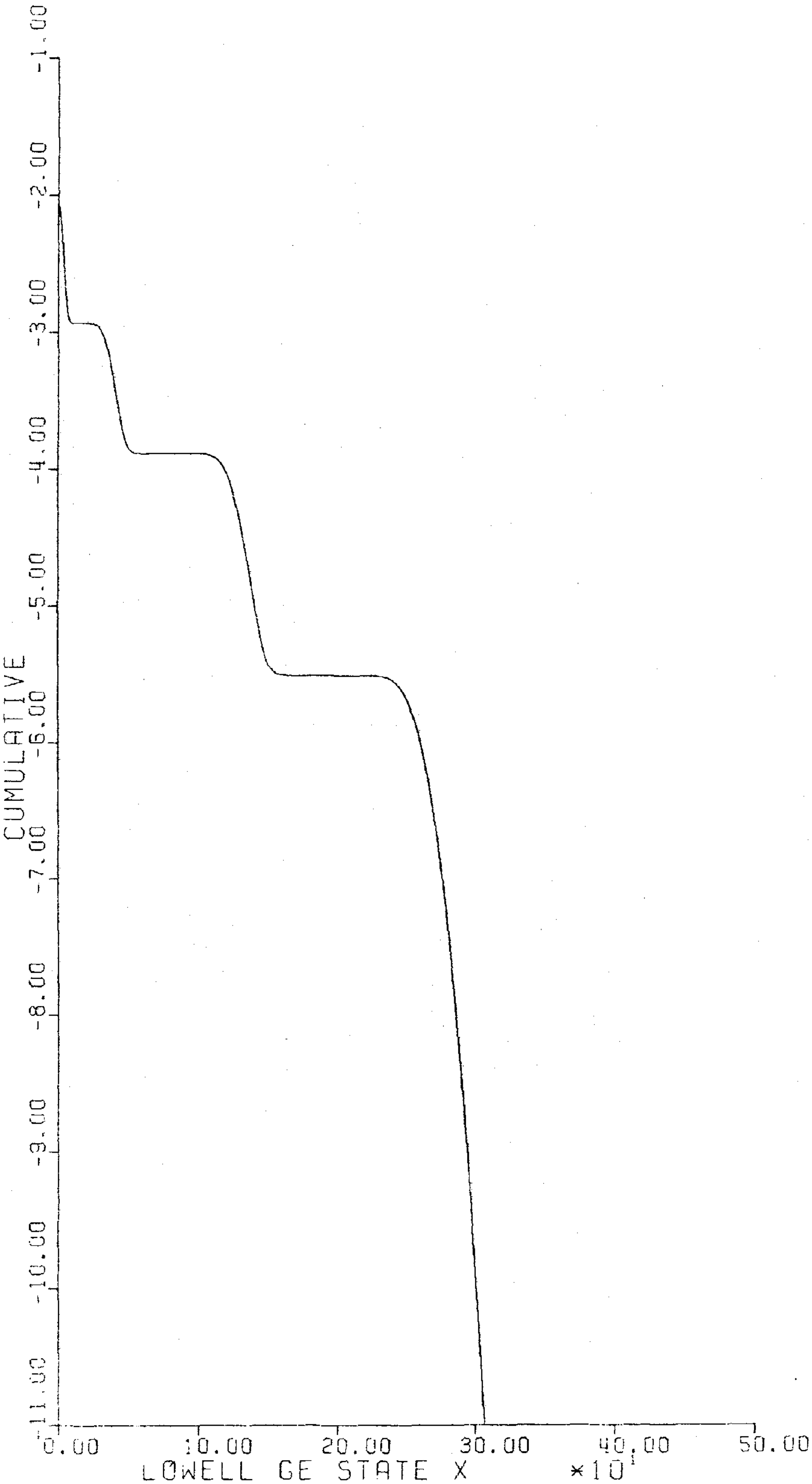
Figure 5.3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LOWELL GE STATE X
*10^1

Figure 5.4

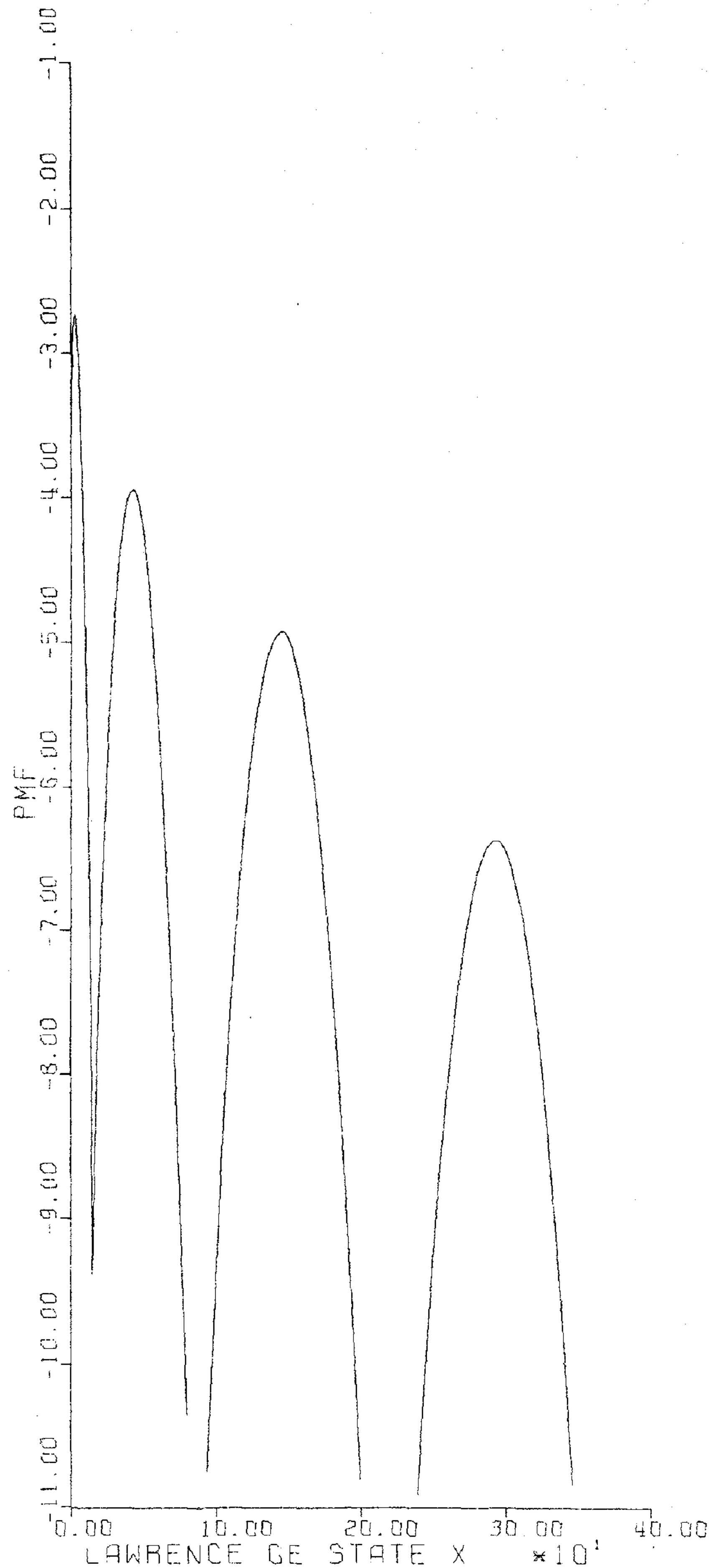
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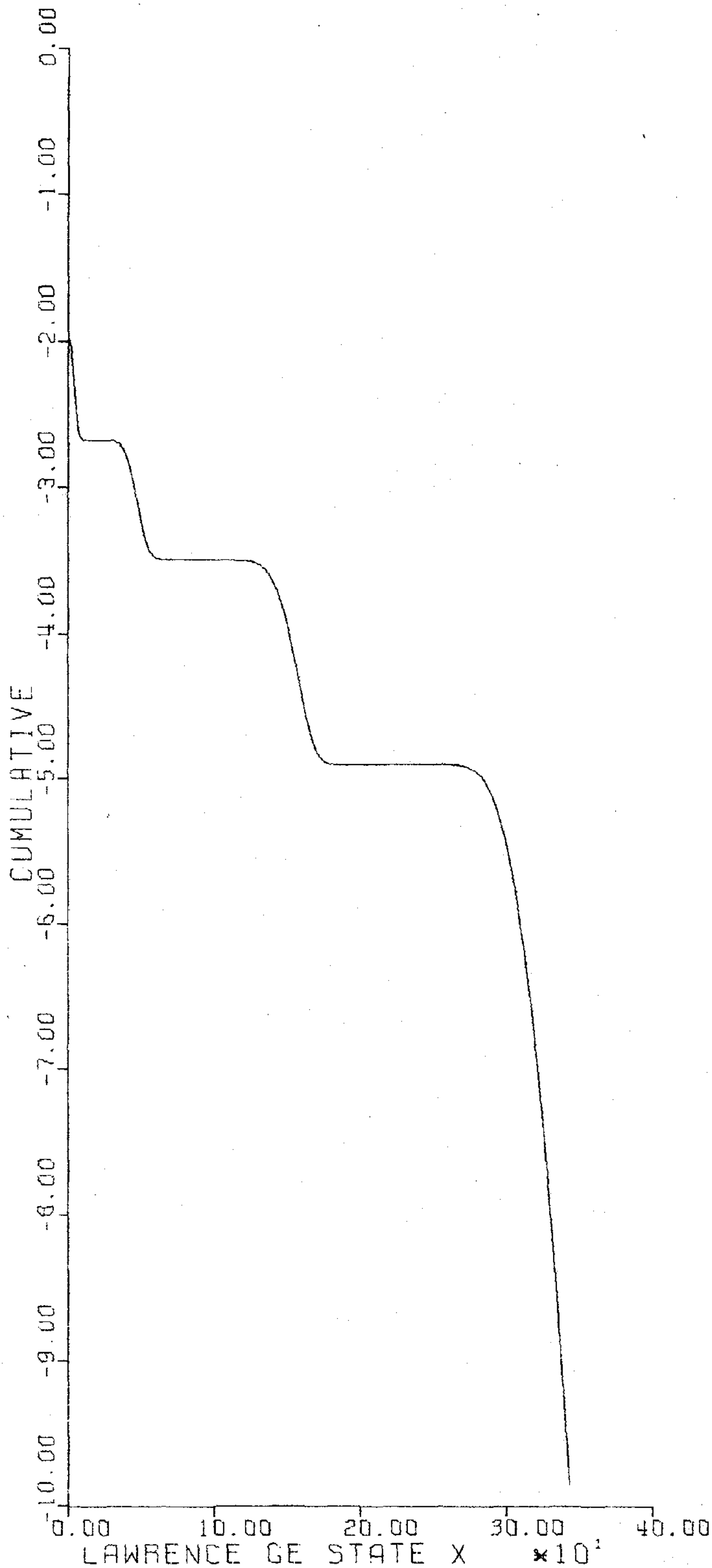
Figure 5.5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10.00
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30.00
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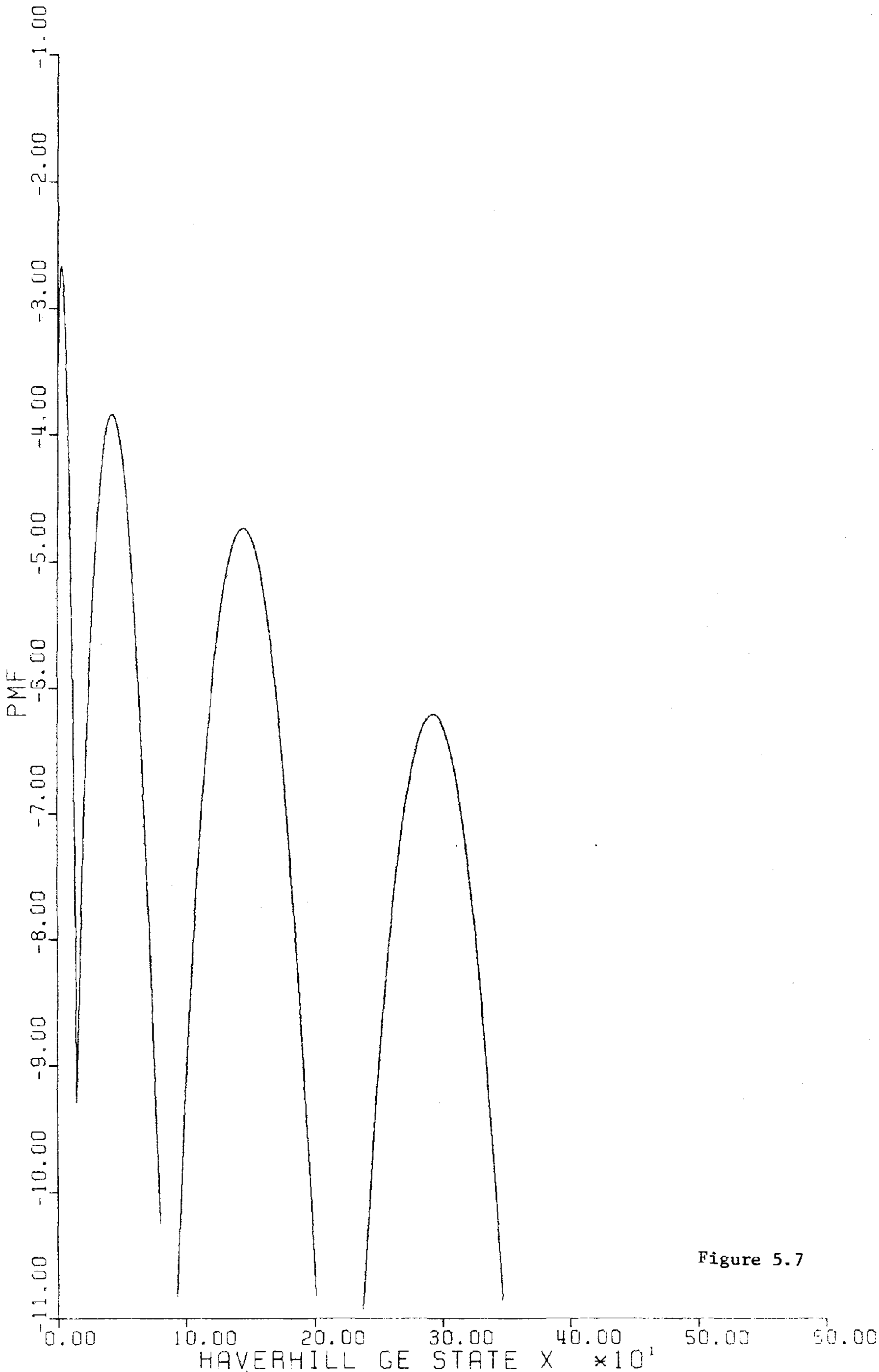
LAWRENCE GE STATE X ×10¹

Figure 5.6



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20.00
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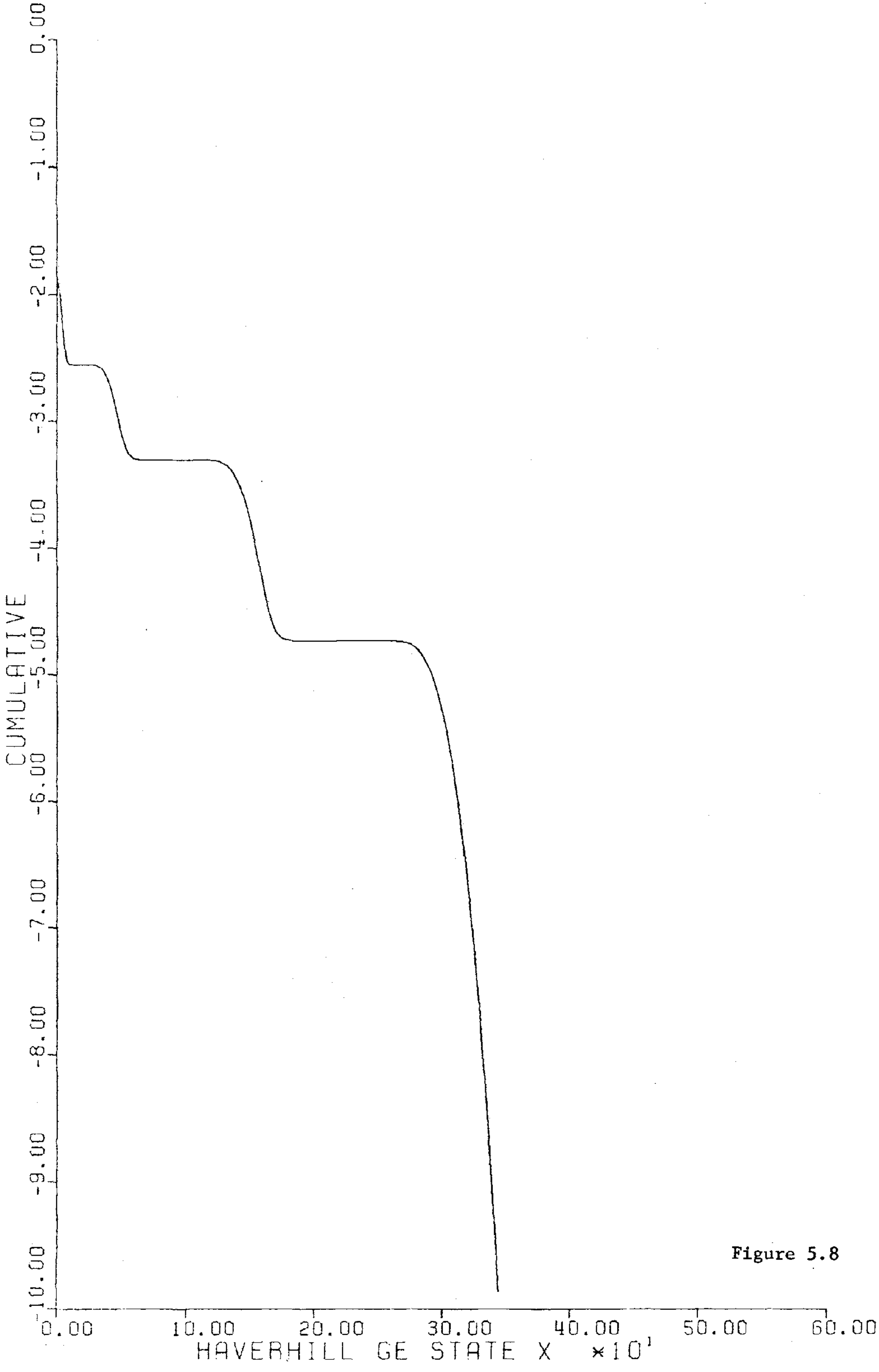
50.00
60.00
HAVERHILL GE STATE X ×10¹

Figure 5.7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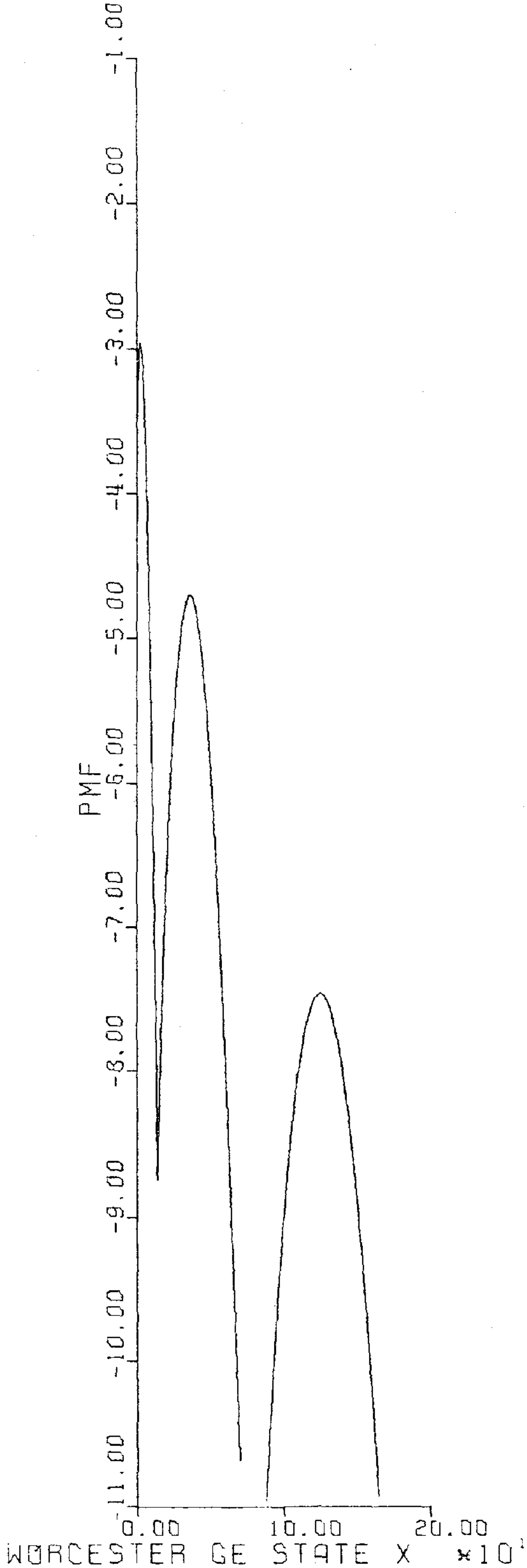
50.00
60.00
HAVERHILL GE STATE X ×10¹

Figure 5.8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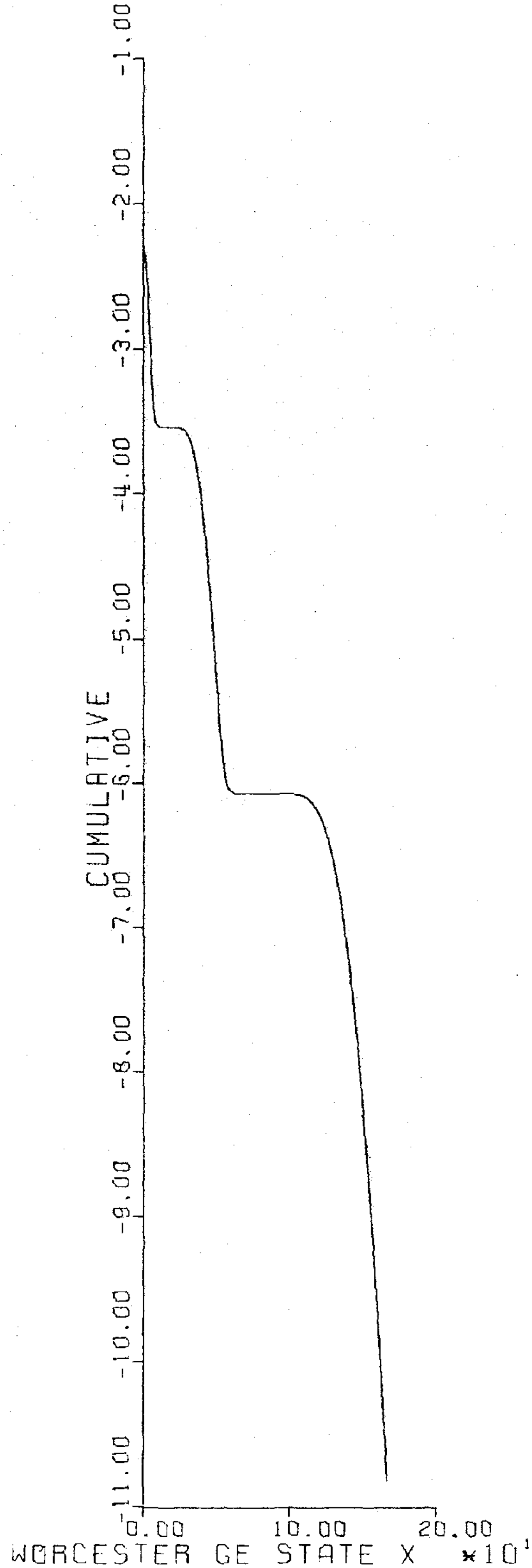
WORCESTER GE STATE X ×10¹

Figure 5.9



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-7.00
-8.00
-9.00
-10.00
-11.00
CUMULATIVE
0.00
10.00
20.00
WORCESTER GE STATE X ×10¹

Figure 5.10

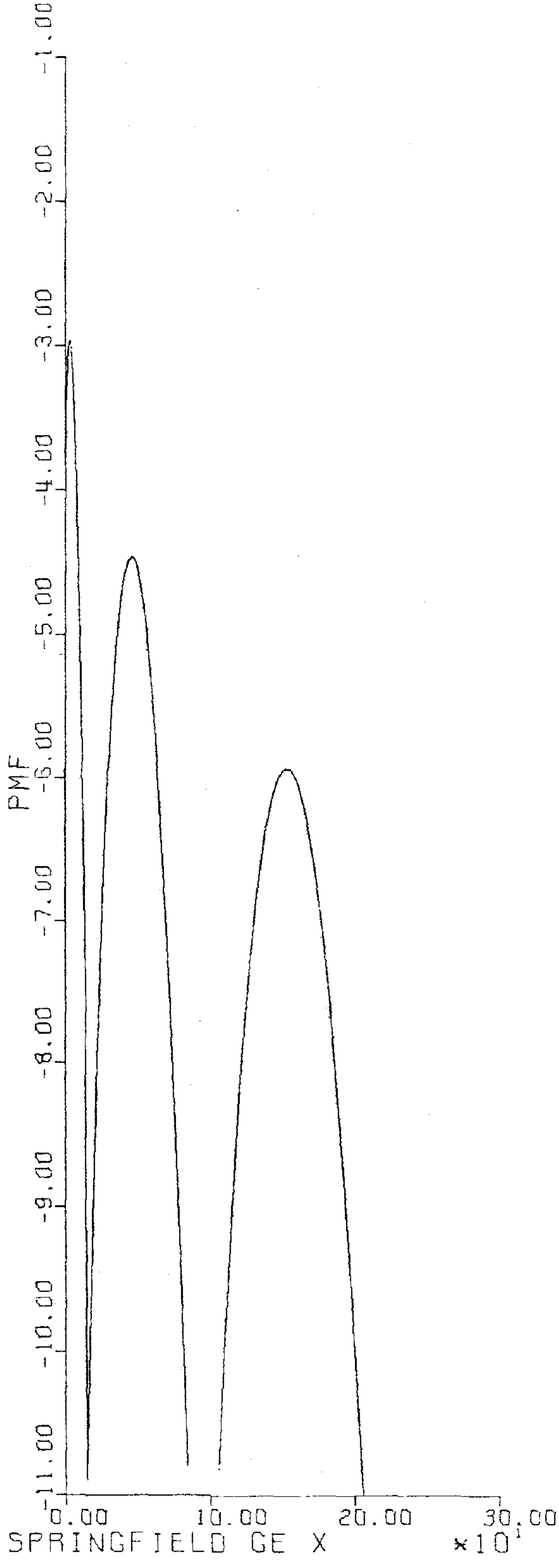

-1.00
-2.00
-3.00
-4.00
-5.00
-6.00
-7.00
-8.00
-9.00
-10.00
-11.00
PMF
0.00
10.00
20.00
30.00
SPRINGFIELD GE X
×10^1

Figure 5.11

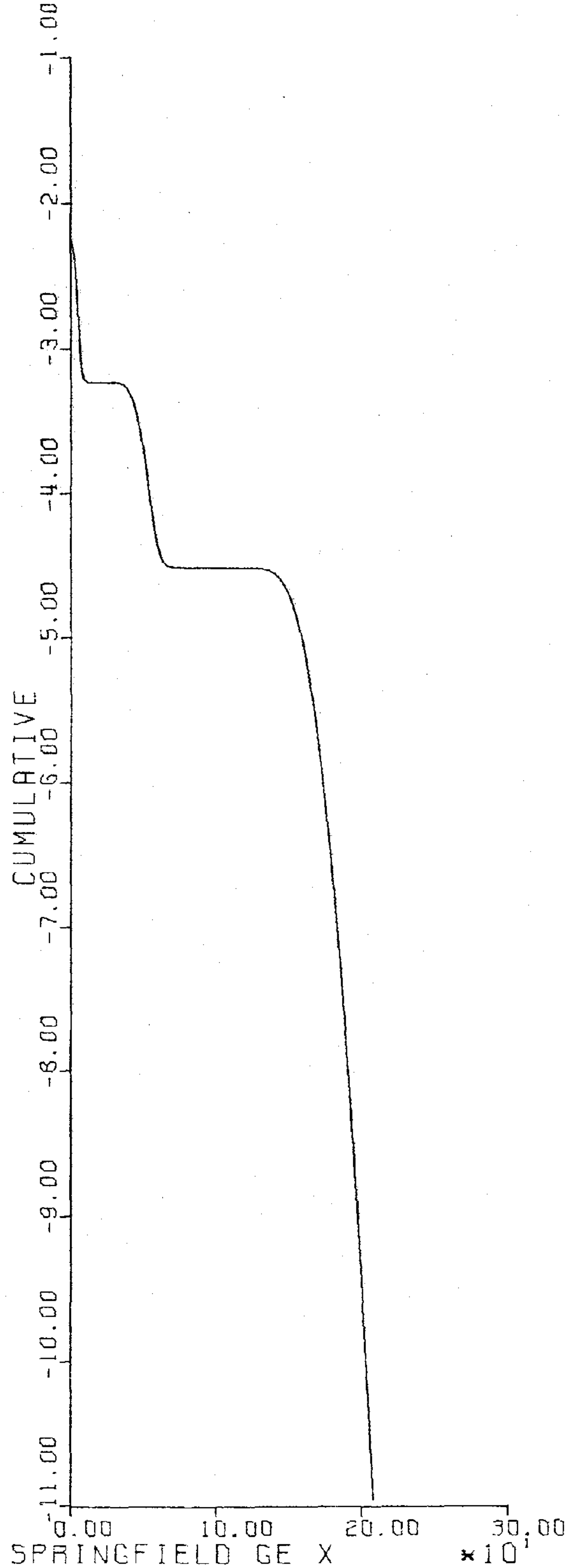

-1.00
-2.00
-3.00
-4.00
-5.00
-6.00
-7.00
-8.00
-9.00
-10.00
-11.00
CUMULATIVE
0.00
10.00
20.00
30.00
SPRINGFIELD GE X
×10^1

Figure 5.12

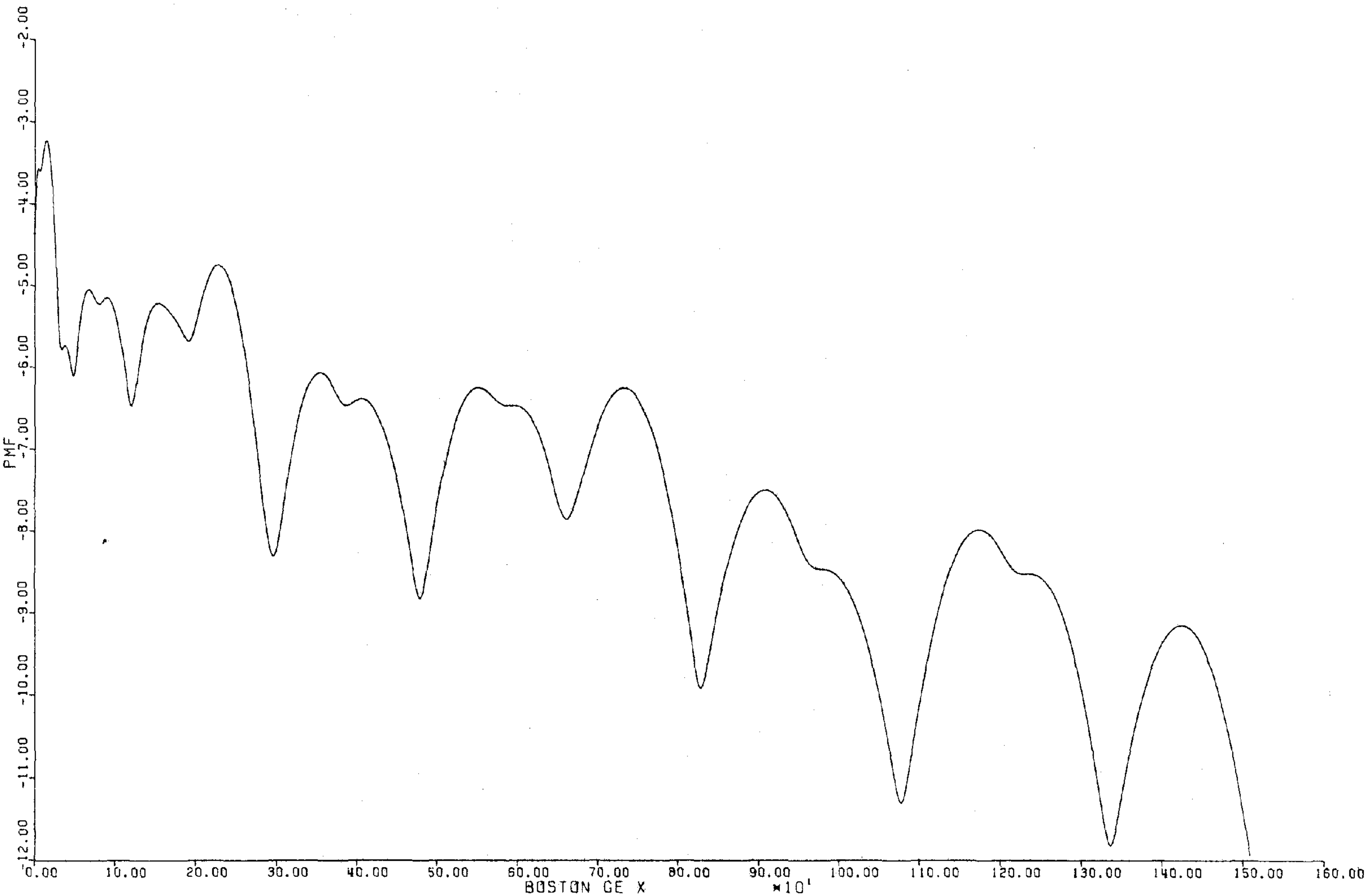

PMF
-2.00
-3.00
-4.00
-5.00
-6.00
-7.00
-8.00
-9.00
-10.00
-11.00
-12.00
0.00
10.00
20.00
30.00
40.00
50.00
60.00
70.00
80.00
90.00
100.00
110.00
120.00
130.00
140.00
150.00
160.00
BOSTON GE X
*10'

Figure 5.13

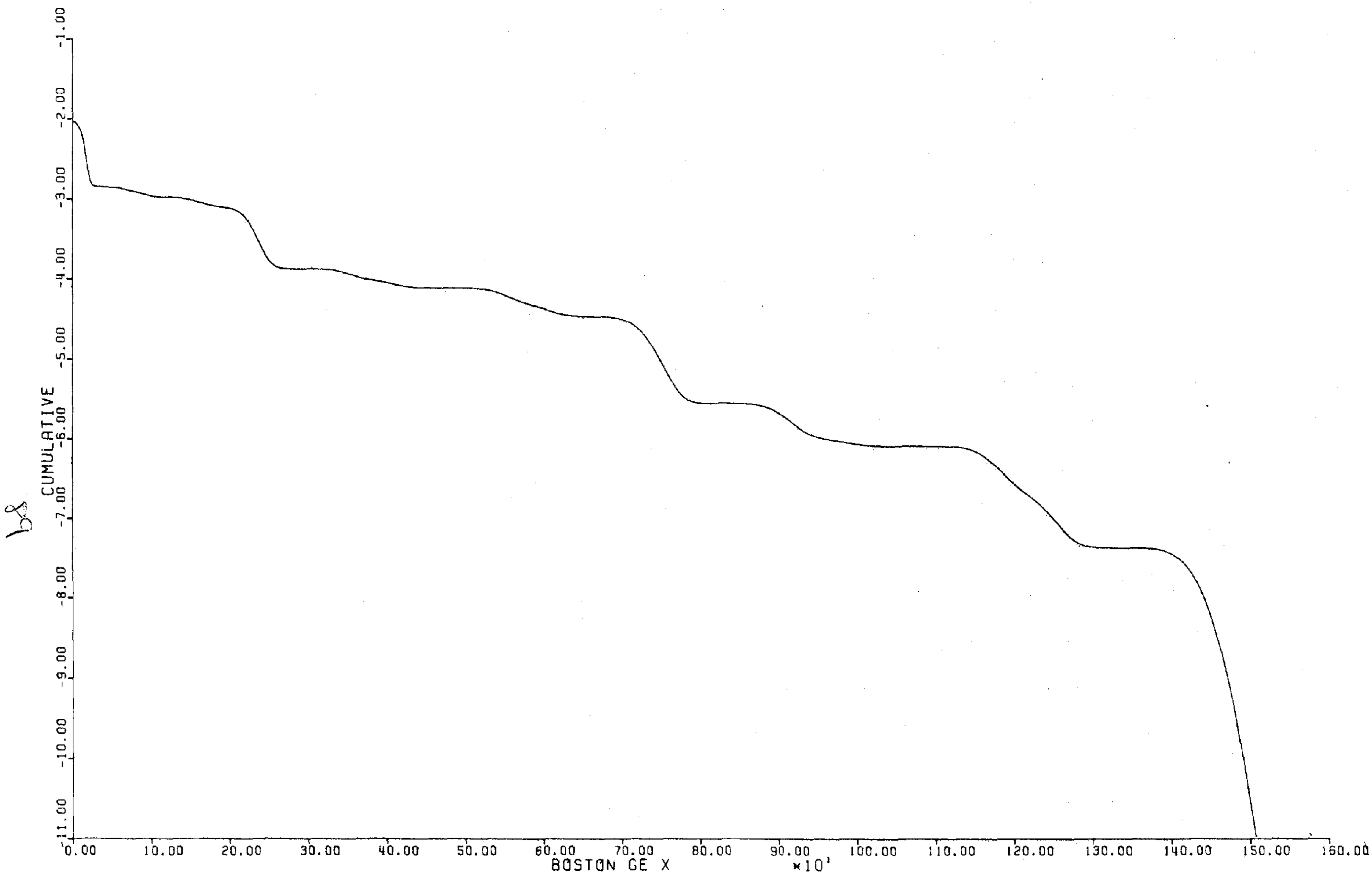

-1.00
-2.00
-3.00
-4.00
-5.00
-6.00
-7.00
-8.00
-9.00
-10.00
-11.00
CUMULATIVE
0.00
10.00
20.00
30.00
40.00
50.00
60.00
70.00
80.00
90.00
100.00
110.00
120.00
130.00
140.00
150.00
160.00
BOSTON GE X
×10¹

Figure 5.14

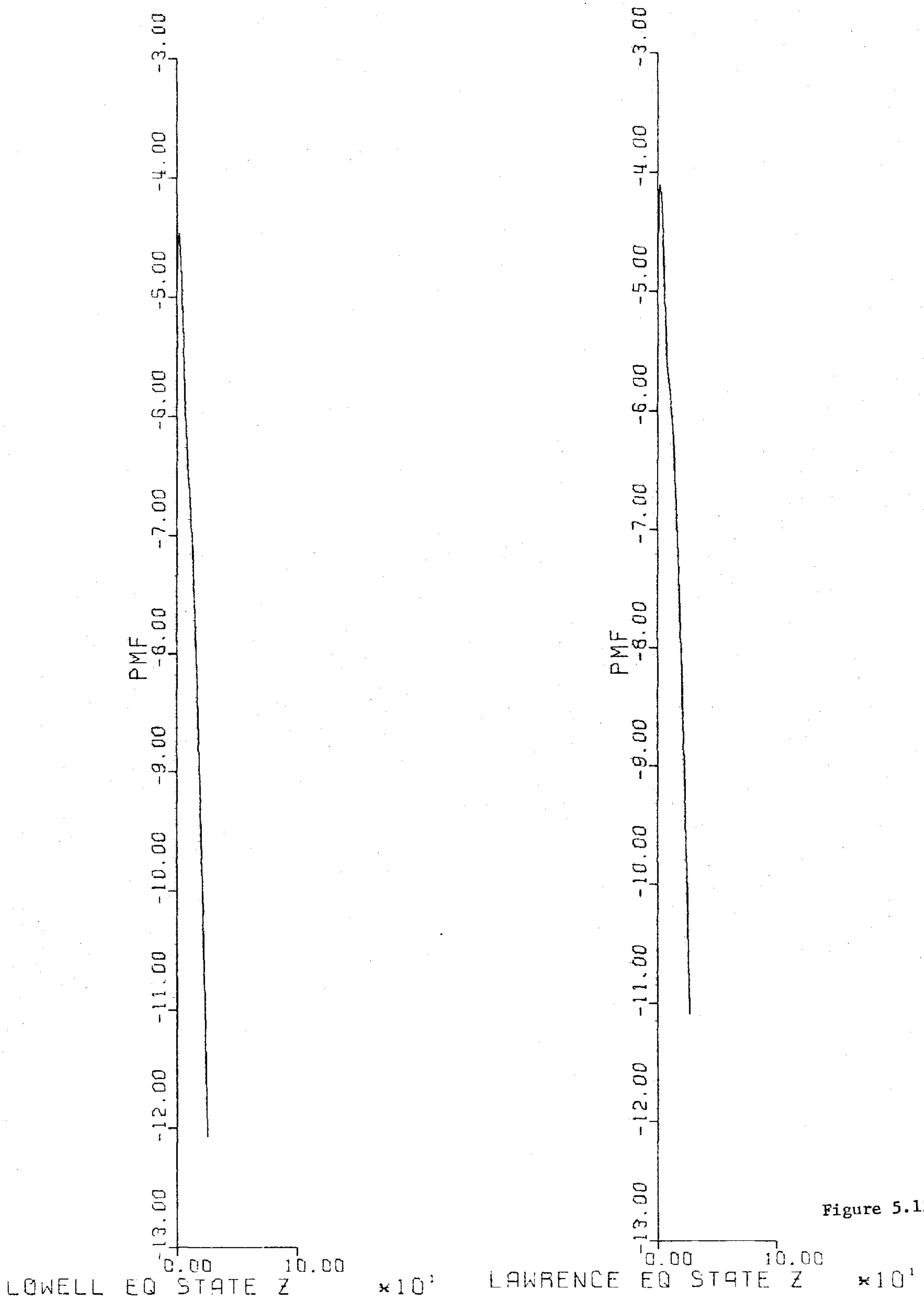
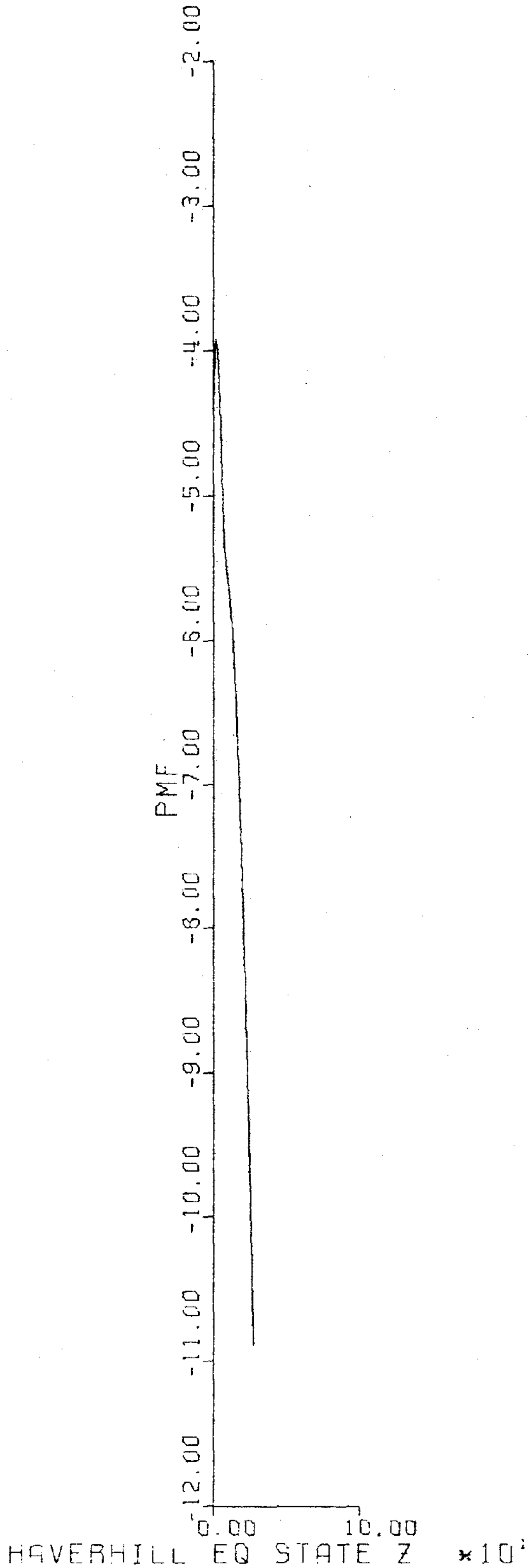
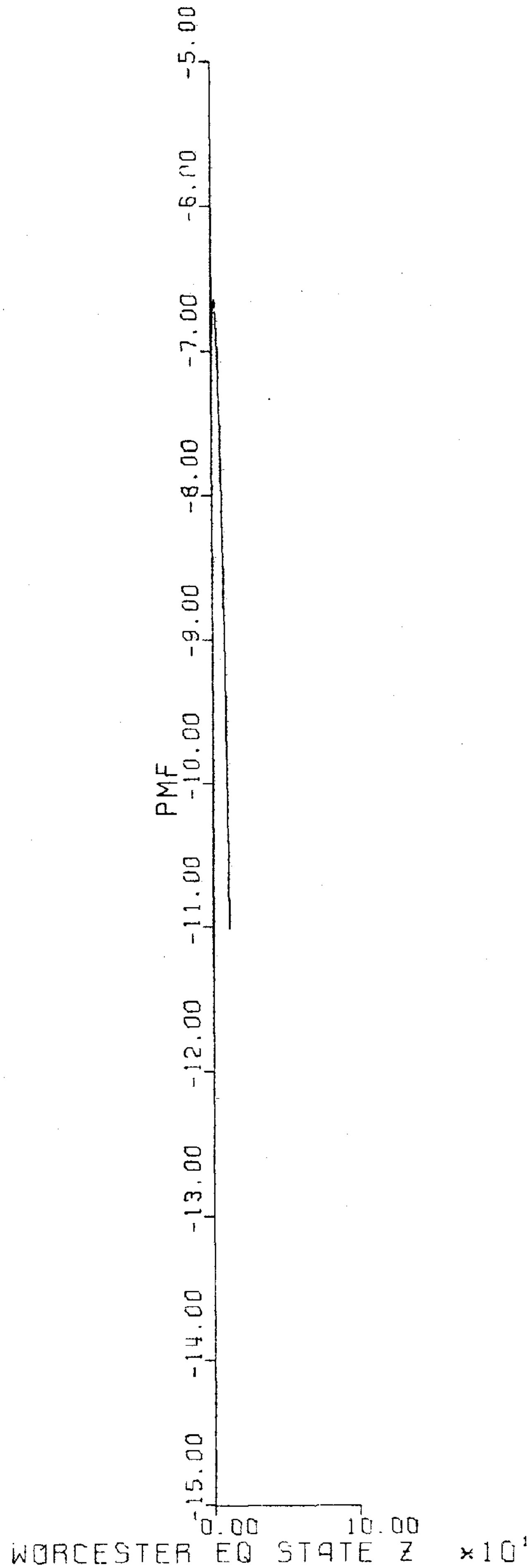
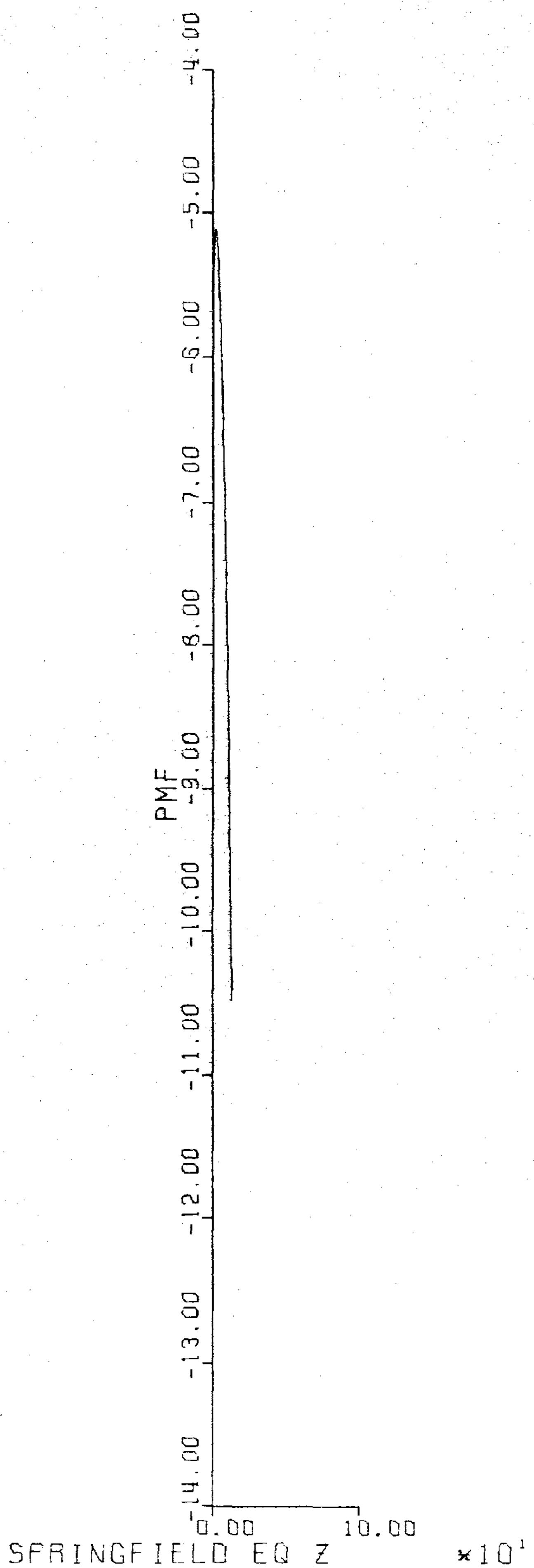
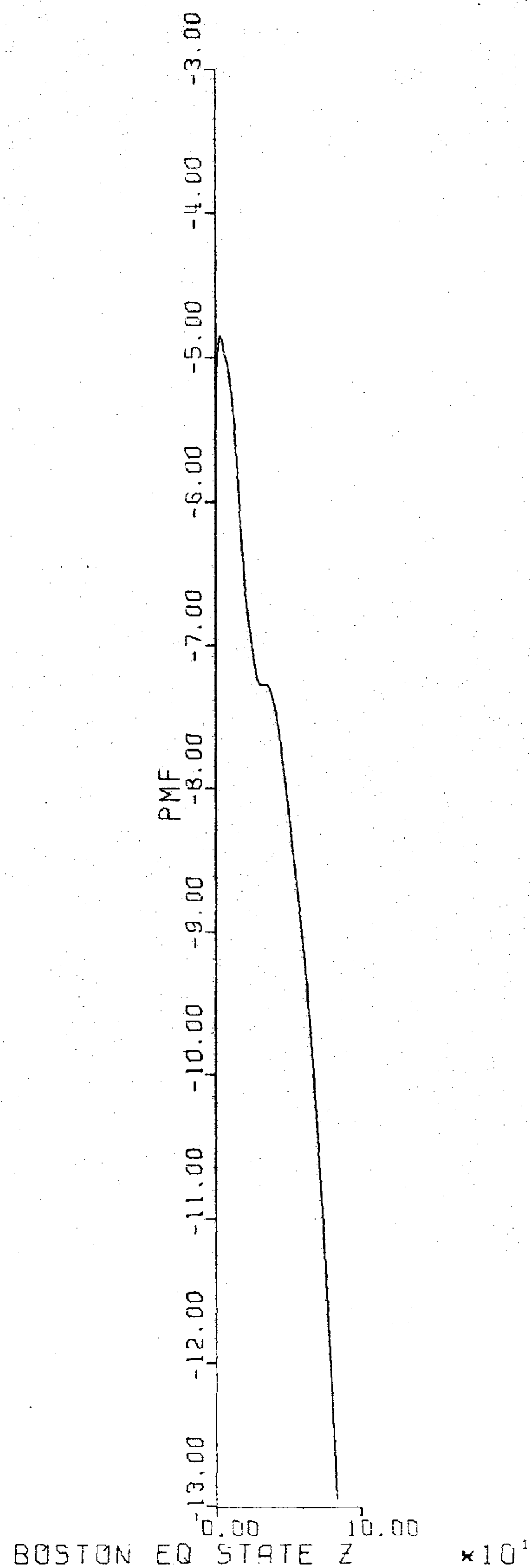

PMF
-3.00
-4.00
-5.00
-6.00
-7.00
-8.00
-9.00
-10.00
-11.00
-12.00
-13.00
0.00
10.00
LOWELL EQ STATE Z
×10¹
PMF
-3.00
-4.00
-5.00
-6.00
-7.00
-8.00
-9.00
-10.00
-11.00
-12.00
-13.00
0.00
10.00
LAWRENCE EQ STATE Z
×10¹

Figure 5.15


-2.00
-3.00
-4.00
-5.00
-6.00
-7.00
-8.00
-9.00
-10.00
-11.00
-12.00
PMF
0.00
10.00
HAVERHILL EQ STATE Z ×10^1


-5.00
-6.00
-7.00
-8.00
-9.00
-10.00
-11.00
-12.00
-13.00
-14.00
-15.00
PMF
0.00
10.00
WORCESTER EQ STATE Z ×10^1


-4.00
-5.00
-6.00
-7.00
-8.00
-9.00
PMF
-10.00
-11.00
-12.00
-13.00
-14.00
0.00
10.00
SPRINGFIELD EQ Z
×10¹


-3.00
-4.00
-5.00
-6.00
-7.00
-8.00
PMF
-9.00
-10.00
-11.00
-12.00
-13.00
0.00
10.00
BOSTON EQ STATE Z
×10¹

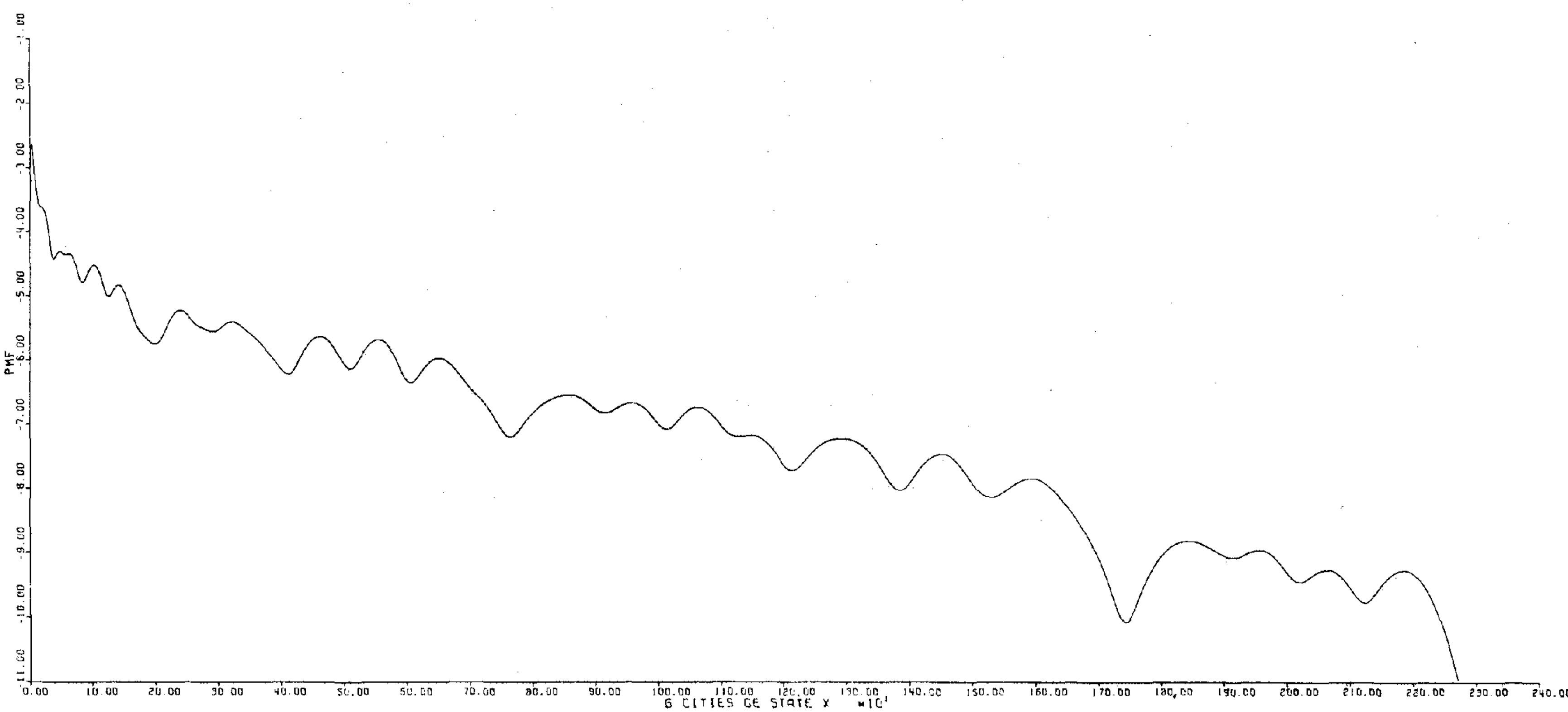


Figure 5.16

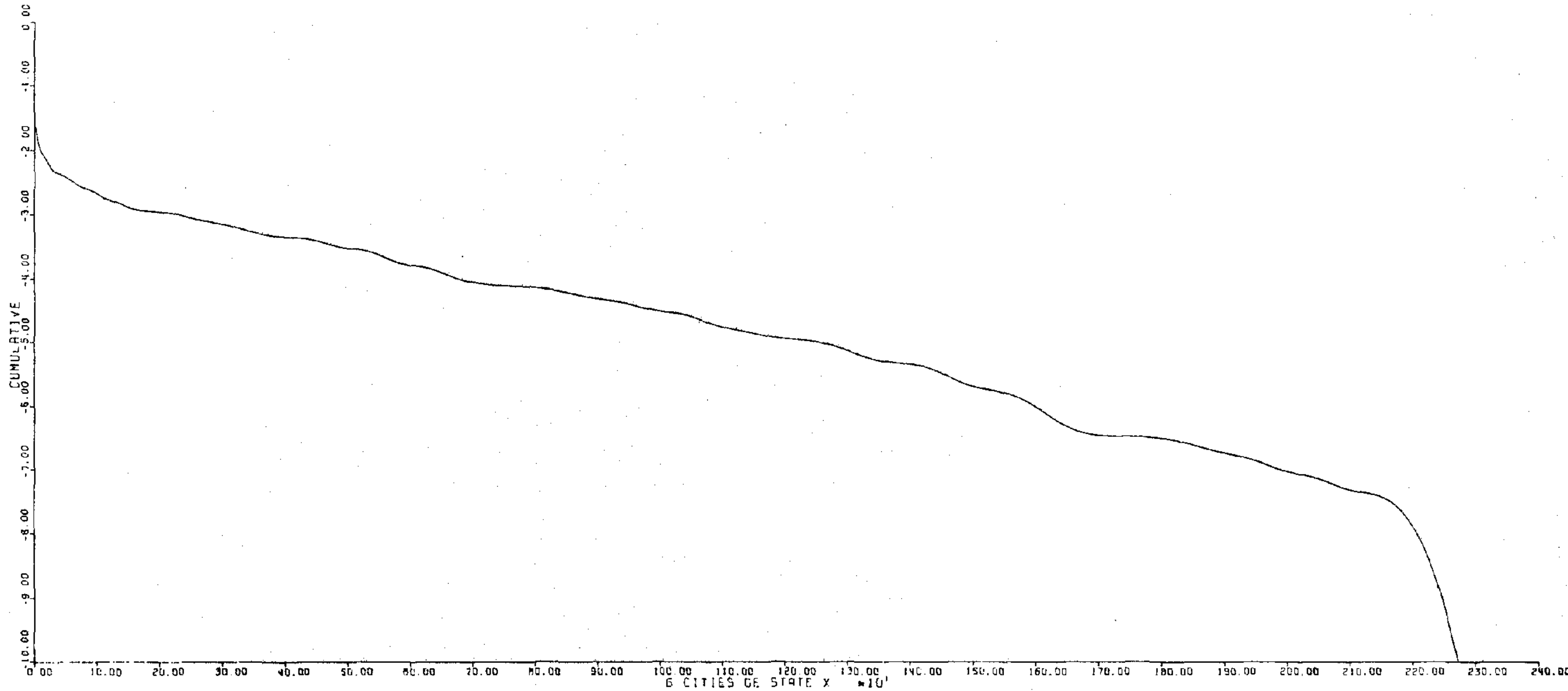
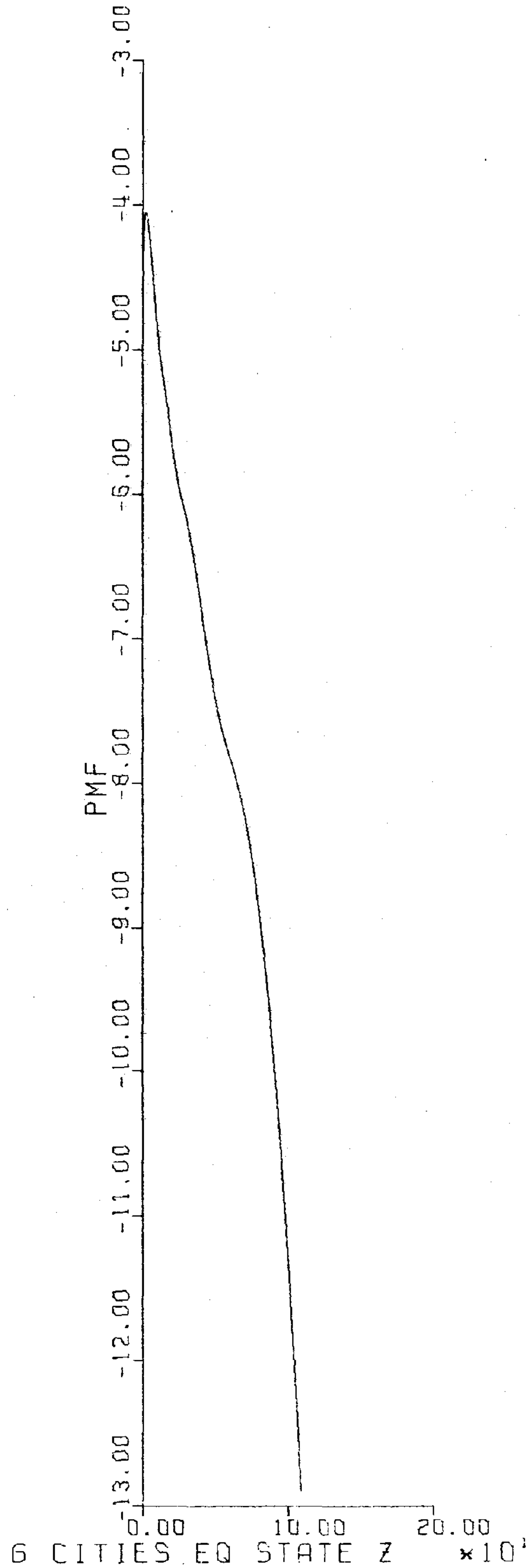

CUMULATIVE
G CITIES OF STATE X *10'
0.00
-1.00
-2.00
-3.00
-4.00
-5.00
-6.00
-7.00
-8.00
-9.00
-10.00
10.00
20.00
30.00
40.00
50.00
60.00
70.00
80.00
90.00
100.00
110.00
120.00
130.00
140.00
150.00
160.00
170.00
180.00
190.00
200.00
210.00
220.00
230.00
240.00

Figure 5.17



PMF
-3.00
-4.00
-5.00
-6.00
-7.00
-8.00
-9.00
-10.00
-11.00
-12.00
-13.00
0.00
10.00
20.00
6 CITIES EQ STATE Z ×10¹

Figure 5.18

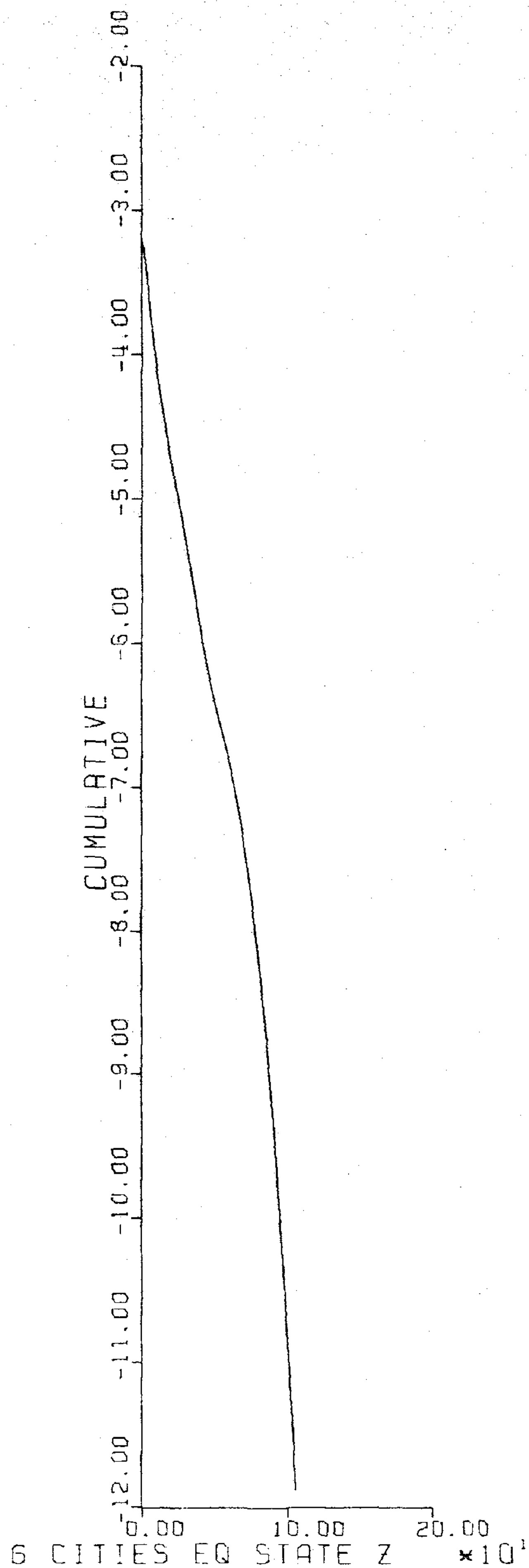
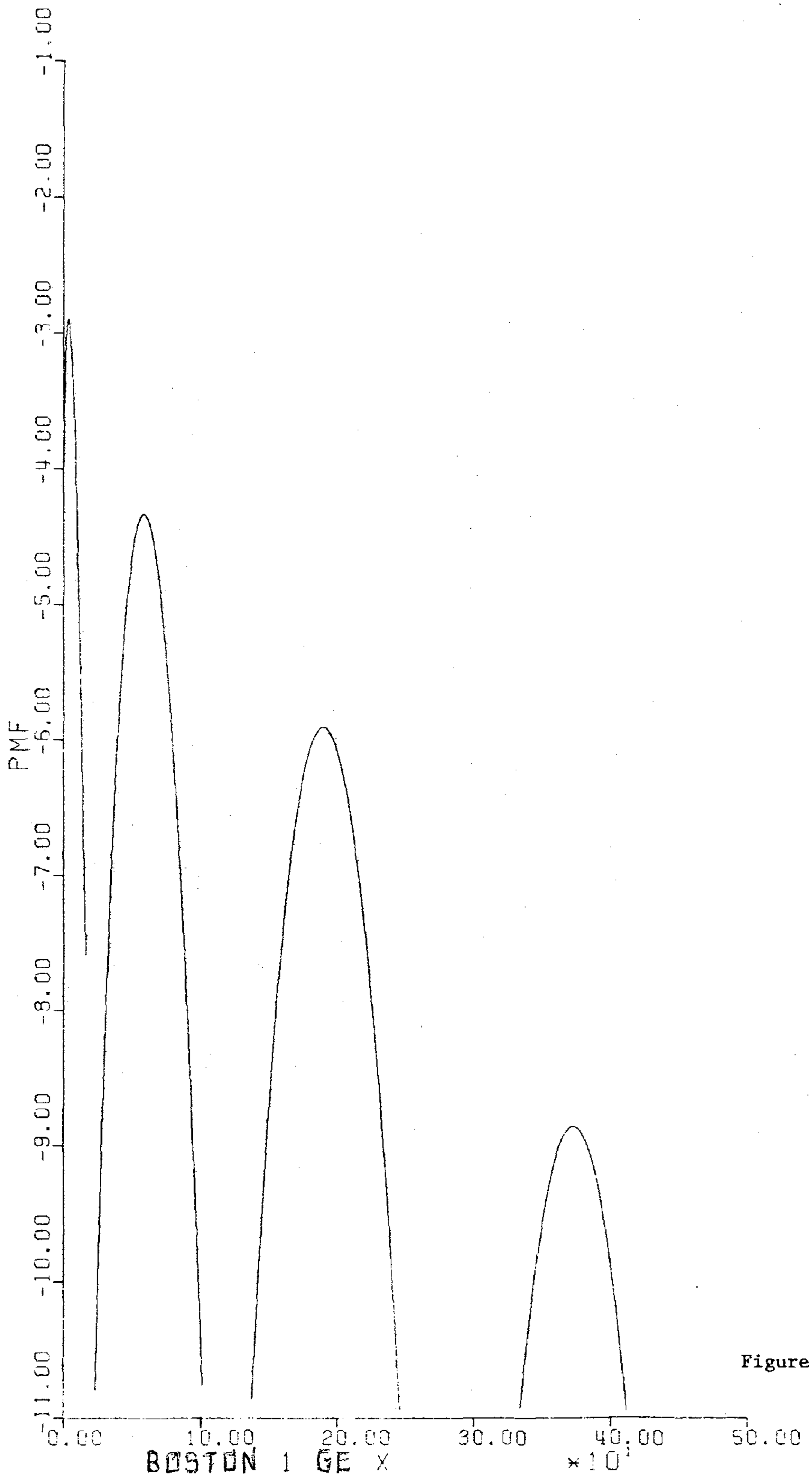

-2.00
-3.00
-4.00
-5.00
-6.00
-7.00
-8.00
-9.00
-10.00
-11.00
-12.00
CUMULATIVE
0.00
10.00
20.00
6 CITIES EQ STATE Z ×10¹

Figure 5.19



-1.00
-2.00
-3.00
-4.00
-5.00
-6.00
-7.00
-8.00
-9.00
-10.00
-11.00
PMF
0.00
10.00
20.00
30.00
40.00
50.00
BØSTON 1 GE X
*10

Figure 5.20

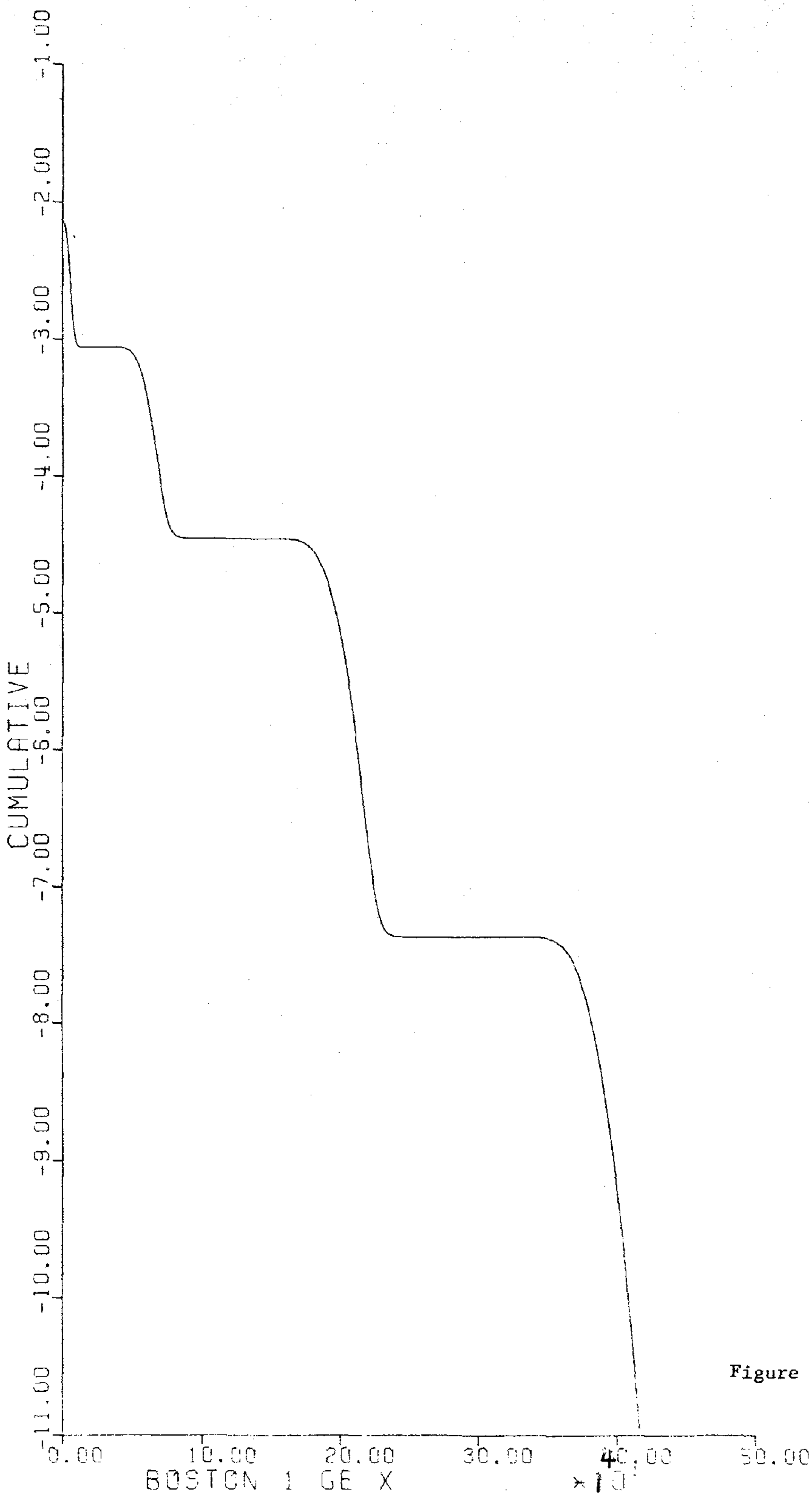

CUMULATIVE
BOSTON 1 GE X
×10
-1.00
-2.00
-3.00
-4.00
-5.00
-6.00
-7.00
-8.00
-9.00
-10.00
-11.00
0.00
10.00
20.00
30.00
40.00
50.00

Figure 5.21

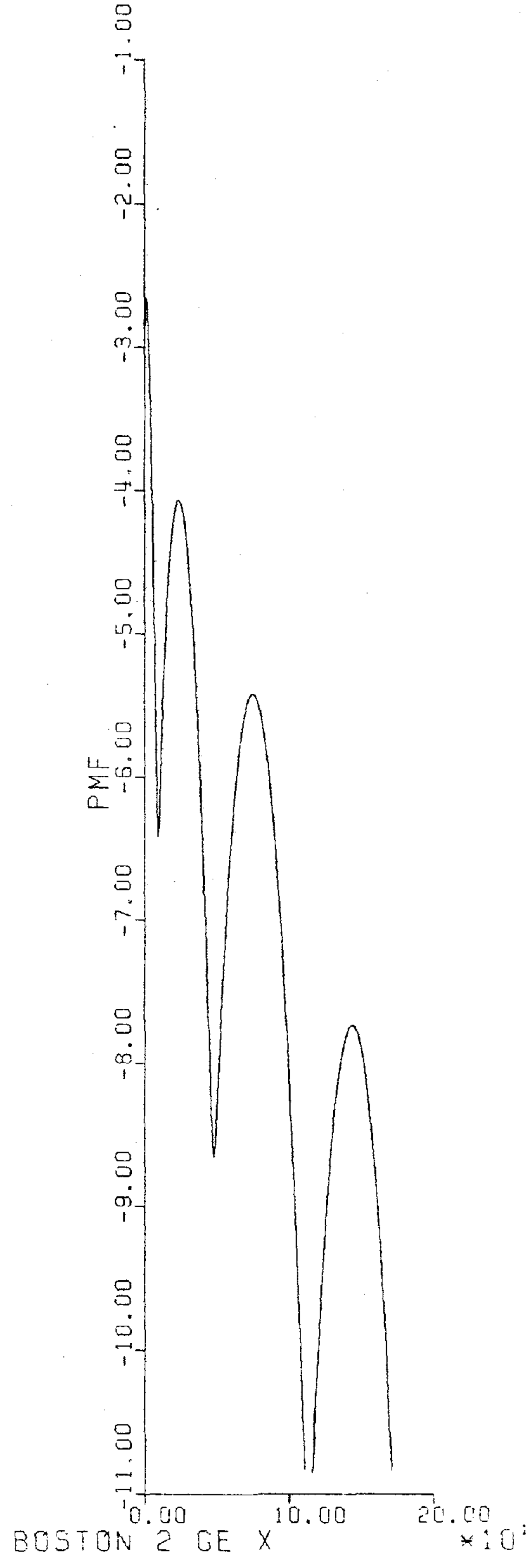

-1.00
-2.00
-3.00
-4.00
-5.00
-6.00
-7.00
-8.00
-9.00
-10.00
-11.00
PMF
0.00
10.00
20.00
BOSTON 2 GE X
*10

Figure 5.22

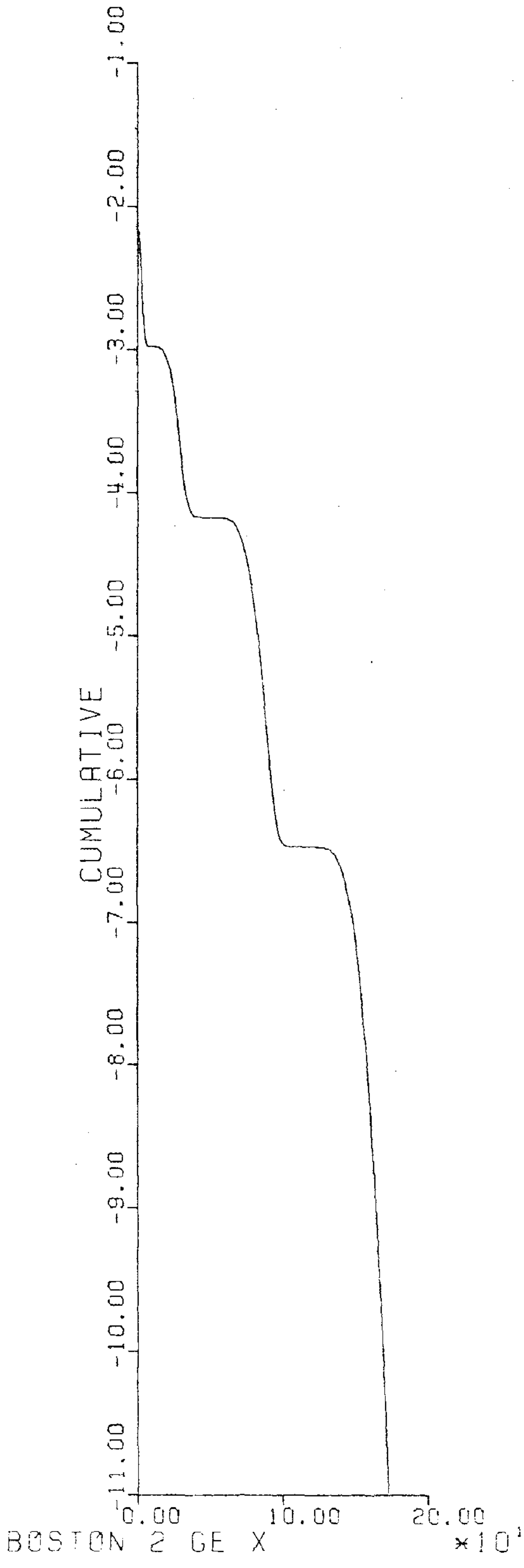
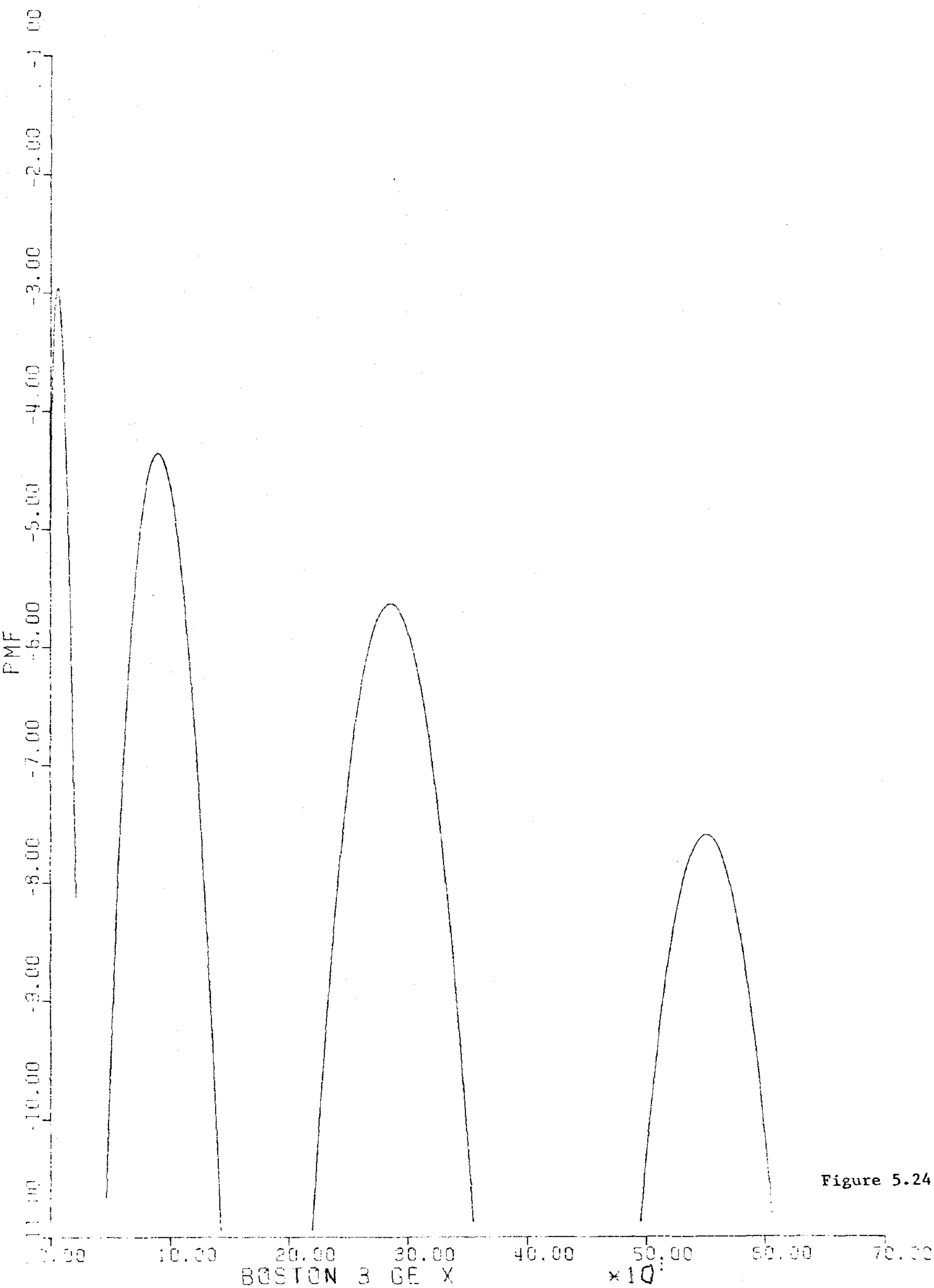
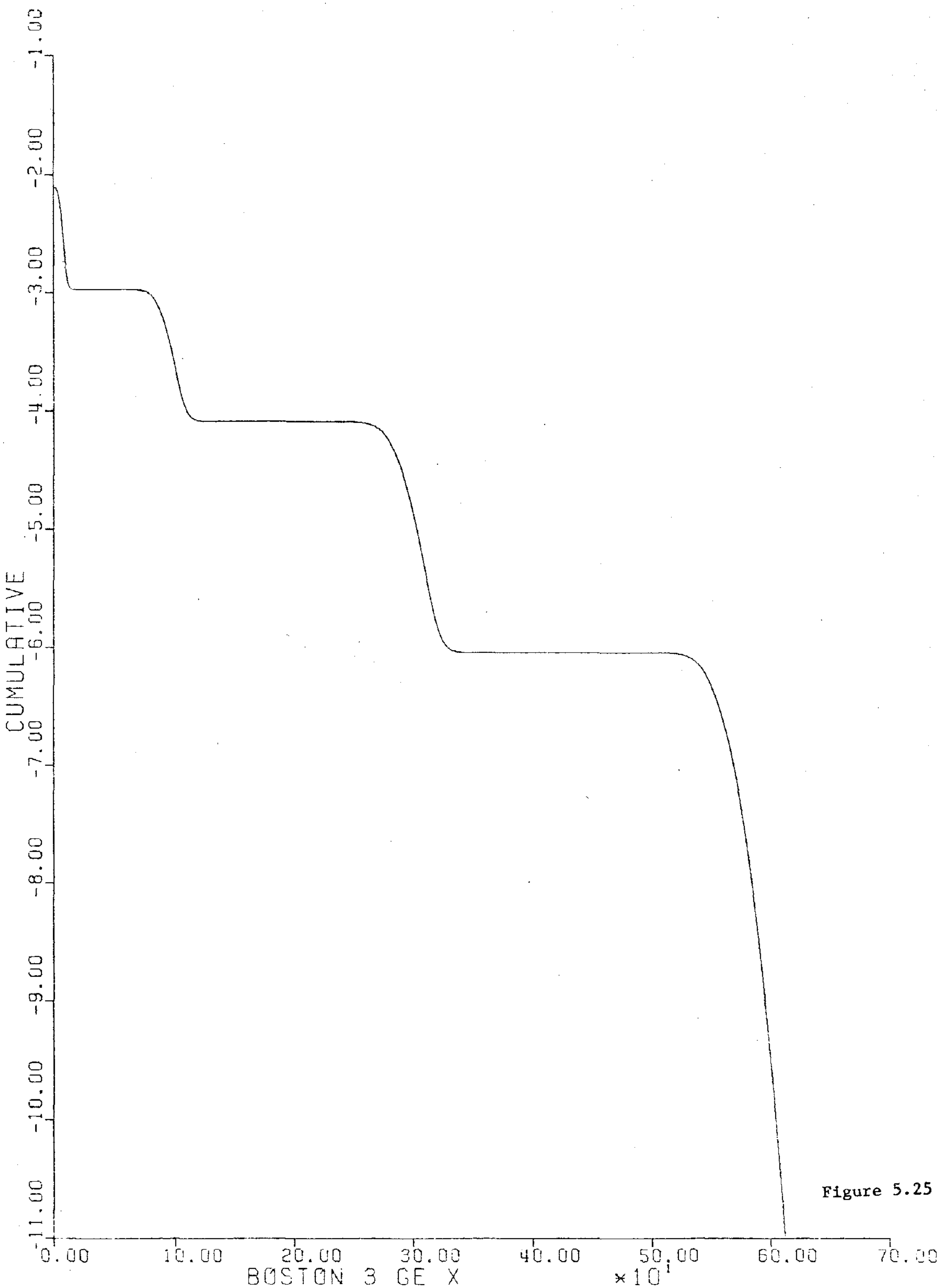

-1.00
-2.00
-3.00
-4.00
-5.00
-6.00
-7.00
-8.00
-9.00
-10.00
-11.00
CUMULATIVE
0.00
10.00
20.00
BØSTON 2 GE X
×10¹

Figure 5.23



PMF
-1.00
-2.00
-3.00
-4.00
-5.00
-6.00
-7.00
-8.00
-9.00
-10.00
0.00
10.00
20.00
30.00
40.00
50.00
60.00
70.00
BOSTON 3 GE X
×10

Figure 5.24



CUMULATIVE
-1.00
-2.00
-3.00
-4.00
-5.00
-6.00
-7.00
-8.00
-9.00
-10.00
-11.00
0.00
10.00
20.00
30.00
40.00
50.00
60.00
70.00
BØSTØN 3 GE X
$\times 10^{1}$

Figure 5.25

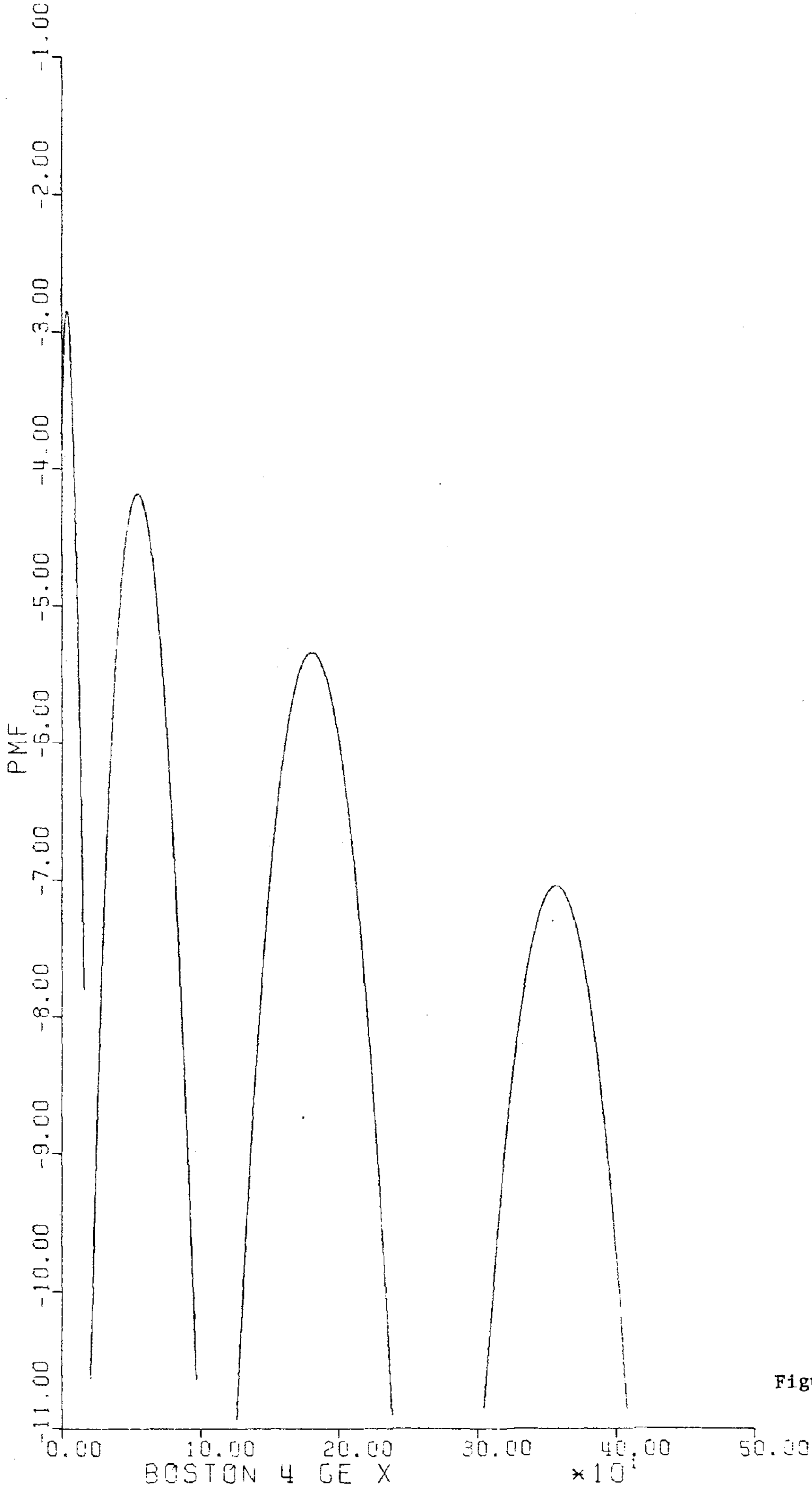

-1.00
-2.00
-3.00
-4.00
-5.00
-6.00
-7.00
-8.00
-9.00
-10.00
-11.00
PMF
0.00
10.00
20.00
30.00
40.00
50.00
BOSTON 4 GE X
×10

Figure 5.26

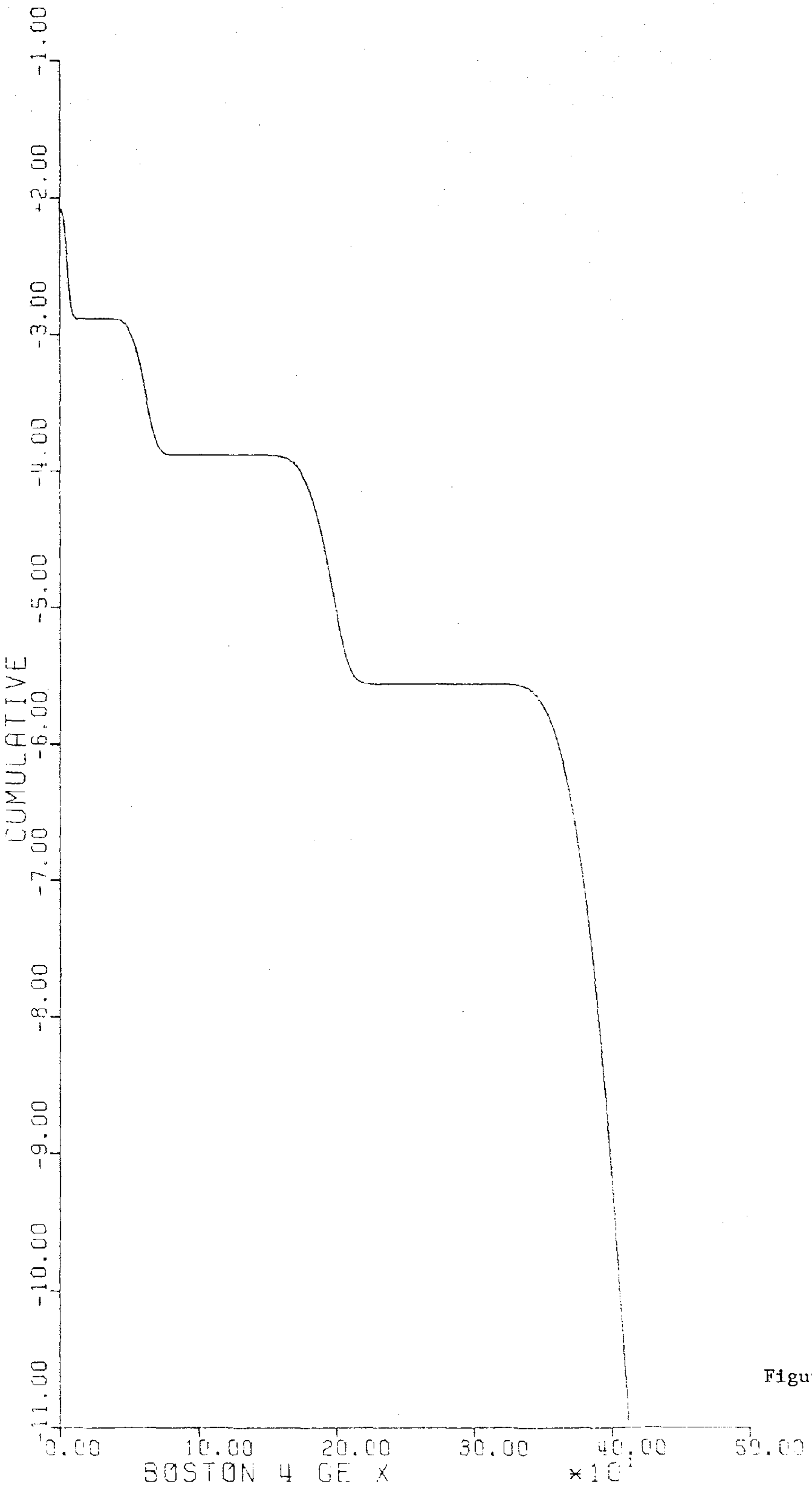
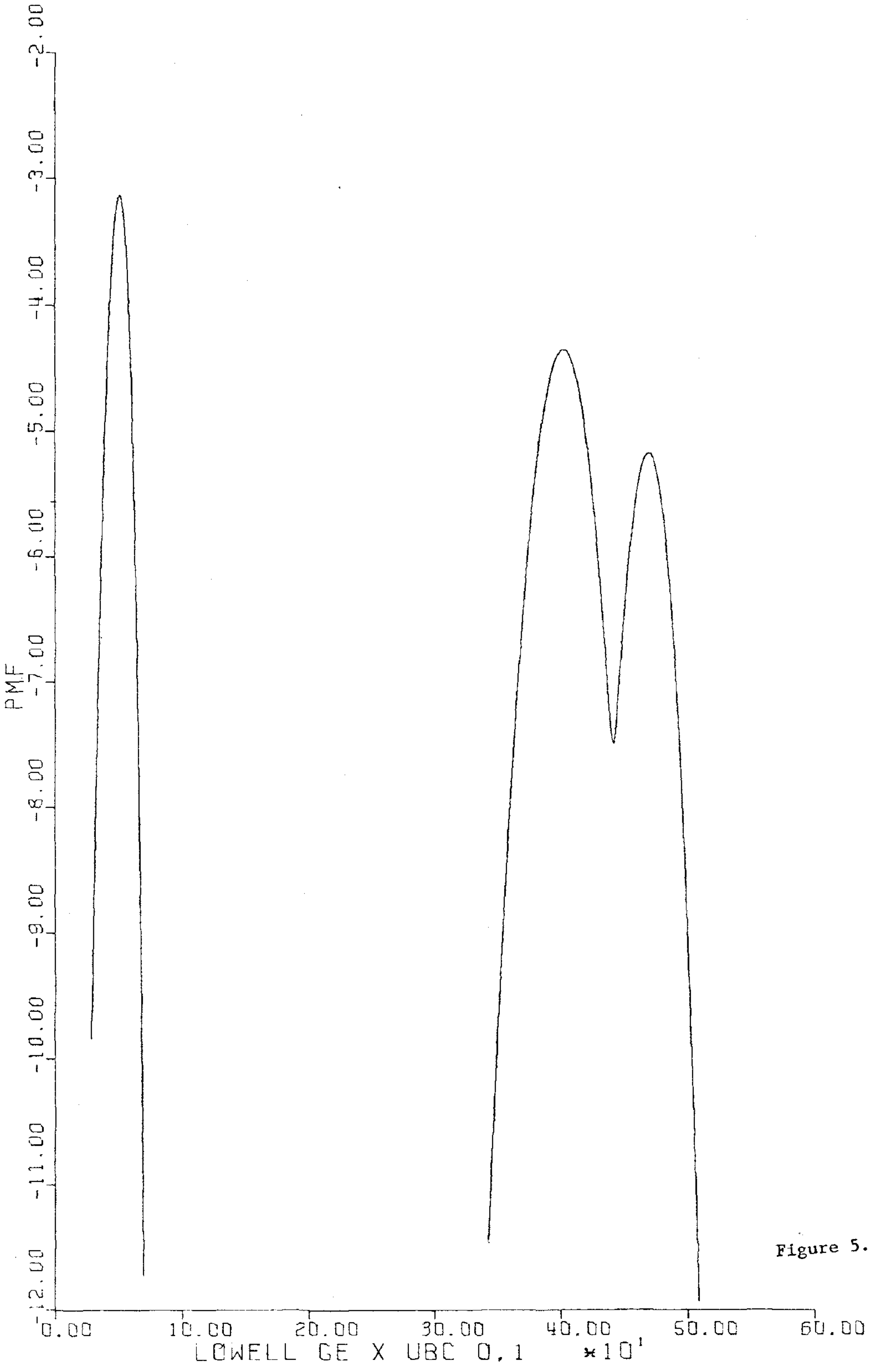

CUMULATIVE
-1.00
-2.00
-3.00
-4.00
-5.00
-6.00
-7.00
-8.00
-9.00
-10.00
-11.00
0.00
10.00
20.00
30.00
40.00
50.00
BOSTON 4 GE X
*10

Figure 5.27



-2.00
-3.00
-4.00
-5.00
-6.00
-7.00
-8.00
-9.00
-10.00
-11.00
-12.00
PMF
0.00
10.00
20.00
30.00
40.00
50.00
60.00
LOWELL GE X UBC 0.1 ×10¹

Figure 5.28

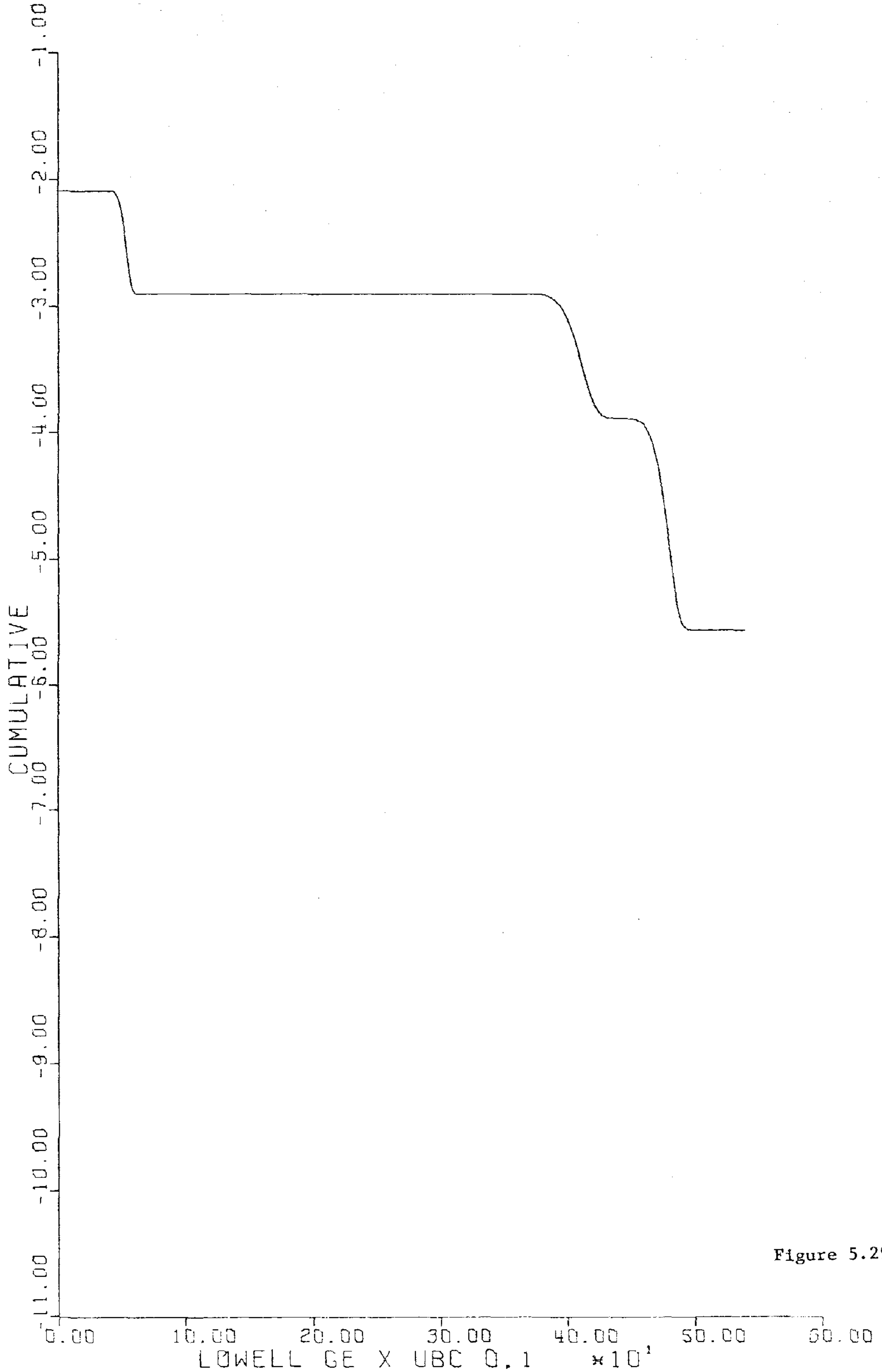
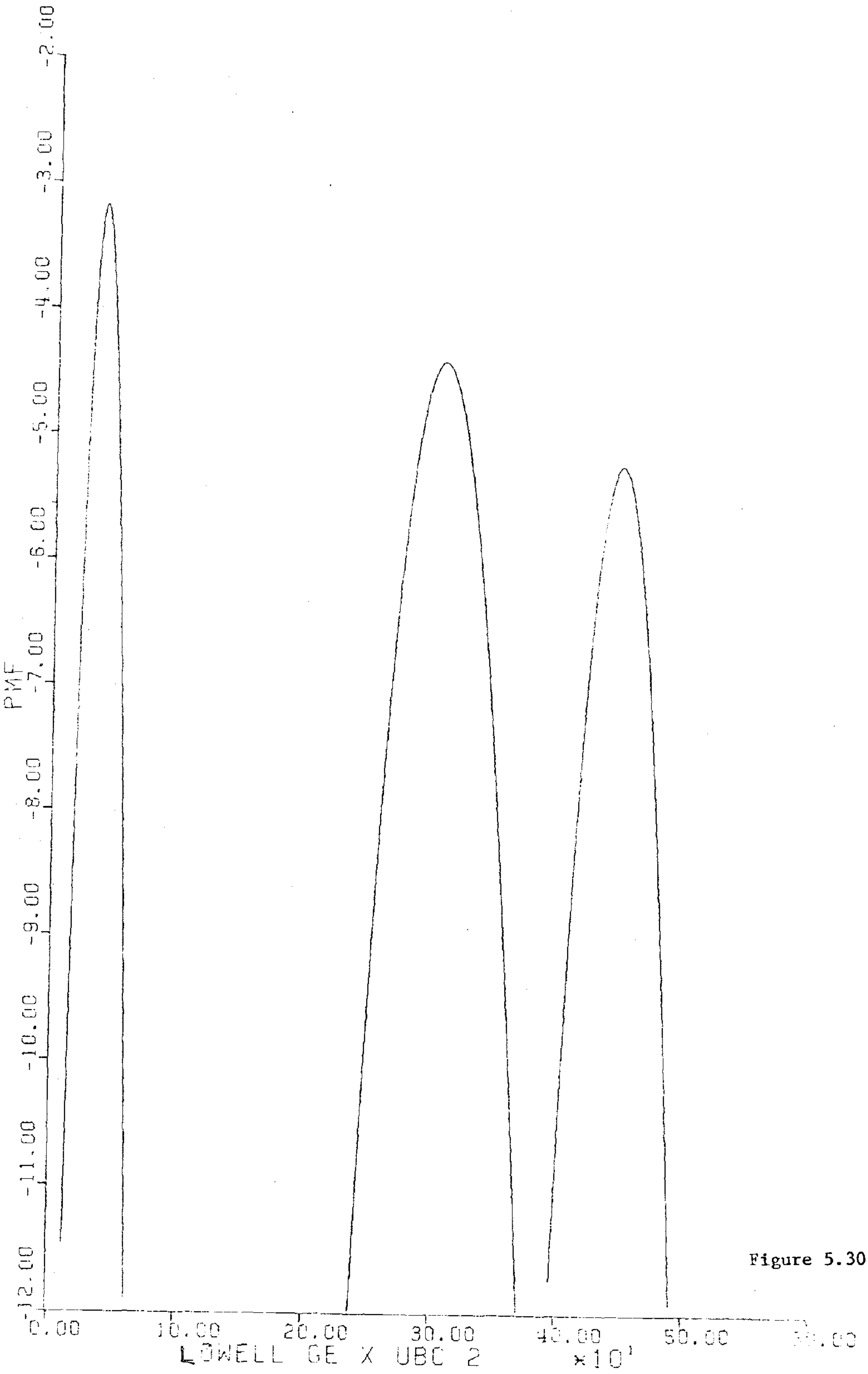
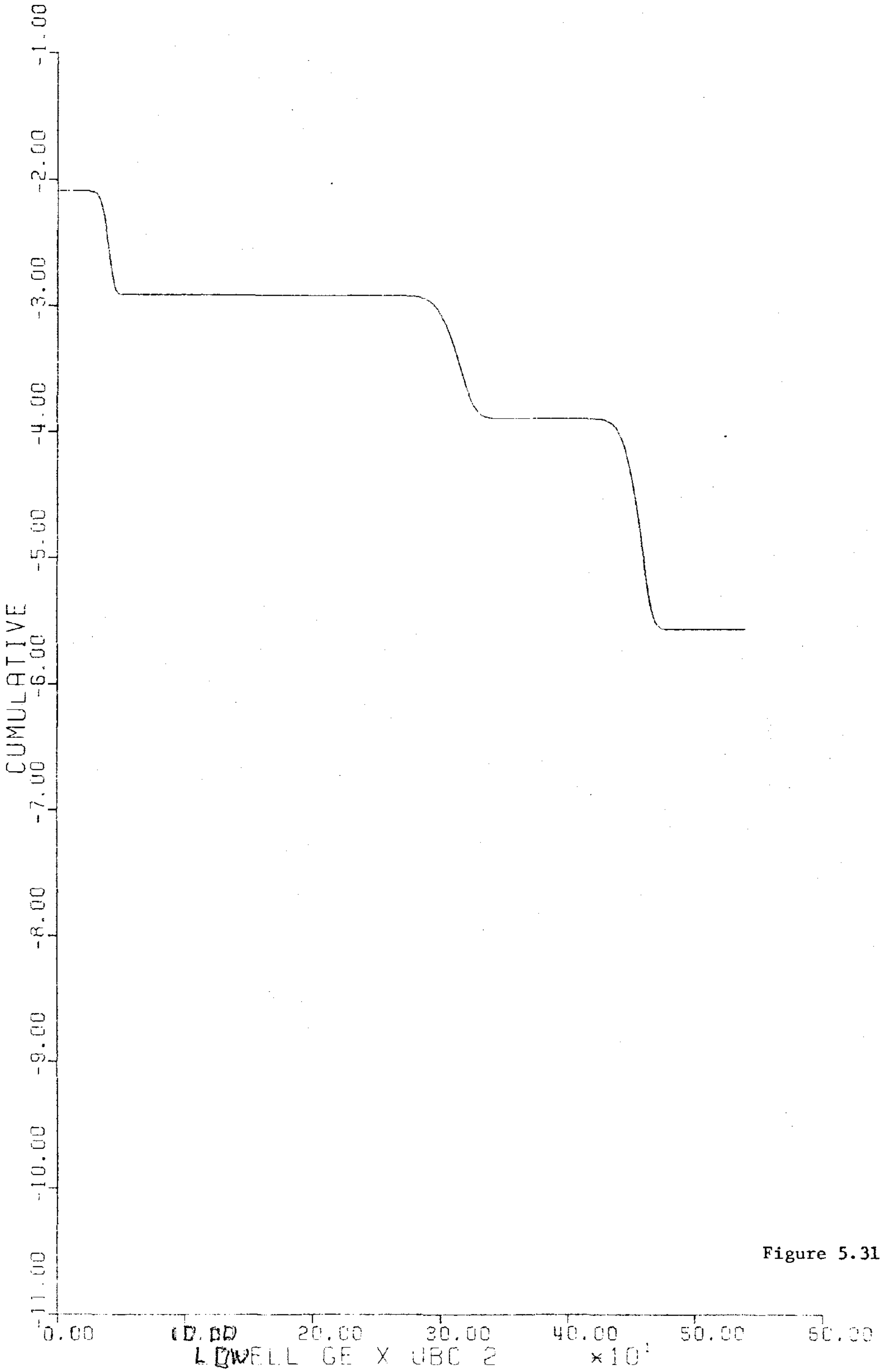

CUMULATIVE
-1.00
-2.00
-3.00
-4.00
-5.00
-6.00
-7.00
-8.00
-9.00
-10.00
-11.00
0.00
10.00
20.00
30.00
40.00
50.00
60.00
LOWELL GE X UBC 0.1 ×10¹

Figure 5.29



-2.00
-3.00
-4.00
-5.00
-6.00
-7.00
-8.00
-9.00
-10.00
-11.00
-12.00
PMF
0.00
10.00
20.00
30.00
40.00
50.00
LOWELL GE X UBC 2
x10'

Figure 5.30



CUMULATIVE
-1.00
-2.00
-3.00
-4.00
-5.00
-6.00
-7.00
-8.00
-9.00
-10.00
-11.00
0.00
10.00
20.00
30.00
40.00
50.00
60.00
LOWELL GE X UBC 2
×10¹

Figure 5.31

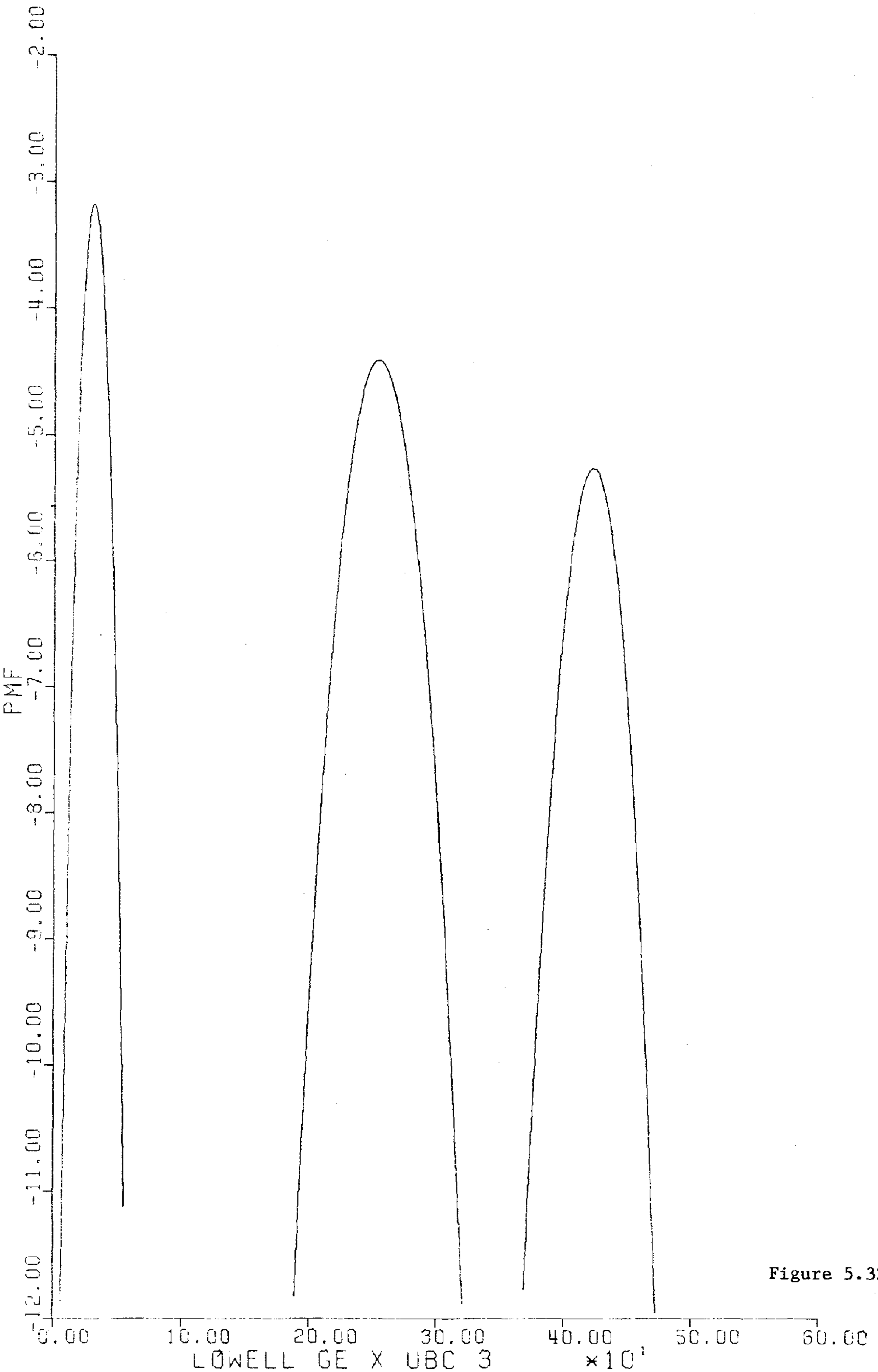

-2.00
-3.00
-4.00
-5.00
-6.00
-7.00
-8.00
-9.00
-10.00
-11.00
-12.00
PMF
0.00
10.00
20.00
30.00
40.00
50.00
60.00
LOWELL GE X UBC 3
*10¹

Figure 5.32

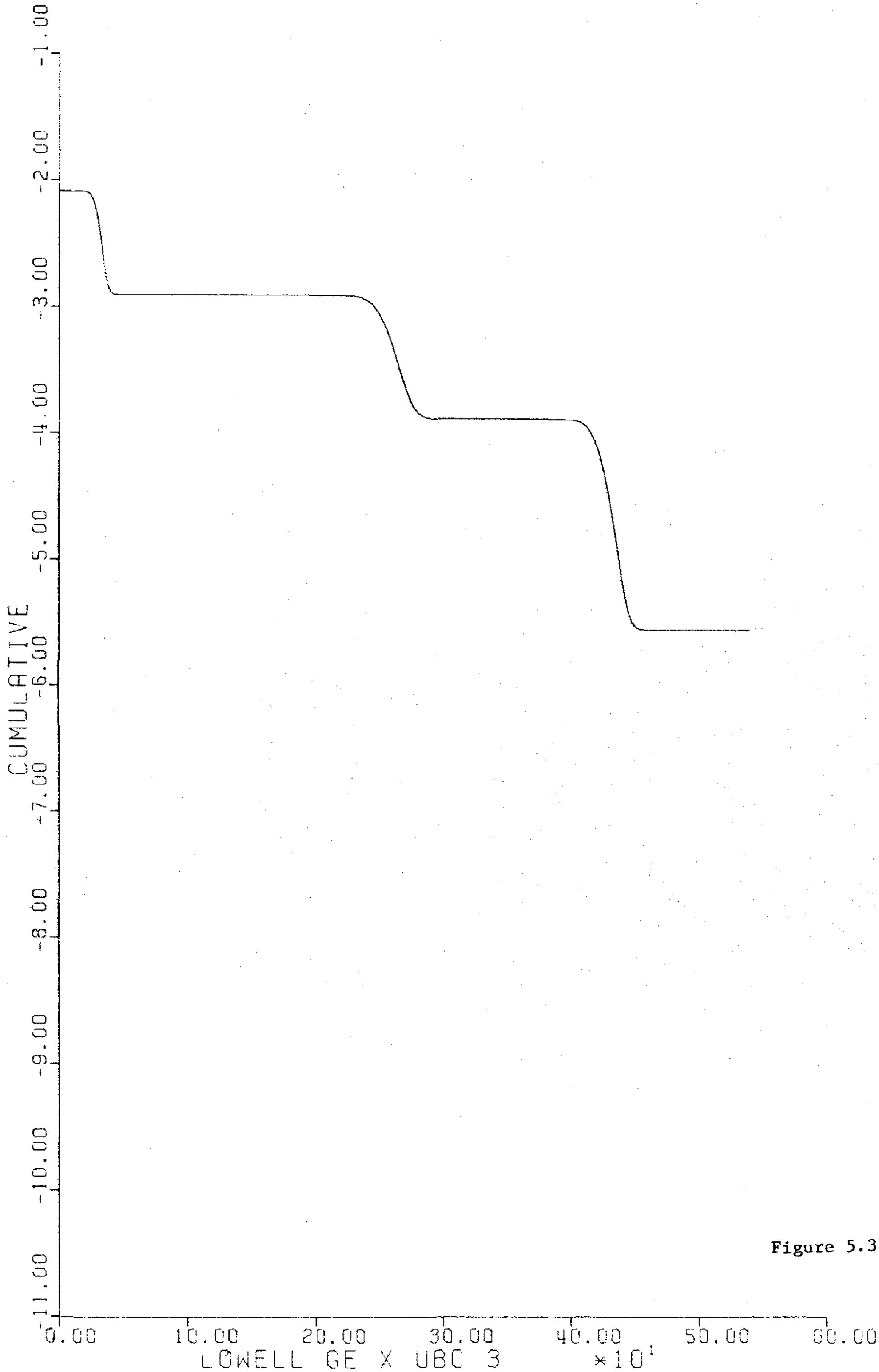

-1.00
-2.00
-3.00
-4.00
-5.00
-6.00
-7.00
-8.00
-9.00
-10.00
-11.00
CUMULATIVE
0.00
10.00
20.00
30.00
40.00
50.00
60.00
LOWELL GE X UBC 3
×10¹

Figure 5.33

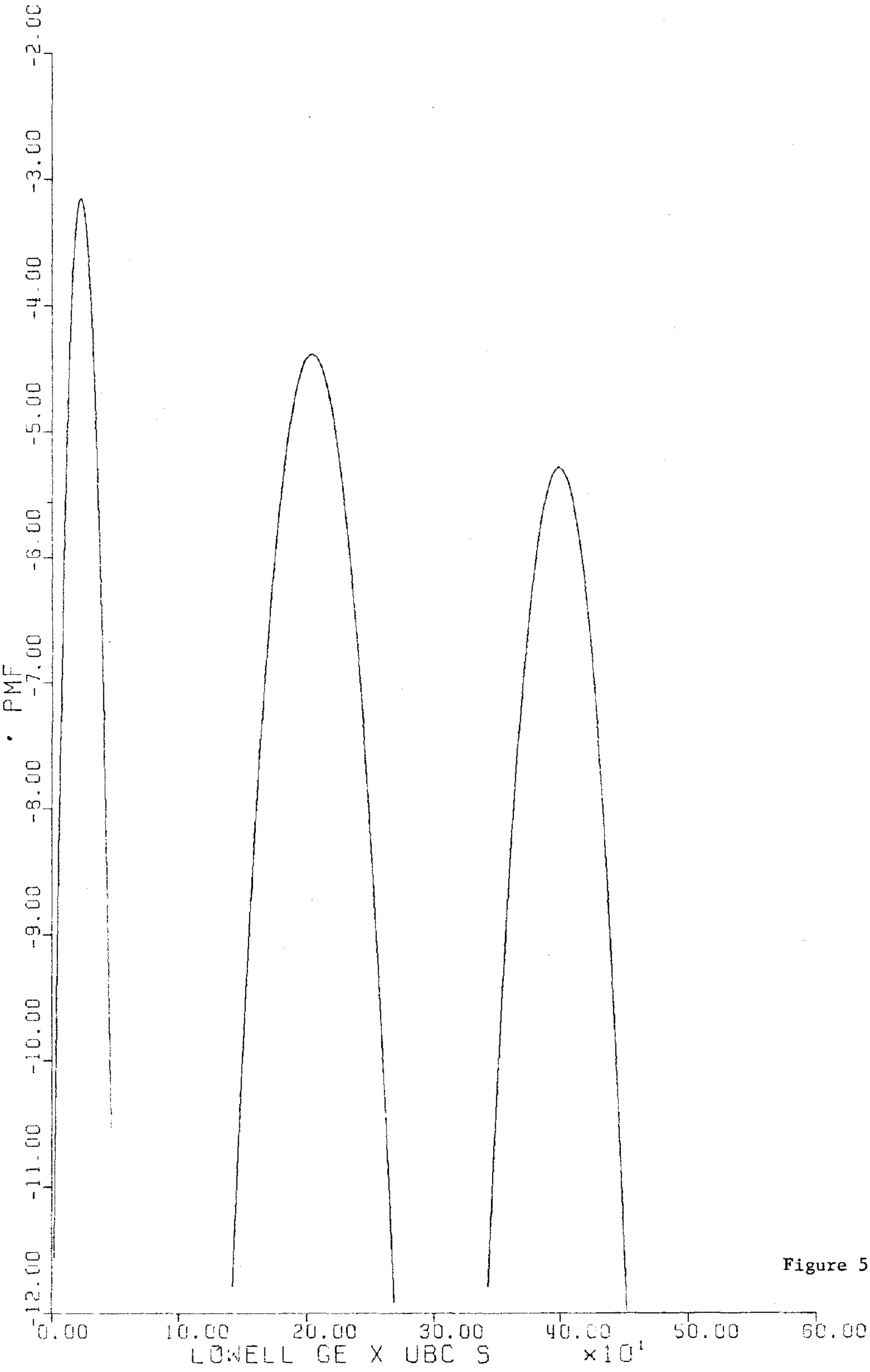

PMF
-2.00
-3.00
-4.00
-5.00
-6.00
-7.00
-8.00
-9.00
-10.00
-11.00
-12.00
0.00
10.00
20.00
30.00
40.00
50.00
60.00
LOWELL GE X UBC S
×10[1]

Figure 5.34

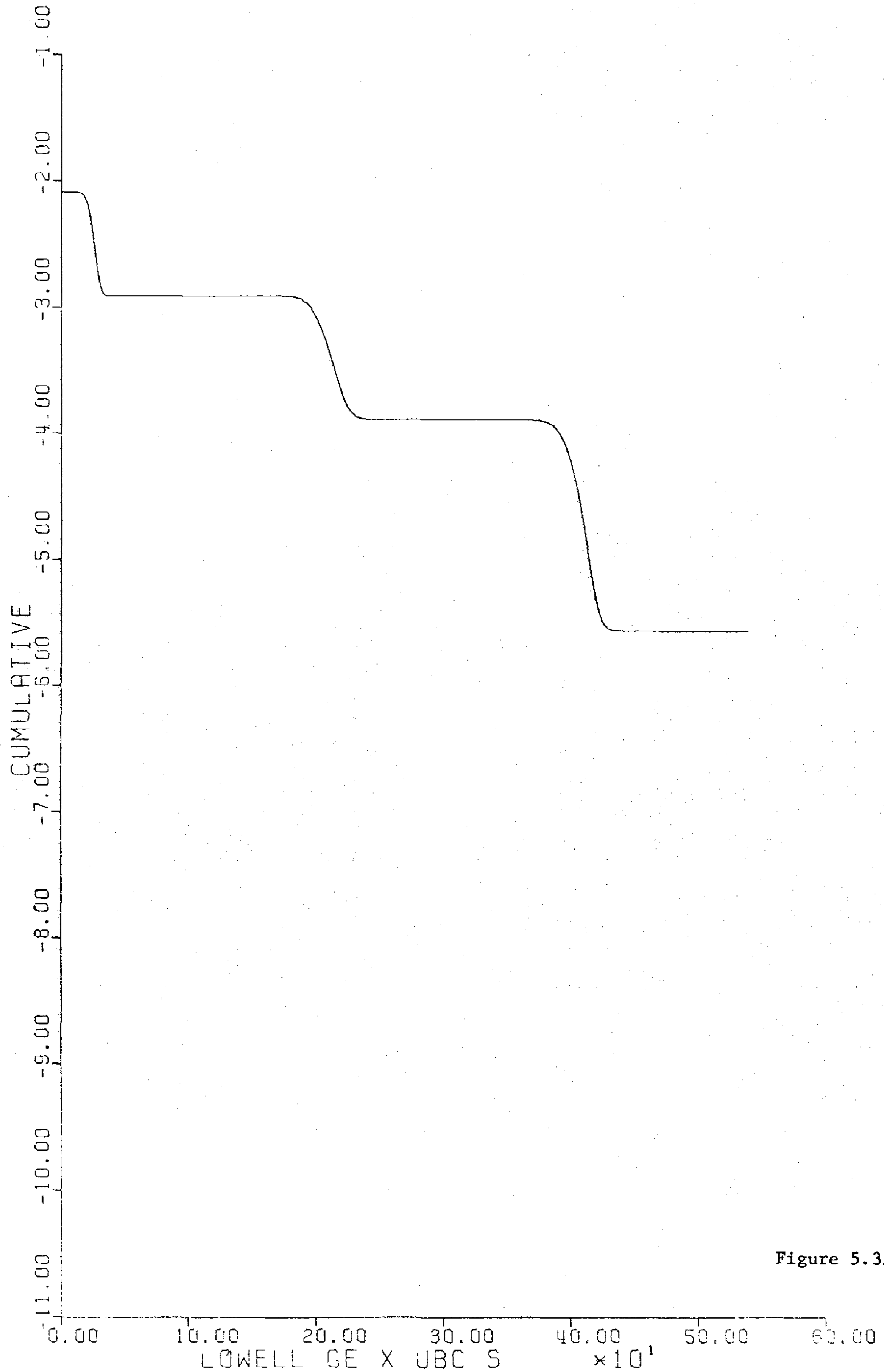

CUMULATIVE
-1.00
-2.00
-3.00
-4.00
-5.00
-6.00
-7.00
-8.00
-9.00
-10.00
-11.00
0.00
10.00
20.00
30.00
40.00
50.00
60.00
LOWELL GE X UBC S
×10¹

Figure 5.35

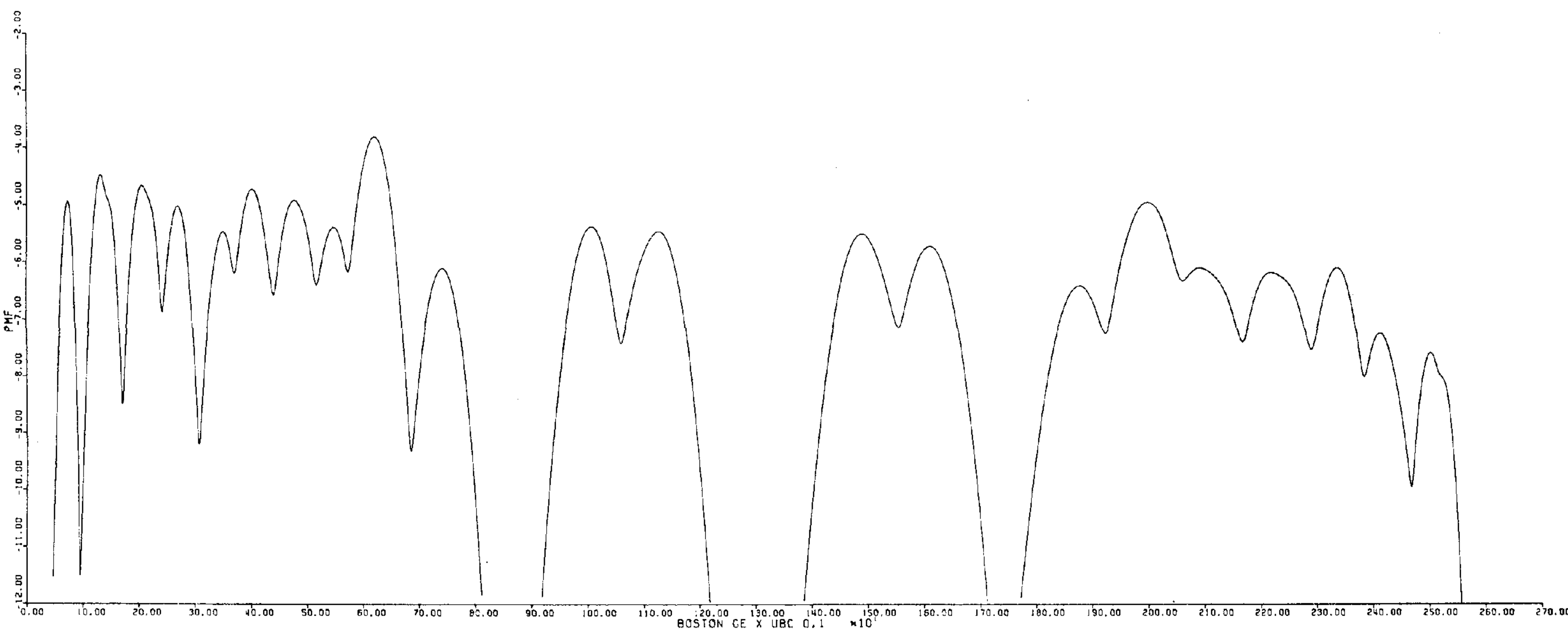
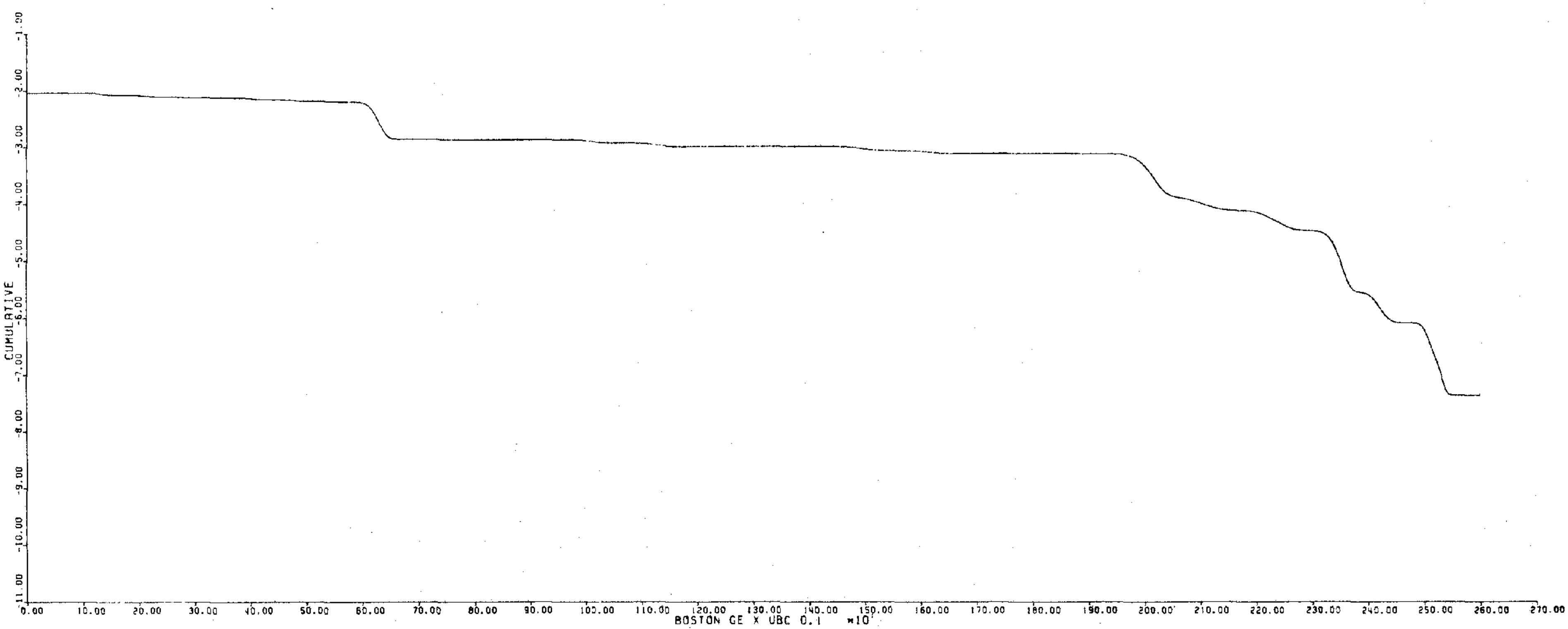

PMF
-2.00
-3.00
-4.00
-5.00
-6.00
-7.00
-8.00
-9.00
-10.00
-11.00
-12.00
0.00
10.00
20.00
30.00
40.00
50.00
60.00
70.00
80.00
90.00
100.00
110.00
120.00
130.00
140.00
150.00
160.00
170.00
180.00
190.00
200.00
210.00
220.00
230.00
240.00
250.00
260.00
270.00
BOSTON GE X UBC 0.1 $\times 10^{1}$

Figure 5.36



CUMULATIVE
-1.00
-2.00
-3.00
-4.00
-5.00
-6.00
-7.00
-8.00
-9.00
-10.00
-11.00
0.00
10.00
20.00
30.00
40.00
50.00
60.00
70.00
80.00
90.00
100.00
110.00
120.00
130.00
140.00
150.00
160.00
170.00
180.00
190.00
200.00
210.00
220.00
230.00
240.00
250.00
260.00
270.00
BOSTON GE X UBC 0.1 *10¹

Figure 5.37

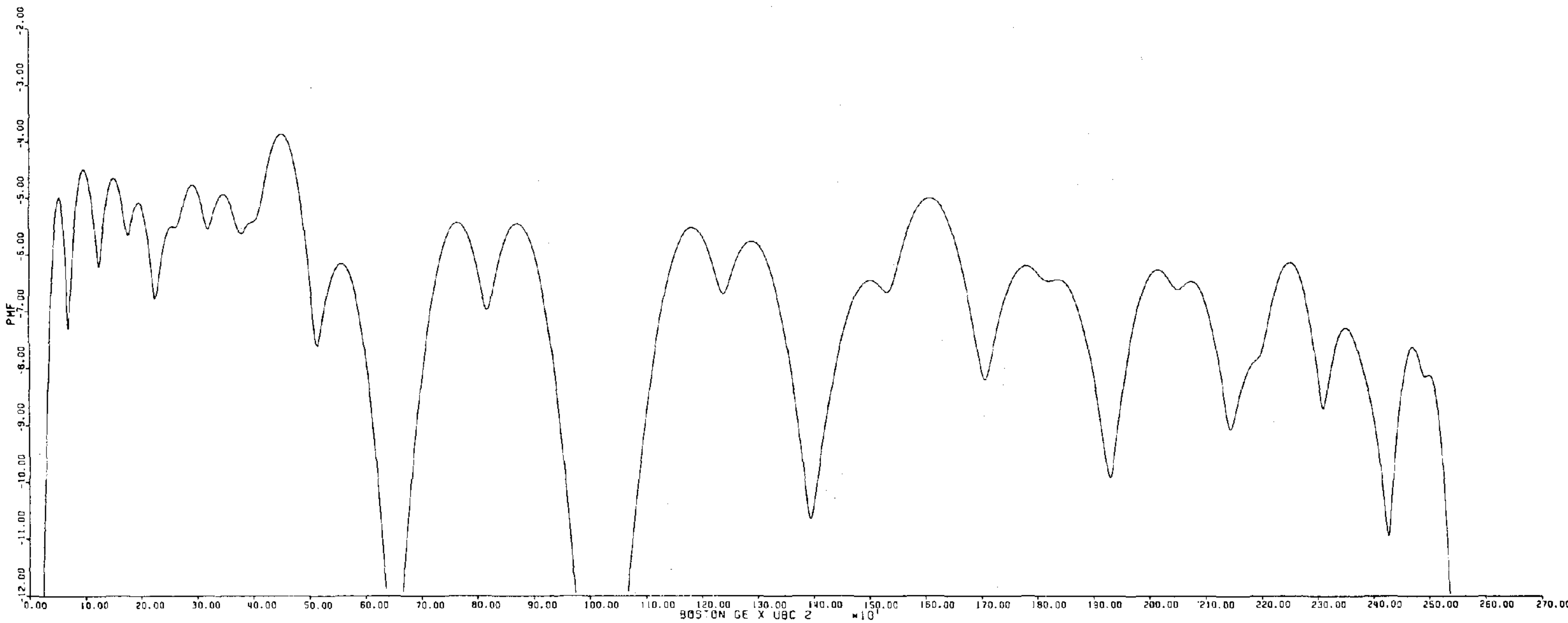

PMF
-2.00
-3.00
-4.00
-5.00
-6.00
-7.00
-8.00
-9.00
-10.00
-11.00
-12.00
0.00
10.00
20.00
30.00
40.00
50.00
60.00
70.00
80.00
90.00
100.00
110.00
120.00
130.00
140.00
150.00
160.00
170.00
180.00
190.00
200.00
210.00
220.00
230.00
240.00
250.00
260.00
270.00
BOSTON GE X UBC 2
$\times 10^{1}$

Figure 5.38

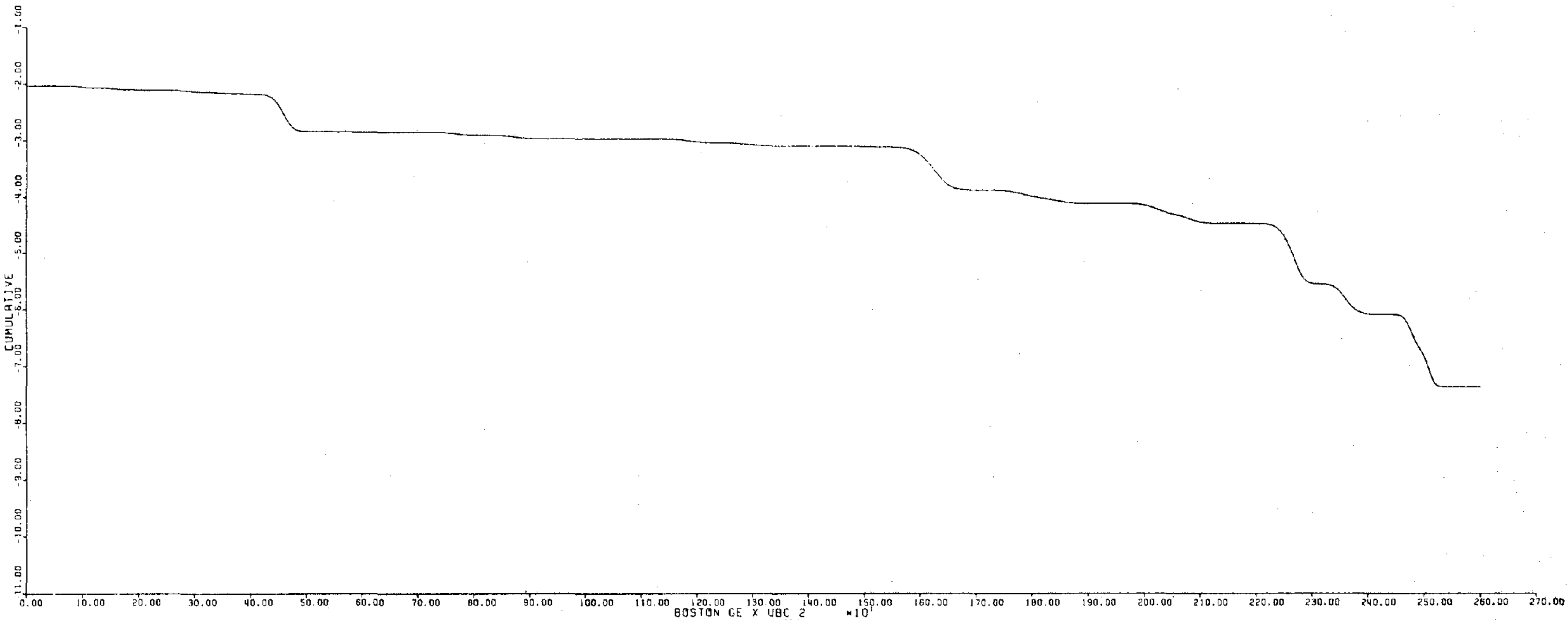

CUMULATIVE
BOSTON GE X UBC 2 *10¹
-1.00
-2.00
-3.00
-4.00
-5.00
-6.00
-7.00
-8.00
-9.00
-10.00
-11.00
0.00
10.00
20.00
30.00
40.00
50.00
60.00
70.00
80.00
90.00
100.00
110.00
120.00
130.00
140.00
150.00
160.00
170.00
180.00
190.00
200.00
210.00
220.00
230.00
240.00
250.00
260.00
270.00

Figure 5.39

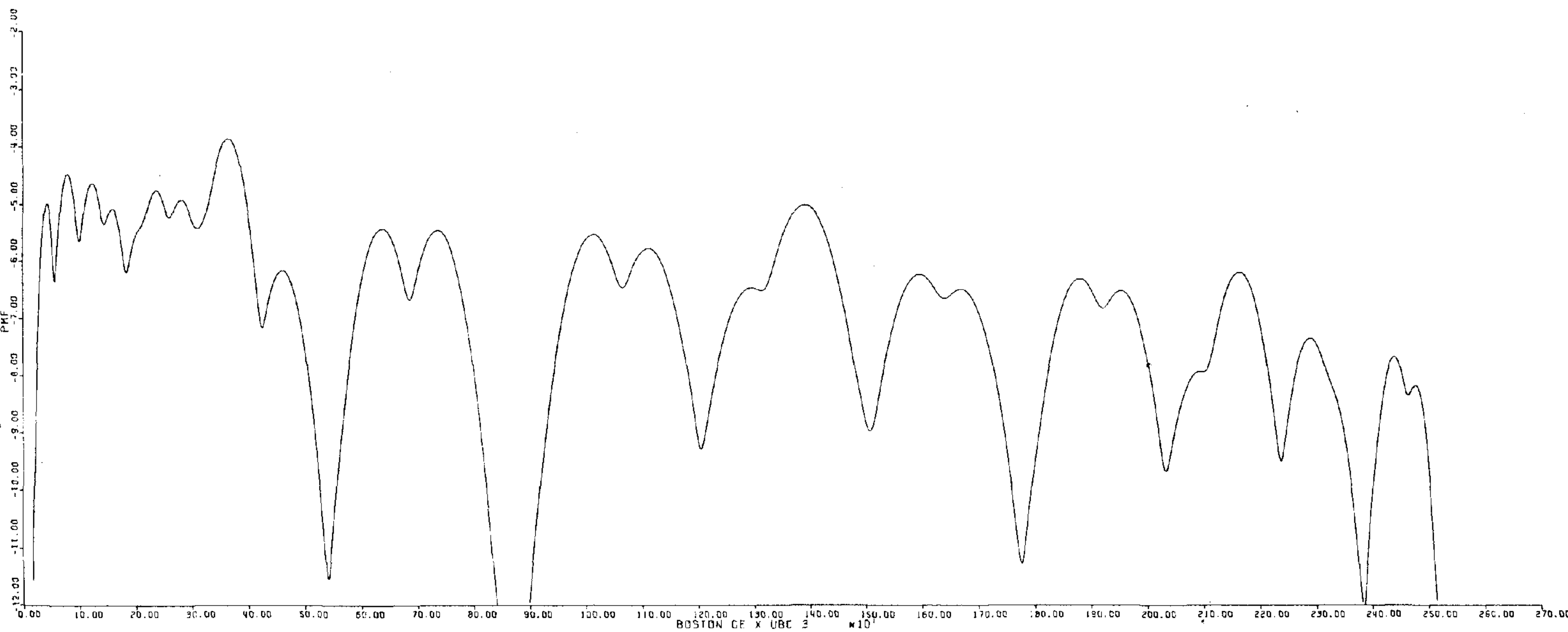

PMF
-2.00
-3.00
-4.00
-5.00
-6.00
-7.00
-8.00
-9.00
-10.00
-11.00
-12.00
0.00
10.00
20.00
30.00
40.00
50.00
60.00
70.00
80.00
90.00
100.00
110.00
120.00
130.00
140.00
150.00
160.00
170.00
180.00
190.00
200.00
210.00
220.00
230.00
240.00
250.00
260.00
270.00
BOSTON GE X UBE 3 *10

Figure 5.40

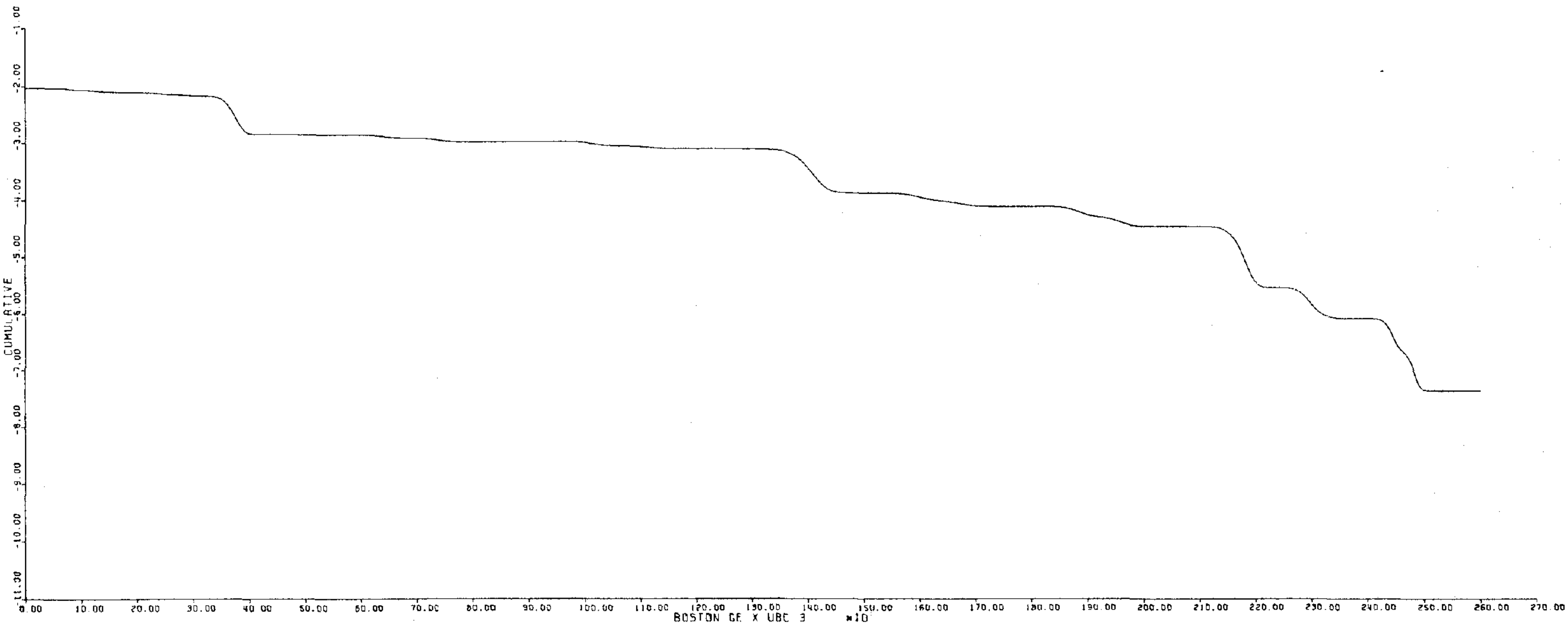

CUMULATIVE
-1.00
-2.00
-3.00
-4.00
-5.00
-6.00
-7.00
-8.00
-9.00
-10.00
-11.00
0.00
10.00
20.00
30.00
40.00
50.00
60.00
70.00
80.00
90.00
100.00
110.00
120.00
130.00
140.00
150.00
160.00
170.00
180.00
190.00
200.00
210.00
220.00
230.00
240.00
250.00
260.00
270.00
BOSTON GE X UBC 3 *10

Figure 5.41

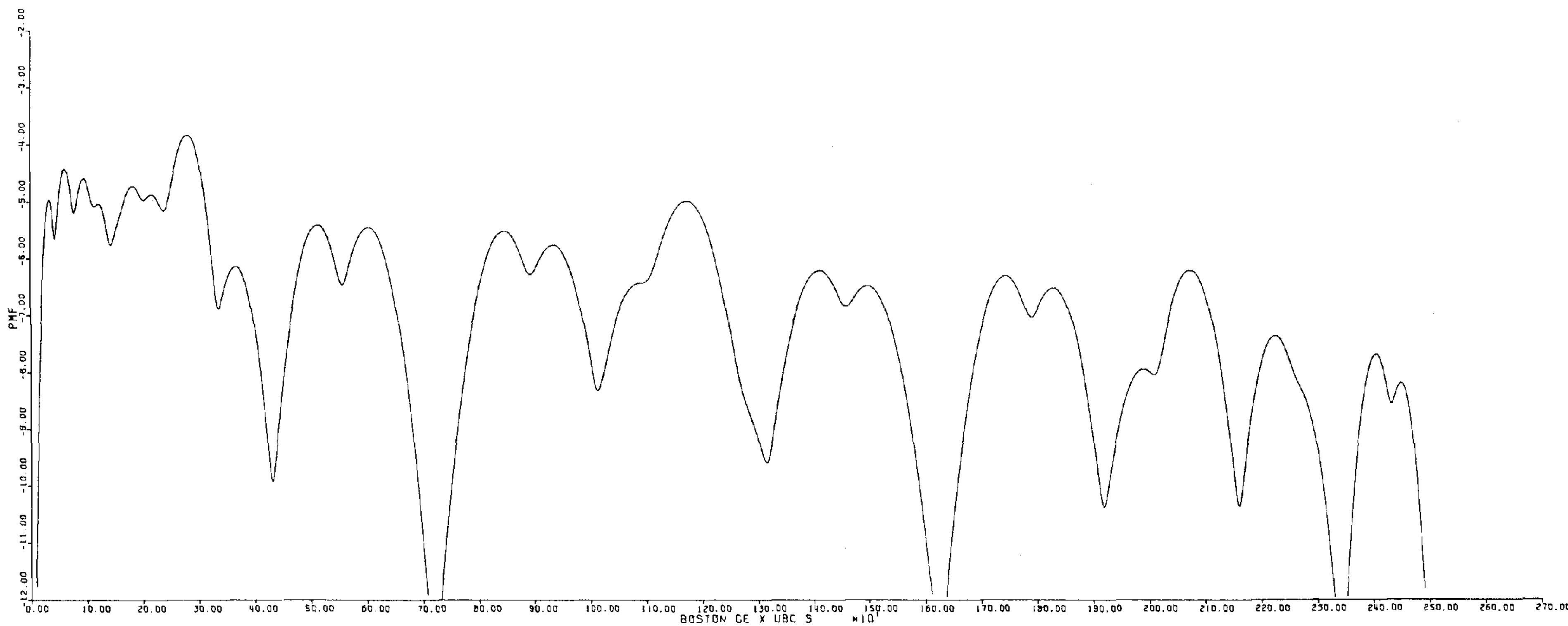

PMF
-2.00
-3.00
-4.00
-5.00
-6.00
-7.00
-8.00
-9.00
-10.00
-11.00
-12.00
0.00
10.00
20.00
30.00
40.00
50.00
60.00
70.00
80.00
90.00
100.00
110.00
120.00
130.00
140.00
150.00
160.00
170.00
180.00
190.00
200.00
210.00
220.00
230.00
240.00
250.00
260.00
270.00
BOSTON GE X UBC S *10¹

Figure 5.42

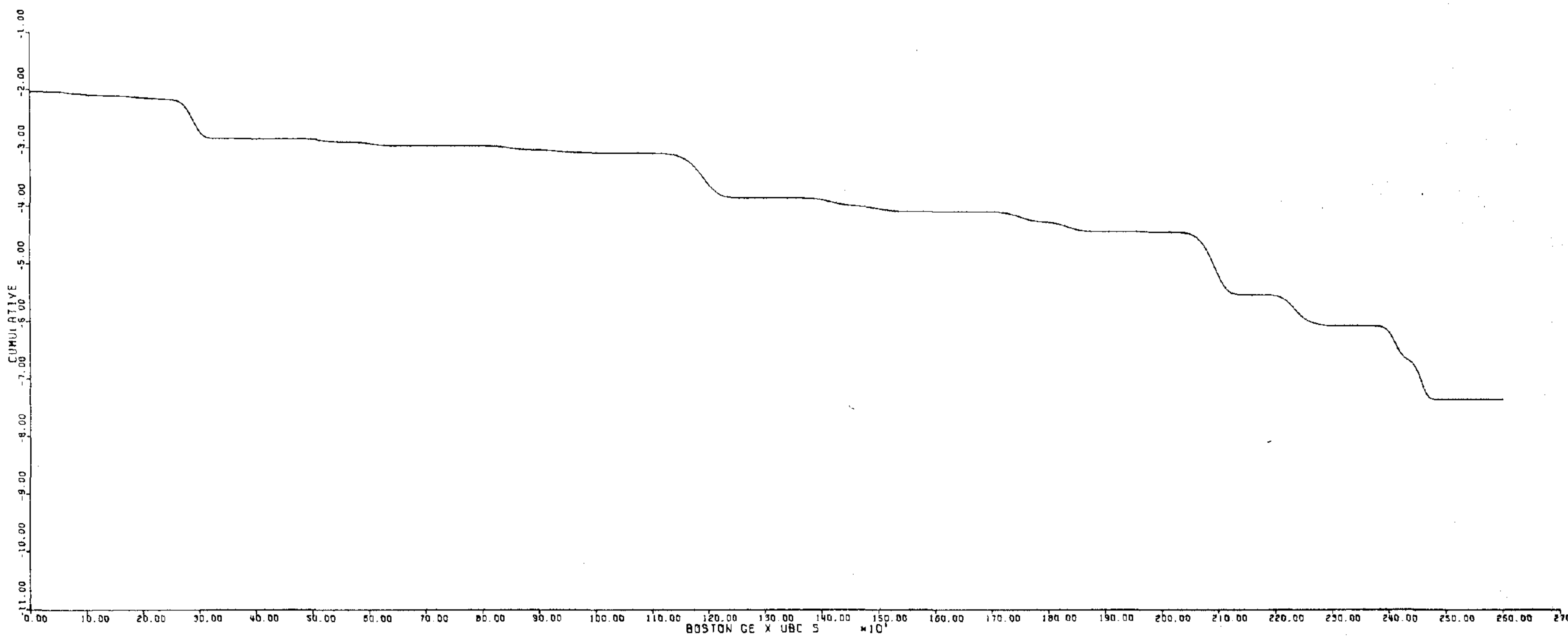

-1.00
-2.00
-3.00
-4.00
-5.00
-6.00
-7.00
-8.00
-9.00
-10.00
-11.00
CUMULATIVE
0.00
10.00
20.00
30.00
40.00
50.00
60.00
70.00
80.00
90.00
100.00
110.00
120.00
130.00
140.00
150.00
160.00
170.00
180.00
190.00
200.00
210.00
220.00
230.00
240.00
250.00
260.00
270.00
BOSTON GE X UBC 5 $\times 10^{1}$

Figure 5.43