Seismic Response of a Buried Pipeline

I. Infinite Medium

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Abstract

A system of equations governing the nonaxially symmetric motion of a circular cylindrical shell embedded in an infinite elastic medium is developed in this paper taking into account the transverse shear and rotary inertia. The object here is to model the general motion of a long straight buried pipeline due to seismic waves. The special case of the axially symmetric motion of a pipeline excited by a travelling longitudinal wave is solved under the assumption that the inertia effects can be neglected.

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1. Introduction

Dynamic response of lifelines excited by seismic waves is a subject of considerable interest. Examples of lifelines are highways, bridges, tunnels, pipelines, etc. Earlier studies (see the review article by Wang, et al. [1]) have shown that buried pipelines have been severely damaged by earthquakes. It has been found that damage to underground pipelines is caused by landslides, liquefaction of soil, faulting and also due to shaking by traveling seismic waves [2].

Several investigators have recently studied the dynamic response of buried pipelines to seismic excitation. References to these works can be found in the review articles [3, 1] (see also [4]). Most of these works have treated the pipeline as a continuous or segmented beam.

The departure from the beam model is in a recent work by Muleski, et al. [5]. The authors have investigated the axisymmetric response of a cylindrical shell on an elastic foundation. The motion of the shell was assumed to be governed by Flügge's equations (which neglects transverse shear and rotatory inertia effects) and the spring constants of the foundation were taken to be known constants.

The full problem of the interactions between the shell and the surrounding elastic medium has been considered recently by El-Akily and Datta [6, 7]. In these papers the plane strain problem of vibration of a shell embedded in an elastic half-space excited by incident plane waves has been considered. The shell equations were taken to be Flügge's. It has been found that the interaction of the shell with the surrounding medium, which is significantly influenced by the depth of the shell, plays an important role in the displacement of the shell and stresses in it.

In this connection mention should be made of the paper by Parmelee and Ludtke [8], who considered the plane strain problem of a buried pipe in an elastic semi-infinite medium. In this analysis the pipe is treated as rigid and on an elastic foundation. The spring constant of the foundation was calculated from the ratio of the point force and the average static displacement around the pipe-wall obtained from the static solution given by Mindlin. The dynamic interaction problem of the shell and the elastic half-space was not considered.

In the present paper we have examined this interaction problem for the three dimensional motion of the shell. Our object here has been threefold: (1) to develop the three dimensional equations of motion of the shell taking into account the effects of transverse shear and rotatory inertia, (2) to formally solve these equations of motion coupled with those of the surrounding medium, and finally (3), to use the low frequency approximate expression of the traction force on the shell due to the surrounding medium for evaluating the spring constants of the elastic foundation approximating the infinite surrounding medium. It is shown that these spring constants are dependent on the wavelength of the exciting waves, the mean radius of the shell and the material constants of the surrounding medium. In order to facilitate our analysis we have considered the surrounding medium to be infinite in extent and the interaction problem has been solved for the case of axisymmetry. In a subsequent paper we will present the solution to the nonaxisymmetric problem of motion of a shell in a semi-infinite elastic medium.

2. Equations of Motion of the Shell

In this section the equations of motion of a shell element are developed following the analysis of Mirsky and Herrmann [9].

Consider an infinite circular cylindrical shell of mean radius R and thickness h. Let x, θ, z be the coordinates measured in the axial, circumferential and normal to the middle surface directions, respectively. We shall also consider a cylindrical polar coordinate system r, θ, x with r measuring the distance from the axis of the shell. Thus

$$r = R + z \, , \quad -\frac{h}{2} < z < \frac{h}{2} \tag{1}$$

The components of the displacement of a point in the shell are denoted by u_x, u_θ, u_z, which will be approximated by the following equations.

$$\begin{aligned} u_x (r, \theta, x, t) &= u(\theta, x, t) + z\Psi_x (\theta, x, t) \\ u_\theta (r, \theta, x, t) &= v(\theta, x, t) + z\Psi_\theta (\theta, x, t) \\ u_z (r, \theta, x, t) &= w(\theta, x, t) \end{aligned} \tag{2}$$

Here t is the time and u, v, w are the displacement components of a point on the middle surface, $z = 0$. Ψ_x, Ψ_θ measure the angles of rotation of a normal to the middle surface in the xz- and $z\theta$-planes, respectively.

The shell stress-resultants and displacements are then connected by the approximate equations (see Eqns. (3)) of [9]. There is a misprint in the equation for $N_{\theta x}$.),

$$N_{xx} = E_p \frac{\partial u}{\partial x} + \frac{D}{R} \frac{\partial \Psi_x}{\partial x} + \frac{E_p \nu}{R} \left(\frac{\partial v}{\partial \theta} + w \right)$$

$$\frac{1}{G} N_{\theta x} = h \frac{\partial v}{\partial x} + \frac{h}{R} \left(1 + \frac{I}{hR^2}\right) \frac{\partial u}{\partial \theta} - \frac{I}{R^2} \frac{\partial \Psi_x}{\partial \theta}$$

$$M_{xx} = \frac{D}{R}\left(\nu \frac{\partial \Psi_\theta}{\partial \theta} + R \frac{\partial \Psi_x}{\partial x} + \frac{\partial u}{\partial x}\right)$$

$$\frac{1}{G} M_{\theta x} = \frac{I}{R}\left(- \frac{1}{R} \frac{\partial u}{\partial \theta} + \frac{\partial \Psi_x}{\partial \theta} + R \frac{\partial \Psi_\theta}{\partial x}\right)$$

$$\frac{1}{G} N_{x\theta} = h \frac{\partial v}{\partial x} + \frac{1}{R} \frac{\partial \Psi_\theta}{\partial x} + \frac{h}{R} \frac{\partial u}{\partial \theta}$$

$$N_{\theta\theta} = \frac{1}{R}\left(E_p + \frac{D}{R^2}\right)\left(\frac{\partial v}{\partial \theta} + w\right) - \frac{D}{R^2} \frac{\partial \Psi_\theta}{\partial \theta} + E_p \nu \frac{\partial u}{\partial x} \tag{3}$$

$$\frac{1}{G} M_{x\theta} = \frac{I}{R}\left(\frac{\partial \Psi_x}{\partial \theta} + \frac{\partial v}{\partial x} + R \frac{\partial \Psi_\theta}{\partial x}\right)$$

$$M_{\theta\theta} = \frac{D}{R}\left(\frac{\partial \Psi_\theta}{\partial \theta} - \frac{1}{R} \frac{\partial v}{\partial \theta} - \frac{w}{R}\right) + D\nu \frac{\partial \Psi_x}{\partial x}$$

$$\frac{1}{G} Q_x = k_x^2 h \left(\frac{\partial w}{\partial x} + \Psi_x\right)$$

$$\frac{1}{G} Q_\theta = k_\theta^2 h \left(1 + h^2/12R^2\right)\left(\frac{1}{R} \frac{\partial w}{\partial \theta} + \Psi_\theta - \frac{1}{R} u_\theta\right)$$

Here

$$E_p = \frac{Eh}{1 - \nu^2} \quad , \quad I = \frac{h^3}{12} \quad , \quad D = \frac{Eh^3}{12(1 - \nu^2)}$$

E is the Young's modulus, G the shear modulus and ν the Poisson's ratio of the shell, k_x, k_θ are the shear correction factors and will be taken as $\pi^2/12$, although for a shell vibrating in an elastic medium these may be somewhat different. The corrections, however, will not be significant for long wavelengths.

Substituting the stress-displacement relations (3) into the equations of motion we obtain the displacement equations of motion of the shell as

$$\left[E_p \frac{\partial^2}{\partial x^2} + \frac{Gh}{R^2}\left(1 + I/hR^2\right) \frac{\partial^2}{\partial \theta^2} - \rho_s h \frac{\partial^2}{\partial t^2}\right] u +$$

$$\left[\frac{D}{R} \frac{\partial^2}{\partial x^2} - \frac{GI}{R^3} \frac{\partial^2}{\partial \theta^2} - \frac{\rho_s I}{R} \frac{\partial^2}{\partial t^2}\right] \Psi_x + \frac{E_p(1 + \nu)}{2R} \frac{\partial^2 v}{\partial x \partial \theta} + \frac{E_p \nu}{R} \frac{\partial w}{\partial x} + p_1^* = 0$$

$$[\frac{D}{R}\frac{\partial^2}{\partial x^2} - \frac{GI}{R^3}\frac{\partial^2}{\partial\theta^2} - \frac{\rho_s I}{R}\frac{\partial^2}{\partial t^2}]u + [D\frac{\partial^2}{\partial x^2} + \frac{GI}{R^2}\frac{\partial^2}{\partial\theta^2} -$$

$$k_x^2 Gh - \rho_s I\frac{\partial^2}{\partial t^2}]\Psi_x + \frac{D(1+\nu)}{2R}\frac{\partial^2\Psi_\theta}{\partial x\partial\theta} - k_x^2 Gh\frac{\partial w}{\partial x} + p_2^* = 0$$

(4)

$$\frac{E_p(1+\nu)}{2R}\frac{\partial^2 u}{\partial x\partial\theta} + [Gh\frac{\partial^2}{\partial x^2} + \frac{1}{R^2}(E_p + D/R^2)\frac{\partial^2}{\partial\theta^2} -$$

$$k_\theta^2 G(h + I/R^2)/R^2 - \rho_s h\frac{\partial^2}{\partial t^2}]v + [\frac{GI}{R}\frac{\partial^2}{\partial x^2} -$$

$$\frac{D}{R^3}\frac{\partial^2}{\partial\theta^2} + \frac{k_\theta^2 G}{R}(h + I/R^2) - \frac{\rho_s I}{R}\frac{\partial^2}{\partial t^2}]\Psi_\theta +$$

$$[\frac{1}{R^2}(E_p + D/R^2) + \frac{k_\theta^2 G}{R^2}(h + I/R^2)]\frac{\partial w}{\partial\theta} + p_3^* = 0$$

$$\frac{D(1+\nu)}{2R}\frac{\partial^2\Psi_x}{\partial\theta\partial x} + [\frac{GI}{R}\frac{\partial^2}{\partial x^2} - \frac{D}{R^3}\frac{\partial^2}{\partial\theta^2} + \frac{k_\theta^2 G}{R}(h + I/R^2) -$$

$$\frac{\rho_s I}{R}\frac{\partial^2}{\partial t^2}]v + [GI\frac{\partial^2}{\partial x^2} + \frac{D}{R^2}\frac{\partial^2}{\partial\theta^2} - k_\theta^2 G(h + I/R^2) -$$

$$\rho_s I\frac{\partial^2}{\partial t^2}]\Psi_\theta - [\frac{D}{R^3} + \frac{k_\theta^2 G}{R}(h + I/R^2)]\frac{\partial w}{\partial\theta} + p_4^* = 0$$

$$-\frac{E_p\nu}{R}\frac{\partial u}{\partial x} + k_x^2 Gh\frac{\partial\Psi_x}{\partial x} - [\frac{k_\theta^2 G}{R^2}(h + I/R^2)\frac{\partial}{\partial\theta} +$$

$$\frac{1}{R^2}(E_p + D/R^2)\frac{\partial}{\partial\theta}]v + [\frac{D}{R^3}\frac{\partial}{\partial\theta} + \frac{k_\theta^2 G}{R}(h + I/R^2)\frac{\partial}{\partial\theta}]\Psi_\theta +$$

$$[-\frac{1}{R^2}(E_p + D/R^2) + \frac{k_\theta^2 G}{R^2}(h + I/R^2)\frac{\partial^2}{\partial\theta^2} + k_x^2 Gh\frac{\partial^2}{\partial x^2} -$$

$$\rho_s h\frac{\partial^2}{\partial t^2}]w + p_5^* = 0$$

Here each p_i^* is composed of two parts. One part is due to the forces exerted by the surrounding medium on the shell. This part will be denoted by p_i^{1*}, which is given by

$$p_1^{1*} = (1 + \frac{h}{2R})\tau_{rx}^* \quad , \quad p_2^{1*} = \frac{h}{2} p_1^{1*}$$

$$p_3^{1*} = (1 + \frac{h}{2R})\tau_{r\theta}^* \quad , \quad p_4^{1*} = \frac{h}{2} p_3^{1*}$$

$$p_5^{1*} = (1 + \frac{h}{2R})\tau_{rr}^* \tag{5}$$

ρ_s is the density of the shell material, τ_{rx}^*, $\tau_{r\theta}^*$ and τ_{rr}^* are the components of the traction force of the surrounding medium on the outer surface of the shell, $r = R + h/2$. These are to be obtained by solving the equations of motion of the surrounding medium simultaneously with Eqns. (4) and satisfying the appropriate displacement boundary conditions on the outer surface of the shell. The other part of p_i^* will be due to the externally applied forces. This part will be denoted by p_i^{2*}.

Assuming that the shell is perfectly bonded with the surrounding medium the displacement boundary conditions at $r = R + h/2$ are

$$\begin{aligned} u + \frac{h}{2}\Psi_x &= u_x^* \\ w &= u_r^* \\ v + \frac{h}{2}\Psi_\theta &= u_\theta^* \end{aligned} \tag{6}$$

where $\underset{\sim}{u}^*$ is the displacement of a point of the surrounding medium adjacent to the shell, i.e.

$$\underset{\sim}{u}^* = \underset{\sim}{u}(R + \frac{h}{2}, \theta, x, t) \tag{7}$$

3. Equations Governing the Motion of the Outer Medium and Their Solutions

The surrounding medium will be assumed to be homogeneous, isotropic and linearly elastic with Lamé constants λ, μ and density ρ. Then the displacement $\underset{\sim}{u}(r, \theta, x, t)$ of a point satisfies the equation of motion

$$\tau^2 \underset{\sim}{\nabla}\underset{\sim}{\nabla} \cdot \underset{\sim}{u} - \underset{\sim}{\nabla} \wedge \underset{\sim}{\nabla} \wedge \underset{\sim}{u} = \frac{1}{c_2^2} \frac{\partial^2 \underset{\sim}{u}}{\partial t^2} \tag{8}$$

where

$$\tau = \sqrt{\frac{\lambda + 2\mu}{\mu}} = \sqrt{\frac{2(1-\sigma)}{1-2\sigma}} \quad , \quad c_2 = \sqrt{\frac{\mu}{\rho}}$$

σ is the Poisson's ratio and c_2 is the shear wave speed.

Since the object of this paper is to study the dynamic response of the shell to seismic waves originating from sources outside the shell the displacement field outside the shell will be composed of two parts, the ground displacement field, $\underset{\sim}{u}^{(i)}$, in the absence of the shell and the scattered field, $\underset{\sim}{u}^{(s)}$.

If $\underset{\sim}{u}^{(i)}$ and $\underset{\sim}{u}^{(s)}$ are assumed to have harmonic time dependence, $e^{-i\omega t}$, then $\underset{\sim}{u}^{(s)}$ can be expressed in cylindrical coordinates as [10]

$$\begin{aligned} u_r^{(s)} &= \frac{\partial \Phi}{\partial r} + \frac{1}{r} \frac{\partial H_x}{\partial \theta} - \frac{\partial H_\theta}{\partial x} \\ u_\theta^{(s)} &= \frac{1}{r} \frac{\partial \Phi}{\partial \theta} + \frac{\partial H_r}{\partial x} - \frac{\partial H_x}{\partial r} \\ u_x^{(s)} &= \frac{\partial \Phi}{\partial x} + \frac{1}{r} \frac{\partial}{\partial r}(rH_\theta) - \frac{1}{r} \frac{\partial H_r}{\partial \theta} \end{aligned} \tag{9}$$

where

$$\Phi = f(r, x)\, e^{in\theta - i\omega t} \tag{10}$$

$$H_r = iH_\theta = g_1(r, x)\, e^{in\theta - i\omega t} \tag{11}$$

$$H_x = g_3(r, x)\, e^{in\theta - i\omega t} \tag{12}$$

The functions f, g_1 and g_3 are solutions of the equations

$$(\nabla^2 - \frac{n^2}{r^2} + k_1^2)\, f = (\nabla^2 - \frac{n^2}{r^2} + k_2^2)\, g_3 = (\nabla^2 - \frac{(n+1)^2}{r^2} + k_2^2)\, g_1 = 0 \tag{13}$$

Here

$$\nabla^2 \equiv \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial x^2} \quad , \quad k_2 = \omega/c_2 \quad , \quad k_1 = k_2/\tau$$

Further, assuming traveling wave solutions in the x-direction the functions f, g_3 and g_1 will have the forms

$$f(r, x) = f(r)\, e^{i\xi x} \;, \quad g_1(r, x) = g_1(r)\, e^{i\xi x} \;, \quad g_3(r, x) = g_3(r)\, e^{i\xi x} \tag{14}$$

where $c(= \omega/\xi)$ is the wave speed of the traveling disturbance in the x-direction.

The functions f, g_1 and g_3 will be given by

$$\begin{aligned} f(r) &= A\, H_n(\alpha r) \;, & \alpha &= \sqrt{k_1^2 - \xi^2} \\ g_1(r) &= B\, H_{n+1}(\beta r) \;, & \beta &= \sqrt{k_2^2 - \xi^2} \\ g_3(r) &= C\, H_n(\beta r) \end{aligned} \tag{15}$$

where $H_n(z)$ is the Hankel function of the first kind.

The components of the scattered displacement $\underset{\sim}{u}^{(s)}$, can now be obtained from (9). These are

$$u_r^{(s)} = [A\{\frac{n}{r} H_n(\alpha r) - \alpha H_{n+1}(\alpha r)\} - \xi B H_{n+1}(\beta r) + \frac{in}{r} C H_n(\beta r)] e^{i\xi(x-ct)+in\beta} \qquad (16)$$

$$u_\theta^{(s)} = [\frac{in}{r} A H_n(\alpha r) + i\xi B H_{n+1}(\beta r) + C\{\beta H_{n+1}(\beta r) - \frac{n}{r} H_n(\beta r)\}] e^{i\xi(x-ct)+in\theta} \qquad (17)$$

$$u_x^{(s)} = [\, i\xi A H_n(\alpha r) - i\beta B H_n(\beta r)] e^{i\xi(x-ct)+in\theta} \qquad (18$$

The components of displacement, u_r^*, u_θ^* and u_x^*, of a point of the surrounding medium adjacent to the shell outer surface will be obtained by putting $r = R + h/2$ in (16) - (18) and adding to them the contributions from $\underset{\sim}{u}^{(s)}$. Substitution of these in equation (6) gives three equations for the determination of A, B, C in terms of the components of displacement and rotation of the middle surface of the shell and the amplitude of the ground displacement field. Stresses at any point of the outer medium can now be determined. In particular the traction components τ_{rr}^*, $\tau_{r\theta}^*$ and τ_{rx}^* at $r = R + h/2$ can be expressed in terms of u, v, w, Ψ_x and Ψ_θ. Substituting these in Eqs. (4) one obtains a set of five linear equations in these five unknown quantities, which then can be determined for particular values of ξ, ω, n and particular material and geometrical parameters of the shell and its surrounding medium.

In this paper attention will be focused on the case of axisymmetry ($n = 0$). Further, the interest here is to obtain a set of simplified equations when k_1R and k_2R are very small.

4. Axisymmetric Vibration

Eqs. (4) simplify considerably for axisymmetric motions. In that case motions in the axial plane and transverse to it are independent. In the former case $u_\theta = 0$ and u_r and u_x are given by

$$u_r = u_r^{(i)} - \alpha A H_1(\alpha r) - i\xi B H_1(\beta r) \tag{19}$$

$$u_x = u_x^{(i)} + i\xi A H_0(\alpha r) + \beta B H_0(\beta r) \tag{20}$$

The factor $e^{-i\omega t + i\xi x}$ has been suppressed in the above expressions.

For torsional oscillation, $u_r = u_x = 0$ and u_θ is given by

$$u_\theta = u_\theta^{(i)} + i\xi C H_1(\beta r) \tag{21}$$

In the following we shall consider these two cases separately.

4a. Motion in the axial plane. Using (20) and (21) in the first two equations of (6) and solving for A and B we obtain

$$A = A_1 u_0 + A_2 \Psi_x + A_3 w_0 \tag{22}$$

$$B = B_1 u_0 + B_2 \Psi_x + B_3 w_0 \tag{23}$$

where

$$A_1 = \frac{i\xi H_1(\beta R^*)}{\Delta}, \quad A_2 = \frac{h}{2} A_1, \quad A_3 = \frac{\beta H_0(\beta R^*)}{\Delta} \tag{24}$$

$$B_1 = \frac{\alpha H_1(\alpha R^*)}{\Delta}, \quad B_2 = \frac{h}{2} B_1$$

$$B_3 = \frac{i\xi H_0(\alpha R^*)}{\Delta}$$

$$\Delta = -[\xi^2 H_0(\alpha R^*) H_1(\beta R^*) + \alpha\beta H_1(\alpha R^*) H_0(\beta R^*)]$$

$$u_0 = u - u_x^{(i)*}, \quad w_0 = w - u_r^{(i)*} \tag{25}$$

Here $R^* = R + h/2$.

The expressions for p_i^* $(i = 1, 2, \ldots, 5)$ are given by

$$p_1^* = \mu[-2i\xi\alpha A H_1(\alpha R^*) + (\xi^2 - \beta^2) B H_1(\beta R^*)] \times (1 + h/2R) + \tau_1 \tag{26}$$

$$p_2^* = \frac{h}{2} p_1^* \tag{27}$$

$$p_3^* = p_4^* = 0 \tag{28}$$

$$p_5^* = \mu[\{(\xi^2 - \beta^2) H_0(\alpha R^*) + \frac{2\alpha}{R^*} H_1(\alpha R^*)\} A - 2i\xi\beta\{H_0(\beta R^*) - \frac{1}{\beta R^*} H_1(\beta R^*)\} B](1 + \frac{h}{2R}) + T_3 \tag{29}$$

Here τ_1 and τ_3 are due both to the externally applied forces and the stresses arising from the displacement field $\underset{\sim}{u}^{(i)}$.

Since $p_3^* = p_4^* = 0$, $\underset{\sim}{p}^*$ can be considered as a three dimensional vector whose components will be labelled as p_i. Using (22), (23) in (26) - (29) p_i can be written as

$$p_i = p_{ij} \overline{U}_j + \tau_i \tag{30}$$

where

$$\overline{U}_1 = u_0, \quad \overline{U}_2 = \Psi_x, \quad \overline{U}_3 = w_0$$

$$\tau_2 = \frac{h}{2} \tau_1$$

$$p_{1i} = \mu[-2i\xi\alpha A_I H_1(\alpha R^*) + (\xi^2 - \beta^2) B_i H_1(\beta R^*)] \times (1 + h/2R)$$

$$p_{2i} = \frac{h}{2} p_{1i}$$

$$p_{3i} = \mu [\{(\xi^2 - \beta^2) H_0 (\alpha R^*) + \frac{2\alpha}{R^*} H_1 (\alpha R^*)\} A_i -$$

$$2i\xi\beta \{H_0 (\beta R^*) - \frac{1}{\beta R^*} H_1 (\beta R^*)\} B_i] (1 + \frac{h}{2R}) \tag{31}$$

Substitution of the expressions for A_i and B_i from (24) and (25) in (31) gives

$$p_{11} = \frac{\mu}{\Delta} \{(\xi^2 + \beta^2) \alpha H_1 (\alpha R^*) H_1 (\beta R^*)\} (1 + \frac{h}{2R})$$

$$p_{12} = p_{21} = \frac{h}{2} p_{11}$$

$$p_{13} = - p_{31} = - \frac{i\xi\mu}{\Delta} [2\alpha\beta H_1 (\alpha R^*) H_0 (\beta R^*) +$$

$$(\xi^2 - \beta^2) H_1 (\beta R^*) H_0 (\alpha R^*)] (1 + \frac{h}{2R})$$

$$p_{22} = (\frac{h}{2})^2 p_{11}$$

$$p_{23} = - p_{32} = - \frac{h}{2} p_{31}$$

$$p_{33} = \frac{\mu}{\Delta} [- \beta (\xi^2 + \beta^2) H_0 (\alpha R^*) H_0 (\beta R^*) + \frac{2}{R^*} \times$$

$$\{\alpha\beta H_1 (\alpha R^*) H_1 (\beta R^*) + \xi^2 H_1 (\beta R^*) H_0 (\alpha R^*)\}] \times (1 + \frac{h}{2R}) \tag{32}$$

For the motion under consideration the third and fourth equations of (4) are identically satisfied. The remaining three equations can be written, using Eq. (30), as

$$(A_{ij} + \omega^2 B_{ij}) U_j = p_{ij} U_j^{(i)} - \tau_i \tag{33}$$

where

$$U_1 = u , \quad U_2 = \Psi_x , \quad U_3 = w$$

and

$$U_1^{(i)} = u_x^{(i)} , \quad U_2^{(i)} = 0 , \quad U_3^{(i)} = u_r^{(i)}$$

The matrix elements A_{ij} and B_{ij} are

$$A_{11} = -\xi^2 E_p + p_{11}$$

$$A_{12} = A_{21} = -\frac{D}{R}\xi^2 + \frac{h}{2} p_{11}$$

$$A_{13} = -A_{31} = \frac{E_p \nu}{R} i\xi + p_{13}$$

$$A_{22} = -\xi^2 D - k_x^2 Gh + \frac{h}{2} p_{12}$$

$$A_{23} = -A_{32} = -k_x^2 Ghi\xi + \frac{h}{2} p_{13}$$

$$A_{33} = -\frac{1}{R^2}(E_p + D/R^2) - k_x^2 Gh\xi^2 + p_{33} \quad ; \tag{34}$$

$$B_{11} = \rho_s h$$

$$B_{12} = B_{21} = \rho_s I/R$$

$$B_{22} = \rho_s I$$

$$B_{23} = B_{32} = B_{31} = B_{13} = 0$$

$$B_{33} = \rho_s h \tag{35}$$

Equations of free vibrations of the shell in vacua are obtained from (33) by setting $p_{ij} = \tau_i = 0$. This problem was discussed by Herrmann and Mirsky [11], who obtained the frequencies of vibration as functions of $\xi (>0)$. These frequencies are real and correspond to the first three flexural modes. When the shell vibrations freely in an elastic medium the frequencies will no longer be real for finite ξ and are obtained by solving the transcendental equation

$$|A + \omega^2 B| = 0 \tag{36}$$

The solutions to Eqs. (33) and (36) for finite ξ and k_2 will be discussed in a later communication. This paper will be concerned with the case of small k_2R. Section 5 of this paper is devoted to this case.

4B. Torsional Oscillation. From (21) and the last equations of (6) it is found that

$$C = C_1 v_0 + C_2 \Psi_\theta \tag{37}$$

where

$$C_1 = -\frac{i}{\xi H_1(\beta R^*)}, \qquad C_2 = \frac{h}{2} C_1$$

$$v_0 = v - u_\theta^{(i)*} \tag{38}$$

The traction components $\tau_{r\theta}^*$ may then be written as

$$\tau_{r\theta}^* = -i\beta\xi\mu H_2(\beta R^*) + \tau_{r\theta}^{(i)*} \tag{39}$$

Substitution of (39) in the third and fourth equations of (4) then gives

$$(C_{ij} + \omega^2 D_{ij}) V_j = s_{ij} V_j^{(i)} - \tau_i$$

$$(i, j = 1, 2) \tag{40}$$

where

$$C_{11} = -\xi^2 Gh - \frac{k_\theta^2 G}{R^2}(h + I/R^2) + s_{11}$$

$$C_{12} = C_{21} = -\xi^2 \frac{GI}{R} + \frac{k_\theta^2 G}{R}(h + I/R^2) + \frac{h}{2} s_{11}$$

$$C_{22} = -\xi^2 GI - k_\theta^2 G(h + I/R^2) + \left(\frac{h}{2}\right)^2 s_{11} \tag{41}$$

$$D_{11} = \rho_s h, \quad D_{12} = D_{21} = \frac{\rho_s I}{R}, \quad D_{22} = \rho_s I \tag{42}$$

$$\tau_1 = \tau_{r\theta}^{(i)*}\left(1 + \frac{h}{2R}\right), \quad \tau_2 = \frac{h}{2}\tau_1 \tag{43}$$

Here,

$$V_1 = v \ , \quad V_2 = \Psi_\theta \ , \quad V_1^{(i)} = u_\theta^{(i)*} \ , \quad V_2^{(i)} = 0$$

$$s_{11} = - i\beta\xi\mu\, C_1\, H_2\,(\beta R^*)\ (1 + \frac{h}{2R})$$

In section 5 Eq. (41) will be specialized for low frequency and long wavelength.

5. Low Frequency-Long Wavelength Approximation

Since Eqs. (33) and (40) are rather complicated it would be advantageous to make certain simplifications suggested by observations made by several workers. These observations and their conclusions have been summarized in [12]. It has been observed that in earthquakes that originated far from the site the long period components were predominant in ground motions and the pipelines move nearly with the ground and that the inertia effects do not play significant roles in this case. The neglect of inertia in ground and pipe motion simplifies the p_{ij} and C_{ij}.

If it is assumed that $\xi R >> k_2 R >> k_1 R$ then $H_n(\alpha R)$, $H_n(\beta R)$ may be approximated by

$$H_n(\alpha R) \approx H_n(\beta R) \approx -\frac{2i}{\pi} (-i)^n K_n(\xi R) \qquad (44)$$

If a further assumption is made that ξh is very small then the shear deformation may be neglected. This implies that $\Psi_x = -w'$, $\Psi_\theta = \frac{v}{R} - \frac{1}{R}\frac{\partial w}{\partial \theta}$. Thus there will be three unknowns, u, v and ω. For axisymmetric motion, the equation for v is obtained from Eqs. (4) and $(4)_4$ by setting neglecting rotatory inertia and eliminating Ψ_θ. It is found that v satisfies the equation

$$Gh \frac{\partial^2 v}{\partial x^2} - \rho_s h \frac{\partial^2 v}{\partial t^2} = -p_3^* \qquad (45)$$

On the other hand the equations of axisymmetric bending motion of the shell are

$$(E_p \frac{\partial^2}{\partial x^2} - \rho_s h \frac{\partial^2}{\partial t^2}) u + (-\frac{D}{R} \frac{\partial^3}{\partial x^3} + \frac{E_p \nu}{R} \frac{\partial}{\partial x} + \frac{\rho_s I}{R} \frac{\partial^3}{\partial t^2 \partial x}) w + p_1^* = 0$$

$$(\frac{D}{R} \frac{\partial^3}{\partial x^3} - \frac{E_p \nu}{R} \frac{\partial}{\partial x} - \frac{\rho_s I}{R} \frac{\partial^3}{\partial t^2 \partial x}) u - (D \frac{\partial^4}{\partial x^4} + \frac{1}{R^2} (E_p + \frac{D}{R^2}) +$$

$$\rho_s h \frac{\partial^2}{\partial t^2} - \rho_s I \frac{\partial^4}{\partial t^2 \partial x^2}) w + p_5^* + \frac{\partial p_2^*}{\partial x} = 0 \qquad (46)$$

If the forcing terms are set equal to zero then these equations reduce to those due to Flügge [13].

Following the same procedure as in section 4 a set of equations for the solutions of u and ω can be obtained from Eqs. (46). As an illustrative example let

$$u_x^{(i)} = u_0 I_0 (\xi r) \sin \xi (x - ct) , \quad u_r^{(i)} = - u_0 I_1 (\xi r) \cos \xi (x - ct) \qquad (47)$$

These have been chosen to satisfy the equilibrium equations. Then the solutions of (46) may be taken as

$$u = \bar{u} \sin \xi (x - ct) , \quad w = \bar{w} \cos \xi (x - ct) \qquad (48)$$

The equations for the determination of $\bar{u}$ and $\bar{\omega}$ are, in the matrix form

$$(A + \omega^2 B) U = u_o P \qquad (49)$$

where A and B are 2×2 matrices

$$A = \begin{bmatrix} D\xi^4 + \frac{1}{R^2} (E_p + D/R^2) - T_{11} & \frac{\xi}{R} (D\xi^2 + E_p \nu) - T_{12} \\ \frac{\xi}{R} (D\xi^2 + E_p \nu) - T_{12} & E_p \xi^2 - T_{22} \end{bmatrix}$$

$$B = \begin{bmatrix} -\rho_s h & 0 \\ 0 & -\rho_s h \end{bmatrix} \qquad (50)$$

and U and P are 2×1 matrices

$$U = \begin{bmatrix} \bar{\omega} \\ \bar{u} \end{bmatrix} \qquad P = \begin{bmatrix} -T_{12} I_0(\xi R) + T_{11} I_1(\xi R) - 2\mu\xi I_1'(\xi R) \\ -T_{22} I_0(\xi R) + T_{12} I_1(\xi R) + 2\mu\xi I_1(\xi R) \end{bmatrix} \tag{51}$$

In deriving Eqs. (49) - (51) inertia effects and terms of $O(h/R)$ in the expressions for the traction forces acting on the shell have been neglected. The elements T_{ij} are given by

$$T_{11} = \frac{2\mu}{R}\left[\frac{(K_0(\xi R))^2}{C_2^2 D} - 1\right], \qquad T_{22} = \frac{2\mu}{R}\,\frac{(K_1(\xi R))^2}{C_2^2 D}$$

$$T_{12} = -2\mu\xi\left[1 + \frac{K_0(\xi R)\,K_1(\xi R)}{\xi R \cdot C_2^2 D}\right] \tag{52}$$

where

$$C_2^2 D = \frac{1}{2(1-\sigma)}\left[(K_1(\xi R))^2 - (K_0(\xi R))^2 - 4(1-\sigma)\,K_0(\xi R)\,K_1(\xi R)/\xi R\right]$$

Note that $K_n(\xi R)$ is the modified Bessel function of the second kind.

The coefficients T_{ij} may be interpreted as spring constants of an effective spring replacing the surrounding medium. It may be appropriate to point out here that a large number of papers has been published in which the problem of a vibrating pipe in an elastic medium has been modelled as that of a pipe restrained by springs, the stiffnesses of which have been taken to be constants known a priori. However, as the preceding analysis has shown, these spring constants depend on the wavelength and frequency, the elastic constants and density of the surrounding medium and on the geometrical parameters of the shell (in this case the radius of the shell).

T_{ij}'s can be determined by considering the model problem of a shell of the same geometrical parameters vibrating in a prescribed manner and measuring the resistive forces experienced by the shell. Once the T_{ij}'s are known as functions of frequency and wavelength they can be used in Eqs. (49) for the determination of U.

The limiting values of T_{ij} when the wavelength tends to infinity are

$$T_{11} \doteq -\frac{2\mu}{R} , \qquad T_{12} \doteq -\mu\xi , \qquad T_{22} \doteq \frac{\mu}{R\,\ell n\,\xi R} \tag{53}$$

It is seen that for very long wavelengths T_{12} and T_{22} are negligible and T_{11} becomes independent of the wavelength. Figure 1 shows the variation of T_{ij} with ξR for two different values of σ.

The matrix P given by Eq. (51) represents the total exciting force on the pipe due to the ground movement in the axial direction. It shows that a movement in the axial or radial direction generates both axial and radial forces on the pipe.

Eqs. (49) have been solved for $\bar{u}/u_0$ and $\bar{w}/u_0$ for different values of $M(=\mu/G)$ and ξR. Two different values of σ have been considered, viz., $\sigma = .25$ and $.4$. ν has been taken to be 0.3 and $m(= h/R)$ has been taken to be 0.05. Results have been plotted in Figures 2 - 4. It is seen that for a given Poisson's ratio σ both $\bar{u}/u_0$ and $\bar{w}/u_0$ increase with increasing M. It may be noted that the changes in $|\bar{w}/u_0|$ and $|\bar{u}/u_0|$ with σ is not very pronounced. For small M both $|\bar{w}/u_0|$ and $\bar{u}/u_0$ increase with increasing σ. This is reversed as M increased. For large M the rate of increase of $|\bar{w}/u_0|$ and $\bar{u}/u_0$ with $\ell\,(=\xi R)$ accelerates as ℓ increases. In fact, they become exponentially large. However, for large ℓ the inertial effects will be important and would have to be taken into account. Note that for very small ℓ, $\bar{u}/u_0 \approx 1$ and $\bar{w}/u_0 \approx 0$. It is interesting to note that for small M, $\bar{u}/u_0$ decreases as ℓ increases, the decrease being very sharp for very small M. (see Fig. 4)

6. Conclusion

Formal solutions have been presented for the motion of a long cylindrical pipe surrounded by an elastic isotropic homogeneous medium. Since these solutions are rather complicated for arbitrary wavelengths and frequencies a particular simple case, that of quasi-static motion, has been examined in detail for various axial wavelengths and soil properties. It is shown that the deflection of (hence the state of stress in) the pipe depends critically on the two parameters, (1) the radius to wavelength ratio: $\frac{R}{\lambda} = \frac{\xi R}{2\pi}$; (2) the rigidity ratio of the soil and the pipe: $M = \mu/G$; and to a lesser extent on the Poisson's ratio of the soil. It would appear that these parameters would play even more important roles if dynamic (inertia) effects are included. A treatment of the dynamic problem will be published later. In the quasi-static case it has been found that large radial deflections may be caused by axial ground motion only if M and ξR are large. Now for a typical case of steel pipe in rocky material M is about 0.3. For a steel pipe in sandy silt M is about 7×10^{-4}. It is seen from the numerical results presented that in the former case large radial deflections may occur as the wavelength decreases whereas, in the second situation radial deflections will not be appreciable.

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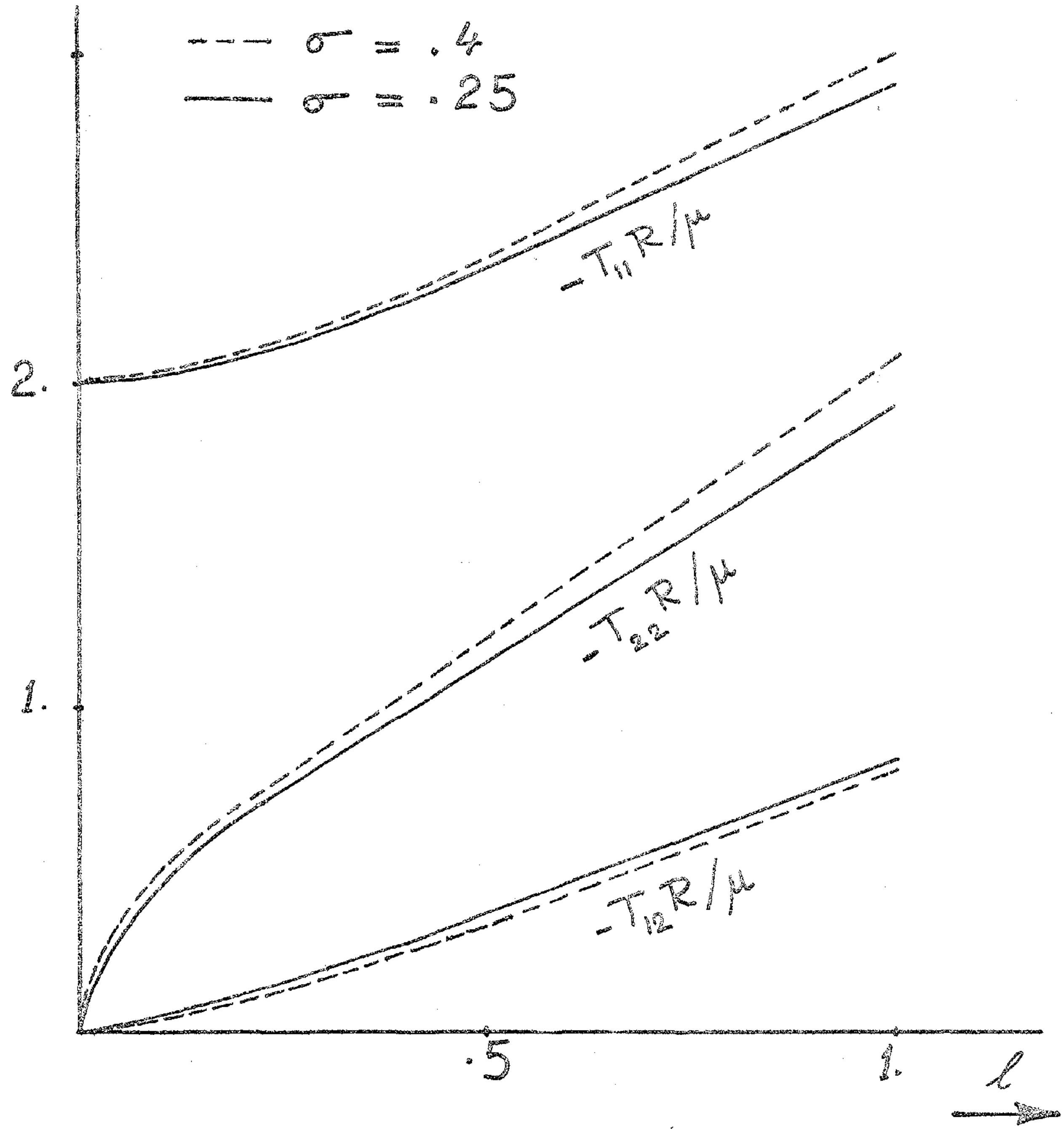

Figure 1

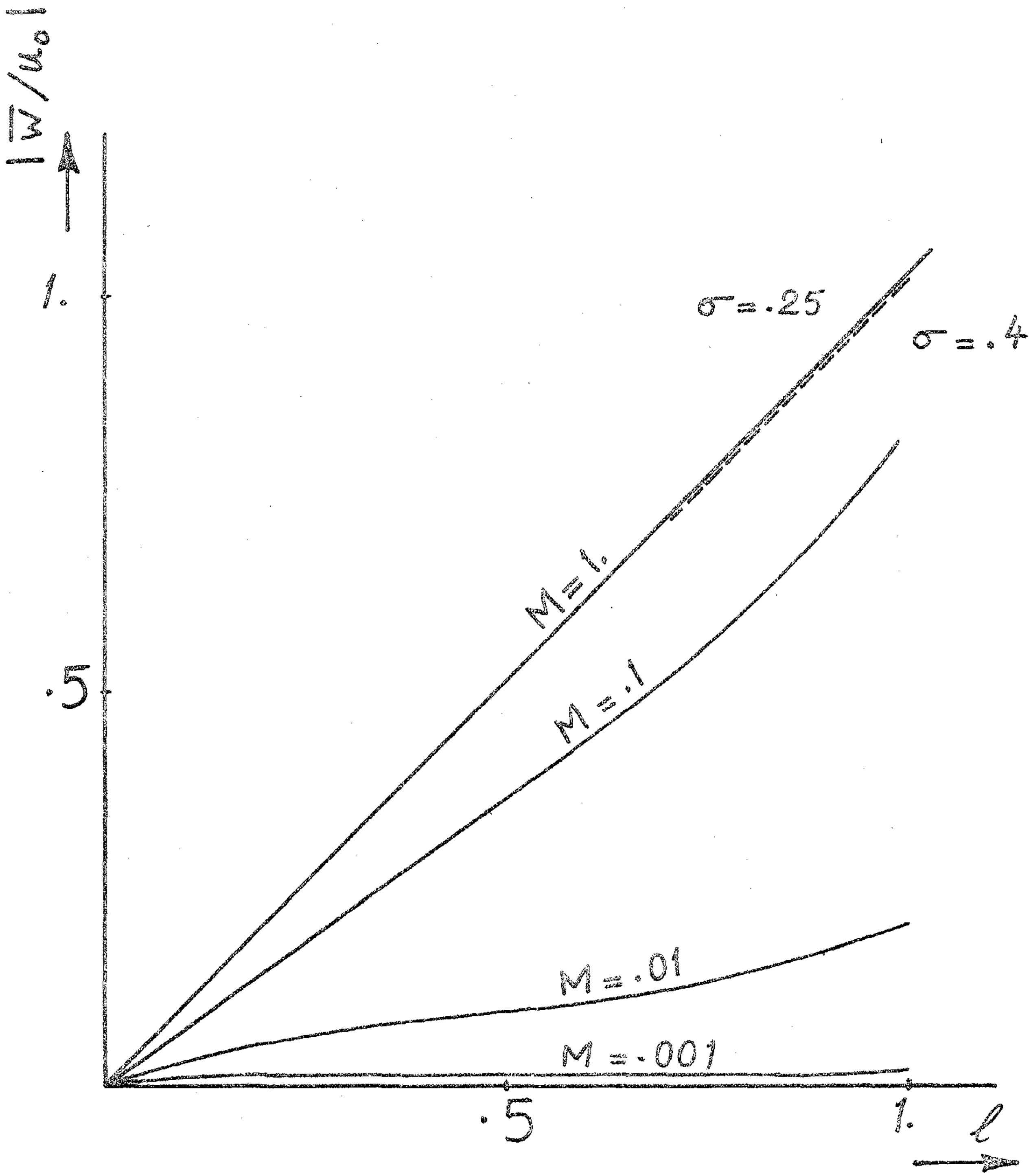

Figure 2

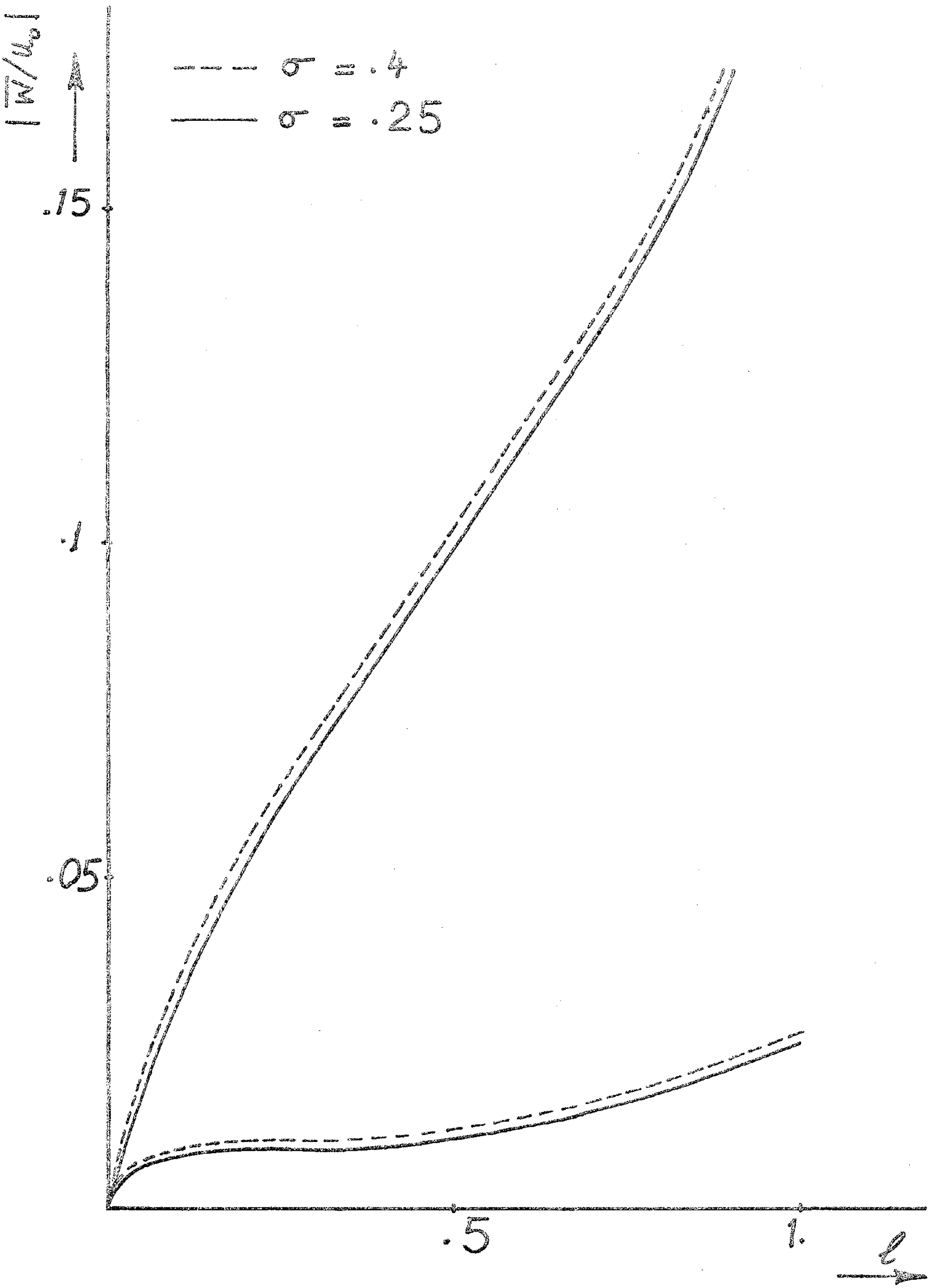

Figure 3

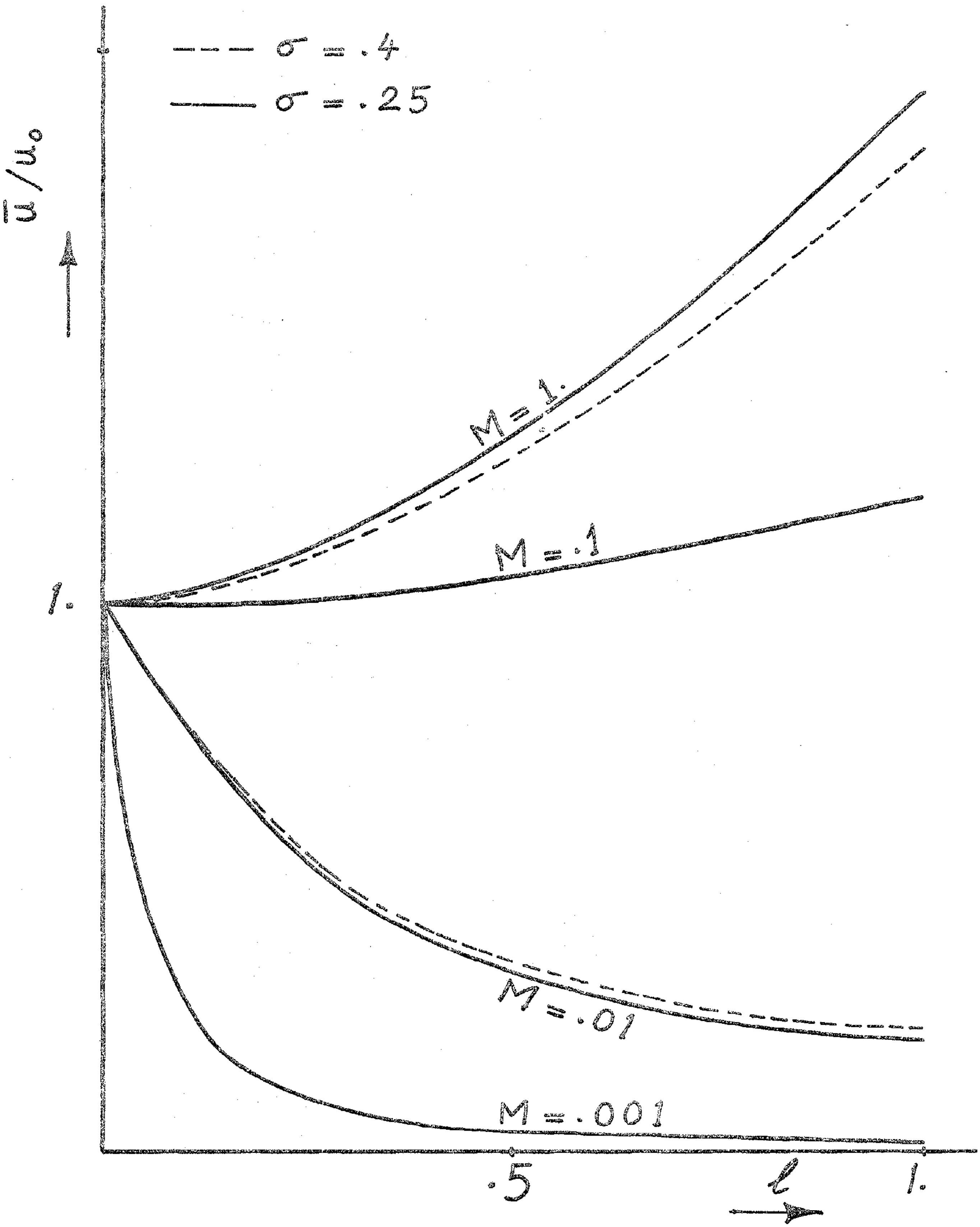

Figure 4