DERIVATION OF EQUATIONS OF MOTION

OF A

FREE RIGID BODY STEAM GFNERATOR

SUPPORTED BY

VERTICAL ELASTIC RODS

Prepared by

H. Lo and J.L. Bogdanoff

School of Aeronautics and Astronautics
Purdue University
West Lafayette, Indiana

Submitted to

THE NATIONAL SCIENCE FOUNDATION

October, 1977

50272-101

REPORT DOCUMENTATION PAGE	1. REPORT NO. NSF/RA-770920	2.	3. Recipient's Accession No. PB80-222912

4. Title and Subtitle
Derivation of Equations of Motion of a Free Rigid Body Steam Generator Supported by Vertical Elastic Rods

5. Report Date October 1977

6.

7. Author(s)
H. Lo, J. L. Bogdanoff

8. Performing Organization Rept. No.

9. Performing Organization Name and Address
Purdue University
School of Aeronautics and Astronautics
West Lafayette, Indiana 47907

10. Project/Task/Work Unit No.

11. Contract(C) or Grant(G) No.
(C)
(G) ENV7401575

12. Sponsoring Organization Name and Address
Engineering and Applied Science (EAS)
National Science Foundation
1800 G Street, N.W.
Washington, D.C. 20550

13. Type of Report & Period Covered

14.

15. Supplementary Notes

16. Abstract (Limit: 200 words)

Linear equations were derived for the motion of a steam generator supported by a number of vertical elastic rods attached to a fixed support. These derivations were performed because current computer codes do not account for the potential energy encountered in this type of setup. Three dimensional motions and gravity were considered. Vertical rods were treated as pin-ended with different lengths and axial spring constants. The arrangement of the supporting points in the plane of the top fixed support was arbitrary. The free motion of the rigid body was that of a complex pendulum-bifilar type. The principal thrust concerned deriving equations to calculate the potential energy (V) in terms of the three components of the linear motion of the mass center (G) and three components of rigid body rotation. These six coordinates were assumed to be small. Geometry of motion was considered in detail since customary first order analysis proved inadequate in obtaining the potential energy (V). Results were applied to TVA unit #3, a 1200 MW coal-fired steam generator in Paradise, Kentucky.

17. Document Analysis a. Descriptors

Boilers
Earthquakes
Structural analysis
Kentucky
Mathematical models
Rods
Potential energy
Equations of motion
Linearity
Linear algebraic equations

b. Identifiers/Open-Ended Terms

Earthquake Hazards Mitigation

c. COSATI Field/Group

18. Availability Statement	19. Security Class (This Report)	21. No. of Pages
NTIS	20. Security Class (This Page)	22. Price

(See ANSI-Z39.18) *See Instructions on Reverse* **OPTIONAL FORM 272 (4-77)** (Formerly NTIS-35) Department of Commerce

CAPITAL SYSTEMS GROUP, INC.
11301 ROCKVILLE PIKE
KENSINGTON, MD. 20795

NOTATION

SG: steam generator treated as a rigid body;

OXYZ: rectangular cartesian coordinates fixed in space;

$\vec{I}$, $\vec{J}$, $\vec{K}$: unit vectors along OXYZ coordinates in usual order; gravity in $\vec{K}$ direction; in equilibrium position, c.g. G of SG is on OZ axis.

0_1 is point at which OZ axis intersects roof of SG in equilibrium position;

$\overline{00_1} = \ell_o$;

$\overline{0_1G_1} = c$;

Gxyz: rectangular cartesian axes fixed in SG and parallel to OXYZ axes in equilibrium position;

jth rod is attached to fixed space A_{jo} with coordinates x_j, y_j, ρ_j relative to OXYZ axes;

jth rod is attached to SG at A_j with coordinates x_j, y_j, $-(c + \lambda_j)$ relative to Gxyz axes.

ℓ_j: length of jth rod;

t_j: tension (force) in jth rod in equilibrium position;

k_j: spring constant (force/unit length) of jth rod;

e_j: extension in jth rod measured from equilibrium position;

$c_j = c + \lambda_j$;

m: mass of SG;

$G\xi\eta\zeta$: rectangular cartesian axes fixed in SG that coincide with Gxyz axes used to specify mass distribution of SG;

A,B,C,F,G,H: moments and products of inertia of SG with respect to $G\xi\eta\zeta$ axes;

e_{jo}: initial extension of jth rod in equilibrium position;

u, v, w: displacement components of G, with respect to OXYZ axes, from equilibrium position;

ϕ, θ, ψ: angular rotations of SG about Gxyz axes in order given;

A_j': displaced position of A_j.

λ_j: distance j^{th} vertical rod attachment point is above top of SG

ρ_j: distance j^{th} vertical rod attachment point is below reference plane of top girder system

$\vec{R}_{A_j'}$: position vector of A_{j_o}

$\vec{R}_{A_j}$: $\vec{R}_{A_j'} - \vec{R}_{A_{j_o}}$

1. Introduction

The purpose of this report is to derive the linear equations of motion of a rigid body (steam generator) supported by a number of vertical elastic rods attached to a fixed top support. Three dimensional motion is considered. The vertical rods are considered as pin-ended; their lengths and axial spring constants may be different; and the arrangement of the supporting points in the plane of the top fixed support is arbitrary. Gravity acts. A schematic of the system considered is shown in Figure 1.1.

The free motion of the rigid body is obviously of a complex pendulum-bifilar type. The principal task is to get the potential energy V in terms of the three components of the linear motion of the mass center G and of the three components of rotation of the rigid body. These six coordinates are assumed small. To obtain V requires that the geometry of the motion be considered in some detail as the usual straight forward first order analysis is not adequate.

The principal reason for this derivation is that current computer codes do not account for the type of potential energy encountered in this probalm. We have found no prior derivations up to this time.

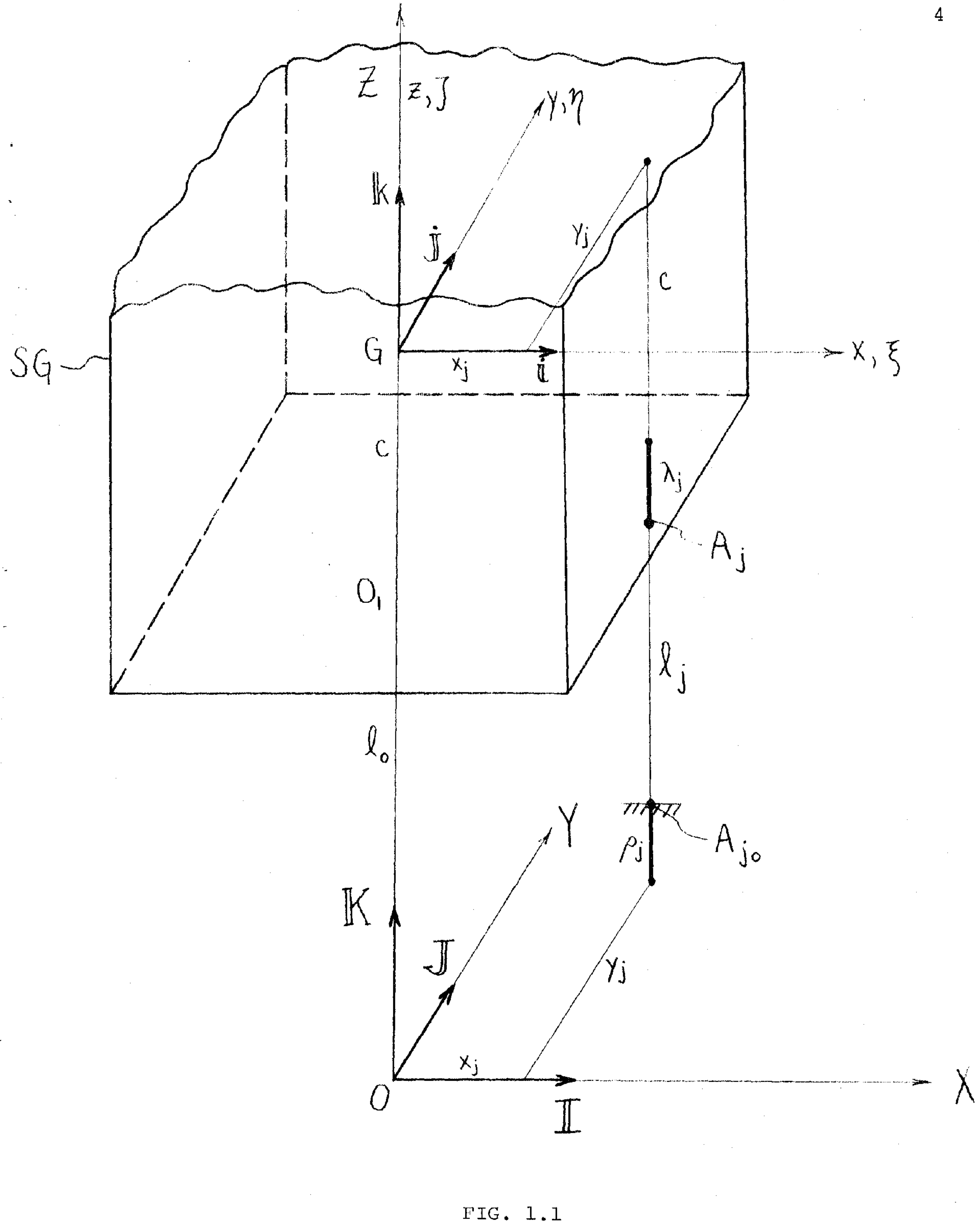


FIG. 1.1

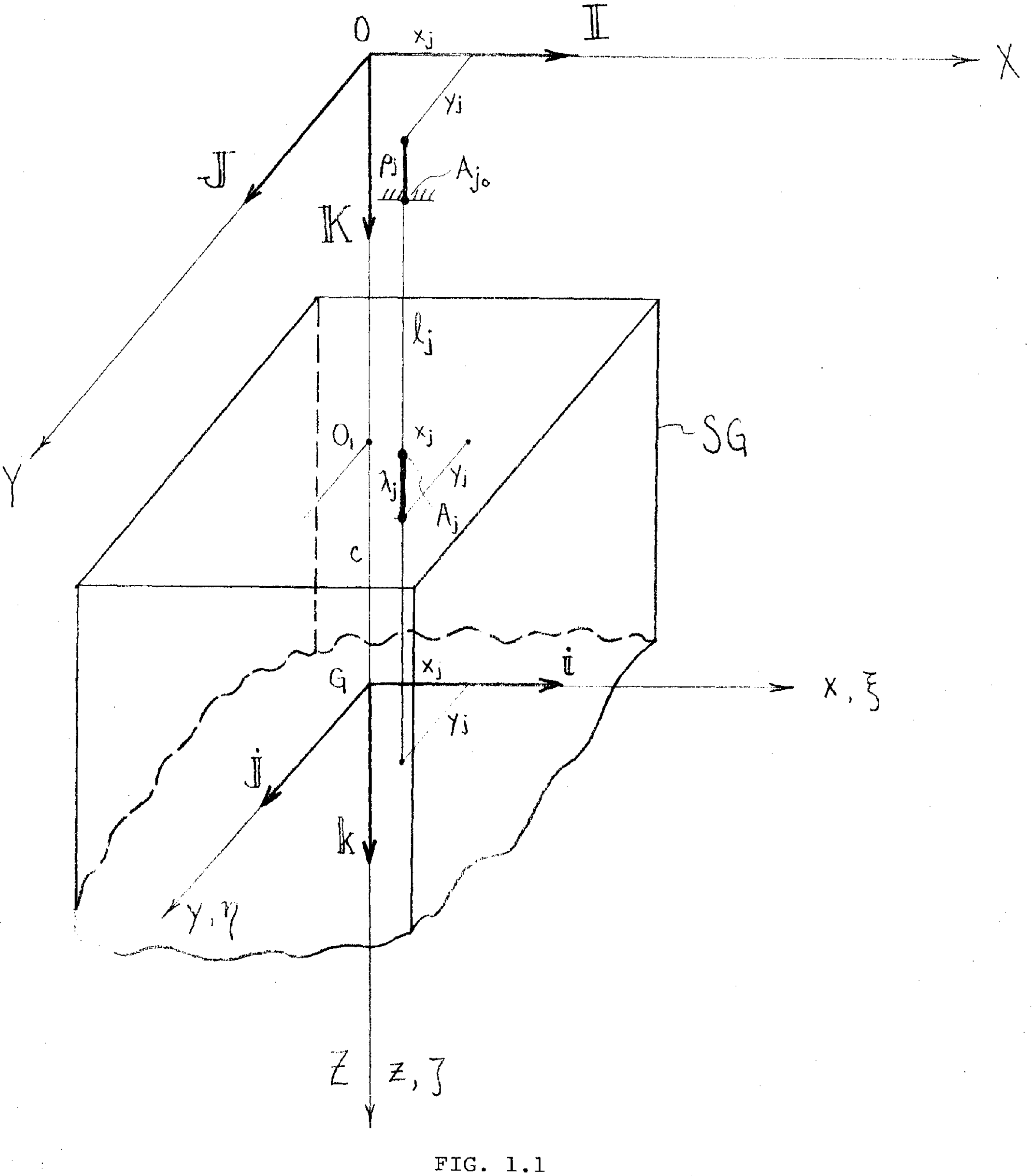

O
x_j
𝕀
X
y_j
𝕁
ρ_j
A_{j_0}
𝕂
ℓ_j
Y
O_1
x_j
y_j
SG
λ_j
A_j
c
G
x_j
𝕚
x, ξ
y_j
𝕛
𝕜
y, η
Z
z, ζ

FIG. 1.1

2. Some Preliminaries

We need the formulas connecting $\vec{i}$, $\vec{j}$, $\vec{k}$ in the displaced position with respect to $\vec{I}$, $\vec{J}$, $\vec{K}$.

Figure 2.2a shows the rotation ϕ about Gx; we see that

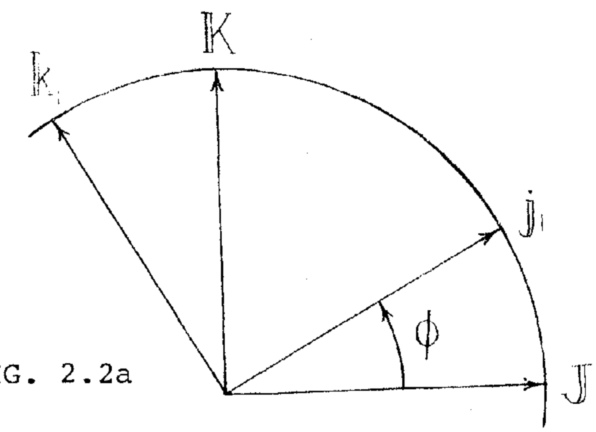


FIG. 2.2a

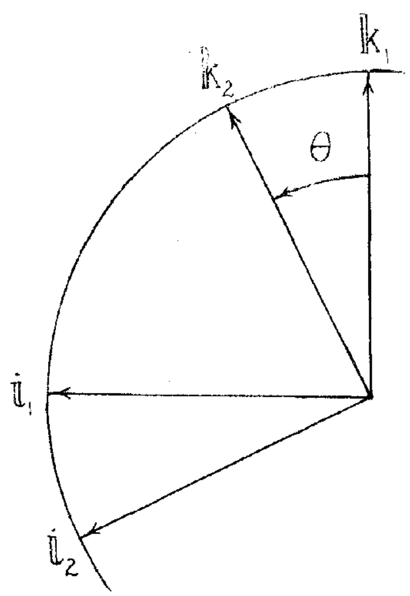
$$\vec{i}_1 = \vec{I}$$

$$\vec{j}_1 = \cos\phi\ \vec{J} + \sin\phi\ \vec{K} \tag{2.1}$$

$$\vec{k}_1 = -\sin\phi\ J + \cos\phi\ \vec{K},$$

where $\vec{i}_1$, $\vec{j}_1$, $\vec{k}_1$ are intermediate unit vectors after rotation ϕ about Gx

We next have the rotation θ about j_1; the next diagram shows that



$$\vec{i}_2 = \cos\theta\ \vec{i}_1 - \sin\theta\ \vec{k}_1$$

$$\vec{j}_2 = \vec{j}_1 \tag{2.2}$$

$$\vec{k}_2 = \sin\theta\ \vec{i}_1 + \cos\theta\ \vec{k}_1$$

The unit vectors $\vec{i}_2$, $\vec{j}_2$ $\vec{k}_2$ are the second set of intermediate unit vectors.

FIG. 2.2b

Finally, we have the rotation ψ about $\vec{k}_2$; we see from Figure 2.2c that

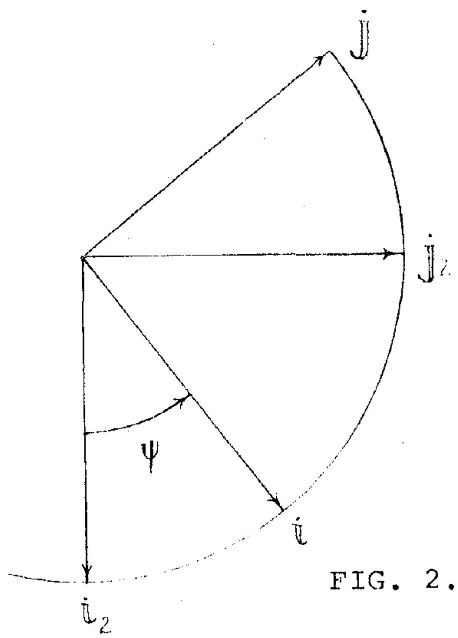


FIG. 2.2c

$$\vec{i} = \cos\psi\,\vec{i}_2 + \sin\psi\,\vec{j}_2$$
$$(2.3)\quad \vec{j} = -\sin\psi\,\vec{i}_2 + \cos\psi\,\vec{j}_2$$
$$\vec{k} = \vec{k}_2$$

Here, $\vec{i}$, $\vec{j}$, $\vec{k}$ are the final displacement position of the Gxyz axes.

The formulas we want connecting $\vec{i}$, $\vec{j}$, $\vec{k}$ in the displacement position with $\vec{I}$, $\vec{J}$, $\vec{K}$ are

$$\vec{i} = \ell_{11}\vec{I} + \ell_{12}\vec{J} + \ell_{13}\vec{K}$$
$$(2.4)\quad \vec{j} = \ell_{21}\vec{I} + \ell_{22}\vec{J} + \ell_{23}\vec{K}$$
$$\vec{k} = \ell_{31}\vec{I} + \ell_{32}\vec{J} + \ell_{33}\vec{K}$$

where the direction cosines ℓ_{rs} are given by

$$\ell_{11} = \cos\theta\cos\psi$$
$$\ell_{12} = \cos\phi\sin\psi + \sin\phi\sin\theta\cos\psi$$
$$\ell_{13} = \sin\phi\sin\psi - \cos\phi\sin\theta\cos\psi$$
$$(2.5)\quad \ell_{21} = -\cos\theta\sin\psi$$
$$\ell_{22} = \cos\phi\cos\psi - \sin\phi\sin\theta\sin\psi$$
$$\ell_{23} = \sin\phi\cos\psi + \cos\phi\sin\theta\sin\psi$$
$$\ell_{31} = \sin\theta$$

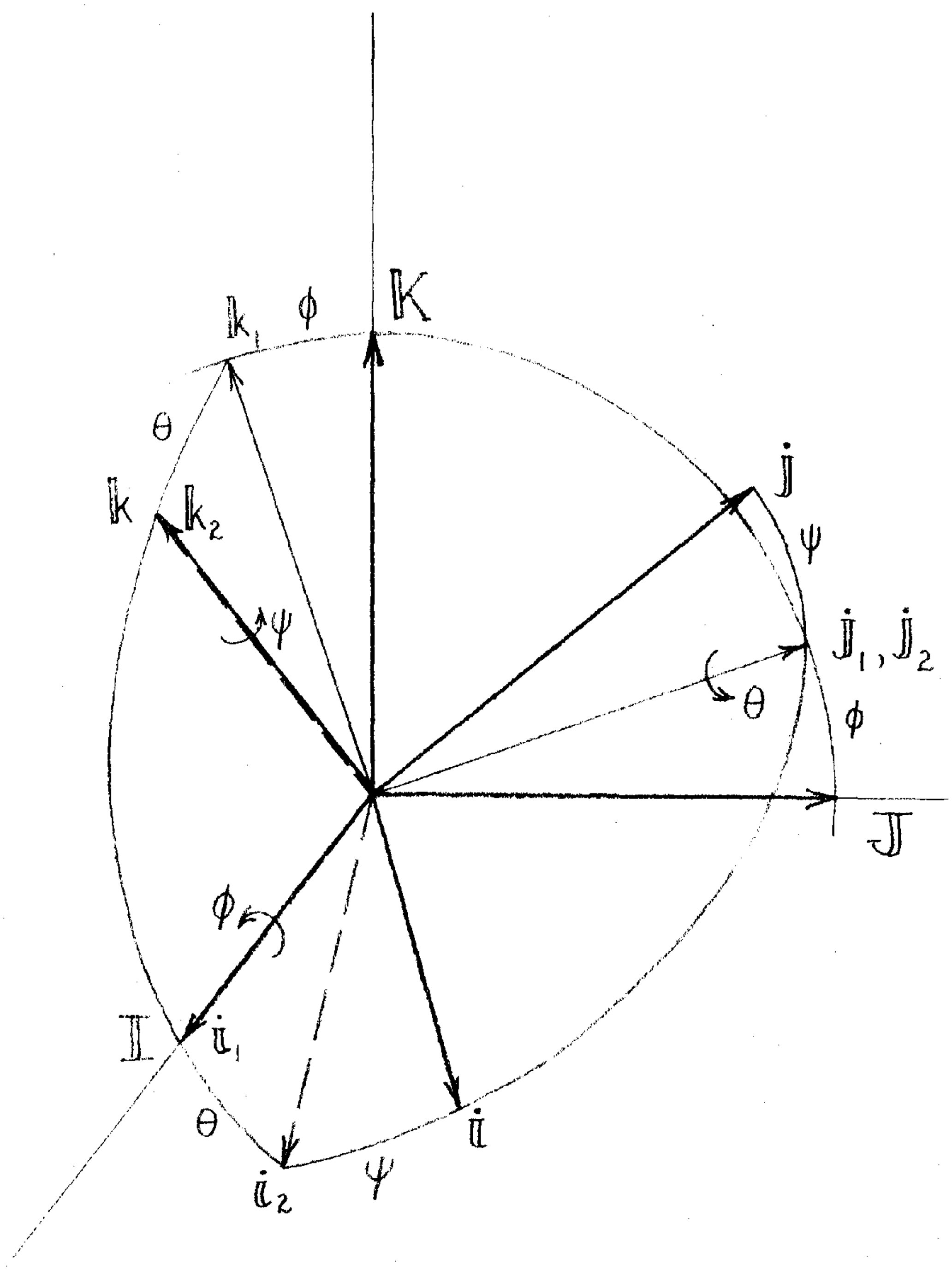


FIG. 2.3

$$\ell_{32} = - \sin \phi \cos \theta$$

$$\ell_{33} = \cos \phi \cos \theta$$

The special arrangement of the various sets of vectors and axes is shown in Figure 2.3.

If ϕ, θ, ψ are assumed small, then to within 2nd order of small quantities, (2.5) became

$$\begin{aligned}
\ell_{11} &\overset{2}{=} 1 - \frac{\theta^2 + \psi^2}{2} \\
\ell_{12} &\overset{2}{=} \Psi + \phi\theta \\
\ell_{13} &\overset{2}{=} \phi\psi - \theta \\
\ell_{21} &\overset{2}{=} - \psi \\
\ell_{22} &\overset{2}{=} 1 - \frac{\phi^2 + \psi^2}{2} \\
\ell_{23} &\overset{2}{=} \phi + \theta\psi \\
\ell_{31} &\overset{2}{=} \theta \\
\ell_{32} &\overset{2}{=} - \phi \\
\ell_{33} &\overset{2}{=} 1 - \frac{\phi^2 + \theta^2}{2}
\end{aligned} \tag{2.6}$$

It can be shown that the angular velocity of the body is given by

$$\vec{\omega} = (\dot{\phi} \cos \theta \cos \psi + \dot{\theta} \sin \psi)\vec{i} + (-\dot{\phi} \cos \theta \sin \psi + \dot{\theta} \cos \psi)\vec{j} + (\dot{\phi} \sin \theta + \dot{\psi})\vec{k} \tag{2.7}$$

3. Derivation # 1 of Potential Energy V of System

Let us first determine the potential energy V_j of the jth rod. The position vector of A_{jo} is

$$(3.1) \quad \vec{R}_{A_{jo}} = x_j\vec{I} + y_j\vec{J} + \rho_j\vec{K}$$

The position vector A_j', the displaced position of A_j, is given by

$$(3.2) \quad \vec{R}_{A_j'} = u\vec{I} + v\vec{J} + (\ell_o + c + w)\vec{K} + x_j\vec{i} + y_j\vec{j} - (c_j)\vec{k}$$

$\vec{i}$, $\vec{j}$, $\vec{k}$ are related to $\vec{I}$, $\vec{J}$, $\vec{K}$ through (2.4); thus

$$(3.3) \quad \vec{R}_{A_j'} = (u + \ell_{11}x_j + \ell_{21}y_j - \ell_{31}c_j)\vec{I} + (v + \ell_{12}x_j + \ell_{22}y_j - \ell_{32}c_j)\vec{J} + (\ell_o + c + w + \ell_{13}x_j + \ell_{23}y_j - \ell_{33}c_j)\vec{K}$$

The vector $\underline{\vec{R}}_{A_j}$ from A_{j_o} to A_j' is

$$(3.4) \quad \underline{\vec{R}}_{A_j} = \vec{R}_{A_j'} - \vec{R}_{A_{j_o}}$$
$$= (u - x_j + \ell_{11}x_j + \ell_{21}y_j - \ell_{31}c_j)\vec{I}$$
$$+ (v - y_j + \ell_{12}x_j + \ell_{22}y_j - \ell_{32}c_j)\vec{J}$$
$$+ (\ell_o + c + w - \rho_j + \ell_{13}x_j + \ell_{23}y_j - \ell_{33}c_j)\vec{K}$$

To within 2nd order of small quantities

$$(3.5) \quad \underline{\vec{R}}_{A_j} \stackrel{2}{=} (u - \frac{\theta^2 + \psi^2}{2}x_j - \psi\, y_j - \theta\, c_j)\vec{I}$$
$$+ (v + \{\psi + \phi\theta\}\, x_j - \frac{\phi^2 + \psi^2}{2}y_j + \phi c_j)\vec{J}$$
$$+ (w + \{\phi\psi - \theta\}x_j + \{\phi + \theta\psi\}y_j + \frac{\phi^2 + \theta^2}{2}c_j + \ell_j)\vec{K}$$

Let e_j be the extension of the jth rod due to motion from the equilibrium position. Then,

$$(3.6) \quad (\ell_j + e_j)^2 = \vec{\underline{R}}_{A_j} \cdot \vec{\underline{R}}_{A_j}$$

In what follows, we only need e_j to within 1st order of small quantities; we obtain, on combining (3.5) and (3.6)

$$(3.7) \quad e_j \doteq w - x_j\theta + y_j\phi$$

The magnitude of the tensile force in the jth rod is $t_j + k_j e_j$, where t_j is load supported by the jth rod in the equilibrium position and k_j is the spring constant of the jth rod. Clearly, in equilibrium

$$(3.8) \quad \sum t_j = mg, \qquad \sum x_j t_j = 0, \qquad \sum y_j t_j = 0$$

The force acting on the body due to the jth rod is

$$(3.9) \quad \vec{F}_j = -(t_j + k_j e_j) \frac{\vec{\underline{R}}_{A_j}}{\ell_j + e_j}$$

The virtual displacement of A_j' is obtained from (3.5); we find that

$$\begin{aligned}(3.10) \quad \delta\vec{\underline{R}}_{A_j} = {} & (\delta u - y_j\delta\psi - c_j\delta\theta - \{\theta\delta\theta + \psi\delta\psi\}x_j)\vec{I} \\ & + (\delta v + x_j\delta\psi + c_j\delta\phi + \{\phi\delta\theta + \theta\delta\phi\}x_j - \{\phi\delta\phi + \psi\delta\psi\})\vec{J} \\ & + (\delta w - x_j\delta\theta + \delta\phi y_j + \{\phi\delta\psi + \psi\delta\phi\}x_j + \{\theta\delta\psi + \psi\delta\theta\}y_j \\ & + \{\phi\delta\phi + \theta\delta\theta\}c_j)\vec{K}\end{aligned}$$

The virtual work δW_j of the force F_j acting on the body due to

the jth rod in the virtual displacement $\vec{R}_{A_j}$

$$(3.11)\quad \delta W_j = -(t_j + k_j e_j)\,\frac{\vec{R}_{A_j}}{\ell_j + e_j} \bullet \delta\vec{R}_{A_j}$$

We have from (3.5)

$$(3.12)\quad \frac{R_{A_j}}{\ell_j + e_j} \doteq \frac{u - c_j\theta - y_j\psi}{\ell_j}\vec{I} + \frac{v + c_j\phi + x_j\psi}{\ell_j}\vec{J} + \vec{K}$$

Equation (3.11) now becomes

$$(3.13)\quad \delta W_j = -\Bigg\{\left(\frac{t_j}{\ell_j}u - \frac{t_jc_j}{\ell_j}\theta - \frac{t_jy_j}{\ell_j}\psi\right)\delta u$$

$$+\left(\frac{t_j}{\ell_j}v + \frac{t_jc_j}{\ell_j}\phi + \frac{t_jx_j}{\ell_j}\psi\right)\delta v$$

$$+ (k_jw + k_jy_j\phi - k_jx_j\theta + t_j)\delta w$$

$$+\Bigg[\frac{t_jc_j}{\ell_j}v + k_jy_jw + \left(t_jc_j + \frac{t_jc_j^{\,2}}{\ell_j} + k_jy^2\right)\phi$$

$$-k_jx_jy_j + \left(t_jx_j + \frac{t_jc_jx_j}{\ell_j}\right)\psi\Bigg]$$

$$+\Bigg[-\frac{t_jc_j}{\ell_j}u - k_jx_jw - k_jx_jy_j\phi + \Bigg(t_jc_j + \frac{t_jc_j^{\,2}}{\ell_j}$$

$$+ k_jx_j^{\,2}\Bigg)\theta + \left(t_jy_j + \frac{t_jc_jy_j}{\ell_j}\right)\psi\Bigg]\delta\theta$$

$$+ \left[- \frac{t_j y_j}{\ell_j} u + \frac{t_j x_j}{\ell_j} v + \left(t_j x_j + \frac{t_j c_j x_j}{\ell_j} \right) \phi \right.$$

$$\left. + \left(t_j y_j + \frac{t_j c_j y_j}{\ell_j} \right) + \frac{t_j (x_j + y_j)}{\ell_j} \psi \right] \delta\psi \Bigg\}$$

The total virtual work W_t of the forces acting on the body in the virtual displacement is

(3.14) $\delta W_t = mg\ \delta w + \Sigma \delta W_j$

Using (3.8) and (3.13), we find

$$(3.15)\quad \delta W_t = - \Bigg\{ \left(\sum \frac{t_j}{\ell_j} u - \sum \frac{t_j c_j}{\ell_j} \theta - \sum \frac{t_j y_j}{\ell_j} \psi \right) \delta u$$

$$+ \left(\sum \frac{t_j}{\ell_j} v + \sum \frac{t_j c_j}{\ell_j} \phi + \sum \frac{t_j x_j}{\ell_j} \psi \right) \delta v$$

$$+ \left(\sum k_j w + \sum k_j y_j \phi - \sum k_j x_j \theta \right) \delta w$$

$$+ \left[\sum \frac{t_j c_j}{\ell_j} v + \sum k_j y_j w + \sum \left(t_j c_j + \frac{t_j c_j{}^2}{\ell_j} + k_j y_j{}^2 \right) \phi \right.$$

$$\left. - \sum k_j x_j y_j \theta + \sum \frac{t_j c_j x_j}{\ell_j} \psi \right] \delta\phi$$

$$+ \left[-\sum \frac{t_j c_j}{\ell_j} u - \sum k_j x_j w - \sum k_j x_j y_j \phi \right.$$

$$+ \sum \left(t_j c_j + \frac{t_j c_j^2}{\ell_j} + k_j x_j{}^2 \right) \theta + \left. \sum \frac{t_j c_j y_j}{\ell_j} \psi \right] \delta\theta$$

$$+ \left[- \sum \frac{t_j y_j}{\ell_j} u + \sum \frac{t_j x_j}{\ell_j} v + \sum \frac{t_j c_j x_j}{\ell_j} \phi \right.$$

$$\left. \left. - \sum \frac{t_j c_j y_j}{\ell_j} \theta + \sum \frac{t_j (x_j{}^2 + y_j{}^2)}{\ell_j} \right] \delta\psi \right\}$$

Since

(3.16) $\delta V = - \delta W_t$

we find from (3.15) that

$$(3.17) \quad 2V = c_{11}u^2 + 2c_{15}u\theta + 2c_{16}u\psi$$
$$+ c_{22}v^2 + 2c_{24}v\phi + 2c_{26}v\psi$$
$$+ c_{33}w^2 + 2c_{34}w\phi + 2c_{35}w\theta$$
$$+ c_{44}\phi^2 + 2c_{45}\phi\theta + 2c_{46}\phi\psi$$
$$+ c_{55}\theta^2 + 2c_{56}\theta\psi + c_{66}\psi^2$$

where

$$c_{11} = c_{22} = \sum \frac{t_j}{\ell_j} , \quad -c_{15} = c_{24} = \sum \frac{t_j c_j}{\ell_j}$$

$$c_{16} = -\sum\frac{t_j y_j}{\ell_j}\ , \quad c_{26} = \sum\frac{t_j x_j}{\ell_j}$$

$$c_{33} = \sum k_j\ , \quad c_{34} = \sum k_j y_j\ , \quad c_{35} = -\sum k_j x_j$$

$$(3.18) \qquad c_{44} = \sum\left(t_j c_j + \frac{t_j c_j^2}{\ell_j} + k_j y_j{}^2\right)\ , \quad c_{45} = -\sum k_j x_j y_j,$$

$$c_{46} = \sum\frac{t_j x_j c_j}{\ell_j}\ , \quad c_{55} = \sum\left(t_j c_j + \frac{t_j c_j{}^2}{\ell_j} + k_j x_j{}^2\right.$$

$$c_{56} = \sum\frac{t_j y_j c_j}{\ell_j}\ , \quad c_{66} = \sum\frac{t_j(x_j^2 + y_j^2)}{\ell_j}$$

4. Derivation # 2 of Potential Energy V of System

The potential energy V of the system can be written as

$$(4.1)\quad V = \tfrac{1}{2} \sum k \left[(e_j + e_{jo})^2 - e_{jo}^2 \right] - mgw$$

However, now e_j must be evaluated up to 2nd order terms since $k_j e_{jo} e_j$ appears in V. Thus, the form of e_j given by (3.7) cannot be used.

Let us write (3.5) as

$$(4.2)\quad \underline{R}_{A_j} = u_j \vec{I} + v_j \vec{J} + (w_j + \ell_j)\vec{K}$$

where

$$u_j \overset{2}{=} u - c_j\theta - y_j\psi - x_j \frac{\theta^2 + \psi^2}{2}$$

$$(4.3)\quad v_j \overset{2}{=} v + c_j\phi + x_j\psi + x_j\phi\theta - y_j \frac{\phi^2 + \psi^2}{2}$$

$$w_j \overset{2}{=} w + t_j\phi - x_j\theta + x_j\phi\psi + y_j\theta\phi + c_j \frac{\phi^2 + \theta^2}{2}$$

Now

$$(4.4)\quad \ell_j + e_j = \left[u_j^2 + v_j^2 + (w_j + \ell_j)^2 \right]^{\frac{1}{2}} ,$$

$$\approx \ell_j \left[1 + \frac{w_j}{\ell_j} + \frac{u_j^2 + v_j^2}{2\ell_j^2} \right];$$

Here we have neglected the term w_j^2 in comparison with $2\ell_j w_j$. Thus,

$$(4.5)\quad e_j = w_j + \frac{u_j^2 + v_j^2}{2\ell_j}$$

The substitution of (4.5) into (4.1) yields

$$(4.6)\quad V = \tfrac{1}{2}\sum k_j e_j + \sum k_j e_{jo} w_j + \sum k_j e_{jo}\frac{u_j^2 + v_j^2}{2\ell_j} - mgw$$

However,

$$(4.7)\quad k_j e_{jo} = t_j$$

Thus, (4.6) may be written as

$$(4.8)\quad V = \tfrac{1}{2}\sum k_j w_j^2 + \sum t_j w_j + \sum t_j \frac{u_j^2 + v_j^2}{2\ell_j} - mgw$$

We use (4.3) to obtain

$$(4.9)\quad V = \tfrac{1}{2}\sum k_j (w + y_j\phi - x_j\theta)^2 + \sum t_j\left[w + y_j\phi - x_j\theta + x_j\phi\psi + y_j\theta\psi + c_j\frac{\phi^2 + \theta^2}{2}\right] + \sum t_j\frac{(u - c_j\theta - y_j\psi)^2 + (v + c_j\phi + x_j\psi)^2}{2\ell_j} - mgw$$

Remembering (3.8), we finally obtain

$$\begin{aligned}2V = {} & \sum\frac{t_j}{\ell_j}u^2 - 2\sum\frac{t_jc_j}{\ell_j}u\theta - 2\sum\frac{t_jy_j}{\ell_j}u\psi \\ & + \sum\frac{t_j}{\ell_j}v^2 + 2\sum\frac{t_jc_j}{\ell_j}v\phi + 2\sum\frac{t_jx_j}{\ell_j}v\psi \\ & + \sum k_j w^2 + 2\sum k_j y_j w\phi - 2\sum k_j x_j w\theta \\ & + \sum\left[t_jc_j + \frac{t_jc_j^2}{\ell_j} + k_jy_j^2\right]\phi^2 - 2\sum k_jx_jy_j\phi\theta + \sum\frac{t_jc_jx_j}{\ell_j}\phi\psi\end{aligned}$$

$$+ \sum \left(t_j c_j + \frac{t_j c_j^2}{\ell_j} + k_j x_j^2 \right) \theta^2 + 2 \sum \frac{t_j c_j y_j}{\ell_j} \theta\psi$$

$$+ \sum \frac{t_j (x_j^2 + y_j^2)}{\ell_j} \psi^2$$

This is the same result as obtained in (3.17).

5. Derivation of Kinetic Energy T.

The principal moments and products of inertia of the body with respect to the $G\xi\eta\zeta$ axes are defined as

$$
\begin{aligned}
A &= \sum m_j \ (\eta_j^2 + \zeta_j^2) \\
B &= \sum m_j \ (\zeta_j^2 + \xi_j^2) \\
C &= \sum m_j \ (\xi_j^2 + \eta_j^2) \\
F &= \sum m_j \eta_j \zeta_j \\
G &= \sum m_j \zeta_j \xi_j \\
H &= m_j \xi_j \eta_j
\end{aligned}
\tag{5.1}
$$

where m_j is a generic mass particle, and ξ_j, η_j, ζ_j are its coordinates with respect to $G\xi\eta\zeta$ axes.

We shall assume that the supporting structure is fixed in space. The velocity of G is then

$$(5.2) \quad \vec{v} = u\vec{I} + v\vec{J} + w\vec{K}$$

The angular velocity ω_1 of the body is found in the usual way making use of (2.5); we find that

$$(5.3) \quad \vec{\omega} = \omega_1\vec{i} + \omega_2\vec{j} + \omega_3\vec{k},$$

where

$$\omega_1 = \dot{\phi} \cos \theta \cos \psi + \dot{\theta} \sin \psi \overset{2}{=} \dot{\phi} + \dot{\theta}\psi \overset{1}{=} \dot{\phi}$$

$$(5.4) \quad \omega_2 = -\dot{\phi}\cos\theta\sin\psi + \dot{\theta}\cos\psi \overset{2}{=} -\dot{\phi}\psi + \dot{\theta} \overset{1}{=} \dot{\theta}$$

$$\omega_3 = \dot{\phi}\sin\theta + \dot{\psi} \overset{2}{=} \dot{\phi}\theta + \dot{\psi} \overset{1}{=} \dot{\psi}$$

We now use the theorem on kinetic energy of a rigid body; keeping only 2nd order term; we obtain

$$(5.5) \quad 2T = a_{11}\dot{u}^2 + a_{22}\dot{v}^2 + a_{33}\dot{w}^2$$
$$+ a_{44}\dot{\phi}^2 + 2a_{45}\dot{\phi}\dot{\theta} + 2a_{46}\dot{\phi}\dot{\psi}$$
$$+ a_{55}\dot{\theta}^2 + 2a_{56}\dot{\theta}\dot{\psi}$$
$$+ a_{66}\dot{\psi}^2$$

Here

$$(5.6) \quad \begin{aligned} &a_{11} = a_{22} = a_{33} = m, \quad a_{44} = A, \quad a_{55} = B, \quad a_{66} = C \\ &a_{45} = -H, \quad a_{46} = -G, \quad a_{56} = -F \end{aligned}$$

6. Equations of Free Motion

We use Lagrange's equation and T and V to derive the equations of free motion.

$$a_{11}\ddot{u} + c_{11}u + c_{15}\theta + c_{16}\psi = 0 \quad ,$$

$$a_{22}\ddot{v} + c_{22}v + c_{24}\phi + c_{26}\psi = 0 \quad ,$$

$$a_{33}\ddot{w} + c_{33}w + c_{34}\phi + c_{35}\theta = 0 \quad ,$$

$$c_{24}v + c_{34}w + a_{44}\ddot{\phi} + c_{44}\phi + a_{45}\ddot{\theta} + c_{45}\theta$$

(6.1) $$+ a_{46}\ddot{\psi} + c_{46}\psi = 0 \quad ,$$

$$c_{15}u + c_{35}w + a_{45}\ddot{\phi} + c_{45}\phi + a_{55}\ddot{\theta} + c_{55}\theta$$

$$+ a_{56}\ddot{\psi} + c_{56}\psi = 0 \quad ,$$

$$c_{16}u + c_{26}v + a_{46}\ddot{\phi} + c_{46}\phi + a_{56}\ddot{\theta} + c_{56}\theta$$

$$+ a_{66}\ddot{\psi} + c_{66}\psi = 0$$

7. Results for Unit # 3, Paradise

Table 7.1 shows results obtained for Unit # 3 of Paradise.

Table 7.1

Mode Numbers	1	2	3	4	5	6
N.f. in Hz	.125	.209	.214	3.17	3.96	9.83
Mode						
u	30.4	0	15.9	0	-.03	0
v	0	64	0	-.36	0	0
w	0	-.08	0	16	0	-62
ϕ	0	0	0	-.97	0	.24
θ	0	0	0	-.97	0	.24
ψ	-1.87	0	.07	0	.019	0

The constants on which these results are based are as follows:

$A = 3.19 \times 10^6$ k. ft. sec^2. , $m = 750$ k. sec^2./ft.

$B = 3.87 \times 10^6$ " " "

$C = 2.75 \times 10^6$ " " "

$F = .44 \times 10^6$ " " "

$G = H = 0$

$$c_{11} = c_{22} = \sum \frac{t_j}{\ell_j} = 1309.13 = 1.31 \times 10^3 \text{ k/ft}$$

$$c_{15} = c_{24} = \sum \frac{t_j c_j}{\ell_j} = 112{,}387.61 = 1.12 \times 10^9 \text{ k}$$

$$c_{16} = -\sum \frac{t_j y_j}{\ell_j} = 13,780.20 = 1.38 \times 10^4 \text{ k}$$

$$c_{26} = \frac{t_j x_j}{\ell_j} = 0.00$$

$$c_{33} = \sum k_j = 2,712,163.40 = 2.71 \times 10^6 \text{ k/ft}$$

$$c_{34} = \sum k_j y_j = 39,964,044.78 = -40 \times 10^6 \text{ k}$$

$$c_{35} = -\sum k_j x_j = 0.00$$

$$c_{44} = \sum \left(t_j c_j + \frac{t_j c_j^{\,2}}{\ell_j} + k_j y_j^{\,2} \right) = 9.649 \times 10^6 + 1.590 \times 10^6$$
$$+ 1.918 \times 10^9 = 1.929 \times 10^9$$
$$= 1.93 \times 10^9 \text{ k.ft.}$$

$$c_{45} = -\sum k_j x_j y_j = 0.00$$

$$c_{46} = \sum \frac{t_j x_j c_j}{\ell_j} = 0.00$$

$$c_{55} = \sum \left(t_j c_j + \frac{t_j c_j^{\,2}}{\ell_j} + k_j x_j^{\,2} \right) = 2.35 \times 10^9 \text{ k.ft.}$$

$$c_{56} = \sum \frac{t_j y_j x_j}{\ell_j} = -1,169,196.26 = -\ 1.17 \times 10^9 \text{ k.ft.}$$

$$c_{66} = \sum \frac{t_j (x_j^2 + y_j^{\,2})}{\ell_j} = 1,914,630.12 = 1.91 \times 10^6 \text{ k.ft.}$$