TSUNAMIS

Proceedings of the National Science Foundation Workshop

May 1979

Organized and Edited
by
Li - San Hwang and Y. Keen Lee

Tetra Tech Inc.
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PREFACE

A tsunami workshop was held at the Coto de Caza, Trabuco Canyon, Southern California on 7-9 May 1979. The workshop was organized by Tetra Tech, Inc., under National Science Foundation sponsorship, with Dr. Li-San Hwang of Tetra Tech as the principal investigator of the grant (No. PFR-7805646 A01).

The purpose of the workshop was to provide a forum for a critical review of the status of tsunami research. Participation was limited to about sixty invited persons from industry, government agencies, and academia. The workshop was divided into a number of sessions with presentations of critical reviews of selected topics by the chairmen of the workshop sessions. Each review served as a focus and guide for further discussions by workshop participants. In addition, certain individuals were designated as recorders of the various workshop sessions, whose task was to prepare a summary of the deliberations of the assigned sessions. These were to be contributed, together with the text of the chairmen's presentation, to a report on the proceedings of the workshop.

This volume of the proceedings of the tsunami workshop was produced in the hope that it will be a medium through which the findings and deliberations in the workshop will reach a wider community of interested parties. The editor of the proceedings is Dr. Y. Keen Lee, of Tetra Tech, Inc.

The following pages contain an account of the presentations and deliberations during the tsunami workshop. The contributions to each session of the workshop were the responsibility of the chairman and recorder of the particular session. The instructions to them regarding their contributions to this volume of the proceedings was deliberately vague in order to accommodate the various styles and inclinations of the contributors as well as the tenor of the actual discussions. It was the implicit function of the recorder of each session to make notes regarding the discussions following the chairman's presentation, and then work with the chairman on the form and content of their contribution to this volume.

Due to the widely varying form of the resulting contributions to this volume, the editor was confronted with a problem of acknowledgements to the authors of the various contributions. This was necessary in order to give proper credit to the authors and also to facilitate references to specific texts within this volume by future researchers. It was decided that for those sessions in which a chairman had also submitted a text of his presentation, his specific authorship of the text would be acknowledged. Otherwise, joint contribution to the text by chairman and recorder is implied without specific attribution of authorship. Accounts of the discussions during each session are generally authored by the recorder with minor modifications and additions by the editor in some cases.

Workshop participants will probably notice that the actual proceedings during the workshop differ in some detail from their representation in this volume. In particular, it will be noticed that certain presentations from the chair have received more extensive treatment in this volume than was actually delivered at the workshop. On the other hand, omissions of certain impromptu presentations from the floor are also evident. The former is due to the fact that the chairmen had decided to depart from their prepared texts, submitted to the editor, in order to allow more discussions from the floor and/or to avoid repetition of material covered by others in earlier sessions.

The efforts of all the contributors to this volume are greatly appreciated. In many instances, their efforts made the editorial task exceedingly simple. Unfortunately, a number of people had other commitments which did not allow them to prepare their contributions in time for publication despite valiant attempts to do so. The editor has taken the liberty to prepare summaries of those sections thus affected with the aid of an audio-tape of the Workshop proceedings. The sessions concerned were those on "Fault Mechanics and Frequencies of Occurrence" and "Shore Protection and Flood Plain Management." Any misrepresentations are entirely the fault of the editor.

The organizers wish to express their special appreciation to Professor Fredric Raichlen, Dr. Michael Gaus and Dr. S.C. Liu for their valuable suggestions on the conduct and organization of the Workshop.

INAUGURAL ADDRESS

BY
S. C. LIU

It gives me great pleasure to kick off this important Workshop on tsunami research. Three important factors have made this Workshop possible. The first factor is the research community, which all of you represent. The second factor is the National Science Foundation, which provided the financial support. The third factor is the people who organized this Workshop.

The research community, as represented by all of you, has demonstrated that the tsunami problem, complicated as it is, can be tackled by scientific means. This is evidenced by the tremendous progress made in the last decade in various aspects of the problem, such as tsunami generation, wave propagation, the topographic effect on waves in coastal areas, and numerical computational modeling techniques. We at NSF feel that we are nearing the goal of resolving the tsunami problem through the integrated efforts of the scientists and engineers involved in tsunami research. Because of the broad range of issues involved in this problem, and because of the rapid discovery of new knowledge and techniques, it is important that researchers active in this field get together, as on this occasion, to exchange information, to assess the status of knowledge, and to identify future research needs and research directions. These are the major purposes of this Workshop.

It has become clear that tsunamis can no longer be considered as a problem only by scientists. The tsunami problem is of concern to scientists, engineers, planners, architects, government regulatory bodies, city and state officials, and the general public. From the hazard mitigation viewpoint, the problem can be characterized as multidisciplinary and multidimensional. Today's meeting was conceived in recognition of the common interests of all those involved.

The National Science Foundation, whose function is to support and facilitate the Nation's scientific research, is delighted to be able to play a role in this Workshop. I am particularly happy to see many leading foreign scientists at this meeting. The commingling of scientists across national boundaries is quite consistent with the basic nature of the problem, since tsunamis ignore distances and national boundaries.

This Workship will produce a document summarizing all its important findings so that scientists can use this document in the future to develop and to guide their research efforts. Hopefully, through this means tsunami hazards can be mitigated more efficiently.

Finally, I would like to say a few words concerning the person who organized this conference, a person who had to sacrifice many of his other interests to devote time to this Workship. Dr. Li-San Hwang deserves special thanks from all of us for organizing this Workshop.

I welcome you and look forward to a very productive meeting.

1 TSUNAMIGENIC EARTHQUAKES

1. FAULT MECHANISMS AND FREQUENCIES OF OCCURRENCE
   CHAIRMAN : G. PLAFKER
   RECORDER : R. GELLER

2. CHARACTERISTICS OF GROUND MOTIONS INFERRED FROM SEISMIC WAVES
   CHAIRMAN : H. KANAMORI
   RECORDER : J. KELLEHER

FAULT MECHANISMS AND FREQUENCIES OF OCCURRENCE

BY
G. PLAFKER

George Plafker's discussion consisted of the following three major topics:

1. A review of the tectonic aspects and mechanism of thrust-type tsunamigenic earthquakes;

2. The relationship between ground deformation and seismic sea waves; and

3. Repeat times for tsunamigenic earthquakes based on geologic, historic and plate tectonic data.

The discussion was based largely on data obtained from the 1960 Chilean and the 1964 Alaskan earthquakes.

The locations of tsunamigenic earthquakes were shown to be closely associated with major volcanic arcs around the Pacific. Rupture zones of earthquakes were outlined by considering the distribution of large aftershocks for the earthquakes. This gave dimensions of the order of 800 km long and 300 km wide for the Alaskan earthquake and similarly large dimensions for the Chilean one. These areas were each comparable to 2/3 the size of California. Teleseismic data defined the dipping fault planes of the earthquakes. The magnitudes of the ground deformations and the shapes of the deformations were traced by surveys of the barnacle line on exposed marine terraces created by crustal uplift. Maximum uplifts of the order of 11 m at Montague Island, Alaska were attained and these were extrapolated to the edge of the continental shelf 50 km offshore where, on Middleton Island, the uplift was about 4 m. Subsidence inland was evident from the inundation of coastal roads and leveling studies. Magnitudes of 2 1/2 m were recorded at Kodiak. At places where the fault broke the surface, fault slips of 6 to 7 m were measured. Much larger horizontal displacements were detected from triangulation networks. In areas outside the region of stability, 20 m horizontal seaward displacements were recorded for the Alaskan earthquakes. These values represent a minimum.

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The same pattern and magnitude of displacements were also in evidence from the 1960 Chilean event.

These studies were consistent with a model of earthquakes generated by thrust-type faulting. The picture which emerges is that of an oceanic plate underlying an upper plate (continental or oceanic) with strain accumulation due to plate convergence. Sudden shear between plate boundaries represents the earthquake. The rebound of the upper plate causes the uplift. This accounts for the initial wave of the seismic sea wave being positive. The measured deformations imply more complicated possibilities such as imbricate faults on the upper plate as well. The alternative faulting mechanism of normal faulting within the downgoing slab is also possible, such as occurred for the Rat Island tsunami in the Aleutian Trench. However the thrust-type faulting seems to be responsible for the larger tsunamis.

Terminology in the literature needs clarification to avoid confusion. Plafker suggested that tsunami be applied to any long period wave generated by earthquakes whereas seismic sea waves be applied to the class of tsunamis (generated by large scale tectonic displacements of the sea floor) which have propagated over hundreds of km. Therefore tsunamis generated by subaqueous and subaerial landslides and volcanic explosions are to be excluded. Thus, it is incorrect to compare the Alaskan and Chilean tsunamis with the Lituya Bay tsunami caused by a landslide, generated by strike-slip faulting. During the Alaskan event, about 35 people in the U.S. were killed by the seismic sea wave but more were killed by the tsunamis created locally in the generation region.

At Middleton Island, a series of 5 marine terraces are clearly defined. The oldest of these is 50 m above sea level. After the Alaskan earthquake, the uplift created a new layer. It can be hypothesized that each terrace is created by the uplift due to a major earthquake or a closely spaced sequence of major earthquakes. Radiocarbon dating has established the age of each terrace, the oldest being 4300 years old while the youngest (prior to the Alaskan event, the bottom layer) 1350 years old. Each terrace averaged 8 to 12 m thick. A plot of uplift versus age showed the average slope (the mean uplift rate) to be 1 cm per year. In recent years, the slope is 1/2 cm per year thus indicating either an uplift rate change or the possibility of another large earthquake in the future so that the uplift rate conforms to the long term average: The Alaskan earthquake created a 4 m uplift in the terrace and this only represents half the long term average. Another comparable earthquake with another 4 m uplift in the terraces is possible.

Besides using geologic date such as the above to estimate repeat times, it is also possible to use the rates of relative motions between the plates together with measured strains to deduce recurrence intervals. In the Alaskan earthquake, at least 20 m of slip occurred. This corresponds to a strain rate of 5 cm per year with at least 400 years neeeded to accumulate the strain equivalent to that released in the Alaskan earthquake. For the Chilean region, with surface slips of the order of 20 m and slips of 30 m from seismic wave data, convergence rate of plates of the order of 9 cm per year maximum implies 300 years minimum to account for the horizontal surface displacement in the Chilean earthquakes. In Japan, shorter repeat times are present: With a convergence rate of 9 to 10 cm per year and the Nankaido events occurring at 110 years interval. The resulting strain gives rise to 10 m of slip.

Both from marine terrace geologic studies and plate tectonic considerations, it is evident that the repeat times of large tsunamigenic earthquakes is of the order of centuries and not decades. This is a comforting thought.

CHARACTERISTICS OF GROUND MOTIONS

BY
H. KANAMORI

The session was opened by H. Kanamori with a discussion of the source mechanisms of great earthquakes and the generation of tsunamis. The spatio-temporal characteristics of the ground deformation associated with large earthquakes can be inferred from the analysis of long-period seismic body and surface waves. For several major tsunamigenic earthquakes the inferred ground deformations are consistent, at least to the first order of approximation, with the estimated tsunami height at the source. A number of exceptional events, however, called tsunami earthquakes have been found. Recent studies indicated that these earthquakes generate very long-period (larger than 300 sec) surface waves which are disproportionately large for their earthquake magnitudes. This anomalous excitation of long-period surface waves and tsunamis has been interpreted in terms of either very slow source process, secondary faulting such as imbricate faults, or a combination of these effects. In view of these results, Kanamori recommended use of a magnitude scale based on very long-period surface waves for the tsunami warning system.

In fact, Abe (1979) had recently examined the size of great earthquakes from 1837 to 1974 inferred from tsunami data. He defined a new magnitude scale M_t, based on far field tsunami waves and by experimentally adjusting the M_t scale to the M_w scale introduced by Kanamori (1977), Abe found that the M_t scale measures the seismic moment of a tsunamigenic earthquake as well as the overall size of the tsunami at the source.

Several speakers pointed out the importance of the time history of ground deformation toward determining both near field and far field tsunami amplitudes. Kanamori discussed how reasonable estimates of the ground deformation could be made by using dislocation theory together with estimates of seismic source parameters including dimensions of the fault plane, displacement and focal plane mechanisms. In fact, tsunami waves have been simulated by this method for some Japanese earthquakes. Abe (1973) examined a fairly complete set of seismological and tsunami data for the Kurile Islands earthquake of 1969 and the Tokachi-Oki earthquake of 1968. He found good agreement between the sea bottom deformation calculated from seismic data and the tsunami source area obtained from inverse refraction diagrams, the first motion of tsunami waves and the height of sea level disturbance at the source.

The use of seismic gaps along subduction boundaries to predict potential tsunami source areas was discussed by J. Kelleher. Some recent successes were achieved by this method in estimating earthquake locations, several of which were accompanied by small or moderate tsunamis. For evaluating tsunami potential, the importance of understanding and interpreting the morphologic features of the inner wall of the trench were discussed, with particular emphasis on the ridges, scarps and perched basins that result from imbricate thrust faulting. These relationships were discussed by Kelleher with reference to a summary map of tsunami source areas (Figure 1).

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2. Abe, K. 1979. Size of great earthquakes of 1837–1974 inferred from tsunami data. J. Geophys. Res., 84:1561.

3. Kanamori, H. 1977. The energy release in great earthquakes. J. Geophys. Res., 82:2981.

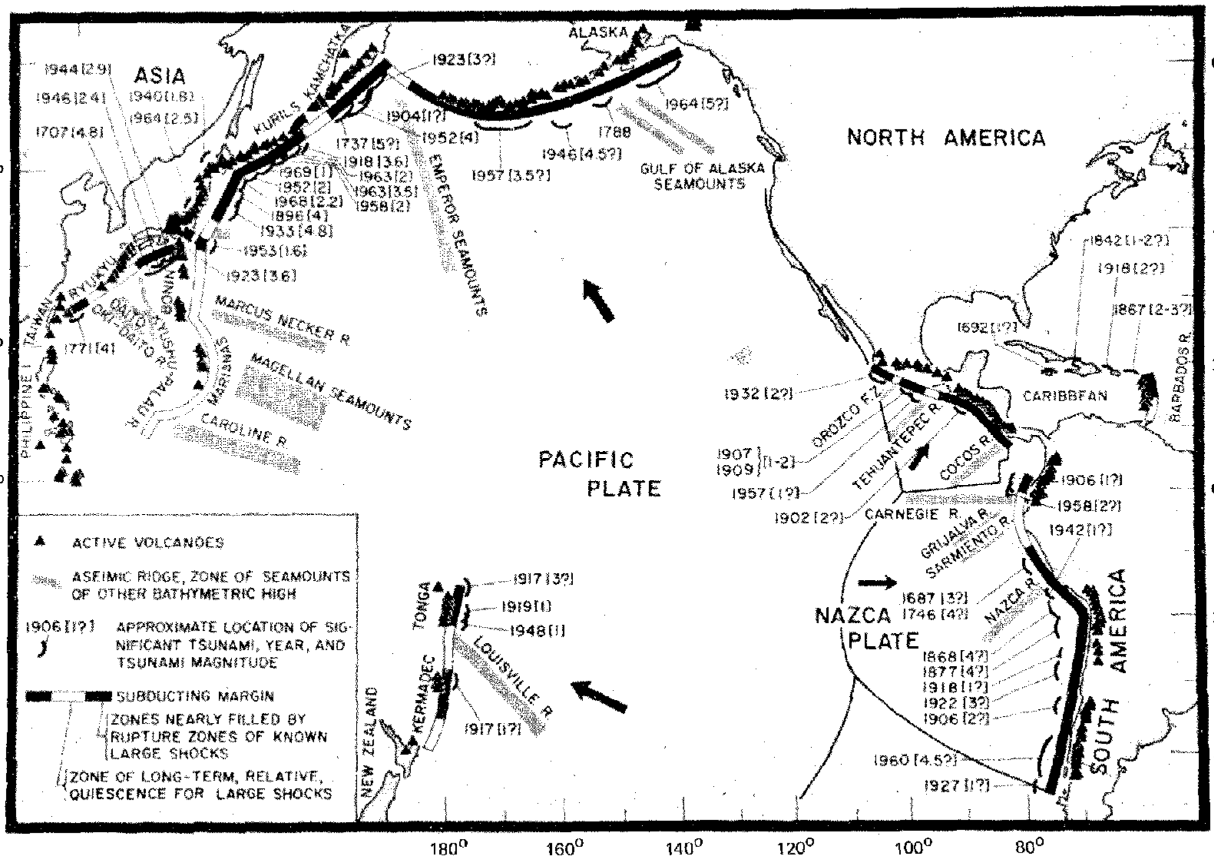

FIGURE 1 SUMMARY MAP OF TSUNAMI SOURCE AREA (J. KELLEHER)

2 TSUNAMI GENERATION

CHAIRMAN : K. KAJIURA
RECORDER : Y. KEEN LEE

TSUNAMI GENERATION

BY
KINJIRO KAJIURA

Tsunamis can be generated by many different mechanisms. Instances can be cited from historical records of tsunami generation by earthquakes, volcanic or nuclear explosions, landslides, rock falls and submarine slumps.

For example, a very large volcanic eruption occurred in 1883 at Krakatoa which created a very large wave in the Sunda Straits, with a very long period of an hour or more. But the exact mechanism by which the wave was generated is still not clear. In the case of landslides, there is a very well known example in 1792 in Shimabara, Japan on Kyushu Island. A large mass of the mountain slid into Ariake Bay and caused gravity waves which reached a height of 10 m in some places, killing a large number of people. Yet another example is the rockfall into Lituya Bay in 1959 which splashed waters up to a height of 500 meters and also generated solitary waves about 30 meters high. During the 1964 Alaskan earthquake, submarine slumping caused local tsunamis in many places. Not much is known with regard to the relation of waves to turbidity currents. The characteristics of the waves generated by the processes mentioned above are quite different and are the source of much confusion in discussions of tsunamis.

The subject of the present discussion is tsunami generation by earthquakes or tectonic movements. Even so, there are significant differences in wave characteristics. Figure 1 presents the wave data for some previous tsunamis which can be used to illustrate these differences. For example, the Kamchatka (1952), Chilean (1960) and Alaskan (1964) tsunamis were generated on the shallow continental shelves and had main waves with periods of, say, more than 40 minutes. In contrast, the main waves associated with the 1946 and 1957 Aleutian events had very short periods, 10-15 minutes and 7-10 minutes, respectively (Stoneley 1963). These period differences may not be explained simply by the differences in earthquake magnitudes. It is probably caused by the differences of source locations. Tsunamis generated by sources located on

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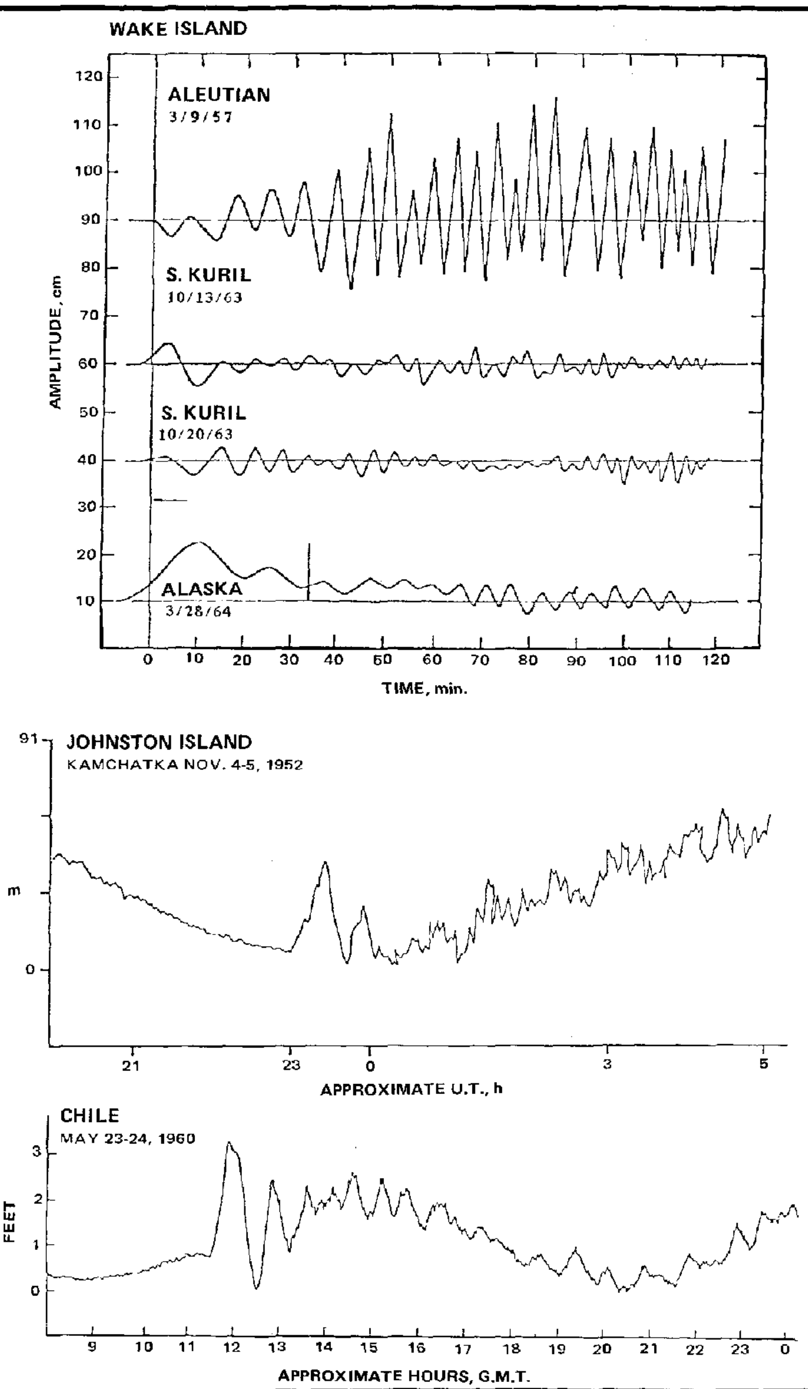

FIGURE 1
TSUNAMI SIGNATURES OBSERVED AT ISLAND STATIONS. UPPER FIGURE AT WAKE ISLAND (VAN DORN, 1965), LOWER FIGURES AT JOHNSTON ISLAND AND CHILE.

the shallower continental shelves have periods that are longer than the periods of tsunamis generated by sources in the deep trenches. Furthermore, the coastal response to short period and long period waves are also quite different; thus, run-up and resonance effects are quite different.

Parameters of Ground Motion Relevant to Tsunami Generation

Figure 2 shows the pattern of the permanent ground displacements associated with the Alaskan earthquake of 1964. Short and sharp ground displacements are superimposed on the gradual topographic chart shown. A cross section of the vertical surface displacement due to the fault is also shown. The scale of the ground surface disturbances is of the order of the width of the fault, and the horizontal scale of water surface motions can be scaled by the width of the fault except for secondary features.

The average length to width ratio L/W of earthquake faults is about 2. Studies of the variation of fault width W and fault slip D as a function of seismic moments M_0 show that $M_0 \sim W^3 \sim D^3$, with a resultant approximately constant ratio of $D/W \sim 3x10^{-5}$. This ratio can be used to characterize the resultant water wave steepness. The rise time, generally less than a few seconds, is the time in which slip is completed locally, and the time scale of the vertical motion in the ground surface is of the order of 3 to 4 times the rise time. Rupture velocity is slightly less than the dilatational wave velocity of the crust. Based on these parameters, the approximate rupture times and vertical velocity of the ground motion can be computed. In this case, since the vertical velocity is of the order of a few tens cm/sec, there is insignificant overshoot in the ground displacement. This order of magnitude of the vertical velocity is, of course, characteristic of only the overall average ground displacement and not that associated with seismic waves. The magnitudes of these parameters are shown in Table 1.

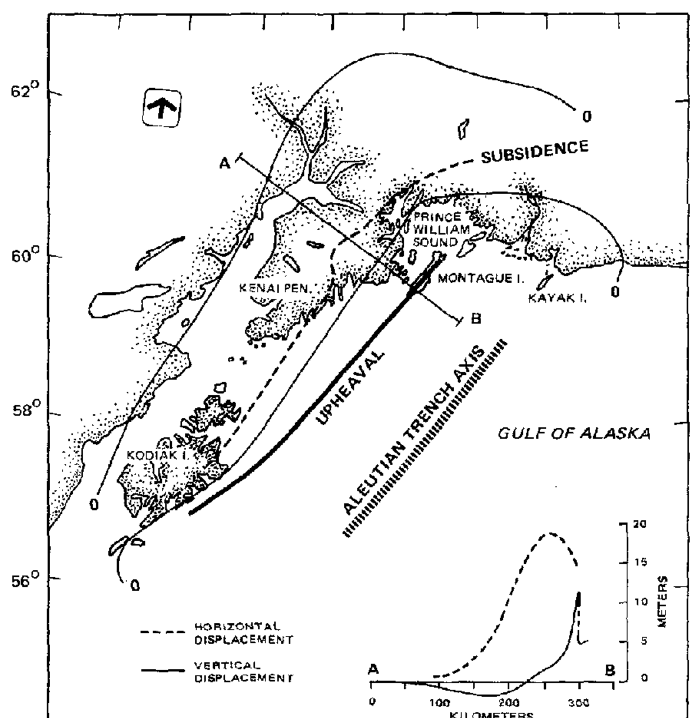

FIGURE 2 CRUSTAL DEFORMATIONS IN SOUTH—CENTRAL ALASKA ASSOCIATED WITH THE ALASKA EARTHQUAKE OF 1964 (FROM PLAFKER, 1969). SOLID AND DASHED LINES INDICATE THE AXIS OF THE MAXIMUM UPHEAVAL AND SUBSIDENCE RESPECTIVELY.

TABLE 1 TYPICAL MAGNITUDES OF FAULT PARAMETERS AND THE GROUND SURFACE MOTION FOR THE TSUNAMI GENERATION PROBLEM

Fault Parameters	$L \lesssim 500$ km, $W \sim L/2$ $D \sim 3 \times 10^{-5} W$ Rupture velocity ~ 3 km/s Rise time $\lesssim 5$ sec
Ground Motion	Time Scale of ground motion $T_C \lesssim 20$ sec Rupture time $\lesssim 3$ min Velocity of the ground $W_B < 1$ m/sec

TABLE 2 POWERS OF r IN THE DECAY OF THE LEADING WAVE (FOR $r >> 1$ AND $t \sim r$)

	Deformation (finite volume)		Impulse or Asymmetric Deformation (dipole type)	
	$P_a \sim 3$	$P_a < 1$	$P_a \sim 3$	$P_a < 1$
One-dimensional	0	-1/3	-1/3	-2/3
Two-dimensional	-2/3	-1	-1	-4/3

Waves in Liquid-Solid Coupled System

Important parameters in the problem are the density of water $\rho_1 \sim 1 gm/cm^3$, the shallow water wave speed $c_0 \lesssim 0.2$ km/s, and the speed of sound in water $c_s \sim 1.5$ km/s. Corresponding solid earth parameters are: density of the upper crust $\rho_2 \sim 2.7$ gm/cm^3, primary (compressional) wave speed $V_p \sim 7$ km/s, and secondary (dilatational) wave speed $V_s \sim 4$ km/s.

Most of the literature on tsunami generation contains discussions of the problem in the context of the assumptions of incompressible water over a rigid bottom. The resulting gravity wave on the water's free surface is the tsunami. However, the assumptions in these models can be relaxed to introduce more complicated effects. For example, allowing for the compressibility of the water, both sound and gravity waves coexist. Considering the bottom to be part of an elastic solid introduces Rayleigh waves to the solid-liquid interface. The adequacy of the conventional treatments in the context of these additional effects can be investigated by considering the dispersion relation of the waves in the most general case. This can be written as:

$$\left[\text{G.W.};\ (c_0/c_s)^2,\ (c_0/c_s)^4/(kh)^2\right] \times \left[\text{R.W.};\ (c_0/c)^2/(kh)^4\right]$$

$$+\ \rho_1/\rho_2\ \left[\text{coupling},\ (c_0/c)^4/(kh)^2\right] = 0;\ c = \sigma/k$$

where k is the wave number, σ is the angular frequency, and the first term represents the gravity wave dispersion relation G.W. modified by terms of, at most, a few percent when applied to tsunami problems. In the second term R.W. represents the ordinary Rayleigh wave dispersion relation, and the third term is the liquid-solid coupling term of the Rayleigh wave. These are modified by terms which are also small in most tsunami applications except for extremely long waves ($kh \rightarrow 0$), when they may become appreciable. Since the phase and group velocities of water waves and elastic waves are quite different, each mode of motion can be treated independently. In general, it can be concluded that for tsunami problems, the dynamics in the water is the most important part of the problem and the bottom can be assumed rigid. In passing, it may be mentioned that the effect of density stratification in water can be neglected for tsunami generation by tectonic movement of the bottom, since the generation mode is mainly barotropic and, also, the surface and internal wave velocities are quite different.

Effect of Compressibility

When the bottom is displaced upwards with any velocity, compressional waves will be generated which will propagate towards the free surface and create disturbances there. The simple two-dimensional case with a rigid horizontal bottom moved upward a distance H_B abruptly has been considered by Sells (1965). His results for the free surface displacement η over the bottom as a function of non-dimensionalized time τ (= t/t_c with $t_c = h/c_s$ where h is the depth and c_s the acoustic velocity in water) is shown in Figure 3. The sea surface response can be characterized as sea shocks which occur immediately over the area of tectonic displacement and, at the same time, the mean surface level is raised by an amount equal to the bottom displacement. This mean level change is the source of a gravity wave spreading outwards. The period of the oscillations depends on the depth of the water. For a non-rigid bottom, energy will be lost as the compressional waves return from the free surface and partially reflect off the bottom so that the sea shocks will decay. The case with a gradual bottom displacement (Kajiura, 1970) shows qualitatively similar behavior with a less abrupt surface disturbance.

Water Wave Generation

The starting point for the consideration of tsunamis as gravity waves is normally the inviscid, irrotational theory. Natural length and time scales for non-dimensionalization are the depth h and $\sqrt{h/g}$, respectively. The water is considered to be initially at rest until some time t_0 when there is a vertical bottom velocity $W_B(\underset{\sim}{r}_0, t_0)$ where $\underset{\sim}{r}_0$ is the horizontal position vector. Subsequent free surface motions $\eta(\underset{\sim}{r}, t)$ can be analyzed either by

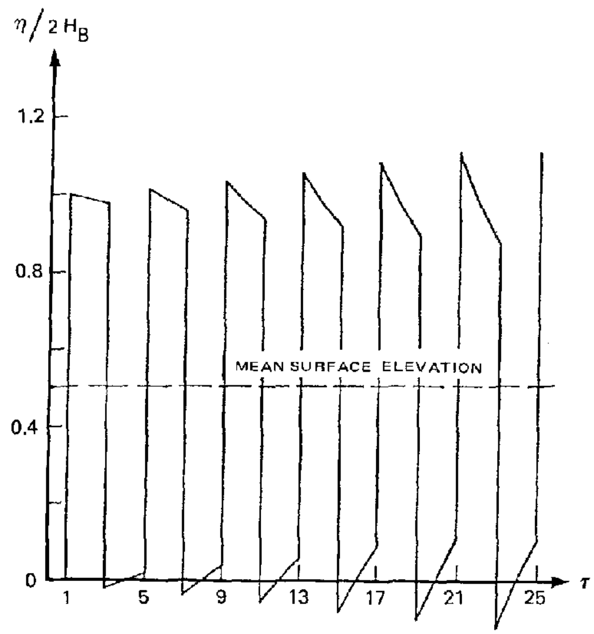

FIGURE 3 SEA SHOCK ABOVE THE INSTANTANEOUSLY DISPLACED BOTTOM.
$\tau = h/c_s$, $(c/c_s)^2 = 1/50$. (ADAPTED FROM SELLS, 1965)

Green function or by Fourier-Laplace transform techniques and can be expressed in general as:

$$\eta(\underset{\sim}{r},t) = \int_0^t \iint_S R(\bar{r},\bar{t})\, W_B(\underset{\sim}{r}_0,t_0)\, dS_0\, dt_0$$

with $$R = \frac{1}{2\pi}\int_0^\infty \frac{\cos\sigma\bar{t}}{\cosh k} J_0\,(k\bar{r})\, dk$$

and $$\sigma^2 = k \tanh k,\ \bar{r} = |\underset{\sim}{r} - \underset{\sim}{r}_0|,\ \bar{t} = t - t_0$$

For a sudden displacement of the bottom at time $t_0 = 0$,

$$W_B(\underset{\sim}{r}_0, t_0) = H_B(\underset{\sim}{r}_0)\ \delta\,(t_0)$$

$$\eta(\underset{\sim}{r},t) = \iint_S RH_B\, dS_0$$

For an abrupt point source displacement at the bottom, the initial surface displacement (the Green function) is shown in Figure 4 for one and two dimensional waves (Kajiura, 1963). The initial surface displacement has a width of about one or two depths, in contrast to the case of long wave approximation in which the surface disturbance is concentrated on a point immediately above the point source. A closed form solution for the initial elevation of a one dimensional wave due to an abrupt displacement of a block of width L is given by Sells (1965). The character of the solution is similar. The surface disturbance extends only one or two depths beyond the edge of the source, however large the width L is. Thus, for very large values of L, the non-hydrostatic effect can be neglected within the error of O(h/L). The resulting evolution of the wave with time for the case of L/h = 1/2 (rather small extent of the source) is shown in Figure 5.

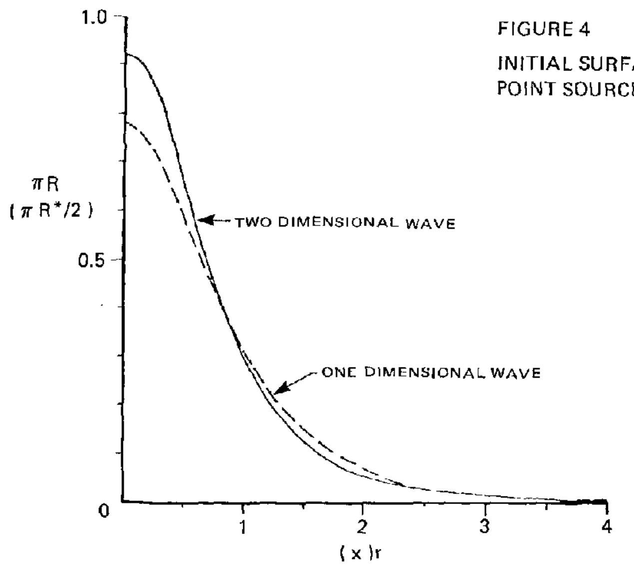

FIGURE 4

INITIAL SURFACE DISPLACEMENT DUE TO A POINT SOURCE AT THE BOTTOM

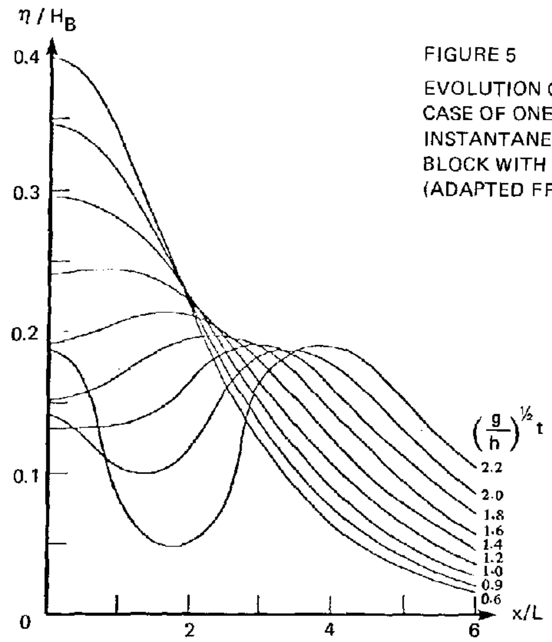

FIGURE 5

EVOLUTION OF A GRAVITY WAVE FOR THE CASE OF ONE DIMENSIONAL PROPAGATION. INSTANTANEOUS DISPLACEMENT H_B OF A BLOCK WITH A HALF LENGTH L (L/h = 1/2) (ADAPTED FROM SELLS, 1965)

Long Wave Approximation

The long wave approximation is very often used in numerical models. In this approximation, there is no dispersion and the Green function takes on a much simpler form:

$$R = \partial S/\partial \bar{t}$$

where $$S(\bar{r},\bar{t}) \begin{cases} = \frac{1}{2\pi} (\bar{t}^2 - \bar{r}^2)^{-1/2} & \bar{t} > \bar{r} \\ = 0 & \bar{t} \leq \bar{r} \end{cases}$$

for the two dimensional wave,

and $$R = \frac{1}{2} \{\delta(\bar{t}+\bar{x}) + \delta(\bar{t}-\bar{x})\}$$

for the one dimensional wave.

In this approximation, the initial surface disturbance caused by the abrupt displacement of the bottom takes exactly the same form as the bottom deformation. The longwave approximation is quite valid for most tsunami applications where the horizontal scales of motion are much larger than the water depth.

Energy Transfer*

Assuming a hydrostatic pressure distribution which is consistent with the long-wave approximation, the total energy E transferred to water can be computed as:

$$E = \int_0^T \iint_S p_B\, w_B\, dS_0\, dt_0$$

*All physical quantities are expressed in their dimensional form.

where the bottom pressure p_B is given by:

$$p_B = \rho g(h-h_B+\eta), \; W_B = \frac{\partial h_B}{\partial t}$$

The time dependent bottom displacement h_B satisfies the following:

$$h_B = H_B \qquad \text{for } t \geq T$$

and $$\iint_S H_B \, dS_0 = V$$

H_B is the final static displacement, T the duration of the ground motion and V the volume of water displaced by the ground.

The energy transfer can be written into a duration dependent part E_D and duration independent part E_0 with the latter being irrelevant for tsunami applications.

$$E = E_0 + E_D$$

where $$E_0 = \rho g(hV - \frac{1}{2} \iint_S H_B{}^2 \, dS_0)$$

$$E_D = \rho g \int_O^{T'} \iint_S W_B \quad \eta dS_0 \, dt_0$$

$$\sim \begin{cases} \rho g \iint_S H_B{}^2 \, dS_0 = E_D & t \to 0 \\ 0 & T \to \infty \end{cases}$$

The limit $T \to 0$ corresponds to instantaneous surface displacement and for slow movement ($T \to \infty$) E_D is negligible. The former is a good approximation for most tsunamis. A very important parameter in the evaluation is cT/A where c is the long wave velocity and A some horizontal length scale of the ground motion. Figure 6 shows the geometry of bottom displacement in a uniform depth h considered by Kajiura (1970). The resulting energy of the tsunami as a function of duration of bottom motion was computed and is shown in Figure 7. The limiting case of $B/A \to \infty$ corresponds to the one-dimensional wave. Even if the duration T is a minute or so and A several hundred kilometers, $cT/A \sim 0.1$, it can be seen that the assumption of instantaneous displacement is a good approximation from this point of view. Although based on a different approach, a similar conclusion was also obtained by Hammack (1973) for the case of the one-dimensional propagation of waves.

Directivity of Energy Radiation

The horizontal energy flux of diverging gravity waves per unit length at a distance R can be computed approximately by:

$$E_f(R,\theta) = \rho g \int_0^\infty \eta^2(R,\theta)\ c\ dt$$

and a directivity coefficient defined by:

$$Q(R,\theta) = 2\pi R\ E_f(R,\theta)/E_D$$

For a very long source, near the source the one-dimensional approximation is good until the edge effects arrive. Therefore a measure of approach to one-dimensionality is $R^* = R/X$ where X is the source dimension transverse to the direction of propagation ($X = A$ when $\theta = \pi/2$ and $X = B$ when $\theta = 0$). For very small R^* the situation is approximately one-dimensional so E_f is constant and Q increases with R. Far away, $R^* >> 1$, two-dimensional geometric spreading requires E_f to be inversely proportional to R so that Q is approximately constant. Figure 8 from Kajiura (1970) demonstrates this for specific

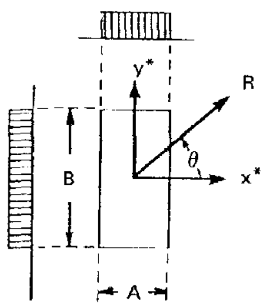

FIGURE 6 GEOMETRY OF THE BOTTOM DISPLACEMENT

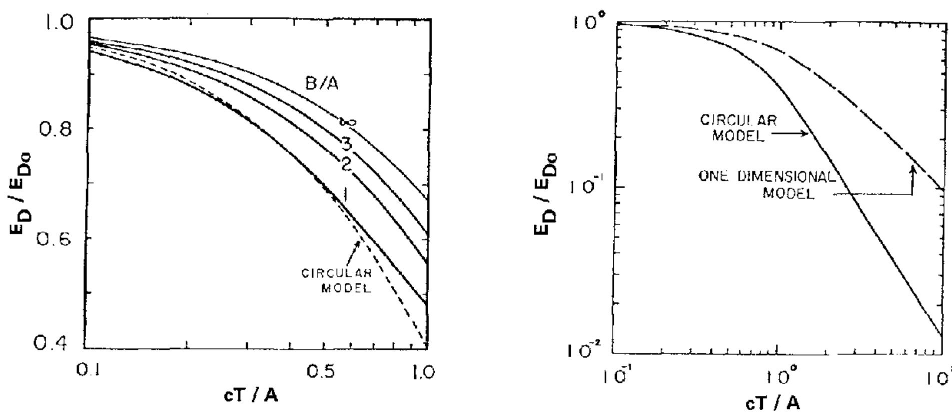

FIGURE 7 EFFICIENCY OF TSUNAMI GENERATION E_D/E_{DO} AS A FUNCTION OF THE DURATION OF BOTTOM MOVEMENT, cT/A. E_D IS THE GENERATED GRAVITY ENERGY AND E_{DO} IS THE ENERGY FOR THE CAST OF T→O.

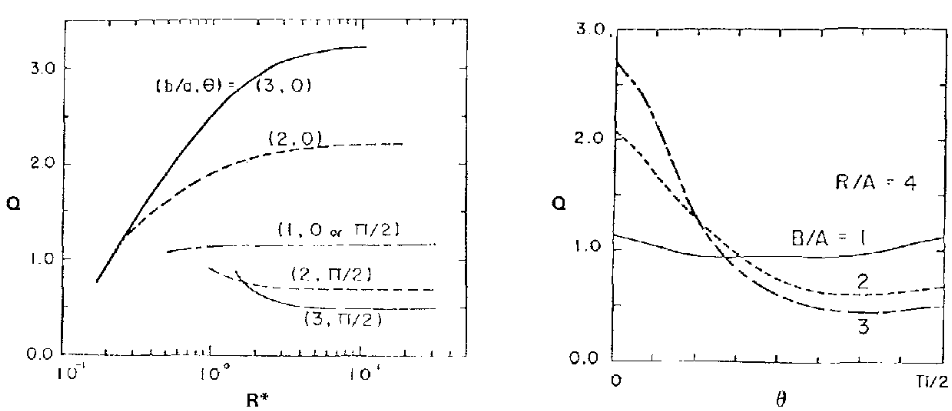

FIGURE 8 LEFT: DIRECTIVITY COEFFICIENT Q AS A FUNCTION OF A RELATIVE DISTANCE R*. RIGHT: DIRECTIVITY COEFFICIENT Q AS A FUNCTION OF θ. IN BOTH CASES, cT/A = 0.2.

cases, with two dimensionality being important at R* about 2 or 3. The figure of Q as a function of Θ shows almost isotropic radiation at R/A = 4 for B/A = 1. For more elongated sources, there is preferential concentration of energy in the direction transverse to the major axis.

Edge Wave Generation

Another factor to be considered is the generation of edge waves. Consider the situation in Figure 9 with the bottom motion of length 2a confined to a shelf of width l. Then it can be shown (Kajiura, 1972) that as the length of the source increases in comparison to the shelf width, the proportion of energy going into the edge waves decreases, or the proportion of energy going into the deep ocean increases. Figure 10 shows this for a few particular cases. Thus for the particular depth ratio shown, when the length of the source is much smaller than the shelf width (2a/l is small), then more than fifty percent of the energy is trapped on the shelf. But when the source is large, say $2a/l \sim 5$, most of the energy is radiated into the ocean.

Far Field Propagation

So far the discussion has been confined to processes within the generation region, or the near field. In the far-field ($t \gg 1$, $r \gg 1$) and near the front ($t \sim r$), the wave length is very long ($k \gg 1$) because of dispersion, then σ can be expanded as:

$$\sigma \simeq k - \frac{1}{6} k^3$$

and the function becomes (Kajiura, 1963):

$$R = \partial S/\partial \bar{t}$$

$$= \frac{1}{2\pi} (\pi \bar{r})^{-1/2} (6/\bar{t})^{-1/2} \partial T/\partial p$$

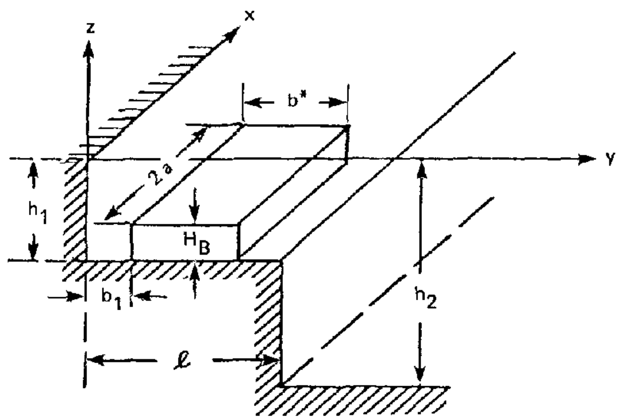

FIGURE 9 GEOMETRY OF A TSUNAMI SOURCE ON THE SHELF

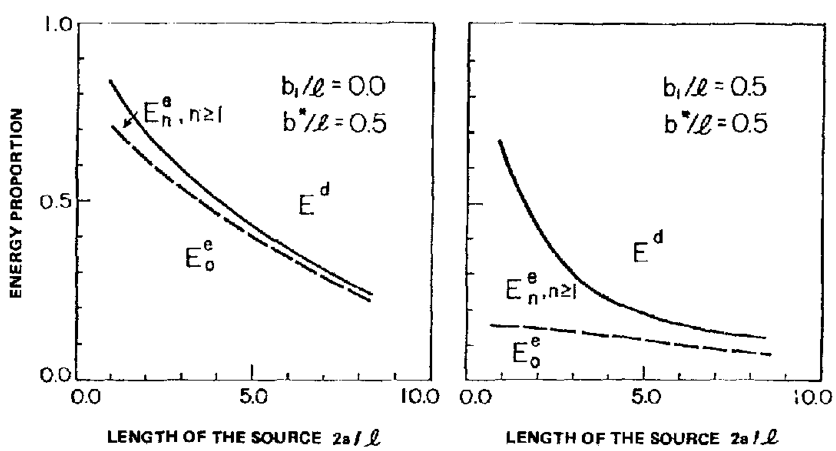

FIGURE 10 ENERGY PARTITION AMONG WAVES OF TRAPPED AND LEAKY MODES. E_n^e; EDGE WAVE MODES, E^d; LEAKY MODE, $h_1/h_2 = 0.05$

and $$S = \frac{1}{2\pi} (\pi \bar{r})^{-1/2} (6/\bar{t})^{-1/6} \quad T(p)$$

with $$T(p) = \text{Re} \left[(1+i) \int_0^\infty \exp i \, (u^6 + pu^2) \, du \right]$$

where $$p = \left(\frac{6}{\bar{t}}\right)^{1/3} \quad (\bar{r} - \bar{t})$$

T(p) has to be computed numerically once and for all. The behavior of $\partial T/\partial p$ is somewhat analogous to the Airy function. The idea is that the long distance asymptotic solution is derived for a point source at first within the relative error of $O(1/\bar{r})$, and then construct a solution for a given source by the method of superposition.

In particular, for the abrupt displacement of the bottom confined between $|x_0| < a$, $|y_0| < b$ (see Figure 11), the solution far away in the x-direction (a/r, $b/r \ll 1$) can be written as:

$$\eta = \frac{1}{2\pi} (\pi r)^{-1/2} (6/t)^{1/6} \, U$$

where $$U(p^*) = \int_{-p_a}^{-p_a} \tilde{H}_B \, (p_0) \, (-\partial T/\partial p) \, dp_0$$

and $$\tilde{H}_B = \int_{-b}^{b} H_B \, dy_0$$

with $$p^* = (6/t)^{1/3} \xi \, , \; p_0 = (6/t)^{1/3} x_0, \; p_a = (6/t)^{1/3} \, a$$

and $$\xi = r - t, \; p = p^* - p_0$$

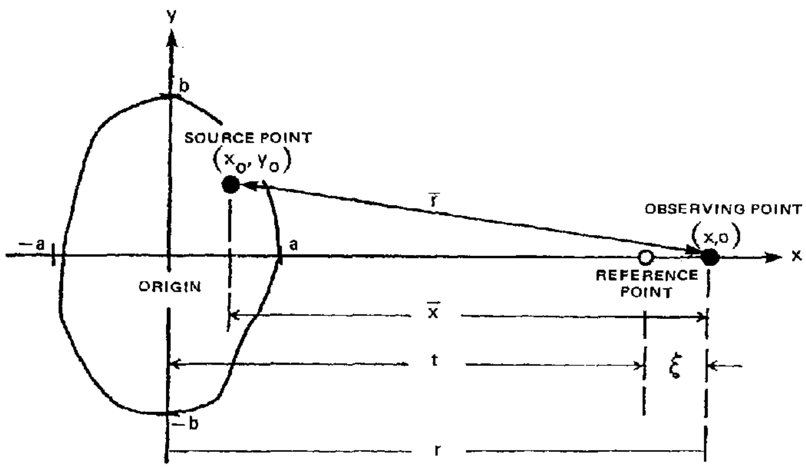

FIGURE 11 GEOMETRY OF SOURCE AND FIELD POINTS FOR THE FAR FIELD ASYMPTOTICS PROBLEM

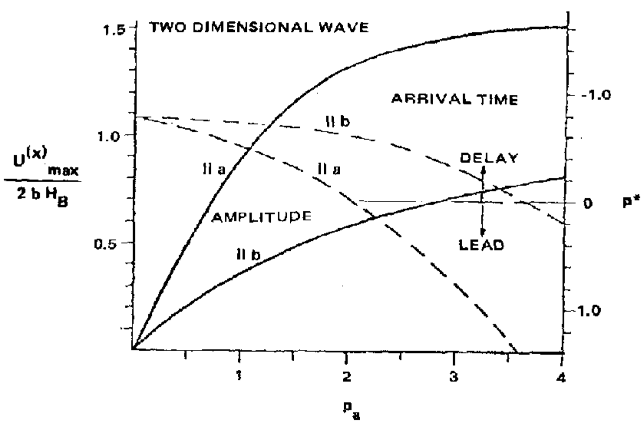

FIGURE 12 MAXIMUM AMPLITUDE (SOLID LINE) AND THE ARRIVAL OF THE MAXIMUM (BROKEN LINE) AS A FUNCTION OF p_a FOR MODEL IIa AND IIb

Similar approximations can be developed (Kajiura 1963) for the one-dimensional wave and for the case of an initial impulse as well.

For the bottom deformation of the following form:

Model IIa: H_B = constant $|x_0| < a, |y_0| < b$

Model IIb: $H_B = H_{B0}\left\{1 - (y_0/b)^2 - (x_0/a)^2\right\}$

the amplitude decay with distance is shown in Figure 12 (solid lines). From these curves, one can see the importance of the parameter p_a for the decay of the leading wave at large distances. The parameter p_a and its impact on the decay with distance is summarized in Table 2. Note that the example given here is relevant for a simple deformation with finite volume only. In general, a higher order asymmetry in the forcing function leads to higher powers in the decay rates (see Braddock et al. 1973).

Range of Applicability of Long Wave Approximation

A comparison of tsunami time histories at a large distance r from the source using the dispersive and non-dispersive theories will also show the importance of p_a. Consider the case of Model IIb: an instantaneous bottom displacement of a parabolic form over an elliptical area. The results are shown in Figure 13 (Kajiura, 1970), with the scaled time t* and elevation ζ^* in the x-direction defined by:

$$t^* = (a-\zeta)/a$$

and $$\eta/H_{BO} = \frac{b}{\sqrt{ar}}\,\zeta^*$$

The remarkable similarity of results between dispersive and nondispersive theories is evident for $p_a \gtrsim 4$. For $p_a \lesssim 2$ it is also evident that the dispersive theory must be used. These results can be summarized by the plot of

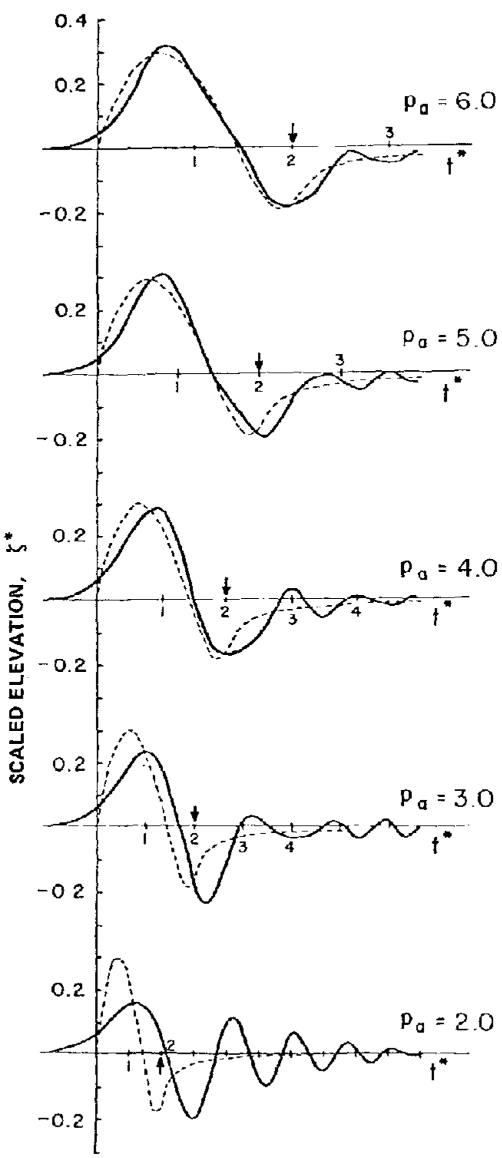

FIGURE 13 COMPARISON OF THE WAVE TIME HISTORIES FOR MODEL IIb IN THE x–DIRECTION. SOLID LINES FOR DISPERSIVE THEORY AND BROKEN LINES FOR NONDISPERSIVE THEORY.

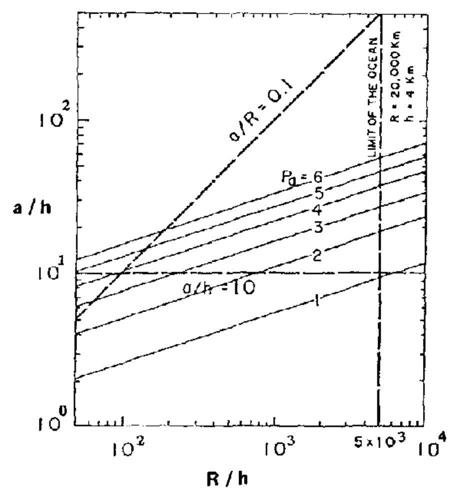

FIGURE 14

DIAGRAM INDICATING THE RANGE OF VALIDITY OF THE NONDISPERSIVE ASSUMPTION WITH RESPECT TO a/h AND R/h.

Figure 14 indicating the range of validity of the non-dispersive assumption. Assuming a mean depth of 4 km for the Pacific Ocean, the maximum distance of $R = 20{,}000$ km imposes an upper limit of $R/h = 5 \times 10^3$. A ground displacement with a value of $a/h = 10$ will require dispersive theory when $R/h \gtrsim 750$ (i.e., $R \gtrsim 3{,}000$ km for $h = 4$ km). If the source with the total width L in the offshore direction is located on the continental shelf of depth h', the deep water scale a/h is given by:

$$a/h = (L/2) / \sqrt{h\,h'}$$

For example, take $h' = 200$ m and $L = 50$ km. Then, $a/h \sim 28$ and the nondispersive theory is safely applicable ($p_a \gtrsim 4$) up to $R/h \lesssim 2000$ ($R \lesssim 8000$ km).

Similar comparison of wave forms between the dispersive and nondispersive theories based on the analytical solution was presented by Carrier (1970) for the case of one-dimensional propagation of the initial Gaussian wave form. If the scale "a" is chosen suitably (say, the distance from the center of the Gaussian wave form to the position where the elevation is 1/10 of the maximum), the acceptable limit of the long wave approximation can also be defined by a certain value of p_a around 4.

There are other aspects of tsunamis which remain to be discussed, such as nonlinearity. These will probably be taken up by others in the following sessions.

The author wishes to express his sincere thanks to Dr. Y. Keen Lee for his great efforts to summarize my talk at the seminar. Without his help this paper would not be completed.

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DISCUSSION

The following discussions ensued during the session. These are grouped according to the topic dealt with.

1. Length Scales of Ground Displacements for Tsunami Generation

E. Tuck - You mentioned 2:1 for the eccentricity of the lateral dimensions of the uplift region. The really big earthquakes people have been talking about earlier, such as the Alaskan and Chilean ones, have far greater eccentricities.

K. Kajiura - The 2:1 ratio was obtained from Kelleher's work, but it is relevant for most tsunamis and earthquakes. 2:1 is very good for tsunamis.

R. Geller - Abe has also shown similar results for other earthquakes.

K. Kajiura - The Chilean and Alaskan earthquakes are quite exceptional. Kamchatka too. Those three are the largest in a few hundred years.

E. Tuck - A comment about this ratio, which should be relevant to all tsunamis: You can't really take just the total area of the uplift but the area of maximum uplift. Those earthquakes are extremely elongated because the maximum uplift is very, very narrow.

K. Kajiura - This 2:1 is just for a first approximation.

2. Internal Waves

Anonymous - A question on the internal waves. You said that the Coriolis force will always be important. Isn't that dependent on the latitude of the source location?

K. Kajiura - The relevant scale is the ratio between Rossby's deformation radius and the horizontal length scale of the upthrust.

J. Hammack - The relevant scale should be the ratio of the period of the wave and the inertial period at the site of the source region.

K. Kajiura - But my discussion is for the generated wave. Yours is for the free waves. There are two scales: a time scale and a space scale. What I mentioned was for an instantaneously generated wave which is then radiated. When we have a disturbance over a large horizontal area then the Coriolis force is important. The radius of deformation is the scale c/f, where c is the internal wave speed, about 2 m/s.

Anonymous - But that ratio goes to infinity near the equator. So it is dependent on latitude.

K. Kajiura - Yes. What I really want to point out in my talk is not the importance of the Coriolis force but the fact that the forcing is very small so the energy generated is very small for the internal waves.

3. Sea Shocks

T. Wu - What is the boundary condition at the sea bottom for the sea-shock computations? Perfect reflection?

K. Kajiura - A rigid bottom.

T. Wu - Have you tried any impedance condition?

K. Kajiura - I have never seen the problem treated in this way with impedance included. Seismologists are always interested in the far-field while this is directly above the ground motions which are not seismic waves but a mean displacement.

G. Carrier - If you put an impedance at the bottom such that you have a 10 percent transmission coefficient, for example, then that is the only fact you need to tell you that you have an e-folding time ten times that.

4. The Initial Wave

T. Wu - I don't see your surprise that the trough in the evolution of the initial wave given by Sells does not go negative. The excess mass for this case must be constant - the original water above the undisturbed water surface must be constant. So it may take some time yet.

K. Kajiura - This is for a small disturbance only.

5. Dispersive Effects

C. Mei - You compared, for a very large tectonic area, two theories: one is for dispersive waves and one is the non-dispersive longwave theory. Did you use the full dispersive theory or the linearized Boussinesq approximation?

K. Kajiura - I took the first two terms of the dispersion relation so that is like the Boussinesq theory.

C. Mei - But that theory applies to only the far field near the wave front. I wonder how good the non-dispersive shallow water theory is in the near field if the tectonic area is very large. The dispersive theory you used for comparison is not the exact, not the most accurate possible, dispersive theory.

K. Kajiura - In that theory, taking only two terms, the higher frequency components are exaggerated.

C. Mei - Does that affect the accuracy of your comparison in the near field very near the source region?

G. Carrier - If the Boussinesq theory says that in the region under consideration dispersion is not important, then it is not important. I think that's the answer to the question asked. I know it is a correct statement.

K. Kajiura - Near the origin, there is a complete dispersive theory available which can be evaluated numerically. The comparison with the long-wave theory gives very good agreement.

3 TSUNAMI PROPAGATION

1. MODELS FOR PREDICTING TSUNAMI PROPAGATION
   CHAIRMAN : E. O. TUCK
   RECORDER : PHILIP L. F. LIN

2. EVALUATION OF EXISTING MODELS
   CHAIRMAN : T. Y. WU
   RECORDER : J. J. LEE

MODELS FOR PREDICTING TSUNAMI PROPAGATION

BY
E. O. TUCK

The detailed paper, following this summary, contains a review and extension of theoretical and numerical models for tsunami generation, propagation and reception. Here the main conclusions are summarized, and discussed in sequence.

The linear long-wave equations are adequate to describe most of the tsunami generation, propagation and reception processes

This is clearly a potentially controversial conclusion, but is moderated by the words "adequate" and "most" in its statement. It is not disputed that both non-linearity and frequency dispersion can play a role in parts of the tsunami story; the thesis advanced here is that these effects never play a major role for significant tsunamis.

Various new and old arguments are put forward to support this conclusion. It is clear that the conclusion is necessarily valid if the tsunami wave is "sufficiently" low in height and long in length. The only argument that can arise concerns the criteria for sufficiency, and it is recognized that establishment of such criteria is a most difficult task. Nevertheless, criteria extracted from the recent literature and new estimates based on parameters of the important 1964 Alaskan tsunami, suggest that the errors involved in neglect of non-linearity and frequency dispersion are indeed extremely small over the whole time and space history of such significant tsunamis.

Another basis for this conclusion is the purely pragmatic one, that when this approximation is made, the predicted results are "adequate," i.e., acceptable within engineering bounds commensurate with the accuracy of the input data and the uses to which the results are to be put. Again, evidence for the truth of this conclusion is provided both from recent publication and in the

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form of illustrative computations for the 1964 tsunami, presented here for the first time.

It could, of course, be argued, also from a pragmatic point of view, that given present computing capabilities and advances in numerical methods, there is no need to make the approximations recommended, even if they are justified, since we can simply let the computer grind away forever with the exact potential flow equations - or perhaps even with the full Navier-Stokes equation! Although such a point of view has some merit, it is also clear that a line has to be drawn somewhere, and the first conclusion is really advocacy of a suitable place to draw that line. The residual numerical task when the line is drawn at this point is far from trivial.

The input data, especially for the upthrust, are seldom likely to be good enough to allow accurate tsunami prediction

This is hardly a surprising or controversial conclusion, but nevertheless needs to be stated and constantly kept in mind when any attempts are being made to provide theoretical or computational predictions of tsunami behavior. Tsunamis are caused by undersea earthquakes whose upthrust temporal and spatial characteristics are almost completely unknown during the event itself, and determinable subsequently only in the crudest outline. It is hardly conceivable that a seismic event has ever been or will ever be better documented after the event than the 1964 Alaskan earthquake; yet even then the data that can be used as input for numerical work are uncertain by factors of 2 or more in many important aspects.

Numerical solution is particularly sensitive to the spatial discretization interval, and a too-large interval can produce spurious short-period oscillations

An important detail leading to the first conclusion requires estimation of the dominant periods in the spectrum of a propagating deep-ocean tsunami. Examination of some of the existing literature involving numerical computations from seismic data gives the impression that a significant amount of energy may exist at periods of the order of 10 minutes or less, even for an event of the scale of the 1964 tsunami.

The point is made here that such energy may be spurious, an artifact of the discretization process involved in the numerical solutions. In view of the enormity of the task of preparation of input data, it is natural to use a spatial mesh that is not too small in size, and meshes of sizes of the order of 10 to 20 km are common in the literature. However, it takes the tsunami times of the order of 10 minutes to cross a single such mesh element, and it is possible that spurious partial-standing-wave oscillations are produced with such periods.

This type of error is somewhat analogous to "aliasing" errors in spectral analysis of stationary random processes, and in the field it is accepted that such errors are inevitable, i.e., not in general capable of elimination by clever numerical methods, but rather to be avoided by restricting computations to lower frequencies. The analogy is not quite exact, however, (since most of the element of randomness is missing) and there may indeed be scope for numerical ingenuity in the present problem. Nevertheless, the warning is there, and one should be hesitant to accept without qualification presence of apparent energy at noise-like frequencies, corresponding to the transit times for the numerical mesh.

New sample one-dimensional computations are presented here to illustrate this phenomenon, by varying the mesh size from 20 km down to 1 km. The spurious oscillations reduce in amplitude and period in a consistent manner as the mesh size is reduced, until a satisfactory solution, essentially free of short-period oscillations, is obtained from the 1964 tsunami at a mesh size of 1 km. Unfortunately, it is not reasonable to expect that such a small mesh could be used uniformly for any global tsunami study, so that there is a need for a degree of sophistication in the numerical methods beyond what has been common up to now.

If continental-slope upthrust occurs, it plays an enhanced role in the first-arriving part of the deep-ocean tsunami

Tsunamigenic earthquakes tend to occur on continental shelves. A portion (of the order of half) of the resulting upthrust moves shoreward and may cause local havoc, before being reflected to follow the other half of the upthrust into the deep ocean. So long as the disturbance is on the shelf, it moves relatively slowly, say at 40 meters/second, but in the deep ocean this speed increases to the order of 200 meters/second.

This means that if some upthrust does occur off the shelf, e.g., on the continental slope, the effect of such upthrust will be observed at a distant shore significantly earlier than that for the on-shelf upthrust which has been delayed by a slower over-shelf transit. Not only that, but the height of this first-arriving signal will be magnified relative to the on-shelf portion, because the latter will suffer reflections (e.g., at the shelf-slope boundary) to which the former is immune.

Hence it is clear that if one is most interested in the leading wave of a tsunami (and in general that must be the case for ultimate reception at a far coastline), the portion, if any, of the upthrust that occurs off the continental shelf must be of the greatest importance. Unfortunately, the available seismic data for such upthrust are inevitably poor to nonexistent!

This paradox is illustrated by sample computations for the 1964 tsunami, in which two plausible seismic models, one involving continental-slope upthrust and one not, are compared. The difference in the results for the deep-ocean tsunami is quite profound, and comparison with records at reception sites suggests that, at least for this event, upthrust was essentially confined to the shelf.

One-dimensional strip-wise models of the generation and reception processes are valid for elongated upthrust zones and coastlines, and can be matched with suitable two-dimensional solutions in the deep ocean

The detailed paper also contains a first attempt at a theory capable of bridging the gap between the generation and propagation problems for elongated upthrust zones. It is clear intuitively that when the earthquake upthrust zone is elongated along the relatively-narrow continental shelf, the disturbance will tend to propagate at first in a direction mainly perpendicular to the axis of elongation. This assumption can be given mathematical expression as a formal asymptotic limit when the aspect ratio (length/breadth) of the upthrust zone becomes large. Such elongated upthrust zones are a common feature of (indeed may even be a prerequisite for) generation of a tsunami that is capable of causing significant disturbance at a distant shore.

The simplified one-dimensional theory that results is obviously very attractive from the computational point of view, and one can easily construct a strip-wise procedure for its solution out to the deep ocean, at every value of a coordinate measured along the upthrust axis. However, the validity of such one-dimensional computations must necessarily be confined to distances perpendicular to the axis that are small compared to the axis length, since at greater distances the two-dimensional influence of the finiteness of the axis length must begin to be felt. Also, such a theory cannot ever be accurate near the ends of the axis. Nevertheless, there remains a large portion (extending well out into the deep ocean) of the generation area of a tsunami such as that of 1964, for which a one-dimensional theory appears appropriate.

The ultimate failure of a one-dimensional theory due to two-dimensional dispersion is no cause for rejection of its basis. Instead, what is needed is a procedure to match the one-dimensional results to a two-dimensional deep-ocean propagation model. The analogy is with an elongated loudspeaker in acoustics, or an antenna in electromagnetic propagation, such that a local one-dimensional theory serves to quantify the characteristics of the radiator, as seen in the far field.

An approach to such a matching procedure is provided here, using as a tool the "parabolic approximation" of diffraction theory. This theory assumes that the wave slowly modulates as it propagates in a mainly uni-directional manner, and computes the resulting two-dimensional radiation pattern. This procedure is capable of analyzing the well-known directional effect of elongation of the upthrust zone on the deep-ocean tsunami.

DETAILED TEXT OF MODELS FOR PREDICTING TSUNAMI PROPAGATION

INTRODUCTION

The main original content of this paper is presented in the form of 8 technical appendices. These cover a variety of aspects of the title problem, and each is a separate mathematical or numerical study. These are supplemented by a discursive text, in which the general tsunami problem is surveyed, and various conclusions arrived at, based on intuitive reasoning, previously published work, and results taken from the appendices.

The important conclusions are:

(1) The linear long-wave equations are adequate to describe most of the tsunami generation, propagation and reception processes.

(2) The input data, especially for the upthrust, is seldom likely to be good enough to allow accurate tsunami prediction.

(3) Numerical solution is particularly sensitive to the spatial discretization interval, and a too-large interval can produce spurious short-period oscillations.

(4) If continental-slope upthrust occurs, it plays an enhanced role in the first-arriving part of the deep-ocean tsunami.

(5) One-dimensional strip-wise models of the generation and reception processes are valid for elongated upthrust zones and coastlines, and can be matched with suitable two-dimensional solutions in the deep ocean.

Tsunamis are very long waves, normally of extremely low steepness. Although an attempt (probably futile) is being made by the scientific community to educate the general public to use the term "tsunami" rather than the common English-language expression "tidal wave," perhaps it is a pity entirely to dismiss the mental association with tides. Of course, the physical mechanism has nothing to do with astronomical tides, but the common use of the term "tidal wave" surely arose because most tsunamis are quite satisfactorily described as giving the appearance of "fast-rising tides." Instead of being offended by a terminology which may appear to associate a phenomenon with an incorrect cause, we should rather be pleased that it provides a reasonably-accurate picture (to those of the general public who think about the meaning of words!) of what actually happens. A near-vertical moving wall of water, as in movies like "The Poseidon Adventure," is unlikely to occur in the open ocean, and is the exception rather than the rule for coastal impact of actual tsunamis.

Even within the community of oceanographically-oriented scientists, there appears some confusion as to just how long tsunamis are. A common answer to my questions, "What is a typical tsunami period?", asked recently of a representative sample was 10 minutes. Although obviously there is no universally-correct answer, 10 minutes is rather far from the true mean. Answers between 20 minutes and 2 hours would have been more reasonable. One problem, certainly in the United States, is that most concern has been with places like Hilo, Hawaii, which has a highly-tuned harbor resonance at about 18 minute period. Hilo "sings" at this period, no matter what longer-period excitation hits it, and the extent of devastation there depends less on the frequency content, than of the direction of propagation of the tsunami.

In this paper I shall pay greatest attention to the tsunami of the 1964 Alaskan earthquake, mainly because of the large amount of available data, especially as collected together in References (7) and (8). However, other destructive tsunamis have similar characteristics. Major tsunamigenic earthquakes tend to occur on the continental shelves of Alaska, Chile, or Japan.

The nature of the faulting mechanisms driving these earthquakes demands a somewhat-elongated generating zone, with the long axis running along the shelf, and with the "short" axis more or less normal to the coastline and encompassing a significant fraction of the whole shelf. Such is certainly the pattern for the major 1964 Alaskan and 1960 Chilean tsunamis.

The length of the short axis of the earthquake uplift area is the main determinant of the length of the outgoing tsunami, with the width of the continental shelf also playing a major role. Major tsunamis may be major largely because these two length scales are close in value. The continental shelf is a partially resonant chamber for oscillations normal to its contours, and necessarily responds greatest to excitations with length scales close to its natural modes of oscillation. Because of the large ratio (~50) between continental-shelf and deep-ocean depths, there is a significant amount of reflection (Lamb amplitude coefficient ~0.75) of long waves at the continental slope, and hence a relatively-small radiation damping, so that the resonance is highly tuned.

The 25 percent or so of the wave which does get across the continental slope constitutes the deep-ocean tsunami, which then travels more or less unimpeded, dispersing gradually due to two-dimensional effects, until it meets a far coast where it may still have sufficient energy to wreak havoc. Meanwhile, the original wave continues to bounce back and forth over the continental shelf, sending out a new tsunami each time it meets the continental slope, and losing energy gradually, both by this mechanism and by dissipation and dispersion on the coastline. This can last for days, although of course most of the energy goes within the first few hours.

Indeed, just what time scales are we talking about? For the 1964 Alaskan tsunami, information regarding periods was gathered from tide-gauge data all around the Pacific, by Wilson and Torum and by Van Dorn and Cox, and is summarized in the paper of the latter authors in Reference (7). There is not the slightest doubt that the main 1964 tsunami period is in the range 1 1/2 to 2 hours, with a fairly reliable mean value of 1.7 hours. There are, of course, shorter waves present in the tide-gauge data, but there seems no urgency to invoke mechanisms other than local resonances to explain these. The fact is, in March 1964, the Pacific Ocean was subjected to an excitation of dominant period 1.7 hours. Its border responded to this excitation in a manner dependent on local topography.

Instead of looking for shorter waves, it is instructive to examine evidence for longer waves. The extensive collection of observational evidence on the Alaskan coast by Wilson and Torum (1972) certainly seems to suggest up to 4 hours periodicities, and these authors even attempt (quite unnecessarily) to explain (p. 475) the 4-hour period as an astronomical-tide harmonic. In fact, the fundamental Alaskan continental shelf period is about 4 hours. Hence, one should expect a situation in which Alaskan observers report recurrence of signifiant events on that time scale. Of course, this is such a long period that one is inhibited from describing it as a wave, and also is such that there is inevitably observational confusion with the astronomical tide. Also, in order to excite this mode preferentially, the upthrust would have had to have been greatest near the shore (which was certainly not true), and essentially one-signed, which was not quite the case.

On the other hand, the first-harmonic shelf period is about 1.4 hours and is clearly an important contributor to the main tsunami wave. The nature of the upthrust pattern in 1964 was such that this mode (with two nodes, one near the edge of the continental shelf and one somewhere between the shore and that point) was preferentially excited, as indicated by some model computations in Appendix C.

If the continental shelf was of exactly-uniform depth, with perfect open-end reflection at its seaward edge, and closed-end reflection at the shore, the 1st-harmonic/fundamental period ratio would be exactly 1/3, and the internal node would be 1/3 of the shelf width from shore. The 1st-harmonic is in practice a little longer, and the internal node is nearer to the shore because of the slow depth increase over the shelf, and the smooth but fast transition over the slope. For example, in Appendix B, we indicate how, for a special idealized nonuniform depth, the period ratio approaches 1/2, and the internal node is at 1/8th of the shelf width from shore.

The 2nd-harmonic (3-noded) period of the Alaskan shelf is about 50 minutes. This is a little longer than the value (1/5 of the fundamental) that is predicted for a perfect uniform shelf, and appears to be only weakly excited by the 1964 upthrust pattern. Higher harmonics are even more difficult to excite.

The "half-wavelength" of the dominant wave on the shelf is of course the length scale of the earthquake uplift zone, which, for the 1964 tsunami, is close to the half-width of the shelf itself. Thus a representative number for the on-shelf full wavelength is 160 km. This increases to the order of 1200 km in the deep ocean, corresponding to the main period 1.7 h in water of depth about 4 km, i.e., wave speed about 200 ms^{-1}.

This is an enormous wavelength. For example, two waves span the whole distance between the generating area and an important reception point at Crescent City, California, and even Australia is only ten wavelengths away. For such waves, dispersion due to finite depth effects is utterly negligible, during every phase of generation, propagation and reception.

These are also waves of very low steepness. The case is clearest in the open ocean, where a maximum elevation of about 1 metre is reasonable, giving a steepness less than 10^{-6}; it is not easy to conceive of a more gentle disturbance. Even on the generating shelf itself, where the wavelength is less and the full upthrust (in 1964 of the peak order of 9 metres) is present, the mean steepness of the major wave is still only of the order of 10^{-4} or less. On a receiving shelf, where the amplitude is again much less, the steepness is not much more than the deep-ocean value, until very near the shoreline. These gentle disturbances are amply describable by linear equations; no non-linear effects are necessary to account for the main tsunami, which is therefore determined by solving the linear long-wave or shallow-water equations.

The above arguments are broad generalizations and intuitive deductions, based on the extreme smallness of the appropriate measures. More-precise estimation procedures have recently been outlined by Hammack and Segur (1978), with similar conclusions. Hammack and Segur base their arguments on an a priori assumption that (at least for h constant and $\partial/\partial y \equiv 0$) uni-directional wave propagation is adequately modelled by the well-known Korteweg-deVries equation. While this may still be a matter of some controversy, it is true that this equation does at least contain terms which separately measure the effects of non-linearity and of frequency dispersion. The orders of magnitude of these terms (compared to those retained in the linearized shallow-water theory) are (amplitude/depth) and $(\text{depth/wavelength})^2$, respectively, and thus these effects are both quite negligible for tsunamis. The so-called Ursell number, which measures the relative importance of these two small terms, conveys no information by itself about their separate negligibility.

No equation purporting to describe a physical situation is ever exact. As soon as one has removed one's pen from the paper, one has made an approximation to reality. It is almost always the case that formal justification for such approximations, if attempted at all, is based on estimation of the orders of magnitude of the neglected terms in the equation compared to those retained. Justification based on solution of the (unapproximated) equation is less often available, since if one could solve the complete equation, one would not normally have made the approximation in the first place. However, where special exact solutions exist, they may serve to confirm validity of the approximation, and the analysis of Hammack and Segur is based on some such special solutions of the Korteweg-deVries equation.

Justification based on solutions can also take place a posteriori. That is, if one goes ahead and makes the approximations anyway, is the solution of the resulting approximate equation "satisfactory?" In the present context, evidence for the validity of (1) as an adequate model for tsunami generation, propagation, and reception has been accumulating steadily of late, as computations such as those of Houston (1978) continue to give agreement with observation that is as satisfactory as any one could wish, in view of the limitations of both input and output data. Some additional, but more limited, computations of this nature are presented in Appendix G of the present paper. Although some non-linear effects are included, the good agreement with observation shown by Hwang and Divoky (1975) confirms that unimportance of frequency dispersion.

Of course, one must not over-sell this simplication. In particular, in the generating zone, both very close to the point of maximum upthrust, and at the nearest shoreline points, there is a possibility of significant non-linear and dispersive effects. Indeed, non-linearity is inevitable at the final stage of run-up on any slope, as the water depth finally vanishes, and non-linearity must be included in any complete analysis of this stage. However, the seaward extent of the zone where shoreline non-linearity is important cannot be measured in more than some hundreds of metres, and is unlikely to influence the overall generation and propagation phenomenon. Breaking can occur (but need not), and this is a nonlinear phenomenon which can have far-field effects due to energy dissipation. Breaking and non-breaking dissipation can be modelled empirically, by use of appropriately-reduced reflection coefficients at the shoreline, as in Appendix A.

Extremely localized non-linear and dispersive effects are also possible at the very centers of the earthquake activity. These occur with periods of the order of the earthquake time, i.e., finish in a matter of seconds, and hence are definitely dispersive. The non-linearity is also striking, but very local, as the graphic descriptions of the 1964 destruction of Valdez and Seward (Reference 7) testify. Once these local and immediate crisis pass, the main linear tsunami still runs its inevitable course, and adds to the peril for hours to follow. Although the prediction of the instantaneous disturbance is clearly a major problem, it is unlikely that what happens locally during these first seconds has any serious effect on the evolution of the subsequent linear tsunami.

Suppose that, in spite of some reservations as above, we are prepared to accept the use of the linear long-wave equations to describe at least the dominant phenomena of tsunami generation, propagation and reception. Is the problem then reduced to a trivial exercise? Far from it.

To fix ideas, let us write down the equation of interest, neglecting for the moment the curvature of the earth, and assuming that the ocean is describable in terms of its depth contours $h = h\ (x,y)$ for some set of axes (x,y) on the assumed plane of the earth. If the water-surface elevation is $\eta(x,y,t)$, the linear long-wave equation can be written (Stoker 1957):

$$\frac{\partial}{\partial x}\ (h\eta_x) + \frac{\partial}{\partial y}\ (h\eta_y) = \frac{1}{g}\ \frac{\partial^2}{\partial t^2}\ (\eta - \eta_o) \qquad (1)$$

where $\eta_o(x,y,t)$ is the upthrust of the ocean floor, due to the earthquake. This partial differential equation must be solved subject to suitable initial conditions immediately prior to the earthquake, and boundary conditions at shorelines. If we wish to confine attention to a manageable segment of the ocean, not entirely bounded by land, we shall also need appropriate radiation conditions at the artificial truncation boundary.

Virtually every aspect of the above problem specification is non-trivial and fraught with uncertainity. Of course, the biggest uncertainty of all is associated with $\eta_o(x,y,t)$. The earthquake upthrust must be specified fully, as a function of space and time, before we can commence solving (1). The fact is, even for the most thoroughly studied earthquake of all time, in 1964, our knowledge of $\eta_o(x,y,t)$ is in many respects unsatisfactory.

In particular, we know nothing of the time history of the earthquake upthrust. The best we can expect to be able to measure is $\eta_o(x,y,\infty)$, i.e., the permanent deformation. We can only guess at how this deformation was arrived at. We hope that the fine detail of this time history, occurring over a period of at most a few minutes, is not significant in the generation process. This hope can be quantified (e.g. Tuck and Hwang 1972 and Appendix E) and it would appear that, with errors comparable to those already made in accepting (1) as the governing equation, we may approximate the ground motion as a step function, i.e.:

$$\eta_o(x,y,t) = \begin{cases} 0 , & t < 0 \\ \eta_o(x,y,\infty), & t > 0. \end{cases} \tag{2}$$

This approximation is equivalent to saying that when the shaking stops, the water surface has simply been elevated to $\eta_o(x,y,\infty)$, as if by an instantaneous step, and then the tsunami generation begins, commencing with a state of rest. One would expect to be permitted a small amount of smoothing of the upthrust, very steep local displacements being dispersed during the earthquake.

But even $\eta_o(x,y,\infty)$ is hardly known satisfactorily for most earthquakes. The data available for the 1964 earthquake (summarized by Plafker in (8) far exceeds that for any previous case, and yet is still not good enough. The deep-ocean extent of the uplift zone is a matter of pure speculation, and, as we shall see, this is a rather important detail.

Even if $\eta_o(x,y,t)$ were adequately known, there would still be many profound issues to settle. The equation itself is not too hard to solve numerically with present techniques and computers, but it does demand as input, depth contours $h(x,y)$ that are not always known very well. Just how much precision in $h(x,y)$ and $\eta_o(x,y,t)$ is significant?

What size of discretized mesh in space is acceptable, and what time step should we use? Although a number of successful computer programs have been developed (e.g., Hwang and Divoky in (7) and (5), Houston (1978) little effort has been devoted to sensitivity studies, i.e., to the question of just what is important, what approximations are acceptable, and how much of what one computes is computational artifact and how much is significant physically.

One other aspect of use of (1) is worth comment. Equation (1) is a form of the classical wave equation, and, for example has the property that information is transmitted at the local wave speed $\sqrt{gh(x,y)}$. One can therefore estimate the time of arrival of the tsunami by use of this speed, and, (when h(x,y) is known accurately) this can be done very systematically and accurately (see e.g., Braddock (1972). The remarkable agreement between times of arrival computed this way and the actual times measured is indirect evidence for validity of (1); indeed, where there are discrepancies, the discrepancy is adequately explainable as arising from uncertainties over h(x,y) and coastline reflection effects, rather than from physical causes such as non-linear or dispersive effects.

The idea of so determining times of arrival can also be thought of as a process of tracing "rays." However, this is quite dangerous and incorrect. Ray tracing is a process justifiable as a short-wave asymptotic expansion of the governing equation (1), and we are not interested in such short waves. As Hammack and Segur (1978) have observed, linear asymptotics do not apply to tsunamis, and no reliance can be placed on such short-wave concepts as ray tracing or refraction diagrams to tsunami work.

Nevertheless, in view of all of the other uncertainties in this field, some form of approximate solution of (1) appears justified, at least in order to enable rapid investigation of the qualitative and sensitivity questions posed earlier. Ray tracing may serve such a role in the absence of other candidates.

However, one obvious alternative simplification is reduction of the number of space dimensions from two to one. In situations where there is greater variation in the x than the y direction, (1) may be simplified by omitting the term $(h\eta_y)_y$. Note that y-variation is not eliminated, since both h and η_o may still depend on y, but the role of the coordinate y has been relegated to that of a parameter.

The beneficial effects of this simplification are many. The numerical task is simplified by a very large factor. For example (Appendix A) the frequency-response problem may now be solved as an initial-value problem in the space coordinate x as well as the time corodinate t, whereas the original two-dimensinal problem demanded solution as a boundary-value problem in space. In the special (but not untypical) case of uniform depth, the exact analytic solution may be written down (Appendix C), and this provides valuable insight into the qualitative features to be expected of any fuller numerical treatment. The task of understanding phenomena is much simplified by a reduction in the number of dimensions, thereby concentrating attention on a smaller range of varying parameters.

But is the one-dimensional approximation appropriate for tsunamis? Fortunately, "yes." This is so in the near-field generation problem, because of the elongated nature of tsunamigenic earthquakes. If y is a coordinate measured along the long axis of the earthquake, with x perpendicular to this axis, we may expect that the outgoing wave at least begins to propagate mainly in the x direction, with a slow modulation of its amplitude as a function of y.

By itself, elongation of the upthrust zone would not justify a one-dimensional approximation, unless accompanied by a corresponding character to the depth contours. Here again, the location of important tsunamigenic earthquakes on the continental shelf comes to our aid. Even though there may be quite pronounced apparent local depth variations with y, the overall trend on continental shelves is for depth contours to run parallel to the coastline, and thus also parallel to the y axis of elongation of the upthrust zone.

Appendix F gives some details of an illustrative one-dimensional data preparation for the 1964 tsunami in this way. In no way should this treatment be viewed as definitive, but rather as an example of what can be done, the subsequent computations of Appendix G being simple enough to be performed at negligible cost on a TRS-80 micro-computer. There is reasonable qualitative agreement with previous computations, and with near-field observations at Seward, at least sufficient to add further weight to the contention that physical effects such as disperson and non-linearity are of minor importance.

For the present paper, I have used this program in Appendix G for some of the possible sensitivity studies suggested earlier, especially in analyzing the influence of the (largely-unknown) seaward extent of the upthrust zone. It is clear from the computations that the very first wave observed in the deep ocean is due to that portion, if any, of the upthrust which extends over the continental slope. This is of course to be expected, since any such upthrust moves off first, with higher initial velocity and less topographic reflection than those portions of the upthrust generated in shallower water. If such upthrust did exist (and the tide gauge observations in 1964 at reception sites such as the Hawaiian Islands cast serious doubts on its existence) the task of predicting the leading wave of any tsunami would be almost impossible, since its extent must remain largely unknown.

Apart from continental-slope upthrust, the only parameter to which the computations are highly sensitive is a numerical one, namely the spatial discretization interval Δx. If this is too high, notably comparable to the length scale of the upthrust, spurious short-period oscillations can appear in the computed results. For the 1964 data, values of Δx at least as small as one or two kilometres are needed to avoid this phenomenon, unless the input upthrust is artificially and subjectively smoothed.

The one-dimensional assumption may also be valid for deep-ocean propagation of, and shoreline response to, tsunamis, most directly for the latter. If the coastline of reception is reasonably straight, with depth contours largely parallel to the coast, as is the case for example with the Californian coast, the problem is again representable one-dimensionally. It is not necessarily true now, however, that y-derivatives of η are negligible, since the incident tsunami may not arrive from a direction normal to the coast. No essential difficulty occurs in incorporation of a general incident angle into the one-dimensional model, but only normal incidence has been treated in the Appendices of this paper. Appendix D gives some results for the frequency-response characteristics of Crescent City, California, on this basis.

One-dimensional deep-ocean propagation is valid only over distances that are small compared to the long axis of the upthrust. Once the wave has propagated beyond such a distance, two-dimensional geometrical dispersion becomes significant. This does not violate the original one-dimensional near-field assumption, but rather demands a theory to account for the dispersion process in the far field.

Remarkably, no such theory appears to exist. In Appendix H, I outline an approach to such a theory, using matched asymptotic expansions and the parabolic approximation of scattering theory. In essence, the upthrust zone appears in its own far field as a line of "acoustic" sources, whose strength (as a function of the coordinate y measured along the line) has already been determined, by solving a sequence of one-dimensional problems, with y as a parameter. This is then matched to a two-dimensional solution which allows curvature of the outgoing wave fronts, and enables computation of the "polar diagram" of the equivalent acoustic radiator.

The principal mathematical difficulty in such a theory lies in incorporating a "moderately high-frequency" character. This is because if, as assumed, the tsunami wave-length is comparable with the shelf width, it is necessarily short compared to the upthrust elongation distance. Thus the propagation problem is (in this special respect only) a short-wave asymptotic problem, in spite of the huge size of the propagating wavelength.

As an aside, it may be of interest to note that the above-outlined problem bears a remarkable resemblance to that of ship motions in waves. A heaving and pitching ship is an elongated wave generator, which makes nearly one-dimensional waves at each section. Such waves then propagate into the far field of the ship, where dispersion ultimately curves their wave fronts. In ship hydrodynamics, there has existed for many years a very successful "one-dimensional" theory (Korvin-Kroukovsky's "Strip Theory" (1970) which, in spite of almost universal use in the practical prediction of ship seakeeping, has defied rigorous proof of validity by mathematicians).

In summary, I have attempted to argue in this paper for the use of the linear long-wave equation as an adequate model for most of the tsunami problems. In particular, to an accuracy commensurate with the quality of the input data and the output measurements, there appears little need to include non-linear or frequency-dispersion effects.

Neglect of these effects leaves the task as one of solving an equation such as (1), except that any truly global analysis would demand inclusion of further terms due to the curvature of the earth. This can be done quite easily in numerical studies, and has been done by a number of authors, e.g., (11). Inclusion of curvature terms is necessary in order to explain refocusing, such as appeared to occur in Japan due to the 1960 Chilean earthquake. There may be scope for further qualitative (i.e. non-numerical) study of this phenomenon, preferably divorced from the inaccurate and possibly misleading context of ray tracing.

I have also argued here for the validity of a "matched one-dimensional" approach to the tsunami modelling problem. This is a further simplification to the equations, which may appear unnecessary in circumstances where an adequate computational treatment of the full equation (1) is already available. However, there are some doubts as to the adequacy of existing computations, especially in view of potential difficulties regarding the spatial discretization interval, and hence the one-dimensional approach may have computational value. One merit of the one-dimensional approximation is that it concentrates attention on a smaller range of parameters, which are localized in such a way that the generation, propagation and reception problems are separated. This is of great value for qualitative understanding of the main features of the tsunami problem.

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APPENDIX A

A General Class of Linear One-Dimensional Generation or Response Problems

We suppose a configuration such as that illustrated in Figure A.1. A region of non-uniform depth $h = h(x)$, $0 < x < L$, separates constant-depth regions $h = h_0$ in $x < 0$, and $h = h_\infty$ in $x > L$. Ground motion, if any, occurs in $0 < x < L$, and represents a small upward displacement $\eta_0(x,t)$ of the bottom surface. The resulting free-surface displacement $\eta(x,t)$ satisfies the linear shallow-water equation (1) in one space dimension, i.e.:

$$\left[\frac{\partial}{\partial x} \; h(x) \; \frac{\partial \eta}{\partial x}\right] = \frac{1}{g} \; \frac{\partial^2}{\partial t^2} [\eta - \eta_0] \; . \tag{A.1}$$

We suppose that the fluid motion starts from rest, with initial conditions:

$$\eta(x,0_+) = \eta_t(x,0_+) = 0 \; , \tag{A.2}$$

and similar initial conditions for the ground motion η_0.

The boundary conditions at $x = 0$ and L depend on whether or not there is any incident energy from these directions, in addition to, or instead of, the ground-motion excitation. In general, we suppose that any such incident wave is from $x = +\infty$, so that for $x < 0$ we have only a transmitted (left-going) wave, i.e.:

$$\eta = \eta_L \left(t + \frac{x}{\sqrt{gh_0}}\right) \; , \; x < 0 \; , \tag{A.3}$$

for some function $\eta_L(t)$. For $x > L$, we allow the possibility of a given incident wave:

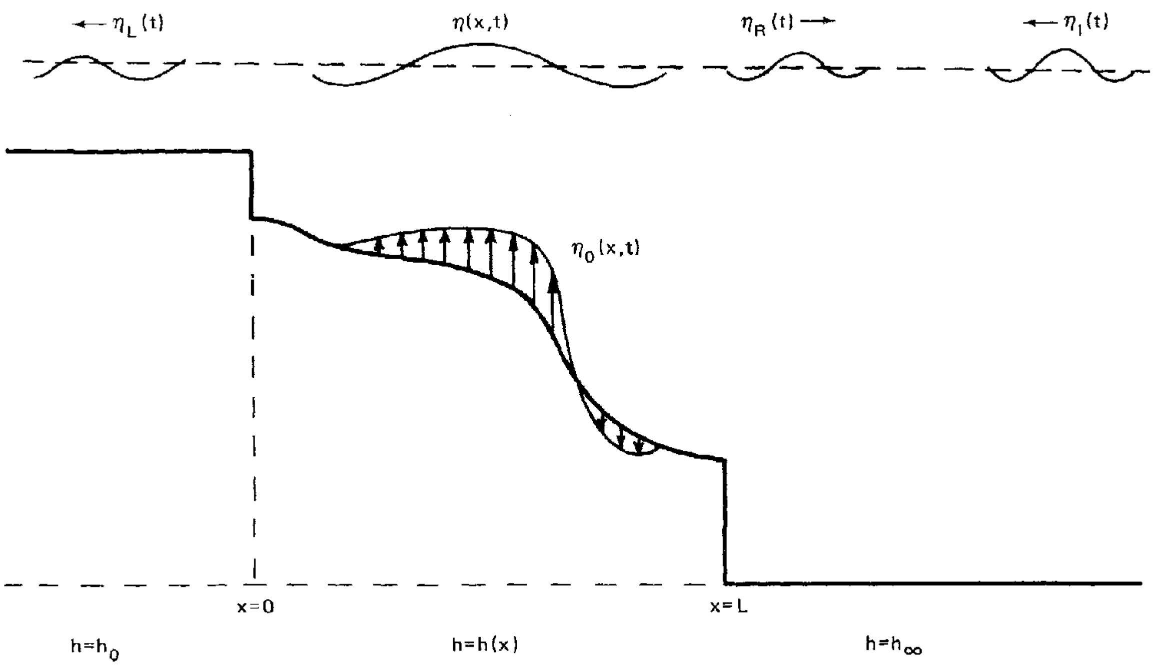

FIGURE A.1 SKETCH OF A GENERAL TWO–DIMENSIONAL SHALLOW–WATER WAVE PROBLEM

$$\eta_I\left(t + \frac{x-L}{\sqrt{gh_\infty}}\right) ,$$

writing
$$\eta = \eta_I\left(t + \frac{x-L}{\sqrt{gh_\infty}}\right) + \eta_R\left(t - \frac{x-L}{\sqrt{gh_\infty}}\right) , \quad x > L , \tag{A.4}$$

where η_R is a right-going wave, produced either by partial reflection of η_I, or by transmission of the ground-motion-generated wave, or both.

We do not necessarily demand continuity of bottom depth at $x = 0$ or $x = L$. The matching conditions (continuity of η and $h\eta_x$) at such a discontinuity enable elimination of the function η_L, leaving a mixed boundary condition at $x = 0_+$, i.e.:

$$\sqrt{gh(0_+)}\,\eta_x(0_+,t) = \alpha\eta_t(0_+,t) , \tag{A.5}$$

where
$$\alpha = [h_o/h(0_+)]^{1/2} . \tag{A.6}$$

Similarly, at $x = L_-$, we can eliminate η_R, giving

$$\beta\sqrt{gh(L_-)}\,\eta_x(L_-,t) = -\eta_t(L_-,t) + 2\eta'_I(t) , \tag{A.7}$$

where
$$\beta = [h(L_-)/h_\infty]^{1/2} . \tag{A.8}$$

Equations (A.5), (A.7) may be used as boundary conditions on (A.1) at $x = 0,L$ respectively, completing the specificiation of the initial-boundary-value problem in $0 \le x \le L$. The parameters α and β vanish when there is perfect reflection, and take a unit value when there is no discontinuity. For example, the Lamb amplitude reflection coefficient at the shore $x = 0$ is $(1-\alpha)/(1+\alpha)$.

Note, however, that the most important output quantities are precisely the functions η_L, η_R just eliminated. Thus:

$$\eta_L(t) = \eta(0_+,t) \tag{A.9}$$

is the shoreline or edge-of-shelf generated amplitude, important for the local tsunami in the generation problem, and acting as the generator for the final run-up on the beach in the response problem. Similarly, in the generation problem, we are most interested in the seaward-going wave:

$$\eta_R(t) = \eta(L\_,t) - \eta_I(t) \quad , \tag{A.10}$$

with $\eta_I \equiv 0$ in pure ground motion without incident wave.

If $\bar{\eta}(x,s)$ denotes the Laplace transform of $\eta(x,t)$, and similarly for $\eta_o(x,t)$, we have from (A.1):

$$\frac{d}{dx}\left[h(x)\,\frac{d\bar{\eta}}{dx}\right] - \frac{s^2}{g}\,\bar{\eta} = -\,\frac{s^2}{g}\,\eta_o \;. \tag{A.11}$$

This is an ordinary differential equation to determine $\bar{\eta}$, subject to the two-point boundary conditions:

$$\sqrt{gh}\;\bar{\eta}_x = \alpha s\bar{\eta}, \quad \text{at } x = 0_+ \tag{A.12}$$

and $$\beta\,\sqrt{gh}\;\bar{\eta}_x = -s\bar{\eta} + 2s\bar{\eta}_I \;, \quad \text{at } x = L\_ \;. \tag{A.13}$$

If there were no ground motion, but instead an initial elevation $\eta\,(x,0\_)$ at time $t = 0\_$, we should obtain the same problem, but with an apparent ground motion given by:

$$\bar{\eta}(x;s) = \frac{\eta(x,0\_)}{s} \tag{A.14}$$

i.e. $$\eta_o(x,t) = \begin{cases} 0 \;, & t < 0 \\ \eta(x,0\_) \;, & t > 0 \end{cases} \tag{A.15}$$

Thus an initial water elevation is entirely equivalent to a step-function ground motion. Conversely, any ground motion, such as that produced by an earthquake, which takes place over a time scale (seconds) which is extremely short compared to the time scales of the generated tsunami (many minutes), produces effects indistinguishable, on the latter scale, from that for an initial surface elevation.

If h(x) is piecewise constant, (A.11) can be solved explicitly. However, for general h(x), (A.11) must be solved numerically for a large range of values of s, and a numerical inverse Laplace transformation performed. If the complete time history of the response is wanted, it is preferable to solve the original time-dependent equation (A.1) directly, for a range of values of t. This is done for the 1964 Alaskan tsunami in Appendix G.

However, for some purposes, it is desirable to determine the system frequency response, i.e., the steady-state outputs η_L, η_R where the inputs η_o, η_I are pure sinusoidal in time, and in that case numerical solution of (A.11) is the preferred approach. Thus if:

$$\eta_o(x,t) = R\bar{\eta}_o(x)e^{-i\omega t} \, , \quad (A.16)$$

and $$\eta_I(t) = R\bar{\eta}_I e^{-i\omega t} \, . \quad (A.17)$$

Then $$\eta(x,t) = R\bar{\eta}(x)e^{-i\omega t} \quad (A.18)$$

where $$\bar{\eta}(x) = \bar{\eta}(x;-i\omega) \quad (A.19)$$

satisfies (A.11) with $s = -i\omega$. (A.20)

APPENDIX B

Response for a Special Non-Uniform Depth

There is one case, other than uniform depth, for which a simple analytic solution exists, namely, when the depth varies as the 4/3 power of distance. Thus the expression:

$$\eta(x,t) = x^{-1/3}F(t\pm x^{1/3}) \tag{B.1}$$

satisfies (A.1) with $\eta_o \equiv 0$, if

$$h(x) = \frac{9}{g\gamma^2} \cdot x^{4/3} . \tag{B.2}$$

This solution may be exploited in a number of response problems, by appropriate shifts of the x-axis and choices of the parameter γ and function F. As a simple example, suppose we contrast (i) a steep cliff at x = 0, in uniform depth $h = h_o$, dropping suddenly to $h = h_\infty >> h_o$ at x = L, with (ii) a beach described by (B.2), reaching the depth h_o at x = L and then also dropped to h_∞ suddenly.

In each case, we examine the natural frequencies of the almost-isolated shelf, assuming as a first approximation for $h_\infty >> h_o$ that the end x = L appears open, i.e., x = L is a node, with:

$$\eta\,(L\_,t) = 0 . \tag{B.3}$$

The appropriate constant-depth solution is just:

$$\eta\,(x,t) = A \cos\left(\frac{\omega x}{\sqrt{gh_o}}\right) e^{-i\omega t} , \tag{B.4}$$

for some constant A. This solution gives zero velocity, as required, at the shore (cliff) x = 0, and satisfies (B.3) if:

$$\frac{\omega L}{\sqrt{gh_o}} = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots \qquad \text{(B.5)}$$

That is, the natural frequencies for a uniform shelf are odd interger multiples of $\frac{\pi}{2}\sqrt{gh_o/L}$. In particular, the first harmonic has a period 1/3 of that of the fundamental.

The appropriate solution for the "4/3 power" depth is:

$$\eta(x,t) = \frac{A}{\gamma\omega} x^{-1/3} \sin(\gamma\omega x^{1/3}) e^{-i\omega t} . \qquad \text{(B.6)}$$

Note that as $x \rightarrow 0$,

$$\eta(x,t) \rightarrow Ae^{-i\omega t} - \frac{1}{6} A\gamma^2 \omega^2 x^{2/3} e^{-i\omega t} + 0(x^{5/3}) \qquad \text{(B.7)}$$

and $$h\eta_x \rightarrow -A \frac{\omega^2}{g} xe^{-i\omega t} \rightarrow 0 \qquad \text{(B.8)}$$

i.e., there is no net volume flux past the shoreline $x = 0$ as required. On the other hand, the open-end condition (B.3) at $x = L$ requires that:

$$\gamma\omega L^{1/3} = \pi, 2\pi, 3\pi, \ldots \qquad \text{(B.9)}$$

That is, the natural frequencies for such a specially-shaped beach are all integer multiples of the fundamental $\pi/(\gamma L^{1/3})$. In particular, the first harmonic has half the fundamental period. The internal node of the first harmonic is at $x/L = \frac{1}{8}$.

The spectrum of shelf natural frequencies for actual topographies cannot be expected to match either of these two cases, but may be somewhere between. Thus we may expect the first-harmonic period to be between one-half and one-third of the fundamental, and the first-harmonic node to be between $\frac{1}{8}$ and $\frac{1}{3}$ of the shelf width from the shoreline.

APPENDIX C

Uniform Depth Case

An important special case of (A.11) is when the depth is constant, $0 < x < L$. The general solution is, with $c = \sqrt{gh}$:

$$\bar{\eta}(x;s) = -\frac{s}{c}\int_0^x \bar{\eta}_o(\xi;s)\sinh\left[\frac{s(x-\xi)}{c}\right]d\xi$$

$$+\ \bar{\eta}(0;s)\cosh\frac{sx}{c} + \frac{c}{s}\,\bar{\eta}_x(0;s)\sinh\frac{sx}{c}\,, \qquad \text{(C.1)}$$

where $\bar{\eta}(0;s)$, $\bar{\eta}_x(0;s)$ are arbitrary. The boundary conditions (A.12), (A.13) provide two equations to determine the unknown quantities, completing the solution, namely:

$$\frac{c}{s}\,\bar{\eta}_x(0;s) = \alpha\bar{\eta}(0;s) \qquad \text{(C.2)}$$

and

$$\left[(1+\alpha\beta)\cosh\frac{sL}{c} + (\alpha+\beta)\sinh\frac{sL}{c}\right]\bar{\eta}(0;s)$$

$$= 2\bar{\eta}_I(s) + \frac{s}{c}\int_0^L \bar{\eta}_o(\xi;s)\left[\sinh\frac{s(L-\xi)}{c} + \beta\cosh\frac{s(L-\xi)}{c}\right]d\xi\ . \qquad \text{(C.3)}$$

When both ground motion η_o and an incident wave η_I are present, the response is the sum of contributions from each, weighted by the transfer function:

$$K(s) = \left[(1+\alpha\beta)\cosh\frac{sL}{c} + (\alpha+\beta)\sinh\frac{sL}{c}\right]^{-1} \qquad \text{(C.4)}$$

of the shelf. Although, in practice, we are only interested in the distant generation problem with $\eta_I \equiv 0$, and response problem with $\eta_o \equiv 0$, it is important to note that in each case the function K plays a role, and that in particular, the output is magnified where K(s) possess its maxima.

In particular, the frequency response is controlled by the maxima of:

$$|K(-i\omega)| = \left[(1+\alpha\beta)^2\cos^2\frac{\omega L}{c} + (\alpha+\beta)^2\sin^2\frac{\omega L}{c}\right]^{-1/2} . \quad \text{(C.5)}$$

Since α and β are both quantities between 0 and 1, the term in $\sin^2\frac{\omega L}{c}$ tends to be somewhat small compared to that in $\cos^2\frac{\omega L}{c}$. Hence the maximum of $|K|$ occurs near to the points where $\cos\frac{\omega L}{c} = 0$, i.e., where ω is an odd integer multiple of the natural shelf frequency:

$$\omega_o = \frac{c}{2L} . \quad \text{(C.6)}$$

In the limit as α and β both tend to zero, i.e., when the beach depth is very much less than the shelf depth, which in turn is very much less than the open-sea depth, this is an exact result, and we predict an unbounded resonance, in both the generation and response problems, at these natural frequencies. If either α or β is non-zero, the resonance is bounded, and there is an amplification factor of approximate value $(\alpha+\beta)^{-1}$ at the natural frequencies.

The transfer function $K(s)$ may be defined for non-uniform depths h, as the Laplace transform of the response at the shelf edge $x = 0$, when there is no ground motion $\eta_o \equiv 0$, and when $\eta_I = 1/2$. This is the response due to a δ-function incident pulse. The corresponding frequency-response function $K(-i\omega)$ corresponds to an incident sine wave. This is accompanied by a corresponding reflected wave η_R in $x > L$, and hence the apparent flow field at $x = +\infty$ is a partial standing wave. Indeed, if $\alpha = 0$, there is total reflection of the incident energy at $x = 0$, and the solution corresponds to a perfect standing wave at $x = +\infty$.

It is important to emphasize that, although K(s) is so interpreted in terms of the response problem, it also provides information about shelf effects on the generation problem, at least qualitatively. Only in the constant-depth case is the correspondence quantitatively established, but in all cases we may expect to obtain enhanced tsunami generation whenever the forcing frequency is close to a natural frequency of the shelf, as defined by maxima of $|K(-i\omega)|$.

In the generation problem, with $\eta_I \equiv 0$, we are most interested in the outgoing wave η_R, obtained by setting x = L in (C.1), and we find:

$$\bar{\eta}_R(s) = \frac{\beta}{c} sK(s) \int_0^L \bar{\eta}_o(\xi;s) \left[\cosh \frac{s\xi}{c} + \alpha \sinh \frac{s\xi}{c}\right] d\xi \ , \tag{C.7}$$

displaying explicitly the dependence of the generation problem on the response transfer function K(s). As a particular example, consider the step function (in space and time) ground motion given by:

$$\bar{\eta}_o(x;s) = \begin{cases} A/S \ , & a<x<b, \quad (0<a<b<L) \\ 0 \ , & \text{otherwise} \end{cases} \tag{C.8}$$

for which (C.7) reduces to:

$$\bar{\eta}_R = \frac{\beta KA}{s} \left[\sinh \frac{s\xi}{c} + \alpha \cosh \frac{s\xi}{c}\right]_{\xi = a}^{\xi = b} . \tag{C.9}$$

For instance, if there is perfect reflection at the shoreline ($\alpha = 0$):

$$\frac{|\eta_R(-i\omega)|}{A} = \frac{\left|\sin \frac{\omega b}{c} - \sin \frac{\omega a}{c}\right|}{\omega\sqrt{1-(1-\beta^2)\sin^2 \frac{\omega L}{c}}} \tag{C.10}$$

In Figure C.1 we show plots of this quantity for $\beta = 0.2$ and b = L, against $\omega L/c$, for various values of a/L.

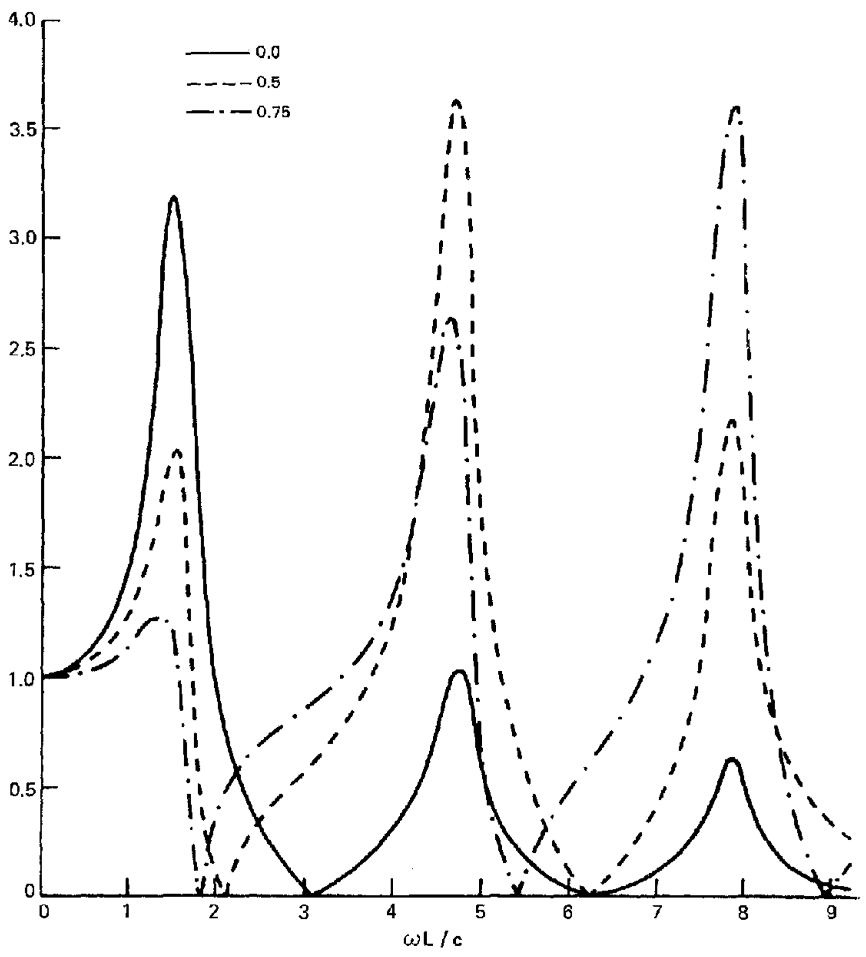

FIGURE C.1 OUTPUT SPECTRUM AS A FUNCTION OF FREQUENCY, FOR UPTHRUST ON A CONSTANT–DEPTH SHELF. THE SHELF IS OF WIDTH L AND DEPTH 1/25 OF THE DEEP–WATER DEPTH. THE UPTHRUST EXTENDS FROM X = a TO X = L, AND IS UNIFORM OVER THAT DISTANCE, WITH THE SAME TOTAL VOLUME AS a/L VARIES.

The ordinate is actually rescaled by dividing by its zero-frequency limit. This enables us to compare the influence of location of the upthrust on the shelf, for a fixed total volume of upthrust. The results shown in Figure C.1 correspond to an upthrust extending from a variable position $x = a$, right to the edge $b = L$ of the shelf, the peak amplitude of upthrust increasing as $a \to L$, to keep the total volume fixed.

The influence of the natural response frequencies of the shelf is clearly demonstrated in Figure C.1. The energy of the generated tsunami is concentrated at peaks corresponding to the natural frequencies of the shelf. The main effect of varying the extent of upthrust, relative to the shelf width, is in changing the balance between the various harmonics. Thus if $a = 0$, i.e., there is upthrust over the whole shelf, naturally the fundamental is strongly excited, and all harmonics are small. If $a/L = 1/2$, the 1st harmonic is highly excited, having an amplitude of about twice that of both the fundamental and the 2nd harmonic. This is approximately the situation that occurred with the 1964 Alaskan tsunami.

APPENDIX D

One-Dimensional Numerical Solution-Frequency Response

Our task is to solve (A.11), with $s = -i\omega$, subject to (A.12) and (A.13), with $\bar{\eta}_o$ and $\bar{\eta}_I$ given by (A.16), (A.17). It is appropriate to separate the response and generation problems, writing $\eta_1(x)$ for the solution with η_I a given non-zero constant but $\eta_o(x) \equiv 0$, and η_2 for the solution with $\eta_o(x)$ given but $\eta_I = 0$.

The problem for η_1 is to solve the homogeneous ordinary differential equation:

$$\frac{d}{dx}[h(x)\eta'_1(x)] + \frac{\omega^2}{g}\eta_1(x) = 0 \tag{D.1}$$

subject to the inhomogeneous mixed boundary conditions:

$$\sqrt{gh}\,\eta'_1 + i\omega\alpha\eta_1 = 0\ , \qquad \text{at } x = 0\ , \tag{D.2}$$

and

$$\beta\sqrt{gh}\ \eta'_1 - i\omega\eta_1 = -2i\omega\eta_I\ , \quad \text{at } x = L\ . \tag{D.3}$$

Since the problem is linear, and we are really only interested in the ratio η_1/η_I, we can solve it as an initial-value problem, starting at the shoreline $x = 0$ by assigning an arbitrary value, say $\eta_1 = 1$, to the shoreline amplitude. Thus, we replace (D.2) by the pair of initial conditions:

$$\begin{cases} \eta_1 = 1 \\ \sqrt{gh}\ \eta'_1 = -i\omega\alpha\ , \end{cases} \tag{D.4}$$

at x = 0. The solution can thus be obtained very quickly as an initial-value problem, and we then treat (D.3) as an equation that evaluates the quantity η_I. For example, if we are interested in the transfer function $K(-i\omega)$ defined in Appendix B, we require the shoreline amplitude when $\eta_I = 1/2$. This just corresponds to $K(-i\omega)$ times the solution η_1 so computed, and hence:

$$K(-i\omega) = \frac{0.5}{\eta_I} \tag{D.5}$$

where η_I is computed using (D.3), from this solution η_1.

It should be noted that the mixed boundary condition (D.2) is appropriate only for cases where $h(0) \neq 0$, i.e., where the shelf ends in a depth $h(0)$, which then changes at x = 0 to h_o, with $\alpha = \sqrt{h_o/h(0)}$. If $h(0) = 0$, i.e., the shore ends in a beach, the appropriate condition at x = 0 is the same as that for $\alpha = 0$, i.e., there is perfect reflection. If $\alpha \neq 0$, some energy is transmitted, in principle to $x = -\infty$, but in practice lost in run-up. Thus the coefficient α may be used as an empirical measure of absorption of energy at the shore, providing we terminate the computation a short distance seaward of the actual coastline, with $h(0) \neq 0$.

The generation problem for $\eta_2(x)$ is specified by solving the inhomogeneous equation:

$$\frac{d}{dx}[h(x)\eta'_2(x)] + \frac{\omega^2}{g}\eta_2(x) = \frac{\omega^2}{g}\eta_o(x) \ , \tag{D.6}$$

subject to the homogeneous boundary conditions:

$$\sqrt{gh}\ \eta'_2 + i\omega\alpha\eta_2 = 0 \ , \quad \text{at } x = 0 \tag{D.7}$$

$$\beta\sqrt{gh}\ \eta'_2 - i\omega\eta_2 = 0 \ , \quad \text{at } x = L \tag{D.8}$$

This problem may also be solved as an initial-value problem, providing we add a suitable multiple of the response solution $\eta_1(x)$. Thus we write:

$$\eta_2(x) = \eta_3(x) + A\eta_1(x)$$

where $\eta_1(x)$ is already computed, η_3 satisfies (D.6) subject to:

$$\begin{cases} \eta_3 = 0 \\ \sqrt{gh}\eta'_3 = 0 , \quad \text{at } x = 0 , \end{cases} \tag{D.9}$$

and A is an arbitrary constant, identifiable as the shoreline amplitude (since $\eta_1(0) = 1$). Now (D.8) is satisfied if:

$$A = [\beta\sqrt{gh}\ \eta'_3(L) - i\omega\eta_3(L)]/(2i\omega\eta_I) , \tag{D.10}$$

where η_I is computed by (D.3). Note again the fundamental connection between the response transfer function $K(-i\omega) = 1/(2\eta_I)$, and the generation characteristics.

The actual procedure for discretization and numerical solution of the initial-value problems to which we have reduced the problem is of little concern to us in the present paper, and one can assume that some efficient technique can be borrowed from the vast literature on this aspect of numerical analysis.

For illustrative purpose, I have used a simple "leap frog" procedure, e.g., have written:

$$h_j = h((j-1/2)\Delta x) \tag{D.11}$$

$$Q_j = h_j\eta'_1((j-1/2)\Delta x) \tag{D.12}$$

and $$P_j = \eta_1(j\Delta x) \tag{D.13}$$

whereupon (D.1) gives as an approximation that:

$$Q_j = Q_{j-1} - \frac{\omega^2}{g} \Delta x P_{j-1} \ . \qquad \text{(D.14)}$$

On the other hand, using a centered difference for $\eta_1{}'$ in (D.12) we have:

$$P_j = P_{j-1} + \frac{\Delta x}{h_j} Q_j \ . \qquad \text{(D.15)}$$

Successive use of (D.14), (D.15) enables us to march from the shoreline, starting with $P_o = 1$ and $Q_1 = -i\omega a/\sqrt{gh(0_+)}$, from (D.4). Somewhat improved estimates of Q_1 may be devised. A similar procedure is used for the generation problem defined by (D.6).

A useful feature of the fact that the solution is obtained by solving an initial-value problem, is that we can easily test various assumptions regarding the deep-ocean termination, by evaluating η_I from (D.3) continuously at every x location. That is, as we march outward, we have available for all $x = j\Delta x$, $j = 1,2,\ldots$ the response of a shelf which transits at that x to a uniform-depth deep ocean of any desired depth.

For example, Figure D.1 shows the frequency response function $|K(-i\omega)|$ for the shelf near Crescent City, California, to normally-incident sinusoidal waves, using $\Delta x = 1.7$ km, with $\alpha = 0$. The input data for the depth is prepared in a manner similar to that described in Appendix F. The deep-ocean depth is taken as $h_\infty = 4$ km. The figure shows computations for two different values of β in (D.3), corresponding to the assumption that the depth drops suddenly to 4 km from either 2 km (at x = 60 km) or 3 km (at x = 80 km). The weak dependence on β indicates that the exact detail of the approach to the deep ocean is not too important.

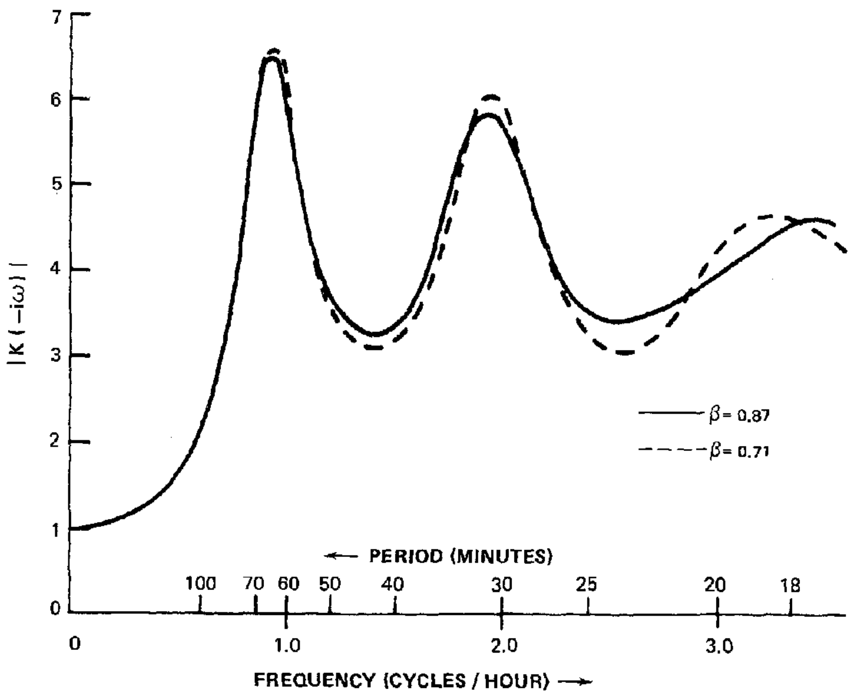

FIGURE D.1 COMPUTED FREQUENCY RESPONSE TO A NORMALLY-INCIDENT UNIT-AMPLITUDE SINUSOIDAL WAVE NEAR CRESCENT CITY. THE FAR-FIELD DEPTH IS ASSUMED TO BE 4 KM, AND IS ATTAINED ARTIFICIALLY BY A SHARP DROP FROM A DEPTH OF 3 KM (β = 0.87) OR 2 KM (β = 0.71), THE DEPTH VARIATION SHOREWARD FROM THAT POINT BEING SMOOTH.

Figure D.1 indicates bounded but sharp resonant peaks at periods of 63, 31 and 18 minutes. However, as is partly indicated by the increasing discrepancy between the two curves, the results with a spacing as large as Δx = 1.7 km are somewhat suspect for periods less than about 25 minutes, since there are insufficient data points near the shore per wavelength. However, the first two peaks lie in a region where a spacing of 1.7 km appears quite adequate. The actual amplification factor at the peaks is somewhat sensitive to β and to the assumed deep-ocean depths, but lies within a range of about 5 to 10. The fundamental and 1st harmonic are about equally amplified, both giving significantly greater response than the 2nd harmonic. Thus one should expect to see about 63- and 31-minute response periods at Crescent City to any input tsunami.

The resonant periods vary quite considerably along the coast near Crescent City. For example, just 4 km north, at Point St. George, the first two peaks lie at 50 and 23 minutes whereas a further 15 km north they are at 75 and 31 minutes. This particular variation is due to the reef at Point St. George. Further work, along the lines of Appendix H, is needed to test whether variations such as this can influence the magnitude of response at particular locations such as Crescent City itself. Features such as the Point St. George reef and (even more important on an "outer" scale) the Mendocino fracture system further south, are likely to be more significant in this respect than the gently-curving crescent shape of the local coastline, to which Wilson and Torum ([7], p. 490) attribute the large response at Crescent City.

It should be noted that the peaks in the present one-dimensional model, correspond to limits as the eccentricity becomes large of the "fundamental" and "fifth" of the modes described by Wilson and Torum for a constant-depth elliptical-basin approximation to the coastline near Crescent City. The other modes of the elliptic basin are "two-dimensional" in the high eccentricity limit, involving significant motion parallel to shore, with nodal lines intersecting the coast at near right angles, and would not be significantly excited by normally-incident waves. The fundamental and fifth-mode periods quoted by Wilson and Torum are in the same range as the present first two peak periods.

APPENDIX E

One-Dimensional Numerical Solution - Time Domain

Using the same notation as in Appendix D, the space-discretized form of (A.1) is:

$$\ddot{P}_j = \ddot{P}_j^o + g\,\frac{Q_{j+1}-Q_j}{\Delta x}\,, \tag{E.1}$$

Q_j being given by (D.15), and all quantities evaluated at time t. If we choose a time step Δt, and use the central difference approximation:

$$\ddot{P}_j(t) = \frac{P_j(t+\Delta t)-2P_j(t)+P_j(t-\Delta t)}{\Delta t^2}\,, \tag{E.2}$$

then (E.1) enables computation of the unknown $P_j(t+\Delta t)$, given previous values $P_j(t)$, $P_j(t-\Delta t)$. The upthrust acceleration P_j may be evaluated from the upthrust time history $\eta_o(j\Delta x,t)$ by a similar formula. Thus the final algorithm is of the form:

$$\begin{aligned} P_j(t+\Delta t) = {} & 2P_j(t)-P_j(t-\Delta t) \\ & + P_j^o(t+\Delta t)-2P_j^o(t)+P_j(t-\Delta t) \\ & + g\,\frac{\Delta t^2}{\Delta x}\,[Q_{j+1}(t)-Q_j(t)] \end{aligned} \tag{E.3}$$

The boundary conditions (A.5), (A.7) at $x = 0,L$ may be implemented by suitable finite-difference approximations. For example, $P_o(t+\Delta t)$ may be determined by the simple first-order equivalent of (A.5) as:

$$P_o(t+\Delta t) = P_o(t) + \Delta t\sqrt{\frac{g}{h_o}}\,Q_1(t) \; . \tag{E.4}$$

If $h_o = h_1$, i.e., $\alpha = 1$, and there is no depth discontinuity at the shore, this is equivalent to allowing the wave to move to the left at speed $\sqrt{gh_o}$ without change of form over the end interval. If $\alpha = 0$, i.e., there is a solid wall at $x = 0$, (E.4) cannot be used directly, and a somewhat preferable procedure is to introduce an artificial value $Q_o = -Q_1$, then determining P_o from (E.3) with $j = 0$.

Initial conditions may be implemented by suitable choice of $P_j(0)$ and $P_j(-\Delta t)$. Thus if $P_j(0)$ is a given vector and we set $P_j(-\Delta t) = P_j(0)$, we have an approximation to motion starting from rest with a given initial elevation $\eta(j\Delta x,0) = P_j(0)$. A somewhat better approximation is obtained by choosing $P_j(-\Delta t)$ so that the quantity $P_j(\Delta t)$ computed by (E.3) at $t = 0$ is equal and opposite to $P_j(-\Delta t)$.

It is notable that the equivalence between a rapid upthrust and an initial elevation is preserved in this discretized form of the equations. For example, suppose we have a model earthquake in which the permanent deformation $\eta_o(x,\infty)$ is attained linearly in a time T, i.e.:

$$\eta_o(x,t) = \begin{cases} 0 & , \quad t < 0 \\ \eta_o(x,\infty)t/T & , \; 0 < t < T \; . \\ \eta_o(x,\infty) & , \quad t > T \end{cases} \tag{E.5}$$

If $T = N\Delta t$, i.e., the earthquake takes N time steps, then the upthrust terms of (E.3) give:

$$\overset{o}{P}_j(t+\Delta t) - 2\overset{o}{P}_j(t) + \overset{o}{P}_j(t-\Delta t) = \begin{cases} \eta_o(x,\infty)/N & t = 0 \\ -\eta_o(x,\infty)/N & t = N\Delta t \\ 0 & t = k\Delta t \; , \\ & k \neq 0,N \; . \end{cases} \tag{E.6}$$

Thus at time $t = 0$, there is a forcing term equivalent to a non-zero initial elevation and velocity. The elevation then builds up more-or-less linearly in N time steps, each 1/N of the final elevation, until the earthquake ends at $t = N\Delta t$, then a negative impulse reduces the upward velocity to near zero, the subsequent motion being without ground motion. Indeed, if $N = 1$, i..e, the time step is chosen as equal to the earthquake duration, the numerical values for a motion due to the model earthquake (E.5) starting from rest at zero initial elevation at time $t = 0$, are identical to those for an initial elevation $\eta_o(x,\infty)$, starting from rest at time $t = \Delta t$, in the absence of ground motion.

APPENDIX F

Data for a Study of the 1964 Tsunami

For this study I have selected a strip of the Alaskan shelf and slope, with dimensions 300 km in x by 100 km in y. The end x = 0 is near to an averaged position of the shoreline near Seward, and the long x axis of the strip is nearly perpendicular to the long upthrust axis of the earthquake. The line "BB'" shown in Figure 3 of Wilson and Torum ([7], p. 366) runs almost down the middle of the strip, and the strip's northeastern edge just touches the southwest tip of Montague Island. Figure F.1 shows (dashed) the boundaries of the strip, together with smoothed depth contours and coastline features, data being taken from available charts and figures, several of which were reproduced in papers of [7] and [8].

The continental-slope depth contours run almost directly across much of the strip, and even in the continental-shelf area, where this is not quite the case, the depth variations across the strip are not extreme. Figure F.2 shows (dashed) shelf-depth values, averaged across the strip, together with maximum and minimum depths, as a function of x. The variance in the depth is normally much less than the maximum-minimum range. Figure F.3 shows the mean depth and mean (permanent) upthrust for the whole length of the strip. The upthrust data is taken directly from Plafker's ([3], p. 118) Plate 1, and has considerably less variation across the strip than the depth data.

Figure F.3 also shows (dashed) an "assumed" upthrust curve, taken from Wilson and Torum's ([7], p. 408) Figure 52a. The dashed curve has a smooth and wide peak of 6 m rather than the "measured" very sharp 10 m peak, and exhibits no continental-slope upthrust. Wilson and Torum presumably base the "assumed" curve on their subjective allowance for smoothing of the upthrusted water mass during the period of the earthquake. Indeed, even the "measured" figure is a little smoother at the seaward edge of the main peak than the near-vertical discontinuity suggested by Plafker. Other authors (e.g. Hwang and Divoky in [7]) have also used various subjective degrees of smoothing. There is little possibility of objectivity here, and one can only compare the effects of various subjective choices on the final results.

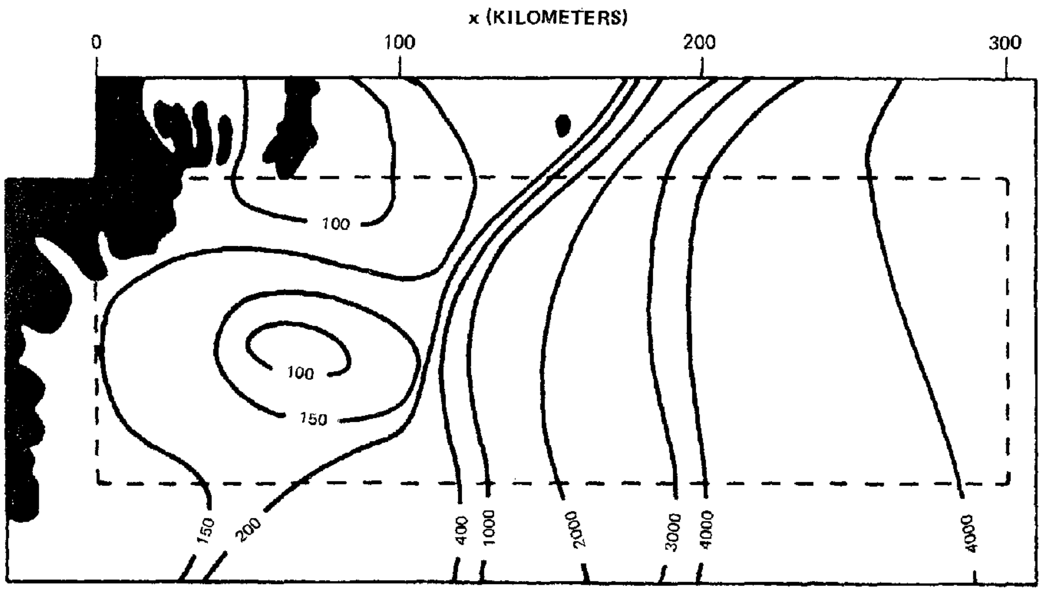

FIGURE F.1 SMOOTHED DEPTH AND COASTLINE CONTOURS FOR A 300 KM X 100 KM STRIP NORMAL TO THE ALASKAN SHELF NEAR SEWARD. DEPTHS ARE IN METRES.

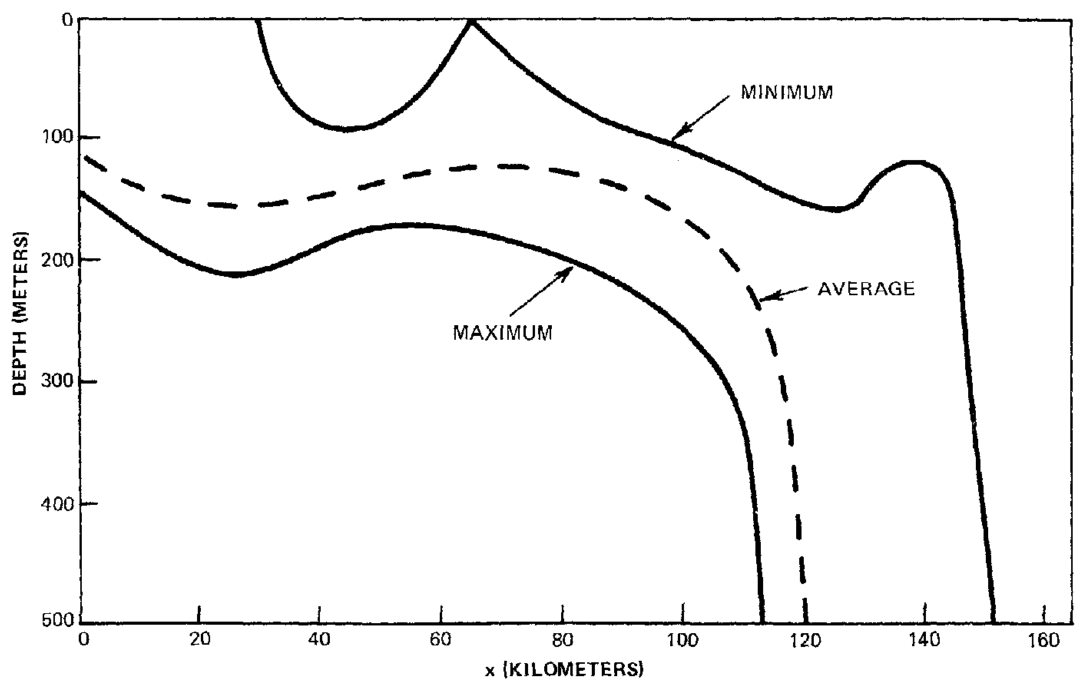

FIGURE F.2 MAXIMUM, AVERAGE AND MINIMUM DEPTHS ACROSS THE STRIP OF FIGURE F.1, AS A FUNCTION OF THE COORDINATE X ALONG THE STRIP.

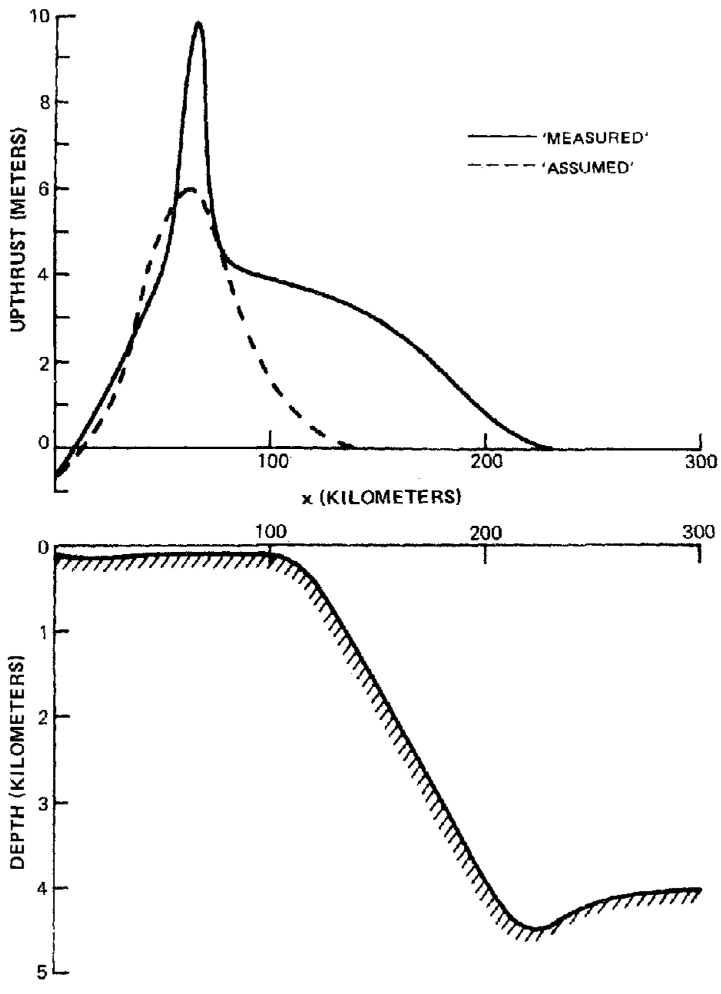

FIGURE F.3
UPTHRUST AND AVERAGE DEPTHS, AS A FUNCTION OF THE COORDINATE X ALONG THE STRIP OF FIGURE F.1. THE "MEASURED" UPTHRUST IS FROM PLAFKER IN (5) AND THE "ASSUMED" UPTHRUST FROM WILSON AND TORUM IN (1).

The presence or non-presence of continental-slope upthrust is a major question that probably cannot be resolved directly. The far offshore data is admitted by Plafker to be conjectural, based on plausible extrapolation to the deep ocean of measurements at its borders. The observed 4 m elevation at Middleton Island is one of the few "hard" pieces of evidence, but this is just at the shelf-slope boundary. Plafker suggests that there is upthrust as far out as the Aleutian trench, and conjectures in his Plate 1 a more-or-less linear decay with distance along the slope. However, in the text ([8], p. 138), he states: "Nor is it known whether the uplift seaward from Middletown Island dies out gradually toward the toe of the continental slope, as inferred on the profiles on Plate 1, or whether it terminates abruptly in one of more faults or flexures on the slope." Again, in the absence of observational evidence, the best that one can do is to compare the effects of various assumptions. The two upthrust curves in Figure F.3 are representative of such widely-differing, but in each case entirely plausible assumptions.

The data in Figure F.3 will now be used in one-dimensional tsunami generation studies, based on the numerical method of Appendix E. In fact, a more sensible strategy for use of such one-dimensioal models would be to construct a large number of much narrower strips, across which depth-contour and upthrust averaging would be more accurate. Then, subsequent to solution of the one-dimensional problem for each separate strip, the two-dimensional propagation could be studied in the manner of Appendix H, or even more crudely, by averaging among all such strips the predicted one-dimensional outgoing tsunamis. However, this is a too-ambitious program for the present study, and hence I have used a rather wide strip, such that (in effect) this averaging takes place prior to solution of the wave-propagation problem.

This particular strip has been chosen to include the highest measured upthrusts, and hence to give an upper bound on the outgoing tsunami. Strips located further southwest will involve (a) a lower maximum upthrust, (b) more coastal absorption (e.g. into Cook Inlet), (c) more two-dimensional geometric dispersion, and (d) a deeper Aleutian trench. All of these factors indicate that effect of such strips on the outgoing tsunami will be less than that of the strip chosen. Similar conclusions apply to the small region of upthrust to the northeast of the present strip.

The shoreward edge x = 0 of the strip is a little (say 10 km) offshore from an averaged measure of the convoluted coastline near Seward. This is not a significant distance on the total scale of the strip, but can be allowed for by appropriate choice of the parameter α in equation (A.5), if desired. The deep-ocean edge x = 300 km is just beyond the Aleutian trench, and at a depth of 4 km. This 4 km depth is taken as representative of the deep-ocean value, so that I have set $\beta = 1$ in Equation (A.7).

For use in the numerical analysis, I have discretized the mean depth and upthrust data of Figure F.3 at 5 km intervals in x. This is already a much finer interval than is normally used (e.g. 16 km by Hwang and Divoky in [7] and $1/3° \sim 10$-18 km quoted by Houston [4], etc.) but, even so, is only just capable of describing the main upthrust peak. For example, on this scale there are only 4 data points with ("measured") up-thrust exceeding 5 m. On intervals coarser than 10 km, there is a serious possibility of failing to incorporate the actual main peak of the measured data at all. However, this is not a problem with the "assumed" upthrust of Figure F.3, for which even a 20 km spacing characterizes the peak adequately. For use in numerical work at spacings finer than 5 km, linear interpolation is adequate.

APPENDIX G

Parametric Study of the 1964 Tsunami

The one-dimensional wave equation (A.1) has been solved numerically, subject to various boundary and initial conditions, with the data presented in Appendix F. The aim is not so much to provide firm practical tsunami predictions, which are impossible because of the inadequacy of the input data, but rather to illustrate effects of changes in assumed data and conditions.

It is convenient to state immediately a number of the negative conclusions; that is, to identify those parameters which have a negligible (generally $< 2\%$) effect on the outgoing tsunami. These include:

(1) The Aleutian trench, which is admittedly not a pronounced feature at the location of this particular strip. Smoothing it out changes the results by a negligible amount.

(2) Continental-shelf depth variations. Assuming a uniform 120 m depth out to $x = 100$ km changes the results negligibly.

(3) Earthquake time history, at least if the "simple-ramp" model (E.5) is used. There is negligible difference between results for assumed earthquake times up to 8 minutes, and those for an initial water elevation (at zero initial velocity) of magnitude equal to the permanent deformation, the $t = 0$ instant being chosen at the end of the earthquake.

(4) The time interval chosen for the numerical computations, providing it is small enough for numerical stability. This means that, since the highest wave speed is about 200 m/s, any time interval smaller than $\Delta x/200$ is satisfactory for an explicit numerical scheme, such as outlined in Appendix E.

(5) The value of the shoreline reflection parameter α, at least for the first hour. This is a priori obvious, since it is not until the main upthrust has been reflected from the shore that such a parameter can influence the outgoing tsunami.

The factors that are of fundamental significance are:

(6) The spatial interval Δx chosen, and

(7) The nature of the assumed upthrust curve.

Neither of these conclusions is at all surprising, but the extent and nature of the resulting effects may be worth discussion.

The question of choice of Δx is in one sense a purely computational one, since, given any data at all, one can always in principle refine the mesh by interpolation, until convergence is attained. However, there may be practical and cost difficulties with any such refinement, if carried too far. Most computations to be presented here were carried out at essentialy-zero cost on a TRS-80 micro-computer, for Δx down to 2.5 km. However, for more sophisticated (e.g. two dimensional) models, such a small interval could lead to unacceptably-large computation times and costs on large computers, and storage limitations would also become significant.

Nevertheless, it is impossible to resolve an upthrust as sharp as the "measured" curve of Figure F.3, unless one is prepared to go down to a mesh of the order of one or two kilometres at the largest. This conclusion seems not to be dependent on any degree of skill in programming, or sophistication in numerical method. For example, an implicit method, instead of the simple explicit method of Appendix E, gave essentially the same answers, at a fixed Δx. The problem is one of data resolution - if there are insufficient points, the computation inevitably predicts spurious effects.

Figure G.1 shows these effects, for the "measured" input data at Δx = 10, 5 and 2.5 km. The quantity plotted is the water surface elevation in metres at x = 300 km, against time in minutes. The results represent the outgoing (one-dimensional) tsunami shape, in both space and time, for all greater values of x. This is because d'Alembert's solution applied, on the reasonable approximation of constancy of the deep-ocean depth for x > 300 km. The results are converging to a final wave shown in Figure G.2. This convergence was checked by a "benchmark" computation with the extremely-fine grid Δx = 1 km, for which 41 seconds of CYBER-173 computer time had to be used.

The obvious feature of the unconverged results is a spurious short-period oscillation, superposed on the main tsunami. This false period is approximately the time for a wave to travel back and forth over one interval Δx on the shelf, and thus becomes shorter as Δx is reduced. At the same time, the amplitude of the error decreases. For Δx = 10 km, the apparent period induced by discretization error is about 15 minutes, and the amplitude is large enough to be easily confused with real features of the tsunami. The height of the main peak is also underestimated by about 20 percent at this interval, although this feature is much improved by a reduction to Δx = 5 km.

What can we learn from this? Certainly, no confidence can be placed in predictions of generated tsunami energy at periods comparable with the time taken to transit one or two mesh intervals. Energy at the order of 10 minutes period in the 1964 tsunami is contained in the main upthrust peak only. Apparent oscillations with such short periods should be interpreted as numerical error, and the final converged results approximated by fairing a smooth curve through these oscillations. In view of the difficulty of achieving a sufficiently well-refined mesh, and especially since the fine detail of the continental-slope depth contours is neither known nor likely to be significant, the strategy adopted by Garcia and Houston [2] of using a semi-analytical technique to propagate the wave off the shelf seems appropriate and worthy of further investigation.

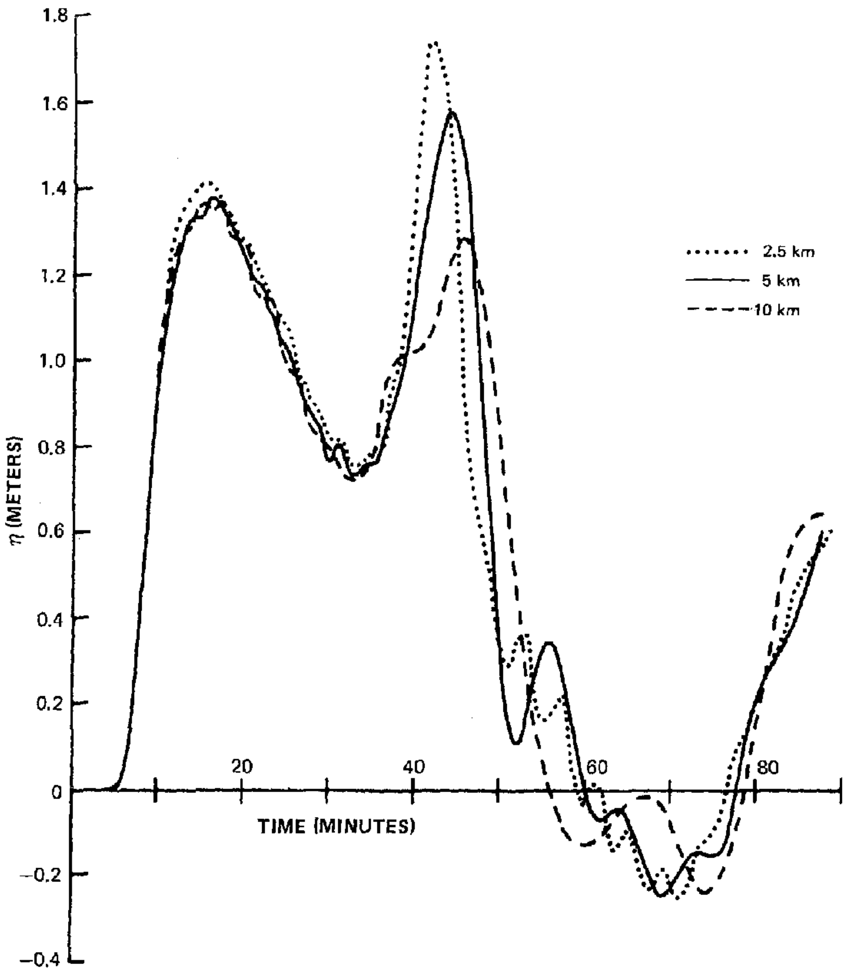

FIGURE G.1 COMPUTED OUTGOING TSUNAMI AT X = 300 KM, BASED ON "MEASURED" UPTHRUST OF FIGURE F.3, FOR VARIOUS MESH SIZES △X.

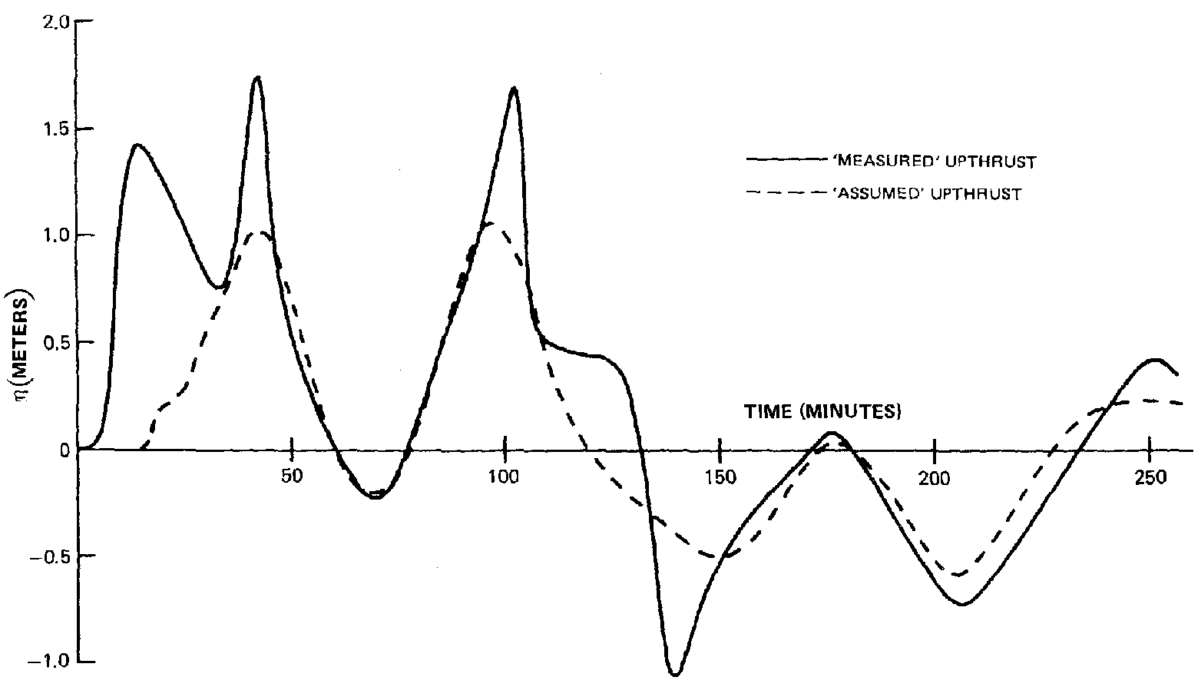

FIGURE G.2 CONVERGED COMPUTATIONS OF OUTGOING TSUNAMI AT X = 300 KM.

The fact that it is the sharpness of the upthrust peak that is responsible for this type of error, is illustrated by corresponding results with Δx = 20 and 10 km (not shown) for the smoother "assumed" upthrust curve of Figure F.3. In that case, the spurious oscillations seem not to occur at all, even with a step size as coarse as 20 km. There is good monotone convergence to a final result shown in Figure G.2, which was computed with Δx = 5 km to better than 2 percent accuracy. In view of the fact that the observational status of the extremely-sharp "measured" data is not entirely secure, it seems that another alternative procedure to accelerate convergence and avoid spurious oscillations, rather than unrealistic mesh refinement, is artificially to smooth the initial data before computation. There is almost as much a priori justification (at least for the portion on the shelf) for use of the smooth (dashed) data as the sharp (solid line) data of Figure F.3.

Figure G.2 thus shows the converged computed outgoing tsunami, based on the two possible sets of upthrust assumptions. The computations are carried on longer in time than those of Figure G.1, and display the waves that are reflected from the shoreline, arriving at distant points nearly two hours after the first tsunami wave. These computations were made with α = 0 (perfect reflection), and indicate that the main peak (at t = 102 minutes) of the reflected wave, is essentially of the same height as the main peak (at t = 43 minutes) of the first direct wave. Computations with non-zero α values show progressive reductions in the size of this second peak, and at α = 1 (perfect absorption at the shore) it is entirely absent.

Aside from a reduction in the size of the main peak due to (and of the same order of magnitude as) the assumed reduction in the upthrust input, the most profound difference between the two curves in Figure G.2 is the complete absence for the dashed curve of the first peak, at t = 15 minutes. This peak in the computed results for the "measured" data is, of course, entirely due to that portion of the "measured" upthrust that occurs on the continental-slope, which is absent in the "assumed" data.

The size of the peak in the outgoing tsunami produced by this portion of the upthrust is necessarily greatly exaggerated by hydrodynamic effects, compared to the upthrust data. This is because the water raised by such upthrust immediately runs off to infinity, at a speed higher than that at which the main upthrust peak first moves along the continental shelf. In addition that main peak suffers a large reflection when it encounters the shelf-slope boundary, whereas the portion of the upthrust which is already on the slope is largely free of such an amplitude-reducing effect. This is confirmed by the relatively weak energy in the shore-reflected signal at t = 125 minutes, due to the continental-slope part of the "measured" upthrust.

Thus, continental-slope upthrust (if any) produces a water displacement which quickly rushes out to sea, and constitutes the first part of the deep-ocean tsunami wave, but plays almost no role after the first half hour. The sharper peak which follows at t = 43 minutes is due to the main earthquake upthrust, and this is then subject to successive reflections from shore and shelf-slope boundaries.

It is disturbing that, if the conjectured extent of onslope upthrust were to be accepted, the very first (and hence presumably the most significant) large wave of the deep-ocean tsunami would be due to a portion of the upthrust whose details are, to say the least, obscure! The absence of the split-peaked character in tsunami observations from tide gauges at distant stations such as the Hawaiian Islands, tends to cast indirect doubt on the existence of on-slope upthrust during the 1964 earthquake, and it is this author's opinion that it did not occur at all.

The actual magnitude (1-1 1/2 metre peaks) and shape of the deep-ocean tsunami predicted by these computations is in reasonable qualitative agreement with what has been observed and computed previously. For example, once the spurious 10-15 minute oscillations are smoothed out of the computations of Hwang and Divoky ([7], p. 203, Points 4 and 5), who also include Plafker's conjectural continental-slope upthrust, the agreement is good enough to suggest that neither non-linearity nor two-dimensional effects (both of which are included in their model) are particularly significant in determining the out-going tsunami characteristics. There is some reduction, of the order of 20-30 percent, in peak amplitudes, perhaps due to these causes. The shape of the dashed curve (but not the solid curve) is also quite close to that for the deep-water tsunami assumed by Houston [4] in successful attempts to predict the effect on the Hawaiian Islands, the amplitude being at that great distance reduced by a factor of order 10 by two-dimensional effects.

As a by-product of the present computations, we also obtain the near-field tsunami wave, and in particular show in Figure G.3 the shoreline amplitude at $x = 0$ for both "measured" and "assumed" upthrust. These results are of course quite sensitive to the value of α, here taken as zero.

Immediately after the earthquake, a wave containing roughly one half of the upthrust travels shoreward, and a corresponding half-wave travels seaward. If $\alpha = 0$, that shore-travelling half-wave is perfectly reflected, as if by a vertical cliff at $x = 0$, thereby producing a doubling of its effective wave height, and thus a shoreline elevation comparable to the original upthrust magnitude, as confirmed by Figure G.3. If $\alpha = 1$, there is perfect absorption and the amplitude is about equal to the arriving wave, i.e., half of the initial upthrust. Thus, in general, one should expect shore amplitudes between 50 and 100 percent of the upthrust, but nearly always closer to 100 percent, since perfect reflection is a much more tenable assumption than perfect absorption.

The first peak positive elevation of 9 metres in the present case should have been at 1/2 hour after the earthquake, with a positive elevation maintained at more than 3 metres for another 1/2 hour, followed by a sudden fall in level to 5 metres below zero. The level should then have stayed negative for about 2 hours, before the next positive wave arrived. This description seems to be in general accord with observations at Seward (Wilson and Torum in [7], Table 7, p. 415).

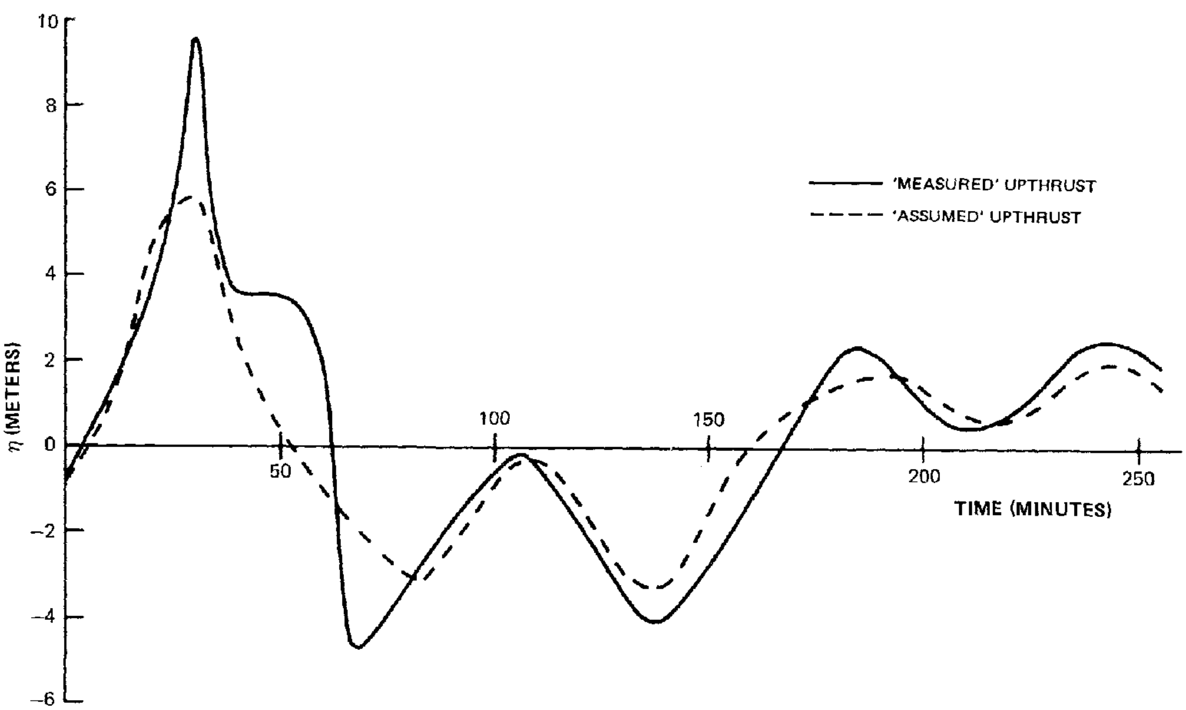

FIGURE G.3 CONVERGED COMPUTATIONS OF SHORELINE TSUNAMI AT X = 0.

Figure G.3 shows a small downward movement at first; this is the local downthrust during the earthquake. Since the origin $x = 0$ for the present computations is slightly off-shore from Seward, a somewhat larger downthrust is appropriate as the effect at Seward during the first few minutes, including the actual earthquake time. It was during that period that dramatic local nonlinear effects, involving catastrophic destruction of the Seward docks and slumping of the harbor bed took place. The subsequent chaos prevented other than rather sparse observations of the tsunami which followed, but a 30 foot wave is described as arriving at 20-30 minutes, and very large positive elevations are mentioned until up to one hour after the earthquake. The absence of observations for the next 2-1/2 hours is consistent with the computations, indicating no attention-gaining positive peaks in that time, and the next observation, of a positive 14-15 foot wave at 4 hours after the earthquake, is presumably the second of the subsequent two positive peaks, perhaps amplified by nonlinear run-up effects. Later observations include a rather doubtful 40 foot negative draw-down, and a recurrence of waves at 1-1/4 hour intervals, consistent with the first-harmonic shelf periods.

Again, in view of the great uncertainties in data and observations, the ability of a drastically-simplified theory such as the present one to come as close as it does to prediction of events of such great scale, is indirect evidence for the insignificance of nonlinearity, frequency dispersion and geometrical (two-dimensional) dispersion for the tsunami-generation problem.

APPENDIX H

1D-2D Connection

If the upthrust zone has x-wise extent w, and y-wise extent ℓ, where $w \ll \ell$, then a one-dimensional model of the tsunami generation problem is appropriate, at least for x values which are not too large. Such a model will lead to a separate prediction at each value of y, of an outgoing wave, moving entirely in the x-direction, whose form will be a slowly-varying function of y.

Such near-field one-dimensional analysis can be carried out using the procedures outlined in the previous Appendices. For the present, we shall restrict attention to steady-state time-sinusoidal problems, to which the numerical method of Appendix D applies. Thus we may assume that, for every separate value of the coordinate y, we have solved a stripwise one-dimensional problem, to determine an outgoing wave in $x \gg w$ of the form:

$$\eta(x,y,t) \rightarrow \mathcal{R}[A_0(y)e^{ikx-i\omega t}] \ . \qquad (H.1)$$

In (H.1), $k = \omega/\sqrt{gh_\infty}$ is the wave number at "infinity" where we assume the depth has settled down to the constant value h_∞, and $A_0(y)$ is the computed complex-valued amplitude. Thus the outgoing wave appears as a pure sine wave in the x-direction, whose amplitude and phase vary (slowly) with respect to the label parameter y for the strip being currently considered.

The picture (H.1) can be expected to apply in an "intermediate" range, with $x \gg w$, but must fail for a sufficiently large value of x, certainly for x as large as the length ℓ of the upthrust zone. Also, we must envisage that the wavelength $2\pi/k$ is comparable with w, but is not as great as ℓ.

We now attempt to solve for the subsequent propagation of the wave whose initial generation is prescribed by (H.1), into a region $x = O(\ell) \gg O(w)$ where two-dimensional effects are significant. This region is many wavelengths from the upthrust axis. We assume for simplicity that the depth is the uniform value h_∞ in this region. Thus we must solve the full wave equation (1) including y-variation, but can make use of the simplifying feature that the wavelength is short compared to the fundamental length scale of the region.

This enables us to use the "parabolic approximation" of diffraction theory [1], [6], which involves retention of part of the near-field assumption of a pure-sinusoid outgoing wave. Thus, if we substitute into (1) the expression:

$$\eta(x,y,t) = R[A(x,y)e^{ikx-i\omega t}] \ , \tag{H.2}$$

(which differs from (H.1) only in allowing the amplitude A to vary with x), we find the (exact) equation for A(x,y) to be:

$$\frac{\partial^2 A}{\partial x^2} + 2ik\frac{\partial A}{\partial x} + \frac{\partial^2 A}{\partial y^2} = 0 \ . \tag{H.3}$$

The parabolic approximation consists of neglect of the first term of (H.3), giving:

$$2ik\frac{\partial A}{\partial x} + \frac{\partial^2 A}{\partial y^2} = 0 \ . \tag{H.4}$$

This is justified on the basis of a slowly-varying character to A.

Equation (H.4) is a Schrodinger equation, or a heat equation with imaginary conductivity, and enables direct solution in many cases. In particular, a useful class of solutions is given by:

$$A(x,y) = \frac{1}{1+i}\sqrt{\frac{k}{\pi x}}\int_0^{\ell} dy' A_o(y')e^{ik(y-y')^2/2x} \tag{H.5}$$

which has the property that

$$\lim_{x \to 0_+} A(x,y) = A_o(y) \ . \tag{H.6}$$

Again, the practical interpretation of the limit in (H.6) is that (H.6) applies when $x \ll O(\ell)$. Thus (H.6) applies when (H.1) is valid, and hence matching between the 1D and 2D solutions can take place in the common or overlap domain of validity.

Thus $A(x,y)$ is now a known quantity, given by the integral (H.5) with a known form for $A_o(y)$, and the two-dimensionally propagating wave (H.2) is fully determined. This wave still propagates "mainly" in the x direction, but account is now being taken of two-dimensional dispersion, and (H.5) can be used to predict the steady loss of amplitude due to this cause.

If we move even further away from the generation zone, i.e. allow $x \gg \ell$, the propagation becomes less and less oriented in the x direction, and, if the depth remains at the constant value h_∞, ultimately the tsunami wave fronts will become circular. Again, it is possible to apply matching principles to determine the polar diagram of the resulting pattern, as follows:

If $x = r \cos \Theta$ and $y = r \sin \Theta$, we expect that, for $r \gg \ell$, the spreading tsunami takes the form:

$$\eta \rightarrow R\left[\frac{F(\Theta)}{\sqrt{r}}\, e^{ikr-i\omega t}\right] \tag{H.7}$$

for some complex amplitude function $F(\Theta)$. Now if we let x and y become large in (H.5) we obtain:

$$\eta \rightarrow R\left[\frac{1}{1+i}\sqrt{\frac{k}{\pi x}}\, e^{ikw+iky^2/2x-i\omega t} \cdot \int_0^{\ell} dy' A_o(y') e^{-ikyy'/x}\right] , \quad (H.8)$$

which is the approximate form for y<<x of (H.7), with:

$$F(\theta) = \frac{1}{1+i}\sqrt{\frac{k}{\pi}} \int_0^{\ell} dy' A_o(y') e^{-iky'\tan\theta} . \quad (H.9)$$

Thus, finally, by Equation (H.9) we are able to relate the far-field polar pattern $F(\theta)$ of the propagating tsunami, to the computed one-dimensional outgoing amplitude $A_o(y)$. Note that the energy of the tsunami is still strongly concentrated along the axis $\theta = 0$ normal to the upthrust line, indicating the residual effect on the propagating tsunami, of the original elongated character of the upthrust.

The above is little more than an outline of the procedure for executing a transition from the one-dimensional near field to the two-dimensional far-field propagation. Much more work is needed to fill in the details, even for the steady-state pure-sinusoidal case treated here. The more interesting transient problem, with general time dependence, has not yet been analyzed at all on this basis.

DISCUSSION

W. Van Dorn - Wanted a confirmation from E. Tuck that the ground deformation model with continental slope upthrust was measured whereas the model without slope upthrust was just "guesswork."

E. Tuck - Both models were essentially "guesswork", expecially the section over the continental slope. The first model was given by Plafker and the second by Wilson. Stressed that the models were used to show that if there was continental slope upthrust then the hydrodynamics would exaggerate the contribution to the wave from the slope upthrust vis-a-vis the wave coming off the shelf (Figure G.2). Since this effect was not seen in the 1964 tsunami, continental slope upthrust probably did not exist.

W. Van Dorn - Wanted to know more precisely why Tuck claimed that the waveform represented by the continental slope upthrust model was not seen.

G. Carrier - Nobody has seen anything which they could identify with a particular part of the bottom deformation.

E. Tuck - Van Dorn's gauge measurements at Wake Island showed something which you could interpret as looking like the waveform without slope upthrust. If slope upthrust did exist then it would feature prominently in the tide gauge records.

G. Carrier - It's been postulated by many people for a long time that the propagation over the open ocean is certainly linear. It is believed quite widely that whether or not in the generation region the tsunami is linear, it makes no sense to do anything more elaborate while trying to get general understanding because the input details are far too vaguely known to justify detailed study. However, in the run-up stage, the phenomenon is certainly non-linear (defining run-up to be on dry land). Questioned E. Tuck whether he would subscribe to that.

E. Tuck - Agreed but stressed that linear theory is still good for taking the tsunami from the deep ocean up and over the continental shelf until very close to the shoreline.

G. Carrier - Disagreed strongly with E. Tuck on his statement regarding dispersion. The importance of dispersion cannot be decided by comparing depth and length. You must also have the travel distance. Any wavelength will have important dispersive effects if it travels far enough. If the depth over the main transmission path is 3 miles, and the distance travelled is 3,000 miles (which are characteristic scales in the Pacific) then a wave of length of the order of 50 miles will disperse very rapidly.

E. Tuck - The wavelength isn't that short.

G. Carrier - There is a good deal of energy in that wavelength regime and you want to know where it ends up and when.

E. Tuck - Agreed. But stressed that the dominant energy is in much longer wavelengths than that.

G. Carrier - Pointed out that Tuck's calculation has something with much shorter wavelengths with considerable energy in them, these cannot be thrown out because the topography which is generating them has a 100 km length-scale on the shelf.

E. Tuck - Pointed out that in the deep ocean the wavelength goes up by a factor of 10 or 20.

G. Carrier - Stated that if E. Tuck's results are correct then the structure therein is in the wavelength regime which would disperse rapidly.

E. Tuck - Disagreed.

G. Carrier - Restated Kajiura's rule presented earlier in a different form: Divide the travel distance by the depth and take the cube root, if the result exceeds 3 or 4, which it does, then dispersion is important.

E. Tuck - Objected and pointed out that it is a very longwave so that there are only two wavelengths from Alaska to Crescent City and only ten to Australia.

G. Carrier - Posed the problem of a Gaussian disturbance started out in deep water with a 50 mile half-width at the edge of the continental shelf but in deep water. The result is that there is considerable dispersion by the time it travels 3,000 miles. The amplitude is reduced by a factor far greater than 2. The problem was solved and the solution published a long time ago. Dispersion is important - this is a fact which has been calculated with precision.

E. Tuck - Conceded the point made by Carrier with regard to a 50 mile half-width Gaussian pulse and travelling 3,000 miles over deep water.

G. Carrier - Suggested the following alternative to avoid numerical problems in the propagation problem. If there is an initial strip in which you have a displacement you want to propagate, a very straightforward thing to do is to make up that initial displacement by a series of cylindrical Gaussians. The wiggles in that composite disturbance are not important because dispersion will filter them out. Since linearity is assumed, the solution is obtained by superposition of the disturbances resulting from each individual cylindrical Gaussian. The procedure is very simple and you can do without the numerical methods which give so many problems. However, the procedure does not take the disturbance down from the shelf on which it is generated.

E. Tuck - Stated that he was merely pointing out possible traps in the numerical schemes.

C. Mader - Stated that these were not essential traps.

G. Carrier - Stated that as everyone has emphasized, we don't know the initial ground motions which generate the tsunamis. Except something about the length, width and general intensities of these disturbances. The question that has always intrigued him was why the run-up in Hawaii differs so much from that in Wake. Is it the topographic features along the transmission path which creates waveguides or reflective interference or is it the receiver or terminal impedance of the target area? He thinks he now knows the answer in most instances viz. the latter. Therefore he really does not worry about the appropriate, fairly, meticulous description of the wave at the edge of the generating shelf before it starts out across the oceans. He is willing to settle for anything that has qualitatively the right macroscopic properties at that place. Instead he wants to know, when the wave gets across the ocean, how different are the waves at Wake and Hawaii etc. In the absence of initial data, the details of the early transmission down the slope may not mean much because we may not be using the right input. He does not deprecate the attempt to investigate the initial waveform but emphasizes that the propagation over the ocean is much more important than what happens locally at the slope, which he even considers to be part of the generation problem.

E. Tuck - But you must know what to propagate and the waveform resulting from the continental slope upthrust is not the one.

G. Carrier - I would not dare rule out wiggles with a 100 km length scale because the topography has them.

H. Loomis - Referred to E. Tuck's two-dimensional result showing amplitude decaying as $r^{-1/2}$. This would imply energy varying as r^{-1} or that the energy is moving with the wavefront. Nevertheless, if the problem is solved in a uniform depth ocean, what happens is, as the wave front moves, it keeps changing shape and leaving a disturbed portion of the ocean behind. Therefore, he would argue that energy cannot be in the wave front.

G. Carrier - The discrepancy is in the next asymptotic term.

T. Wu - Thought that it lies somewhere between $r^{-1/2}$ and r^{-1}. The front wave is still dispersing at large distances so that the wavelength is still increasing as $t^{1/3}$. Energy and excess mass will be preserved. There is probably some criteria for which E. Tuck's equations hold.

E. Tuck - Does not accept what Carrier has said about dispersion since the wavelength is 500 km and not 50 km.

G. Carrier - There are many other tsunamis in which the wavelength is not that large.

ON TSUNAMI PROPAGATION – EVALUATION OF EXISTING MODELS

BY
T. Y. WU

Introduction

Tsunami waves, generated by the tsunamigenic type of earthquakes, are unique in many characteristics of all ocean waves. Large tsunamis can be attributed to a rapidly occurring tectonic displacement of the ocean bottom (usually near the coast) over a large horizontal dimension (of hundreds to over a thousand km^2) during strong earthquakes, causing vertical displacements of tens of meters. Various source mechanisms have been studied as possible models, including sudden uplifting, slumping, tilting and avalanching of the ocean bottom. The detailed source motion of a specific tsunami is generally difficult to determine, though the technique of 'inverse radiation' (from the observed radiation field to assess the source motion) has been used (Van Dorn 1961) to evaluate the source disturbance history from the observed spectrum of the wave-amplitude envelope using the Fourier inversion. The large size of source area implies on hydrodynamic theory that the 'new born' waves would be initially long and the energy contained in the large wave-number part (k, nondimensionalized with respect to the local ocean depth, h) would be unimportant.

Soon after leaving the source region, the large wave-number components of the source spectrum are further dispersed effectively by the factor sech kh to the low wave-number parts. Tsunamis thus evolve into a train of long waves, with wavelength continually increasing from about 50 km to as high as 250 km, but with a quite small amplitude, typically of 1/2 m or smaller, as they travel across the Pacific Ocean at a speed of 650-760 km/h.

There is experimental evidence indicating that tsunamis continually, though slowly, evolve due to dispersion while propagating in the open ocean. In observing the data taken at Wake Island of the March 9, 1957 Aleutian tsunami, Van Dorn (1961) reported that "commencing with a low trough 277 minutes after the initial earthquake shock, the record is characterized by a long train of waves of ever-increasing frequency....256 minutes after the first (wave) arrival, 71 crests have been observed." If modelled according to linear dispersive wave theory, the wavelength of the leading wave would increase with time like $t^{1/3}$ or with distance travelled like $x^{1/3}$. Thus the wavelength of 45 km observed at Wake Island would continually increase to more than 200 km after a single traverse across the Pacific Ocean.

Attenuation of tsunamis due to dissipation and absorption by harbors, inland seas, volcanic islands and coral reefs during propagation is still unclear, but is generally known to be small. This contention is based on the rough estimate that the particle-velocity to wave-velocity ratio, being of the order of the amplitude to water-depth ratio, i.e, $u/c \simeq a/h$, is very small ($\sim 10^{-4}$, corresponding to $u \sim 1$ cm/s). Based on observations, Munk (1961) suggests the figure that the 'decay time' (intensity decays to $1/e$) is about 1/2 day, and the 'reverberation time' (intensity decays to 10^{-6}) is about a week.

One of our primary interests is, of course, the evolution of tsunamis in coastal waters and their terminal effects. Their amplitude can magnify many-fold as they climb up the continental slope and propagate into shallower waters. Large tsunamis (from the initial potential energy as high as 10^{15} -10^{16}J in the source region) can amplify to devastating waves up to 20 m or higher upon arriving at a beach. The terminal amplification is further affected by three-dimensional configuration of the coastal environment en-route to the coast. These factors dictate the transmission, reflection, rate of growth, and trapping of tsunamis in their terminal stage.

Although the physical quantities of a tsunami may undergo a fairly wide range of variations through its life span, it is useful to fix ideas by providing the following table of pertinent characteristics and scaling parameters.

TABLE I

		Deep Ocean	Shallow Ocean		Coastal Water
water depth,	h	2.5–4 km (deep ocean)	1 km (shallow ocean)		<300 m (coastal water)
phase velocity,	c	550–750 km/hr	decreasing	$(\sim h^{1/2})$	
wave length,	λ	40–200 km	decreasing	$(\sim h^{1/2})$	
period,	T	3.5 – 20 min & longer	ray invariant		
amplitude-depth ratio	a/h	10^{-4}	increasing	$(\sim h^{-5/4})$	
depth-wave length ratio	h/λ	10^{-1} – 5×10^{-3}	decreasing	$(\sim h^{1/2})$	
$U_r = a\lambda^2/h^3$		10^{-2} – 5	increasing	$(\sim h^{-9/4})$	
continental slope		0.02–0.2			
source potential energy		10^{15} – 10^{16} J			
decay time		1/2 day			
reverberation time		1 week			
reflection frequency (across the Pacific)		1.7/day			

It is of significance to note that while the Ursell number of large tsunamis is generally small in the deep ocean, typically of order 10^{-2}, it can increase by 10^3 upon arriving in near-shore waters. This indicates that the effects of nonlinearity (amplitude dispersion) are practically nonexistent in the deep ocean, but gradually become more important and can no longer be neglected when the Ursell number increases to order unity or greater during the terminal stage in which the coastal effects manifest. The overall evolution of tsunamis after having arrived in coastal waters, as only crudely characterized in Table I, depends in fact on many factors such as the direction of incidence, the three-dimensional configuration of the coastal region, converging or diverging passage for the waves, local reflection and absorption, etc. Further, the small values of the dimensionless wave number, $kh = 2\pi h/\lambda$ being generally of order 0.6 - 0.03, suggests that a slight dispersive effect is still present and this effect, though small, is important in predicting the phase position over very large distances of travel. This point seems to be well supported by experimental observations.

In view of the wide range of variation of the pertinent physical factors that characterize a tsunami, a primary problem is therefore to establish the optimum model(s) which can most effectively (from its accuracy and effort cost points of view) describe specific tsunamis. The present discussion and survey will concentrate on the three-dimensional (with propagation in two horizontal dimensions) effects under various conditions and a comparison between different wave models in different circumstances.

Three-dimensional Water-layer Transport Equations

For applications to general tsunami problems, it is useful to include the feature of ocean bottom motion in the model. We thus consider the motion of three-dimensional waves of finite amplitude and arbitrary wave number (in two horizontal dimensions x,y) in the ocean whose surface, if undisturbed, is at $z = 0$ and whose bottom is prescribed by $z = -h(x,y,t)$ measured along the upward vertical axis. The ocean water is assumed to be incompressible and inviscid, for which case the Euler equations of motion are:

$$\nabla_o \cdot \underset{\sim}{u}_o = 0 \ , \tag{1}$$

$$\frac{d\underset{\sim}{u}_o}{dt} = \frac{\partial \underset{\sim}{u}_o}{\partial t} + \underset{\sim}{u}_o \cdot \nabla_o \underset{\sim}{u}_o = -\frac{1}{\rho} \nabla_o (p + \rho g z) \tag{2}$$

with the boundary conditions

$$w = d\zeta/dt \qquad \text{on} \qquad z = \zeta(x,y,t) \ , \tag{3}$$

$$p = p_o(x,y,t) \qquad \text{on} \qquad z = \zeta(x,y,t) \ , \tag{4}$$

$$w = -dh/dt \qquad \text{on} \qquad z = -h(x,y,t) \tag{5}$$

Here $\underset{\sim}{u}_o = (u,v,w)$ denotes the flow velocity, $\nabla_o = (\partial/\partial x,\ \partial/\partial y,\ \partial/\partial z)$ the vector operator, ρ is the water density, p the pressure, g the gravity constant, $z = \zeta(x,y,t)$ the free water surface. Irrotational motion is assumed so that $\underset{\sim}{u}_o = \nabla_o \phi$, $\nabla_o^2 \phi = 0$, and we have the Bernoulli equation:

$$p/\rho + \partial\phi/\partial t + \frac{1}{2} u_o^2 + gz = 0 \tag{6}$$

The following transport theorem is useful: for a function $f = f(x,y,z,t)$, we have

$$[\frac{df}{dt}] = \frac{\partial}{\partial t}[f] + \nabla \cdot [\underset{\sim}{u}\ f] \ , \tag{7}$$

where $\underset{\sim}{u} = (u,v)$ is the vector with the two horizontal velocity components, $\nabla = (\partial/\partial x,\ \partial/\partial y)$ the corresponding two-dimensional vector operator, and

$$[F] \equiv \int_{-h}^{\zeta} F(x,y,z,t)dz \equiv (\zeta+h)\overline{F} \ , \tag{8}$$

So [F] is the integral of F across the entire vertical water layer $-h<z<\zeta$ at a given horizontal position $\underset{\sim}{r} = (x,y)$ and at time t, and $\overline{F}$ signifies the average of F over the vertical layer. This theorem has been used in one form or another by Laplace and many others. It can readily be proved by considering a material volume V which at time t coincides with a vertical column having a horizontal cross-sectional area S_c so that

$$\frac{d}{dt} \int_{V(t)} f dV = \int_V \frac{df}{dt} dV = \int_{S_c} dS \int_{-h}^{\zeta} (\frac{\partial f}{\partial t} + \nabla_o \cdot (\underset{\sim}{u}_o f)) dz$$

$$= \int_{S_c} [\frac{\partial}{\partial f} \int_{-h}^{\zeta} f dz + \nabla \cdot \int_{-h}^{\zeta} \underset{\sim}{u}\ f\ dz]\ dS$$

where in the last step use has been made of (3) and (5). Since S_c is otherwise arbitrary, the result (7) is therefore immediate.

The quantities of particular interest are $f = 1$, $\underset{\sim}{u} = (u,v)$, w, and

$$f = H \equiv \frac{1}{2}\rho(u^2 + v^2 + w^2) + \rho g z = (p_o - p) - \rho\partial\phi/\partial t \tag{9}$$

we note that

$$[H] = \int_{-h}^{\zeta} \frac{1}{2}\rho u_o^2 \, dz + \frac{1}{2}\rho g(\zeta^2 - h^2) = e_k + e_p \tag{10}$$

is the sum of the kinetic and potential energy densities of the water layer. By taking these quantities in turn for f in (7) and using Equations (1) - (6), we obtain the following equations

$$\eta_t + \nabla \cdot (\eta\overline{\underset{\sim}{u}}) = 0 \qquad (\eta = h + \zeta) \; , \tag{11}$$

$$(\eta\overline{\underset{\sim}{u}})_t + \nabla \cdot (\eta\overline{\underset{\sim}{u}\underset{\sim}{u}}) = -\eta\overline{\nabla p}/\rho \tag{12}$$

$$(\eta\overline{w})_t + \nabla \cdot (\eta\overline{w\underset{\sim}{u}}) = -g\eta - (p_o - p_b)/\rho , \tag{13}$$

$$(\eta\overline{H})_t + \nabla \cdot [\eta\overline{(H + p)\underset{\sim}{u}}] = -(p_o\zeta_t + p_b h_t) \; , \tag{14}$$

where $\overline{F}$ is defined by (8), $\eta = h + \zeta$ is the total water layer thickness, $\overline{\underset{\sim}{u}\underset{\sim}{u}}$ denotes a tensor in the two horizontal dimensions ($x = x_1$, $y = x_2$) such that its ijth component is $T_{ij} = \overline{u_i u_j}$ and the ith component of $\nabla \cdot T$ is $\partial T_{ji}/\partial x_j$, and $p_b = p(x,y,-h)$ is the pressure at the ocean floor. These equations are exact.

From these equations we can prove a set of conservation laws under the assumptions that (1) the free surface spans over the entire xy-plane, (2) all the physical quantities in (11) - (14) fall off sufficiently fast at large distances from the source region such that the integrals over the entire horizontal plane are convergent, (3) $h = h_o(x,y) + h_1(x,y,t)$, h_1 is the only time dependent component and is assumed integrable over the water plane S. Then for the excess mass m:

$$m \equiv \rho \int_S (\zeta + h_1) dS = \text{const.} \tag{15}$$

For the center of excess mass, $\underset{\sim}{R}$, defined by

$$m\underset{\sim}{R} = \rho \int_S (\zeta + h_1)\underset{\sim}{r}\, dS, \qquad \underset{\sim}{r} = (x,y), \tag{16}$$

we have the impulse equation

$$m\dot{\underset{\sim}{R}} = \rho \int_V \underset{\sim}{u}\, dV \equiv \underset{\sim}{I} \quad \text{(impulse)}, \tag{17}$$

$\dot{\underset{\sim}{R}}$ denoting $d\underset{\sim}{R}/dt$, and V the entire flow region. Further, we have the conservation of horizontal momentum:

$$\dot{\underset{\sim}{I}} = \int_S [p_o\, \nabla\zeta + (p_b - \rho g h)\, \nabla h]\, dS\ , \tag{18}$$

and the conservation of vertical momentum,

$$\dot{I}_3 \equiv \frac{d}{dt} \int_V \rho w dV = \int_S (p_b - p_o - \rho g \eta) dS. \tag{19}$$

Finally, conservation of the total energy is given by the relation,

$$\dot{E} = \dot{E}_k + \dot{E}_p = - \int_S (p_o \zeta_t + p_b h_t) dS. \tag{20}$$

Three-dimensional Long Wave Models

We observe that since the irrotationality condition ($\nabla_o \times \underset{\sim}{u}_o = 0$) has not been used in deriving (10) - (14), the transport equations (11) - (14) in the present exact form apply equally well to rotational flows of an inviscid fluid (such as in sheared flows when the viscous diffusion effects are unimportant). On this more general premise, the transport equations (11) - (14) are four leading moments of a set of hierarchy equations since each new moment gives rise to additional new unknowns. For irrotational motions, however, these unknowns are in fact inter-related by the field equation $\nabla_o^2 \phi = 0$ for the velocity $\underset{\sim}{u}_o = \nabla_o \phi$ and by the Bernoulli equation (6) which relates p to ϕ. Their relationships are especially easy to find for long waves. In fact, only the first two transport equations, (11) and (12) are needed; (13) - (14) can be used afterwards for evaluating the vertical momentum and the energy transport.

For the irrotational long wave motion, the following nondimensional variables are used:

$$(x^*,y^*) = (x,y)/\lambda, \quad z^* = z/h_o, \quad t^* = ct/\lambda, \quad \zeta^* = \zeta/h_o, \quad h^* = h/h_o,$$
$$(u^*,v^*,w^*) = (u,v,w)/c, \quad \phi^* = \phi/c\lambda, \quad p^* = p/\rho g h_o, \quad p^*_o = p_o/\rho g h_o, \tag{21}$$

where * indicates a dimensionless variable, λ is a characteristic wavelength, h_o a constant representative water depth, $c = \sqrt{gh_o}$ a typical wave speed. In addition, there are two important parameters associated with long waves, namely:

$$\varepsilon = h_o/\lambda \, ; \qquad \alpha = a/h_o \, , \tag{22}$$

where a is a representative wave amplitude. As usual, $\varepsilon << 1$ for all long-wave theories by definition. The magnitude of α differentiates between the linear long wave, the weakly nonlinear (of the Boussinesq class) and the fully nonlinear (the Airy class) theories according as $\alpha = o(\varepsilon^2)$, $O(\varepsilon^2)$ or $> O(\varepsilon)$. This relative magnitude is conveniently incorporated by the Ursell number.

$$Ur = \alpha/\varepsilon^2 = \alpha\lambda^2/h_o^3 \, . \tag{23}$$

Substituting (21) in (1), (11) and (12) and immediately omitting *, we obtain

$$\frac{\partial w}{\partial z} = -\varepsilon \nabla \cdot \underset{\sim}{u} \, , \qquad \underset{\sim}{u} = \nabla \phi \, , \qquad w = \frac{1}{\varepsilon} \frac{\partial \phi}{\partial z} \, , \tag{24}$$

$$\eta_t + \nabla \cdot (\eta \bar{\underset{\sim}{u}}) = 0 , \qquad (\eta = h + \zeta) \tag{25}$$

$$(\eta \bar{\underset{\sim}{u}})_t + \nabla \cdot (\eta \overline{\underset{\sim}{u}\underset{\sim}{u}}) = -\eta \overline{\nabla p} , \tag{26}$$

where $\bar{\underset{\sim}{u}}$ and similar layer-mean quantities $\bar{F}$ have the same expression as for the original definition (8). The kinematic condition at the ocean floor, (5) now becomes:

$$w = -\varepsilon (h_t + \underset{\sim}{u} \cdot \nabla h) \qquad \text{at } z = -h(x,y,t) . \tag{27}$$

Further combination of the equations in (24) gives

$$\frac{\partial^2 \phi}{\partial z^2} = -\varepsilon^2 \nabla^2 \phi , \tag{28}$$

which is the new nondimensional form of the original Laplace equation for ϕ.

Clearly, $\zeta = O(\alpha)$ and $\eta = O(1)$, Hence $\bar{\underset{\sim}{u}} = O(\alpha)$ by (25), $w = O(\alpha\varepsilon)$ by (24), and $p + z = O(\alpha)$ by (26). We thus assume for ϕ the expansion:

$$\phi = \alpha \sum_{n=0}^{\infty} \varepsilon^{2n} \Phi_{2n}(\underset{\sim}{r},z,t) . \tag{29}$$

Upon substituting (29) in (28), we obtain Φ_{2n} in the form:

$$\Phi_0 = \phi_0(\underset{\sim}{r},t) ,$$

$$\Phi_2 = \phi_2(r,t) + z\, \phi_3(\underset{\sim}{r},t) - \frac{z^2}{2} \nabla^2 \phi_0 , \tag{30}$$

$$\Phi_{2n} = \phi_{2n}(\underset{\sim}{r},t) + z\phi_{2n+1}(\underset{\sim}{r},t) - \nabla^2 \int dz \int dz_1 \, \Phi_{2(n-1)}(\underset{\sim}{r},z_1,t)$$

$(n = 1, 2, \ldots)$. A possible additive term $z\phi_1(\underset{\sim}{r},t)$ to $\phi_o(\underset{\sim}{r},t)$ for ϕ_o is discarded in order to satisfy the order estimate of w. Then we have for $\underset{\sim}{u}$ and w the expansion:

$$\underset{\sim}{u} = \nabla\phi = \alpha \left\{\underset{\sim}{u}_o + \varepsilon^2 \underset{\sim}{u}_1 + \varepsilon^4 \underset{\sim}{u}_2 + \ldots\right\} \quad ,$$
$$w = \frac{1}{\varepsilon}\frac{\partial\phi}{\partial z} = \alpha\varepsilon \left\{w_1 + \varepsilon^2 w_2 + \ldots\right\} . \qquad (31)$$

Note that $\underset{\sim}{u}_o = \nabla\phi_o(r,t)$ is independent of z, and for $n = 1,2,\ldots$, $\underset{\sim}{u}_n = \nabla\Phi_{2n}$ and $w_2 = \partial\Phi_{2n}/\partial z$ may vary with z. In consequence, we assume (see Eq. 27) that $h_t = O(\alpha)$ and $\nabla h = O(1)$. Then substitution of (31) in (27) yields:

$$\phi_3 = -\left\{h_t + \nabla \cdot (hu_o)\right\} . \qquad (32)$$

This is the only ϕ_n which is determined from given boundary condition, while the other ϕ_n's are yet left free so that (26) can be satisfied. To do so, we first derive from the Bernoulli equation (6) (now in nondimensional form) the expansion:

$$p + z = -\phi_t - \frac{1}{2}(\underset{\sim}{u} \cdot \underset{\sim}{u} + w^2)$$
$$= -\alpha \left\{\dot{\phi}_o + \varepsilon^2 \dot{\phi}_2 + \frac{\alpha}{2} u_o^2 + O(\varepsilon^4, \alpha\varepsilon^2)\right\} \quad , \qquad (33)$$

where $\dot{\phi} = \partial\phi/\partial t$. Next we apply the boundary condition (4) to (33), giving:

$$\zeta + p_o = -\alpha \left\{\dot{\phi}_o + \varepsilon^2 \hat{\dot{\phi}}_2 + \frac{\alpha}{2} u_o^2 + O(\varepsilon^4, \alpha\varepsilon^2)\right\} \quad , \qquad (34)$$

where $\hat{}$ denotes the variable evaluated at the free surface, $z = \zeta$.

From (33) and (34) we readily deduce that:

$$\overline{\nabla p} - \nabla\zeta = \nabla p_o + \alpha\varepsilon^2 \left\{ \nabla\hat{\dot{\Phi}}_2 - \overline{\nabla\dot{\Phi}_2} + O(\varepsilon^2,\alpha) \right\}$$

$$= \nabla p_o + \alpha\varepsilon^2 \left\{ -\frac{1}{2}h \frac{\partial}{\partial t} \nabla(h_t + \nabla \cdot (h\, \underset{\sim}{u}_o)) + \frac{1}{6}h^2 \frac{\partial}{\partial t} \nabla^2 \underset{\sim}{u}_o + O(\varepsilon^2,\alpha) \right\} . \tag{35}$$

Also we immediately see from (31) that, since $\partial u_o / \partial z = 0$,

$$\overline{\underset{\sim}{u}\ \underset{\sim}{u}} - \bar{\underset{\sim}{u}}\ \bar{\underset{\sim}{u}} = \alpha^2 \varepsilon^4 (\overline{\underset{\sim}{u}_1 \underset{\sim}{u}_1} - \overline{\underset{\sim}{u}_1}\ \overline{\underset{\sim}{u}_1}) , \tag{36}$$

so the difference between the two terms on the left side is negligible. Finally, by substituting (35) and (36) in (26) and rewriting (25) we obtain:

$$\zeta_t + \nabla \cdot ((h + \zeta)\, \bar{\underset{\sim}{u}}) = - h_t , \tag{37}$$

$$\overline{\underset{\sim}{u}_t} + \bar{\underset{\sim}{u}} \cdot \nabla\bar{\underset{\sim}{u}} + \nabla\zeta = - \nabla p_o + \frac{h}{2} \frac{\partial}{\partial t} \nabla(h_t + \nabla\cdot(h\, \bar{\underset{\sim}{u}})) - \frac{h^2}{6} \frac{\partial}{\partial t} \nabla^2 \bar{\underset{\sim}{u}} , \tag{38}$$

after suppressing the order parameters α and ε. This is the set of two equations, of the Boussinesq class, for the two unknowns ζ and $\bar{\underset{\sim}{u}}$ for weakly nonlinear three-dimensional long waves traveling in a layer of water of variable depth and with the possibility of the ocean floor being in motion. Equation (37) is exact while the momentum equation (38) has an error term of order $O(\alpha\varepsilon^4, \alpha^2, \varepsilon^2)$.

We should, however, note that $\bar{u}(x,y,t) = \overline{\nabla\phi}$, which is generally different from $\nabla(\bar{\phi})$, is in fact a rotational vector field in the xy-plane wherever $\nabla h \neq 0$, and consequently (38) does not possess in the case of $\nabla h \neq 0$ a first integral as its 'Bernoulli equation' for the two-dimensional layer motion. The apparent vorticity $\nabla \times \bar{u}$ that may arise in the layer-mean-velocity $\bar{u}$ may be attributed to the presence of the last two terms on the right of (38); they represent the frequency diffusion effects in an inhomogeneous medium (due to the varying depth) and act as a source or sink of vorticity through the variation in the vertical acceleration. It is nevertheless possible to define a new velocity associated with the two-dimensional layer flow as:

$$\underset{\sim}{u}' = \nabla\bar{\phi} \quad \text{so that} \quad \nabla \times \underset{\sim}{u}' = 0. \tag{39}$$

The difference between $\bar{\underset{\sim}{u}}$ and $\underset{\sim}{u}'$ can be found by using (29) and (32) as:

$$\bar{\underset{\sim}{u}} - \underset{\sim}{u}' = \alpha\varepsilon^2 \left\{- \frac{1}{2}(h_t + \nabla\cdot(h\ \nabla\phi_o)) + \frac{h}{3}\nabla^2\ \phi_o\right\} \nabla h, \tag{40}$$

Here we note that $\bar{\underset{\sim}{u}} = \underset{\sim}{u}'$ up to $O(\alpha\varepsilon^2)$ when $\nabla h = 0$. We may also recast (34) in terms of $\bar{\phi}$ as:

$$(\bar{\phi})_t + \frac{1}{2}(\nabla\bar{\phi})^2 + \zeta + p_o = (\bar{\phi})_t + \frac{1}{2}(\nabla\bar{\phi})^2 - \hat{\dot{\phi}} - \frac{1}{2}\widehat{(\nabla\phi)^2}$$

$$= \alpha\varepsilon^2 \left\{\frac{h}{2}\frac{\partial}{\partial t}(h_t + \nabla\cdot(h\ \nabla\phi_o)) - \frac{h^2}{6}\frac{\partial}{\partial t}\nabla^2\ \phi_o + O(\varepsilon^2,\alpha)\right\} .$$

Substituting (40) in (37) and writing $\alpha\phi_o \simeq \bar{\phi}$ in the above equations, we obtain:

$$\zeta_t + \nabla\cdot((h+\zeta)\nabla\bar{\phi}) = -h_t + \nabla\cdot\left\{\frac{h}{2}(h_t+\nabla\cdot(h\nabla\bar{\phi})) - \frac{h^2}{3}\nabla^2\bar{\phi}\right\} \nabla h , \tag{41}$$

$$(\bar{\phi})_t + \frac{1}{2}(\nabla\bar{\phi})^2 + \zeta + p_o = \frac{h}{2}\frac{\partial}{\partial t}(h_t + \nabla\cdot(h\,\nabla\bar{\phi})) - \frac{h^2}{6}\frac{\partial}{\partial t}\nabla^2\bar{\phi} \quad . \tag{42}$$

This pair of equations may be regarded as a variation of the original set (37) and (38) within the Boussinesq class since the error terms of (41) and (42) remain in the same order, namely of $O(\alpha\varepsilon^4, \alpha^2\varepsilon^2)$, as that of (38). They, however, appear to be superior to (37) and (38) for calculating three-dimensional long waves since only two scalar unknowns, ζ and $\bar{\phi}$, are involved instead of three in the pair (37) and (38). Even more significantly, ζ can immediately be eliminated by substituting (42) for ζ in (41). After $\bar{\phi}$ is so determined, we can deduce ζ from (42) and $\bar{u}$ from (40).

Various long-wave models can be obtained directly from (41) and (42). If the nonlinear terms in (37) and (38) are neglected, we have:

(1) Linear dispersive long-wave model

$$\zeta_t + \nabla\cdot(h\,\nabla\bar{\phi}) = -h_t + \nabla\cdot\left\{\frac{h}{2}(h_t + \nabla\cdot(h\,\nabla\bar{\phi})) - \frac{h^2}{3}\nabla^2\bar{\phi}\right\}\nabla h \ , \tag{43}$$

$$(\bar{\phi})_t + \zeta + p_o = \frac{h}{2}\frac{\partial}{\partial t}(h_t + \nabla\cdot(h\,\nabla\bar{\phi})) - \frac{h^2}{6}\frac{\partial}{\partial t}\nabla^2\bar{\phi} \ . \tag{44}$$

If, on the other hand, the dispersion effects are neglected, we have

(2) Nonlinear nondispersive long-wave model

$$\zeta_t + \nabla\cdot((h+\zeta)\,\nabla\bar{\phi}) = -h_t \tag{45}$$

$$\bar{\phi}_t + \frac{1}{2}(\nabla\bar{\phi})^2 + \zeta = -p_o \ . \tag{46}$$

This system is the generalized Airy wave model. When both the nonlinear and dispersive effects are neglected, we have the simplest case:

(3) Linear Nondispersive Long-Wave Model

$$\zeta_t + \nabla\cdot(h\nabla\bar{\phi}) = -h_t \tag{47}$$

$$\bar{\phi}_t + \zeta = -p_o \ . \tag{48}$$

At this stage, our primary interest is of course in establishing the criteria for choosing the optimum long wave model under a specific condition. In the case of two-dimensional motion in water of uniform depth, it seems that the general criteria can be sought from the value of the local Ursell number (or its extended definition by Hammack 1972), as supported by our accumulated experience. Crudely speaking, the linear theory is sufficiently accurate if $UR \ll 1$; whether or not the dispersion effect is negligible depends on how small is the value of $\varepsilon = h/\lambda$. To the other extreme, the nonlinear nondispersive theory is appropriate for $UR \gg 1$. When Ur is of O(1), the full Boussinesq equation has no comparable substitute in its overall performance. In a more refined study by Hammack and Segur (1978), it has been further pointed out that the choice should also depend on the initial excess mass and an "initial Ursell number" based on the amplitude and length of the initial wave.

However, several recent case-studies have indicated that the general criteria may further be complicated by such factors as reflection and transmission due to substantial changes in the overall water depth, refraction and diffraction processes arising in two-dimensional propagation, as well as self-focusing and regional resonance in three-dimensional motion. Some of these aspects will be commented in the sequel.

(4) Behavior near the Front of a Longwave Train

For small values of the amplitude-to-depth ratio ($\varepsilon = a/h \ll 1$), as generally is always the case for tsunamis propagating in the open ocean, linear theory is valid provided the Ursell number remains small. In the case when the water is of uniform depth (h = const), it is well known (see Jeffreys and Jeffreys 1956) that the asymptotic behavior near the wave front of the one-dimensional motion is:

$$\zeta = \frac{m}{2\rho(\gamma t)^{1/3}} \, Ai\left\{\frac{x - ct}{(\gamma t)^{1/3}}\right\} \qquad (c = \sqrt{gh}, \quad \gamma = \tfrac{1}{2}ch^2) \;, \tag{49}$$

m being the excess mass. The wave thus decays exponentially ahead of x = ct and becomes oscillatory behind. When observed in the wave frame the amplitude decays like $t^{-1/3}$ while the wave length increases by $t^{1/3}$. The modification of both the amplitude and wavelength are thus rather gradual and slowly varying. At a fixed time, the wavelength continually decreases towards the rear and finally merge with that of a uniform wave train. This means that at a fixed station, the waves will arrive with ever-increasing frequency, as observed by Van Dorn (1961).

If the initial ocean floor disturbance is dipole like (such as with an anti-symmetric tilting, or an uprising adjacent to a subsidence of the ocean floor), with zero net excess mass in the resulting wave, the large-time asymptotic behavior becomes:

$$\zeta \simeq \frac{\ell^2}{2h}\left(\frac{1}{\gamma t}\right)^{2/3} \; Ai'\left\{\frac{x - ct}{(\gamma t)^{1/3}}\right\} \;, \tag{50}$$

where Ai'(z) = dAi(z)/dz. Thus by linear dispersive theory, the waves originated from a dipole source motion decay at a faster rate, $t^{-2/3}$, than do the waves from a monopole generation. The dispersive features in the two cases are, however, very much alike since for long waves they depend only on the properties of the medium.

Another factor that can affect wave attenuation is the number of dimensions in the wave propagation. For the diverging cylindrical waves, originally concentrated at $r = r_o$, the asymptotic behavior for large time is:

$$\zeta \simeq \frac{m}{4(r_o r)^{1/2}} \frac{1}{(\gamma t)^{1/3}} \left\{Ai\left(\frac{r - r_o - ct}{(\gamma t)^{1/3}}\right) + Gi\left(\frac{r_o + r - ct}{(\gamma t)^{1/3}}\right)\right\} \;, \tag{51}$$

where $$Gi(z) = \frac{1}{\pi} \int_0^\infty \sin(zt + \tfrac{1}{3}t^3)dt \tag{52}$$

and m is the excess mass. The behavior of Gi(z) is quite similar to the Airy function Ai(z). With the radial spreading, the amplitude now decays like $t^{-1/3}$ for fixed r and like $r^{-1/2}$ for fixed t. Hence near the wave front (the leading waves at $r = r_o + ct$ and $r = ct-r_o$), the waves decay like $t^{-5/6}$, which is a rate considerably faster than in the case of one-dimensional propagation. This asymptotic behavior also holds for converging cylindrical waves at large radial distances ($r \gg h$) if $(r-r_o-ct)$ is replaced by (r_o-r-ct) in the argument of Ai and the function Gi disregarded; the accurate asymptotic behavior of ζ near the focus (r h) is, however, very complex, as will be discussed later.

The foregoing few examples can serve as reference cases for making comparisons with the other long-wave models. A simpler view of comparison is to note that the dispersive waves (49) and (50) satisfy the equation:

$$\zeta_t + c\zeta_x + \frac{1}{3}\gamma\zeta_{xxx} = 0 \tag{53}$$

which corresponds to linear dispersive right-going long waves. The corresponding equation of the Boussinesq class is the Korteweg-deVries equation:

$$\zeta_t + c(1 + \frac{3}{2}\frac{\zeta}{h})\zeta_x + \frac{1}{3}\gamma\zeta_{xxx} = 0 \ . \tag{54}$$

For the one-dimensional motion (49), the magnitude of the nonlinear term, $\zeta\zeta_x$ in (54), relative to the dispersion term is:

$$O(c\frac{\zeta}{h}\zeta_x)/O(\gamma\zeta_{xxx}) = O(\frac{\alpha}{\varepsilon^2}(\varepsilon^2 t^*)^{1/3}) \tag{55}$$

in which t* is the dimensionless time and the excess mass m has been taken to be:

$$m = O(a\lambda) \ . \tag{56}$$

This estimate shows that the neglected nonlinear terms cause a cumulative error that can become appreciable (relative to the dispersion term) for:

$$t^* > \varepsilon^4/\alpha^3 = \frac{1}{U_r^{\,3}\,\varepsilon^2} \tag{57}$$

Thus, for the waves to amplify with the Ursell number increasing to order of unity, the time required is about $t^* = \varepsilon^{-2}$, or in physical dimensions:

$$t = t_c = \lambda/c\varepsilon^2 \ . \tag{58}$$

For the typical values of the tsunami parameters given in Table I, t_c is at least after 5-10 hours of travel in the open ocean before the nonlinear effects may become appreciable. By similar estimation, it can be seen that for diverging cylindrical waves, the nonlinear effects always diminish with increasing time due to the radial decay factor $r^{-1/2}$ in (52). However, the situation is drastically different for converging cyclindrical waves, as will be discussed later.

The prediction of one-dimensional long waves in water of uniform depth by linear nondispersive theory is simply:

$$\zeta = f(x-ct) \quad \text{for} \quad \zeta(x,o) = f(x), \quad \zeta_t(x,o) = -cf'(x) \tag{59}$$

as given initial conditions. For the same initial conditions, the solution based on nonlinear nondispersive theory (see (54) with the term ζ_{xxx} neglected) is:

$$\zeta(x,t) = f(x-V(\zeta)t), \qquad V(\zeta) = c(1 + \frac{3}{2}\frac{\zeta}{h}) \tag{60a}$$

which determines (x,t) implicitly, or, if written in terms the characteristic variable ξ:

$$\zeta(x,t) = f(\xi), \qquad x = \xi + V(f(\xi))t \ . \tag{60b}$$

This solution is continuous and one-valued in x either when $f'(x)>0$ for all x or otherwise for:

$$t < -(\frac{3}{2}\frac{c}{h}f'(\xi))^{-1} = t_b \text{ say.} \tag{60c}$$

The time limit t_b of 'wave breaking' for typical tsunamis progressing in the ocean of uniform depth is also very long; based on Table I figures, $t_b \approx 25$ hours.

The primary test on the validity of the nondispersive models is thought to lie in their accuracy in predicting (1) whether the total number of waves emitted from the source region will increase appreciably over the long distance of travel, (2) whether the wavelength will increase with time, and (3) whether the phase (the location of wave crests) is amplitude-independent (see (60a)). Conceivably, definitive answers to these questions are not easy to obtain in view of the exceedingly small α for any possible field observation and the limited length of wave propagation for practical laboratory tests. Before we proceed to seek other means for assessing these and additional aspects of tsunami phenomena, it should be stressed that the nondispersive models cannot provide any of the salient features of dispersing waves as given by (49)-(52).

Converging Cylindrical Long Waves

As a simple case to study the effects of number of dimensions on the validity of long-wave models, we consider the motion of converging cylindrical long waves for some specific depth variations. The nonlinear dispersive model of the Boussinesq class is in this case given by the equations ($\partial h/\partial t = 0$ in the present case):

$$\zeta_t + \frac{1}{r}\frac{\partial}{\partial r}[(h+\zeta)r\phi_r] = \frac{1}{r}\frac{\partial}{\partial r}\left\{[\frac{h^2}{6}\frac{\partial}{\partial r}(r\phi_r) + \frac{h}{2}r\phi_r h_r]h_r\right\} \tag{61}$$

$$\phi_t + \frac{1}{2}\phi_r^2 + \zeta = \frac{h^2}{3}\frac{1}{r}\frac{\partial}{\partial r}(r\phi_{rt}) + \frac{h}{2}h_r\phi_{rt} . \tag{62}$$

This model has been adopted by Chwang and Wu (1976) to calculate converging cylindrical long waves for the cases of self-focusing in the wave of uniform depth, climbing up a conical island and transmission and reflection over a submerged cylindrical mount. The boundary condition at $r = 0$ is $\bar{u} = 0$.

The first example is the self-focusing and reflection in water of uniform depth ($h = 1$) of the initial wave:

$$\zeta = \alpha \operatorname{sech}^2 \frac{\sqrt{3\alpha}}{2} (r+ct-r_o) \qquad \text{at } t = 0 \ , \tag{63}$$

which is shown in Figure 1 for $\alpha = 0.1$ and $r_o = 30$ together with the initial velocity. The time sequence of wave evolution, as plotted from the numerical result in Figure 2, shows that before the crest reaches the center $r = 0$, the wave profile remains essentially similar, with the maximum amplitude increasing like $r^{-1/2}$. After the wave converges at the center ($r = 0$), where the wave height reaches its maximum, the wave is reflected to go outward, leaving a negative trailing wave which then evolves into a train of oscillatory waves. In Figure 3, the theoretical results are compared with the experimental wave elevation measured by four wave gauges located at $r = 1.6, 7, 15$, and 30. A good part of the small discrepancy between theory and experiment can be attributed to the viscous effects which are neglected in the theory.

A simple example to show that a solitary-like cylindrical wave, after self-focusing and being reflected in water of uniform depth, will maintain its solitary feature is given by considering a particular case of the Levi-Civita solution.

$$\phi = \int_0^\infty f(f \pm \frac{r}{c} \cosh u) du, \qquad c = (gh)^{1/2} \ , \tag{64}$$

which satisfies the linear nondispersive wave equation in the cylindrical coordinates with axisymmetry provided that $f(z)$ vanishes sufficiently fast as $z \to \pm \infty$ such that the integrals for ϕ and its first two derivatives exist. A simple function with this required behavior is:

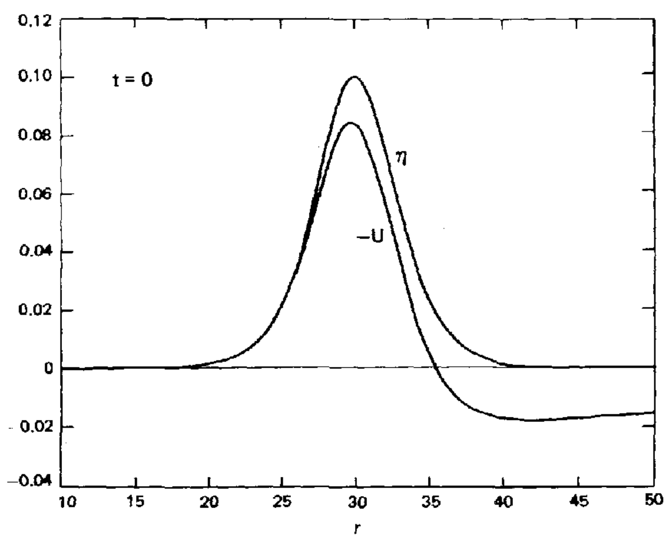

FIGURE 1
THE INITIAL CYLINDRICAL SOLITARY WAVE PROFILE AND THE CORRESPONDING MEAN RADIAL VELOCITY WITH a = 0.1 AND r_o = 30.

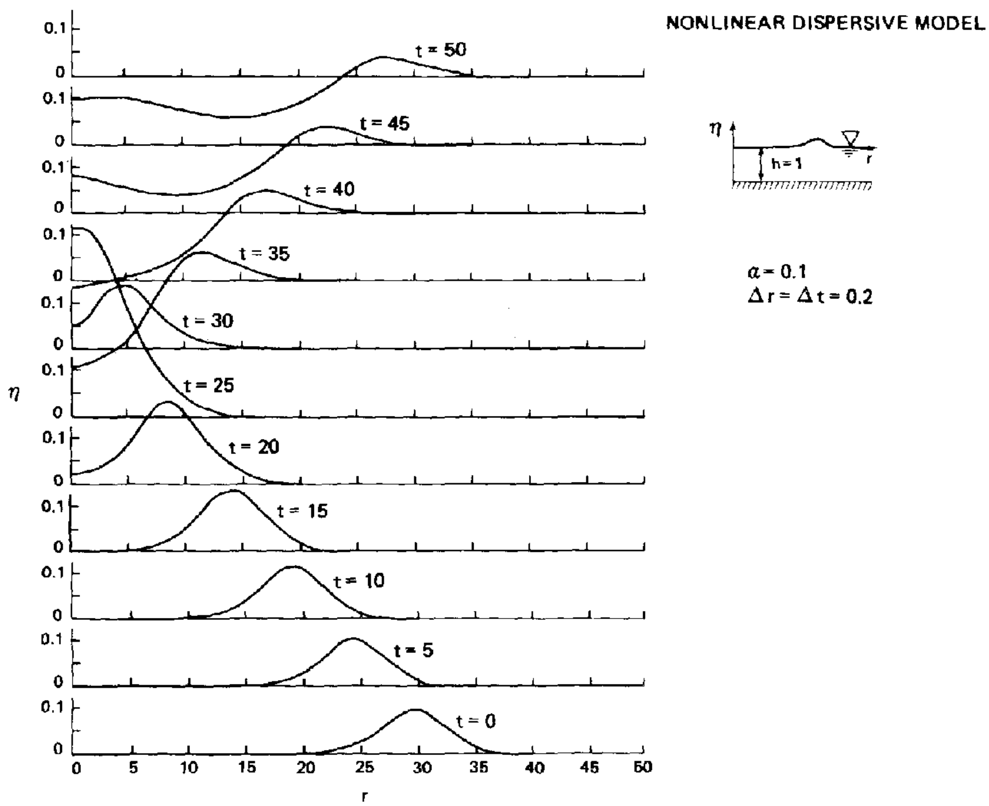

FIGURE 2 PROPAGATION OF A CYLINDRICAL SOLITARY WAVE OVER A REGION OF CONSTANT WATER DEPTH WITH $\alpha = 0.1$, $r_o = 30$ AND $\Delta t = \Delta r = 0.2$.

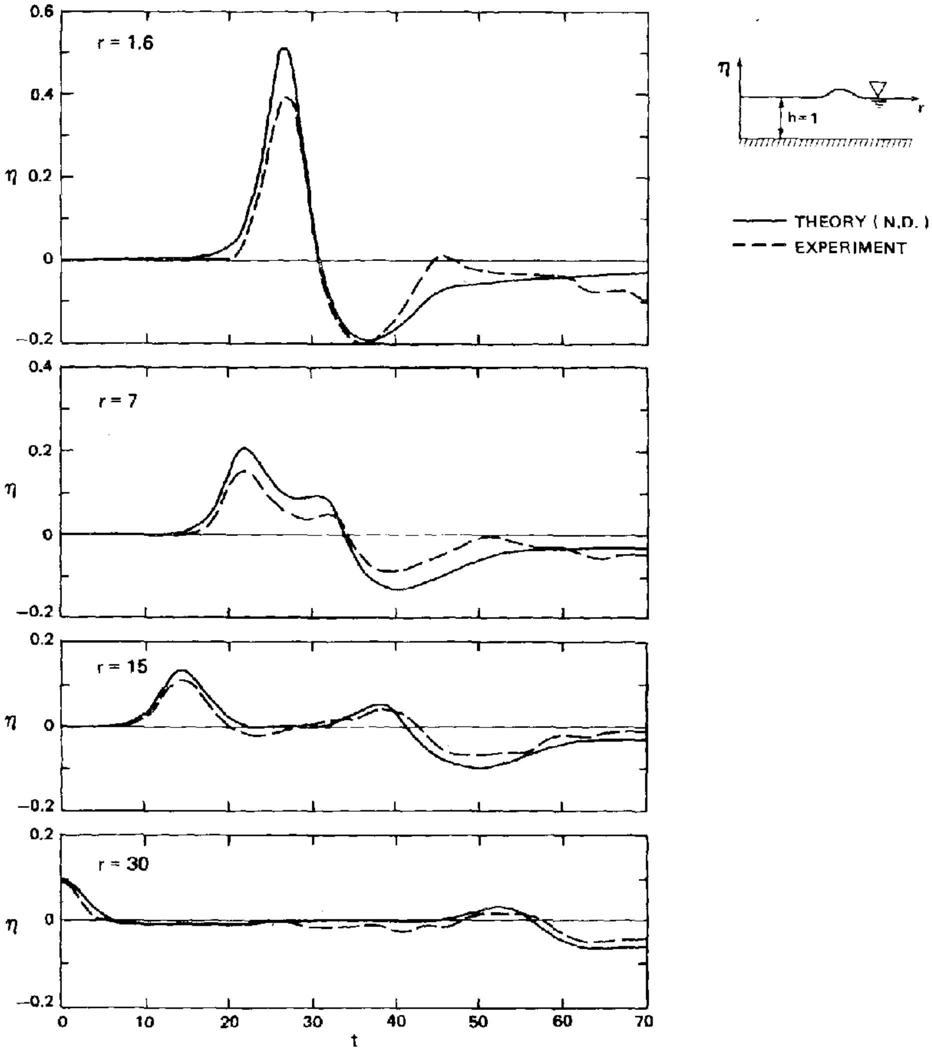

FIGURE 3 COMPARISON OF THEORETICAL RESULTS WITH EXPERIMENTS AT DIFFERENT LOCATIONS

$$f = A\ \mathrm{Im}\left\{\frac{1}{t - \frac{r}{c}\,\mathrm{chu} + i\tau} + \frac{1}{t + \frac{r}{c}\,\mathrm{chu} + i\tau}\right\} \quad . \tag{65}$$

where A, τ are real constants. This f represents an impulse which is even in r and behaves like $2A\tau/(t^2 + \tau^2)$ at $r = 0$ for $-\infty < t < \infty$, τ being a measure of its half breadth. The corresponding water surface elevation:

$$\xi = -\frac{1}{g}\frac{\partial\phi}{\partial t}\ , \tag{66}$$

is shown in Figure 4 over a sequence of time $-10 < t < 10$ (normalized with $h = 1$, $c = 1$ and $A = 1/2\pi$). The converging wave started with a solitary peak, preceded by a negative shallow valley (which is an intrinsic property of the simple form of the f chosen, and which can probably be eliminated by adding to (65) some higher moments of the form (65)). Both the crest and the valley magnify while traveling toward the center and later recover nearly the original profile after having traversed a distance of 10 depths of the water away from the center. It may be noted that the eventual recovery of wave form depends on the property that the medium is homogeneous i.e., in water of uniform depth.

In general, evolution of long waves propagating in water of variable depth is predicted by all the different long-wave models. A comparative study has been carried out for the reflection of a converging cylindrical solitary wave by a submerged cylindrical sea mount of radius 10h and height $\frac{1}{2}h$ rising from an otherwise flat ocean bottom. Figure 5 exhibits comparison between the non-linear dispersive theory (Equations 61 and 62) and experiment, in which the numerical results were obtained by the same method described in Chwang and Wu (1976). The theory is thus found to be quite satisfactory in predicting the important features of wave propagation and reflection; much of the small discrepancies can again be ascribed to the viscous effects. In contrast, the numerical results of linear dispersive and linear nondispersive long wave models both become quite inaccurate for the (nondimensional) time $t > 20$, as shown in Figures 6-8 in which the wave form evolved from the same initial

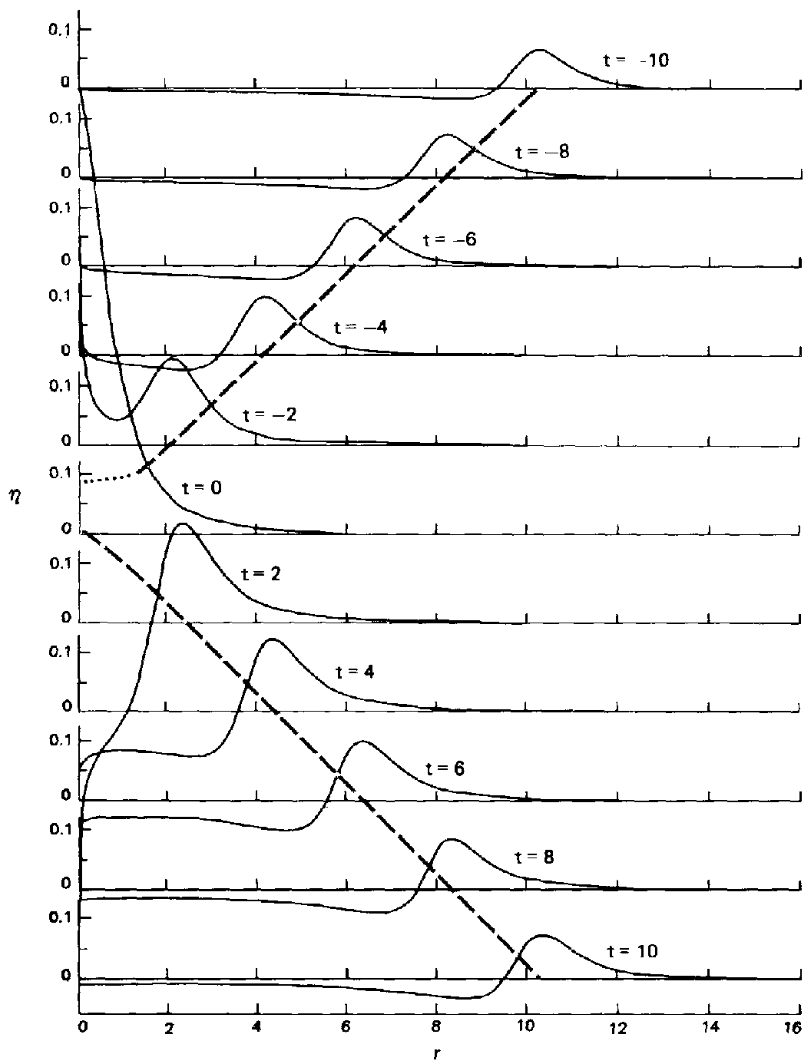

FIGURE 4 WAVE SURFACE ELEVATION OVER $-10 < t < 10$

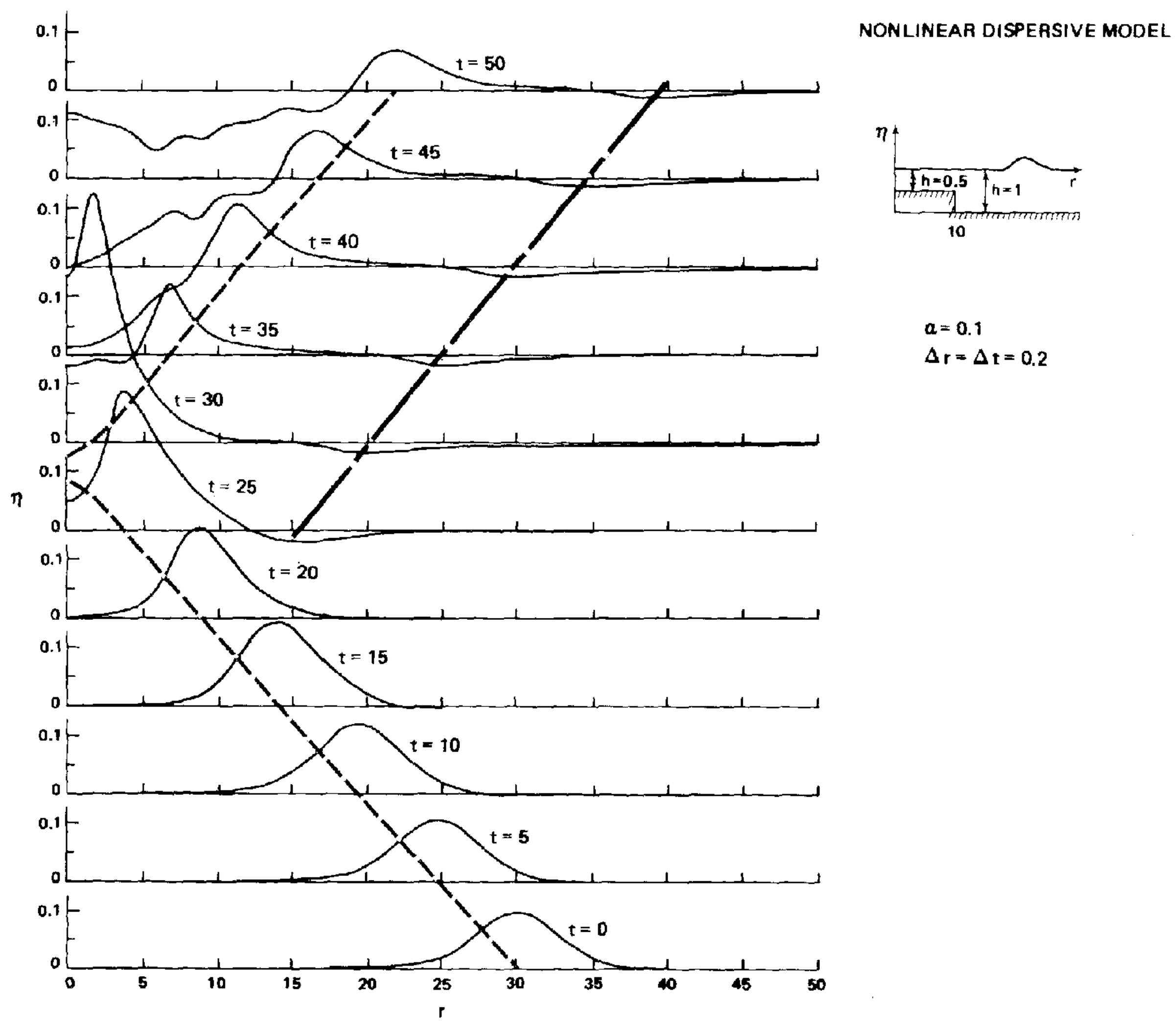

FIGURE 5 COMPARISON OF NONLINEAR DISPERSIVE MODEL WITH EXPERIMENTS

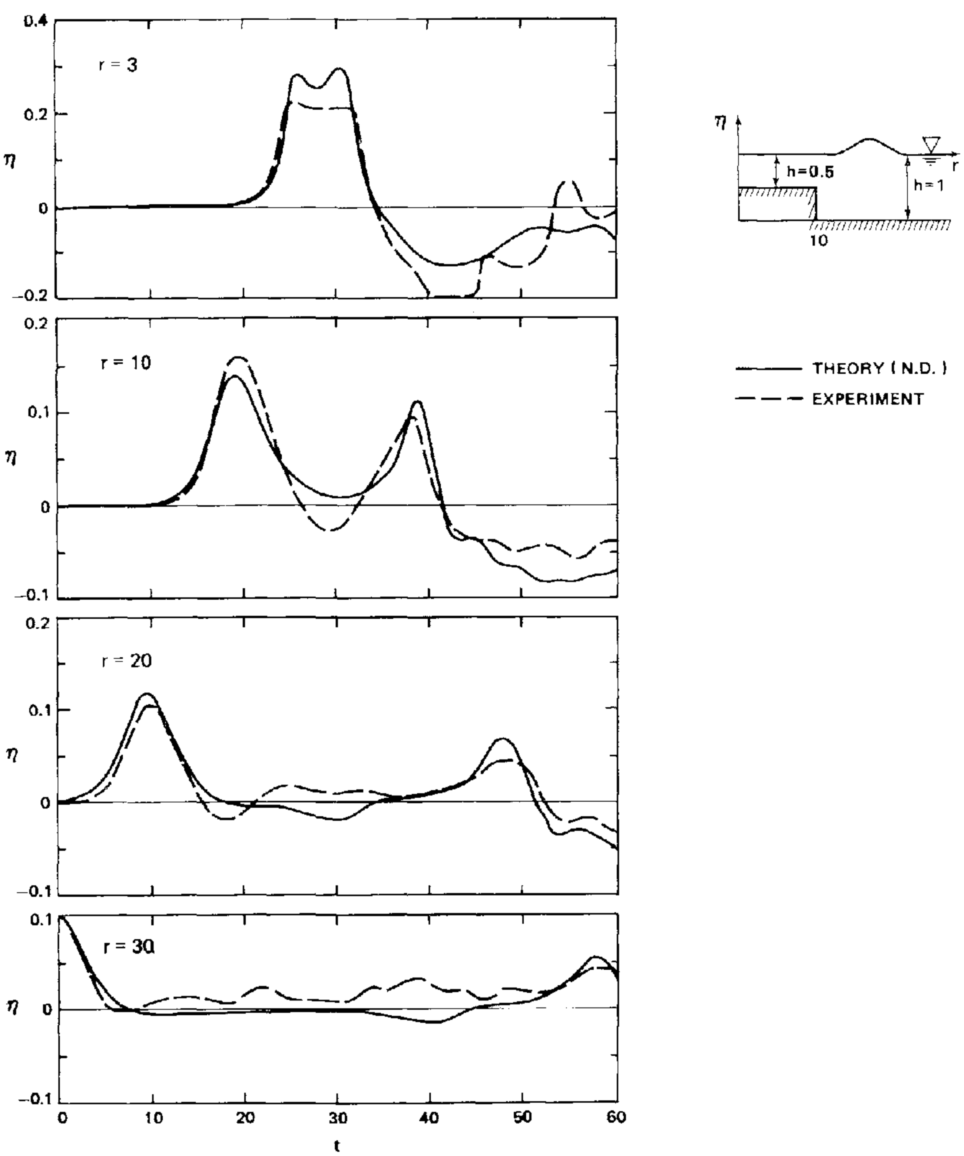

FIGURE 6 COMPARISON OF NONLINEAR DISPERSIVE THEORY WITH EXPERIMENTS AT DIFFERENT LOCATIONS

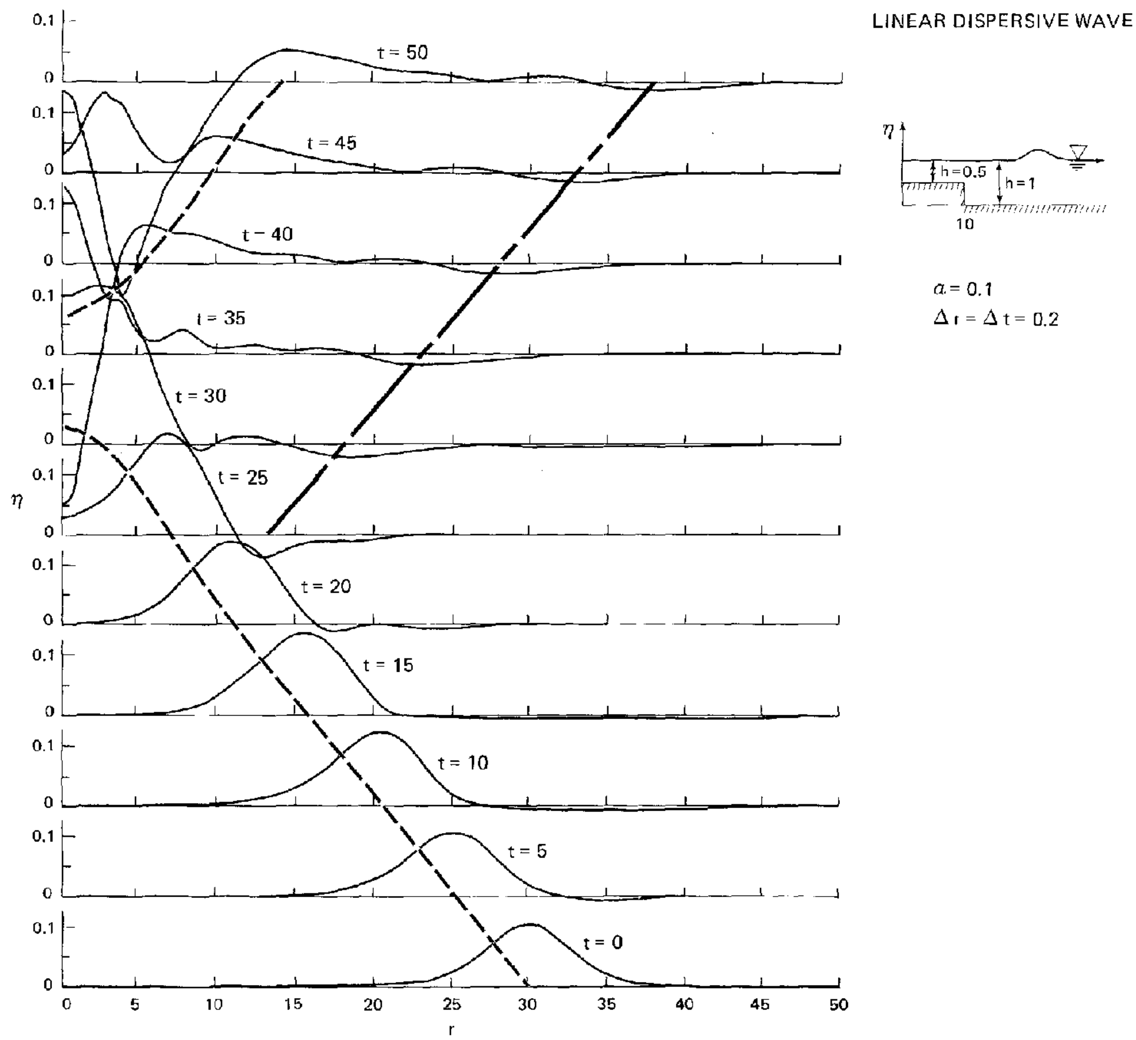

FIGURE 7 COMPARISON OF LINEAR DISPERSIVE WAVE WITH EXPERIMENTS AT DIFFERENT LOCATIONS

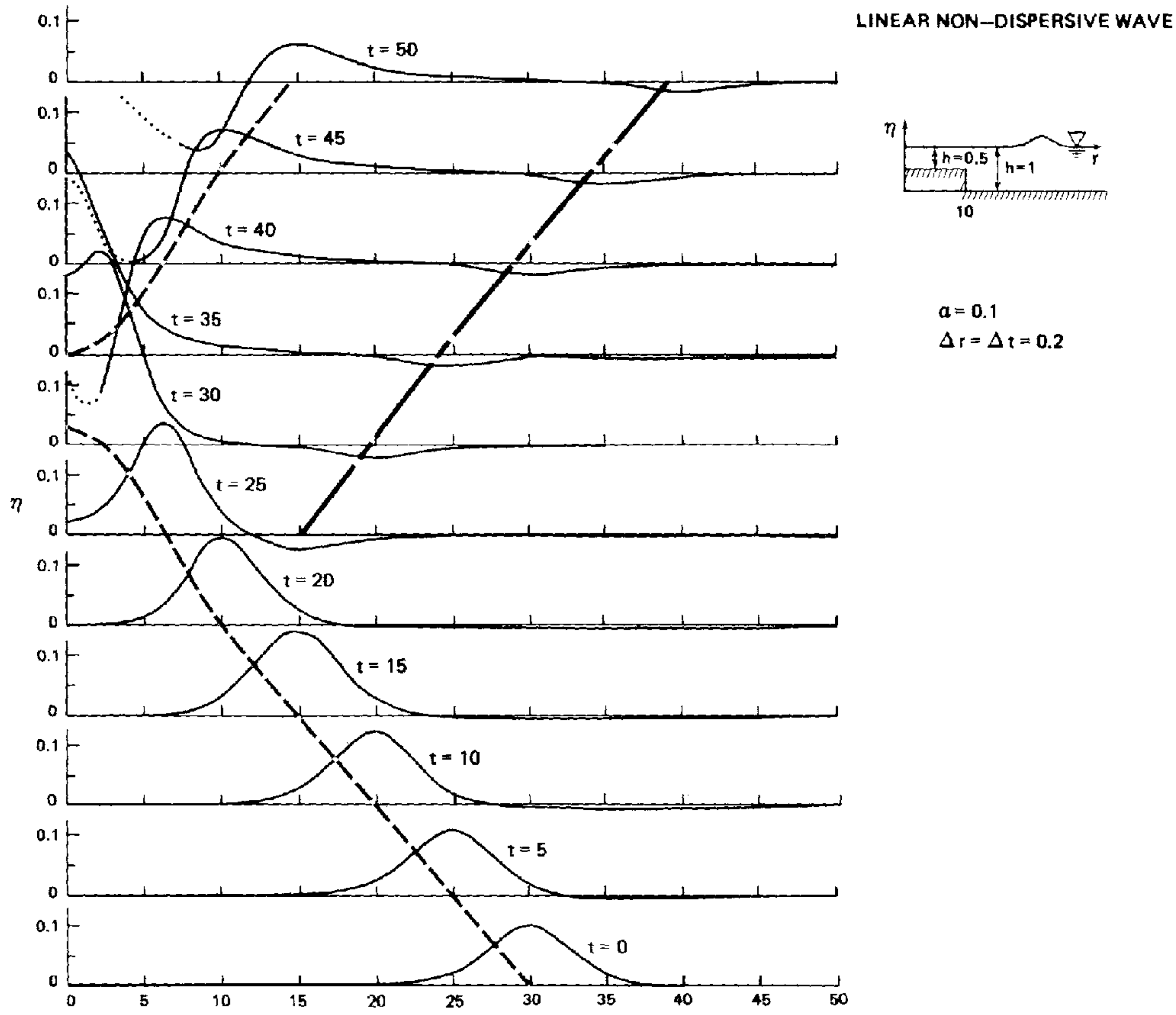

FIGURE 8 COMPARISON OF LINEAR NON–DISPERSIVE WAVE WITH EXPERIMENTS AT DIFFERENT TIMES

cylindrical solitary wave is plotted over a time sequence for the three theories. The two linear theories under-estimate the phase velocity, which is expected, and overpredict the phase jump between the inward and outward propagation. They are also grossly inaccurate in predicting the undular trailing wave train following the diverging wave front after reflection. However, all three theories predicted a small negative leading wave which precedes the reflected main crest; this leading wave is evidently the first wave reflected by the vertical face of the sea mount. As a further remark, we state that the numerical results (not shown) based on the equation of Korteweg-deVries type (that can be readily derived from (61) and (62) for one directional propagation only, either converging or diverging) appear to be the poorest of all these models, the reason being clearly due to the limitation that the incoming wave and the reflected outgoing wave cannot occur simultaneously in this model.

Reflection and Transmission of Long Waves

In classical literature, reflection and transmission of long waves have been evaluated on linear nondispersive theory (cf. Lamb, Art. 176, 185) for propagation over either abrupt or very gradual slopes. For the two-dimensional motion over a submerged step, the reflection and transmission coefficients are:

$$C_R = \frac{1-\beta}{1+\beta} \qquad C_T = \frac{2}{1+\beta}, \qquad \beta = \frac{b_2}{b_1}\left(\frac{h_2}{h_1}\right)^{1/2}, \tag{67}$$

where b_1, b_2 are the breadths at the surface, and h_1, h_2 are the mean depths of water across the step, towards which the incident wave (of arbitrary shape) propagates on the side designed by b_1 and h_1. The energy is conserved in this process, since $C_R^2 + \beta C_T^2 = 1$.

On the other hand, for a gradual slope (i.e. a slope over which the depth and breadth are slowly varying functions relative to the scale of wave length), consideration of either momentum or energy yields for the wave amplitude the relation:

$$\zeta \propto b^{-1/2} h^{-1/4} \tag{68a}$$

which is Green's law. Hence, after traversing a finite gentle slope terminated with breadths b_1, b_2 and depths h_1, h_2, the wave experiences, when arriving at the end designated by b_2 and h_2, the transmission coefficient:

$$C_T = \beta^{-1/2} \tag{68b}$$

Rayleigh pointed out that if the slope dimension is but a moderate multiple of a wavelength there is practically no reflection. On this basis, the case of abrupt step (either up or down in the direction of the primary wave) may be called 'strong reflection' and the case of gradual slope 'weak reflection.' From the energy consideration it is therefore obvious that the transmission coefficient for the weak reflection case is greater, for the same value of β, than that for the strong reflection case. In fact, we have:

$$C_{T(WR)} - C_{T(SR)} = \beta^{-1/2} - \frac{2}{1+\beta} = \frac{(1-\beta^{1/2})^2}{\beta^{1/2}(1+\beta)} > 0 \ . \tag{69}$$

Between these two extreme cases, the general problem of reflection and transmission of waves over a slope of arbitrary inclination has been investigated by many authors. With special interest for tsunami applications, this subject has been reviewed and exemplified by Kajiura (1961) on the basis of linear nondispersive theory. In another direction, the method first developed by Carrier and Greenspan (1958) based on the nonlinear nondispersive model has been applied by Tuck and Hwang (1972), Kajiura (1976) and others to investigate long wave motions on a sloping beach. Recently, a joint experimental and theoretical study has been carried out by Goring (1978) with detailed comparison between the linear nondispersive theory, nonlinear dispersive theory and his experimental measurements. Before we give a brief summary, it can first be said that since the above classical results of the coefficients are based on linear nondispersive theory, they also imply that both the transmitted and reflected waves undergo no change in phase from that of the incident wave (aside from the jump to an opposite phase for the reflected wave when $\beta > 1$). It can also be said that as the reflection coefficient in the weak reflection regime (for slopes of moderate to small inclination over a multiple of a wavelength) is small, $C_R = \varepsilon$ say, then by energy consideration, $C_R^2 + \beta C_T^2 = 1$, or $\beta C_T^2 = (1 - \varepsilon^2)$ so C_T has an error term one order higher than C_R.

In the regime of strong reflection, Goring observed in laboratory experiment that both the reflected and transmitted waves undergo appreciable evolution in wave form near the step. Such evolutions, though cannot be predicted by linear nondispersive theory, can be expected from a basic observation by Hammack and Segur (1974) with the following conclusion. If the initial wave has a net positive excess mass, at least one solitary wave will emerge followed by a train of oscillatory waves. If the excess mass is nonpositive, appearance of solitary waves depends on the form of the initial wave. If the initial wave is entirely negative, no solitary waves will emerge.

Only a few analytic solutions are available to illustrate the above principle of Hammack and Segur. One of these, obtained by Whitham (1974, p. 597) using the inverse scattering method, is for the evolution of the initial wave:

$$\eta(x,o) = A \operatorname{sech}^2 kx \tag{70}$$

where A is the wave amplitude and k the 'wave number.' This will be a permanent wave propagating with velocity $c = [g(h + A/2)]^{1/2}$ through water of uniform depth h if:

$$kh = (3A/4h)^{1/2}, \quad \text{or} \quad k = (3A/4h^3)^{1/2} \equiv k_o \text{ say} \tag{71}$$

When k does not satisfy this relation, the number of solitary waves eventually emerging as $t \to \infty$ is given by:

$$N = \text{largest integer} < \frac{1}{2}\left\{(1 + 8\ Ur)^{1/2} + 1\right\} \equiv P\ , \tag{72}$$

where $$Ur = \eta_{max}^3/h^3 \; |\frac{3}{2}\,\eta_x|_{max}^2 = (k_o/k)^2. \tag{73}$$

This definition of the Ursell number is after Hammack (1972) with only a modification of the constant factor. The height of the emerging solitary waves is given by Whitham (1974) as:

$$A_n = \frac{A}{4}(P - 2n)^2 \qquad (n = 0,1,\ldots N\text{-}1). \tag{74}$$

Thus, the initial solitary wave will 'fission' to become at least two solitary waves when $Ur > 1$. The solitary wave fission phenomena as predicted by the inverse scattering theory have been confirmed experimentally, and are of interest to tsunami applications. As long tsunami waves climb up a series of continental and near coast slopes, their local Ursell number may increase from very small values in the open ocean (of order 10^{-2} or less) to order unity and greater, hence it is quite possible for each wave to split into a number of waves. If such fission should occur, the frequency and amplitude data obtained from tide gauges could not be used directly to trace back the tsunami characteristics in the open ocean, let alone to the source region.

In the same respect, as the 'new born waves' emanating from a near coast source region enter the deep ocean, the transmitted waves will have their amplitude reduced, thereby evolving, according to Hammack and Segur's principle, into a leading wave followed by a train of oscillatory waves. This argument is well supported by the experimental results of Goring (1978), who also found that linear dispersive theory is required (whereas the nondispersive theories will fail) to predict the conspicuous dispersive behavior of the waves resulting from a solitary wave off the shelf into deep water.

We have thus seen two interesting aspects of tsunami propagation in which the dispersion effects play a significant role in predicting (from specific source motion) or back-tracking (from coastal data on arriving waves) the tsunami behavior in the open ocean. When these transient evolutions of the waves during transmission are appropriately accounted for, the classical formulas for the reflection and transmission coefficients are found to be valid. Nevertheless, the point of importance is that the same theoretical basis (i.e,, on linear nondispersive model) may not be valid in predicting the frequency and phase through the processes of reflection and transmission which would in turn affect predictions of the tsunami behavior during the long journey across the ocean.

Acknowledgements

I would like to thank Professor Fred Raichlen for stimulating discussions and Dr. George Yates and Helene Schember for the valuable assistance they have so kindly provided.

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DISCUSSION

Dr. Carrier commented that for tsunamis in the deep sea, the ratio of wave amplitude to wave length is so small that the propagation in the deep ocean is essentialy linear and, therefore, soliton or other nonlinear features do not enter into the discussion. Can one suggest that as the waves propagate across the shelf the nonlinear effect can become increasingly so important that solitons might develop in the shelf region? If we look at the horizontal scale of the shelf region, it may be too short to allow the wave to be sorted out and to allow the emergence of solitons.

Dr. Wu replied that if we refer to Table 1, we can indeed conclude that for generation and oceanic propagation the process is linear and they should be treated by linear dispersive theory. However, if we look at the coastline tsunami record, we cannot help being astonished by the tremendous large amplitudes that have been recorded (some as high as 33 meters). This certainly cannot be explained solely by refractive effects due to decreasing water depth (with amplitude $\sim h^{-1/4}$). Therefore, the three-dimensional effects such as the focusing of horizontal plane form in addition to depth changes may indeed play an important role.

As for the duration of wave propagation in the shelf region, Dr. Wu agreed that it would be somewhat short if the waves are perpendicular to the shelf and the coastline. However, if the waves are propagating obliquely in climbing over the shelf, the ray path in the shelf region can be considerably longer than the distance between the shelf and the coastline.

This mode of propagation can conceivably lend sufficient time for the nonlinear effects to grow and give rise to nonlinear waves.

Dr. Miles commented that the long wave equation can also be derived by using his Hamiltonian averaging method. He questioned if all source motions should be of a dipole type since a consideration of the conservation of mass including both the earth solid mass and the overlaying ocean water would suggest this to be the case. It was pointed out by several participants that this would be true if the uplift and subsidence of the ocean bottom occurs within an oceanic region but is not necessarily the general truth such as in the 1964 Alaska earthquake in which case the dipole axis was parallel to and very near the coastline and the subsidence occurred on the landside.

Dr. Miles also commented that if the initial displacement is of the dipole type (a positive displacement balanced precisely by a negative displacement), then it can be calculated that there will always emerge a solitary wave.

Dr. Carrier reported his two preliminary studies on any possible topographical effects on tsunamic propagation.

(1) Is there any systematic feature in the ocean maps (for example, from Aleutian Island to Hawaii) that could allow the transmission line to act as a wave guide and to significantly modify the waves in a particular region? His conclusion is negative.

(2) Does the bottom irregularity in the ocean interfere (or significantly affect) the propagating waves? Two preliminary studies were done on this aspect: one is to look at the bottom irregularity to vary like a periodic or almost periodic fashion. His conclusion was that there existed an exceedingly narrow frequency bandwidth through which the wave profile could be changed. Therefore, waves are propagating essentially unaffected. Another one is to treat the bottom variation as randomly distributed. There the reflection or backscattering is uniformly distributed across the frequency range and that effect is also small compared with other uncertainties. In this latter point, Dr. Miles also reached a similar conclusion in his study about three years ago.

Another participant further pointed out that if the direction of wave propagation is along the trench, a significant effect on the propagating wave could be expected. Further investigation may be warranted.

Dr. Hammack presented the following comments on the proper modeling equations for tsunami propagation.

For one-dimensional wave propagation, we know that in deep oceans, the ratio of water depth to wave length (h/L) is very small compared with unity ($h/L << 1$) and that the ratio of wave amplitude to water depth is also very small ($a/h << 1$). The controlling factors as to which equation should be used are:

(1) A parameter representing the size of the domain of propagation;

(2) An initial-volume parameter related to excess mass is defined as:

$$V = \frac{3}{2} \frac{|a| \cdot L}{h^2}$$

where $|a|$ is the amplitude of the disturbance (or initial wave amplitude) L is the initial wave length, h is the water depth. This is a parameter mostly responsible for the determination of the linearity of the problem.

(3) Ursell's parameter

$$U_r = \frac{3}{2} \cdot \frac{|a| L^2}{h^3} = \frac{L}{h} V .$$

This parameter gives a measure of the nonlinear effects compared with the dispersive affects. For one-dimensional wave propagation, three different possibilities exist:

(1) Linear and non-dispersive wave propagation
(2) Linear and dispersive wave propagation
(3) Non-linear and dispersive wave propagation.

The following guidelines would be useful in determining the criteria of the appropriate modeling equations:

(1) For the case of $U_r << 1$, $\forall << 1$, the wave propagates according to linear non-dispersive theory for some time until it reaches a time, t_1. Beyond the time t_1, the wave propagation should be governed by linear and dispersive wave theory. This theory should then be valid until a later time, t_2, is reached. After the time t_2, the wave should be modeled by nonlinear dispersive theory. The two time parameters, t_1 and t_2, are given as follows:

$$t_1 = 2 \sqrt{\frac{h}{g}} \left(\frac{U_r}{\forall}\right)^2 = 2 \sqrt{\frac{h}{g}} \left(\frac{L}{h}\right)^2$$

$$t_2 = 2 \sqrt{\frac{h}{g}} \left(\frac{U_r}{\forall}\right)^3 = 2 \sqrt{\frac{h}{g}} \left(\frac{L}{h}\right)^3$$

Thus, for the case of

$$0 < t \leq t_1 ,$$

we can use linear, non-dispersive theory and when

$$t_1 \leq t \leq t_2 ,$$

we have to switch to linear and dispersive theory; and finally, for $t > t_2$, non-linear and dispersive theory should be used.

(2) For the case of $\forall << 1$, and $U_r \sim 1$, we should not use linear and dispersive wave theory, but should start out using linear non-dispersive theory until the time reaches t_1. Beyond t_1, we should use non-linear, dispersive wave theory. The time parameter t_1 is obtained as follows:

$$t_1 = 2\sqrt{\frac{h}{g}}\frac{h}{|a|}$$

where h is the water depth and $|a|$ is the absolute value of the wave amplitude.

Dr. Hammack further provided examples concerning the values of these physical parameters.

Case 1:

A typical representation is the 1964 Alaskan earthquake: $a \sim 1$ ft., $h \sim 10^4$ ft., $L \sim 10^6$ ft. Under these conditions, U_r is of order unity and V of order 10^{-2}. Then we start out using linear non-dispersive theory until time reaches approximately 100 hours. Beyond that time, we are supposed to use the non-linear, dispersive wave theory. However, to reach 100 hours of wave propagation, it would require a distance of propagation as gigantic as 40,000 kilometers! Obviously, under these conditions, the 1964 Alaskan earthquake can be treated almost entirely by linear, non-dispersive wave theory.

Case 2:

Consider another possible extreme case: $a \approx 10$ ft., $h \approx 1.5 \times 10^4$ ft., $L = 2 \times 10^5$ ft. Then, the excess volume parameter V will be of order 10^{-2} and the Ursell's parameter, $U_r \sim 10^{-1}$.

Then the time parameters proper to linear, non-dispersive wave theory would be $t_1 = 1.2$ hours. This corresponds to approximately a propagation distance of 600 miles. Beyond this time (or propagation distance), we should switch to linear, dispersive wave theory until $t_2 = 12$ hours. Thus, roughly in propagation distance of 600 miles $< x <$ 6000 miles, the linear- and dispersive wave theory should be used.

Of course, we should be reminded that all of these discussions only apply to one-dimensional wave propagation. For two-dimensional propagation, the modeling criteria must be modified accordingly. At the present time, we do not have a well-defined parameter for these more realistic and more complicated cases.

Dr. Wiegel commented that there are many types of tsunami waves, some are quite dispersive, others are not. We should not just concentrate on the tsunami generated by the Alaskan earthquake as a basis to draw a general conclusion that tsunamis are not dispersive.

4 COASTAL TRANSFORMATIONS AND TERMINAL EFFECTS

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3. ISLAND RESPONSE TO TSUNAMIS
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4. BAY AND HARBOR RESPONSES TO TSUNAMIS
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COASTAL TRANSFORMATIONS

BY
G. CARRIER

For purposes of discussion consider a tsunami which has propagated over long distances from its source region across a deep or moderately deep ocean of relatively uniform depth. (Tsunami interactions with land masses near the source region are specifically excluded.) This wave transmission process can be described by a linear dispersive theory or, perhaps, a linear nondispersive theory if the waves are extremely long. As these waves encounter an island or continental land mass where the water depth rapidly decreases, energy is crowded into less and less water. At some stage, the increasing intensity of the waves will necessitate a nonlinear description of motion. Further, dispersion effects can be neglected here since, in general, transformation occurs too rapidly for dispersion to become significant. Hence, we should adopt a nonlinear, shallow-water theory to describe tsunami transformation during impingement on coastal regions.

In order to more clearly delineate the transformation process, first consider the simplistic case of plane waves normally incident on a region of uniformly decreasing depth. The nonlinear mathematics becomes tractable for this case and exhibits the remarkable and fortuitous property that one can define new independent variables, analogous to distance and time in the nonlinear problem, for which the nonlinear description becomes linear (see Carrier and Greenspan, J. Fluid Mech., 1958, 4:97-109). In addition, the new linear problem is precisely equivalent to a linear nondispersive wave model if the artificial independent variables are interpreted as distance and time! Hence, we can solve the linear problem for wave transformation and extract the nonlinear description (for this one-dimensional case) simply by interpreting the linear solution properly, i.e., according to the relation between actual distance and time and their analogous form needed to linearize the nonlinear description. When this strategy is followed, one finds that the resulting solutions are not a strong function of the sea floor slope. On the basis of results for this simple case, one would not expect the response of different

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land masses to differ appreciably. Hence, the great variability of response observed between coastal sites to tsunami forcing must result from lateral properties of the land mass geometry.

Even though the above analysis cannot be formally generalized to three-dimensional geometries, it does suggest the possibility that linear solutions may still be applicable to these nonlinear phenomena if they are interpreted appropriately. This conjecture is further supported by the fact that the contributions to the results by nonlinear terms for the preceding case are significant only in the very last shallow portion of water. Lautenbacher (J. Fluid Mech., 1970, 41:655-672) exploited these ideas to infer the effects of the lateral large-scale topography on tsunami refraction around islands. For islands with elliptical depth contours rising over a distance L from a uniform depth ocean and monochromatic incident waves of length λ, Lautenbacher found a very strong variation in wave amplification around the island when $\lambda/L = O(1)$. The existence of such profound variation for a simple elliptical geometry leaves one with little surprise at the variation observed for the complicated geometrics of actual islands.

Finally, it is conjectured that the excitation of small-scale responses by local geometries such as run-up in stream beds and harbor oscillations is highly nonlinear. In particular, the ringing of harbors for long time intervals after excitation by a tsunami suggests a strong nonlinear mechanism. Long-term ringing requires the excitation of oscillation modes with small transmission coefficients ("high Q"). These modes cannot be excited in a linear manner without a long-term energy source. Nonlinearity would permit the excitation of high Q modes during a short forcing interval. However, once these modes are excited, their subsequent behavior may be described well by linear models.

Edge Waves

It is well known that gradients in water depth can lead to the trapping of wave energy in localized regions of large fluid domains. Edge waves are one class of trapped wave motions which occur near the shoreline of a uniformly sloping beach. First discovered theoretically by Stokes in 1846, these unusual waves propagate in the alongshore direction with crests pointing in the offshore direction; crest amplitude is maximum at the shoreline and decays exponentially offshore. Both field and laboratory measurements have confirmed the reality of these wave modes. Recent theoretical developments have provided insight into the nonlinear mechanism for exciting edge waves by both obliquely and normally incident waves from deep water. For example, a normally incident and reflected wave of frequency ω will preferentially feed energy into any small background noise in the alongshore direction of frequency 2ω--termed subharmonic generation (see Guza and Davis, J. Geophy. Res., 1974, 79-9:1285-1291 or Whitham and Minzoni, J. Fluid Mech. 1977, 79-2:273-287). Both the growth rate and steady-state amplitudes of the resulting edge wave modes are predictable (theoretically) in terms of incident wave properties and beach slopes. It is not clear in general if the period of tsunami forcing is sufficiently long to excite significant edge wave activity. However, it is important to note that tsunami energy is concentrated at long periods often coincident with potential (subharmonic) edge wave modes. In particular, lateral geometrical features such as headlands can give rise to standing edge wave modes where energy remains trapped in both the offshore and alongshore direction. (In fact, experiments suggest that any type of barrier pointing offshore and penetrating the surf zone may be sufficient to support standing edge waves.) Once tsunami energy is fed into these (linear) edge wave modes by nonlinear coupling, it can remain there for periods much longer than that of the tsunami and act as the forcing for other nearshore responses (e.g., harbor oscillations).

ENGINEERING METHODS: RUN-UP, SURGE ON DRY BED, ENERGY DISSIPATION OF TSUNAMI WAVES

BY
BERNARD LE MEHAUTE

Definition of Engineering Problems - Probabilistic Approaches

This presentation addresses the engineering methods with particular emphasis on coastal problems and wave run-up. The effects of earthquake at sea come in a variety of forms depending upon the location of the engineering structure. One must consider separately underwater systems (pipelines, vehicles), floating structures in deep and shallow water, structures attached to the sea bottom (offshore drilling towers) and, finally, onshore structures.

Undersea vehicles, submarines, underwater pipelines, etc., are deeply affected by earthquakes at the location of the earthquake, even by earthquakes of small magnitude. Indeed, if we assume that ground motion is characterized by a vertical acceleration $\frac{dv}{dt}$ beyond acoustic frequency, the entire water column above it (including the neutrally buoyant submarine) is subjected to the same acceleration. Subsequently, the pressure at any given elevation z is no longer $\rho g z$ but $\rho(g+\frac{dv}{dt})z$ as a result of the inertia of the water column. (Similarly we feel added pressure on our feet when an elevator starts moving upwards.)

Consequently, assuming that $\frac{dv}{dt} = \pm\frac{1}{2}g$ for example, any underwater body at 400 foot depth will be subjected to pressure fluctuations between 200 to 600 feet at earthquake frequency, which must be quite a shaking experience.

A floating structure will follow the free surface movement, but will be immune to these pressure fluctuations. High frequency effects in the acoustic range are also damped by impedance discontinuities of the media. A floating structure located in relatively deep water is almost immune from earthquake as well as tsunami effects.

By opposition, a structure fixed at the bottom subjected to earthquake displacement is very vulnerable: structure-water interaction generates high forces related to ground motion. Cases of soil liquefaction have also been reported. Fixed structures at depths larger than 50 feet, however, will not be vulnerable to tsunami generated at a distance, as induced particle velocities are too small to create significant forces.

Whether due to near field or far field effects, most of the engineering problems are located on the coastline and are results of tsunami run-up.

The effects of tsunami run-up at times result into simple flooding with little dynamic destruction, but at times the force due to fast moving water edges induces large destruction forces. Sometimes, the run down is also a source of concern as cooling water intake dries out.

Since all engineering decisions are measured in terms of cost, the problem consists of assessing risk vs. cost, requiring as necessary input the probability of exceedance of run-up (or run-down). At each time that one of the axes is a probability scale, the error which is done along this axis far overcomes the error which is done in determining deterministically the run-up at a given point from a given earth displacement. This is particularly true in assessing the low probability range of extreme events. Indeed, 1) one is dealing with relatively low sample population, and 2) tsunami events are not ergodic. Successions of seismic correlated tremors cluster under limited time periods on the human time scale.

From an engineering standpoint, it is comfortable to know, for example, that dispersive terms may lead to a small error, but the subject matter is rather academic at this time considering the large margin of error on the probability axis.

It is then attempted to collect all possible historical information at a given location, or nearby, in view of establishing the probability curve (Wiegel, 1965), Figure 1. But as previously mentioned, the risk of exceedance cannot

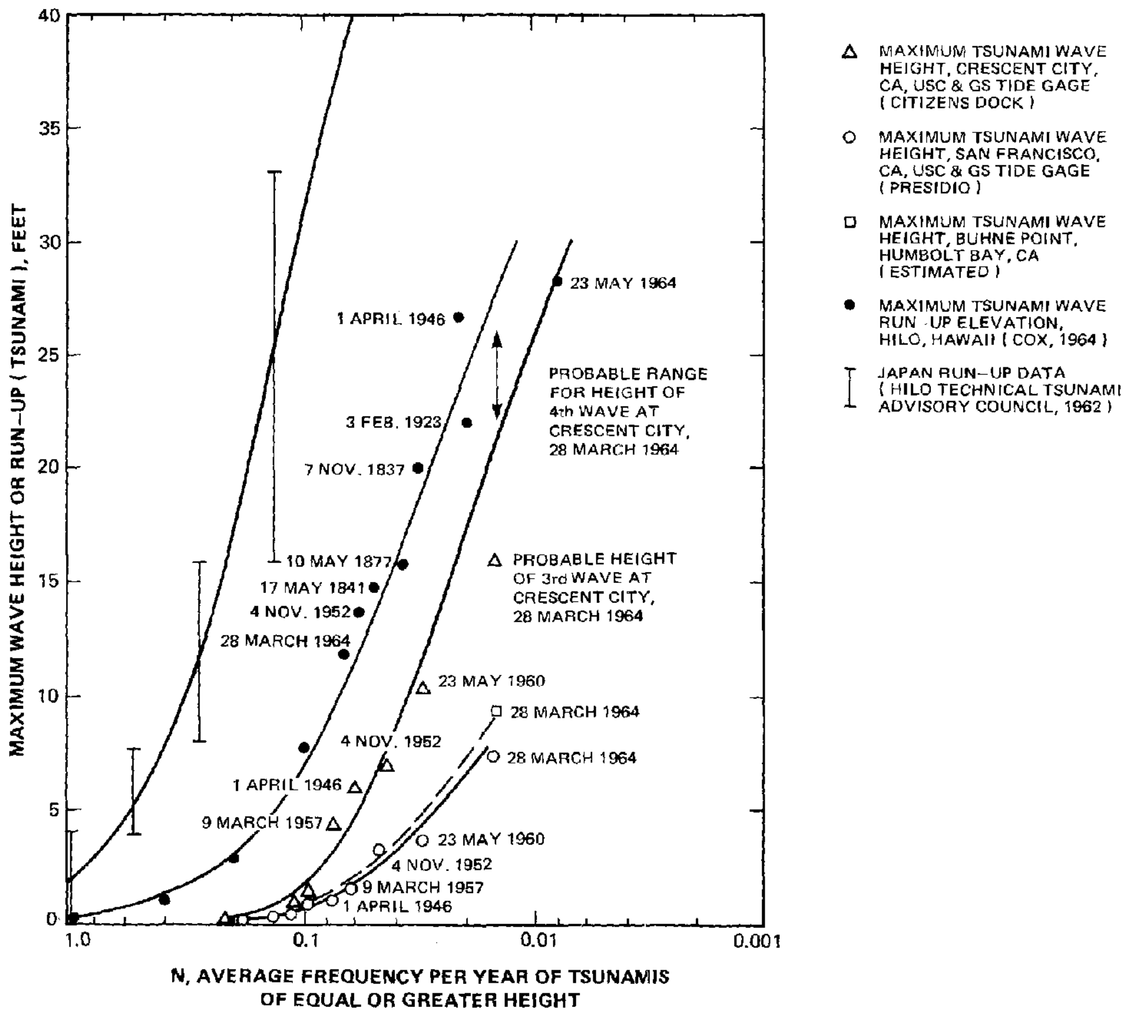

FIGURE 1 DISTRIBUTION FUNCTION FOR MAXIMUM TSUNAMI WAVES (ADAPTED FROM WIEGEL, 1965a)

be determined easily. Another method may consist of considering all possible seismic events around the Pacific and their probability in a given time period. Then to calculate from a number of synthetic ground displacements the run-up at the location under consideration, rank run-up magnitude in descending order and subsequently calculate their frequency of exceedance from the compilation of recurrence frequency of each seismic event. (Somewhat in anology with the method applied to storm surge due to hurricane.)

Transformation of Tsunami Wave Nearshore

In order to carry out the calculation of wave run-up, one must be able to determine the near field or far field wave system, depending upon relative locations. Many of these subject matters have already been covered elsewhere, and we will concentrate our discussion to the very shallow wave propagation and run-up.

It would then appear that if the theoretical treatment of wave generation, deep water and even relatively shallow water propagation obeys a set of well-defined equations under a set of well-determined approximations, it is far from being the case where the wave reaches the shoreline.

In contrast with the deep water wave problem, in shallow water the tsunami problem has all the characteristics to make it mathematically untractable; it is a free surface flow, non periodic, highly nonlinear with vertical acceleration. It is highly dissipative with forced turbulence from the solid boundaries and free turbulence where a tidal bore forms. Finally, all 3-D effects are amplified. These complex phenomena are all superimposed: Mach stem, edge wave, resonance, etc., are all present, and sometimes superimposed on wind waves. Let us concentrate on the simple case of a very long wave (deep water wave steepness in the 10^{-5} to 10^{-6} range) arriving on a gentle slope.

The first question which arises consists of determining whether a bore will form or not. The formula of Keller (1961)

$$H \geq \left(\frac{2\alpha}{\pi}\right)^{1/2} \frac{\alpha^2}{3\pi} LK_s$$

has been verified with some degree of success experimentally (LeMehaute, et al., 1968), Figure 2. H is the wave height, L the wave length on the shallow shelf before α, K_s is the shoaling coefficient on the shelf.

If a bore forms, the nonlinear long wave (NLLW) equation has been used to describe the tidal bore. Actually, the NLLW will tend to predict a bore sooner than actually occurring. It is the long wave paradox. Second, the NLLW equations combined with the bore equation indicate that the bore collapsed at zero depth (Ho and Meyer 1961), Figure 3. The collapsing slope even tends to infinity when the depth tends to zero. It is evident that under such conditions the vertical acceleration term is no longer negligible. Even more, the bore equations are no longer valid when the bore travels over a very thin layer of fluid. The bore is actually transformed into a rarefaction wave which runs up the shore as a surge over a dry bed.

The dissipative effect due to bottom friction is then more important than the dissipative effect due to tidal bore. Even though the wave is no longer a shock wave, in a fluid mechanics sense, it tends to present the physical aspect of a bore, i.e., near vertical wall, high rate of turbulence. This is particularly true when wind waves are superimposed on the tsunami wave.

Since so little attention has been given in the past to the problem of surge on dry bed, it may be worthwhile at this time to present various existing approaches and to propose some new ones, which one may characterize by order of approximation. One will limit it to the one-dimensional case.

Surge on Dry Bed

Surge on dry bed occurs when the tsunami runs up the shore. The word "dry" does not necessarily imply that the soil is effectively dry, but from a fluid mechanics standpoint, the momentum transfer from the surge to the layer of fluid ahead is negligible, which is not the case of tidal bore or moving hydraulic jump as previously mentioned. From a fluid mechanics viewpoint, a surge on a dry bed is not a shock wave but rather a rarefaction wave.

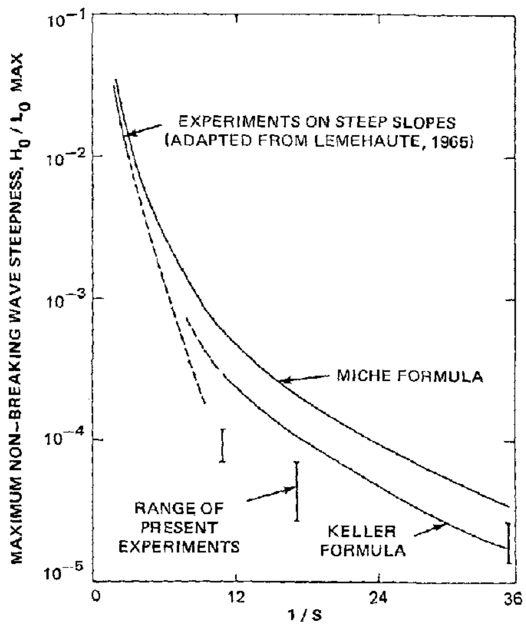

FIGURE 2 A COMPARISON OF BREAKING CRITERIA AND EXPERIMENTAL RESULTS FOR PERIODIC WAVES

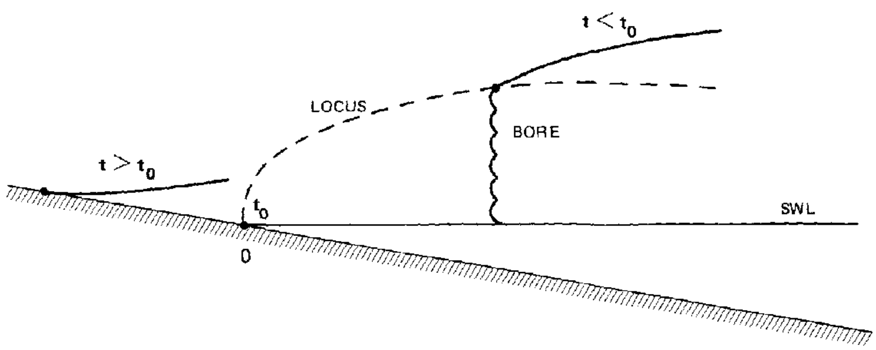

FIGURE 3 BORE COLLAPSE PARADOX

Surge on dry bed also occurs in the dam break problem or when a storm surge rises over a very flat land in the uprush of a breaking wave on a beach.

In general, the subject has been treated as if the motion were frictionless as in the dam break or wave run-up problem (see for example, Stoker 1957, p. 517, or Ho and Meyer, 1961). Consequently, the water edge has a depth which tends to zero as shown on Figure 3.
Experiments by Schoklitsch, as old as 1917, actually showed that due to friction the water edge is rounded, as also shown on Figure 4. According to Keulegan (1949), the maintenance of a constant flow at the rate q at a given section of an originally dry channel will produce a surge front of height y which travels approximately with the velocity

$$U \simeq 1.5\ [g\ q]^{1.3} \simeq \sqrt{gy}$$

One also finds an experimental investigation by R.F. Dressler (1952) expanding some of these results. On the theoretical side, the only investigation known to the writer is by Whitham (1955), which has also investigated the problem of a tidal bore travelling on a small layer of fluid (1958). Freeman and Le Mehaute (1969) simply assumed a linear relationship between the water edge velocity U and $C = \sqrt{g\ y}$, y being the surge front height. An experimental study of forces due to tsunami surge on cylindrical pile has also been done by Wiegel. A mathematical formulation could be incorporated into various computer programs used in the study of tsunami run-up and improve our knowledge of forces in structures due to tsunami waves.

As a first approximation, the treatment of surge over dry bed, in analogy with storm surge, could be treated by finite difference from the NLLW equations. At the water edge, a moving boundary condition is to be defined. If the slope is relatively steep, the slope is replaced by a succession of vertical walls separated by interval Δx (Figure 5). One applies the NLLW equation with these vertical walls as boundary conditions and when the water level exceeds a given contour, one assumes initially that the water level extends horizontally over the next Δx interval. In case of a very gentle slope, or near horizontal

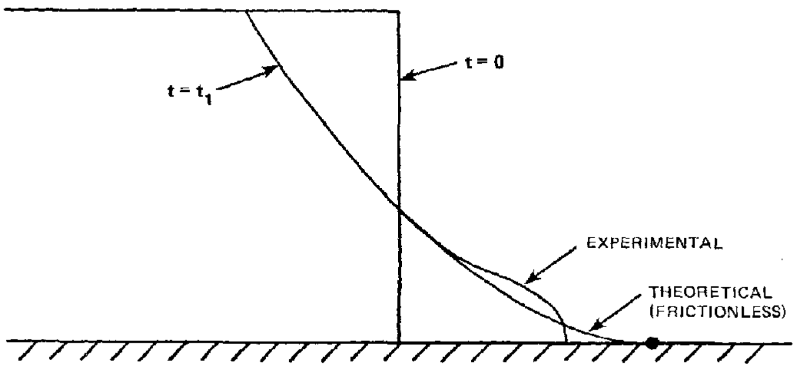

FIGURE 4 THE DAM BREAK PROBLEM

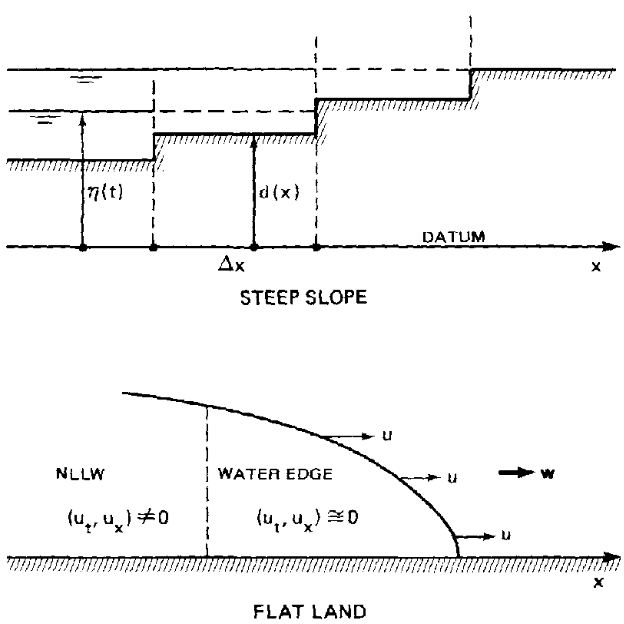

FIGURE 5 CALCULATION PROCEDURES
a) STEEP SLOPE
b) FLAT LAND

bottom, the interval Δx between bottom contours is too long, as the water edge does not travel fast enough for the free surface to be considered horizontal in the interval x. Then it has been assumed that the weir equation applies, i.e., the discharge Q is proportional to (water depth h)$^{3/2}$ in the preceding interval. Instead, the following method is proposed. Near the water edge, the dominating forces are the friction and gravity force. The inertial force is quite small by comparison. Furthermore, the water edge moves "en masse" at uniform velocity throughout, i.e., $u_x \simeq 0$ (Conventional notation, Figure 5).

Subsequently the NLLW equation for a horizontal bed at the water edge reduces to:

$$0 = g\eta_x + \frac{gu|u|}{C_c^2 \eta}$$

$$\eta_t + u\eta_x = 0$$

where $u_t + uu_x$ in the momentum equation and u_x in the continuity equation are neglected. Inserting $\eta_x = -\frac{\eta_t}{u}$ and equating the speed u to the speed of the water edge W, yields:

$$W = \left[C_c^2\right] \eta_t^{\ 1/2}$$

C_c is then expressed in terms of η such as given by the Manning formula. Such expression is easily converted in a finite difference scheme, using η and its derivative at the first interval Δx behind the water edge.

The very same method applies over a very gentle slope. Then one finds (s is the slope, d is the bottom elevation with respect to a horizontal datum)

$$\eta_t + (d+\eta)_x \, u = 0$$

$$-g \, (d+\eta)_x \pm gs - \frac{g \, u|u|}{C_c^2 (d+\eta)} = 0$$

which also gives W = u from the cubic equation:

$$\frac{\eta_t}{u} \pm gs - \frac{g|u|u}{C_c^2 (d+\eta)} = 0$$

This new boundary condition could then be applied to the NLLW equation (including inertia term) for investigating the flow behind the water edge.

This approach is particularly suitable to investigate the penetration of flooding over relatively flat land. The problem of impact forces due to tsunami waves on structures requires further analysis. For this, a much refined investigation on the water edge needs to be done. The NLLW are no longer valid. The vertical acceleration terms and boundary layer effects are too important to be neglected. The following approach is then proposed.

Hydrodynamics of Water Edge

The theoretical approach can be done with either a fixed coordinate system, as in the case of a real surge on a dry bed, or in a moving coordinate system moving at the speed of the surge by a Galilean transformation. An analogous experiment would reproduce similar conditions by making the surge stationary on a moving belt (Figure 6).
Since one can translate the results from one to the other by a single vectorial addition of the velocity vectors, it is more convenient to develop the theory in the case of the fixed surge with respect to a moving coordinate system similar to the conditions of the described experiment. It is then

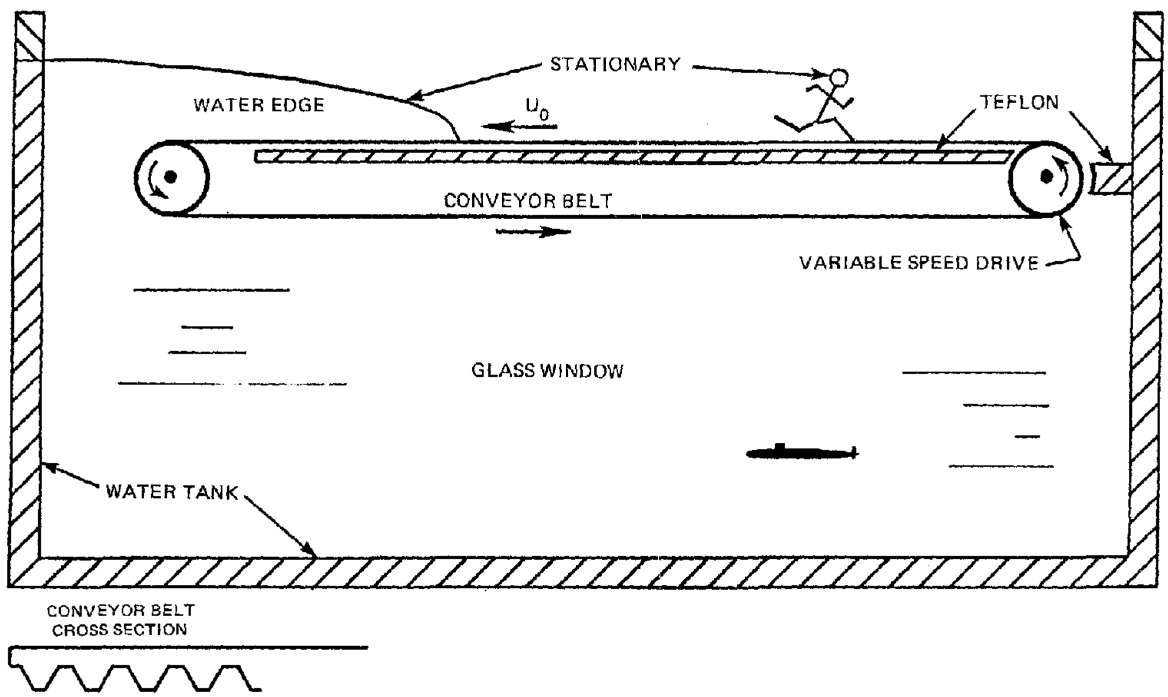

FIGURE 6 EXPERIMENTAL INSTALLATION

sufficient to add the velocity of the coordinate system to the measured or theoretical velocity flow field to describe the case of a movable surge over a dry bed. It is recalled that the Galilean transformation permits us to transform an unsteady motion into a steady one and is possible only in case of steady wave profile. It will be assumed that the water edge satisfies this condition, so that one can cancel all local inertia terms.

Even though the case of a viscous surge is of great academic interest to solve, it will only be considered as incidental, the case of a turbulent flow being more conformed with the real cases. Accordingly, the flow patterns as shown in Figure 6, can then be intuited and the average fluid flow could be divided into three domains:

(1) The lower domain (1), (See Figure 7) where the flow is deeply influenced by the friction on the moving belt. The flow is rotational and the problem is similar to a boundary layer problem.

(2) The upper domain (2) where the flow could be considered as irrotational with a free surface.

(3) A separation zone (3) defined by $y = y_s$ where the fluid from the domain (2) is entrained into the domain (1) and which could actually reach the free surface.

In the case where the separating zone (3) is considered as a line (1st approximation), one has a singular point (4) at the extreme water edge defined by circle of infinitely small radius ε as shown in Figure 8.

It is recalled that in domain (1), the boundary layer assumptions are:

$$U \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} << u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y}$$

and the pressure is hydrostatic in the boundary layer. Also $\mu\Delta^2 v$ is small compared to $\mu\Delta^2 u$.

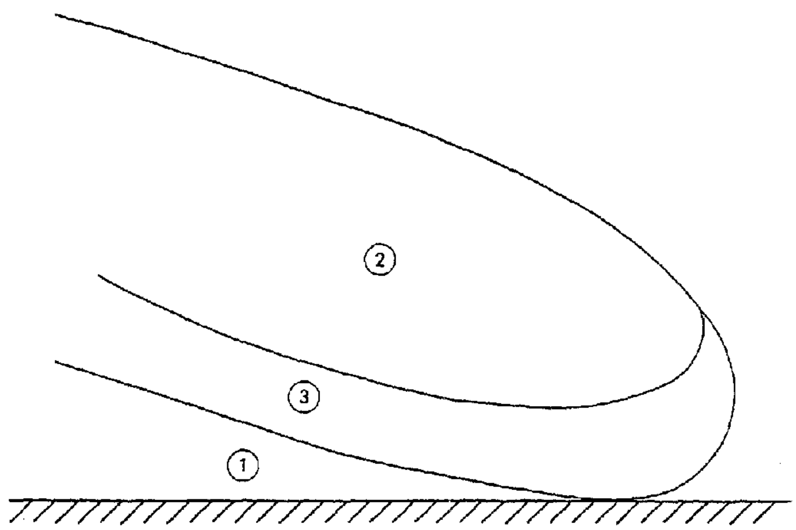

FIGURE 7 DEFINITION OF VARIOUS DOMAINS AND ASSUMPTIONS

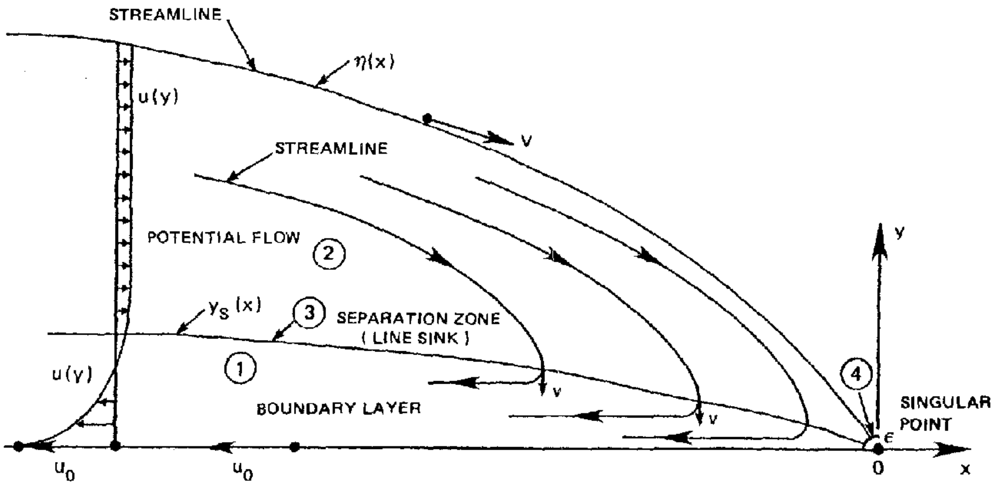

FIGURE 8 1ST ORDER APPROXIMATION

However, in contrast with the case of a boundary layer on a flat plate, the pressure also varies with the free surface elevation $\eta(x)$ i.e., p is a function of x.

The principle of similarity of velocity profile still applies, i.e.,

$$\frac{u}{u_o} = f\left(\frac{y}{y_s}\right)$$

and most of the concepts and principles which have been developed to investigate the flow over a flat plate also apply, as function of the Reynolds number $\frac{u_o|x|}{\nu}$.

In particular, if one accepts the Prandtl mixing length theory

$$\tau_o = \rho \ell^2 \left|\frac{du}{dy}\right| \frac{du}{dy}$$

and $\ell = Kx$, (K is the Karman constant) the universal logarithmic velocity profile is obtained.

When the function $u(x,y)$ is determined, it is then possible to obtain the value of $v(x)$ at $y = y_s$.

Indeed for continuity one has:

$$\int_o^{y_s} u \, dy = \int_o^{x} v\,(y_s)\, dx$$

which means that all of the fluid in the boundary layer has to go across the limit $y = y_s$.

Therefore, $v\,(y_s) = \frac{d}{dx} \int_o^{y_s} u(y)\, dx.$

This function yields the line sink distribution which must be applied to the irrotational flow defined by domain (2).

Also for the sake of continuity:

$$\int_{0}^{y_s} |u|dy = \int_{y_s}^{\eta} |u|dy$$

It is interesting to note that in the case of a viscous flow, $\tau_o = \mu\frac{\partial u}{\partial y}$, and a parabolic velocity profile is obtained which extends to the free surface η as shown in Figure 9. In this case the line u = 0 is obtained from the continuity equation above.

In the case of a turbulent flow, one has assumed that the flow is irrotational above the boundary layer. At a distance from the water edge, the curvature and vertical acceleration is small, the pressure is hydrostatic and the velocity u (y) is uniform.

The free surface is then such that the momentum flux

$\int_o^{\eta} (p + \rho u^2)\, dy$ is equal to the external force $\int_o^{x} \tau_o\, dx$ applied at

y = 0, τ_o is the wall shear stress. Since the pressure is hydrostatic:

$$\tau_o = \frac{d}{dx} \int_o^{\eta} (\rho g \eta + \rho u^2)\, dy$$

Therefore, knowing u(y) in the boundary layer, both η and u in the domain (2) could be determined.

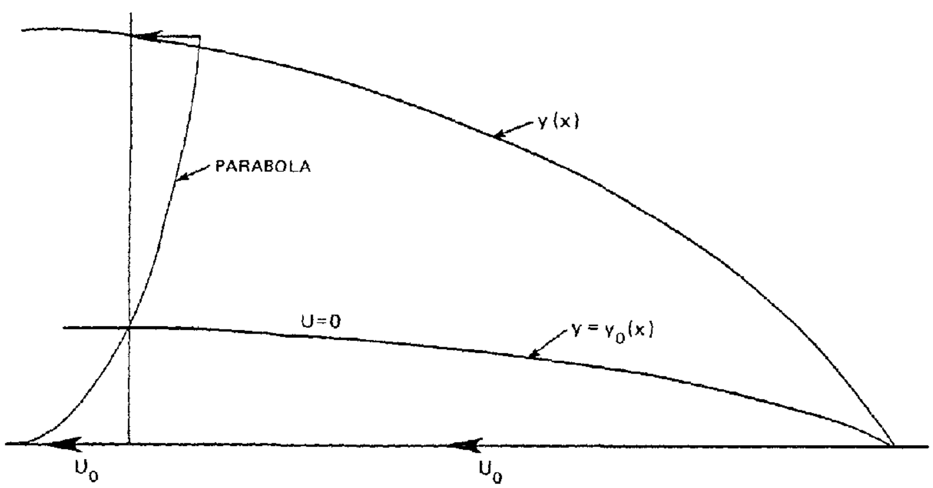

FIGURE 9 VISCOUS FLOW SOLUTION

Near the water edge, the pressure is no longer hydrostatic, as vertical accelerations become more and more important.

The free surface is a streamline which is continued by the moving boundary through the singular point defining the water edge (1st approximation) or a sharply curved overhanging free surface (2nd approximation).

The pressure gradient which makes the free surface overhang is due to the viscous shear effect taking place in zone (3) near the water edge. At the water edge, the free surface is vertical, at some point, since the particle velocity changes direction. Along the moving boundary $\frac{\partial u}{\partial x} = 0$ (since $u = u_o$ = const). Therefore, for sake of continuity $\frac{\partial v}{\partial y} = 0$ and $v \neq V = 0$.

Applying the Bernoulli equation to the streamline at the free surface and continued by the moving boundary one has:

$$\frac{V^2}{2g} + g\eta(x) = \frac{u_o{}^2}{2g} + \frac{p(x)}{\rho} + F(x)$$

V is the particle speed, at any point and in particular at the free surface boundary, u_o is the velocity of the moving boundary and F(x) is a friction term.

At the water edge (x = 0)

$\eta = 0$

$p = 0$

$F(x) \simeq 0$

Therefore $V\ (x = 0) = u_o$

and subsequently, along the free surface

$$V = u_o - \sqrt{2g\eta}$$

The velocity at the water edge is u_o, but suddenly changes direction as a result of the friction force τ_o. In our first approximation procedure, the acceleration due to this sudden change is infinity and applies during an infinitely small amount of time over an infinitely small amount of fluid within a quadrant of radius ε. The problem is very much analogous to the slamming of a free surface flow with a rigid structure, which, assuming incompressible fluid and no elasticity, yields similar results.

The irrotational gravitational flow is now entirely defined by its boundary conditions. It could be represented by a flow net. The effect of the boundary layer being simply represented by a line sink in analogy with the theory of water jet. The strength of the line sink has been determined from past experiments which go back to G.I. Taylor (1958).

Therefore the method which applies to free surface steady flow governed by gravity can be used including the graphical method (flow net) and numerical methods as shown in Figure 10.

A formal solution for the domain (2) from the Eulerian equation can also be considered.

It is interesting to note that at any point in the domain (2) $\frac{v^2}{2g} + \frac{p}{\rho} = z$ is equal to the Bernoulli constant $\frac{U_o^2}{2g}$.

Conclusion

This short survey of engineering problems related to tsunami run-up indicates that one needs to better understand the hydrodynamics of tsunami wave reaching very shallow depth, and surging over a dry land after or without bore formation. One also needs to understand better the hydrodynamics of the water

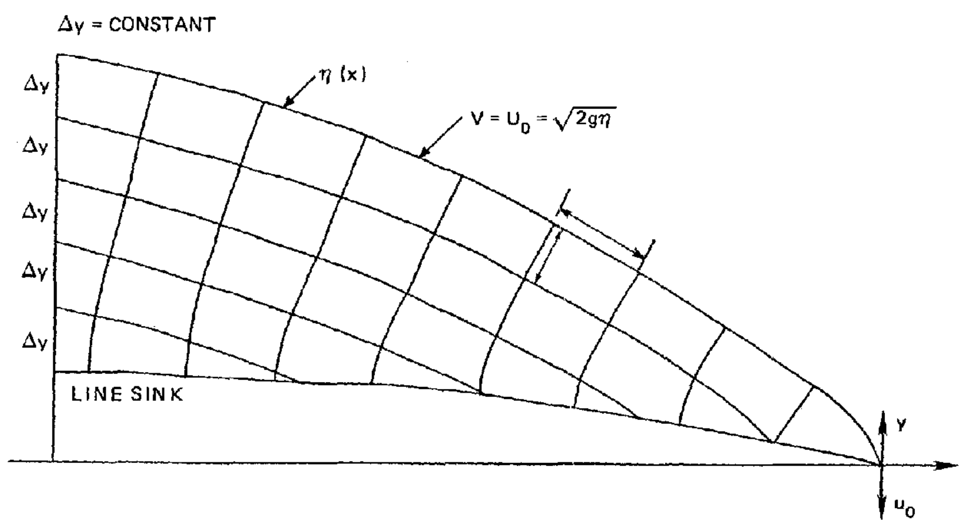

FIGURE 10 IRROTATIONAL FLOW SOLUTION

edge in order to be able to evaluate the impact force of tsunami wave on structures. Two possible approaches have been proposed for that purpose.

Despite all progress - past and future - it would appear that, due to the extreme complexity of shallow water effects, scale model investigations still remain an invaluable tool when engineers have to deal with the problem of tsunami run-up over complex 3-D bathymetry and topography.

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ISLAND RESPONSE TO TSUNAMIS

BY
ROBERT O. REID

Since the leading waves in a tsunami event (at least far from the source) are generally of a length large compared to oceanic depths, it follows that profound changes of amplitude can be produced at an island by the entire submerged structure of the island or island system through the usual mechanisms of refraction, contraction of wave length, diffraction in the lee of the island and scattering of wave energy. Perhaps equally or even more important is the possibility of incident tsunami waves exciting trapped "hedge" waves or quasiresonant modes (Longuet-Higgins, 1967). The necessary condition for the existence for such wave modes for a round island of bathymetry h(r) is that a region $d(r^{-2}h(r))/dr > 0$ exists. Longuet-Higgins' analytical study for round islands and a recent extension of this work by Lozano and Meyer (1976) indicate that such quasiresonant modes are characterized by extremely small energy leakage rates (radiation), which implies that the response to a sustained forcing at the resonant frequency can attain extremely large magnitude. Put in another way, the frequency response (transfer function) for sustained periodic forcing can contain many resonant peaks which are of extremely narrow frequency band width. This raises a very serious question as to the adequacy of any numerical approach in estimating the true response of a given island system to incident waves of given frequency spectrum. Aside from the analytical approach, which is feasible only for very regular geometry (e.g., axially symmetric), two different numerical approaches have been employed in past work.

One approach is to solve for the response to a sustained input of fixed frequency employing a numerical algorithm of the linearized long wave equations with variable depth. By repeating this for many frequencies, an estimate of the response versus frequency is obtained. Another approach is to estimate the response (by similar numerical methods) to a statistically-stationary input having a broad band spectrum, the transfer function being determined from the ratio of the spectrum of the response at a shore point to that of the input. Examples of the first approach include studies by Vastano and Reid (1967, 1970), Lautenbacher (1970), and Houston (1978); examples of the second approach include studies by Knowles and Reid (1970), and Bernard and Vastano (1977). The study of Knowles and Reid points up the difficulties in getting a good rendition of the transfer function for a case which has trapped "hedge" waves. The studies by Vastano and Reid (1967) and Bernard and Vastano (1977), while indicating reasonable agreement between model and analytical results for a parabolic island, are not definitive since the test cases are ones for which $d(r^{-2}h)dr \leq 0$. The study by Houston for the Hawaiian Islands is unique among these in that it employs a finite element numerical approach of a generalized Helmholtz equation, with extremely good spatial resolution near the islands. Unfortunately, however, a critical comparison of this method against known analytical solutions with trapped "hedge" modes does not exist to assess its adequacy with respect to frequency response.

The above capsulization of the state of the art in assessing the linear transformation from deep water to the nearshore regions of an island emphasizes that, while we have come a long way (in the last 30 years) from deductions based only upon simple refraction analysis, some important problems still remain to be resolved. I list here some specific problems which I would hope will be subjects of discussion by the participants of the workshop:

(1) Is it important that one be able to establish in the transfer function possible resonant peaks whose band width is extremely narrow, considering the fact that full resonance is achieved only for an input duration inversely proportional to the frequency band width? Houston's spectacular results for two actual tsunamis of record may suggest otherwise.

(2) What is the best approach in principle to determining the frequency response via a numerical simulation of the linearized long wave equations?

(3) What is the most efficient and accurate method for allowing for radiation of scattered wave energy in time marching numerical models at an open boundary?

(4) What is the most appropriate boundary condition for numerical models at the island boundary, if a beach situation is to be modeled?

(5) While the condition $d(r^{-2}h(r))dr > 0$ is necessary and sufficient for the existence of trapped waves for a round island, is this a sufficient condition for islands of more general shape? It is possible that variations in topography in the azimuthal direction tend to de-tune the potential resonant modes.

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DISCUSSION

Following Professor Reid's lecture the following discussion ensued:

H. Loomis - You mentioned that ray theory could give approximations on the amplitudes of the first or second waves. I wondered, if we are talking about wave lengths comparable with the island size, is ray theory appropriate.

R. Reid - I mentioned ray theory as another technique to contrast to our finite difference model. We have looked at ray theory to identify first wave arrivals, but the theory becomes very complicated for secondary and tertiary waves and their reflections.

H. Loomis - I was thinking about this as a transient phenomena using ray theory as a means of indicating areas of energy convergence and divergence for estimating amplitudes at island shorelines. I see this as a means of indicating a single transfer of energy from the generating area to the shoreline not in the context of steady state.

R. Reid - Certainly the convergence of wave rays is an important aspect of the problem. Another aspect is that, by examining the wave equation, a relationship may exist between the Q values and the different azimuthal modes and radial modes.

T. Wu - How much wave energy is trapped by the island?

R. Reid - The amount of energy trapped at the islands with a continuous monochromatic wave depends critically on the island bathymetry, as the examples indicate, and on the wave frequency.

T. Wu - Does the energy converge at higher frequencies?

R. Reid - The sum of all modes converges to a finite level as shown in the Wake Island case. If you inject a broad band pulse into the island system, each mode excitable will ring to some degree. The energy trapped in these resonance modes will then gradually radiate away. Energy levels for the very narrow peaks may not be high because of the short time to excite these modes.

R. Wiegel - Are you asking, that, if the pronounced peaks occur, won't they be smeared out by nature?

R. Reid - I believe the peaks are real, but what I'm really asking is - Do you want to be able to resolve these sharp peaks?

Let me digress to the finite difference, time-stepping model. We input a pulse whose spectrum is broad banded. We then spectrally analyze the time history of the resulting waves at the shorelines. We are limited by the Δt of the model for frequency resolution and by duration for the lowest frequency examined. So again, is it really important to resolve these extremely narrow peaks?

G. Carrier - I would say the answer to your question is no. I feel that if you have an extremely high Q, it will receive very little energy unless you force a continued excitation on it. I think that excitation is not present with tsunami-like waves, therefore what you suggest about islands is unimportant. Unless there is a very special case where the topography creates a non-linear interaction with wave inputs, I don't feel these things exist. My intuition is that for island topography situations these sharp peaks are unimportant, but for harbors just the opposite is true.

R. Weigel - The types of resonances you show are as high as ten. In nature very few systems have such Q's because of damping. In practice, dissipation may well eliminate the type of resonance you suggest.

R. Reid - I guess the bottom line is islands can have resonances the same as harbors but we do not yet have suitable data for assessing how important such resonances are.

BAY AND HARBOR RESPONSE TO TSUNAMIS

BY
F. RAICHLEN

F. Raichlen - I'm going to talk about the problem of bay and harbor response. I will break the discussion into three parts. First, I will show real data and I hope to provoke some discussion. Then, I'd like to review where we have been in harbor resonance, and, finally finish up with where we are with regard to the excitation of harbors by tsunamis.

Figure 1 is the area which extends from north of Santa Monica Bay south to the San Diego area. I want to present a figure of tide gage records of tsunami response at three locations in this area: One is Santa Monica Bay -- a very wide open bay, another is within Los Angeles Harbor and the last is at La Jolla. This figure provides the locations for these tide gage records.

Now let me make a few comments about the bathymetry. This region of the California Bight and the Continental Shelf break has depth changes on the order of 4-5 to 1 from the deep ocean to the nearshore. In addition, there are an interconnected series of basins which we will see in another map, and islands in this region.

Tide gage records from the Alaskan tsunami at three locations -- Santa Monica, Los Angeles Harbor, and La Jolla -- are presented in Figure 2. There are several things I want to show here, to begin the discussion of the response of harbors to tsunamis.

Look at the record for Santa Monica Bay, and note this covers a period of 22 hours from the arrival of the first wave. There are major changes in water level at the beginning of the record, of on the order of seven feet. Twenty-two hours later we have fluctuations of about three feet, and there are about 35 oscillations occurring during that interval. If you compute the Q of the system -- assuming it to be a linear oscillator -- the Q comes out to be about 100 to 150. (It will also be on the order of 100 for the other two locations if one were to do the calculation.)

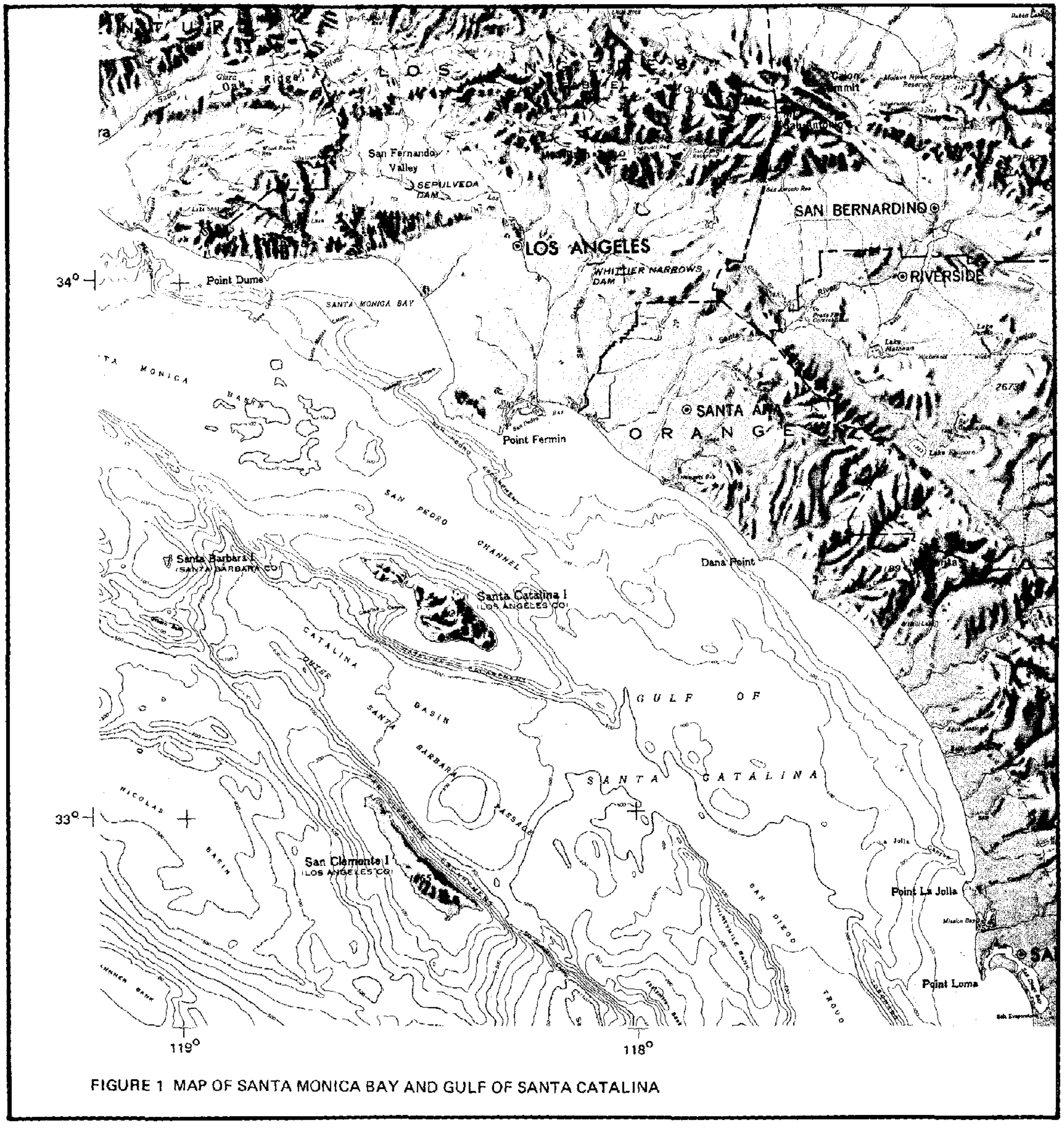

FIGURE 1 MAP OF SANTA MONICA BAY AND GULF OF SANTA CATALINA

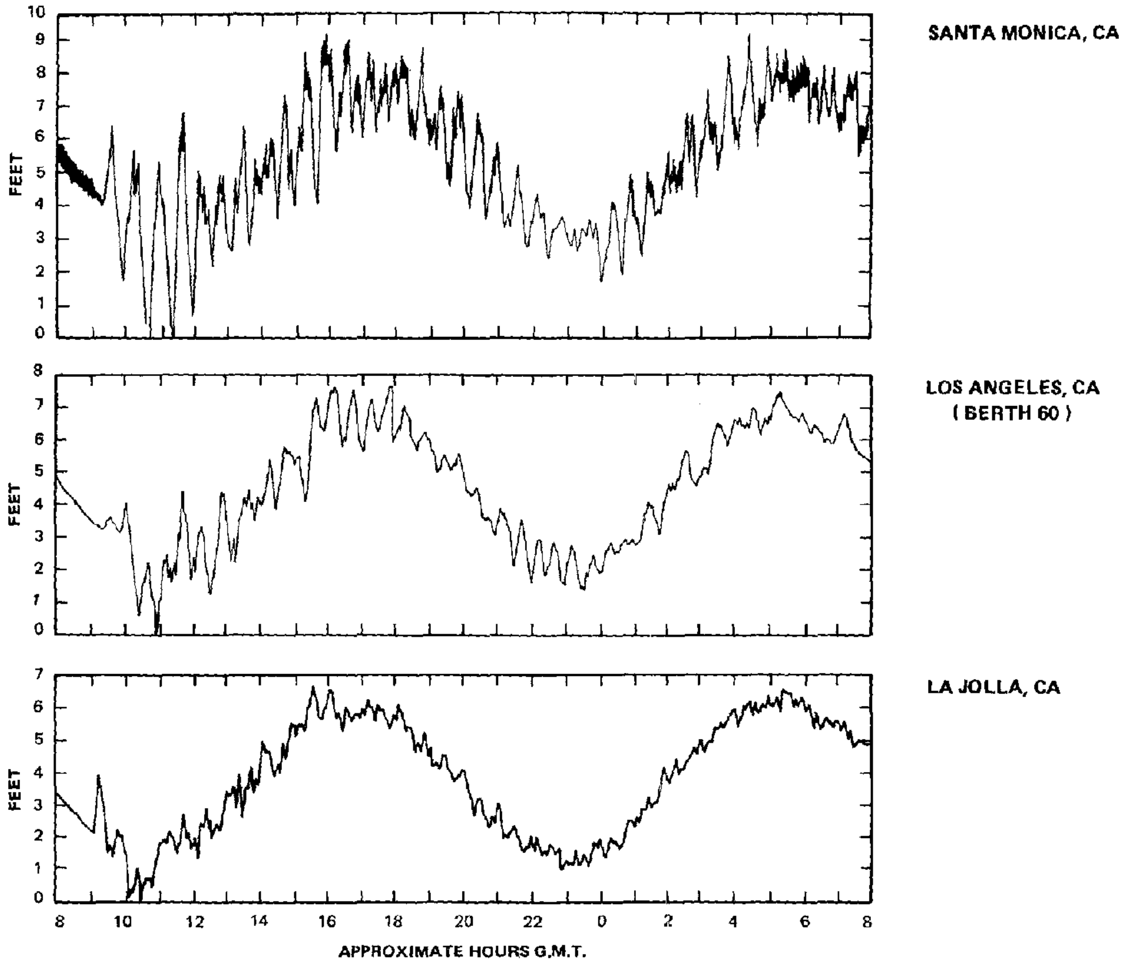

FIGURE 2 TIDE GAGE SHOWING ALASKA TSUNAMI, MARCH 28 – 29, 1964

Now Santa Monica Bay is extremely wide and open with a depth change on the order of five or so, and you begin to wonder what are you really seeing? Are you seeing a ringing of the harbor? Or, are you seeing a persistent incoming wave -- which is very hard to believe -- really, what is it which is being measured?

If you look at tide gage records from other sites in the Pacific you may or may not see this type of behavior. You see it at locations in the Hawaiian Islands and in Japan, but at many places you do not see this long ringing. This is the first question I will offer for discussion.

The spectra of the tide gage records at these three locations is presented in Figure 3. I normalized with respect to the mean square of the signal. The similarity in the same general shape for each case is apparent -- two peaks, one at about 0.5 hours^{-1} and the other at about 1.5 hours^{-1}. (In this figure, the resolution is purposely low to smooth out the spectra and show the major concentrations of energy.)

W. Van Dorn - Two of them look the same, but Santa Monica certainly doesn't.

F. Raichlen - Well, they are at different locations so you expect some differences, but there is a similarity in terms of concentration of energy. However, it is true that there are differences between the spectrum at Santa Monica and the other two locations.

In Figure 4, I have included the Chilean tsunami of 1960 and compare the spectra from two tsunamis at each of two locations. At a given location, there are surprising similarities between the spectra for the two tsunamis. There are, certainly, differences, but from this, you might form an opinion that perhaps the tsunamis are quite similar, since they each resulted from major earthquakes. On the other hand, perhaps what you really see in the signal is just the excitation of the offshore waters with little real information apparent relating to the tsunamis themselves. These are questions I do not have an answer for, but are one of the reasons why the local tsunami

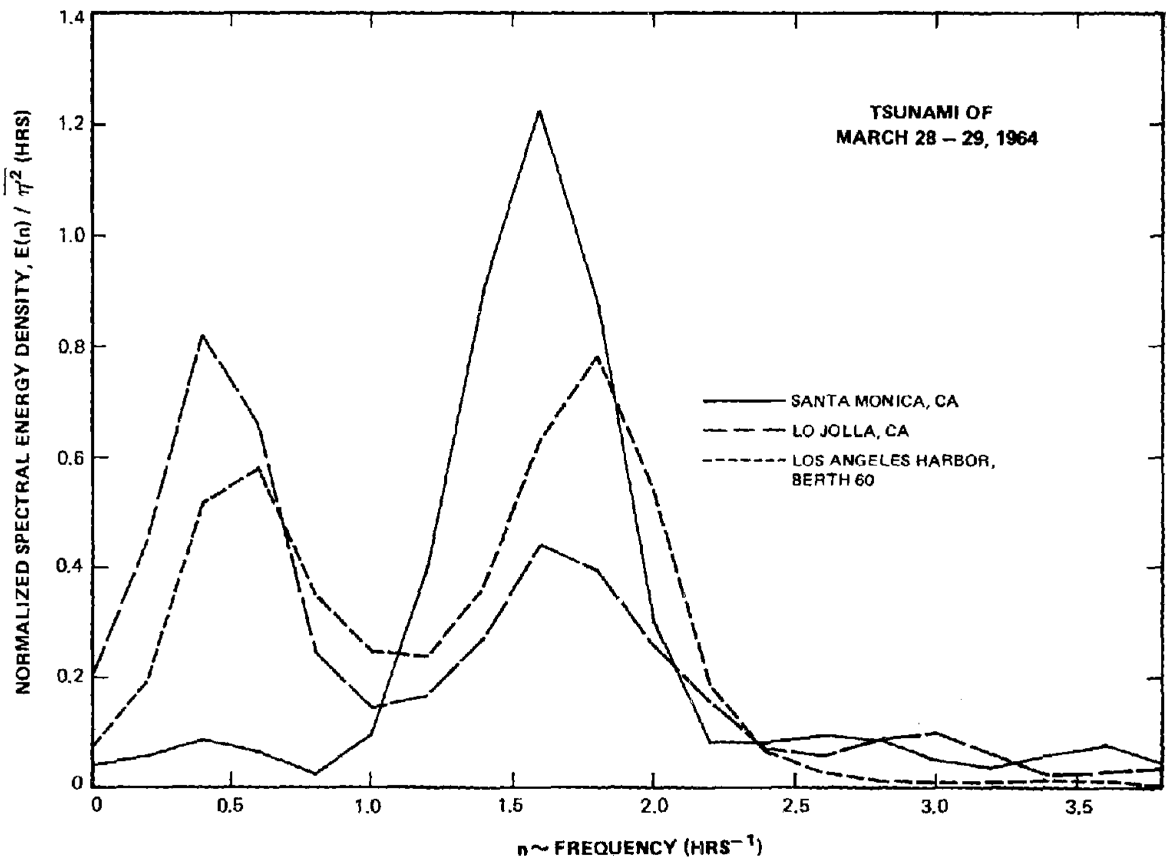

FIGURE 3 SPECTRA OF THE TIDE GAGE RECORDS

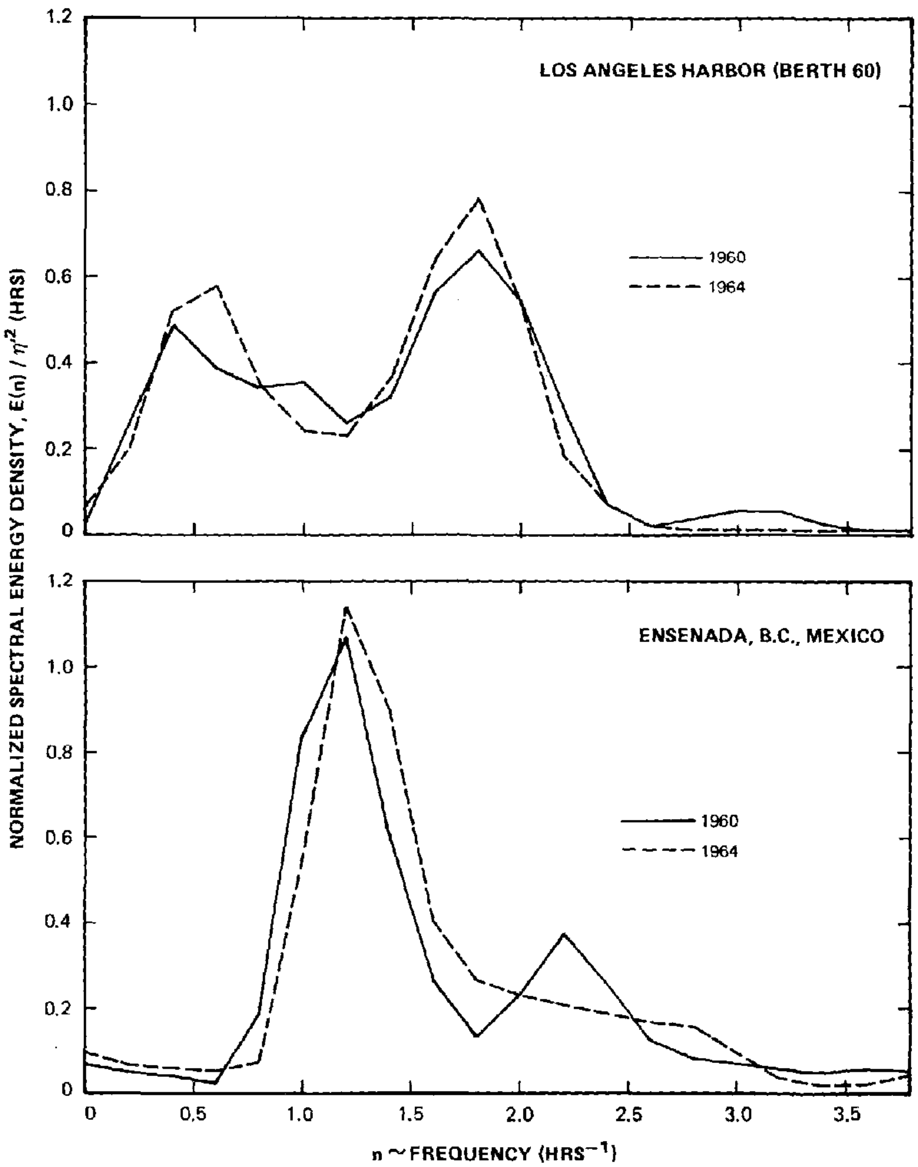

FIGURE 4 COMPARISON OF SPECTRA OF CHILEAN AND ALASKAN TSUNAMIS AT TWO DIFFERENT LOCATIONS

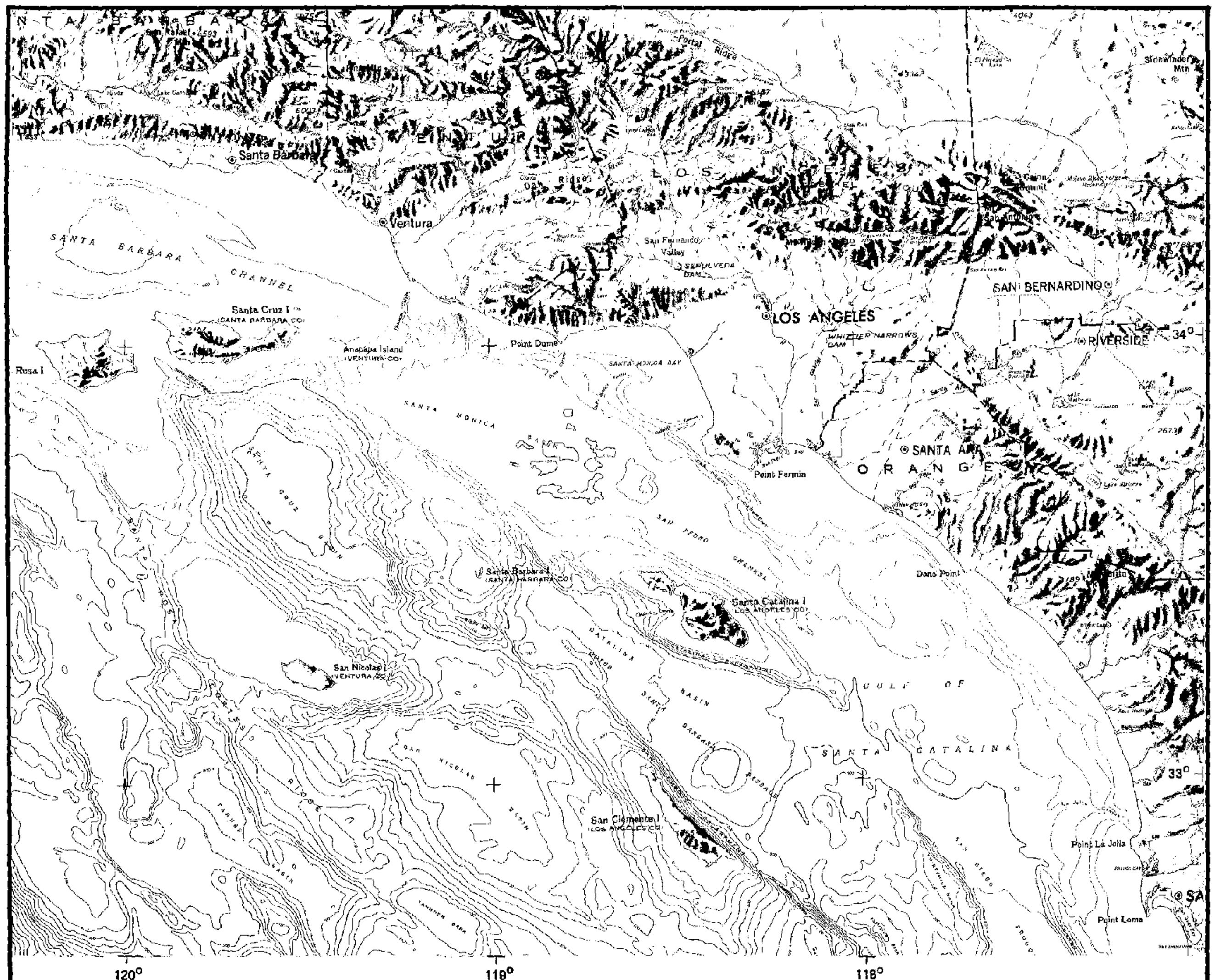

FIGURE 5 MAP OF OFFSHORE BATHYMETRY

response of nearshore waters is of interest i.e., the use of the resultant transfer function to interpret the causative wave system.

The offshore bathymetry for this area is presented in Figure 5. The islands of Southern California are shown along with Santa Monica Bay, Los Angeles Harbor and La Jolla; the very, very tortured bathymetry offshore is apparent here. I would agree, as Basil Wilson has proposed, that there must be some sort of energy trapping in this whole region in order to create a ringing of the duration and the intensity which is seen in tide gage records in the area which include tsunamis.

I will start my discussion of harbor resonance some years ago and work forward in terms of what we have learned about wave induced harbor excitation and try to develop some questions about needed research dealing with the interaction of harbors and tsunamis.

I am going to give examples from several papers dividing the field into five general areas. I will discuss first the linear inviscid 2-D steady state approach, (by this I mean constant depth); then a few papers that address the problem of linear, inviscid, steady-state, 3-D problems; linear with viscous effects; one paper that has looked at the non-linear viscous problem; and then a couple of papers that look at the transient linear inviscid case. These topics are presented in Figure 6.

In Figures 7, 8 and 9 I have listed some papers -- I will not talk about all of them -- which demonstrate the sorts of things I would like to discuss in this state-of-the-art summary.

I will start with the paper by Miles and Munk (1961) which really set the pace for looking at the harbor problem in terms of real effects.

An important contribution of Miles and Munk (1961) was the realization that there was a close acoustical analogy to the harbor, and the concept of radiation, i.e., a radiative loss, could be applied directly from acoustics to the

1. Steady-State; Linear; Inviscid; 2-D.
2. Steady-State; Linear; Inviscid; 3-D.
3. Steady-State; Linear; Viscous.
4. Steady-State; Nonlinear; Viscous.
5. Transient; Linear; Inviscid.

FIGURE 6 FIVE DIFFERENT APPROXIMATIONS TO THE HARBOR RESPONSE PROBLEM

1. Steady-State; Linear; Inviscid; 2-D.

Miles, J. W. and Munk, W. H., "Harbor Paradox," Proc. ASCE, J. Waterways Harbors Div., 1961, 87:111-130.

LeMéhauté, B., "Theory of Wave Agitation in a Harbor," Proc. ASCE, J. Hydr. Div., 87:172-185, 1961.

Ippen, A. T. and Goda, Y., "Wave-Induced Oscillations in Harbors: The Solution of a Rectangular Harbor Connected to the Open Sea," Report No. 59, Hydrodynamics Lab, MIT, Cambridge, MA, 1963.

Raichlen, F. and Ippen, A. T., "Wave-Induced Oscillations in Harbors," Proc. ASCE, J. Hydr. Div., 91(HY2), March 1965.

Hwang, L. S. and LeMéhauté, B., "On the Oscillation of Harbors of Arbitrary Shape," Report TC-123A, Tetra Tech, Inc., Oct. 1968.

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Lee, J.-J., "Wave-Induced Oscillations in Harbors of Arbitrary Geometry," J. Fluid Mech., 1971, 45:375-394.

Carrier, G. F., Shaw, R. P. and Miyata, M., "The Response of Narrow-Mouthed Harbors in a Straight Coastline to Periodic Incident Waves," J. Appl. Mech., Vol. 38, Series E, No. 2, 1971.

Lee, J.-J. and Raichlen, F., "Oscillations in Harbors with Connected Basins," Proc. ASCE, JWWH, 98:WW3, Aug. 1972.

Miles, J. W. and Lee, Y. K., "Helmholtz Resonance of Harbors," J. Fluid Mech., 1975, 67:445-464.

FIGURE 7 PAPERS COVERED UNDER APPROXIMATION 1

2. Steady-State; Linear; Inviscid; 3-D.

Leendertse, J. J., "Aspects of a Computational Model for Long-Period Water Wave Propagation," Memo RM-5294-PR, Rand Corp., Santa Monica, CA, 1967.

Olsen, K. and Hwang, L. S., "Oscillations in a Bay of Arbitrary Shape and Variable Depth," J. Geophys. Res., 76:5048-5064, 1971.

Chen, H. S. and Mei, C.-C., "Oscillations and Wave Forces in an Offshore Harbor," Report 190, R. M. Parsons Lab. for Water Res. & Hydro., MIT, 1974.

Raichlen, F. and Naheer, E., "Wave-Induced Oscillations of Harbors with Variable Depth," Proc. 5th ICCE, Hawaii 1976.

3. Steady-State; Linear; Viscous.

Mei, C. C., Liu, P. L.-F., Ippen, A. T., "Quadratic Loss and Scattering of Long Waves," Proc. ASCE, J. Waterways, Harbors, and Coastal Engr. Div., 100:WW3, Aug. 1974.

Shaw, R. P. and Lai, C.-K., "Channel Friction and Slope Effects on Harbor Resonance," Proc. ASCE, J. Waterways, Harbors, and Coastal Engr. Div., 100:WW3, Aug. 1974.

Ünlüata, Ü. and Mei, C.-C., "Effects of Entrance Loss on Harbor Oscillations," Proc. ASCE, J. Waterways, Harbors, and Coastal Engr. Div., 101:WW2, May 1975.

FIGURE 8 PAPERS COVERED UNDER APPROXIMATIONS 2 AND 3

4. Steady-State; Nonlinear; Viscous.

Rogers, S. R. and Mei, C.-C., "Nonlinear Resonant Excitation of a Long and Narrow Bay," J. Fluid Mech., 88:1, 1978.

5. Transient; Linear; Inviscid.

Carrier, G. F. and Shaw, R. P., "Response of Narrow-Mouthed Harbors to Tsunamis," Proc. Int'l Symposium on Tsunamis, IUGG, Oct. 1969.

Wilson, B. W., "Tsunami-Responses of San Pedro Bay and Shelf, Calif.," Proc. ASCE, J. Waterways, Harbors and Coastal Div., 97:WW2, May 1971.

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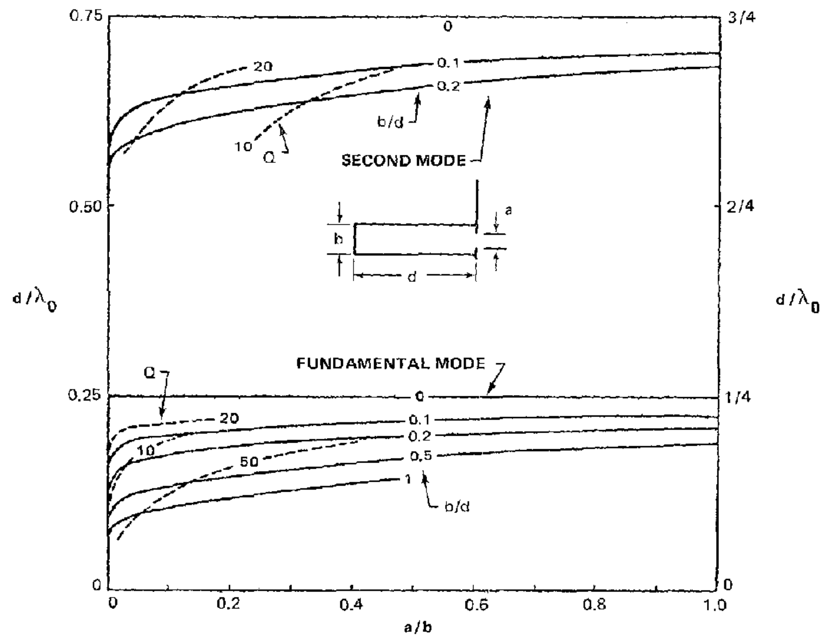
FIGURE 9 PAPERS COVERED UNDER APPROXIMATIONS 4 AND 5 harbor oscillation problem. They looked at a harbor of simple planform and applied the idea of the Q of the response. Figure 10 shows the ratio of the harbor length to the wave length at resonance as well as the so-called Q of the harbor as a function of the aspect ratio of the harbor (b/d) and the width of the entrance to the harbor width. The Q represents the "peakedness" of the response curve and hence gave rise to the "harbor paradox" title of the paper. Figure 10 shows this paradox for a small aspect ratio: as the entrance closes down the Q increases. Figure 11 will show this more concisely.

Figure 11 shows some work by Ippen and Goda (1963) dealing with the response in a harbor, which is defined as the amplitude at a point inside normalized by the amplitude of the standing wave at the entrance with the entrance closed. The response is presented for the second mode of oscillation of a square harbor and as a function of the ratio of the opening width to the width of the harbor. As the basin is closed progressively, the same effect that Miles indicated is seen, i.e., the increase in the amplification of incoming wave energy as the basin entrance is closed. In the inviscid theory, as the entrance is closed, even though less energy enters the harbor, the energy is trapped. The other feature shown by the experimental results, is what is negating the harbor paradox: although for a wide open harbor there is apparently good agreement between the linear theory and the experiments, as the entrance is narrowed there is not. Damping greatly reduces the response with some corresponding shift in the frequency of the peak. However, the major effect is in the Q. So as we reduce the entrance width from the wide open condition, the harbor paradox causes an increase in the response until a point is reached where the dissipation at the entrance becomes a controlling factor and the response drops down.

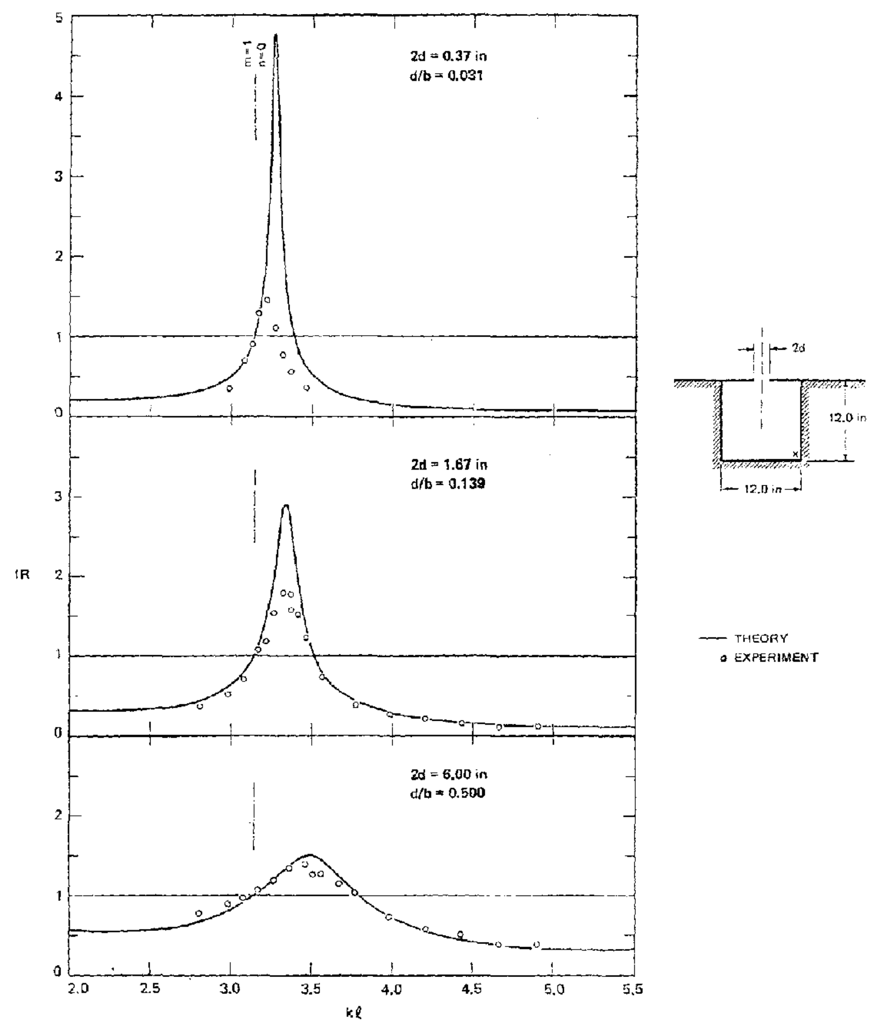
Question from Audience - What was the depth in these experiments?

F. Raichlen - These were deep water experiments so the only boundary dissipation is along the sidewalls. The primary dissipation occurs at the entrance.



MILES, J.W., AND MUNK, W.H., 'HARBOR PARADOX,' PROC. ASCE, J. WATERWAYS HARBORS DIV., 1961, 87 : 111–130.

FIGURE 10 DEPENDENCE OF RESONANCE ON HARBOR OPENING WIDTH



IPPEN, A.T. AND GODA, Y., 'WAVE–INDUCED OSCILLATIONS IN HARBORS: THE SOLUTION OF A RECTANGULAR HARBOR CONNECTED TO THE OPEN SEA,' REPORT NO. 59, HYDRO. LAB., MIT, CAMBRIDGE, MASS, 1963.

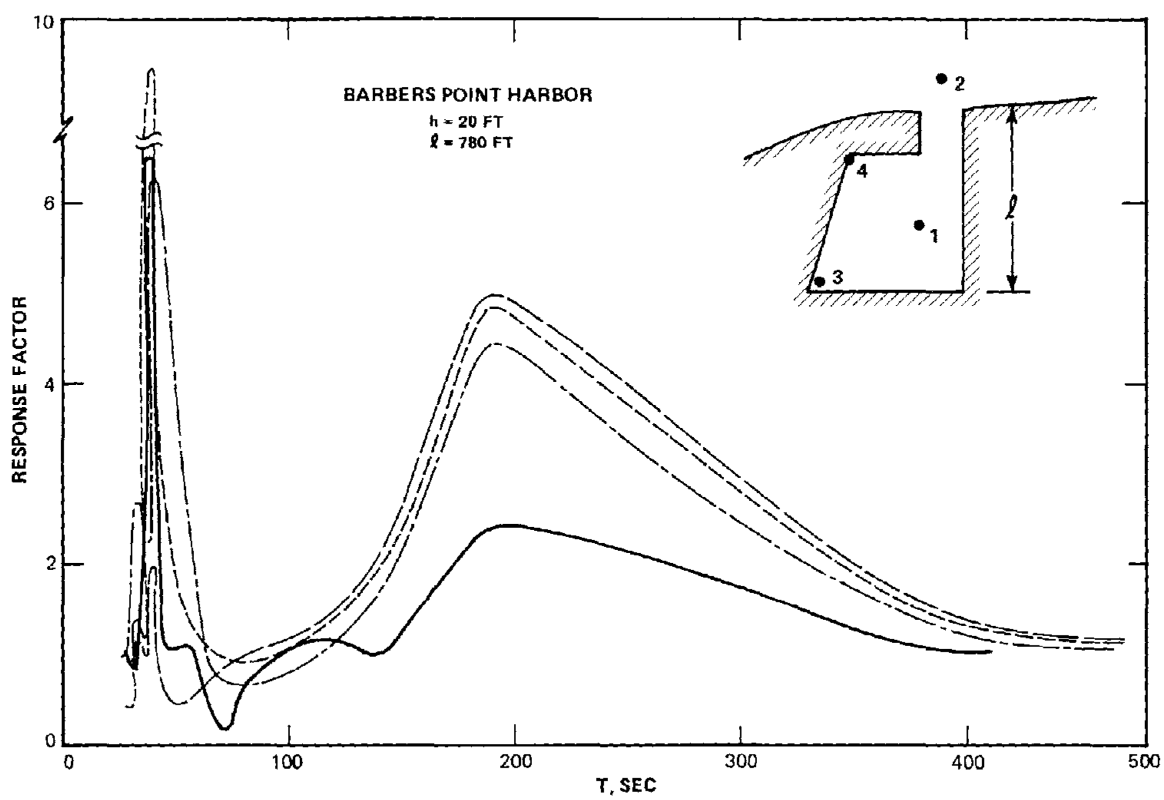
FIGURE 11
FREQUENCY RESPONSES OF SQUARE HARBORS WITH PARTIAL OPENINGS

An example of a response curve for a real harbor is presented in Figure 12 from Hwang and Le Mehaute (1968) who considered the constant depth case for harbors of arbitrary shape. Using an integral equation technique they solved the problem of an arbitrary shape boundary by using one domain -- that is, the harbor and the open sea were treated as a single domain. This figure shows computed response at several points within Barbers Point Harbor in Hawaii. As one expects, it shows great variability in steady-state response with wave period, and also a variation from point to point. (So if we try, for example, to consider the California Bight as one whole area -- with the indentations of harbors being rather small on the scale of things -- then as you travel up and down the coast you expect some difference in response, but sensitivity in the same frequency regions.)

In Figure 13 certain results of J. J. Lee (1971), performed at about the same time, are presented. He also used the integral equation technique but matched two domains -- the outside ocean and the inside harbor -- at the harbor entrance. He performed some careful experiments in the laboratory for various simple geometric shapes as well as for the configuration shown in Figure 13 which is at Long Beach Harbor. This is constant depth and the experiments are deepwater experiments so the dissipation again is associated primarily with the entrance and the side boundaries. The agreement of the theory with the experiments is quite good.

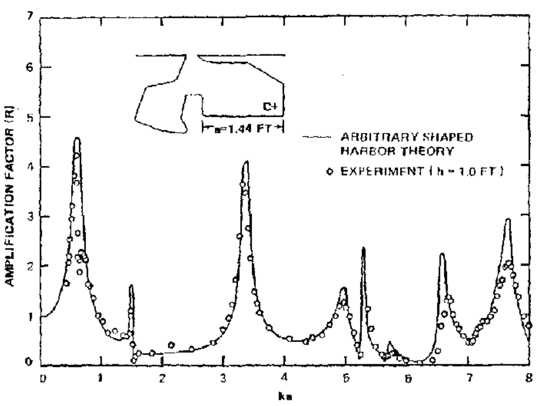
Comment from Audience - While you have that on the screen, may I point out that in contrast to the case of Ippen and Goda which had two sharp plates at the entrance, here you have only one plate with a rounded edge, so that there is much less dissipation.

F. Raichlen - That's very true; in fact very good agreement with the theory was also obtained with measurements of velocities at the entrance. Returning to Figure 13, we see the computed distribution of wave amplitude within the harbor for a particular frequency. This is compared with a measured distribution obtained in a laboratory model by Knapp and Vanoni at Caltech in 1945. As you can see, there is generally similar behavior and one might think "well,

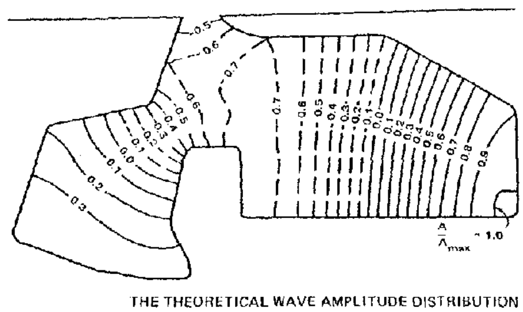


HWANG, L.S. AND LE MÉHAUTÉ, 'ON OSCILLATIONS OF HARBORS OF ARBITRARY SHAPE,' REPORT TC-123A, TETRA TECH, OCT. 1968.

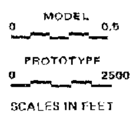
FIGURE 12
RESPONSE FUNCTION AT BARBERS POINT HARBOR

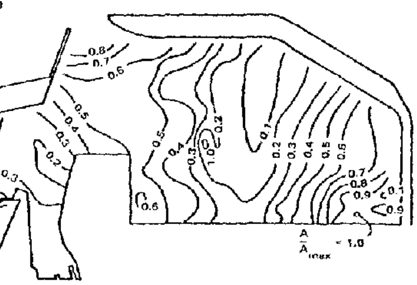


RESPONSE CURVE AT POINT C OF THE LONG BEACH HARBOR MODEL



THE THEORETICAL WAVE AMPLITUDE DISTRIBUTION IN THE LONG BEACH HARBOR MODEL (ka = 3.38).





WAVE AMPLITUDE DISTRIBUTION INSIDE THE HARBOR MODEL MODEL OF KNAPP AND VANONI (1945) FOR SIX MINUTE WAVES (ka = 3.30).

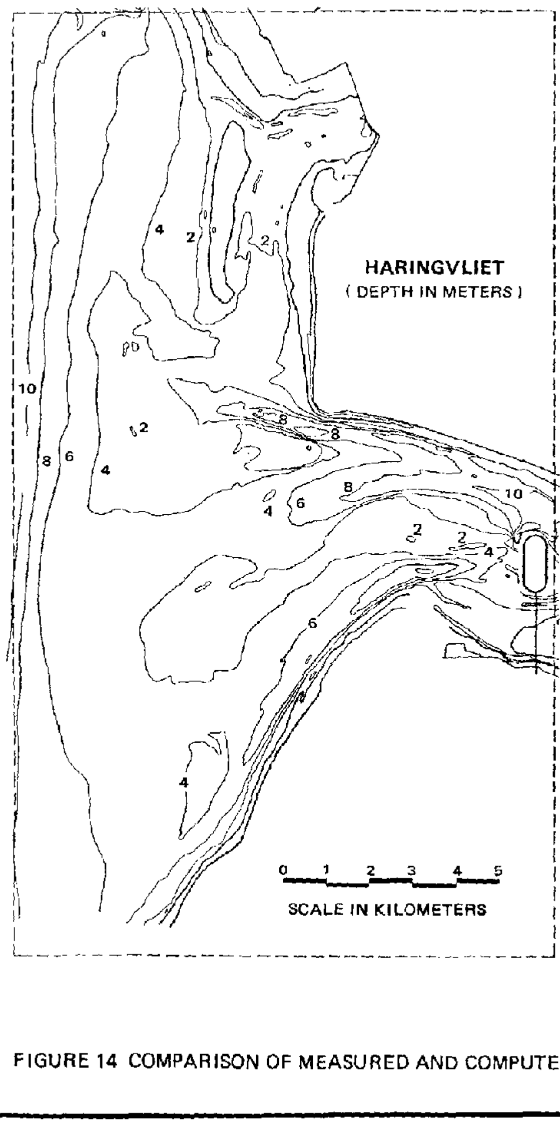
LEE, J.-J., ' WAVE-INDUCED OSCILLATIONS IN HARBORS OF ARBITRARY GEOMETRY', J. FLUID MECH., 1971, 45:375--394.

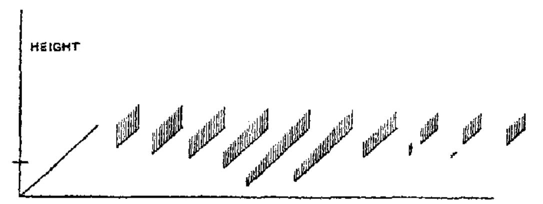
FIGURE 13 RESPONSE CURVE AND WAVE AMPLITUDE DISTRIBUTION AT LONG BEACH HARBOR we don't have to worry about hydraulic models anymore, we have the analytical solution." I feel that one way an analyical model can best be used is as a guide for performing experimental studies as another tool to assist in a problem solution.

Leendertse (1967) developed a longwave model which was a nonlinear nondispersive model using a finite difference technique in which he imposed an input condition at the seaward edge of the boundary. Some results are shown in Figure 14 for the amplitudes at various locations in the harbor at a given time; the lower portion of the figure shows both computed and measured velocity vectors at various locations. You see reasonably good agreement; one problem in this approach is that one has to be able to specify input conditions along an outer boundary. (This was one of the first models to allow for three dimensional variations.)

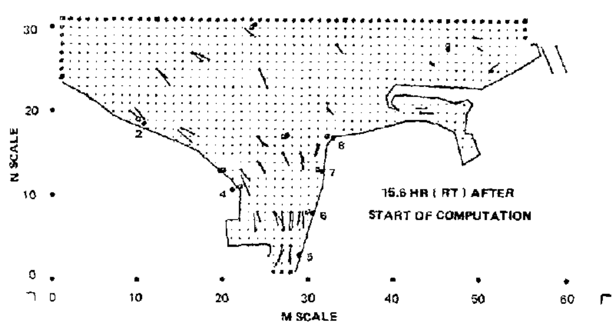
Olsen and Hwang (1971) considered a three-dimensional harbor in Hawaii where field measurements were available. The harbor is shown in the upper part of Figure 15. They used a finite difference model for the harbor and some distance outside, and match this up with an open sea integral equation to determine the response defined in terms of the power density. In the lower part of Figure 15, spectra are presented. The long-dash curve is the measurement obtained offshore, the solid curve is the measurement inside the harbor at Station 3, and the short-dash curve is obtained by taking the transfer function and applying it to the offshore data. It is observed that this approach does a reasonably good job of reproducing the trend of the distribution of energy.

Coming now to 1974, Figure 16 shows an example from Chen and Mei (1974). This work incorporated a hybrid finite-element model of an offshore harbor which was to house a floating nuclear power plant near Atlantic City, New Jersey. The question was, is the wave environment inside acceptable in terms of the motion of the large barges? The model was developed with this in mind and here we see a response curve for this harbor.





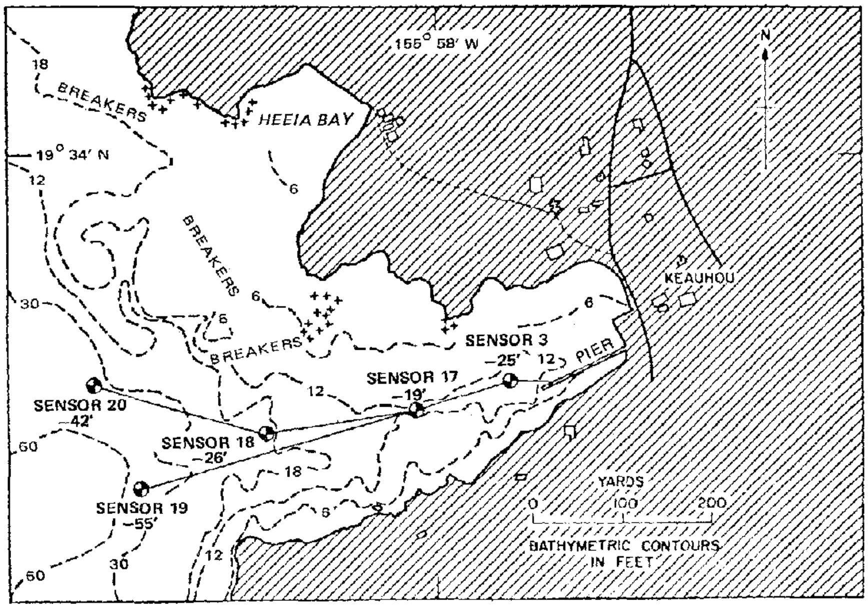
ISOMETRIC SECTIONS REPRESENTING COMPUTED WATER LEVELS ON EVERY FIFTH LINE OF THE GRAPH BELOW.



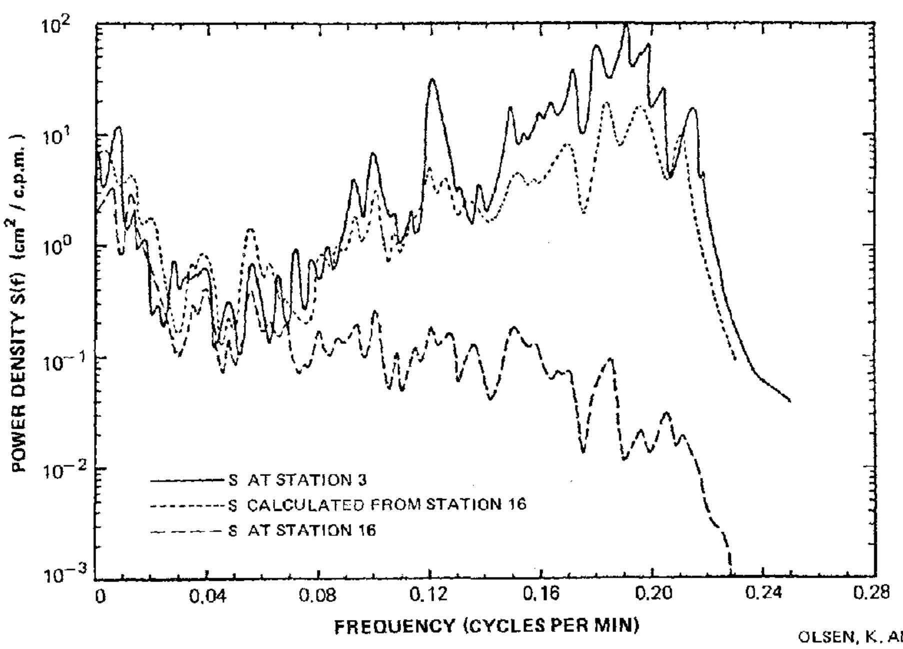
FIELD OF THE COMPUTATION AND COMPARISON BETWEEN MEASURED VELOCITIES AND COMPUTED VELOCITIES. MEASURED VELOCITIES ARE AVERAGE VELOCITIES OF SIMILAR TIDAL CYCLES AT A PARTICULAR LOCATION. COMPUTED VELOCITIES ARE THE AVERAGE VELOCITY IN AN 800–BY–800–METER AREA OF A PARTICULAR TIDAL CYCLE.

LEENDERTSE, J.J. 'ASPECTS OF A COMPUTATIONAL MODEL FOR LONG–PERIOD WATER WAVE PROPAGATION,' MEMO RM–5294–PR RAND CORP., SANTA MONICA, CA, 1967.

FIGURE 14 COMPARISON OF MEASURED AND COMPUTED VELOCITIES



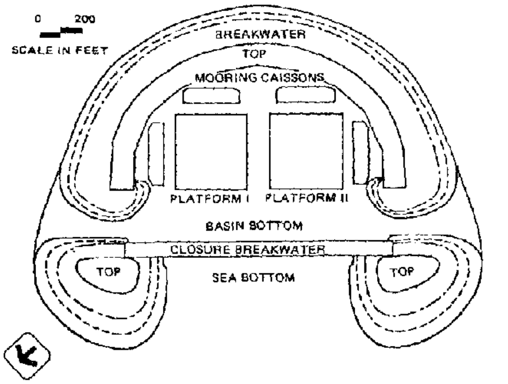
WAVE–SENSOR ARRAY IN KEAUHOU BAY, HAWAII, SHOWING SENSOR LOCATIONS AND APPROXIMATE CABLE RUNS. ALL DEPTHS ARE IN FEET.



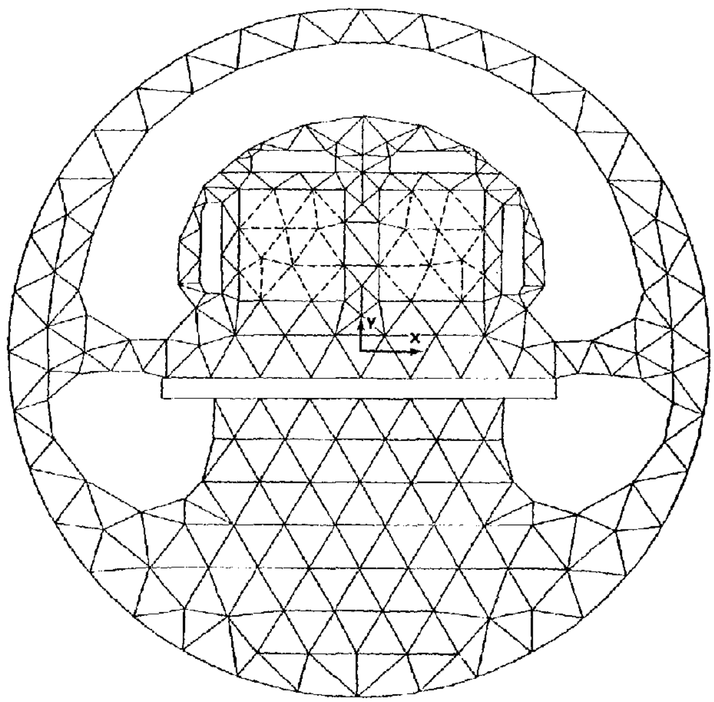
COMPARISON OF MEASURED AND CALCULATED WAVE SPECTRA AT STATION 3. THE CALCULATED SPECTRUM IS BASED ON THE MEASURED (INPUT SPECTRUM) AT STATION 16.

OLSEN, K. AND HWANG, L.S., 'OSCILLATIONS IN A BAY OF ARBITRARY SHAPE AND VARIABLE DEPTH,' J. GEOPHYS. RES., 76: 5048–5064, 1971.

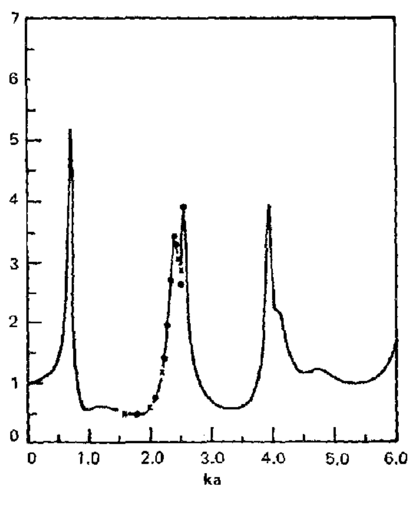
FIGURE 15 COMPARISON OF MEASURED AND CALCULATED WAVE SPECTRA



GENERAL PLAN OF ATLANTIC OFFSHORE HARBOR



NETWORK OF FINITE ELEMENTS FOR ATLANTIC CITY OFFSHORE HARBOR FOR $0 < ka < 2.5$



MEAN HARBOR RESPONSE

CHEN, H.S. AND MEI C.–C., ' OSCILLATIONS AND WAVE FORCES IN AN OFFSHORE HARBOR, REPORT 190, R.M. PARSONS LAB. FOR WATER RES. AND HYDRO., MIT, 1974, PROC. 10th NAVAL HYDRODYNAMICS SYMPOSIUM, JUNE 1974.

FIGURE 16 HARBOR RESPONSE CALCULATED BY FINITE ELEMENTS METHOD

This is a way to obtain the response of a harbor in three dimensions -- how important the variation in three dimensions is, I think remains to be seen for the simple reason that in terms of the magnitude of the peaks I wonder if one is really that concerned whether, say, this peak is five or whether it is three. For most engineering purposes you are really interested in whether this particular shape or this particular location is bad in terms of response. There are certainly other simplifications, for example in dissipation at the entrances, which could modify the amplification considerably.

B. Le Mehaute - Were the floating barges treated as a free-surface flow?

C. Mei - I think it's with the barge. The calculations are two dimensional using linear shallow water wave theory with the approximation of Fritz John taking care of the depth underneath the barges.

Audience Question - What is the wavelength relative to the gap opening?

F. Raichlen - Well, there is the whole range. I'm not sure what the dimension "a" was in this case.

C. Mei - I think "a" was 800 feet. The range is down to something like 8-second waves.

F. Raichlen - Of course, the accuracy declines for the shorter waves.

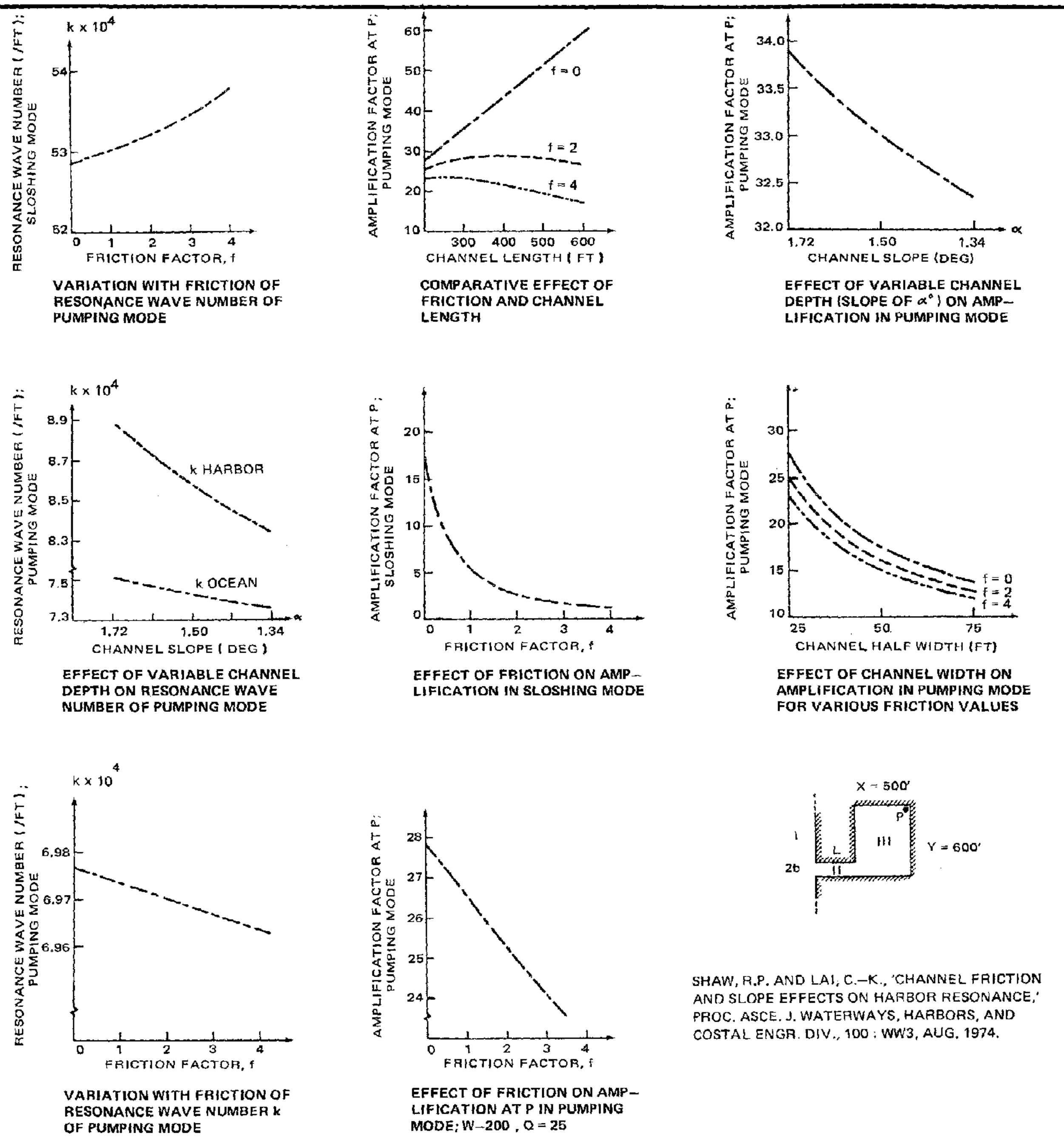
J.J. Lee - What are those dots on the response curve?

C. Mei - Those are points representing results from two different grids -- a fine grid for a range of short waves and a coarse grid for a range of longer waves. We wanted to check that in the overlapping region the two grids give the same answer. So these dots are not physical experiments, they are numerical results.

F. Raichlen - Now I would like to show a couple of examples of the effects of friction.

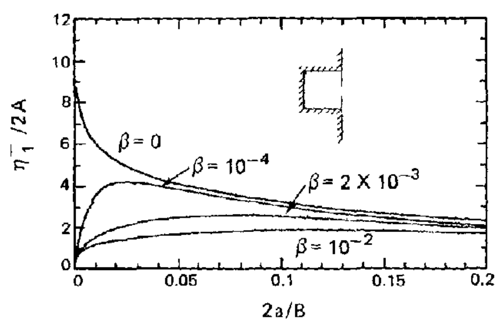
Shaw and Lai (1974) discuss the effect of dissipation in an entrance channnel of a simple shape harbor. Some selected results are presented in Figure 17. I want to use this simply to indicate some trends. In the first part of the figure we see the amplification factor for the pumping mode as a function of friction factor in the entrance channel only. This is a boundary friction and it does not include entrance dissipation. Although this may not be the major dissipation mechanism for this sort of problem, it does illustrate the general effect of friction. For example, in the fifth figure we see the effect of channel length. For zero friction, increasing the length amounts to closing the harbor off, so we end up with the harbor paradox. But if we put in dissipation, a point is reached after which the response falls off with greater channel length. There are several other things shown here. For example, in the second figure, we see the effect of friction on resonant wave number. You notice here that as in the case of the linear harmonic oscillator, we can put in a lot of friction and the frequency of resonance does not change significantly but the amplification does.

Another example of the effect of dissipation on the response is presented in Figure 18 from work of Unluata and Mei (1975). This was an analytical study which included the entrance dissipation in a manner similar to that of Ito whose work was concerned with a tsunami breakwater in Japan, i.e., as a quadratic loss. Figure 18 corresponds to the first harmonic mode of oscillation; in the original paper the pumping mode was also shown. Figure 18 shows a measure of the amplification within the harbor for different friction parameters and as a function of the ratio of the width of the entrance to the width of the harbor. Beta equal to zero represents the undamped case, and demonstrates the harbor paradox as the entrance is closed down. As beta increases, dissipation increases and, for example, at $\beta = 10\text{-}4^{-4}$ the harbor paradox controls until a certain point and entrance dissipation takes over. In the more damped cases shown here, there is no harbor paradox. Note though that for an entrance width of 0.2 -- which is really rather narrow -- there is really not a great deal of friction loss. The significant effect comes only at extremely small entrances. These are effects which should be investigated more fully.



SHAW, R.P. AND LAI, C.–K., 'CHANNEL FRICTION AND SLOPE EFFECTS ON HARBOR RESONANCE,' PROC. ASCE. J. WATERWAYS, HARBORS, AND COSTAL ENGR. DIV., 100 : WW3, AUG. 1974.

FIGURE 17 CHANNEL FRICTION AND SLOPE EFFECTS ON HARBOR RESONANCE



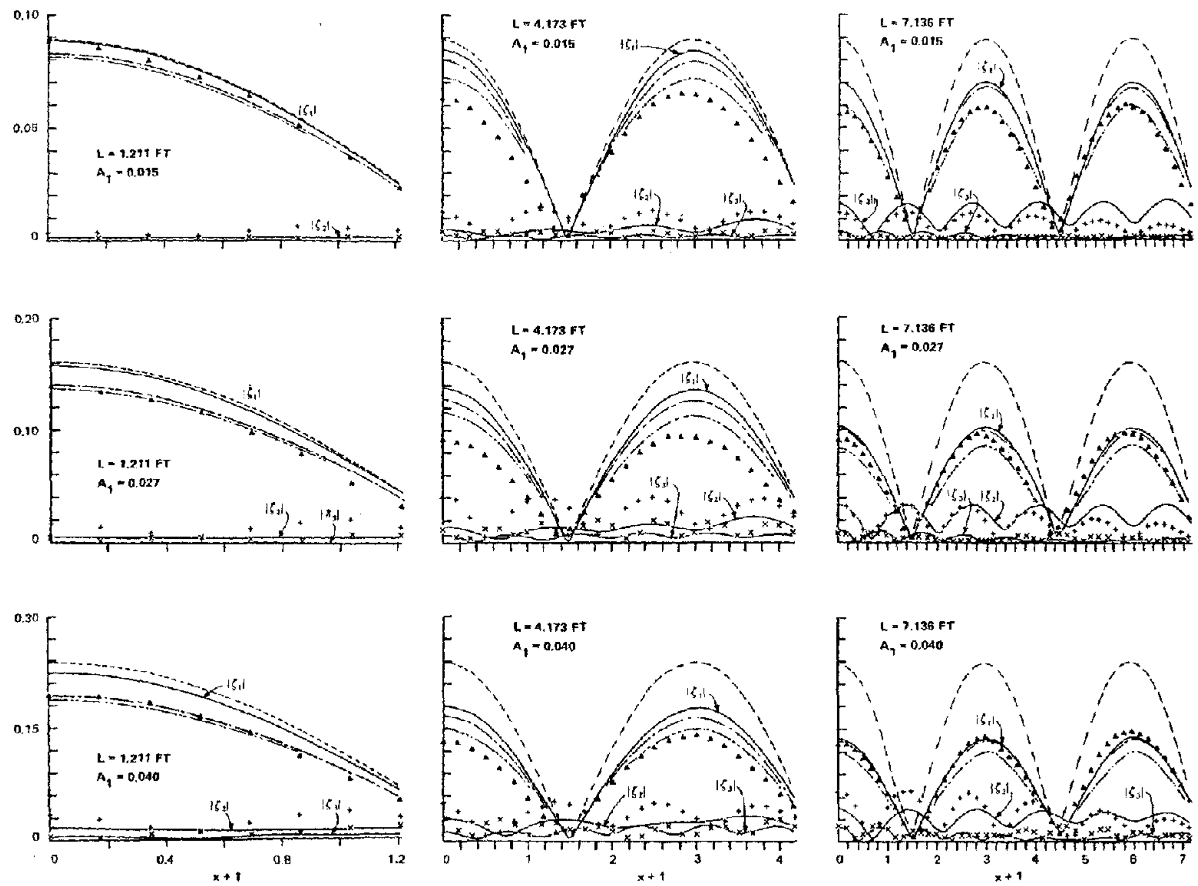
ÜNLÜATA, Ü. AND MEI, C.–C., 'EFFECTS OF ENTRANCE LOSS IN HARBOR OSCILLATIONS,' PROC. ASCE, JWWH, 101 : WW2, MAY 1975.

FIGURE 18

FIRST HARMONIC RESONANT AMPLIFICATION AT CORNER
$x = -B$, $y = B/2$ AS FUNCTION OF 2a/B FOR FIRST MODE $k_{0,1}B = \pi$
[OR $\eta_1 \propto \cos(2\pi y)/B$]

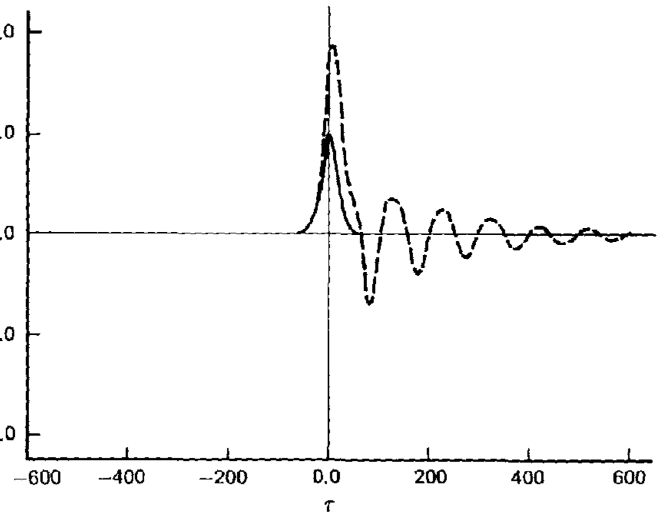
An example from a recent paper by Roger and Mei (1978) is presented in Figure 19 with regard to the steady-state nonlinear response problem with viscous effects. These are results of both analysis and experiments for a rectangular harbor, fully open, in which three different modes of oscillation are shown. (The length of the harbor and the width of the harbor were changed in these experiments to insure a constant length to width ratio.) The curves are for the first three harmonics. The wave heights were quite small in an absolute sense, being only a couple of millimeters. The agreement between theory and experiment is quite good for the fundamental mode for the case in which entrance dissipation is included, entrance dissipation being more important than nonlinear effects in this case. On the other hand, as you go to a higher mode of oscillation, the non-linear effects become more important than viscous effects. So there is a trade-off between these two factors affecting a real harbor. Now, whether we are really concerned that much with the Helmholtz mode in, for example, Southern California where we have an interaction with this large offshore basin, I think remains to be seen. So I am not sure that we can really neglect nonlinear effects for the tsunami problem; that is, for the type of harbor resonance we are talking about here, not considering runup, not considering the regeneration of waves, but considering only the basic problem of harbor resonance.

In Figure 20, an example of the transient response of a harbor is presented. This figure is from some work by Carrier and Shaw (1969) in which they investigated the linear problem analytically for conditions with and without the entrance channel shown here. The treatment was inviscid. What this shows is the amplitude as a function of time at a point inside the harbor caused by an incident wave which has the form of a pulse. This truly describes resonance in the sense that we have an impulse and then the energy is radiated out with a certain amount trapped. This leads to the rather long duration of the oscillations compared to the short duration of the input wave. The upper part of the figure is for a zero channel length while the lower part corresponds to a finite length entrance channel. It is seen that the action of the channel is similar to the effect of closing down the entrance and trapping the energy. This is the linear inviscid case; viscous effects would cause a more rapid decay of the oscillations.

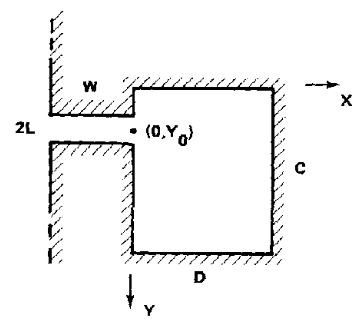


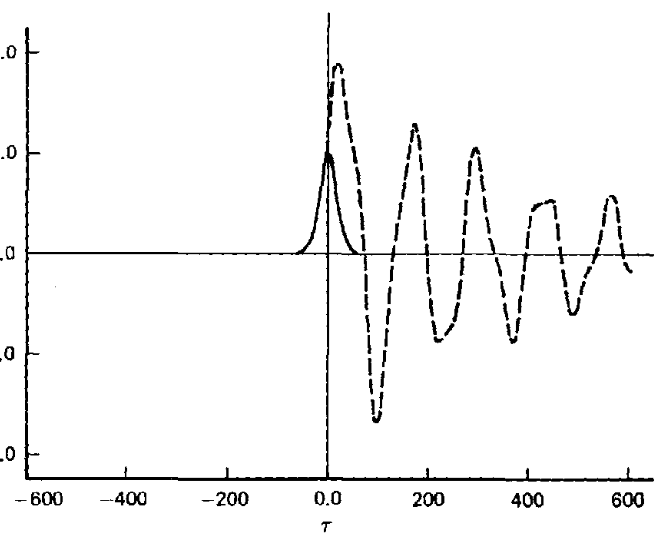
ROGERS, S.R. AND MEI, C.C., 'NONLINEAR RESONANT EXCITATION OF A LONG AND NARROW BAY,' J. FLUID MECH., 88:1, 1978.

FIGURE 19 NONLINEAR RESONANT EXITATION OF A LONG AND NARROW BAY



TRANSIENT RESPONSE TO SINGLE–CREST WAVE FOR D.B. WITH $\alpha = 20$ AND W = 0.





TRANSIENT RESPONSE TO SINGLE–CREST WAVE FOR D.B. WITH $\alpha = 20$ AND W = 1000.

CARRIER, G.F. AND SHAW, R.P., 'RESPONSE OF NARROW–MOUTHED HARBORS TO TSUNAMIS,' PROC. INT'L SYMPOSIUM ON TSUNAMIS, IUGG, OCT. 1969.

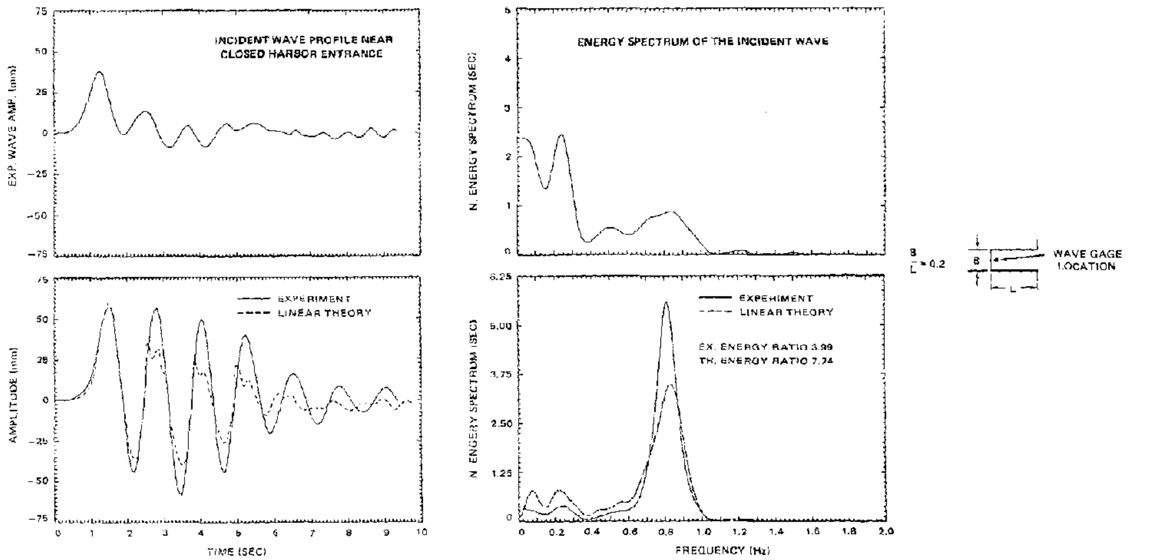
FIGURE 20 TRANSIENT RESPONSE TO SINGLE–CREST WAVE

Figure 21 shows some recent work of Lepelletier (1978) in connection with an investigation of the transient response of a harbor. (In his investigations both nonlinear and linear, viscous and inviscid, effects are investigated.) The example shown is for a fully open rectangular harbor with a width to length ratio of 0.2. The upper set of figures is for an incoming transient wave system with a maximum amplitude to depth ratio of 0.16. We see the shape and the spectrum of the incident wave system in the top pair of figures. Then we see both experimental observations and the predictions using a linear theory for the wave history at the backwall of the harbor, and, finally, the corresponding spectra. This is for a particular harbor length -- 30 cm -- such that primarily the second mode of oscillation was excited. The second set of figures is for the case where the harbor length was increased to 100 cm in order to extract the first mode of oscillation and the amplitude to depth ratio was about 0.5. The agreement between experiment and the linear theory can be seen. Considering the amplitude to depth ratio shown, the agreement is quite good. (However, it remains to be seen how good such agreement would be for a large Ursell number). We can consider this an impulsive-type wave and we see that with radiative and dissipative losses, the response dies-off quite rapidly.

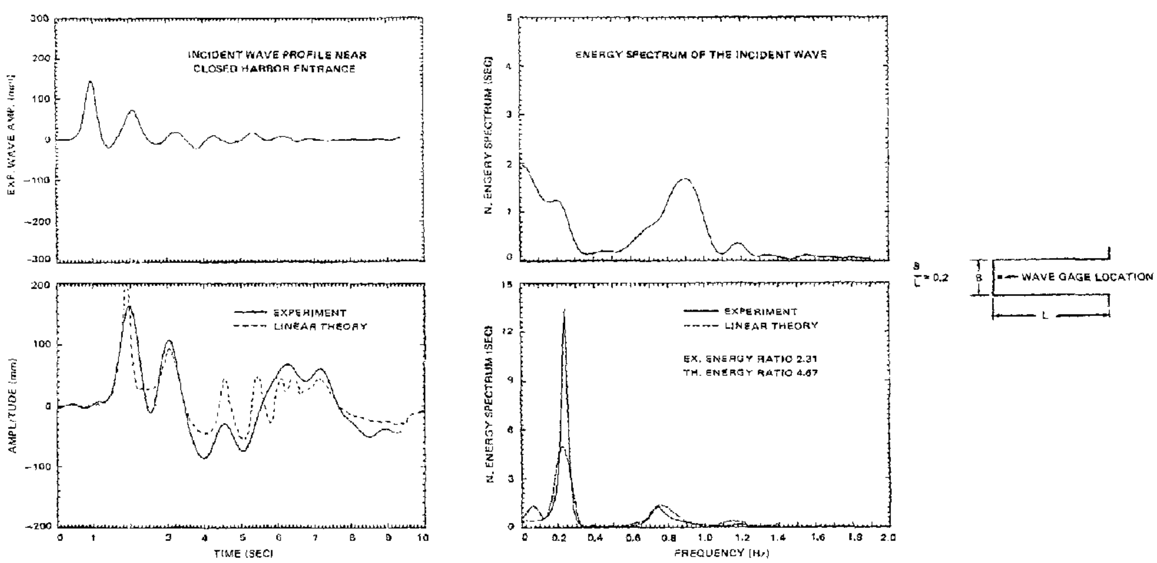
DISCUSSION

J. Miles - Fred, I would like to expound on what Munk and I thought we meant by the term "harbor paradox" because I think it is being used in various senses here. It was not simply that as you narrow down the entrance to the harbor the response would increase. The argument was a little more delicate than that -- as we originally gave it, it was not delicate enough, as it turned out.

Namely, if you have a simple oscillator and you excite it with a sinusoidal wave, then the intensity of the response -- the square of the amplitude -- will go like Q squared. On the other hand, if you excite it with a broadband excitation, then since you have superimposed the broadband spectrum of the input over the resonance curve, it turns out that the input is inversely proportional to the Q, so that the net intensity of the response -- we argued -- is proportional to the Q. In effect, therefore, by increasing the Q by



L = 30 cm, h = 14 cm, a/h = 0.16



L = 100 cm, h = 14 cm, a/h = 0.52

LEPELLETIER, T.G., 'RESPONSE OF HARBORS TO TRANSIENT WAVES: A PROGRESS REPORT,' TECH. MEMO. 78–3, W.M. KECK LAB. OF HYDR. AND WATER RES., CALTECH, MARCH 1978.

FIGURE 21 RESPONSE OF HARBORS TO TRANSIENT WAVES narrowing the harbor mouth, you increase the response. We were well aware of the fact that you also of course take longer, but if you have a broadband input then by definition if it lasts long enough to have the low frequencies in it, it lasts long enough to excite these low frequencies. So I think that that argument against the paradox is something of a red herring.

Now in fact we did not even look carefully enough at what our own mathematics showed us, that for the Helmholtz mode the paradox as we envisioned it, without any other damping, was true -- that is to say the response went up, the intensity, like Q. But for the higher modes it does not happen that way, and the reason there is a little more subtle. The Helmholtz mode, viewing it as I would as an ex-electrical engineer, is a simple series resonant circuit, just like a simple oscillator, an LC circuit, and it responds as a simple resonant circuit.

The higher modes of a harbor with an open mouth, radiating out, respond like a series resonance and a parallel resonance in close proximity. That is, as I would envision it, you have an LC parallel circuit and in series with it, the mouth of the harbor, another impedence that can make that whole thing go series resonant. The proximity of these two resonant circuits makes things work out in such a way that the response of the higher modes, other than the Helmholtz mode, is independent of the Q. That is without wave damping -- when you put in the real damping as opposed to the radiation damping, then, of course, you lower the response further. So narrowing the mouth of the harbor, in reality actually decreases the response of all modes except the Helmholtz mode.

G. Carrier - I know it's been a long day but I have one more harbor-like phenomenon that is very different in it's emphasis. If I may I'll take five minutes and tell you about it.

The phenomenon came to my attentation in 1969 when I was visiting Gaylord Miller and the Hawaii group, and he called it to my attention as being something quite odd. Immediately after the Alaskan earthquake -- long before any tsunami could have gotten there -- the tide gage in Nawiliwili Bay started to show a five minute oscillation which persisted, if I remember correctly, for about three days. He gave me a piece of paper -- either a report or a reprint, I remember a reprint but I may be wrong and I cannot find it or any reference to it. If anyone knows where that thing might be, what the repository is, I would really appreciate getting it because I would like to make sure that three days is accurate. That means a 2000-cycle e-folding time -- really a high Q system!

You can look at it as a rather well defined not-quite rectangular channel with an open mouth, there is no constriction, yet it has this very high Q. Now the five minute period is not at all mysterious -- Lautenbacher showed that that is the period of the first sloshing mode. The fascinating conjecture -- which has nothing to do with what else I am going to say -- is that the whole island moved about a centimeter laterally. Take a cake pan, put some water in it, jerk it, and it will slosh. But if it is open at the end, then you would expect the energy to leak out very, very quickly -- in a couple of $L/\sqrt{gh}$'s. But it does not.

The thing you forget is that although it widens up to the sea, it also gets deeper. So you can sneak up on what is going on in two steps, or at least I like to do it this way. You can ask, first, what would happen if we had no depth change, started the sloshing, and let it go? The energy would leak out extremely rapidly and nothing like what we really saw would happen. You can do the analysis by doing a separation of variables -- you put in the trigonometric dependence, you put in the $e^{i\omega t}$, and you get an ordinary differential equation. Maybe you have to take a transform or something since you are doing a transient problem, but that does not really matter.

In fact, the other thing it pays to do if you are going to play this game is to forget this back boundary and pretend you have a wave coming in from the back at an oblique angle going seaward so that it's almost a sloshing mode, and you ask what is the reflection coefficient. If you can answer that question for real frequencies, you can extend to complex frequencies rather trivially. So the transmission coefficient problem is completely equivalent to the decay time problem.

Now, if it gets deeper near the entrance in an appropriate way, there is a turning point, and the reflection coefficient is precisely unity. If we just had straight walls and a deepening, that would account for the whole thing. It isn't quite that simple, but it's basically that. Although you have to do some moderately elegant things in order to show it, if the size of what tends to make it leak does not compete successfully with the rate at which things deepen, then the reflection coefficient is still unity. Now, for real systems, of course, the deepening stops while the widening continues and therefore the leakage always eventually wins. What that means is that it becomes a two turning-point problem.

The mathematics must be very closely related to that trapping stuff of Longuet-Higgins somebody was talking about earlier -- but by no means precisely, of course. Nevertheless, it becomes a two turning-point problem, but as long as they are far enough apart, the reflection coefficient is awfully close to unity. The fact is, of course, there are all kinds of open-mouthed things, some of which must deepen "faster" than they widen, and it seems to me that the long time ringing of these somewhat non-deep depressions in the landscape, may well be accounted for by that peculiar very small transmission coefficient -- very large reflection coefficient -- so that the decay is primarily frictional. I suspect this is pertinent to the ringing of some things after tsunamis, but of course I don't know -- it's a conjecture.

B. LeMehaute - I think this may explain the ringing observed at Santa Monica Bay.

F. Raichlen - Perhaps so, but these are such small indentations in a very long coastline, that I wonder if it's not the whole offshore area oscillating, and that the harbor is really not doing anything selective.

C. Mei - It seems that the problem just described is another way of generating "ledge" waves by linear excitation, by resonance. From the shallow side to the deep side there is a trapped wave very similar to edge waves.

G. Carrier - Let me point out, parenthetically, it sure isn't the Helmholtz mode.

J. Miles - It's very much like the open-ended organ pipe which is a very inefficient radiator since its mouth is so narrow compared to the wavelength.

R. Wiegel - George, if you've excited this, if you've got it oscillating, and you've got a spectrum out in the ocean say for a week with energy distributed all over -- would the little bit of energy available at that frequency keep it moving at that frequency?

G. Carrier - Yes -- well, I'm not sure I understand but let me say this. When a tsunami hits a harbor it dumps some water in it. That means there's some energy in there. A lot of it's going to leak out fast, the nonlinearities are going to switch it around a lot, but after a while, for practical purposes, we're operating the various normal modes of the thing including this high Q mode. There's no reason in the world why that shouldn't keep going just as though it were all by itself, even though superimposed on it are the consequences of all the other things around the place and incident on the harbor. To be sure, there are obviously some small non-linear interactions which may degrade or upgrade or something, this kind of thing, but I'm not looking for a meticulous explanation, I'm just asking, how can energy in that part of the spectrum hang around so long? It's just possible this topographic business could account for it.

F. Raichlen - The problem with all these measurements is that usually we have only one location, and we try to infer a great deal over a wide area, with no phase information, at all.

G. Carrier - Yes, absolutely.

B. Wilson - In my post-mortem study of the Alaskan tsunami, I investigated the oscillating properties of the shelf off Crescent City and came up with the identical periodicity found by Bob Wiegel in an analysis of the energy peaks in the tsunami. So it seems like the 1960 tsunami was exciting shelf-oscillations. We analyzed tsunami records for Los Angeles-Long Beach -- three locations -- and came up with numerous energy peaks. Most of these seemed to correlate quite well with studies I had made of the oscillating properties of San Pedro Bay and shelf which is a sort of complicated shelf, dropping-off sharply. One can analyze that situation approximately, at least by a normal analysis which involves Bessel functions and so on. What comes out of it is that there are prominent bay periodicities of 60 minutes, 33 minutes, 22 minutes, and so on. These are periodicities which we also pick up in the spectral analysis of the tide gage records. Sixty minutes seems to be one of the prominent periods at which the inner harbor can resonate so there is undoubtedly some resonance, but by and large the harbor seems to be responding to the bay oscillations. There were residual peaks of longer period which couldn't be explained -- of the order of 1.7 or 1.8 hours for the Alaskan tsunami and 2.5 hours for the Chilean tsunami. Those have not been explained. They could be related to the offshore island basins, or they could be a residual longwave train reaching the harbor from the earthquake source.

F. Raichlen - I guess one question which could be raised is, is it only of academic interest to try to find out what these responses are? In a physical sense, what can one do?. When you're talking about waves of periods on the order of hours or tens of minutes, there's very little one can do to prevent this. Only in one case that I am aware of in Japan was a significant modification made to the entrance to a bay to specifically handle the tsunami problem.

R. Wiegel - Don't restrict yourself to where we have things right now -- we do build new things. We build new terminals and these are exactly the sorts of things that are important. It may be very important for the design of an iron-ore terminal, or a bauxite, whether or not you get trapping of long-period waves which may not be tsunami induced, but if you've got these modes you have the same problem. For example we did some work for a terminal in Peru and this was one thing that had to be looked at. It has a harbor oscillation and every third or fourth time you'd bring in a 120,000 ton bulk carrier and moor it, you'd break the lines or move so much you can't load the ore.

F. Raichlen - You're certainly correct.

Question from Audience - There is a lot of energy in the ocean at those low frequencies for an extended time after a tsunami, so couldn't the long harbor ringing occur even for a low Q if it's continually driven? If the Q were, say, a 100 you'd think the harbor would be ringing all the time from things that are happening, the tides and so forth.

F. Raichlen - If there were energy of sufficient magnitude at long periods, that would certainly be true. And it opens up the important question of the forcing function.

5 NUMERICAL ASPECTS OF TSUNAMI MODELING

CHAIRMAN : C. C. MEI
RECORDER : J. R. HOUSTON

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ASPECTS OF NUMERICAL METHOD FOR LONG WAVE DIFFRACTION

BY
CHIANG C. MEI

Outline of Method

As the principle of superposition can be applied in a linearized theory, the sea response to a transient incident wave of amplitude spectrum $A_I(\omega)$ may be expressed as:

$$\Phi(x,\vec{t}) = \int_{-\infty}^{\infty} d\omega\, e^{-i\omega t} \quad A_I(\omega)\phi(\vec{x},\omega) \tag{1}$$

where ϕ is the simple harmonic response to an incident wave of unit amplitude. Assuming long waves in shallow water, the boundary value problem for ϕ is specified by:

$$\nabla \cdot (h\nabla\phi) + \frac{\omega^2}{g}\,\phi = 0 \qquad \text{in the fluid A} \tag{2}$$

and

$$h\,\frac{\partial\phi}{\partial n} = 0 \qquad \text{on the coast B} \tag{3}$$

Breaking loss is ignored for tsunamis. For convenience only, islands in an unbounded ocean are considered; modifications for a mainland are possible. Let the region of variable depth be limited to a finite neighborhood of the coastline in question so that $h \rightarrow h_o$ = constant sufficiently far away from B. The incident wave may then be expressed as:

$$\phi_I = e^{ik_o r\cos(\theta-\alpha)} \tag{4}$$

with $k_o = \omega(gh_o)^{-1/2}$ and α being the direction of the incident wave. The scattered waves which must be outgoing at infinity then satisfies:

$$\sqrt{k_o r}\ (\frac{\partial}{\partial r} - ik_o)\ (\phi - \phi_I) \rightarrow 0 \qquad k_o r >> 1 \qquad (5)$$

Integral equations methods of various kinds have been applied to this problem. However, when part of the sea depth is variable, it is not possible to construct a Green's function. Past methods by using an unnatural fundamental solution for a constant depth requires the discretization of the entire area of variable depth, hence lead to an integral equation whose approximate matrix equation is very large and consists of matrix elements relatively complicated because of the singularity of the fundamental solution, (Lautenbacher (1970), Mattioli (1978)). On the other hand, direct application of discrete method is also uneconomical for a problem with an infinitely large domain. Therefore a first step toward computation economy is to use direct discretization only over the region of constant depth. The second step is to seek a scheme which ensures the continuity across the (artificial) boundary of the two regions with the least number of fallacies which are not inherent in the original boundary value problem.

The hybrid element method (Chen and Mei 1974) is devised to meet these goals. In the region of constant depth a contour C is drawn. We denote the fluid within C by A and the potential by ϕ, and the fluid outside C by $\bar{A}$ and the potential by $\bar{\phi}$. In the region $\bar{A}$ the potential which satisfies (2) and (5) may be expressed analytically as:

$$\bar{\phi} = \phi_I + \sum_{n=0}^{\infty} (\alpha_n \cos n\Theta + \beta_n \sin n\Theta) H_n^{(1)}(k_o r) \qquad (6)$$

Then the stationarity of the following functional:

$$F(\phi,\bar{\phi}) = \iint_A \frac{1}{2}\left(h(\nabla\phi)^2 - \frac{\omega^2}{g}\phi^2\right) + \int_C \frac{1}{2}(\bar{\phi}-\phi_I)\frac{\partial}{\partial n}(\bar{\phi}-\phi_I)$$

$$- \int_C h\phi \frac{\partial}{\partial n}(\bar{\phi}-\phi_I) - \int_C h\phi \frac{\partial\phi_I}{\partial n} + h\int_C \phi_I \frac{\partial}{\partial n}(\bar{\phi}-\phi_I) \qquad (7)$$

-----

is the equivalent to the original boundary value problem.

Indeed, the first variation of F due to arbitrary but small variations of ϕ and $\bar{\phi}$ may be manipulated to give:

$$\delta F = - \iint_A (\nabla \cdot h\nabla\phi + \frac{\omega^2}{g} \phi)\delta\phi + \int_C h(\frac{\partial\phi}{\partial n} - \frac{\partial\bar{\phi}}{\partial n})\delta\phi + h \int_C (\phi - \bar{\phi}) \frac{\partial\delta\bar{\phi}}{\partial n}$$

$$+ \frac{1}{2} \int_C h[\delta\bar{\phi} \frac{\partial}{\partial n} (\bar{\phi} - \phi_I) - (\bar{\phi} - \phi_I) \frac{\partial\delta\bar{\phi}}{\partial n}] \qquad (8)$$

Clearly from the first integral, Equation (2) is seen to be the Euler-Lagrange equation, the second and third integral imply that continuity of ϕ and $\partial\phi/\partial n$ are the natural boundary conditions. The last integral may be shown to vanish if Green's theorem is applied over $\bar{A}$ using the fact that $\phi - \bar{\phi}_I$ and $\delta\bar{\phi}$ both satisfy the Helmholtz equation and the radiation condition.

Since all the integrals in Equation (7) are within C or on C, the finite region A may be divided into a limited number of finite elements. A finite number of series terms for $\bar{\phi}$ is also needed. Let ϕ in A be represented by the interpolating functions $N_i(\vec{x})$:

$$\phi = \sum_i \phi_i N_i(\vec{x}) \quad \text{in A} \qquad (9)$$

where ϕ_i are the nodal unknowns. Equation (7) may be extremized to give a matrix equation:

$$\begin{Bmatrix} [K_1] & & \\ & & [K_2] \\ & [K_2^T] & [K_3] \end{Bmatrix} \begin{bmatrix} \phi_1 \\ \phi_2 \\ \alpha_1 \\ \beta_1 \end{bmatrix} = \begin{bmatrix} q_1 \\ q_2 \\ \\ \\ \end{bmatrix} \qquad (10)$$

The special features of the present variational formulation are:

(i) The stiffness matrix is symmetric and banded.

(ii) If the contour is a closed circle, K_3 is diagonal; otherwise K_3 is full but the number of finite elements may be reduced.

A number of complex geometry and topography have been studied by this method including the offshore harbor of the Atlantic Generation Station planned but not constructed by New Jersey Public Gas and Electricity Co. (Chen and Mei, 1974) and the tsunami response near the Hawaiian Islands to the earthquakes of Chile and Alaska, Houston et al (1977). Substantial economy is reported by those who have also experimented with other methods (Houston 1976; Matioli 1978).

A Theoretical Property Regarding the Critical Frequencies

As Lamb (1932) has shown, ordinary integral equation methods for an exterior problem involving the Helmholtz equation may break down at certain critical frequencies. Take the example of a uniformly pulsating circular cyclinder. The governing equations are:

$$\nabla^2\phi + k^2\phi = 0 \qquad r > a \tag{11}$$

$$\frac{\partial\phi}{\partial n} = U \qquad r = a \tag{12}$$

$$\phi \text{ outgoing} \qquad kr \to \infty \tag{13}$$

The exact solution is:

$$\phi = \frac{U}{k}\frac{H_o^{(1)}(kr)}{H_o^{(1)'}(ka)} \tag{14}$$

is perfectly well-behaved for all k. However, if the source distribution method is used then the integral equation is:

$$-U = \lim_{r \to a} \int_{+C} \sigma(\vec{r}') \frac{\partial}{\partial n'} \frac{i}{4} H_o^{(1)} (k|\vec{r}-\vec{r}'|) d\vec{r}' \tag{15}$$

Formally, the solution for the source strength can be found to be:

$$\sigma = -2iU[\pi ka\, J_o(ka) H_o^{(1)'}(ka)]^{-1} \tag{16}$$

and the potential may be found subsequently to be just Equation (14). Nevertheless, at the zeroes of $J_o(ka)$, σ is infinite. If a discrete approximation is made of the integral equation (15), the resulting matrix is ill-conditioned. This numerical difficulty is the fault of the integral equation method and is caused by the fact that the homogeneous versions of Equation (15) has nontrivial solutions at these critical frequencies which correspond to the Eigen-frequencies of the interior Dirichlet problem. Although methods can be devised to cure this difficulty, it is nevertheless another positive feature of the present hybrid element method that its solution is always unique for all frequencies, implying that there are no critical frequencies, hence no ill-conditioning. The proof of uniqueness if given in Aranha, Mei, and Yue (1979).

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TSUNAMI NUMERICAL MODELING: AN OVERVIEW

BY
JAMES R. HOUSTON

Several numerical models have been used in recent years to simulate tsunami propagation and interaction with land masses. These models usually solve similar equations but often employ different numerical techniques and are applied to different segments of the total problem of tsunami propagation from generation regions to distant areas of runup. For example, several numerical models have been used to simulate the interaction of tsunamis with islands. These models have used finite difference, finite element, and boundary integral methods to solve the linear long wave equations. However, more important than the particular numerical technique that is used is the question of whether these models that solve relatively simple equations provide reasonable simulations of tsunamis for engineering purposes and, if so, what are some of the limitations that can be expected.

Some of the earliest numerical modeling of tsunamis involved models that were used to generate tsunamis and to propagate them across the deep ocean. These models use finite difference methods to solve the linear long wave equations on a spherical coordinate grid that covers a section of the Pacific Ocean. Transmission boundary conditions are used on open boundaries to allow waves to escape from the grid instead of reflecting back into the region of computations. One of the models (Hwang et al 1972) uses an implicit-explicit formulation developed by Leendertse (1967) and the others (Chen 1973 and Garcia 1976) use explicit formulations. These models use as an initial condition an uplift of the water surface in the source region that is identical to the permanent vertical ground displacement produced by the tsunamigenic earthquake. Hammack (1972) has demonstrated that it is this permanent vertical ground displacement and not the transient motions that occur during the earthquake that determine the far-field characteristics of the resulting tsunami. In addition, Hammack (1972) has shown that the small-scale details of the permanent ground deformation produce waves that are not significant far from the source region. Thus, distantly generated tsunamis can be modeled if the major features of the permanent vertical ground deformation are known. Hwang et al (1972) used data of the permanent vertical ground displacement of the 1964 Alaskan tsunami collected by Plafker (1964) in a simulation of the 1964 tsunami. They demonstrated good agreement between the initial portion of a recording of the 1964 tsunami off the coast of Wake Island (Van Dorn 1970) and a numerical model simulation of this tsunami.

Tsunami destruction in the Hawaiian Islands has directed interest toward the development of numerical models to simulate the interaction of tsunamis with islands. These models all solve the linear long wave equations. Vastano and Reid (1967) used a transformation of coordinates technique to map an arbitrary shoreline of an island into a circle in the image plane. The finite difference solution employed a grid that allowed greater resolution in the vicinity of the island than in the deep ocean. Vastano and Bernard (1973) extended the technique to a multiple island system. Since this transformation of coordinate method allows only a single island to be represented in detail, Bernard and Vastano (1977) developed a model that employs a standard rectilinear finite difference grid that covers all of the Hawaiian Islands. Lautenbacher (1970) developed a numerical model that solved an integral equation. Finally, Houston (1978) used a finite element numerical model based upon a model developed by Chen and Mei (1970) for harbor oscillation studies to calculate the interaction of tsunamis with the Hawaiian Islands.

The finite element model used by Houston (1978) employed a finite-element grid (Figure 1) that telescoped from a large cell size in the deep ocean to a very small size in shallow coastal waters. The grid covered a region that included the eight major islands of the Hawaiian Islands. Although time periodic motion was assumed in the solution, the interaction of an arbitrary tsunami waveform with the islands was easily determined within the framework of a linear theory by superposition. Using a generation and deep ocean propagation numerical model and historical data of ground uplifts for the 1960 Chilean tsunami and for the 1964 Alaskan tsunami, Houston (1978) determined deep-ocean waveforms for these two tsunamis. These waveforms were used

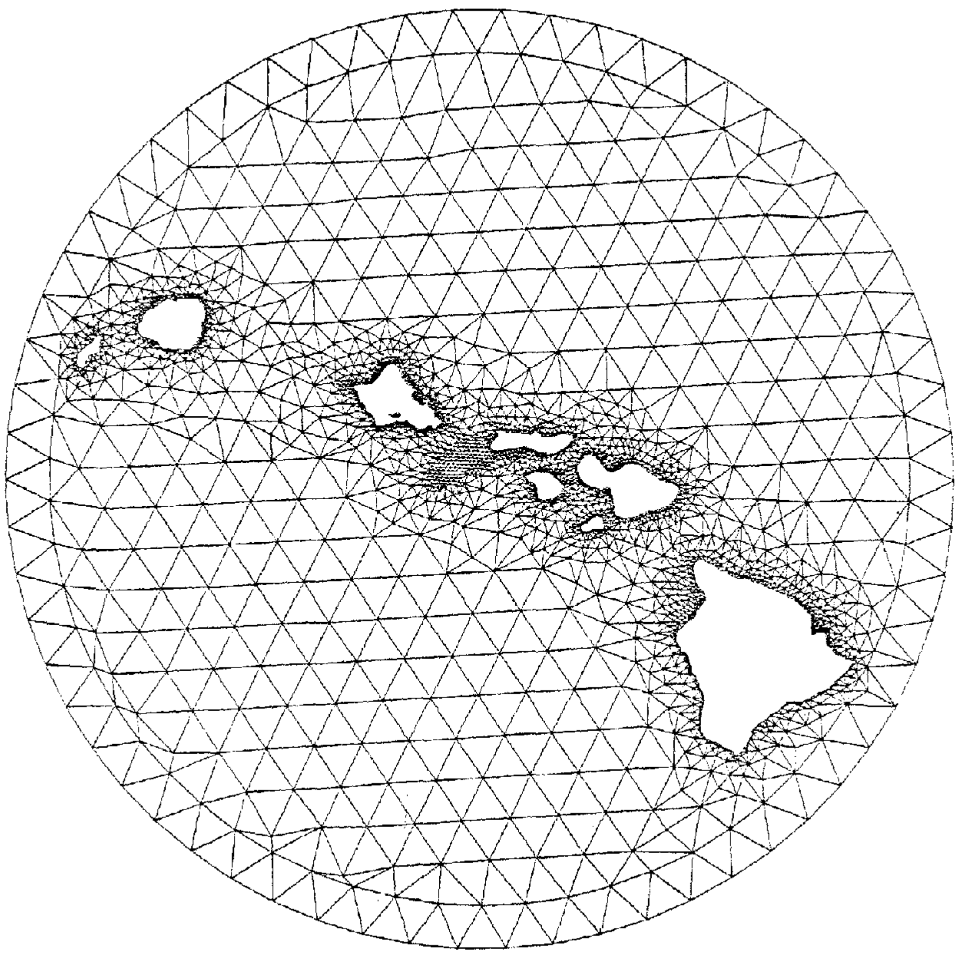

FIGURE 1 FINITE ELEMENT GRID FOR HAWAII

as input to the finite element model that propagated the tsunamis to shore. Figures 2 through 5 show comparisons between the first few major waves calculated by the finite element model and historical tide gage recordings of these tsunamis at Kahului, Hilo, and Honolulu, Hawaii.

Numerical models have been developed to calculate tsunami interaction with continental coastlines. Aida (1969) has developed a two-dimensional explicit finite difference numerical model that solves the linear long wave equations. More recently, Aida (1978) applied a similar model that used a telescoping finite difference grid to study historical tsunamis off the coast of Japan. A crude general agreement was shown between the numerical model calculations and historical tide gage recordings of these simulated tsunamis. Differences between the recorded and measured tsunamis were attributed to inaccuracies in the seismic fault model used to determine the vertical displacement of the sea bottom. Houston and Garcia (1978) used a two-dimensional finite difference numerical model based upon the original formulation of a tidal hydraulics numerical model by Leendertse (1967) to study tsunami interaction with the west coast of the United States. This model solves long wave equations that include nonlinear and dissipative terms. To verify the model, Houston and Garcia (1978) used a generation and deep ocean propagation numerical model to generate the 1964 Alaskan tsunami and propagate it to the west coast of the United States. The resulting waveform was used as input to this nearshore numerical model that propagated the tsunami to the shoreline. Good agreement (Figures 6 and 7) was demonstrated between tide gage recordings of the 1964 tsunami at Crescent City and Avila Beach, California, and the numerical model calculations. A time-stepping two-dimensional finite element numerical model has been recently developed by Kawahara et al (1978). Unlike most finite element models that are implicit and require costly matrix inversions at each time step, this model uses a two-step explicit formulation. A simulation of a historical Japanese tsunami was performed by Kawahara et al (1978) and a crude general agreement was demonstrated between tide gage recordings and numerical calculations (with differences attributable to lack of knowledge concerning the ground displacement that generated the tsunami). Finally, Chen et al. (1978) have developed

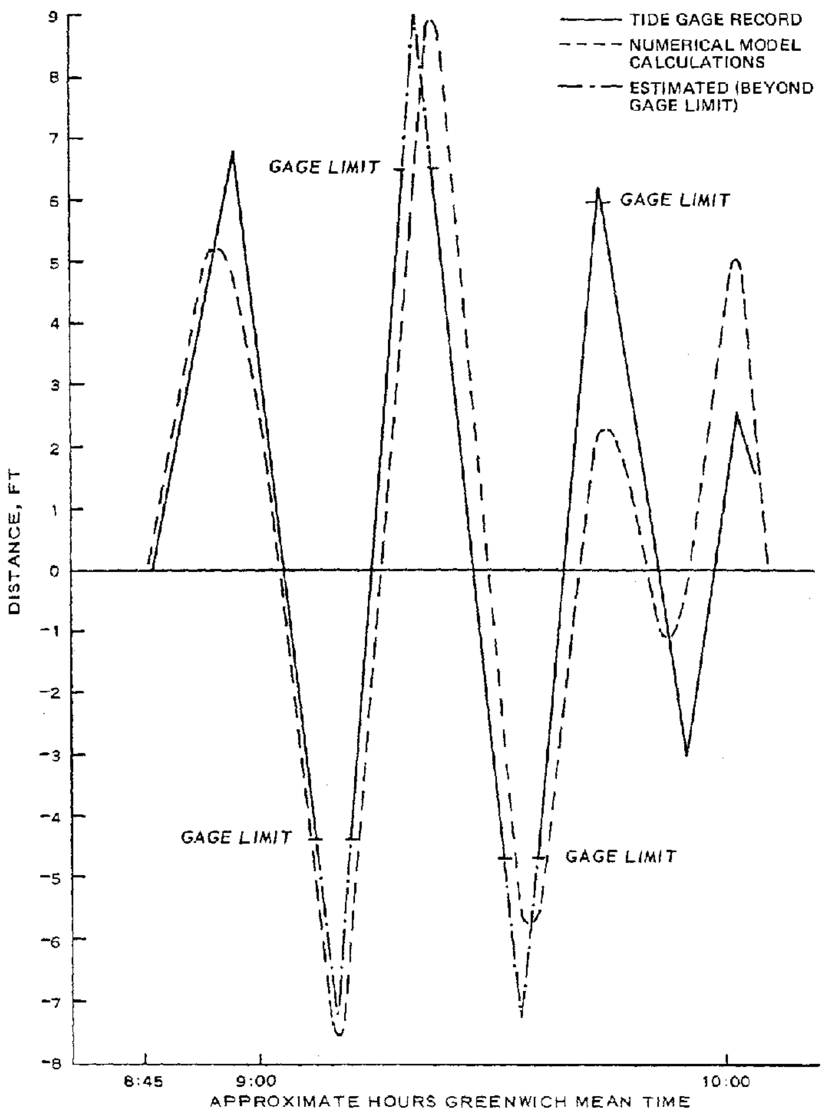

FIGURE 2 COMPARISON AT KAHULUI, HAWAII, FOR 1964 TSUNAMI

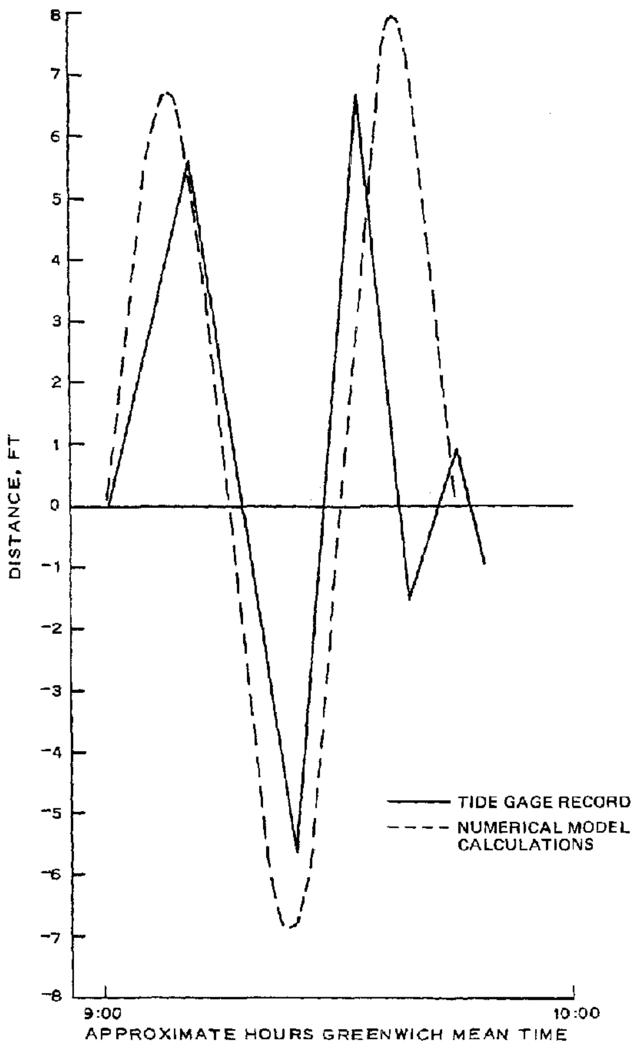

FIGURE 3 COMPARISON AT HILO, HAWAII, FOR 1964 TSUNAMI

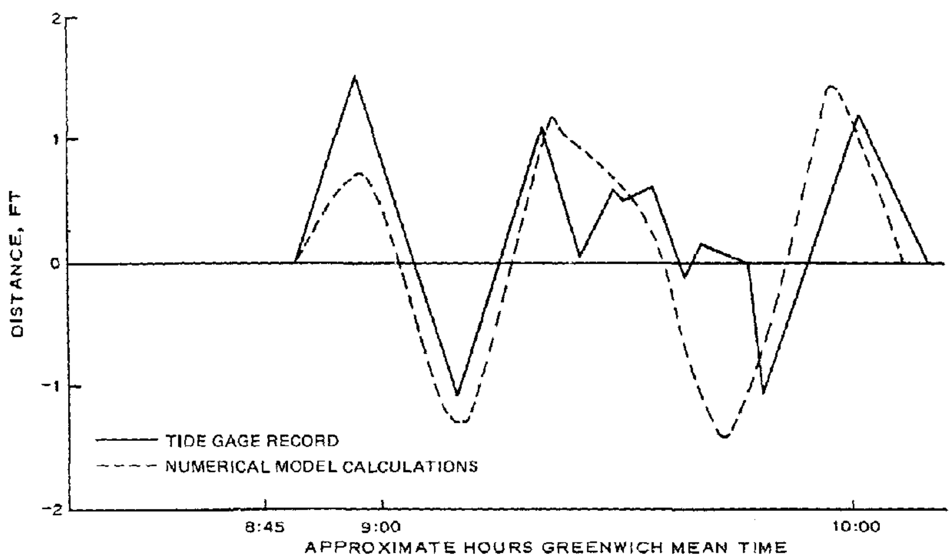

FIGURE 4 COMPARISON AT HONOLULU, HAWAII, FOR 1964 TSUNAMI

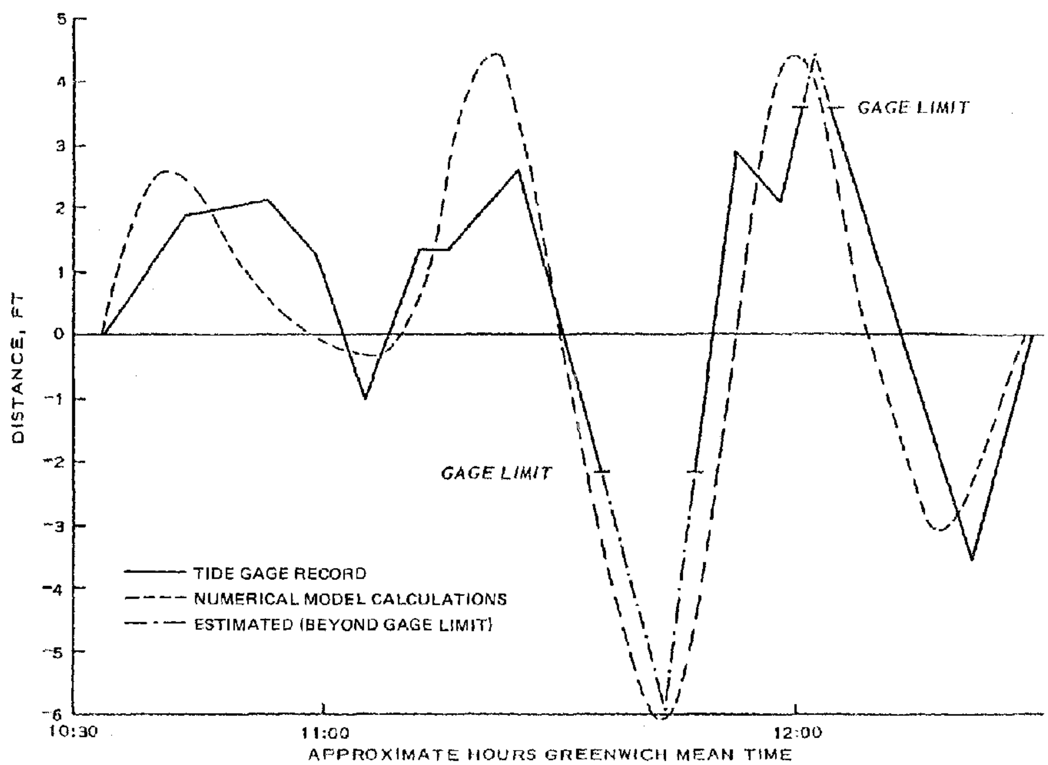

FIGURE 5 COMPARISON AT HONOLULU, HAWAII, FOR 1960 TSUNAMI

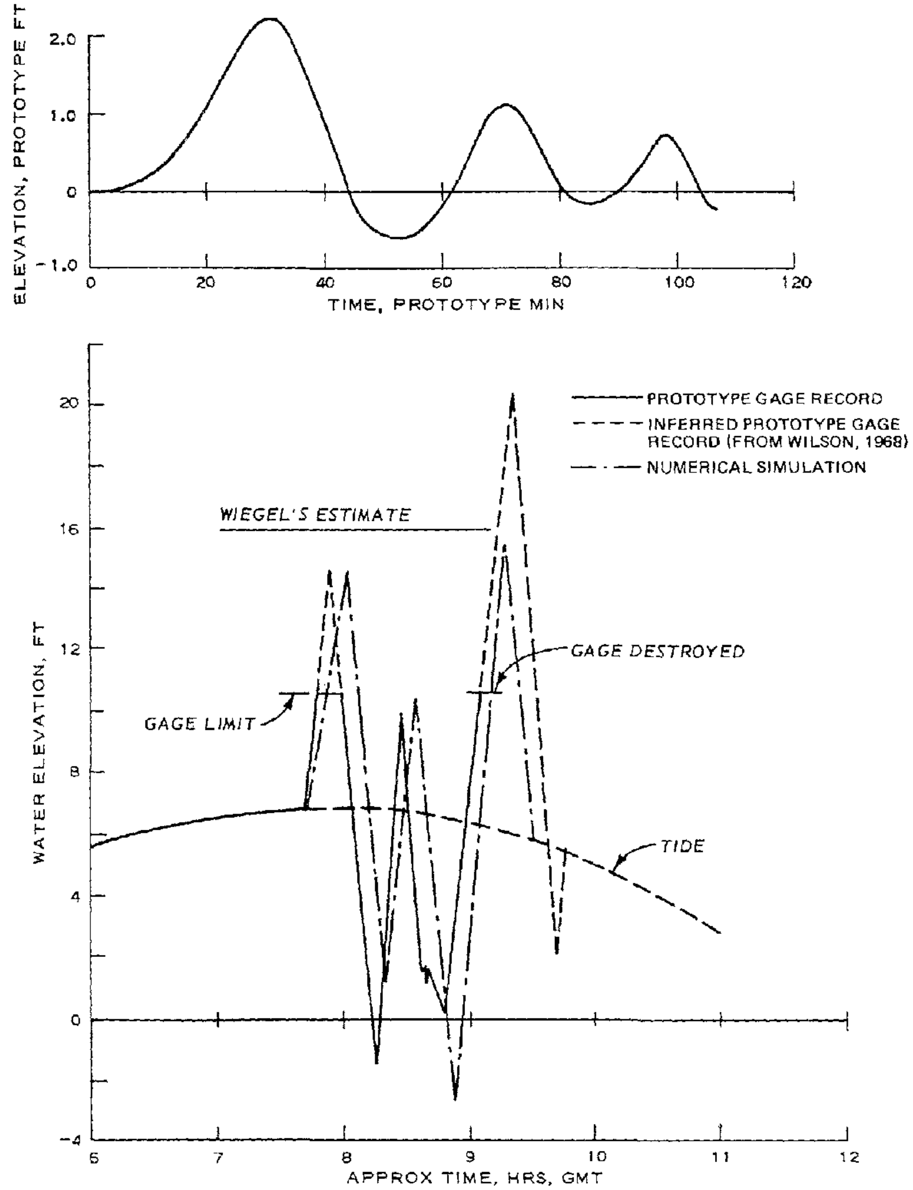

FIGURE 6 COMPARISON AT CRESCENT CITY, CALIFORNIA, FOR 1964 TSUNAMI

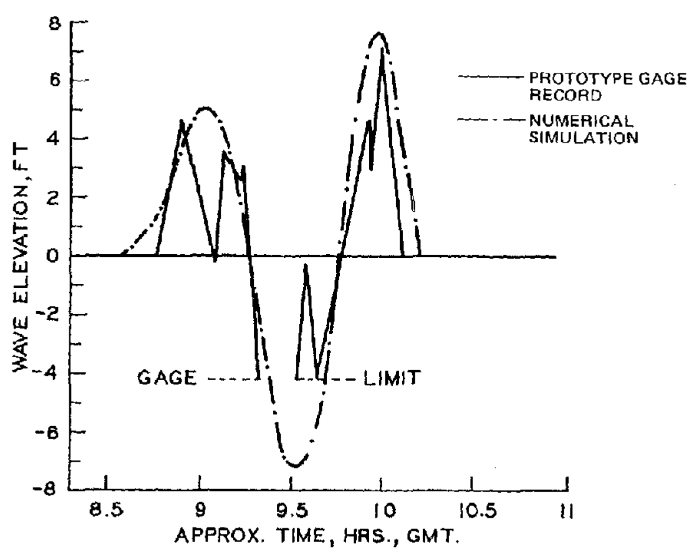

FIGURE 7
1964 TSUNAMI FROM ALASKA RECORDED AT AVILA BEACH, CALIFORNIA

a two-dimensional finite difference numerical model that solves Boussinesq type equations. In a simulation of a tsunami off the coast of California, Chen et al (1978) found that a numerical model solving long wave equations including nonlinear terms calculated a waveform almost identical to the waveform calculated by the model solving the Boussinesq equations.

The final phase of tsunami propagation involves the inundation of previously dry land. Tsunamis usually appear in the form of rapidly rising water levels and rarely in the form of bores. Small bores do form at the leading edge of a tsunami during propagation over flat land; however, the extent of inundation is governed by the large (with a fairly flat surface slope) mass of water behind the bore face. Bretschneider and Wybro have developed a one-dimensional numerical model to calculate tsunami inundation. Frictional effects, but not time dependent or flow divergence and convergence effects, are included in the model calculations. Houston and Butler (1979) have developed a two-dimensional and time-dependent numerical model that calculates land inundation of a tsunami. The model solves long wave equations that include bottom friction terms. A coordinate transformation was used to allow the model to employ a smoothly varying grid (Figure 8) that permits cells to be small in the inundation region and large in the ocean. The model was verified by simulating the 1964 Alaskan tsunami at Crescent City, California. Figure 9 shows a comparison between recorded water levels in the developed area of Crescent City and the numerical model calculations. Good agreement also was demonstrated between high water marks recorded by Magoon (1965) for this tsunami and the numerical model calculations.

In the preceding paragraphs, it was shown that numerical models solving long wave equations (often linear long wave equations) have been successfully used to simulate tsunami generation and propagation across the deep ocean, tsunami propagation from the deep ocean to the shoreline, and tsunami inundation of previously dry land. Good comparisons between historical measurements of tsunamis and numerical simulations of these tsunamis have been demonstrated for the first few waves of very long period tsunamis such as the 1960 Chilean tsunami and the 1964 Alaskan tsunami. Using an asymptotic solution of the

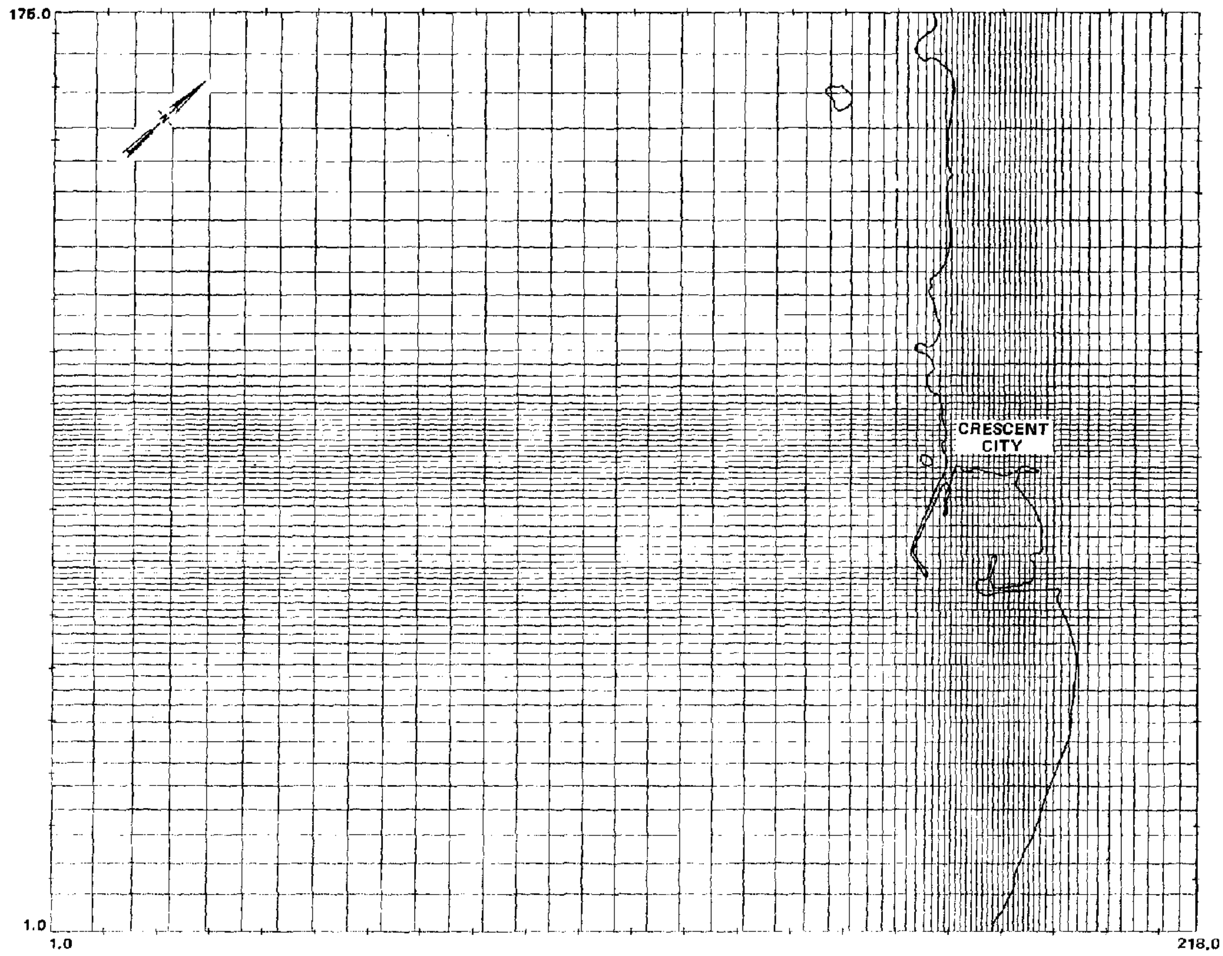

FIGURE 8 VARIABLE GRID FOR CRESCENT CITY, CALIFORNIA

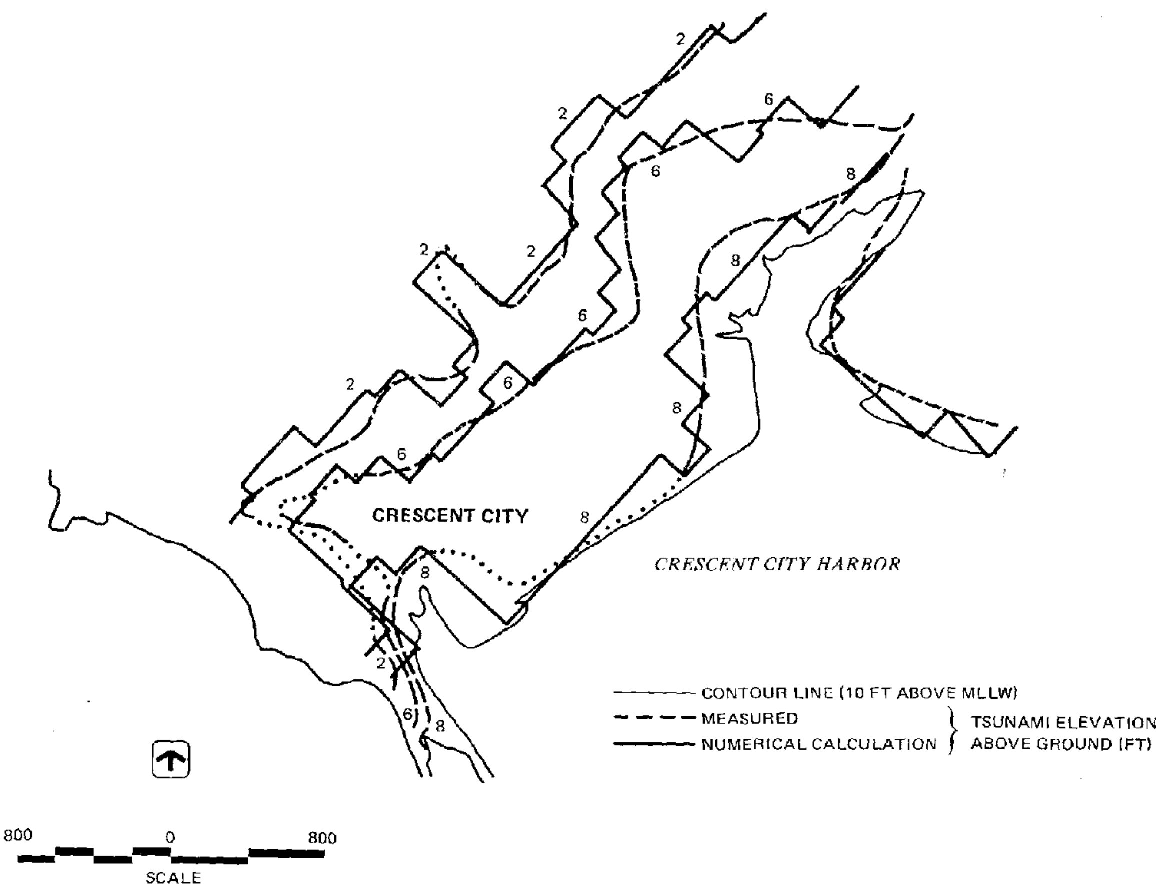

FIGURE 9 INUNDATION LINES 1964 TSUNAMI (ADAPTED FROM MAGOON, 1965)

Korteweg-deVries equation (nonlinear-dispersive equation), Hammack and Segur (1978) have shown that the propagation of all tsunamis is eventually governed by nonlinear-dispersive equations. However, for a very long period tsunami (e.g., the 1960 or 1964 tsunamis), Hammack and Segur (1978) show that neither nonlinearity nor frequency dispersion has any effect on the lead wave as it propagates across any ocean (the lead wave would have to propagate distances several times the length of the Pacific Ocean before nonlinearity or frequency dispersion became significant). Furthermore, for areas with continental shelves with lengths less than approximately 200 miles, Hammack and Segur (1978) have shown that linear-nondispersive theory holds for tsunami propagation across the continental shelf. Goring (1978) also used a solution of nonlinear-dispersive equations to show that for long period tsunamis, the propagation from the "deep ocean to the continental shelf-break and for some distance onto the shelf will be predicted as well by the linear nondispersive theory as by the nonlinear theories." Finally, Tuck (1979) has similarly concluded that "the linear long-wave equations are adequate to describe most of the tsunami generation, propagation, and reception processes." These observations explain the success of numerical models that solve long wave equations. For example, the Hawaiian Islands have a very short continental shelf and thus, there is not sufficient time for nonlinearity and frequency dispersion to become significant during tsunami propagation from the deep ocean to the shorelines of these islands. Hence, the simulations of historical tsunamis using a model solving the linear long wave equations (1978) have produced reasonable agreement between historical measurements and numerical calculations.

Numerical models have been shown to provide reasonable simulations of tsunamis for engineering purposes. Long wave equations govern the propagation of the leading waves of long period tsunamis such as the 1960 Chilean tsunami and the 1964 Alaskan tsunami. These equations may not govern the propagation of short period tsunamis or the trailing waves of long period tsunamis. Hammack and Segur (1978) discuss the criteria for determining the governing equations for tsunami propagation. However, numerical models can be used to simulate the deep-ocean and nearshore propagation and dry land inundation of a wide class of tsunamis of importance to shorelines of the United States.

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DISCUSSION

R. Shaw (Submitted to Editor) - There seems to be some question as to the validity of the numerical models presently being used (or not used). I personally find it difficult to understand why Houston's model has not been checked against other tsunamis or against measured runup in other locations. I have heard criticism of that model, but it must be checked further and the criticism made specific; otherwise an outsider might very well conclude on the basis of that model that we are able to predict real time events (not withstanding the fact that the input to that model took several months to compile after the event).

6 COASTAL PROTECTION

1. SHORE PROTECTION AND FLOOD PLAIN MANAGEMENT
   CHAIRMAN : R. WIEGEL
   RECORDER : O. MAGOON

2. TSUNAMI FLOODING RISK ANALYSIS
   CHAIRMAN : Y. KEEN LEE
   RECORDER : H. LOOMIS

SHORE PROTECTION AND FLOOD PLAIN MANAGEMENT

BY
R. WIEGEL

R. Wiegel discussed the importance and application of damage information in the on-shore, off-shore and near-shore regimes for engineering design purposes. He stressed the need for better knowledge of the various types of possible damages caused by a tsunami in the belief that the knowledge obtained would allow for better design to mitigate or alleviate future destruction. In considering tsunamis, he applied the term to all long waves generated by tectonic displacements, rockfalls, landslides, underwater slumps and horizontal motions. He pointed out that large locally generated tsunamis may have insignificant transoceanic effects and appealed for more consideration of locally generated tsunamis.

The nature of tsunami damages can be classified into three types. The first is flooding due to the rapidly rising tide. Aside from the water damage caused by flooding, there are also others not often mentioned. One important effect of flooding very common to wooden buildings is due to the fact that these buildings are often only constrained laterally. Therefore with flooding, they are floated off their foundations and displaced laterally. Resettlement of the building with even a slight off-set can cause the building to crack in a "hogging" mode. The second type of tsunami damage occurs in the other extreme and is the result of dynamic loadings. These are the effects caused by flows with great velocities within tsunamis. Direct forces are one aspect though they are often small. The more common aspect of this type of damage is debris impact. The flotation of objects weighing a few tons, such as logs and vehicles, combined with flow speeds of a couple of meters per second creates very destructive projectiles. Erosion around foundations is another consequence of high flow velocities. Intermediate between flooding and debris impact damages are those due to such phenomena as drawdown and overtopping. The former is very important for the design of power plant cooling-water intake structures. Knowledge of the duration of drawdown is also important as this will give guidelines for shutdown procedures. The mooring of ships is also affected by drawdown as well as high flows.

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Examples of the various types of damages were presented in a series of slides of previous tsunami damages. Wiegel advocated the need for structural and soils engineers in the make-up of damage survey teams because of their special expertise in identifying possible causes of damages from the on-site surveys. He also suggested a need for more detailed study of damage scenarios to answer such questions as how long it will take a floating object to get up to speed, what is the likelihood of a tsunami generated locally by a landslide or the possibility of overtopping of dams and the resulting erosion mechanism on the dam face.

Finally, he tackled the problem of risk analysis and stressed the need for a systematic study of upperbounds set by physical limits to physical variables. He warned against the extrapolation of a handful of data by curve fitting, stressing the futility of debates with regard to the use of one form of theoretical probability distribution over another, without proper consideration of physical upper bounds.

DISCUSSION

L. Hulman - Wanted to know whether there was any evidence of short-period wave damage.

R. Wiegel - Did not know but thought it was quite possible.

W. Van Dorn - Gave a couple of examples of damages in Hawaii due either to short-period waves or bores.

O. Magoon - Cited the photographic evidence in the 1946 tsunami in which high-waves could be seen riding on top of tsunamis. Also mentioned large (4-5 foot diameter) corals on top of the runway and suggested they may have been due to the action of high waves.

TSUNAMI RISK ANALYSIS

BY
Y. KEEN LEE

Introduction

There is a large variety of activities that call for an assessment of the risks associated with tsunamis. Among such activities are coastal flood insurance studies, siting and design of power plants, breakwaters, harbors and other shore protection or coastal structures wherein safety and cost considerations have to be carefully weighed and therefore the levels of risks quantified. A primary task in the decision process is the assignment of probabilities to a physical variable (or set of variables) of interest. Most studies associated with tsunami risks have concentrated on making probabilistic statements about one physical variable only, namely, the water levels, i.e., run-up or drawdown. In many instances (flooding and overtopping) this is by far the most directly relevant and certainly the simplest variable of concern. However, in considerations of structural integrity and foundation stability, other variables such as wave and surge forces and/or currents surely play an important role as well. In such cases, these variables are sought as inputs to further analysis of scouring rates and structural failure (neglecting the structural damage due to debris impact). This discussion deals primarily with the problem of the analysis of water level exceedance probabilities. The analysis of inertia and drag forces, as well as currents, implies considerations of the accelerations and velocities induced by the tsunamis. Conceptually, a knowledge of the transfer function giving accelerations and velocities in terms of free surface displacements is all that is necessary. Within the linear hydrodynamic regime, such as for offshore underwater storage tanks, this is relatively simple. However, for many cases encountered in practice, the transfer function is not readily accessible.

Methodologies

The need to consider transfer functions and other theoretical hydrodynamic results is largely due to the fact that data on observed tsunami forces and currents are virtually nonexistent. With regard to tsunami elevations, however, there are a few locations (such as Crescent City, San Francisco, and Hawaii in the U.S.) for which a sufficiently long history of observed maximum tsunami levels exists. For such locations, a purely statistical analysis of the data is sufficient to provide probabilistic statements regarding the maximum water levels (for example, Wiegel 1965 and Adams 1970). For the vast majority of coastal locations around the Pacific Ocean, with the possible exception of some in Japan and Chile, one is compelled by the sparsity of local tsunami data to obtain tsunami elevations by other means before reliable probabilistic statements can be made concerning them. An attempt to lump together tsunami elevations for an extended stretch of coastline to provide more data simply ignores the fact that bathymetric influences are very strong and lumping together different locations blurs these bathymetric influences. Thus, Bayesian statistics not withstanding (see Rascon and Villareal 1975, for an example), when local data are scarce, the purely statistical approach is not reliable for probabilistic statements about local tsunami levels. As for considerations of drawdown, tsunami induced currents and other variables which may be of interest, the purely statistical approach is not possible because the vast body of historical data is concerned only with observed maximum elevations.

An alternative procedure is to compute tsunami behavior around the Pacific due to an ensemble of tsunami sources whose sizes (horizontal dimensions and vertical uplift magnitudes) as well as locations are dictated by what is considered possible in the light of present day knowledge of the hydrodynamics of tsunamis and the physics of faulting mechanisms. By expanding the universe of tsunami sources to include what is possible in contrast to merely what has happened historically, probabilistic statements can be made concerning all sorts of tsunami variables besides maximum water levels. The keys to this synthetic approach to tsunami risk analysis are reliable numerical models of tsunami behavior and reliable probabilistic statements concerning tsunamigenic source parameters.

Current versions of the synthetic approach have one common limitation. At present, we can only consider the population of tsunamis generated by distant sources. The population of tsunamis generated by local sources cannot be considered as yet, because neither hydrodynamic nor geophysical theories are sufficiently refined to allow accurate or economical computations of tsunami behavior in the source region. Many other generation mechanisms are involved in local tsunamis besides block tectonic uplift such as landslides, submarine slumping, boundary oscillations (horizontal shaking) and sea shocks. For most of these other mechanisms, probabilistic statements are difficult to construct due to lack of sufficient data.

Outline of the Synthetic Approach

To provide a framework for further discussion, a brief outline of the synthetic approach will be given. A fairly general model is presented here within which a number of variations may be discussed.

Assume that a number S of independent source regions is clearly identified. Each such source region will have a dimension L_i (i=1 to S). Within each source region, individual tsunami sources will be located with their centers at coordinates X. These sources will be characterized by horizontal spatial scales A, B in the longitudinal and lateral directions, respectively, as well as a vertical uplift scale of H. A generic shape function for the ocean bottom deformations in source region i will be assumed known, namely $G_i(\underset{\sim}{x}';A,B,H)$ with $\underset{\sim}{x}'$ the horizontal position vector about the center of the individual tsunami source. All, or none, of the source parameters A,B,H and X can be assumed to be random variables represented by the vector $\underset{\sim}{R}$, so that the probability of joint occurrence of a particular set of values lying between $\underset{\sim}{r}$ and $\underset{\sim}{r}+d\underset{\sim}{r}$ for the random variables $\underset{\sim}{R}$ is given by:

$$p_i = f_{i,\underset{\sim}{R}}(\underset{\sim}{r})d\underset{\sim}{r}$$

The source is now completely specified by $\underset{\sim}{R}$. Each tsunamigenic event within source region i is one realization $\underset{\sim}{r}$ of $\underset{\sim}{R}$ with probability p_i of occurrence given that there is a tsunamigenic event. Next, assume that given the tsunamigenic event, known deterministic models exist which can predict reliably the tsunami variable (or variables) of interest. The resulting variable at station j due to a source within region i can be represented by:

$$Z_{ij} = Z_{ij}(\underset{\sim}{R})$$

Note that the dependence on shape function G_i and observation point j is represented by the indexing.

For all possible sources in source region i, the corresponding probability that at station j the tsunami variable Z_{ij} exceeds the value z is given by:

$$P_{ij}\ (Z_{ij}>z) = \int_{R_{ij}} f_{i,\underset{\sim}{R}}(\underset{\sim}{r})d\underset{\sim}{r}$$

where R_{ij} is the region of the parameter space R wherein $Z_{ij}(\underset{\sim}{R})$ exceeds z.

If the mean frequency of occurrence of tsunamigenic events in source region i is σ_i, then the frequency of events in which the tsunami variable at station j, Z_{ij} exceeds z is $\sigma_i P_{ij}$. This is often termed somewhat loosely as the exceedance frequency. Assuming independence of source regions, the exceedance frequency at station j is $\sum_i \sigma_i P_{ij}$. The reciprocal of this last quantity is the return period of events with the tsunami variable of interest at station j, Z_j exceeding z.

An ensemble of tsunamigenic sources is generated either by Monte-Carlo simulation of the joint distribution function $f_{i,\underset{\sim}{R}}$ to determine the source parameters $\underset{\sim}{R}$ of each member of the ensemble, or by selection of appropriate combinations of source parameters with the probability of each combination known from $f_{i,\underset{\sim}{R}}$. The latter approach usually reduces the computational effort considerably.

However, some skill is needed in selecting a representative set of combinations of source parameters. The criterion used for such selection is usually based on considerations of sensitivity of tsunami signatures to variations in the source parameters.

Frequency of Occurrence

The return period for an event is defined as the ensemble average (expected) waiting time between occurrences of the event. Therefore, one would expect the type of stochastic process concerned to be rather important for its determination. In particular, for the event of interest in the previous paragraph (namely the exceedance of a value z by the tsunami variable Z_{ij}) a formula $\sigma_i P_{ij}$ was given. That formula, widely used in engineering, has been strictly proven only for Poisson processes and the trivial case of non-random periodic processes. It may also be true for other processes but one can certainly find some processes for which it is false.

The assumption of a Poisson process for the occurrence of tsunamigenic earthquakes in a given source region has never been conclusively verified. The usual justification for the assumption is that the Poisson process holds (besides the other conditions of stationarity and nonmultiplicity) if the number of incidents in any interval of time is independent of the number in any other (nonoverlapping) interval of time. Present views of the earthquake mechanism as a release of accumulated strain energy would suggest that this justification is invalid. However, Rosenblueth (1973) has suggested that for large earthquakes it may still be consistent with the elastic rebound theory to assume a Poisson process. Recent work on seismic space-time trends for large, shallow focus earthquakes (Mogi 1968, Fedotov 1969, Kelleher 1970, Sykes 1971 and Kelleher et al. 1973) would suggest that the process is closer to being periodic than Poisson.

It is clear that the question is far from settled. However, as the Poisson process contains many other useful properties for theoretical analysis (such as the existence of the Gamma distribution as the conjugate prior distribution for σ_i - which is most convenient for Bayesian estimation of σ_i), it will remain the favored choice of theoreticians. The parameter σ_i can be derived from tsunami catalogs (Iida et al. 1967 and Soloviev and Go 1969). Alternatively, catalogs of shallow focus earthquakes together with empirical tsunami-earthquake relationships such as those developed by Iida (1958) may have to be used for other areas.

Source Specification

Source regions are defined at the boundaries between subducting oceanic lithospheric plates and overriding lithospheric plates. These regions are normally characterized by the presence of oceanic trenches bordering continental land masses or island arcs. These considerations give a belt of possible sources around the Pacific. Natural subdivisions of the circum-Pacific belt would be at the plate junctions (nodes). Further restrictions to regions with a history of tsunamigenic earthquakes or large, shallow focus earthquakes with dip-slip thrust faulting would serve to distinguish the Aleutian-Alaskan, Kamchatka-Kurile, Peru-Chile and Japanese regions from other regions of lesser importance for transoceanic tsunamis. Such analyses will provide the data on L_i the length of the source regions.

The horizontal dimensions of the individual tsunami sources within a given source region are related to the characteristics of the generating earthquakes in that source region. For example, tsunami source areas determined from inverse refraction (ray tracing) have been correlated with earthquake aftershock areas (Hatori 1969) and tsunami magnitudes with earthquake magnitudes (Iida 1958). Furthermore, lengths, areas and ellipticities of individual tsunamigenic earthquake sources in a given source region are functions of the earthquake magnitude (Iida 1958, Utsu and Seki 1955, Hatori 1969, and Tocher 1958). Unfortunately, the tsunami magnitude and earthquake magnitude correlation exhibits considerable scatter, otherwise the problem of determining tsunami source parameters (and the problem of tsunami warning) would be much easier. Nevertheless, these empirical relationships do indicate that one can use tsunamigenic earthquake dimensions to supplement tsunami inverse refraction data. This is often necessary because of the poor quality of bathymetric data in the deep oceans. Techniques used to determine large, shallow focus earthquake areas by projections of fault plane areas determined from seismic and geodetic studies are exemplified by the works of Fitch and Scholz (1971), Kanamori (1972), Kelleher et al. (1974), and Plafker (1972).

For simplicity, a good model of an individual tsunami source would be a rectangular or elliptically shaped region. The major and minor axes and the source area are then simply related, enabling one axis to be defined in terms of data on source area and the other axis. Assuming the major and minor axes to be independent random variables, their respective probability distributions can be determined from all available data pertinent to a given source region. It is often assumed that the ellipticity (A/B) is a constant (2 or 3). This would imply that only one of the axes need be regarded as a random variable. That this is not universally true is revealed by a study of the figures in Kelleher et al. (loc. cit.). Some indication of the variation of ellipticity with earthquake magnitude is provided by Hatori (loc. cit.). It should be mentioned in passing that the probability distributions for the source area parameters (A,B) can sometimes be derived quite simply from studies in the literature of seismic-risk analysis. In those studies, a combination of Richter's law of magnitude (Richter 1958) for the probability distribution of earthquake magnitudes and empirical correlations of source area parameters with earthquake magnitudes (for example, Krinitzsky 1974) will give the required probability distributions. However, care must be taken to investigate whether the data base for any of the empirical relations used is applicable to tsunamigenic earthquakes.

Probably the most important and least known source parameter for tsunami problems is the vertical uplift. The general shape can be determined from geodetic surveys (Plafker 1972). The data are sparse but are consistent with that predicted by static elastic dislocation theory (Savage and Hastie 1966), although there are several complications such as the possibility of secondary faulting. Consistent with observations from geodetic surveys, the dislocation theories show that the area of maximum uplift is considerably smaller than the source (aftershock) area and is dependent on the fault plane dip angle δ, slip direction λ and slip magnitude D. This implies that the bottom deformation can generally be written in the form $HG(x/A, y/B)$, wherein the form G of the function incorporates δ and λ effects while H, the vertical uplift scale, incorporates δ, λ and slip magnitude effects. The main tsunami at large distances is probably insensitive to the detailed features of the shape function G and, thus, simple readily integrable approximations to G should suffice for any given region. Actual analytical expression for the surface deformation is quite complicated even for the simplest cases. (See, for example, Maruyama 1964 and Mansinha and Smylie 1971). The volume V of displaced water is the result of the integration of the bottom displacement over the source area. For the rectangular area model,

$$V = HAB\int_0^1\int_0^1 G(\xi,\eta)d\xi\ d\eta$$

from which H can be determined if V is known.

The vertical uplift scale H can be defined as the vertical crustal displacement averaged over the source area, in which case the normalization condition:

$$\int_0^1\int_0^1 G d\xi\ d\eta = 1$$

holds. Abe (1973) showed that H is approximately equal to the initial tsunami wave heights averaged along the periphery of the tsunami source areas. His procedure involved use of nearby tide-gauge data and application of Green's law for inverse refraction. The procedure is inapplicable for transoceanic tsunamis and generalized bathymetry since it ignores scattering and diffraction processes. However, more sophisticated tsunami models (see below) can be used iteratively to determine H for historical tsunamis from tide-gauge data. The statistics of H can then be obtained by first relating H and the tsunami intensity i for selected historical tsunamis (such as Kamchatka 1952, Chile 1960, Alaska 1964 and Rat Island 1965), and then using a frequency-intensity distribution law for i similar to Richter's law (loc. cit.) in seismology. A number of tsunami intensity or magnitude scales have been proposed (Iida 1956, Soloviev 1970 and Abe 1979). Soloviev's is probably the most relevant for the present considerations.

Alternatively, the vertical uplift magnitudes can be determined directly from the slip magnitude D required in the elastic dislocation theories. This quantity can be estimated from geodetic and seismic studies for a number of historical earthquakes. Kanamori and Anderson (1975) have shown that large, shallow focus earthquakes at subduction zones conform with a constant stress (or strain) drop hypothesis. Using their results, D can be determined by assuming the stress-drop $\Delta\sigma$ to be 30 bars (Kanamori and Anderson loc. cit.) and the relation

$$\Delta\sigma \simeq \mu D/A$$

where μ is the rigidity and A the fault length. Thus the hypothesis implies that D is proportional to A. Hence the statistics of D are those of A and the random variable D need not appear on the risk analysis. The apparent contradiction between this result and that of the previous paragraph may be resolved by noting that, as discussed earlier, there is a correlation (albeit a poor one) between i and M_s (the earthquake magnitude) and also a correlation between M_s and A. Thus both procedures may produce closely similar statistics.

A final element in the source specifications is the random variable X identifying the location of the center of the source in a given source region. Two hypotheses are possible. The first assigns the probability distribution function of X based on the historical data. In view of the lack of such data for large tsunamigenic earthquakes in some source regions, a uniform probability density function may be justified therein. The second hypothesis assigns the probability density of X according to the predictions of seismic gap theory (see Fedotov 1965, Sykes 1971, and Kelleher et al. 1974 for a discussion of seismic gaps). It should be noted, however, that the two hypotheses are somewhat complementary in the sense that the gap theory assigns a high probability to sources at places which may have a lower frequency of historical events (depending on the length of available records). The latter frequency is, of course, the probablity assigned by the first hypothesis.

Tsunami Models

The second ingredient of the synthetic approach to tsunami risk analysis is a reliable tsunami model. As most of the probabilistic considerations reside in the source parameters, very little will be said with regard to tsunami models. They have been covered more thoroughly in other sessions of this workshop.

The initial condition of instantaneous bottom or free-surface elevation is quite adequate to simulate the generation process if the main interest is in the effects of large tsunamis at trans-oceanic distances. This then excludes considerations of rise time and rupture time of the earthquake models dealt with above. The initial waves of large tsunamis, with significant trans-oceanic disturbances, can be modelled quite accurately by the long-wave equations (see Hwang and Divoky 1972, Houston and Garcia 1978 and Aida 1978 for examples). For smaller tsunamis, mildly dispersive effects are present and the long wave model may not be valid. The 1946 Aleutian event may be just such a case. These anomalous tsunamis must then be accounted for separately. (Note that their geophysical origins are also anomalous, requiring larger uplift values for the estimated fault dimensions than indicated by the constant stress-drop theory--see Kanamori 1972).

The later wave systems of large tsunamis are not predicted reliably by existing models except for special cases. The later wave system is, however, quite important for the determination of the maximum tsunami elevation in the presence of astronomical tides (see below).

Shoreline inundation is also reliably predicted by moving boundary versions of the long-wave equation (e.g., Leendertse 1970 and Yeh 1979), although the numerical algorithms used are appropriate only to extremely long period waves with little dynamical run-up effects. That is, the inundation is determined principally from constraints of mass conservation at the shoreline. Therefore, for smaller period tsunamis (such as the 1946 Aleutian event) the run-up predicted by these models may be inaccurate.

The linearized longwave equation is adequate for the prediction of large period tsunamis arriving from distant sources, provided the observation point is in relatively deep water ($\gtrsim$ 100 m, say). This implies that at a given location within the linear regime, the tsunami elevations are directly proportional to the uplift magnitudes of a tsunamigenic source at a fixed location and fixed horizontal extent. In the synthetic approach discussed here, tsunami elevations have to be determined from sources with various combinations of the three factors: Uplift magnitude, horizontal extent and location. Therefore linearity allows the reduction of computational effort needed for determining the ensemble of tsunami signatures at a given deep water location. Nearer to the shore, where nonlinear (inertial and frictional) effects appear, the nonlinear longwave theory has to be used. It is therefore natural to break the computations into two parts: First, use linear theory and rather large grid sizes for computing the deepwater signatures which are then applied as input boundary conditions for the nearshore nonlinear computations with finer meshes for better resolution of bathymetric shoreline features (apart from possible stability requirements). The appropriate boundary conditions to be applied must be handled with care.

Combined Effects

The ubiquitous role of astronomical tides has so far been ignored. However, near the shoreline, the combination of tsunamis and tides must be considered, especially for studies of flooding risks. A very reasonable hypothesis is that the arrival of a tsunami at a given location is independent of the phase of the astronomical tide. That is, the phase of the tide with respect to the tsunami can be considered a uniformly distributed random variable Φ. It should be noted that the traditional statistical analysis of historical high-water marks gives flood levels which implicitly represent the combination of tides with tsunamis. However, these estimates may be inaccurate since the historical data may represent a biased sample in the sense that the sample tsunamis may have occurred at preferred phases of the tides. This problem is particularly prevalent when the sample size is small. Again, the analysis of historical data based only on destructive tsunamis may have neglected those occurring at low tides with heights less than the normal tidal range.

The tide in the absence of tsunamis can be computed quite reliably from the tidal harmonic constants at a given location (although extremely high tides, such as perigean-syzygial spring tides, can only be predicted with harmonic constants derived from lengthy time series data). The tide can interact non-linearly with the tsunami, especially if their magnitudes are comparable. Then they must be computed simultaneously in the tsunami models. This interesting problem will not be dealt with here. Instead, it will be assumed for simplicity that the tide and tsunami are independent and linear superposition suffices to determine the combined level. The resulting statistics are simply determined (see e.g. Petrauskas and Borgman 1971) provided the tsunami time-history is known. The statistic of interest for flood levels resulting from a combination of tides and tsunamis is:

$$Z = \max_{0<t<\tau} \left\{ A(t+\Phi) + T(t) \right.$$

where $A(t)$ is the astronomical tide, $T(t)$ the tsunami alone and τ the duration of the tsunami. Z is a random variable for a given $T(t)$ by virtue of the fact that the phase Φ of $A(t)$ with respect to $T(t)$ is a uniformly distributed random variable. For a given location, the statistics of the family of functions $T(t)$ reflects those of the source parameters. Z is easily evaluated by machine.

However, as we have seen, current tsunami models do not give very accurate representations of tsunamis at later times. An examination of tide-gauge data for historical tsunamis will reveal that the peak elevation (tsunami plus tide) often occurs with a later wave of the tsunami. Therefore, the problem is to approximate the tail of the tsunami for combination with tides. For relatively open coastal regions, the results of Miller et al. (1962) indicate that the decay of the tsunami energy at a given location can be roughly approximated by an exponential function with an e-folding time of about one-half day. This relaxation time depends, of course, on the bathymetry and geometry of the specific location considered, as well as on the duration of the arriving tsunami input. For example, prolonged ringing with a time constant of the order of days may occur if weakly damped trapped modes are excited. If the tsunami input contains significant energy over a prolonged time span, due perhaps to severe aftershock activity or delayed arrivals from waves reflected off other portions of the oceanic boundaries, the decay may likewise be slower. Miller et al. also showed that the spectral shape during the decay is not significantly altered at least for the open coastal station of their measurements. These results suggest that the tail of the tsunami may be approximated roughly as sinusoidal with period equal to the mean of the zero-crossing periods of the initial waves and with amplitude decaying exponentially with time with a time constant of about one-fourth day, except when the input periods are such that local resonances may be excited. For the latter case, special computations must be made to determine the quality Q of the excited mode (see for example Miles and Munk 1961, Miles 1971, and Miles and Lee 1974). Q is an inverse measure of the relaxation time of the free-oscillations of the resonant mode. The actual decay time is a composite of Q and the duration of the excitation (Carrier 1971).

Summary

The problem of tsunami risk analysis is complicated by a lack of adequate historical tsunami records at most locations of interest. An approach to the estimation of risks in such locations is the synthetic approach outlined in the present paper. The estimation of source parameters of tsunamigenic earthquakes can be performed using a combination of tectonic, geodetic, seismic and tsunami data. However, there is at present no systematic effort made to analyze all such data to provide the appropriate probability distribution functions required by the synthetic approach. When such data analysis is performed, the resulting distribution functions will give tsunami source parameter probabilities which will be better than purely subjective a-priori assumptions. It is unlikely, given the nature of the tsunami data problem, that more sophisticated statistical techniques by themselves can provide better estimates of the resultant risks (except when there is abundant corroborative historical data). Therefore, the synthetic approach should be regarded as one attempt to fulfill the ongoing need for a scientifically based and objective technique for producing risk assessments needed for major economic and legislative enterprises involving the coastal zone. The synthetic approach does not cover locally generated or anomalous (1946 Aleutian) tsunamis for which either the statistical base is inadequate or the causative factors are still largely unexplored. On the other hand, because the synthetic approach is based on the laws of geophysics and hydrodynamics, there seems to be ample hope for improvement as our understanding of the fundamental causative mechanisms of tsunamis progresses.

The author would like to express his indebtedness to Drs. Ian Collins and Aziz Tayfun for stimulating discussions on approximation techniques for computing the statistics of the linear superposition of tsunamis with the astronomical tides. However, he accepts sole responsibility for the particular opinions he has advanced herein on the subject of combining tides with tsunamis.

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DISCUSSION

J. Kelleher - Mentions sparcity of data and suggests that a priori probabilities for tsunamigenic earthquakes be used. Instead of using the idea that any place on the boundary of a tectonic plate is a possible source, one can argue from seismic gap theory that there are actually only a few possible sources. If you look at a particular coast, then only a few sources would be important to that coast, and if you look at the seismic history, there will be still fewer sources that could generate a tsunami that is significant on that particular coast. Take coasts affected by the 1960 Chilean tsunami; it will be many decades before that tsunami is repeated.

R. Wiegel - Agrees with the above. References article in Science about seismic gaps and mentions that Mexican earthquake occurred in one of the gaps. There is a gap just southwest of the 1964 Alaska earthquake and that is the one that scares him. It is oriented to direct a tsunami toward California and a large earthquake there could generate a devastating tsunami. What do we know about calculating the probability of it occurring in 10, 20, or 50 years?

W. Van Dorn - That is a different kind of statistic, Bob.

B. Le-Mehaute - Yes. The process is not ergodic on the human time scale.

L.S. Hwang - There has been a lot of talk about these probabilities but not much to go on. Wishes more people were looking into this.

Unknown - Disagrees with Kelleher's idea that because the 1960 Chile tsunami has occurred, you can now forget about it for 100 years.

J. Kelleher - Well, you can't exactly forget about it.

Unknown - Remember that we are establishing use in the coastal area and once established, that use will continue for 300 years. Therefore, we should not ignore the events that have occurred.

H. Loomis - I used to agree with previous speakers that there was not enough information at any coastal point to make a statement about probabilities. The thing that changed my mind was experience in the Flood Insurance Program. The nature of the problem changes when a law requires that at each shoreline point, the wave height with probability 0.01/year be defined and the use of the affected property is controlled by this designation. The economic consequences of doing this are enormous. There is also the question of fairness. If the government is going to impose restrictions on people, are they going to treat probability distributions casually and draw a line by eye through a few points and estimate probabilities way out on the tail? If a private client employs an engineer to do this, he is just wasting his own money, but the government has the weight of law behind it. So as a government employee I started thinking about this seriously. The first thing that is obvious is that if there is one wave measurement at a point, you know more than if there was none. Similarly, two known heights are better than one, etc. You can ask the classical statistical question, "What is the best estimate of the 0.01/year wave height with whatever information you have?" You have to make the estimate. You might even have to go to court and explain to some smart lawyer how the estimate was made. This is not too different than other disaster predictions and there is a body of work on these questions. One question that I looked at was, "Given that a tsunami had recurred, what is the form of the probability distribution for wave heights at a point on the coast?" This is one of the questions that can be looked at with more intelligence than it has been looked at in the past. The exponential formula $p(x) = \exp(-kx)$ = probability of a wave height exceeding x has been used so much that it is usually accepted without question. It is used in the flood insurance studies. Frequently this gives a poor fit to measured wave heights. The probability 0.01 is way out on the right of the curve and the points being fitted are clustered around the left end of the abscissa. Is there any physical reason for assuming this is the probability law? Syd Wigen's data from Tofina, B.C., that was distributed at the back of the room gives a perfect fit to

$$p(x) = x^{-m} .$$

Chinnery and North in a recent paper in Science use the rule x^{-m} for the distribution of seismic moment to which tsunami size is proportional. A third contender for maximum tsunami wave height distribution is the Gumbel asymptotic distribution of the extreme value of a sample. This is given by

$$1 - \exp(-\exp(-\frac{x-u}{m})) = \text{probability of exceeding } x$$

where u and m are parameters to be determined. This distribution is kind of independent of the population from which the sample is drawn so that it should be a good representation of the probability of an extreme value for a large number of cases.

Any one of the above can be fit by data but the tail of each distribution is different. To people in a conference, a difference of 20 percent in estimating the 0.01/year wave height seems a small matter, but if you own the land there it might be worth hundreds of thousands of dollars to argue about.

Also, the business of fitting a distribution to the data should be examined. Usually one is fitting log p vs. x by least squares which is not the same as fitting p vs. x.

Y.K. Lee - The main objective of choosing a functional fit is to extrapolate the data - otherwise you might just as well use the histograms formed by the data. Whatever function you then choose to fit the data, you have to extrapolate and you don't want to extrapolate too far. I have found, as have many others, that inferences drawn from extrapolations based on sparse data is fraught with dangers. I don't believe that there is any good physical rationale for choosing a functonal form simply because there are quite a few factors such as bathymetry, period of tsunami, energy dissipation and tidal phase to consider. More fundamentally, you simply cannot justify extrapolation using one functional form in favor of another if you have sparse data.

H. Loomis - You might not want to extrapolate too far, but you still might be forced to give the .01/year wave height and if you have a few points then you have a lot of extrapolation. Also, if you don't make the extrapolation, then somebody else will.

In extrapolating from the three curves, you have to keep in mind the variance of the parameters. The parameters of the distribution are estimated so that the k in exp(-kx) should have error bars in it. In fact, with few points, $\Delta k/k$ would be perhaps on the order of one!

F. Camfield - We've got something going on this right now. The subcommittee 8 of the Earthquake Hazards Mitigation program is looking at error bars.

H. Loomis - So the problem is recognized. Another problem concerns the distribution in time of tsunamis. The Poisson assumption is that the probability of occurrence/unit time is the same for any interval of time. The return time for a tsunami of given magnitude can easily be related to the parameter in the Poisson distribution.

There are two competing assumptions that deserve attention: one is that strain accumulation is continuous so that earthquakes (and tsunamis) occur at regular intervals, or that earthquakes come in clusters related to physical things like the orientation of the earth's axis of rotation. For example, was the period 1946-1964 which had five major tsunamis typical or was it a cluster?

In summary, I believe that the two parts of the probability function, time dependence and amplitude distribution, have hardly been looked at and very much need to be.

Unknown - How do you decide which points to include? Syd's (Wigen) data has lots of tsunamis of about 6 cm. Do you include these?

H. Loomis - When you look at cases where you have a lot of data, you find that low amplitude tsunamis lie on a different probability curve than large amplitude tsunamis. Therefore, if I'm interested in extrapolating to large tsunamis, I don't want small tsunamis to have too much to do with the direction of the line of best fit.

Unknown - You have different populations on one piece of paper.

Y.K. Lee - One thing you can do is to make a conditional probability statement. That is, you construct your probablity distributions conditional on events exceeding, say, b meters. There are functional forms which in fact allow for this. For example the probability of exceeding x given by the Weibull distribution:

$$p(x) = \exp - \left\{\frac{x-b}{m}\right\}^k$$

holds for $x \geq b$. Combine this with the frequency of events exceeding b, and you may have solved your problem. This is useful because small tsunami run-up heights probably belong to a different population than the large tsunamis. Besides, you may miss many small tsunamis if historical data is used together with instrumented records. You may create a bias - so a conditional statement is useful. Then you run into the problem of having insufficient data with the large tsunamis and you have to use an alternative, not merely a curve-fitting approach.

R. Wiegel - You mentioned the Poisson and that gets us into the next stage where you have to make a financial decision. You have to take into account the design life of a structure and you have another set of curves of wave heights versus probability of occurrence during the design life. What is the percent risk during the design life? If you are talking about risk to life, then people will take a lot of risk in their automobiles where they have control but don't want any risk from nuclear power plants. I haven't been able to pin anyone down on what risk they are willing to take.

B. Le Mehaute - The risk of exceedence is calculated from the probability distributions assuming uncorrelated events. It is not evident that the succession of first waves are uncorrelated. So what we are applying based on classical laws of probability is not applicable to tsunamis. That is my conclusion.

Y.K. Lee - The main problem is to find the events which would allow you to model the process as stationary or even as a more complicated Markovian process. There simply aren't that many recorded events.

Unknown - Yes. How can you tell that the existing history in 50 years includes what you call the 1 in 3,000 year event.

Y.K. Lee - Loomis says you make the prediction with one or two points.

H. Loomis - It is just that you have to make them and two points is better than none. One might be tempted to combine data points from different coastal locations by scaling each set to the average value and combining. There is a multitude of interesting ways to look at what little data there is.

Let me say that we can do more with this subject than we have. We can use some common sense, classical statistics, seismology and hydrodynamics to make best possible estimates of tsunami wave height predictions. It is possible to pose this problem so that it is respectable intellectually.

L.S. Hwang - We will be coming back to this again. We don't know too much about it. Time for coffee.

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7 INSTRUMENTATION AND OBSERVATIONS

CHAIRMAN : W. VAN DORN
RECORDER : K. OLSEN

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BY
W. VAN DORN

Possibilities for Near Real-Time Determination of Seismic Source Mechanisms for Tsunamigenic Earthquakes

Bill Van Dorn (Scripps Institution of Oceanography) reviewed some recent work - largely by Steven N. Ward (also of Scripps) - on reverberation phases observed on seismograms of suboceanic earthquakes which Van Dorn felt was very promising for rapid determination of tsunamigenic potential. Ward's paper has since been published (see "Ringing P Waves and Submarine Faulting," Journal of Geophysical Research, Vol. 84, p. 3057-3062, June 1979, where details can be found), but a resume of the important facts and conclusions are:

Long-period (~20 second) teleseismic records from shallow focus oceanic earthquakes often exhibit nearly monochromatic, "ringing" wave trains for several minutes following the initial impulsive P, pP and sP phases. Ward has modeled these waveforms with a synthetic seismogram computer program and shows that these ringing waves can be well explained by multiple reflections of compressional waves bouncing between the top and the base of the water layer. On every downward "bounce" seismic energy again enters the solid sediment and crustal layers and is propagated with appropriate time delays to the receiver along nearly the same seismic raypath as the original signal. Such reverberation or leaking-mode excitation of the oceanic waveguide has also been noted on both long-period and short-period seismograms by other investigators. Herrman (BSSA 66, p. 1221-1232, 1976) discusses LP seismograms of a 1966 Aleutian event; Mendrigen (JGR 76, p. 3861-3879, 1971) shows examples of short-period multiple "pulses" from a compressive (mag 5.7) event interior to the Nazca plate, and R. Pearce (unpublished University of Newcastle/ Blacknest Ph.D thesis) has observed and synthetically modeled short-period multiples from a very shallow earthquake in the eastern Gulf of Aden. All these examples are not known to be tsunamigenic.

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Ward's synthetic model studies indicate that the conditions for the generation of conspicuous ringing wave (RW) P phases are: (1) a very shallow focal depth - i.e., the fault deformations occur largely in the sediment and very upper crustal layers immediately below the ocean bottom. The ratio of ringing waves (RW) to body waves (BW), RW/BW, varies inversely with focal depth. (2) a thrust or steep dip-slip focal mechanism. RW/BW ratios from strike slip mechanisms are usually small. (3) the RW/BW ratio is enhanced by the types of shallow crustal structures found in subduction zone areas. (Tsunami source areas are predominantely near subduction zones.) All of these three parameters are common to tsunamigenic earthquakes.

One of the events analyzed by Ward was a shallow (~22 km - ISC) magnitude 6.3 earthquake on 7 May 1964 in the Sea of Japan which gave rise to moderate tsunami run-up (0.3 - 1.0 meter) on the nearby northwest coast of Japan. Ringing P waves were very prominent on WWSSN stations in Europe and North America for this event and strongly suggest that rapid and straightforward observation of ringing waves together with a determination of an epicenter very near or under the ocean could be of great utility to tsunami warning system alerting procedures.

Since the advent of high quality WWSSN-type seismic data in the early 1960's, there have been very few tsunamigenic earthquakes on which to base a detailed evaluation of the utility of rapid ringing P wave analysis. One very important question needing attention is how useful the technique would be for a complex multiple rupture event which is the situation usually pertaining for the great earthquakes having potentially catastrophic tsunami potential. Perhaps synthetic seismogram modeling studies could help clarify several important problems. It is obvious that much additional research needs to be done on this promising technique.

Recent Work at JTRE, University of Hawaii (now JIMAR)

Bill van Dorn also reported on the following activities communicated to him by Marty Vitousek.

(1) Kurile Island Program: For the past several years JTRE has been involved in a joint US-USSR program aimed at measuring a small tsunami within a tsunamigenic area. JTRE supplied bottom mounted tsunami recorders and seismometers, and the Russians supplied a deployment vessel and a dense shoreline seismic network. The first field trip of six weeks' duration in the vicinity of the Kurile islands failed to produce any activity. A second trip in Fall, 1978 near Guam was similarly negative. However, excellent oceanic tide data were obtained. This program was aborted by the loss of Dr. Robert Harvey at sea off Hawaii last spring.

(2) Buoy Telemetry: JTRE has a funded program to develop a real-time tsunami gage system in conjunction with NOAA monster buoys deployed in the Gulf of Alaska. Hardware is currently nearing sea trial stage, and consists of a quartz pressure sensor dropped in a bottom capsule that telemeters pressure data to the surface acoustically for retransmission to shore station (Wallops Island) by satellite. In the normal mode, a reading is taken every hour and once every 24 hours the readings are transmitted in one burst. Upon command from the satellite, the sensor switches to one-minute data sampling, which can be repeated on command as long as required.

(3) Air-Deployable Tsunami-Tide Gages: JTRE has submitted a proposal to NOAA to convert several Kurile-type pop-up bottom gages so that they can be launched and retrieved by aircraft, using snatch recovery techniques from JTRE's PBY-5A amphibian. This would allow rapid deployment of several gages after a tsunami alert, hopefully in time to record all data of interest. They also plan to use the same technique to monitor tides, by rotating air drops so as to keep one gage on the bottom at all times for a protracted period.

<u>Midocean "Microtsunami" Stations on Three Pacific Atolls</u>

Ken Olsen (Los Alamos Scientific Laboratory) reviewed the status of the digital "microtsunami" recording systems originally installed in 1972-1973 under AEC sponsorship at Wake, Johnston and Marcus atolls.

During the late 60's and early 70's when the AEC (now DOE) had programmatic interest in tsunami and explosion wave phenomena, it was felt that the topic for which observations were critically needed for fundamental understanding was the measurment of long-period water waves in midocean. In midocean, wave trains are relatively uncomplicated by shoaling, runup and other transformations that occur near shorelines of continents and large islands. Van Dorn pioneered a practical solution to this difficult measurement problem when he suggested that small, steepsided atolls in the Pacific would induce minimal perturbations on the long wavelength components of tsunami-like waves. Beginning with the IGY in 1957, he made installations of specially designed transducers (which corrected some of the shortcomings of conventional tide gauge instrumentation) at Wake Island. Between 1957 and 1965 when the original station was shut down, the Wake instrumentation provided recordings of about four tsunamis (Van Dorn, Advances in Hydroscience, Vol. 2, 1965) and these data constituted nearly the entire scientific knowledge of the deep water characteristics of tsunamis. In an attempt to extend and improve this data base, the "microtsunami" instruments were installed in 1972-1973 at Wake, Marcus and Johnston atolls by J. McNeil (Delco Electronics), M. Vitousek (Hawaii Institute of Geophysics) and their coworkers. Modern electronic transducers and digital electronics were used to improve dynamic range and signal-to-noise ratios. The objective was to attempt to gather midocean observations on small non-destructive tsunamis that are more frequent than the rare but catastrophic events; hence the term "microtsunami system."

The general layout of sensor positions and cable runs for Wake, Johnston, and Marcus are shown in Figures 1, 2, and 3. Schematics of the underwater transducer appear in Figure 4. The Wake sensor was the only one installed (by ship) in water sufficiently deep ($\sim$310 meters) such that the short-period (3-15 sec) pressure variations from sea and swell were adequately attenuated and an absolute pressure (Vibrotron) transducer could be used for longer period measurements. Because of operational constraints, the Johnston and Marcus underwater installations had to be made by divers at shallower depths. To achieve adequate long-period (>15 sec) sensitivity, back-biased

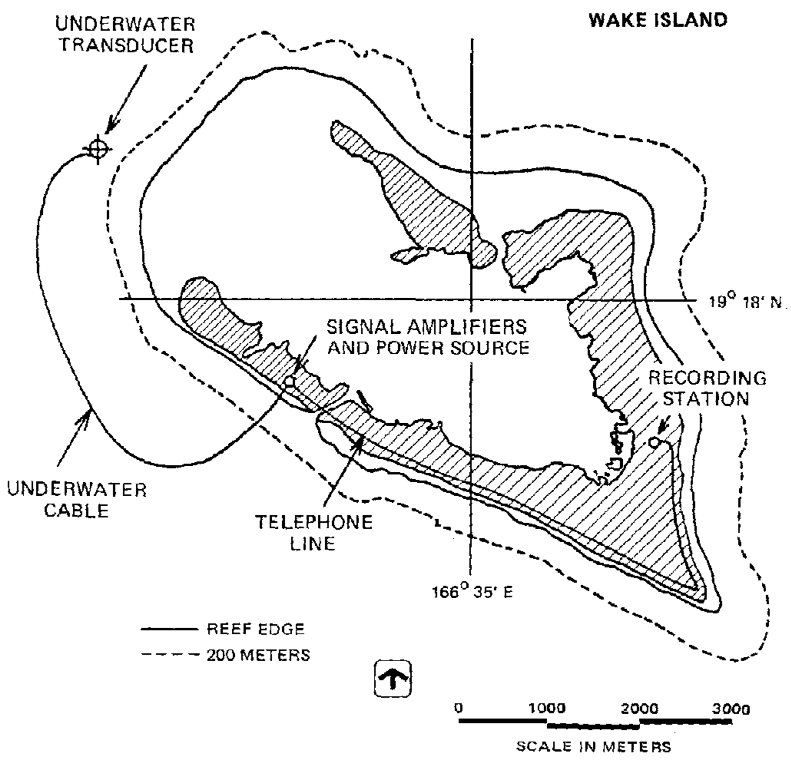


FIGURE 1 MAP OF WAKE ISLAND

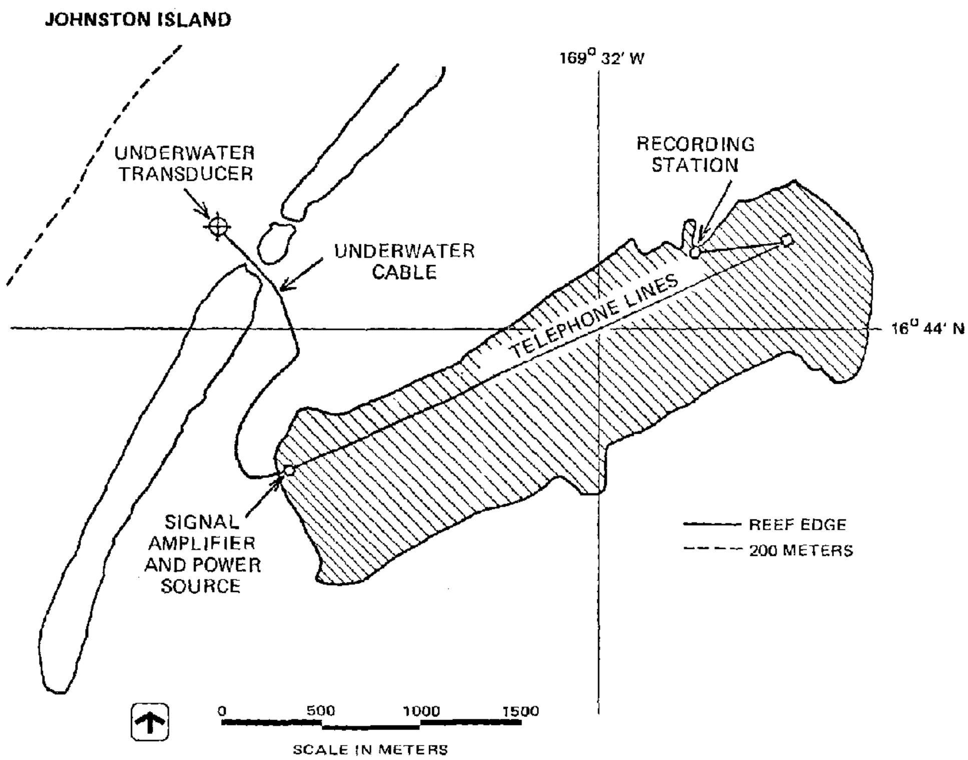


FIGURE 2 MAP OF JOHNSTON ISLAND

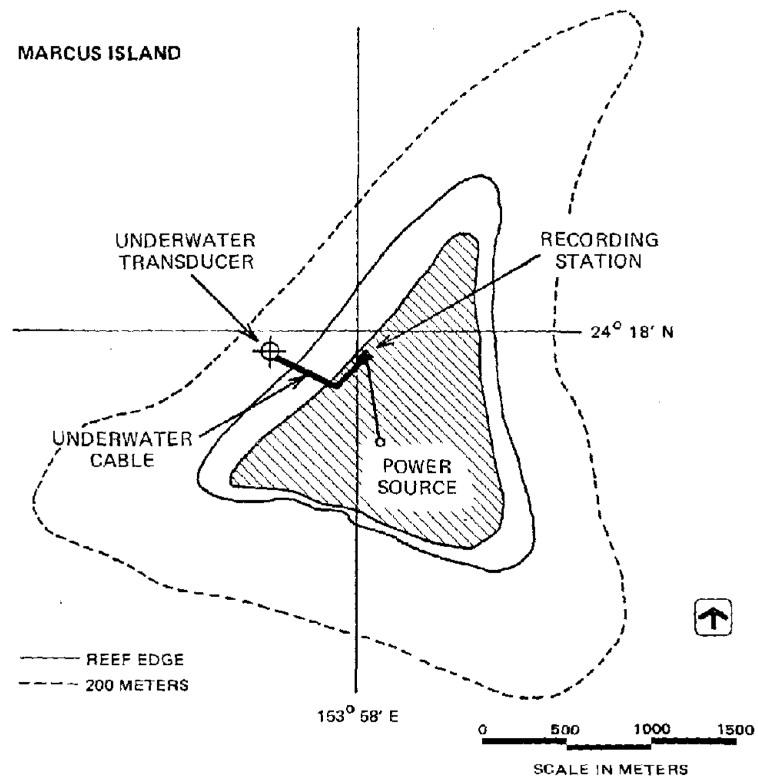
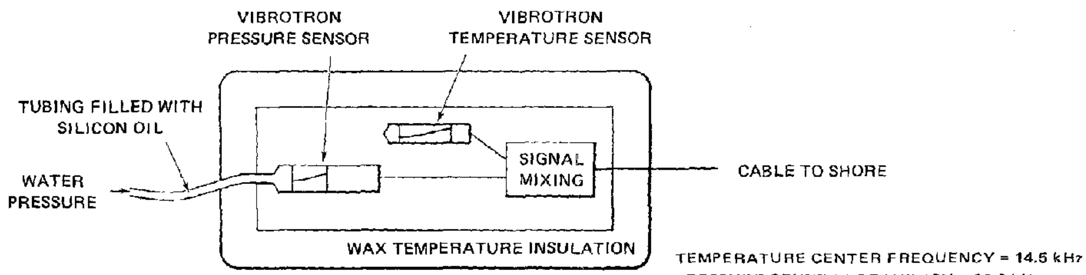


FIGURE 3 MAP OF MARCUS ISLAND



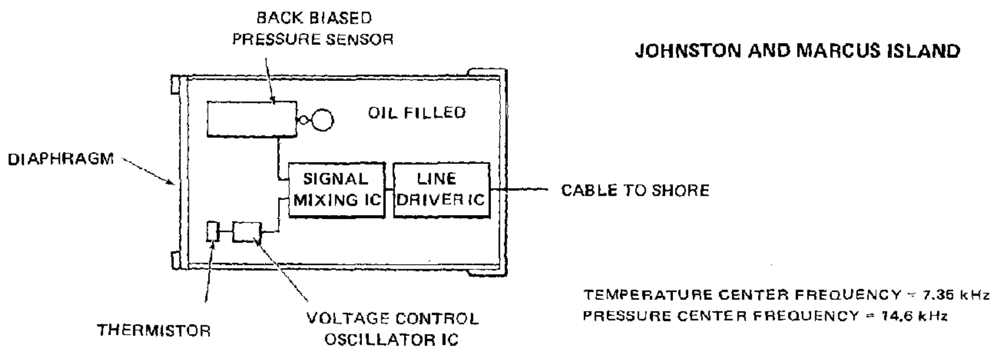


FIGURE 4 SCHEMATICS OF UNDERWATER TRANSDUCER

differential pressure transducers were employed along with hydraulic filters to suppress sea and swell variations.

The Wake station was installed in January 1972, Marcus in June 1972, and Johnston in September 1973. Apart from occasional interruptions of up to several weeks duration (the time to prepare and dispatch repair crews from stateside for major problems, e.g., typhoon damage) the instruments were operated by on-island Weather Service or Coast Guard personnel until February 1976. Digital tapes and analog strip charts were returned to Delco and LASL for analysis. In early 1976, AEC funding was terminated but the recording equipment and cable installations at Wake and Johnston were mothballed in anticipation that these stations could later be reactivated under NOAA/Tsunami Warning System sponsorship. The Marcus installation was deactivated and the recording equipment returned to the U.S., however.

During the operational period, records of three tsunamis (10 June 1975, 31 October 1975, and 29 November 1975 - Figures 5, 6, and 7) were obtained on these instruments. Plots of the digital data are shown in Figures 5, 6, and 7. Because of a recorder malfunction, no digital tape exists at Johnston for the November 1975 event - only an unfiltered analog monitor chart exists.

Inspection of the tsunami records in Figures 5-7 indicates the presence of considerable energy in the 1 to 10 minute period range at all three islands. These dominant periods are somewhat shorter than those usually obtained with the previous Van Dorn instruments and suggest interesting research questions regarding source mechanisms and near-atoll interactions. Bernard, Olsen and Vastano (paper in preparation) have compared the observed spectra at Wake for the 1975 Kurile and Philippine events with those calculated from a numerical model of Wake and the underwater topography within a 200 km radius. They find evidence of several low-Q resonance peaks in the 3 to 25 minute period range which correlate well and suggest the numerical technique is very promising for modeling tsunami-island interaction phenomena.

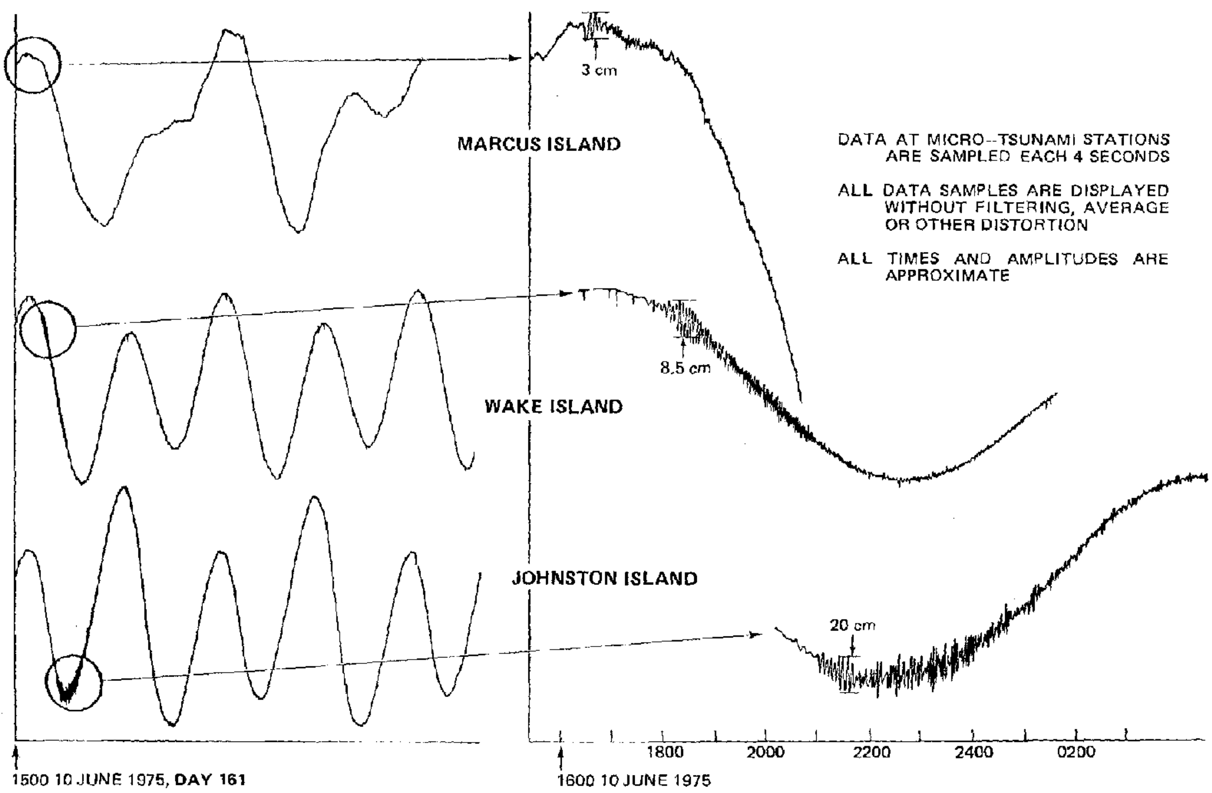


FIGURE 5 PACIFIC OCEAN MICRO - TSUNAMI STATION DATA
SHOWING THE TSUNAMI CAUSED BY THE 10 JUNE 1975 EARTHQUAKE (HOKKAIDO, JAPAN)

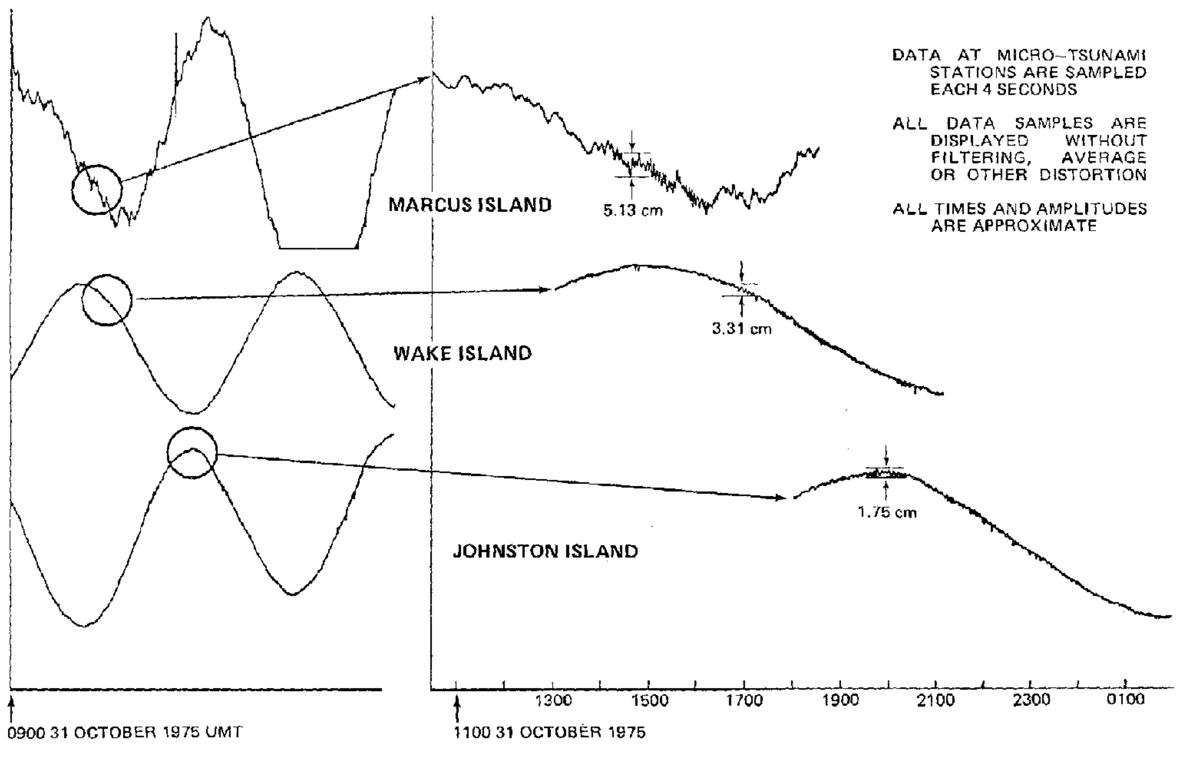


FIGURE 6 PACIFIC OCEAN MICRO - TSUNAMI STATION DATA
SHOWING THE TSUNAMI CAUSED BY THE 31 OCTOBER 1975 EARTHQUAKE (PHILLIPINE ISLANDS)

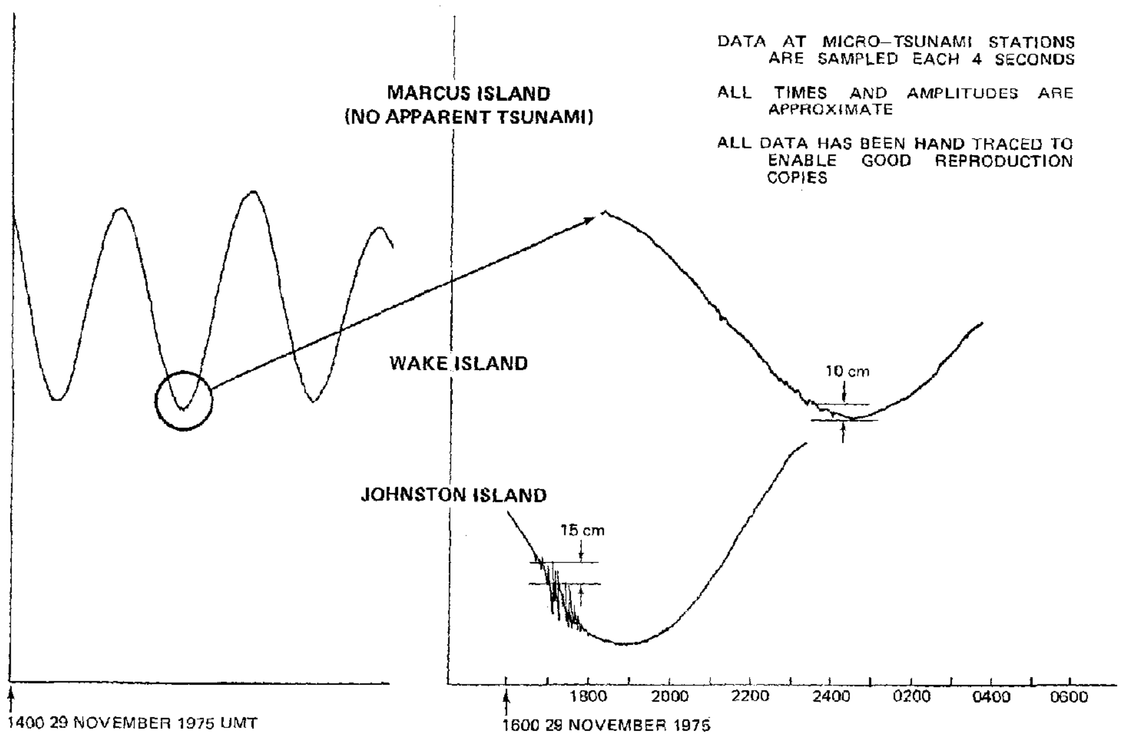


FIGURE 7 PACIFIC OCEAN MICRO - TSUNAMI STATION DATA
SHOWING THE DATA TSUNAMI CAUSED BY THE 29 NOVEMBER 1975 EARTHQUAKE (HAWAIIAN ISLANDS)

Soon after termination of DOE funding for operation of the Wake and Johnston stations, efforts were started by interested parties at LASL, HIG, and the NOAA Pacific Tsunami Warning Center (National Weather Service) to seek support to reactivate the stations and provide, in addition, a real time telemetry link to the warning center through the Geostationary Orbiting Environmental Satellite (GOES) network. This effort has progressed intermittently and slower than desired, but during the summer and fall of 1979, personnel from LASL, HIG and NWS visited Wake and reconditioned the shore-based recording and data handling equipment. In late September, the field party found that the underwater Vibrotron transducer had failed during the summer and could not be reactivated. In October, an attempt was made to install a new sensor and cable to shore but this was unsuccessful when the ship-laid deep water cable end could not be picked up at the outer reef and brought to shore. Another attempt to install a new transducer at Wake is planned. No activity is scheduled to reactivate Johnston until work on the Wake station is completed.

Tsunami Alerting Systems

Harold Clark of the USGS Albuquerque Seismological Laboratory described the concept and design of the tsunami seismic and tide systems designed to operate with the GOES network (Figure 8). In cooperation with NOAA, the USGS Albuquerque instrumentation laboratory has recently built the electronic components of five TS-4 tsunami-seismic systems and one TT-3 tsunami-tide system for the National Weather service. These microprocessor based units are designed to detect, store and transmit abbreviated seismic and/or tide data to the GOES net for near real time use in the tsunami alerting system. A detailed description of these units has recently appeared in the July-August 1979 (Vol. 11, Number 4, p. 132-137) issue of Earthquake Information Bulletin published by the USGS.

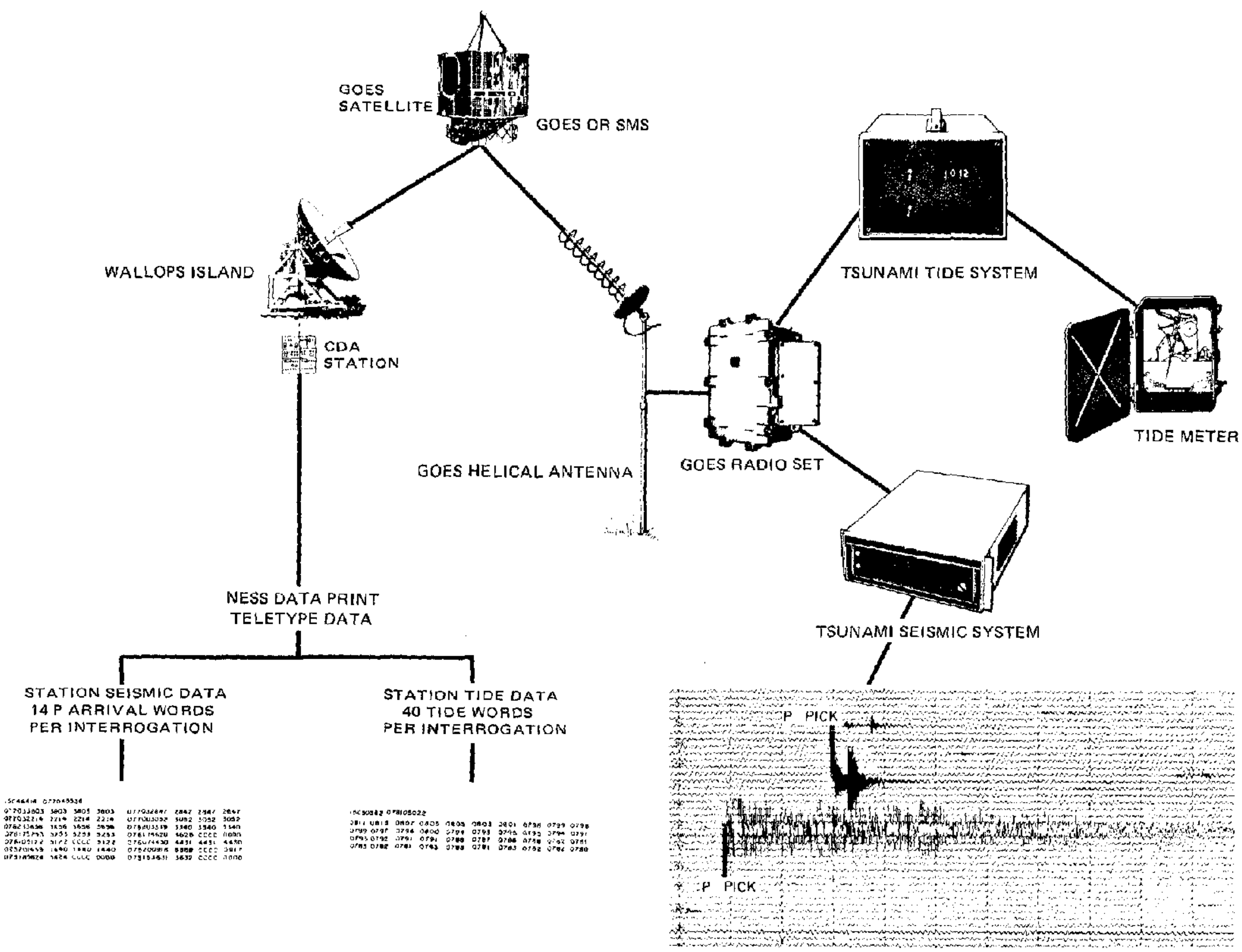


FIGURE 8 TSUNAMI SYSTEMS FLOW CHART

Photos of Historic Tsunamis

James Lander of the NOAA Environmental Data Service (Boulder, Colorado) announced that the EDS/WDC-A (World Data Center - A) has collected a file of over 500 black and white photos of tsunami events (between 1946 and 1975). Prints and slides are obtainable from WDC-A; a catalog describing individual photos will be issued later in 1979. Those wishing to contribute additional photos to this collection are urged to contact WDC-A.

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SUMMARY AND DISCUSSIONS

BY
LI-SAN HWANG

Introduction

In this final session of the Workshop, I will attempt to give a summary of the deliberations of the last two and one-half days. In order to allow time for others who have not had the opportunity to speak during the last few days, I will be brief. The purpose of my summary is to provoke discussions, which can then be put together into some recommendations for the National Science Foundation with regards to the direction of future tsunami research.

Tsunamigenic Earthquakes

With regard to tsunamigenic earthquakes, one of the things which Plafker did, which I consider to be very important, is the radiocarbon dating of marine terraces to determine the recurrence intervals of major episodes of co-seismic uplift. These studies give some indication of the magnitude of the recurrence intervals for major tsunamigenic earthquakes, at least for the Alaskan region. The recurrence times he mentions are of the order of 500 to 1500 years. Such studies give an indication of the maximum bound to possible ground displacements, and thus defines also the maximum possible tsunami. The circumPacific distribution of tsunamigenic earthquakes is another useful study since the impact of tsunamis can be felt across the ocean.

Plafker has already indicated how complicated the spatial distribution of ground deformations in an earthquake can be. We know only the general variations, but not the detailed features. Our calculations have indicated that far away, the detailed features of the bottom deformation is unimportant. However, if you are interested in the near field, in the generation region of the tsunami, large amplitude local-scale bottom deformations can be important. Another aspect is the time history of the ground displacements in a given earthquake event. It really depends on how far away you are. Certainly, some distance away, the time history of the ground displacement is

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not important. But, if we are considering measured oscillatons of a 5-10 second period in the near field, then the time dependent part of the ground displacement should be considered.

Another aspect which is important is related to tsunami warning. Namely, how fast can we use all the information available in real time to make an analysis and predict where the earthquake has occurred, and what sort of ground deformation is involved. Do we currently have sufficient knowledge or capability to predict those features sufficiently accurately for tsunami warning purposes? This area deserves more consideration by Kanamori and other seismologists. I hope we can better define the problem of the analysis of seismic data to compute rapidly the resulting wave with ample time for public response to any warnings issued.

We should really attempt to communicate to the geophysicists, geologists, and seismologists the things we are interested in. Hopefully, this Workshop will stimulate more interest in our problems by people in their community. This cooperation will enable us to solve our problems faster and more efficiently.

Tsunami Generation

Tsunami generation from the perspective of the hydrodynamicist is the topic I will now consider. Certainly, both the near field and far field problems are important. For the far field tsunami, from the discussions we have had, it is my interpretation that linear long wave theory is sufficient to describe the behavior of tsunamis. The generating ground motion can be modeled by considering the vertical component of the ground displacement. In the near field, with large amplitude displacements, non-linear and dispersive aspects may have to be considered. In addition to the time dependence of the ground movements, consideration of the horizontal component of ground movements is also necessary. This is particularly important for the California area because of the strike slip faults involved. If bays and harbors along the coast are subjected to an impulsive horizontal displacement, sloshing occurs. In the recent San Fernando earthquake, water sloshed around and even splashed out of many swimming pools and similar water bodies. So, there is a sloshing effect when you are within the generation area.

Tsunami Propagation

Tsunami propagation has received a lot of discussion. Some people consider the problem solved. I think there are still a few things to look into. When the wave length is large, the linear long wave theory is good enough. However, higher frequency components are always present. If your main interest is in the maximum flooding, consideration of the main longwave component is probably adequate. On the other hand, if your interest is in the sloshing of harbors, mooring problems, and other higher frequency aspects, the tail of the tsunami is also important. So, if you want to look into the propagation of the higher frequency components, such as the tail, you might have to consider dispersive theories.

Coastal Transformation

We had quite a bit of discussion on the coastal transformation of tsunamis and their effects. One effect is tsunami run-up and run-down. The latter, as Professor Wiegel has also pointed out, has not received much attention but is important in relation to design of water intake structures of nuclear power plants. The topic of three dimensional effects at the shoreline is certainly very important. Laboratory studies on idealized beaches with S-shaped plan forms, show run-up behavior which is very different from the usual one-dimensional result. Even for the traditional one-dimensional wave run-up, as LeMehaute and Hammack pointed out, the problem is still generally three-dimensional. That is the second aspect; i.e., strictly speaking in terms of beach run-up, the one-dimensional treatment is inadequate. A one-dimensional wave incident on the beach becomes three-dimensional. This is very clearly demonstrated in the movie shown by Hammack.

The surge on a dry bed may not be important in practice. The reason is because the rise in water level of tsunamis at the shoreline is still very slow. In determining the maximum surge level, it is the later part of the tsunami wave which contributes to it. What is happening at the leading tip of the tsunami (the surge on the dry bed) is not expected to affect the behavior of the main part of the tsunami following behind. There are certainly some arguments on that. Numerical experiments on the sensitivity of maximum surge levels, to variations in friction coefficients, demonstrate that frictional roughness is not a deciding factor. Perhaps more studies will be needed to point that out. Perhaps analytical or other studies can show the indications given by the numerical studies to be generally true.

Edge wave and Mach stem effects have also been discussed. Certainly, more study is needed, in particular, on their effects on the responses of bays and harbors.

The topic of tsunami signatures was brought up by F. Raichlen. He showed some slides which indicated that spectral responses of different tsunamis at the same location didn't seem to be that much different. This certainly creates the impression that the location has a lot of influence on the response. This is certainly possible because while the behavior of the water is dominated by the input tsunami initially, after a couple of waves, the basin characteristics begin to exert themselves--the free oscillations are determined by the basin geometry. This is especially evident if you remove the initial waves from the tsunami records and perform a spectral analysis of the tail.

Numerical Modeling

This has to encompass generation, propagation, and terminal effects. For far field tsunami calculations, the long wave theory seems to be adequate. There are a number of aspects that need to be considered further. One is the arrival time. If one uses the implicit scheme the arrival time is dependent on grid-size. If you have a larger grid, the arrival will be delayed. This is important in regions where the tsunami would arrive from different paths. The phase lag between the two different paths can, of course, affect the tsunami elevation computed. The other aspect is the boundary conditions. The current practice is to assume total reflection (or total transmission sometimes). This is certainly untrue because the bays, channels, and beaches absorb part of the energy.

Numerical studies of terminal effects need considerably more study. More progress is certainly needed in modeling surges and beach run-up realistically. Harbor and island response computations have progressed tremendously, by comparison, and I will not go into details here.

Coastal Protection

The problem is to identify (1) the things that we need to protect from tsunamis, and (2) the nature of the damage, for example, whether broaching or inundation. When the nature of damages is identified, coastal management, through improved design will reduce the resultant damages.

Risk analysis is important. We don't have sufficient data or analyses of data to perform the statistics. Especially, for some design purposes such as nuclear power plants, which require knowledge of events with extremely low probabilities, we don't have the time to wait for the data base to be generated.

Instrumentation and Observations

We certainly need more deep water data to verify the various numerical schemes and theories about the tsunami signature. There are, of course, two major difficulties. One is the difficulty of deploying deepwater gauges. The other is the infrequency of tsunamis. Considerable effort has to be expended in maintaining the gauges while waiting for the large tsunamis to occur! If you really want to understand the generation mechanism and the correlation of tsunamis with the ground motion, you need deepwater gauges which are not contaminated by the local response. Putting them on mid-ocean islands may eliminate contamination by island response if the islands are sufficiently small compared to the tsunami wavelength.

With regard to the tsunami warning system, it is pretty clear what the problem is: more accurate warnings. We have had many false alarms. But, the improved capability in the research areas we have been talking about these last few days will certainly minimize the false alarms.

General Discussion

R. Wiegel - Mentioned that in very shallow water substantial horizontal currents are excited by tsunamis in harbors. Even with small amplitude tsunamis, the currents are strong.

O. Magoon - Called for the preparation of a methodology and organization for prototype tsunami data gathering. This should emerge from this Workshop and cover (1) the coordination and summary of current activities in various national governing bodies and universities; (2) preparation of a field observation manual or a program to obtain data, for example, the organization of a field team equipped to obtain data.

L. Hulman - (1) Wanted to encourage the work of J. Kelleher who claimed that at present, the location of future tsunami sources can be predicted a lot better than in the past. The use of the so-called "gap theory" can aid all forms of research and application of tsunami warning. In particular, being able to predict the locations of future tsunamis with a relatively high likelihood of success offers the researcher the ability to collect much needed data, and the predictor with the opportunity to provide adequate warning. (2) Suggested that NSF should be a clearinghouse for ocean programs in the various federal agencies (such as the Navy, DOE, and NASA) and attempt to integrate the programs. Tsunami data collection can then piggyback on some existing programs. Knowledge of what are the existing programs would help the tsunami effort. (3) Announced that the discussion of risk can benefit from technology emerging from the nuclear industry, and suggested that we should look into that.

R. Shaw - Noted the lack of discussion of physical models in the Workshop sessions.

R. Wiegel - Indicated that physical models can be used to check numerical models and not merely used to simulate a particular coastline.

F. Raichlen - Expressed concern about the scale effects in physical models.

G. Carrier - Concerned about Wiegel's suggestion: it is easy to make experiments that agree superbly with numerical or analytical models. However, neither the physical nor the mathematical model may have any bearing on the prototype. Cited the example of Carrier and Munk's theory on the Gulf Stream which was verified by the Woods Hole physical model but neither had anything to do with the real ocean.

R. Wiegel - Remarked that in earthquake physical models, the scale effects were not a problem but the materials used were.

G. Carrier - Declared that his earlier remarks still hold.

B. Le Mehaute - Claimed that the scale effects have been studied and resolved many years ago and we know exactly how to take care of them. However, people have forgotten these scale effect studies.

Members of the audience expressed some disagreement with the claim.

G. Carrier - Pointed out that some effects cannot be modeled.

B. Le Mehaute - Agreed but claimed that what cannot be done is known.

G. Carrier - Expressed the thought that while Le Mehaute may know the limitations, not many other people do.

S. Wigen - (1) Pointed out that more people died from tsunamis in the period 1975-1979 than in the preceeding 20 years. Indicated that this should put some perspective on the importance of the tsunamis between 1946 and 1964. (2) Reaffirmed the need for more data but indicated that much is available.

Recommended a publication by the International Tsunami Information Center, titled, "Historical Studies of Tsunamis" for data references. (3) Discussed the Canadian need for coastal zone management: As a result of the 1964 earthquake, low lying areas of Port Alberni were inundated. Subsequently, development in the area was restricted. Requested that Workshop participants communicate with him by letter with suggestions as to appropriate guidelines for coastal zone regulations. (4) Announced that breakthroughs in instrumentation technology has enabled self-contained deep water tsunami gauges to be installed at the sea bottom without servicing for one year. Water levels at 1 sec. intervals can be obtained. The question to be posed now is how to place a finite number of instruments to most efficiently record tsunamis and deep sea tides.

D. Moore - Remarked that as a theoretician with experience in other fields of oceanography, he has never seen so much modeling with so little data base as exists in the tsunami field. Stressed the need for deep ocean tide gauges both for improving theoretical studies and for tsunami warning. Questioned whether real-time data were obtainable from the gauges and suggested real time recovery by satellite telemetry as a possibility.

G. Pararas-Carayannis - Remarked that the discussions centered on the 1946 and 1964 tsunamis indicated to him that there is a need to consider more data. For example, in the last couple of years, large tsunamis occurred in the vicinity of the Sunda Islands, Indonesia and Mindanao, Philippines. These were very suitable for tsunami studies but were inadequately surveyed.

L.S. Hwang - Agreed with the need for data but pointed out that other agencies besides the NSF should do more. Since there was a large contingent present from other agencies, suggested that the collective resources of agencies represented by those present should be tapped.

G. Hebenstreit - Agreed with Hwang. Referred to his own interest in tsunamis as arising from a desire to do something useful. However, noted the low priority of tsunamis in funding agencies, e.g., tsunami warning system in NOAA.

H. Loomis - Pointed out that the present tsunami data base arose through coincidence. In the 1946, 1952, 1957, 1960 tsunamis, people without actual responsibilities for the problem but who happened to be present did the data collecting. Even the 1964 tsunami was studied on an ad-hoc basis. Furthermore, other records of tsunamis available to the research community are from tide gauges.

L.S. Hwang - Remarked that most of those present at the Workshop are not supported by tsunami research or other tsunami related funding. Their efforts were therefore essentially piggybacking on other activities. However, a few present such as George Pararas-Carayannis, Eddie Bernard, Jim Lander, John Nelson, and Hal Loomis are in some sense more directly responsible. He suggested that they should be encouraged to pull together to do something about the data problem.

H. Loomis - Noted that there was more tsunami data in Hawaii from the Hawaii Sugar Planters Association than in the tsunami community.

R. Wiegel - Pointed out that the Japanese have done many of the things being discussed. For example, they have much more near-field data, including ground displacements, and extensive gauging networks. He would like to see as a matter of high priority some testing of numerical models against the Japanese data.

K. Kajiura - Described briefly the Japanese data. Indicated that actually only 2 tsunami gauges exist, the rest being tide gauges. Reported that Japanese have tested against available data the analytical earthquake fault model in combinaton with a tsunami generation numerical mode. The results indicate that as a first approximation the fault model is good. Some discrepancies with data occur mostly according to geographical location. Stated that deepwater gauges deserve high priority but that Japan is too close to the source area to perform deepwater tsunami gauging studies.

L. Hulman - Inquired whether a Japan-U.S. joint enterprise is feasible.

K. Kajiura - Indicated an attempt between Japan and U.S.S.R. without much success.

L. Hulman - Suggested that the U.S.-Japan enterprise would probably be more fruitful.

S.C. Liu - Stated the NSF position - Noted that the NSF is a small agency and one of many involved in tsunami studies, e.g., DOE, NASA, and USGS. Suggested that the NSF can help coordinate efforts by identifying funding sources and the interest and capabilities of people in the tsunami community. Mentioned that the tsunami research at NSF is funded under the Earthquake Hazard Mitigation Program which has an $18 million annual budget. Within this program, a subactivity involving siting studies with an annual budget of $10 million covers tsunami research among others. In recent years tsunami research funding averaged $0.5 million a year out of the $10 million. However, NSF responds to proposals and he suggests that the participants send proposals to the NSF. With regard to the problem of instrumentation, he noted that support is difficult because of the long term nature of an instrumentation program which would tie up the available budget for a long time. He stressed that suggestions on what is needed, their priorities and the amount of funding needed would be topics the NSF could deal with, and indicated his willingness to help as much as he can.

R. Wiegel - Suggested that much of the activity could be subsumed under those of other organizations. For example, for risk analysis, he indicated that the ASCE has a committee on Reliability Analysis for offshore structures with earthquake, wave and wind loading. He suggested that they could also consider tsunamis. He pointed out the multi-faceted and complicated nature of the considerations which need to be taken in real problems and suggested a tie-in with a major group.

E. Bernard - Suggested a vote on certain recommendations regarding objectives and organizations.

After some discussion, the participants agreed that there are already a number of existing organizations currently involved in tsunami activities (such as the AGU, NOAA, JTRE, ASCE, MTS) and that the creation of yet another group would be futile. Also, a debate on specific recommendations to the NSF would involve much more time than available in the remaining time at the Workshop. At Dr. Liu's suggestion, a committee was formed to synthesize recommendations to be submitted by Workshop participants, at a later date to the committee. Those present elected the folowing committee members:

W. Van Dorn, Chairman
E. Bernard
G. Carrier
L. Hulman
L.S. Hwang

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APPENDIX I

SUMMARY REPORT AND
RECOMMENDATIONS OF
AD HOC COMMITTEE

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SUMMARY REPORT AND RECOMMENDATIONS OF AD HOC COMMITTEE

TSUNAMI RESEARCH
ADVISORY COMMITTEE

Introduction

The Tsunami Research Advisory Committee was elected as an ad hoc recommendation during the terminal discussions of the NSF-sponsored Tsunami Workshop, organized by Tetra Tech, and held at El Toro California May 7-9, 1979. The committee's charge was to recommend to the NSF directions and priorities to guide further support of tsunami research.

After solicitation and examination of written recommendations from committee members, and at the invitation of Dr. Dennis Moore, a two day meeting was held at the offices of JIMAR (Joint Institute for Marine and Atmospheric Research), University of Hawaii, October 15-16, 1979. The meetings were also attended by Dr. Moore, and by Dr. S.C. Liu, Program Manager, Division of Program-Focused Research, National Science Foundation.

This report, which hopefully reflects the views of the Tsunami Workshop attendees, contains a subject outline, a summary of pertinent discussion and recommendations, and suggestions for future action.

In a dual sense, our recommendations for a National Tsunami Plan appear fortuitously timely. The recent passage of the Earthquake Hazard Reduction Act, and the establishment of a Federal Emergency Management Agency provide a precedent which should facilitate national support for tsunami research, per se.

Discussion and Recommendations

Introduction

Because all of the committee members attended the Tsunami Workshop, the Chairman's introduction comprised a summary review of the current status of tsunami research. It was pointed out that, while substantial progress has been made in some areas within the past decade, there remain many important problems that have received little attention. On the whole, we are still unable to make satisfactory estimates of terminal effects for purposes of warnings, engineering guidance, or risk analysis.

It was therefore proposed that presentation and discussion be addressed in the context of the following outline, so as to provide the broadest overview, and to identify problem areas requiring additional work.

Tsunami sources

1. Temporal and spatial distribution
2. Source mechanisms
3. Source geometry

Propagation and shelf interaction

1. Near field
2. Far field
3. Wave dynamics

Shoreline effects

Human interaction

Sea Wave Verification

While the technology of present sea wave sensing and real-time data transmittal is considered adequate for warning purposes, additional stations are needed in the South Pacific.

There is a great need for improved communications in remote areas. In most cases, this is a socio-political problem which may require State Department assistance.

Arrival time predictions for distant tsunamis exhibit persistent discrepancies with observations. Propagation models should be reexamined.

Shoreline Effects

There is a need for better real-time estimates of maximum wave amplitudes and/or run-up in susceptible localities, in order to quantify warnings.

Additionally, some means of estimating the duration of hazardous conditions is desirable.

Human Interaction

There is a need for education programs for civil and military authorities, as well as for the general public, in order that tsunami warnings be utilized most effectively, with least disruptive impact.

Engineering Design

As opposed to warnings, tsunami engineering involves the long-term assessment of environmental effects that, together with tides and storm waves, must be considered in the design of coastal structures.

While federally-sponsored research has resulted in substantial improvements in our understanding of the gross phenomenology of tsunami generation and deep sea propagation, the complexity of coastal effects still resists useful description. The engineer has little guidance other than historical experience in specific localities.

Because of the 16-year hiatus in destructive events along U.S. coastlines, and the rapid developments in numerical modeling, without adequate experimental verification, there exists a dangerous social philosophy that "things are under control." In fact, nothing could be farther from the truth. Just as current inactivity along the San Andreas fault portends a major earthquake in

San Bernardino County, major tsunamis will inevitably recur in the Pacific and devastate new developments that are inadequately designed.

Committee Recommendations

Seismic Sources

Closer ties should be maintained with seismologists and tectonophysicists at all levels, so as to identify as reliably as possible the location, geometry, motion predisposition, and temporal recurrence probabilities of potential tsunami sources.

The development of numerical source models consistent with the above information should be continued. The use of the gap theory for predicting possible locations of tsunami sources and for the placement of instrumentation should be pursued.

Propagation and Shelf Interaction

For a set of representative sources ringing the Pacific, compute unit-normalized deep water wave histories throughout the circumpacific spherical grid matrix. This work has already been conducted for special cases of interest. The above data should be stored in a master file for general use.

Compute modification of representative wave systems by coastal shelf interaction and/or local response factors at specific locations, as needed.

Design tests or experiments for verification of numerical models, and upgrade the latter as improvements are developed. This may require construction of a special model facility, since none presently exists that is capable of satisfying the many restrictive similitude laws that apply to tsunamis.

Existing hydraulic modeling techniques and their applications should be reassessed, since it is manifest that numerical methods cannot provide essential detail over complex shallow water topography.

Shoreline Effects

The objectives here are the best estimates of magnitude, direction, and duration of water motion at arbitrary locations, and their worst-credible and most-probable extremes for any physically probable tsunami.

A realistic and useful data bank for making the above estimates will comprise many types of information.

(1) Historical evidence should be collected, collated, and reexamined for accuracy.

(2) Special observer teams should be established, and trained to make a variety of wave effect observations in the event of future tsunamis. Such teams have been used with some success in Hawaii, but there has been no systematic effort to prepare for rapid deployment of observer teams to any site in the Pacific. Appointment of locals for this purpose is seldom effective because of the long intervals between events, and because substantial expertise is required to assure data uniformity and adequacy.

(3) Coastal and mid-ocean monitoring of tsunamis should be reinstituted and expanded. Through a recent unfortunate division of federal authority, there no longer exists a single instrument in the U.S. that is capable of recording tsunami waves unambiguously for research purposes. Can one imagine seismometry without a single seismograph?

To correct this situation, key tide gages should be reestablished, or tsunami gages should be modified to also provide unambiguous data for research purposes.

At least three appropriately located small island tsunami gages should be established on a permanent basis, and maintained principally for research purposes. Such stations provide the only present approximation to undistorted deep sea tsunami signatures.

Efforts to develop deep-ocean monitoring of tsunamis should be expanded. Such data are required for ultimate verification of numerical models.

The responsiblity of World Data Center A to process and store existing tide gage records of past tsunamis should be reexamined in the light of modern analysis techniques. The present files are inadequate for most research purposes.

(4) Continued development of modeling techniques and their applications should be encouraged. Relevant results should be phrased in standard format and stored for general accessibility.

(5) Experiments should be conducted on the resistance of structures to representative tsunami wave effects, leading to a body of design criteria that can be applied wherever such effects can be estimated.

Risk Analysis

The committee admits the lack of expertise in this area. It feels the need for more advice on the following questions:

(1) Is the existing data base sufficient for realistic analysis?

(2) What types of analyses are most appropriate for the end uses envisaged?

Without answers to these questions, we feel that there is danger that inadequate methods now being employed will lead to inappropriate regulation. In our opinion, risk analysis is an exercise leading to probabalistic statements about tsunami hazard at a given area or location, obtained by combining information on sources, propagation, and shoreline effects.

Some statistician interested in tsunamis should pose the probabilistic questions that should be answered, and then assess the possibility of answering them with the existing data base.

Future Action

Although the committee feels that it has fulfilled its function in developing these summary recommendations, the question was raised by Dr. Liu as to the most effective means of ensuring their implementation. After some discussion, the following points were agreed upon.

(1) Tsunami impact on society is great, perhaps comparable in terms of life and property damage to major earthquakes.

(2) There is a tendency for federal agencies to underestimate the tsunami hazard, and to erroneously assume that it is being appropriately handled.

(3) For lack of direction, basic and applied research on tsunamis has been largely ad hoc, and as necessary, with many overlaps and omissions.

(4) Appropriate implementation of the Committee's recommendations, their amplification, and assignment of priorities can probably succeed only if they result from a comprehensive National Tsunami Plan.

Such a plan is conceived of as analogous to that which has resulted in the National Earthquake Hazards Reduction Program, now jointly sponsored by some sixteen federal agencies and departments.

While the tsunami problem is briefly addressed in the EHRP, the committee feels that its importance deserves separate consideration. Tsunamis should not be treated under the umbrella of earthquakes, or flooding from hurricane storm surge, although all have common elements.

(5) The National Science Foundation is an appropriate agency to sponsor the formulation of such a plan.

In response to Dr. Liu's opinion that endorsement of the committee's recommendations by concerned federal agencies would be an important factor in initiating action in support of a National Tsunami Plan, the committee composed a letter, which was sent to a broad list of addressees. Their respective responses were in the main, strongly favorable.

To avoid possible interagency partisanship, the committee further recommends that the formulation of a National Tsunami Plan should be remanded to an appropriate working group, to be selected by an impartial agency, such as the National Academy of Sciences. It is anticipated that implementation of the plan will necessarily involve participation by many agencies already having some responsibilities concerning tsunamis.

APPENDIX II

TSUNAMI WORKSHIP AGENDA
AND LIST OF PARTICIPANTS

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TSUNAMI WORKSHOP AGENDA

MAY 7 TO MAY 9, 1979

1.	Introduction Welcoming remarks and review of workshop objectives and procedures	National Science Foundation
2.	Tsunamigenic Earthquakes	
	2.1 Fault Mechanisms and Frequencies of Occurrence	*G. Plafker, USGS **R. Geller, Stanford Univ.
	2.2 Characteristics of Ground Motions Inferred from Seismic Waves	*H. Kanamori, Caltech **J. Kelleher, NRC
3.	Tsunami Generation	*K. Kajiura, Japan **K. Lee, Tetra Tech
4.	Tsunami Propagation	
	4.1 A Survey of Fundamental Features	*E. Tuck, Australia **P. Liu, Cornell Univ.
	4.2 Evaluation of Existing Models	*T. Wu, Caltech **J.J. Lee, USC
5.	Coastal Transformations and Terminal Effects	
	5.1 Coastal Transformation--Survey of Fundamental Features and Evaluation	*G. Carrier, Harvard **J. Hammack, UC Berkeley
	5.2 Engineering Methods--Run-up, Surge on Dry Bed, and Energy Dissipation	*B. LeMehaute, Univ. of Miami **G. Pararas-Carayannis, ITIC
	5.3 Island Response to Tsunamis	*R. Reid, Texas A&M **E. Bernard, PTWC
	5.4 Bay and Harbor Responses to Tsunamis	*F. Raichlen, Caltech **D. Divoky, Tetra Tech
6.	Numerical Aspects of Tsunami Modeling	*C. Mei, MIT **J. Houston, WES

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7.	Coastal Protection	
	7.1 Shore Protection and Flood Plain Management	*R. Wiegel, UC Berkeley **O. Magoon, COE
	7.2 Combined Effects and Tsunami Flooding Risk Analysis	*K. Lee, Tetra Tech **H. Loomis, JTR, NOAA
8.	Instrumentation and Observations	*W. Van Dorn, UC San Diego **K. Olsen, LASL
9.	Discussions and Conclusions	*L. Hwang, Tetra Tech

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* Chairman

** Recorder

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