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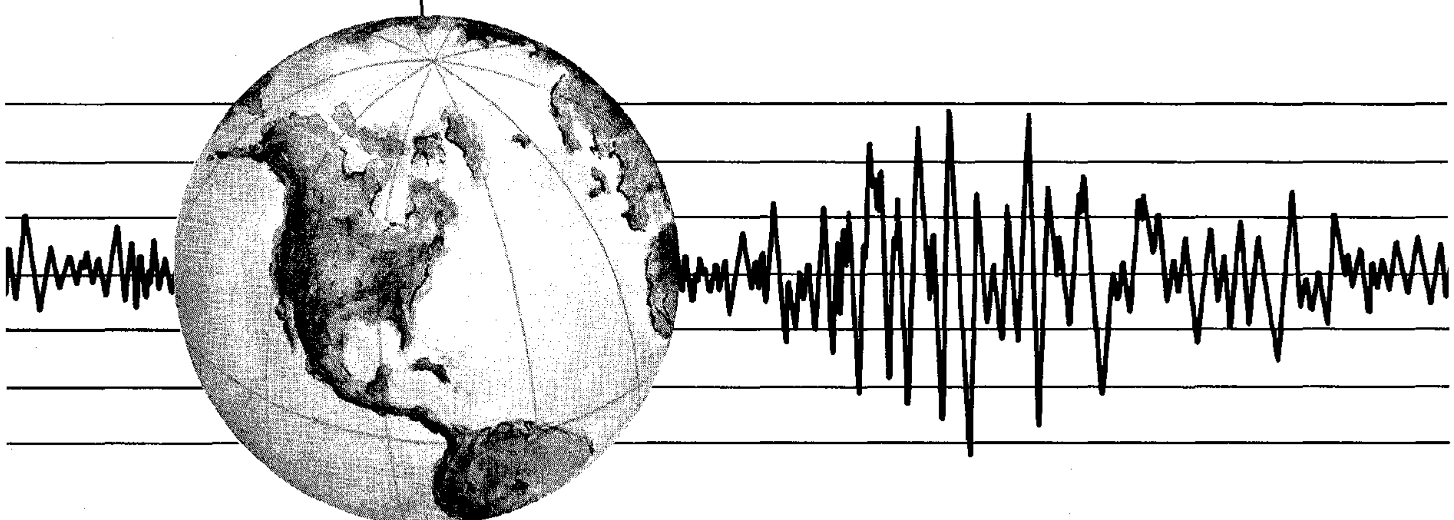
INELASTIC TORSIONAL RESPONSE OF STRUCTURES SUBJECTED TO EARTHQUAKE GROUND MOTIONS

by
YUTAKA YAMAZAKI

Report to
National Science Foundation



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A single-story structure with a rectangular deck and four resisting elements was used to examine these objectives. First, dimensionless equations of motion were formulated containing the basic parameters which control earthquake response and, then, parametric studies were carried out to determine the effects of such parameters on elasto-plastic coupled translational-torsional response.

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Al Klash is responsible for the drafing and the manuscript was typed by Ruth Horning.

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LIST OF SYMBOLS

$\underline{a}$	dimensionless diagonal yield shear force matrix
$\underline{a}^{(1,3)}$	dimensionless diagonal yield shear force matrix
$\underline{a}_i$	dimensionless diagonal yield shear force matrix
a_{x0}, a_{y0}	dimensionless yield shear forces
$a_{\theta 0}$	dimensionless yield moment
A_1, A_2, A_3	parameters to define a damping matrix
$\underline{b}_i$	dimensionless matrix to define rate of dimensionless plastic work
$\underline{B}_i$	matrix to define rate of plastic work
$\underline{c}_i$	dimensionless matrix to define rate of dimensionless shear force during yielding
c_0	parameter to determine dimensionless yield shear force
$\underline{C}_i$	matrix to define rate of shear force during yielding
C_M	symbol representing the center of mass
C_R	symbol representing the center of rigidity
d_{ix}, d_{iy}	dimensionless distances
$\underline{D}$	orthogonal damping matrix
e_x, e_y	static eccentricities
E	symbol denoting elastic response
EP	symbol denoting elasto-plastic response without force interaction
EPI	symbol denoting elasto-plastic response with force interaction
$\underline{F}$	restoring force vector
g	gravitational constant
H	story height
i	subscript denoting a resisting element number
I	moment of inertia

LIST OF SYMBOLS (cont.)

$\underline{k}_i$	diagonal translational stiffness matrix
k_{ix}, k_{iy}	translational stiffnesses
$\underline{K}$	translational stiffness matrix
K_x, K_y	translational stiffnesses
K_θ	torsional stiffness
M	mass
$\underline{M}$	diagonal mass matrix
M_{ix}, M_{iy}	bending moments
M_{ix0}, M_{iy0}	yield moments
n	dimensionless natural circular frequency
$\underline{n}$	dimensionless natural circular frequency matrix
p	superscript denoting plastic state
$\underline{p}$	dimensionless restoring force vector
$\underline{p}_i$	dimensionless restoring force vector
p_{ix}, p_{iy}	dimensionless restoring forces
p_x, p_y, p_θ	dimensionless restoring forces
q_{ix0}, q_{iy0}	dimensionless yield shear forces
$q_{i\theta x0}$, $q_{i\theta y0}$	dimensionless yield moments
$\underline{Q}$	restoring force vector
$\underline{Q}_i$	restoring force vector
Q_{ix}, Q_{iy}	restoring forces
Q_{ixb}, Q_{iyb}	restoring forces for a bending failure system
Q_{ix0}, Q_{iy0}	yield shear forces
Q_x, Q_y	restoring forces
Q_θ	torsional moment

LIST OF SYMBOLS (cont.)

Q_{x0}, Q_{y0}, $Q_{\theta 0}$	yield shear forces
r	radius of gyration
$\underline{r}$	radius of gyration matrix
$\underline{r}_i$	dimensionless diagonal matrix to define dimensionless shear forces during yielding
r_{k_i}	stiffness ratio
r_{Q_i}	yield shear force ratio
s_{ix}, s_{iy}	dimensionless translational stiffnesses
$s_{i\theta x}$, $s_{i\theta y}$	dimensionless torsional stiffnesses
t	time
T	uncoupled natural period
T_x, T_y	uncoupled natural periods
$\underline{u}$	displacement vector
$\underline{\ddot{u}}_g$	ground acceleration vector
$\ddot{u}_{gx}$, $\ddot{u}_{gy}$	ground accelerations
$\lvert\ddot{u}_{gx}\rvert max$, $\lvert\ddot{u}_{gy}\rvert max$	the maximum ground accelerations
$\underline{u}_i$	displacement vector
$\underline{\dot{u}}_i^p$	plastic part of velocity vector
u_{ix}, u_{iy}	displacements
u_{ix0}, u_{iy0}	elastic limit displacements
u_x, u_y	displacements
u_θ	rotation
u_{x0}, u_{y0}, $u_{\theta 0}$	elastic limit displacements
$\underline{U}$	displacement vector

LIST OF SYMBOLS (cont.)

$\underline{v}$	dimensionless displacement vector
$\underline{v}_i$	dimensionless displacement vector
v_{ix}, v_{iy}	dimensionless displacements
v_{ix0}, v_{iy0}	dimensionless elastic limit displacements
$v_{i\theta x0}$, $v_{i\theta y0}$	dimensionless parameters to define a transfer matrix
v_{i0}	dimensionless elastic limit displacement matrix
v_x, v_y, v_θ	dimensionless displacemnts
w_i^p	dimensionless plastic work
w_{ix}^p, w_{iy}^p	dimensionless plastic works
$\dot{w}_i^p$	rate of plastic work
$\dot{W}_{ix}^p$, $\dot{W}_{iy}^p$	rates of plastic works
x, y	coordinates
x_i, y_i	distances
$\underline{z}_i$	dimensionless transfer matrix
$\underline{z}_i^{(1)}$, $z_i^{(2)}$	row matrices of $\underline{z}_i$
$\underline{Z}_i$	transfer matrix
$\underline{Z}_i^{(1)}$, $\underline{Z}_i^{(2)}$	row matrices of $\underline{Z}_i$
$\underline{\alpha}_g$	dimensionless ground acceleration vector
α_{gx}, α_{gy}	dimensionless ground accelerations
β_i	parameter to define rate of dimensionless plastic work
B_i	parameter to define rate of plastic work
γ_i	parameter to define rate of dimensionless shear forces during yielding
Γ_i	parameter to define rate of shear forces during yielding
δ_1, δ_2, δ_3	parameters to define a dimensionless damping matrix

LIST OF SYMBOLS (cont.)

$\underline{\Delta}$	orthogonal damping matrix
$\underline{\varepsilon}$	dimensionless eccentricity matrix
$\underline{\zeta}$	uncoupled frequency ratio matrix
$\zeta_{xy}, \zeta_{x\theta}, \zeta_{y\theta}$	uncoupled frequency ratios
η	dimensionless variable
$\underline{\iota}$	constant matrix
$\underline{\lambda}$	dimensionless stiffness matrix
$\underline{\Lambda}$	stiffness matrix
$\underline{\mu}_i$	ductility factor vector
μ_{ix}, μ_{iy}	ductility factors
$\underline{\nu}$	dimensionless damping matrix
$\underline{\xi}$	modal damaping factor vector
ξ_1, ξ_2, ξ_3	modal damping factors
$\underline{\pi}$	dimensionless restoring force vector
$\underline{\rho}$	elastic limit displacement ratio matrix
$\rho_{x\theta}, \rho_{y\theta}$	elastic limit displacement ratio
$\underline{\sigma}$	frequency ratio matrix
τ	dimensionless time
$\ddot{U}_{gx}, \ddot{U}_{gy}$	dimensionless ground accelerations
$\lvert\ddot{U}_{gx}\rvert max$, $\lvert\ddot{U}_{gy}\rvert max$	the maximum dimensionless ground accelerations
Φ_i	yield surface function
$\underline{\omega}$	frequency matrix
$\omega_x, \omega_y, \omega_\theta$	uncoupled circular frequencies
$\omega_1, \omega_2, \omega_3$	natural ciruclar frequencies

LIST OF SYMBOLS (cont.)

$\lvert \quad \rvert$	symbol denoting absolute values
$\langle \, , \, \rangle$	symbol denoting inner product of vectors
$\begin{bmatrix} \ddots \end{bmatrix}$	symbol denoting a diagonal matrix
$\dot{}$	symbol denoting differentiation with respect to time t
$'$	symbol denoting differentiation with respect to dimensionless time τ

LIST OF FIGURES

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1. INTRODUCTION

It is important in earthquake resistant design of structures to know the effect of eccentricity on seismic response. The coupled lateral and torsional response of structures has been investigated by many researchers during the past ten years or so. These studies have considered the response of both linear and non-linear systems subjected to both single- and multi-dimensional earthquake excitations.

Coupling between lateral and torsional motions of a primary structure and an attachment, such as a penthouse or a tower, has been studied using linear systems by Dempsey[9], Douglas and Trabert[10], Kan and Chopra[20,21,22], Medeario [28], Müller and Keintzel[33], Penzien[43] and Skinner, Skilton and Laws[51].

Lateral-torsional response of a single-story and multi-story elastic structure subjected to two-dimensional ground motions has been investigated by Housner and Outinen [18], Jurukovski and Bickovski[19], Mazilu, Sandi and Teodorescu[27], Newmark[34], Shepherd and Donald[48], Shiga[50] and Tso and Biswas[54]. Mazilu, Sandi and Teodorescu[27] and Newmark[34] have investigated torsional response of linear systems subjected to torsional ground motions caused by wave propagation on the ground surface. Differing from the above studies using discrete parameter models, Gibson Moody and Ayre[13,14] and Hoerner[17] have investigated the torisional response of continuous models.

Non-linear response of three-dimensional structures subjected to multi-component ground motions has been investigated by many researchers, that is, by Anagnostopoulos, Roesset and Biggs[4], Clough, Bennuska and Wilson[6], Erdik[12], Kan and Chopra[23], Kobori, Minai and Fujiwara[24], Koh, Takase and Tsugawa[25], Meyer and Oppenheim[29], Morris[32], Nigam [35,36,37], Okada, Murakami, Udagawa, Nishikawa, Osawa and Tanaka[38], Pecknold[42], Porter and Powell[44], Prasad and Jagadish[45], Onose and Shiga[49], Takizawa[52], Toridis and Khozeimeh[53], Wen and Farhoomand [57]and Yamazaki[58].

Non-linear behavior of simple resisting elements, such as columns or beams, under multi-dimensional forces also has been investigated by Aktan, Pecknold and Sozen[1,2], Aktan and Pecknold[3], Chen and Atsuta [5], Darwin and Pecknold[8], Hodge[16], Liu, Nilson and Slate[26], Morris and Fenves[30,31], Okada, T., Seki, Asai, Paku and Okada, K.[39], Okada, T., Seki, Paku and Okada, K.[40,41], Rosenblueth and Contreras[46], Santathadaporn and Chen[47], Warner[55] and Wen and Beylerian[56].

Two basic approaches have been used to investigate the inelastic behavior of resisting elements under multi-dimensional forces. One approach uses plasticity theory leading one to the concept of a yield surface on the whole section of a resisting element. Stable inealstic material defined by Drucker[11] is usually assumed in this case. The other approach has similarities to the finite element method in that the section of a resisting element is divided into sub-elements possessing a uniaxial stress-strain relationship. This approach has been applied many times to the analyses of the inelastic behavior of reinforced concrete columns, e.g. Aktan, Pecknold and Sozen[1,2], Aktan and Pecknold [3], Okada, T., Seki, Asai, Paku and Okada, K.[39], Okada, T., Seki, Paku and Okada, K.[40,41] and Warner[55].

The objectives of this paper are (1) to identify the basic parameters which control the earthquake response of torsionally coupled systems composed of resisting elements providing interaction during yielding; (2) to clarify differences in response between systems subjected to single-component ground motion and systems subjected to double-component ground motion; (3) to clarify differences in response among an elastic system, an elasto-plastic system without force interaction and an elasto-plastic system with force interaction (EPI-system) and (4) to evaluate the effects of magnitude of eccentricity and magnitudes of yield shear forces on the response of EPI-systems. To fulfill these objectives, a structural system was used having the following characteristics:

i. A single-story system is used having a rectangular deck and four resisting elements supported on a rigid base.

ii. The floor deck is assumed to be rigid and the resisting elements are assumed to be rigidly clamped at top and bottom.

iii. The rigid base is excited by two orthogonal horizontal components of ground motion in the x- and y- directions.

iv. Axial deformations of the resisting elements are neglected and the inter-story lateral displacements are assumed to be small compared with the dimensions of the system.

v. Elasto-plastic hysteretic model is assumed for the relation between shear force and shear deformation of the resisting elements.

vi. The interaction effect between two orthogonal components of shear force acting on sections of the resisting elements during yielding is taken into account using an assumed circular yield surface.

2. EQUATIONS OF MOTION

2.1 Linear Systems

Let k_{ix} and k_{iy} represent the translational stiffnesses of the i-th resisting element (column or wall) along the principal axes of resistance, x and y, respectively. Then,

$$K_x = \sum_i k_{ix}$$

and (2.1)

$$K_y = \sum_i k_{iy}$$

are the translational stiffnesses of the structure in the x- and y-directions, respectively. Also, let x_i and y_i be the distances of the i-th resisting element from the center of mass along the x- and y-axes, as shown in Fig. 2.1. Then,

$$K_\theta = \sum_i k_{ix} \; y_i^2 + \sum_i k_{iy} \; x_i^2 \tag{2.2}$$

is the torisonal stiffness of the structure about the center of mass. The torsional stiffnesses of the individual resisting elements about their own centroidal axes can be neglected.

For a system of discrete resisting elements, the center of resistance is located at distances e_x and e_y (the static eccentricities) along the x- and y- axes, respectively, where

$$e_x = \frac{1}{K_y} \sum_i k_{iy} \, x_i$$

and (2.3)

$$e_y = \frac{1}{K_x} \sum_i k_{ix} \, y_i$$

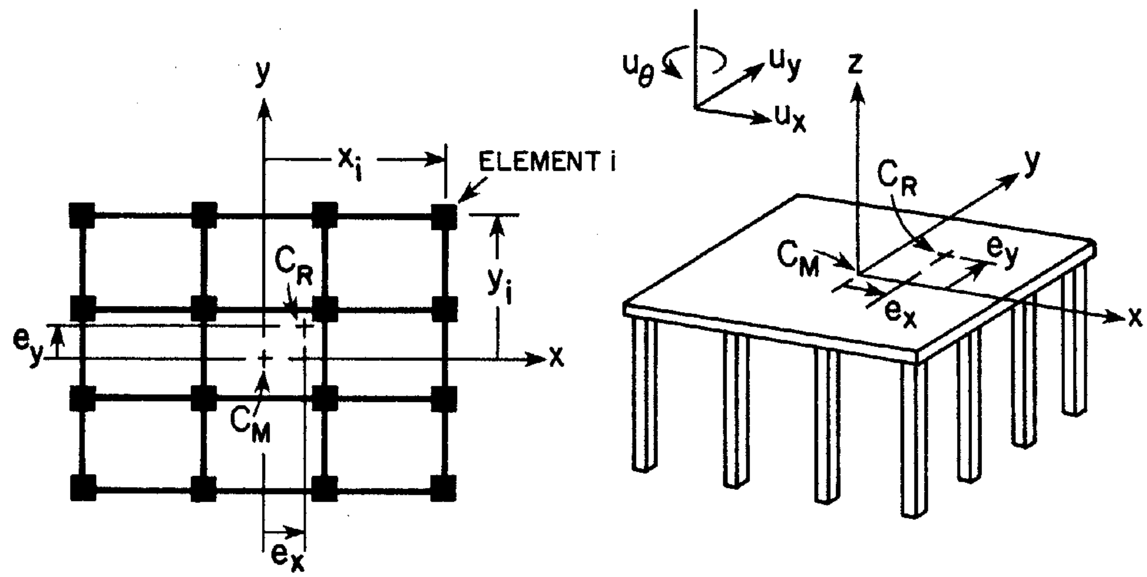


Fig. 2.1 Single-Story System

Let the earthquake ground motion be defined by accelerations $\ddot{u}_{gx}(t)$ and $\ddot{u}_{gy}(t)$ along the x- and y-axes. The equations of motion of the system shown in Fig. 2.1, without damping, can then be written in the standard form

$$\left.\begin{array}{l} M\ddot{u}_x + K_x u_x - e_y K_x u_\theta = -M\ddot{u}_{gx} \\ I\ddot{u}_\theta - e_y K_x u_x + K_\theta u_\theta + e_x K_y u_y = 0 \\ M\ddot{u}_y + e_x K_y u_\theta + K_y u_y = -M\ddot{u}_{gy} \end{array}\right\} \quad (2.4)$$

in which M and I are the mass of the deck and the inertia of rotation of the deck about a vertical axis through the center of mass, respectively, and u_x, u_y and u_θ are displacement components of the center of mass relative to the base in the x-, y- and θ-directions, respectively.

Equation 2.4 can be rewritten in the matrix form

$$\underline{M}\,\underline{\ddot{U}} + \underline{K}\,\underline{U} = -\underline{M}\,\underline{\ddot{u}}_g \quad (2.5)$$

where

$$\underline{M} = \begin{bmatrix} M & & \\ & I & \\ & & M \end{bmatrix} \quad (2.6)$$

$$\underline{K} = \begin{bmatrix} K_x & -e_y K_x & 0 \\ -e_y K_x & K_\theta & e_x K_y \\ 0 & e_x K_y & K_y \end{bmatrix} \quad (2.7)$$

$$\underline{U} = \begin{Bmatrix} u_x \\ u_\theta \\ u_y \end{Bmatrix} \quad (2.8)$$

and

$$\underline{\ddot{u}}_g = \begin{Bmatrix} \ddot{u}_{gx} \\ 0 \\ \ddot{u}_{gy} \end{Bmatrix} \quad (2.9)$$

Equation 2.5 is for an undamped system. For a damped system, an orthogonal damping matrix[7], $\underline{D}$, can be used in the form

$$\underline{D} = \underline{M}\ (A_1\ \underline{K}^{-1}\ \underline{M} + A_2 + A_3\ \underline{M}^{-1}\ \underline{K}) \tag{2.10}$$

where

$$\begin{Bmatrix} A_1 \\ A_2 \\ A_3 \end{Bmatrix} = 2\ \underline{\omega}^{-1}\ \underline{\xi} \tag{2.11}$$

in which

$$\underline{\omega} = \begin{bmatrix} \frac{1}{\omega_1^3} & \frac{1}{\omega_1} & \omega_1 \\ \frac{1}{\omega_2^3} & \frac{1}{\omega_2} & \omega_2 \\ \frac{1}{\omega_3^3} & \frac{1}{\omega_3} & \omega_3 \end{bmatrix} \tag{2.12}$$

and

$$\underline{\xi} = \begin{Bmatrix} \xi_1 \\ \xi_2 \\ \xi_3 \end{Bmatrix} \tag{2.13}$$

Quantities ω_i and ξ_i $(i = 1,2,3)$ are the circular frequencies and specified modal damping factors, respectively, for the i-th natural mode of the system. The equations of motion of the damped system now become

$$\underline{M}\ \ddot{\underline{U}} + \underline{D}\ \dot{\underline{U}} + \underline{K}\ \underline{U} = -\ M\ \ddot{\underline{u}}_g \tag{2.14}$$

which can be changed into the normalized form

$$\ddot{\underline{u}} + \underline{\Delta}\ \dot{\underline{u}} + \underline{\Lambda}\ \underline{u} = -\ \ddot{\underline{u}}_g \tag{2.15}$$

where

$$\underline{\Delta} = A_1\,\underline{\Lambda}^{-1} + A_2 + A_3\,\underline{\Lambda} \tag{2.16}$$

$$\underline{\Lambda} = \begin{bmatrix} \omega_x^2 & -\frac{e_y}{r}\,\omega_x^2 & 0 \\ -\frac{e_y}{r}\,\omega_x^2 & \omega_\theta^2 & \frac{e_x}{r}\,\omega_y^2 \\ 0 & \frac{e_x}{r}\,\omega_y^2 & \omega_y^2 \end{bmatrix} \tag{2.17}$$

$$\underline{u} = \begin{Bmatrix} u_x \\ r\,u_\theta \\ u_y \end{Bmatrix} = \underline{r}\;\underline{U} \tag{2.18}$$

in which

$$\left.\begin{aligned} \omega_x^2 &= K_x/M \\ \omega_y^2 &= K_y/M \\ \omega_\theta^2 &= K_\theta/I \end{aligned}\right\} \tag{2.19}$$

and

$$\underline{r} = \begin{bmatrix} 1 & & \\ & r & \\ & & 1 \end{bmatrix} \tag{2.20}$$

Quantity r in Eq. 2.20 is the radius of gyration of the deck about a vertical axis through the center of mass as defined by

$$I = M\,r^2 \tag{2.21}$$

2.2 Non-Linear Systems

The equations of motion of the nonlinear system can be written in the form

$$\underline{M}\,\ddot{\underline{U}} + \underline{D}\,\dot{\underline{U}} + \underline{F} = -\,M\,\ddot{\underline{u}}_g \tag{2.22}$$

or

$$\ddot{\underline{u}} + \underline{\Delta}\,\dot{\underline{u}} + \underline{M}^{-1}\,\underline{Q} = -\,\ddot{\underline{u}}_g \tag{2.23}$$

where the corresponding restoring force vectors are given by

$$\underline{F} = \begin{Bmatrix} Q_x \\ Q_\theta \\ Q_y \end{Bmatrix} = \underline{r} \begin{Bmatrix} \sum_i Q_{ix} \\ -\sum_i Q_{ix}\, d_{iy} + \sum_i Q_{iy}\, d_{ix} \\ \sum_i Q_{iy} \end{Bmatrix} \tag{2.24}$$

and

$$\underline{Q} = \begin{Bmatrix} Q_x \\ r\,Q_\theta \\ Q_y \end{Bmatrix} = \underline{r}\,\underline{F} = \underline{r}^2 \begin{Bmatrix} \sum_i Q_{ix} \\ -\sum_i Q_{ix}\, d_{iy} + \sum_i Q_{iy}\, d_{ix} \\ \sum_i Q_{iy} \end{Bmatrix} \tag{2.25}$$

in which

$$\begin{Bmatrix} d_{ix} \\ d_{iy} \end{Bmatrix} = \begin{Bmatrix} x_i/r \\ y_i/r \end{Bmatrix} \tag{2.26}$$

In the above equations Q_x, Q_y and Q_θ denote restoring shear forces of the system in the x- and y-directions and the restoring torsional moment of the system about the vertical axis through the center of mass, respectively, and Q_{ix} and Q_{iy} denote restoring shear forces of the i-th resisting element in the x- and y-directions, respectively. The restoring shear forces Q_{ix} and Q_{iy} of the i-th resisting element are, if the system is linear, given by

$$\underline{Q}_i = \underline{k}_i\,\underline{u}_i \tag{2.27}$$

where

$$\underline{Q}_i = \begin{Bmatrix} Q_{ix} \\ Q_{iy} \end{Bmatrix} \tag{2.28}$$

$$\underline{k}_i = \begin{bmatrix} k_{ix} & \\ & \ddots & k_{iy} \end{bmatrix} \tag{2.29}$$

$$\underline{u}_i = \begin{Bmatrix} u_{ix} \\ u_{iy} \end{Bmatrix} \tag{2.30}$$

in which u_{ix} and u_{iy} are, as shown in Fig. 2.2, displacements of the i-th resisting element relative to the base in the x- and y-directions, respectively. They are related to the displacements of the center of mass relative to the base, u_x and u_y, and the rotation of the deck about the vertical axis through the center of mass, u_θ, by

$$\underline{u}_i = \underline{Z}_i \, \underline{u} \tag{2.31}$$

where

$$\underline{Z}_i = \begin{bmatrix} 1 & -d_{iy} & 0 \\ 0 & d_{ix} & 1 \end{bmatrix} \tag{2.32}$$

For the nonlinear system, conditional equations must be defined which consider interaction between forces acting on the section of elements during yielding. A general theory considering yielding in structures with interaction in terms of generalized forces and displacements has been presented by Nigam [35] (see Appendix I). Using this theory to take account of the interaction effects between shear forces Q_{ix} and Q_{iy} in the x- and y-directions acting on an elasto-plastic resisting

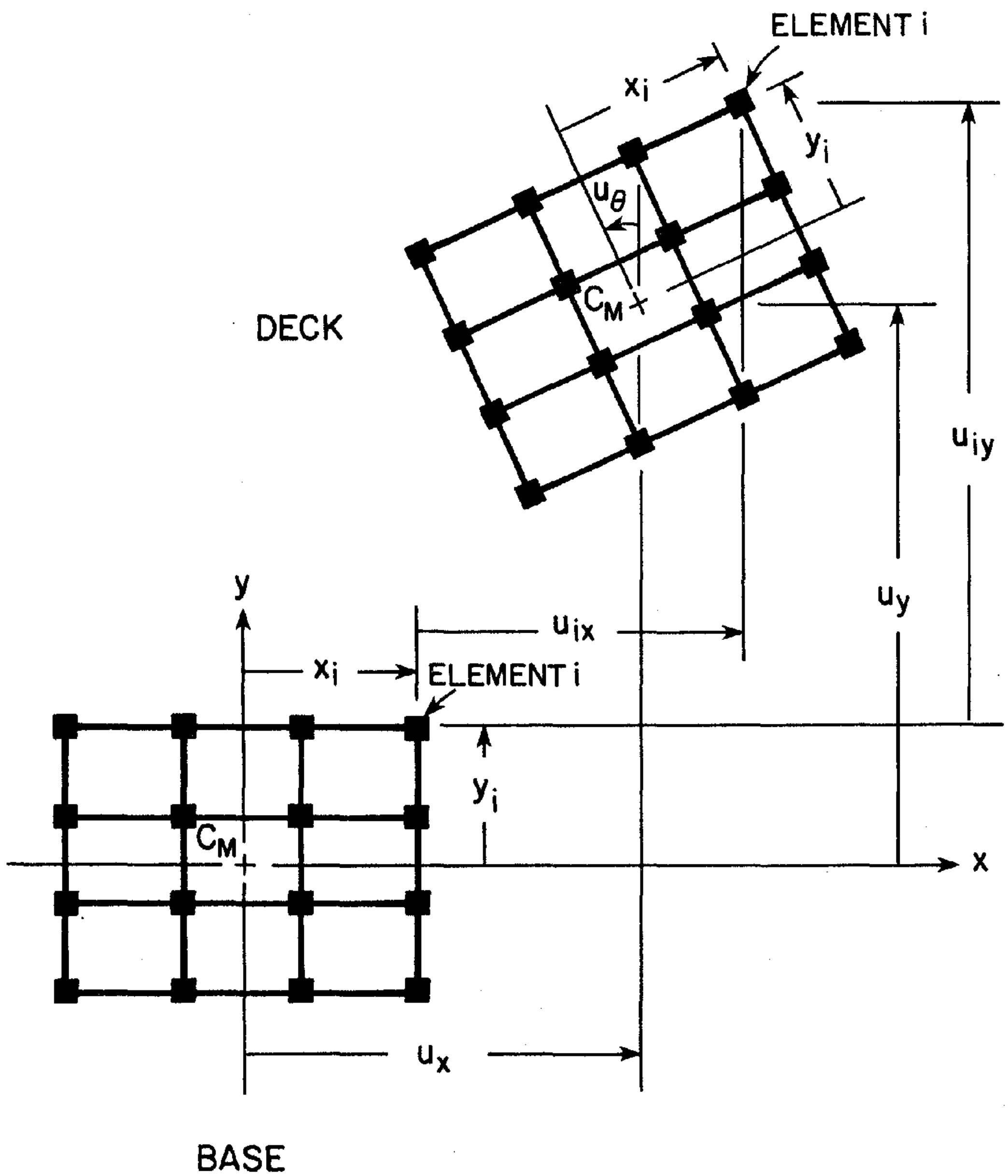


Fig. 2.2 Displacement of Resisting Elements

element during yielding, the force-displacement relationship of the i-th resisting element can be expressed in terms of a yield surface define by

$$\Phi_i(\underline{Q}_i) = 1 \tag{2.33}$$

; thus, giving

$$\dot{Q}_i = \underline{k}_i \, \dot{\underline{u}}_i \tag{2.34}$$

which applies when

$$\Phi_i(\underline{Q}_i) < 1$$

or

$$\Phi_i(\underline{Q}_i) = 1$$

$$\dot{W}_i^p < 0$$

and

$$\dot{\underline{Q}}_i = \underline{k}_i \, (\dot{\underline{u}}_i - \dot{\underline{u}}_i^p) \tag{2.35}$$

which applies when

$$\Phi_i(\underline{Q}_i) = 1$$

$$\dot{W}_i^p \geq 0$$

In these equations, $\dot{\underline{u}}_i^p$ is the plastic part of velocity vector $\dot{\underline{u}}_i$ representing the i-th resisting element. It can be evaluated using the relation

$$\dot{\underline{u}}_i^p = \frac{\left\langle \underline{k}_i \, \dot{\underline{u}}_i \, , \, \frac{\partial \Phi_i}{\partial \underline{Q}_i} \right\rangle}{\left\langle \underline{k}_i \, \frac{\partial \Phi_i}{\partial \underline{Q}_i} \, , \, \frac{\partial \Phi_i}{\partial \underline{Q}_i} \right\rangle} \cdot \frac{\partial \Phi_i}{\partial \underline{Q}_i} \tag{2.36}$$

where the symbol <,> denotes inner product of two vectors. The quantity $\dot{W}_i^p$ in the inequalities above is rate of plastic work of the i-th resisting element and is defined by

$$\dot{W}_i^p = \left\langle \underline{Q}_i \, , \, \dot{\underline{u}}_i^p \right\rangle \tag{2.37}$$

Equations 2.36 and 2.37 can be rearranged and finally placed in the forms

$$\dot{\underline{u}}_i^p = B_i \, \underline{B}_i \, \underline{k}_i \, \underline{Z}_i \, \dot{\underline{u}} \tag{2.38}$$

and

$$\dot{W}_i^p = B_i \, \underline{Q}_i^T \, \underline{B}_i \, \underline{k}_i \, \underline{Z}_i \, \dot{\underline{u}} \tag{2.39}$$

where

$$B_i = \frac{1}{k_{ix}\left(\frac{\partial \Phi_i}{\partial Q_{ix}}\right)^2 + k_{iy}\left(\frac{\partial \Phi_i}{\partial Q_{iy}}\right)^2} \tag{2.40}$$

and

$$\underline{B}_i = \begin{bmatrix} \left(\frac{\partial \Phi_i}{\partial Q_{ix}}\right)^2 & \frac{\partial \Phi_i}{\partial Q_{ix}} \frac{\partial \Phi_i}{\partial Q_{iy}} \\ \frac{\partial \Phi_i}{\partial Q_{ix}} \frac{\partial \Phi_i}{\partial Q_{iy}} & \left(\frac{\partial \phi_i}{\partial Q_{iy}}\right)^2 \end{bmatrix} \tag{2.41}$$

Now, the force-displacement relationships given by Eqs. 2.34 and 2.35 for the i-th resisting element can be expressed in the forms

$$\dot{\underline{Q}}_i = \underline{k}_i \, \underline{Z}_i \, \dot{\underline{u}} \tag{2.42}$$

for

$$\Phi_i(\underline{Q}_i) < 1$$

or

$$\Phi_i(\underline{Q}_i) = 1$$

$$\dot{W}_i^p < 0$$

and

$$\dot{\underline{Q}}_i = \Gamma_i \, \underline{C}_i \, \underline{Z}_i \, \dot{\underline{u}} \tag{2.43}$$

for

$$\Phi_i(\underline{Q}_i) = 1$$

$$\dot{W}_i^p \geq 0$$

where

$$\Gamma_i = \frac{1}{\frac{1}{k_{iy}}\left(\frac{\partial\Phi_i}{\partial Q_{ix}}\right)^2 + \frac{1}{k_{ix}}\left(\frac{\partial\Phi_i}{\partial Q_{iy}}\right)^2} \tag{2.44}$$

$$\underline{C}_i = \begin{bmatrix} \left(\frac{\partial\Phi_i}{\partial Q_{iy}}\right)^2 & -\frac{\partial\Phi_i}{\partial Q_{ix}} \frac{\partial\Phi_i}{\partial Q_{iy}} \\ -\frac{\partial\Phi_i}{\partial Q_{ix}} \frac{\partial\Phi_i}{\partial Q_{iy}} & \left(\frac{\partial\Phi_i}{\partial Q_{ix}}\right)^2 \end{bmatrix} \tag{2.45}$$

It is useful at this point to consider the equations of motion of an elasto-plastic system without interaction between shear forces in the x- and y-directions. In this case, the force-displacement relationship of the i-th resisting element in the x-direction can be written as

$$\dot{Q}_{ix} = k_{ix} \, \underline{Z}_i^{(1)} \, \dot{\underline{u}} \tag{2.46}$$

for the conditions

$$|Q_{ix}| < Q_{ix0}$$

and

$$|Q_{ix}| = Q_{ix0}$$

$$\dot{W}_{ix}^p < 0$$

Note that

$$\dot{Q}_{ix} = 0 \tag{2.47}$$

when

$$|Q_{ix}| = Q_{ix0}$$

$$\dot{W}^p_{ix} \geq 0$$

Similarly, the force-displacement relationship in the y-direction becomes

$$\dot{Q}_{iy} = k_{iy}\, \underline{Z}_i^{(2)} \dot{\underline{u}} \tag{2.48}$$

when

$$|Q_{iy}| < Q_{iy0}$$

or

$$|Q_{iy}| = Q_{iy0}$$

$$\dot{W}^p_{iy} < 0$$

Again, note that

$$\dot{Q}_{iy} = 0 \tag{2.49}$$

when

$$|Q_{iy}| = Q_{iy0}$$

$$\dot{W}^p_{iy} \geq 0$$

Quantities $\underline{Z}_i^{(1)}$ and $\underline{Z}_i^{(2)}$ are row matrices corresponding to the first and second rows, respectively, of the matrix $\underline{Z}_i$; that is,

$$\underline{z}_i^{(1)} = \begin{Bmatrix} 1 \\ -\ d_{iy} \\ 0 \end{Bmatrix}^T$$

$$\underline{z}_i^{(2)} = \begin{Bmatrix} 0 \\ d_{ix} \\ 1 \end{Bmatrix}^T \tag{2.50}$$

and Q_{ix0} and Q_{iy0} are yield shear forces, in the x- and y-directions, respectively, of the i-th resisting element. Quantities $\dot{W}^p_{ix}$ and $\dot{W}^p_{iy}$ in the inequalities above are rates of plastic work, in the x- and y-directions, respectively, of the i-th resisting element and are represented by

$$\dot{W}^p_{ix} = Q_{ix}\, \underline{z}_i^{(1)} \underline{\dot{u}} \tag{2.51}$$

and

$$\dot{W}^p_{iy} = Q_{iy}\, \underline{z}_i^{(2)} \underline{\dot{u}} \tag{2.52}$$

2.3 Dimensionless Equations of Motion

Consider the overall system as represented by

$$\begin{Bmatrix} Q_{x0} \\ r\ Q_{\theta 0} \\ Q_{y0} \end{Bmatrix} = \underline{r}^2 \begin{Bmatrix} \sum_i Q_{ix0} \\ \sum_i Q_{ix0}\left|d_{iy}\right| + \sum_i Q_{iy0}\left|d_{ix}\right| \\ \sum_i Q_{iy0} \end{Bmatrix} \tag{2.53}$$

$$\begin{Bmatrix} u_{x0} \\ r\ u_{\theta 0} \\ u_{y0} \end{Bmatrix} = \begin{Bmatrix} Q_{x0}/K_x \\ r\ Q_{\theta 0}/K_\theta \\ Q_{y0}/K_y \end{Bmatrix} \tag{2.54}$$

$$\underline{v} = \begin{Bmatrix} v_x \\ v_\theta \\ v_y \end{Bmatrix} = \begin{Bmatrix} u_x/u_{x0} \\ r\ u_\theta/r\ u_{\theta 0} \\ u_y/u_{y0} \end{Bmatrix} \tag{2.55}$$

$$\underline{p} = \begin{Bmatrix} p_x \\ p_\theta \\ p_y \end{Bmatrix} = \begin{Bmatrix} Q_x/Q_{x0} \\ r\ Q_\theta/r\ Q_{\theta 0} \\ Q_y/Q_{y0} \end{Bmatrix} \tag{2.56}$$

$$\begin{Bmatrix} \zeta_{x\theta} \\ \zeta_{xy} \\ \zeta_{y\theta} \end{Bmatrix} = \begin{Bmatrix} \omega_x/\omega_\theta \\ \omega_x/\omega_y \\ \omega_y/\omega_\theta \end{Bmatrix} = \begin{Bmatrix} \sqrt{K_x\ r^2/K_\theta} \\ \sqrt{K_x/K_y} \\ \sqrt{K_y\ r^2/K_\theta} \end{Bmatrix} \tag{2.57}$$

$$\begin{Bmatrix} \rho_{x\theta} \\ \rho_{y\theta} \end{Bmatrix} = \begin{Bmatrix} u_{x0}/r\ u_{\theta 0} \\ u_{y0}/r\ u_{\theta 0} \end{Bmatrix} \tag{2.58}$$

and

$$\begin{Bmatrix} a_{x0} \\ a_{\theta 0} \\ a_{y\theta} \end{Bmatrix} = \begin{Bmatrix} Q_{x0}/M\ g \\ r\ Q_{\theta 0}/I\ \underset{\sim}{g} \\ Q_{y0}/M\ g \end{Bmatrix} \tag{2.59}$$

having resisting elements as represented by

$$\begin{Bmatrix} s_{ix} \\ s_{iy} \end{Bmatrix} = \begin{Bmatrix} k_{ix}/K_x \\ k_{iy}/K_y \end{Bmatrix} \tag{2.60}$$

$$\begin{Bmatrix} s_{i\theta x} \\ s_{i\theta y} \end{Bmatrix} = r^2 \begin{Bmatrix} k_{ix} \; d_{iy}^2/K_\theta \\ k_{iy} \; d_{ix}^2/K_\theta \end{Bmatrix} \tag{2.61}$$

$$\begin{Bmatrix} q_{ix0} \\ q_{iy0} \end{Bmatrix} = \begin{Bmatrix} Q_{ix0}/Q_{x0} \\ Q_{iy0}/Q_{y0} \end{Bmatrix} \tag{2.62}$$

$$\begin{Bmatrix} q_{i\theta x0} \\ q_{i\theta y0} \end{Bmatrix} = r^2 \begin{Bmatrix} Q_{ix0} \; d_{iy}/r \; Q_{\theta 0} \\ Q_{iy0} \; d_{ix}/r \; Q_{\theta 0} \end{Bmatrix} \tag{2.63}$$

$$\begin{Bmatrix} u_{ix0} \\ u_{iy0} \end{Bmatrix} = \begin{Bmatrix} Q_{ix0}/k_{ix} \\ Q_{iy0}/k_{iy} \end{Bmatrix} \tag{2.64}$$

$$\underline{v}_i = \begin{Bmatrix} v_{ix} \\ v_{iy} \end{Bmatrix} = \begin{Bmatrix} u_{ix}/u_{ix0} \\ u_{iy}/u_{iy0} \end{Bmatrix} \tag{2.65}$$

$$\underline{p}_i = \begin{Bmatrix} p_{ix} \\ p_{iy} \end{Bmatrix} = \begin{Bmatrix} Q_{ix}/Q_{ix0} \\ Q_{iy}/Q_{iy0} \end{Bmatrix} \tag{2.66}$$

$$r_{k_i} = k_{ix}/k_{iy} \tag{2.67}$$

$$r_{Q_i} = Q_{ix0}/Q_{iy0} \tag{2.68}$$

$$w_i^p = W_i^p/\frac{Q_{ix0}^2}{k_{ix}} \tag{2.69}$$

and

$$\tau = \omega_x \, t \tag{2.70}$$

The dimensionless equations of motion of the single-story non-linear system can be placed in the form (see Appendix II)

$$\underline{\ddot{v}} + 2 \, \underline{\nu} \, \underline{\dot{v}} + \underline{\pi} = - \underline{\alpha}_g \tag{2.71}$$

where

$$\underline{\nu} = \delta_1 \, (\underline{\sigma} - \frac{1}{\eta} \, \underline{\rho}^{-1} \, \underline{\varepsilon} \, \underline{\iota} \, \underline{\varepsilon} \, \underline{\rho}) + \delta_2 + \delta_3 \, \underline{\rho}^{-1} \, \underline{\lambda} \, \underline{\rho} \tag{2.72}$$

$$\underline{\pi} = \underline{\zeta}^{-2} \, \underline{p} \tag{2.73}$$

$$\underline{\alpha}_g = \begin{Bmatrix} \alpha_{gx} \\ 0 \\ \alpha_{gy} \end{Bmatrix} = \begin{Bmatrix} \ddot{u}_{gx} \, (\frac{\tau}{\omega_x})/g \; a_{x0} \\ 0 \\ \ddot{u}_{gy} \, (\frac{\tau}{\omega_y \, \zeta_{xy}})/g \; a_{y0} \, \zeta_{xy}^2 \end{Bmatrix} \tag{2.74}$$

and

$$\begin{Bmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \end{Bmatrix} = \underline{n}^{-1} \, \underline{\xi} \tag{2.75}$$

in which

$$\underline{n} = \begin{bmatrix} \left(\frac{\omega_x}{\omega_1}\right)^3 & \left(\frac{\omega_x}{\omega_1}\right) & \left(\frac{\omega_1}{\omega_x}\right) \\ \left(\frac{\omega_x}{\omega_2}\right)^3 & \left(\frac{\omega_x}{\omega_2}\right) & \left(\frac{\omega_2}{\omega_x}\right) \\ \left(\frac{\omega_x}{\omega_3}\right)^3 & \left(\frac{\omega_x}{\omega_3}\right) & \left(\frac{\omega_3}{\omega_x}\right) \end{bmatrix} \tag{2.76}$$

$$\underline{\sigma} = \begin{bmatrix} 1 & & \\ & 0 & \\ & & \zeta_{xy} \end{bmatrix} \tag{2.77}$$

$$\underline{\rho} = \begin{bmatrix} \rho_{x\theta} & & \\ & 1 & \\ & & \rho_{y\theta} \end{bmatrix} \tag{2.78}$$

$$\underline{\varepsilon} = \begin{bmatrix} e_y/r & & \\ & 1 & \\ & & e_x/r \end{bmatrix} \tag{2.79}$$

$$\underline{\iota} = \begin{bmatrix} 1 & 1 & -1 \\ 1 & 1 & -1 \\ -1 & -1 & 1 \end{bmatrix} \tag{2.80}$$

$$\underline{\lambda} = \begin{bmatrix} 1 & -\frac{e_y}{r} & 0 \\ -\frac{e_y}{r} & \frac{1}{\zeta_{x\theta}^2} & \frac{e_x}{r}\frac{1}{\zeta_{xy}^2} \\ 0 & \frac{e_x}{r}\frac{1}{\zeta_{xy}^2} & \frac{1}{\zeta_{xy}^2} \end{bmatrix} \qquad (2.81)$$

$$\underline{\zeta} = \begin{bmatrix} 1 & & \\ & \zeta_{x\theta} & \\ & & \zeta_{xy} \end{bmatrix} \qquad (2.82)$$

$$\underline{p} = \begin{Bmatrix} p_x \\ p_\theta \\ p_y \end{Bmatrix} = \begin{Bmatrix} \sum_i p_{ix} \; q_{ix0} \\ -\sum_i p_{ix} \; q_{i\theta x0} + \sum_i p_{iy} \; q_{i\theta y0} \\ \sum_i p_{iy} \; q_{iy0} \end{Bmatrix} \qquad (2.83)$$

and

$$\eta = \left(\frac{e_y}{r}\right)^2 + \left(\frac{e_x}{r}\right)^2 \frac{1}{\zeta_{xy}^2} - \frac{1}{\zeta_{x\theta}^2} \qquad (2.84)$$

The symbol ' denotes differentiation with respect to dimensionless time, i.e. $d/d\tau$.

The dimensionless forms of the force-displacement relationships corresponding to Eqs. 2.42 and 2.43 of the i-th resisting element of the elasto-plastic system having force interaction become

$$p_i = \underline{z}_i \, \underline{v}' \qquad (2.85)$$

for

$$\Phi_i(p_i) < 1$$

or

$$\Phi_i(\underline{p}_i) = 1$$

$$\dot{w}_i^p < 0$$

and

$$\underline{\dot{p}}_i = \gamma_i \; \underline{c}_i \; \underline{r}_i \; \underline{z}_i \; \underline{\dot{v}} \tag{2.86}$$

for

$$\Phi_i(\underline{p}_i) = 1$$

$$\dot{w}_i^p \geq 0$$

Quantities γ_i, $\underline{c}_i$, $\underline{r}_i$ and $\underline{z}_i$ are defined by

$$\gamma_i = \frac{1}{r_{k_i}\left(\frac{\partial \Phi_i}{\partial p_{ix}}\right)^2 + r_{Q_i}^2\left(\frac{\partial \Phi_i}{\partial p_{iy}}\right)^2} \tag{2.87}$$

$$\underline{c}_i = \begin{bmatrix} \left(\frac{\partial \Phi_i}{\partial p_{iy}}\right)^2 & -\frac{\partial \Phi_i}{\partial p_{ix}} \frac{\partial \Phi_i}{\partial p_{iy}} \\ -\frac{\partial \Phi_i}{\partial p_{ix}} \frac{\partial \Phi_i}{\partial p_{iy}} & \left(\frac{\partial \Phi_i}{\partial p_{ix}}\right)^2 \end{bmatrix} \tag{2.88}$$

$$\underline{r}_i = \begin{bmatrix} r_{Q_i}^2 & \\ & \ddots & \\ & & r_{k_i} \end{bmatrix} \tag{2.89}$$

and

$$\underline{z}_i = \begin{bmatrix} v_{ix0}^{-1} & -\,v_{i\theta x0}^{-1} & 0 \\ 0 & v_{i\theta y0}^{-1} & v_{iy0}^{-1} \end{bmatrix} \tag{2.90}$$

where

$$\begin{Bmatrix} v_{ix0} \\ v_{iy0} \end{Bmatrix} = \begin{Bmatrix} q_{ix0}/s_{ix} \\ q_{iy0}/S_{iy} \end{Bmatrix} \tag{2.91}$$

and

$$\begin{Bmatrix} v_{i\theta x0} \\ v_{i\theta y0} \end{Bmatrix} = \begin{Bmatrix} q_{i\theta x0}/s_{i\theta x} \\ q_{i\theta y0}/s_{i\theta y} \end{Bmatrix} \tag{2.92}$$

The rate of dimensionless plastic work, $\dot{w}_i^p$, in the inequalities above is represented by

$$\dot{w}_i^p = \beta_i \, \underline{p}_i^T \, \underline{b}_i \, \underline{z}_i \, \underline{\dot{v}} \tag{2.93}$$

where

$$\beta_i = \frac{r_{k_i}}{r_{k_i}\left(\frac{\partial\Phi_i}{\partial p_{ix}}\right)^2 + r_{Q_i}^2\left(\frac{\partial\Phi_i}{\partial p_{iy}}\right)^2} \tag{2.94}$$

and

$$\underline{b}_i = \begin{bmatrix} \left(\dfrac{\partial\Phi_i}{\partial p_{ix}}\right)^2 & \dfrac{\partial\Phi_i}{\partial p_{ix}} \dfrac{\partial\Phi_i}{\partial p_{iy}} \\ \dfrac{\partial\Phi_i}{\partial p_{ix}} \dfrac{\partial\Phi_i}{\partial p_{iy}} & \left(\dfrac{\partial\Phi_i}{\partial p_{iy}}\right)^2 \end{bmatrix} \tag{2.95}$$

Further, the dimensionless forms of the force-displacement relationships represented by Eqs. 2.46 - 2.49 for the i-th resisting element of the elasto-plastic system without force interaction become

$$\dot{p}_{ix} = \underline{z}_i^{(1)} \, \underline{\dot{v}} \tag{2.96}$$

for

$$|p_{ix}| < 1$$

or

$$|p_{ix}| = 1$$

$$\dot{w}^p_{ix} < 0 ,$$

$$\dot{p}_{ix} = 0 \tag{2.97}$$

for

$$|p_{ix}| = 1$$

$$\dot{w}^p_{ix} \geq 0$$

$$\dot{p}_{iy} = \underline{z}_i^{(2)} \, \underline{\dot{v}} \tag{2.98}$$

for

$$|p_{iy}| < 1$$

or

$$|p_{iy}| = 1$$

$$\dot{w}_{iy}^{p} < 0$$

and

$$\dot{p}_{iy} = 0 \qquad (2.99)$$

for

$$|p_{iy}| = 1$$

$$\dot{w}_{iy}^{p} \geq 0$$

Quantities $\underline{z}_i^{(1)}$ and $\underline{z}_i^{(2)}$ denote row matrices corresponding to the first and second rows, respectively, of the matrix, $\underline{z}_i$, i.e.

$$\left.\begin{aligned} \underline{z}_i^{(1)} &= \begin{Bmatrix} \dfrac{s_{ix}}{q_{ix0}} \\ -\dfrac{s_{i\theta x}}{q_{i\theta x0}} \\ 0 \end{Bmatrix}^T \\ \underline{z}_i^{(2)} &= \begin{Bmatrix} 0 \\ \dfrac{s_{i\theta y}}{q_{i\theta y0}} \\ \dfrac{s_{iy}}{q_{iy0}} \end{Bmatrix}^T \end{aligned}\right\} \qquad (2.100)$$

The rates of dimensionless plastic work, $\dot{w}_{ix}^{p}$ and $\dot{w}_{iy}^{p}$, in the x- and y-directions, respectively, are represented by

$$w'^{p}_{ix} = p_{ix}\, \underline{z}_i^{(1)}\, \underline{v}' \tag{2.101}$$

and

$$w'^{p}_{iy} = p_{iy}\, \underline{z}_i^{(2)}\, \underline{v}' \tag{2.102}$$

where dimensionless plastic works w^p_{ix} and w^p_{iy} in the x- and y-directions, respectively, are defined by

$$w^p_{ix} = W^p_{ix} \Big/ \frac{Q^2_{xi0}}{k_{ix}} \tag{2.103}$$

$$w^p_{iy} = W^p_{iy} \Big/ \frac{Q^2_{iy0}}{k_{iy}} \tag{2.104}$$

Solving the above equations of motion, dimensionless quantities of response are obtained. Dimensional response can be obtained by simply transforming the dimensionless quantities consistent with their definitions, thus one obtains

$$\left.\begin{aligned}
\underline{u} &= \begin{Bmatrix} u_x \\ r\, u_\theta \\ u_y \end{Bmatrix} = \frac{g}{\omega_x^2}\, \underline{\zeta}^2\, \underline{a}\, \underline{v} \\
\dot{\underline{u}} &= \begin{Bmatrix} \dot{u}_x \\ r\, \dot{u}_\theta \\ \dot{u}_y \end{Bmatrix} = \frac{g}{\omega_x}\, \underline{\zeta}^2\, \underline{a}\, \underline{v}' \\
\ddot{\underline{u}} &= \begin{Bmatrix} \ddot{u}_x \\ r\, \ddot{u}_\theta \\ \ddot{u}_y \end{Bmatrix} = g\, \underline{\zeta}^2\, \underline{a}\, \underline{v}''
\end{aligned}\right\} \tag{2.105}$$

where

$$a = \begin{bmatrix} a_{x0} & & \\ & a_{\theta 0} & \\ & & a_{y0} \end{bmatrix} \tag{2.106}$$

$$\underline{F} = \begin{Bmatrix} Q_x \\ Q_\theta \\ Q_y \end{Bmatrix} = M\ g\ \underline{r}\ \underline{a}\ \underline{p} \tag{2.107}$$

$$\left.\begin{aligned} \underline{u}_i &= \begin{Bmatrix} u_{ix} \\ u_{iy} \end{Bmatrix} = \frac{g}{\omega_x^2} \{\underline{\zeta}^{(1,3)}\}^2\ \underline{a}^{(1,3)}\ \underline{v}_{i0}\ \underline{v}_i \\ \underline{\dot{u}}_i &= \begin{Bmatrix} \dot{u}_{ix} \\ \dot{u}_{iy} \end{Bmatrix} = \frac{g}{\omega_x} \{\underline{\zeta}^{(1,3)}\}^2\ \underline{a}^{(1,3)}\ \underline{v}_{i0}\ \underline{\dot{v}}_i \\ \underline{\ddot{u}}_i &= \begin{Bmatrix} \ddot{u}_{ix} \\ \ddot{u}_{iy} \end{Bmatrix} = g\ \{\underline{\zeta}^{(1,3)}\}^2\ \underline{a}^{(1,3)}\ \underline{v}_{i0}\ \underline{\ddot{v}}_i \end{aligned}\right\} \tag{2.108}$$

where

$$\underline{\zeta}^{(1,3)} = \begin{bmatrix} 1 & \\ & \ddots & \\ & & \zeta_{xy} \end{bmatrix} \tag{2.109}$$

$$\underline{a}^{(1,3)} = \begin{bmatrix} a_{x0} & \\ & \ddots & \\ & & a_{y0} \end{bmatrix} \tag{2.110}$$

and

$$\underline{v}_{i0} = \begin{bmatrix} v_{ix0} & \\ & \ddots & \\ & & v_{iy0} \end{bmatrix} \tag{2.111}$$

$$\underline{Q}_i = \begin{Bmatrix} Q_{ix} \\ Q_{iy} \end{Bmatrix} = M\ g\ \underline{a}_i\ \underline{p}_i \tag{2.112}$$

$$\underline{a}_i = \begin{bmatrix} a_{x0}\, q_{ix0} & \\ & a_{y0}\, q_{iy0} \end{bmatrix} \tag{2.113}$$

and finally

$$\underline{\mu}_i = \begin{Bmatrix} \mu_{ix} \\ \mu_{iy} \end{Bmatrix} = \underline{v}_i = \underline{z}_i\, \underline{v} \tag{2.114}$$

2.4 Relation among Dimensionless Variables.

The dimensionless variables defined in section 2.3 are composed of both independent and dependent variables. The independent dimensionless variables are as follows;

ξ_1, ξ_2, ξ_3	modal damping factors for the 1st, 2nd and 3rd natural modes, respectively, of the system
ζ_{xy}	circular frequency ratio, defined by Eq. 2.57, of the system
a_{x0} , a_{y0}	coefficients of yield shear forces, defined by Eq. 2.59, of the system
d_{ix} , d_{iy}	dimensionless distances, defined by Eq. 2.26, of the i-th resisting element from the center of mass
s_{ix} , s_{iy}	contribution ratios, defined by Eq. 2.60, of translation stiffnesses of the i-th resisting element to the system

and

q_{ix0} , q_{iy0}	contribution ratios, defined by Eq. 2.62, of yield shear forces of the i-th resisting element to the system

The dependent variables are transformed into dimensionless forms through the relations

$$\begin{Bmatrix} \dfrac{e_x}{r} \\ \dfrac{e_y}{r} \end{Bmatrix} = \begin{Bmatrix} \sum_i s_{iy}\, d_{ix} \\ \sum_i s_{ix}\, d_{iy} \end{Bmatrix} \tag{2.115}$$

$$\begin{Bmatrix} \rho_{x\theta} \\ \rho_{y\theta} \end{Bmatrix} = \frac{1}{a_{\theta 0}} \begin{Bmatrix} a_{x0}/\zeta_{x\theta}^2 \\ a_{y0}/\zeta_{y\theta}^2 \end{Bmatrix} \tag{2.116}$$

$$\begin{Bmatrix} s_{i\theta x} \\ s_{i\theta y} \end{Bmatrix} = \begin{Bmatrix} s_{ix}\, d_{iy}^2\, \zeta_{x\theta}^2 \\ s_{iy}\, d_{ix}^2\, \zeta_{y\theta}^2 \end{Bmatrix} \tag{2.117}$$

$$\begin{Bmatrix} q_{i\theta x0} \\ q_{i\theta y0} \end{Bmatrix} = \frac{1}{a_{\theta 0}} \begin{Bmatrix} a_{x0}\, q_{ix0}\, d_{iy} \\ a_{y0}\, q_{iy0}\, d_{ix} \end{Bmatrix} \tag{2.118}$$

$$\left.\begin{aligned} \zeta_{x\theta}^2 &= 1.0/(\sum_i s_{ix}\, d_{iy}^2 + \sum_i \frac{s_{ix}}{r_{k_i}}\, d_{ix}^2) \\ \zeta_{y\theta}^2 &= \zeta_{x\theta}^2/\zeta_{xy}^2 \end{aligned}\right\} \tag{2.119}$$

$$a_{\theta 0} = a_{x0} \sum_i q_{ix0}\left|d_{iy}\right| + a_{y0} \sum_i q_{iy0}\left|d_{ix}\right| \tag{2.120}$$

$$r_{k_i} = \frac{s_{ix}}{s_{iy}}\, \zeta_{xy}^2 \tag{2.121}$$

and

$$r_{Q_i} = \frac{q_{ix0}}{q_{iy0}}\, \frac{a_{x0}}{a_{y0}} \tag{2.122}$$

The dimensionless frequencies ω_1/ω_x, ω_2/ω_x and ω_3/ω_x in Eq. 2.76 can be numerically obtained by solving the dimensionless eigen value problem

$$\begin{vmatrix} 1-n^2 & -\frac{e_y}{r} & 0 \\ -\frac{e_y}{r} & \frac{1}{\zeta_{x\theta}^2}-n^2 & \frac{e_x}{r}\frac{1}{\zeta_{xy}^2} \\ 0 & \frac{e_x}{r}\frac{1}{\zeta_{xy}^2} & \frac{1}{\zeta_{xy}^2}-n^2 \end{vmatrix} = 0 \tag{2.123}$$

where

$$n = \omega/\omega_x \tag{2.124}$$

2.5 Yield Functions of Shear Failure System

Based on Drucker's postulate [11] concerning stable elastic-plastic material, the yield surface must be closed and convex. The lowest bound, under the postulate, being able to be mathematically considered is, as shown in Fig. 2.3a, a closed surface formed by the four straight lines $|p_{ix}| + |p_{iy}| = 1$ on the $p_{ix} - p_{iy}$ plane in which p_{ix} and p_{iy} are normalized shear forces acting on the i-th resisting element in the x- and y-directions, respectively. The uppermost bound, based on Drucker's postulate, which can be mathematically considered is, as shown in Fig. 2.3b, a closed surface formed by the four straight lines, $|p_{ix}| = 1$ and $|p_{iy}| = 1$. The yield surface having the uppermost bound corresponds to the one for elasto-plastic systems without interaction between shear forces acting on a resisting element during yielding.

The most popular yield surface as shown in Fig. 2.4 for each resisting element subjected to shear forces, Q_{ix} and Q_{iy}, in the x- and y-directions will now be taken into account. In this case, the yield surface function Φ_i is defined by

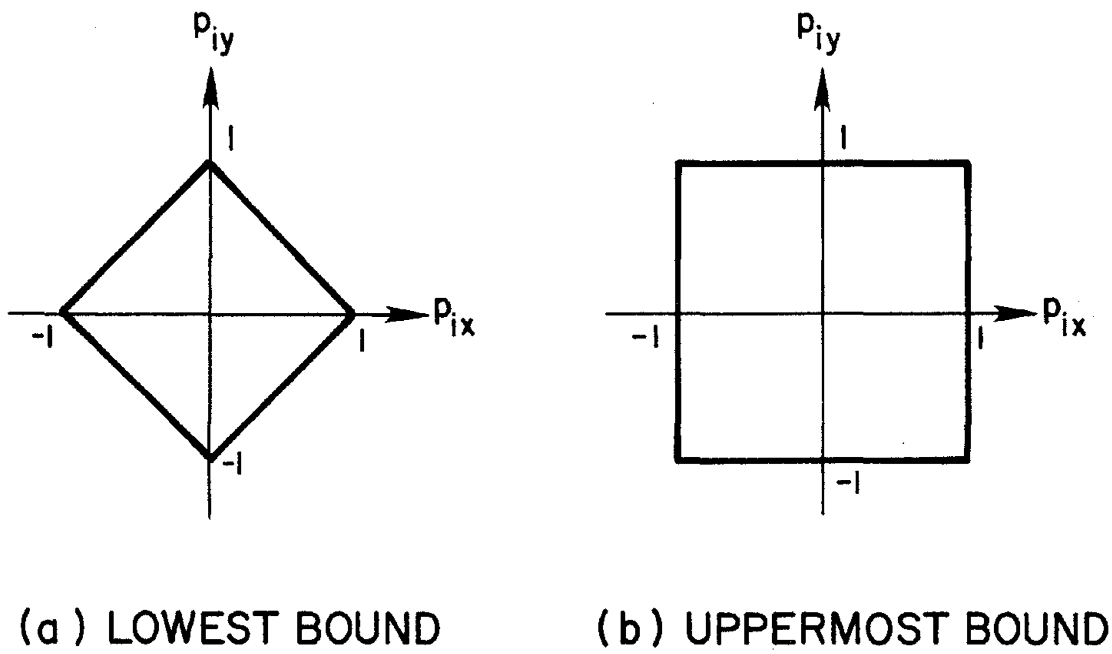


Fig. 2.3 The Lowest and Uppermost Bounds of Yield Function

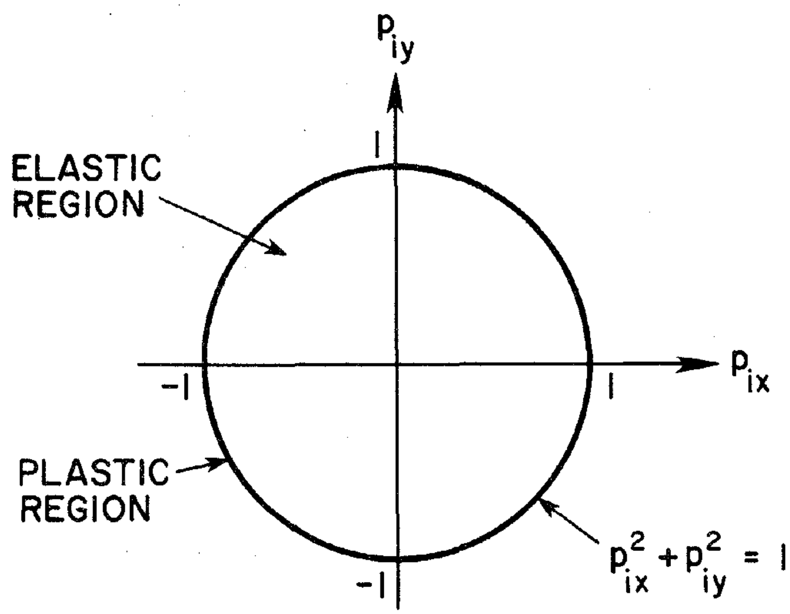


Fig. 2.4 Yield Surface for Elements with Circular Section under Shear Forces

$$\Phi_i(Q_{ix}, Q_{iy}) = \left(\frac{Q_{ix}}{Q_{ix0}}\right)^2 + \left(\frac{Q_{iy}}{Q_{iy0}}\right)^2 = 1 \quad (2.125)$$

which in a dimensionless form becomes

$$\Phi_i(p_{ix}, p_{iy}) = p_{ix}^2 + p_{iy}^2 = 1 \quad (2.126)$$

where

$$\begin{Bmatrix} p_{ix} \\ \\ p_{iy} \end{Bmatrix} = \begin{Bmatrix} Q_{ix}/Q_{ix0} \\ \\ Q_{iy}/Q_{iy0} \end{Bmatrix} \quad (2.127)$$

in the x- and y-directions, respectively. The bound of this circular yield surface lies between both the lowest and uppermost bounds described above.

A yield surface function for elliptical sections subjected to bending moments, M_{ix} and M_{iy}, in the x- and y-directions, respectively, is represented by [34];

$$\left(\frac{M_{ix}}{M_{ix0}}\right)^2 + \left(\frac{M_{iy}}{M_{iy0}}\right)^2 = 1 \quad (2.128)$$

in which M_{ix0} and M_{iy0} are yield moments of the sections in the x- and y-directions, respectively. Considering a bending failure system in which yielding occurs at the top and bottom sections of each resisting element at the same time and the element is rigidly clamped at the top and bottom to the deck and base, respectively, of the system, the relation between shear forces acting on the element and bending moments acting on the top and bottom sections of the element becomes

$$\begin{Bmatrix} Q_{ix} \\ Q_{iy} \end{Bmatrix} = \frac{2}{H} \begin{Bmatrix} M_{ix} \\ M_{iy} \end{Bmatrix} \tag{2.129}$$

where "H" denotes story height. Shear forces Q_{ixb} and Q_{iyb} corresponding to yield moments of the bending failure system are given by

$$\begin{Bmatrix} Q_{ixb} \\ Q_{iyb} \end{Bmatrix} = \frac{2}{H} \begin{Bmatrix} M_{ix0} \\ M_{iy0} \end{Bmatrix} \tag{2.130}$$

Substitution of Eqs. 2.129 and 2.130 into Eq. 2.128 gives

$$\left(\frac{Q_{ix}}{Q_{ixb}}\right)^2 + \left(\frac{Q_{iy}}{Q_{iyb}}\right)^2 = 1 \tag{2.131}$$

which is similar to Eq. 2.125; therefore, when Q_{ixb} and Q_{iyb} are chosen instead of the yield shear forces, Q_{ix0} and Q_{iy0}, the yield function for the shear failure system can be applied to the bending failure system.

3. CASE STUDIES

3.1 Choice of Parameters

3.1.1 Systems

Four element systems shown in Fig. 3.1 were considered. Parameter studies have been carried out for the following properties:

(1) Uncoupled period in the x-direction, $T_x = 2\pi/\omega_x$

Several values in the range of 0.3 to 2.2 sec. were considered.

(2) Frequency ratio, $\zeta_{xy} = \omega_x/\omega_y = T_y/T_x$

Values, 1.0, 1.5 and 2.0, were considered. This means that $T_y \geq T_x$ in all cases.

(3) Distribution of resisting elements, d_{ix} and d_{iy}

Resisting elements are located at each corner of the square deck and the center of mass is located at the geometrical center of the deck itself. Hence, dimensionless distances of the elements from the center of mass are

$$|d_{ix}| = |d_{iy}| = (3/2)^{1/2} = 1.2247 \quad (i = 1,2,3,4)$$

(4) Distribution of stiffnesses of resisting elements, s_{ix} and s_{iy}

Case a) Models with eccentricity in only the y-direction (Fig. 3.1a)

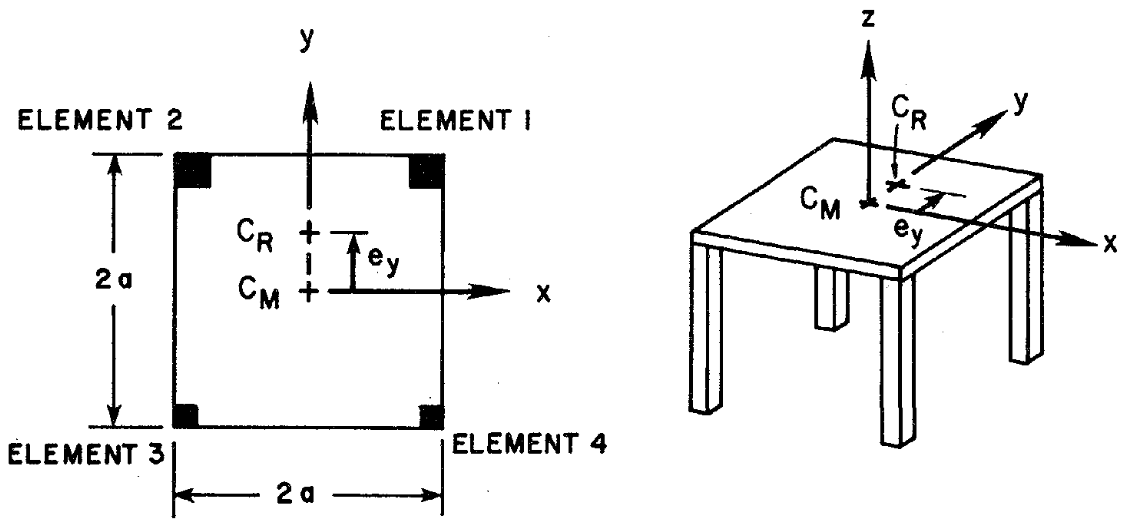
Dimensionless stiffnesses of elements in the x-direction were assumed to be as follows:

$$s_{1x} = s_{2x}, \; s_{3x} = s_{4x}$$

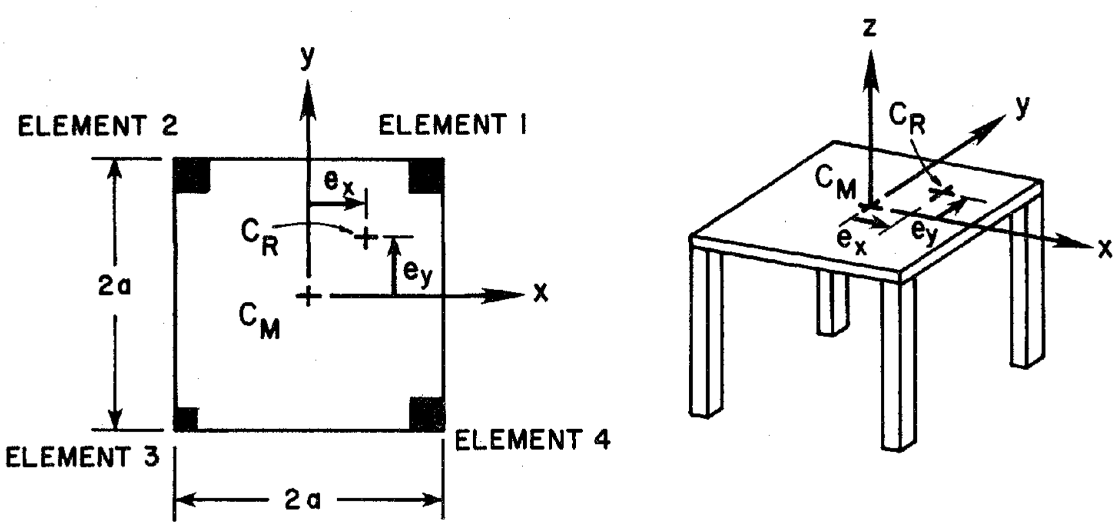
Relation between the normalized eccentricity in the y-direction e_y/r and the dimensionless stiffnesses is

$$e_y/r = 2(3/2)^{1/2} \; (s_{1x} - s_{3x}) = 2.4495 \; (s_{1x} - s_{3x})$$

Normalized eccentricity values equal to 0, 0.2 and 0.4 were considered.



a) System with Eccentricity in the y - Direction



b) System with Eccentricities in the x - and y - Directions

Fig. 3.1 Four Element Single-Story Systems

Dimensionless stiffnesses of elements in the y-direction were assumed the same, i.e.;

$$s_{1y} = s_{2y} = s_{3y} = s_{4y}$$

hence,

$$e_x/r = 0.$$

Case b) Models with eccentricities in both the x- and y-directions (Fig. 3.1b).

Dimensionless stiffnesses of elements in the x-direction were assumed the same as Case a. Dimensionless stiffnesses in the y-direction were assumed to follow the relations

$$s_{1y} = s_{4y},\ s_{2y} = s_{3y}.$$

The relation between normalized eccentricity in the x-direction e_x/r and the dimensionless stiffnesses in this case is

$$e_x/r = 2(3/2)^{1/2}\ (s_{1y} - s_{2y}) = 2.4495\ (s_{1y} - s_{2y})$$

Normalized eccentricity values equal to 0, 0.2 and 0.4 were considered.

(5) Yield shear forces of systems, a_{x0} and a_{y0}

Dimensionless yield shear forces in the x- and y-directions, a_{x0} and a_{y0}, respectively, were given by the following relation:

$$\begin{aligned} a_{x0} \text{ and } a_{y0} &= 2.5\ c_o \qquad T \leq 0.8 \text{ sec.} \\ &= \frac{c_0}{T-0.4} \qquad T \geq 0.8 \text{ sec.} \end{aligned} \tag{3.1}$$

in which T denotes uncoupled natural period, T_x or T_y. Values of parameter, c_0, considered were 0.06, 0.09 and 0.12. The value 0.06 was determined to be standard by referring to the response spectrum (Fig. 3.2) for inelastic systems with 2% of critical damping when subjected to the El Centro, California, earthquake using a ductility factor $\mu = 5$. The dimensionless yield shear forces concerned with uncoupled natural period

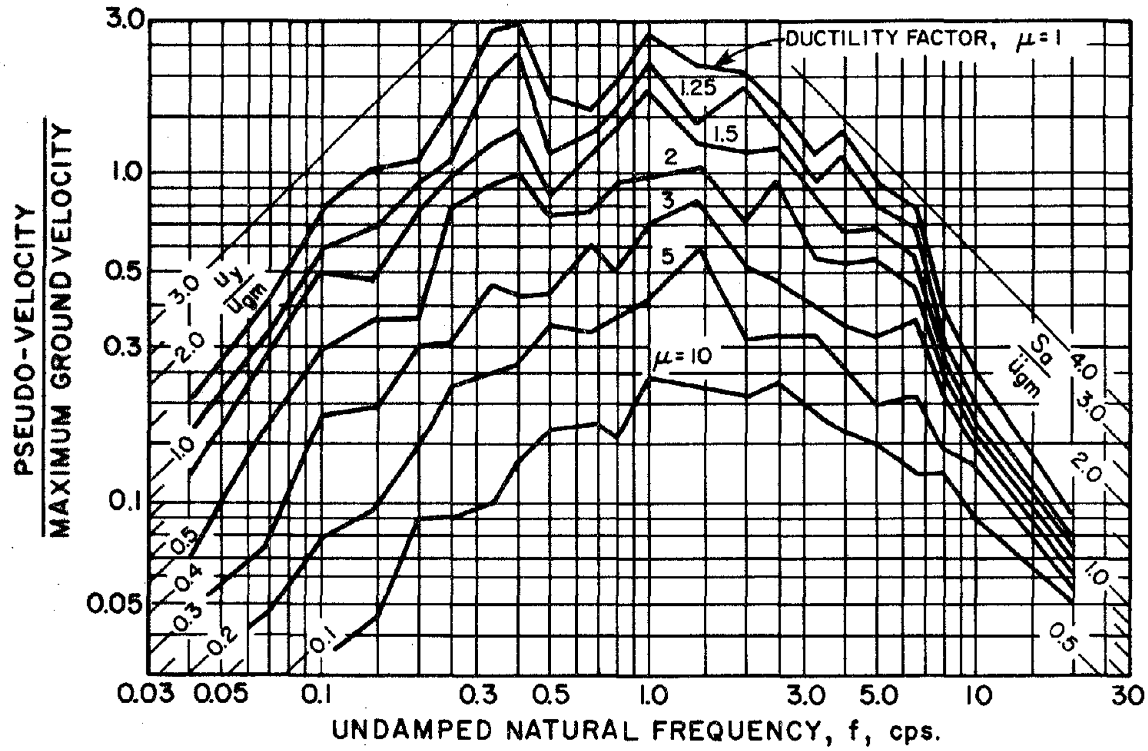


Fig. 3.2 Response Spectra for Elasto-Plastic Systems with 2% of Critical Damping Subjected to the El Centro Earthquake

for different values of parameter c_0, i.e. 0.06, 0.09 and 0.12, are illustrated in Fig. 3.3.

The values of dimensionless yield shear forces for inelastic systems to be considered are listed in Table 3.1.

(6) Distribution of yield shear forces, q_{ix0} and q_{iy0}

Yield shear forces of resisting elements were assumed to be proportional to their stiffnesses. Hence, dimensionless yield shear forces of the elements in the x- and y-directions, q_{ix0} and q_{iy0}, respectively, are equal to their dimensionless stiffnesses s_{ix} and s_{iy}, respectively, i.e.;

$$q_{ix0} = s_{ix} \text{ and } q_{iy0} = s_{iy}$$

(7) Shape of yield function

A circular type yield function was assumed for each elasto-plastic resisting element of the EIP-system.

(8) Damping factors, ξ_1 , ξ_2 and ξ_3

Modal damping factors for the first, second and third natural modes of the system, ξ_1 , ξ_2 and ξ_3, respectively, were assumed equal to 0.05.

3.1.2 Ground Motions

The ground motions considered were the first 30 seconds of the El Centro accelerogram recorded during the Imperial Valley earthquake of May 18, 1940, and the first 30 seconds of the Taft Lincoln School Tunnel record of July 21, 1952. These ground acceleration histories presented in Figs. 3.4a and 3.4b were digitized using the "standard" base line correction [15]. A uniform time interval of 0.02 seconds was used in the digitization process. The maximum accelerations of these records are

El Centro	S00E (NS)	:	341.7 gals
	S90E (EW)	:	210.1 gals

Table 3.1 Dimensionless Yield Shear Forces for Inelastic Systems and Dimensionless Maximum Ground Accelerations

Uncoupled Period T_x or T_y (sec.)	Parameter c_0	Dimensionless Yield Shear Forces a_{x0} or a_{y0}	Maximum Dimensionless Ground Accelerations	
			$\lvert\ddot{v}_{gx}\rvert max$	$\lvert\ddot{v}_{gy}\rvert max$
0.3	0.06	0.15	2.32	1.43
	0.09	0.225	1.55	0.95
	0.12	0.30	1.16	0.71
0.4	0.06	0.15	2.32	1.43
	0.09	0.225	1.55	0.95
	0.12	0.30	1.16	0.71
0.6	0.06	0.15	2.32	1.43
	0.09	0.225	1.55	0.95
	0.12	0.30	1.16	0.71
0.8	0.06	0.15	2.32	1.43
	0.09	0.225	1.55	0.95
	0.12	0.30	1.16	0.71
1.0	0.06	0.10	3.49	2.14
	0.09	0.15	2.32	1.43
	0.12	0.20	1.74	1.07
1.4	0.06	0.06	5.81	3.57
	0.09	0.09	3.87	2.38
	0.12	0.12	2.91	1.79
1.8	0.06	0.0429	8.13	5.00
	0.09	0.0643	5.42	3.33
	0.12	0.0857	4.07	2.50
2.2	0.06	0.0333	10.47	6.44
	0.09	0.05	6.97	4.29
	0.12	0.0667	5.23	3.21

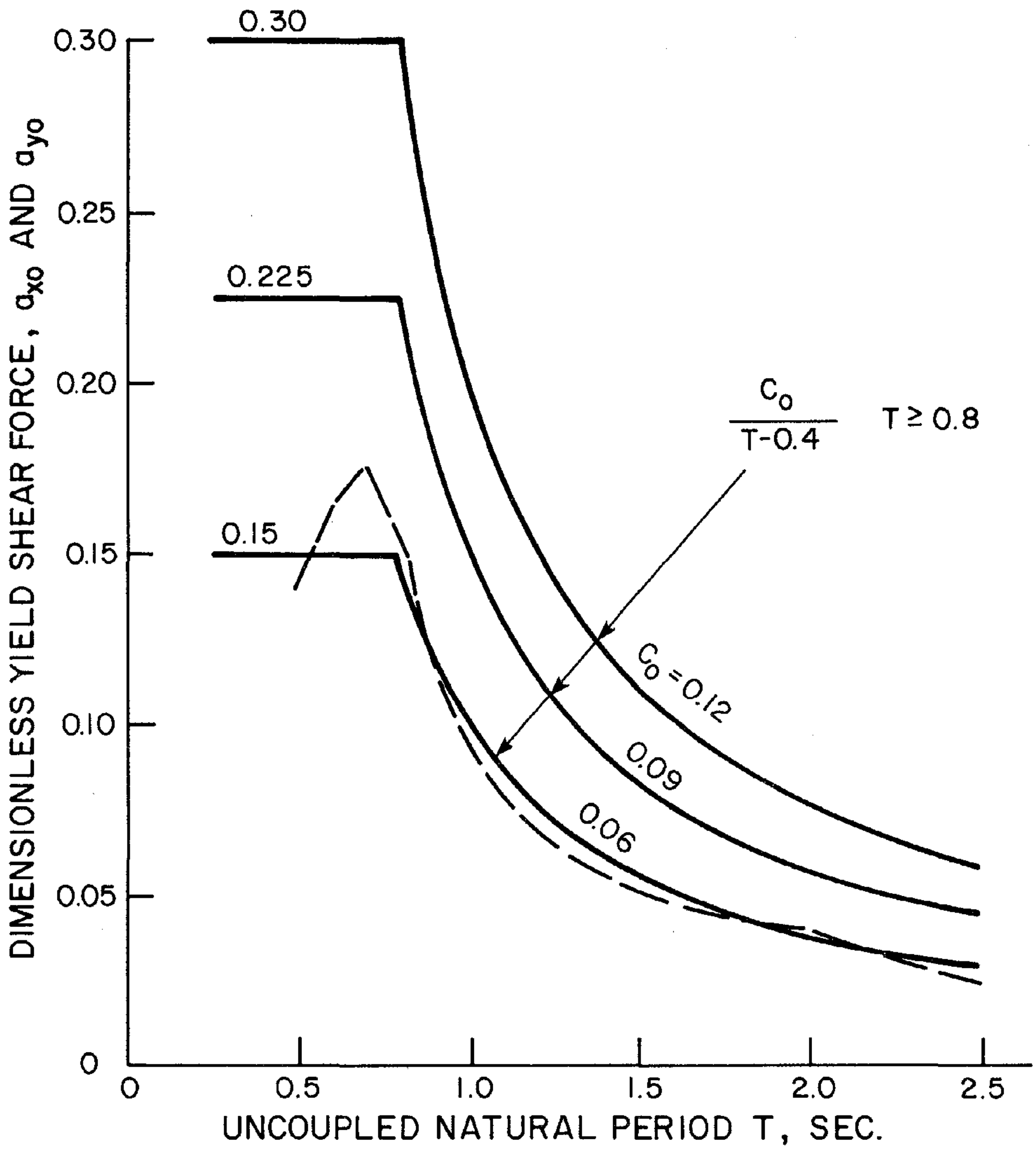


Fig. 3.3 Dimensionless Yield Shear Force-Uncoupled Natural Period Dotted Line: Yield Shear Force Resulting Ductility Factor $\mu = 5$ to A System when Subjected to the El Centro Earthquake

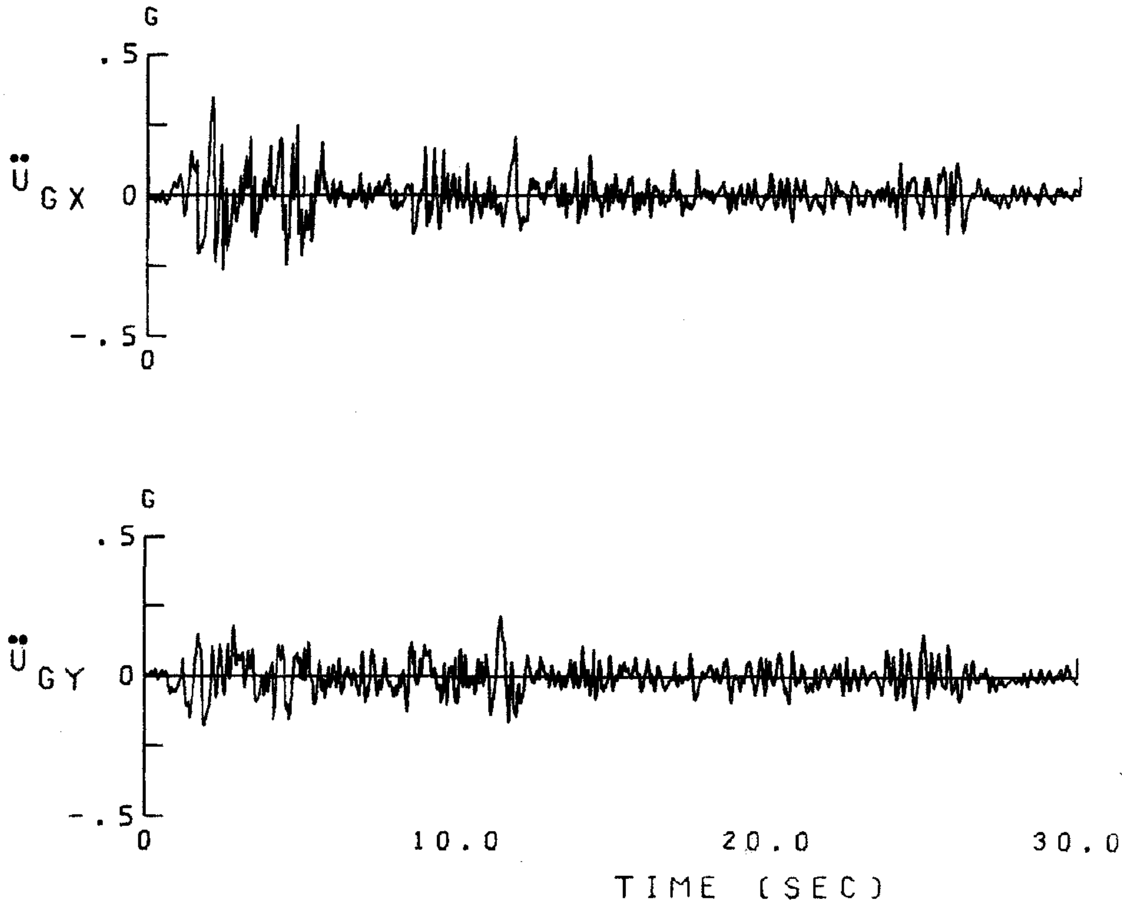


Fig. 3.4a Time History of Ground Accelerations of the El Centro Record Obtained During the Imperial Valley Earthquake of May 18, 1940

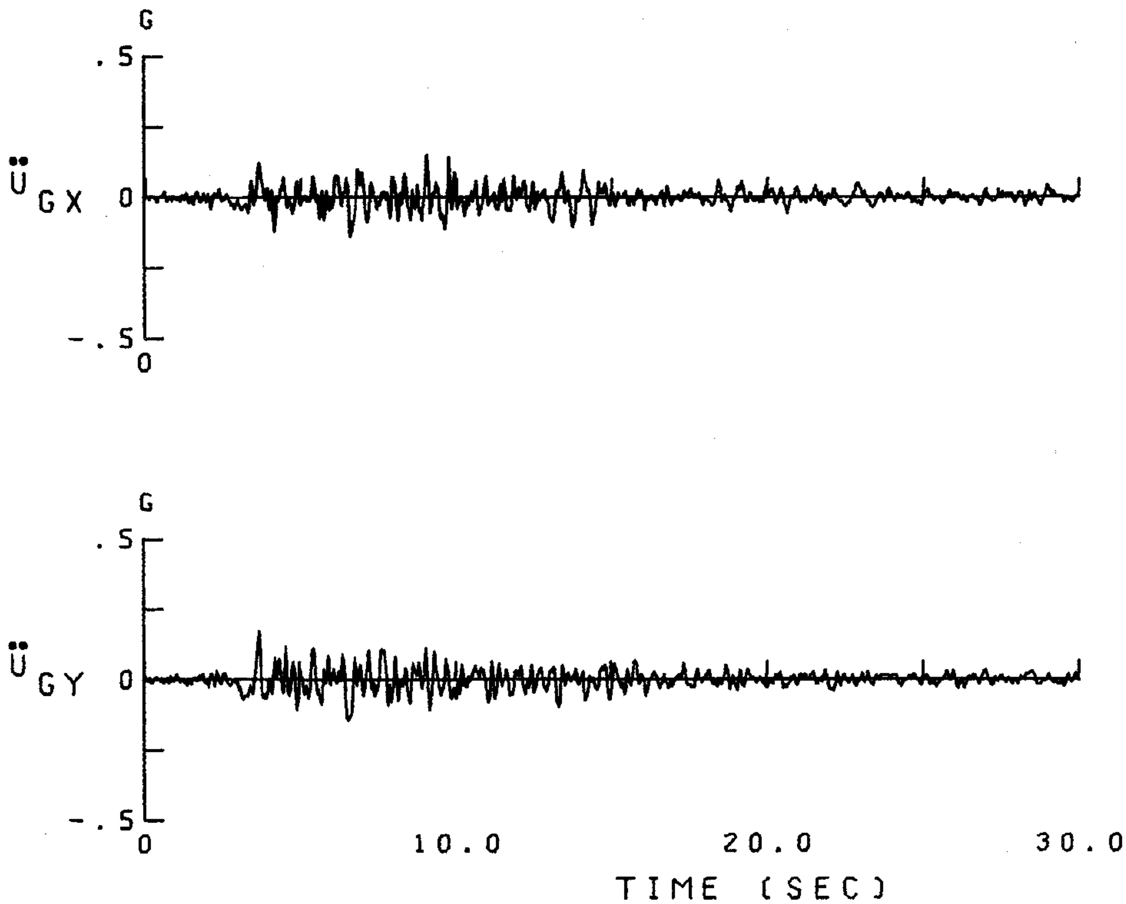


Fig. 3.4b Time History of Ground Accelerations of the Taft Lincoln School Tunnel Record of July 21, 1952

Taft	S00E (NS)	:	152.7 gals
	S90E (EW)	:	175.9 gals

For convenience of numerical comparison between response of systems subjected to the El Centro records and the response of systems subjected to the Taft records, the Taft records were normalized to the same intensity levels as those of the corresponding El Centro records. Components NS and EW of the ground motions were considered as excitations to systems in the x- and y-directions, respectively.

It is convenient to consider dimensionless ground motions instead of absolute ones. Referring to Eq. 2.74, the dimensionless ground accelerations $\ddot{U}_{gx}$ and $\ddot{U}_{gy}$ in the x- and y-directions, respectively, are defined as:

$$\begin{Bmatrix} \ddot{U}_{gx} \\ \ddot{U}_{gy} \end{Bmatrix} = \begin{Bmatrix} \ddot{u}_{gx}/g\ a_{x0} \\ \ddot{u}_{gy}/g\ a_{y0} \end{Bmatrix} \tag{3.2}$$

The maximum dimensionless ground accelerations also become

$$\begin{Bmatrix} |\ddot{U}_{gx}|max \\ |\ddot{U}_{gy}|max \end{Bmatrix} = \begin{Bmatrix} |\ddot{u}_{gx}|max/g\ a_{x0} \\ |\ddot{u}_{gy}|max/g\ a_{y0} \end{Bmatrix} \tag{3.3}$$

In this report dimensionless yield shear forces a_{x0} and a_{y0} in the x- and y-directions, respectively, were determined by Eq. 3.1. Substitution of Eq. 3.1 into Eq. 3.3 gives

$$\begin{Bmatrix} |\ddot{U}_{gx}|\max \\ |\ddot{U}_{gy}|\max \end{Bmatrix} = \frac{1}{2.5\ g\ c_0} \begin{Bmatrix} |\ddot{u}_{gx}|\max \\ |\ddot{u}_{gy}|\max \end{Bmatrix} , \ T\leq 0.8 \text{ sec.}$$

$$= \frac{T-0.4}{g\ c_0} \begin{Bmatrix} |\ddot{u}_{gx}|\max \\ |\ddot{u}_{gy}|\max \end{Bmatrix} , \ T\geq 0.8 \text{ sec.} \tag{3.4}$$

The maximum dimensionless accelerations, hence, are changed depending on the uncoupled natural periods of the systems. The maximum values of the dimensionless accelerations taken into account are listed in Table 3.1.

3.1.3 Method of Analysis

Integration of the equations of motion given by Eq. 2.71 is carried out using the third order Runge-Kutta method described in Appendix III.

Figure 3.5 shows dimensionless displacement - time response at the center of mass of an EPI-system having parameters ξ_i = 5%, T_x = 0.6 sec., ζ_{xy} = 1 , e_x/r = 0 and e_y/r = 0.2 subjected to the El Centro earthquake in which the maximum values of the dimensionless ground motions in the x- and y-directions, $|\ddot{U}_{gx}|\max$ and $|\ddot{U}_{gy}|\max$, are 2.32 and 1.43, respectively. Figures 3.6 and 3.7 show dimensionless displacement - time response of the elements 1 and 3, respectively, of the EPI-system subjected to the ground motions. A locus of dimensionless displacement response and dimensionless hysteresis curves at the center of mass of the system are shown in Fig. 3.8 and 3.9, respectively. Locusci of dimensionless displacement response and dimensionless hysteresis curves of the elements 1 and 3 of the system are shown in Figs. 3.10 to 3.13. The locusci of shear force response, shown in Figs. 3.10b and 3.12b,

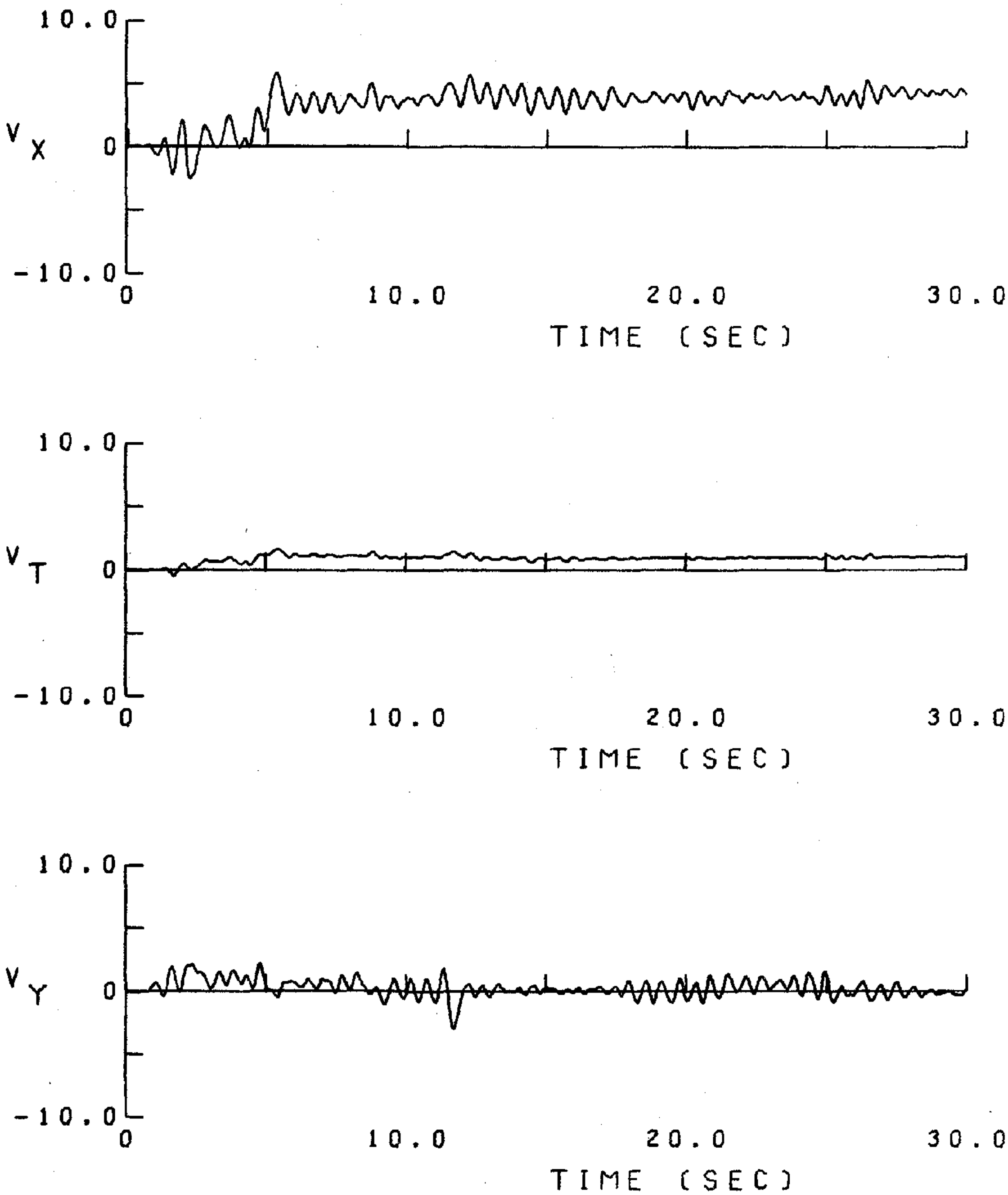
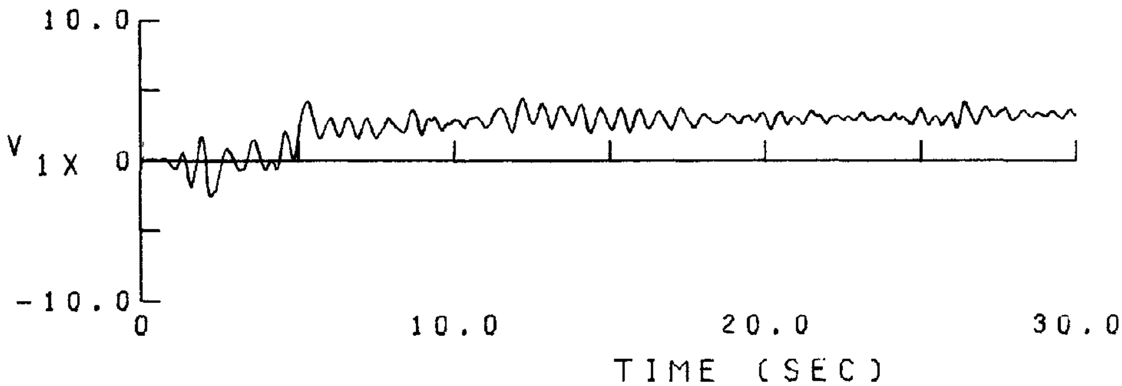


Fig. 3.5 Displacement-Time Response at the Center of Mass of EPI-System Subjected to the El Centro Earthquake. System Parameters: $\xi_i = 5\%$, $T_x = 0.6$ sec., $\zeta_{xy} = 1.0$, $e_x/r = 0$, $e_y/r = 0.2$, $|\ddot{U}_{gx}|\max = 2.32$ and $|\ddot{U}_{gy}|\max = 1.43$



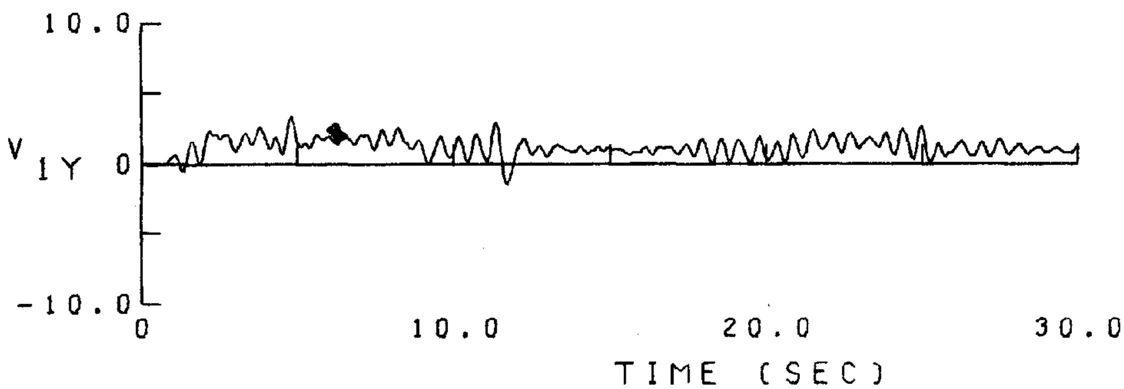


Fig. 3.6 Displacement-Time Response of Element 1 of EPI-System Subjected to the El Centro Earthquake. System Parameters: $\xi_i = 5\%$, $T_x = 0.6$ sec., $\zeta_{xy} = 1.0$, $e_x/r = 0$, $e_y/r = 0.2$, $|\ddot{U}_{gx}|max = 2.32$ and $|\ddot{U}_{gy}|max = 1.43$

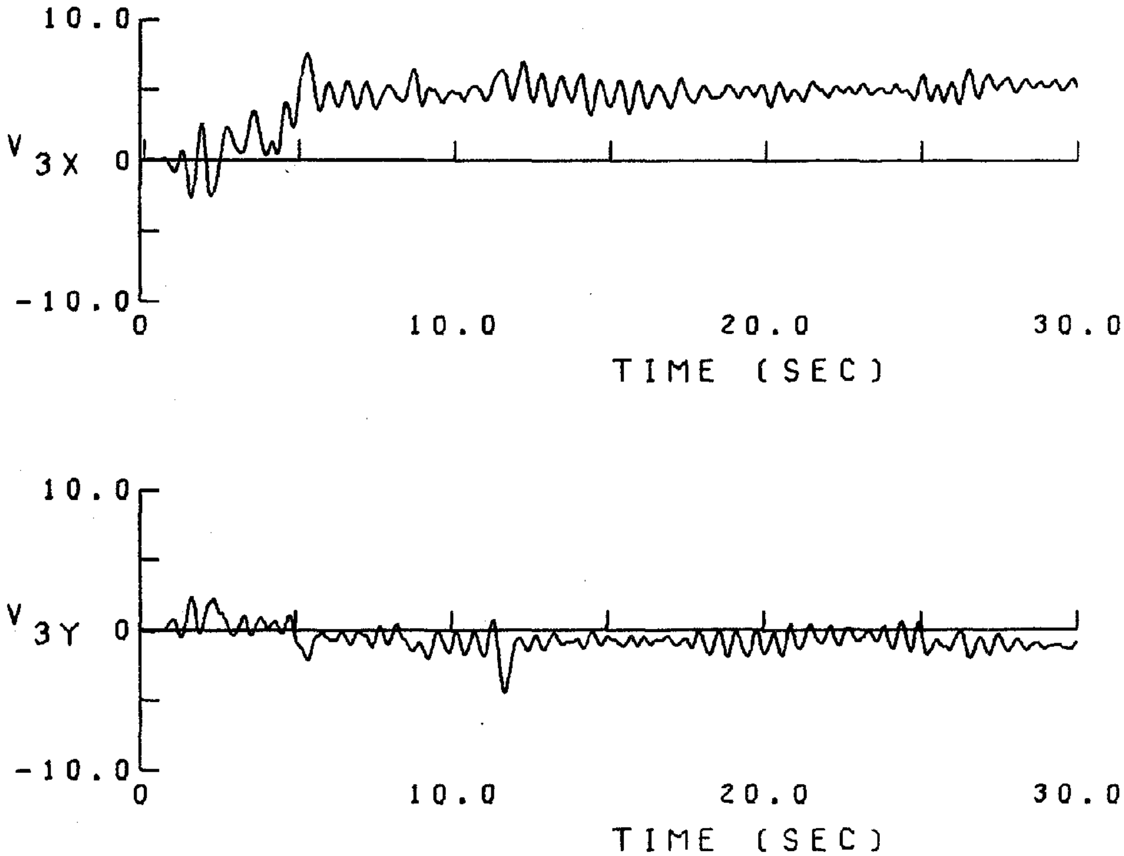


Fig. 3.7 Displacement-Time Response of Element 3 of EPI-System Subjected to the El Centro Earthquake. System Parameters: $\xi_i = 5\%$, $T_x = 0.6$ sec., $\zeta_{xy} = 1.0$, $e_x/r = 0$, $e_y/r = 0.2$, $|\ddot{U}_{gx}|\max = 2.32$ and $|\ddot{U}_{gy}|\max = 1.43$

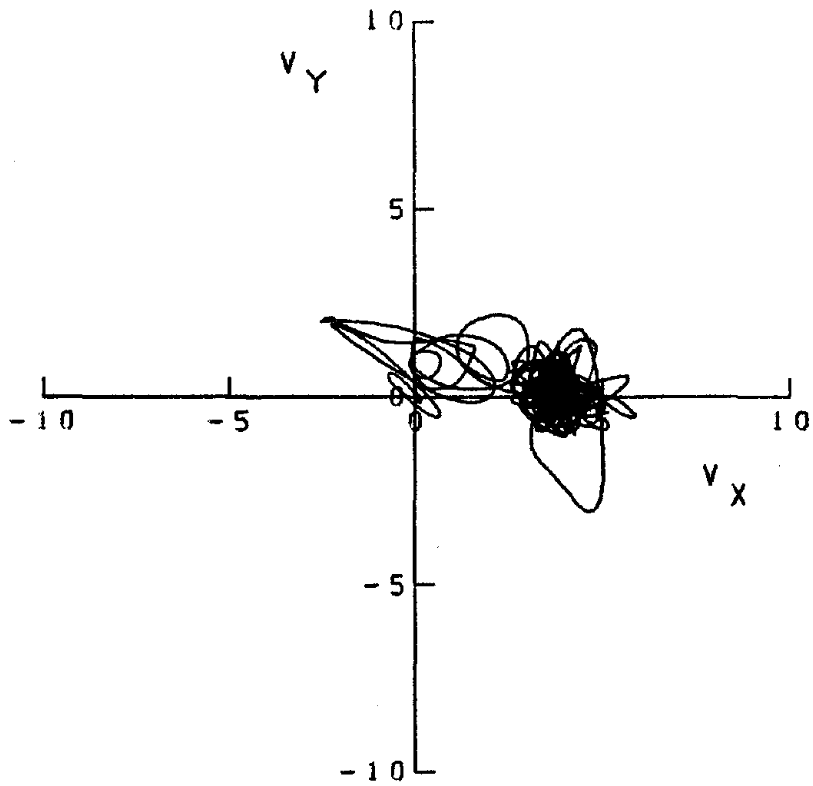


Fig. 3.8 Locus of Displacement Response at the Center of Mass of EPI-System Subjected to the El Centro Earthquake. System Parameters: $\xi_i = 5\%$, $T_x = 0.6$ sec., $\zeta_{xy} = 1.0$, $e_x/r = 0$, $e_y/r = 0.2$, $|\ddot{U}_{gx}|\max = 2.32$ and $|\ddot{U}_{gy}|\max = 1.43$

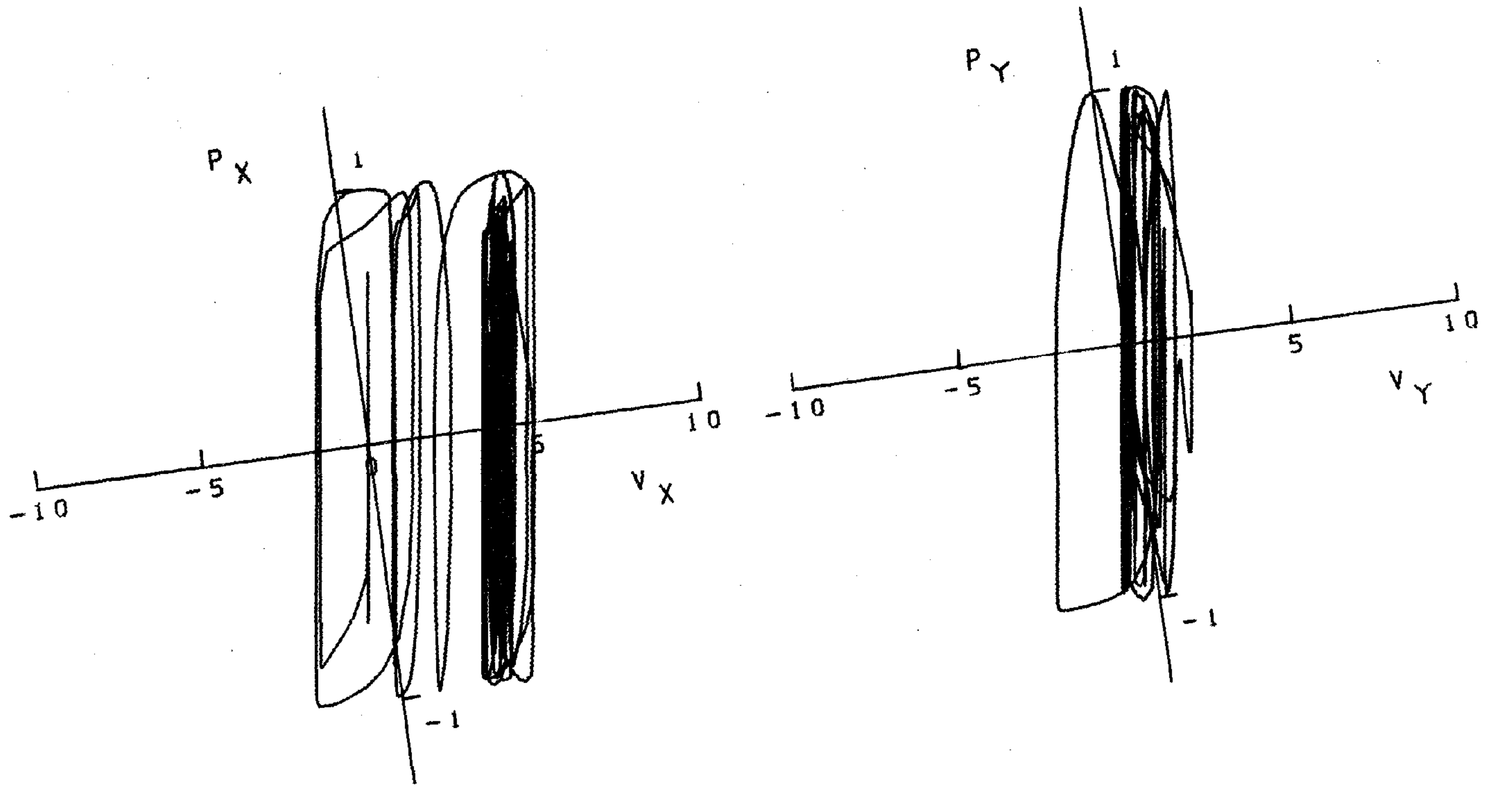
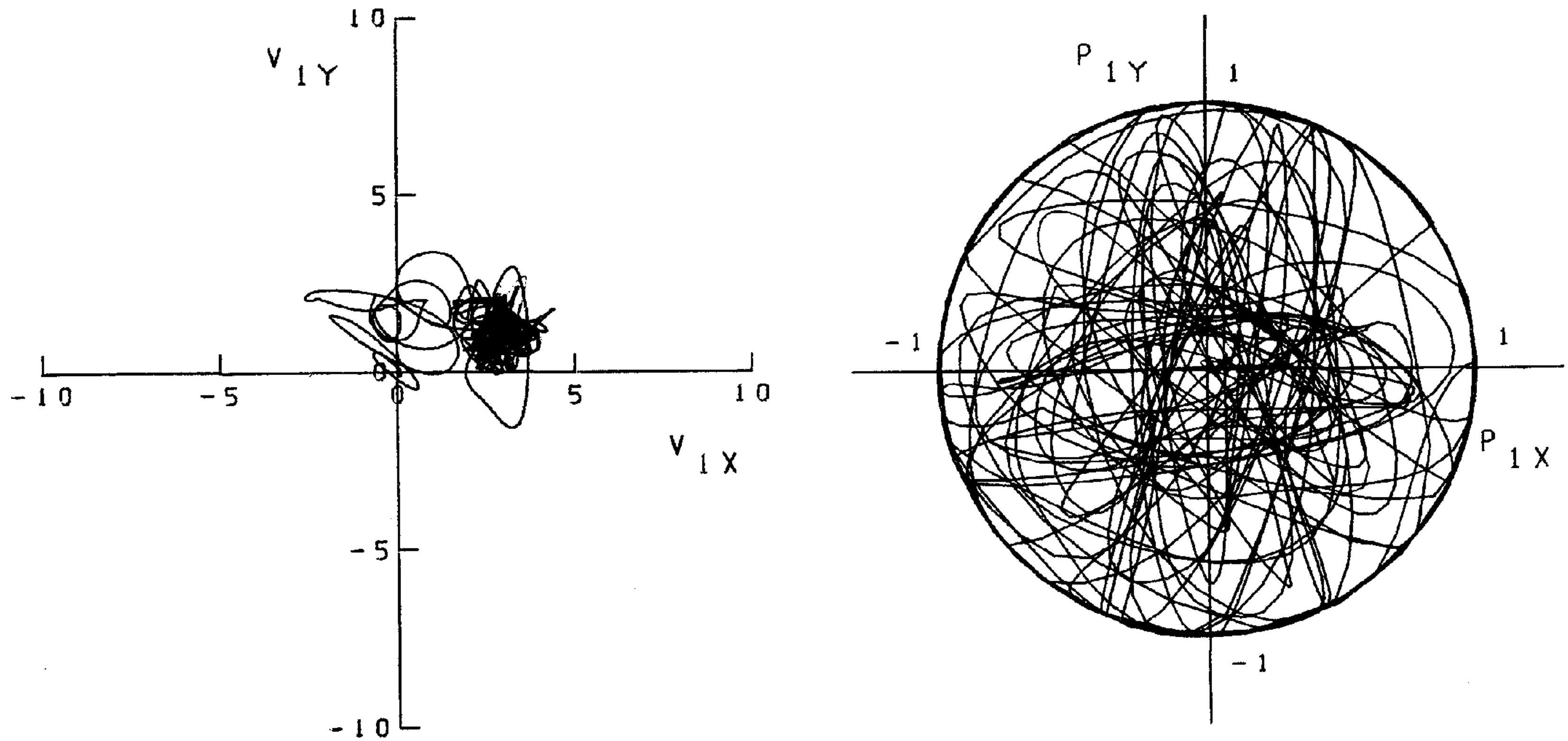


Fig. 3.9 Hysteresis Curves of Response at the Center of Mass of EPI-System Subjected to the El Centro Earthquake. System Parameters: $\xi_i = 5\%$, $T_x = 0.6$ sec., $\zeta_{xy} = 1.0$, $e_x/r = 0$, $e_y/r = 0.2$, $|\ddot{U}_{gx}|_{max} = 2.32$ and $|\ddot{U}_{gy}|_{max} = 1.43$



a) Locus of Displacement Response

b) Locus of Shear Force Response

Fig. 3.10 Locusci of Displacement Response and Shear Force Response of Element 1 of EPI-System Subjected to the El Centro Earthquake. System Parameters: ξ_i = 5%, T_x = 0.6 sec., ζ_{xy} = 1.0, e_x/r = 0, e_y/r = 0.2, $|\ddot{U}_{gx}|$max = 2.32 and $|\ddot{U}_{gy}|$max = 1.43

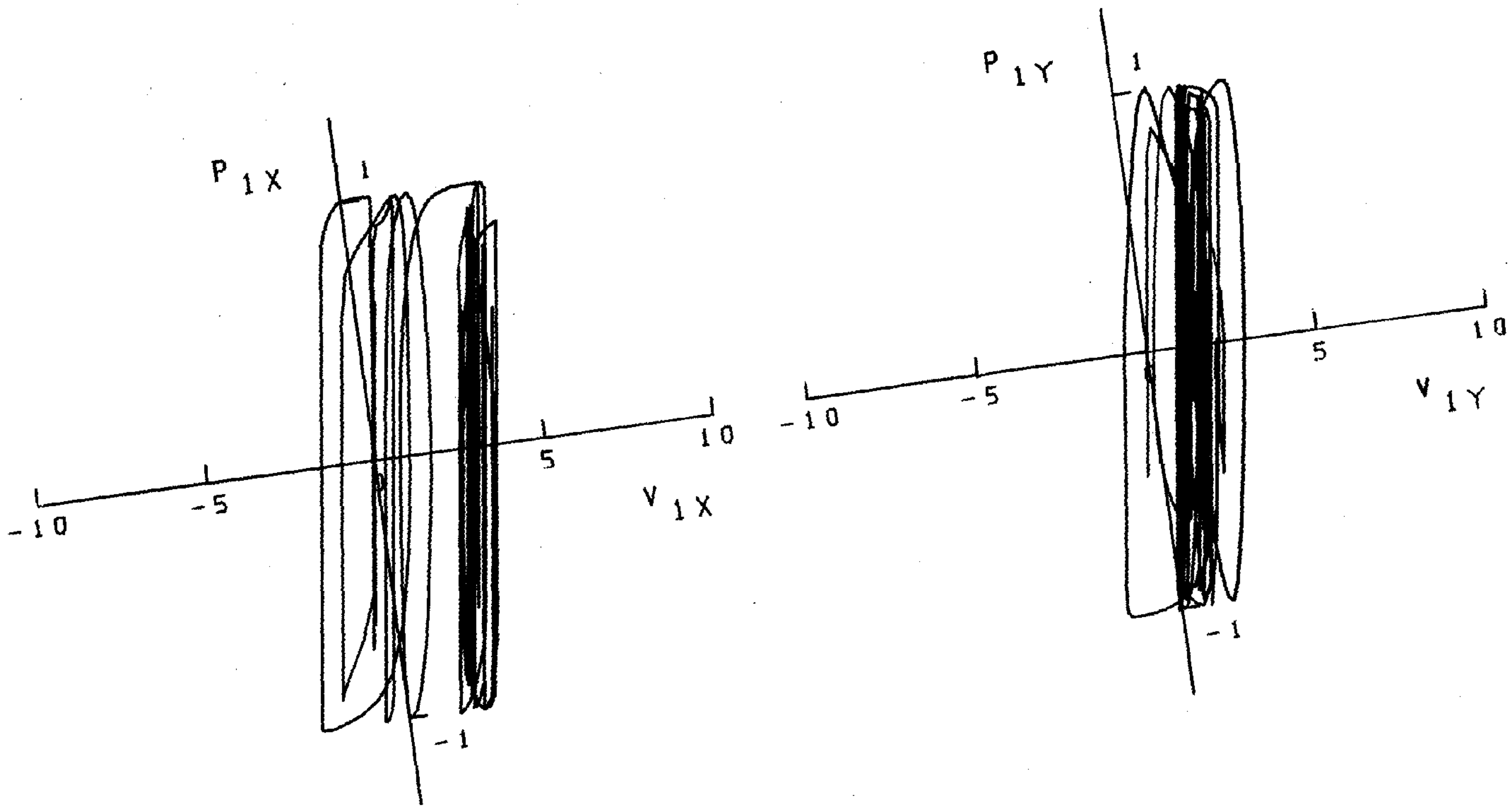
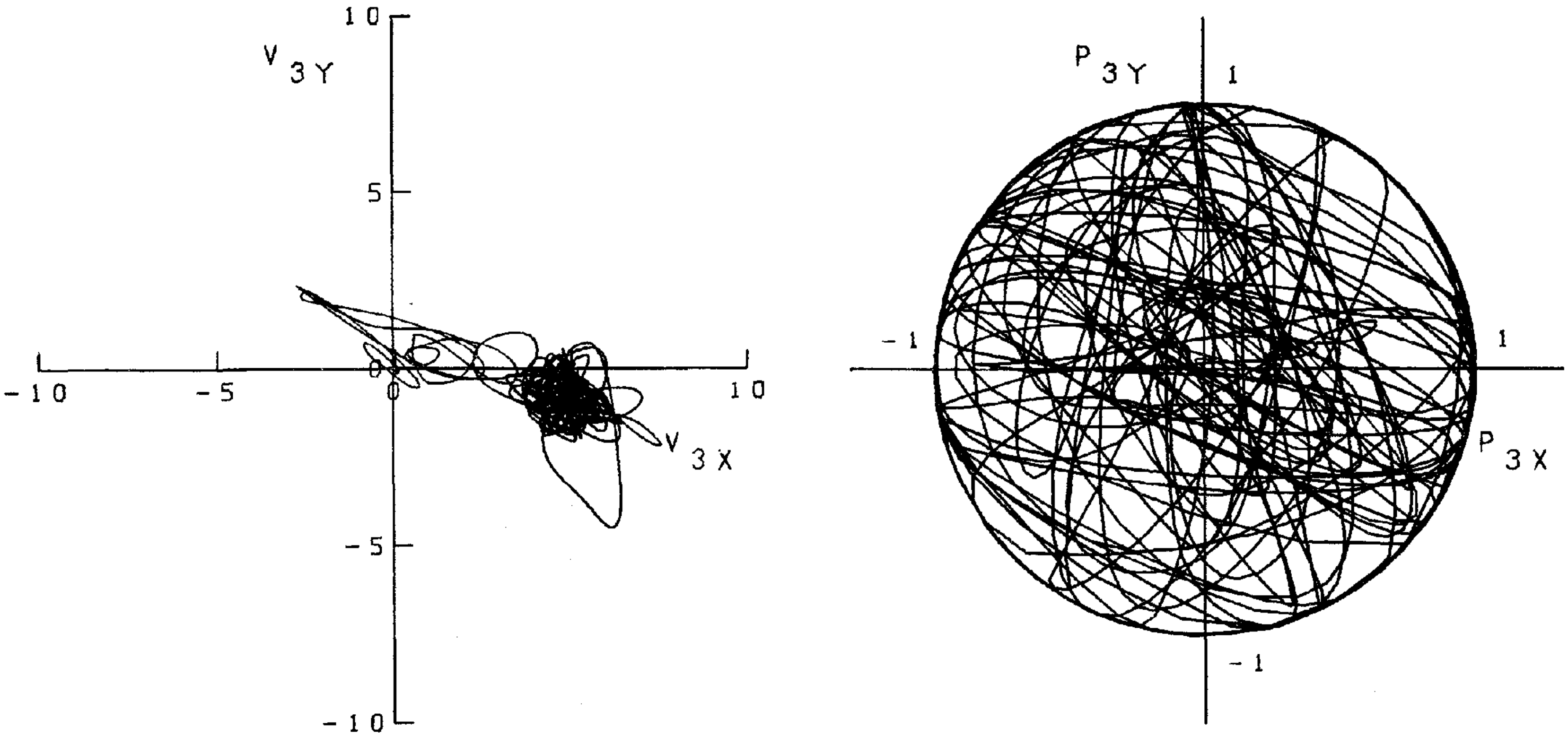


Fig. 3.11 Hysteresis Curves of Response of Element 1 of EPI-System Subjected to the El Centro Earthquake. System Parameters: $\xi_i = 5\%$, $T_x = 0.6$ sec., $\zeta_{xy} = 1.0$, $e_x/r = 0$, $e_y/r = 0.2$, $|\ddot{U}_{gx}|max = 2.32$ and $|\ddot{U}_{gy}|max = 1.43$



a) Locus of Displacement Response

b) Locus of Shear Force Response

Fig. 3.12 Locusci of Displacement Response and Shear Force Response of Element 3 of EPI-System Subjected to the El Centro Earthquake. System Parameters: ξ_i = 5%, T_x = 0.6 sec., ζ_{xy} = 1.0, e_x/r = 0, e_y/r = 0.2, $|\ddot{U}_{gx}|max$ = 2.32 and $|\ddot{U}_{gy}|max$ = 1.43

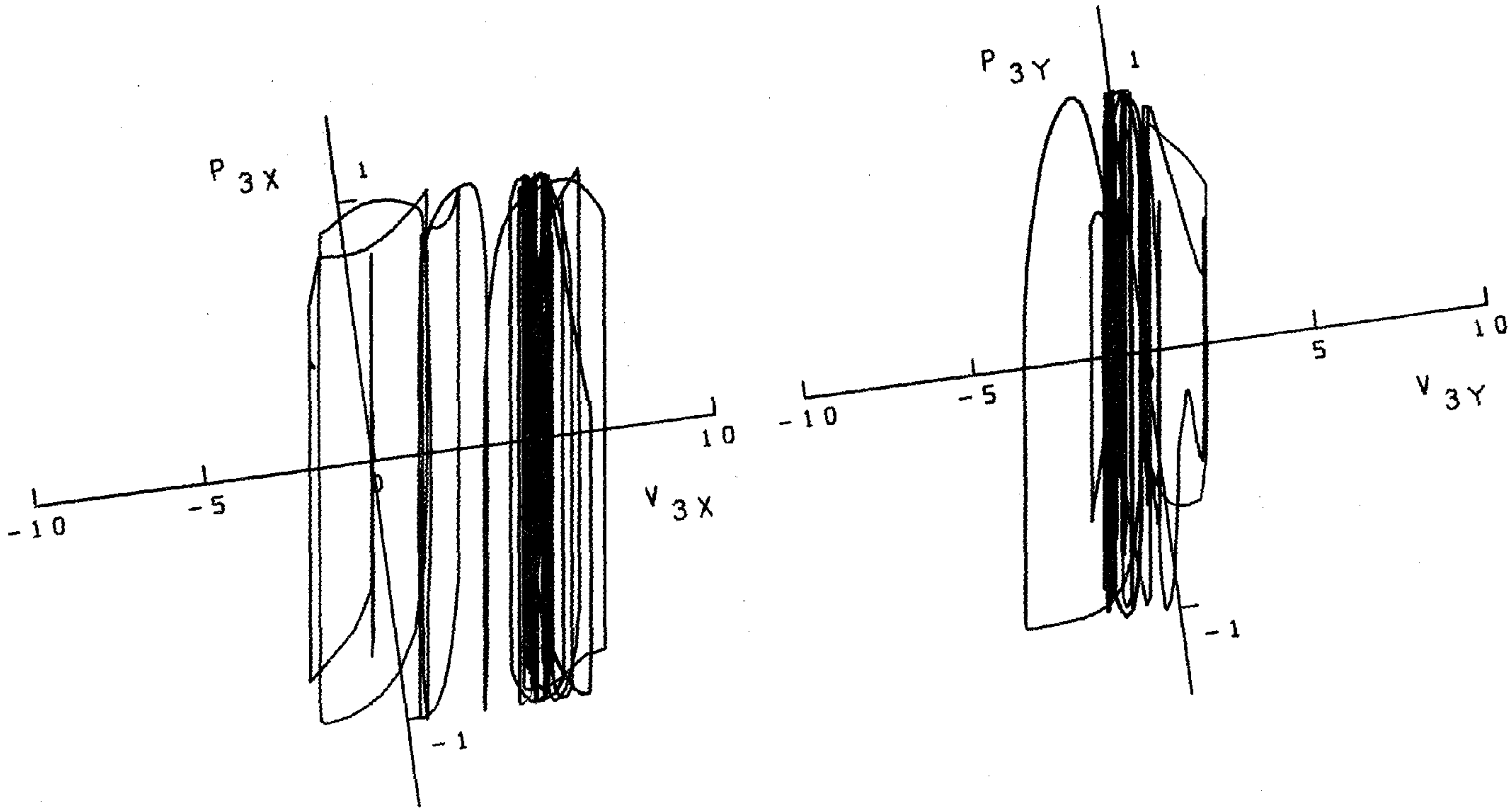


Fig. 3.13 Hysteresis Curves of Response of Element 3 of EPI-System Subjected to the El Centro Earthquake. System Parameters: ξ_i = 5%, T_x = 0.6 sec., ζ_{xy} = 1.0, e_x/r = 0, e_y/r = 0.2, $|\ddot{U}_{gx}|max$ = 2.32 and $|\ddot{U}_{gy}|max$ = 1.43

for the elements 1 and 3 follow very well the rule concerning force interaction of elements with a circular yield surface. Shear force drops are also seen in the hysteresis curves shown in Figs. 3.11 and 3.13. These drops are caused by interaction between the shear forces in the x- and y-directions.

3.2 Effects of Ground Motions

3.2.1 Response to the El Centro and the Taft Earthquakes

Response spectra of dimensionless displacements of EPI-systems subjected to the El Centro and the Taft earthquakes are presented in Figs. 3.14 to 3.16 for the three sets of values of eccentricities e_x/r and e_y/r. The main features of these response spectra are summarized as follows:

1. Response spectra of dimensionless displacements at the center of mass in the x-direction, v_x, for EPI-systems when subjected to the El Centro earthquake are of the same level as those produced by the Taft earthquake (Figs. 3.14a, 3.15a and 3.16a).
2. Response spectra of dimensionless displacements at the center of mass in the y-direction, v_y, are relatively flat for both earthquakes in the range $T_x \geq 0.8$ sec.; however, the spectral values for the El Centro earthquake are approximately two times greater than those for the Taft earthquake (Figs. 3.14a, 3.15a and 3.16a).
3. Response spectra of dimensionless rotation about the center of mass, v_θ, for the El Centro earthquake are approximately the same as those for the Taft earthquake. Note however that the response spectra for the El Centro earthquake have predominent peaks at $T_x = 0.6$ sec. which show spectral values for the

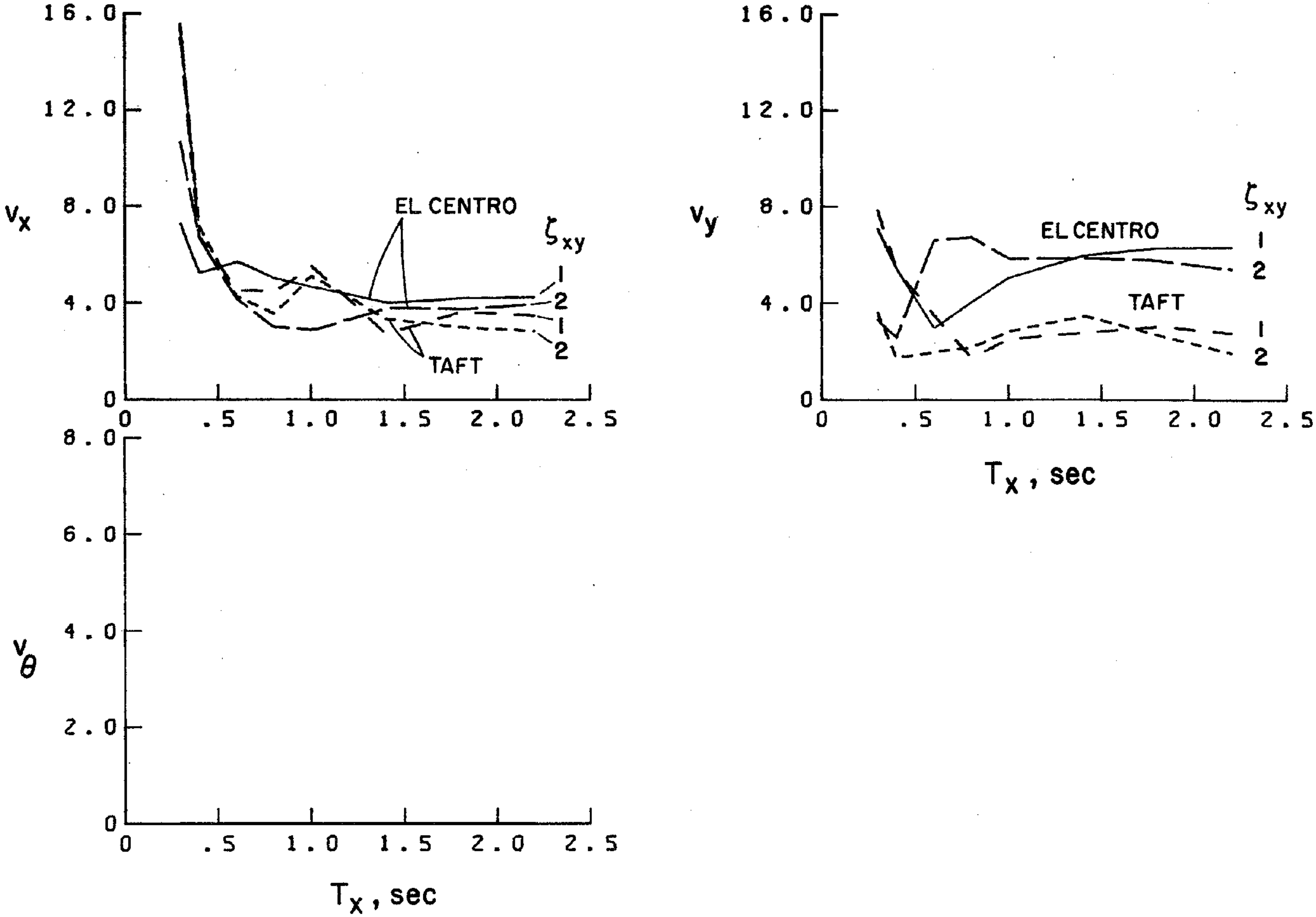


Fig. 3.14a Maximum Displacements of the Center of Mass of EPI-Systems Subjected to Double-Component Input. System Parameters: ξ_i = 5%, $e_x/r = 0$, $e_y/r = 0$ and $c_0 = 0.06$

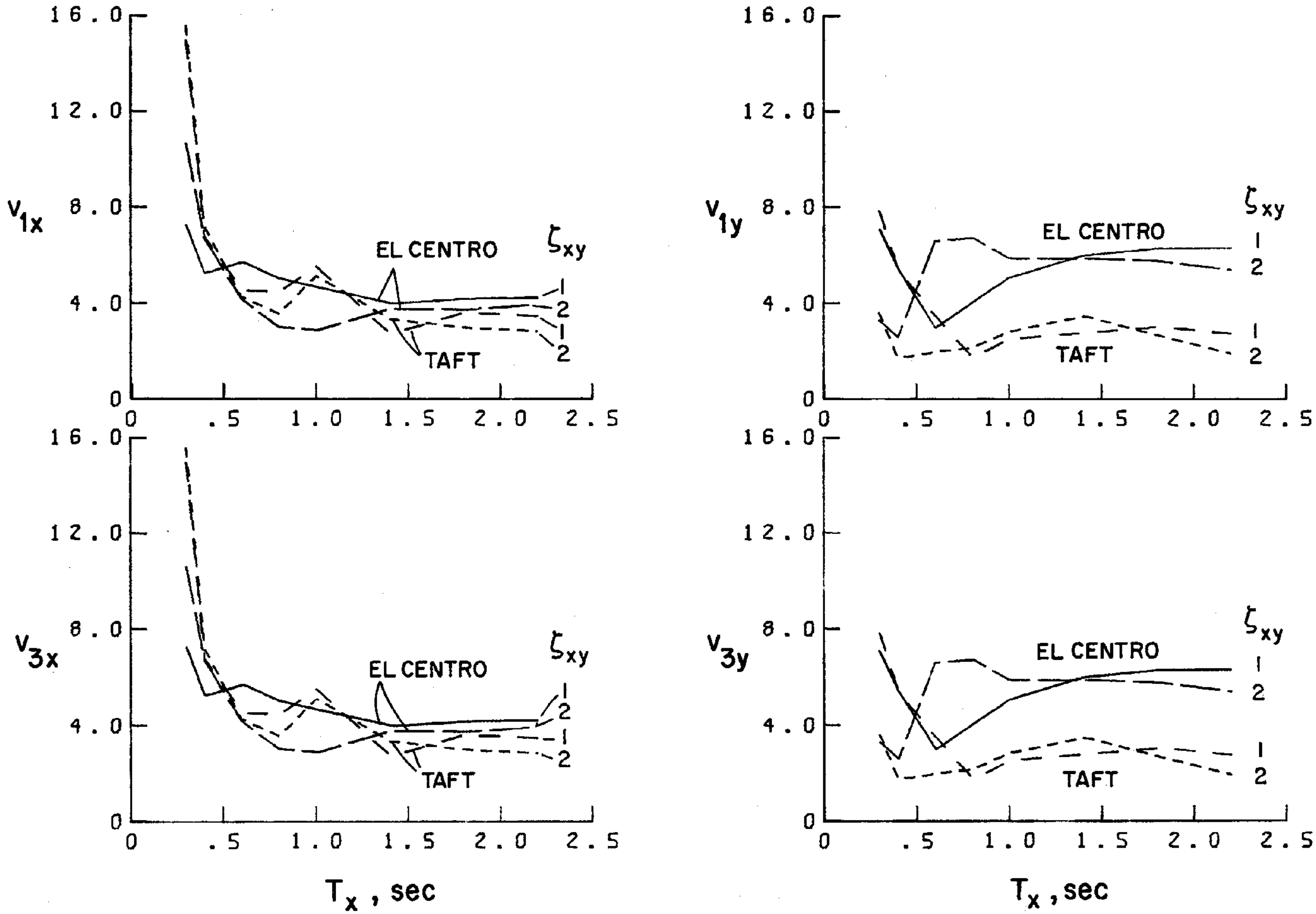


Fig. 3.14b Maximum Displacements (Ductility Factors) of Elements 1 and 3 of EPI-Systems Subjected to Double-Component Input. System Parameters: ξ_i = 5%, e_x/r = 0, e_y/r = 0 and c_0 = 0.06

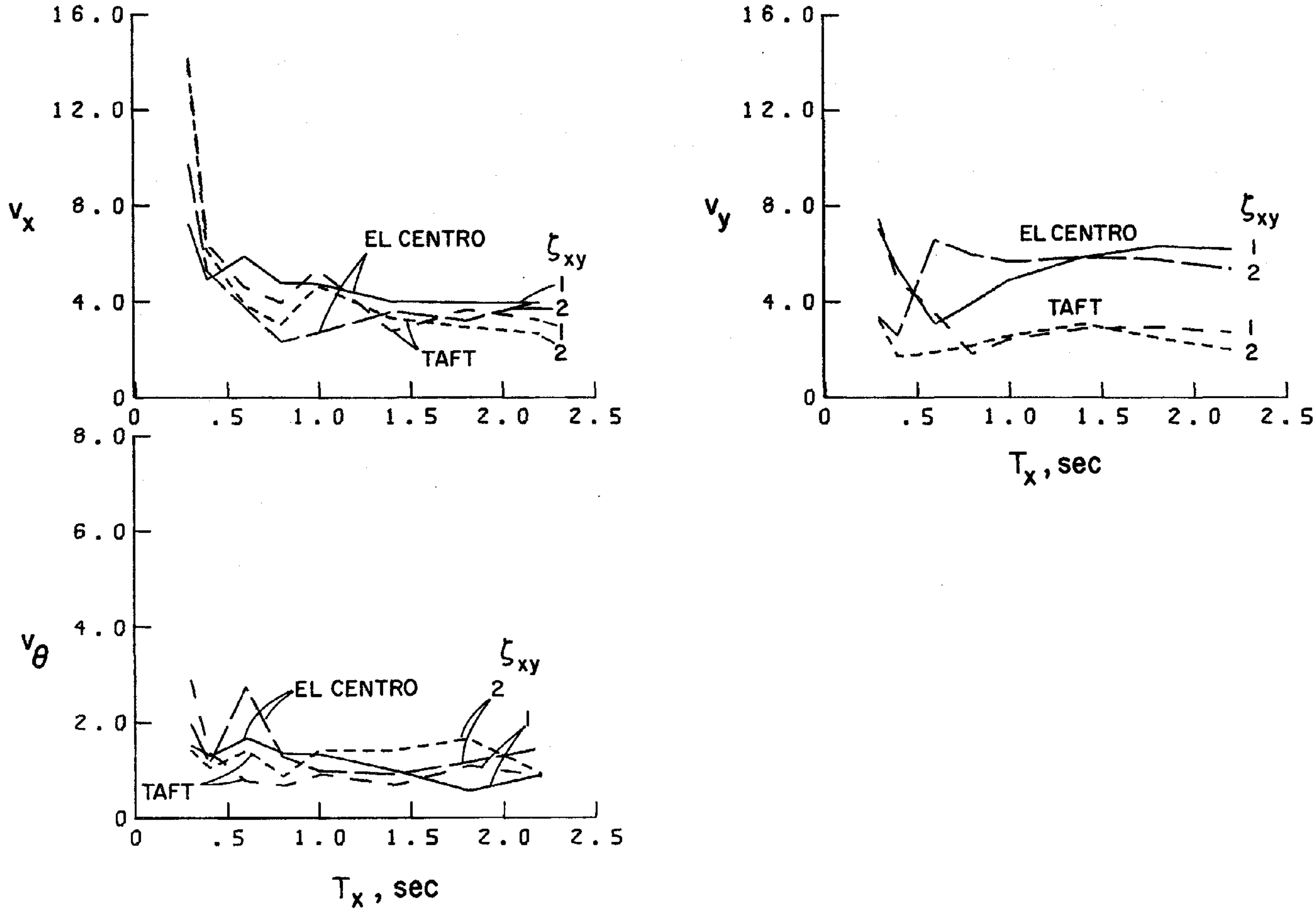


Fig. 3.15a Maximum Displacements of the Center of Mass of EPI-Systems Subjected to Double-Component Input. System Parameters: ξ_i = 5%, e_x/r = 0, e_y/r = 0.2 and c_0 = 0.06

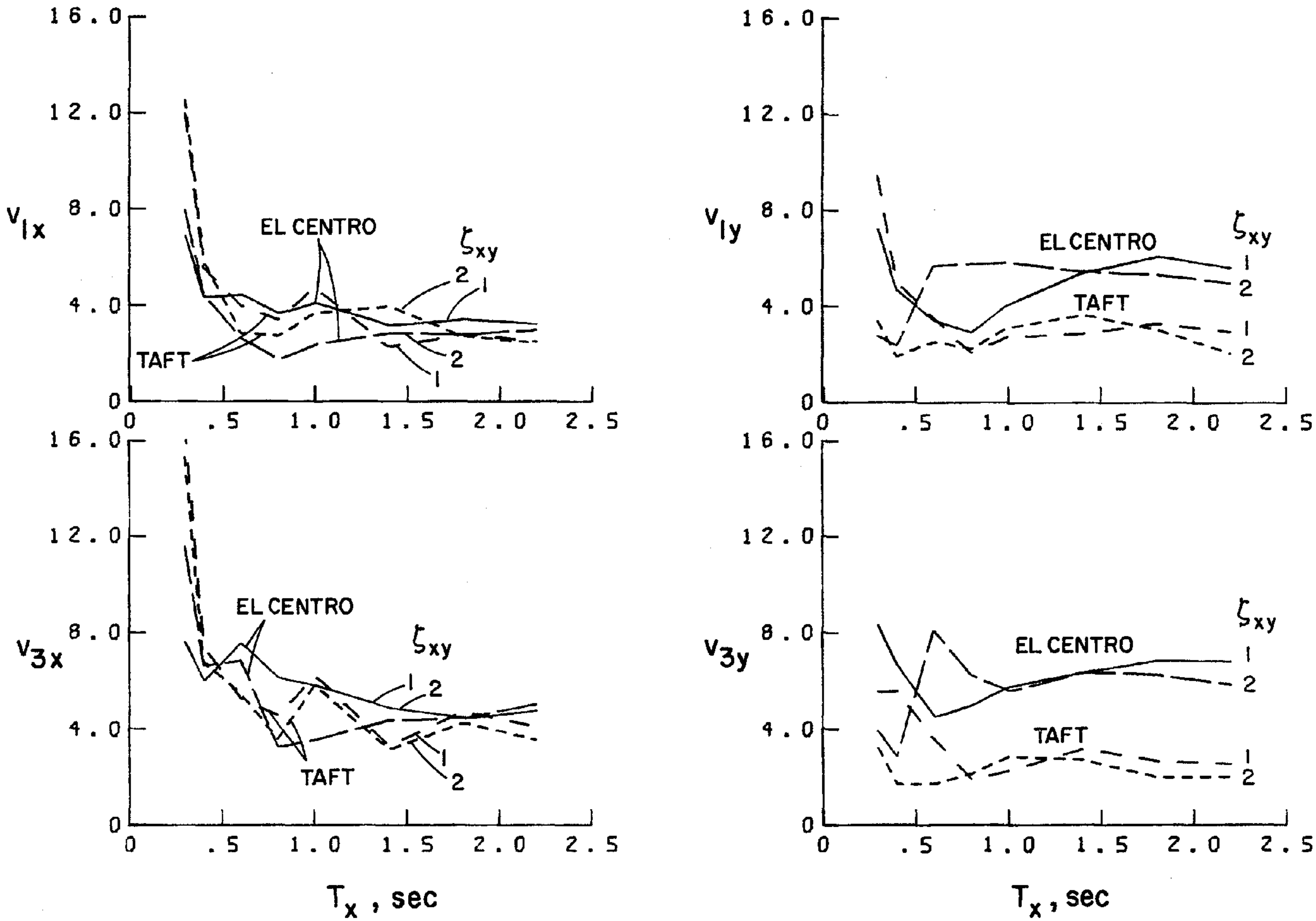


Fig. 3.15b Maximum Displacements (Ductility Factors) of Elements 1 and 3 of EPI-Systems Subjected to Double-Component Input. System Parameters: ξ_i = 5%, e_x/r = 0, e_y/r = 0.2 and c_0 = 0.06

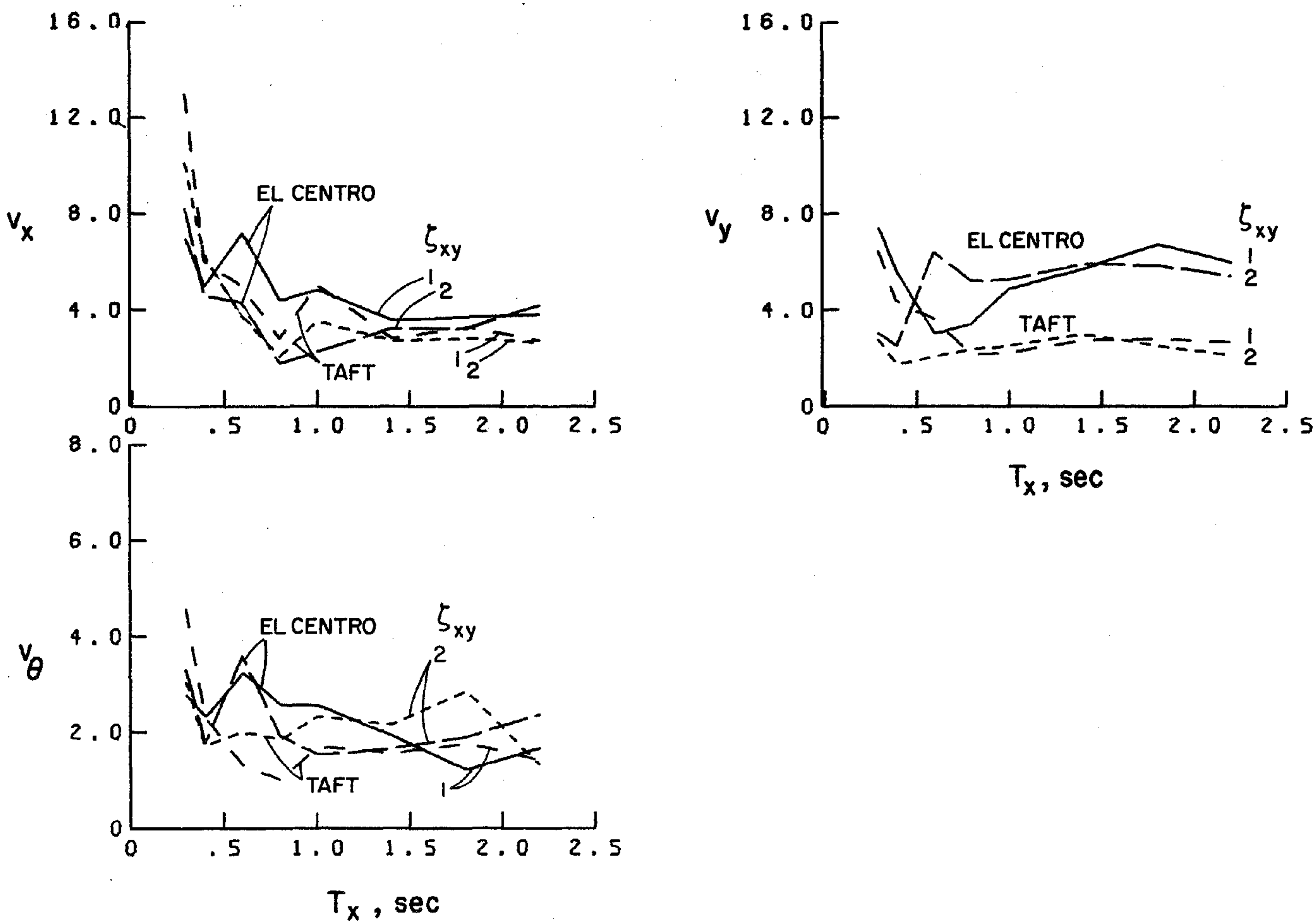


Fig. 3.16a Maximum Displacements at the Center of Mass of EPI-Systems Subjected to Double-Component Input. System Parameters: ξ_i = 5%, e_x/r = 0, e_y/r = 0.4 and c_0 = 0.06

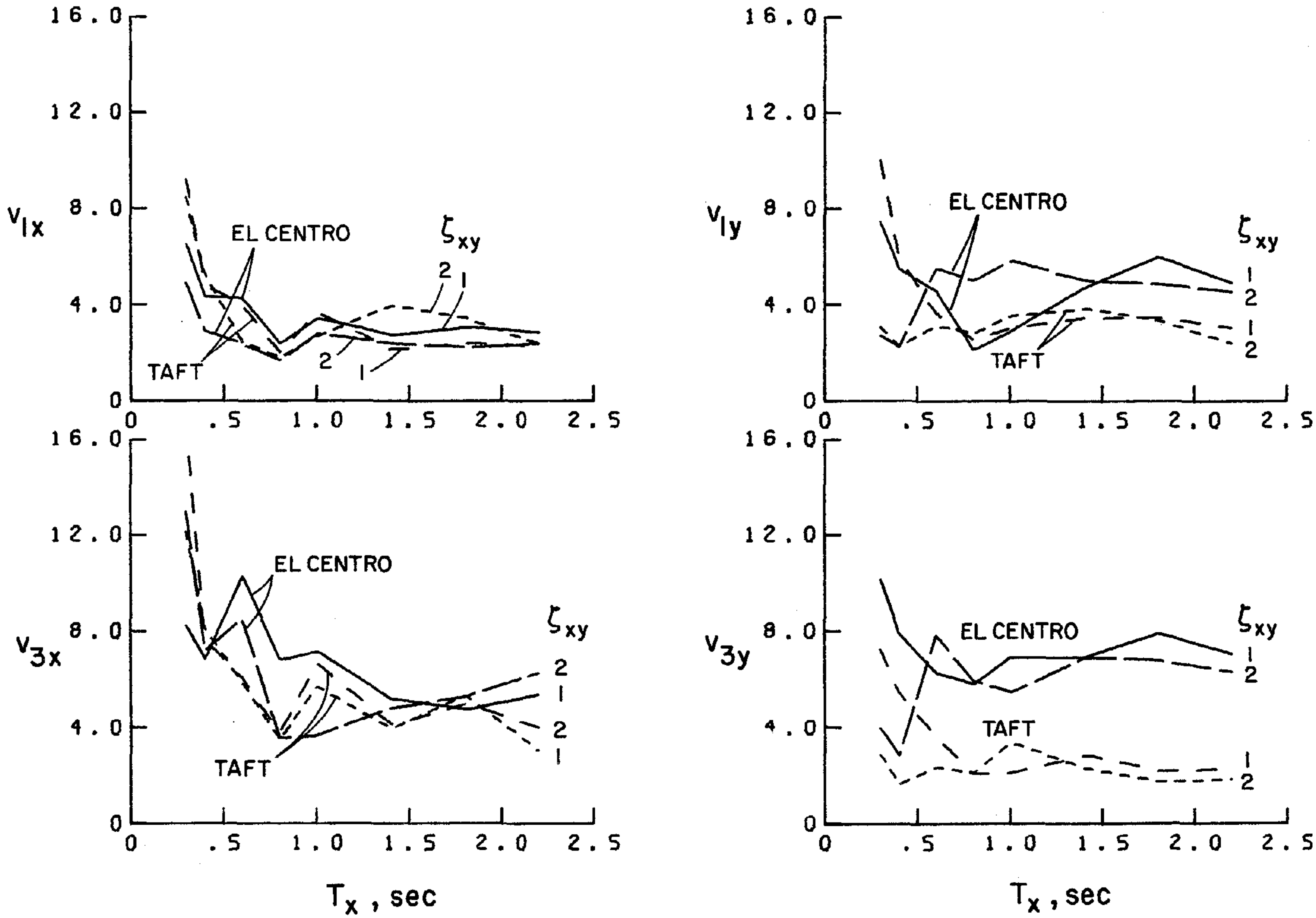


Fig. 3.16b Maximum Displacements (Ductility Factors) of Elements 1 and 3 of EPI-Systems Subjected to Double-Component Input. System Parameters: $\xi_i = 5\%$, $e_x/r = 0$, $e_y/r = 0.4$ and $c_0 = 0.6$

El Centro earthquake to be approximately twice those of the Taft earthquake (Figs. 3.14a, 3.15a and 3.16a).

4. Response spectra of dimensionless displacements of resisting elements in the x-direction for the El Centro earthquake are roughly the same as those for the Taft earthquake. However, the corresponding spectra in the y-direction show different features between the two earthquakes, i.e., the curves for the El Centro earthquake show decreasing and increasing spectral values with increasing values of eccentricity e_y/r for resisting elements 1 and 3, respectively, while the corresponding curves for the Taft earthquake show very little change with increasing values of the eccentricity (Figs. 3.14b, 3.15b and 3.16b).

Response spectra for the uncoupled EPI-systems having no eccentricities in both the x- and y-directions show different features between the El Centro and the Taft earthquakes as shown in Fig. 3.14. Further as indicated above, the coupled translational-torsional systems show different spectral response variations with frequency ratio ζ_{xy} and with eccentricities e_x/r and e_y/r for the different types of earthquake ground motions used.

3.2.2 Response to Single- and Double-Component Ground Motions

The maximum dimensionless displacement responses for coupled EPI-systems subjected to single-component ground motion (NS-component of the El Centro earthquake in the x-direction) and double-component ground motion (NS- and EW-components of the El Centro earthquake in the x- and y-directions, respectively) are compared in Figs 3.17 to 3.20 for different values of uncoupled natural period T_x. The different features shown for

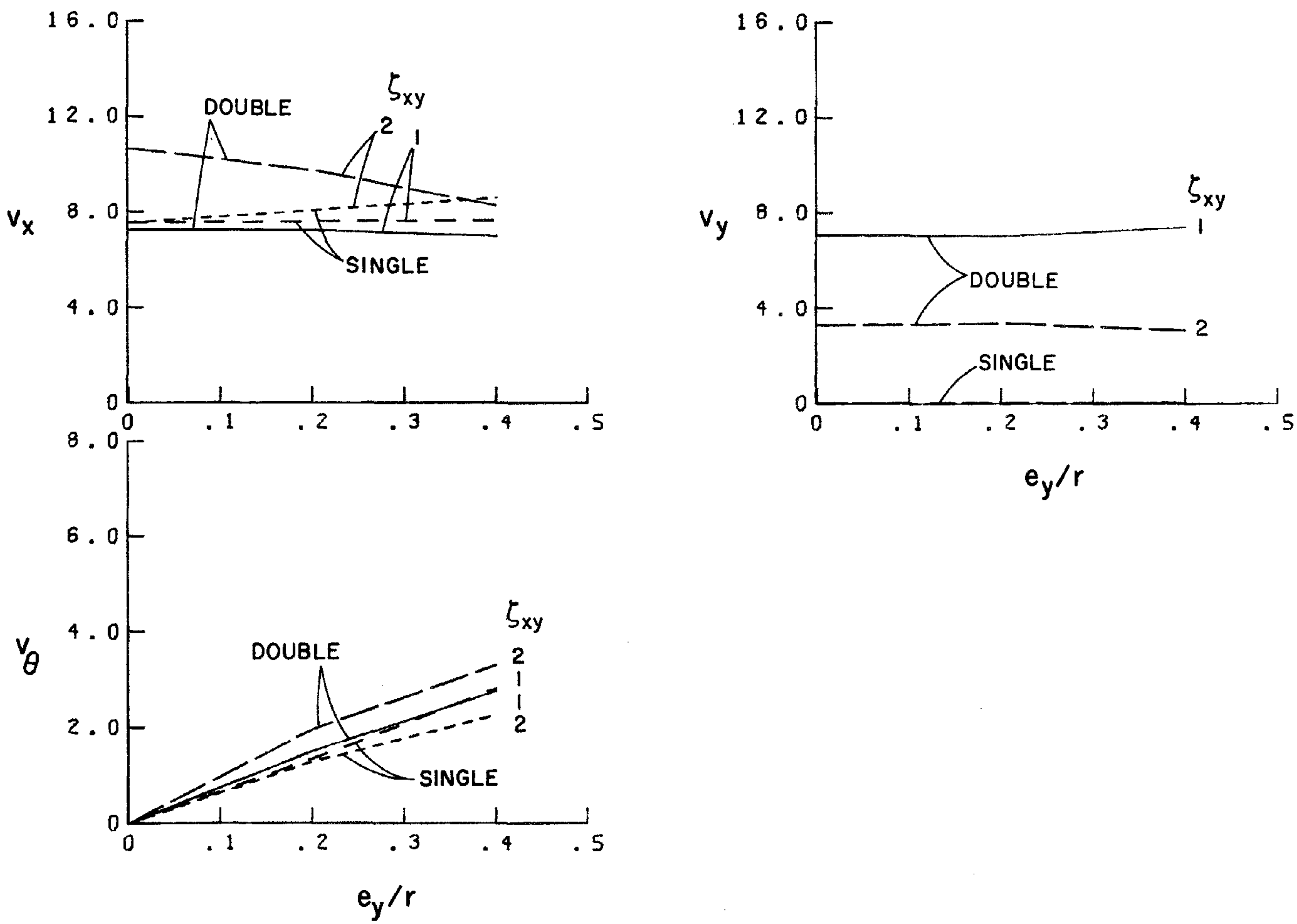


Fig. 3.17a Maximum Displacements at the Center of Mass of EPI-Systems Subjected to Single-Component Input (El Centro NS) and Those Subjected to Double-Component Input (El Centro NS and EW). System Parameters: ξ_i = 5%, T_x = 0.3 sec., e_x/r = 0 and c_0 = 0.6

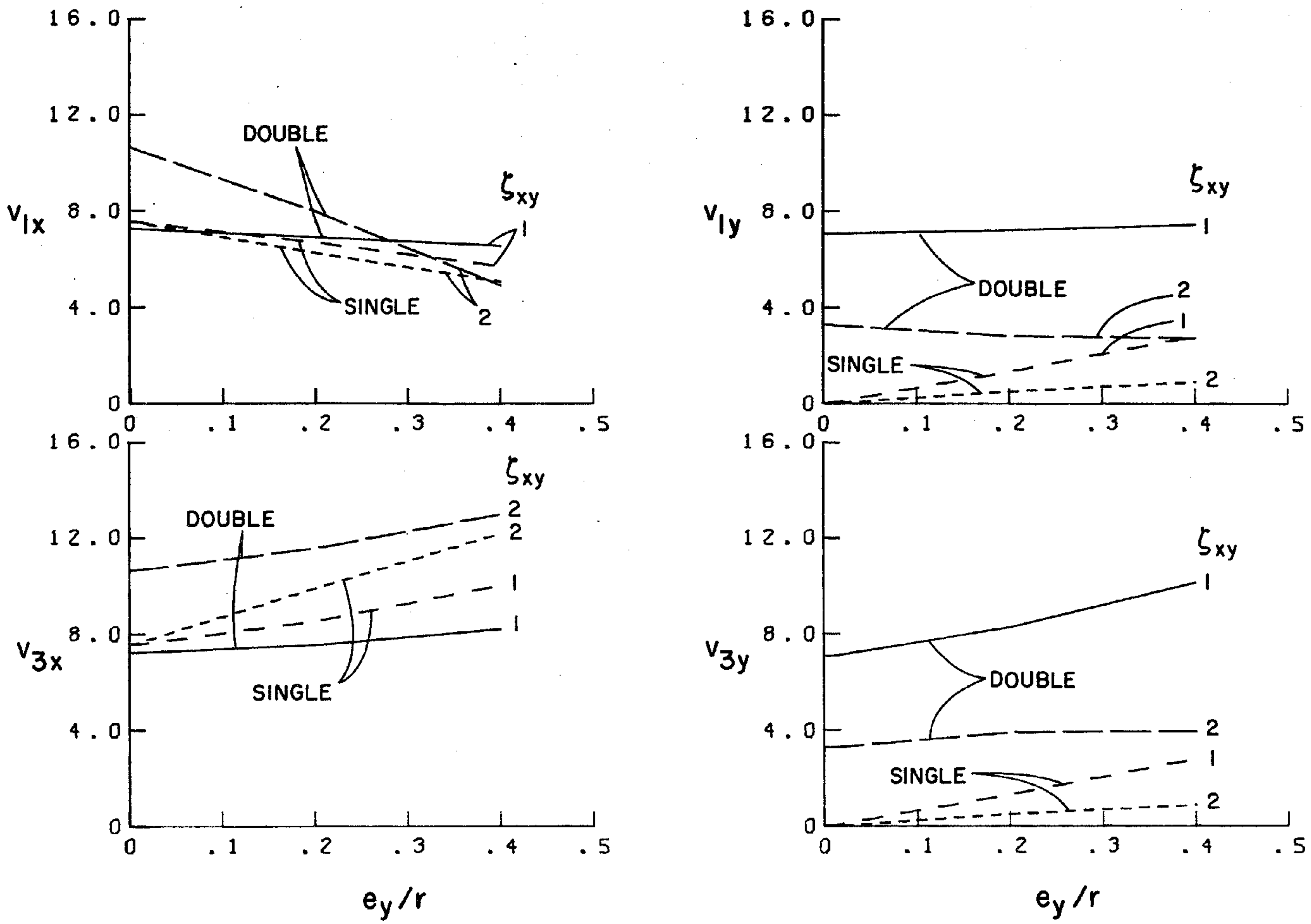


Fig. 3.17b Maximum Displacements (Ductility Factors) of Elements 1 and 3 of EPI-Systems Subjected to Single-Component Input (El Centro NS) and Those Subjected to Double-Component Input (El Centro NS and EW). System Parameters: ξ_i = 5%, T_x = 0.3 sec., e_x/r = 0 and c_0 = 0.06

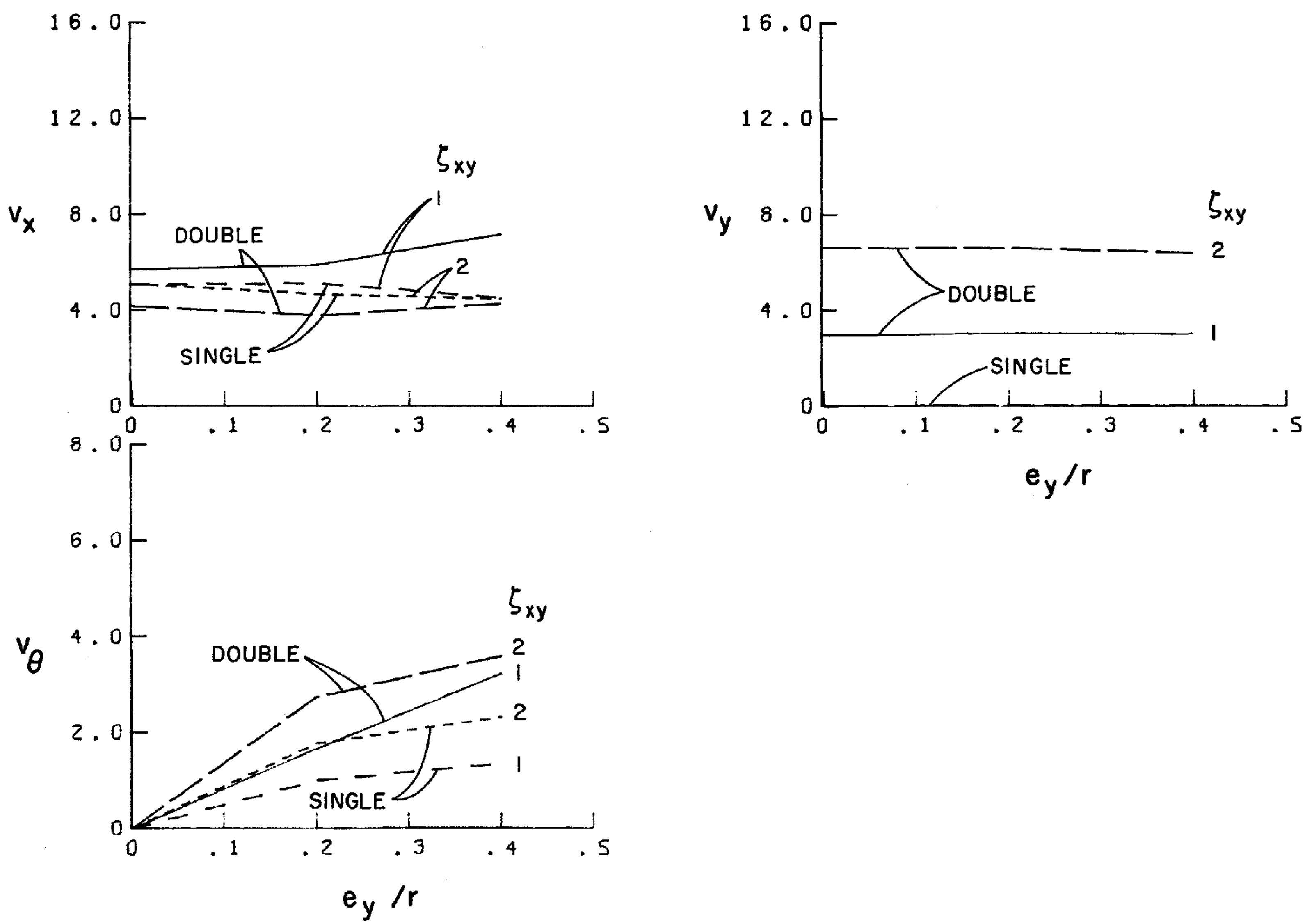


Fig. 3.18a Maximum Displacements at the Center of Mass of EPI-Systems Subjected to Single-Component Input (El Centro NS) and Those Subjected to Double-Component Input (El Centro NS and EW). System Parameters: ξ_i = 5%, T_x = 0.6 sec., e_x/r = 0 and c_0 = 0.06

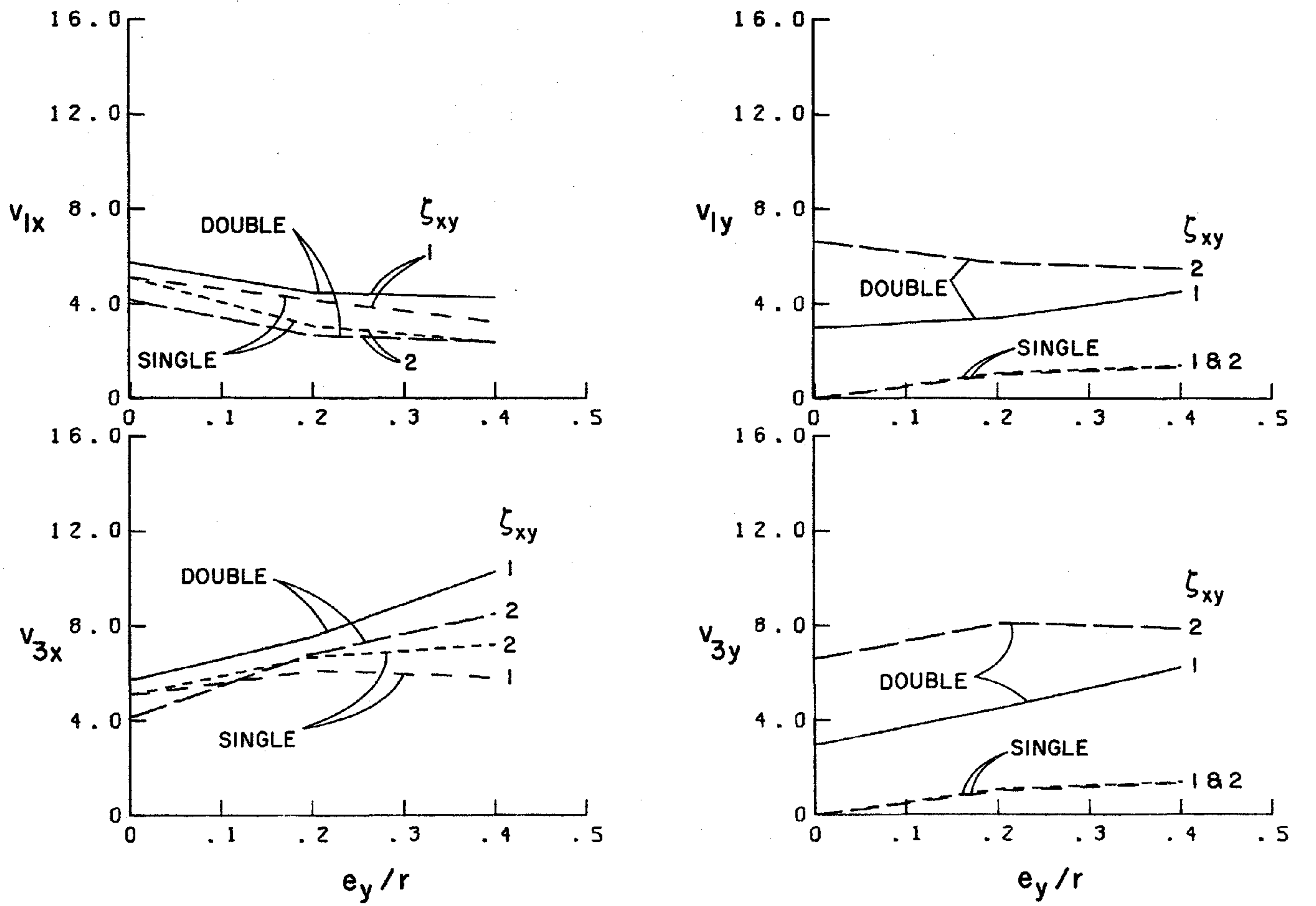


Fig. 3.18b Maximum Displacements (Ductility Factors) of Elements 1 and 3 of EPI-Systems Subjected to Single-Component Input (El Centro NS) and Those Subjected to Double-Component Input (El Centro NS and EW). System Parameters: ξ_i = 5%, T_x = 0.6 sec., e_x/r = 0 and c_0 = 0.06

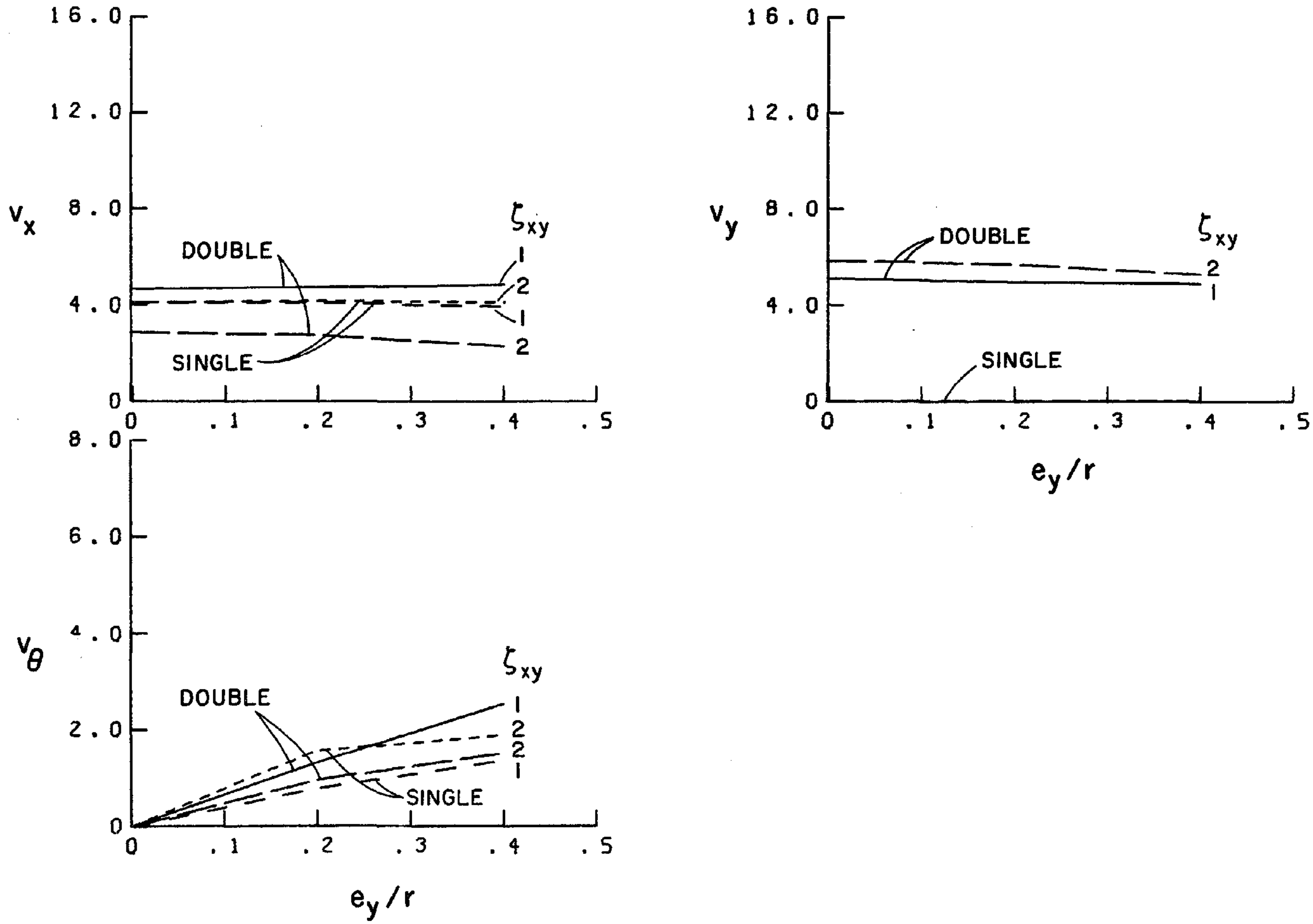


Fig. 3.19a Maximum Displacements at the Center of Mass of EPI-Systems Subjected to Single-Component Input (El Centro NS) and Those Subjected to Double-Component Input (El Centro NS and EW). System Parameters: ξ_i = 5%, T_x = 1.0 sec., e_x/r = 0 and c_0 = 0.06

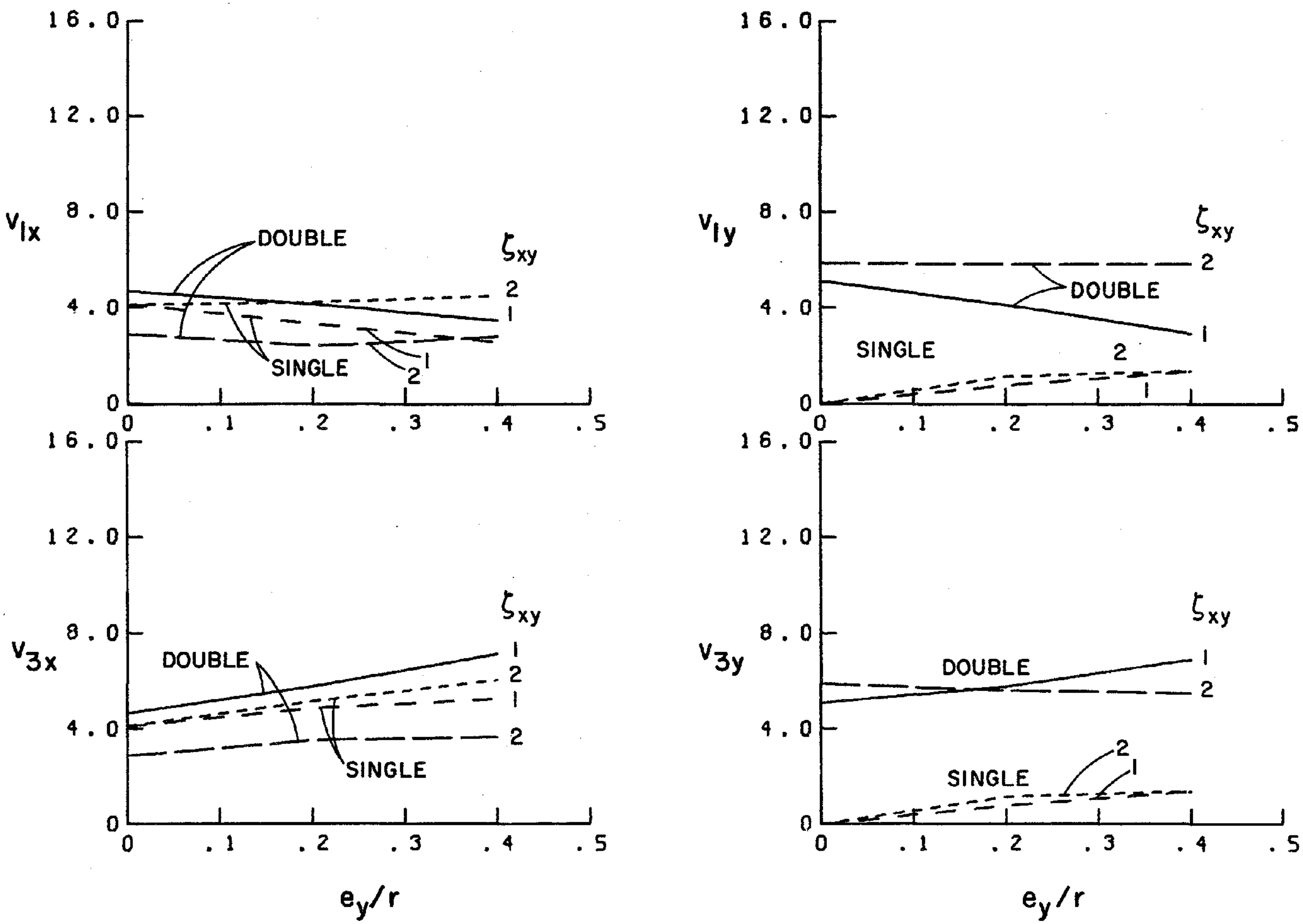


Fig. 3.19b Maximum Displacements (Ductility Factors) of Elements 1 and 3 of EPI-Systems Subjected to Single-Component Input (El Centro NS) and Those Subjected to Double-Component Input (El Centro NS and EW). System Parameters: $\xi_i = 5\%$, $T_x = 1.0$ sec., $e_x/r = 0$ and $c_0 = 0.06$

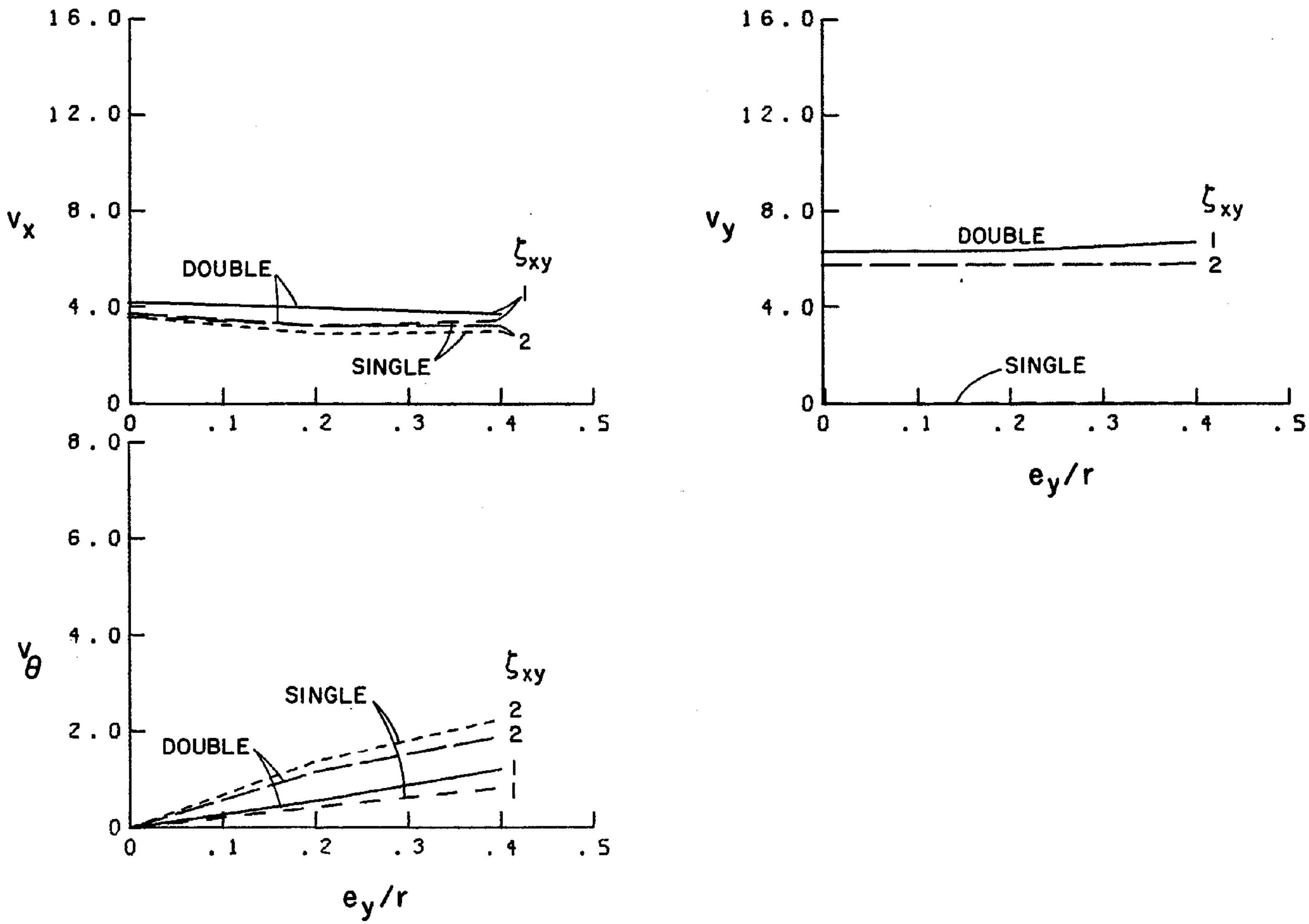


Fig. 3.20a Maximum Displacements at the Center of Mass of EPI-Systems Subjected to Single-Component Input (El Centro NS) and Those Subjected to Double-Component Input (El Centro NS and EW). System Parameters: ξ_i = 5%, T_x = 1.8 sec., e_x/r = 0 and c_0 = 0.06

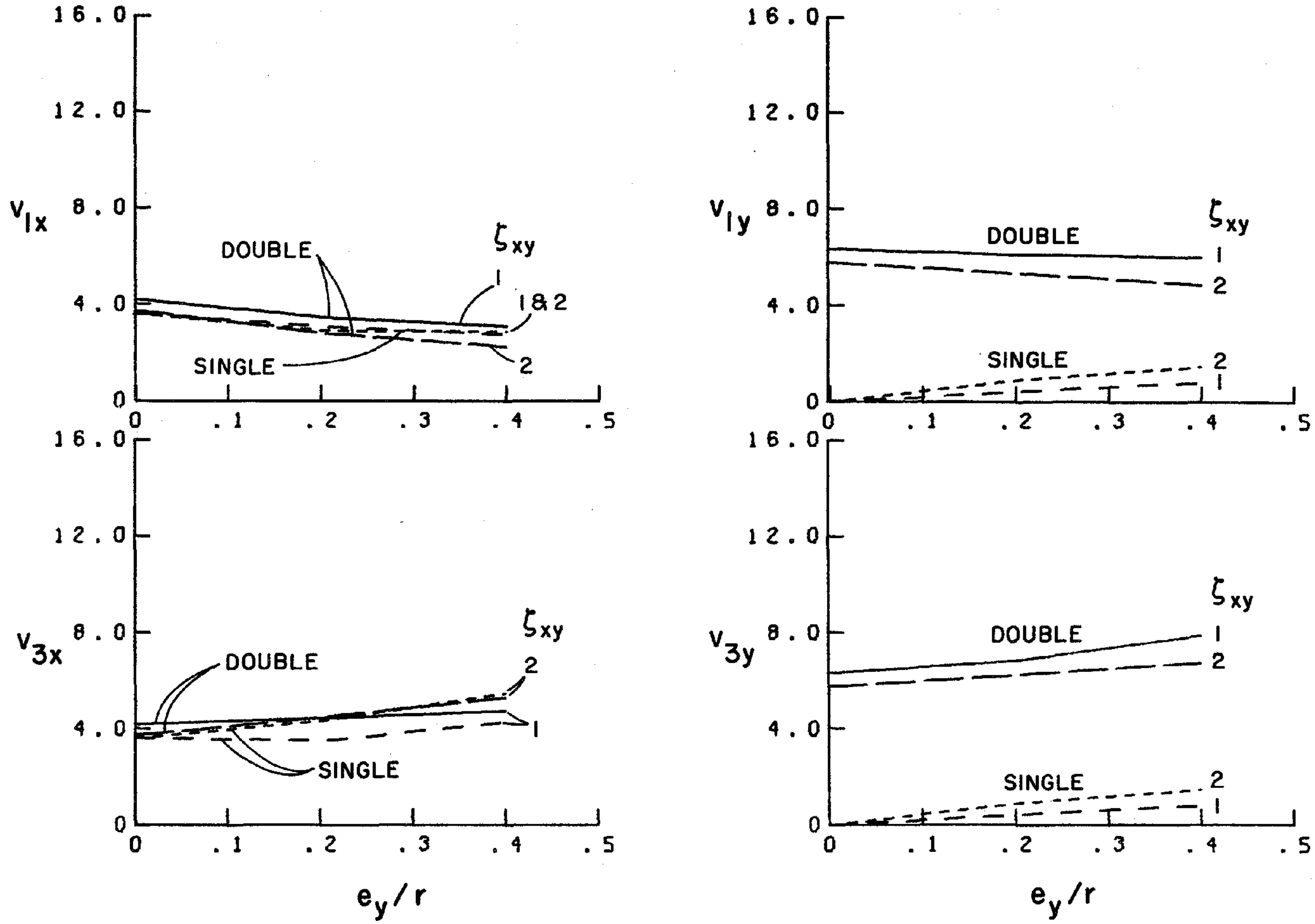


Fig. 3.20b Maximum Displacements (Ductility Factors) of Elements 1 and 3 of EPI-Systems Subjected to Single-Component Input (El Centro NS) and Those Subjected to Double-Component Input (El Centro NS and EW). System Parameters: $\xi_i = 5\%$, $T_x = 1.8$ sec., $e_x/r = 0$ and $c_0 = 0.06$

responses at the center of mass in the x-direction for EPI-systems when subjected to both single-component and double component ground motions are caused primarily by the differences in force interaction. Likewise, differences in the x-direction shown for uncoupled frequency ratio ζ_{xy} equal to 1 and 2 are caused by differences in force interaction. It can be recognized from Figs. 3.17a, 3.18a, 3.19a and 3.20a that the differences between the maximum dimensionless displacements at the center of mass in the x-direction, v_x, for systems with $\zeta_{xy} = 1$ and with $\zeta_{xy} = 2$ for double-component ground motion are much greater than those for single-component ground motion. This observation indicates that the effects of force interaction on the response of EPI-systems to single-component ground motion are much less than the corresponding effects on the response of similar systems subjected to double-component ground motion. The reason why such effects are relatively small for single-component ground motion even for systems having large values of eccentricities is that the response is primarily in the direction of the input motion.

Torsional responses about the center of mass of EPI-systems, v_θ, when subjected to both single-component and double-component ground motions show different tendencies with changing values of both eccentricity e_y/r and uncoupled frequency ratio ζ_{xy} as shown in Figs. 3.17a, 3.18a, 3.19a and 3.20a.

3.3 Effects of Force Interaction

Response spectra for E-, EP- and EPI-systems with $\zeta_{xy} = 1$ when subjected to the double-component ground motion of the El Centro earthquake are presented in Figs. 3.21 to 3.23 for different values of eccentricity. It can be observed that the response spectra for EPI-systems are generally smooth function of natural period T_x which is in contrast with the

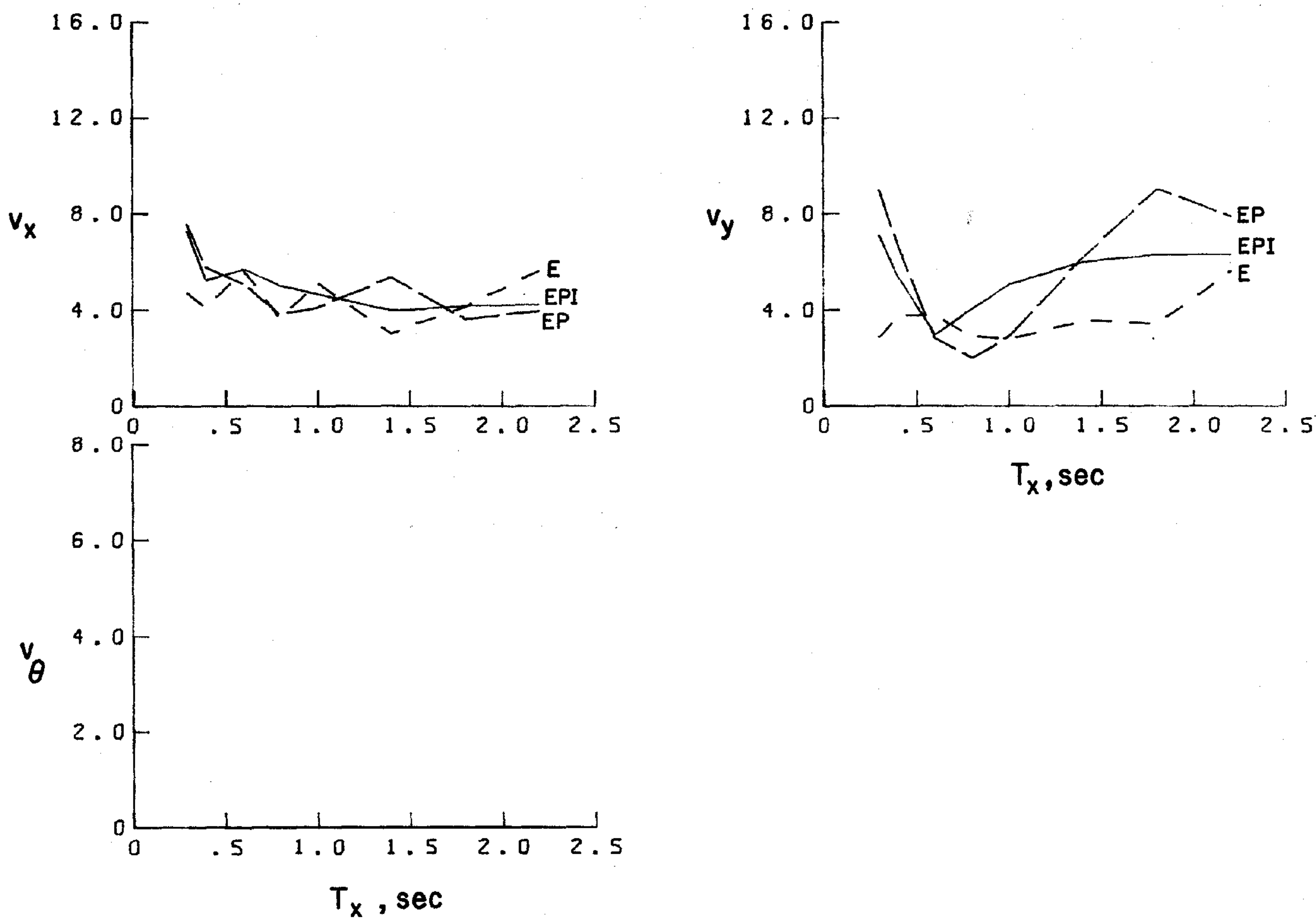


Fig. 3.21a Maximum Displacements at the Center of Mass of E-, EP- and EPI-Systems Subjected to the El Centro Earthquake (Double-Component Input). System Parameters: $\xi_i = 5\%$, $\zeta_{xy} = 1.0$, $e_x/r = 0$, $e_y/r = 0$ and $c_0 = 0.06$

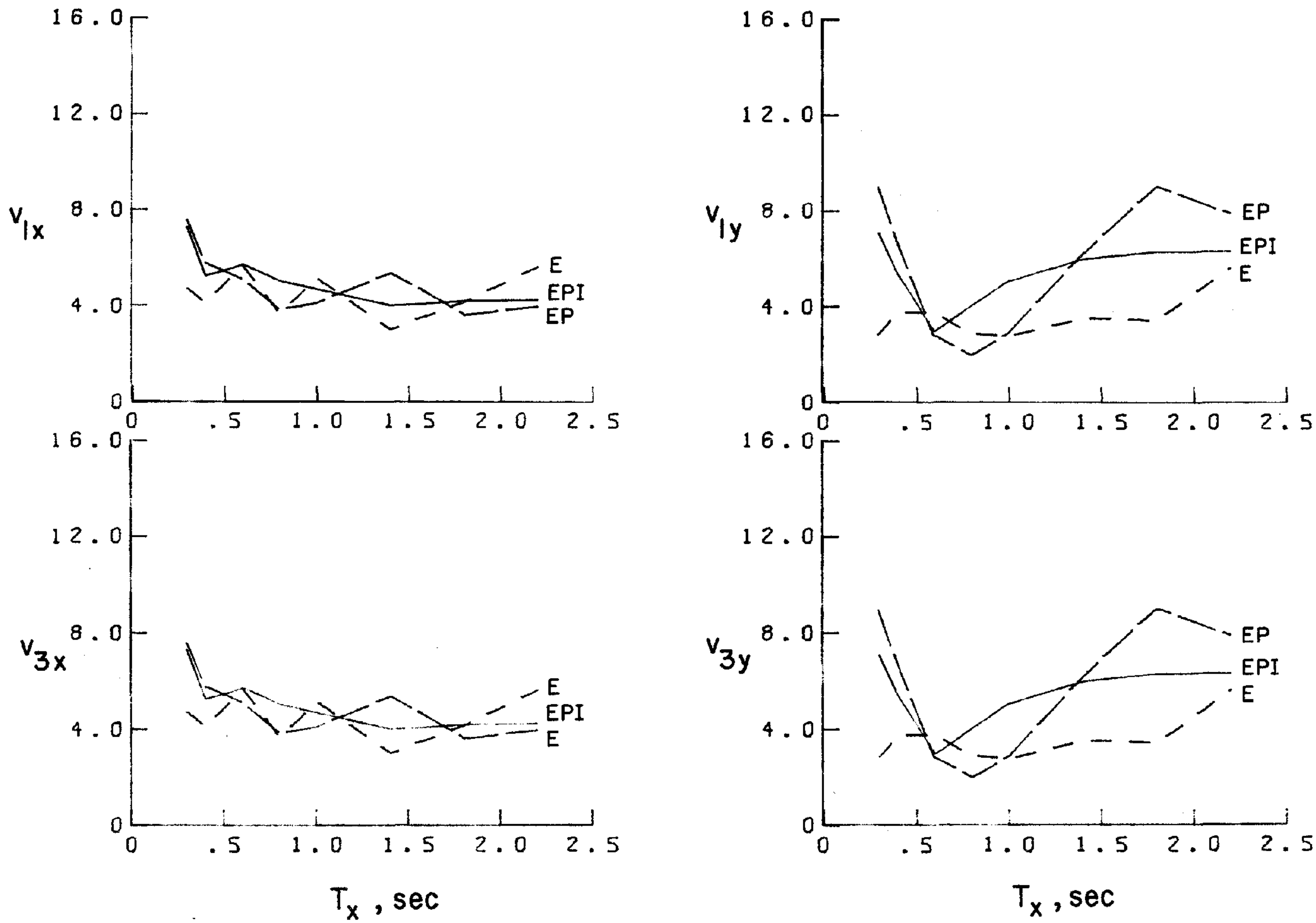


Fig. 3.21b Maximum Displacements (Ductility Factors) of Elements 1 and 3 of E-, EP- and EPI-Systems Subjected to the El Centro Earthquake (Double-Component Input). Systems Parameters: $\xi_i = 5\%$, $\zeta_{xy} = 1.0$, $e_x/r = 0$, $e_y/r = 0$ and $c_0 = 0.06$

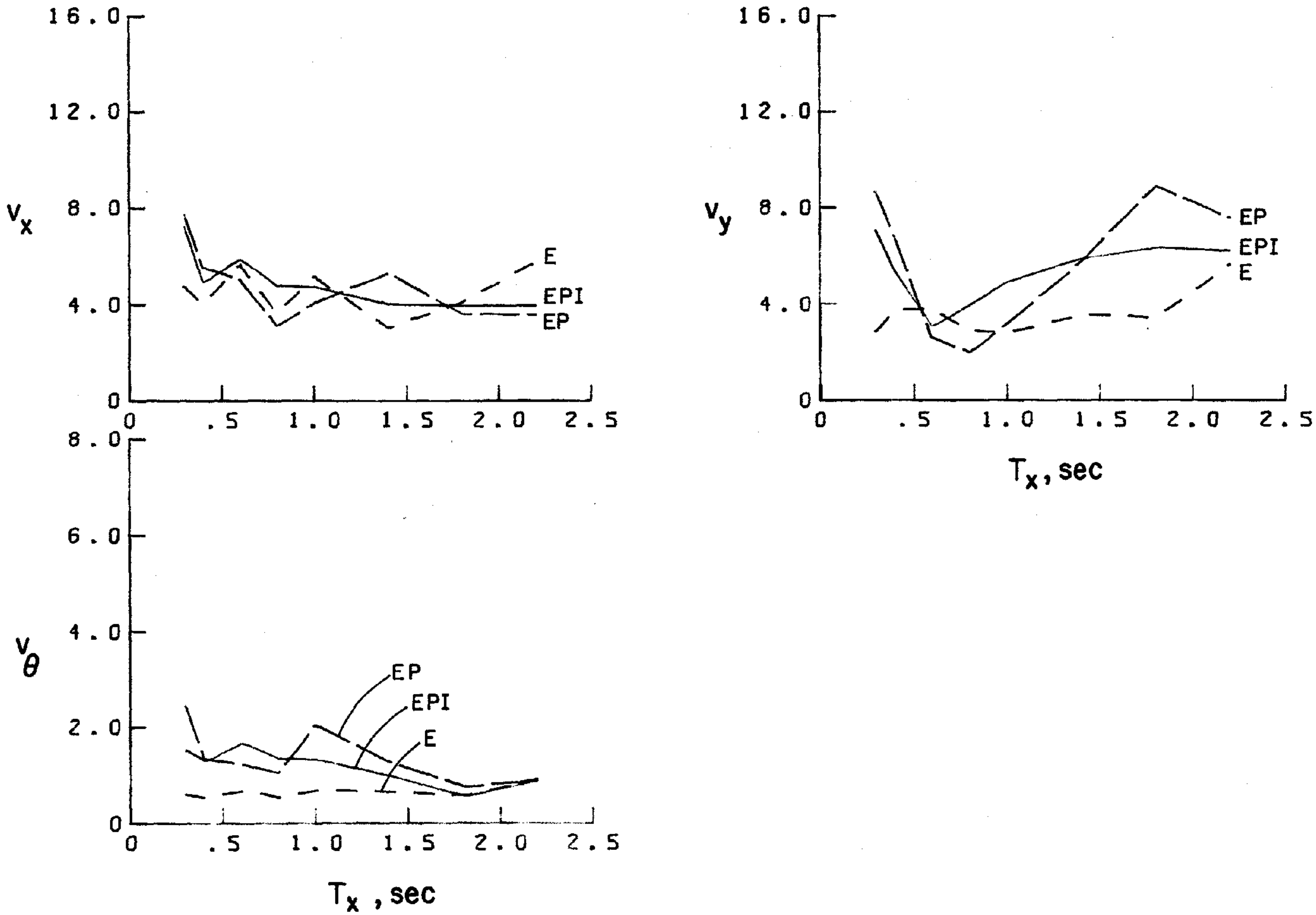


Fig. 3.22a Maximum Displacements at the Center of Mass of E-, EP- and EPI-Systems Subjected to the El Centro Earthquake (Double-Component Input). System Parameters: ξ_i = 5%, ζ_{xy} = 1.0, e_x/r = 0, e_y/r = 0.2 and c_0 = 0.06

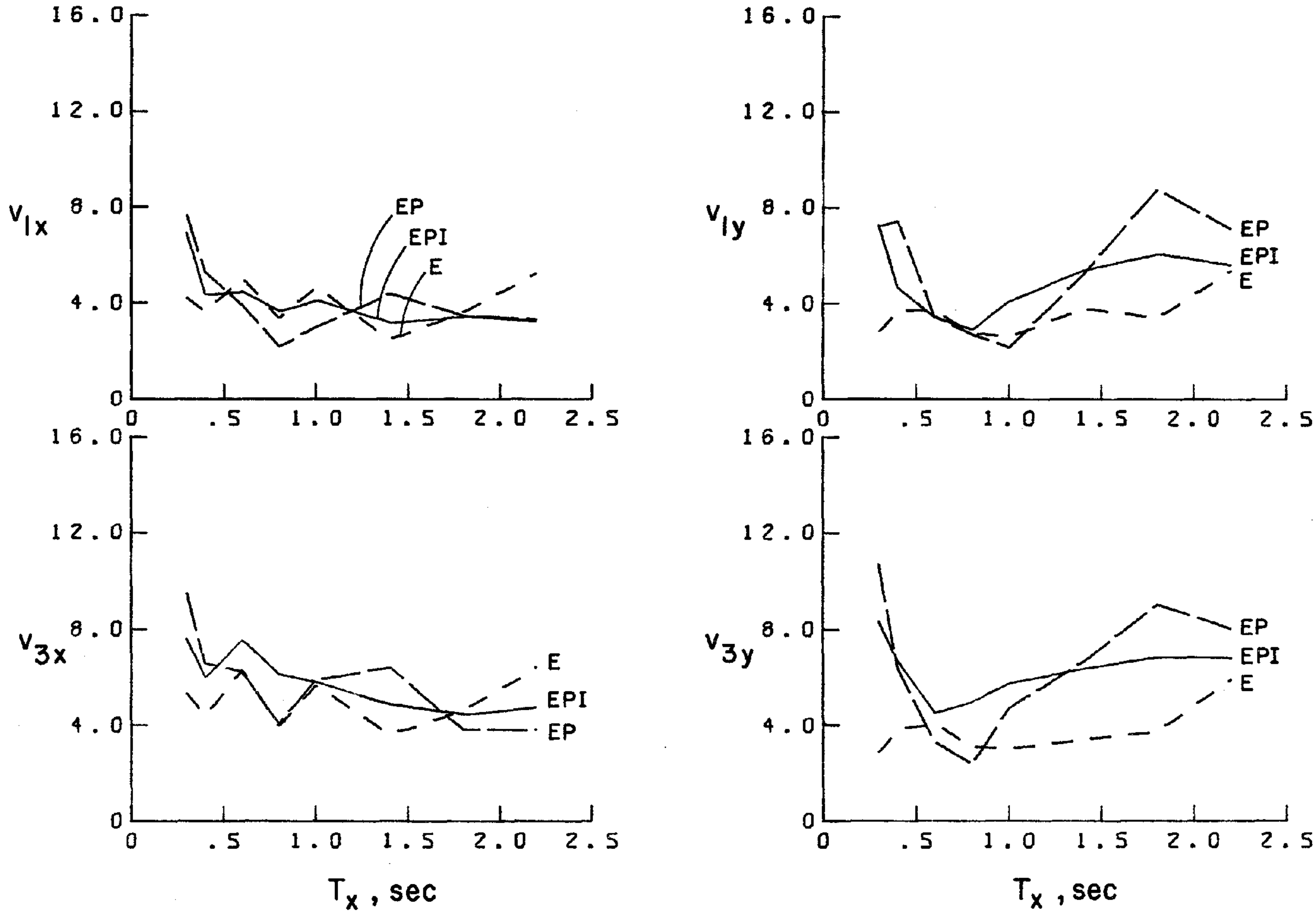


Fig. 3.22b Maximum Displacements (Ductility Factors) of Elements 1 and 3 of E-, EP- and EPI-Systems Subjected to the El Centro Earthquake (Double-Component Input). System Parameters: ξ_i = 5%, ζ_{xy} = 1.0, e_x/r = 0, e_y/r = 0.2 and c_0 = 0.06

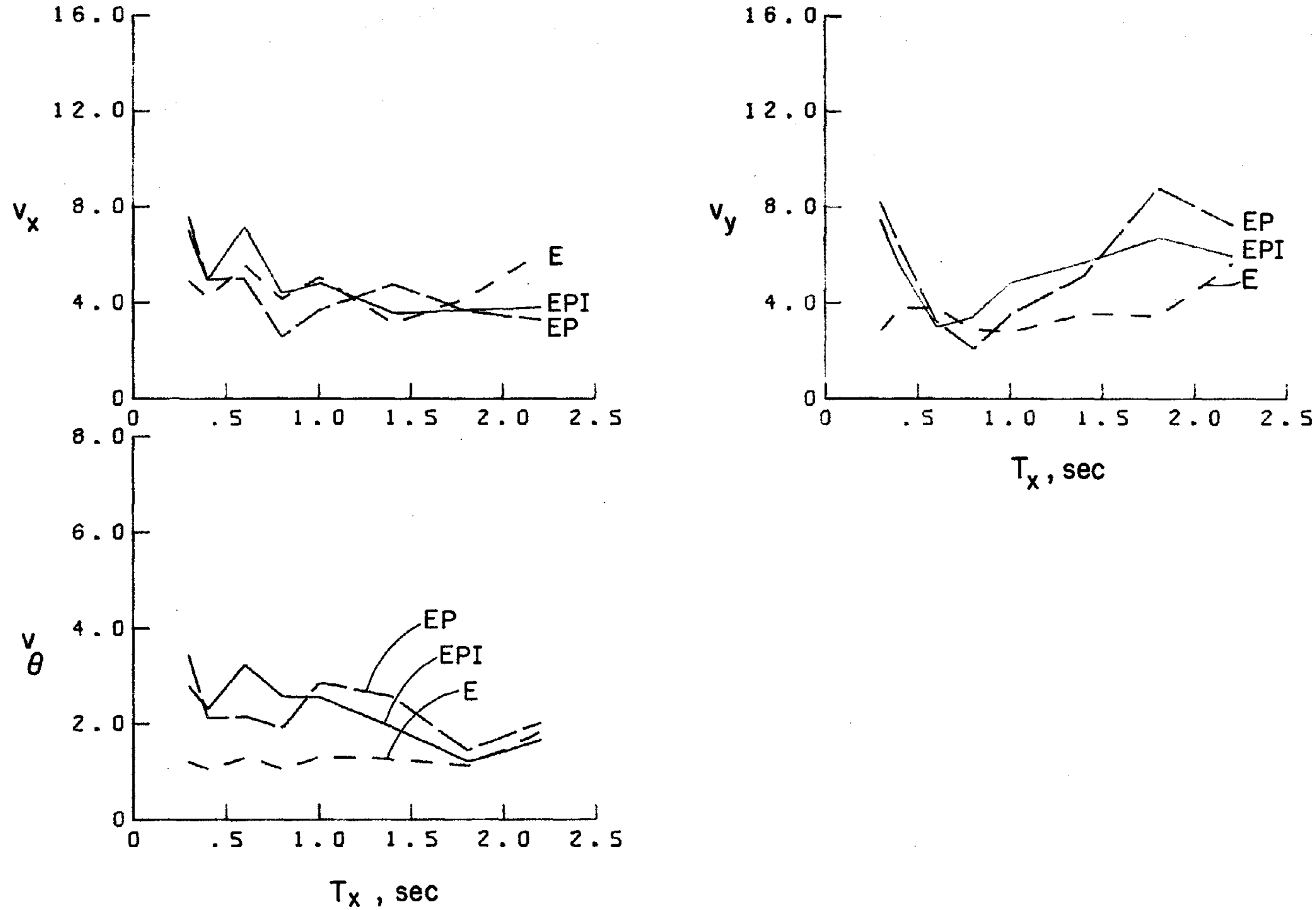


Fig. 3.23a Maximum Displacements at the Center of Mass of E-, EP- and EPI-Systems Subjected to the El Centro Earthquake (Double-Component Input). System Parameters: ξ_i = 5%, ζ_{xy} = 1.0, e_x/r = 0, e_y/r = 0.4 and c_0 = 0.06

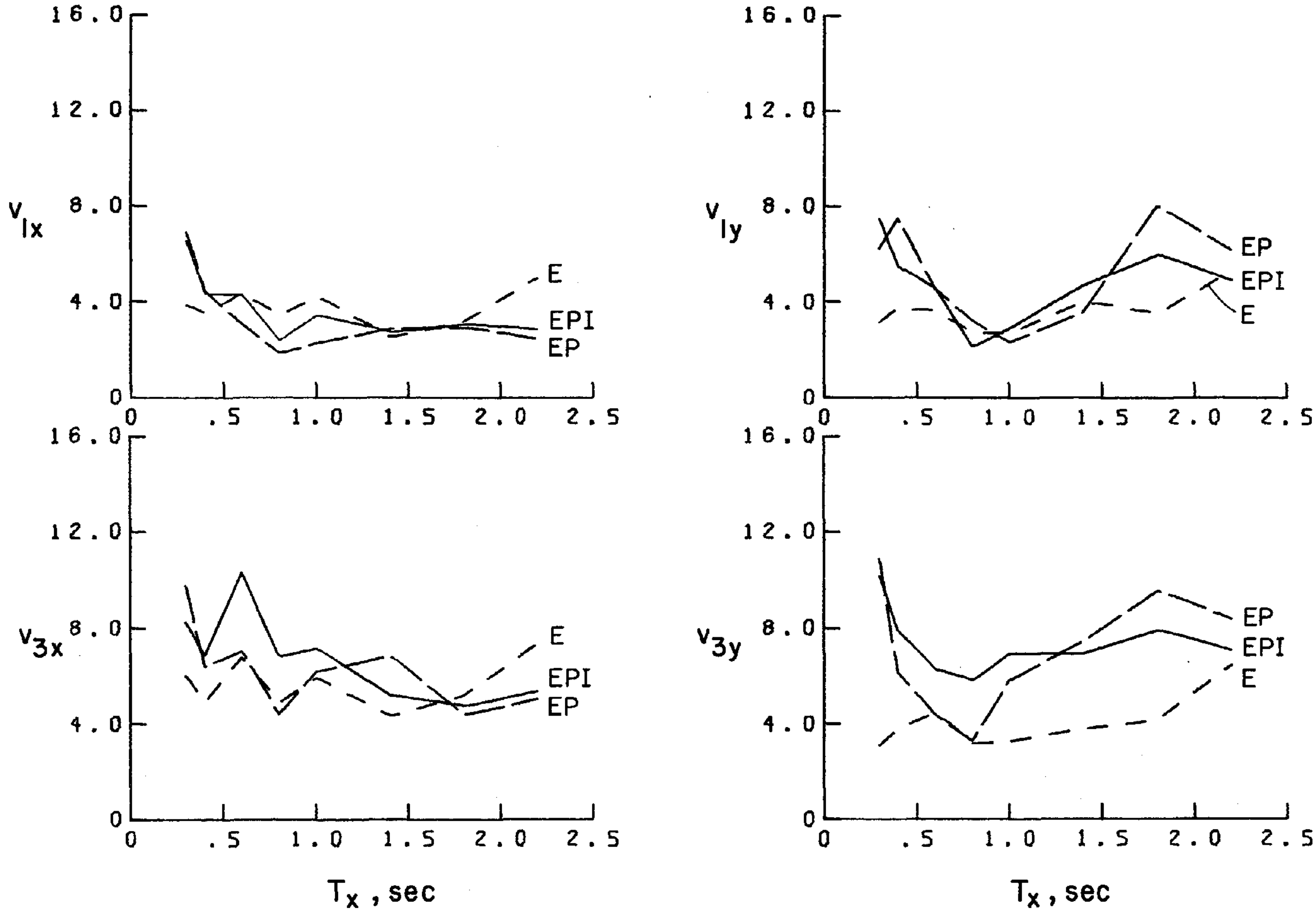


Fig. 3.23b Maximum Displacements (Ductility Factors) of Elements 1 and 3 of E-, EP- and EPI-Systems Subjected to the El Centro Earthquake (Double-Component Input). System Parameters: ξ_i = 5%, ζ_{xy} = 1.0, e_x/r = 0, e_y/r = 0.4 and c_0 = 0.06

response spectra for E- and EP-systems which are quite irregular. Differences between the maximum responses for the EP- and EPI-systems are produced by force interaction effects. The results in Figs. 3.21 to 3.23 show that the maximum responses of the EPI-systems are larger in certain ranges of T_x and smaller in other ranges than those of EP-systems. This observation, which is consistent with the results of Kobori [24], indicates that force interaction effects do not always produce larger maximum displacements.

Kobori indicates also that force interaction effects have a tendency to balance the two components of ductility ratio response in column members for those cases involving large plastic deformations. This same tendency is confirmed by the results presented herein, especially for those systems having long uncoupled natural periods T_x. The results shown in Figs. 3.21 to 3.23 are for those systems having an uncoupled frequency ratio $\zeta_{xy} = 1$. As previously mentioned, force interaction effects on inelastic response are variable with the value of uncoupled frequency ratio.

3.4 Effects of Uncoupled Natural Frequencies

The maximum dimensionless displacement responses for EPI-systems having no eccentricities in the x-direction i.e. $e_x/r = 0$, are presented in Figs. 3.24 to 3.27 for different values of uncoupled natural period T_x. In these figures, the variations in the maximum dimensionless displacement response at the center of mass in the x-direction, v_x, with uncoupled natural frequency ratio ζ_{xy} are caused primarily by force interaction. Torsional effects in this case are small. The degree of force interaction effects is related to the value of uncoupled frequency ratio ζ_{xy} (Figs. 3.24a, 3.25a, 3.26a and 3.27a). Note that the maximum

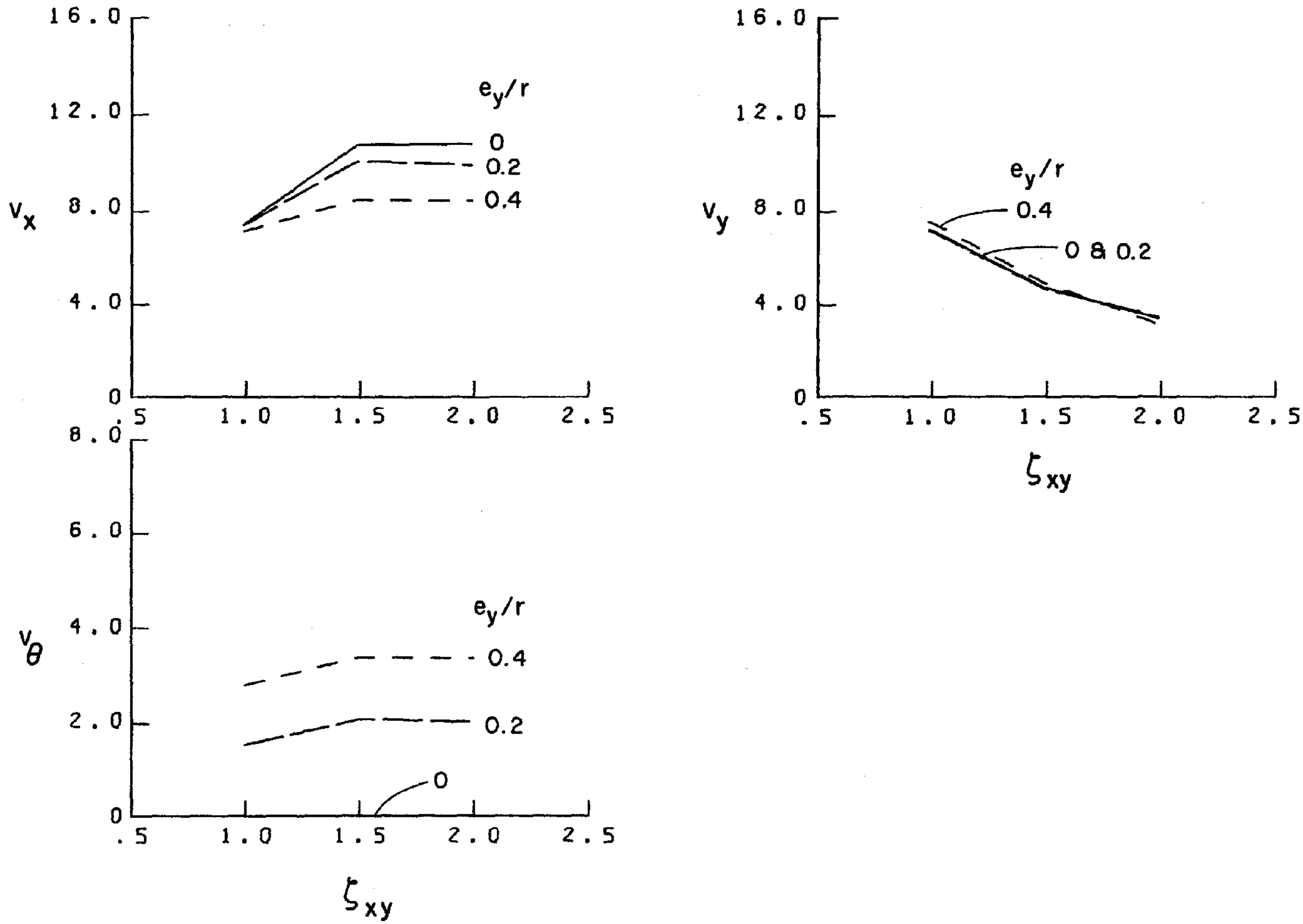


Fig. 3.24a Maximum Displacements at the Center of Mass of EPI-Systems Subjected to the El Centro Earthquake (Double-Component Input). System Parameters: ξ_i = 5%, T_x = 0.3 sec., e_x/r = 0 and c_0 = 0.06

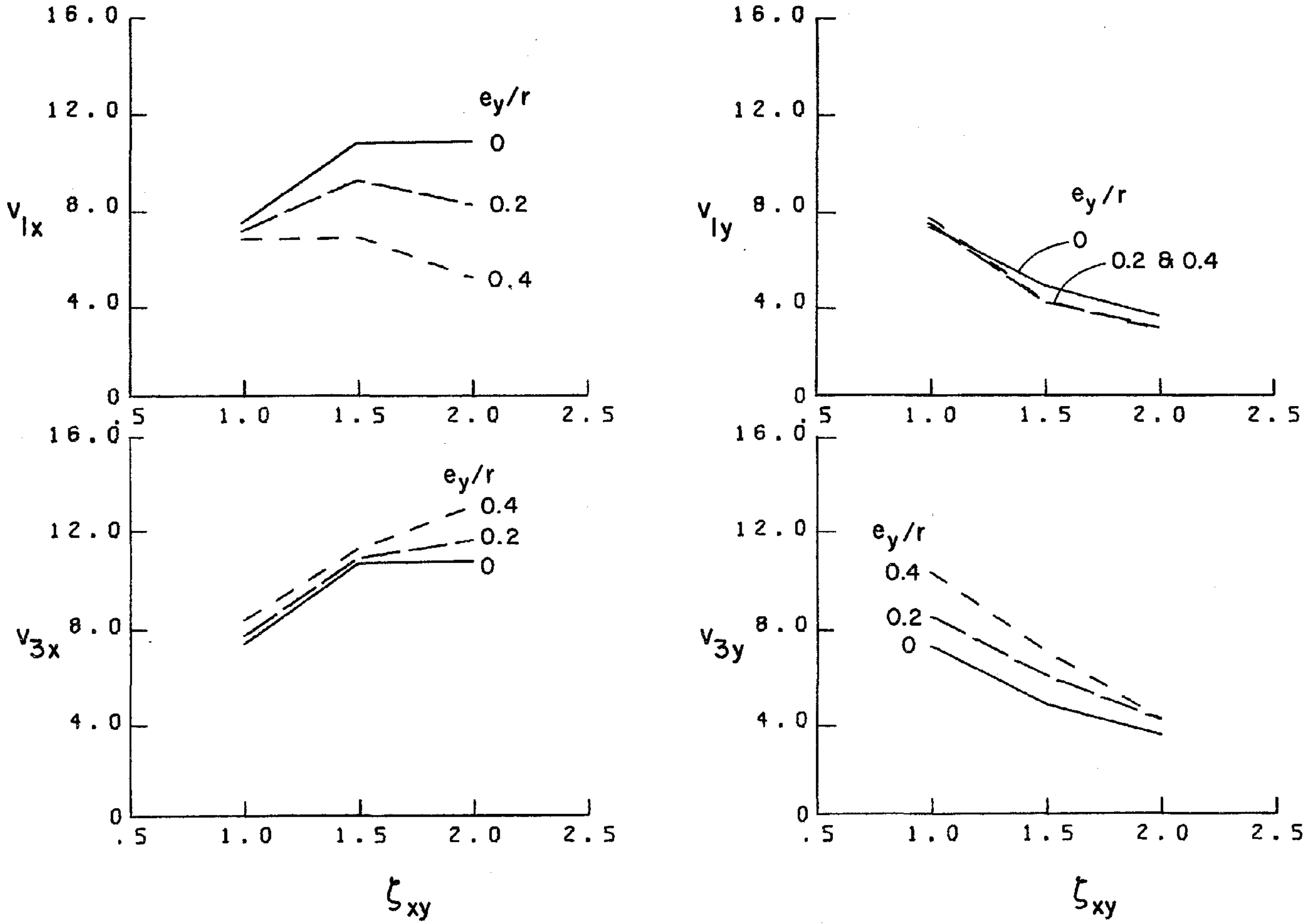


Fig. 3.24b Maximum Displacements (Ductility Factors) of Elements 1 and 3 of EPI-Systems Subjected to the El Centro Earthquake (Double-Component Input). System Parameters: $\xi_i = 5\%$, $T_x = 0.3$ sec., $e_x/r = 0$ and $c_0 = 0.06$

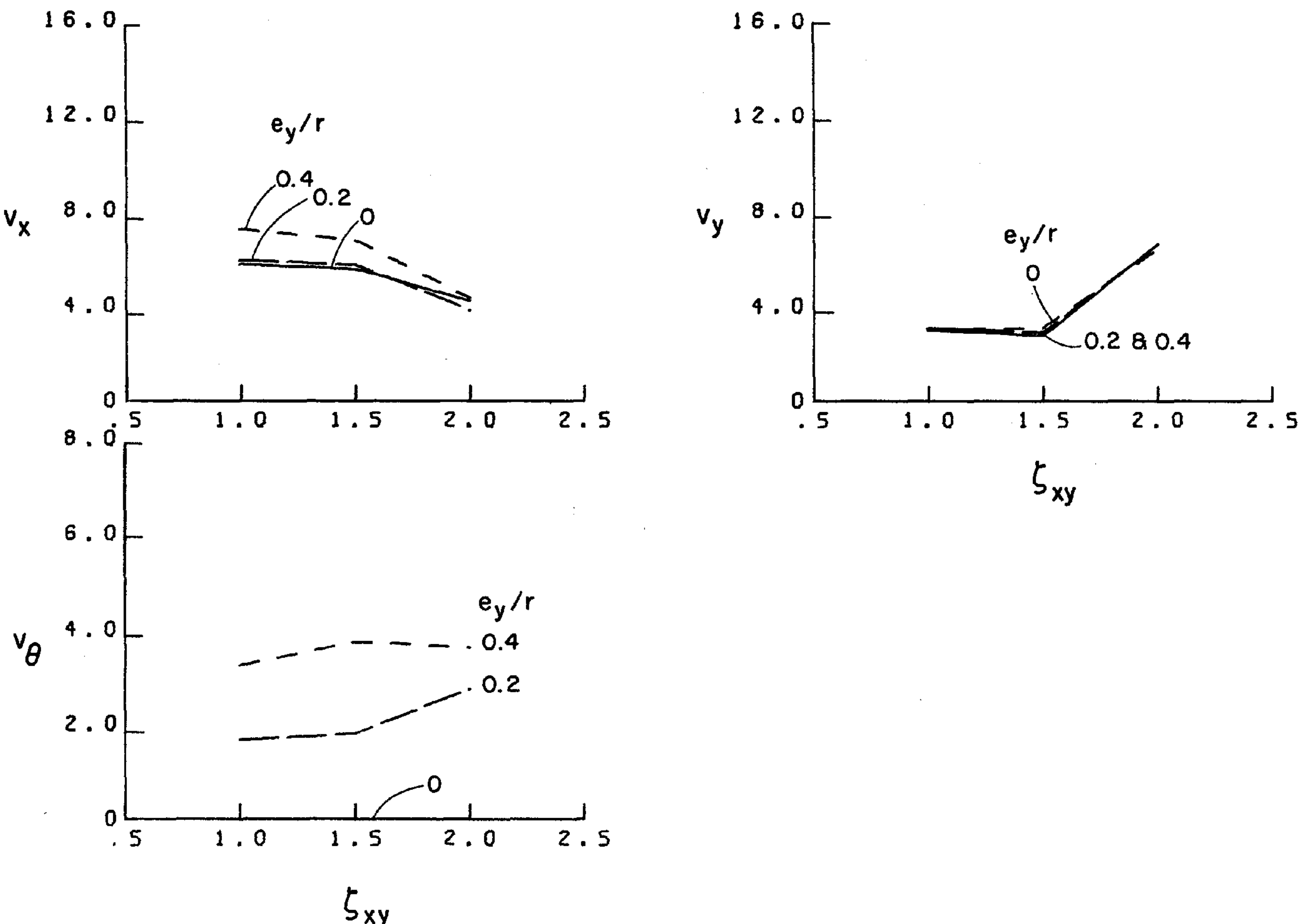


Fig. 3.25a Maximum Displacements at the Center of Mass of EPI-Systems Subjected to the El Centro Earthquake (Double-Component Input). System Parameters: ξ_i = 5%, T_x = 0.6 sec., e_x/r = 0 and c_0 = 0.06

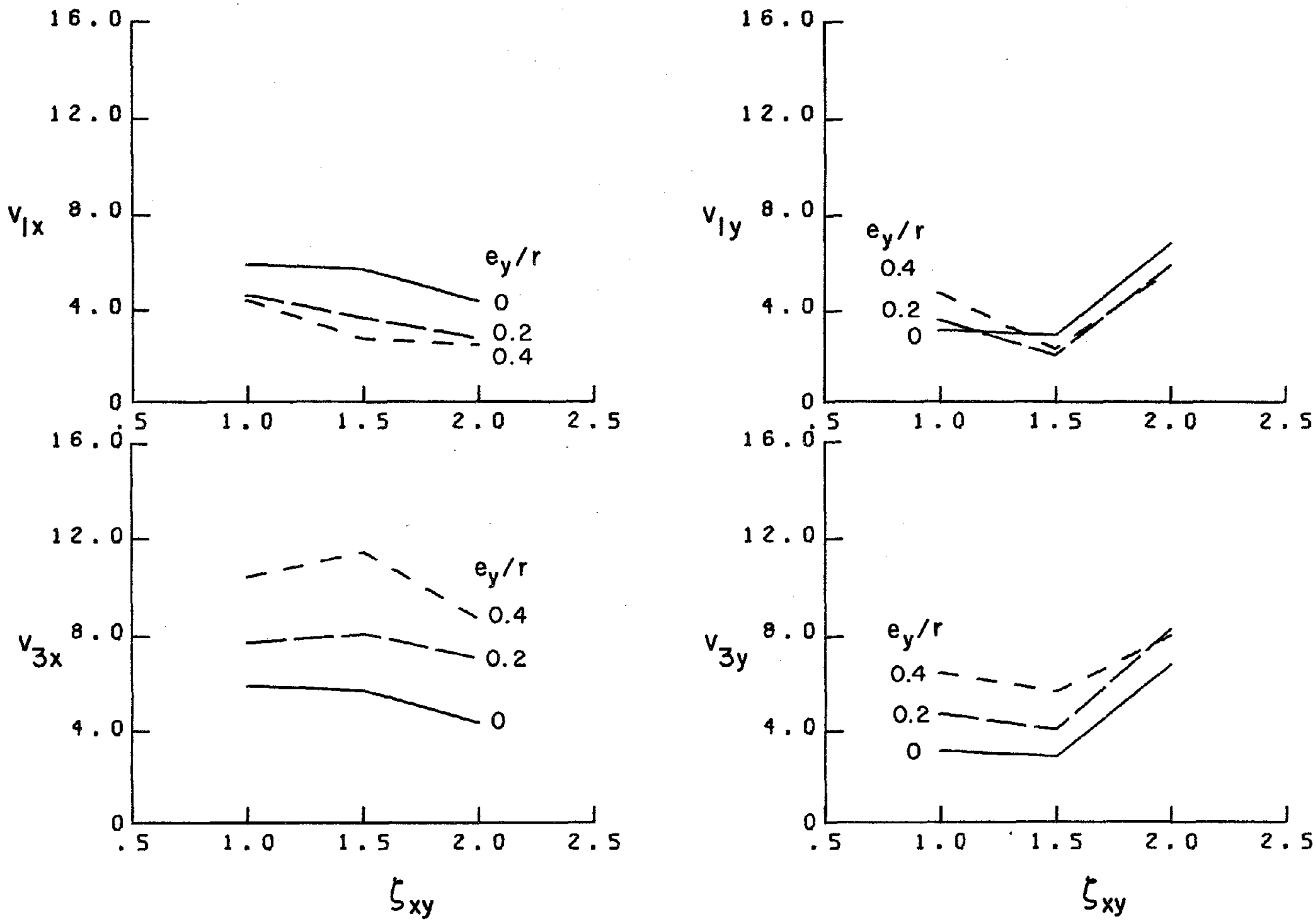


Fig. 3.25b Maximum Displacements (Ductility Factors) of Elements 1 and 3 of EPI-Systems Subjected to the El Centro Earthquake (Double-Component Input). System Parameters: $\xi_i = 5\%$, $T_x = 0.6$ sec., $e_x/r = 0$ and $c_0 = 0.06$

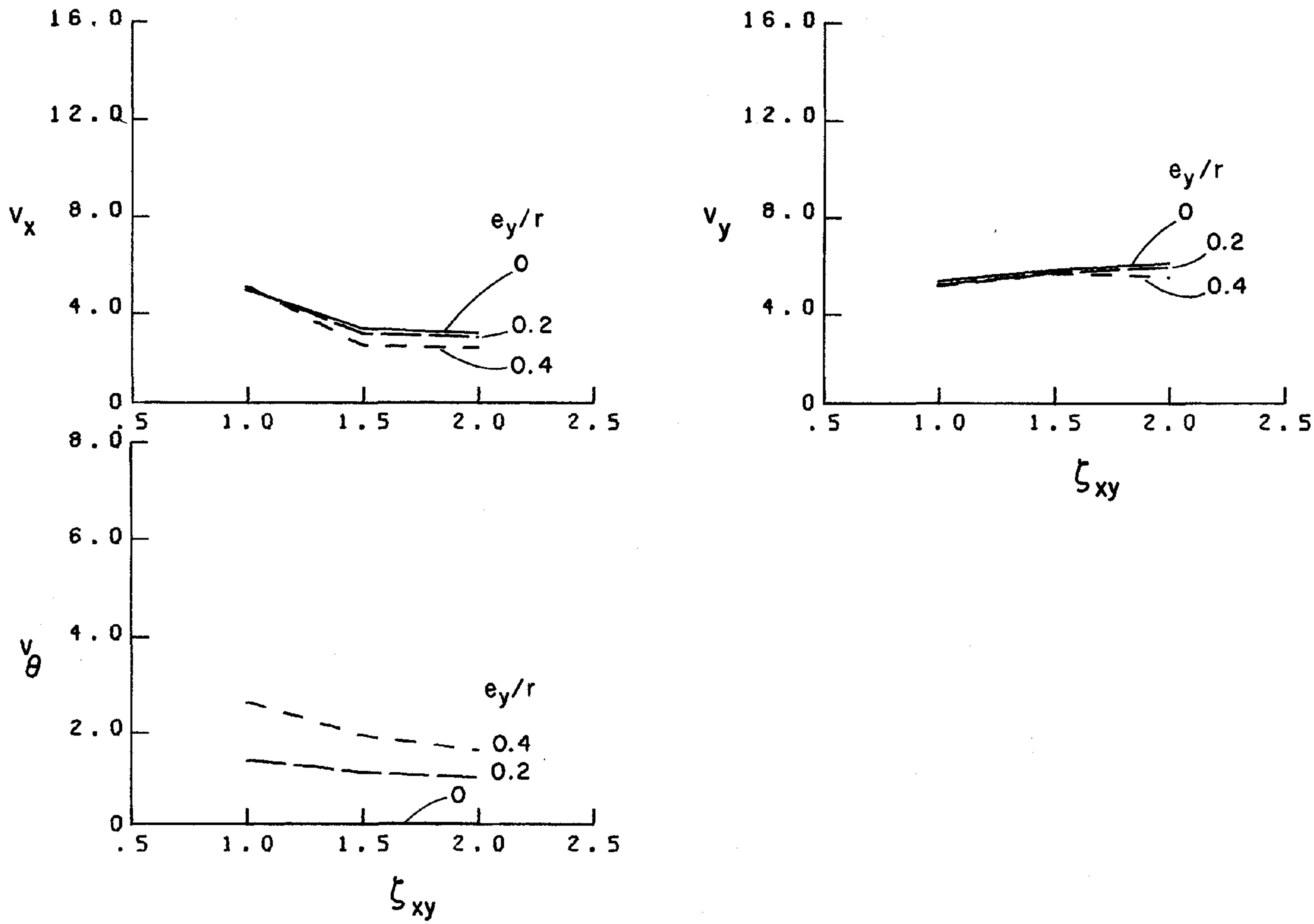


Fig. 3.26a Maximum Displacements at the Center of Mass of EPI-Systems Subjected to the El Centro Earthquake (Double-Component Input). System Parameters: ξ_i = 5%, T_x = 1.0 sec., e_x/r = 0 and c_0 = 0.06

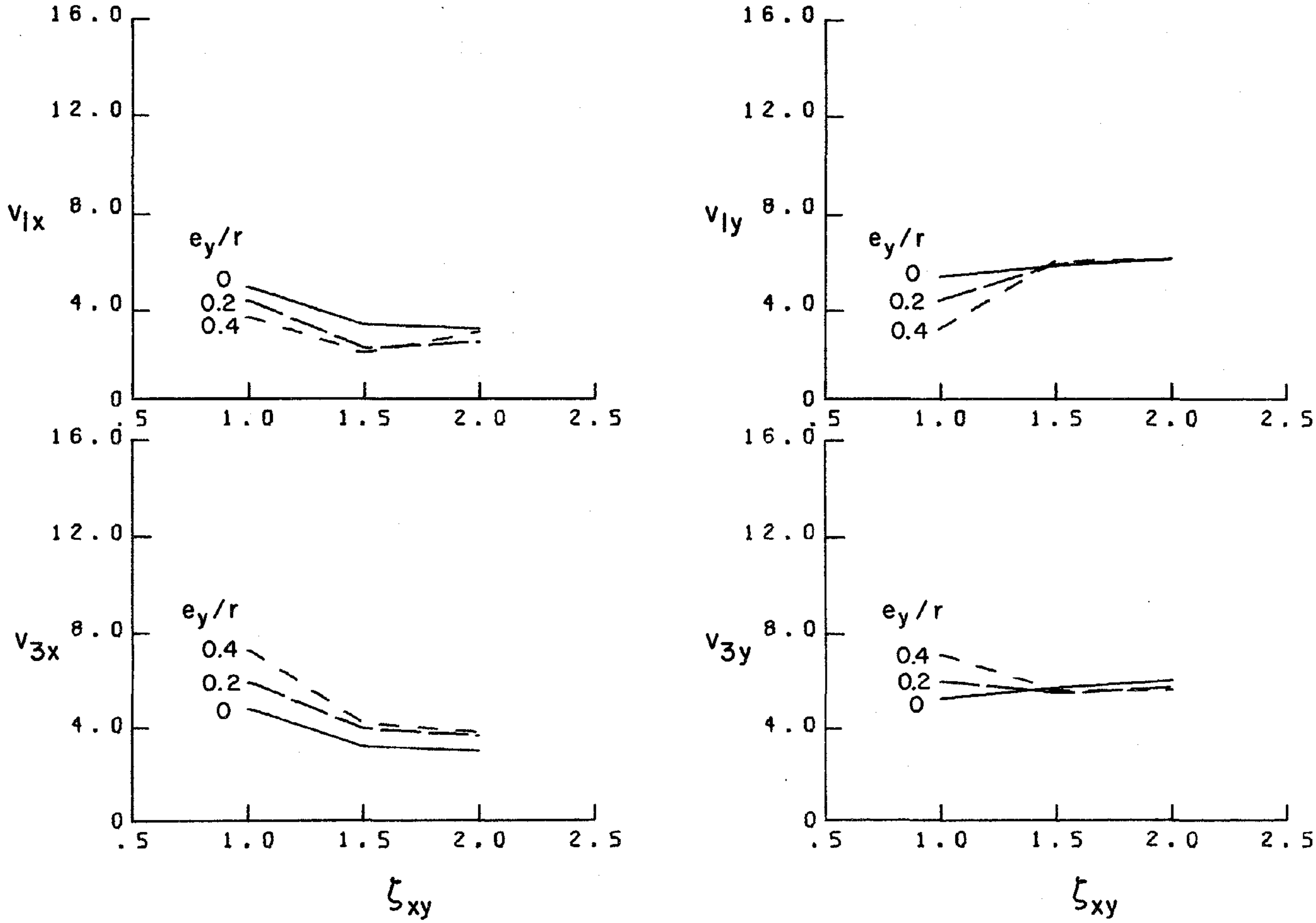


Fig. 3.26b Maximum Displacements (Ductility Factors) of Elements 1 and 3 of EPI-Systems Subjected to the El Centro Earthquake (Double-Component Input). System Parameters: $\xi_i = 5\%$, $T_x = 1.0$ sec., $e_x/r = 0$ and $c_0 = 0.06$

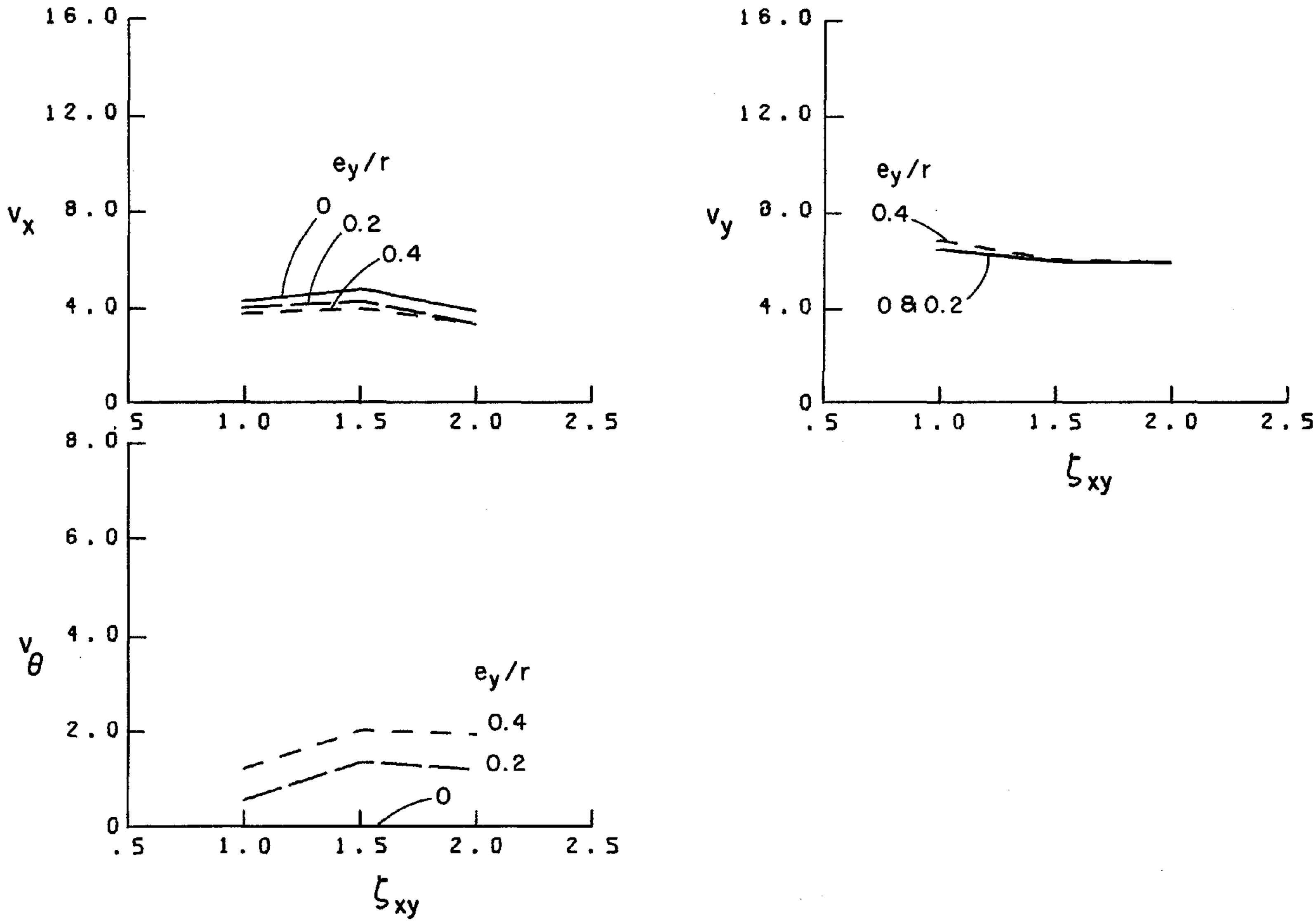


Fig. 3.27a Maximum Displacements at the Center of Mass of EPI-Systems Subjected to the El Centro Earthquake (Double-Component Input). System Parameters: $\xi_i = 5\%$, $T_x = 1.8$ sec., $e_x/r = 0$ and $c_0 = 0.06$

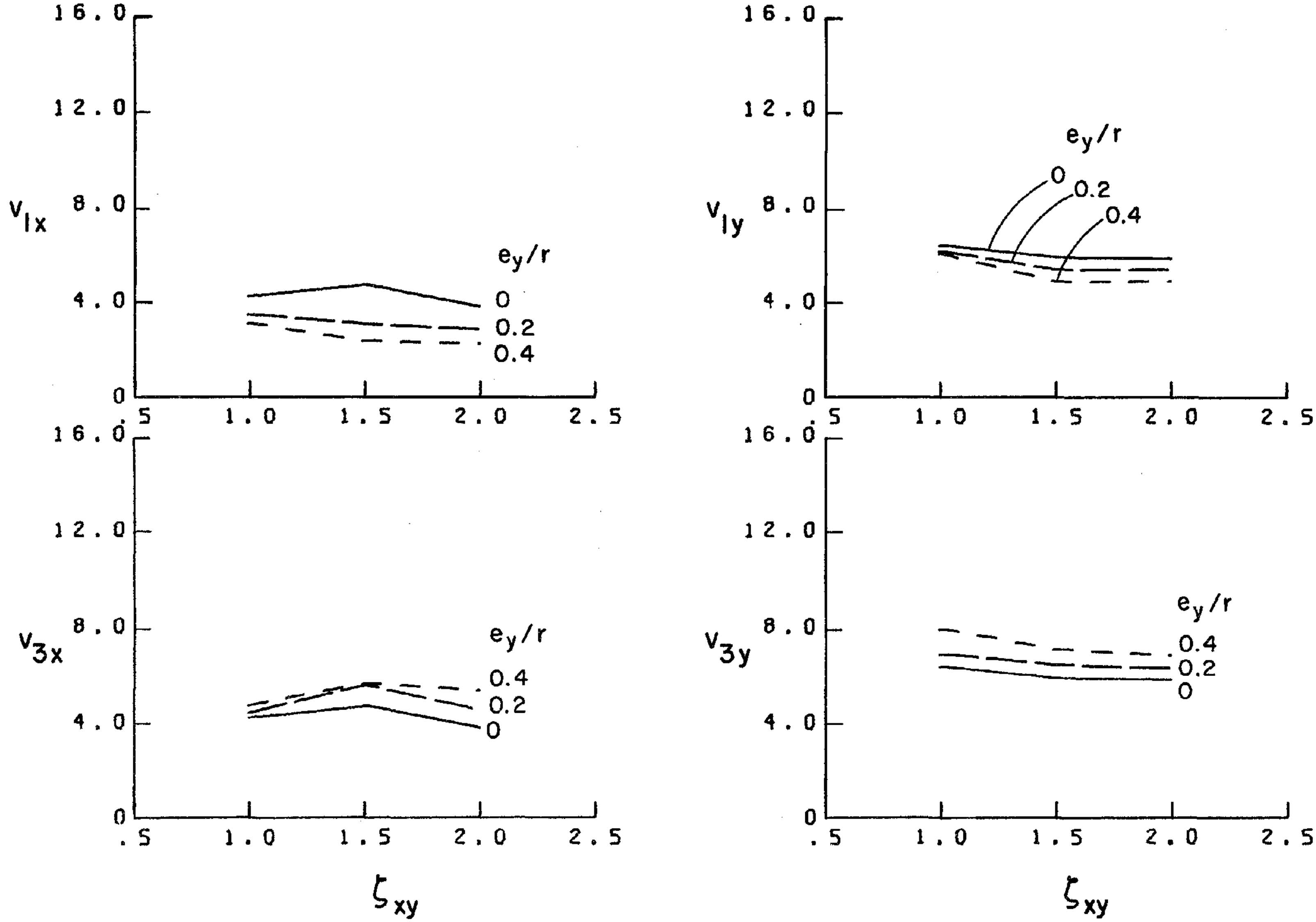


Fig. 3.27b Maximum Displacements (Ductility Factors) of Elements 1 and 3 of EPI-Systems Subjected to the El Centro Earthquake (Double-Component Input). System Parameters: ξ_i = 5%, T_x = 1.8 sec., e_x/r = 0 and c_0 = 0.06

dimensionless displacements at the center of mass of the EPI-systems in the y-direction, v_y, have been affected not only by force interaction, but also by changes in the value of the uncoupled natural period T_y.

3.5 Effects of Eccentricities

The maximum dimensionless displacement responses for EPI-systems with eccentricity in only the y-direction and with eccentricities in both the x- and y-directions are presented in Figs. 3.28 and 3.29, respectively. These results show that translational displacement responses at the center of mass of the EPI-systems, v_x and v_y, are insensitive to changes in the values of eccentricities, especially, for the systems having long uncoupled natural periods T_x. They also show that torsional response about the center of mass, v_θ, increases almost linearly with increasing values of the eccentricities. This means that translational displacement response at the center of mass is not related to the eccentricities of the system differing completely with torsional response which is directly related to the eccentricities of the system.

Torsional response about the center of mass, v_θ, for those systems with eccentricities in both the x- and y-directions produces larger displacements than for those systems with eccentricity in only the y-direction. However translational displacement responses at the center of mass, v_x and v_y, for both the systems having not large values of eccentricities give almost the same values in the x- and y-directions, respectively.

3.6 Effects of Yield Shear Forces

The maximum dimensionless responses for EPI-systems with different values of yield shear forces a_{x0} and a_{y0} but for fixed values of uncoupled frequency ratio ζ_{xy} are presented in Figs. 3.30 to 3.33 for

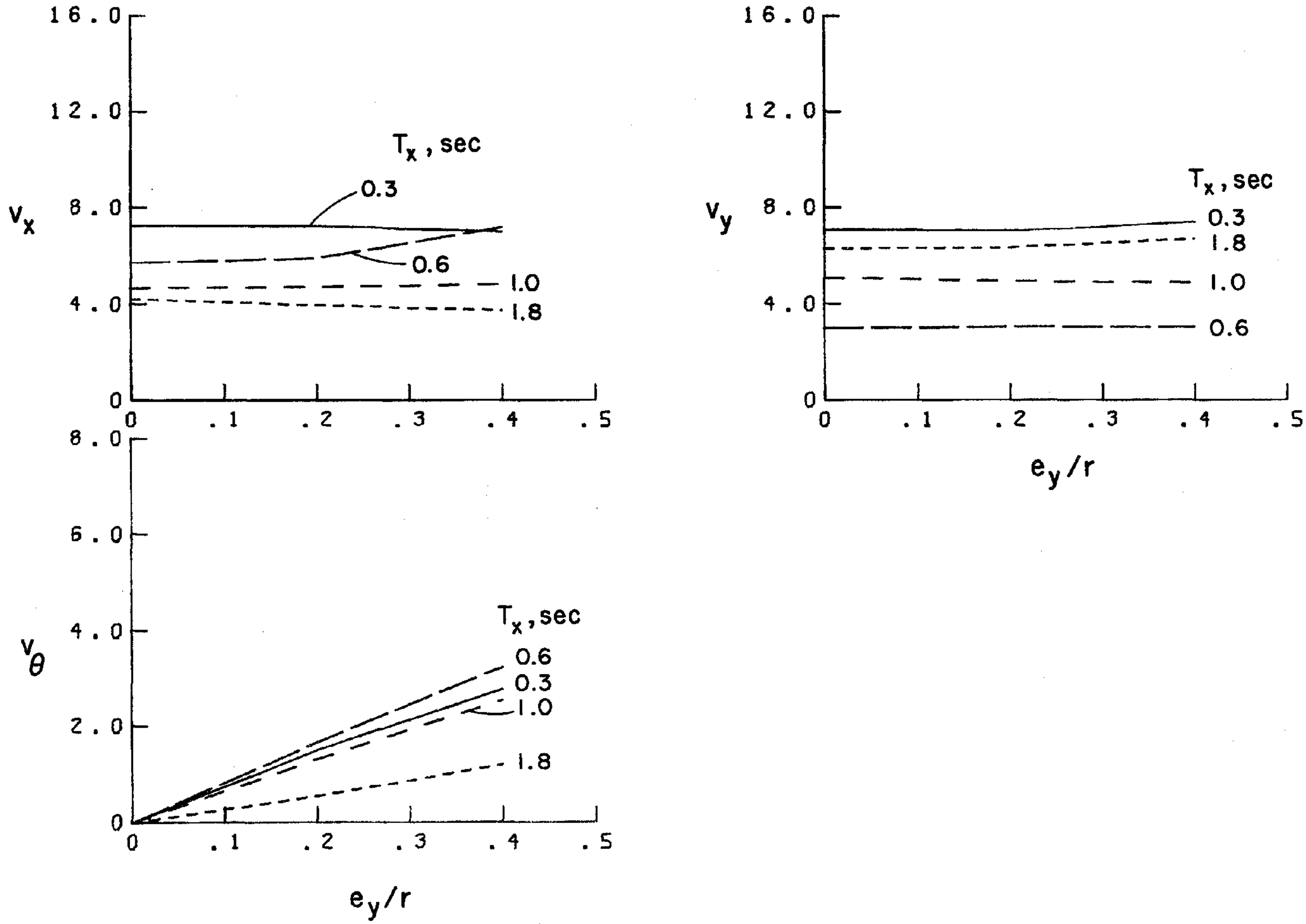


Fig. 3.28a Maximum Displacements at the Center of Mass of EPI-Systems Subjected to the El Centro Earthquake (Double-Component Input). System Parameters: $\xi_i = 5\%$, $\zeta_{xy} = 1.0$, $e_x/r = 0$ and $c_0 = 0.06$

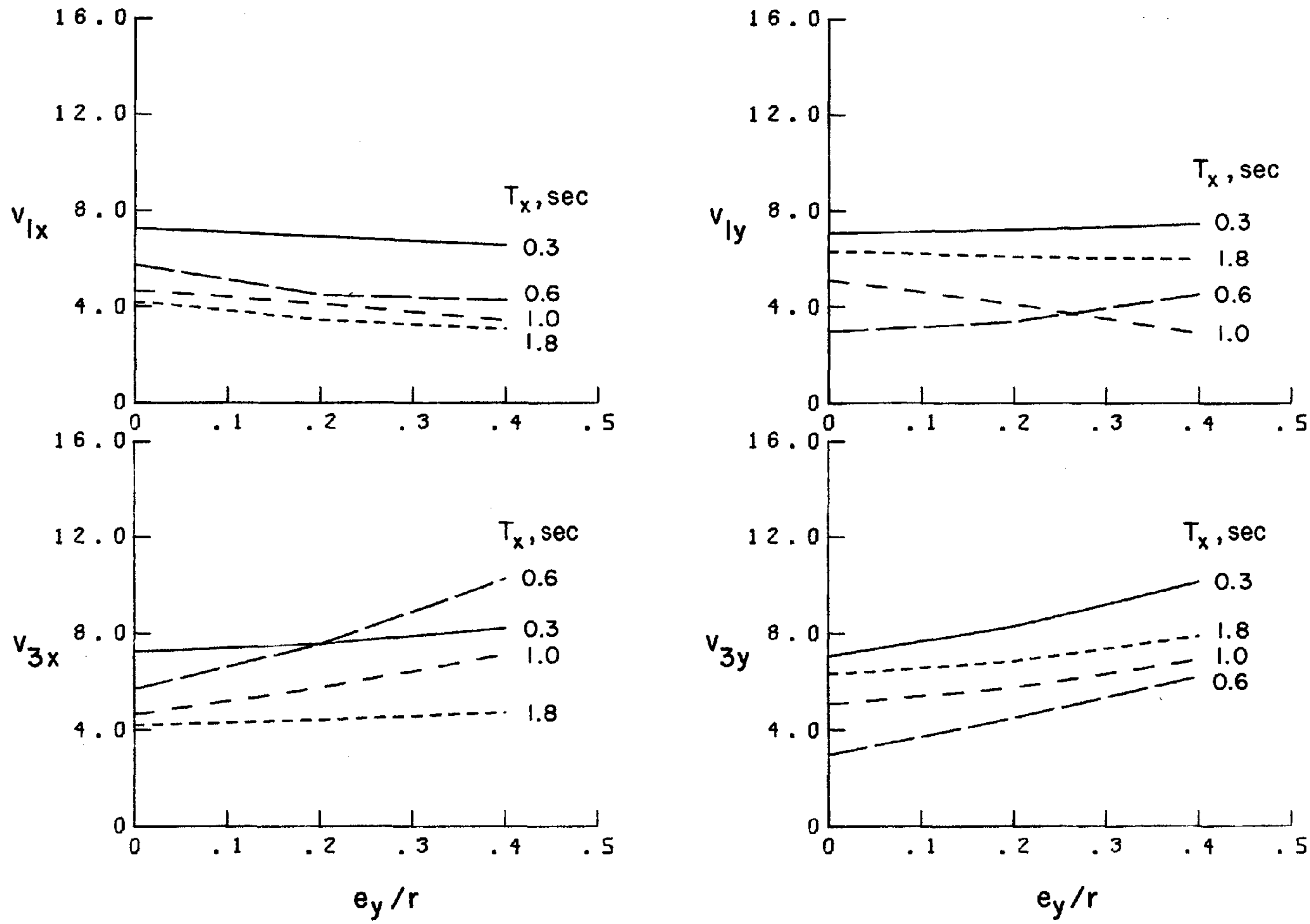


Fig. 3.28b Maximum Displacements (Ductility Factors) of Elements 1 and 3 of EPI-Systems Subjected to the El Centro Earthquake (Double-Component Input). System Parameters: ξ_i = 5%, ζ_{xy} = 1.0, e_x/r = 0 and c_0 = 0.06

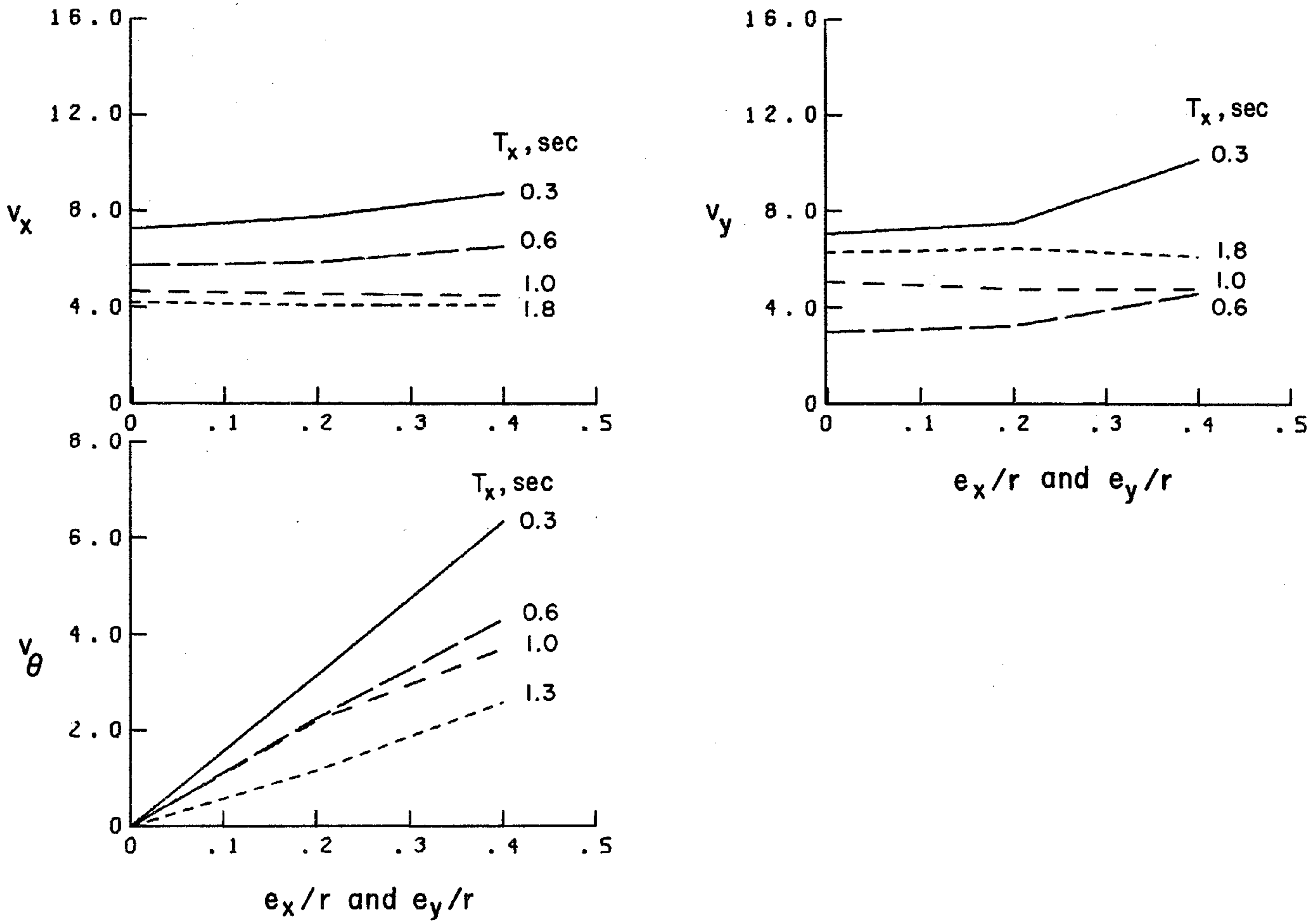


Fig. 3.29a Maximum Displacements at the Center of Mass of EPI-Systems Subjected to the El Centro Earthquake (Double-Component Input). System Parameters: $\xi_i = 5\%$, $\zeta_{xy} = 1.0$ and $c_0 = 0.06$

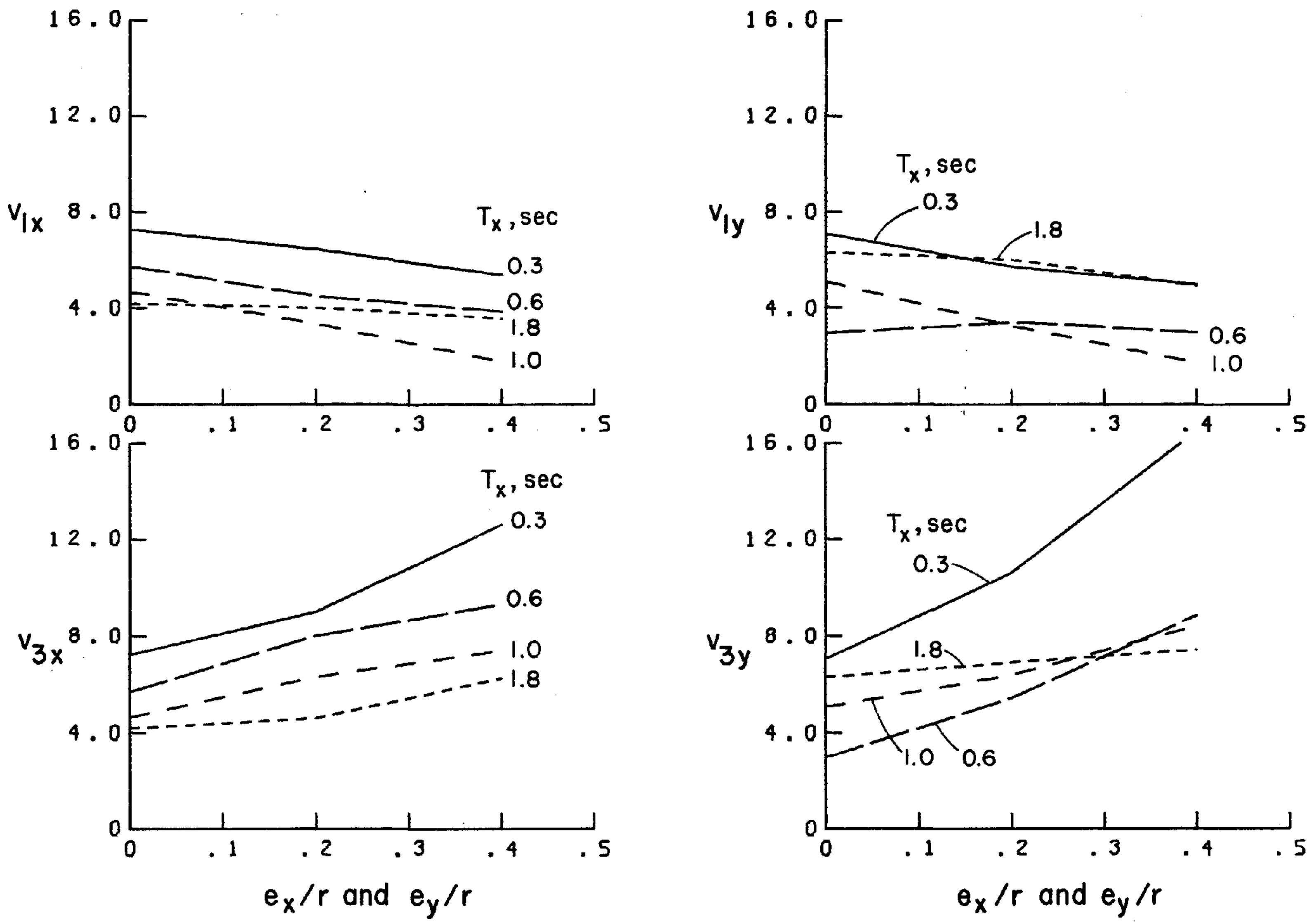


Fig. 3.29b Maximum Displacements (Ductility Factors) of Elements 1 and 3 of EPI-Systems Subjected to the El Centro Earthquake (Double-Component Input). System Parameters: ξ_i = 5%, ζ_{xy} = 1.0 and c_0 = 0.06

different values T_x. In these figures, c_0 is the parameter used to define yield shear forces a_{x0} and a_{y0} as given by Eq. 3.1. It is easily noted that the translational and torsional displacement responses decrease with increasing values of the yield shear forces (Figs. 3.30 to 3.33).

The maximum dimensionless displacement responses of the resisting elements of the EPI-systems for small values of parameter c_0, e.g. c_0 = 0.06, fluctuate with changing values of eccentricity. This fluctuation decreases however with increasing values of c_0 (Figs. 3.30b, 3.31b, e.32b and 3.33b). This means that excessive torsional response due to eccentricities can be controlled by increasing the yield shear forces appropriately.

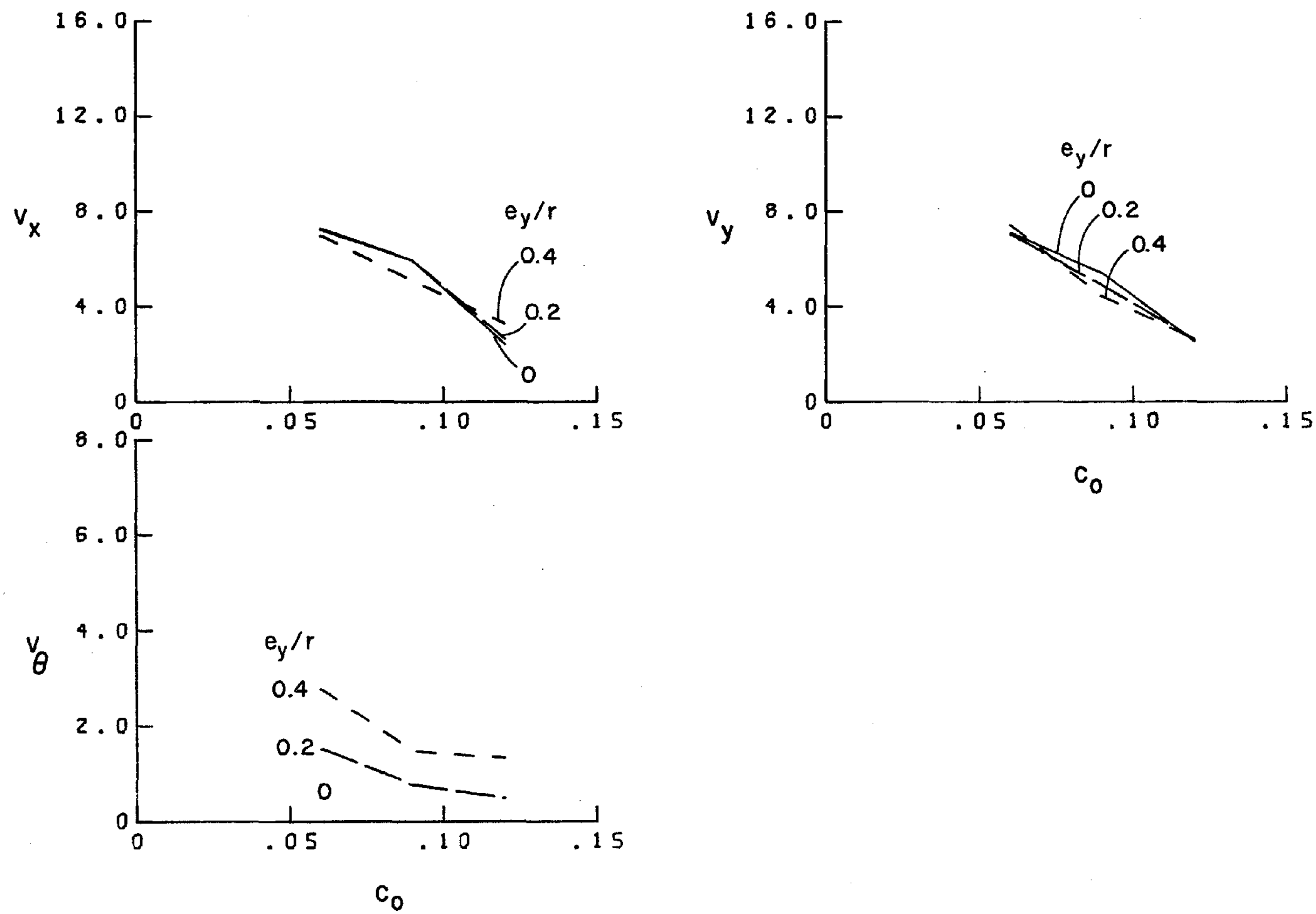


Fig. 3.30a Maximum Displacements at the Center of Mass of EPI-Systems Subjected to the El Centro Earthquake (Double-Component Input). System Parameters: ξ_1 = 5%, T_x = 0.3 sec. and ζ_{xy} = 1.0

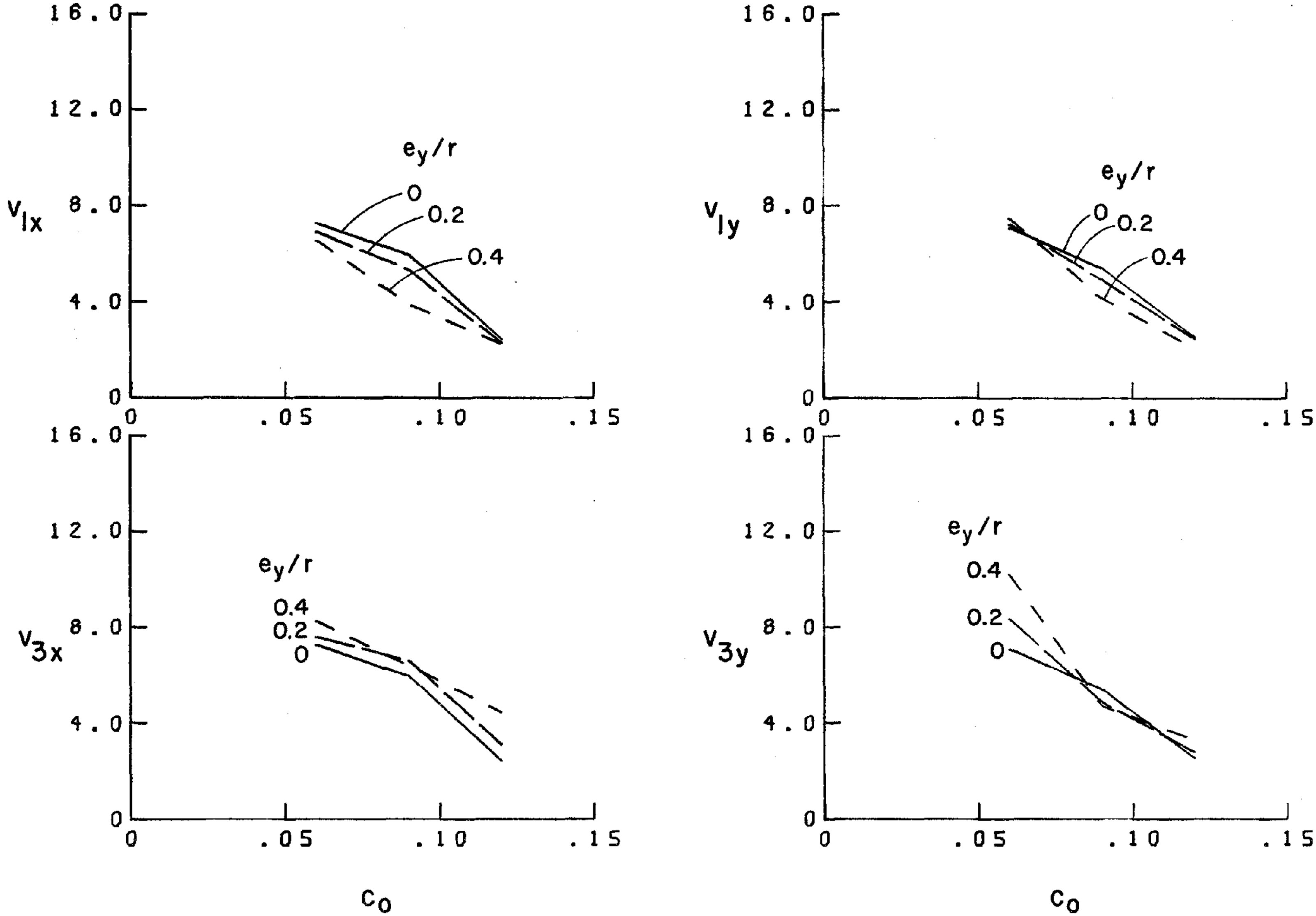


Fig. 3.30b Maximum Displacements (Ductility Factors) of Elements 1 and 3 of EPI-Systems Subjected to the El Centro Earthquake (Double-Component Input). System Parameters: $\xi_i = 5\%$, $T_x = 0.3$ sec. and $\zeta_{xy} = 1.0$

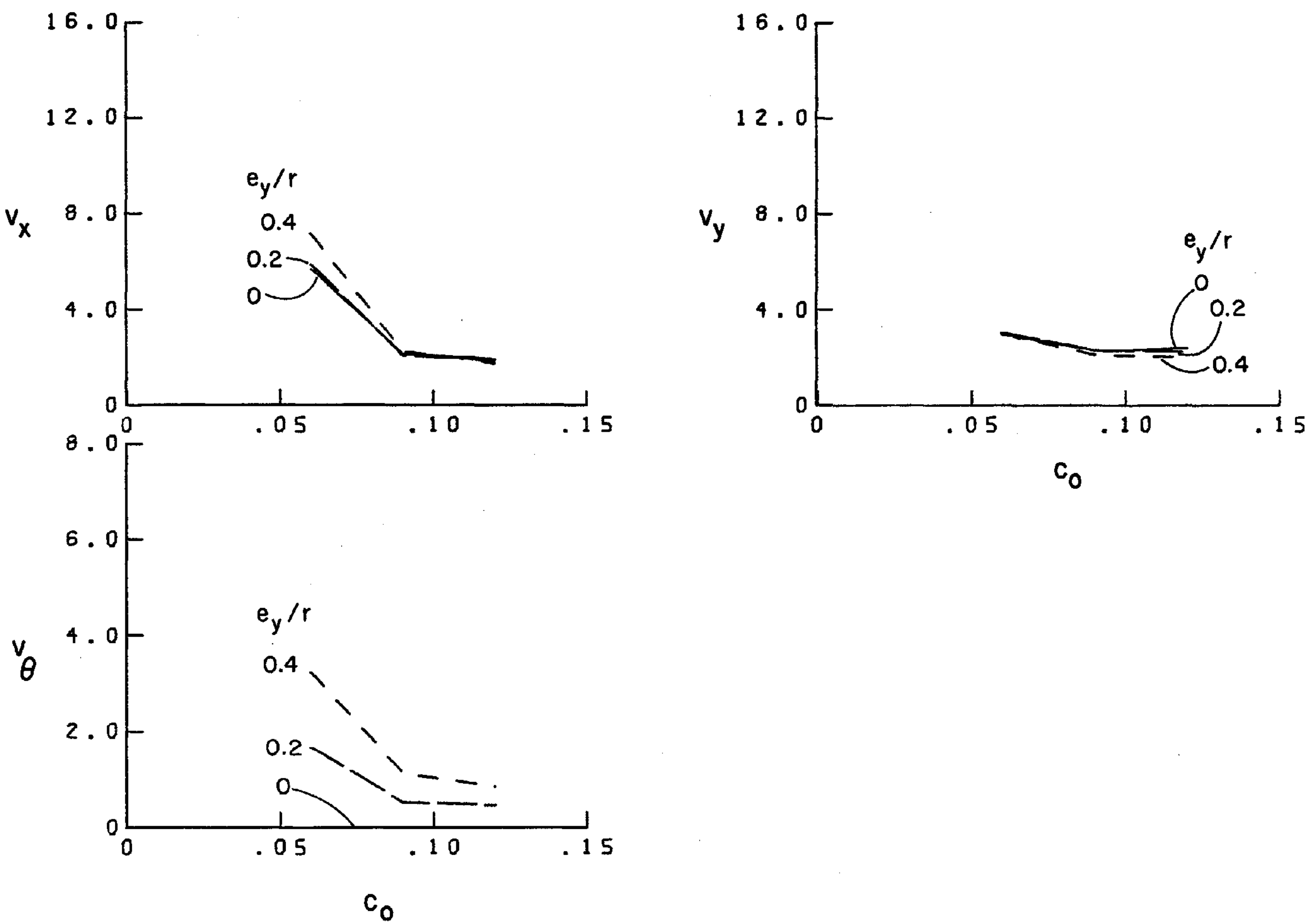


Fig. 3.31a Maximum Displacements at the Center of Mass of EPI-Systems Subjected to the El Centro Earthquake (Double-Component Input). System Parameters: ξ_i = 5%, T_x = 0.6 sec. and ζ_{xy} = 1.0

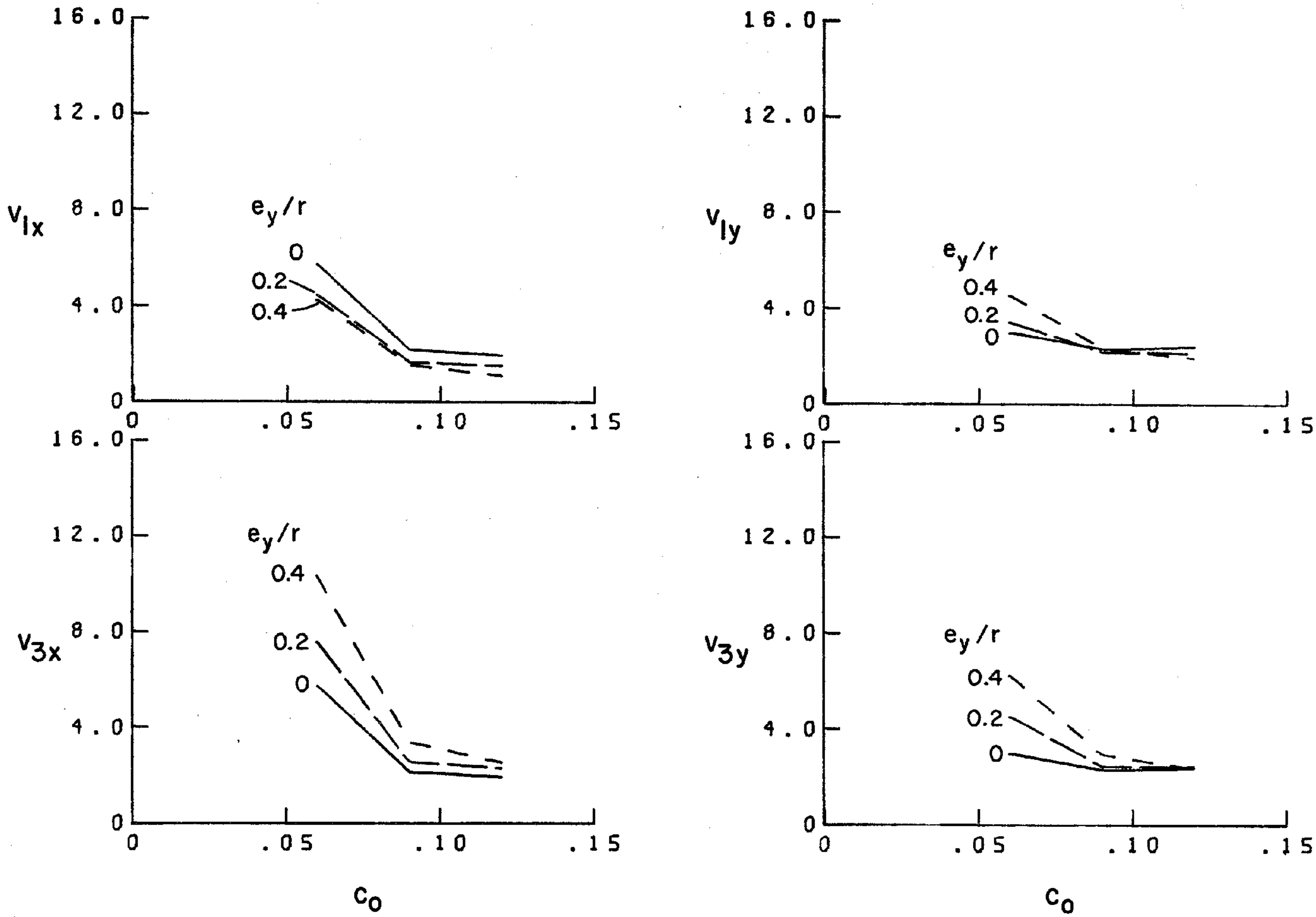


Fig. 3.31b Maximum Displacements (Ductility Factors) of Elements 1 and 3 of EPI-Systems Subjected to the El Centro Earthquake (Double-Component Input). System Parameters: $\xi_i = 5\%$, $T_x = 0.6$ sec. and $\zeta_{xy} = 1.0$

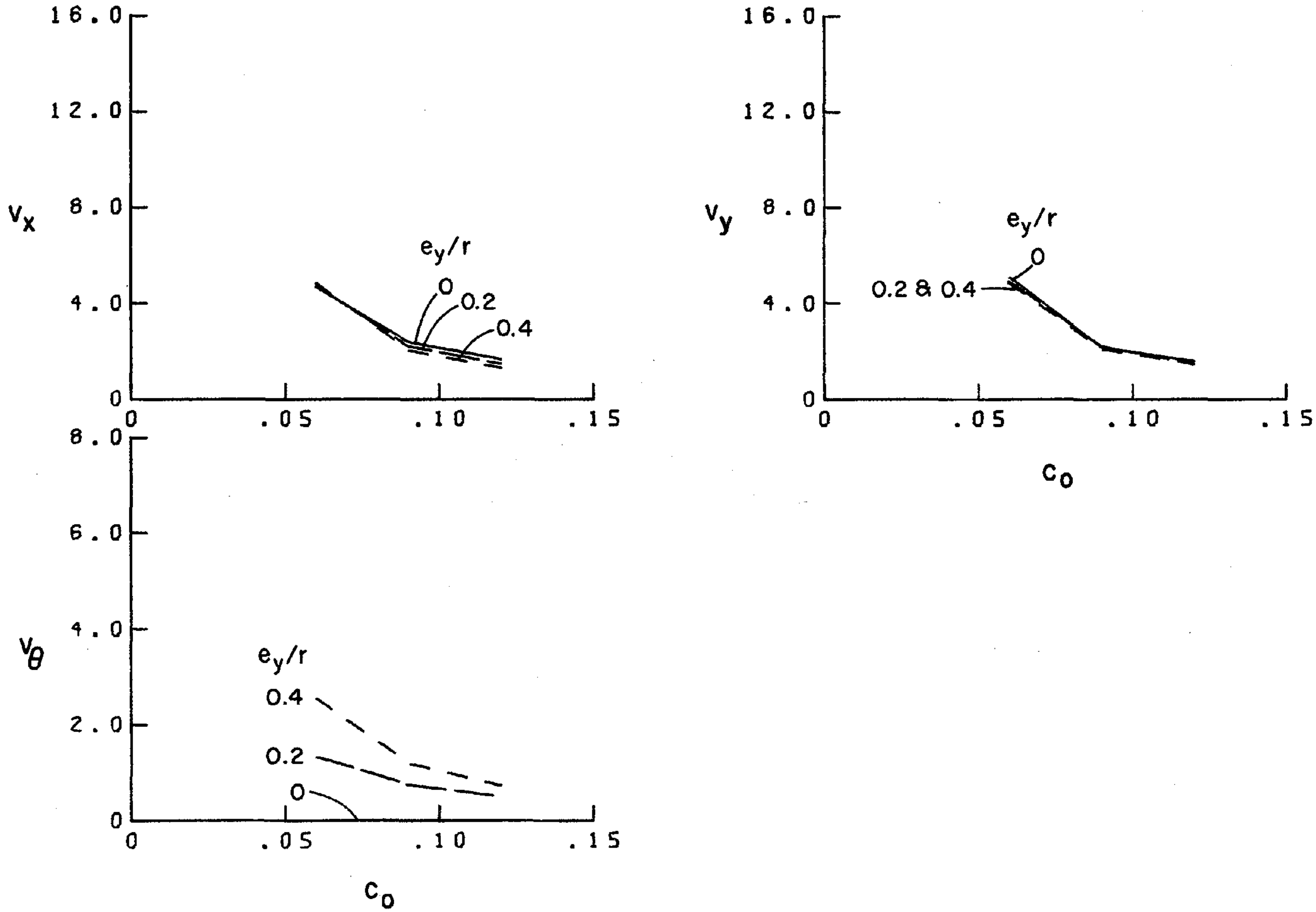


Fig. 3.32a Maximum Displacements at the Center of Mass of EPI-Systems Subjected to the El Centro Earthquake (Double-Component Input). System Parameters: ξ_i = 5%, T_x = 1.0 sec. and ζ_{xy} = 1.0

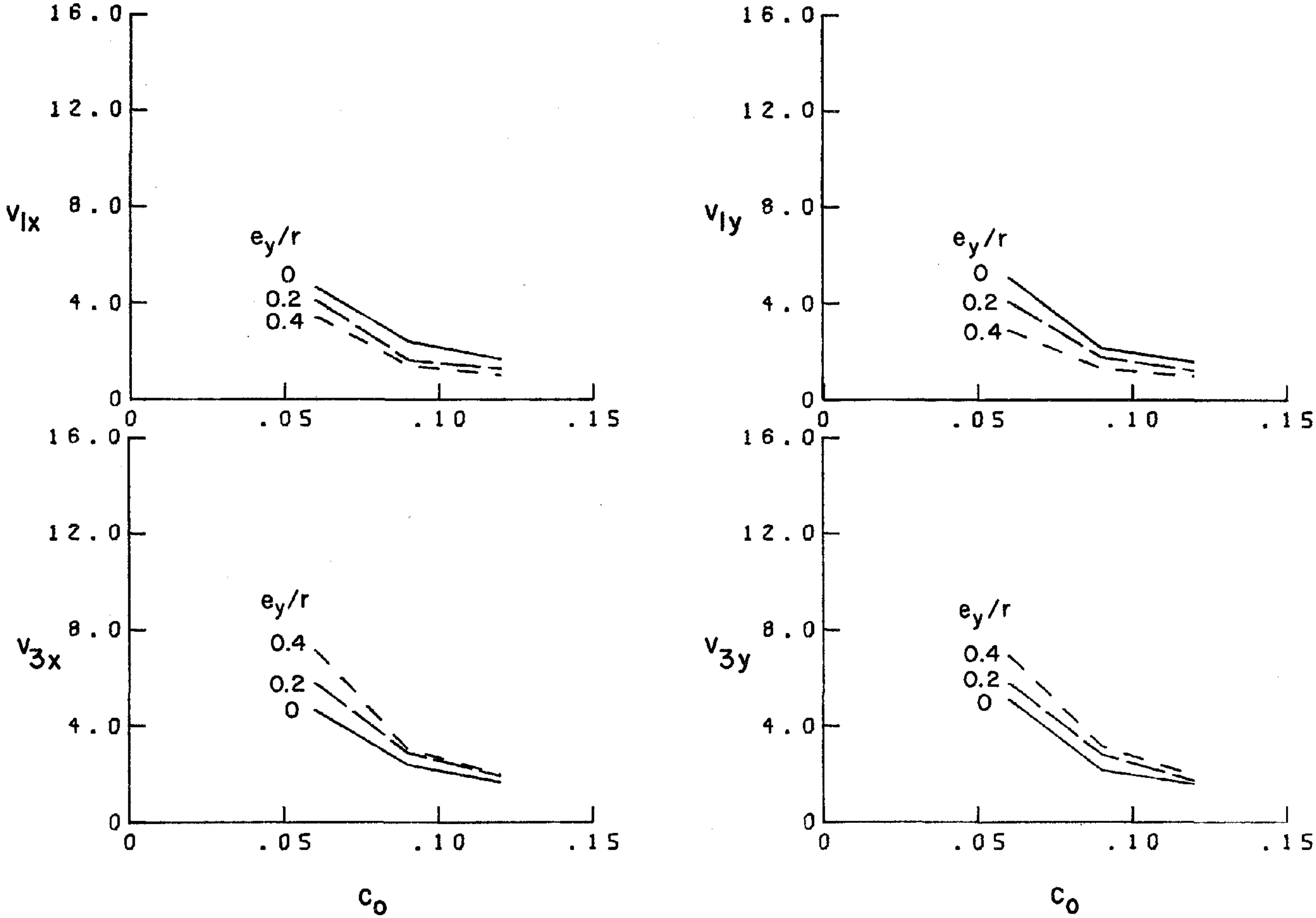


Fig. 3.32b Maximum Displacements (Ductility Factors) of Elements 1 and 3 of EPI-Systems Subjected to the El Centro Earthquake (Double-Component Input). System Parameters: $\xi_i = 5\%$, $T_x = 1.0$ sec. and $\zeta_{xy} = 1.0$

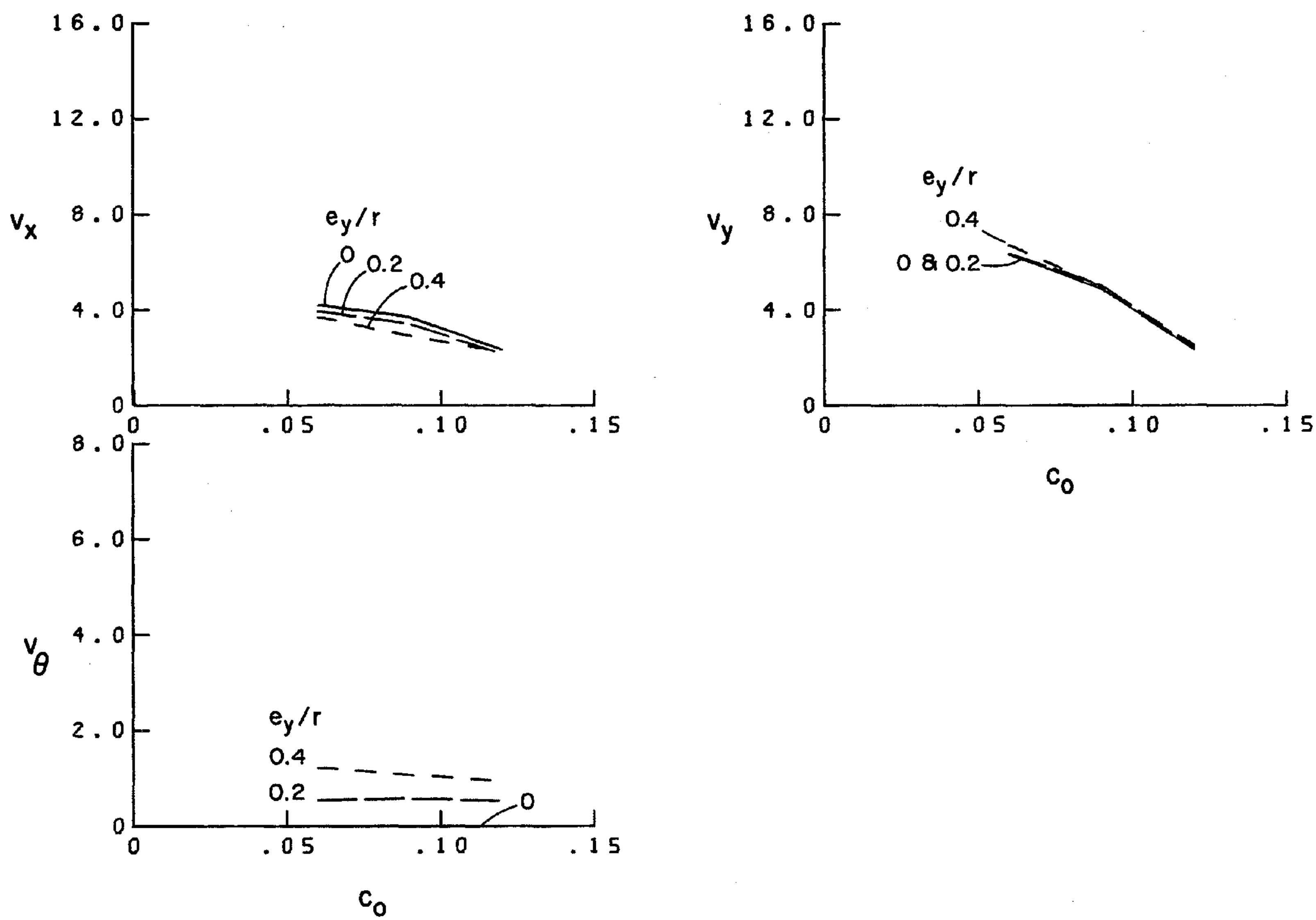


Fig. 3.33a Maximum Displacements at the Center of Mass of EPI-Systems Subjected to the El Centro Earthquake (Double-Component Input). System Parameters: $\xi_i = 5\%$, $T_x = 1.8$ sec. and $\zeta_{xy} = 1.0$

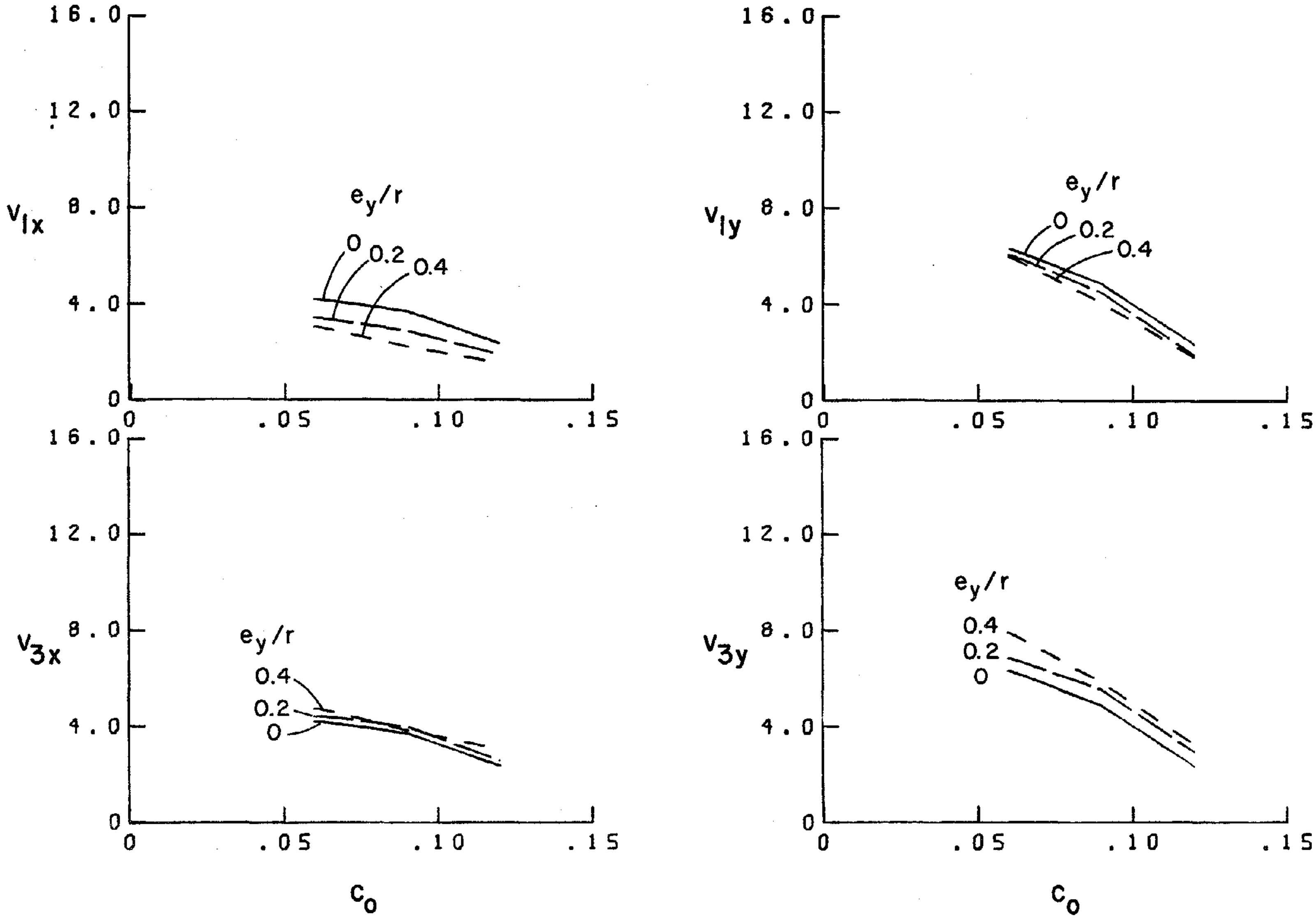


Fig. 3.33b Maximum Displacements (Ductility Factors) of Elements 1 and 3 of EPI-Systems Subjected to the El Centro Earthquake (Double-Component Input). System Parameters: $\xi_i = 5\%$, $T_x = 1.8$ sec. and $\zeta_{xy} = 1.0$

4. CONCLUSIONS

The principal conclusions of this study concerning the coupled translational (x and y) - torisonal (θ) response of single-story force-interacting elasto-plastic (EPI) systems subjected to earthquake ground motions are the following:

1. The coupled translational-torsional systems show different spectral response variations with frequency ratio ζ_{xy} and with eccentricities e_x/r and e_y/r for the different types of earthquake ground motions used.
2. The effects of force interaction on the response of EPI-systems to single-component ground motion are much less than the corresponding effects on the response of similar systems subjected to double-component ground motion.
3. The response spectra for EPI-systems are generally smooth functions of natural period T_x which is in contrast with the response spectra for E- and EP-systems which are quite irregular.
4. Force interaction effects do not always produce larger maximum displacements.
5. Force interaction effects on inelastic response have a tendency to balance the two components of displacement response for those cases involving large plastic deformations, especially for systems having long uncoupled natural periods T_x.
6. Translational displacement responses at the center of mass of the EPI-systems, v_x and v_y, are insensitive to changes in the values of eccentricities, especially, for those systems having long uncoupled natural periods T_x; however, torsional response

about the center of mass, v_θ, increases almost linearly with increasing values of the eccentricities.

7. Torsional response about the center of mass, v_θ, for those systems with eccentricities in both the x- and y-directions produces larger displacements than for those systems with eccentricity in only the y-direction. However translational displacement responses at the center of mass, v_x and v_y, for both the systems having not large values of eccentricities give almost the same values in the x- and y-directions, respectively.

8. Excessive torsional response due to eccentricities can be controlled by increasing the yield shear forces appropriately.

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APPENDIX I

YIELDING IN ELEMENTS

Fundamental relations describing the interaction between forces acting on sections of elements during yielding are described herein [34].

The internal work of a linear elastic element can be expressed in the form

$$W = \frac{1}{2} \langle \bar{Q} \, , \, \bar{q} \rangle \tag{I.1}$$

in which $\bar{Q}$ and $\bar{q}$ represent generalized n-dimensional force and displacement vectors, respectively, and the symbol $\langle , \rangle$ denotes inner product of two vectors. Vector $\bar{Q}$ in a linear elastic element can be written as

$$\bar{Q} = \bar{\bar{S}} \, \bar{q} \tag{I.2}$$

in which $\bar{\bar{S}}$ is a constant stiffness matrix of the element. The limit yield surface for a perfectly elasto-plastic system, a closed surface enclosing the origin, can be defined through a scalar function of the generalized forces of the form

$$\Phi(\bar{Q}) = 1 \tag{I.3}$$

Postulating stable inelastic material [11] and assuming that the coordinate axes of the generalized forces $\bar{Q}$ and the displacement increments $\Delta\bar{q}$ coincide, the yield surface will be convex and the plastic displacement vector $\Delta\bar{q}^p$ will lie along the outer normal to the yield surface at a regular point. Hence, the normal to the surface will be in the direction of the gradient and

$$\Delta\bar{q}^p = \lambda \frac{\partial \Phi}{\partial \bar{Q}} \tag{I.4}$$

where λ is a positive scalar.

During yielding the force vector $\bar{Q}$ moves on the yield surface. In plasticity theory this is called "loading" and is characterized by the relation

$$\Phi(\bar{Q}) = 1$$

and

$$d\Phi = 0$$

The change from plastic behavior to elastic behavior occurs if

$$\Phi(\bar{Q}) = 1$$

and

$$d\Phi < 0$$

This is called "unloading". The work done during an increment of yielding is given by

$$dW^p = \left\langle \bar{Q} \, , \, \Delta\bar{q}^p \right\rangle \tag{I.5}$$

It follows from Eqs. I.4 and I.5 that during loading

$$\Phi(\bar{Q}) = 1$$

and

$$dW^p \geq 0$$

since the yield surface encloses the origin. Unloading must occur when

$$\Phi(\bar{Q}) = 1$$

and

$$dW^p < 0$$

The criteria for elastic and inelastic behavior at a section can now be expressed as the section is linearly elastic for

$$\Phi(\bar{Q}) < 1$$

or

$$\left.\begin{aligned} (\bar{Q}) &= 1 \\ dW^{p} &< 0 \end{aligned}\right\} \text{(Unloading)}$$

and the section is yielding for

$$\left.\begin{aligned} \Phi(\bar{Q}) &= 1 \\ dW^{p} &\geq 0 \end{aligned}\right\} \text{(Loading)}$$

and

$$\Phi(\bar{Q}) \ngtr 1$$

The displacement increments can be decomposed into elastic and plastic parts, $\Delta\bar{q}^{e}$ and $\Delta\bar{q}^{p}$, respectively, so that

$$\Delta\bar{q} = \Delta\bar{q}^{e} + \Delta\bar{q}^{p}. \tag{I.6}$$

During yielding the tip of the force vector moves on the yield surface and the incremental generalized force vector $\Delta\bar{Q}$ is related to the elastic part of the incremental displacement vector by the relation

$$\Delta\bar{Q} = \bar{S}\ \Delta\bar{q}^{e} \tag{I.7}$$

Since the plastic incremental displacement vector $\Delta\bar{q}^{p}$ is normal to the yield surface (Eq. I.4) and the force vector moves on the yield surface during yielding, incremental vectors $\Delta\bar{Q}$ and $\Delta\bar{q}^{p}$ must be orthogonal, i.e.

$$\left\langle \Delta\bar{Q}\ ,\ \Delta\bar{q}^{p} \right\rangle = 0 \tag{I.8}$$

Substitution of Eqs. I.6 and I.7 into Eq. I.8 finally gives an equation for the positive scalar λ, that is

$$\lambda = \frac{\left\langle \bar{S}\ \Delta\bar{q}\ ,\ \frac{\partial\Phi}{\partial\bar{Q}} \right\rangle}{\left\langle \bar{S}\ \frac{\partial\Phi}{\partial\bar{Q}}\ ,\ \frac{\partial\Phi}{\partial\bar{Q}} \right\rangle} \tag{I.9}$$

The plastic incremental displacement vector $\Delta\bar{q}^{p}$ given by Eq. I.4 can now be represented as

$$\Delta\bar{q}^{p} = \frac{\left\langle \bar{S}\ \Delta\bar{q}\ ,\ \frac{\partial\Phi}{\partial\bar{Q}} \right\rangle}{\left\langle \bar{S}\ \frac{\partial\Phi}{\partial\bar{Q}}\ ,\ \frac{\partial\Phi}{\partial\bar{Q}} \right\rangle} \cdot \frac{\partial\Phi}{\partial\bar{Q}} \tag{I.10}$$

and the elastic incremental displacement vector $\Delta\bar{q}^{e}$ becomes

$$\Delta\bar{q}^{e} = \Delta\bar{q} - \frac{\left\langle \bar{S}\ \Delta\bar{q}\ ,\ \frac{\partial\Phi}{\partial\bar{Q}} \right\rangle}{\left\langle \bar{S}\ \frac{\partial\Phi}{\partial\bar{Q}}\ ,\ \frac{\partial\Phi}{\partial\bar{Q}} \right\rangle} \cdot \frac{\partial\Phi}{\partial\bar{Q}} \tag{I.11}$$

The incremental generalized force vector $\Delta\bar{Q}$ given by Eq. I.7 can also be expressed in the form

$$\Delta\bar{Q} = \bar{S} \left\{ \Delta\bar{q} - \frac{\left\langle \bar{S}\ \Delta\bar{q}\ ,\ \frac{\partial\Phi}{\partial\bar{Q}} \right\rangle}{\left\langle \bar{S}\ \frac{\partial\Phi}{\partial\bar{Q}}\ ,\ \frac{\partial\Phi}{\partial\bar{Q}} \right\rangle} \cdot \frac{\partial\Phi}{\partial\bar{Q}} \right\} \tag{I.12}$$

Dividing Eq. I.12 by time increment Δt and taking the limit as $\Delta t \to 0$, there results

$$\dot{\bar{Q}} = \bar{S} \left\{ \dot{\bar{q}} - \frac{\left\langle \bar{S}\ \dot{\bar{q}}\ ,\ \frac{\partial\Phi}{\partial\bar{Q}} \right\rangle}{\left\langle \bar{S}\ \frac{\partial\Phi}{\partial\bar{Q}}\ ,\ \frac{\partial\Phi}{\partial\bar{Q}} \right\rangle} \cdot \frac{\partial\Phi}{\partial\bar{Q}} \right\} \tag{I.13}$$

where the dot shown above quantities $\bar{Q}$ and $\bar{q}$ denotes differentiation with respect to time, i.e., d/dt. This equation defines the force-displacement relationship at a section when yielding is taking place. The corresponding relationship at a section under the most general condition of loading can be written as

$$\dot{\bar{Q}} = \bar{S}\,\dot{\bar{q}} \tag{I.14}$$

for

$$\Phi(\bar{Q}) < 1$$

or

$$\left.\begin{array}{r} \Phi(\bar{Q}) = 1 \\ \dot{W}^p < 0 \end{array}\right\} \quad \text{(Unloading)}$$

and

$$\dot{\bar{Q}} = \bar{S}\left\{\dot{\bar{q}} - \frac{\left\langle \bar{S}\,\dot{\bar{q}}\,,\ \dfrac{\partial\Phi}{\partial\bar{Q}}\right\rangle}{\left\langle \bar{S}\,\dfrac{\partial\Phi}{\partial\bar{Q}}\,,\ \dfrac{\partial\Phi}{\partial\bar{Q}}\right\rangle}\cdot\frac{\partial\Phi}{\partial\bar{Q}}\right\} \tag{I.15}$$

for

$$\left.\begin{array}{r} \Phi(\bar{Q}) = 1 \\ \dot{W}^p \geq 0 \end{array}\right\} \quad \text{(Loading)}$$

where the rate of plastic work for an element, $\dot{W}^p$, is obtained from Eq. I.5, i.e.

$$\dot{W}^p = \left\langle \bar{Q}\,,\ \dot{\bar{q}}^p \right\rangle \tag{I.16}$$

in which the plastic part of the velocity vector, $\dot{\bar{q}}^p$, is obtained by dividing Eq. I.10 by Δt and taking the limit as $\Delta t \to 0$; thus one obtains

$$\dot{\bar{q}}^{p} = \frac{\left\langle \bar{\bar{S}}\,\dot{\bar{q}}\ ,\ \frac{\partial\Phi}{\partial\bar{Q}} \right\rangle}{\left\langle \bar{\bar{S}}\frac{\partial\Phi}{\partial\bar{Q}}\ ,\ \frac{\partial\Phi}{\partial\bar{Q}} \right\rangle} \cdot \frac{\partial\Phi}{\partial\bar{Q}} \qquad \text{(I.17)}$$

APPENDIX II

DIMENSIONLESS EQUATIONS OF MOTION

It is desirable to transform the equations of motion, Eq. 2.23, into dimensionless form. Since the inverse of stiffness matrix $\underline{\Lambda}$ defined by Eq. 2.17 can be represented as

$$\underline{\Lambda}^{-1} = \frac{1}{\omega_x^2} \left(\underline{\sigma}^2 - \frac{1}{\eta} \underline{\varepsilon}\, \underline{\iota}\, \underline{\varepsilon} \right) \tag{II.1}$$

in which

$$\underline{\sigma} = \begin{bmatrix} 1 & & \\ & \ddots \; 0 & \\ & & \ddots \; \zeta_{xy} \end{bmatrix} \tag{II.2}$$

$$\underline{\varepsilon} = \begin{bmatrix} \frac{e_y}{r} & & \\ & 1 & \\ & & \frac{e_x}{r} \end{bmatrix} \tag{II.3}$$

$$\underline{\iota} = \begin{bmatrix} 1 & 1 & -1 \\ 1 & 1 & -1 \\ -1 & -1 & 1 \end{bmatrix} \tag{II.4}$$

and

$$\underline{\eta} = \left(\frac{e_y}{r}\right)^2 + \left(\frac{e_x}{r}\right)^2 \frac{1}{\zeta_{xy}^2} - \frac{1}{\zeta_{x\theta}^2} \tag{II.5}$$

, the damping matrix $\underline{\Delta}$ defined by Eq. 2.16 can be rewritten in the form

$$\underline{\Delta} = \frac{A_1}{\omega_x^2} (\underline{\sigma}^2 - \frac{1}{\eta} \underline{\varepsilon}\ \underline{\iota}\ \underline{\varepsilon}) + A_2 + A_3\ \underline{\Lambda} \tag{II.6}$$

Then the dimensional equations of motion become

$$\ddot{\underline{u}} + \left\{ \frac{A_1}{\omega_x^2} (\underline{\sigma}^2 - \frac{1}{\eta} \underline{\varepsilon}\ \underline{\iota}\ \underline{\varepsilon}) + A_2 + A_3\ \underline{\Lambda} \right\} \dot{\underline{u}}$$

$$+ \underline{M}^{-1}\ \underline{Q} = - \ddot{\underline{u}}_g \tag{II.7}$$

Differentiation with respect to time t, $(\dot{\ })\equiv d(\ )/dt$, can be transformed into differentiation with respect to dimensionless time τ, $(\ ')\equiv d(\ )/d\tau$, using the relations

$$\begin{aligned} \dot{\underline{u}} &= \omega_x (r\ u_{\theta 0})\ \underline{\rho}\ \underline{v}' \\ \ddot{\underline{u}} &= \omega_x^2 (r\ u_{\theta 0})\ \underline{\rho}\ \underline{v}'' \end{aligned} \tag{II.8}$$

where

$$\underline{\rho} = \begin{bmatrix} \rho_{x\theta} & & \\ & 1 & \\ & & \rho_{y\theta} \end{bmatrix} \tag{II.9}$$

Substitution of Eq. II.8 into Eq. II.7 gives

$$\omega_x^2 (r\ u_{\theta 0})\ \underline{\rho}\ \underline{v}'' + \omega_x (r\ u_{\theta 0}) \left\{ \frac{A_1}{\omega_x^2} (\underline{\sigma} - \frac{1}{\eta} \underline{\varepsilon}\ \underline{\iota}\ \underline{\varepsilon}) \right.$$

$$\left. + A_2 + A_3\ \underline{\Lambda} \right\} \underline{\rho}\ \underline{v}' + \underline{M}^{-1}\ \underline{Q} = - \ddot{\underline{u}}_g \tag{II.10}$$

Pre-multiplying Eq. II.10 by

$$\frac{1}{\omega_x^2 (r\ u_{\theta 0})} \quad \underline{\zeta}^2\ \underline{\rho}^{-1}$$

leads to

$$\underline{\zeta}^2\ \ddot{\underline{v}} + \left\{ \frac{A_1}{\omega_x^3} \underline{\zeta}^2\ (\underline{\sigma} - \frac{1}{\eta}\ \underline{\rho}^{-1}\ \underline{\varepsilon}\ \underline{\iota}\ \underline{\varepsilon}\ \underline{\rho}) \right.$$

$$\left. + \frac{A_2}{\omega_x} \underline{\zeta}^2 + \frac{A_3}{\omega_x} \underline{\zeta}^2\ \underline{\rho}^{-1}\ \underline{\Lambda}\ \underline{\rho} \right\} \dot{\underline{v}}$$

$$+ \frac{1}{\omega_x^2 (r\ u_{\theta 0})} \underline{\zeta}^2\ \underline{\rho}^{-1}\ \underline{M}^{-1}\ \underline{Q} = - \frac{1}{\omega_x^2 (r\ u_{\theta 0})} \underline{\zeta}^2\ \underline{\rho}^{-1}\ \ddot{\underline{u}}_g \qquad \text{(II.11)}$$

, since

$$\frac{A_3}{\omega_x} \underline{\zeta}^2\ \underline{\rho}^{-1}\ \underline{\Lambda}\ \underline{\rho} = \omega_x\ A_3\ \underline{\zeta}^2\ \underline{\rho}^{-1}\ \underline{\lambda}\ \underline{\rho}\ ,$$

$$\frac{1}{\omega_x^2 (r\ u_{\theta 0})} \underline{\zeta}^2\ \underline{\rho}^{-1}\ \underline{M}^{-1}\ \underline{Q} = \underline{p}$$

and

$$\frac{1}{\omega_x^2 (r\ u_{\theta 0})} \underline{\zeta}^2\ \underline{\rho}^{-1}\ \ddot{\underline{u}}_g = \begin{Bmatrix} \ddot{u}_{gx} \left(\frac{\tau}{\omega_x}\right) \big/ g\ a_{x0} \\ 0 \\ \ddot{u}_{gy} \left(\frac{\tau}{\omega_y\ \zeta_{xy}}\right) \big/ g\ a_{y0} \end{Bmatrix}$$

Substituting these relations into Eq. II.11 and pre-multiplying by $\underline{\zeta}^{-2}$ give

$$\ddot{\underline{v}} + 2\ \underline{\nu}\ \dot{\underline{v}} + \underline{\pi} = - \underline{\alpha}_g \qquad \text{(II.12)}$$

where

$$\underline{\nu} = \delta_1 \ (\underline{\sigma} - \frac{1}{\eta} \underline{\rho}^{-1} \ \underline{\varepsilon} \ \underline{\iota} \ \underline{\varepsilon} \ \underline{\rho}) + \delta_2 + \delta_3 \ \underline{\rho}^{-1} \ \underline{\lambda} \ \underline{\rho} \tag{II.13}$$

$$\underline{\pi} = \underline{\zeta}^{-2} \ \underline{p} \tag{II.14}$$

$$\underline{\alpha}_g = \begin{Bmatrix} \alpha_{gx} \\ 0 \\ \alpha_{gy} \end{Bmatrix} = \begin{Bmatrix} \ddot{u}_{gx}(\frac{\tau}{\omega_x}) \ / \ g \ a_{x0} \\ 0 \\ \ddot{u}_{gy}(\frac{\tau}{\omega_y \ \zeta_{xy}}) \ / \ g \ a_{y0} \ \zeta_{xy}^2 \end{Bmatrix} \tag{II.15}$$

and

$$\underline{\delta} = \begin{Bmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \end{Bmatrix} = \frac{1}{2} \begin{Bmatrix} A_1/\omega_x^3 \\ A_2/\omega_x \\ A_3 \cdot \omega_x \end{Bmatrix} = \underline{n}^{-1} \ \underline{\xi} \tag{II.16}$$

in which

$$\underline{n} = \begin{bmatrix} \left(\frac{\omega_x}{\omega_1}\right)^3 & \frac{\omega_x}{\omega_1} & \frac{\omega_1}{\omega_x} \\ \left(\frac{\omega_x}{\omega_2}\right)^3 & \frac{\omega_x}{\omega_2} & \frac{\omega_2}{\omega_x} \\ \left(\frac{\omega_x}{\omega_3}\right)^3 & \frac{\omega_x}{\omega_3} & \frac{\omega_3}{\omega_x} \end{bmatrix} \tag{II.17}$$

APPENDIX III

NUMERICAL INTEGRATION PROCEDURE

III.1 Third Order Runge-Kutta Method

Integration of the equations of motion given by Eq. 2.71 have been carried out using a third order Runge-Kutta method. To develope this method for a system of n first order differential equations, consider the system of differential equations with respect to time t written in the form

$$\dot{\bar{u}} = \bar{f}\ (t\ ,\ \bar{u}) \tag{III.1}$$

in which $\bar{u}$ and $\bar{f}$ are an n-dimensional vector and an n-dimensional vector function, respectively. Then, the Runge-Kutta formula is given by

$$\bar{u}_{s+1} = \bar{u}_s + \frac{1}{4}\ (\bar{M}_0 + 3\bar{M}_2) + \bar{0}\ (\Delta t^4) \tag{III.2}$$

where

$$\left.\begin{aligned} \bar{M}_0 &= \Delta t \cdot \bar{f}\ (t_s\ ,\ \bar{u}_s) \\ \bar{M}_1 &= \Delta t \cdot \bar{f}\ \{(t_s + \tfrac{1}{3}\ \Delta t),\ (\bar{u}_s + \tfrac{1}{3}\ \bar{M}_0)\} \\ \bar{M}_2 &= \Delta t \cdot \bar{f}\ \{(t_s + \tfrac{2}{3}\ \ t),\ (\bar{u}_s + \tfrac{2}{3}\ \bar{M}_1)\} \end{aligned}\right\} \tag{III.3}$$

and

in which $\bar{M}_0$, $\bar{M}_1$ and $\bar{M}_2$ are n-dimensional vectors, Δt is the interval of integration, s and s+l are subscripts denoting integration steps, and $\bar{0}$ denotes an n-dimensional error vector proportional to Δt^4.

III.2 Application of Runge-Kutta Method to Equations of Motion

The equations of motion can now be expressed as

$$\begin{Bmatrix} \ddot{u}_x \\ r\ \ddot{u}_\theta \\ \ddot{u}_y \end{Bmatrix} + \underline{\Delta} \begin{Bmatrix} \dot{u}_x \\ r\ \dot{u}_\theta \\ \dot{u}_y \end{Bmatrix} + \underline{M}^{-1} \begin{Bmatrix} Q_x \\ r\ Q_\theta \\ Q_y \end{Bmatrix} = - \begin{Bmatrix} \ddot{u}_{gx} \\ 0 \\ \ddot{u}_{gy} \end{Bmatrix} \tag{III.4}$$

where the restoring forces of the system are

$$\begin{Bmatrix} Q_x \\ r\, Q_\theta \\ Q_y \end{Bmatrix} = \underline{r}^2 \begin{Bmatrix} \sum_i Q_{ix} \\ -\sum_i Q_{ix}\, d_{iy} + \sum_i Q_{iy}\, d_{ix} \\ \sum_i Q_{iy} \end{Bmatrix} \qquad \text{(III.5)}$$

Rates of restoring forces of the i-th resisting element of EPI-systems are given by

$$\begin{Bmatrix} \dot{Q}_{ix} \\ \dot{Q}_{iy} \end{Bmatrix} = \underline{k}_i\ \underline{Z}_i \begin{Bmatrix} \dot{u}_x \\ r\, \dot{u}_\theta \\ \dot{u}_y \end{Bmatrix} \qquad \text{(III.6)}$$

for

$$\Phi_i\ (Q_{ix}\ ,\ Q_{iy}) < 1$$

or

$$\left.\begin{array}{r} \Phi_i\ (Q_{ix}\ ,\ Q_{iy}) = 1 \\ \dot{W}_i^p < 0 \end{array}\right\} \text{(Unloading)}$$

and

$$\begin{Bmatrix} \dot{Q}_{ix} \\ \dot{Q}_{iy} \end{Bmatrix} = \Gamma_i\ \underline{C}_i\ \underline{Z}_i \begin{Bmatrix} \dot{u}_x \\ r\, \dot{u}_\theta \\ \dot{u}_y \end{Bmatrix} \qquad \text{(III.7)}$$

for

$$\left.\begin{array}{r} \Phi_i\ (Q_{ix}\ ,\ Q_{iy}) = 1 \\ \dot{W}_i^p \geq 0 \end{array}\right\} \text{(Loading)}$$

where

$$\dot{W}_i^p = B_i \begin{Bmatrix} Q_{ix} \\ Q_{iy} \end{Bmatrix}^T \underline{B}_i\ \underline{k}_i\ \underline{Z}_i \begin{Bmatrix} \dot{u}_x \\ r\, \dot{u}_\theta \\ \dot{u}_y \end{Bmatrix} \qquad \text{(III.8)}$$

Rates of restoring forces of the i-th resisting element of EP-systems in the x-direction are given by

$$\dot{Q}_{ix} = k_{ix} \, \underline{Z}_i^{(1)} \begin{Bmatrix} \dot{u}_x \\ r \, \dot{u}_\theta \\ \dot{u}_y \end{Bmatrix} \tag{III.9}$$

for the conditions

$$|Q_{ix}| < Q_{ix0}$$

and

$$|Q_{ix}| = Q_{ix0}$$

$$\dot{W}^p_{ix} < 0$$

Note that (III.10)

$$\dot{Q}_{ix} = 0$$

when

$$|Q_{ix}| = Q_{ix0}$$

$$\dot{W}^p_{ix} \geq 0$$

The corresponding relationships for the y-direction become

$$\dot{Q}_{iy} = k_{iy} \, \underline{Z}_i^{(2)} \begin{Bmatrix} \dot{u}_x \\ r \, \dot{u}_\theta \\ \dot{u}_y \end{Bmatrix} \tag{III.11}$$

when

$$|Q_{iy}| < Q_{iy0}$$

or

$$|Q_{iy}| = Q_{iy0}$$

$$\dot{W}^p_{iy} < 0$$

Again, note that

$$\dot{Q}_{iy} = 0 \tag{III.12}$$

when

$$|Q_{iy}| = Q_{iy0}$$

$$\dot{W}^p_{iy} \geq 0$$

Quantities $\dot{W}^p_{ix}$ and $\dot{W}^p_{iy}$ in the inequalities above are given by

$$\dot{W}^p_{ix} = Q_{ix}\ \underline{Z}_i^{(1)} \begin{Bmatrix} \dot{u}_x \\ r\ \dot{u}_\theta \\ \dot{u}_y \end{Bmatrix} \tag{III.13}$$

and

$$\dot{W}^p_{iy} = Q_{iy}\ \underline{Z}_i^{(2)} \begin{Bmatrix} \dot{u}_x \\ r\ \dot{u}_\theta \\ \dot{u}_y \end{Bmatrix} \tag{III.14}$$

In continuing this development, let

$$\left.\begin{aligned} u(1) &= u_x \\ u(2) &= r\ u_\theta \\ u(3) &= u_y \\ u(4) &= \dot{u}_x \\ u(5) &= r\ \dot{u}_\theta \\ u(6) &= \dot{u}_y \\ q(1,i) &= Q_{ix} \\ q(2,i) &= Q_{iy} \end{aligned}\right\} \tag{III.15}$$

Then, the set of first order differential equations becomes

$$\begin{Bmatrix} \dot{u}(1) \\ \dot{u}(2) \\ \dot{u}(3) \end{Bmatrix} = \begin{Bmatrix} u(4) \\ u(5) \\ u(6) \end{Bmatrix} \qquad \text{(III.16)}$$

and

$$\begin{Bmatrix} \dot{u}(4) \\ \dot{u}(5) \\ \dot{u}(6) \end{Bmatrix} = - \begin{Bmatrix} \ddot{u}_{gx} \\ 0 \\ \ddot{u}_{gy} \end{Bmatrix} - \underline{\Delta} \begin{Bmatrix} u(4) \\ u(5) \\ u(6) \end{Bmatrix} - \underline{M}^{-1} \underline{r}^2 \begin{Bmatrix} \sum_i q(1,i) \\ -\sum_i q(1,i) \cdot d_{iy} + \sum_i q(2,i) d_{ix} \\ \sum_i q(2,i) \end{Bmatrix} \qquad \text{(III.17)}$$

The relationships among rates of restoring forces and displacements for EPI-systems can be rewritten as

$$\begin{Bmatrix} \dot{q}(1,i) \\ \dot{q}(2,i) \end{Bmatrix} = \underline{k}_i \ \underline{Z}_i \begin{Bmatrix} u(4) \\ u(5) \\ u(6) \end{Bmatrix} \qquad \text{(III.18)}$$

for

$$\Phi_i\{q(1,i) \ , \ q(2,i)\} < 1$$

or

$$\Phi_i\{q(1,i) \ , \ q(2,i)\} = 1$$

$$\dot{W}_i^p < 0$$

and

$$\begin{Bmatrix} \dot{q}(1,i) \\ \dot{q}(2,i) \end{Bmatrix} = \Gamma_i \ \underline{C}_i \ \underline{Z}_i \begin{Bmatrix} u(4) \\ u(5) \\ u(6) \end{Bmatrix} \qquad \text{(III.19)}$$

for

$$\Phi_i\{q(1,i) \ , \ q(2,i)\} = 1$$

$$\dot{W}_i^p \geq 0$$

where

$$\dot{W}_i^p = B_i \begin{Bmatrix} q(1,i) \\ q(2,i) \end{Bmatrix}^T \underline{B}_i \, \underline{k}_i \, \underline{Z}_i \begin{Bmatrix} u(4) \\ u(5) \\ u(6) \end{Bmatrix} \qquad \text{(III.20)}$$

The corresponding relationships for EP-systems can be similarly written. First, the relationships for the x-direction become

$$\dot{q}(1,i) = k_{ix} \, \underline{Z}_i^{(1)} \begin{Bmatrix} u(4) \\ u(5) \\ u(6) \end{Bmatrix} \qquad \text{(III.21)}$$

for the conditions

$$|q(1,i)| < Q_{ix0}$$

and

$$|q(1,i)| = Q_{ix0}$$

$$\dot{W}_{ix}^p < 0$$

Note that

$$\dot{q}(1,i) = 0 \qquad \text{(III.22)}$$

when

$$|q(1,i)| = Q_{ix0}$$

$$\dot{W}_{ix}^p \geq 0$$

Similarily, the relationships for the y-direction become

$$\dot{q}(2,i) = k_{iy} \, \underline{Z}_i^{(2)} \begin{Bmatrix} u(4) \\ u(5) \\ u(6) \end{Bmatrix} \qquad \text{(III.23)}$$

for

$$|q(2,i)| < Q_{iy0}$$

or

$$|q(2,i)| = Q_{iy0}$$

$$\dot{W}_{iy}^p < 0$$

and

$$\dot{q}(2,i) = 0 \tag{III.24}$$

when

$$|q(2,i)| = Q_{iyo}$$

$$\dot{W}^p_{iy} \geq 0$$

Quantities $\dot{W}^p_{ix}$ and $\dot{W}^p_{iy}$ in the inequalities above are given by

$$\dot{W}^p_{ix} = q(1,i)\ \underline{Z}_i^{(1)} \begin{Bmatrix} u(4) \\ u(5) \\ u(6) \end{Bmatrix} \tag{III.25}$$

and

$$\dot{W}^p_{iy} = q(2,i)\ \underline{Z}_i^{(2)} \begin{Bmatrix} u(4) \\ u(5) \\ u(6) \end{Bmatrix} \tag{III.26}$$

III.3 Application of Runge-Kutta Method to Dimensionless Equations of Motion

The dimensionless equations of motion are

$$\begin{Bmatrix} v''_x \\ v''_\theta \\ v''_y \end{Bmatrix} + 2\underline{\nu} \begin{Bmatrix} v'_x \\ v'_\theta \\ v'_y \end{Bmatrix} + \underline{\pi} = - \begin{Bmatrix} \alpha_{gx} \\ 0 \\ \alpha_{gy} \end{Bmatrix} \tag{III.27}$$

where the dimensionless restoring forces are given by

$$\underline{\pi} = \underline{\zeta}^{-2} \begin{Bmatrix} p_x \\ p_\theta \\ p_y \end{Bmatrix} = \zeta^{-2} \begin{Bmatrix} \sum_i p_{ix}\, q_{ix0} \\ -\sum_i p_{ix}\, q_{i\theta x0} + \sum_i p_{iy}\, q_{i\theta y0} \\ \sum_i p_{iy}\, q_{iy0} \end{Bmatrix} \tag{III.28}$$

The rates of dimensionless restoring forces of the i-th resisting element for EPI-systems are given by

$$\begin{Bmatrix} \dot{p}_{ix} \\ \dot{p}_{iy} \end{Bmatrix} = \underline{z}_i \begin{Bmatrix} \dot{v}_x \\ \dot{v}_\theta \\ \dot{v}_y \end{Bmatrix} \tag{III.29}$$

for

$$\Phi_i(p_{ix}, p_{iy}) < 1$$

or

$$\Phi_i(p_{ix}, p_{iy}) = 1$$

$$\dot{w}_i^p < 0$$

and

$$\begin{Bmatrix} \dot{p}_{ix} \\ \dot{p}_{iy} \end{Bmatrix} = \underline{\gamma}_i \; \underline{c}_i \; \underline{r}_i \; \underline{z}_i \begin{Bmatrix} \dot{v}_x \\ \dot{v}_\theta \\ \dot{v}_y \end{Bmatrix} \tag{III.30}$$

for

$$\Phi_i(p_{ix}, p_{iy}) = 1$$

$$\dot{w}_i^p \geq 0$$

where

$$\dot{w}_i^p = \beta_i \begin{Bmatrix} p_{ix} \\ p_{iy} \end{Bmatrix}^T \underline{b}_i \; \underline{z}_i \begin{Bmatrix} \dot{v}_x \\ \dot{v}_\theta \\ \dot{v}_y \end{Bmatrix} \tag{III.31}$$

Likewise , the rates of dimensionless restoring forces of the i-th resisting element for EP-systems in the x-direction are given by

$$\dot{p}_{ix} = \underline{z}_i^{(1)} \begin{Bmatrix} \dot{v}_y \\ \dot{v}_\theta \\ \dot{v}_y \end{Bmatrix} \tag{III.32}$$

for the conditions

$$|p_{ix}| < 1$$

and

$$|p_{ix}| = 1$$

$$\dot{w}_{ix}^{p} < 0$$

Note that

$$\dot{p}_{ix} = 0 \tag{III.33}$$

when

$$|p_{ix}| = 1$$

$$\dot{w}_{ix}^{p} \geq 0$$

Similarly, the corresponding dimensionless force-displacement relationships in the y-direction become

$$\dot{p}_{iy} = \underline{z}_i^{(2)} \begin{Bmatrix} \dot{v}_x \\ \dot{v}_\theta \\ \dot{v}_y \end{Bmatrix} \tag{III.34}$$

when

$$|p_{iy}| \leq 1$$

or

$$|p_{iy}| = 1$$

$$\dot{w}_{iy}^{p} < 0$$

and

$$\dot{p}_{iy} = 0 \tag{III.35}$$

when

$$|p_{iy}| = 1$$

$$\dot{w}^{p}_{iy} \geq 0$$

Qunatities $\dot{w}^{p}_{ix}$ and $\dot{w}^{p}_{iy}$ in the inequalities above are given by

$$\dot{w}^{p}_{ix} = p_{ix}\, \underline{z}_i^{(1)} \begin{Bmatrix} \dot{v}_x \\ \dot{v}_\theta \\ \dot{v}_y \end{Bmatrix} \tag{III.36}$$

and

$$\dot{w}^{p}_{iy} = p_{iy}\, \underline{z}_i^{(2)} \begin{Bmatrix} \dot{v}_x \\ \dot{v}_\theta \\ \dot{v}_y \end{Bmatrix} \tag{III.37}$$

To continue this development, let

$$\left.\begin{aligned} v(1) &= v_x \\ v(2) &= v_\theta \\ v(3) &= v_y \\ v(4) &= \dot{v}_x \\ v(5) &= \dot{v}_\theta \\ v(6) &= \dot{v}_y \\ p(1,i) &= p_{ix} \\ p(2,i) &= p_{iy} \end{aligned}\right\} \tag{III.38}$$

Then, the set of first order differential equations becomes

$$\begin{Bmatrix} \dot{v}(1) \\ \dot{v}(2) \\ \dot{v}(3) \end{Bmatrix} = \begin{Bmatrix} v(4) \\ v(5) \\ v(6) \end{Bmatrix} \tag{III.39}$$

and

$$\begin{Bmatrix} \dot{v}(4) \\ \dot{v}(5) \\ \dot{v}(6) \end{Bmatrix} = -\begin{Bmatrix} \alpha_{gx} \\ 0 \\ \alpha_{gy} \end{Bmatrix} - 2\underline{\nu}\begin{Bmatrix} v(4) \\ v(5) \\ v(6) \end{Bmatrix} - \underline{\zeta}^{-2}\begin{Bmatrix} \sum_i p(1,i)\, q_{ix0} \\ -\sum_i p(1,i)\, q_{i\theta x0} + \sum_i p(2,i)\, q_{i\theta y0} \\ \sum_i p(2,i)\, q_{iy0} \end{Bmatrix} \quad \text{(III.40)}$$

The relationships among rates of dimensionless restoring forces and dimensionless displacements for EPI-systems can be rewritten as

$$\begin{Bmatrix} \dot{p}(1,i) \\ \dot{p}(2,i) \end{Bmatrix} = \underline{z}_i \begin{Bmatrix} v(4) \\ v(5) \\ v(6) \end{Bmatrix} \quad \text{(III.41)}$$

for

$$\Phi_i\{p(1,i)\ ,\ p(2,i)\} < 1$$

or

$$\Phi_i\{p(1,i)\ ,\ p(2,i)\} = 1$$

$$\dot{w}_i^p < 0$$

and

$$\begin{Bmatrix} \dot{p}(1,i) \\ \dot{p}(2,i) \end{Bmatrix} = \gamma_i\ \underline{c}_i\ \underline{r}_i\ \underline{z}_i \begin{Bmatrix} v(4) \\ v(5) \\ v(6) \end{Bmatrix} \quad \text{(III.42)}$$

for

$$\Phi_i\{p(1,i)\ ,\ p(2,i)\} = 1$$

$$\dot{w}_i^p \geq 0$$

where

$$\dot{w}_i^p = \beta_i \begin{Bmatrix} p(1,i) \\ p(2,i) \end{Bmatrix}^T \underline{b}_i\ \underline{z}_i \begin{Bmatrix} v(4) \\ v(5) \\ v(6) \end{Bmatrix} \quad \text{(III.43)}$$

Likewise, the relationships among rates of dimensionless restoring forces and dimensionless displacements for EP-systems can be obtained. First, the relationships for the x-direction become

$$\dot{p}(1,i) = \underline{z}_i^{(1)} \begin{Bmatrix} v(4) \\ v(5) \\ v(6) \end{Bmatrix} \qquad \text{(III.44)}$$

for the conditions

$$|p(1,i)| < 1$$

and

$$p(1,i) = 1$$

$$\dot{w}_{ix}^{p} < 0$$

note that

$$\dot{p}(1,i) = 0 \qquad \text{(III.45)}$$

when

$$|p(1,i)| = 1$$

$$\dot{w}_{ix}^{p} \geq 0$$

Similarly, the relationships for the y-direction become

$$\dot{p}(2,i) = \underline{z}_i^{(2)} \begin{Bmatrix} v(4) \\ v(5) \\ v(6) \end{Bmatrix} \qquad \text{(III.46)}$$

for

$$|p(2,i)| < 1$$

or

$$|p(2,i)| = 1$$

$$\dot{w}_{iy}^{p} < 0$$

and

$$\dot{p}(2,i) = 0 \qquad \text{(III.47)}$$

when

$$|p(2,i)| = 1$$

$$\dot{w}_{iy}^{p} \geq 0$$

Quantities $\dot{w}_{ix}^{p}$ and $\dot{w}_{iy}^{p}$ in the inequalities above are given by

$$\dot{w}^{p}_{ix} = p(1,i)\ \underline{z}_i^{(1)} \begin{Bmatrix} v(4) \\ v(5) \\ v(6) \end{Bmatrix} \tag{III.48}$$

and

$$\dot{w}^{p}_{iy} = p(2,i)\ \underline{z}_i^{(2)} \begin{Bmatrix} v(4) \\ v(5) \\ v(6) \end{Bmatrix} \tag{III.49}$$

III.4 Time Interval and Tolerances on Numerical Integration

Dimensionless time τ is related to absolute time t by the equation

$$\tau = \omega_x\ t \tag{2.70}$$

The integration interval $\Delta\tau$ was similarly determined by

$$\Delta\tau = \frac{\omega_x \cdot \Delta t}{N} \tag{III.50}$$

in which time increment Δt was taken equal to the interval of digitization of the earthquake ground motion, i.e. $\Delta t = 0.02$ seconds. A proper choice of the value of integer N depends upon the values of the uncoupled periods, T_x and T_y, of the system. A few test cases for $T_x = T_y = 0.5$ seconds showed that changes in displacement response were less than 1% for N = 1 and 2. For $T_x = T_y = 0.3$ seconds, the changes in displacement response were less than 2% for N = 2 and 4. Hence, choice of values for integer N was determined as follows:

$$N = 1 \text{ for } T_x \text{ and } T_y \geq 0.5 \text{ seconds}$$
$$2 \text{ for } T_x \text{ and } T_y < 0.5 \text{ seconds}$$

For elasto-plastic systems, transitions from elastic to plastic state and from plastic to elastic state must be carried out with sufficient accuracy. The following criteria were used for such transitions:

$$\text{Elastic state if} \quad \Phi_i < 1$$

$$\text{Plastic state if} \quad \begin{cases} 1 \leq \Phi_i \leq 1 + 10^{-2} \\ 0 < \dot{w}_i^p \end{cases}$$

and

$$\begin{array}{r} \text{Transition from plastic} \\ \text{to elastic state if} \end{array} \quad \begin{cases} 1 \leq \Phi_i \leq 1 + 10^{-2} \\ -10^{-2} \leq \dot{w}_i^p \leq 0 \end{cases}$$

The integration interval initially selected was subdivided if the criteria described above were not satisfied during numerical integration.

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