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# RELIABILITY CONSIDERATIONS FOR FATIGUE ANALYSIS AND DESIGN OF STRUCTURES

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## RELIABILITY CONSIDERATIONS FOR FATIGUE ANALYSIS AND

## DESIGN OF STRUCTURES\*

#### Ъy

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#### 1. INTRODUCTION

The subject area of structural fatigue is concerned with the mechanical behavior of structures, which are subject to repeated load applications. The number of load applications causing structural failure is an important factor, the values of which may range from one to infinity. When a structure fails with relatively few load applications, the phenomenon has been called low-cycle fatigue [1,2]. The lowest possible number of load applications causing structural failure is one, which corresponds to the "static" or one-cycle test [3]. For small steel or aluminum alloy specimens, one-cycle test results can be used to estimate the low-cycle fatigue behavior of such axially loaded specimens [3]. When a structure does not fail after several million load applications, the loading level is called endurance limit, below which structures of certain materials are supposedly to last forever. Therefore, the fatigue behavior of structures included low-cycle fatigue starting with "static" test and highcycle fatigue ending with infinite number of cycles.

Most available experimental data are for small-sized specimens subjected to constant cyclic stresses. It has been well known that the fatigue life at a given stress level is a random phenomenon. Various probability distributions are being used for the random variable denoting the fatigue life. Because of the limited number of test specimens in most cases, it is difficult to verify the tail portions of each distribution function statistically. Frequently, two or more distributions cannot be rejected at a given level of significance using a statistical goodness-of-fit test [4].

Using data from constant cyclic-stress fatigue tests, a cumulative damage theorem can be applied to estimate the life of a specimen subjected to a known variable-stress history. Many such cumulative damage theorems exist. The earliest as well as the simplest one is the linear rule which is known as the Palmgren-Miner cumulative damage theorem [5,6]. Results of recent variable-stress fatigue tests of full-size beams indicate that the linear rule can be used to give reasonable and practical estimates of fatigue life [7].

The estimation of fatigue life on the basis of constant cyclic stress tests of small specimens and cumulative damage theorems provides a useful tool in analysis and design of structures. Meanwhile, more fatigue test results of full-size structures are becoming available to help engineers in

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## their understanding of structural fatigue [8].

Recently, more emphasis is being placed on the interpretation of inspection results of existing structures. As an example, the reliability aspects of several nondestructive inspection techniques were discussed [9]. However, the decision process utilizing inspection and testing results in the classification of an appropriate damage state remains to be studied [10-12].

The objective of this paper is to critically review and briefly summarize the available literature concerning various reliability considerations for fatigue analysis and design of structures. The basic problem of structural reliability involving repeated load applications is formulated. Several important probability distributions for the fatigue life with constant cyclic stress are discussed. Then, the use of linear cumulative damage theorem in structural fatigue is reviewed along with available results of variablestress experiments. In view of relating test results of relatively small and simple specimens to the fatigue behavior of full-scale structures, the work of Baldwin et al [8] is reviewed and discussed. Finally, the reliability aspects of nondestructive inspection and testing procedures are explored along with the problem of damage assessment of existing structures.

## 2. STRUCTURAL RELIABILITY INVOLVING REPEATED LOADS

The reliability function for a structure is denoted by  $L_T(t)$ , which is defined as the probability that the useful life of the structure is greater than a specified time period t, i.e.,

$$L_{m}(t) \equiv P(T > t)$$
(1)

The probability of failure,  $F_{\pi}(t)$ , in time interval [0,t] is given by

$$F_{m}(t) \equiv P(T \leq t) = 1 - L_{m}(t)$$

The corresponding probability density function,  $f_{\rm T}(t)$  and hazard function  $h_{\rm r}(t)$  are such that

$$f_{T}(t)dt \equiv P(t < T \leq t + dt)$$
(3)

 $h_{T}(t)dt \equiv P(t < T \leq t + dt | T > t)$ 

Equations (1) through (4) are described in detail by Freudenthal, Garrelts, and Shinozuka in a committee report of the American Society of Civil Engineers [13].

The reliability function as defined in Equation (1) can also be expressed in terms of random processes R(t) and S(t) representing respectively the resistance and the applied force as follows [14,15]:

$$\mathbf{L}_{m}(\mathbf{t}) = \mathbf{P}(\mathbf{R}(\tau) > \mathbf{S}(\tau); \ \mathbf{0} \leq \tau \leq \mathbf{t})$$
(5)

Because of possible wear and cumulative damage as well as the increasing chance of encountering extreme loading conditions with long time period [0,t],

2

(2)

(4)

the reliability function usually decreases with time. For fatigue analysis, the resistance process R(t) may be considered as a function of previous loading history [16].

Alternatively, the reliability function can be given as a function of cumulative damage D(t) as follows:

$$\mathbf{L}_{m}(t) = \mathbf{P}(\mathbf{D}(t) < \mathbf{C})$$

where C is a random variable denoting the limiting capacity of the cumulative damage. Whenever the lifetime T can be related to the number of load applications N, the reliability function can be expressed as follows:

$$L_{M}(n) \equiv P(N > n) = P(D(n) < C)$$

$$(7)$$

where D(n) is a random variable denoting the cumulative damage at n-th load application. Frequently, the limiting value (for a specific failure definition) of cumulative damage is set to be unity [6]. Conceivably, if the statistical characteristics of D(n) and C are known, the reliability function can be computed accordingly. In reality, however, cumulative damage is an abstract quantity, the probabilistic description of which is not easy to obtain in general.

## 3. CONSTANT CYCLIC-STRESS TEST DATA

In many fatigue tests, several specimens are tested at each of several stress levels. Therefore, it is possible to describe the interrelationships of the reliability function, the cyclic-stress amplitude, and the fatigue life. Weibull [17] listed the following three graphical representations of these interrelationships: (a) empirical distribution function vs. fatigue life for various cyclic-stress amplitudes; (b) cyclic-stress amplitudes vs. fatigue life with specified survival probabilities; and (c) empirical distribution function vs. cyclic-stress amplitudes with given fatigue lives.

Many mathematical functions have been used for the statistical description of the fatigue life of specimens subjected to constant cyclic stresses. One popular model is the lognormal (logarithmic-normal) distribution function [18]. As it was observed by Gumbel [19], the hazard function for the lognormal distribution decreased with increasing life, which is contradictory to the expected fatigue behavior of materials.

A reasonable distribution function is due to Weibull [20]. The application of the Weibull distribution to fatigue analysis is summarized by Freudenthal and Gumbel [21]. The so-called three-parameter Weibull distribution function and its corresponding reliability function are given as follows:

$$F_{N}(n) = 1 - \exp \left[ -\left( \frac{n - n_{os}}{v_{s} - n_{os}} \right)^{\alpha} s \right]$$

(8)

(6)

$$L_{N}(n) = 1 - F_{N}(n) = \exp \left[ -\left(\frac{n - n_{OS}}{v_{s} - n_{OS}}\right)^{\alpha} \right]$$

where  $\alpha_{a}$  = shape parameter at cyclic stress level s

v = scale parameter or characteristic life

n = location parameter, sensitivity limit, or minimum life

Frequently, it is reasonable to set  $n_{0,s} = 0$ . Then Equations (8) and (9) reduce to the following two-parameter Weibull distribution function and reliability function, respectively.

$$F_{N}(n) = 1 - \exp\left[-\left(\frac{n}{v_{s}}\right)^{\alpha}s\right]$$
(10)  
$$L_{N}(n) = \exp\left[-\left(\frac{n}{v_{s}}\right)^{\alpha}s\right]$$
(11)

Several statistical methods for the estimation of these parameters and the testing of goodness-of-fit are summarized and compared by Wirsching and Yao [4]. Two numerical examples using graphical method, method of moments, maximum likelihood and several other methods were also given for the purpose of illustration.

## 4. CUMULATIVE DAMAGE THEOREMS

In reality, the time-history of stresses (or strains) is usually variable [22-24]. Therefore, results of constant cyclic-stress tests cannot be directly used in the prediction of fatigue lives in cases where the stress histories are variable or random. Many cumulative damage theorems are available to-date [25-27]. The earliest and simplest one is the linear cumulative damage theorem due to Palmgren [5] and Miner [6]. Although many investigators reject this rule for various reasons and modified it to obtain better fit to particular sets of test data, the Palmgren-Miner linear damage theorem remains the most widely used one in fatigue design at present because of its simplicity and practicality.

The Palmgren-Miner damage theorem is given as follows:

$$\sum_{\substack{i=1\\i=1}}^{k} \frac{n_i}{N_1} = 1$$

where n denotes the number of cycles at s, which is ith i stress level in a k-level test,

N denotes the fatigue life in a single stress level test with cyclic stresses s<sub>i</sub>.

Tang and Yao [28] treated the denominators  $N_i$  as random variables, and let the sum be another random variable, D, representing the cumulative damage, i.e.,

(12)

(9)

 $n_i = n$ , then the random variable D as given in Equation (13) If we let  $\Sigma$ 

is equal to D(n) in Equation (7). The cumulative damage D may be considered as a demand to the structure. On the other hand, a "damage index at failure", c, is considered to be a capacity, the exceedance of which is called failure. Thus, the probability of failure can be obtained as follows:

$$P_{c} = P(D \ge c)$$

This damage index c is a realized value of the random variable C as given in Equation (7). It is assumed that the random variables  $N_i$  follow Weibull dis-

tributions, sample design curves using the test data of Corten, Sinclair and Dolan [29] were computed and presented graphically.

In 1974, Yao [30] Proposed a simple basis for fatigue design using constant cyclic-stress test data and the linear damage theorem. Let

$$p = P(N_i \leq n_{in}), i = 1, 2, ..., k$$
 (15)

where  $n_{in}$  = the realized value of the random variable  $N_i$  as defined previously with probability p of having a shorter life. Also, from the recorded stress histories such as those given by Cudney [24] and Drew [23], we find the fractions g, of the total number of load applications with stress amplitudes s, respectively, i.e.,

 $n_i = g_i n$ 

where n is the total number of load applications as described earlier. According to Shinozuka [31], a structural member which is weak under one stress level is likely to remain weak when it is subjected to repeated loads at other stress amplitudes. Using the Palmgren-Miner theorem, the total life n can be related to n ip as follows:

$$\sum_{i=1}^{k} \frac{n_i}{n_{ip}} = n \sum_{i=1}^{k} \frac{g_i}{n_{ip}} = 1$$
(17)

The distribution of the total life N can be found as follows:

$$F_{N}(n) = P(N \leq n) = P\left(N \leq \frac{1}{k} + \frac{g_{i}}{g_{i}}\right)$$
  
$$i=1 \quad n_{ip}$$

5

(13)

(14)

(16)

(18)

Assuming that,

$$p = P \left( N \leq \frac{1}{k} g_{i} \\ \Sigma & \frac{1}{n_{ip}} \right)$$
$$i=1 \quad ip$$

the distribution function  $F_N(n)$  can be evaluated using various values of p.

The fatigue test data of Corten, Sinclair, and Dolan [29] were used for the purpose of illustration. Using the log-normal distribution, the median life n and standard deviation  $\sigma_{\log N}$  can be computed accordingly. These computed

values are plotted against observed values. Because the data points are found to lie on both sides of the 45° line, it seems that the result of using the linear cumulative damage criterion neither over-estimate nor underestimate the experimental fatigue life. However, more comparisons are needed before such a method can be used in practice. Ang and Munse [32-34] proposed a method, in which they assumed that (a) the fatigue life follows a Weibull distribution, (b) the coefficient of variation is a constant, and (c) the linear damage theorem is applicable. This method can be applied to various problems of random fatigue including those involving offshore structures [34].

Recently, Bogdanoff et al [35,36] presented a history-dependent phenomenological model to account for the cumulative damage in fatigue studies. He defined the "duty cycle" as a repetitive period of operation in the life of a structural component [37]. The following assumptions are made: (a) the increment in damage at the end of a duty cycle depends in a probabilistic manner only on the amount of damage present at the start of the duty cycle and on that duty cycle itself (Markov assumption); (b) the damage accumulation in a duty cycle is non-negative and can only increase by one unit in any one dutycycle; (c) damage states are given as y = 1, 2, ..., b, where state b denotes failure or that replacement is required; and (d) the initial damage present in a component arises from material defects and from defects during manufacturing and fabrication procedures. Let

$$= \begin{bmatrix} P_1 & q_1 & 0 & 0 & \cdots & 0 & 0 \\ 0 & P_2 & q_2 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & P_{b-1} & q_{b-1} \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Q

(20)

- where p = probability of the cumulative damage remaining in state j in a
   duty cycle given that the damage was in state j at the beginning
   of the duty cycle.

and

$$p_n = \left\{ p_n(1), p_n(2), \dots, p_n(b) \right\}$$
 (21)

where  $p_n(j)$  = probability of the cumulative damage being in state j at n th duty cycle, and n=0 represents the initial state.

Assuming that,

$$\sum_{j=1}^{D} p_n(j) = 1, n=0,1,2,...$$
(22)

then,

$$p_n = p_o Q^r$$

It can be shown that

$$F_{N}(n) = p_{n}(b)$$

and

$$E[N] = \sum_{n=0}^{\infty} [1 - F_N(n)]$$

$$\operatorname{Var}[N] = \sum_{n=0}^{\infty} [1 - F_N(n)]$$

+2 
$$\sum_{n=1}^{\infty} n[1-F_N(n)] - (E[N])^2$$

Bogdanoff and Krieger [36] applied this method to six sets of fatigue and wear life test data [37,38], and found excellent agreements between the mathematical model and observed data. Nevertheless, the authors did not describe in detail how they estimated these parameters. It was briefly mentioned that they used the methods of moments, least squares, and maximum likelihood. In addition, they mentioned that their extensive experience with this model played a significant part in the estimation of these parameters. It is also noted that the

(24)

(23)

damage is indexed on "state", which is not directly related to the number and size of flaws and cracks.

## 5. RANDOM LOAD AND FULL-SCALE FATIGUE TESTS

In many applications, the applied stress histories can be represented with random processes [39-41]. Earlier experimental investigations of random fatigue were reviewed by the writer in 1974 [30].

Schilling and Klippstein [7] used an effective stress range to represent the variable-amplitude spectrum. The effective stress range as given in Equation (26) is defined as the constant-amplitude stress range which would cause the same fatigue life as the one resulting from the variableamplitude spectrum.

$$\mathbf{s}_{e} = \begin{bmatrix} \mathbf{k} & & \\ \boldsymbol{\Sigma} & \boldsymbol{g}_{i} & \boldsymbol{s}_{i} \\ \mathbf{i}-1 & & \mathbf{i} \end{bmatrix}^{\frac{1}{m}}$$

where  $g_i$  is the frequency of currence of the <u>ith</u> stress range  $s_i$ , and the exponent m is the slope of cyclic crack growth rate, which can also be equal to the absolute value of the inverse slope of the mean regression line for constant-amplitude fatigue test data. The effective stress range is called "RMS stress range" when m=2, because it corresponds to calculations using the Rayleigh distribution. It is called the "Miner stress range" when m=3. In both cases, the use of such effective stress ranges agrees well with experimental results of testing some 156 welded beams with partial-length cover plates and 60 welded beams without cover plates. It was concluded that the RMS stress range (i.e., m=2) provides a somewhat better fit of the test data.

Yamada and Albrecht [42] recommended a design method for variableamplitude fatigue with the consideration of the fatigue-limit effect. Abrecht and Friedland [43] reported on the data of 38 constant cyclic-stress and 41 variable amplitude fatigue tests of axially-loaded specimens with welded transverse stiffeners. The stress-range histogram of the variable amplitude tests included up to 97.2% of the stress cycles below the fatigue limit. They concluded that the Miner stress range is a good transfer function as long as its value is above the fatigue limit. When it is lower than the fatigue limit, the method of Yamada and Albrecht [42] yields good estimates of the variable-amplitude fatigue lives. However, there exist test results indicating that one must be careful in correlating constant cyclic-stress and variable amplitude test data [44].

Because of the large size and huge cost involved in the construction of civil engineering structures, there are relatively few full-scale fatigue tests to-date. In 1978, Baldwin et al [8] reported on a comprehensive fatigue test program which was conducted with a full-scale three-span (72 ft/93 ft/72 ft) continuous composite bridge. The test structure was designed in 1962 using the 1961 AASHO Specifications and built in 1963. It was scheduled for removal due to a certain flood control project. Investigators were able to conduct a series of experiments in 1975 prior to its final removal. The fatigue loading was applied through the use of a moving-mass closed-loop electrohydraulicactuator system. During certain intervals of the fatigue test, the bridge was inspected with the application of eight inspection mehtods including visual,

(26)

ultrasonic, radiographic, acoustic emission, and several dynamic-signature techniques. Among many significant results reported therein, two of eight critical regions at the ends of welded coverplates failed at approximately

 $4.5 \times 10^{2}$  cycles. The nominal stress range at these critical regions was approximately 8.7 ksi with estimated fatigue lives in the neighborhood of

2x10<sup>6</sup> cycles. Authors stated that "it may not be possible to accurately predict the total fatigue life on the basis of a simple relationship involving only stress range and type of detail". In addition, it was concluded that (a) a pulse-echo ultrasonic unit was the most reliable device for the device for the detection of cracks, and (b) visual inspection was considered as being the second most reliable method of detection. Although radiography was found to be nearly as reliable as ultrasonic inspection, more than half of the locations to be inspected was inaccessible for the use of radiography.

## 6. FATIGUE RELIABILITY OF EXISTING STRUCTURES

In 1921, Griffith [45] derived an equation giving a critical crack size, above which the brittle material would fracture. This critical size depend on the state of stress, the properties and environment of the material, and the geometry of the crack. Since then, it has become known that (a) subcritical cracks can grow, and (b) not all of the cracks exist from the beginning. Therefore, it is desirable to conduct nondestructive inspection and/or testing procedures to determine whether the structure contain defects that would prevent its use. Various techniques for such nondestructive evaluation are reviewed and summarized recently [26].

A major problem in making such nondestructive evaluation is the inability to measure the seriousness of a defect. In a recent review article, Robinson [46] stated that "Researchers emphasize that there can never be a 100 percent certain answer to the question: Will a part last the designed service life or not? Thus, in view of an inevitable uncertainty, some of the decisionmaking must be based on nonquantitative values, such as how badly we want to avoid DC-10 crashes". Pilot studies have been conducted to evaluate such nonquantitative values and to relate measurable quantities such as the number and size of cracks to abstract factors such as fatigue damage [47,48]. However, more work needs to be done before these methods can become practical.

Packman et al [49] found the following distribution function for the detection of a fatigue crack of size x:

if x>a,

$$F_{X}(n) = 0, \qquad \text{if } x < a_{1}$$

$$= \left( \frac{x - a_{1}}{a_{2} - a_{1}} \right)^{W} \qquad \text{if } a_{1} \leq x \leq a_{2}$$

1

(27)

where a, = minimum crack size below which the crack cannot be detected

a<sub>2</sub> = maximum crack size beyond which the crack can be detected with certainty

w = an empirical parameter

These parameters  $a_1$ ,  $a_2$ , and w are to be determined from experimental results of a particular nondestructive inspection method. As an example, a reasonable representation to the dye penetrant method can be obtained by using  $a_1 = 0.5$  mm,  $a_2 = 7.6$  mm, and w = 1. Equation (27) was used by Yang and Trapp [16] in their reliability analysis of structures. Another possible distribution function for crack detection was given by Davidson [50] as follows:

 $F_{X}(x) = \begin{cases} 0, & \text{if } x \leq a \\ c \left[ 1 - e^{-\beta(x-a_{1})} \right], & \text{if } x > a_{1} \end{cases}$ 

where c = a value slightly less than unity.  $\beta = an$  empirical constant.

It is to be noted that the constant c in Equation (28) indicates the fact that there is always a small chance of missing even a large crack during an inspection.

In the Bogdanoff formulation [35], he used a term  $\tau_j$  to represent the probability that the damage state j is detected during an inspection after  $n_1$  duty cycles. The distribution of damage after inspection and the replacement of rejected components with new ones is given by

$$p_{o}^{(1)} = \left\{ p_{o}^{(1)}(1), p_{o}^{(1)}(2), \dots, p_{o}^{(1)}(b) \right\}$$

$$p_{o}^{(1)}(j) = (1 - \tau_{j}) p_{n_{j}}(j)$$

where p<sup>(1</sup>

Packman [9] defined a false or error call as the nondestructive inspection indication that a defect was present when, in fact, there was no such defect. The effect of having such false call to the number of components to be replaced was formulated using the Bayes theorem.

In the state of nature, the crack either exists or does not exist. The inspector using certain nondestructive inspection techniques either detects or does not detect a crack. The consequences are listed as follows:

jÐ

(28)

(29)

	State of Nature			
		<u>Crack Exists</u>	Crack Does Not Exist	
Inspection Result	Crack Detected	Good Inspection	False Call	
	Crack Not Detected	Inspection Error	Good Inspection	

In the case of inspection error or false call, the effect of human errors should be considered [52].

## 7. SUMMARY AND CONCLUDING REMARKS

An attempt is made herein to review and summarize the available literature concerning reliability aspects for fatigue analysis and design of structures. It is noted that the list of references is not exhaustive partly because of the citation of earlier review papers. Moreover, the writer admits that he is biased during the preparation of this paper in the sense that emphases are made on relatively simple methods with practical implications.

The basic formulation of the structural reliability problem involving repeated load applications is presented. Other approaches in this regard can be found elsewhere [e.g., 53]. Several probability distribution functions for fatigue lives are discussed, and the use of linear cumulative damage theorem in structural fatigue is reviewed. Some random-load and/or full-scale fatigue tests are discussed. Finally, a few comments are presented on fatigue reliability of existing structures. For detailed discussion of nondestructive inspection techniques, the reader should consult Reference [26]. In the following, the overall problem of structural fatigue is reviewed from an individual viewpoint.

Various activities of the structural engineering practice can be summarized in terms of the state-of-nature (the way things are) and the state-of-the-art (the body of knowledge) [e.g., 54]. In 1978, Liu and Yao [55] presented a schematic diagram as shown in Figure 1 for the damage assessment and reliability evaluation of existing structures. The main point of this diagram is that all the mathematical formulations in structural analysis and design are results of idealizations and generalizations from available knowledge and past experience. Once the structure is built, the mathematical representation of the behavior of a particular structure needs further modification. Then, results of nondestructive inspection and testing can be used for such purposes. Generally, structural identification consists of the following two parts: (a) the assessment of structural damage at the time of inspection, and (b) the evaluation of the reliability of the structure in future years following the inspection. One major problem in this regard is to classify the structural damage on the basis of measured data and field observation. In other words, structural damage as such is not clearly defined for various types of civil

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Figure 1. Schematic Diagram for Structural Identification

л Х engineering structures, which are generally large in size and complex in composition.

Although experimental data have been available for load-deformation relations of many types of members and connections [e.g., 56,57], the emphasis of these studies appears to be on the mathematical representation of these constitutive relations. It is usually difficult to obtain information concerning failure criteria from such test data. Recently, worthy attempts have been made to study the dynamic behavior of real structures [8,58-63]. Nevertheless, the techniques for the assessment and classification of structural damage remain to be priviledged information of relatively few experts in the structural engineering profession [10].

In recent years, structures more flexible than traditional ones are being designed and built because (a) better analytical methods are available, (b) costs of materials are more expensive, and (c) taller buildings and longer bridges are being attempted. In addition, more structures are fabricated and constructed with welded joints, which tend to cause more fatigue problems. The writer feels that the consideration of structural fatigue will become even more important in future years when these modern and flexible structures become relatively old. Meanwhile, the concept of structural control is being examined and implemented by various investigators [5i]. One consequence of controlling the structural response is to reduce the cyclic amplitudes but possible increase the number of load applications. It seems to be timely to study the interrelationship of structural control and fatigue reliability of modern structures at present.

In conclusion, our knowledge concerning fatigue to-date is primarily concerned with the behavior of relatively small and simple specimens subjected to a large number of repeated load applications. Although much progress has been made in the subject areas of structural reliability and random fatigue, more studies are needed to understand the fatigue behavior of real structures before significant improvements can be obtained in fatigue reliability and design.

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#### REFERENCES

- [1] Coffin, L. F., Jr., "The Stability of Metals Under Cyclic Plastic Strain", <u>Journal of Basic Engineering</u>, ASME, Series D, Vol. 82, No. 3, September 1960, p. 671.
- [2] Yao, J. T. P., and Munse, W. H., "Low-Cycle Fatigue of Metals Literature Review", <u>Welding Journal</u>, Research Supplement, Vol. 41, April 1962, p. 182s.
- [3] Yao, J. T. P., and Munse, W. H., "Low-Cycle Axial Fatigue Behavior of Mild Steel", <u>ASTM Special Technical Publications No. 338</u>, 1962, pp. 5-24.

- [4] Wirsching, P. H. and Yao, J. T. P., "Statistical Methods in Structural Fatigue", Journal of the Structural Division, ASCE, Vol. 96, No. ST6, June 1970, pp. 1201-1219.
- [5] Palmgren, A., "The Endurance of Ball Bearing", Zeitschrift des Vercines Deutschor Ingenienro, Vol. 68, Apríl 5, 1924, pp. 339-341.
- [6] Miner, M. A., "Cumulative Damage in Fatigue", Journal of Applied Mechanics, ASME, Vol. 12, 1945, pp. A159-A164.
- [7] Schilling, C. G., and Klippstein, K. H., "Fatigue of Steel Beams by Simulated Bridge Traffic", <u>Journal of the Structural Division</u>, ASCE, Vol, 103, No. ST8, August 1977, pp. 1561-1575.
- [8] Baldwin, J. W., Jr., et al., Fatigue Test of a Three-Span Composite Highway Bridge, Study 73-1, Department of Civil Engineering, University of Missouri, Columbia, Missouri, 1978.
- [9] Packman, P. F., "Advanced Nondestructive Inspection Techniques as Applied to Fracture Mechanics Design for Turbine Engine Components", <u>Fatigue Life</u> <u>Technology</u>, ASME, 27-31 March 1977, pp. 95-116.
- Yao, J. T. P., "Damage Assessment and Reliability Evaluation of Existing Structures", Invited Paper, presented at the Symposium Honoring Professor T. V. Galambos, Washington University in St. Louis, Missouri, 17 April 1979; also Engineering Structures, Vol. 1, No. 5, October 1979, pp. 245-251.
- [11] Yao, J. T. P., <u>Reliability of Existing Buildings in Earthquake Zones</u> -<u>Final Report</u>, Technical Report No. CE-STR-79-6, School of Civil Engineering, Purdue University, West Lafayette, Indiana, December 1979.
- [12] Yao, J. T. P., "Damage Assessment of Existing Structures", <u>Journal of the Engineering Mechanics Division</u>, ASCE, Vol. 106, No. EM4, August 1980 (in press).
- [13] Freudenthal, A. M., Garrelts, J. M., and Shinozuka, M., "The Analysis of Structural Safety", <u>Journal of the Structural Division</u>, ASCE, Vol. 92, No. ST1, February 1966, pp. 267-325.
- [14] Lin, Y. K., Probabilistic Theory of Structural Dynamics, McGraw-Hill, 1967.
- [15] Task Committee on Structural Safety, "Structural Safety A Literature Review", <u>Journal of the Structural Division</u>, Vol. 98, No. ST4, April 1972, pp. 845-884.
- [16] Yang, J. N., and Trapp, W., "Reliability Analysis of Aircraft Structures Under Random Loading and Periodic Inspection", <u>AIAA Journal</u>, Vol. 12, No. 12, December 1974, pp. 1623-1630.
- [17] Weibull, W., <u>Fatigue Testing and Analysis of Results</u>, Pergamon Press, Oxford, 1961.
- [18] Freudenthal, A. M., "Planning and Interpretation of Fatigue Tests", <u>ASTM</u> <u>Special Technical Publication</u> No. 121, June 1951, pp. 3-13.

- [19] Gumbel, E. J., "Parameters in Distribution of Fatigue Life", <u>Journal</u> of the Engineering Mechanics Division, ASCE, Vol. 89, No. EM5, October 1963, pp. 45-63.
- [20] Weibull, W., "A Statistical Theory of Strength of Materials", Proceedings, Royal Academy of Engineering Sciences, No. 15, 1939.
- [21] Freudenthal, A. M., and Gumbel, E. J., "On the Statistical Interpretation of Fatigue Tests", <u>Proceedings</u>, The Royal Society of London, Series A, Vol. 216, 1953, pp. 309-332.
- [22] Reemsnyder, H. S., and Fisher, J. W., "Service Histories and Laboratory Testing", Journal of the Structural Division, ASCE, Vol. 94, No. ST12, December 1968, pp. 2699-2712.
- [23] Drew, F. P., "Recorded Stress Histories in Railroad Bridges", Journal of the Structural Division, ASCE, Vol. 94, No. ST12, December 1968, pp. 2713-2724.
- [24] Cudney, G. R., "Stress Histories of Highway Bridges". Journal of the Structural Division, ASCE, Vol. 94, No. ST12, December 1968, pp. 2725-2737.
- [25] Stallmeyer, J. E., and Walker, W. H., "Cumulative Damage Theories and Application", Journal of the Structural Division, ASCE, Vol. 94, No. ST12, December 1968, pp. 2739-2750.
- [26] ASCE Committee on Fatigue and Fracture Reliability, "Fatigue and Fracture Reliability: A State of the Art Review", to appear in the Journal of the Structural Division, ASCE.
- [27] Impellizzeri, L. F., "Cumulative Damage Analysis in Structural Fatigue," <u>Special Technical Publication</u> No. 462, American Society for Testing and Materials, 1970, pp. 40-68.
- [28] Tang, J. P., and Yao, J. T. P., "Fatigue Damage Factor in Structural Design", <u>Journal of the Structural Division</u>, ASCE, Vol. 98, No. ST1, January 1972, pp. 125-134.
- [29] Corten, H. T., Sinclair, G., and Dolan, T. J., "On Fatigue Life of Aluminum", Proceedings, ASTM, Vol. 54, 1954, p. 753.
- [30] Yao, J. T. P., "Fatigue Reliability and Design", <u>Journal of the Structural</u> <u>Division</u>, ASCE, Vol. 100, No. ST9, September 1974, pp. 1827-1836.
- [31] Shinozuka, M., "Application of Stochastic Processes to Fatigue Creep, and Catastrophic Failures," Department of Civil Engineering and Engineering Mechanics, Columbia University, New York, N.Y., Nov., 1966.
- [32] Ang, A. H-S., and Munse, W. H., "Practical Reliability Basis for Structural Fatigue", presented at ASCE Structural Engineering Conference, New Orleans, LA, April 1975, Preprint No. 2494.
- [33] Ang, A. H-S., "Basis for Reliability Approach to Structural Fatigue", <u>Proceedings</u>, The Second International Conference on Structural Reliability, Munich, Germany, September 1977.

- [34] Munse, W. H., "Predicting the Fatigue Behavior of Weldments for Random Loads", Proceedings, Offshore Technology Conference, OTC 3300, 1978.
- [35] Bogdanoff, J. L., "A New Cumulative Damage Model Part 1", <u>Journal of</u> Applied Mechanics, ASME, V. 45, June 1978, pp. 246-250.
  - [36] Bogdanoff, J. L., and Krieger, W., "A New Cumulative Damage Model Part 2", Journal of Applied Mechanics, ASME, V. 45, June 1978, pp. 251-257.
  - [37] Birnbaum, Z. W., and Saunders, S. C., "A New Family of Life Distributions", Journal of Applied Probability, Vol. 6, No. 2, 1969, pp. 319-327.
  - [38] Parker, R. J., Zaretsky, E. V., and Dietrich, M. W., <u>Rolling-Element</u> <u>Fatigue Life of Four M - Series Steels</u>, NASA TN D-7033, February 1971.
  - [39] Crandall, S. H., and Mark, W. D., <u>Random Vibration in Mechanical Systems</u>, Academic Press, 1963.
  - [40] Lin, Y. K., Probabilistic Theory of Structural Dynamics, McGraw-Hill, 1967.
  - [41] Bolotin, V. V., <u>Statistical Methods in Structural Mechanics</u>, Holden-Day Publishing Co., San Francisco, CA, 1969.
  - [42] Yamada, K., and Albrecht, P., "Fatigue Design of Welded Bridge Details", <u>Transportation Research Record No. 607</u>, Transportation Research Board, National Research Council, April 1977.
  - [43] Albrecht, P., and Friedland, I. M., "Fatigue-limit Effect on Variable-Amplitude Fatigue of Stiffeners", <u>Journal of the Structural Division</u>, ASCE, Vol. 105, No. ST12, December 1979, pp. 2657-2675.
  - [44] Stephens, R. I., Benner, P. H., Mauritzson, G., and Tindall, G. W., "Constant and Variable Amplitude Fatigue Behavior of Eight Steels", <u>Journal of Testing and Evaluation</u>, ASTM, Vol. 7, No. 2, March 1979, pp. 68-81.
  - [45] Griffith, A. A., "The Phenomena of Rupture and Flow in Solids", <u>Phil.</u> Trans. Roy. Soc., London, Series A, Vol. 221, 1921, p. 163.
  - [46] Robinson, A. L., "Making Nondestructive Evaluation a Science", <u>Science</u>, AAAS, Vol. 205, No. 3, August 1979, pp. 477-479.
  - [47] Blockley, D. I., and Ellison, E. G., "A New Technique for Estimating System Uncertainty in Design", <u>Proceedings</u>, The Institution of Mechanical Engineers, London, Vol. 193, No. 5, 1979, pp. 159-164.
  - [48] Yao, J. T. P., "Application of Fuzzy Sets in Fatigue and Fracture Reliability", Presented at the ASCE EMD Specialty Conference on Probabilistic Mechanics and Structural Reliability, Tucson, AZ, 10-12 January 1979.
  - [49] Packman, P. F., et al., "Definition of Fatigue Cracks Through Nondestructive Testing", Journal of Materials, ASTM, Vol. 4, No. 3, 1969, pp. 666-700.

- [50] Davidson, J. R., "Reliability After Inspection", <u>Fatigue of Composite</u> <u>Materials</u>, ASTM Special Technical Publication No. 569, 1975, pp. 323-334.
- [51] Yao, J. T. P., "Identification and Control of Structural Damage", <u>Structural Control</u>, Edited by H. H. E. Leipholz, North-Holland Publishing Company and SM Publications, 1980, pp. 757-777.
- [52] Beuner, P. M., Sorenson, K. G., and Johnson, D. P., "A Workable Approach for Extending the Life of Turbine Rotors", <u>Fatigue Life Technology</u>, ASME, 27-31 March 1977, pp. 53-71.
- [53] Ferry Borges, J., "Safety Concepts for Non-Repeated and Repeated Loadings", Reports of the Working Commissions, IABSE, Vol. 12, 1973, pp. 101-124.
- [54] Galambos, T. V., and Yao, J. T. P., "Additional Comments on Code Formats", <u>Proceedings</u>, Second International Workshop on Code Formats, Mexico City, 3-5 January 1976, pp. 26-26.
- [55] Liu, S. C., and Yao, J. T. P., "Structural Identification Concept", Journal of the Structural Division, ASCE, Vol. 104, No. ST12, December 1978, pp. 1845-1858.
- [56] Park, R., "Theorization of Structural Behavior with a View to Defining Resistance and Ultimate Deformability", Reports of the Working Commissions, IABSE, Vol. 12, 1973, pp. 1-26.
- [57] Popov, E. P., and Bertero, V. V., "On Seismic Behavior of Two R/C Structural Systems for Tall Buildings", <u>Structural and Geotechnical Mechanics</u>, Edited by W. J. Hall, Prentice-Hall, 1977, pp. 117-140.
- [58] Hudson, D. E., "Dynamic Tests of Full-scale Structures", <u>Journal of the</u> <u>Engineering Mechanics Division</u>, ASCE, Vol. 103, No. EM6, December 1977, pp 1141-1157.
- [59] Hart, G. C., and Yao, J. T. P., "System Identification in Structural Dynamics", Journal of the Engineering Mechanics Division, ASCE, Vol 103, No. EM6, December 1977, pp. 1089-1104.
- [60] Freeman, S. A., Honda, K. K., and Blume, J. A., "Dynamic Response Investigations of Real Buildings", <u>Proceedings</u>, Workshop on Earthquake - Resistant Reinforced Concrete Building Construction, University of California, Berkeley, CA, July 11-15, 1977, pp. 1517-1536.
- [61] Shepherd, R., and Jennings, P. C., "Experimental Investigations Correlation with Analysis", <u>Proceedings</u>, Workshop on Earthquake - Resistant Reinforced Concrete Building Construction, University of California, Berkeley, CA, Vol. III, July 11-15, 1977, pp. 1537-1554.
- [62] Mayes, R. L., and Galambos, T. V., "Large-Scale Dynamic Shaking of 11-Story Reinforced Concrete Building", <u>Proceedings</u>, Workshop on Earthquake -Resistant Reinforced Concrete Building Construction, University of California, Berkeley, CA, Vol. III, July 11-15, 1977, pp. 1555-1587.
- [63] Sozen, M. A., "Earthquake Simulation in the Laboratory", Proceedings, Workshop on Earthquake - Resistant Reinforced Concrete Building Construction, University of California, Berkeley, CA, July 11-15, 1977, pp. 1606-1629.

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