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MEASUREMENTS OF ON-SITE DYNAMIC PARAMETERS FOR SEISMIC EVALUATIONS

by

Roger M. Zimmerman
Professor of Civil Engineering
New Mexico State University

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Kenneth R. White
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Submitted to:

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Earthquake Hazards Mitigation
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In an effort to develop a low cost and relatively simple method to determine the sensitivity of existing bridges to earthquakes, this study developed a non-contact electromagnetic induction device to measure bridge vibration and damping properties. Three small coils were oriented orthogonally to each other and attached to a bridge to see if a static magnetic field could be brought in proximity, to determine three dimensional motion. This tri-coil sensor was used to test two bridges in New Mexico: Rio Grande Canal Bridge in Fairacres and Big Dry Creek Bridge in Glenwood. The major disadvantage of the device is that the bridge vibrations are qualitative in nature and all data is reduced from relative vibrational magnitudes. Therefore, displacements and stresses cannot be measured. However, this pilot test demonstrated the feasibility of the tri-coil sensor and of the relative ease in performing non-destructive forced vibration tests.

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Field tests	Vibration
Bridges (structures)	Measuring

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Earthquake Hazards Mitigation

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I. INTRODUCTION

The purpose of this study was to develop and field test a new method to measure the resonant frequencies, modal shapes, and damping ratios for in-service bridges. These measurements are to be made in an effort to develop a low cost and relatively simple method to determine the sensitivity of existing bridges to earthquake. This information can serve two purposes. First the small data base used for AASHTO code formulation could be expanded and second bridges can be modified or retrofitted if they show particular earthquate sensitivity. In particular damping is a property that can only be determined through field data while vibrational modes can be modeled.

A relatively limited number of field tests have been conducted on bridges [1-19]. In some cases the bridges were carefully instrumented and tested to failure. Efforts have not been directed towards developing a mobile nondestructuve forced vibration testing technique, although such methods are needed. In particular, there is a need to be able to induce controlled vibrational energy such that amplitudes can be varied and the resulting vibrational responses studied. Such field data is extremely useful because it has been shownthat damping varies with the magnitude of the amplitude [15]. More damping data can be used in improving modelliing efforts.

Vibration sensing equipment has been developed around conventional force, acceleration, velocity, or displacement measuring devices. Each have their advantages and disadvantages. Accelerometers are probably the most popular, but they are expensive. A three-dimensional accelerometer can cost over $300. If six of these were placed on a bridge, the resulting cost would be significant. Economics suggested a study to develop a cheaper and simpler method to measure the key dynamic properties of vibrational magnitude, modes, and damping ratios.

It is known that coils moving in a static magnetic field induce currents. It was proposed that three small coils could be oriented orthogonal to each other and attached to a bridge and, if a static magnetic field were brought in proximity, three dimensional motion could be determined. This led to the proposal that was submitted and later funded under the NSF program titled, "Research Initiation in Earthquake Hazards Mitigation." This paper covers the study conducted under contract to New Mexico State University. The study objectives were:

1. Develop non-contact electromagnetic induction device to measure bridge vibration and damping properties.

2. Field test the device using forced vibrations on an existing bridge.

II. PROGRAM REQUIREMENTS

A. Experimental Developments

The program required the development of a shaker to induce the forced vibrations, a sensor to monitor the vibrations and an external magnet to activate the sensor. The program was funded as a research initiation effort and activities were directed solely towards fabricating equipment and conducting sufficient field studies to demonstrate the validity of the methods and not to perform a documented research study on bridge vibrations.

Bridge shakers have been developed around various principles. The double rotating mass and electrohydraulic oscillating mass shakers have been the most popular outside the United States [8,9,12-15,19]. Most U. S. researchers have used traffic actuated vibrations using tractor-trailer combinations. It was felt that a precisely refined force generator was not necessary as the main requirement was to induce a vibration with controlled frequencies and amplitudes. This suggested the use of a single eccentric mass being driven by a variable speed motor. Older model pecan tree shakers operate on the principle that a cable is attached to a tree and to an eye bar that is connected to the housing enclosing a near spherical shaped ball bearing, approximately $6\frac{1}{2}$ inches in diameter, which is eccentrically connected to a $1\frac{1}{2}$ inch diameter shaft. The shaft can be connected to a power takeoff on a tractor with a belt and the frequency of shaking controlled by the tractor operator. This eccentric bearing unit was removed from its tree shaking apparatus and became the basic fixture which the forced vibration shaker was developed. The bearing provided a radial eccentric distance of 13/16 inch.
The pecan shaker shaft was fixed to the top of twin channel beams which had been sectioned from a school bus used in an earlier study. The shaker unit was energized by placing a three-horsepower Century D. C. Motor on the channel beams. The motor operated at 120V and drew 22.6 Amp from a AC power generator that was rectified to DC voltage and oper, etc. The motor had compound windings to give a variable speed control of $\pm 10\%$ for any pulley combination. Different size pulleys were coupled with the pecan shaker to give a range of speeds. For example a 3.5 inch pulley produced a speed of 810 RPM (13.5 Hz) and a 6.5 inch pulley produced a speed of 1520 RPM (25.3 Hz).

The overall shaker was designed such that the maximum vertical force due to the eccentric mass was less than that of the dead weight of the shaker itself. The addition of lead to the bus frame caused the total weight of the shaker unit to be approximately 1250 lbs.

A simple spring mass system analysis shows that the inertial force generated by eccentric masses is linear with the rotating mass and varies with the square of the frequency. This meant that the dead weight limit required matching of the frequency and weight. The eccentric bearing was connected to a 26 lb. steel basket that was guided to reciprocate in a vertical direction. This weight was adequate for high frequencies, and up to 150 lbs of lead could be added to the basket for the lower frequencies. Thus the shaker was formed around a rigid frame containing a mass vibrating along a single axis. It was necessary to transport the shaker nearly 180 miles for a bridge test. The frame was attached to a trailer axle for this transporting. Upon arrival at the test site, the axle and wheel were removed and the entire unit lowered on two 4" x 40" channels which served as footings. One inch expanded PVC padding was placed under the footings for a cushioning effect. Figure 1 shows the shaker in place on the highway bridge.

The sensor, which was the central new feature to be incorporated in the study, was proposed in an effort to develop a simple and economical method that could be used to detect and measure the three desired properties. Figure 2 shows two sensors in place with the accompanying electromagnet. Three coils ¾ inches in diameter and 7/8 inches long manufactured as Miniature Type Magnetic Pickups by Power Instruments Inc., were rigidly attached to an aluminum block with nominal dimensions of 1" x 1" x 2". Aluminum was selected as it has approximately the same magnetic permeability as air. The aluminum coil unit was attached to a ½" x 2" x 3" aluminum plate contaning a 6-lead electrical plug and which could be bonded to a bridge with 3M contact cement. The sensor unit was inexpensive as each coil cost approximately $10 and the entire sensor could be fabricated for under $50.

The coils are oriented orthogonal to each other such that three dimensional vibrations can be monitored and the individual directional components identified. A schematic is shown in Figure 3. The induced signals are small and require amplification. Signal amplification consisted of a 48dB gain through the system. Calibration was with a sine wave at 100 Hz and signals were filtered above this frequency. Data was displayed on a Tekronix Type 564 B Storage Scope with a four trace Type 3A74 plug-in unit. Thus, four signals could be displayed and stored on the screen simultaneously. Permanent records of traces could be obtained with a polaroid camera.

The procedure was to activate the three coils of any one sensor by bringing a magnet in proximity and plugging in the leads from the oscilloscope and displaying the signal on the screen. The fourth channel was connected to a coil located on the shaker that recorded the RPM by sensing the passing of a magnet on the rotating eccentric shaft. Each time the rotating magnet passed the coil a sharp signal was induced and the time required for one cycle could be accurately observed on the scope. Typical signals for the three coils and the RPM indicator are shown in Figure 4. The signals, as displayed in Figure 4, were manually triggered for storage on the scope by visually monitoring it during

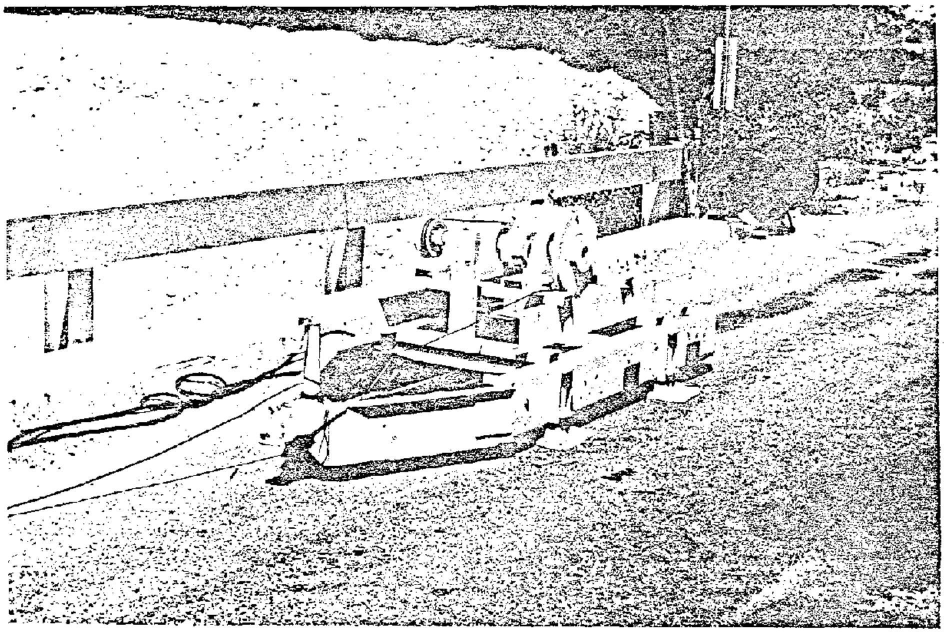


Figure 1. Shaker on Big Dry Creek Bridge

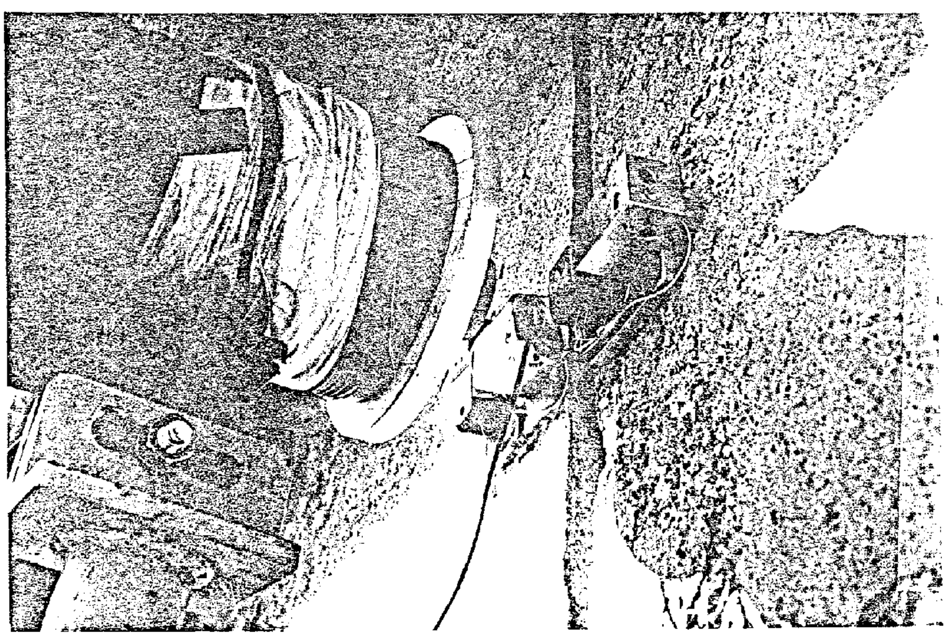


Figure 2. Sensors D & E on Girder Ends with Electromagnet

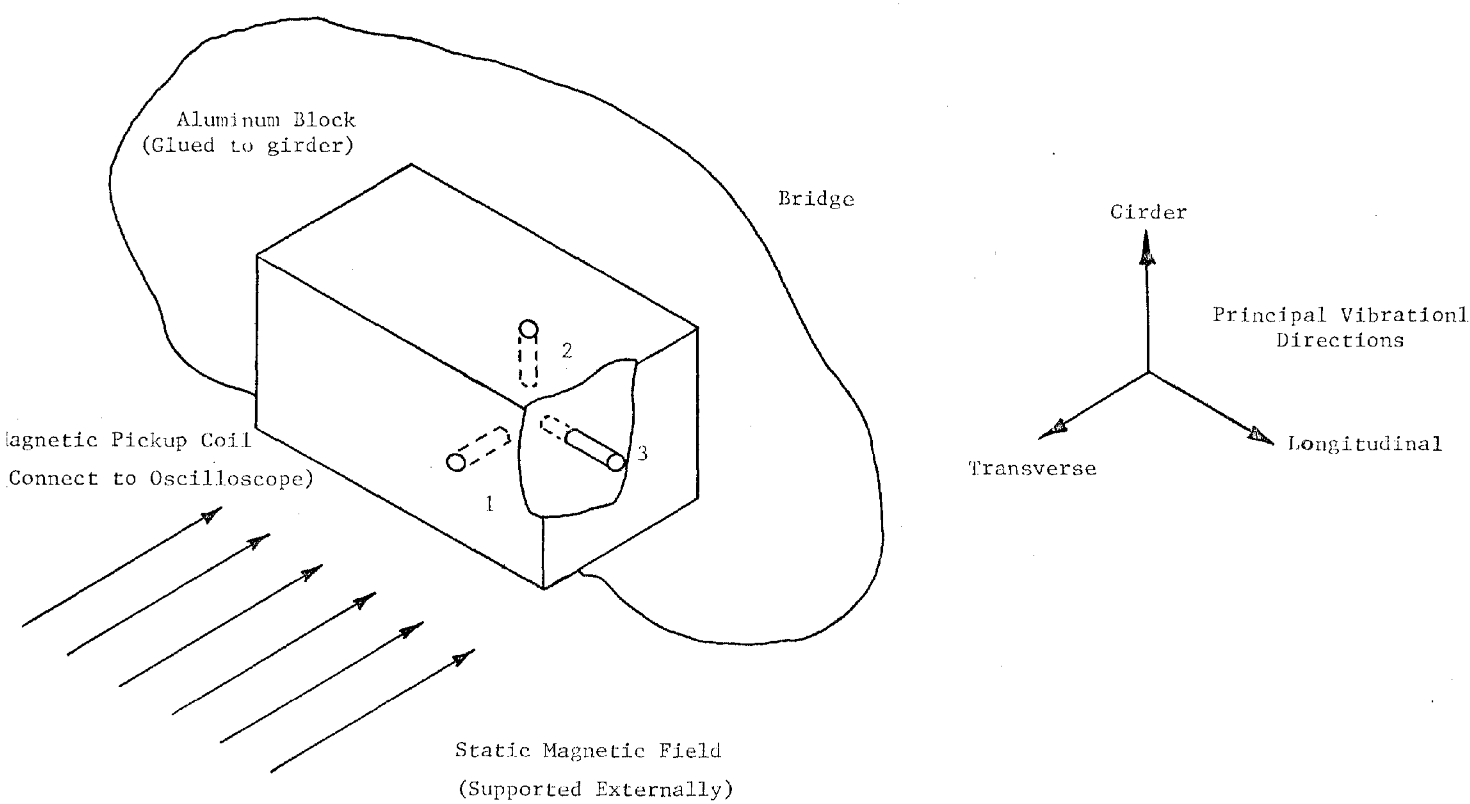
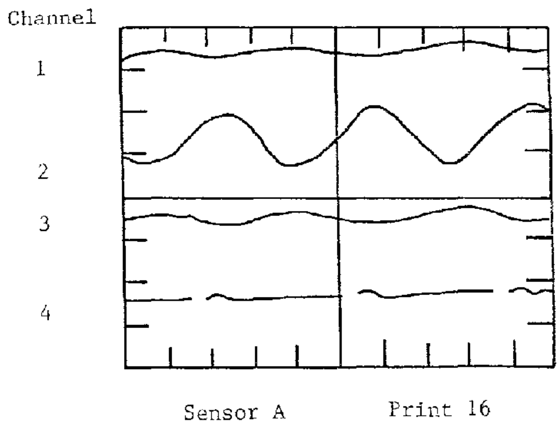


Figure 3. Tricoil Sensor Principle



Electromagnet

1 orientation: vertical

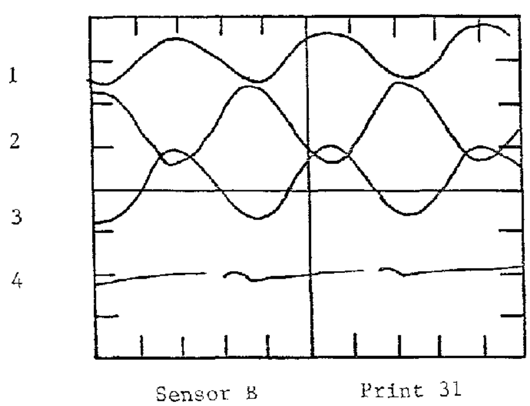
1 distance: 3/4"

Oscilloscope

Vertical Scale: 50mV/$_{cm}$ channel 1 & 3

100mV/$_{cm}$ channel 2

Horizontal Scale: 20 ms/$_{cm}$



Electromagnet

1 orientation: vertical

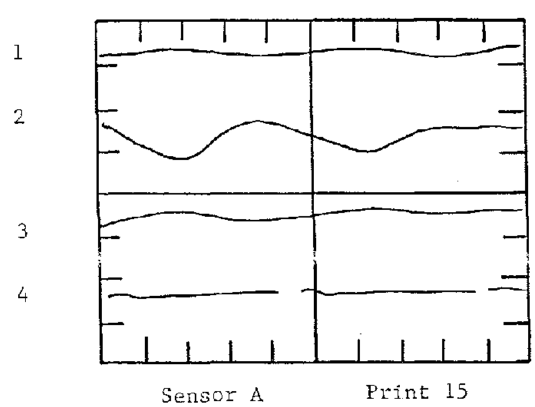
1 distance: 1/2"

Oscilloscope

Vertical Scale: 20 mV/$_{cm}$ channel 1 & 3

50 mV/$_{cm}$ channel 2

Horizontal Scale: 20 ms/cm



Electromagnet

1 orientation: vertical

1 distance: 3/4"

Oscilloscope

Vertical Scale: 50 mV/$_{cm}$ channel 1 & 3

100 mV/$_{cm}$ channel 2

Horizontal Scale: 20 ms/cm

Figure 4. Typical Oscillograph Traces

bridge vibration. A single electromagnet was used to induce the current in the coils in the figure, but the same result could have been obtained with a permanent magnet. A permanent magnet having a flux density of approximately 1100 gauss at the surface could energize the coils up to a distance of one inch.

An electromagnet was fabricated around a 2 inch diameter cold rolled steel core. Thirty pounds of #18 gage enamel coated copper wire were wrapped around this core over a 7 inch length. This provided a coil having approximately 35 layers of wire. The electromagnet operated at 110 volts DC and a 1.75 amps with minor heat vise over long periods of time. Electrical energy was provided in the field by rectifying 110 AC from a portable generator to DC. This magnet had a field density of approximately 2500 gauss at the center of the core. The electromagnet could activate the sensors for distances up to 5 inches.

The tri-coil sensor has an advantage and disadvantage in this mode of operation. The advantage is that the desired vibrational properties can be found as long as a signal can be received on the oscilloscope. The disadvantage is that there is no calibration as to the magnitudes of the vibrations. There must be controlled in the design of the force generating system.

B. Theoretical Developments

The program was designed as an experimental field oriented effort. The bridges to be tested were amenable to analytical modeling and vibrational modes and magnitudes were computed in an effort to validate the field measurements. ICES - STRUDL - II computational capabilities were available and contained dynamic analysis features. This computer package contains three analytical techniques appropriate for bridge modeling. The first is the plane frame solution in which all loads and deformations are in the same plane. In this case a bridge can be modeled as an equivalent beam. The second method is the plane grid. Here the loads and deformations are orthogonal to the plane of the structure. Finally, there is the finite element approach in which the total structure is defined by combining sub-elements with definable force-deformation properties.

Each of these programs has provisions whereby the structural properties of the materials and the corresponding structure geometry can be inputted and a dynamic model analysis requested. The computed results are eigenvalues and eigenvectors describing the model behavior. It should be pointed out that the plane frame solution was limited to planar eigenvectors.

The first three modal shapes for a typical simply supported bridge are illustrated in Figure 5. The torsional aspects can cause the second mode to be very near the first in magnitude. The modal shapes show that the first and second modes can be distinguished by observing the relative motions of points at the center and on the outside edges.

III. Field Tests

A. Test Progression

The test plan involved a progressive development of the test equipment. The first requirement was to verify that the tri-coil sensor principle would work on a bridge in the field. A bridge on Interstate 25 over Missouri Street in Las Cruces, New Mexico was selected for the first test. In this case a single tri-coil sensor was glued to the center of an end span and a permanent magnet brought in proximity by supporting it on the slope of the abutment fill under the end support. Vibrations were actuated by traffic and the tri-coil sensor displayed adequate signal strength. No quantitative data was recorded, Electrical grounding of the sensor became the only new variable to consider.

Two bridges were selected for analysis and field testing. These bridges were made available for field testing by the Elephant Butte Irrigation District and the New Mexico State Highway Department. The bridges were selected because of their accessibility and minimum inconvenience to users for the periods of testing. The general properties are given in Table 1. Both bridges were simply supported.

The next phase involved the testing of the shaker and verification that the basic system worked. The bridge selected was the short span over the Rio Grande Canal. The test involved the first field operation of the shaker and most efforts were directed towards matching the shaker to the relatively light bridge. The shaker was operated without the basket and the only eccentric weight was that of the eccentric bearing and housing. The housing of the bearing was connected to a short piece of pipe to keep it from rotating. The sensor was glued to the center of an outside Tee leg, and the magnet was attached to an external beam simply supported on the abutments. The magnet support beam was relatively light and had a much lower fundamental frequency than the bridge. Therefore, the measured vibrations were that of the bridge relative to the magnet--both being supported on the same abutments. The bridge has been modeled as a plane frame and the first mode vibration predicted. Field measurements included measuring the periodic motion of the sensor by taking measurements on the memory oscilloscope. The theoretical resonant frequency was 21 Hz and the measured was 20 Hz. It was estimated that the deviator

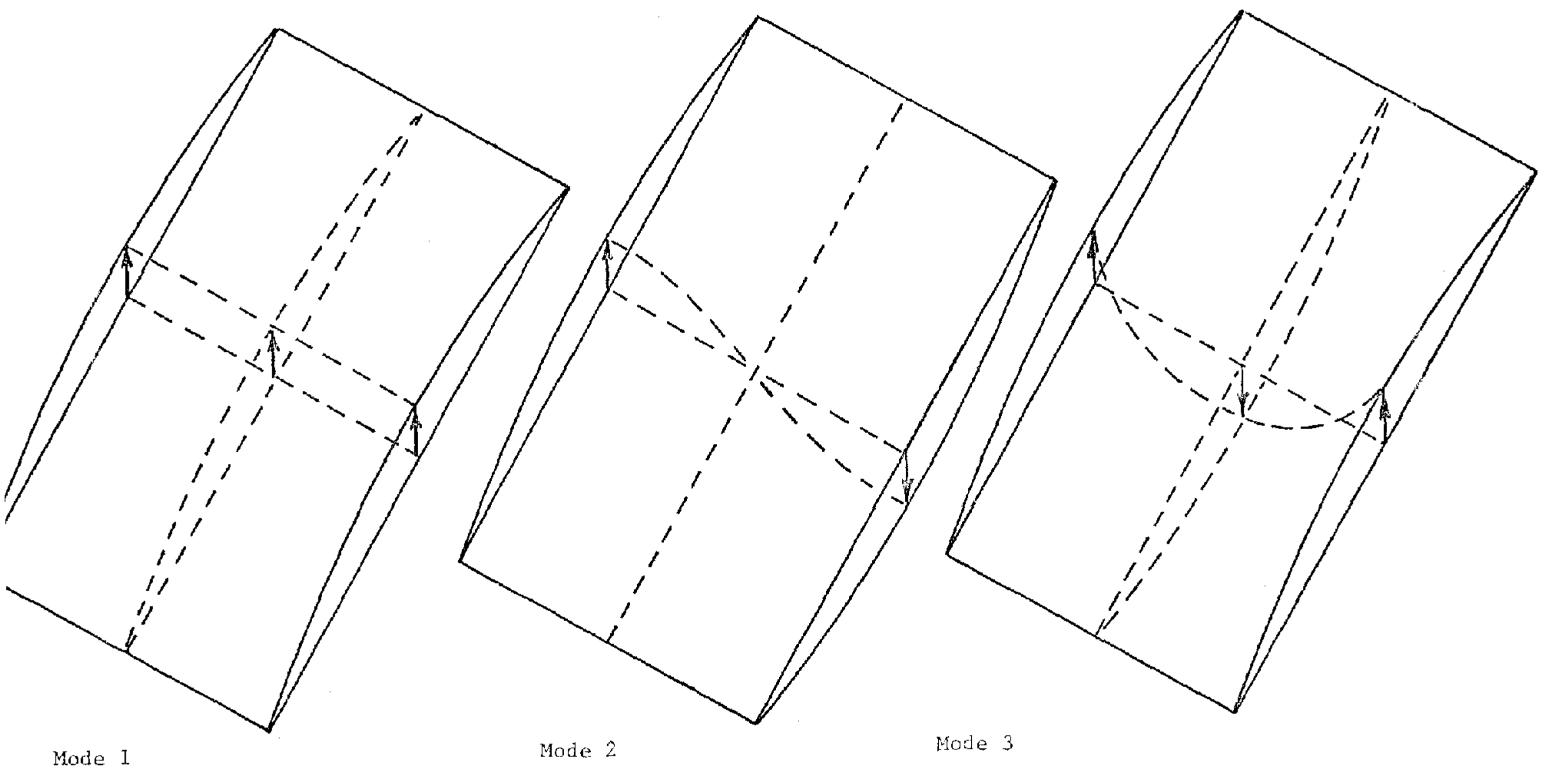


Figure 5. Typical Mode Shapes

Table I

Rio Grande Canal Bridge, Fairacres, New Mexico

Length: 20' 0" 1 span

Width: 16' 5"

Construction: 6 prestressed twin tee sections with no deck surface

Girder Depth: 15"

Support: Simple

Assume 28 day concrete strength of 4500 psi.

Big Dry Creek Bridge, Glenwood, New Mexico, US180

Length: 33' 7" each of 3 spans

Width: 20' 0" at roadway

Construction: Monilithic deck and 5 girders - reinforced concrete

Girder Depth: 28"

Supports: Abutments - with 2 intermediate reinforced concrete piers

Assume 28 day concrete strength of 300 psi.

force caused by the shaker was ±600 lbs. The preliminary tests on the Rio Grande Canal bridge verified that the forced vibration driver and sensor system would work. The passing of this stage led to the tests on the highway bridge where higher order vibrational modes and damping were to be measured.

The testing on the Bid Dry Creek bridge involved some logistics. The bridge was located approximately 180 miles from Las Cruces which meant that all equipment had to be transported there. It was planned that the sensor system would be installed, and the electrical sensor and recording system would be checked out using traffic induced vibrations within one day. This method required the development of a procedure such that the triggering of the oscilloscope would coincide with the vibration induced by the passing vehicles. Many trials were necessary to obtain ten vibration traces. Of these ten, eight were taken with the permanent magnet and two with the electromagnet. No damping was observed. The traffic actuated measurements could be conducted without impeding normal traffic flow. Arrangements had been made with the New Mexico State Highway Department to close one lane of traffic for the second day of testing. The eastbound lane was closed and the shaker placed in the center of the lane on the end span as shown in Figure 1. This span was selected to facilitate the placement of the magnets under the sensors.

Six sensors were attached to the bridge. Sensor B was placed on the bottom of the center girder at midspan. Sensor F was placed on the lower side of this girder, again at midspan. These two sensors were placed near each other to verify the directional properties of the two sensors because they were orthogonal to each other in this placement. Sensor A was located on the bottom of the east end girder at midspan in order that distinction could be made between the first and second vibrational modes. Sensors D and E were located at mid depth on the ends of the end and center girders over the pier. These two are shown in Figure 2. This was done to detect relative longitudinal motion of the two girders, if any. Sensor C was placed on the end of the pier cap under the ends of the girders containing the other two sensors.

The test procedure involving the shaker and sensors consisted of locating the electromagnet near the sensor in either a vertical or horizontal alignment. The magnet would be activated and the shaker started. The vibration would be visually monitored on the oscilloscope and then the frequency manually adjusted to a resonance. When an acceptable wave form was formed, it was stored on the scope and the shaker turned off. A polaroid picture was made of the oscilloscope trace in order that numerical data could later be obtained. In the case of measuring the damping ratio, the scope image was stored after the shaker had been shut off and the signal was decaying. The shaker stopped motion in just a few cycles and the bridge continued to vibrate for a few cycles longer. A total of 30 records were made using the shaker.

The damping ratio was calculated using the logarithmic decrement of succeeding cycles. Data from as many cycles as possible were calculated from any one print and the results averaged for that particular run. A number of shaker runs were made to see if the damping ratio could be calculated by measuring the amplitudes of the received signals on both sides of resonance, and plotting these in order that the frequency equation could be used to calculate this quantity. This method did not work and all damping ratios were calculated using the logarithmic decrement method.

B. Test Results

The field tests on the bridge over Big Dry Creek exercized the entire system and the results are summarized in this section. Table II summarizes the first mode test data that was reduced to find the vibrational frequencies, modes, and damping ratios. The data is organized by sensor to show variation within the data and between the sensors. Damping ratio data is separated for clarity. The first set of data pertains to the fundamental resonant frequencies. The table shows that the 18 measurements taken with the shaker system were reasonably consistent with an average of 13.75 Hz. The two clean readings obtained with traffic actuations show more scatter and an average of 13.52 Hz. Multiple wheel effects caused a number of the traffic actuated prints to be voided. Although this data is easier and cheaper to obtain, it was found that the problems in triggering the system and the difficulties in making proper frequency identification make this a much less desirable method.

Buried in Table II are data from the ends of the girder and the pier cap. Small vibrations in the longitudinal direction, parallel to the girder, were observed for the pier cap. Even smaller vibrations in this same direction were noted for the end of the girder. This suggests some longitudinal freedom of motion of the pier cap. This observation was supported by a review of the prints of the pier cap motion under traffic actuated conditions. In one case a truck passed over the pier cap and a highly damped vertical vibration of the pier cap at a frequency of 20 Hz was observed. A review of the physical restraints affecting the pier cap suggests that these were the dominant conditions. The channel under the bridge has been cleaned out to a level near the footing of the pier, which in turn was resting on piles. The lack of fill material around the

Table II

First Mode Resonance Measurements

Sensor	Print	Location	Frequency	Actuation	Magnet
A	16	Outside Girder	13.94 Hz	Shaker	Electromagnet
	37	"	14.02	"	"
	38	"	14.08	"	"
	39	"	13.30	"	"
	14	"	13.79	"	"
B	11	Center Girder	13.44	"	"
	25	"	13.37	"	"
	26	"	13.39	"	"
	27	"	13.70	"	"
	28	"	13.61	"	"
	31	"	13.92	"	"
C	22	Pier Cap	13.42	"	"
	23	"	13.70	"	"
E	19	Girder End	13.57	"	"
F	32	Center Girder	14.29	"	"
	33	"	14.13	"	"
	34	"	13.89	"	"
	35	"	13.94	"	"
		Ave	13.75		
B	2	Center Girder	11.90	Traffic	Permanent
B	10	"	15.15		Electromagnet
		Ave	13.52		

Damping Ratio

Sensor	Print	Location	Damping Ratio	Actuation	Magnet
B	29	Center Girder	0.06	Shaker	Electromagnet
A	15	Outside Girder	0.07	"	"
B	3	Center Girder	0.07	Traffic	Permanent

Table III

Computer Method

Mode	Plane Frame	Plane Grid	Finite Element	Field Meas.
1	12.9 Hz	12.9 HZ	12.7 HZ	13.8 Hz
2	-	14.3	14.7	15.7
3	-	19.2	20.1	-

pier and the inherent longitudinal freedom found in simply supported spans appeared to allow the pier cap to vibrate vertically and longitudinally.

The damping ratio could be clearly measured on three prints. Ratios of all succeeding amplitudes were taken and an average established for a particular print. The table shows that there is reasonable consistency between the shaker and traffic actuated values. It was felt that the shaker method gave the best results and would be more consistent.

Table III summarizes the modal data for the three theoretical methods and the average field data. The table shows that the field data is approximately 9 percent higher for the first mode and 10 percent higher for the second mode. Third mode vibrations were not measured in the field. Several factors may explain the fact that the field measurements were higher. All properties of the bridge were assumed and only physical dimensions were measured. The bridge railing was omitted in the calculations. The bridge wearing surface was added as mass only. It was noted that sand had accumulated in the gap between the girders and the abutment and center span girders and the possibility is that this stiffened the net structural system. The net conclusion is that the different modeling methods compared well with each other that the field tests verified the models adequately.

IV. SUMMARY AND CONCLUSIONS

The objectives of this study were to develop a non-contact electromagnetic induction device that could be used to measure bridge vibration and damping properties and to demonstrate this device in the field using forced vibration. These objectives were fulfilled. The tri-coil sensor proved to be an economical and relatively simple device to operate. A single bridge was tested in one day illustrating the relative ease of operation. The major disadvantages of the device is that the bridge vibrations are qualitative in nature and all data is reduced from relative vibrational magnitudes. Thus, displacements and stresses cannot be measured.

This pilot test demonstrated the feasibility of the tri-coil sensor and of the relative ease in performing non-destructive forced vibration tests. This leads to two possible future efforts. The first is to improve the sensor to be able to measure displacements accurately. The second is to develop a mobile unit that can be used to excite bridges to low level vibrations and to be able to measure the desired dynamic properties.

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