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ARESPONSE SPECTRUM METHOD FOR RANDOM VIBRATIONS

by

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Report to National Science Foundation

UNIVERSITY OF CALIFORNIA . Berkeley, **California**

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A **RESPONSE SPECTRUM METHOD FOR RANDOM VIBRATIONS**

By

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SUMMARY

A response spectrum method for stationary random vibration analysis of linear structures is developed. The method is based on the assumption that the input excitation is a wide-band, stationary Gaussian process and the response is also stationary. However, it can also be used as a good approximation for the response to a transient stationary Gaussian input with a duration several times longer than the fundamental period of the structure. Various response quantities, including the mean-squares of the response and its time derivative, the response mean frequency, and the cumulative distribution and the mean and variance of the peak response are obtained in terms of the ordinates of the mean response spectrum of the input excitation and the modal properties of the structure. The formulation includes the cross-correlation between modal responses, which are shown to be significant for modes with closely spaced natural frequencies.

The proposed procedure is demonstrated for an example structure that is subjected to earthquake induced base excitations. Computed results based on the response spectrum method are in close agreement with simulation results obtained from time-history dynamic analysis. The significance of closely spaced modes and the error associated with a conventional method that neglects the modal correlations are also demonstrated through this example.

INTRODUCTION

In random vibration analysis of structures subjected to stationary excitations, a description of the input excitation in terms of a power spectral density function is commonly used. For linear systems, it is well known that the power spectral density of the stationary response is the product of the system transmittancy function and the power spectral density of the input process (8). It is also known that most response quantities of engineering interest can be obtained in terms of the first few moments of the response power spectral density taken about the frequency origin (16). For example, the zeroth moment, i.e. the area under the power spectral density function, is the mean-square response, whereas the second moment, i.e. its moment of inertia, is the mean-square of the response rate. In the special case when the response process is Gaussian, it has been shown that the first three spectral moments, *i.e.*, the zeroth, the first, and the second, are also sufficient to determine the cumulative distribution and the mean and variance of the peak response over a specified duration $(16,5)$. These response quantities are fundamental to and, for the most part, are adequate for safety assessment of structural systems subjected to random excitations.

In certain applications, such as in earthquake engineering, the specification of the input excitation in terms of a power spectral density function is not the most convenient method. A description that is often found to be more expedient is in terms of the *mean response spectrum.* This is a function describing the mean of the peak response of an oscillator of varying frequency and damping to a given input excitation. In structural engineering practice, a responsespectrum description of an input is preferred because of a variety of reasons. Chief among these is, perhaps, tradition; many existing structural codes and specifications are based on the response-spectrum method, and most structural engineers are accustomed with this idea. Another reason is convenience in generating design response spectra rather than power spectral densities from existing data. Finally, as shown in this paper, for certain critical response quantities, such as the mean of the peak response, a formulation based on the response spectrum method is computationally simpler than that in terms of the power spectral density function.

It is shown in this paper that, under a general set of conditions, the first three moments of the response power spectral density for a linear structure can approximately be obtained in terms of the mean response spectrum of the input excitation and the modal properties of the structure. These conditions are that: (a) the structure have classical modes, (b) the input excitation be a stationary Gaussian process, and (c) the input be a wide-band process, i.e. have a smoothly varying power spectral density over a range of frequencies covering the significant modes of vibration of the structure. The resulting expressions for the three spectral moments lead to a set of modal combination rules whereby each statistical quantity of the response, including the mean-squares of the response and its time derivative, the response mean frequency and the mean and variance of the peak response, is expressed as a combination of the mean values of maximum modal responses, where each maximum modal response is obtained in terms of the ordinate of the mean response spectrum, associated with the corresponding modal frequency and damping, and the modal properties of the the structure. This analysis also yields the cumulative distribution of the peak response in terms of the input response spectrum. An important feature of this formulation is that it accounts for the cross-correlation between modal responses. It is shown that this correlation can be highly significant for structures with closely spaced natural frequencies.

In practice, the restrictions under which the method is applicable can be considerably relaxed. Specifically, the method may be used for the transient response of structures to wideband Gaussian inputs, provided the peak response occurs during a stationary phase of the input with a duration several times longer than the fundamental period of the structure. Within this context, the method can be particularly useful in earthquake engineering in determining the response of structures to transient base inputs. Such an application is demonstrated for an example structure with closely spaced frequencies subjected to an ensemble of artificially generated earthquake motions. Computed results based on the proposed response spectrum method are shown to be in close agreement with Monte Carlo results obtained through timehistory analyses of individual motions. The example also serves to demonstrate the significance of the cross-correlation between responses in modes with closely spaced frequencies.

RESPONSE OF MOF SYSTEMS TO STATIONARY EXCITATION

Consider an n-degree-of-freedom, viscously damped, linear structure. Assume that the structure has classical modes with ω_i , ζ_i , $i=1, 2, ..., n$, denoting its modal frequencies and damping coefficients, respectively. It is well known (2) that, using a mode-superposition procedure, any response $R(t)$ of such a system can be expressed in terms of its modal responses as

$$
R(t) = \sum_{i} R_i(t) = \sum_{i} \Psi_i S_i(t)
$$
 (1)

where $R_i(t) = \Psi_i S_i(t)$ is the response in mode i, in which Ψ_i is the effective participation factor for mode *i* and is a constant in terms of the i -th modal vector and the mass matrix (it is in general the product of the conventional participation factor and a linear combination of the elements of the *i*-th modal vector (2)), and $S_i(t)$ is the *i*-th normal coordinate, representing the response of an oscillator of frequency ω_i and damping coefficient ζ_i to the given input. Consider the stationary response of the system to a stationary input $F(t)$, described through a onesided power spectral density $G_F(\omega)$. With no loss of generality, let $F(t)$ be a zero-mean process. Then, the response is also a zero-mean process. Its one-sided power spectral density is given by

$$
G_R(\omega) = \sum_i \sum_j \Psi_i \Psi_j G_F(\omega) H_i(\omega) H_j^*(\omega)
$$
 (2)

where $H_i(\omega) = 1/(\omega_i^2 - \omega^2 + 2i\zeta_i\omega_i\omega)$ is the complex frequency-response function (for displacement response) of mode *i* and the asterisk denotes a complex conjugate. Using Eq. 2, moments of the response power spectral density about the frequency origin are obtained as

$$
\lambda_m = \int_0^\infty \omega^m G_R(\omega) d\omega = \sum_i \sum_j \Psi_i \Psi_j \lambda_{m,ij} \tag{3}
$$

where

$$
\lambda_{m,ij} = \text{Re}\left[\int_0^\infty \omega^m G_F(\omega) H_i(\omega) H_j^*(\omega) d\omega\right]
$$
 (4)

are cross-spectral moments of the normal coordinates associated with modes i and $j(5)$. It is noted that because of symmetry $G_R(\omega)$ is always real-valued; therefore, only the real parts of the cross-spectral moments are of interest. Introducing coefficients $\rho_{m,ij} = \lambda_{m,ij}/\sqrt{\lambda_{m,ii}\lambda_{m,jj}}$, Eq. 3 can be written in terms of the spectral moments of the individual normal coordinates as

$$
\lambda_m = \sum_i \sum_j \Psi_i \Psi_j \rho_{m,ij} \sqrt{\lambda_{m,ii} \lambda_{m,jj}} \tag{5}
$$

It is well known that $\lambda_0 = \sigma_R^2$ and $\lambda_2 = \sigma_R^2$ are the mean squares of the response, R (t), and its time derivative, $\dot{R}(t)$, respectively, whereas $\lambda_{0,ii}$ and $\lambda_{2,ii}$ are the mean squares of the *i*-th normal coordinate $S_i(t)$ and its time derivative, $\dot{S_i}(t)$, respectively. Also, from the preceding definition of $p_{m,ij}$, it should be clear that $p_{0,ij}$ and $p_{2,ij}$ are cross-correlation coefficients between $S_i(t)$ and $S_i(t)$ and between their time derivatives, $S_i(t)$ and $S_i(t)$, respectively. The moments λ_1 and $\lambda_{1,ii}$ are related to the envelope of the response process, as described in Ref. 16. The corresponding coefficients, $\rho_{1,ii}$, have no obvious physical interpretation. However, their behavior is also similar to a correlation coefficient, as shown in Ref. 5.

Closed-form solutions for $\lambda_{m,ij}$ and $\rho_{m,ij}$ for $m = 0, 1, 2,$ i.e., for the first three spectral moments, are given in Ref. 5 for responses to white-noise and filtered white-noise inputs. For these classes of inputs, the power spectral densities are of the form

$$
G_F(\omega) = G_0 \tag{6}
$$

and

$$
G_F(\omega) = \frac{\omega_g^4 + 4\zeta_g^2 \omega_g^2 \omega^2}{(\omega_g^2 - \omega^2)^2 + 4\zeta_g^2 \omega_g^2 \omega^2} G_0
$$
\n(7)

respectively, where G_0 is a scale factor and ω_g and ζ_g are the filter frequency and damping coefficient. By proper selection of these parameters, a variety of input power spectral density shapes can be studied. In particular, the filter frequency, ω_{g} , determines the dominant range of input frequencies, whereas ζ_g determines the smoothness of the power spectral density shape, see Fig. 1. Observe that spectral amplitudes for frequencies greater than ω_g rapidly diminish with increasing frequency. As a result of this, responses in modes with frequencies much greater than ω_{μ} will generally be small and insignificant in comparison to responses in modes with frequencies within the dominant range. This fact plays an important role in the subsequent development, since results obtained in the response spectrum method for modes within the dominant range of frequencies will be more accurate leading to accurate estimation of the response. Also observe in Fig. 1 that the spectra are smoother for larger values of ζ_g . For the purpose of this study, a filtered white-noise input with $\zeta_g \ge 0.6$ will be considered as a wideband input. It is noted in passing that a filtered white noise with $\omega_g = 5\pi$ and $\zeta_g = 0.6$ is commonly used in earthquake engineering to model the ground acceleration process $(2,7)$.

It is shown in Ref. 5 that whereas $\lambda_{m,ij}$ are sensitive to the shape of input power spectral density (as described by parameters ω_g and ζ_g), the coefficients $\rho_{m,ij}$ remain relatively indifferent for wide-band inputs, i.e. for $\zeta_g \geq 0.6$. As an example, Fig. 2 shows a comparison of $\rho_{0,i}$ for responses to white-noise and filtered white-noise inputs with $\zeta_g = 0.6$. Similar results are given in Ref. 5 for $\rho_{1,ij}$ and $\rho_{2,ij}$. Observe in this figure that the correlation coefficient rapidly diminishes as the two modal frequencies ω_i and ω_j move further apart. This is especially true at small damping values that are typical of structures. Thus, for wide-band inputs cross terms in Eq. 5 are only significant for modes with closely spaced frequencies. Also note that the coefficients for responses to the two types of inputs are nearly the same as long as the modal frequencies are not far beyond the dominant range of input frequencies, as it happens at the right end of the lower graph in Fig. 2 where both ω_i and ω_j are much greater than ω_g . Since the latter case is not critical, it follows that cross-correlation coefficients based on response to a white-noise input are good approximations to the corresponding coefficients for responses to wide-band inputs. Exact solutions of $\rho_{m,ij}$, $m = 0, 1, 2$, for response to white-noise input are given in Ref. 5. A set of approximate expressions obtained from these results are

$$
\rho_{0,ij} = \frac{2\sqrt{\zeta_i\zeta_j} \left[(\omega_i + \omega_j)^2 (\zeta_i + \zeta_j) + (\omega_i^2 - \omega_j^2) (\zeta_i - \zeta_j) \right]}{4(\omega_i - \omega_j)^2 + (\omega_i + \omega_j)^2 (\zeta_i + \zeta_j)^2}
$$
\n(8)

$$
\rho_{1,ij} = \frac{2\sqrt{\zeta_i\zeta_j} \left[(\omega_i + \omega_j)^2 (\zeta_i + \zeta_j) - 4(\omega_i - \omega_j)^2 / \pi \right]}{4(\omega_i - \omega_j)^2 + (\omega_i + \omega_j)^2 (\zeta_i + \zeta_j)^2}
$$
(9)

$$
\rho_{2,ij} = \frac{2\sqrt{\zeta_i\zeta_j} \Big[(\omega_i + \omega_j)^2 (\zeta_i + \zeta_j) - (\omega_i^2 - \omega_j^2) (\zeta_i - \zeta_j) \Big]}{4(\omega_i - \omega_j)^2 + (\omega_i + \omega_j)^2 (\zeta_i + \zeta_j)^2}
$$
(10)

These expressions, which are simpler but good approximations of the exact results, are plotted in Fig. 3 for selected values of damping. On the basis of the preceding discussion, these expressions will be used here for responses to wide-band inputs with arbitrary power spectral density shapes. With these expressions given, to evaluate Eq. 5 it is only necessary to compute the spectral moments for individual normal coordinates given by

$$
\lambda_{m,ii} = \int_0^\infty \omega^m G_F(\omega) |H_i(\omega)|^2 d\omega \tag{11}
$$

These moments for $m = 0$, 1, and 2 are subsequently obtained in terms of the ordinates of the mean response spectrum.

STATISTICS OF PEAK RESPONSE FOR GAUSSIAN EXCITATION

It is well known that the response of a linear structure to a zero-mean Gaussian input is also zero-mean and Gaussian (8). Vanmarcke (17) has derived a distribution for the firstcrossing time of a symmetric barrier for a zero-mean, stationary Gaussian process in terms of its first three spectral moments. Using his formulation, the cumulative distribution of the peak absolute response over a duration τ , defined as

$$
R_{\tau} = \max |R(t)| \tag{12}
$$

can be expressed as

$$
F_{R_{\tau}}(r) = \left[1 - \exp(-s^2/2)\right] \exp\left[-\nu \tau \frac{1 - \exp(-\sqrt{\pi/2} \delta_c s)}{\exp(s^2/2) - 1}\right], \quad r > 0 \tag{13}
$$

in which $s = r/\sigma_R = r/\sqrt{\lambda_0}$ is a normalized barrier level,

$$
\nu = \frac{\sigma_{\hat{R}}}{\pi \sigma_R} = \frac{1}{\pi} \sqrt{\frac{\lambda_2}{\lambda_0}}
$$
(14)

is the mean zero-crossing rate of the process, and $\delta_e = \delta^{1.2}$, where

$$
\delta = \sqrt{1 - \frac{\lambda_1^2}{\lambda_0 \lambda_2}}
$$
 (15)

is a shape factor for the response power spectral density with a value between zero and unity. (A small value for δ denotes a narrow-band process whereas a value near unity denotes a wideband process (17).) The mean and standard deviation of $R_τ$ may in general be obtained as $\overline{R}_{\tau} = p\sigma_R$ and $\sigma_{R_{\tau}} = q\sigma_R$, respectively, where p and q are peak factors given in terms of the three spectral moments and the duration τ . For $10 \le \nu \tau \le 1000$ and $0.11 \le \delta \le 1$, which are of interest in earthquake engineering, approximate expressions for p and q from Ref. 5 are

$$
p = \sqrt{2\ln \nu_e \tau} + \frac{0.5772}{\sqrt{2\ln \nu_e \tau}}
$$
\n(16)

$$
q = \frac{1.2}{\sqrt{2\ln \nu_e \tau}} - \frac{5.4}{13 + (2\ln \nu_e \tau)^{3.2}}
$$
(17)

where

$$
\nu_e = \begin{cases} (1.63\delta^{0.45} - 0.38)\nu, & \delta < 0.69\\ \nu, & \delta \geq 0.69 \end{cases}
$$
(18)

is an equivalent rate of statistically independent zero crossings. Fig. 4 shows plots of p and q versus $\nu\tau$ for selected values of δ . Shown in this figure is also the ratio q/p which is the coefficient of variation of the peak response.

For large values of $\nu\tau$ (say, $\nu\tau \ge 5000$), which may be of interest in some applications such as in wind and ocean engineering, asymptotic expressions of the peak factors given by Davenport (3) may be used

$$
p = \sqrt{2\ln \nu \tau} + \frac{0.5772}{\sqrt{2\ln \nu \tau}}
$$
\n(19)

$$
q = \frac{\pi}{\sqrt{6}} \frac{1}{\sqrt{2\ln \nu \tau}}
$$
\n(20)

These expressions, which are independent of δ , disregard the dependence between the crossings of the process. For this reason they can only be used for large $\nu\tau$ for which the influence of such dependence on the mean and variance is negligible (17) .

Results similar to the above also apply to each normal coordinate $S_i(t)$. It suffices to replace *R* by S_i and λ_m by $\lambda_{m,ii}$ in each of Eqs. 12-15. For notational purposes, the parameters ν , δ , ρ , and q for the *i*-th normal coordinate will be denoted in the subsequent analysis by ν _i, δ_i , p_i , and q_i , respectively.

DEVELOPMENT OF THE RESPONSE SPECTRUM METHOD

Let $\bar{S}_\tau(\omega,\zeta)$ represent the mean value of the maximum absolute response of an oscillator of frequency ω and damping ζ to a stationary input excitation, $F(t)$, over a duration τ . The function $\bar{S}_{\tau}(\omega,\zeta)$ for variable ω and ζ is defined herein as the mean response spectrum associated with the input $F(t)$ and the duration τ . It is the objective in this section to develop a procedure for evaluating the response of a multi-degree-of-freedom structure when the input is a zero-mean, wide-band, stationary Gaussian process specified through its mean response spectrum.

From solutions in Ref. 5, it can be shown that the mean zero-crossing rate, $\nu_i=\sqrt{\lambda_{2,ii}/\lambda_{0,ii}}/\pi$, and the shape factor, $\delta_i=\sqrt{1-\lambda_{1,ii}^2/\lambda_{0,ii}\lambda_{2,ii}}$, associated with the response of an oscillator of frequency ω_i and damping coefficient ζ_i , are not too sensitive to the shape of the input power spectral density, provided that the input is wide-band and that the oscillator frequency is not far beyond the significant range of input frequencies. As an example, Fig. 5 shows the ratios of these quantities for response to a filtered white-noise input with $\zeta_g = 0.6$ to those for response to a white-noise input. Observe that the ratio of the mean zero-crossing rates is always very near unity. Also, the ratio of shape factors is near unity for values of the oscillator frequency that are within the dominant range of input frequencies. From this argument, and the fact that the response is not overly sensitive to small variations in the mean zero-crossing rate and the shape factor (as it is evident in Fig. 4 for the mean and variance of the peak response), it follows that values of ν_i and δ_i that are based on a white-noise input are good approximations for the corresponding values for response to a wide-band input with arbitrary power spectral density shape. Thus, using results for a white-noise input (5,17),

$$
\nu_i = \frac{\omega_i}{\pi} \tag{21}
$$

and

$$
\delta_i = \left[1 - \frac{1}{\sqrt{1 - \zeta_i^2}} \left(1 - \frac{2}{\pi} \tan^{-1} \frac{\zeta_i}{\sqrt{1 - \zeta_i^2}}\right)^2\right]^{1/2} \approx 2 \left(\frac{\zeta_i}{\pi}\right)^{1/2}
$$
 (22)

where the approximation is valid for small damping. These expressions are used in Eqs. 16-18 (or 19-20) to compute the peak factors p_i and q_i for each normal coordinate in terms of the corresponding modal frequency and damping coefficient.

From the definition of the mean response spectrum, it is clear that $\bar{S}_{\tau}(\omega_i,\zeta_i)$ is the mean of the absolute maximum of the *i*-th normal coordinate, $S_i(t)$. Thus, using the relation $\overline{S}_{\tau}(\omega_i, \zeta_i) = p_i \sqrt{\lambda_{0,i}},$ one obtains

$$
\lambda_{0,ii} = \frac{1}{p_i^2} \overline{S}_\tau^2 (\omega_i, \zeta_i)
$$
 (23)

Furthermore, using the relations $\nu_j = \sqrt{\lambda_{2,ii}/\lambda_{0,ii}}/\pi$ and $\delta_j = \sqrt{1 - \lambda_{1,ii}/\lambda_{0,ii}\lambda_{2,ii}}$ together with the above expression and Eqs. 21-22, the first and second spectral moments for the $\dot{+}$ th normal coordinate are obtained as

$$
\lambda_{1,ii} = \frac{\omega_i \sqrt{1 - 4\zeta_i/\pi}}{p_i^2} \overline{S}_\tau^2(\omega_i, \zeta_i)
$$
 (24)

and

$$
\lambda_{2,ii} = \frac{\omega_i^2}{p_i^2} \overline{S}_\tau^2 (\omega_i, \zeta_i)
$$
 (25)

respectively. Eqs. 23-25 give the first three spectral moments of the i-th normal coordinate in terms of the corresponding modal frequency and damping coefficient and the ordinate of the input response spectrum. These results are, of course, only valid for responses to wide-band inputs and for modes whose frequencies are within the dominant range of the input frequencies.

Using Eqs. 23-25 together with Eqs. 8-10 in Eq. 5, the moments λ_0 , λ_1 , and λ_2 of the response power spectral density are computed in terms of the response spectrum ordinates. These moments can be used to evaluate the cumulative distribution of the peak response or the various statistical quantities of the response as described in the previous section (Eqs. 13-20). In particular, denoting $\overline{R}_{i\tau}=\Psi_i\overline{S}_{\tau}(\omega_i,\zeta_i)$ as the mean of the maximum response in mode i, this analysis yields the following modal combination rules for the response quantities:

Root-mean-square of response:

$$
\sigma_R = \left[\sum_i \sum_j \frac{1}{p_i p_j} \rho_{0,ij} \overline{R}_{i\tau} \overline{R}_{j\tau} \right]^{V_2}
$$
 (26)

Root-mean-square of response rate:

$$
\sigma_{\vec{R}} = \left(\sum_{i} \sum_{j} \frac{\omega_{i} \omega_{j}}{p_{i} p_{j}} \rho_{2,ij} \overline{R}_{i\tau} \overline{R}_{j\tau} \right)^{\nu_{i}}
$$
(27)

Mean of peak response:

$$
\overline{R}_{\tau} = p \sigma_R = \left(\sum_{i} \sum_{j} \frac{p^2}{p_i p_j} \rho_{0,ij} \overline{R}_{i\tau} \overline{R}_{j\tau} \right)^{\frac{1}{2}}
$$
(28)

Standard deviation of peak response:

$$
\sigma_{R_{\tau}} = q \sigma_R = \left[\sum_{i} \sum_{j} \frac{q^2}{p_i p_j} \rho_{0,ij} \overline{R}_{i\tau} \overline{R}_{j\tau} \right]^{\gamma_i}
$$
 (29)

where p and q in the last two equations are the peak factors for the response and are obtained from Eqs. 16-18 (or 19-20) in terms of the mean crossing rate (Eq, 14) and the shape factor (Eq. 15) of the response process. Another quantity that is of practical interest is the response mean frequency, denoted by $\bar{\omega}$, which is given by

$$
\overline{\omega} = \pi \nu = \frac{\sigma_{\hat{R}}}{\sigma_R} = \left(\frac{\sum_{i} \sum_{j} \frac{\omega_i \omega_j}{p_i p_j} \rho_{2,ij} \overline{R}_{i\tau} \overline{R}_{j\tau}}{\sum_{i} \sum_{j} \frac{1}{p_i p_j} \rho_{0,ij} \overline{R}_{i\tau} \overline{R}_{j\tau}} \right)^{1/2}
$$
(30)

This frequency determines the average number of response cycles over a unit duration and is useful in certain studies such as for fatigue related failures. Observe that, since $\rho_{0,ij}$ and $\rho_{2,ij}$ are nearly the same (see Fig. 3), the mean response frequency is a weighted root-mean-square of the modal frequencies of the structure.

It is important to note in the preceding expressions that since $\overline{S}_{\tau}(\omega_{i},\zeta_{i})$ is by definition positive, the sign of $\overline{R}_{i\tau}$ is the same as that of the corresponding effective participation factor, Ψ_i . The sign of this factor depends on the modal characteristics of the structure and on the direction of input. Since $\rho_{0,ij}$ and $\rho_{2,ij}$ are always positive, it follows that the cross terms in Eqs. 26-30 are negative when Ψ_i and Ψ_j assume opposite signs. It is easy to show, however, that the double summations inside parentheses in these expressions are always positive.

In many practical applications, the mean of the peak response is all that is required. Therefore, a close examination and possible simplification of Eq. 28 is of special interest. It is first noted from substituting Eq. 21 in Eq. 30 that the mean zero-crossing rate of the response process, ν , is a weighted root-mean-square of the mean zero-crossing rates, ν_i , of the normal coordinates. On the other hand, the shape factor, δ , of the response process would usually tend to be greater than δ_i , since the response being contributed by all modes is usually a broaderband process than each of the normal coordinates. This, however, may not be true when the response is mainly contributed by one mode or two closely spaced modes, in which case δ can be equal to or smaller than δ_i . In any case, since the peak factor is only slightly dependent on the shape factor (see Fig. 4), it is easy to see that, because of the relation of ν to ν_i , p would generally tend to be some sort of an average of p_i . It follows, then, that the ratios p/p_i are around unity and, since the peak factor monotonically increases with the frequency, they tend to decrease with increasing mode number. The role of these ratios in Eq. 28 for the mean response, therefore, is to enhance the contributions from the lower modes and to reduce those from the higher modes of the structure. Since the peak factor has slow variation with the frequency, as is evident from its logarithmic relation to the mean-crossing rate (see Fig. 4), for most structures the ratios p/p_i would tend to be near unity. Thus, these ratios in the expression for the mean response can be discarded without much loss of accuracy. With this simplification, Eq. 28 reduces to

$$
\overline{R}_{\tau} = \left(\sum_{i} \sum_{j} \rho_{0,ij} \overline{R}_{i\tau} \overline{R}_{j\tau}\right)^{V_2}
$$
\n(31)

The advantage gained from this simplification is that the mean response is now given directly in terms of the maximum modal responses and the coefficients $\rho_{0,i,j}$; i.e., there is no need to compute the spectral moments from Eq. 5. Also note that this expression for the mean response is independent of the duration (except that which is implied through the input response spectrum).

A similar simplification of the expression for the response mean frequency, $\overline{\omega}$, is also possible, which after multiplying the numerator and denominator in Eq. 30 by p and neglecting the ratios p/p_i , reduces to

$$
\overline{\omega} = \left(\frac{\sum_{i} \sum_{j} \omega_{i} \omega_{j} \rho_{2,ij} \overline{R}_{i\tau} \overline{R}_{j\tau}}{\sum_{i} \sum_{j} \rho_{0,ij} \overline{R}_{i\tau} \overline{R}_{j\tau}} \right)^{V_{2}}
$$
(32)

Observe that, with this simplification, $\overline{\omega}$ becomes the root-mean-square of the modal frequencies as weighted by the maximum modal responses. Because of this, this frequency is a good indicator of the significance of contributions from various modes of a structure to a particular response, i.e. a larger $\overline{\omega}$ would indicate larger contributions from higher modes.

For structures with well separated frequencies the coefficients $\rho_{m,ij}$ vanish; see Fig. 3. As a result, all cross terms in the response expressions, i.e., Eqs. 5 and 26-32, can be neglected for such structures. (As a simple rule, such terms can be dropped when ω_i/ω_j is less than $0.2/(\zeta_i+\zeta_j)$, which approximately corresponds to $\rho_{m,ij}$ less than 0.1 (5).) In particular, Eq. 31 in this case reduces to

$$
\overline{R}_{\tau} = \left(\sum_{i} \overline{R}_{i\tau}^{2}\right)^{\frac{1}{2}}
$$
 (33)

This is the well known square-root-of-sum-of-squares (SRSS) rule for modal combination. It is clear from this derivation that the SRSS rule for the peak response is only adequate for structures with well spaced frequencies. The error associated with the SRSS method for neglecting the modal cross-correlations can be significant, as illustrated in the subsequent example.

APPLICATION TO EARTHQUAKE LOADING

The proposed modal combination procedure should be particularly useful in earthquake engineering, where a response spectrum description of the ground motion is widely used. However, for such an application, it is important to examine the validity of the basic assumptions of the method relative to earthquake excitations. Specifically, assumptions to be examined are: (a) that the ground motion is a stationary Gaussian process with a wide-band power spectral density, and (b) that the response of the (linear) structure is a stationary process. Whereas earthquake-induced ground motions are inherently nonstationary, the strong phase of such motions is usually nearly stationary. Since the peak response generally occurs during this phase, it is reasonable, at least for the purpose of developing a response spectrum method, to assume it to be a stationary process. This assumption would clearly become less accurate for short-duration, impulsive earthquakes. The assumption of Gaussian excitation is acceptable on the basis of the central limit theorem, since the earthquake ground motion is the accumulation of a large number of randomly arriving pulses (2). The wide-band assumption for the earthquake motion has been verified based on recorded motions and is generally accepted $(2,7)$. Finally, for the assumption of stationary response, it is well known (e.g., Ref. 8) that the response of a not-too-lightly damped oscillator to a wide-band input reaches stationarity in just a few cycles. Thus, this assumption should be acceptable for structures whose fundamental periods are several times shorter than the strong-phase duration of the ground motion. These considerations also suggest that the strong-phase duration of the ground motion is the appropriate value for the parameter τ to be used in the response spectrum method.

It is clear from the above discussion that the response spectrum method for earthquake loading will be most accurate for earthquakes with long, stationary phases of strong shaking and for not-too-lightly damped structures whose fundamental periods are several times shorter than the duration of earthquake. Through a number of example studies, it has been found that the procedure is quite accurate for typical structures and earthquakes (see the example below). It has also been found that Eq. 31 for the mean response closely approximates the maximum response for a deterministic ground motion with a non-smooth response spectrum. For this reason, it has been proposed as a replacement for the SRSS method in deterministic analysis (18). Based on the coefficient of variation of the peak response, i.e. the ratio q/p in Fig. 4, maximum errors in such applications are expected to range within 10 to 30 percent, depending on the response frequency.

Several formulations for the mean of the peak response to earthquake excitations have previously been proposed (10,11,14). These are similar to Eq. 31 of the present formulation, except for the difference in expressions given for the cross-correlation coefficient. The best known among these methods is that of Rosenblueth et al. (10). In their formulation, which has somewhat heuristic bases, the cross-correlation coefficients are given in terms of the modal frequencies and damping coefficients and the duration of motion. For earthquake-type excitations, this method appears to give good results if the duration is known (see Refs. 4 and 13 for comparisons of the method with exact solutions). However, since the duration is often unknown, Eq. 31, which is independent of duration, provides a better method to be used in such applications. It must be pointed out that none of the existing response spectrum methods provide any means for computing the variance or the cumulative distribution of the peak response.

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This aspect of the present formulation, therefore, is unique and a furtherance of the state of the art.

EXAMPLE APPLICATION

As an example application of the proposed response spectrum method, the responses of a 5-story building structure to a set of 20 simulated ground motions are studied. The building is assumed to have rigid floors with uniform mass and stiffness along the height. A typical floor plan and the properties are shown in Fig. 6. The structure is subjected to ground motions in the *x* direction only. However, because of asymmetry about the *x* axis, the center of mass at each floor has a rotational as well as a translational degree of freedom. This results in modes with closely spaced frequencies, as described in the table in Fig. 6.

The ground motions used in this study were obtained using a simulation method by Ruiz and Penzien (12). In this method, ground acceleration records are generated as samples of a filtered Gaussian white-noise process as modulated by an intensity function. The filter parameters for the power spectral density were selected to be $\omega_g = 5\pi$ and $\zeta_g = 0.6$. An intensity function similar to that of a type-B earthquake, as defined by Jennings et al. (6), was used for this purpose. It includes a stationary strong-motion phase of 11 seconds (between 4 and 15 seconds), yielding $\tau = 11$ for the response spectrum analysis. The power spectral density scale factor, Go (see Eq. 7), was selected such as to produce a mean peak ground acceleration of *0.5g* over the ensemble of records. A sample of the simulated ground motions is illustrated in Fig. 7.

Pseudo-velocity response spectra associated with each individual ground motion for 0, 2, 5, 10, and 20 percent damping were computed. Means, standard deviations, and coefficients of variation of these spectra are shown in Fig. 8. For comparison, the coefficient of variation, q/p , based on Eqs. 16-18 and 21-22, are also shown in Fig. 8(c) for the spectra with non-zero damping (smooth curves). Observe that for such spectra the analytical estimates of the coefficient of variation closely agree with simulation results. It is interesting to note that the coefficient of variation is relatively insensitive to damping. Also, note that it increases with increasing period ranging from about 0.1 at short periods (0.05 seconds) to about 0.3 to 0.4 at long periods (5 seconds). This implies that the peak response of low-frequency oscillators is more sensitive to details of the ground motion than that of high-frequency oscillators. This is in agreement with Trifunac's results in a study of digitization noise for recorded accelerograms (15). It is also interesting to note in the simulation results of Fig. 8(c) that for an undamped oscillator the coefficient of variation in the peak response is much larger than that for a damped oscillator, especially for short period oscillators. (Note that for an undamped oscillator the response does not reach stationarity and the analytical expressions in Eqs. 16-18 do not hold.) A somewhat surprising result in Fig. 8(c) is the small magnitude of the coefficient of variation for damped spectra. Studies based on recorded accelerograms, such as that by Newmark (9), generally indicate much larger variability. This can be explained by the fact that acceleration records used in such studies are for different earthquakes and site conditions and do not represent members of a single stochastic process. The larger coefficient of variation is a consequence of this added variability.

Using numerical integration, peak responses of the example structure to each of the 20 ground motions were computed. These samples are subsequently compared with response estimates based on the proposed method using the mean response spectrum in Fig. 8(a). Table 1 shows a comparison of the means and standard deviations of peak responses. (The word "simulation" in this table denotes results based on numerical integration). Included in this table are estimates of the mean peak response based on Eq. 28, the simplified method of Eq. 31, and the SRSS method of Eq. 33, and estimates of the standard deviation of peak response based on Eq. 29. Estimates of the root-mean-square response based on Eq. 26 are also included in the last column of the table. It can be observed in this table that Eqs. 28 and 29 for the mean and standard deviation of the peak response are in close agreement with the simulation results. The simplified expression for the mean response, Eq. 31, also appears to give good results. However, the SRSS method, Eq. 33, is in gross error reflecting the significance of the correlation between modal responses which is neglected in this approach. Note that this method of modal combination underestimates the translational responses by as much as 27 percent and overestimates the rotational responses by a factor greater than 2.

Fig. 9 shows a plot of the mean frequency, $\overline{\omega}$, for various responses of the structure. As can be observed, this frequency has a small value (close to the frequencies of the first two modes) for floor displacements and rotations and for lower story shears and torques, whereas it has a larger value for upper story shears and torques and for floor pseudo-accelerations. Since $\overline{\omega}$ is the root-mean-square of modal frequencies as weighted by the corresponding modal responses, it follows from Fig. 9 that, as expected, floor displacements and rotations and lower story shears and torques are mainly contributed by the first two modes, whereas upper-story shears and torques and floor pseudo-accelerations have significant contributions from higher modes.

Finally, Fig. 10 shows comparisons between the cumulative histograms of the simulated samples and the cumulative distribution of Eq. 13, based on the response spectrum method, for selected response quantities. In all cases, the theoretical distribution is acceptable based on the Kolmogorov-Smirnov test (1) at all significance levels.

CONCLUSIONS

The principal results and conclusions of this study can be summarized as follows:

- (1) A response spectrum method for stationary random vibration analysis of linear structures subjected to wide-band, stationary Gaussian excitations is developed. Modal combination rules are derived for the mean-squares of the response and its time derivative, the mean and variance of the peak response and the mean frequency of the response. The cumulative distribution of the peak response is also obtained in terms of the input response spectrum. The analysis properly accounts for the cross-correlation between modal responses.
- (2) The cross-correlation between modal responses is significant for modes with closely spaced frequencies. The conventional SRSS method of modal combination, which neglects this correlation, can lead to gross errors in estimating the peak response when the structure

frequencies are closely spaced.

- (3) The proposed response spectrum method can be used for structures subjected to transient Gaussian wide-band inputs, such as earthquake induced base excitations.' In such applications, the method would be more accurate when the excitation has a long, stationary phase of strong motion, and the structure is not too lightly damped and has a fundamental period which is several times shorter than the duration of excitation.
- (4) In an example application, results based on the proposed response spectrum method are in close agreement with simulation results based on time-history computations. It is shown that the coefficient of variation in the peak response of an oscillator increases with decreasing oscillator frequency and damping. This coefficient is found to range between 0.1 to 0.4 for oscillators of period 0.05 to 5 seconds. It is also shown that the response mean frequency, which is given as the root-mean-square of modal frequencies as weighted by the corresponding modal responses, is a good indicator of the significance of higher modes to a particular response.

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NOTATION

 $\mathcal{A}^{\text{max}}_{\text{max}}$

 $\mathcal{A}^{\text{max}}_{\text{max}}$

 $\sim 10^6$

 $\mathcal{A}^{\mathcal{A}}$

Table 1. Summary of Results for Example Structure

Note: \overline{R}_7 = mean of peak response; σ_{R_7} = standard deviation of peak response; $\sigma_{R=}$ root-meansquare response.

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Figure 1. Power Spectral Density Shapes for White-Noise and Filtered White-Noise Inputs.

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Figure 2. Comparison of Correlation Coefficients for Responses to White-Noise and Filtered White-Noise Inputs.

Figure 3. Coefficients $\rho_{m,ij}$ for Response to White Noise.

Figure 4. Peak Factors for Response to Stationary Gaussian Input.

Figure 5. Ratios of Mean-Crossing Rates and Shape Factors for Responses to Filtered White-Noise (FWN) and White-Noise (WN) Inputs.

TYPICAL FLOOR PLAN ELEVATION

Properties of Typical Floor and Story:

Mass = 100,000 *kg.* Mass Radius of Gyration ⁼ 5.56 *m.* Stiffness: ${\bf k_x}$ = ${\bf k_y}$ = 205900 *kN/m*. $e = 0.30 \ m.$ ² a= 5.56 *m.*

Modal Properties

Figure 6. Properties of Example Structure.

Figure 7. Sample of Simulated Earthquake Ground Motion.

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 $-28-$

Figure 8. Pseudo-Velocity Spectra for 20 Generated Earthquakes.

Figure 9. Mean Frequency for Responses of Example Structure.

Computed and Simulated Cumulative Distributions for Figure 10. Selected Peak Responses of Example Structure.

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